Household Consumption and Inattention to House Prices

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Abstract

In a quantitative life-cycle model of housing choice, consumption and saving, I introduce households that are inattentive to house price information. In the model, households' inattention results in household-specific, state- and time-dependent consumption wedges. I determine that inattention, and its interaction with liquidity and borrowing constraints, is a significant driver of the documented housing wealth effect heterogeneity in the literature. I find that the model-estimated impact of inattention and the housing wealth effect magnitude vary cyclically, providing an explanation to the literature's open question on why housing wealth effect estimates differ considerably between house price events.

Keywords: inattention, beliefs, housing wealth effect, financial constraints

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1 Introduction

In recent years, there has been a lot of analysis of evidence on households' subjective beliefs about macroeconomic variables and asset prices[1]. The topic has garnered wider interest outside of the academic literature as well (Zitner et al. (2024), Harris and Sojourner (2023), Aliaga (2024)), as households' perceptions and expectations about economic variables drive household financial decisions, with significant implications for asset markets and the real economy when perceptions deviate from actual values. In papers on this topic, such deviation between perceived and actual economic variables has been achieved by modeling inattentive agents. Abel et al. (2007), Abel et al. (2013), Alvarez et al. (2012) and Reis (2006) introduce observation costs to otherwise rational households, inducing inattention to the stock market, to agents' liquid wealth or consumption plan[2]. However, there has been less focus on whether households are attentive to house prices and house price changes, and whether their degree of attention has any effect on their consumption. A household's house is the largest asset the median household will own over its lifetime, and its perceived value is of great importance to how households consume. In addition to this, the magnitude of the housing wealth effect, or how households consume out of changes in house prices, remains an open question in the literature[3].

This paper addresses this open question by studying households' attention to house price information, and its implications for the housing wealth effect. I develop a stylized model of optimal attention, and find that attention is either state- or time-dependent, subject to households' housing adjustment plans, and that actual household attention behavior agrees with model predictions.

[1]Papers that have presented and studied empirical evidence of household attention include Weber et al. (2025), Roth and Wohlfart (2020), Sicherman et al. (2015), Dahlquist and Martinez (2015), Coibion and Gorodnichenko (2015), Gorodnichenko (2008) and Mackowiak and Wiederholt (2012). With respect to house prices, Bailey et al. (2018) examines households' perceptions about local house prices.

[2]Other papers have also examined household selective attention driven by loss aversion (Pagel (2018), Kőszegi and Rabin (2009)).

[3]Estimates of the housing wealth effect vary significantly between papers. Some examples of relevant papers are Mian and Sufi (2011), Mian et al. (2013),Saiz (2010), Gyourko et al. (2008), Aladangady (2017), Guren et al. (2020), Kessel et al. (2019), Graham and Makridis (2023), Campbell and Cocco (2007), Attanasio et al. (2009), Sinai and Souleles (2005), Case et al. (2000), Berger et al. (2017), Midrigan and Gorea (2017) and Kaplan et al. (2020).

To examine the implications of household attention on the housing wealth effect, I build a rich quantitative life-cycle model of housing choice, consumption and saving. In the life-cycle model, I introduce inattentive agents, whose attention function is calibrated to match the attention data. The model simulation reveals that a wedge develops between households' house price beliefs and actual house prices. These belief wedges create household-specific, state- and time-dependent consumption wedges. Therefore, the model introduces a separate mechanism for housing wealth effect heterogeneity to the ones that have been described in related papers until now. Furthermore, I show that, over time, the impact of inattention and the housing wealth effect magnitude vary cyclically, providing an explanation to the literature's open question on why housing wealth effect estimates differ significantly between house price events. I find that the main driver behind these results is the interaction of households' attention to house price information with their exposure to financial constraints.

To develop intuition, I begin with a stylized model of optimal attention, where households choose when to observe house price information, whether to adjust their housing, remain owners, or remain renters and how much to consume. Households considering housing adjustments jointly decide when to next adjust their housing, as well as when to next observe house price information. This leads to a state-dependent rule of optimal attention. Specifically, the probability of adjustment and observation is convex in wealth over housing. First, the probability increases as households come closer to their perceived current borrowing constraint, and second, when they approach some optimally chosen upper threshold of wealth over housing, and they decide to move to a larger house. For households who choose to remain owners or renters respectively, a time-dependent attention rule emerges, that does not vary by wealth over housing. These model predictions for optimal attention are compared to empirical data on household attention to house price information from the New York Fed Survey of Consumer Expectations (SCE). I find that the data on household attention behavior agrees with the theoretical predictions.

I then build a rich quantitative life-cycle model of housing choice, consumption and saving in

order to examine the implications of household attention on their financial decisions. The novel feature of the model is that households are inattentive to house prices and house price changes. The attention function governing the observation frequency of house price information is calibrated to match the SCE empirical data, making attention household-time specific. This approach allows the model to incorporate the state- and time-dependency of households' attention, and brings together the literature on optimal attention and the housing wealth effect literature on life-cycle models (Campbell and Cocco (2007) and Guren et al. (2020))[4]. Furthermore, this paper extends state-of-the-art literature on infrequent information updating (Auclert et al. (2020) and Carroll et al. (2020), originating from Gabaix and Laibson (2002), Mankiw and Reis (2002) and Mankiw and Reis (2007)), where agents update their information sets with some constant probability. In each period in the model, households choose whether to be homeowners or renters, as well as their non-housing and housing consumption, and derive utility from bequests to their offspring. Household earnings are subject to aggregate and idiosyncratic permanent and transitory income shocks, housing is illiquid and the agents face borrowing constraints, based on their housing wealth and labor income. Because the aim of this model is to determine the partial equilibrium response of household consumption to house price changes, all asset prices are exogenous.

In addition to agents' attention, the house price belief formation process in the model is informed by complimentary stylized facts on household perceptions and expectations, determined using a number of data sources on U.S. households. Initially, I confirm that perceived house prices and house price growth deviate from the corresponding actual values. Even more, I show that households exhibit a degree of 'stickiness' in their perceptions, by believing that both positive and negative house price changes are smaller in absolute value than they actually are. Based on these facts, households in the model form beliefs about house prices during the period of inattention using their latest perceived house price information. The model can thus capture the 'stickiness' in agents'

[4]Theoretical papers have also developed general equilibrium models in order to examine the changes in consumption within the context of a recession, such as is the case with Midrigan and Gorea (2017) and Kaplan et al. (2020).

perceptions. From the empirical data, I also find that households extrapolate their latest perceived house price growth to form expectations for future house prices. I use the same assumption for expectation formation in the model[5].

The model is calibrated in two stages. The first stage calibrates all parameters that can be identified without solving the model, using macroeconomic data or values from the literature. These include the parameters that define the dynamics of macroeconomic variables, household income and survival probabilities, as well as institutional characteristics relevant to non-housing consumption, housing and borrowing. The parameters that cannot be directly observed in the data require moments produced from the life-cycle model in order to be identified. I calibrate all preference parameters in the model by matching empirical moments along the life-cycle of households from the Survey of Consumer Finances for the years 2013-2019. The targeted moments capture households' precautionary savings, risk aversion, the preference for bequests, the strength of the bequest motive and the preference of consumption over housing implied from the micro-data.

I start by using the model to examine the impact of households' inattention on their beliefs about house prices and the impact of inattention on consumption. In terms of the impact of inattention on beliefs, I find that the wedges between perceived and actual house prices decrease with increasing probability of attention. Then, with respect to the impact of inattention on consumption, I proceed in two steps. First, I examine how inattention impacts the inaction region of housing adjustment, by solving the model for the optimal housing choice of households with no, -10% and +10% house price perception errors. Each scenario produces different optimal choices, driven by the interaction of perceptions and household liquidity[6]. Second, I determine the effect of attention on consumption conditional on housing choice, by presenting the model-predicted average consumption wedges by

[5] Agents' deviation from rationality in their expectations about the evolution of macroeconomic variables has been examined in many settings. Such examples include Carroll (2003), Barberis et al. (2018), Bordalo et al. (2021), and for house prices Kindermann et al. (2021) and Kuchler and Zafar (2019).

[6] I find that, when households believe house prices are lower than they actually are, they adjust their housing, to take advantage of the lower prices. When they believe house prices are higher, the lower-liquid wealth households choose to be renters, and higher-liquid wealth households remain or become owners to take advantage of the benefits of the perceived to be appreciated asset in terms of debt capacity, pure wealth and endowment effects.

probability of attention. Similarly to the belief wedge results, the inattention-driven consumption differences decrease as the probability of attention increases.

The following sections study in detail the impact of inattention on the housing wealth effect. First, the marginal propensity to consume out of housing (MPCH) is calculated for households that expect house prices to increase, decrease or remain the same in the future. I present the results along the liquid wealth, housing, house price and age distributions, to examine the housing wealth effect's variation along the cross-section of households. Overall, for owners who expect house prices to increase, consumption today is substituted for consumption tomorrow, when they will be able to borrow more. The reverse is true for households who believe that house prices will decrease. Aside from the expected change in borrowing capacity, households may also plan to take advantage of lower house prices in the future and increase their housing units, thus anticipating they will have to consume less in order to adjust their housing. Whatever their expectations, households with mid-level liquid wealth, high housing levels, low house prices and younger households change their consumption the most after a house price shock through their consumption.

To explicitly determine what drives the model-predicted heterogeneity in MPCH, I decompose the total effect of inattention along a set of metrics that capture constraints households are exposed to. The aim of this application is to understand how inattention interacts with financial constraints and impacts the housing wealth effect of different households. Households' debt constraints are considered by plotting the MPCH against the debt level, debt capacity and loan-to-value, and liquidity constraints are quantified by the ratios of liquid wealth over income and liquid wealth over housing wealth. In addition to the MPCH, the impact of inattention along these distributions is captured by taking the ratio of the predicted MPCH in the counterfactual simulation where households are always fully attentive to house price information, over the baseline MPCH estimates. Throughout these cross-sections, the same relationship emerges: as households approach their liquidity or borrowing constraints, they become more attentive to house price information. Therefore, the impact of inattention on consumption decreases and the magnitude of the housing wealth effect increases.

Altogether, these results quantify how inattention, interacting with financial constraints, creates an additional driver in MPCH heterogeneity to the ones previously examined in the literature.

Using the stylized model of optimal attention, I showed that the properties of household attention are different when households are considering adjusting their house, where attention is state-dependent and varies by their wealth over housing, from when they plan to remain owners or renters, where attention is time-dependent and remains constant over wealth/housing. No matter their housing adjustment plans, households choose whether to pay attention by trading off the costs and benefits of observing house price information. From a policy perspective, to reduce the overall consumption wedge caused by inattention, the observation cost could be decreased and households' responsiveness to house price changes therefore could be increased. To identify this 'reducible' component of the consumption wedge, I run the 'Plan to Move' counterfactual simulation, by calibrating the attention function to match the empirical attention data of households that plan to move. The residual distance between the predicted MPCH under this simulation, and the simulation assuming full attention depicts the 'reducible' component of the consumption wedge. This decomposition is performed along the household problem state variables and along the variables used earlier to capture financial constraints.

In summary, the aforementioned model applications have analyzed the state-dependence of the MPCH and revealed how it is driven by the interaction of households' attention to house price information with households' financial constraints. However, these findings have broader implications for the housing wealth effect, when unexpected changes in house prices occur under different economic conditions. In particular, at different points in the business cycle, such as during an expansion or a contraction, many households move further away or closer to their financial constraints than before, and will adapt their attention to house prices accordingly. This would result in the impact of attention to house prices, and the average MPCH, to depend on the state of the economy and co-move with the business cycle.

The model is applied to simulate an expansion and subsequent contraction of the economy and through a series of experiments, the model produces households whose attention consumption wedge is cyclical and co-moves with the business cycle. Specifically, first the baseline economy is simulated for 190 periods, followed by 5 periods of expansion and 5 periods of contraction. For each year during the boom and bust period in the simulation, a separate experiment is ran, where house prices are shocked by +10%. Therefore, 10 separate experiments are available, each giving an estimate for the housing wealth effect under different economic conditions. To examine the results, the average MPCH is plotted by experiment, as well as the total attention consumption wedge, or the difference in MPCH between the baseline simulation and the full attention counterfactual.

The model predicts that the MPCH decreases during the periods of expansion, as households see both their incomes and house prices rising, and are less attentive. The opposite occurs during the bust, where households' incomes and house prices decrease, LTV ratios increase, and households' attention to house prices increases. The state-dependency of the MPCH is examined in detail by also running the 'Plan to Move' counterfactual simulation and decomposing the attention consumption wedge. During the expansion and contraction, more households enter and then exit their inaction region of housing adjustment. This drives the average MPCH to approximately 0 late in the expansion, to very high values at the end of the contraction. These elevated values match the magnitude of the housing wealth effect for highly debt constrained households under full attention, as estimated in the previous section of the paper.

These model experiments exhibit that households' attention to house prices and its impact on the housing wealth effect is cyclical, which has significant implications for the estimation of the magnitude of the housing wealth effect. Based on these model results, the value of the average MPCH will vary depending on the house price event incurred or more specifically, when a house price shock occurs in the business cycle. This finding provides an explanation for a key puzzle in the housing wealth effect literature, which is the large range of estimates for the average magnitude of the housing wealth effect. Because attention to house prices varies by household and between

an expansion or a contraction of the economy, the attention consumption wedges, and hence the MPCH estimates, will be subject to an additional source of variation. This variation can only be captured if attention is explicitly incorporated in the analysis, in order to pin down this attention mechanism through which the housing wealth effect is affected differentially over the business cycle.

