Borrowing Constraints, Markups, and Misallocation*

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Abstract

We document that less constrained firms in an industry have higher markups. This connection between markups and borrowing constraints has important allocative implications: it lowers the TFP losses from markup dispersion, because looser borrowing constraints help higher markup firms access capital, moving their market shares closer to the efficient level. We further document that this relationship is stronger in industries where firms rely more on earnings to borrow, and that markup dispersion is also higher in these industries. We explain all of these patterns using a standard Kimball demand model augmented with borrowing against assets and earnings, and show that the connection between markups and borrowing constraints quantitatively lowers TFP losses from markup dispersion substantially, especially when firms rely on earnings to borrow.

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1 Introduction

A growing literature has provided key insights into how markup dispersion contributes to misallocation (Peters, 2020; Edmond, Midrigan, and Xu, 2023; Baqaee, Farhi, and Sangani, 2024a,b). In these studies, variations in firms' markups are driven by differences in productivity, and markup dispersion leads to substantial TFP losses. Firms' production activities and markups, however, can also depend on their ability to raise financing, such as constraints on borrowing emphasized in macro-finance research (Bernanke, Gertler, and Gilchrist, 1999; Ottonello and Winberry, 2020). In this paper, we present empirical and theoretical analyses showing that variations in firms' markups are linked to differences in the tightness of their borrowing constraints, with less constrained firms having higher markups.

This connection between markups and borrowing constraints has important allocative efficiency implications: it substantially reduces the TFP losses from markup dispersion. Because higher markup firms face looser borrowing constraints, they can grow larger, moving their market shares closer to the efficient level. This effect is especially pronounced when firms rely on earnings to borrow.

We first provide new empirical evidence that links firms' markups with their borrowing constraints, using both U.S. Compustat data and Census of Manufacturing data. We present the following findings. We start by documenting that, within an industry, less constrained firms have higher markups (Fact 1a). For markups, we use several estimates from De Loecker, Eeckhout, and Unger (2020) in Compustat data and the measure based on the ratio of sales over intermediate inputs plus wage bill in Census data, as well as translog markups following De Ridder (2024) in both datasets. For constraint tightness, we use cash holdings (normalized by book assets) in Compustat data, which is a common approach in the literature (Gilchrist et al., 2017): firms with abundant cash holdings can use internal funds and are less constrained, while those with little cash need to rely on external borrowing and are more constrained. We also consider alternative measures of constraint tightness, such as the distance to violating financial covenants in debt contracts and the log capital wedge, based on log production worker hours over capital in Census data, which aligns with our model.

Furthermore, we find that the relationship between markups and constraint tight-

ness is stronger in industries where firms have low liquidation values of pledgeable assets and therefore rely more on earnings to borrow (Fact 1b). Markup dispersion is also higher in these industries that rely more on earnings to borrow (Fact 2). These industry-level relationships hold in both Compustat data and Census of Manufacturing data.

We then show how these firm-level and industry-level results can be quantitatively explained by a standard Kimball demand model augmented with borrowing against assets and earnings. In this model, the competitive final good producer aggregates differentiated intermediate goods through a Kimball aggregator. Monopolistically competitive intermediate goods producers use labor and capital and face asset-based and earnings-based borrowing constraints. Specifically, firms can pledge assets on a standalone basis and borrow up to the liquidation value of the assets (asset-based borrowing constraints), or pledge the earnings from their operations and borrow up to a multiple of the earnings (earnings-based borrowing constraints). The overall debt limit is the greater value of the two options. Earnings-based borrowing constraints are affected by firms' productivity and markups, whereas asset-based borrowing constraints are driven by how generic the production assets are, which varies across industries due to the production assets' physical characteristics as shown by Kermani and Ma (2023a). Intermediate good producers not only optimally choose the amount of labor and capital used for production but also endogenously choose their saving.

Two key mechanisms in the model explain the negative correlation between constraint tightness and markups. First, looser borrowing constraints lower marginal costs, allowing firms to attain larger market shares and, because of Kimball demand, charge higher markups. This applies in general for both asset-based and earnings-based borrowing constraints. Second, when firms rely on earnings to borrow, higher productivity firms tend to have looser constraints because of their higher earnings. These firms also tend to have higher markups, again because of their larger market shares and Kimball demand. This further generates a negative correlation between constraint tightness and markups. This mechanism is, however, absent under asset-based borrowing constraints, because higher productivity firms instead face tighter constraints due to their greater borrowing needs, and their higher earnings and markups do not help relax asset-based constraints. Both reasons explain Fact 1a. Because the second reason operates only under earnings-based borrowing constraints, the negative relationship be-

tween markups and constraint tightness is stronger in industries that rely more heavily on earnings to borrow, thereby explaining Fact 1b. By the same token, in these industries, the second reason enables high markup firms to borrow more, further expand their market share, and thus raise their markups even further, increasing overall markup dispersion, thereby explaining Fact 2.

The interaction between markups and borrowing constraints has important allocative efficiency implications: in particular, it reduces the overall TFP losses from markup dispersion. To illustrate further, we rewrite the Hsieh and Klenow (2009) formula for TFP losses (extending it to the case with Kimball demand) to express the TFP losses as an increasing function of the variance in log markup and constraint tightness (measured in log capital wedge, defined as in Hsieh and Klenow (2009)), along with the covariance between log markup and constraint tightness.

$$\begin{aligned}\text{Total TFP loss} \quad \approx \quad & \underbrace{\text{log markup dispersion}}_{>0} \quad + \quad \text{dispersion in borrowing constraint tightness} \\ & + \underbrace{\text{covariance of log markup \& borrowing constraint tightness}}_{<0 \text{ if higher markup firms less constrained}} \end{aligned} \tag{1}$$

This formula illustrates how introducing borrowing constraints mitigates the TFP loss from markup dispersion. Without borrowing constraints, markup dispersion leads to TFP losses, as summarized by the first term in (1). With borrowing constraints, the negative last term in the TFP formula (1) mitigates the TFP losses from markup dispersion because high markup firms are less constrained (Fact 1a), as explained by the two mechanisms above. When high markup firms face looser constraints, their market share moves closer to the planner's counterpart, thereby reducing TFP losses. Furthermore, how much the borrowing constraint channel mitigates TFP losses from markup dispersion also varies across industries, given the empirical findings in Fact 1b and Fact 2. In industries that rely more on earnings to borrow, markup dispersion is higher, but markups are also more negatively correlated with constraint tightness. Indeed, in our model, the latter channel dominates and total TFP losses are smaller in those industries.

Finally, the TFP gains from size-dependent subsidies aimed at removing markup dispersion are smaller compared to the substantial gains from such policies without borrowing constraints (Edmond, Midrigan, and Xu, 2023). The reason is that borrowing constraints already mitigate the TFP losses from markup dispersion, especially if firms

rely more on earnings to borrow.

Several studies examine how firms set prices in response to *temporary negative shocks*, in the presence of financial constraints. Some find that firms with tighter constraints set higher prices (Chevalier and Scharfstein, 1995; Gilchrist et al., 2017; Meinen and Soares, 2022; Minseog and Park, 2024; Renkin and Züllig, 2024). The interpretation is that firms intertemporally substitute their investment in building customer base, and they cut back on such investment when their constraints tighten. Meanwhile, Kim (2021) and Lenzu et al. (2024) find that firms cut prices in response to credit tightening (e.g., due to banks' exposure to the Lehman crisis or the European debt crisis). Our analyses instead focus on long-run steady-state variations in markups across firms, rather than how prices respond to shocks. For example, constrained firms cannot always charge higher markups and underinvest in customer capital in the steady state (even though they can cut back in bad times and engage in intertemporal substitution). Eventually, they will lose market share (Renkin and Züllig, 2024), and correspondingly their long-term markups will fall.

Related Literature A large literature has analyzed misallocation arising from financial frictions (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014; Ottonello and Winberry, 2024) and information frictions (David, Hopenhayn, and Venkateswaran, 2016; David and Venkateswaran, 2019). Recent research offers key insights into misallocation due to markup dispersion (Peters, 2020; Baqaee and Farhi, 2020; Edmond, Midrigan, and Xu, 2023; Boar and Midrigan, 2024). The focus has been markup dispersion due to productivity differences. An earlier version of Boar and Midrigan (2019) and Peruffo (2025) study misallocation arising from markup dispersion when firms face asset-based borrowing constraints, but do not analyze earnings-based borrowing constraints. Using both novel empirical evidence and a model that quantitatively fits these facts, we show that the possibility of borrowing against both assets and earnings generates unique interactions between firms' markups and constraint tightness, and correspondingly distinct implications for allocative efficiency. In contemporaneous work, Ebsim (2025) performs theoretical and quantitative analyses where firms charge variable markups and face only earnings-based borrowing constraints: he compares welfare losses from financial frictions with and without markup distortions and shows that for a planner constrained by financial frictions, the optimal policy does not fully eliminate

the markup distortions.

The empirical and theoretical connection between markups and borrowing constraints we highlight builds on macro-finance research, especially the work on earnings-based borrowing constraints. A number of papers document that borrowing against earnings is important among U.S. nonfinancial firms, and firms are subject to debt limits as a function of their earnings (Greenwald, 2019; Lian and Ma, 2021; Li, 2022; Drechsel, 2023; Caglio, Darst, and Kalemli-Ozcan, 2024; Adler, 2025; Zhao, 2025; Su, 2026). Such borrowing is especially prevalent when firms' pledgeable assets have low liquidation values on a standalone basis (Kermani and Ma, 2023a), in which case the traditional borrowing constraints based on asset liquidation values a la Kiyotaki and Moore (1997) are too tight and borrowing against earnings improves debt capacity if earnings are high. Borrowing against earnings leads to a close connection between markups and financial frictions.

2 Motivating Facts

In this section, we present new empirical facts at the firm level and the industry level. In Sections 3 and 4, we develop a standard Kimball demand model augmented with borrowing against assets and earnings to explain the empirical facts documented here. In Section 5, we study the implications for allocative efficiency.

2.1 Baseline Empirical Facts

In Section 2.1.1, we present baseline empirical facts using Compustat data, which cover public firms in all industries. In Section 2.1.2, we present further evidence using Census of Manufacturing data, which are among the best data to measure markups in the U.S. and cover not only public companies, though this sample only covers manufacturing in Economic Census years (years ending in 2 and 7) and has limited information about firms' financial conditions.

2.1.1 Compustat Data

In the following, we lay out the key empirical facts using Compustat data.

Fact 1a. Less constrained firms have higher markups We start with the firm-level relationship between markups and constraint tightness in Panel A of Figure 1. This firm-year level binscatter plot uses the log firm-level markup from De Loecker, Eeckhout, and Unger (2020) on the y-axis, and we confirm similar results with a number of other markup estimates in Table 1. The x-axis uses a common proxy for firms' constraint tightness, namely firms' cash holdings (normalized by book assets), following Gilchrist et al. (2017) for example. This measure reflects the abundance of firms' internal funds and is inversely related to how constrained they are. The sample covers U.S. Compustat firms annually from 1987 (we start in 1987 because the Census of Manufacturing data we use in Section 2.1.2 begin in 1987) to 2016 (we end in 2016 because the output elasticity estimates from De Loecker, Eeckhout, and Unger (2020) stop in 2016). We use industry (3-digit NAICS code) by year fixed effects to pinpoint the heterogeneity among firms in the same industry at a given point in time.

We perform several robustness checks for alternative measures of constraint tightness at the end of this section. First, we use the distance to violating earnings-based financial covenants in debt contracts constructed following Lian and Ma (2021). This measure links directly to borrowing limits, whereas Farre-Mensa and Ljungqvist (2016) find that firms classified as constrained according to traditional financial constraint proxies in the literature such as the Kaplan and Zingales (1997) index or the Whited and Wu (2006) index have no trouble raising debt. Second, we also check that the main empirical results hold using the log capital wedge in Census of Manufacturing data, based on log production worker hours relative to capital as an alternative proxy for constraint tightness, which maps to the model in Section 3. Finally, Altomonte et al. (2024) study a policy reform in France that led to variations in firms' liquidity, and find that higher liquidity contributes to higher markups.

Figure 1, Panel A, shows a strong positive relationship between firms' markups and cash holdings. We confirm the statistical significance of this relationship in column (1) of Table 1, Panel A. This suggests that less constrained firms have higher markups. The remaining columns perform further checks using several other markup proxies. Column (2) uses markups from De Loecker, Eeckhout, and Unger (2020) where the production function includes overhead as a factor of production. Column (3) uses accounting markups from De Loecker, Eeckhout, and Unger (2020), which is the ratio of sales to cost

of good sold scaled by the industry cost share. Column (4) uses translog markups following De Ridder (2024). We exclude observations with markup less than one, which would be abnormal and could arise from measurement issues (this issue seems more severe for accounting markups). We use industry (3-digit NAICS code) by year fixed effects, and double cluster standard errors by both industry and year. The magnitude of the regression coefficient implies that a one standard deviation change in cash over assets is associated with about a 0.2 standard deviation change in firm markups.

In Table IA1, we show that the results are robust to controlling for firm age. Since Compustat does not have a direct measure of firm age, we use the older of the year since incorporation (from Datastream) and the year since IPO (from Compustat). Indeed, the coefficients on cash holdings in Table IA1 are very similar to those in Table 1. The age variable is generally not correlated with firm markups.

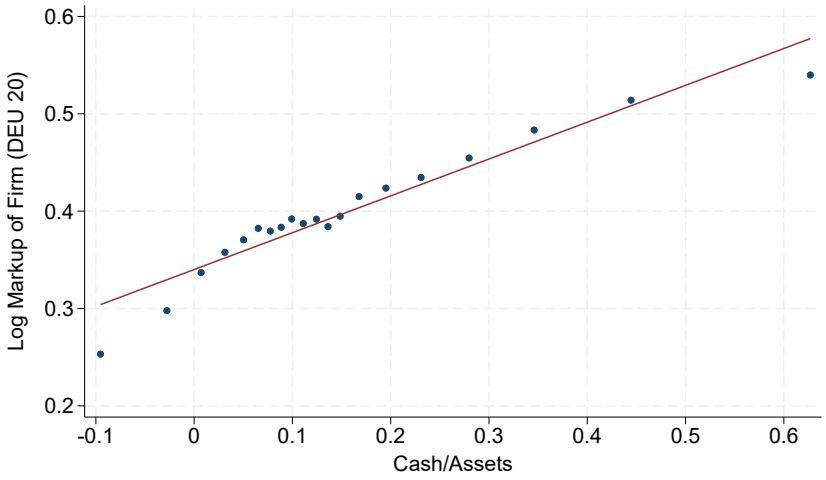
One concern is that the revenue-based markup estimates in De Loecker, Eeckhout, and Unger (2020) are not informative of the level of markups (Bond et al., 2021). As De Ridder, Grassi, and Morzenti (2026) point out, although the average level of markups can be affected by biases, these measures still capture the differences in markups across firms and hence markup dispersion reasonably well, which is our focus. Also, it is worth noting that our markup estimates remain valid in the presence of borrowing constraints (or, more generally, the existence of capital wedges, as pointed out by De Ridder, Grassi, and Morzenti (2026)).[1]

Finally, the relationship between firms' markups and constraint tightness does not need to be interpreted causally in either direction to be of interest. In our subsequent analysis, we explain how this relationship arises naturally from a standard Kimball demand model augmented with borrowing against assets and earnings. Moreover, their univariate relationship (without controls) is of particular interest, as the univariate covariance between markup and constraint tightness directly enters the TFP loss (25).

Fact 1b. The relationship between markups and constraint tightness is stronger in industries that rely more on earnings to borrow Furthermore, we find that the relationship between markups and constraint tightness is stronger among industries where

[1] Within the context of our model, this point can be seen from (13), which determines markups based on the intermediate good producer's optimal choice of labor. Because choices of labor are not subject to borrowing constraints, the presence of such constraints does not alter (13).

