General Equilibrium Effects of Carbon Offsets

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Abstract

We construct an analytical general equilibrium model of an economy with carbon offsets. We solve for closed-form solutions and find that raising the price of carbon offsets increases production in the clean sector while having an ambiguous effect on production in the dirty sector. Also, aggregate emissions may increase or decrease when the offset price increases. We then assess the model's results using two definitions of carbon offset additionality, and show that they are over-credited under most parameterizations relative to the aggregate emissions change they induce. Furthermore, additionality is not a sufficient statistic for welfare, and welfare is non-monotonic in additionality. Welfare effects depend on the relative size of the consumption good decrease versus the aggregate emissions change. Our results suggest that market spillover effects warrant consideration when evaluating carbon offset policies.

JEL Codes: H23, D58, D62

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1 Introduction

Demand for carbon offsets from policymakers, corporations, and nonprofits is growing around the world (Harrison, 2024). As the size of the offset market increases, there will be heightened impacts from offset policies on other markets, emissions, and the broader economy. However, the general equilibrium effects of carbon offsets are not well studied. Furthermore, the (non-)additionality of carbon offsets is a well-known problem in offset markets (Bushnell, 2010; Zhang and Wang, 2011; Calel et al., 2025; Aspelund and Russo, 2026), where the true additionality of offsets—that is, the actual reduction in emissions caused by the offset policy—is difficult to measure. Therefore, an analytical general equilibrium model is well-suited to examine the effect of offset policies on additionality. In this study, we examine the economy-wide impacts of offsets, how these impacts influence offset additionality, and we detail how additionality is not a sufficient statistic for welfare changes.

We construct a tractable general equilibrium model of an economy with carbon offsets. The model has two types of inputs—clean and dirty—and the clean input can be subsidized by the sale of carbon offsets. For simplicity, we refer to these inputs as renewable (clean) energy and fossil fuel (dirty) energy, but our model can be applied to non-energy settings such as agricultural carbon offsets. Consumers gain utility from a composite final consumption good but lose utility from the emissions externality. Emissions are produced by both the fossil fuel energy and final goods sectors. To isolate the effect of offsets in the economy, we specify that the government is the sole purchaser of carbon offsets and uses a lump-sum tax to raise the necessary revenue.[1]

From the analytical general equilibrium model, we solve for closed-form solutions and find that raising the price of carbon offsets increases clean energy production and decreases final consumption good production. However, dirty energy production may increase and aggregate emissions can increase or decrease. While a reduction in aggregate emissions is a necessary condition for an increase in welfare, it is not sufficient. Rather, welfare effects depend on the relative size of the consumption good decrease (i.e., the policy's cost) versus the value of the emissions change.

Next, we assess the model's results using two separate definitions of carbon offset additionality and crediting. Additionality is the idea that the emissions reductions from an offset-generating project would not have occurred in the absence of the offset payment. First, we examine what we call "conventional additionality." Conventional additionality credits offsets with the direct emissions reductions they cause; this aligns with most practitioners' definitions. Second, we examine what we call "aggregate additionality." Aggregate additionality is the change in aggregate, economy-wide emissions due to the existence of a specific offset-generating project, including both direct and indirect changes. We then demonstrate the difference between the two measures and how that difference changes as key parameters shift. Importantly, we highlight a type of rebound effect: conventional additionality often over-credits offsets when compared to the aggregate emissions decreases they induce, due to market- and economy-wide responses. To focus on general equilibrium effects and not the non-additionality of offsets, we assume that all offsets are both real and additional as measured by traditional, project-level screening tools. Therefore, our results suggests a further problem

[1] This assumption abstracts from other issues such as distortionary taxation.

with the way that carbon offset additionality is currently measured, as we find that even real and additional offsets can increase aggregate emissions.

In the economics literature, the term "additionality" is used in ways that can align with either of our definitions. For example, Calel et al. (2025) examine additionality of wind projects in India at the project-level, determining that at least 52 percent of the projects are non-additional. In our context, these projects would all be what we call "non-additional by traditional, project-based assessments," and thus excluded from our analysis. Furthermore, the wind projects' emissions reductions were credited using our definition of conventional additionality, as they only consider direct changes. On the flip side, other economists have taken a system-wide view of additionality in line with our definition of aggregate additionality. To that end, Zhang and Wang (2011) examine the additionality of power sector Clean Development Mechanism (CDM) projects in China. They find there is no impact of CDM projects on co-pollutants (which they use as a proxy for less observable carbon emissions), suggesting that they were what we call "non-additional in aggregate". Zhang and Wang (2011) are unable to pinpoint the additionality of specific projects, instead focusing on economy-wide emissions, as we do in our definition of aggregate additionality. Bushnell (2010) is one of the few to clearly make the distinction between conventional and aggregate additionality, albeit in a less formal manner than our treatment.

There is a small but growing literature that empirically assesses carbon offset additionality (Aspelund and Russo, 2026; Calel et al., 2025; Schneider, 2011; Zhang and Wang, 2011), and also a small theoretical literature on offset additionality. The later focuses mainly on screening and other solutions to additionality concerns (Globus-Harris, 2020; Mason and Plantinga, 2013; Montero, 2000; Van Benthem and Kerr, 2013). Again, we assume that all offsets are real and additional as measured by traditional, project-level screening tools, and the general equilibrium model provides a direct relationship between the change in clean energy and both dirty energy and emissions (as a function of the offset subsidy).

There is a related literature on carbon policy leakage. Much of this literature focuses more broadly on emissions leakage under carbon taxes, cap-and-trade systems, or other regulations (see, for example, McGuinness and Ellerman (2008); Fowlie et al. (2016); Guo (2023); Wang and Kuusi (2024). A more limited literature examines leakage in the context of carbon offsets (Filewod and McCarney, 2023). Specifically, Filewod and McCarney (2023) discuss various types of emissions leakage from carbon offsets, including what they call "market" leakage that operates via price effects. The bulk of this literature consists of empirical work quantifying the extent of carbon offset leakage (see, for example, Alix-Garcia et al. (2012) and Heilmayr et al. (2020)). We extend the literature on carbon offset leakage by examining offsets and their net impact on emissions in a general equilibrium model, more thoroughly accounting for all possible channels of market-based leakage.

We contribute to a growing literature that uses general equilibrium models to examine environmental policies. Previous literature has used these models to examine pollution taxes (Fullerton and Heutel, 2007, 2011; Fullerton and Karney, 2018; Garnache and Mérel, 2022; Ren et al., 2011), renewable portfolio standards (Bento et al., 2018; Fullerton and Ta, 2025), emissions trading schemes (Konishi and Tarui, 2015), leakage (Böhringer et al., 2025; Karney et al., 2025), and environmental mandates (Fullerton and Heutel, 2010; Fullerton and Ta, 2020). We expand this literature by exam-

ining the general equilibrium impacts of a type of environmental subsidy—carbon offsets—and we examine the implications for offset additionality and carbon emissions.

Environmental subsidies can be implemented at multiple points in a production economy. We study carbon offsets, which can be interpreted as an abatement subsidy or clean input subsidy. In the simplest partial equilibrium setting, this can sometimes be equivalent to a Pigouvian tax on emissions. However, in a more general setting there are several important differences between a carbon offsets (an abatement subsidy) and a Pigouvian tax on carbon emissions (Sallee, 2025). First, in our model, carbon offsets only partially subsidize abatement due to regulatory constraints: producers are subsidized for producing more renewable energy (thereby avoiding emissions), but are not subsidized for reducing emissions in other sectors, making it an incomplete abatement subsidy. Second, the policymaker must generate revenue for the subsidy, which has a welfare cost in a general equilibrium setting. This revenue generation effect on welfare has similarities to the double dividend literature (see Bento (2024) for an overview of this literature and its history): the double dividend refers to the possibility that a revenue-generating Pigouvian tax can correct externalities while also decreasing the burden of a distortionary tax system. In our context, we see the reverse. Because carbon offsets are a type of subsidy, they require revenue generation and are themselves (potentially) distortionary. The revenue-generating taxes in our model are equivalent to lump-sum taxes, but this revenue generation to pay for offsets still comes at the cost of consumption. Meanwhile, the incomplete environmental subsidy changes relative prices and thus introduces a distortion in the economy. Thus, carbon offsets both partially correct an externality while introducing a distortionary cost and a revenue-generation cost into the economy. These costs can be seen in our welfare analysis: carbon offsets may decrease emissions, but they also decrease final good consumption.

Using a welfare metric known as the marginal value of public funds (MVPF), Hahn et al. (2024) finds that subsides for clean production have higher MVPF values compared to other environmental subsidies in a framework with learning-by-doing spillovers. We model offsets as a subsidy in this study but do not have learning-by-doing spillovers. Regardless, we find that the MVPF is monotonic with welfare in our model. Also, while Hahn et al. (2024) empirically estimates MVPF values for a variety of climate policies—including clean production subsidies—we provide a theoretical framework that can be used to examine the relationship between the MVPF and additionality. We find that additionality is not a sufficient statistic for the MVPF as the two do not always move in the same direction. This result emphasizes the potential disconnect between the focus on additionality and the welfare outcomes in the context of emission offsets.

Environmental subsidies have previously been studied in general equilibrium models, typically in the context of a green production subsidy, often an energy efficiency subsidy or mandate (Fullerton and Wolverton, 1997; Fullerton and West, 2002; Fullerton and Mohr, 2003). In these settings, there is the need for a two-part instrument (the second instrument being a tax on output) to achieve optimal outcomes, because the subsidy alone incentivizes over-production. Similar to our setting, energy efficiency subsidies are incomplete regulations that do not fully align social and private incentives. However, the incentives created by energy efficiency subsidies create distortions via a direct rebound effect (which is not always seen in our setting), as our setting has the incomplete regulation imposed on the "correct" input (abatement). Note that for some parameter values,

we find a backfire effect, which is similar to the rebound effect seen in the two-part instrument literature. Furthermore, emissions in our model come from two sectors: energy and consumption good production. Thus, a two-part instrument (with a carbon offset and a tax on energy output) would not generally achieve first-best, as it would fail to account for emissions from the consumption good sector.[2] Also, the optimal two-part instrument literature often operates in settings without budget constraints (Fullerton and Wolverton, 1997), while we consider the case of a policymaker who must raise revenue via taxation to fund a carbon offset subsidy. Thus, our examination of carbon offsets provides new insights on the role of environmental subsidies and incomplete emissions regulations. There is also a literature on second-best environmental subsidies. For example, Schubert et al. (2026) examine using green subsidies as a second-best complement to a lower-than-optimal carbon tax, finding that larger subsidies are needed to counteract smaller carbon taxes for a given carbon allowance, generating welfare costs. Similarly, we find that carbon offsets also have welfare costs in the form of lowered consumption.

Our general equilibrium model closely follows that of Fullerton and Ta (2025) in its use of clean and dirty energy industries and a single consumption good. However, we extend the model to further differentiate between the clean and dirty industries by modeling a fixed factor of production in the clean industry (e.g., land suitable for wind turbines), following Karney et al. (2025) who similarly model a renewable energy industry with a fixed factor of production.

We find that carbon offsets are often over-credited because they do not cause emissions to decrease by the full quantity for which they are credited under the conventional definition of additionality. This is true even when offset projects would not have occurred in the absence of the offset payment—that is, the offsets are real and additional by traditional measures—and arises due to downstream effects of carbon offsets on other prices and quantities in the economy. Further, we find that under certain conditions, carbon offsets can actually *increase* carbon emissions due to these economy-wide effects. This is somewhat similar to what is found in the rebound effect literature, where an extreme rebound effect (referred to as "backfire") means that an improvement in energy efficiency can increase energy use.[3] In our setting, the mechanism is different. In particular, the rebound effect is most strongly seen in partial equilibrium models and tends to be muted in general equilibrium (Fullerton and Ta, 2020), while our "backfire" effect is present only in general equilibrium. This is due to the difference in mechanism, which in our setting is driven by changes in relative prices in the economy (rather than by a single marginal cost change in the partial equilibrium rebound effect literature).

Our findings have important implications for carbon offset policy as such policies can have general equilibrium effects due to the (potential) size of the carbon market. In particular, policymakers should be aware the even real and additional carbon offsets have spillover effects that are likely to erode their emissions benefits, and in the extreme case can even increase aggregate emissions. Practitioners may wish to re-examine how they credit carbon offsets to account for these market

[2]If we assume zero emissions from the final goods sector, then our model and results more closely mirror that of Fullerton and Wolverton (1997), and a two-part instrument can achieve first-best. However, a two-part instrument cannot generally achieve first-best in our model (see Section 4.3).

[3]The rebound effect is when an improvement in energy efficiency causes an increase in use of the energy efficient durable good, as its marginal operating costs have decreased (Gillingham et al., 2016). For an example of backfire, see Davis et al. (2014).

spillover effects. In this way, we add to the literature on the limitations of carbon offsets as a tool to expand the reach and reduce the costs of incomplete carbon regulation.

The remainder of the paper is organized as follows. Section 2 lays out the general equilibrium model. Section 3 discusses and formalizes our definitions of "conventional" and "aggregate" additionality. Section 4 lays out welfare and MVPF definitions, and discusses optimal policy. Section 5 presents the analytical solution to the model, and Section 6 provides illustrative numerical examples to highlight the relative importance of different parameters as well as a case study of carbon capture and storage subsidies. Section 7 concludes.

2 Model

We model an economy with two types of energy, a single final consumption good, and an emissions externality, following Fullerton and Ta (2025).[4]

Total productive resources in the economy are represented by an aggregate of capital, labor, and materials. This total resource is fixed and denoted by $\bar{K}$. We call this aggregate resource simply "capital" for convenience, but it can be more generally defined as a composite of the primary inputs to production. Capital is employed to produce three goods: an aggregate, final consumption good (X); renewable energy (R); and fossil fuel energy (F). Although we call these inputs "energy", the model works for any clean and dirty inputs; for example, R could be low-carbon emission agriculture (e.g., no-till) while F is conventional agriculture, both of which can be used to produce the consumption good (food).[5]

The resource constraint in the economy is given by $\bar{K} = K_X + K_R + K_F$ where K_i denotes the amount of capital employed in sector $i \in \{X, R, F\}$. The additive form of the resource constraint implies capital is homogeneous and thus paid a uniform price (P_K). We log-linearize the economy as shown in Appendix A. The log-linear version of the resource constraint becomes:

$$0 = \alpha_X \hat{K}_X + \alpha_R \hat{K}_R + \alpha_F \hat{K}_F, \tag{1}$$

where the "hat" notation indicates a small, marginal change; specifically, $\hat{K}_i \equiv \frac{dK_i}{K_i}$. The parameters $\alpha_i \equiv \frac{K_i}{\bar{K}}$ in equation 1 express the initial share of capital employed in each production sector, and thus $1 = \alpha_X + \alpha_R + \alpha_F$. Therefore, an increase in capital use in the renewable energy sector (R) requires a decrease in capital use in the fossil fuel energy sector (F) and/or final goods sector (X). As discussed in Section 4, the potential to decrease capital use in the final goods sector is important because consumers receive their consumption utility directly from good X.