This paper extends the literature that has focused on household house price expectations and their effect on household consumption. However, up to now it is assumed that households change their consumption as a response to actual house price changes. Some papers have incorporated beliefs in their respective theoretical models and presented their impact (Landvoigt (2017) and Kaplan et al. (2020)), have considered how long-run home price growth expectations affect consumption (Chopra et al., 2025), have looked into whether unanticipated changes in home values impact spending and home equity extraction (Andersen and Leth-Petersen, 2020), and have quantified the consumption effect of unexpected housing wealth changes (Browning et al. (2013), Campbell and Cocco (2007), and Paiella and Pistaferri (2017)). House price expectations play an important role in the results presented in the current paper, but the mechanism that is the main driver for producing the findings is households' attention to house price information, and the belief wedges that exist between perceived and actual house prices. The approach in the current paper provides a novel point of view in this area in the literature, by determining the impact of households' attention on how households consume out of house price changes.

2 A Stylized Model of Optimal Attention: Theoretical Predictions and Empirical Evidence

This section first develops a stylized model of optimal attention, where households choose their frequency of observation for house price information, and then provides empirical evidence that validates the theoretical predictions.

2.1 Overview of the Stylized Model

Consider a household that faces a discrete-continuous choice in each period. First, they decide between remaining an owner if they are one currently, remaining a renter if they are one currently, or adjusting their housing. Conditional on this discrete choice, they choose their non-housing consumption $c_{i,t}$ and their housing consumption $h_{i,t}$. The household state variable vector in this stylized model is $z \equiv (a, y, h, P_j^H, \phi_j^H)$, where a is the liquid wealth of the household, y is its income, P_j^H is the price per unit housing and ϕ_j^H is the growth rate of house prices. Between periods t and $t+1$, the following dynamics hold for $a_{i,t}$ and $P_{j,t}^H$:

$$a_{i,t+1} = \begin{cases} (1+r)a_{i,t} + y_{i,t+1} - c_{i,t} - \Xi_{i,t}^H & \text{for owners} \\ (1+r)a_{i,t} + y_{i,t+1} - c_{i,t} - R_{i,t}^H & \text{for renters} \end{cases}$$

$$P_{j,t+1}^H = \phi_{j,t}^H P_{j,t}^H \Psi_{t+1}^H$$

where $\Xi_{i,t}^H$ is the per-period housing depreciation cost, $R_{i,t}^H$ is the per-period rental cost and Ψ_t^H are i.i.d. shocks all house prices are exposed to. Households are also subject to borrowing constraints, where owners can borrow up to a $(1-\theta)$ fraction of their housing wealth $P_{j,t}^H h_{i,t}$, and renters up to σ times their income y:

$$a_{i,t} \geq \begin{cases} -(1-\theta)P_{j,t}^H h_{i,t} & \text{for owners} \\ -\sigma y_{i,t} & \text{for renters} \end{cases}$$

The household's discrete choice is:

$$V(z) = \max\{V^{own}(z), V^{adj}(z), V^{rent}(z)\}$$

where $V^{adj}(z) = V^{adj|own}(z)$ will be the conditional value function for a household that is currently an owner and chooses to adjust their housing, and $V^{adj}(z) = V^{adj|rent}(z)$ will be the conditional value function for a household that is currently a renter and chooses to adjust their housing by

purchasing a home, respectively. We continue by introducing inattentive owners and renters who choose to observe house price information at optimally chosen period intervals[7].

Owner Inattentive to House Price Information: For a household that is inattentive to house price information and has chosen to adjust their housing (the adjuster's case $V^{adj|own}(\cdot)$), their inaction interval τ' coincides with their inattention interval, as we assume that house price information is observed when a housing transaction takes place. The value function of the household is then:

$$V^{adj|own}(a, y, h, P_j^H, \phi_j^H) = \max_{\{c_{i,t}\}, h', \tau'} \mathbb{E}\left[\sum_{s=0}^{\tau'} \beta^s u(c_{i,s}, h) + \beta^{\tau'} V(a', y', h', \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'})\right]$$

$$\begin{aligned}
\text{subject to:} \quad a_{i,t+1} =& (1+r)a_{i,t} + y_{i,t+1} - c_{i,t} - \Xi_{i,t}^H \\
\tilde{P}_{i,j,s+1}^H =& \mathbb{E}[P_{j,s+1}^H | P_j^H, \phi_j^H] = (\phi_j^H)^s P_j^H \Psi_{s+1}^H \quad \text{for } 0 \le s \le \tau' - 1 \\
a_{i,t} \ge& -(1-\theta)\tilde{P}_{i,j,t}^H h \\
a' =& a(\tau') - (\kappa^O + \kappa^T) P_j^H h
\end{aligned}$$

Households today will determine their continuation value using their expected future house price $\tilde{P}_{i,j,s+1}^H = \mathbb{E}[P_{j,s+1}^H | P_j^H, \phi_j^H]$, based on their current perceived house price level and growth. With respect to house price growth, households assume it remains constant at the latest observed level $\tilde{\phi}_{i,j}^{H'} = \phi_j^H$. During the inaction interval τ', a grows as determined by the dynamics of the household's income process, choice of consumption in each period and housing depreciation cost. Wealth is also subject to a borrowing constraint, such that households can borrow up to a portion of their housing wealth. Households plan their future financial decisions based on their perceived borrowing constraint $(1-\theta)\tilde{P}_{i,j,t}^H h$, but when they borrow, they can only do so up to $(1-\theta)P_{j,t}^H h$. At dates when a housing adjustment is made, $a(\tau')$ falls by the combined amounts of the transaction and observation costs.

[7] Throughout this section, for some variable x, x is the current value, x' is the value at the next observation date, $x_{i,t}$ is the value of the variable at time t, between now and the next observation date.

Immediately after adjustment, when a household will plan to remain an owner, h' is not included in the household's set of control variables, as it remains constant. They will have to choose τ', which represents the time interval until the next observation of house prices, as well as their consumption path. The household's problem is then:

$$
\begin{aligned}
V^{own}(a, y, h, P_j^H, \phi_j^H) = \max_{\{c_{i,t}\}, \tau'} \mathbb{E}\left[\sum_{s=0}^{\tau'} \beta^s u(c_{i,s}, h) + \beta^{\tau'} V(a', y', h, \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'})\right] \\
\text{subject to:} \quad a_{i,t+1} =& (1+r)a_{i,t} + y_{i,t+1} - c_{i,t} - \Xi_{i,t}^H \\
\tilde{P}_{i,j,s+1}^H =& \mathbb{E}[P_{j,s+1}^H | P_j^H, \phi_j^H] = (\phi_j^H)^s P_j^H \Psi_{s+1}^H \quad \text{for } 0 \leq s \leq \tau' - 1 \\
a_{i,t} \geq& -(1-\theta)\tilde{P}_{i,j,t}^H h \\
a' =& a(\tau') - \kappa^O P_j^H h
\end{aligned}
$$

At this point, when house price information is next observed, $a(\tau')$ falls by the amount of the observation cost only.

Renter Inattentive to House Price Information: The value function of a household that wants to adjust their housing and is currently a renter is the following:

$$
\begin{aligned}
V^{adj|rent}(a, y, \bar{h}, P_j^H, \phi_j^H) = \max_{\{c_{i,t}\}, h', \tau'} \mathbb{E}_t\left[\sum_{s=0}^{\tau'} \beta^s u(c_{i,s}, \bar{h}) + \beta^{\tau'} V(a', y', h', \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'})\right] \\
\text{subject to:} \quad a_{i,t+1} =& (1+r)a_{i,t} + y_{i,t+1} - c_{i,t} - R_{i,t}^H \\
\tilde{P}_{i,j,s+1}^H =& \mathbb{E}[P_{j,s+1}^H | P_j^H, \phi_j^H] = (\phi_j^H)^s P_j^H \Psi_{s+1}^H \quad \text{for } 0 \leq s \leq \tau' - 1 \\
a_{i,t+1} \geq& -\sigma y_{i,t} \\
a' =& a(\tau') - (\kappa^O + \kappa^T) P_j^H \bar{h}
\end{aligned}
$$

Immediately after adjustment, when a household will plan to remain a renter, they will again have to choose τ', the interval until the next observation of house prices. The household's problem is

then:

$$V^{rent}(a, y, \bar{h}, P_j^H, \phi_j^H) = \max_{\{c_{i,t}\}, \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s u(c_{i,s}, \bar{h}) + \beta^{\tau'} V(a', y', \bar{h}, \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'}) \right]$$

$$\begin{aligned}
\text{subject to:} \quad a_{i,t+1} =& (1+r)a_{i,t} + y_{i,t+1} - c_{i,t} - R_{i,t}^H \\
\tilde{P}_{i,j,s+1}^H =& \mathbb{E}[P_{j,s+1}^H | P_j^H, \phi_j^H] = (\phi_j^H)^s P_j^H \Psi_{s+1}^H \quad \text{for } 0 \leq s \leq \tau' - 1 \\
a_{i,t+1} \geq& -\sigma y_{i,t} \\
a' =& a(\tau') - \kappa^O P_j^H \bar{h}
\end{aligned}$$

Lemma 1: There exists a range of values for the wealth/housing ratio where, at a given perceived current house price level and perceived current house price growth, households will choose to remain inactive, so that they will not adjust their housing and will not observe house price information. The optimal thresholds that define this inaction region is as follows:

$$-(1-\theta)\tilde{P}_{i,j,t}^H \leq b < B < +\infty \quad \text{for owners}$$

$$-\sigma y_{i,t} \leq b < B < +\infty \quad \text{for renters}$$

These thresholds b and B are optimally chosen by the household. There is no upper bound for the thresholds, but there is a lower bound based on the perceived borrowing constraint $a_{i,t}/h_{i,t} \geq -(1-\theta)\tilde{P}_{i,j,t}^H$ for owners and $a_{i,t} \geq -\sigma y_{i,t}$ for renters respectively. Therefore, the ratio $a_{i,t}/h_{i,t}$ has an inaction region which is the open interval (b, B) that is driven by owner's perception of their current housing wealth. *See Proof in Appendix.*

Corollary 1: The inaction region defined for a range of values for the wealth/housing ratio can be expressed in terms of time and defines the optimal period until the next observation of house price information τ'. The household's choice of the optimal period until the next observation and adjustment will be such that:

$$\tau'(a/h, \tilde{P}_{i,j}^H, \tilde{\phi}_{i,j}^H) = \tau'(b, \tilde{P}_{i,j}^H, \tilde{\phi}_{i,j}^H) \wedge \tau'(B, \tilde{P}_{i,j}^H, \tilde{\phi}_{i,j}^H)$$

See Proof in Appendix.

Note therefore, that household perceptions determine households' perceived borrowing limit, and as a result impact households' housing adjustment inaction region as well. This leads to the first main prediction of the stylized model:

Prediction 1: Attention to house price information interacts with households' perceived current borrowing constraints.

I can continue to write the Hamilton-Jacobian-Bellman equation for this problem and characterize the optimal solution for the time interval between observation dates of house price information, $\tau^{\star}$. It is intuitive that the inverse of the value will provide the probability that households pay attention to house price information in a year. Therefore, we can describe the characteristics of the probability of attention, $P^{att}(a/h, \tilde{P}^{H}_{i,j}, \tilde{\phi}^{H}_{i,j})$.

Proposition 1: The policy function that determines the optimal probability that households pay attention to house price information in a year, while in the inaction region, has the following properties:

- *Within the inaction region, the probability of attention is convex*
- *As $a/h \downarrow b$ or $a/h \uparrow B$, the probability of attention increases*

See Proof in Appendix.

The above Proposition describes the properties of τ' in the inaction region and summarizes the state-dependency of τ' and P^{att}. Specifically, when the wealth over housing ratio of a household approaches the threshold defined by the borrowing constraint as perceived by the household, the probability of attention to information about house prices increases. Similarly, when the wealth/housing ratio increases up to some upper boundary determined by the household, the probability of attention also increases, and so does the probability of housing adjustment. This implies a convex relationship between the probability of attention and the ratio of wealth/housing, within

thresholds b and B. This Proposition also implies that for renters, where housing utility is assumed to be received from some constant level $\bar{h}$, the probability of attention will be monotonically increasing in wealth.

Prediction 2: For households planning to adjust their housing in the future, attention to house price information varies with their wealth/housing ratio.

For the value of τ' immediately after adjustment, when a household either plans to remain an owner without adjusting their housing, or when a household plans to remain a renter, we examine the optimal τ' for conditional value functions $V^{own}(a, y, h, P_j^H, \phi_j^H)$ and $V^{rent}(a, y, \bar{h}, P_j^H, \phi_j^H)$ mentioned earlier. Conditional on either the *own* and *rent* choice, households have to decide 2 instead of 3 control variables: their non-housing consumption and the optimal interval until the next observation of house price information. Therefore, the optimal τ' can be solved in closed form.

Proposition 2: The policy function that determines the optimal probability that households pay attention to house price information in a year, immediately after housing adjustment, has the following properties:

- *Immediately after adjustment, the probability of attention does not vary for different values of the household's wealth over housing*
- *The probability of attention depends on household preferences and the level of interest rates and is higher when:*
  - *The uncertainty of a household's wealth is higher*
  - *The uncertainty of house prices is higher*
  - *The housing depreciation cost in the owners' case or the rental cost in the renters' case is lower*
  - *The observation cost is lower*

See Proof in Appendix.