Panel A. Less Constrained Firms Have Higher Markups



Panel B. Relationship Stronger in Industries that Rely More on Earnings to Borrow

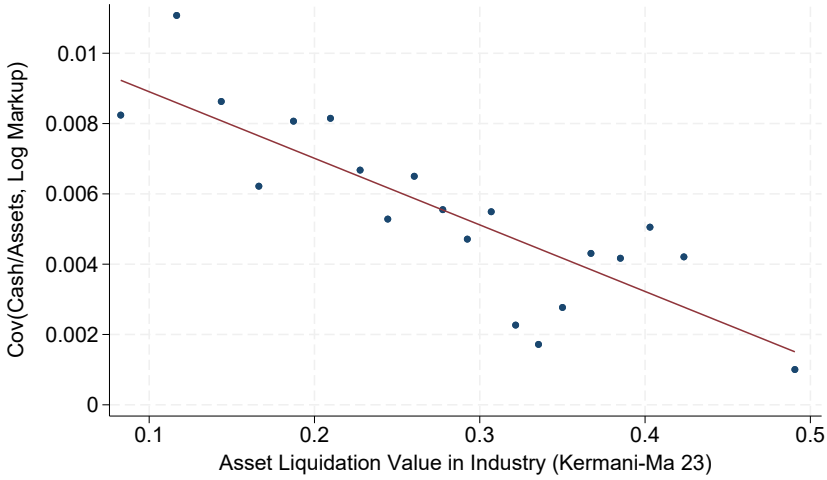


Figure 1: Firm Markups and Constraint Tightness. Panel A shows a binscatter plot of firm-level log markup on the y-axis and firm-level cash holdings (normalized by book assets) on the x-axis, using U.S. Compustat firms annually. Markups use the baseline measure in De Loecker, Eeckhout, and Unger (2020). Industry (3-digit NAICS code) by year fixed effects are included. Panel B shows the covariance between firm-level log markup and cash/assets in each industry-year on the y-axis, and the industry's liquidation value of fixed assets plus working capital (normalized by book assets) on the x-axis. The liquidation value data are from Kermani and Ma (2023a). Year fixed effects are included. We exclude observations with markup less than one.

firms have low liquidation values of pledgeable assets and therefore rely more on borrowing against earnings (Lian and Ma, 2021; Kermani and Ma, 2023a). In the binscatter plot in Figure 1, Panel B, we study the covariance between firm-level log markup and cash over assets (i.e., the covariance of the two variables on the two axes in Panel A) in each industry-year on the y-axis. We use the industry-level liquidation value of pledge-

Table 1: Firm-Level Markups and Constraint Tightness (Compustat)

Panel A. Less Constrained Firms Have Higher Markups

	Firm Log Markup			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Cash/Assets	0.378***	0.333***	0.192***	0.304***
	(0.036)	(0.030)	(0.014)	(0.048)
Fixed Effects		Year		
Observations	121,789	80,937	60,174	95,300
R^2	0.29	0.26	0.19	0.29

Panel B. Relationship Stronger in Industries that Rely More on Earnings to Borrow

	Industry Cov(Log Markup, Cash/Assets)			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.019***	-0.012*	-0.009**	-0.013***
	(0.006)	(0.007)	(0.004)	(0.005)
Fixed Effects		Year		
Observations	2,351	2,223	2,268	2,262
R^2	0.05	0.03	0.02	0.03

Notes. Panel A shows firm-year level regressions $\text{Log Markup}_{it} = \alpha_{ind(i)t} + \beta \text{Cash/Assets}_{it} + \varepsilon_{it}$, among U.S. Compustat firms. Log Markup_{it} for firm i in year t uses the baseline markup in De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input in De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup in De Loecker, Eeckhout, and Unger (2020) in column (3), and translog markup following De Ridder (2024) in column (4). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Panel B shows industry-year level regressions $\text{Cov(Log Markup, Cash/Assets)}_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, among U.S. Compustat firms. $\text{Cov(Log Markup, Cash/Assets)}_{kt}$ is the covariance of firm-level log markup and cash/assets among firms in industry k in year t. The set of markups is the same as the columns in Panel A. The variable Liqval_{kt} is the industry's asset liquidation value from fixed assets and working capital normalized by the industry's total book assets. The liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a). Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

able assets such as property, plant, and equipment and working capital (normalized by book assets) from Kermani and Ma (2023a,b) on the x-axis, which are driven by physical characteristics of assets used in each industry and shape the extent to which firms need to borrow against earnings.[2] When the liquidation value of pledgeable assets is high,

[2]We follow to Kermani and Ma (2023b) and calculate the asset liquidation value of Compustat firms in industry k and year t as: $\text{LiqVal}_{kt} = \sum_j \lambda_k^j A_{kt}^j$, where A_{kt}^j is the book value of asset type j (including

firms can directly pledge them for asset-based borrowing; when the liquidation value is low, firms rely more on earnings to borrow, which we verify in Appendix Figure IA1. Figure 1, Panel B, shows that the relationship between markups and cash over assets in Panel A is especially strong in low liquidation value industries where firms rely more on earnings to borrow. We confirm the statistical significance of this relationship in column (1) of Table 1, Panel B. Columns (2) to (4) use other markup measures (for the covariance between log markup and cash holdings on the left hand side) parallel to those in Panel A. We use year fixed effects, and double cluster standard errors by industry and year. The magnitude of the regression coefficient implies that a one standard deviation change in the industry-level liquidation value is associated with about a 0.1 standard deviation change in the covariance between firm markups and cash over assets in the industry.

Similar regressions in Table IA3, Panel A, show that the positive covariance between markups and cash over assets is also stronger when the industry has a larger share of borrowing against earnings (cash flow-based debt) in its total debt. The magnitude of the regression coefficient implies that a one standard deviation change in the share of cash flow-based debt in the industry is associated with about a 0.1 standard deviation change in the covariance between firm markups and constraint tightness. This magnitude is comparable to that in Table 1, Panel B discussed above.

Fact 2. Markup dispersion is higher in industries that rely more on earnings to borrow
Next, we show that markup dispersion is higher in industries with low liquidation values of pledgeable assets and correspondingly a greater reliance on borrowing against earnings. The binscatter plot in Figure 2 shows the variance of firm-level log markup within an industry-year on the y-axis, and the industry's liquidation value of pledgeable assets on the x-axis (same x-axis as Figure 1, Panel B). We confirm the statistical significance of this relationship in column (1) of Table 2. Columns (2) to (4) use other markup measures parallel to those in Table 1. The magnitude of the regression coefficient implies that a one standard deviation change in the industry-level liquidation value is associated with about a 0.3 standard deviation change in the industry's markup dispersion. Similar regressions in Table IA3, Panel B, show that markup dispersion is also higher when the

property, plant, and equipment, inventory, and receivables, which are commonly pledged for asset-based borrowing) of Compustat firms in industry k and year t, and λ_k^j is the liquidation recovery rate (liquidation value per dollar of book value) for asset type j in industry k from Kermani and Ma (2023a). The value Liqval_{kt} is normalized by total book assets of Compostat firms in industry k and year t.

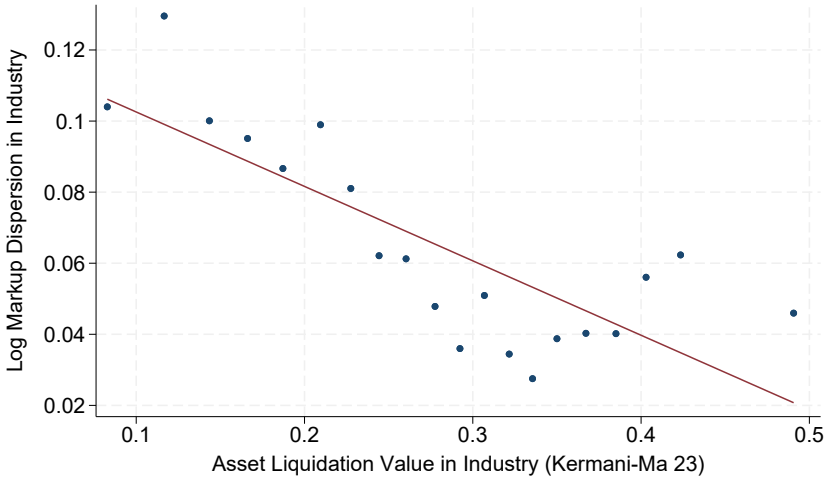


Figure 2: Industry Markup Dispersion (Compustat). This figure shows a binscatter plot of the relationship between the variance of firm-level log markup in each industry-year on the y-axis, and the industry's liquidation value of fixed assets plus working capital (normalized by book assets) on the x-axis. Markups use the baseline measure from De Loecker, Eeckhout, and Unger (2020). The liquidation value data are from Kermani and Ma (2023a). Year fixed effects are included. We exclude observations with markup less than one.

industry has a larger share of borrowing against earnings in total debt. The magnitude of the regression coefficient implies that a one standard deviation change in the share of cash flow-based debt in the industry is associated with about a 0.2 standard deviation change in the industry's markup dispersion. This magnitude is comparable to that in Table 1.

2.1.2 Census of Manufacturing Data

In this section, we examine the key empirical facts using Census of Manufacturing, which is one of the best datasets for markup measurement in the U.S. (Autor et al., 2020). In this dataset, we cover a comprehensive set of firms in the economy (not just public companies), but are restricted to manufacturing firms in Economic Census years (years ending in 2 and 7).[3]

In this dataset, we construct a markup measure using the ratio of sales to the sum of intermediate inputs and wage bill. This ratio only differs by a scalar from the standard markup measurement such as the ratio estimator in De Loecker, Eeckhout, and Unger (2020), which captures the output elasticity. Because we focus on cross-firm differences

[3]We use the same cleaning procedure as the Dispersion Statistics on Productivity (DiSP) database from the Census Bureau (U.S. Census Bureau, 2024), and the sample runs from 1987 to 2017.

Table 2: Industry Markup Dispersion (Compustat)

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.260***	-0.249***	-0.222***	-0.243***
	(0.066)	(0.064)	(0.052)	(0.055)
Fixed Effects	Year			
Observations	3,820	3,578	3,923	3,006
R^2	0.13	0.10	0.10	0.14

Notes. This table shows industry-year level regressions $\text{Var}(\text{Log Markup})_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, among U.S. Compustat firms. $\text{Var}(\text{Log Markup})_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Table 1. The variable Liqval_{kt} is the industry's liquidation value from fixed assets and working capital normalized by the industry's total book assets. The liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a). Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

in log markup and on the dispersion of log markup, the scalar is irrelevant for our analysis.[4] We also construct translog markups following De Ridder (2024). We verify that markups in Census and Compustat data (using the De Loecker, Eeckhout, and Unger (2020) approach) are significantly positively correlated, as shown in Table IA6.

Since financial information like cash is not available in census data, we construct an alternative measure of constraint tightness that is feasible in this dataset. Specifically, we follow the model in Section 3 and use the log capital wedge, based on the log of production worker hours relative to capital.[5] A high capital wedge indicates constraints as firms cannot acquire enough capital, so they use "too little" capital relative to labor. Although capital intensity may differ across industries for other reasons, our analyses only use variations of the capital wedge among firms in the same industry-year.

Fact 1a. Less constrained firms have higher markups In Table 3, Panel A, we examine the firm-level relationship between log markup and log capital wedge, the measure of constraint tightness in census data. We find that firms with higher capital wedge (more constrained) have significantly lower markups. This pattern is consistent with results

[4] This approach is valid if the output elasticity is homogenous within an industry. The translog markup measure relaxes this assumption and demonstrates the robustness of our findings.

[5] Again, the ratio of production worker hours to capital differs from its theoretical counterpart in the definition of capital wedge in (14) by a scalar irrelevant for our purposes.

in Table 1 (the results using log capital wedge in Table 3 have the opposite sign as those using cash holdings in Table 1 because higher capital wedges indicate tighter constraints whereas higher cash holdings indicate looser constraints). We include industry by year fixed effects as in Table 1, so the relationship between markups and capital wedges here reveals within-industry variations at a given point in time.

Fact 1b. The relationship between markups and constraint tightness is stronger in industries that rely more on earnings to borrow In Table 3, Panel B, we show that the above relationship is stronger for industries with low asset liquidation values. We regress the covariance of log markup and log capital wedge on the industry asset liquidation value, and find that the covariance is more negative when asset liquidation values are low (where firms need to rely more on earnings to borrow). Here we merge the industry-level asset liquidation value measure used in Section 2.1.1 with the Census data (based on 3-digit NAICS code), assuming that the liquidation value is an industry characteristic (any measurement noise would work against us finding significant results).

Fact 2. Markup dispersion is higher in industries that rely more on earnings to borrow Finally, Table 4 uses the Census data to verify that markup dispersion (measured using the variance of log firm-level markups within an industry-year) is smaller in industries with high asset liquidation values. Figure IA3 presents a bincatter plot to display this relationship visually.

2.2 Further Discussions

2.2.1 Alternative Proxies of Constraint Tightness

As mentioned above, we use alternative measures of constraint tightness to perform robustness checks for the main results. First, we use the distance to violating earnings-based financial covenants in debt contracts, which is directly linked to how binding debt limits are: these financial covenants in corporate loans impose limits on the maximum total debt to EBITDA (earnings before interest, taxes, depreciation, and amortization) ratio or the minimum debt payments to EBITDA ratio, and firms need to show compliance on a quarterly basis. We calculate the overall distance to the most binding limit in outstanding loans, following the construction in Lian and Ma (2021). In Figure

Table 3: Firm-Level Markups and Constraint Tightness (Census of Manufacturing)

Panel A. Less Constrained Firms Have Higher Markups

	Firm Log Markup	
	(1)	(2)
Markup Measure	Baseline	Translog
Log Capital Wedge	-0.014***	-0.010***
	(0.003)	(0.003)
Fixed Effects	Industry × Year	
Observations	164,000	164,000
R^2	0.10	0.06

Panel B. Relationship Stronger in Industries that Rely More on Earnings to Borrow

	Industry Cov(Log Markup, Log Capital Wedge)	
	(1)	(2)
Markup Measure	Baseline	Translog
Industry Asset Liquidation Value	0.101***	0.079***
	(0.019)	(0.017)
Fixed Effects	Year	
Observations	150	150
R^2	0.28	0.23

Notes. Panel A shows firm-year level regressions $\text{Log Markup}_{it} = \alpha_{ind(i)t} + \beta \text{Log Capital Wedge}_{it} + \varepsilon_{it}$, among Census of Manufacturing firms in Economic Census years. Log Markup_{it} for firm i in year t uses the log of sales over intermediate inputs plus wage bill in column (1) and log translog markup following De Ridder (2024) in column (2). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Panel B shows industry-year level regressions $\text{Cov(Log Markup, Log Capital Wedge)}_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, among Census of Manufacturing firms in Economic Census years. $\text{Cov(Log Markup, Log Capital Wedge)}_{kt}$ is the covariance of firm-level log markup and log capital wedge among firms in industry k in year t. The set of markups is the same as the columns in Panel A. The variable Liqval_{kt} is the industry-level asset liquidation value from Table 1, Panel B. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We drop observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

IA2 and Table IA2, we show that markups are higher among firms that are further from these covenants being binding. This result aligns with Fact 1a, which finds that less constrained firms have higher markups. Since this measure is only available for firms with loans that include financial covenants, it is less suitable for the robustness check for Fact 1b, due to both selection and the limited number of observations in each industry-year.

Second, in our model in Section 3, the constraint tightness will be reflected by the capital wedge. In Section 2.1.2, we measure the log capital wedge in the Census data based on the log of production worker hours relative to capital. We observe consistent

Table 4: Industry Markup Dispersion (Census of Manufacturing)

	Variance of Firm-Level Log Markup in Industry	
	(1)	(2)
Markup Measure	Baseline	Translog
Industry Asset Liquidation Value	-0.093*	-0.091*
	(0.041)	(0.043)
Fixed Effects	Year	
Observations	150	150
R^2	0.30	0.29

Notes. This table shows industry-year level regressions $\text{Var(Log Markup)}_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, among Census of Manufacturing firms in Economic Census years. $\text{Var(Log Markup)}_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Table 3. The variable Liqval_{kt} is the industry-level asset liquidation value from Table 2. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We drop observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

results using this measure as well.

2.2.2 Relevance of Earnings-Based Borrowing Constraints

Our analyses in this paper highlight that earnings-based borrowing constraints can shape variations in markups documented above. A common question is whether companies such as those in Compustat are constrained. Lian and Ma (2021) provide an extensive study of the relevance of earnings-based borrowing constraints among large U.S. nonfinancial firms. They find that almost 70% of large Compustat nonfinancial firms have earnings-based borrowing constraints written explicitly in their debt contracts. In a given year, about 10% of these firms violate the constraints and 25% are within one standard deviation to violation. This importance of earnings-based borrowing constraints is frequently discussed by firms' financial reports. For example, Starwood Hotel's 2011 annual report states: "A substantial decrease in EBITDA ... could make it difficult for us to meet our debt service requirements and restrictive covenants and force us to sell assets and/or modify our operations." GNC's 2012 annual report states: "These restrictions may prevent us from taking actions that we believe would be in the best interest of our business and may make it difficult for us to successfully execute our business strategy or effectively compete with companies that are not similarly restricted."

2.2.3 Covariates with Industry Asset Liquidation Values

One question is whether the asset liquidation value used in Fact 1b and Fact 2 might be correlated with other industry characteristics that can affect markup dispersion. In particular, it is important to check that our results are not due to asset liquidation value being correlated with the volatility of shocks in an industry, which can contribute to markup dispersion as well. First, we follow the literature and capture the volatility of shocks using stock return volatility, measured as the average firm-level stock return volatility in the industry in a given year. Table IA4 presents the results. The coefficients on asset liquidation value are very similar to those in the baseline regressions in Table 2. Second, our model in Section 3 will show that the standard deviation of earnings to debt in an industry is an important reflection of the volatility of shocks, and we control for it in Table IA5. The coefficients on asset liquidation value are again very similar to those in the baseline regressions in Table 2.