The final consumption good (X) is produced via a constant returns to scale (CRTS) production

[4]Fullerton and Ta (2025) examine Renewable Portfolio Standards, while we examine carbon offsets. Note that our model can be interpreted as applying to Renewable Energy Credits, which are a component of many Renewable Portfolio Standard policies. Our model is distinct from Fullerton and Ta (2025) in that it can be used to examine the additionality implications of carbon offsets. Our model also includes an additional component of a fixed factor in the renewable energy sector, which adds an additional complexity that both reflects real-world resource constraints and is necessary for tractability in our setting.

[5]There are many other possible examples that fit the model, including: landfills with and without methane capture technology, traditional cookstoves versus clean cookstove technology, and livestock farms with and without biogas technology.

function using capital (K_X) and total energy (E) as inputs given by $X = X(K_X, E)$, and sold at price P_X. Total energy is the combination of fossil (F) and renewable (R) energy produced by an energy sector E then sold at price P_E to sector X.[6] Totally differentiating sector X's production function gives the condition:

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XE}\hat{E}, \tag{2}$$

where $\hat{X}$ is the change in total final goods production ($\hat{X} \equiv \frac{dX}{X}$), and the other "hat" variables are similarly defined. The θ_{Xj} parameters indicate the initial input costs shares such that $\theta_{XK} \equiv \frac{(1+t)P_K K_X}{P_X X}$ where t is an *ad valorem* tax on capital (equivalent to a lump-sum tax since $\bar{K}$ is fixed and all capital is sold into the input market). This means $(1+t)P_K$ is the per unit cost of capital for firms; the capital tax revenue will be used to pay for carbon offsets. Similarly, $\theta_{XE} \equiv \frac{P_E E}{P_X X}$ is the input cost share for energy and the cost shares must sum to one due to the CRTS assumption meaning $1 = \theta_{XK} + \theta_{XE}$. The CRTS assumption also implies a zero-profit condition for sector X given by $P_X X = (1+t)P_K K_X + P_E E$; totally differentiating yields:

$$\hat{P}_X + \hat{X} = \theta_{XK}(\hat{t} + \hat{P}_K + \hat{K}_X) + \theta_{XE}(\hat{P}_E + \hat{E}), \tag{3}$$

where $\hat{t} \equiv \frac{dt}{1+t}$ is an *ad valorem* tax.[7] Substitution in production between the capital and energy inputs in the final goods sector is summarized by the elasticity of substitution $\sigma_X \geq 0$. Applying the elasticity's definition leads to:

$$\hat{K}_X - \hat{E} = \sigma_X(\hat{P}_E - \hat{t} - \hat{P}_K), \tag{4}$$

such that increasing the energy price ($\hat{P}_E > 0$), all else equal, increases the ratio of capital to energy ($\hat{K}_X - \hat{E} > 0$) when $\sigma_X > 0$, and, conversely, the capital-to-energy input ratio falls if the price of capital or the capital tax increases. As σ_X goes to infinity the inputs tend towards perfect substitution, while $\sigma_X = 0$ implies perfect complements (i.e., Leontief production).

Next, the energy sector's CRTS production function is given by $E = E(F, R)$ and totally differentiating finds:

$$\hat{E} = \theta_{EF}\hat{F} + \theta_{ER}\hat{R}. \tag{5}$$

The input share parameters in equation 5 are defined as $\theta_{EF} \equiv \frac{P_F F}{P_E E}$ and $\theta_{ER} \equiv \frac{P_R R}{P_E E}$ where P_F and P_R are the prices for renewable and fossil fuel energy, respectively. The CRTS assumption means $1 = \theta_{EF} + \theta_{ER}$ and totally differentiating the zero-profits condition ($P_E E = P_F F + P_R R$) recovers:

$$\hat{P}_E + \hat{E} = \theta_{EF}(\hat{P}_F + \hat{F}) + \theta_{ER}(\hat{P}_R + \hat{R}). \tag{6}$$

[6] Alternatively, E could be produced within sector X via a nested sub-production function. Consumers in this model do not directly purchase energy.

[7] Equation 3 can be represented as $\hat{P}_X = \theta_{XK}(\hat{t} + \hat{P}_K) + \theta_{XE}\hat{P}_E$ after subtracting equation 2. In this form, equation 3 says that the change in the output price is solely a function of the changes in the inputs prices and tax rate. The log-linear zero-profit condition for each of the other production sectors can be rewritten and expressed in a similar manner.

Also, it is helpful to define $\theta_{XF} \equiv \frac{P_F F}{P_X X} = \theta_{XE}\theta_{EF}$ and $\theta_{XR} \equiv \frac{P_R R}{P_X X} = \theta_{XE}\theta_{ER}$ as the shares of dirty and clean energy input costs, respectively, in the production of the final consumption good; it follows that $1 = \theta_{XK} + \theta_{XF} + \theta_{XR}$. The elasticity of substitution in energy production, $\sigma_E \geq 0$, summarizes substitution in production in the energy sector, and applying the definition yields:

$$\hat{R} - \hat{F} = \sigma_E(\hat{P}_F - \hat{P}_R). \tag{7}$$

Again, as σ_E approaches infinity then the different types of energy become perfect substitutes, while $\sigma_E = 0$ implies that renewable and fossil fuel energy cannot be substituted in the aggregate energy output (E).

The production of renewable energy (R) requires the primary factor, capital, as well as a sector-specific resource denoted $\bar{Q}$. This sector-specific resource enters the CRTS production function, $R = (K_R, \bar{Q})$, as a fixed factor such that $d\bar{Q} = 0$. The fixed factor accounts for the limited locations suitable for building renewable energy projects such as wind and solar farms (e.g., suitable windy locations and suitable sunny locations). Rents from the sector-specific resource appear in consumer income at the price P_Q. The fixed factor implies a diminishing marginal product of capital in R and, after totally differentiating, the production function is given by:

$$\hat{R} = \theta_{RK}\hat{K}_R, \tag{8}$$

where $\theta_{RK} \equiv \frac{(1+t)P_K K_R}{(P_R+s)R}$ is the input cost share for capital (i.e., primary inputs). Recall that t is the *ad valorem* tax on the homogeneous capital. The revenue from the capital tax pays for the per unit carbon offset subsidy, s, given to renewable energy production.[8] Thus, $(P_R + s)R$ is the net of subsidy revenue in sector R. The CRTS assumption implies the zero-profit condition $(P_R + s)R = (1 + t)P_K K_R + P_Q\bar{Q}$ and totally differentiating finds:

$$(1 - \gamma)\hat{P}_R + \gamma\hat{s} + \hat{R} = \theta_{RK}(\hat{t} + \hat{P}_K + \hat{K}_R) + \theta_{RQ}\hat{P}_Q, \tag{9}$$

where $\hat{s} \equiv \frac{ds}{s}$ is the change in the carbon offset subsidy. Note that the definition of $\hat{s}$ requires an initial, non-zero carbon offset, and then considers a small change in the offset value. We define the fixed factor input cost share as $\theta_{RQ} \equiv \frac{P_Q\bar{Q}}{(P_R+s)R}$, and then by CRTS it follows that $1 = \theta_{RK} + \theta_{RQ}$. Note that price, or rent, of the fixed factor can vary ($\hat{P}_Q$). Also, we define $\gamma \equiv \frac{s}{s+P_R}$ as the share of the clean energy sector's return due to the offset.[9] Regardless of the fixed factor ($\hat{Q} = 0$), the substitution elasticity in R is still defined and given by $\sigma_R \geq 0$, and applying the definition recovers:

$$\hat{K}_R = \sigma_R(\hat{P}_Q - \hat{t} - \hat{P}_K). \tag{10}$$

This σ_R is not the renewable energy supply elasticity (although the two concepts are related). Rather, it measures how the clean energy sector substitutes between inputs.

[8] We note that this modeling structure also applies to other renewable energy subsidy policies, including Renewable Energy Credits and the US Renewable Energy Production Tax Credit.

[9] The capital tax appears in all sectors where capital is employed due to the assumption of homogeneous capital with perfect mobility.

For production of fossil fuel energy, we assume the production is simply $F = K_F$. That is, the production of F only requires use of the primary input (capital), and totally differentiating leads to:

$$\hat{F} = \hat{K}_F. \tag{11}$$

This modeling choice follows from our national-level interpretation of this setting where the U.S. (or another other country or region) has taxing authority to raise revenue to buy offsets that subsidize local renewable energy production. In other words, U.S. tax revenue is subsidizing U.S. renewable energy projects. In the U.S., the number of feasible renewable energy sites is limited, leading to the fixed factor in the production of R. However, production of fossil fuel energy F is not limited by fixed factors as the U.S. can import virtually unlimited quantities of fossil fuels. Prior literature has used both fixed factors of production in the renewable energy sector (see Karney et al. (2025)) and no fixed factors of production (see Goulder and Hafstead (2017) and Fullerton and Ta (2025)).

There is no substitution in production in sector F since there is only one input. However, the zero-profit condition, $P_F F = (1+t)P_K K_F$, due to CRTS still holds and totally differentiating reveals:

$$\hat{P}_F + \hat{F} = \hat{t} + \hat{P}_K + \hat{K}_F. \tag{12}$$

The many identical households in our model face the budget constraint $P_X X \leq I = P_K \bar{K} + P_Q \bar{Q} + B$, where I is total income (equivalent to GDP) and B is the government's net budget given by $B = t(P_K \bar{K}) - sR$. As Appendix A shows, the zero-profit conditions imply the representative consumer's budget constraint. To focus attention on the carbon offsets' effects, we assume a balanced budget constraint that all tax revenue is spent by the government on the offsets (or equivalently on a renewable energy subsidy) such that $B = 0$. In equilibrium, consumers spend all their income on X. Also, since we fix the total amount of capital, the capital tax is equivalent to a lump-sum tax on consumers. Totally differentiating the government's balanced budget constraint finds:

$$\hat{s} + \hat{R} = \psi \hat{t} + \hat{P}_K, \tag{13}$$

where $\psi \equiv \frac{1+t}{t}$, and this parameter scales the *ad valorem* capital tax relative to the per unit clean energy subsidy.

Finally, we close the general equilibrium system by defining X as the numeraire good and setting $P_X \equiv 1$. This numeraire choice is equivalent to setting the consumer price index (CPI) as the numeraire since there is only one consumption good. Totally differentiating P_X simply yields:

$$\hat{P}_X = 0. \tag{14}$$

When $\hat{P}_X = 0$, the existing per-unit subsidy $s > 0$ is held constant in real terms relative to the CPI and this avoids potentially perverse outcomes as demonstrated by Garnache and Mérel (2022).

The linear system defined by equations 1 through 14 includes two policy variables $\{\hat{t}, \hat{s}\}$ along with 13 endogenous price-quantity variables $\{\hat{K}_X, \hat{K}_R, \hat{K}_F, \hat{X}, \hat{E}, \hat{F}, \hat{R}, \hat{P}_K, \hat{P}_X, \hat{P}_E, \hat{P}_F, \hat{P}_R, \hat{P}_Q\}$. We

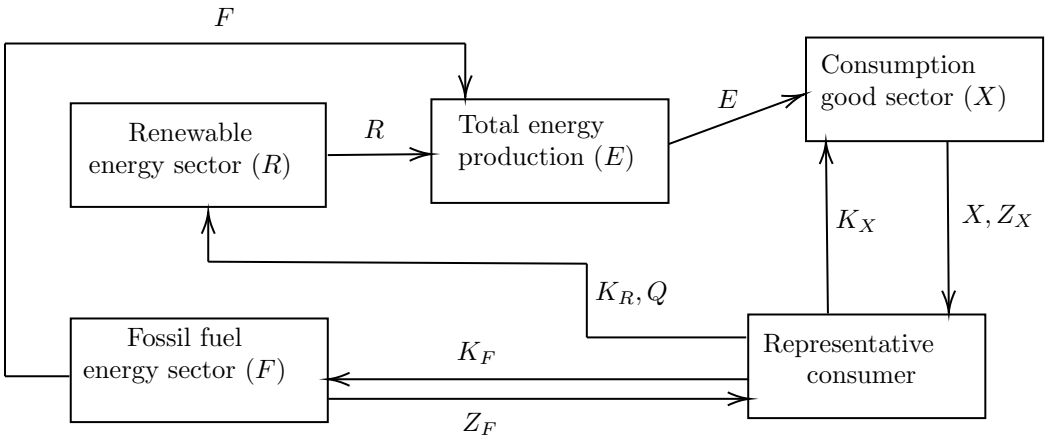


Figure 1: Illustration of the model

study the effect of exogenously increasing the carbon offset or renewable energy subsidy ($\hat{s} > 0$) such that $\hat{t}$ becomes the last endogenous variable (via the balanced-budget condition).

Figure 1 provides an illustration of the modeled economy. Each box represents a production sector as well as a box for the representative consumer. The arrows from the consumer to the production sectors denote the flow of the primary inputs (K_F, K_R, K_X). The arrows return to the consumer show the quantities of the final good (X) and pollution (Z_F, Z_X) that enter the consumers utility function. (Note that Z_F and Z_X are not explicitly solved in the linear system as then are perfectly correlated with F and X, respectively, as discuss in Section 3.) The figure illustrates that sector E simply transforms the clean and dirty energy into a composite energy service for the final goods sector. As we show in Section 6, the elasticity of substitution in sector E given by σ_E is key to determining outcomes.

3 Additionality Definitions

We model carbon offsets as a subsidy on renewable energy production. That is, the policymaker directs offset payments to renewable energy producers because they offset production from the fossil fuel energy sector. This is seen, for example, in carbon offsets sold by wind energy producers in India (Calel et al., 2025) and by hydropower and wind energy producers in China (Zhang and Wang, 2011). Note that our model can also apply to other (non-energy) sectors, such as the agricultural sector, with both clean and dirty inputs and where the clean producers sell offsets.

Carbon offset programs function under the assumption that offsets are both real and additional. In other words, it is assumed that the activity results in an actual reduction in emissions that would not have occurred in the absence of the offset payment. There are multiple methodologies to determine the real-world impacts offset program (Schneider, 2009). However, all methods are applied at the project-level, and tend to inadequately account for market spillover effects and other types of leakage. Millard-Ball and Ortolano (2010) document the leakage assessment requirements of a variety of CDM methodologies, and notes that virtually all energy-sector methodologies ignore

market spillover effects. That is, the renewable energy produced is assumed to directly offset an equivalent amount of fossil fuel energy with no impacts to energy or other market prices. Vöhringer et al. (2006) corroborates this, finding that most CDM projects do not account for leakage due to market and price effects. Calel et al. (2025) also document this for CDM wind farm projects in India, noting that:

> The [offset] revenue is estimated by multiplying the electricity that would be generated by a 'business-as-usual' emissions factor. Most projects use a factor equal to the generation-weighted average carbon dioxide emissions per unit of net electricity generation from all generating power plants serving the same regional grid (tCO_2/MWh). The assumption is that the new project would replace an equivalent amount of 'business-as-usual' generation capacity, avoiding the associated emissions (Calel et al., 2025).

Schneider (2009) provides an overview of additionality assessment in the CDM. He finds that the two most prevalent additionality assessment methods are barrier analysis and investment analysis, both of which are project-level assessments focused on direct emissions reductions. Barrier analysis requires that the offset-producing project demonstrate that there is some sort of barrier preventing the project from moving forward without the offset subsidy. This barrier is often financial, but could be technological or policy-related. Investment analysis involves demonstrating that the offset-generating project is not the most financially attractive option. Furthermore, these projects are then assumed to directly offset an equivalent amount of dirty generation, which is used to determine the quantity of offsets they may sell. These project-level assessments of additionality and crediting for direct emissions reductions align with our definition of conventional additionality.