Proposition 2 reveals the properties of households' attention to house prices when they are not planning to adjust their housing. Regarding testable predictions, we focus on the following:

Prediction 3: For households that do not plan to adjust their housing in the future, attention to house price information does not vary with their wealth/housing ratio.

As households' attention does not vary with their wealth/housing ratio, households that do not plan to adjust their housing follow a time-dependent rule for their optimal attention to house price information.

2.2 Empirical Evidence on Inattention to House Price Information

Next, I use empirical data to examine how household attention compares to the predictions of the stylized model of optimal attention. I utilize the Housing Module of the New York Fed Survey of Consumer Expectations (SCE) (Federal Reserve Bank of New York, Center for Microeconomic Data, 2024), that asks respondents how often they consult websites or other sources that provide information on the estimated current value of their own property or properties in their area, as well as housing characteristics and other data relevant to housing. I also merge wealth data from the Household Finance module and demographics and other characteristics data from the Core questionnaire.

2.2.1 Evidence on Model Prediction 1

Considering first Prediction 1, that attention to house price information interacts with households' perceived current borrowing constraints, I construct a measure of the perceived loan-to-value (LTV) ratio of owners. Importantly, the loan amount used is what households report as the total amount of outstanding loans against the household's primary residence as of the day of the survey, including all mortgages and home equity loans, and the home value corresponds to what households believe their residence is worth today. For renters, the data-equivalent of the borrowing constraint from

the stylized model is households' debt-to-income (DTI) ratio. I run the following specification:

$$y_{i,t} = \gamma_0 + \gamma_1 \text{Perc LTV or DTI}_i + \boldsymbol{\gamma_2}' X_i + \boldsymbol{\gamma_3}' W_i + \xi_j + \zeta_t + \epsilon_{i,t} \quad (1)$$

where $y_{i,t}$ is an index equal to 1 when a household checked house prices at least once in the past year, Perc LTV or DTI_i is the metric for households' borrowing constraints, either perceived LTV for owners or DTI for renters, binned by 20% intervals. X_i are household controls, including the household's credit score, age, gender, ethnicity, race, health state, marital status, risk aversion, employment status and the degree to which the respondent feels confident about their answers to what they believe the value of a typical home in their zip code was one and five years ago. W_i are housing controls including the household's primary residency type and location description, and ξ_j and ζ_t, are commuting zone and year fixed effects. The data is weighed using the survey weights and the specification is ran separately for owners and renters, for the 2014 to 2019 period.

Figure 1 shows the predicted average probability of attention for owners against their perceived LTV in Panel (a) and for renters against their DTI in Panel (b). In both cases, note that the probability that a household pays attention to house price information increases as households are closer to their perceived or actual borrowing constraint. Therefore, the empirical evidence validates Prediction 1, that attention to house price information interacts with households' perceived current borrowing constraints. The figures reveal an additional interesting pattern: households' attention decreases when their debt ratios are higher than permissible to households for increasing their leverage.

2.2.2 Evidence on Model Prediction 2

To study the empirical relationship between the probability of attention to house price information and the ratio of wealth over housing, as in Prediction 2, I run the following specification:

$$y_{i,t} = \gamma_0 + \gamma_1 \text{Wealth/Housing}_i + \gamma_2 (\text{Wealth/Housing})_i^2 + \boldsymbol{\gamma_3}' X_i + \boldsymbol{\gamma_4}' W_i + \xi_j + \zeta_t + \epsilon_{i,t} \quad (2)$$

where $y_{i,t}$ is an index equal to 1 when a household checked house prices at least once in the past year, Wealth/Housing$_i$ is the ratio of total liquid wealth to housing units owned, (Wealth/Housing)$_i^2$ is the same value squared to capture the model-predicted convex relationship between this dependent variable and the probability of attention, if such a relationship does exist in the data, X_i are households controls, including the household's credit score, age, gender, ethnicity, race, health state, marital status, risk aversion, employment status, the degree to which the respondent feels confident about their answers to what they believe the value of a typical home in their zip code was one and five years ago, and the household's perceived current local house price and growth, to match the predicted characteristics of $P^{att}(a/h, \tilde{P}^H_{i,j}, \tilde{\phi}^H_{i,j})$. W_i are housing controls as in 1, and ξ_j and ζ_t, are commuting zone and year fixed effects. The data is weighed using the survey weights and the specification is ran for the 2014 to 2019 period. For renters, I run the same specification, but with Wealth$_i$ and (Wealth)$_i^2$ as the first two dependent variables.

Figure 2 Panel (a) depicts the data-predicted probability that owners pay attention to house price information against their liquid wealth over perceived housing, and Panel (b) depicts the probability that renters pay attention to house price information against their liquid wealth. It is evident that the data reflects the properties highlighted in Proposition 2, and particularly the predicted convex shape for owners, and the monotonically increasing relationship for renters. Note that in Panel (a), at the lower and higher ends of the wealth/housing range, the probability of attention does not reach 1.0, but is approximately .8 and .75 respectively. Since the choice to pay attention involves households trading off the costs and benefits of observing house price information, if full attention is not reached, this would indicate that either the cost is too high for households at the lower end of the wealth/housing distribution, or the benefit of paying attention is perceived to not be high enough to warrant paying attention, particularly for the higher end of the wealth/housing distribution. Additionally, note that the 95% level standard deviation around the mean prediction shown increases with wealth/housing ratio as the available sample of households along the distribution decreases.

2.2.3 Evidence on Model Prediction 3

Next, empirical evidence is examined in relation to the third model prediction. The relevant households are owners and renters who have just adjusted their housing, and therefore do not plan to readjust in the near future. To filter the data for this subset of households, additional data from the SCE is used, that asks households about the percent chance that they will move to a different primary residence. For renters, the more relevant question is a follow-up that specifically asks households if they were to move to a different primary residence over the next 3 years, what is the percent chance that they would buy (as opposed to rent) their new home. I therefore run specification 2 again for owners and renters separately, for the subset of households that provide a percent chance of under 5% that they will move. I also exclude the squared term for the wealth over housing and wealth variable, for owners and renters respectively. Additionally, in the renters' case due to the less data available, there is no commuting zone fixed effect included. Figure 3 shows the results for owners and renters. For owners in Panel (a), the probability of attention remains largely constant for any value of the wealth over housing ratio. This agrees with the third model prediction. For renters, the result is not as stark as the owner case, as there is a slight slope in the predicted attention by wealth graph. However, this slope is much lower than in Figure 2.

2.3 Taking Stock

The stylized model of optimal attention presented in this section described the properties of attention for households that incur an observation cost whenever they decide to pay attention to house price information. I find that, according to the model predictions, households that are considering housing adjustments jointly decide when to next adjust their house and observe house price information. I show that the probability of adjustment and observation is convex in wealth over housing, which leads to a state-dependent rule of optimal attention. First, the probability increases as households come closer to their perceived current borrowing constraint, and second, when they

approach some optimally chosen upper threshold of wealth over housing, and they decide to move to a larger house. Households who choose to remain owners or renters follow a time-dependent attention rule, which does not vary by the ratio of wealth over housing. Moving to the empirical evidence, I find that the survey data from the SCE agrees with these model predictions. Therefore, households appear to make largely optimal decisions when it comes to choosing when to pay attention to house price information. In order to examine the implications of attention on households' financial decisions, in the next section I build a rich quantitative life-cycle model which incorporates inattentive agents.

3 A Quantitative Life-Cycle Model

This section introduces a life-cycle model of housing choice, where households are inattentive to house price information and hold subjective beliefs for current house prices and house price growth, as well as expected future house prices. In each period, households choose whether to remain owners or renters, or to adjust their housing, as well as their non-housing and housing consumption, and derive utility from bequests to their offspring. Households' income is subject to aggregate and idiosyncratic permanent and transitory income shocks, housing is illiquid and households face borrowing constraints, based on their housing wealth and income. As the aim of this model is to determine the partial equilibrium response of household consumption to house price changes, all asset prices are exogenous. I begin by describing the dynamics governing the actual evolution of the model state variables, and continue to define how household inattention to house price information is incorporated in the model.

3.1 State Variables

Households, indexed by i, live for a maximum of T periods, based on some age-varying probability of survival in each period t, equal to $(1 - D_{age})$, where age is the age of the household. They have

two assets: liquid wealth, $a_{i,t}$ and housing wealth, $w^H_{i,t}$. In each period, households choose whether to be renters or owners, or if they want to adjust their housing, when they also chose the units of housing they would like to own $h_{i,t}$. Renters all derive utility from $\bar{h}$ level of housing. Conditional on this, households also choose their consumption $c_{i,t}$. Housing units $h_{i,t}$ are interpreted as the physical size or the quality of the house, and consumption $c_{i,t}$ denotes units of the consumption good with a price normalized and fixed to one. Their choice of $c_{i,t}$ and $h_{i,t}$ is driven by their per-period Constant Relative Risk Aversion utility $u(c_{i,t}, h_{i,t})$, assuming a Cobb-Douglas aggregator over housing and non-housing consumption:

$$u(c_{i,t}, h_{i,t}) = \frac{1}{1-\rho}(c^{\alpha}_{i,t} h^{1-\alpha}_{i,t})^{1-\rho} \tag{3}$$

where ρ is the coefficient of relative risk aversion and α is the non-housing share of consumption.

Households also derive utility from bequests to their offspring, where the bequest function $B(W_{i,t})$ is:

$$B(W_{i,t}) = B_0 \frac{1}{1-\rho^B} W^{1-\rho^B}_{i,t} \tag{4}$$

B_0 captures the strength of the warm-glow bequest motive and $W_{i,t}$ is the total wealth that would be left to the offspring. The total wealth is equal to:

$$W_{i,t} = (1 + r^{\text{save}}) a_{i,t-1} + w^H_{i,t} \tag{5}$$

where $a_{i,t}$ is the liquid wealth and $w^H_{i,t}$ is the housing wealth of the household. Households have access to a savings account with balance $a_{i,t}$, that evolves according to income and their decided level of non-housing and housing consumption. This account earns a rate of return r^{save}, which equals the risk-free rate r plus some spread $g^{\text{save}} > 0$. The household may adjust their liquid wealth without incurring any adjustment costs.

In each period t, households are inelastically supplied with a single unit of labor $y^L_{i,t}$, for which

they are paid a constant, exogenously determined effective wage rate ω. Household income is then:

$$y_{i,t} = \omega y^L_{i,t} \tag{6}$$

Labor units $y^L_{i,t}$ are driven by aggregate and idiosyncratic productivity levels P_t and $p_{i,t}$ respectively, similarly to Carroll et al. (2020). Aggregate productivity P_t is subject to permanent productivity shocks Ψ_t, that are independent and identically distributed with a mean of 1 and standard deviation σ_Ψ. Therefore, aggregate productivity evolves according to:

$$P_{t+1} = P_t \Psi_{t+1} \tag{7}$$

Idiosyncratic productivity $p_{i,t}$ is subject to permanent shocks at the idiosyncratic level, $\psi_{i,t}$, that are independent and identically distributed with a mean of 1 and have a age-varying standard deviation $\sigma_{\psi,age}$, where age is the age of the household. $p_{i,t}$ furthermore grows by the age-specific growth factor γ_{age}, such that idiosyncratic productivity is defined as:

$$p_{i,t+1} = \gamma_{age} p_{i,t} \psi_{i,t+1} \tag{8}$$

Effective labor productivity is also subject to transitory shocks at the idiosyncratic and aggregate level, $\theta_{i,t}$ and Θ_t. Θ_t is independent and identically distributed with a mean of 1 and standard deviation σ_Θ, and $\theta_{i,t}$ is independent and identically distributed with a mean of $\frac{1}{1-u}$ where u is the probability of unemployment and has age-varying standard deviation $\sigma_{\theta,age}$, similarly to Carroll et al. (2017). So, labor units $y^L_{i,t}$ follow the process:

$$y^L_{i,t} = \theta_{i,t} \Theta_t p_{i,t} P_t \tag{9}$$

Upon retirement, households receive retirement income, that is not exposed to any idiosyncratic risk, but is subject to aggregate dynamics, the aggregate permanent productivity shocks and aggregate transitory shocks Ψ_t and Θ_t.

House prices are defined at the local level j, and the house price per unit of housing is $P^H_{j,t}$. House

prices grow by constant growth factor Γ^H and are subject to permanent house price shocks Ψ_t^H. These shocks are mean one i.i.d. and perfectly positively correlated with the aggregate permanent productivity shocks Ψ_t that impact household income. The co-integration with permanent income therefore allows for house prices to co-move with business cycle dynamics. This reflects what is true historically in the data, in agreement with Campbell and Cocco (2007) and Ortalo-Magné and Rady (2006), and has significance for incorporating house price perceptions, as will be further explained in subsection 3.2.

In addition to these components, house price growth in each area j is impacted by dynamics independent of the business cycle, which, significantly for the model setup, are uncorrelated to household income in the model. These dynamics are summarized by the growth rate $\phi_{j,t}^H$ that follows a bounded random walk:

$$\Pr[\phi_{j,t+1}^H = \phi_{j,k}^H | \phi_{j,t}^H = \phi_{j,n}^H] = Z_{n,k} \tag{10}$$

where k and n are state indices for the corresponding finite Markov chain. By incorporating all the aforementioned components, the price of one unit of housing $P_{j,t}^H$ in area j during period t is governed by the following process:

$$P_{j,t+1}^H = \Gamma^H \Psi_{t+1}^H \phi_{j,t+1}^H P_{j,t}^H \tag{11}$$

This transition equation governs the actual value of a house but not the perceived house value for households that do not update their house price perceptions.