The asset liquidation value, by construction, is also related to the degree of investment irreversibility: can irreversibility can affect markup dispersion in other ways, such as investment adjustment costs? However, this channel does not naturally imply a relationship between markups and constraint tightness, and it is not obvious that it makes such a relationship significantly stronger when industry asset liquidation values are low.

Another question is how to think about intangible assets: low asset liquidation values may be correlated with having a large amount of intangible assets in an industry. To the extent that intangible assets (e.g., organizational capital) can be difficult to pledge for asset-based borrowing, so more intangible assets make firms more reliant on pledging earnings for borrowing, then the same mechanisms apply.

3 Model

In this section, we set up an industry equilibrium model featuring a standard Kimball demand aggregator, augmented to allow borrowing against both assets and earnings. Later, we show that it explains the firm-level and industry-level evidence in Section 2, and use it to study implications for TFP losses from markup dispersion.

In this model, the industry's final good is produced by competitive firms that aggre-

gate differentiated intermediate goods through a Kimball aggregator.[6] Monopolistically competitive intermediate good producers use labor and capital and are subject to asset-based and earnings-based borrowing constraints.

Industry final good producers The industry's final good is produced by perfectly competitive producers that aggregate a continuum of intermediate goods $\{y(i)\}_{i\in[0,1]}$ into the final output Y via a Kimball aggregator $\Upsilon(\cdot)$:

$$\max_{\{y(i)\}_{i\in[0,1]}} PY - \int p(i)\,y(i)\,di \quad \text{s.t.} \quad \int \Upsilon\left(\frac{y(i)}{Y}\right) di = 1, \tag{2}$$

where $\Upsilon(1)=1$, $\Upsilon'(\cdot)>0$, $\Upsilon''(\cdot)<0$, and P captures the price of the industry final good. Optimality by final good producers implies that the market share of each intermediate good i, $\tilde{y}(i)=y(i)/Y$, is related to its price $p(i)$ by

$$p(i) = p\left(\tilde{y}(i)\right) \equiv \frac{\lambda}{Y}\Upsilon'\left(\tilde{y}(i)\right), \tag{3}$$

where λ captures the Lagrange multiplier on the constraint in (2). Relationship (3) is the demand curve faced by the intermediate good producer i.

When illustrating key mechanisms of the model and how it quantitatively explains the empirical evidence in the previous section, we use a popular specification of $\Upsilon(\cdot)$ based on Dotsey and King (2005) for the Kimball aggregator,[7]

$$\Upsilon\left(\tilde{y}(i)\right) = \frac{\sigma}{\sigma\left(1-\eta\right)-1}\left(\left[\left(1-\eta\right)\tilde{y}(i)+\eta\right]^{1-\frac{1}{\sigma(1-\eta)}}-1\right)+1, \tag{4}$$

which implies the demand elasticity

$$\sigma\left(\tilde{y}(i)\right) \equiv -\frac{\partial \ln \tilde{y}(i)}{\partial \ln p(i)} = \sigma\left(1+\eta\frac{1-\tilde{y}(i)}{\tilde{y}(i)}\right), \tag{5}$$

where $\sigma>0$ governs the average demand elasticity and $\eta\geq 0$ governs the "superelasticity," i.e., how the demand elasticity of good i decreases with its market share $\tilde{y}(i)$. When $\eta=0$, the demand system corresponds to the standard Constant Elasticity of Substitution (CES) case. Note that this specification is closely related to the specification in

[6] Our mechanism also applies when markup dispersion arises from oligopolistic competition with nested-CES demand, instead of Kimball demand, as in Atkeson and Burstein (2008). Edmond, Midrigan, and Xu (2023) shows the impact of markup dispersion on misallocation is similar under the two market structures.

[7] We rewrote the specification $\Upsilon(\cdot)$ in Dotsey and King (2005) to facilitate parameter interpretation: η here corresponds to $-\eta_o$, where η_o is the original η in Dotsey and King (2005), and σ here corresponds to $\frac{1}{(1+\eta_o)(1-\gamma_o)}$ in the original specification, where γ_o is the original γ in Dotsey and King (2005).

Klenow and Willis (2016) and, in fact, shares the exact same first-order approximation with Klenow and Willis (2016). Due to differences in higher-order terms, Dotsey and King (2005)'s specification generates greater markup dispersion and allows the model to better match the data, facilitating our quantitative analysis. We henceforth use Dotsey and King (2005)'s specification throughout the paper. But the economic lessons we develop here apply to the general formulation of the Kimball aggregator.

Industry intermediate good producers Each industry is populated by a unit measure of intermediate good producers $i \in [0,1]$ operating under monopolistic competition. We write the intermediate good producer's optimization problem recursively and omit the index i for simplicity. The relevant state variables are the pre-production net worth a and the productivity z. Each intermediate good producer produces a differentiated good using a Cobb-Douglas technology in capital k and labor l[8]

$$y = z k^{\alpha} l^{1-\alpha}, \tag{6}$$

where $\alpha \in (0,1)$. Each producer faces idiosyncratic productivity shocks. Its productivity, denoted by z, evolves exogenously according to

$$\log z' = \rho \log z + \epsilon_z, \tag{7}$$

where $\rho \in [0,1)$ and $\epsilon_z \sim \mathcal{N}\left(0, \sigma_e^2\right)$. The idiosyncratic productivity shock, ϵ_z, is independent across time and across firms.

Following Moll (2014), the producer can borrow $b \equiv k - a$ to acquire capital beyond its net worth for production, and the price of capital is normalized to one. Specifically, the producer chooses its capital k and labor l to maximize its post-production net worth

$$n(a,z) \equiv \max_{k,l} \quad p\left(y/Y\right) y - wl - c - (k-a)(1+r) + k(1-\delta), \tag{8}$$

subject to the demand for its goods in (3), the production technology (6), and asset-based and earnings-based borrowing constraints:

$$b \equiv k - a \leq \max\left\{\phi^A k, \phi^{\pi} \pi\right\}, \tag{9}$$

[8]We interpret our model as a model for value added production, but the main analysis and results can be easily extended to a model for gross production in which materials inputs m are explicitly modeled and the firm's production is given by $y = z\left(k^{\alpha} l^{1-\alpha}\right)^{\gamma} m^{1-\gamma}$.

where w is the wage, c is the fixed operating cost, r is the interest rate, δ is the depreciation rate of capital, ϕ^A is the capacity for borrowing against assets (which corresponds to the industry asset liquidation value behind Fact 1a and Fact 2 in Section 2), ϕ^π is the capacity for borrowing against earnings, and

$$\pi \equiv p\left(y/Y\right)\cdot y - wl - c \tag{10}$$

is the producer's earnings.

The producer's overall borrowing capacity in (9) is based on contractual evidence from Lian and Ma (2021). It is determined by the maximum from borrowing against assets, as in Moll (2014), and borrowing against earnings, as in Lian and Ma (2021). Specifically, the firm can pledge assets on a standalone basis and borrow up to the liquidation value of the assets (asset-based borrowing constraints), which is the value of the firm's assets if it is liquidated upon default and the assets are sold off piecemeal. This liquidation value determines the capacity ϕ^A and varies across industries due to differences in the assets' physical characteristics such as transportation costs, customization, and durability as shown by Kermani and Ma (2023a). Alternatively, the firm can pledge earnings from its operations and borrow up to a multiple of the earnings (earnings-based borrowing constraints), which approximates the value from the firm's operations if it undergoes restructuring upon default and continues production. This multiple determines the capacity ϕ^π and is similar across industries (Lian and Ma, 2021).[9]

After production, the producer decides how much dividend d to pay and how much to save a'. Specifically, the producer chooses a' to maximize its value

$$V(a,z) = \max_{a'\geq 0} \quad d + \beta(1-x)\mathbb{E}_z\left[V\left(a',z'\right)\mid z\right] + \beta x a' \tag{11}$$

subject to the budget constraint

$$\psi_d d^2 \mathbb{I}(d<0) + d = n(a,z) - a', \tag{12}$$

where $n(a,z)$ is its post-production net worth in (8), x is the exogenous exit rate, and ψ_d is the cost of equity issuance (namely a negative dividend payment). Exiting firms are replaced by entrants with an initial net worth a^{ent} and productivity $\log z^{ent} = \mathbb{E}\left[\log z\right]$.

[9] If a firm borrows against earnings, the earnings-based constraints always restrict a firm's total debt (including both earnings-based and asset-based borrowing) as a function of its total earnings. Since the firm's assets are pledged to creditors either with their value in liquidation or with their value in continuing operation, the borrowing capacity in (9) is not given by a sum of asset-based borrowing constraints and earnings-based borrowing constraints.

The model closely follows Moll (2014), particularly its discrete time version detailed in Appendix G, with only three key differences. First, the producer is monopolistically competitive rather than perfectly competitive and faces the demand curve in (3). Second, the producer's borrowing capacity in (9) is determined by the maximum implied by asset-based and earnings-based borrowing constraints, rather than by asset-based borrowing constraints alone. Third, instead of modeling the producer as an entrepreneur with log utility, the producer's value is linear in its dividend payment and subject to costly equity issuance and an exogenous probability of exit. The latter assumptions closely follow Ottonello and Winberry (2020, 2024) and better capture large firms subject to earnings-based constraints.

Given the exogenous interest rate r, the wage w, and the demand for the industry final good D, an industry equilibrium consists of a collection of intermediate good producers' decision rules, prices, and quantities $\{a'(a,z), b(a,z), d(a,z), y(a,z), l(a,z), k(a,z), p(a,z)\}$, the industry final good price and quantity P and Y, and the cumulative distribution function of intermediate good producers' states $G(a,z)$ such that: (i) intermediate good and final good producers optimize; (ii) the industry final good price and quantity satisfy industry demand $PY = D$, which follows naturally if the technology aggregates goods produced by different industries in a Cobb-Douglas production form, as in Hsieh and Klenow (2009); (iii) the distribution of intermediate good producers' states $G(a,z)$ is stationary given the saving policy $a'(a,z)$ and the process of z in (7).

4 Borrowing Constraints and Markup Dispersion

In this section, we examine the model's mechanism and performance. In Subsection 4.1, we describe the two key mechanisms that explain why less constrained firms have higher markups. In Subsection 4.2, we calibrate the model to a set of standard data moments. In Subsection 4.3, we show that these two mechanisms can quantitatively explain the empirical facts in Section 2. In Subsection 4.4, we further elaborate the mechanisms and calibration features that deliver the quantitative fit.

4.1 Why Less Constrained Firms Have Higher Markups

What drives markup differences? To explain why less constrained firms have higher markups, we first discuss what drives markup differences across intermediate good producers. In our model, markup differences depend on both intermediate good producers' productivity differences, as in Edmond, Midrigan, and Xu (2023) and Baqaee, Farhi, and Sangani (2024a,b), and on differences in the tightness of their borrowing constraints.

To see this, we first link differences in intermediate good producers' markups in our model to differences in their marginal costs. Each intermediate good producer's optimal choice of labor l implies that

$$p\left(\tilde{y}\right)=\frac{\sigma\left(\tilde{y}\right)}{\sigma\left(\tilde{y}\right)-1}\frac{w}{\left(1-\alpha\right)z\left(k/l\right)^{\alpha}}=\mu\left(\tilde{y}\right)MC, \tag{13}$$

where $MC\equiv\frac{w}{(1-\alpha)z(k/l)^{\alpha}}$ is the marginal cost of producing one additional unit of the intermediate good using additional labor and $\mu\left(\tilde{y}\right)=\frac{\sigma(\tilde{y})}{\sigma(\tilde{y})-1}$ is the markup implied by the demand elasticity in (5). Note that (13) holds regardless of whether the borrowing constraint (9) binds, as it restricts capital choice but not the choice of variable inputs, following Moll (2014). Combining (13) with the demand curve for $p(\tilde{y})$ in (3) and the expression for elasticity $\sigma(\tilde{y})$ in (5) allows us to solve for the intermediate good producer's market share $\tilde{y}$, and thus its markup $\mu(\tilde{y})$, as a function of its marginal cost MC.

We then express the intermediate good producer's marginal cost MC as a decreasing function of its productivity z and an increasing function of the capital wedge τ_k:

$$MC=\frac{\left(r+\delta\right)^{\alpha}w^{1-\alpha}\left(1+\tau_k\right)^{\alpha}}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}z},\quad\text{where}\quad 1+\tau_k\equiv\frac{\alpha}{1-\alpha}\frac{w}{r+\delta}\frac{l}{k}. \tag{14}$$

The capital wedge τ_k is defined as in Hsieh and Klenow (2009). It is positive if the producer uses "too little" capital relative to labor (so $\frac{l}{k}$ is too high), driving the marginal product of capital too high relative to the cost of capital. In our model, a firm has a positive capital wedge τ_k when its borrowing constraint (9) is binding. As a result, τ_k is a measure of constraint tightness in the model, and we will use "constraint tightness" to refer to τ_k in the rest of the draft: the higher the capital wedge τ_k is, the more restrictive the borrowing constraint is for the intermediate good producer's capital choices, and hence, the production outcomes. This constraint tightness is determined jointly by the intermediate good producer's net worth and productivity (a,z), as well as the capacity for borrowing against its assets and earnings (ϕ^A,ϕ^π).

Combining the previous steps, we can express differences in intermediate good producers' markups as the differences in their productivity and constraint tightness. The expressions take a particularly simple form if we take an approximation, around the point where all intermediate goods producers operate efficiently (so their capital wedges $\tau_k = 0$ and markups $\mu = 1$) and have the same productivity z. In fact, this approximation generalizes the Hsieh and Klenow (2009) approach based on log-normal distributions and arrives at the exact same formula for TFP losses in Section 5. Under this approximation, we can express differences in intermediate good producers' markups as follows.

Proposition 1. *Consider two intermediate good producers $i, j \in [0,1]$. Under the specification of $\Upsilon(\cdot)$ in (4), to first order, the differences in their markups are given by*

$$\log\mu(i) - \log\mu(j) \approx \frac{\eta}{(\sigma-1)} \underbrace{\left(\log\tilde{y}(i) - \log\tilde{y}(j)\right)}_{\textit{differences in market share}} \approx -\frac{\sigma\eta}{\sigma-1+\sigma\eta} \underbrace{\left(\log MC(i) - \log MC(j)\right)}_{\textit{differences in marginal cost}} \tag{15}$$

$$\approx \frac{\sigma\eta}{\sigma-1+\sigma\eta} \underbrace{\left(\log z(i) - \log z(j)\right)}_{\textit{differences in productivity}} - \frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta} \underbrace{\left(\log(1+\tau_k(i)) - \log\left(1+\tau_k(j)\right)\right)}_{\textit{differences in constraint tightness}}. \tag{16}$$

(15) captures the standard properties of the Kimball aggregator. Intermediate good producers with lower marginal costs, and hence higher market shares, face less elastic demand (e.g., in (5)) and have higher markups. (16) shows that markup differences in our model depends on differences in both productivity and constraint tightness. As standard in the literature (e.g., Edmond, Midrigan, and Xu, 2023; Baqaee, Farhi, and Sangani, 2024a,b), higher markups can arise because productive firms have lower marginal costs and larger market shares. Unique to our model, higher markups can also arise because less constrained firms (i.e., firms with lower capital wedges τ_k) have lower marginal costs and higher market shares.

Proposition 1 also illustrates how key parameters impact the dispersion of markups across different intermediate goods producers. A higher η (a higher superelasticity) means that the same differences in market share translate into larger differences in demand elasticity and hence larger differences in markups. In turn, the same differences in productivity and constraint tightness also translate into larger differences in markups. In addition, a higher α (greater importance of capital in the production function 6) means that constraint tightness impacts markups more: the same differences in constraint tightness also translate into larger differences in markups.

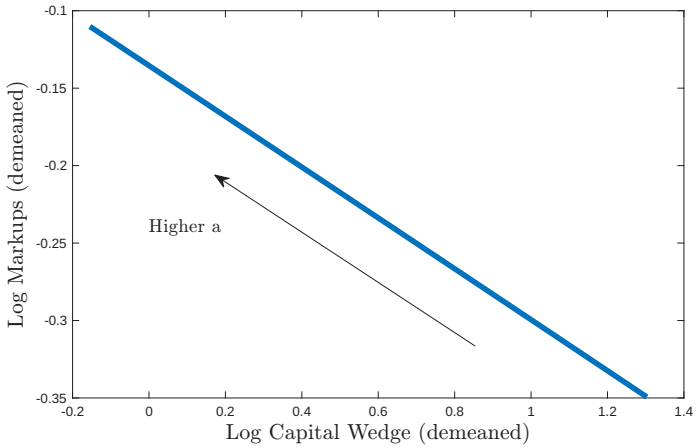
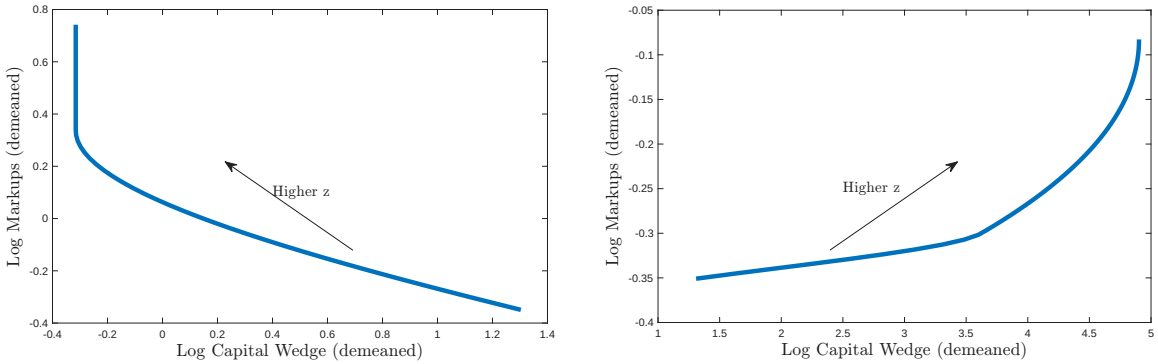


Figure 3: Markups and Constraint Tightness of Firms with Different Net Worth a but the Same Productivity z. This figure plots, for firms with the same productivity ($\log z$ at its mean) but differ in net worth a, their log markup $\log\mu$ (demeaned, x-axis) and their constraint tightness, measured as log capital wedges $\log\left(1+\tau_k\right)$ (demeaned, y-axis). The model parameters are from the baseline calibration explained below.