To isolate the impact of the general equilibrium effects of offsets on aggregate emissions, we limit our attention to offsets that are real and additional as determined by traditional screening tools for additionality. That is, we focus attention on offsets from projects that would not have occurred in the absence of the offset payment, referring to these as "additional by traditional measures". Essentially, by modeling offsets as a subsidy that increases R, our model implicitly assume that the newly incentivized clean energy production ($\hat{R}$) is additional by traditional measures, as it would not have occurred in the absence of the subsidy increase.

In our model, we examine an increase in the offset subsidy ($\hat{s} > 0$). Thus, conventional additionality considers the new quantity of renewable energy ($dR > 0$) induced by the higher subsidy as additional, and then credits those renewable energy projects with the corresponding level of emissions that would have been produced by that quantity of energy under fossil fuel energy production (the direct emissions change). We call this estimate of emissions reductions the "conventional additionality" measure of the offsets, denoted Ω_C. It is defined as $\Omega_C \equiv -\xi_F dR$ and equivalent to:

$$\Omega_C = -(\xi_F R)\hat{R}, \tag{15}$$

where ξ_F is the emission factor of fossil fuel energy (F) such that $\xi_F R$ is the implied total conventional emissions offset prior to the change in the subsidy. Our definition of conditional additionality means $\Omega_C < 0$ when $\hat{R} > 0$. Therefore, a larger increase in renewable energy ($\hat{R}$) or a larger emission factor

for the fossil fuel sector implies more conventional offsets, and hence a larger quantity of emission reductions, all else equal.

Note that there is a subtle distinction between our definition of conventional additionality and the project-level screening tools described above that are used to determine real-world project additionality. In particular, our term "conventional additionality" refers to an estimate of emissions changes caused by an additional offset project, and not the additionality of an offset project itself.

The purported purpose of carbon offset programs is to reduce carbon emissions. If so, then the true interest of policymakers from an emissions-only standpoint is the change in total emissions induced by the increase in the offset price (i.e., subsidy). Total carbon emissions are given by $Z = Z_F + Z_X$ where Z_F denotes emissions from the fossil fuel sector and Z_X represents emissions in the rest of the economy. Thus, the production of fossil fuel energy (F) and the production of the final good (X) both generate emissions but the renewable energy sector (R) does not. Then, following Fullerton and Ta (2025), we define $Z_F \equiv \xi_F F$ and $Z_X \equiv \xi_X K_X$ where ξ_X is the emission factor in the final goods sector related to the primary input K_X. Substituting implies $Z = \xi_F F + \xi_X K_X$ and totally differentiating Z then finds the proportional change in total emissions as follows:

$$\hat{Z} = \rho_F \hat{F} + \rho_X \hat{K}_X. \tag{16}$$

where $\rho_F \equiv \frac{\xi_F F}{Z}$ and $\rho_X \equiv \frac{\xi_X K_X}{Z}$ are the initial shares of carbon emissions in the economy such that $1 = \rho_F + \rho_X$ (see Appendix A for derivations). As shown in Section 5, the signs of $\hat{F}$ and $\hat{K}_X$ are generally ambiguous and thus the sign of $\hat{Z}$ is generally ambiguous.

We define the quantity change in total emissions as the "aggregate additionality" of offsets, denoted Ω_A. Formally, it is defined as $\Omega_A \equiv \xi_F dF + \xi_X dK_X$ and equivalent to:

$$\Omega_A = (\xi_F F)\hat{F} + (\xi_X K_X)\hat{K}_X. \tag{17}$$

Note that Ω_A can be negative or positive (similar to $\hat{Z}$).[10]

In general, it could be that conventional or aggregate additionality is larger for a given increase in the offset subsidy. We measure this difference via the ratio:

$$\begin{aligned} \Delta \equiv \frac{\Omega_A - \Omega_C}{\Omega_C} &= \frac{\Omega_A}{\Omega_C} - 1, \\ &= -\left[\frac{(\xi_F F)\hat{F} + (\xi_X K_X)\hat{K}_X}{(\xi_F R)\hat{R}}\right] - 1. \end{aligned} \tag{18}$$

If $\Omega_C = \Omega_A$ such that conventional additionality perfectly measures aggregate additionality, then $\Delta = 0$. Typically, we find $\Delta < 0$ implying the conventional additionality method overstates emissions reductions relative to the aggregate additionality method. A value of $\Delta = -0.20$ means conventional additionality overstates true emissions reductions by 20 percent while $\Delta = -1$ means there are no aggregate emission reductions ($\Omega_A = 0$) and the conventional additionality definition

[10] Also, $\Omega_A = dZ$, which implies that equations 16 and 17 are related as follows: $\hat{Z} = \frac{\Omega_A}{Z} \Leftrightarrow Z\hat{Z} = \Omega_A$.

records emissions reductions where none exist.[11] In the extreme, $\Delta < -1$ means $\hat{Z} > 0$ (implying $\Omega_A > 0$) and this case is known as "backfire" where total carbon emissions actually increase as result of an offset policy. It is also possible to find $\Delta > 0$ meaning conventional additionality understates true emission reductions in general equilibrium. The ratio in 18 can be used to compare the degree of additionality, or lack thereof, across scenarios with different levels of conventional additionality. Using the closed-form solutions for the endogenous variables reported in Section 5, we examine the extent to which these two additionality measures differ via the measure Δ, and then discuss implications for policymakers and carbon accounting.

4 Welfare

4.1 Change in Welfare

The offset subsidy has real costs as the revenue required to pay for the subsidies comes from taxes. In this model, revenue comes from an *ad valorem* tax on capital (i.e, the primary factor), although it is equivalent to a lump-sum tax due to the fixed amount of capital in the economy. The imposition of the tax means consumers have less money to spend on final consumption goods (X) and this creates a real welfare cost in general equilibrium. In this section, we derive a welfare formula that shows the trade-off between utility from consumption and damages from emissions.

With a large number of households, n, each household does not consider their own impact on aggregate emissions (nZ), and we assume that aggregate emissions enter separably into utility. The household receives direct utility from the consumer of good X with the the representative consumer's utility function given by $U = U(X; nZ)$. We assume that U is continuous, quasiconcave, and twice-differentiable such that U is strictly increasing in X and strictly decreasing in nZ. Aggregate emissions are set apart in the utility function with a semicolon to indicate that the level of aggregate emissions is not a choice variable; that is, households cannot optimize over nZ.

As shown in Appendix B, the change in welfare from an exogenous policy change is found to be:

$$\frac{dU}{\lambda I} = \underbrace{\hat{X}}_{\text{Marginal Cost}} - \underbrace{\mu\left(\frac{nZ}{I}\right)\hat{Z}}_{\text{Marginal Benefit}}, \tag{19}$$

where λ is the marginal utility of income (and recalling that I is total income, or GDP), and thus the left-hand side of equation 19 is the percent change in welfare using the money metric (Bovenberg and De Mooij, 1994). We define $\mu \equiv \frac{-1}{\lambda}\frac{\partial U}{\partial Z} > 0$ as the marginal environmental damage (MED) from a change in Z. The MED is positive since a binding budget constraint implies $\lambda > 0$ and higher pollution reduces utility ($\frac{\partial U}{\partial Z} < 0$). If $\hat{Z} = 0$, then changes in X are equivalent to changes in welfare in this setting and any change in X comes from income effects since there is no substitution in utility.

In a standard model where a negative externality is controlled by an existing tax, τ, the welfare equation will differ from our equation 19 in two important ways (see Fullerton and Karney (2018) as

[11] The ratio Δ is not defined when $\Omega_C = 0$, but $\Omega_C > 0$ when $\hat{R} > 0$ and this always occurs with an increase in the subsidy as shown in Section 5.

an example). First, final consumption goods ($e.g., \hat{X}$) do not appear in the welfare equation as the existing tax internalizes the cost. Second, welfare terms with respect to pollution changes take the form $(\tau - \mu)\frac{nZ}{I}\hat{Z}$ such that a decrease in pollution $\hat{Z} < 0$ increases welfare when the tax is set lower than the MED (meaning the initial τ may not be set optimally). As a result, welfare can increase or decrease depending on whether the pollutant is over- or under-regulated, and welfare is maximized when $\tau = \mu$ (i.e., a Pigouvian tax). That is, τ is the marginal cost of reducing emissions while μ is the marginal benefit, and maximizing welfare requires setting the marginal benefit equal to the marginal cost.

In contrast, our model does not have an explicit cost to producers for pollution so a fixed parameter similar to τ does not appear in equation 19. However, our final consumption good $\hat{X}$ appears in the welfare equation, and decreasing final good consumption is the indirect cost of subsidizing renewable energy production via offsets (and below we show $\hat{X} < 0$ always). This means $\hat{X}$ is the marginal cost of pollution reduction as labeled in equation 19, and this marginal cost is not fixed but instead a function of economic parameters such as the factor shares and production elasticities. Conversely, in equation 19, the term $\mu\left(\frac{nZ}{I}\right)\hat{Z}$ is the marginal benefit of carbon reductions. (The marginal benefit can be negative in cases of "backfire" when subsidizing clean energy increases total emissions.)

Finally, since μ is a free parameter in this model, then for any $\hat{Z} < 0$ there exists a μ^* such that the welfare gain is non-negative meaning the marginal benefit of subsidizing clean energy is greater than or equal to the marginal cost. Formally, when $\hat{Z} < 0$, define μ^* as follows:[12]

$$\mu^* \equiv \left(\frac{I}{nZ}\right)\frac{\hat{X}}{\hat{Z}}, \tag{20}$$

such that μ^* is increasing in income (I) as well as the loss of final good consumption ($\hat{X}$), but decreasing in the initial amount of emissions (Z) and aggregate carbon emission reductions ($\hat{Z}$). As the closed-form solutions in Section 5 make clear, μ^* is not a function of policy change since $\hat{s} > 0$ cancels out from the numerator and denominator of equation 20; rather, μ^* is a function of the economic parameters and initial policy conditions (s, t).

4.2 MVPF

The marginal value of public funds (MVPF) is a benefit-cost ratio for comparing outcomes across policies, and it is defined as the ratio of the benefits to individuals divided by the net cost to government (Bastani, 2024). A larger MVPF means that a spending policy has more "bang-for-buck" to the government. In the context of public goods, Hahn et al. (2024) provides a framework for evaluating the MVPF while emphasizing the importance of externalities as well as (potential) rebound and general equilibrium effects. Our model fully captures both the rebound and general equilibrium effects from a subsidy-based offset policy. In this section, we translate Hahn et al. (2024)'s framework to our setting to evaluate how the MVPF is related to additionality as defined in Section 3.

[12]However, μ^* is not defined when $\hat{Z} > 0$ (i.e., "backfire").

To begin, the MVPF's numerator is the willingness to pay (WTP) for a small increase in the offset subsidy (ds) given by:

$$WTP = \underbrace{Rds}_{\text{Subsidy Value}} + \underbrace{VdR}_{\text{Externality Value}},$$

where V is the marginal externality value per unit of R. The WTP is then comprised of the dollar value of the subsidy and the dollar value of the externality. Rewriting the above equation in our "hat" notation leads to: $WTP = (sR)\hat{s} + (VR)\hat{R}$. The value for V must account for all environmental benefits from a change in R and not just the direct benefits from offsetting F. In other words, V is related to aggregate additionality defined as $\Omega_A \equiv \xi_F dF + \xi_X dK_X$. As shown below, dF and dK_X can be rewritten in terms of dR, so aggregate additionality can be redefined as $\Omega_A = -\Phi dR$, where Φ is a collection of parameters that includes the emission factors (ξ_F, ξ_X) as well as elasticity and share parameters.[13] Then, the dollar value per unit for a change in R is given by $V = \mu\Phi$, and since Ω_A is negative when emissions reductions occur then $\Phi > 0$ (recalling $\mu > 0$ is the MED). Substituting then finds:

$$WTP = (sR)\hat{s} + (\mu\Phi R)\hat{R}, \tag{21}$$

and the closed-form expression for $\hat{R}$ found in Section 5 can be substituted into 21 to find WTP as a function of the exogenous policy variable $\hat{s}$ only.

Next, still following Hahn et al. (2024), the MVPF's denominator is the net cost to the government given by:

$$Cost = Rds + sdR,$$

where dR captures all of the rebound and general equilibrium effects. Again, switching to our hat notation yields:

$$Cost = sR(\hat{s} + \hat{R}). \tag{22}$$

Therefore, the MVPF in our setting is found to be:

$$MVPF = \frac{WTP}{Cost} = \frac{(sR)\hat{s} + (\mu\Phi R)\hat{R}}{sR(\hat{s} + \hat{R})} = \frac{1 + (\mu\Phi/s)\epsilon}{1 + \epsilon}, \tag{23}$$

where $\epsilon = \hat{R}/\hat{s}$ is the general equilibrium elasticity of renewable energy (R) with respect to the subsidy (s). The parameter value ϵ is implicitly defined in equation 24 below. We find $\epsilon > 0$ meaning that an increase in the subsidy always increases R, which implies that MVPF > 0. In our model, a policy is social welfare improving when MVPF > 1 since the willingness to pay exceeds the cost on the margin. The necessary and sufficient condition for MVPF > 1 is $|\mu\Phi| > s$. This condition means that the dollar value of the total externality benefit per unit of R is greater than the

[13] Alternatively, one can calculate Φ as $\Phi = \frac{Z\hat{Z}}{R\hat{R}} = \frac{dZ}{dR}$.

per unit subsidy. The MVPF increases in μ and Φ; that is, as the environmental damage increases, or the aggregate emission offset increases, then the value of the offset program increases. Conversely, the MVPF decreases as the subsidy level increases, all else equal.

4.3 Optimal Policy

The model in Section 2 solves for the change in prices and quantities from one equilibrium to another from a change in the prevailing clean energy subsidy. What is the optimal policy? Regardless of the complexities induces by an explicit clean energy sector with a fixed-factor of production, an optimal policy is straightforward: set a Pigouvian tax on emissions, (t_Z), equal to the marginal environmental damage. That is, set $t_Z = -n\mu > 0$ where t_Z is a per unit tax on $Z = Z_F + Z_X$. As a corollary, the single instrument of a subsidy on R alone cannot induce the socially optimal outcome. We check whether a subsidy on R paired with an output tax recovers the first-best solution (in other words, we search for an optimal two-part instrument like in Fullerton and Wolverton (1997)). We find that a two-part instrument consisting of a subsidy on R and an output tax on X cannot achieve the socially optimal outcome except in the special case where $\xi_F = \xi_X$. In the special case where $\xi_X = 0$ (no externality in the final good sector; emissions only come from F), our results mirror that of Fullerton and Wolverton (1997), and we find that a two-part instrument consisting of a subsidy on R and a tax on E can achieve first best.