The price of rental housing follows house prices, with the ratio of the price of rental housing $R_{j,t}^H$ over the price of housing $P_{j,t}^H$ in area j defined as $\bar{R}$. The rental flow payment required by renters in each period is the following:

$$R_{j,t}^H = \bar{R} P_{j,t}^H \tag{12}$$

Households can rent a median-sized house in their area, and derive utility from that housing equal

to $\bar{h}$ as mentioned earlier. In addition, the rent is higher than the user cost of owner-occupied housing. Therefore, renters will always have an incentive to buy a home, but will only be able to do so if their finances allow it. As my purpose is to study the housing wealth effect, this is the subset of renters I am interested in modeling.

Home owners have housing wealth $w^H_{i,t}$. The value of a house in period t is the product of the price of one unit of housing $P^H_{j,t}$ in the area j where the home owner lives and the number of housing units $h_{i,t}$ the household owns. The actual value of a house depreciates over time by depreciating factor ξ and therefore evolves according to:

$$w^H_{i,t+1} = (1-\xi)h_{i,t}P^H_{j,t+1} \tag{13}$$

The adjustment of housing wealth is costly, rendering it illiquid wealth. Specifically, households incur a cost proportional to the size of their house, captured by their chosen housing units $h_{i,t}$ and by the market price of housing per unit $P^H_{j,t}$. The total adjustment cost is:

$$\kappa^H_{i,t} = \bar{\kappa}^H h_{i,t}P^H_{j,t} \tag{14}$$

where $\bar{\kappa}^H$ is the proportional adjustment cost parameter that reflects the necessary costs involved in buying or selling the property.

Households can increase their consumption by borrowing, but their borrowing limits differ depending on whether they are owners or renters. For owners, the maximum amount they can borrow equals $(1-\theta)$ of the current value of their home before depreciation. Therefore, θ is the equity that owners have in their home and this collateral constraint is equivalent to a LTV constraint on homeowner borrowing. The constraint is:

$$-a_{i,t} \leq (1-\theta)h_{i,t}P^H_{j,t} \tag{15}$$

For renters, as they have no collateral to borrow against, their borrowing constraint is determined against their permanent income and is chosen to be similar to a DTI constraint that households

typically face. The DTI limit is denoted as σ, so for renters the borrowing limit is:

$$-a_{i,t} \leq \sigma p_{i,t} P_t \tag{16}$$

In either the owner or renter case, debt adjustment has no additional cost for the household, as long as they meet their LTV and DTI requirements respectively. The interest rate on debt is common for owners and renters, denoted as $r^{\text{borrow}} > 0$ and equals the risk-free rate r plus some spread g^{borrow}. Note that $g^{\text{borrow}} > g^{\text{save}}$, so that $r^{\text{borrow}} > r^{\text{save}} > r$ as would be expected. Because of $r^{\text{borrow}} > r$ and the existence of the borrowing constraints, the only case in which it would be optimal for households to borrow is if they have depleted their liquid assets, so that the borrowing amount is equal to $-a_{i,t}$.

It is significant for the interpretation of the results to note that homeowners' borrowing limits are determined based on the actual value of their housing wealth, as opposed to their perceived value. So, for example, between a household that is attentive to a house price increase and a household that is not, only the attentive household may seek to borrow or increase its borrowing against it's appreciated house value, but its borrowing amount will be determined based on the actual value of $P^H_{j,t}$.

3.2 House Price Beliefs

Having defined the dynamics that govern the evolution of actual house prices, I move on to discuss how household house price perceptions are incorporated in the model. We have denoted the actual house price level and growth by $P^H_{j,t}$ and $\phi^H_{j,t}$, and I now denote the perceived house price level and growth by household i that lives in area j in time t as $\tilde{P}^{H,IA}_{i,j,t}$ and $\tilde{\phi}^{H,IA}_{i,j,t}$ respectively. Section 2 already discussed household attention to house price information in a stylized model. Here, we further study the properties of attention along all the state variables of the full model, and document additional stylized facts that will guide house price belief formation in the model.

3.2.1 House Price Perceptions

First, house price perceptions and house price growth perceptions are compared to actual values, leveraging three data sources: the Health and Retirement Study (HRS) (Health and Retirement Study, 2024), the American Community Survey (ACS) sample micro data (United States Bureau of Census (2022); Ruggles et al. (2024)) and the FHFA house price index (Federal Housing Finance Agency, 2024).

To compare households' perception of their own house price and the actual value of their house, the HRS is used, a longitudinal data set focusing on older U.S. households, which collects information on home transactions, as well as elicits household beliefs about home values. Specifically, for each respondent that is reported to sell their house, the survey provides information on what the household believes their home is worth before house sale, as well as the actual value at the transaction[8]. This data is unique and highly relevant in this setting, since the actual home value is derived by the sale of the asset, rather than a value assessment or an estimate. The survey also includes data on the original purchase price of the home, which can be used to calculate the perceived versus actual house price growth, from time of purchase up to now.

To further examine house price growth perceptions, the ACS sample micro data is used along with the FHFA house price index. The ACS is a 1-in-100 national random sample of the U.S. population, spanning from 2005 to 2022, and provides house price perceptions by asking home owners how much they believe their property would sell for, if it were for sale at the time of the survey. For households' perceptions of annual house price changes, I use the annual change of the ACS data on house price perceptions. The data counterpart for actual annual house price changes is provided by the county-level FHFA house price index. This is a repeat-sales index calculated using

[8]Because respondents will report the price at which their home was sold at the next survey wave after the sale, there can be up to a 24-month gap between the perceived and actual price. To control for this, I regress the number of months between the survey date for the perceived price and the date for the sale, and use the residual in the remaining analysis. Therefore, the HRS provides both the perceived and the actual price of a house, for the years between 2002 and 2018.

sales transactions of mortgage data and prices from appraisal data, both obtained from Fannie Mae or Freddie Mac.

Figure 4 shows the average relative perception error households make about the value of their house, plotted against the distribution of the corresponding actual house prices. In particular, Panel (a) plots the mean difference between perceived house value and selling price over the price at which the house was sold, along year-specific sale price deciles. Panel (b) is produced using the difference between weighted average perceived house value by county from the ACS and the FHFA county average house price, divided by the FHFA value. These values are then averaged by year-specific FHFA index deciles. The graphs show that households overestimate the value of their house when the price is low in relation to the concurrent housing market, and underestimate how much their house is worth when the actual price is at the higher end of the distribution.

Moving on to house price growth, Figure 5 presents the perception error households make about the price growth of their homes. Panel (a) plots the difference between perceived and actual house price growth since the original purchase date of each home for HRS respondents under two cases: when the actual house price change is positive and when the change is negative. Panel (b) plots the corresponding data from the ACS and FHFA, where the house price growth considered is now annual. In both panels, we see that households perceive the growth of the price of their home to be higher than actual when house prices decrease and they believe house price growth has been lower than what it is for positive house price changes. These two figures lead to the first stylized fact:

Stylized Fact 1*: Households' make perception errors for current house price levels and growth: these errors are higher when actual prices are low and decreasing, and lower when actual prices are high and increasing.*

To develop the model-equivalent to this fact, let's take some set $\mathcal{I}_{i,t}$ to be the set of all information observed by household i in period t. Households' perception of their local house price level

and growth will then be the expected value of $P^H_{j,t}$ and $\phi^H_{j,t}$ given $\mathcal{I}_{i,t}$:

$$\tilde{P}^{H,IA}_{i,j,t} = \mathbb{E}^{IA}[P^H_{j,t}|\mathcal{I}_{i,t}] \tag{17}$$

$$\tilde{\phi}^{H,IA}_{i,j,t} = \mathbb{E}^{IA}[\phi^H_{j,t}|\mathcal{I}_{i,t}] \tag{18}$$

The perception errors depicted in Figures 4 and 5 can be expressed within the model context as belief wedges between actual current house prices and perceived house prices under households' information, and between perceived and actual house price growth:

$$\Delta P^H_{i,j,t} = \tilde{P}^{H,IA}_{i,j,t} - P^H_{j,t} \tag{19}$$

$$\Delta \phi^H_{i,j,t} = \tilde{\phi}^{H,IA}_{i,j,t} - \phi^H_{j,t} \tag{20}$$

Households' perception errors about house price growth are further examined in the data along another dimension: the likelihood of actual house price changes. To create a measure of house price change likelihood, I calculate the standard deviation of house price growth by county in the FHFA data, or standard deviation of the year-on-year house price changes. Actual house price growth values are then expressed as multiples of their corresponding historical local standard deviation. I group the ACS-calculated perceived house price growth by this metric, at .5 intervals, and for each bin, the median difference between perceived and actual growth is calculated. Note that the sign along the year-on-year house price change standard deviation indicates whether the actual house price growth in the period is positive or negative, to capture any asymmetries.

Figure 6 highlights Stylized Fact 1 with respect to house price growth, namely that actual house price growth is higher than perceived house price growth for positive actual house price changes, and the reverse is true for negative actual house price changes. In addition, it shows that this deviation monotonically increases by likelihood, or by size of the actual house price change compared to historical house price changes. Adopting the literature's view that households are Bayesian updaters when forming beliefs about house prices (Kuchler and Zafar, 2019), this result

indicates that households exhibit 'stickiness' in their perceptions, as both positive and negative house price changes are perceived to be smaller in absolute value than they are. Combining this finding with the theoretical predictions and empirical evidence of section 2, households appear to not update their perception of current house price growth continuously, or are only occasionally attentive to actual house price changes. This result brings about the next stylized fact:

***Stylized Fact 2**: Households' perceptions of house price changes are 'sticky'.*

This stylized fact helps in defining the evolution of house price beliefs in the model. Denote τ as the time intervals during which households do not pay attention to house price information. At the end of this interval, households become attentive and observe the actual house price level and growth. Furthermore, it is assumed that households know the aggregate growth factor Γ^H, as it is constant and they only need to observe it once. They also know the permanent house price shocks Ψ_t^H, which are perfectly correlated with shocks to the aggregate component of their income. Therefore, in periods $\tau_{i,t}$ between observation dates, the perceived level and growth rate of house prices $\tilde{P}_{i,j,t}^{H,IA}$ and $\tilde{\phi}_{i,j,t}^{H,IA}$ are the following:

$$\begin{aligned}\{\tilde{P}_{i,j,t}^{H,IA}, \tilde{\phi}_{i,j,t}^{H,IA}\} =& \mathbb{E}_{t-\tau_{i,t}}^{IA}[\{P_{j,t}^H, \phi_{j,t}^H\} | P_{j,t-\tau_{i,t}}^H, \phi_{j,t-\tau_{i,t}}^H] \\ =& \{(\Gamma^H \phi_{j,t-\tau_{i,t}}^H)^{\tau_{i,t}} \Psi_t^H P_{j,t-\tau_{i,t}}^H, \phi_{j,t-\tau_{i,t}}^H\} \end{aligned} \tag{21}$$

The above equation indicates that, when owners perceive house price growth $\phi_{j,t-\tau_{i,t}}^H$ at $\tau_{i,t}$ periods before t, they extrapolate and expect that house prices follow the same annual growth rate as the one they last observed $\tau_{i,t}$ periods ago, hence making household perceptions of house price changes 'sticky'.

3.2.2 House Price Expectations

I now examine how households' perceptions about current house price growth may impact their expectations for house prices in the future. I use the available data from the SCE to trace out the

term structure of expectations for owners and renters. I use survey information about households' perception of the house price of a typical home in the zip code one year ago, at the time of the survey, as well as each household's expectation about the typical house price one year and five years in the future. The specification is the following:

$$\mathrm{HP}_{i,t} = \sum_{l \in \{-1,\ldots,0,\ldots 5\}} \boldsymbol{\beta_l}' D_{i,t+l} + \boldsymbol{\gamma_1}' X_i + \boldsymbol{\gamma_2}' W_i + \xi_j + \zeta_t + \epsilon_{i,t} \tag{22}$$

where $\mathrm{HP}_{i,t}$ is the log perceived typical house price for the zip code the household lives in, by the household a year ago, currently, or its expected zip code-level house price one year or five years in the future. $D_{i,t+l}$ is an indicator variable for whether $\mathrm{HP}_{i,t}$ corresponds to the perception for the house price a year ago, currently, or the expectation 1 year or 5 years in the future, $X_{i,t}$ are household controls including the household's income, credit score, gender, ethnicity, race, age, education, health state, marital status, risk aversion, employment status, and its confidence in its stated perceived value of a typical home in their zip code one and five years ago, W_i are housing controls, including the household's primary residency type and location description. I also control for commuting zone and year fixed effects with ξ_j and ζ_t. The errors are clustered at the commuting zone level, the prices are adjusted for inflation and the observations are weighed by the survey weights, in order to get nationally representative results. The specification is ran separately for owners and renters, and by income and age group. The estimated coefficients β_l for each year around the survey, that capture the difference in each l-house price and the perceived house price at the time of the survey T, are plotted in Figures 7 and 8.

The figures show that both owners and renters extrapolate their latest perceived house price growth into the future. On average over the analysis period, owners perceived a change in local house prices of around 5% and their expected house price change from today's level is extrapolated to about 5% for one year and just over 10% for 5 years out. For renters, Figure 8 shows that households perceived a change in local house prices of just under 10% and their expected house price change from today's level is similarly extrapolated to just under 10% for one year and to

almost 20% for 5 years out. Older renters expect house prices to increase by more after 5 years, at over 20%.