Panel A. Relevant Earnings-Based Constraints

Panel B. No Earnings-Based Constraints

Figure 4: Markups and Constraint Tightness of Firms with the Same Net Worth a but Different Productivity z. This figure plots, for firms with the same net worth ($a = 0.01$) but differ in their productivity z, their log markup $\log\mu$ (demeaned, x-axis) and their constraint tightness, measured as log capital wedges $\log\left(1+\tau_k\right)$ (demeaned, y-axis). The left panel is based on the main calibration explained below, while the right panel reflects the case without earnings-based constraints ($\phi^{\pi} = 0$, holding other parameters fixed).

Two key mechanisms behind the negative correlation between markups and constraint tightness Our model features two key mechanisms that explain the negative correlation between markups and constraint tightness, which helps explain the empirical facts in Section 2. First, looser borrowing constraints lower marginal costs (16), allowing firms to attain larger market shares and thus charge higher markups (15). This applies in general for both asset-based and earnings-based borrowing constraints. Second, when firms rely on earnings to borrow, higher productivity firms tend to have looser constraints because of their higher earnings. These firms also tend to have higher markups,

again through larger market shares and Kimball demand ((15) and (16)). This second mechanism further generates a negative correlation between markups and constraint tightness, but operates only under earnings-based borrowing constraints and is absent under asset-based borrowing constraints.

We now provide simple illustrations of these two mechanisms. The first mechanism is best illustrated by studying firms with the same productivity, z, but different net worth, a. Figure 3 plots their log markup, $\log\mu$ (demeaned, x-axis), and constraint tightness, measured by log capital wedges, $\log(1+\tau_k)$ (demeaned, y-axis).[10] We observe that firms with higher a, which have lower capital wedges and are less constrained, have higher markups.

The second mechanism is best illustrated by studying firms with the same net worth, a, but different productivity, z. Figure 4 plots their log markup $\log\mu$ (demeaned, x-axis) and constraint tightness measured in log capital wedges $\log(1+\tau_k)$ (demeaned, y-axis). Panel A studies the empirically relevant case where earnings-based borrowing constraints are operative: the capacity for borrowing against earnings, ϕ^π, is 3.5, as in our main calibration. We observe that firms with higher z and correspondingly higher markups have lower capital wedges: they are less constrained because they have higher earnings to borrow against. This second mechanism is, however, absent without earnings-based constraints. Panel B plots this case, with $\phi^\pi = 0$, holding other parameters fixed. Firms with higher z and correspondingly higher markups have higher capital wedges and are more constrained: more productive firms seek to borrow more, but under asset-based constraints they have the same borrowing capacity as other firms with the same a and lower z.

Both mechanisms explain Fact 1a in Section 2, namely that less constrained firms have higher markups. Because the second mechanism operates only under earnings-based borrowing constraints, the negative relationship between markups and constraint tightness is stronger in industries with low asset liquidation values that rely more on borrowing against earnings, which explains Fact 1b in Section 2. Furthermore, in these industries, the second mechanism helps high markup firms borrow more to further expand their market share, thereby raising their markups further and increasing overall markup dispersion. This explains Fact 2 in Section 2. In the following, we show that

[10] Figures 3 and 4 are plotted based on the main calibration parameters explained in detail below.

Table 5: Assigned Parameters

Parameter	Description	Source	Value
β	Discount Factor	David and Venkateswaran (2019)	0.95
ϕ^{π}	Capacity for Borrowing against Earnings	Lian and Ma (2021)	3.5
ϕ^{A}	Capacity for Borrowing against Assets	Kermani and Ma (2023a,b)	0.2
δ	Depreciation Rate	David and Venkateswaran (2019)	0.1
r	Interest Rate	Fed Cleveland 10-Year Real Interest Rate	2%
α	Capital Share	David and Venkateswaran (2019)	0.33
ρ	Persistence of Productivity	Sterk, Sedláček, and Pugsley (2021)	0.96
x	Exogenous Exit Rate	Firm Exit Rate Census BDS	9%
σ	Elasticity of Substitution	Broda and Weinstein (2006)	4
w	Wage	Normalize	1

Notes. This table displays parameters that are exogenously assigned in the main calibration.

our model not only accounts for these empirical facts qualitatively, but also reproduces them quantitatively.

4.2 Calibration

We calibrate the model in two steps. First, we exogenously assign a subset of parameters. Second, we choose the remaining parameters in order to match moments in the data. Table 5 lists the parameters that we assign. The model period is one year, so we set the discount rate β to 0.95, a standard value used, for example, in David and Venkateswaran (2019). We set the capacity for borrowing against assets, ϕ^{A}, to 0.2, based on the aggregate liquidation value of fixed assets and working capital for Compustat manufacturing firms, normalized by their aggregate book assets. We set the capacity for borrowing against earnings, ϕ^{π}, to 3.5, following the contractual evidence in Lian and Ma (2021). Capital depreciates at rate $\delta = 0.1$ annually, following David and Venkateswaran (2019). The (real) interest rate is $r = 2\%$, the historical average of the Cleveland Fed 10-Year real interest rate. The capital share $\alpha = 0.33$ is a standard value used, for example, in David and Venkateswaran (2019). The persistence of log idiosyncratic productivity $\log z$ is $\rho = 0.96$, is standard and from Sterk, Sedláček, and Pugsley (2021). The annual exogenous exit rate is $x = 9\%$, calculated based on the Census BDS data for all industries in 2023. We set σ, which governs the average demand elasticity of substitution, to 4—a standard value used, for example, in Broda and Weinstein (2006). We normalize wage w

Table 6: Calibrated Parameters

Parameter	Description	Value
c	Fixed Operating Cost	0.44
D	Industry Demand	1.21
σ_e^2	Var. of Idiosyncratic Productivity Shock	0.10
a^{ent}	Entrants' Initial Net Worth	0.10
ψ_d	Equity Issuance Cost	0.03
η	Governs Superelasticity	0.73

Notes. This table displays parameters that are chosen to match moments in Table 7.

to be 1.[11]

We then choose the parameters listed in Table 6 to match the empirical moments in Table 7. The first set of moments governs how close firms are to their borrowing constraints in the stationary distribution: the ratio of aggregate earnings to aggregate debt ($\mathbb{E}[\pi]/\mathbb{E}[b]$), calculated using Quarterly Financial Report (QFR) firms; the median firm's debt-to-assets ratio (median b/k), calculated using Compustat firms; and the median firm's earnings-to-debt ratio (median π/b), also using Compustat firms.[12] These moments are informative about fixed operating costs c and industry demand $D = PY$, which impact firms' earnings, and the variance of the idiosyncratic productivity shock σ_e^2, which drives the gap between the median firm's earnings-to-debt ratio and the ratio of aggregate earnings to aggregate debt.

The second set of moments governs entrants: the entrant employment share ($l^{ent}/\mathbb{E}[l]$), calculated based on the share of employment in firms less than one year old using the Census Business Dynamics Statistics database for all sector (2%); and the entrant capital relative to the average ($k^{ent}/\mathbb{E}[k]$), taken directly from Ottonello and Winberry (2024) (4%). These moments are informative about entrants with initial net worth a^{ent} and the cost of equity issuance, ψ_d, which governs how much entrants can acquire capital beyond their initial net worth.[13]

Finally, we match the markup elasticity defined as $\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\Big|_{\tilde{y}=1}$, following Beck and

[11] We verified that the normalization does not affect our results.

[12] We use QFR to calculate aggregate earnings relative to aggregate debt, as it covers the full population of firms (including private ones). However, as QFR does not publish firm-level data, we use Compustat (public firms only) to calculate the median firm's debt-to-assets ratio and earnings-to-debt ratio.

[13] Entrants face binding borrowing constraints, so costly equity issuance becomes the relevant margin that governs how much capital they can acquire beyond their initial net worth.

Table 7: Calibration Targets

Calibration Target	Source	Data	Model
Agg Earnings / Debt ($\mathbb{E}[\pi]/\mathbb{E}[b]$)	QFR	0.39	0.38
Median Earnings/Debt (π/b)	Compustat	0.26	0.27
Median Debt/Assets (b/k)	Compustat	0.21	0.20
Entrant Emp Share ($l^{ent}/\mathbb{E}[l]$)	Census BDS	2%	2%
Entrant Relative Capital Share ($k^{ent}/\mathbb{E}[k]$)	Ottonello and Winberry (2024)	4%	5%
Markup Elasticity $\left(\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\Big\vert_{\tilde{y}=1}\right)$	Beck and Lein (2020)	-0.99 IQR: [-2.14, 0.11]	-0.97

Notes. This table displays the empirical moments targeted in our calibration and the corresponding values generated by the model.

Lein (2020). In our model, this corresponds to $-\frac{\sigma\eta}{\sigma-1}$ and is thus informative about η, which governs superelasticity in our model according to (5). Note that this parameter is imprecisely estimated (the 25th to 75th percentile range is $[-2.14, 0.11]$ in Beck and Lein (2020)) and somewhat controversial in the literature (see Beck and Lein (2020) for discussion), so we also assess robustness to alternative values of η below.

Table 7 shows that our model matches the targeted moments well. The calibrated parameters in Table 6 are comparable to existing estimates in the literature. For example, our calibrated variance of the idiosyncratic shock is $\sigma_e^2 = 0.10$ whereas David and Venkateswaran (2019) calibrates a value of 0.08. Table IA9 in the appendix compares the model and data for untargetted moments. The calibrated model qualitatively matches the persistence of firm markups and the dispersion in networth $Var(\log a)$. The model also replicates the small share of employment, sales, and assets of the smallest firms.

4.3 Quantitative Fit

With the two key mechanisms behind the negative correlation between markups and constraint tightness described in Subsection 4.1, our model can quantitatively explain Fact 1a, Fact 1b, and Fact 2 in Section 2, even though it does not directly target any of these facts.

Fact 1a and Fact 1b First, our model explains Fact 1a, which shows that less constrained firms have higher markups. In the benchmark calibration with $\phi^A = 0.2$, we find that firms' markups and their constraint tightness (measured using the capital wedge) are negatively correlated: $Cov\left(\log\mu, \log(1+\tau_k)\right) \approx -0.08$.

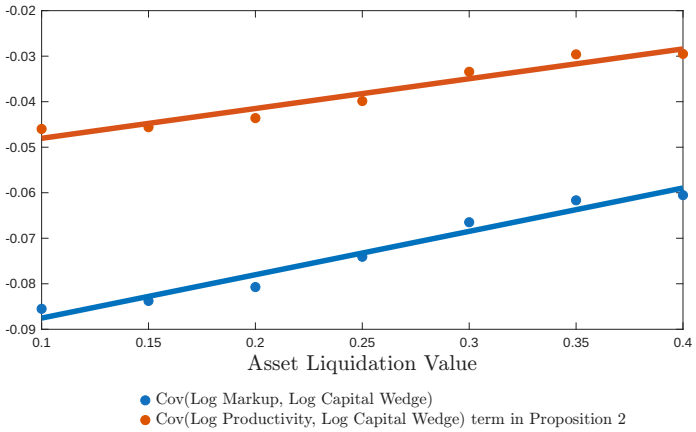


Figure 5: Less Constrained Firms Have Higher Markups, Especially When Firms Rely on Earnings to Borrow. The blue line plots $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ in the model (y-axis) against ϕ^A (x-axis). The red line plots $\frac{\sigma\eta}{\sigma-1+\sigma\eta} Cov\left(\log z, \log\left(1+\tau_k\right)\right)$ in Proposition 2 in the model (y-axis) against ϕ^A (x-axis). When we vary ϕ^A, we recalibrate β while keeping all other parameters fixed, so that aggregate saving $\mathbb{E}[a]$ remains fixed.

Second, our model also explains Fact 1b, which shows that the negative relationship between constraint tightness and markups is stronger in industries with low asset liquidation values (corresponding to a low capacity for borrowing against assets, ϕ^A, in the model) and firms rely more on borrowing against earnings. To apply our model to explain this fact, we solve for the industry equilibria for different values of ϕ^A. When we vary ϕ^A, we recalibrate β such that aggregate saving $\mathbb{E}[a]$ remains fixed for different values of ϕ^A. This ensures that when we vary ϕ^A, we vary the extent of reliance on earnings to borrow, rather than generating systematic variations in aggregate saving and constraint tightness, consistent with the empirical interpretation of ϕ^A in Section 2. We keep other parameters—particularly the capacity for borrowing against earnings, ϕ^π—fixed. This approach is motivated by the evidence in Lian and Ma (2021) and Kermani and Ma (2023a) that ϕ^A differs significantly across industries due to cross-industry variations in asset specificity, whereas ϕ^π remains roughly constant across industries.

In Figure 5, we plot how the covariance between log markup and constraint tightness (measured using log capital wedge), $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$, in the model (y-axis) varies with asset liquidation value ϕ^A (x-axis), shown by the blue line. We find that the negative relationship between firms' markups and constraint tightness is stronger when ϕ^A is lower, so firms rely more on earnings to borrow. This pattern arises because the second mechanism behind the negative correlation between markups and constraint tightness is stronger when ϕ^A is lower. The slope of $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ with respect to ϕ^A in

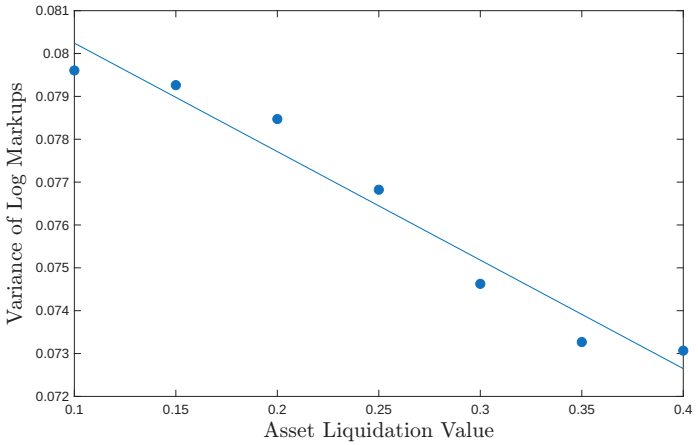


Figure 6: Higher Markup Dispersion When Firms Rely More on Earnings to Borrow. The figure plots $Var\left(\log\mu\right)$ in the model (y-axis) against ϕ^A (x-axis). When we vary ϕ^A, we recalibrate β while keeping all other parameters fixed, so that aggregate saving $\mathbb{E}[a]$ remains fixed.

Figure 5 is 0.10. This magnitude matches the empirical counterpart well: in Column (1) of Panel B of Table 3, the regression coefficient of $Cov\left(\log\mu,\log\left(1+\tau_k\right)\right)$ in an industry (measured using the covariance of log markup with log production worker hours per unit of capital) on the industry asset liquidation value in Census data is around 0.10.[14]

Fact 2 Finally, our model explains Fact 2 in Section 2, which shows that markup dispersion is higher in industries with lower asset liquidation value ϕ^A and greater reliance on earnings to borrow. Figure 6 plots the variance of log markup, $Var\left(\log\mu\right)$, in the model (y-axis) against ϕ^A (x-axis), for the industry equilibrium solved above under different values of ϕ^A. Consistent with Fact 2, we indeed find that markup dispersion is higher when ϕ^A is lower and firms rely more on earnings to borrow. The slope of $Var\left(\log\mu\right)$ with respect to ϕ^A in Figure 6 is -0.03.

The slope of $Var\left(\log\mu\right)$ with respect to ϕ^A in the model (around -0.03) is smaller than its empirical counterpart in the Census data in Table 4 (around -0.09).[15] This smaller magnitude in the model could arise from the well-known fact that markup dispersion in models is often smaller than their empirical counterparts, due to measurement errors of markups in the data which would increase the empirical dispersion (Bils,

[14] Note that we cannot directly compare $Cov\left(\log\mu,\log\left(1+\tau_k\right)\right)$ in the model with the covariance between log markup and the cash/assets ratio studied in Table 1, as the cash/assets ratio does not directly map to $\log\left(1+\tau_k\right)$ in the model.