5 Analytical Solutions

Solving the model detailed in Section 2 yields the following solutions for $\hat{R}$ and $\hat{K}_R$ (see Online Appendix for details):

$$\hat{R} = \theta_{RK}\hat{K}_R = \left[\frac{\gamma\theta_{RK}}{(\theta_{RQ}/\sigma_R) + (AB/C)}\right]\hat{s} = \left(\frac{\gamma\theta_{RK}}{D}\right)\hat{s} = \epsilon\hat{s}, \tag{24}$$

where A and B are a positive collection of share parameters defined by $A \equiv (1-\gamma(1-\theta_{XR})) > 0$ and $B \equiv [\theta_{RK} + \alpha_R\theta_{RQ}] > 0$, respectively. Define $C \equiv [\alpha_X\theta_{ER}\sigma_X + (\alpha_F + \alpha_X\theta_{ER})\sigma_E] \geq 0$ as a non-negative collection of share and elasticity parameters. Finally, let D denote the entire denominator to simplify notation. All parameters and collections are non-negative. Thus, $\hat{R} > 0$ when $\hat{s} > 0$. In words, increasing the subsidy to the clean sector increases the quantity of R. The parameter ϵ (in the final equality) is employed to evaluate the MVPF (as defined in equation 23 below).[14]

The elasticities of substitution play a key role in determining the value of $\hat{R}$. As the elasticities become small, $\hat{R}$ falls. Specifically, $\lim_{\sigma_i \to 0} \hat{R} \to 0 \; \forall i \in \{E, F, X\}$. Intuitively, if the production sectors cannot adjust in response to a change in the clean energy incentive, then the amount of clean energy cannot change. However, the converse does not hold as $\hat{R}$ is finite when any particular

[14]Note that $\hat{R}$ is only defined when (i) $\sigma_R \neq 0$ and (ii) both $\sigma_X \neq 0$ and $\sigma_E \neq 0$. If either condition does not hold, then D is undefined and thus $\hat{R}$ is undefined. The condition $\sigma_R = 0$ cannot hold because that implies K_R and $\bar{Q}$ are perfect complements, but $\bar{Q}$ is fixed and thus R would be fixed too, so the change in R is undefined. The condition $\sigma_X = \sigma_E = 0$ cannot hold because the rest of the economy has no flexibility to adjust to changes in R via substitution (recalling the representative consumer only gets consumption utility from X).

production sector tends towards perfectly substitutable inputs. In Section 6, we numerically calculate $\hat{R}$ and other endogenous variables as a function of the elasticities.

The signs of comparative statics for $\hat{R}$ with respect to the elasticities of substitution are all positive: $\frac{\partial \hat{R}}{\partial \sigma_R}, \frac{\partial \hat{R}}{\partial \sigma_X}, \frac{\partial \hat{R}}{\partial \sigma_E} > 0$. These comparative statics are interpreted as follows. To start, a more elastic substitution in the production of clean energy ($\sigma_R \uparrow$) means production can shift away from the fixed factor and attract capital to increase renewable energy output. Next, a more flexible energy aggregation ($\sigma_E \uparrow$) allows substitution towards the clean energy and away from dirty energy without losing overall energy output. Finally, a more flexible final goods production sector ($\sigma_X \uparrow$) means goods production can utilize the additional energy production coming from the clean energy production. Therefore, more flexible production leads to larger increases in clean energy production, all else equal.

Next, the change in fossil energy production is given by:

$$\hat{F} = \hat{K}_F = \left[1 - \sigma_E \left(\frac{B}{\theta_{RK} C}\right)\right] \hat{R} = \left[1 - \sigma_E \left(\frac{B}{\theta_{RK} C}\right)\right] \left(\frac{\gamma \theta_{RK}}{D}\right) \hat{s}, \tag{25}$$

where we present $\hat{F}$ recursively as a function of $\hat{R}$ and in a closed-form solution. The sign of $\hat{F}$ is generally ambiguous since the bracketed term has ambiguous sign. That is, an increase in the renewable energy subsidy can *either* increase or decrease fossil use depending on parameter values. To provide clarity on the sign of $\hat{F}$, Proposition 1 identifies an if and only if condition where $\hat{s} > 0$ implies $\hat{F} > 0$. Generally speaking, the condition arises when $\sigma_X >> \sigma_E$ (noting σ_X is in C); that is, the final goods sector is much more flexible regarding its inputs than the energy sector. We show such cases numerically in Section 6 where $\hat{F} > 0$ and these cases relate to the issue of "backfire" where increasing the subsidy for renewable energy increases total emissions. Conversely, if σ_E is large enough, then the subsidy induces sector E to aggressively substitute towards R and away from F, pushing the economy towards a reduction in total emissions.

Proposition 1 (Sign of $\hat{F}$)**.** *For $\hat{s} > 0$, if $\sigma_R \neq 0$ and $\sigma_X \neq 0$ (implying $C \neq 0$), then there exist values of $\sigma_E > 0$ such that $\hat{F} > 0$. Furthermore, the range of σ_E values leading to $\hat{F} > 0$ is given by $\frac{\theta_{RK} C}{B} > \sigma_E \geq 0$ such that $\hat{F} < 0$ if and only if $\sigma_E > \frac{\theta_{RK} C}{B} > 0$.*

Proof. Follows from Equation 25. □

Continuing, the change in total energy production is given by:

$$\hat{E} = \left[1 - \sigma_E \left(\frac{B}{\theta_{RK} C}\right) \theta_{EF}\right] \hat{R}, \tag{26}$$

and we omit explicitly substituting for $\hat{R}$ going forward. Since $\hat{F}$ is generally ambiguous, then $\hat{E}$ is generally ambiguous as the production function for energy is given by $E = E(F, R)$. The comparative statics with respect to the elasticities for $\hat{E}$ are scaled versions those for $\hat{F}$, and thus generally ambiguous as well.

The changes in the final goods production sector are found to be:

$$\hat{K}_X = \left(\frac{-1}{\alpha_X \theta_{RK}}\right)\left[\alpha_R + \alpha_F \theta_{RK}\left(1 - \sigma_E\left(\frac{B}{\theta_{RK} C}\right)\right)\right]\hat{R}, \tag{27}$$

$$\hat{X} = \left[\theta_{XE} - \left(\frac{\alpha_R + \alpha_F \theta_{RK}}{\alpha_X \theta_{RK}}\right)\theta_{XK}\right]\hat{R} = -\left(\frac{sR}{I}\right)\hat{R}. \tag{28}$$

For $\hat{K}_X$, if the bracketed term is positive, then an increase in the subsidy (and subsequently $\hat{R}$) leads to a decrease in capital use for final goods production due to the resource constraint in equation 1 and additional capital input use in sector R (as well as sector F, if $\hat{F} > 0$).

As shown in Section 4, $\hat{X}$ is the marginal cost of the offset policy and thus the sign of $\hat{X}$ is important to overall welfare. At first, it appears the sign of $\hat{X}$ is ambiguous given the bracketed term in 28, but it is not. Rather, bracketed terms simplifies to $-sR/I$, where sR is the total offset payment and I is total income (equal to GDP). Therefore, $\hat{X}$ is proportional to the change in R but scaled by the share of offset payments in the economy.

Proposition 2 (Sign of $\hat{X}$)**.** *If $\hat{s} > 0$, then $\hat{X} < 0$.*

Proof. Follows from $\hat{R} > 0$ and Equation 28. □

Proposition 2 means there is always a cost of the offset policy (i.e., no free lunch). This result holds in our model with as-if lump-sum taxes that fund the offset subsidy in an economy with no other distortions. In contrast, the marginal benefit of an offset subsidy can be negative (as discussed next).

Finally, the change in total emissions is $\hat{Z} = \rho_F \hat{F} + \rho_X \hat{K}_X$ and thus can be recursively written:

$$\hat{Z} = \rho_F\left[1 - \sigma_E\left(\frac{B}{\theta_{RK} C}\right)\right]\hat{R} + \rho_X\left(\frac{-1}{\alpha_X \theta_{RK}}\right)\left[\alpha_R + \alpha_F \theta_{RK}\left(1 - \sigma_E\left(\frac{B}{\theta_{RK} C}\right)\right)\right]\hat{R}. \tag{29}$$

The components of $\hat{Z}$ have ambiguous signs and thus $\hat{Z}$ does too since the emission shares (ρ_F, ρ_R) are independent of other parameter values. Therefore, the marginal benefit of the offset policy has an ambiguous sign. In the next section, we numerically demonstrate cases where $\hat{Z}$ is positive meaning the marginal benefit is negative, and in such cases welfare is unambiguously negative since $\hat{X} < 0$. Finally, we note that the sign of $\hat{Z}$ is ambiguous even in the case where $\xi_X = 0$ (that is, the case where there are no emissions from Sector X); this is due to the ambiguous sign of $\hat{F}$, as demonstrated in Proposition 1.

6 Numerical Analysis

This section assigns parameter values for the price-quantity model and welfare formulas shown above. The purpose of conducting the numerical analysis is to generate intuition about the model as well as highlight key features and results and how they change as the underlying economic parameters vary.

To parameterize the model and by way of an illustrative example, we start by setting GDP equal to U.S. nominal GDP of $29.3 trillion in 2024 (U.S. Bureau of Economic Analysis, 2026). Next, we

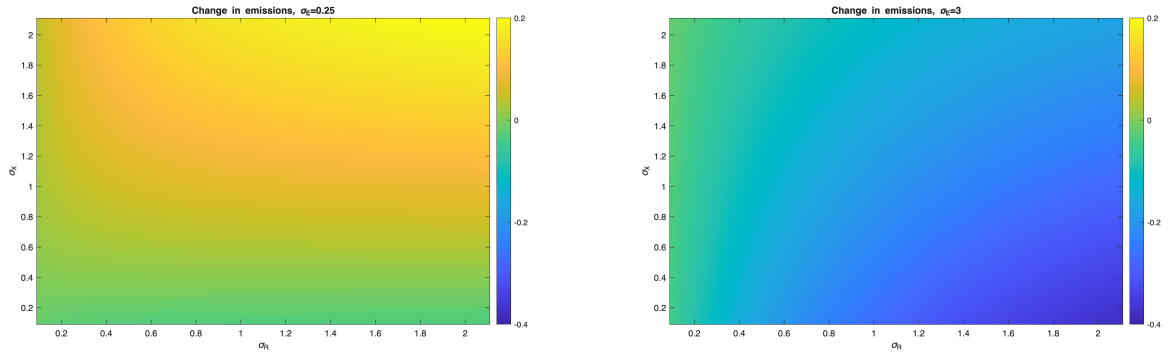
set all initial prices in the economy equation to one and then allocate shares of the primary factor across sectors. Again, by way of example, we set the input shares as $K_X = 0.92$, $K_F = 0.05$, and $K_R = 0.01$ such that $\bar{K} = 0.99$ (implying $\bar{Q} = 0.01$ and total resources sum to one). Thus, sector X uses most of the primary input while the renewable energy sector R uses the least. These shares are not identical to the real input shares for the U.S. economy in 2024, but they are qualitatively accurate and thus means the result here are illustrative of real economic outcomes from a hypothetical carbon offset program.

Next, the initial *ad valorem* tax on capital (i.e., the primary input) is set at $t = 0.005$. The tax on capital must be small since all revenue goes directly to the subsidy and sector R is small relative to the economy. Solving for the initial subsidy finds $s = 0.197$ meaning the subsidized price of clean energy is $P_R + s = 1.197$, so nearly 20 percent higher than the price of good X as the numeraire. All non-pollution share parameters can be recovered from these level values.

Continuing, we set $\rho_F = 0.299$ and $\rho_X = 0.701$ using data from the U.S. Energy Information Administration (2025b) on emissions on U.S. CO_2 emissions by sector. Total U.S. CO_2 in 2024 were 4,772 million metric tons where 1,427 million metric tons were emitted by the electricity sector. Since we model carbon offset focused on the electricity sector, then F is fossil-fuel electricity and $\rho_F = \frac{1427}{4772} = 0.299$, and $\rho_X = 1 - \rho_F$. For μ, we use a social cost of carbon of \$200 per ton, based on the U.S. EPA's previous estimate of \$190 per ton in 2020 dollars (U.S. Environmental Protection Agency, 2022). The simulations in Table 1 each increase the subsidy by 10 percent ($\hat{s} = 10$).

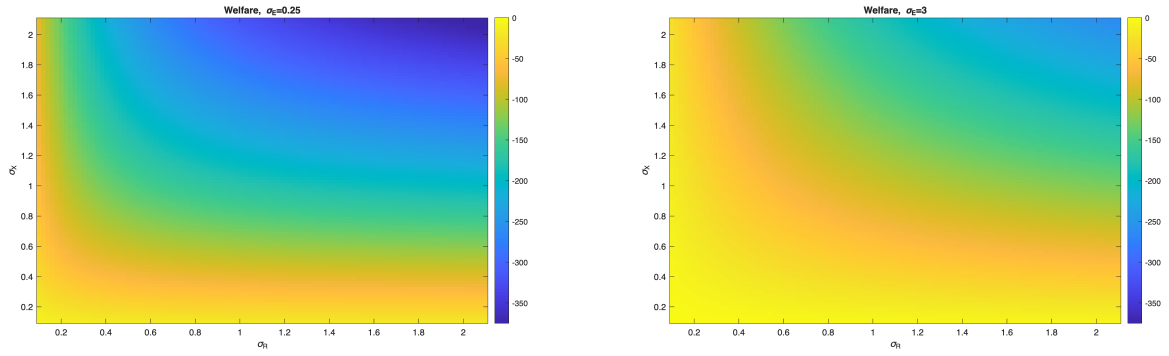
Table 1 reports changes in the quantity outcomes as well as the additionality ratio Δ (from Section 3) and the welfare outcomes (from Section 4), including calculations of μ^*, as the elasticities vary. These results are also illustrated in Figure 2, which shows how emissions vary with the elasticities, and Figure 3, which shows how welfare varies with the elasticities. Each row of the table reports results from a different simulation that varies the elasticity values. Several observations arise from the results. First, $\hat{R} > 0$ in all rows as found in equation 24. Second, the sign of $\hat{F}$ can change. In particular, $\hat{F} > 0$ for $\sigma_X >> \sigma_E$ leading to "backfire" and an increase in emissions (see rows 13-18). For these rows, note that $\Delta < -1$. Third, $\hat{X} < 0$ always, and thus numerically validates Proposition 2. Fourth, comparing Panels A and B shows that a higher σ_E yields higher Δ values, better welfare outcomes, and higher MVPF, although the change in total welfare is negative in all cases where the MVPF is below one. This reflects results in Hahn et al. (2024), who find that subsidies that directly displace dirty energy production tend to have a higher MVPF—in our setting, higher σ_E implies a higher degree of substitution between R and F, meaning increases in R will more effectively displace F. Fifth, μ^* is near \$200 in the initial rows of the table indicating that the baseline $\mu = \$200$ is close to yielding non-negative total welfare. Higher μ values would increase the welfare gains from Z in rows where $\hat{Z} < 0$. Sixth, μ^* does not vary with σ_R.[15] The intuition for this result is that varying σ_R only impacts the partial equilibrium supply of R but varying σ_X and σ_E affects the general equilibrium the supply and demand for R as well as X and Z. Sixth, the MVPF is always less than one since the change in welfare is always negative. The MVPF is monotonic with the welfare value since MVPF is a benefit-cost ratio of the offset policy. Additional figures showing how emissions,

[15] This result is not apparent from equation 20, but arises after substituting the closed-form expression from Section 5; specifically, σ_R only appears directly in $\hat{R}$, but $\hat{R}$ cancels out in the ratio $\hat{X}/\hat{Z}$ using equations 28 and 29.



(a) Low $\sigma_E = 0.25$. (b) High $\sigma_E = 3$.