***Stylized Fact 3**: Households extrapolate their expectations for future house price growth.*

Owners and renters extrapolate their expectations for future house price growth based on their latest perceived house price growth $\phi^H_{j,t-\tau_{i,t}}$, so guided by this empirical fact, the subjective beliefs for $\tilde{P}^{H,IA}_{i,j,t+1}$ and $\tilde{\phi}^{H,IA}_{j,t+1}$ are the following:

$$\begin{aligned}\{\tilde{P}^{H,IA}_{i,j,t+1}, \tilde{\phi}^{H,IA}_{i,j,t+1}\} =& \mathbb{E}^{IA}_{t-\tau_{i,t}}[\{P^H_{j,t+1}, \phi^H_{j,t+1}\} | P^H_{j,t-\tau_{i,t}}, \phi^H_{j,t-\tau_{i,t}}] \\ =& \{(\Gamma^H \phi^H_{j,t-\tau_{i,t}})^{\tau_{i,t}+1} \Psi^H_{t+1} P^H_{j,t-\tau_{i,t}}, \phi^H_{j,t-\tau_{i,t}}\} \end{aligned} \tag{23}$$

This creates a wedge between expected future house prices under full attention and under households' subjective beliefs:

$$\Delta\mathbb{E}_t[P^H_{i,j,t+1}] = \mathbb{E}^{IA}_t[\tilde{P}^H_{i,j,t+1}] - \mathbb{E}^{FA}_t[P^H_{i,j,t+1}]$$

3.2.3 Attention to House Price Information

In the aforementioned belief formation process, $\tau_{i,t}$ is the main variable that will drive the magnitude of both the perception wedge, $\Delta P^H_{i,j,t}$, and expectation wedge $\Delta\mathbb{E}_t[P^H_{i,j,t+1}]$. The properties of this variable $\tau_{i,t}$, and, in turn, the inverse of τ which equals the probability of attention to house price information in each period $P^{att}(\cdot)$, were described in the stylized model in section 2. In that section, I showed that in the model, the optimal value of P^{att} can be determined by the state variables of the household's problem. This was also validated using an empirical analysis based on the SCE data. Therefore, I use a non-parametric policy function of households' attention to house price information, that I calibrate based on the empirical SCE data. The calibration process is explained in the following section.

4 Model Calibration

This section details the procedure followed for the calibration of the model parameters, presents the parameter values and how the model predictions fit the data. The model is solved at an annual frequency and the period used to calibrate the model is between 2013 and 2019. This period is long enough to provide house price variation at the local level, while not coinciding with a house price boom or bust at the national level. Therefore, the attention level of households will relate to what it would be under baseline conditions. The model calibration follows two main stages. The first stage involves the calibration of parameters that can be identified without solving the model, which are all parameters that describe households' attention to house prices, parameters defining the dynamics of macroeconomic variables in the model, household income dynamics and survival probabilities, as well as institutional characteristics relevant to non-housing consumption, housing and borrowing. These values are determined through a reduced form approach or by using values from the literature. For the second stage, all previously determined parameters are inputted in the model to calibrate the remaining parameters, which capture household preferences.

4.1 First Stage: External Parameter Calibration

In the model, agents start their adult life at 25 years old, they retire at 65 and live until they are 120. The survival probabilities are parameterized using data from the life-tables of the United States Social Security Administration, were the 1-year survival probabilities for individuals in the model's age range for 2013 are applied. Households' income process is driven by aggregate and idiosyncratic components, as mentioned in section 3. The standard deviation of the log of permanent and transitory aggregate productivity σ_Ψ and σ_Θ are 0.006 and 0.003 respectively, following Carroll et al. (2020) and the annual aggregate income growth rate is 1%. For the idiosyncratic income components, I use life-cycle income dynamics defined by Cocco et al. (2005), where the estimates used to reproduce the income specification are available through Carroll et al. (2018). Unlike Cocco

et al. (2005), this paper's model does not define households' education level, so I take average values of γ_{age} over education. For the age-specific standard deviations of $\psi_{i,t}$ and $\theta_{i,t}$, I use data from Sabelhaus and Song (2010). The values are adjusted for inflation, for the year 2019. For the wage rate ω, the value captures the steady-state compensation of one unit of labor, defined as $\omega = (1-a)$, where a is the capital's share of income. Using values from Carroll et al. (2020), I take 0.36 for the capital's share of income and hence the steady-state wage rate amounts to 0.64. Finally, the annual probability that a household becomes unemployed while in its working years is 5%.

All parameters that define the dynamics of asset prices in this partial equilibrium model, including interest rates and actual house prices, are calibrated using economic data from the model period 2013 to 2019. The saving rate r^{save} is set to equal 1%, which is approximately the average of the Federal Funds Effective Rate (Board of Governors of the Federal Reserve System (US) (2024b), 2024b) over the period and the borrowing rate is set to 12% from the Commercial Bank Interest Rate on Credit Card Plans (Board of Governors of the Federal Reserve System (US) (2024a), 2024a). The aggregate house price growth rate Γ^H is set to 3.3% to match the growth of real national house prices in the U.S. over the analysis period. As noted in section 3, the local growth rate $\phi^H_{j,t}$ follows a Markov process and defines the house price variations that are independent to business cycle dynamics. The calibration matches data on metro-level house price changes, over and above national house price changes. To first isolate this variation in the data, the annual metropolitan-level house price changes for the 2013-2019 period are calculated using FHFA HPI All-Transactions metro-level indices, controlling for inflation. These growth rates are then regressed against the real national house price changes. The residual captures the dynamics independent of national house price changes that we are after. These values are discretized to summarize a Markov process with 3 states, which produces a process where each of the 3 states represent a decrease in growth, increase in growth, or about the same growth relative to the national average. The transition probabilities are calibrated such that the local house price growth can change once every two years on average. The housing depreciation rate ξ equals 3.6% to match the IRS guidance that

uses a typical depreciation period of 27.5 years, assuming straight line depreciation. The housing adjustment cost parameter $\bar{\kappa}^H$ equals 5%, which is a typical magnitude in the literature (Berger et al., 2017). As explained in section 3, with respect to the renters, I focus on households who want to buy a house at some point in the future. Therefore, the rent-house price ratio $\bar{R}$ is calibrated to reflect US metros with high gross rental yields. Demers and Eisfeldt (2022) calculate rent-to-price ratios between 0 and 0.25, so I use 0.21, which is at the higher-end of their distribution. For the borrowing limits for owners and renters, renters can borrow up to 3.5 times their permanent income and owners can have a maximum LTV of 80%, both of which are broadly consistent with conforming loan requirements according to Chen et al. (2020).

For households' house price attention in the model, as mentioned in section 3, I follow a non-parametric approach, whereby I determine the probability of attention along the joint distribution of the state variables from the data. I discretize the state-space for state variables $m_{i,t}, h_{i,t}, age_{i,t}, P^H_{j,t}, \phi^H_{j,t}$ in 3 quantiles for each, where $m_{i,t}$ is liquid wealth including income, and I run the following logit regression:

$$y_{i,t} = \gamma_0 + \gamma_1 m_i^Q + \gamma_2 h_i^Q + \gamma_3 age_i^Q + \gamma_4 \tilde{P}_i^Q + \gamma_5 \tilde{\phi}_i^Q + \boldsymbol{\gamma_6}' X_i + \boldsymbol{\gamma_7}' W_i + \xi_j + \zeta_t + \epsilon_{j,t} \tag{24}$$

where X_i are household controls, including the household's credit score, gender, ethnicity, race, health state, marital status, employment status and confidence on past house price survey answers, W_i are housing controls including the household's primary residency type and location description, and ξ_j and ζ_t, are commuting zone and year fixed effects. The logit regression includes survey weights as well. These regressions provide the predicted probability that owners and renters are paying attention to house prices in a specific period. The values for these probabilities are inputted in the model to capture state- and time-dependent variation in attention.

4.2 Second Stage: Matching Model-Implied Moments to the Data

The remaining parameters, which are all parameters than cannot be directly observed in the data, are calibrated by matching moments from the Survey of Consumer Finances for the analysis period of 2013-2019 (Board of Governors of the Federal Reserve System (US) (2019), 2013-2019). These parameters are the coefficient of relative risk aversion in the per-period utility ρ, the coefficient of relative risk aversion in the bequest utility ρ^B, the non-housing share of consumption α, the discount factor β and the strength of the warm-glow bequest motive B_0. The target moments are the life-cycle profiles of liquid assets, normalized by the year-specific median income level for owners, home value over income and debt over home value. For each of these ratios, I match median values at 5-year age intervals, from 25 to 70. I also target the average home-ownership rate at the same 5-year age intervals.

In terms of variable definitions, household income is the total household wage income, which is normalized by the median wage income in the data, calculated by taking into account the survey weights. The home value of the household is the sum of the value of their primary residence and the value of other residential real estate held, and liquid assets is the total of all types of transactions accounts, directly held pooled investment funds, directly held stocks, directly held bonds and retirement accounts when they are received. The total of all types of transactions accounts is adjusted upwards by 5% to account for any cash. The home ownership rate is the proportion of owners during the period. All dollar values are adjusted for inflation and all moments are calculated using the appropriate survey weights.

The model is simulated to produce the model-implied moment counterparts to the data moments. The households' initial conditions at birth are determined from the data as follows. The households are first randomly split into owners and renters, according to the proportion of each group in the data. For each group, households are classified as belonging to one of the income quantiles in the data specific to renters or owners. The starting housing and liquid assets equal the

median value for that corresponding group. During the simulation, households' tenure and state variables evolve according to the dynamics presented in section 3. For the simulation, a panel of 10,000 households is created and the model is simulated for 200 annual periods. The simulated data is binned in 5-year age groups and the proportion of owners by age group, the median by age group of housing wealth over income, of normalized liquid wealth and of debt over housing wealth make up the model moments. For the housing wealth over income moment, house prices are adjusted for house price inflation, and the normalized liquid wealth and debt over housing wealth moments are specific to owners only. For the home ownership rate, I count the number of owners over the total number of households in each age bin. For calculating the model moments, I remove the burn-in period, and am left with half a million simulated data points. Overall, 40 moments are used to identify 5 parameters, leading to an over-identified calibration. Refer to the Internet Appendix for more information on the calibration approach and implementation. The model moments are plotted against the corresponding data moments in Figure 9, and as evident from the graphs, the model closely matches the data moment counterparts. The calibrated parameters are included in Table 1.

5 Probability of Attention, Belief Wedges and Consumption

The model is now applied to examine the impact of inattention on household beliefs about house prices, as well as the impact of inattention on household consumption.

5.1 Probability of Attention and Belief Wedges

As discussed in 3.2.1 and Stylized Fact 1, there is a perception error, or belief wedge, between households' perceived current house prices under the baseline model setup IA and actual house prices. The belief wedges with respect to households' perception of current house prices can be

written as follows:

$$\Delta P^H_{i,j,t} = \tilde{P}^{H,IA}_{i,j,t} - P^H_{i,j,t} \tag{25}$$

where $\Delta P^H_{i,j,t}$ is household's i house price perception error at time t. The magnitude of these errors are directly driven by the probability of attention to house prices for each household in each period, P^{att}. To summarize the impact of attention on house price perceptions, the mean of the household-specific differences between perceived house prices and the actual values are calculated by households' attention to house price information, binned in 5% intervals. The results are shown in Figure 10, where the panels plot the house price perception error in USD against the probability of attention, separately for owners and renters, and for when house price growth is positive versus negative.

We observe that the average house price perception errors are positive when actual house prices decrease and negative when actual house prices increase, driven by the assumed 'stickiness' of house price perceptions in the model, discussed in subsection 3.2. Clearly, in all the panels, when the probability of attention to house price information is higher, house price perception errors decrease.

5.2 Probability of Attention and Consumption

As discussed in section 2, attention influences the definition of the owner's inaction region for housing adjustment. Specifically, it was shown that attention to house price information affects households' perceived optimal thresholds that define the inaction region for adjusting their housing. The optimal threshold of wealth/housing for owners is $-(1-\theta)P^H_{j,t} \leq b < B < +\infty$ under full attention, but $-(1-\theta)\tilde{P}^H_{i,j,t} \leq b < B < +\infty$ when households are inattentive. This will, in turn, impact households' choice between $V^{own}(z)$, $V^{adj}(z)$, and $V^{rent}(z)$. Therefore, inattention to house price information and the resulting perceived house price will impact consumption through households' perceived inaction region for adjusting their housing.

In addition, households in the model determine jointly their non-housing and housing consumption, $c_{i,t}$ and $h_{i,t}$, conditional on households' decision of τ' and their choice to be owners, renters or adjust their housing. The consumption policy function is of the following form:

$$c^*_{i,t} = c(a_{i,t}, y_{i,t}, h_{i,t}, \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t})$$

Because non-housing consumption is based on perceived current house price $\tilde{P}^H_{i,j,t}$ and the household's perceived house price growth $\tilde{\phi}^H_{i,j,t}$ which equals their latest observed house price growth, household inattention to house price information will also further affect consumption directly, after households decide whether to be owners, renters or adjust their housing. To illustrate this, consider households that have chosen between $V^{own}(z)$, $V^{adj}(z)$, and $V^{rent}(z)$, and are now looking to determine their non-housing consumption when a house price shock occurs. In the baseline setup of the model defined as IA, optimal non-housing consumption will be:

$$c^*_{i,t} = c(a_{i,t}, y_{i,t}, h_{i,t}, \Delta\tilde{P}^H_{i,j,t} + \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t}) \tag{26}$$

where $\Delta\tilde{P}^H_{i,j,t}$ is the perceived house price shock: it will be observed by the households that are paying attention to house price information in period t, but not by the rest. In the counterfactual where all households observe the house price shock after their housing choice, optimal non-housing consumption will be:

$$c^*_{i,t} = c(a_{i,t}, y_{i,t}, h_{i,t}, \Delta P^H_{j,t} + \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t}) \tag{27}$$

where $\Delta P^H_{j,t}$ is the house price shock, observed by all. In summary, since inattention to house price information results in house price belief wedges, it affects both households' perceived inaction region for housing adjustment, hence their choice between $V^{own}(z)$, $V^{adj}(z)$, and $V^{rent}(z)$, as well as their non-housing consumption conditional on their discrete housing choice.