[15] We compare the slope of markup dispersion with respect to ϕ^A to its Census counterpart in Table 4, rather than to results using Compustat in Table 2, since the Census data provide a better markup measure and broader coverage including both public and private firms.

Klenow, and Ruane, 2021). We hence assess how markup measurement errors affect measures of markup dispersion in the data and construct a version of markup dispersion, free from measurement errors, that is comparable to its model counterpart. Specifically, following the methodology in Bloom and Van Reenen (2007) and Aghion et al. (2024), we use the fact that we have (log) markup measures for the same firm using two different data sources: Census $\log\mu^{Census}$, and Compustat $\log\mu^{Compustat}$ (for Compustat firms that can be matched to the Census data). If the measurement errors in these two datasets are independent—namely,

$$\log\mu^{Census} = \log\mu + \epsilon^{Census} \quad \text{and} \quad \log\mu^{Compustat} = \log\mu + \epsilon^{Compustat},$$

—and if ϵ^{Census} and $\epsilon^{Compustat}$ are independent of the true markup $\log(\mu)$ and of each other, we can recover the true markup dispersion by first regressing $\log\mu^{Compustat}$ on $\log\mu^{Census}$ and obtaining the coefficient

$$\xi = \frac{Cov\left(\log\mu^{Census}, \log\mu^{Compustat}\right)}{Var\left(\log\mu^{Census}\right)} = \frac{Var\left(\log\mu\right)}{Var\left(\log\mu^{Census}\right)}.$$

In this case, the true markup dispersion is

$$Var\left(\log\mu\right) = \xi \cdot Var\left(\log\mu^{Census}\right).$$

Empirically, we merge the firm level markup measures from Census and Compustat, and find that ξ is around 0.4, as shown in Table IA6.

Furthermore, Table IA7 checks that ξ does not vary significantly with the industry asset liquidation value ϕ^A, which is on the x-axis of Figures 2 and IA3 (as well as the model counterpart Figure 6). Table IA7 also checks that ξ does not vary significantly with variances of log markup (measured using either Compustat data or Census data), which are on the y-axis of those figures. In other words, the key relationships in Facts 1b and 2 should not be driven by the degree of markup measurement error somehow being correlated with industry asset liquidation value or markup dispersion.

Accordingly, the slope of how much the true markup dispersion $Var\left(\log\mu\right)$ varies with ϕ^A is given by ξ times the slope of how much the measured markup dispersion $Var\left(\log\mu^{Census}\right)$ varies with ϕ^A, which would be around $-0.036 = 0.4 \times -0.09$, based on estimates for ξ in Table IA6 and the coefficient in Table 4. This is very close to our model counterpart, -0.04, in Figure 6.

4.4 Unpacking the Quantitative Fit

The role of the two key mechanisms The following decomposition further clarifies how the two key mechanisms in Section 4.1 behind the negative correlation between markups and constraint tightness help the model explain Fact 1b and Fact 2 quantitatively. We start with Fact 1b, which finds that the covariance between log markup and constraint tightness (measured in log capital wedge), $Cov\left(\log\mu, \log(1+\tau_k)\right)$, becomes more negative when the asset liquidation value ϕ^A is lower.

Proposition 2. *Under the specification of* $\Upsilon(\cdot)$ *in* (4)*, to second order, the covariance between log markup and constraint tightness (measured in log capital wedge) is given by*

$$Cov\left(\log\mu, \log(1+\tau_k)\right) \approx -\frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta} Var\left(\log(1+\tau_k)\right) + \frac{\sigma\eta}{\sigma-1+\sigma\eta} Cov\left(\log z, \log(1+\tau_k)\right). \quad (17)$$

This decomposition follows from (16) in Proposition 1. The first term in (17) corresponds to the first key mechanism: less constrained firms have lower marginal costs and hence higher markups. As a result, a greater dispersion in constraint tightness (higher $Var\left(\log(1+\tau_k)\right)$) leads to a more negative relationship between markups and constraint tightness (more negative $Cov\left(\log\mu, \log(1+\tau_k)\right)$). The second term in (17) corresponds to the second key mechanism: firms with higher productivity have higher markups. As a result, a more negative relationship between productivity and constraint tightness (more negative $Cov\left(\log z, \log(1+\tau_k)\right)$) leads to a more negative relationship between markups and constraint tightness (more negative $Cov\left(\log\mu, \log(1+\tau_k)\right)$).

How the term $Cov\left(\log z, \log(1+\tau_k)\right)$ in (17) varies with ϕ^A reveals how ϕ^A influences the strength of the second key mechanism. When ϕ^A is lower and firms rely more on earnings to borrow, the second mechanism is stronger: $Cov\left(\log z, \log(1+\tau_k)\right)$ becomes more negative because firms with higher productivity have higher earnings and are less constrained, as illustrated in Panel A of Figure 4. When ϕ^A is higher and firms rely less on earnings to borrow, the second mechanism is weaker: $Cov\left(\log z, \log(1+\tau_k)\right)$ becomes less negative—and can even turn positive—because firms with higher productivity seek to borrow more and therefore may be more constrained, as illustrated in Panel B of Figure 4.

The red line in Figure 5 shows how the $Cov\left(\log z, \log(1+\tau_k)\right)$ term in (17) varies with ϕ^A. Its slope with respect to ϕ^A is 0.07, which accounts for most of the 0.10 slope of

$Cov\left(\log\mu, \log(1+\tau_k)\right)$ with respect to ϕ^A, while the slope of the first term $-\frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta} Var\left(\log(1+\tau_k)\right)$ with respect to ϕ^A is just 0.03. Thus, the slope of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ with respect to ϕ^A is largely driven by how ϕ^A modulates the strength of the second key mechanism.

We have also verified that the approximation formula (17) in Proposition 2 is accurate: the slope of the exact $Cov\left(\log\mu, \log(1+\tau_k)\right)$ with respect to ϕ^A is 0.10, whereas the slope of the approximate $Cov\left(\log\mu, \log(1+\tau_k)\right)$—constructed by adding the two terms in (17)—with respect to ϕ^A is also 0.10.

We now turn to the role of the two key mechanisms in explaining Fact 2, which shows that markup dispersion is higher when ϕ^A is lower and firms rely more on earnings to borrow. To build intuition and connect with the discussion above around Proposition 2, we provide decompositions linking the variance of log markup with (i) the covariance between log productivity and constraint tightness, which determines the strength of the second mechanism and (ii) the covariance between log markup and constraint tightness (measured by the log capital wedge)—the central object in Fact 1b and Proposition 2.

Proposition 3. *Under the specification of* $\Upsilon(\cdot)$ *in* (4)*, to second order, the variance of log markup is given by*

$$Var\left(\log\mu\right) \approx \left[\frac{\sigma\eta}{\sigma-1+\sigma\eta}\right]^2 \left[Var\left(\log z\right) + \alpha^2 Var\left(\log(1+\tau_k)\right) - 2\alpha Cov\left(\log z, \log(1+\tau_k)\right)\right] \quad (18)$$

$$\begin{aligned} &\approx \left[\frac{\sigma\eta}{\sigma-1+\sigma\eta}\right]^2 Var\left(\log z\right) - \left[\frac{\alpha\sigma\eta}{\sigma-1+\sigma\eta}\right]^2 Var\left(\log(1+\tau_k)\right) \\ &\quad \underbrace{-\frac{2\alpha\sigma\eta}{\sigma-1+\sigma\eta} Cov\left(\log\mu, \log(1+\tau_k)\right)}_{>0 \text{ if higher markup firms less constrained}} . \end{aligned} \quad (19)$$

(18) follows from how intermediate good producers' productivity and constraint tightness impact their markups in (16). It links $Var\left(\log\mu\right)$ with the $Cov\left(\log z, \log(1+\tau_k)\right)$ term, which captures the strength of the second key mechanism behind the negative correlation between markups and constraint tightness as explained after Proposition 2. The slope of $Cov\left(\log z, \log(1+\tau_k)\right)$ in (18) with respect to ϕ^A is -0.02 and accounts for most of the -0.03 slope of $Var\left(\log\mu\right)$ with respect to ϕ^A. Similar to Figure 5, how ϕ^A impacts the strength of the second key mechanism is the key to explain the pattern in Figure 6.

(19) uses (17) to replace $Cov\left(\log z, \log(1+\tau_k)\right)$ and directly links $Var\left(\log\mu\right)$ with $Cov\left(\log\mu, \log(1+\tau_k)\right)$—the central object of Fact 1b and Figure 5. It shows that $Var\left(\log\mu\right)$

decreases with $Cov\left(\log\mu, \log(1+\tau_k)\right)$. Hence, from Fact 1b and Figure 5, where a lower ϕ^A (more reliance on earnings to borrow) implies a more negative $Cov\left(\log\mu, \log(1+\tau_k)\right)$, a lower ϕ^A also implies a higher markup dispersion $Var\left(\log\mu\right)$. Intuitively, if larger firms with higher markups are also less constrained, they become even larger through borrowing and hence achieve even higher market share and hence markups. This leads to greater markup dispersion. Because $Cov\left(\log\mu, \log(1+\tau_k)\right)$ is more negative in industries with lower ϕ^A and that rely more on earnings to borrow, as shown in Figure 5, markup dispersion is also higher in those industries.

We have also verified that the approximation in Proposition 3 is accurate. The slope of the exact $Var\left(\log\mu\right)$ with respect to ϕ^A is -0.03. The slope of the approximate $Var\left(\log\mu\right)$, constructed by adding the three terms in (19), with respect to ϕ^A is also -0.03.

Key features of the calibration We now investigate the key features of the calibration that help explain the empirical Facts 1a, 1b, and 2 regarding the connection between constraint tightness and markups in Section 2.

First, we need a sufficiently high η (and hence sufficiently high superelasticity) to ensure that markup varies sufficiently with market share and, consequently, constraint tightness. This generates sufficient markup variation across firms, which enables our two key mechanisms behind the negative correlation between markups and constraint tightness to operate, and is crucial to explain all of Fact 1a, Fact 1b, and Fact 2. In Table 8, we examine how the key results in Figures 5 and 6 vary across different values of η, corresponding to various percentiles of Beck and Lein (2020)'s estimates. Our baseline calibration sets $\eta = 0.73$ so that the implied markup elasticity $\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\Big|_{\tilde{y}=1}$ is close to the mean reported by Beck and Lein (2020). Table 8 shows results for lower values of η corresponding to the median ($\eta = 0.47$) and 75th-percentile markup elasticities ($\eta = 0$, imposing nonnegative η) from Beck and Lein (2020). When we vary η, we aim to isolate the impact of changing superelasticities without inducing systematic variations in average elasticities or markups. Accordingly, we recalibrate σ—which governs average demand elasticities and therefore average markups—while keeping all other parameters fixed, so that aggregate markups remain fixed at their baseline level.

Table 8 shows that a lower superelasticity ($\eta = 0.49$) reduces the magnitudes of the slopes of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ with respect to ϕ^A. This is natural because a lower η reduces markup differences across firms (see Proposition 1), thereby

Table 8: Alternative Values of Parameter Governing Superelasticity η

	Baseline $\eta = 0.73$	Lower $\eta = 0.47$	Zero $\eta = 0$
Slope of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ w.r.t. ϕ^A	0.10	0.09	0
Slope of the $Cov\left(\log z, \log(1+\tau_k)\right)$ Term in (17) w.r.t. ϕ^A	0.07	0.08	0
Slope of $Var\left(\log\mu\right)$ w.r.t. ϕ^A	-0.03	-0.02	0
Slope of the $Cov\left(\log z, \log(1+\tau_k)\right)$ Term in (18) w.r.t. ϕ^A	-0.02	-0.02	0

Notes. The table displays results for alternative values of the parameter η. Specifically, $\eta = 0.47$ and $\eta = 0$ correspond to the median and 75th percentile (imposing nonnegative η) markup elasticities $\frac{d\log\mu(\tilde{y})}{d\log p(\tilde{y})}\Big|_{\tilde{y}=1}$ reported in Beck and Lein (2020). When we vary η, we recalibrate σ while keeping all other parameters fixed, so that aggregate markups remain at their baseline level.

lowering markup dispersion. But this does not fundamentally change the share of the slopes attributable to the slopes of the respective $Cov\left(\log z, \log(1+\tau_k)\right)$ terms in (17) and (18) with respect to ϕ^A, which govern how ϕ^A affects the strength of the second key mechanism, as explained after Propositions 2 and 3. In other words, a lower η does not alter the qualitative properties of Figures 5 and 6 or the economics underlying the two key mechanisms behind the negative correlation between markups and constraint tightness. In the limit case when $\eta = 0$, markup dispersion across firms disappears, so naturally the slopes of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ with respect to ϕ^A are zero.[16]

Second, firms need to have an empirically relevant capacity for borrowing against earnings ($\phi^\pi = 3.5$) to explain the cross-industry variations in Fact 1b and Fact 2. As discussed above, the way $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ vary with ϕ^A hinges on how ϕ^A affects firms' reliance on earnings for borrowing: when ϕ^A is low, firms rely more on earnings to borrow, which strengthens the second key mechanism behind the negative correlation between markups and constraint tightness; when ϕ^A is high, this mechanism is weaker. Without the option of borrowing against earnings ($\phi^\pi = 0$) or with a counterfactually high capacity (e.g., $\phi^\pi = 5$), ϕ^A no longer influences firms' re-

[16] One may wonder whether it is still possible to generate slopes of $Cov\left(\log\mu, \log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ with respect to ϕ^A similar to those empirically documented in Section 2, even with a lower value of η and superelasticity. Our explorations suggest that this is not achievable within the confines of Kimball demand, due to the well-known feature that there exists a choke price that prevents very low-productivity firms from producing, where the choke price is defined as the price at which the market share implied by the demand curve (3) is zero. The existence of such a choke price limits the magnitude of markup dispersion, calculated based on the variance of log markup among producing firms. However, it may be possible to generate the level of dispersion in the data using the nonparametric demand system in Baqaee, Farhi, and Sangani (2024a,b).

Table 9: Alternative Values of Capacity for Borrowing against Earnings ϕ^{π}

	No EBC $\phi^{\pi}=0$	Baseline $\phi^{\pi}=3.5$	Higher EBC Capacity $\phi^{\pi}=5$
Slope of $Cov\left(\log\mu,\log(1+\tau_k)\right)$ w.r.t. ϕ^A	0.02	0.10	0.07
Slope of $Var\left(\log\mu\right)$ w.r.t. ϕ^A	-0.00	-0.03	-0.02

Notes. The table displays results for alternative values of the capacity for borrowing against earnings ϕ^{π}.

liance on earnings to borrow and the strength of the feedback. Table 9 explores how results in Figures 5 and 6 change with different values of ϕ^{π}. Changing ϕ^A leads to significantly smaller variations in both $Cov\left(\log\mu,\log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$, because ϕ^A no longer alters the reliance on borrowing against earnings. When the capacity for borrowing against earnings is counterfactually high ($\phi^{\pi}=5$), firms rely solely on earnings to borrow regardless of ϕ^A, so varying ϕ^A again produces much smaller changes in $Cov\left(\log\mu,\log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$.

Third, a sufficiently large variance of the idiosyncratic productivity shock, σ_e^2, is also important for the model to quantitatively explain empirical Facts 1a, 1b, and 2. A higher σ_e^2 leads to greater heterogeneity across firms in the stationary distribution $G(a,z)$, leading to more markup dispersion and a greater scope for the two key mechanisms behind the negative correlation between markups and constraint tightness. In our baseline calibration, the value $\sigma_e^2=0.1$, is close to that in the literature, e.g., David and Venkateswaran (2019). In Table IA8, we present an alternative calibration where we double the variance of the idiosyncratic productivity shock to $\sigma_e^2=0.2$, while keeping all other parameters fixed. This value is also closer to $\sigma_e^2=0.25$ in Edmond, Midrigan, and Xu (2022). Table IA8 shows that a larger σ_e^2 leads to an even steeper slope of $Cov\left(\log\mu,\log(1+\tau_k)\right)$ and $Var\left(\log\mu\right)$ with respect to ϕ^A in the model: the coefficients increase substantially under this alternative calibration.

5 Allocative Efficiency

5.1 The Planner's Problem

We now study our model's implications for allocative efficiency. The planner allocates the industry's total labor and capital, L and K, across intermediate good producers to

maximize industry final output, subject to the same technology of the final and intermediate good producers, as given in (2) and (6), but not the borrowing constraints.

$$\max_{\{l^P(i),k^P(i)\}_{i\in[0,1]}} Y^P \quad s.t. \quad \int \Upsilon\left(\frac{y^P(i)}{Y^P}\right) di = 1, \quad y^P(i) = z(i)\left(k^P(i)\right)^{\alpha}\left(l^P(i)\right)^{1-\alpha}, \tag{20}$$
$$L = \int l^P(i)\, di, \quad K = \int k^P(i)\, di.$$

As we focus on allocative efficiency, the planner takes the industry's total labor and capital as given, $L = \int l(i)\, di$ and $K = \int k(i)\, di$, from the industry equilibrium studied in Sections 3 and 4.