Figure 2: Change in emissions as elasticities vary.



(a) Low $\sigma_E = 0.25$. (b) High $\sigma_E = 3$.

Figure 3: Change in welfare as elasticities vary.

additionality, and welfare vary with σ_E and σ_X are in Appendix D.

Next, Table 2 reports results from changing the initial emission intensities by varying the underlying emission quantities across sectors (Z_F, Z_X) while maintaining $4772 = Z_F + Z_X$ in all rows. Changing the emission quantities then changes the emission intensities (ξ_F, ξ_X) and the emission shares (ρ_F, ρ_X) leading to different proportional emission changes $(\hat{Z})$ even though the underlying economy is unchanged. That is, all rows in Table 2 have the same change in carbon offsets $(\hat{R})$, fossil fuels $(\hat{F})$, total energy $(\hat{E})$, and final goods $(\hat{X})$. (Recall that the change in carbon emissions $(\hat{Z})$ is an auxiliary variable as defined in Section 2.) Since $\hat{X}$ is fixed, then the change in welfare due to X is the same in all rows. Rows 1-7 find a positive total welfare change despite $\Delta < 0$. In other words, the welfare change can be positive despite conventional additionality falling short of aggregate additionality. These rows also have an MVPF greater than one. Row 8 solves for the emission intensities required such the costs of the subsidy equal the benefits, and this finds $\mu^* = 200$ by construction. Relatedly, since there is no change in total welfare, the MVPF equals one as the marginal willingness to pay (WTP) for offset program equals the marginal cost. Rows 9-14 have negative total welfare as found in Table 1 and thus the MVPF is less than one. Interestingly, rows 12-14 report $\Delta > 0$ meaning aggregate emission reductions are larger than the conventional calculation, but these rows

Table 1: Numerical Results Varying the Production Elasticities

Row	Elasticities σ_R	σ_X	Quantities (% Change) $\hat{R}$	$\hat{F}$	$\hat{E}$	$\hat{X}$	$\hat{Z}$	Add. Δ	Welfare ($billion) X	Z	Total	($) μ^*	(Index) MVPF
Panel A: $\sigma_E = 3.00$ (high value)													
1	0.25	0.25	0.686	-0.310	0.022	-0.003	-0.097	-0.06	-99.5	92.2	-7.3	215.9	0.995
2	0.75	0.25	1.537	-0.695	0.050	-0.008	-0.216	-0.06	-223.0	206.5	-16.5	215.9	0.990
3	1.50	0.25	2.228	-1.007	0.072	-0.011	-0.314	-0.06	-323.2	299.3	-23.9	215.9	0.987
4	0.25	0.75	0.694	-0.244	0.069	-0.003	-0.080	-0.23	-100.7	75.9	-24.8	265.3	0.984
5	0.75	0.75	1.579	-0.555	0.157	-0.008	-0.181	-0.23	-229.0	172.6	-56.4	265.3	0.966
6	1.50	0.75	2.316	-0.814	0.230	-0.011	-0.265	-0.23	-335.9	253.2	-82.7	265.3	0.954
7	0.25	1.50	0.705	-0.158	0.130	-0.003	-0.057	-0.46	-102.2	54.8	-47.5	373.3	0.969
8	0.75	1.50	1.635	-0.367	0.301	-0.008	-0.133	-0.46	-237.1	127.0	-110.0	373.3	0.935
9	1.50	1.50	2.438	-0.548	0.448	-0.012	-0.199	-0.46	-353.7	189.5	-164.2	373.3	0.909
Panel B: Low $\sigma_E = 0.25$ (low value)													
10	0.25	0.25	0.302	-0.009	0.094	-0.001	-0.009	-0.79	-43.7	8.9	-34.9	984.3	0.977
11	0.75	0.25	0.399	-0.012	0.125	-0.002	-0.012	-0.79	-57.8	11.7	-46.1	984.3	0.969
12	1.50	0.25	0.433	-0.013	0.136	-0.002	-0.013	-0.79	-62.9	12.8	-50.1	984.3	0.967
13	0.25	0.75	0.400	0.147	0.232	-0.002	0.029	-1.49	-58.0	-28.0	-86.0	-	0.943
14	0.75	0.75	0.591	0.218	0.342	-0.003	0.043	-1.49	-85.7	-41.4	-127.0	-	0.917
15	1.50	0.75	0.671	0.247	0.388	-0.003	0.049	-1.49	-97.3	-47.0	-144.2	-	0.907
16	0.25	1.50	0.494	0.297	0.362	-0.002	0.066	-1.90	-71.6	-63.1	-134.7	-	0.911
17	0.75	1.50	0.820	0.493	0.602	-0.004	0.110	-1.90	-119.0	-104.9	-223.9	-	0.857
18	1.50	1.50	0.983	0.591	0.722	-0.005	0.132	-1.90	-142.6	-125.7	-268.3	-	0.832

Notes: These simulations normalize all initial prices equal to one with $\hat{s} = 10$. The MED is set at $\mu = 200$. Also, the input resource share parameters are fixed at $\alpha_R = 0.020$, $\alpha_F = 0.051$, and $\alpha_X = 0.929$.

have the worst total welfare outcomes as a result of the policy change. Therefore, focusing just on additionality is a misleading metric for the welfare effect of a carbon offset policy. Portions of these results are also shown in Figure 4, which shows how Δ varies as the emissions intensity changes, with very high and positive values for Δ when Sector F's initial emissions share is small, before rapidly decreasing and becoming negative as its share of emissions grows.

The results in Table 2 and Figure 4 suggest that Δ can only be positive as $Z_F \to 0$, but this is not the case. We repeat the numerical calculation in Row 1 (with $Z_F = 4772$ and $Z_X = 0$), but change the elasticity of substitution in the energy sector from $\sigma_E = 3$ to $\sigma_E = 10000$ (or nearly perfectly elastic). In this case, $\hat{F} < 0$ and sufficiently negative to generate aggregate emission reductions larger than the conventional calculation using $\hat{R} > 0$, and leading to $\Delta = 0.015$. This result is driven by the ease that sector E can switch from F to R. This example numerically demonstrates the importance of σ_E to the economic outcomes of an offset policy as well as further emphasizes the role of σ_E in equations 25 and 26 that solve for $\hat{F}$ and $\hat{E}$, respectively.

Similarly, from Tables 1 and 2, one might infer that the backfire (i.e., $\hat{Z} > 0$) can occur only if there are emissions from Sector X, but this is not the case either. To start, if $\xi_X = 0$, then $\hat{Z} = \rho_F \hat{F}$ by Equation 16. Next, recall that Proposition 1 finds that values of σ_E always exist such that $\hat{F} > 0$. Therefore, backfire can occur even if the rest of the economy (X) emits zero pollution.

Finally, Table 3 and Figure 5 show results from changing the initial capital tax and subsidy rates. In the prior tables, the initial *ad valorem* capital tax is set at $t = 0.005$ implying the per unit subsidy is 0.197 (and repeated in row 5 here). Note that the initial capital tax is small in absolute terms because the capital tax base is large relative to the renewable sector. Table 3 row 1 changes

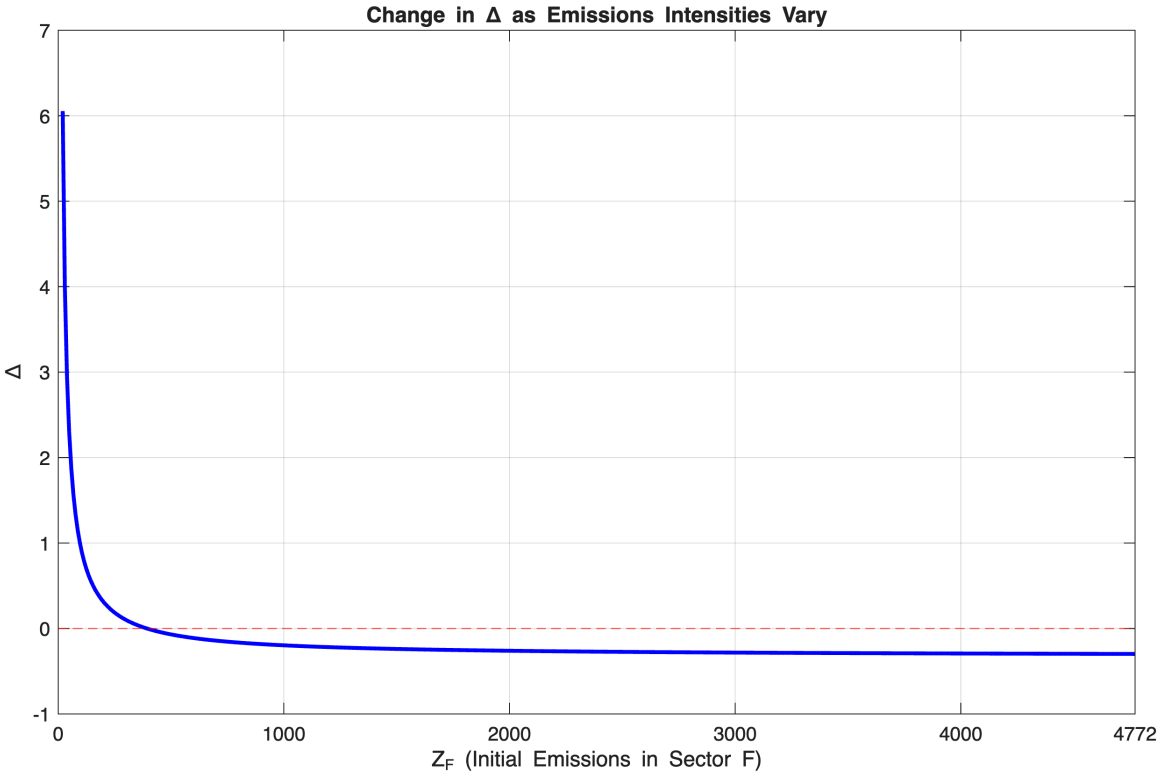


Figure 4: Change in Δ as emissions intensities vary.

Table 2: Numerical Results Varying the Emission Intensities

	Emissions		Emission Parameters & Change (%)					Add.	Welfare ($billion)			($)	(Index)
Row	Z_F	Z_X	ξ_F	ξ_X	ρ_F	ρ_X	$\hat{Z}$	Δ	X	Z	Total	μ^*	MVPF
1	4772	0	3257.3	0.0	1.000	0.000	-0.244	-0.298	-100.7	233.0	132.3	86.5	1.085
2	4500	272	3071.7	10.1	0.943	0.057	-0.231	-0.296	-100.7	220.2	119.5	91.5	1.077
3	4000	772	2730.4	28.6	0.838	0.162	-0.206	-0.292	-100.7	196.7	96.0	102.4	1.062
4	3500	1272	2389.1	47.2	0.733	0.267	-0.182	-0.288	-100.7	173.3	72.5	116.3	1.047
5	3000	1772	2047.8	65.7	0.629	0.371	-0.157	-0.282	-100.7	149.8	49.1	134.5	1.032
6	2500	2272	1706.5	84.3	0.524	0.476	-0.132	-0.273	-100.7	126.3	25.6	159.5	1.017
7	2000	2772	1365.2	102.8	0.419	0.581	-0.108	-0.261	-100.7	102.8	2.1	195.9	1.001
8	1955	2817	1334.4	104.5	0.410	0.590	-0.106	-0.259	-100.7	100.7	0.0	200.0	1.000
9	1500	3272	1023.9	121.4	0.314	0.686	-0.083	-0.239	-100.7	79.3	-21.4	253.8	0.986
10	1000	3772	682.6	139.9	0.210	0.790	-0.059	-0.197	-100.7	55.9	-44.8	360.5	0.971
11	500	4272	341.3	158.5	0.105	0.895	-0.034	-0.069	-100.7	32.4	-68.3	621.9	0.956
12	250	4522	170.6	167.8	0.052	0.948	-0.022	0.186	-100.7	20.7	-80.1	975.3	0.948
13	100	4672	68.3	173.3	0.021	0.979	-0.014	0.952	-100.7	13.6	-87.1	1480.2	0.944
14	10	4762	6.8	176.7	0.002	0.998	-0.010	12.434	-100.7	9.4	-91.3	2146.9	0.941

Notes: These simulations normalize all initial prices equal to one with $\hat{s} = 10$. The initial emission intensities vary by row. Total emissions are fixed such that $4772 = Z = Z_F + Z_X$. The elasticity parameters are set at $\sigma_R = 0.25$, $\sigma_X = 0.75$, and $\sigma_E = 3.00$. All other parameters are set to the values in Table 1. For all row, the quantity changes are $\hat{R} = 0.694$, $\hat{F} = -0.244$, $\hat{E} = 0.069$, and $\hat{X} = -0.003$.

the initial tax to its smallest value, $t = 0.001$, and finds a positive total welfare change—despite a negative additionality ratio. Also, $\mu^* = 48.1$ means the per unit benefit of abatement would not need

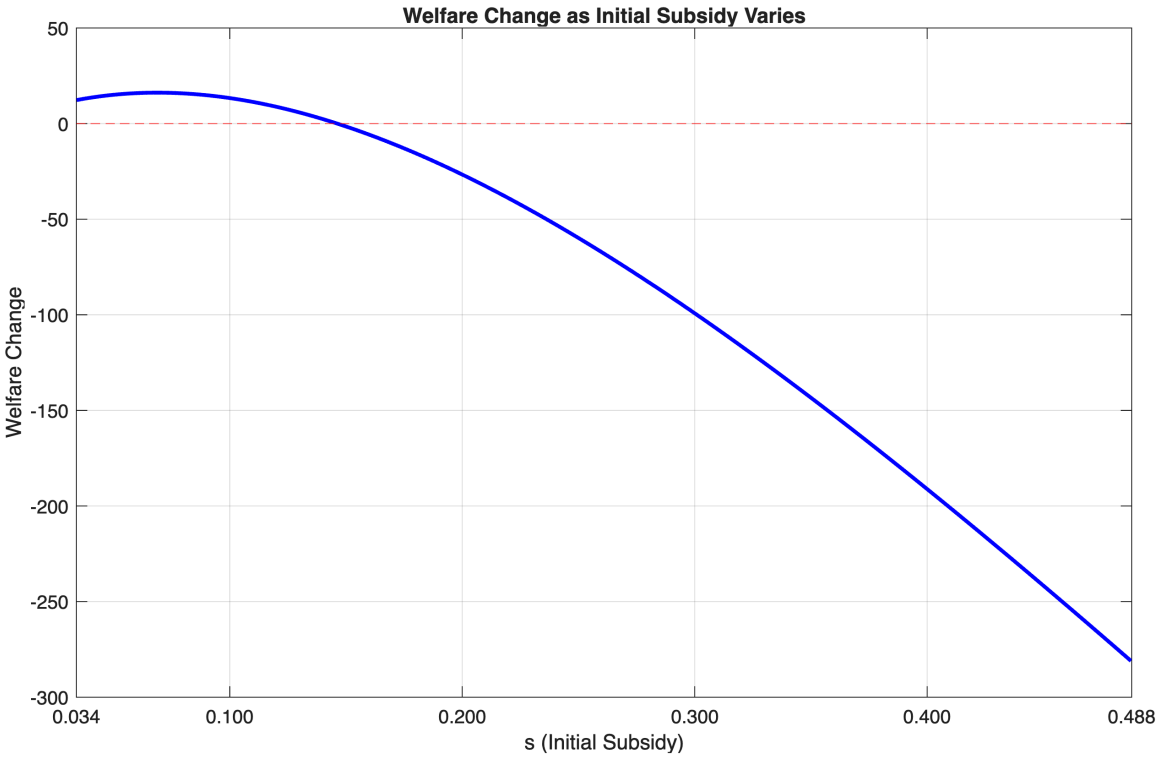


Figure 5: Change in welfare as initial subsidy level changes.

to be close to the social cost of carbon in our study ($200) to yield positive welfare gains. This result arises as the marginal cost of increasing the subsidy ($\hat{X}$) is small when the initial tax is small; that is, the height of the deadweight loss triangle is initially small. In contrast, row 10 has an initial tax ten-times larger than row 1, implying the initial deadweight loss is larger and leading to larger costs from increasing the subsidy. In this case, the welfare change is negative and $\mu^* = 607.3$, notwithstanding the proportional reduction in emissions being much larger in the $t = 0.010$ case. Table 3 further demonstrates the monotonic relationship between MVPF and welfare with MVPF > 1 implying positive welfare changes and MVPF < 1 implying negative welfare changes. When interpreting Table 3, one should take caution when comparing results across rows as the initial cost shares vary as the initial tax and subsidy vary. In other words, each row represents a different economy.