To examine this using the model, I run two separate analyses. First, I consider how attention and house price perceptions impact housing choice, and second, I determine how attention impacts consumption, conditional on housing choice.

5.2.1 Impact on Housing Choice

I determine the optimal choice between owning, renting or housing adjustment under 3 scenarios: full attention, where households' perceived house prices are equal to the actual values, inattention with households believing house prices to be 10% lower than they actually are and inattention with households believing house prices to be 10% higher than they actually are. Figure 11 shows the optimal housing choice for a household that is 45 years old and the last time they observed house prices, they remained steady. For the choice to own, we take households that are considering a house of 2 housing units.

Panel (a) of the graph plots the optimal choice against normalized liquid wealth and normalized actual house price level for households that pay full attention to house price information, Panel (b) plots the results for households that believe house prices to be 10% lower than they actually are and Panel (c) plots the results for households that believe house prices to be 10% higher than they actually are. Note that there are significant differences between the graphs, driven only by households' perceptions of current house prices. In all panels, when house prices are lower, most 45-year old households own and as house prices increase, households at the lower end of the liquid wealth distribution rent. The choice between renting or adjusting and owning or adjusting varies depending on house price perceptions. When households believe house prices are lower than they actually are, they look to adjust their housing, to take advantage of the lower prices. However, when they think house prices are higher, the lower-liquid wealth households choose to be renters thinking they can't afford otherwise, and higher-liquid wealth households remain or become owners to take advantage of the benefits that the appreciating asset can offer them, such as higher borrowing and owning a more valuable asset that makes them feel more wealthy (pure wealth effect) and that they can eventually pass on to their offspring.

5.2.2 Impact on Consumption, Conditional on Housing Choice

As discussed in section 2, households' inattention to house price information further affects household consumption, conditional on their housing choice. Hence, I use the model to quantify the following:

$$\Delta c^*_{i,t} = c(a_{i,t}, y_{i,t}, h_{i,t}, age_{i,t}, \Delta\tilde{P}^H_{i,j,t} + \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t}) - c(a_{i,t}, y_{i,t}, h_{i,t}, age_{i,t}, \Delta P^H_{j,t} + \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t}) \quad (28)$$

where note that $\Delta\tilde{P}^H_{i,j,t}$ is the house price shock as perceived by households, so this will not be observed by households that are not paying attention to house price information in period t, whereas $\Delta P^H_{j,t}$ will be observed by all households. The first component on the right hand side of expression 28 is determined by simulating the baseline model, equivalent to the IA setup defined in section 2, where a positive 10% shock to house prices is applied, after households have made their housing choice and are deciding their non-housing consumption. This shock will be perceived only by households that are paying attention in the current period. The second component is determined by simulating the baseline model, but assuming that all households observe the $\Delta P^H_{j,t}$ shock after deciding their housing choice.

Figure 12, Panels (a) and (b) present the average $\Delta c^*_{i,t}$ by the probability of attention, grouped in 5% intervals. The bars represent the average difference in non-housing consumption over a year after a house price shock, between the baseline simulation and the counterfactual where all households perceive a house price shock, after they have already made their housing choice. We observe that the impact on consumption decreases as the attention probability increases, and the direct impact of attention to renters is of very small magnitude.

6 Impact of Inattention on the Housing Wealth Effect

Having inspected how the probability of attention to house price information impacts household beliefs and consumption, the model is now applied in order to examine the housing wealth effect in detail. For the purposes of this analysis, I quantify the housing wealth effect through the Marginal Propensity to Consume out of Housing (MPCH), defined as the $ change in consumption a year after a positive 10% house price shock over the $ change in house prices. From section 3, the solution for the optimal consumption in each period is:

$$c^*_{i,t} = c(a_{i,t}, y_{i,t}, h_{i,t}, age_{i,t}, \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t}) \tag{29}$$

Therefore, the definition of the MPCH is the following:

$$\tilde{MPCH}^{IA}_{i,t} = \frac{c(a_{i,t}, y_{i,t}, h_{i,t}, age_{i,t}, \Delta\tilde{P}^H_{i,j,t} + \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t}) - c(a_{i,t}, y_{i,t}, h_{i,t}, age_{i,t}, \tilde{P}^H_{i,j,t}, \tilde{\phi}^H_{i,j,t})}{\Delta P^H_{i,j,t}} \tag{30}$$

Note that, since I find that the average MPCH for renters is of the magnitude of $1e^{-}3$ cents, which is very small, I continue by focusing on the MPCH of owners. All references to liquid wealth in the following section of the paper refer to liquid wealth including income.

6.1 MPCH Along Households' Cross-Section

In order to present the housing wealth effect along households' cross-section, the mean MPCH is plotted for the distribution of each of the state variables of the households' problem. In the graphs in Figure 13, the blue lines correspond to the average value of MPCH by decile of the state variable annotated along the x-axis. The orange and green lines show the average value of MPCH for the subset of households that perceive local house price growth to be negative and positive respectively. These results therefore reveal the model predictions for how household consumption responds to a house price shock overall, as well as when they expect house prices to decrease or increase in the future.

According to Panel (a), households with mid to high liquid wealth including income are the most responsive to a house price shock. Although owners at the lower end of the wealth distribution are the ones who have the highest relative effect after an increase in house prices, the effect will be the largest in dollar terms for middle-wealth households who can increase their consumption more than their peers that are lower along these distributions. Households at the top of the liquid wealth distribution are less impacted by the house price shock, given that they are further away from the liquidity constraints, so they are less responsive. As seen in the figure, when households expect house prices to increase in the future they substitute consumption today for tomorrow. At higher future house prices, they expect to be able to borrow more in the future and therefore also consume more. In contrast, when they expect house prices to decrease in the future, they consume more today. Since they believe house prices will fall, their borrowing capacity will decrease. Additionally, at lower house prices, they may choose to adjust their housing and increase their housing units, which would also require from them to consume less in the future in order to afford the down payment of their new house and cover the transaction cost.

Looking to Panel (b) for households' MPCH along the housing units distribution, the housing wealth effect is higher in magnitude for households with larger houses. These households stand to gain the most from a house price appreciation, and therefore increase their consumption by more. In Panel (c), the MPCH is decreasing by house price per unit. Households that own a home at lower house prices will be the ones facing the largest collateral constraints, by unit of housing owned. As a result, they are very sensitive to an appreciation of house prices and react the most to the shock. The variation of the MPCH along the housing and house price distributions for households that expect house prices to decrease or increase in the future is worth noting. As explained before, households have higher MPCH values when they expect house prices to decrease and lower when they expect house prices to appreciate further. However, these differences occur only for owners of mid to large sized houses, who stand to lose or, respectively, gain the most from house price changes. These differences are also more pronounced when house prices are lower and household

expect future house prices to appreciate, in Panel (c). Finally in this figure, Panel (d) highlights that MPCH values decrease with age. Over time in this life-cycle model, households accumulate liquid wealth and move further away from their liquid constraints, and hence respond less to a house price shock.

6.2 MPCH, Attention and Constraints

In discussing how the MPCH varies with respect to the model's state variables, many results appear to be driven by constraints, such as liquidity constraints, borrowing constraints, or constraints that occur with age. To investigate this explicitly, the model is used to estimate the MPCH, for groups of households under different degrees of constraints. To isolate the impact of inattention to house price information, I use as a metric the ratio of the mean MPCH for households simulated under full attention, the FA setup, for a household group g with similar characteristics, over the corresponding average MPCH for households under the baseline inattentive setup.

$$\frac{FA}{IA} = \frac{MPCH_{g,t}^{FA}}{\tilde{MPCH}_{g,t}^{IA}} \tag{31}$$

To produce the $MPCH_{g,t}^{FA}$ values, an additional model simulation is run, where the probability of attention for all households at all times is set to 100%. The purpose of these estimations is to determine how constraints interact with attention, and how this impacts households' responsiveness to a house price shock. The constraints considered are in terms of borrowing, captured by the level of debt households take out, the available debt capacity, and the loan-to-value ratio, in terms of age groups (not deciles as in Panel (d) of Figure 13), and in terms of liquidity, using the ratio of liquid wealth to income, and liquid wealth to housing wealth. The graphs in Figure 14 present the results, where the blue line shows the average MPCH by decile in the baseline model, the red line plots the average MPCH by decile in the full attention counterfactual, and the green shaded area that corresponds to the right vertical axis is the ratio of the values under FA over the values under IA.

Panels (a), (b) and (c) show how debt constraints interact with household attention. Households with more debt and higher LTV values, who face the highest borrowing constraints, are more responsive to a shock in house prices, increasing their consumption more than others. The owners with high levels of debt stand to gain the least by paying full attention to house price information, implying that they already are attentive in the baseline scenario. This is as expected, since they are subject to the highest borrowing constraints. In terms of debt capacity, households at the lower and higher end of the distribution have lower MPCH values and stand to gain the most by being attentive to house price changes. Both these groups are less incentivized to pay attention: households with low debt capacity know that it is unlikely that they will be able to increase their borrowing or adjust their housing, and households with high debt capacity are further from their debt constraints and therefore can afford to be slower in their reactions.

Panel (d) presents the housing wealth effect at different age groups. We note that households have the highest values of MPCH around middle-age, when most have accumulated enough liquid wealth in order to be able to plan for purchasing a house. The housing wealth effect decreases for young and old households, but with an important difference. Young households appear to be highly attentive to house price information, with the ratio between average MPCH under the FA and IA case close to 1. On the contrary, FA/IA for old households is high, at approximately 1.8, indicating that they consume less than they would have if their attention levels were higher.

Finally in this section, Panels (e) and (f) show how attention interacts with liquidity constraints, and impacts the housing wealth effect. In terms of liquid wealth over income, most responsive are owners at the lower end to the middle of the distribution. These owners are the more liquidity constrained, and are more sensitive to an increase in house prices through which they can increase their consumption. Households with high liquid wealth/income ratios are not liquidity constrained, and less responsive. Note that the households with the lowest liquid wealth/income ratios are also less responsive, driven by their inattention. The FA/IA ratio is very high for them, implying whatever cost is associated with observing house price information is not affordable for these low

liquidity households. Otherwise, the attention impact on consumption is approximately the same along the distribution, exhibiting a gradual decrease with liquid wealth/income, again in alignment to households' decrease in liquidity constraints. In terms of the ratio of liquid wealth to housing wealth, the impact of attention is very similar to that observed in Panel (e). However, the households with the highest housing wealth effect are the ones with higher liquid/housing wealth values. As these households have lower housing wealth relative to their liquid wealth, they are able to consider adjusting their housing and therefore will respond more to house price shocks and will be more attentive to house price information.

Throughout these cross-sections, the predicted MPCH in the counterfactual, full-attention case are elevated, and higher than housing wealth effect values typically seen in the literature. As detailed in sections 3 and 4, the baseline model and its preference parameters has been calibrated to match moments in the SCF data, and the model explicitly incorporates households' inattention to house price information. Therefore, in the baseline estimates for the housing wealth effect, household preferences and households' attention determine the MPCH values. The simulation of the full attention counterfactual uses the same preferences parameters, but ignores the potential impact of households' attention levels. As a result, these high values of the housing wealth effect predicted under the FA counterfactual again highlight the significance of incorporating attention in a life-cycle model in this setting and serve for us to quantify the attention consumption wedge.

6.3 Attention Impact Decomposition and Policy Implications

In the previous section, the ratio of MPCH assuming full attention over the baseline inattention setup captures the consumption impact of households' attention. As outlined in section 2, households face observation cost $\kappa^O P_j^H h$ when they pay attention to house price information. The tradeoff between what they may gain by updating their house price perceptions and this observation cost drives their attention decisions, and the model predicts that household attention to

house price information is state-dependent for households planning to adjust their housing in the future (Prediction 2). This prediction was also validated in the empirical data in section 2, where households appear to follow the optimal attention behavior predicted by the model. The properties of attention are different for households who are planning to remain owners or renters, where attention does not vary by wealth/housing, but follows a time-dependent rule.

In addition, whether the probability of attention for households that plan to adjust their housing reaches full attention will depend on the relative value of the observation cost $\kappa^O P_j^H h$ and the benefit of house price information observation. From a policy perspective, if the observation cost could be decreased, households' responsiveness to house price changes could be increased, therefore reducing the overall consumption wedge caused by inattention. To identify this 'reducible' component of the consumption wedge, I can run a counterfactual simulation, where all households plan to buy a house, or are certain future adjusters. The residual distance between the predicted MPCH under this simulation, and the simulation assuming full attention would then quantify the 'reducible' component of the consumption wedge.