Consider two intermediate good producers, $i, j \in [0,1]$. Under the specification of $\Upsilon(\cdot)$ based on Dotsey and King (2005) in (4), the differences in their market shares in the planner's problem, $\tilde{y}^P(i) = y^P(i)/Y^P$ and $\tilde{y}^P(j) = y^P(j)/Y^P$, are then linked by their relative productivity,

$$\log\left(\tilde{y}^P(i) + \frac{\eta}{1-\eta}\right) - \log\left(\tilde{y}^P(j) + \frac{\eta}{1-\eta}\right) = \sigma(1-\eta)\left(\log(z(i)) - \log(z(j))\right). \tag{21}$$

This differs from the equilibrium differences in their relative market shares in two ways, as the latter also depend on their relative markups and relative constraint tightness.

$$\log\left(\tilde{y}(i) + \frac{\eta}{1-\eta}\right) - \log\left(\tilde{y}(j) + \frac{\eta}{1-\eta}\right) = \sigma(1-\eta)\left(\log z(i) - \log z(j)\right) \tag{22}$$
$$-\sigma(1-\eta)\left(\log\mu(i) - \log\mu(j) + \alpha\left(\log(1+\tau_k(i)) - \log(1+\tau_k(j))\right)\right).$$

The following proposition summarizes the difference between the intermediate good producer's equilibrium market share and its market share in the planner's problem.

Proposition 4. *Consider the intermediate good producer $i \in [0,1]$. Under the specification of $\Upsilon(\cdot)$ in (4), to first order, the difference between its equilibrium market share and market share in the planner's problem is given by*

$$\log\tilde{y}(i) - \log\tilde{y}^P(i) \approx -\sigma\underbrace{\left(\log\mu(i) - \int\log\mu(j)\, dj\right)}_{\text{markup relative to avg}} - \sigma\alpha\underbrace{\left(\log(1+\tau_k(i)) - \int\log(1+\tau_k(j))\, dj\right)}_{\text{constraint tightness relative to avg}}. \tag{23}$$

As is standard in the literature (e.g., Edmond, Midrigan, and Xu, 2023; Baqaee, Farhi, and Sangani, 2024a,b), the first term in (23) captures the force that firms with above-average markups are too small ($\tilde{y}(i) < \tilde{y}^P(i)$). However, as emphasized in our paper, these firms with higher markups are less constrained. As a result, the second term in

(23), stemming from constraint tightness, moves these higher-markup firms' market share closer to its planner counterpart.[17] This force has important allocative efficiency implications: it significantly lowers the TFP loss from markup dispersion.

5.2 TFP Losses from Markup Dispersion

To illustrate this, we calculate equilibrium TFP relative to the planner's TFP. For this purpose, we define the equilibrium TFP and its counterpart in the planner's problem as

$$Z = \frac{Y}{K^{\alpha} L^{1-\alpha}} \quad \text{and} \quad Z^{P} = \frac{Y^{P}}{K^{\alpha} L^{1-\alpha}}. \tag{24}$$

The (value-added) TFP loss $\log Z^{P} - \log Z$ in our model serves as a measure of allocative efficiency: it captures the misallocation of labor and capital across intermediate good producers generated by the dispersion in markups and borrowing constraints, while leaving aside distortions stemming from the aggregate markup level that influences the industry's total demand for capital and labor. This is consistent with Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Peters (2020), and Aghion et al. (2023).

We now express this TFP loss as a function of the covariance between the log markup and constraint tightness (measured in the log capital wedge) and their variances, the key objects we studied in Section 4.

Proposition 5. *Under the specification of* $\Upsilon(\cdot)$ *in* (4)*, to second order, the TFP loss is*

$$\underbrace{\log Z^{P} - \log Z}_{\text{Total TFP loss}} \approx \underbrace{\frac{\sigma}{2} Var\left(\log \mu\right)}_{\text{markup dispersion}} + \underbrace{\frac{\sigma\alpha^{2} + \alpha(1-\alpha)}{2} Var\left(\log(1+\tau_{k})\right)}_{\text{dispersion in constraint tightness}} + \underbrace{\sigma\alpha Cov\left(\log \mu, \log(1+\tau_{k})\right)}_{<0 \text{ if higher-markup firms less constrained}} \tag{25}$$

The TFP loss formula (25) is essentially the same formula for the TFP loss in Hsieh and Klenow (2009), re-expressed for our purposes. Compared to Hsieh and Klenow

[17] There is a possibility that the second term in (23) is so strong that it dominates, making higher-markup firms too large relative to the planner. Given our calibration in Section 4.2, for firms that differ in productivity z but share the same net worth a, high markup firms are those with high productivity and remain too small (the first term in (23) dominates). Conversely, when we consider firms with the same productivity z but different net worth a, high markup firms face looser constraints and are instead too large (the second term in (23) dominates). Averaging across all firms, overall higher-markup firms still appear too small, $Cov\left(\log \tilde{y} - \log \tilde{y}^{P}, \log \mu\right) < 0$; thus the first term continues to dominate the second, yet the second term is strong enough to significantly reduce the TFP loss from markup dispersion, as shown below.

(2009), we extend in two dimensions. First, we generalize the formula beyond the CES demand case in the original derivation.[18] Second, we consider a second-order approximation, so we do not need the underlying variables to be log-normally distributed.

(25) illustrates how introducing borrowing constraints mitigates TFP losses from markup dispersion. Without borrowing constraints, markup dispersion causes TFP losses: the first term in (25) is positive because high markup firms are too small (the first term in (23)). With borrowing constraints, high markup firms also face looser constraints (Fact 1a) as a consequence of the two key mechanisms behind the negative correlation between markups and constraint tightness. These forces help high markup firms become relatively larger, moving their market shares closer to the planner's allocation. This mitigates TFP losses from markup dispersion: the third term in (25) is negative.

In the benchmark calibration where the capacity for borrowing against assets is set at $\phi^A = 0.2$, we can decompose the TFP loss in (25) as:

$$\underbrace{12.7\%}_{\text{TFP loss}} \approx \underbrace{15.7\%}_{\text{markup dispersion}} + \underbrace{7.6\%}_{\text{dispersion in constraint tightness}} - \underbrace{10.6\%}_{\text{higher markup firms less constrained}}. \tag{26}$$

In other words, the net TFP loss from markup dispersion—defined as the sum of the first and third terms in (25), is just $15.7\% - 10.6\% = 5.1\%$. The fact that high markup firms are less constrained (the third term in (25)) mitigates the TFP loss from markup dispersion by $10.6\%/15.7\% \approx 67.7\%$.

Although our focus is the latter fraction, which illustrates how much the two key mechanisms behind the negative correlation between markups and constraint tightness can mitigate the TFP loss from markup dispersion, the level of value-added TFP loss from markup dispersion is within the range estimated in the literature. In the above decomposition, TFP loss from markup dispersion is 15.7% before and 5.1% after netting out the covariance term. These numbers are smaller than the 20% loss in Baqaee and Farhi (2020) because we use markup dispersion in the model instead of the empirical dispersion. Edmond, Midrigan, and Xu (2023) also uses the model implied TFP loss and reports 2 to 9% loss depending on the assumed level of aggregate markups. Our number is close to their 9% number, which assumes a cost-weighted aggregate markup of 1.45. The level of markup is difficult to estimate due to the lack of firm level price deflators.

[18] The proof focuses on the (4) specification of $\Upsilon(\cdot)$, but we have also verified that the same formula applies to the case of general Kimball demand.

Table 10: TFP Losses from Markup Dispersion and Mitigation for Different Values of ϕ^A

	$\phi^A = 0.1$	$\phi^A = 0.2$	$\phi^A = 0.3$	$\phi^A = 0.4$
Total TFP Loss in (25)	12.7%	12.7%	12.9%	12.9%
Gross TFP loss from Markup Dispersion in (25)	15.9%	15.7%	14.9%	14.6%
Covariance term in (25)	-11.3%	-10.7%	-8.8%	-8.0%
Net TFP loss from Markup Dispersion in (25)	4.6%	5.0%	6.1%	6.6%
% of TFP Loss from Markup Dispersion Mitigated: $\frac{\text{Covariance Term in (25)}}{\text{Markup Dispersion Term in (25)}}$	70.9%	67.9%	58.8%	54.7%
Dispersion of Constraint Tightness Term in (25)	8.1%	7.6%	6.7%	6.3%

Notes. The table shows how TFP losses and its decompositions based on (25) vary with ϕ^A. When we vary ϕ^A, we recalibrate β while keeping all other parameters fixed, so that aggregate saving $\mathbb{E}[a]$ remains fixed.

De Ridder, Grassi, and Morzenti (2026) uses price data from France and estimates a cost-weighted aggregate markup of 1.45.

Furthermore, the cross-industry differences documented in Fact 1b and Fact 2—and studied in Figures 5-6—also affect the extent to which the borrowing constraint channel mitigates TFP losses from markup dispersion. In Table 10, we study this question under different values of ϕ^A, based on the industry equilibria under different values of ϕ^A behind Figures 5 and 6.

When the capacity for borrowing against assets (ϕ^A) is lower, firms rely more on earnings to borrow: markup dispersion, $Var\left(\log\mu\right)$, is higher (Fact 2); consequently, the gross TFP loss from markup dispersion represented by the first term in (25) is higher. However, markups are also more negatively correlated with constraint tightness, in that $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ is more negative (Fact 1b). Together, the latter effect dominates, so the net TFP loss from markup dispersion which is the sum of the first and third terms in (25) becomes smaller, since the $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ term in (25) offsets a larger fraction of the TFP loss from markup dispersion.

Conversely, when ϕ^A is higher, firms rely less on earnings to borrow: $Var\left(\log\mu\right)$ is lower, but $Cov\left(\log\mu, \log\left(1+\tau_k\right)\right)$ is also less negative. Consequently, the net TFP loss from markup dispersion is higher, even though the gross TFP loss from markup dispersion is lower, since the third term in (25) offsets a smaller fraction of the loss. Finally, as the dispersion of constraint tightness in (25) is quantitatively small, the total TFP loss in Table 10 effectively tracks the net TFP loss from markup dispersion.

The possibility of borrowing against earnings is crucial for the borrowing constraint

channel to mitigate the TFP loss from markup dispersion. Without the option of borrowing against earnings ($\phi^{\pi}=0$), as shown in Table 9, the sign of $Cov\left(\log\mu,\log\left(1+\tau_k\right)\right)$ becomes positive, and high markup firms are now more financially constrained. Specifically, in this case ($\phi^{\pi}=0$), the TFP loss decomposition in (25) is:

$$\underbrace{15.5\%}_{\text{TFP loss}} \approx \underbrace{12.5\%}_{\text{markup dispersion}} + \underbrace{6.8\%}_{\text{dispersion in constraint tightness}} - \underbrace{3.7\%}_{\text{higher-markup firms less constrained}}. \tag{27}$$

The third term in (25) is now smaller and only offsets 29.9% of the TFP loss from markup dispersion: the net TFP loss from markup dispersion is now higher as 12.5%-3.7% = 8.8%. Consequently, the net TFP loss from markup dispersion and the total TFP losses are considerably higher than in (26).

5.3 TFP Gains from Size-Dependent Subsidies

When borrowing constraints mitigate TFP losses from markup dispersion, a natural consequence is that size-dependent subsidies to remove markup dispersion become less valuable. Specifically, following Edmond, Midrigan, and Xu (2023), consider the following size-dependent subsidies to the intermediate goods producer:

$$T\left(y/Y;\lambda\right)\equiv\frac{\lambda\Upsilon\left(y/Y\right)}{\mathcal{M}}-\lambda\Upsilon'\left(y/Y\right)y/Y=\frac{\lambda\Upsilon\left(y/Y\right)}{\mathcal{M}}-p\left(y/Y\right)y, \tag{28}$$

where $\mathcal{M}$ is the aggregate (cost-weighted) markup and we use the demand curve (3) for the latter equality. As in Edmond, Midrigan, and Xu (2023), these size-dependent subsidies are designed to fully remove markup dispersion while keeping the aggregate markup $\mathcal{M}$ fixed. The TFP gain from such subsidies depends on the TFP loss from markup dispersion.

Given the subsidies in (28), the intermediate good producer's objective in (8) is then

$$\begin{aligned} n\left(a,z\right)\equiv\max_{k,l}\quad & p\left(y/Y\right)y+T\left(y/Y;\lambda\right)-wl-c-\left(k-a\right)\left(1+r\right)+k\left(1-\delta\right)\\ =\max_{k,l}\quad & \frac{\lambda\Upsilon\left(y/Y\right)}{\mathcal{M}}-wl-c-\left(k-a\right)\left(1+r\right)+k\left(1-\delta\right), \end{aligned} \tag{29}$$

subject to (3), (6), and (9). In other words, the size-dependent subsidies in (28) allows the intermediate good producer to value output from the competitive final goods producer's perspective (adjusted by the aggregate markup), $\frac{\lambda\Upsilon(y/Y)}{\mathcal{M}}$, rather than from the

Table 11: TFP Gains from Size-Dependent Subsidies

	Baseline	No Constraint	Baseline	No Constraint
	Fixed $G(a,z)$		Endogenous $G(a,z)$	
TFP gain with subsidies (28)	5.8%	11.7%	8.8%	11.7%
Total TFP loss without subsidies (28)	12.7%	11.7%	12.7%	11.7%
Total TFP loss with subsidies (28)	6.9%	0.0%	3.9%	0.0%

Notes. The table shows the TFP gains from the size-dependent subsidies (28). "Baseline" corresponds to the baseline calibration with $\phi^A = 0.2$. "No constraint" corresponds to the case without borrowing constraints ($\phi^A = \infty$), while holding other parameters fixed.

perspective of its own revenue $p\left(y/Y\right)y$.[19]

With such size-dependent subsidies, the intermediate good producer's optimal choice of labor l then implies

$$p\left(y/Y\right) = \frac{\lambda}{Y}\Upsilon'\left(y/Y\right) = \mathscr{M}\frac{w}{(1-\alpha)\,z\,(k/l)^{\alpha}} = \mathscr{M}\cdot MC.$$

Compared to (13), we can see that the size-dependent subsidies shut down markup dispersion by making it optimal for every intermediate good producer to set its markup at the aggregate level $\mathscr{M}$.

By removing markup distortions, these size-dependent subsidies generate TFP gains. Such gains are substantial in the absence of borrowing constraints (Edmond, Midrigan, and Xu, 2023). When borrowing constraints are present, however, the TFP gains from size-dependent subsidies are moderated, because the net TFP loss from markup dispersion is already lower since borrowing constraints mitigate that loss.

To illustrate this point, Table 11 reports TFP gains from size-dependent subsidies (28), comparing our baseline calibration ($\phi^A = 0.2$) with the case without borrowing constraints ($\phi^A = \infty$). The first row reports TFP gains from size-dependent subsidies, defined as the difference between the following two rows across the various cases. The second row reports total TFP losses without size-dependent subsidies. The third row reports total TFP losses with size-dependent subsidies, which eliminate markup dispersion (so the first and third terms in (25) are zero) yet still permit TFP losses due to the

[19] Note that the size-dependent subsidies $T\left(y/Y;\lambda\right)$ does not directly enter the earnings in (10), which determine the capacity for borrowing against earnings, as captured by (9). This is because, in practice, such borrowing capacity is based on firms' EBITDA (earnings before interest, taxes, depreciation, and amortization), which is calculated before any taxes or subsidies are applied.

dispersion of constraint tightness (the second term in (25)).[20]

We find that TFP gains from size-dependent subsidies are smaller with borrowing constraints than without borrowing constraints. This result holds whether we fix the stationary distribution of firms, $G(a,z)$, at its pre-size-dependent-subsidy counterpart to isolate the allocative efficiency implications of the subsidies (the first two columns), or allow the subsidies to influence saving behavior and correspondingly the stationary distribution, $G(a,z)$ (the last two columns). The reason is the mechanism we emphasize: under earnings-based borrowing constraints, the two key mechanisms behind the negative correlation between markups and constraint tightness make higher-markup firms less constrained, which mitigates TFP losses from markup dispersion even without size-dependent subsidies and renders them less effective.

6 Conclusion

In this paper, we document new facts that link firms' markups to borrowing constraints: (1) less constrained firms have higher markups, especially in industries where assets are difficult to borrow against and firms rely more on earnings to borrow; (2) markup dispersion is also higher in industries where firms rely more on earnings to borrow. We explain these relationships using a standard Kimball demand model augmented with borrowing against assets and earnings. The key mechanisms are as follows. First, less constrained firms have lower marginal costs and, hence, higher market shares and markups. This applies to both asset-based and earnings-based borrowing constraints. Second, when firms rely on earnings to borrow, higher productivity firms tend to have looser constraints because of their higher earnings; these firms also tend to have higher markups. This channel is stronger in industries with more reliance on borrowing against earnings. These mechanisms help our model quantitatively explain the empirical facts. They also have important implications for allocative efficiency: borrowing constraints lower the overall TFP losses from markup dispersion, especially in industries that rely more on borrowing against earnings.