6.1 Case Study: Carbon Capture and Storage (CCS)

The Energy Improvement and Extension Act of 2008 established the U.S. federal tax credit known as "45Q"—in reference to its section in the federal tax code—to incentivize carbon capture and storage (CCS). Both the Inflation Reduction Act (IRA) of 2022 and the One Big Beautiful Bill (OBBB) of 2025 expanded the 45Q policy, suggesting that CCS is politically popular. In contrast, direct carbon pricing by the U.S. federal government has not occurred to date. At its core, 45Q is a

Table 3: Numerical Results Varying the Initial Tax and Subsidy Values

	Subsidy/Tax		Quantities (% Change)					Add.	Welfare ($billion)			($)	(Index)
Row	s	t	$\hat{R}$	$\hat{F}$	$\hat{E}$	$\hat{X}$	$\hat{Z}$	Δ	X	Z	Total	μ^*	MVPF
1	0.034	0.001	0.135	-0.053	0.016	-0.0001	-0.017	-0.274	-3.9	16.2	12.3	48.2	1.042
2	0.071	0.002	0.271	-0.104	0.031	-0.0005	-0.033	-0.265	-15.7	31.9	16.2	98.7	1.027
3	0.110	0.003	0.410	-0.153	0.044	-0.0012	-0.049	-0.255	-35.7	47.1	11.4	151.6	1.013
4	0.152	0.004	0.551	-0.200	0.057	-0.0022	-0.065	-0.245	-64.0	61.8	-2.2	207.0	0.998
5	0.197	0.005	0.694	-0.244	0.069	-0.0034	-0.080	-0.235	-100.7	75.9	-24.8	265.3	0.984
6	0.246	0.006	0.840	-0.286	0.080	-0.0050	-0.094	-0.224	-146.1	89.5	-56.7	326.7	0.970
7	0.299	0.007	0.987	-0.325	0.089	-0.0068	-0.107	-0.212	-200.4	102.4	-98.0	391.4	0.956
8	0.356	0.008	1.136	-0.361	0.097	-0.0090	-0.120	-0.199	-263.7	114.7	-149.0	459.9	0.942
9	0.419	0.009	1.288	-0.395	0.104	-0.0115	-0.132	-0.186	-336.3	126.3	-210.0	532.4	0.929
10	0.488	0.010	1.442	-0.425	0.110	-0.0143	-0.144	-0.171	-418.3	137.3	-281.0	609.4	0.915

Notes: These simulations normalize all initial prices equal to one with $\hat{s} = 10$. The MED is set at $\mu = 200$, and the elasticities are $\sigma_R = 0.25$, $\sigma_X = 0.75$, and $\sigma_E = 3.00$. Also, the input resource share parameters are fixed at $\alpha_R = 0.020$, $\alpha_F = 0.051$, and $\alpha_X = 0.929$. The initial cost shares vary by row.

subsidy for carbon-free, fossil-fuel-based electricity generation.[16] Thus, the 45Q tax credit fits well in our model to assess the additionality of CCS policy.

The 2025 U.S. Annual Energy Outlook (AEO) provides projections of CCS outcomes in the electricity market under the IRA's 45Q policy, and we use the AEO projections to parameterize the model (U.S. Energy Information Administration, 2025a). In this case study, sector R is all fossil-fuel fired electric generating units (EGUs) with CCS and F is all fossil-fuel fired EGUs without carbon sequestration, so E is total fossil electricity production and X is all other sectors. In this setting, $\bar{Q}$ can be interpreted as the specific geological features required to make CCS feasible. Currently, CCS is not a prominent feature of the U.S. electric power sector, so we use 2025's AE0 2040 forecast for this case study.

Under the 2025 AEO's reference scenario forecast, total 2040 CO_2 emissions from fossil-fuel EGUs is 490.5 million metric tons (MMmt) while CSS-enabled EGUs capture 38.0 MMmt. We model R as emission neutral and thus 452.5 MMmt in net emissions are attributed to F. For context, these carbon emissions arise from a base of 2,232 gigawatts (GW) of total electric power sector capacity generating 5,472 billion kWh.[17] However, fossil-fuel based generation is only 1,284 billion kWh or 23.4% of total electricity production. Total, economy-wide U.S. CO_2 emissions are projected to be 3456.3 MMmt and thus implying sector X (i.e., rest of the economy) accounts for 3002.8 MMmt.

In terms of GDP and sectoral revenue, the 2040 GDP projection is $38.97 trillion (where all dollar values in this case study are in 2024 dollars). The 2040 average electricity price is 11.5 cents/kWh, so 1,284 billion kWh from fossil-fuel based generation implies $149.7 billion in revenue (i.e., $p_E E = 149.7$). For R (i.e., CCS-enabled EGUs), the subsidy payment is $85 per metric ton CO_2 sequestered (Congressional Budget Office, 2023), and thus leading to a total subsidy payment

[16] Additionally, 45Q subsidizes enhanced oil recovery using CO_2, but we focus on 45Q's electricity market effects in this study. Also, we abstract from issues regarding the long-term stability of carbon sequestration.

[17] For comparison, the 2027 forecast is 1,153.5 MMmt from E (with zero CCS emission reductions) from 1,300 GW of capacity that generates 4,322 billion KWh. Thus, the 2025 AEO projects significant de-carbonization of the U.S. electric power sector without considering the net reductions from CCS.

Table 4: Carbon Sequestration Case Study Results

	Elasticities		Quantities (% Change)				Add.	Welfare ($billion)			($)	(Index)
Row	σ_R	σ_X	$\hat{R}$	$\hat{F}$	$\hat{E}$	$\hat{Z}$	Δ	X	Z	Total	μ^*	MVPF
1	0.25	0.25	0.955	-0.073	0.007	-0.010	-0.08	-3.1	6.7	3.6	92.4	1.102
2	0.75	0.25	2.299	-0.176	0.016	-0.023	-0.08	-7.4	16.1	8.7	92.4	1.218
3	1.50	0.25	3.548	-0.272	0.024	-0.036	-0.08	-11.5	24.8	13.4	92.4	1.305
4	0.25	0.75	0.957	-0.059	0.020	-0.008	-0.25	-3.1	5.4	2.4	113.4	1.067
5	0.75	0.75	2.309	-0.143	0.047	-0.019	-0.25	-7.5	13.1	5.7	113.4	1.143
6	1.50	0.75	3.570	-0.221	0.073	-0.029	-0.25	-11.5	20.3	8.8	113.4	1.201
7	0.25	1.50	0.959	-0.039	0.038	-0.005	-0.50	-3.1	3.7	0.6	169.5	1.016
8	0.75	1.50	2.322	-0.094	0.093	-0.013	-0.50	-7.5	8.9	1.4	169.5	1.034
9	1.50	1.50	3.603	-0.146	0.144	-0.020	-0.50	-11.6	13.7	2.1	169.5	1.048

Notes: These simulations normalize all initial prices equal to one with $\hat{s} = 10$. The MED is set at $\mu = 200$. Also, the input resource share parameters are fixed at $\alpha_R = 0.0003$, $\alpha_F = 0.0035$, and $\alpha_X = 0.9962$. We set $\sigma_E = 3$ for all rows.

of $3.2 billion to sequester 38.0 MMmt.[18] The AEO does not provide generation projections from CCS-enabled EGUs separately from total total fossil-fuel based generation, so we assume $p_E E$ is split between R and F by their share of (absolute) emissions. Under these assumptions, total revenue in R, denoted $(P_R + s)R$, is $14.8 billion.[19] Then, as a residual, $P_F F = 138.1$. Finally, we assume that one-third of the CCS unit's cost come from the fixed factor ($P_Q \bar{Q}$) to secure sequestration sites. The implied per unit subsidy is $s = 0.279$ while the *ad valorem* tax (t) on capital is only a 0.008% since sector R is small.[20]

Table 4 reports numerical results when varying the production elasticities (in a similar manner as Table 1). However, we fix $\sigma_E = 3$ at for Table 4 to reflect the high degree of substitution between CCS-enabled and non-CCS EGUs.[21] For this case study, the change in welfare is always small and positive and, relatedly, the MVPF is greater than one and $\mu^* < \mu = 200$. This net increase in welfare is due to the small initial tax rate (t) such that increasing the subsidy by 10 percent does not lead to large distortions in X. The low absolute value of the increase in welfare comes from the small size of sector R such that increasing the subsidy has a small total effect on the economy. Regardless, the emission reductions in sector F are counteracted by an increase in emissions from sector X as shown by $\Delta < 0$; that is, not all of the offsets are additional in aggregate. For rows 7-9, $\Delta = -0.50$ meaning half of the emission reductions in R due are erased by a corresponding increase in emissions elsewhere.

[18] The Congressional Budget Office (CBO) report finds a similar per year cost estimate and states that the subsidy will be paid to a CCS-enabled EGU if "Construction Beginning Prior to 1/1/2033". The subsidy program is politically popular currently, so it is reasonable to assume that all of CCS-enabled EGUs in 2040 will be receiving the subsidy.

[19] Specifically, the calculation is $149.7 \times (|-38|/490.5) + 3.2 = 14.8$ (after independent rounding).

[20] As result, $\hat{X}$ in the numerical analysis is quite small and thus omitted from Table 4.

[21] We also note that σ_R is likely to be relatively low for this case study, as carbon capture and storage requires specific geological features that are not easily substitutable with technology (National Energy Technology Laboratory, n.d.).

7 Conclusion

This study examines carbon offset policies in general equilibrium using an analytical general equilibrium (AGE) model to find closed-form solutions while focusing on emissions, additionality, and welfare effects. We model offsets as a subsidy to clean production and assume that they are real and additional by traditional measures (meaning, the offset projects would not have occurred in the absence of the offset payment). The general equilibrium setting allows for the comparison between the direct effect on emissions (i.e., "conventional additionality") and the change in economy-wide emissions (i.e., "aggregate additionality") for a given offset policy. This setting also allows for the measurement of aggregate welfare, and we show that neither conventional nor aggregate additionality is a sufficient statistic for welfare and that welfare is not monotonic in additionality. In other words, additionality does not inform policymakers about the welfare effects of an offset policy. This result arises both because the real cost of the offset policy is forgone consumption determined by the complicated demand-supply relationships in general equilibrium, and because offsets have an ambiguous impact on aggregate emissions changes.

What are the takeaways for policymakers? First, conventional additionality is not a good proxy measure for welfare changes since carbon offsets can decrease welfare, particularly, when the initial offset price is high, when the initial emissions from the dirty sector are low, and when clean and dirty energy are relatively complementary.[22] Instead of using additionality as a goal metric, policymakers concerned about social welfare could instead use the marginal value of public funds (MVPF), which is a good proxy for welfare changes in this setting.

Second, conventional additionality is also not a good proxy measure for emissions changes. We find that, even when all offsets are additional by traditional measures, aggregate emissions levels can in fact increase due to general equilibrium effects. This "backfire" effect can occur via two separate mechanisms. For the first mechanism, the price of energy relative to the price of capital can increase, leading to substitution away from energy and towards emissions-producing "capital" in the final goods sector. For example, if an offset policy leads to a relative increase in the price energy services from an electric heat pump relative to the price of energy services from a natural gas furnace, consumers may shift away from the electric heat pump and towards the emissions-producing natural gas furnace. This first type of backfire requires that capital and energy are relatively substitutable in the final goods sector. For the second mechanism, when renewables and fossil fuels are highly complementary, an increase in renewables will lead to a corresponding increase in fossil fuels, potentially leading to backfire. For example, increases in renewable energy may lead to an increased need for ramping of fossil fuel plants during the times of day when renewables go offline (e.g., sunset); because ramping is relatively more emissions intensive, this can erode some of the emissions benefits of renewables or even reverse the effect. We find that this second type of backfire is most likely when clean and dirty production are relatively complementary production

[22] We note that many carbon offset prices are currently low. Offset industry sources report average 2024 market prices around \$4 to \$6 per ton of CO_2 equivalent for voluntary offsets (Carbon Credits, 2025), with higher rated offsets (as determined by external rating companies) commanding higher prices that are still typically far below the social cost of carbon (Sylvera, 2026). This suggests that, all else equal, increases in offset prices may be welfare-improving due to their low initial price.

technologies, and less likely when they are more substitutable.

We note that the substitutability of clean and dirty energy production has likely changed over time and by location, implying that general equilibrium effects are likely to vary across time and space. For example, when penetration rates of renewable energy are low in electricity markets, renewable and fossil fuel energy can be relatively substitutable, even in the absence of energy storage technologies; in the absence of energy storage, intermittency is relatively easy for grid managers to deal with up until 25-50% penetration of renewables (Heal, 2017; Leonard et al., 2020). However, as the penetration of renewables in the energy grid increases, their intermittency becomes more challenging to grid managers, decreasing the substitutability of the two energy sources (lowering σ_E in our model). In the future, as energy storage technologies become more cost-effective, the intermittency of renewables will be less of a concern to grid operations, and thus renewables will become more substitutable with fossil fuels (raising σ_E).[23] Finally, in the absence of backfire, these same mechanisms described above can erode some of the emissions benefits of carbon offsets and lead to the over-crediting of carbon offsets.

We model carbon offsets as a subsidy on renewable energy output. In addition to applying to carbon offsets, our model also applies to green production credits, such as Renewable Energy Credits and the Renewable Energy Production Tax Credit in the US, and our results can be extrapolated to those settings as well. Conversely, some existing (non-offset) policies instead subsidize green capital, K_R. For example, the US also subsidizes renewable energy investment via the Residential Clean Energy Tax Credit and the Advanced Energy Product Credit. In our model, the inclusion of the fixed factor of production causes a distinction between a subsidy on output (R) and a subsidy on green capital (K_R). However, when σ_R is small (i.e., highly complementary), then the difference is likely to be small. Thus, to the extent that capital and the fixed factor are relatively complementary in green energy production, many of our results are likely to apply to a green capital subsidy. However, our model abstracts away from some of the real-world complexities that cause disparate outcomes between green output and green investment subsidies (see Aldy et al. (2023) for a more complete discussion of these differences).