In order to determine the 'reducible' component of the total difference between the MPCH estimates under FA and the corresponding estimates under IA, I simulate the model under the 'Plan to Move' counterfactual. In this counterfactual, the attention probabilities used to calibrate the non-parametric attention function are based on households that plan to move. Specifically, I leverage the SCE housing module that asks households what the percent chance is that over the next 12 months they will move to a different primary residence. I can then run the same specification used to calibrate the baseline model, Equation 24, with an additional dependent variable termed 'Plan to Move' that indicates the self-reported probability of moving over the next year.

$$y_{i,t} = \gamma_0 + \gamma_1 \text{Plan to Move} + \gamma_2 m_i^Q + \gamma_3 h_i^Q + \gamma_4 age_i^Q + \gamma_5 \tilde{P}_i^Q + \gamma_6 \tilde{\phi}_i^Q + \boldsymbol{\gamma_7}' X_i + \boldsymbol{\gamma_8}' W_i + \xi_j + \zeta_t + \epsilon_{j,t} \quad (32)$$

For the calibration, I follow the same approach as before, now also setting Plan to Move = 100% for the estimated probabilities. Figures 15 and 16 plot the average MPCH under the IA, FA

and the 'Plan to Move' model setups along household state variables and along the measures of constraints used in subsection 6.2. The figures show that, as expected, households' consumption is overall more responsive to a shock in house prices when they plan to move houses compared to under the baseline model. However, it is also evident that there is a wedge between how households would consume out of house price changes if they were fully attentive, compared to how they would consume if they all planned to move within the next year. This wedge, depicted in gray in the graphs, quantifies the 'reducible' error along each of the depicted distributions. Policy makers can use such simulations to determine how the possible range of MPCH values would change if their observation costs were eliminated. The shaded area in green, or the distance between the estimated MPCH under the IA and 'Plan to Move' scenarios cannot be directly targeted as it is driven by the change in the perceived benefit of being attentive to house price information by households on average and only by households who plan to adjust their housing in the near term.

7 Cyclical MPCH and Cyclical Attention Consumption Wedges

The model applications in section 6 showed that the MPCH and its heterogeneity along the cross-section of households is driven by the interaction of households' attention to house price information with households' distance from their financial constraints. However, these findings have broader implications for the role of inattention in the consumption of households after unexpected changes in house prices, when these changes occur under different economic conditions. At different points in the business cycle, such as during an expansion or a contraction, many households move further away or closer to their financial constraints than before, and will adapt their attention to house prices accordingly. This would result in the impact of attention to house prices to depend on the state of the economy and co-move with the business cycle.

Note that the state-dependent nature of attention has been discussed in the literature in separate settings to the one in this paper. Coibion and Gorodnichenko (2015), Gorodnichenko (2008) and

Mackowiak and Wiederholt (2012) show that agents' implied sensitivity to information rigidity varies with business cycle conditions or shocks to the aggregate economy. Additionally, a subset of papers in the housing wealth effect literature have presented empirical evidence on the time variation of the MPCH (Calomiris et al. (2013) and Guren et al. (2020)), and Guerrieri and Iacoviello (2017) have estimated a DSGE model that produces consumption elasticities to housing wealth changes that are asymmetrical over an economic activity boom and bust. Regarding the housing wealth effect literature in particular, the magnitude of MPCH estimates vary significantly and hence the size of this effect remains an open question. Therefore, I apply my model in order to determine whether incorporating the role of attention in predicting the housing wealth effect under different economic conditions can provide an explanation to the varying estimates of MPCH across papers.

7.1 MPCH and Attention Consumption Wedges under Different Economic Conditions

I run a series of model experiments where I shock house prices under different economic conditions. I begin by simulating the baseline economy for 190 periods, and then model a business cycle boom, followed by a business cycle bust. Specifically, the economy goes through a period of growth for 5 periods, whereby aggregate income is subject to large and positive permanent productivity shocks. In the next 5 periods, the economy contracts, with aggregate income subject to large and negative permanent productivity shocks. These shocks impact households through their income, as well as through the component of national house prices that is correlated to aggregate productivity. These shocks also impact households' attention to house prices, as the probability of them updating their perception of house prices and house price growth is a function of the level of perceived house prices and their income. For each year during the boom and bust period in the simulation, I run a separate experiment, where I shock house prices by +10%. Therefore, I end up with 10 separate experiments, each giving an estimate for the MPCH of households under different economic conditions.

Figure 17 presents the results of the experiment with respect to the housing wealth effect. Panel (a) shows the MPCH of each experiment with the solid line, as well as the baseline estimate of the average MPCH before the boom and bust, depicted using the dashed line. The MPCH decreases during the periods of expansion, as households see both their incomes and house prices rising, and are less attentive. This behavior is to be expected based on Figure 15, where households during higher house prices have lower MPCH values and are less attentive. The response reverses when entering the bust, where now households' incomes and house prices decrease, and their attention to house prices increases. In addition to this pattern being driven by decreasing incomes and house prices, households' LTV will also be impacted, which further increases owners' attention. Note that at the later stages of the simulated bust, the average value of MPCH is very elevated, at approximately 25 to 35 cents. This magnitude agrees with the average MPCH that the model predicts for highly debt constrained households under full attention, as was discussed in subsection 6.2. During these periods, household attention to house price information is very high, and as this attention interacts with households' financial constraints, the average MPCH rises. The next subsection will discuss further household attention during these later stages of the bust. Panel (b) presents the attention consumption wedge, the average difference between the MPCH estimate under the baseline scenario and the MPCH estimate produced by running the counterfactual full attention scenario, for each experiment. Note that the graph highlights the changing impact of attention on the housing wealth effect.

7.2 Decomposition Under Different Economic Conditions

The attention consumption wedge is examined in further detail, in order to determine what drives the change in the average MPCH estimated for each of the experiments ran. Panel (a) of Figure 18 plots the average house price perception error by experiment, and therefore summarizes the variation of attention over time. These house price perception errors are belief wedges $\Delta P^H_{i,j,t}$, where $\Delta P^H_{i,j,t} = \tilde{P}^{H,IA}_{i,j,t} - P^{H,FA}_{i,j,t}$. Households start off with a small average perception error, with

some households being attentive to house price information and others not. As the economy enters and stays in the boom regime for longer, the magnitude of the average household $\Delta P^H_{i,j,t}$ increases significantly. During these periods, more households are moving further away from their financial constraints, which results in them becoming more inattentive. In addition to this, as the economy continues to expand, house prices increase further, making houses more expensive and housing transaction costs higher, also decreasing the probability of adjusting housing and paying attention to house price information. As the economy starts to contract, households adjust their house price perceptions accordingly. In the beginning of the bust regime, they still perceive house prices to be slightly higher than they actually are, but quickly adjust their beliefs. Later during the contraction of the economy, they hold accurate perceptions about current house prices. During these periods, more households approach their financial constraints and as a result become more attentive to house price information.

Panel (b) decomposes the attention consumption wedge plotted in Figure 17 by using the same type of counterfactual simulations as in subsection 6.3. This figure reveals a number of different drivers behind the estimated changes in average MPCH. During the boom, if all households paid as much attention as those who have housing adjustment plans, the average MPCH would be significantly higher. However, it would still be lower than the counterfactual full attention case. This is because by the fifth period of the expansion, most households that would like to adjust their housing have already done so. These households are now in their inaction region, and also are less attentive. At the same time, at this point in the expansion, households are further away from their financial constraints, again leading them to pay less attention. But under the FA setup, these households could have taken advantage of the appreciated value of their house, borrowed more against this collateral and increased their consumption. As a result of their inattention, their consumption wedge reaches its most negative point among the experiments that were ran at the end of the boom. Moving into the bust regime, households consume more under the baseline simulation, than what they would have in the FA and 'Plan to Move' counterfactuals. On one hand, at this

point some households have not updated their house price perceptions to reflect the drop in house prices. On the other hand, as inattentive households did not optimally increase their consumption enough during the expansion, now they are less able to immediately adjust their spending. For the last two periods of the contraction, households could consume more if their attention was higher. In Figure 17 Panel (a), we saw that during this time the MPCH appears to remain within the 25-35 cent range. As household defaults increase, households are limited with respect to how much they can change their consumption, even after a positive house price shock. So although households now have accurate house price beliefs, they cannot afford the transaction costs involved in downsizing their house, which could relieve them from defaulting, due to their reduced liquidity. In the FA counterfactual, and to a smaller degree in the 'Plan to Move' counterfactual, households would have been able to smooth their consumption more efficiently over time. This would have left them in a better liquidity position during the contraction and thus able to consume more.

These results emphasize the significance of households' attention to house price information for determining the housing wealth effect. As a key puzzle in the housing wealth effect literature is the large range of estimates for the magnitude of the effect, my model provides an explanation: attention to house price information has significant implications for the estimation of the magnitude of the housing wealth effect. Attention interacts with households' financial constraints, such that the impact of attention on the MPCH co-moves with the business cycle. These results highlight the need to incorporate households' attention to house prices in analyses that aim to determine the housing wealth effect.

8 Conclusion

I examine the impact of households' inattention to house price information on their financial decisions, and in particular on the housing wealth effect, or how households change their consumption after a change in house prices. The existing literature on the housing wealth effect highlights the

heterogeneity in the magnitude of the effect between households and finds significantly different average values by paper. The results of this paper reveal that attention gives rise to a separate mechanism to the ones documented in the literature that leads to the heterogeneity in MPCH along the cross-section of households. Importantly, the paper also provides an explanation as to why estimates of the average MPCH vary in the literature: that households' changing attention levels lead to cyclical average MPCH values. I find that the key behind these results is the interaction of households' attention to house price information with their exposure to financial constraints.

Furthermore, this paper reveals that households' attention to house price information varies, but their attention agrees with the theoretical predictions of a stylized model of optimal attention. By incorporating state- and time-dependent attention rules in a quantitative model, an additional driver behind the heterogeneity in perceptions emerges. Because these beliefs will determine financial decisions, the impact of attention can be decomposed to provide a new explanation for the variation in these decisions between households and over time. While this paper examines house prices and consumption, future work can examine attention with respect to other observed household financial decisions that deviate from the predictions of a full-information setting.

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Tables

Table 1: Model Parameters

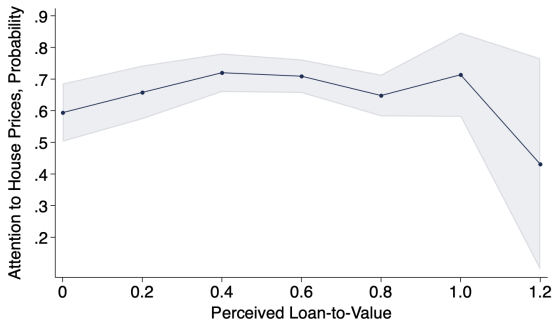
Parameter	Description	Value	Calibration Source/Method
First Stage Calibration			
γ_{age}	Age-Specific Growth Factor		Cocco et al. (2005)
σ^{Ψ}	St. Dev. Log Permanent Aggregate Productivity	0.006	Carroll et al. (2020)
σ^{Θ}	St. Dev. Log Transitory Aggregate Productivity	0.003	Carroll et al. (2020)
σ^{ψ}_{age}	Age-Specific St. Dev. Log Permanent Idiosyncratic Productivity		Sabelhaus and Song (2010)
σ^{θ}_{age}	Age-Specific St. Dev. Log Transitory Idiosyncratic Productivity		Sabelhaus and Song (2010)
a	Steady-State Capital's Share of Income	0.36	Carroll et al. (2020)
ω	Steady-State Compensation of One Unit of Labor	0.64	Carroll et al. (2020)
Γ^H	Aggregate Real House Price Growth Rate	3.3%	FHFA HPI All-Transactions US Index
$\phi^H_{j,low}$	Low Additional Growth State	-3.53%	FHFA HPI All-Transactions Metro Index
$\phi^H_{j,no}$	No Additional Growth State	0.00	FHFA HPI All-Transactions Metro Index
$\phi^H_{j,high}$	High Additional Growth State	4.04%	FHFA HPI All-Transactions Metro Index
$\text{prob}^H_{j,low}$	Unconditional Low State Probability	24.6%	FHFA HPI All-Transactions Metro Index
$\text{prob}^H_{j,no}$	Unconditional No Growth State Probability	58.0%	FHFA HPI All-Transactions Metro Index
$\text{prob}^H_{j,high}$	Unconditional High State Probability	17.4%	FHFA HPI All-Transactions Metro Index
r^{save}	Saving Interest Rate	1%	Federal Funds Effective Rate
r^{borrow}	Borrowing Interest Rate	12%	Commercial Bank Interest Rate on Credit Card Plans
ξ	Housing Depreciation Rate	3.6%	IRS Guidelines
$\bar{\kappa}^H$	Housing Adjustment Cost	5%	Berger et al. (2017)
$\bar{R}$	Rent-to-Price Ratio	0.21	Demers and Eisfeldt (2022)
$1-\theta$	Loan-to-Value Ratio	80%	Chen et al. (2020)
σ	Debt-to-Income Ratio	3.5	Chen et al. (2020)
Second Stage Calibration			
ρ	CRRA	5.35	Matched SCF Data Moments
ρ^B	CRRA for Bequest Utility	5.3	Matched SCF Data Moments
α	Non-Housing Share of Consumption	0.9	Matched SCF Data Moments
Ψ	Strength of Warm-Glow Bequest Motive	98	Matched SCF Data Moments
β	Discount Factor	0.95	Matched SCF Data Moments

Note: The table presents the calibrated parameters used to solve the model. The 'First Stage Calibration' section includes parameter values calibrated based on economic data or other papers in the literature. The 'Second Stage Calibration' section of the table includes the parameters calibrated within the model. The values were chosen so that model moments match the corresponding data moments for home ownership, housing wealth/income and normalized liquid assets/income and debt/housing wealth over the life cycle, as shown in Figure 9.