[20] As discussed above, since TFP losses in (25) capture allocative efficiency, in the case without borrowing constraints, size-dependent policies (28) that remove markup dispersion result in a total TFP loss (25) of zero, as the aggregate markup per se does not lead to TFP losses.

In sum, borrowing constraints provide new insights for understanding the dispersion of markups and its macroeconomic impact. Our work complements studies that examine markup dispersion due to other features of production, products, or pricing (Aghion et al., 2024; Eeckhout and Veldkamp, 2023; Salgado et al., 2025; Bornstein and Peter, 2025; Pellegrino, 2025; Di Tella, Malgieri, and Tonetti, 2025). Future work can further investigate the potential interaction between financial constraints and various determinants of markup dispersion, or the interaction with firm growth through the life cycle (Sedláček and Sterk, 2017; Sterk, Sedláček, and Pugsley, 2021).

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Supplemental Appendix (For Online Publication)
Borrowing Constraints, Markup Dispersion, and Misallocation

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This Appendix contains further material for the article "Borrowing Constraints, Markup Dispersion, and Misallocation." Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."—"B." refer to the main article.

A Supplementary Theoretical Details

We use superscript $*$ to denote the point around which we take an approximation. We approximate around the point where all intermediate goods producers operate efficiently (so their capital wedges $\tau_k^*(i)=0$ and markups $\mu^*(i)=1$) and have the same productivity ($z^*(i)=z$) and hence market share ($\tilde{y}^*(i)=1$). From intermediate goods producers' optimal choice of labor (13), we know that $p^*(i)=\frac{w^{1-\alpha}(r+\delta)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}/z$. From the final good producers' problem (2), and its implied demand curve (3) we know that $P^*=\frac{w^{1-\alpha}(r+\delta)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\frac{1}{z}$ and $\tilde{\lambda}^*\equiv\frac{\lambda^*}{Y^*}=P^*$, where we use

$$\Upsilon'\left(\tilde{y}(i)\right)=\left[\left(1-\eta\right)\tilde{y}(i)+\eta\right]^{-\frac{1}{\sigma(1-\eta)}} \tag{A.1}$$

based on (4). We use a hat over a variable to denote its log deviation from the point of approximation, e.g., $\hat{p}(i)=\ln\left(\frac{p(i)}{p^*(i)}\right)$, with the exception $\hat{\tau}_k(i)=\ln(1+\tau_k(i))$, to accommodate the fact that $\tau_k^*(i)=0$.

Proof of Proposition 1. From the demand elasticity under Dotsey and King (2005)'s specification, we know that

$$\mu(i)=\frac{\sigma\left(\tilde{y}(i)\right)}{\sigma\left(\tilde{y}(i)\right)-1}=\frac{\sigma\left(\eta+\left(1-\eta\right)\tilde{y}(i)\right)}{\sigma\eta+\left(\sigma\left(1-\eta\right)-1\right)\tilde{y}(i)},$$

so, to first order,

$$\hat{\mu}(i)-\hat{\mu}\left(j\right)\approx\frac{\eta}{\sigma-1}\left(\hat{\tilde{y}}(i)-\hat{\tilde{y}}\left(j\right)\right).$$

From (13) and (14), we know that

$$\hat{MC}(i)=\alpha\hat{\tau}_k(i)-\hat{z}(i)=\hat{p}(i)-\hat{\mu}(i).$$

From the demand curve (3), we know that, to first order,

$$\hat{p}(i)-\hat{\tilde{\lambda}}\approx-\frac{1}{\sigma}\hat{\tilde{y}}(i),$$

where $\tilde{\lambda} \equiv \frac{\lambda}{Y}$. Together, we have, to first order,

$$\begin{aligned}\hat{\hat{y}}(i) - \hat{\hat{y}}(j) &\approx -\sigma\left(\hat{\mu}(i) - \hat{\mu}(j) + \alpha\left(\hat{\tau}_k(i) - \hat{\tau}_k(j)\right) - \left(\hat{z}(i) - \hat{z}(j)\right)\right), \\ \hat{\mu}(i) - \hat{\mu}(j) &\approx -\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\left[\alpha\left(\hat{\tau}_k(i) - \hat{\tau}_k(j)\right) - \left(\hat{z}(i) - \hat{z}(j)\right)\right],\end{aligned} \tag{A.2}$$

which proves Proposition 1.

Proof of Proposition 2. Directly from (16).

Proof of Proposition 3. From (16), we have, to second order,

$$Var\left(\log\mu(i)\right) = Var\left(\hat{\mu}(i)\right) \approx \left(\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\right)^2 \left(\alpha^2 Var\left(\hat{\tau}_k(i)\right) + Var\left(\hat{z}(i)\right) - 2\alpha Cov\left(\hat{\tau}_k(i), \hat{z}(i)\right)\right).$$

Together with Proposition 2, we have, to second order,

$$Var\left(\hat{\mu}(i)\right) \approx \left(\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\right)^2 Var(\hat{z}(i)) - \alpha^2\left(\frac{\sigma\eta}{\sigma - 1 + \sigma\eta}\right)^2 Var(\hat{\tau}_k(i)) - 2\alpha\frac{\sigma\eta}{\sigma - 1 + \sigma\eta} Cov(\hat{\mu}(i), \hat{\tau}_k(i)).$$

Proof of Proposition 4. From (21), to first order,

$$\hat{\hat{y}}^P(i) - \hat{\hat{y}}^P(j) \approx \sigma\left(\hat{z}(i) - \hat{z}(j)\right)$$

From (22), to first order,

$$\hat{\hat{y}}(i) - \hat{\hat{y}}(j) \approx \sigma\left(\hat{z}(i) - \hat{z}(j) - \left(\hat{\mu}(i) - \hat{\mu}(j) + \alpha\left(\hat{\tau}_k(i) - \hat{\tau}_k(j)\right)\right)\right).$$

Because $\int \hat{\hat{y}}^P(j)\,dj = \int \hat{\hat{y}}(j)\,dj = 0$, integrating the previous two equations over j leads to (23) to first order.

Proof of Proposition 5. We will use the property that, for any $\{\hat{x}_i\}_{i\in[0,1]}$, to second order, we have

$$\begin{aligned}\ln\left(\int e^{\hat{x}_i} di\right) &\approx \ln\left(\int\left(1 + \hat{x}_i + \frac{1}{2}(\hat{x}_i)^2\right) di\right) \approx \int\left(\hat{x}_i + \frac{1}{2}(\hat{x}_i)^2\right) di - \frac{1}{2}\left(\int \hat{x}_i di\right)^2 \\ &= \mathbb{E}[\hat{x}_i] + \frac{1}{2}\mathrm{Var}(\hat{x}_i),\end{aligned} \tag{A.3}$$

where $\mathbb{E}[\cdot]$ and $\mathrm{Var}(\cdot)$ is over i. From the intermediate good producers' technology (6) and the definition of capital wedge (14), we have

$$\begin{aligned}l(i) &= \frac{y(i)}{z(i)}\left(\frac{k(i)}{l(i)}\right)^{-\alpha} = \frac{y(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{w}{(r+\delta)}\right)^{-\alpha}(1+\tau_k(i))^{\alpha} \\ k(i) &= l(i)\frac{k(i)}{l(i)} = \frac{y(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{w}{(r+\delta)}\right)^{1-\alpha}(1+\tau_k(i))^{\alpha-1}\end{aligned}$$

where $\tilde{\lambda} \equiv \frac{\lambda}{Y}$. From the definition of TFP in (24), we have

$$Z = \frac{1}{\left(\int \frac{\tilde{y}(i)}{z(i)} (1+\tau_k(i))^{\alpha} di\right)^{1-\alpha} \left(\int \frac{\tilde{y}(i)}{z(i)} (1+\tau_k(i))^{\alpha-1} di\right)^{\alpha}} \tag{A.4}$$

$$\hat{Z} = -(1-\alpha)\ln\left(\int e^{\hat{\tilde{y}}(i)-\hat{z}(i)+\alpha\hat{\tau}_k(i)} di\right) - \alpha\ln\left(\int e^{\hat{\tilde{y}}(i)-\hat{z}(i)+(\alpha-1)\hat{\tau}_k(i)} di\right). \tag{A.5}$$

From (2), (3), and (A.1), we have

$$\int \Upsilon\left(\Upsilon'^{-1}\left(\frac{p(i)}{\tilde{\lambda}}\right)\right) di = 1 \quad \text{where} \quad \left(\Upsilon'\right)^{-1}(x) = \frac{x^{-\sigma(1-\eta)} - \eta}{1-\eta}.$$

Together with (4), this means

$$\tilde{\lambda} = \left(\int p(i)^{1-\sigma(1-\eta)} di\right)^{\frac{1}{1-\sigma(1-\eta)}}$$

$$\hat{\tilde{\lambda}} = \frac{1}{1-\sigma(1-\eta)} \ln\left(\int e^{(1-\sigma(1-\eta))\hat{p}_i} di\right)$$

Together with (A.3), we have, to second order,

$$\hat{\tilde{\lambda}} \approx \mathbb{E}[\hat{p}_i] - \frac{\sigma(1-\eta)-1}{2}\mathrm{Var}(\hat{p}_i).$$

From the demand curve (3) and the function form of Dotsey and King (2005) for $\Upsilon'(\cdot)$ in (A.1), we have

$$\tilde{y}(i) = -\frac{\eta}{1-\eta} + \frac{1}{1-\eta}\left(\frac{p(i)}{\tilde{\lambda}}\right)^{-\sigma(1-\eta)}.$$

To second order, we have

$$\begin{aligned}\hat{\tilde{y}}(i) &\approx -\sigma\left(\hat{p}(i) - \hat{\tilde{\lambda}}\right) - \frac{1}{2}\sigma^2\eta\left(\hat{p}(i) - \hat{\tilde{\lambda}}\right)^2 \\ &\approx -\sigma\left(\hat{p}(i) - \mathbb{E}[\hat{p}(i)] + \frac{\sigma(1-\eta)-1}{2}\mathrm{Var}(\hat{p}(i))\right) \\ &\quad -\frac{1}{2}\sigma^2\eta\left(\hat{p}(i) - \mathbb{E}[\hat{p}(i)] + \frac{\sigma(1-\eta)-1}{2}\mathrm{Var}(\hat{p}(i))\right)^2.\end{aligned} \tag{A.6}$$

As a result, to second order, we have

$$\mathbb{E}[\hat{\tilde{y}}(i)] \approx -\frac{\sigma(\sigma(1-\eta)-1)}{2}\mathrm{Var}(\hat{p}(i)) - \frac{\sigma^2\eta}{2}\mathrm{Var}(\hat{p}(i)) = -\frac{\sigma(\sigma-1)}{2}\mathrm{Var}(\hat{p}(i)). \tag{A.7}$$

From the function form of Dotsey and King (2005) for demand elasticity in (5) and a second order approximation, we obtain

$$\mu(i) = \frac{\sigma(\tilde{y}(i))}{\sigma(\tilde{y}(i))-1} = \frac{\sigma(\eta+(1-\eta)\tilde{y}(i))}{\sigma\eta+(\sigma(1-\eta)-1)\tilde{y}(i)}$$

$$\hat{\mu}(i) \approx \frac{\eta}{\sigma-1}\hat{\tilde{y}}(i) + \frac{\eta(1+\sigma(2\eta-1)-\eta)}{2(\sigma-1)}\left(\hat{\tilde{y}}(i)\right)^2.$$

From (A.3) and (A.5), we know that, to second order,

$$\begin{aligned}\hat{Z} \approx & -(1-\alpha)\left(\mathbb{E}\left[\hat{\tilde{y}}(i)-\hat{z}(i)+\alpha\hat{\tau}_k(i)\right]+\frac{1}{2}\mathrm{Var}\left(\hat{\tilde{y}}(i)-\hat{z}(i)+\alpha\hat{\tau}_k(i)\right)\right)\\ & -\alpha\left(\mathbb{E}\left[\hat{\tilde{y}}(i)-\hat{z}(i)+(\alpha-1)\hat{\tau}_k(i)\right]+\frac{1}{2}\mathrm{Var}\left(\hat{\tilde{y}}(i)-\hat{z}(i)+(\alpha-1)\hat{\tau}_k(i)\right)\right)\\ \approx & -\mathbb{E}\left[\hat{\tilde{y}}(i)-\hat{z}(i)\right]-\frac{1}{2}\mathrm{Var}\left(\hat{\tilde{y}}(i)-\hat{z}(i)\right)-\frac{\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right)\\ \approx & \frac{\sigma(\sigma-1)}{2}\mathrm{Var}\left(\hat{p}(i)\right)-\frac{1}{2}\mathrm{Var}\left(\sigma\hat{p}(i)+\hat{z}(i)\right)-\frac{\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right),\end{aligned}$$

where we use (A.6), (A.7), and the fact that $\mathbb{E}[\hat{z}(i)]=0$ based on (7) in the stationary distribution. From (13) and (14), we know that

$$\hat{p}(i)=\hat{\mu}(i)+\alpha\hat{\tau}_k(i)-\hat{z}(i).$$

As a result, to second order,

$$\begin{aligned}\hat{Z} &\approx \frac{\sigma(\sigma-1)}{2}\mathrm{Var}\left(\hat{\mu}(i)+\alpha\hat{\tau}_k(i)-\hat{z}(i)\right)-\frac{1}{2}\mathrm{Var}\left(\sigma\left(\hat{\mu}(i)+\alpha\hat{\tau}_k(i)-\hat{z}(i)\right)+\hat{z}(i)\right)-\frac{\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right)\\ &\approx \frac{\sigma-1}{2}\mathrm{Var}\left(\hat{z}(i)\right)-\frac{\sigma}{2}\mathrm{Var}\left(\hat{\mu}(i)\right)-\frac{\sigma\alpha^2+\alpha(1-\alpha)}{2}\mathrm{Var}\left(\hat{\tau}_k(i)\right)-\sigma\alpha\mathrm{Cov}\left(\hat{\mu}(i),\hat{\tau}_k(i)\right). \qquad \text{(A.8)}\end{aligned}$$

Now consider the planner's problem in (20). Let λ_Y^P be the Lagrange multiplier on $\int_0^1 \Upsilon\left(\frac{y^P(i)}{Y^P}\right)di=1$, λ_L^P be the Lagrange multiplier on $L=\int l^P(i)\,di$, and λ_K^P the Lagrange multiplier on $K=\int k^P(i)\,di$. Planner optimality implies that

$$\lambda_Y^P\frac{1-\alpha}{Y^P}\Upsilon'\left(\frac{y^P(i)}{Y^P}\right)z(i)\cdot\left(\frac{l^P(i)}{k^P(i)}\right)^{-\alpha}=\lambda_L^P \quad\text{and}\quad \lambda_Y^P\frac{\alpha}{Y^P}\Upsilon'\left(\frac{y^P(i)}{Y^P}\right)z(i)\cdot\left(\frac{l^P(i)}{k^P(i)}\right)^{1-\alpha}=\lambda_K^P,$$

which means

$$\frac{l^P(i)}{k^P(i)}=\frac{(1-\alpha)\lambda_K^P}{\alpha\lambda_L^P} \quad\text{and}\quad z(i)\Upsilon'\left(\frac{y^P(i)}{Y^P}\right)=\frac{(\lambda_K^P)^\alpha(\lambda_L^P)^{1-\alpha}}{\lambda_Y^P\alpha^\alpha(1-\alpha)^{1-\alpha}}Y^P.$$

From technology (6), we have

$$l^P(i)=\frac{y^P(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{\lambda_L^P}{\lambda_K^P}\right)^{-\alpha} \quad\text{and}\quad k^P(i)=\frac{y^P(i)}{z(i)}\left(\frac{\alpha}{1-\alpha}\frac{\lambda_L^P}{\lambda_K^P}\right)^{1-\alpha}.$$

From the definition of TFP in (24), we have

$$Z^P=\frac{1}{\int\frac{\tilde{y}^P(i)}{z(i)}di},$$

where $\left\{\tilde{y}^P(i)\equiv\frac{y^P(i)}{Y}\right\}_{i\in[0,1]}$ is uniquely pinned down by the conditions that $z(i)\Upsilon'\left(\tilde{y}^P(i)\right)$ is constant in i and $\int\Upsilon\left(\tilde{y}^P(i)\right)di=1$. Such a solution coincides with the equilibrium TFP when

$\mu(i) = 1$ and $\tau_k(i) = 0$ for all i. This is because, if $\mu(i) = 1$ and $\tau_k(i) = 0$ for all i, the equilibrium TFP in (A.4) is given by $\frac{1}{\int \frac{\tilde{y}(i)}{z(i)} di}$, where $\tilde{y}(i)$ is uniquely pinned down by the conditions that $z(i)\Upsilon'\left(\tilde{y}(i)\right)$ is constant in i (from (3), (13), and (14) when $\mu(i) = 1$ and $\tau_k(i) = 0$ for all i) and $\int \Upsilon\left(\tilde{y}(i)\right) di = 1$. By (A.8), to second order, we henceforth have

$$\hat{Z}^P \approx \frac{\sigma - 1}{2} \text{Var}\left(\hat{z}(i)\right).$$

As a result, to second order, we have

$$\hat{Z}^P - \hat{Z} \approx \frac{\sigma}{2} \text{Var}\left(\hat{\mu}(i)\right) + \frac{\sigma\alpha^2 + \alpha(1-\alpha)}{2} \text{Var}\left(\hat{\tau}_k(i)\right) + \sigma\alpha \text{Cov}\left(\hat{\mu}(i), \hat{\tau}_k(i)\right),$$

which proves Proposition 5.