Our numerical analysis provides intuition for our results and highlights how, ever in a relatively simple AGE model, additionality is not equivalent to changes in welfare. A more detailed set of results using a computational general equilibrium (CGE) model with many disaggregated production and consumption goods could generate additional guidance to policymakers about a specific offset policy. That is, the model in this study provides the theoretical foundation for future empirical and numerical studies on the aggregate effects of offset policies.

The model in this study abstracts from several important issues related to carbon offsets. To start, our model assumes that an offset policy is equivalent to a clean production subsidy that easily interacts with market prices in the general equilibrium setting without considering additional complications such as voluntary offsets. Next, we assume that all offsets are real and additional, as determined by traditional measures. Even under this conservative, best-case scenario assumption,

[23] Minor backfire has been empirically documented; for example, Sexton et al. (2021) empirically estimate that an additional solar panel installation leads to a slight increase in $PM_{2.5}$ emissions in some parts of the United States due to its impacts on other electricity generating units.

we find that emissions levels may increase under offset policies. If some offsets are not real or are non-additional by traditional measures, emissions increases could be even larger. The consideration of non-additional offsets complicates the analysis for both AGE and CGE models when considering the general equilibrium effects of offset policies, and is left for future work.

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APPENDIX

A Appendix: Log-Linear Equations

This appendix derives the log-linear equations that appear in Section 2.

To begin, totally differentiating the primary-factor resource constraint finds:

$$d[\bar{K} = K_X + K_R + K_F] \Rightarrow 0 = dK_X + dK_R + dK_F$$
$$\Leftrightarrow 0 = \frac{K_X}{\bar{K}}\frac{dK_X}{K_X} + \frac{K_R}{\bar{K}}\frac{dK_R}{K_R} + \frac{K_F}{\bar{K}}\frac{dK_F}{K_F} \Leftrightarrow 0 = \alpha_X\hat{K}_X + \alpha_R\hat{K}_R + \alpha_F\hat{K}_F,$$

where α_i is the share of primary factors used in Sector i. Note that all parameter definitions are found in Section 2.

We continue with Sector X. Totally differentiate the production function and simplify as follows:

$$d[X = X(K_X, E)] \Rightarrow dX = \frac{\partial X}{\partial K_X}dK_X + \frac{\partial X}{\partial E}dE$$
$$\Leftrightarrow dX = \frac{(1+t)P_K}{P_X}dK_X + \frac{P_E}{P_X}dE \text{ [by profit-max FOCs]} \Leftrightarrow \frac{dX}{X} = \frac{(1+t)P_KK_X}{P_XX}\frac{dK_X}{K_X} + \frac{P_EE}{P_XX}\frac{dE}{E}$$
$$\Leftrightarrow \hat{X} = \frac{(1+t)P_KK_X}{P_XX}\hat{K}_X + \frac{P_EE}{P_XX}\hat{E} \Leftrightarrow \hat{X} = \theta_{XK}\hat{K}_X + \theta_{XE}\hat{E},$$

and $\theta_{XK} + \theta_{XE} = 1$, where $\theta_{XK} \equiv \frac{(1+t)P_KK_X}{P_XX}$ and $\theta_{XE} \equiv \frac{P_EE}{P_XX}$. Next, totally differentiating the zero-profit condition for Sector X finds:

$$d[P_XX = (1+t)P_KK_X + P_EE]$$
$$\Rightarrow dP_XX + P_XdX = dtP_KK_X + (1+t)dP_KK_X + (1+t)P_KdK_X + dP_EE + P_EdE$$
$$\Leftrightarrow dP_XX + P_XdX = (1+t)P_KK_X\left(\frac{dt}{1+t} + \frac{dP_K}{P_K} + \frac{dK_X}{K_X}\right) + dP_EE + P_EdE$$
$$\Leftrightarrow \hat{P}_X + \hat{X} = \frac{(1+t)P_KK_X}{P_XX}(\hat{t} + \hat{P}_K + \hat{K}_X) + \frac{P_EE}{P_XX}(\hat{P}_E + \hat{E})$$
$$\Leftrightarrow \hat{P}_X + \hat{X} = \theta_{XK}(\hat{t} + \hat{P}_K + \hat{K}_X) + \theta_{XE}(\hat{P}_E + \hat{E}),$$

where $\hat{t} = dt/(1+t)$. Finally, we define the elasticity of substitution in sector X as:

$$\sigma_X \equiv -\frac{d(K_X/E)/(K_X/E)}{d((1+t)P_K/P_E)/((1+t)P_K/P_E)} \geq 0.$$

Then, differentiating through the definition finds:

$$\Rightarrow \sigma_X = -\frac{((dK_XE - K_XdE)/E^2)\cdot(E/K_X)}{((dtP_KP_E + (1+t)dP_KP_E - (1+t)P_KdP_E)/P_E^2)\cdot(P_E/((1+t)P_K))}$$
$$\Leftrightarrow \sigma_X = -\frac{\frac{dK_X}{K_X} - \frac{dE}{E}}{\frac{dt}{1+t} + \frac{dP_K}{P_K} - \frac{dP_E}{P_E}} \Leftrightarrow \sigma_X = -\frac{\hat{K}_X - \hat{E}}{\hat{t} + \hat{P}_K - \hat{P}_E} \Leftrightarrow \hat{K}_X - \hat{E} = \sigma_X(\hat{P}_E - \hat{t} - \hat{P}_K).$$

Similar derivations to those in sector X apply to sectors E and F, so those derivation are omitted here. The only differences are that sector E does not employ "capital" directly (and thus its inputs are not subject to the tax) and sector F has only one input (and thus there is no substitution in production). However, since sector R has a fixed factor and its output receives the subsidy, then we provide the full set of derivations below. To start, totally differentiate the production function and simply as follows:

$$d[R = R(K_R, \bar{Q})] \Rightarrow dR = \frac{\partial R}{\partial K_R} dK_R \text{ [since } d\bar{Q} = 0]$$
$$dR = \frac{(1+t)P_K}{P_R + s} dK_R \text{ [by profit-max FOCs]} \Leftrightarrow \frac{dR}{R} = \frac{(1+t)P_K K_R}{(P_R+s)R}\frac{dK_R}{K_R} \Leftrightarrow \hat{R} = \theta_{RK}\hat{K}_R,$$

and $\theta_{RK} + \theta_{RQ} = 1$, where $\theta_{RK} \equiv \frac{(1+t)P_K K_R}{(P_R+s)R}$ and $\theta_{RQ} \equiv \frac{P_Q \bar{Q}}{(P_R+s)R}$. For the zero-profits condition, the derivation goes:

$$\begin{aligned}
d[(P_R + s)R &= (1+t)P_K K_R + P_Q \bar{Q}] \\
\Rightarrow (dP_R + ds)R + (P_R + s)dR &= dt P_K K_R + (1+t)dP_K K_R + (1+t)P_K dK_R + dP_Q \bar{Q} \\
\Rightarrow (P_R + s)R\left(\frac{dP_R + ds}{P_R + s} + \frac{dR}{R}\right) &= (1+t)P_K K_R\left(\frac{dt}{1+t} + \frac{dP_K}{P_K} + \frac{dK_R}{K_R}\right) + P_Q\bar{Q}\frac{dP_Q}{P_Q} \\
\Rightarrow (P_R + s)R\left(\frac{P_R}{P_R + s}\frac{dP_R}{R} + \frac{s}{P_R + s}\frac{ds}{s} + \frac{dR}{R}\right) &= (1+t)P_K K_R\left(\frac{dt}{1+t} + \frac{dP_K}{P_K} + \frac{dK_R}{K_R}\right) + P_Q\bar{Q}\frac{dP_Q}{P_Q} \\
\Rightarrow \frac{P_R}{P_R + s}\hat{P}_R + \frac{s}{P_R + s}\hat{s} + \hat{R} &= \frac{(1+t)P_K K_R}{(P_R+s)R}(\hat{t} + \hat{P}_K + \hat{K}_R) + \frac{P_Q\bar{Q}}{(P_R+s)R}\hat{P}_Q \\
\Rightarrow (1-\gamma)\hat{P}_R + \gamma\hat{s} + \hat{R} &= \theta_{RK}(\hat{t} + \hat{P}_K + \hat{K}_R) + \theta_{RQ}\hat{P}_Q,
\end{aligned}$$

where $\hat{s} = ds/s$ is the change in the per unit subsidy and $\gamma \equiv \frac{s}{P_R+s}$ is the share of the initial subsidy in the total price of R. Despite the fixed factor, firms in Sector R want to substitute according to the equation:

$$\sigma_R \equiv -\frac{d(K_R/\bar{Q})/(K_R/\bar{Q})}{d((1+t)P_K/P_Q)/((1+t)P_K/P_Q)} \geq 0.$$

Applying the differentiation through the definition (as shown above) then finds:

$$\hat{K}_R = \sigma_R(\hat{P}_Q - \hat{t} - \hat{P}_K),$$

since $\bar{Q}$ is fixed.

Totally differentiating the lump-sum rebate (B) and apply government's budget constraint finds:

$$\begin{aligned}
&d[B = t(P_K\bar{K}) - sR] \Rightarrow dB = dtP_K\bar{K} + tdP_K\bar{K} + tP_Kd\bar{K} - dsR - sdR \\
&\Leftrightarrow dB = dtP_K\bar{K} + tdP_K\bar{K} - dsR - sdR \Leftrightarrow dB = tP_K\bar{K}\left(\frac{dt}{t} + \frac{dP_K}{P_K}\right) - sR\left(\frac{ds}{s} + \frac{dR}{R}\right) \\
&\Leftrightarrow \frac{dB}{B} = \frac{tP_K\bar{K}}{B}\left(\frac{1+t}{t}\frac{dt}{1+t} + \frac{dP_K}{P_K}\right) - \frac{sR}{B}\left(\frac{ds}{s} + \frac{dR}{R}\right) \\
&\Leftrightarrow \hat{B} = \frac{tP_K\bar{K}}{B}\left(\frac{1+t}{t}\hat{t} + \hat{P}_K\right) - \frac{sR}{B}\left(\hat{s} + \hat{R}\right) \\
&\Leftrightarrow \hat{B} = \phi_t\left(\psi\hat{t} + \hat{P}_K\right) - \phi_s\left(\hat{s} + \hat{R}\right).
\end{aligned}$$

If the government spends all the tax revenue on the subsidy, then $B = 0$ and $\phi_t = \phi_s$, and

$$0 = \psi\hat{t} + \hat{P}_K - \hat{s} - \hat{R} \Leftrightarrow \hat{s} + \hat{R} = \psi\hat{t} + \hat{P}_K,$$

where $\psi = \frac{1+t}{t}$ and this parameter links the *ad valorem* capital tax to the per unit subsidy.

We conclude this appendix by deriving the change in total emission in log-linear form as follows:

$$\begin{aligned}
&d[Z = \xi_F F + \xi_X K_X] \Rightarrow dZ = \xi_F dF + \xi_X dK_X \Rightarrow \frac{dZ}{Z} = \frac{\xi_F F}{Z}\frac{dF}{F} + \frac{\xi_X K_X}{Z}\frac{dK_X}{K_X} \\
&\hat{Z} = \frac{\xi_F F}{Z}\hat{F} + \frac{\xi_X K_X}{Z}\hat{K}_X \Rightarrow \hat{Z} = \rho_F\hat{F} + \rho_X\hat{K}_X,
\end{aligned}$$

where $\rho_F + \rho_X = 1$. This last equation assists in the additionality and welfare calculations but if not part of the log-linear general equilibrium system.

B Appendix: Welfare Deviation

This appendix derives the welfare formula in Section 4. To begin, totally differentiate the utility function to find:

$$dU = \frac{\partial U}{\partial X}dX + n\frac{\partial U}{\partial Z}dZ.$$

Next, substitute the first-order conditions (FOCs) from the representative consumer's utility maximization problem, $\frac{\partial U}{\partial X} = \lambda P_X$, where λ is the marginal utility of income (i.e., the Lagrange multiplier on the budget constraint) to get:

$$dU = \lambda P_X dX + n\frac{\partial U}{\partial X}dZ.$$

Continuing, divide through by λ and multiply the terms on the right-hand side by appropriate values of one to yield:

$$\frac{dU}{\lambda} = P_X X\hat{X} - \mu(nZ)\hat{Z},$$

where $\mu \equiv \frac{-1}{\lambda}\frac{\partial U}{\partial Z} > 0$ is the marginal environmental damage (MED) from a change in Z. Finally, divide through total income I to find equation 19 noting $P_X X = I$ at the optimum.

C Appendix: Optimal Policy

C.1 Social Planner's Problem

This section sets up the social planner's problem and its FOCs that appear in Section 4.3. The social planner maximizes utility (including emissions) subject to the resource constraint:

$$\max_{K_R,K_F,K_X} \mathcal{L} = U(X,Z) + \lambda\left[\bar{K} - K_R - K_F - K_X\right].$$

Substitute in the emissions equation, and Sector X, E, F, and R production functions:

$$\max_{K_R,K_F,K_X} \mathcal{L} = U\left[X\left(K_X, E\left(K_F, R\left(K_R,\bar{Q}\right)\right)\right), \xi_F K_F + \xi_X K_X\right] + \lambda\left[\bar{K} - K_R - K_F - K_X\right],$$

such that the above expression is only a function of K_R,K_F,K_X.

The FOCs are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_x} &\Rightarrow U_X X_K + U_Z \xi_X = \lambda,\\
\frac{\partial \mathcal{L}}{\partial K_F} &\Rightarrow U_X X_E E_F + U_Z \xi_F = \lambda,\\
\frac{\partial \mathcal{L}}{\partial K_F} &\Rightarrow U_X X_E E_R R_K = \lambda.
\end{aligned}$$

C.2 Private Market Outcome

Consumers maximize utility taking emissions as given (we offset emissions with a semicolon to denote this) and subject to the budget constraint:

$$\begin{aligned}
&\max_X \mathcal{L} = U\left(X;Z\right) + \gamma\left[I - P_X X\right],\\
\Rightarrow &\max_X \mathcal{L} = U\left(X;Z\right) + \gamma\left[P_K\bar{K} - (P_X + t_X)X\right],
\end{aligned}$$

where t_X is a tax on consumption of good X. Also, set capital as the numeraire, meaning $P_K = 1$:

$$\max_X \mathcal{L} = U\left(X;Z\right) + \gamma\left[\bar{K} - (P_X + t_X)X\right].$$

The FOC is:

$$\frac{\partial \mathcal{L}}{\partial X} \Rightarrow U_X = (P_X + t_X)\gamma.$$

Next, we solve the producer problems. For Sector X, producers maximize profits for their choice

of inputs:

$$\max_{K_X,E} \Pi = P_X X(K_X, E) - (P_K + t_K)K_X - (P_E + t_E)E - t_Z \xi_X K_X,$$
$$\max_{K_X,E} \Pi = P_X X(K_X, E) - (1 + t_K)K_X - (P_E + t_E)E - t_Z \xi_X K_X,$$

where t_K is a tax on capital use, t_E is a tax on consumption energy, and t_Z is a tax on emissions. Good X is already taxed in the private consumer's problem. (Note that any per unit tax can be recast as an *ad valorem* tax to yield the same equilibrium outcome.) The FOCs are:

$$\frac{\partial \Pi}{\partial K_X} \Rightarrow P_X X_K = (1 + t_K) + t_Z \xi_X,$$
$$\frac{\partial \Pi}{\partial E} \Rightarrow P_X X_E = (P_E + t_E).$$

Sector E is only a "pass through" sector:

$$\max_{F,R} \Pi = P_E E(F, R) - P_F F - P_R R,$$

and its FOCs are:

$$\frac{\partial \Pi}{\partial F} \Rightarrow P_E E_F = P_F,$$
$$\frac{\partial \Pi}{\partial R} \Rightarrow P_E E_R = P_R.$$

This implies that the dirty and clean energy have the same gross-of-tax prices.