B Additional Results

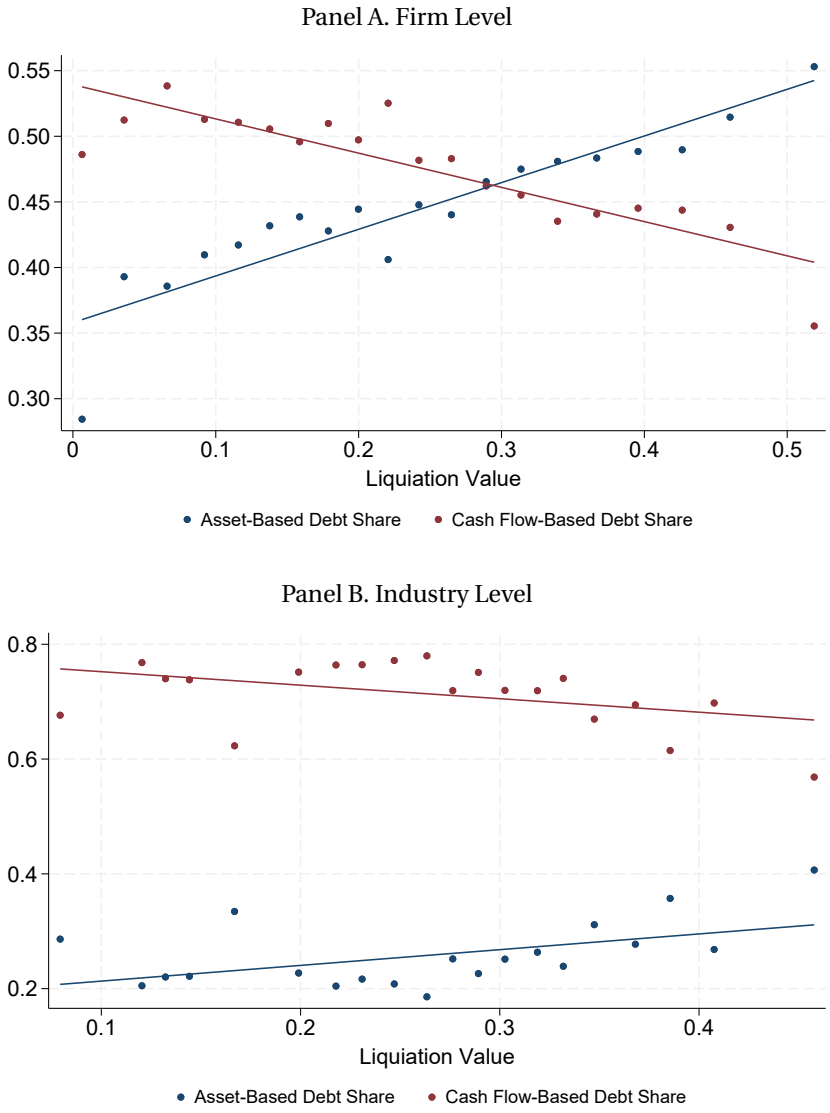


Figure IA1: Liquidation Value and Firm Borrowing. Panel A shows binscatter plot of firm-year level share of debt that pledge the liquidation value of assets (asset-based debt) and debt that pledges earnings (cash flow-based debt) in total debt (measured using CapitalIQ data available from 2002 following Lian and Ma (2021)), as a function of the firm's liquidation value of fixed assets and working capital. Firm-level liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a), normalized by book assets. Panel B shows binscatter plot of industry-year level share of asset-based debt and cash flow-based debt in total debt (measured using CapitalIQ data available from 2002 following Lian and Ma (2021)) against the industry's liquidation value of fixed assets and working capital. Industry-level debt share is calculated using total asset-based debt and cash flow-based debt in the industry divided by total debt in the industry. Industry-level liquidation value is the book value of property, plant, and equipment, inventory, and receivables multiplied by their respective industry-level liquidation recovery rates from Kermani and Ma (2023a), normalized by total book assets.

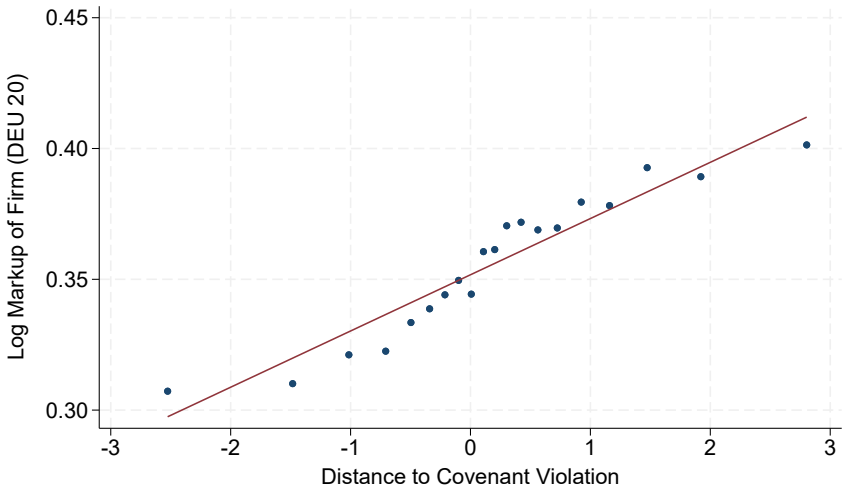


Figure IA2: Firm Markups and Constraint Tightness: Alternative Measure. This figure shows a binscatter plot of the relationship between log markup and firms' distance to violating earnings-based financial covenants, using U.S. Compustat firms annually that have outstanding DealScan loans with earnings-based financial covenants. The x-axis represents firms' distance to violation, which is the standard deviation to violating at least one earnings-based covenant using covenant thresholds in DealScan, and the y-axis represents the log firm-level markup from De Loecker, Eeckhout, and Unger (2020). Industry (3-digit NAICS code) by year fixed effects are included. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

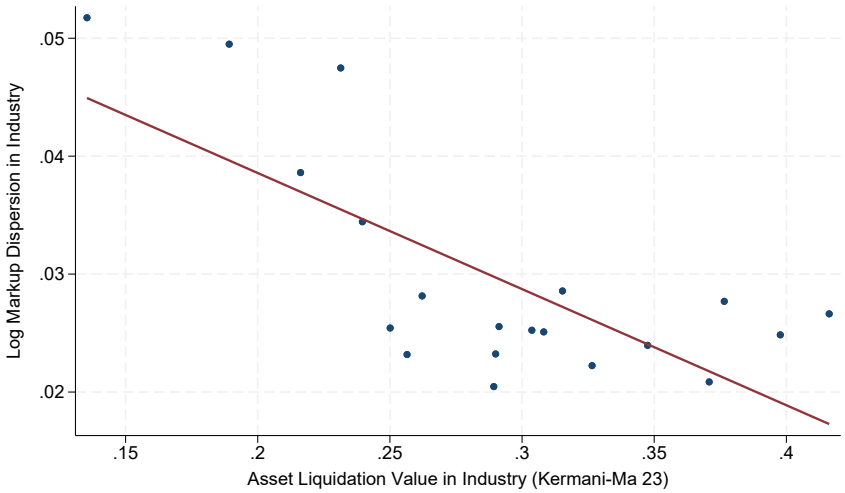


Figure IA3: Industry Markup Dispersion (Census of Manufacturing). This figure shows a binscatter plot of the relationship between the variance of firm-level log markup (based on sales over intermediate inputs plus wage bill in Census of Manufacturing data) in each industry-year on the y-axis, and the industry asset liquidation value from Figure 2 on the x-axis. Year fixed effects are included. We exclude observations with markup less than one.

Table IA1: Firm-Level Markups and Constraint Tightness, Controlling for Firm Age

	Firm Log Markup			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Cash/Assets	0.358***	0.306***	0.175***	0.296***
	(0.037)	(0.030)	(0.015)	(0.046)
Log Firm Age	-0.026***	-0.031***	-0.022***	-0.007
	(0.005)	(0.005)	(0.004)	(0.004)
Observations	109,425	73,050	54,609	86,957
R^2	0.30	0.27	0.20	0.30

Notes. This table shows firm-year level regressions $\text{Log Markup}_{it} = \alpha_{ind(i)t} + \beta \text{Cash/Assets}_{it} + \gamma \text{Log Age}_{it} + \varepsilon_{it}$, among U.S. Compustat firms. Log Markup_{it} for firm i in year t uses the baseline markup from De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input from De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup from De Loecker, Eeckhout, and Unger (2020) in column (3), and the translog markup following De Ridder (2024) in column (4). Compustat does not have a direct measure of firm age. We calculate age using the larger value between time since incorporation (we collect incorporation year data from Datastream) and time since IPO (IPO year is from Compustat). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA2: Firm-Level Markups and Constraint Tightness: Alternative Measure

	Firm Log Markup			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Distance to Covenant Violation	0.021***	0.017***	0.010***	0.021***
	(0.003)	(0.004)	(0.002)	(0.003)
Observations	25,226	16,261	13,076	23,581
R^2	0.34	0.33	0.24	0.36

Notes. This table shows firm-year level regressions Log Markup$_{it} = \alpha_{ind(i)t} + \beta$Distance to Violation$_{it} + \varepsilon_{it}$, among U.S. Compustat firms that have outstanding DealScan loans with earnings-based financial covenants. Distance to violation is the standard deviation to violating at least one earnings-based covenant, using covenant thresholds in DealScan. Log Markup$_{it}$ for firm i in year t uses the baseline markup from De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input from De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup from De Loecker, Eeckhout, and Unger (2020) in column (3), and translog markup following De Ridder (2024) in column (4). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA3: Relationship with Borrowing against Earnings

Panel A. Covariance between Markups and Cash/Assets

	Industry Cov(Log Markup, Cash/Assets)			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Cash Flow-Based Debt Share	0.008**	0.008***	0.008**	0.004**
	(0.003)	(0.003)	(0.003)	(0.002)
Observations	1,145	1,086	1,080	1,062
R^2	0.02	0.03	0.01	0.01

Panel B. Markup Dispersion

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Cash Flow-Based Debt Share	0.056**	0.025**	0.051**	0.031**
	(0.020)	(0.010)	(0.018)	(0.014)
Observations	1,146	1,091	1,087	1,064
R^2	0.04	0.03	0.03	0.03

Notes. Panel A shows industry-year level regressions $\text{Cov(Log Markup, Cash/Assets)}_{kt} = \alpha_t + \beta X_{kt} + \varepsilon_{kt}$, among U.S. Compustat firms. $\text{Cov(Log Markup, Cash/Assets)}_{kt}$ is the covariance of firm-level log markup and cash/assets among firms in industry k in year t. Firm-level markup uses the baseline markup from De Loecker, Eeckhout, and Unger (2020) in column (1), the markup with overhead in production input from De Loecker, Eeckhout, and Unger (2020) in column (2), the accounting markup from De Loecker, Eeckhout, and Unger (2020) in column (3), and translog markup following De Ridder (2024) in column (4). The variable X_{kt} is the fraction of debt that pledges earnings (cash flow-based debt) in total debt in industry k in year t; cash flow-based debt is measured using CapitalIQ data (available from 2002) constructed following Lian and Ma (2021). Panel B shows industry-year level regressions $\text{Var(Log Markup)}_{kt} = \alpha_t + \beta \text{Liqval}_{kt} + \varepsilon_{kt}$, among U.S. Compustat firms. $\text{Var(Log Markup)}_{kt}$ is the variance of firm-level log markup among firms in industry k in year t. The set of markups is the same as the columns in Panel A. The variable X_{kt} is the fraction of debt that pledges earnings (cash flow-based debt) in total debt in industry k in year t. Year fixed effects are included (α_t). Standard errors are double clustered by industry and year. We exclude observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA4: Industry Stock Return Volatility

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.216***	-0.161***	-0.091***	-0.118***
	(0.047)	(0.038)	(0.019)	(0.027)
Industry Stock Price Volatility	0.026*	0.019	0.009	0.004
	(0.015)	(0.013)	(0.007)	(0.011)
Fixed Effects			Year	
Observations	2,225	2,148	2,173	2,153
R^2	0.21	0.15	0.13	0.16

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA5: Industry EBITDA/Debt Dispersion

	Variance of Firm-Level Log Markup in Industry			
	(1)	(2)	(3)	(4)
Markup Measure	DEU Baseline	DEU w/ Overhead	DEU Accounting	Translog
Industry Asset Liquidation Value	-0.189***	-0.151***	-0.084***	-0.103***
	(0.044)	(0.036)	(0.018)	(0.027)
Industry EBITDA/Debt Dispersion	0.004***	0.003**	0.002***	0.002**
	(0.001)	(0.001)	(0.001)	(0.001)
Fixed Effects			Year	
Observations	2,340	2,220	2,264	2,256
R^2	0.22	0.15	0.14	0.16

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA6: Census Markup and Compustat Markup

	Log Markup (Compustat, DEU)	
	(1)	(2)
Log Markup (Census)	0.396***	
	(0.062)	
Log Markup (Census, Translog)		0.396***
		(0.060)
Fixed Effects	Industry × Year	
Observations	7,300	7,300
R^2	0.28	0.28

Notes. This table shows firm level regressions Log Markup in Compustat$_{it}$ = $\alpha_{ind(i)t}$ + βLog Markup in Census$_{it}$ + ε_{it}, for Compustat firms matched to Census of Manufacturing. Log Markup in Compustat$_{it}$ is the log markup in Compustat data following De Loecker, Eeckhout, and Unger (2020). Log Markup in Census$_{it}$ for firm i in year t uses log markup based on sales over intermediate inputs plus wage bill in column (1) and log translog markup following De Ridder (2024) in column (2). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We drop observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA7: Measurement Error Checks

	Log Markup (Compustat, DEU)		
	(1)	(2)	(3)
Log Markup (Census)	0.592**	0.208**	0.263***
	(0.197)	(0.074)	(0.061)
Log Markup (Census) × Industry Asset Liquidation Value	-0.767		
	(0.577)		
Log Markup (Census) × Markup Dispersion (Census)		4.014	
		(2.008)	
Log Markup (Census) × Markup Dispersion (Compustat)			2.376
			(1.539)
Fixed Effects		Industry × Year	
Observations	7,300	7,300	7,300
R^2	0.29	0.29	0.29

Notes. This table shows firm level regressions Log Markup in Compustat$_{it} = \alpha_{ind(i)t} + \beta$Log Markup in Census$_{it} \times Z_{ind(i)t} + \varepsilon_{it}$, for Compustat firms matched to MCensus of Manufacturing. Log Markup in Compustat$_{it}$ is the log markup for firm i in year t in Compustat data following De Loecker, Eeckhout, and Unger (2020). Log Markup in Census$_{it}$ for firm i in year t uses log markup based on sales over intermediate inputs plus wage bill in Census data. Industry characteristic $Z_{ind(i)t}$ is the asset liquidation value in firm i's industry in column (1), the dispersion of log markup in Census data in column (2), and the dispersion of log markup in Compustat data in column (3). Industry (3-digit NAICS code) by year fixed effects are included ($\alpha_{ind(i)t}$). Standard errors are double clustered by industry and year. We drop observations with markup less than one. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table IA8: Alternative Values of Variance of the Idiosyncratic Productivity Shock σ_e^2

	Baseline $\sigma_e^2 = 0.1$	Higher $\sigma_e^2 = 0.2$
Slope of the $Cov\left(\log\mu, \log(1+\tau_k)\right)$ w.r.t. ϕ^A	0.10	0.16
Slope of $Var\left(\log\mu\right)$ w.r.t. ϕ^A	-0.03	-0.05

Notes. The table displays results for alternative values of variance of the idiosyncratic productivity shock, σ_e^2, while keeping other parameters fixed.

Table IA9: Comparison of Model and Untargetted Moments

Untargetted moment	Source	Data	Model
Persistence of log markups $Var[\log(\mu)]$	Compustat	0.89	0.78
Variance of log book equity $Var[\log(a)]$	Compustat	4.34	4.00
Bottom 10% asset share	SOI manufacturing 2012	0.0%	0.6%
Bottom 10% sales share	SOI manufacturing 2012	0.0%	2.4%
Bottom 10% employment share	Census of Manufactures 2012	0.1%	1.5%

Notes. The table compares the model and data for moments not used in the calibration.