For Sector F:

$$\max_{K_F} \Pi = (P_F - t_F)K_F - (1 + t_K)K_F - t_Z \xi_F K_F,$$

recalling that $F = K_F$, and where t_F is a tax on fossil fuel use. The FOC is:

$$\frac{\partial \Pi}{\partial K_F} \Rightarrow (P_F - t_F) = (1 + t_K) + t_Z \xi_F.$$

For Sector R:

$$\max_{K_R} \Pi = (P_R - t_R)R(K_R, \bar{Q}) - (1 + t_K)K_R - P_Q \bar{Q},$$

recalling $R = R(K_R, \bar{Q})$, and where t_R is a subsidy (or tax) on renewable use. The FOC is:

$$\frac{\partial \Pi}{\partial K_R} \Rightarrow (P_R - t_R)R_K = (1 + t_K).$$

C.3 The Standard Pigouvian Tax

The following equations characterize the problem with for any set of taxes, $\{t_X, t_K, t_E, t_F, t_R, t_Z\}$:

$$U_X X_K + U_Z \xi_X = \gamma, \tag{C-1}$$
$$U_X X_E E_F + U_Z \xi_F = \gamma, \tag{C-2}$$
$$U_X X_E E_R R_K = \gamma, \tag{C-3}$$
$$U_X = (P_X + t_X)\lambda, \tag{C-4}$$
$$P_X X_K = (1 + t_X) + t_Z \xi_X, \tag{C-5}$$
$$P_X X_E = (P_E + t_E), \tag{C-6}$$
$$P_E E_F = P_F, \tag{C-7}$$
$$P_E E_R = P_R, \tag{C-8}$$
$$(P_F - t_F) = (1 + t_K) + t_Z \xi_F, \tag{C-9}$$
$$(P_R - t_R) R_k = (1 + t_K). \tag{C-10}$$

For a standard Pigouvian tax, we put a tax on all emissions $t_Z \neq 0$ (and set all other taxes equal to zero). The equations then simplify to:

$$\begin{aligned}
U_X X_K + U_Z \xi_X &= \gamma, \\
U_X X_E E_F + U_Z \xi_F &= \gamma, \\
U_X X_E E_R R_K &= \gamma, \\
U_X &= P_X \gamma, \\
P_X X_K &= 1 + t_Z \xi_X, \\
P_X X_E &= P_E, \\
P_E E_F &= P_F, \\
P_E E_R &= P_R, \\
P_F &= 1 + t_Z, \xi_F \\
P_R R_K &= 1.
\end{aligned}$$

Solving for t_z yields:

$$t_z = -\frac{U_z}{\gamma} = \mu > 0,$$

which is the standard Pigouvian tax equal to marginal damages.

C.4 Special Case: No emissions from Sector X

If $\xi_X = 0$, the set up simplifies to:

$$\begin{aligned}
U_X X_K &= \gamma, \\
U_X X_E E_F + U_Z \xi_F &= \gamma, \\
U_X X_E E_R R_K &= \gamma, \\
U_X &= (P_X + t_X)\lambda, \\
P_X X_K &= (1 + t_X), \\
P_X X_E &= (P_E + t_E), \\
P_E E_F &= P_F, \\
P_E E_R &= P_R, \\
(P_F - t_F) &= (1 + t_K) + t_Z \xi_F, \\
(P_R - t_R) R_k &= (1 + t_K).
\end{aligned}$$

If we assume that $t_R, t_E \neq 0$ and all other taxes are zero, then it further simplifies to:

$$\begin{aligned}
U_X X_K &= \gamma, \\
U_X X_E E_F + U_Z \xi_F &= \gamma, \\
U_X X_E E_R R_K &= \gamma, \\
U_X &= P_X \lambda, \\
P_X X_K &= 1, \\
P_X X_E &= (P_E + t_E), \\
P_E E_F &= P_F, \\
P_E E_R &= P_R, \\
P_F &= 1, \\
(P_R - t_R) R_k &= 1.
\end{aligned}$$

Solving for t_R and t_E yields the following two-part instrument, similar to Fullerton and Wolverton (1997):

$$\begin{aligned}
t_E &= -\frac{U_Z \xi_F}{\gamma} > 0, \\
t_R &= \frac{U_Z \xi_F}{\gamma} < 0.
\end{aligned}$$

C.5 Special Case: Equal Emissions Factors in Both Sectors

In this section, we solve for another two-part instrument inspired by Fullerton and Wolverton (1997): the case where $t_X, t_R \neq 0$ and all other taxes are set to zero. Although we do not start with the

assumption that $\xi_X = \xi_F$, we find that this two-part instrument only achieves first-best when that condition is met.

We start with the same set up as the general set up in Equations C-1 through C-10. We then set all taxes other than t_R and t_X equal to zero, yielding:

$$\begin{aligned}
U_X X_K + U_Z \xi_X &= \gamma, \\
U_X X_E E_F + U_Z \xi_F &= \gamma, \\
U_X X_E E_R R_K &= \gamma, \\
U_X &= (P_X + t_X)\lambda, \\
P_X X_K &= 1 + t_X, \\
P_X X_E &= P_E, \\
P_E E_F &= P_F, \\
P_E E_R &= P_R, \\
P_F &= 1, \\
(P_R - t_R) R_k &= 1.
\end{aligned}$$

This yields the following two equations:

$$\begin{aligned}
\left[(1 + t_X) - \frac{1}{(1 - t_R)}\right] &= -\frac{U_Z \xi_X}{\gamma}, \\
\left[(1 + t_X) - \frac{1}{(1 - t_R)}\right] &= -\frac{U_Z \xi_F}{\gamma},
\end{aligned}$$

which can only hold if $\xi_F = \xi_X$. If we assume that $\xi_F = \xi_X$, then it follows that:

$$\begin{aligned}
t_X &= \frac{U_X X_K}{\gamma} - 1, \\
t_R &= 1 - \frac{X_K}{X_E E_R R_K}.
\end{aligned}$$

D Appendix: Additional Figures

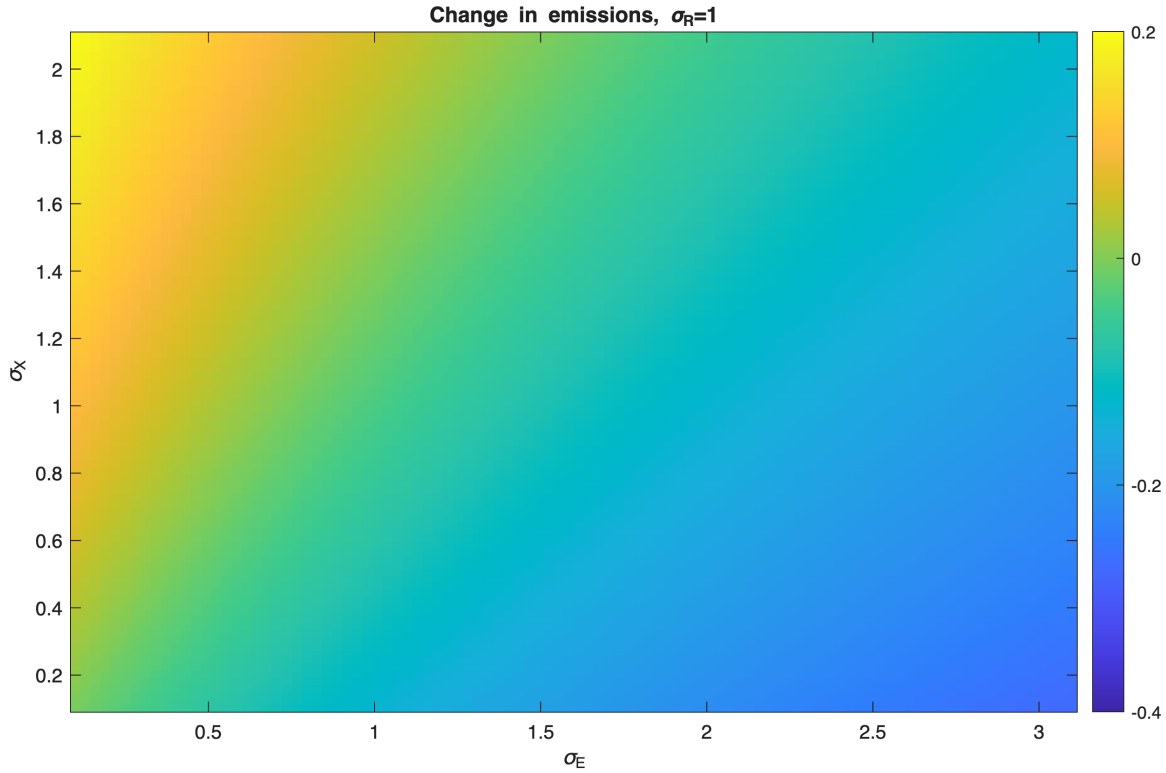


Figure 6: Change in emissions as σ_E and σ_X vary.

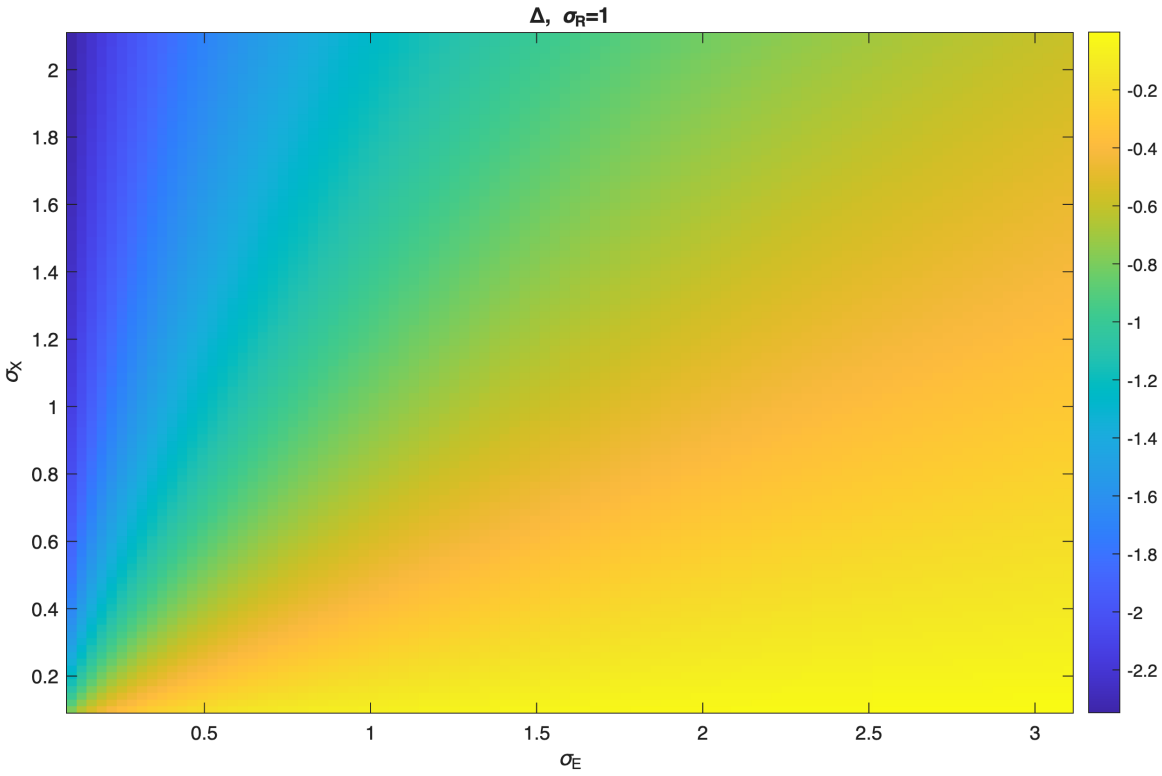


Figure 7: Change in Δ as σ_E and σ_X vary.

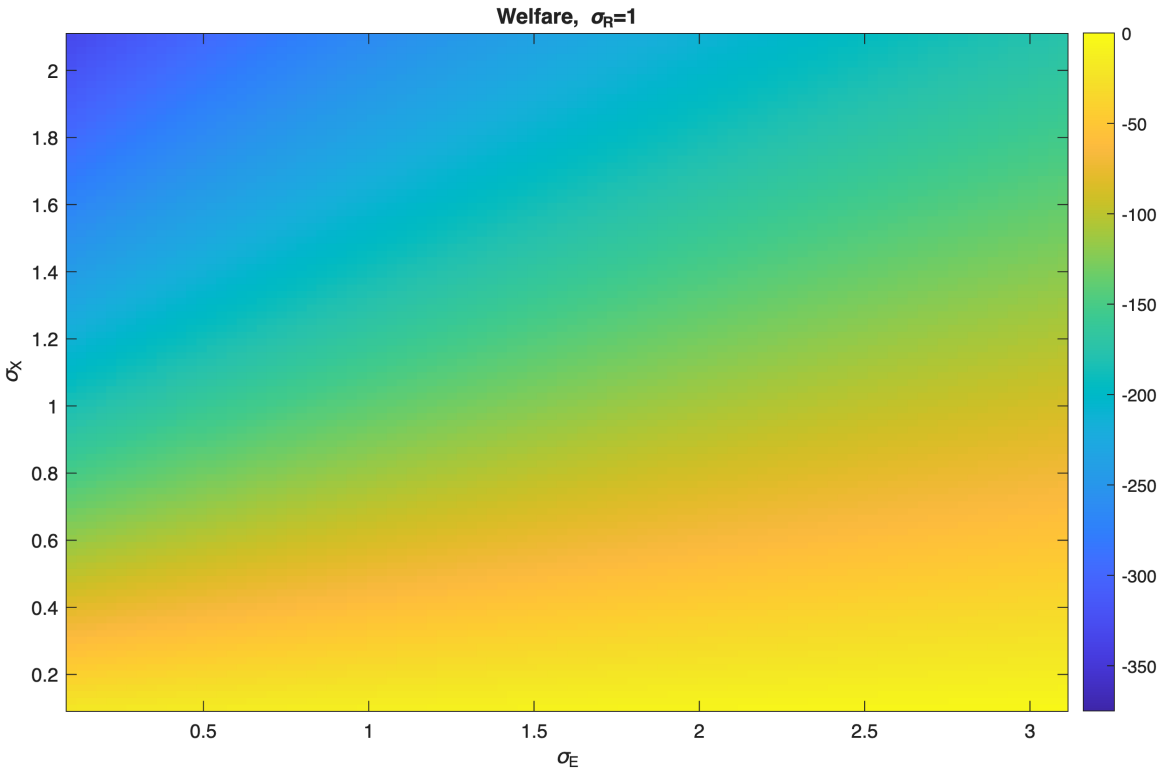


Figure 8: Change in welfare as σ_E and σ_X vary.