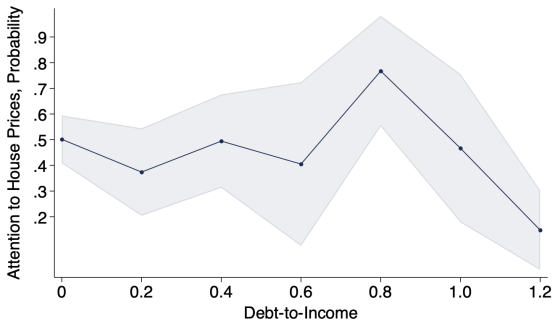
Figures

Figure 1: Attention to House Price Information and Borrowing Constraints

(a) Owner



(b) Renter



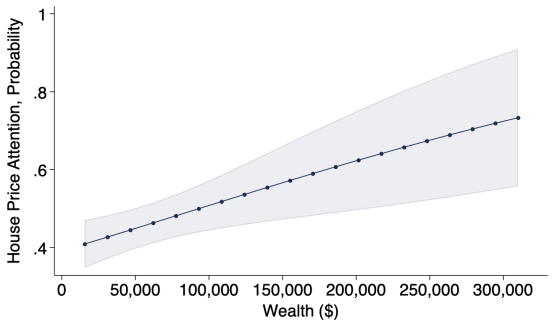
Note: The figure presents empirical evidence that confirms the first prediction of the stylized model of optimal attention presented in section 2, that attention to house price information interacts with households' perceived current borrowing constraints. Specifically, Panel (a) plots the predicted average probability of owners' attention to house price information against their perceived LTV. Panel (b) plots the predicted average probability of renters' attention to house price information against renters' DTI. The estimates are attained by running specification 1 using data from the SCE. Confidence intervals for 95% statistical significance.

Figure 2: Attention to House Price Information and Liquid Wealth/Housing

(a) Owner



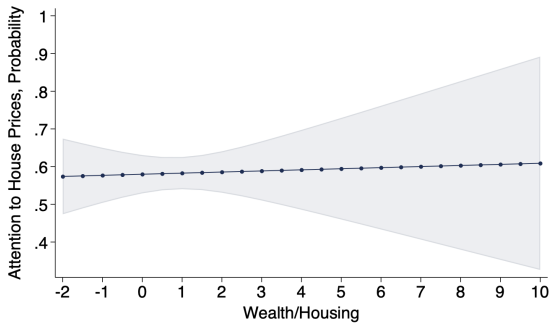
(b) Renter



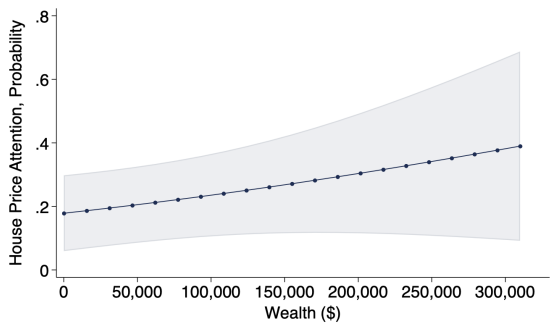
Note: The figure presents empirical evidence that confirms the second prediction of the stylized model of optimal attention presented in section 2, that attention to house price information varies with households' wealth/housing ratio. Specifically, Panel (a) plots the empirically-estimated probability that owners pay attention to house price information against their liquid wealth over perceived housing, and Panel (b) plots the empirically-estimated probability that renters pay attention to house price information against their liquid wealth. The estimates are attained by running specification 2 using data from the SCE. Confidence intervals for 95% statistical significance.

Figure 3: Attention to House Price Information and Liquid Wealth/Housing, Households without Housing Adjustment Plans

(a) Owner



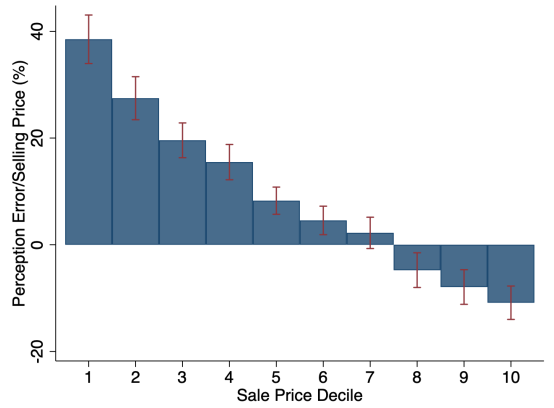
(b) Renter



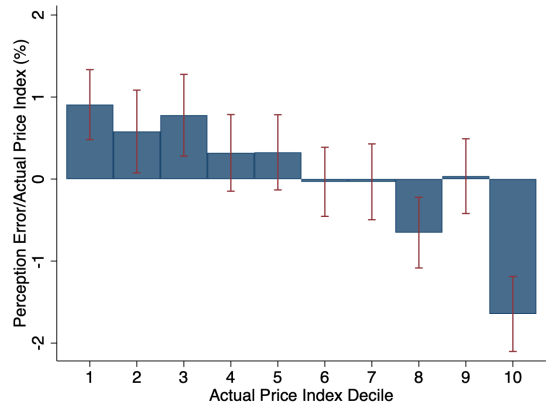
Note: The figure presents empirical evidence that confirms the third prediction of the stylized model presented in section 3, that attention to house price information does not vary for different values of the household's wealth over housing when households are not looking to adjust their housing. Specifically, Panel (a) plots the empirically-estimated probability that owners who have no plan to adjust their housing pay attention to house price information against their liquid wealth over perceived housing wealth, and Panel (b) plots the empirically-estimated probability that renters who have no plan to move pay attention to house price information against their liquid wealth. The estimates are attained by running specification 2 using data from the SCE. Confidence intervals for 95% statistical significance.

Figure 4: Perceived vs. Actual House Prices

(a) HRS

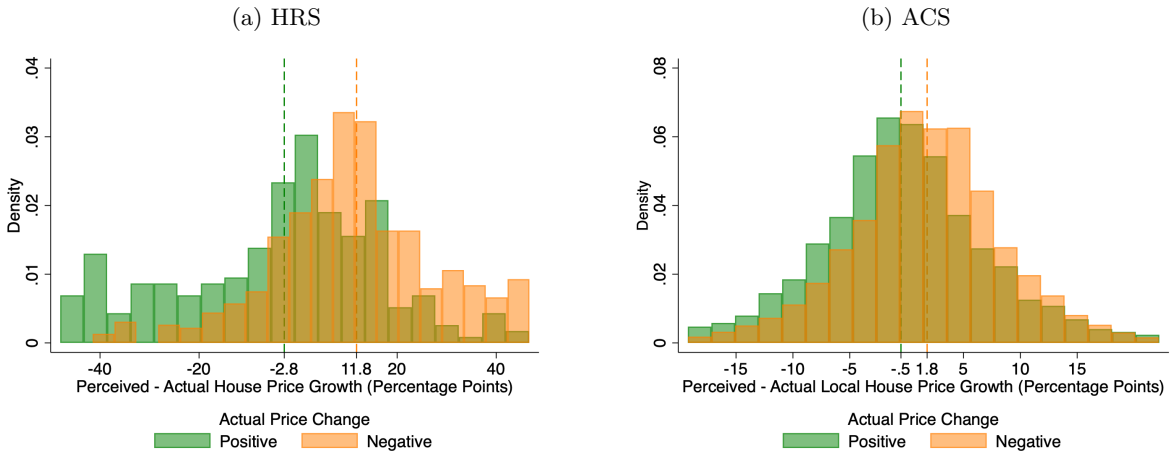


(b) ACS



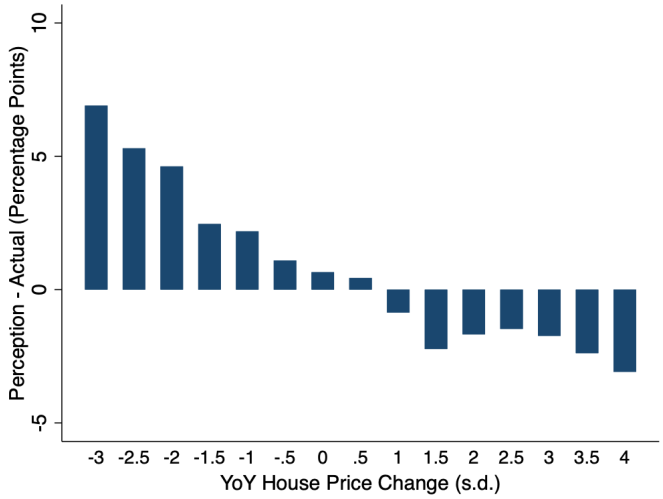
Note: The figure presents the relative perception error households make about the value of their house, plotted against the year-specific distribution of actual house prices. Specifically, Panel (a) shows the relative perception error of HRS respondents whose house is subsequently sold. Panel (b) uses ACS sample micro data to show the difference between weighted average perceived house value by county from the ACS and the FHFA county average house price, divided by the FHFA value. The perception error that each bin plots is the average error by selling price decile for Panel (a) and by FHFA county-level value decile for Panel (b). Confidence intervals for 95% statistical significance.

Figure 5: Perceived vs. Actual House Price Growth



Note: The figure presents the distribution of relative perception errors households make about the growth of the value of their house. Specifically, Panel (a) shows the difference between perceived and actual house price growth since the original purchase date of each home in the HRS, separately for when the actual house price change is positive and when the change is negative. Panel (b) plots the corresponding data in terms of annual house price growth, comparing self-reported values from the ACS and the actual counterpart from the FHFA.

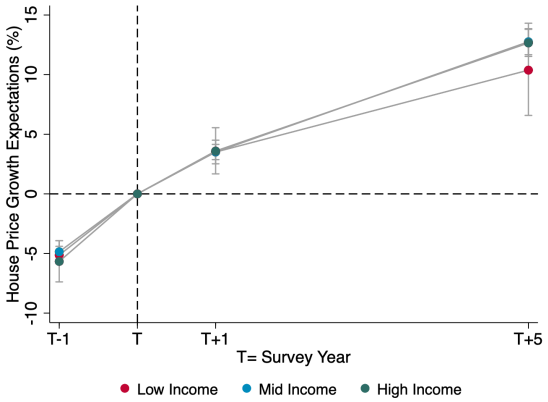
Figure 6: House Price Growth Perception Error by Likelihood of Actual House Price Growth



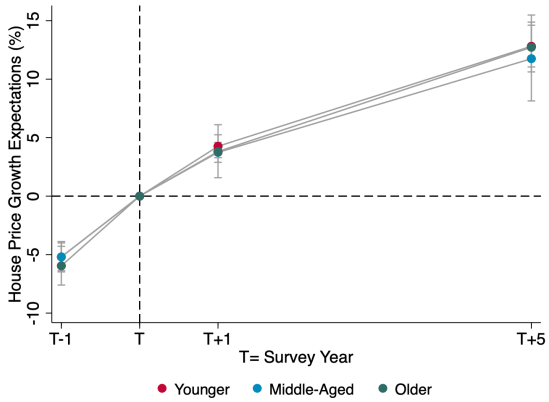
Note: The figure plots empirical evidence showing that household' perceptions of house price changes are 'sticky'. I create a measure of house price change likelihood, by calculating the standard deviation of house price growth by county in the FHFA data. The x-axis of the figure shows actual house price growth values expressed as multiples of their corresponding historical local standard deviation, grouped at .5 intervals. Each bar is the median difference between perceived and actual growth for each .5 group of the calculated likelihood metric.

Figure 7: House Price Growth Perceptions and Expectations, Owners

(a) By Income



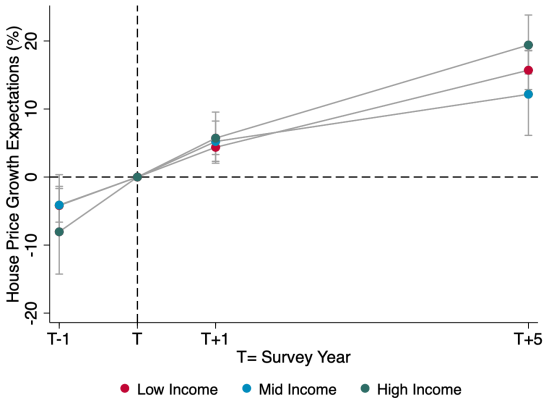
(b) By Age



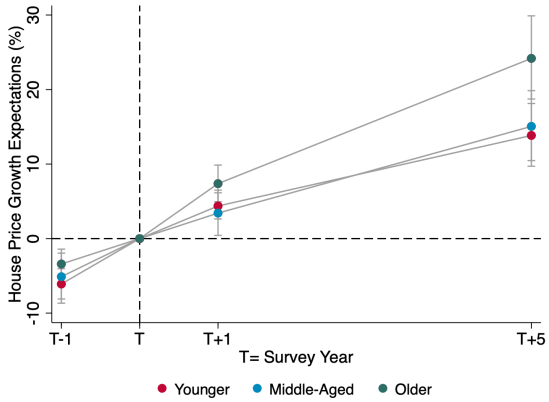
Note: The figure depicts owners' perception of house price growth over the last year, and their expected future house price growth in 1 and 5 years from the time of the survey, T. The figure plots estimated coefficients β_l of specification 22 for households of low, mid and high income and age. The data used for the estimation is SCE, including households' perception of the house price of a typical home in the zip code one year ago, at the time of the survey, as well as each household's expectation about the typical house price one year and five years in the future. Confidence intervals for 95% statistical significance.

Figure 8: House Price Growth Perceptions and Expectations, Renters

(a) By Income

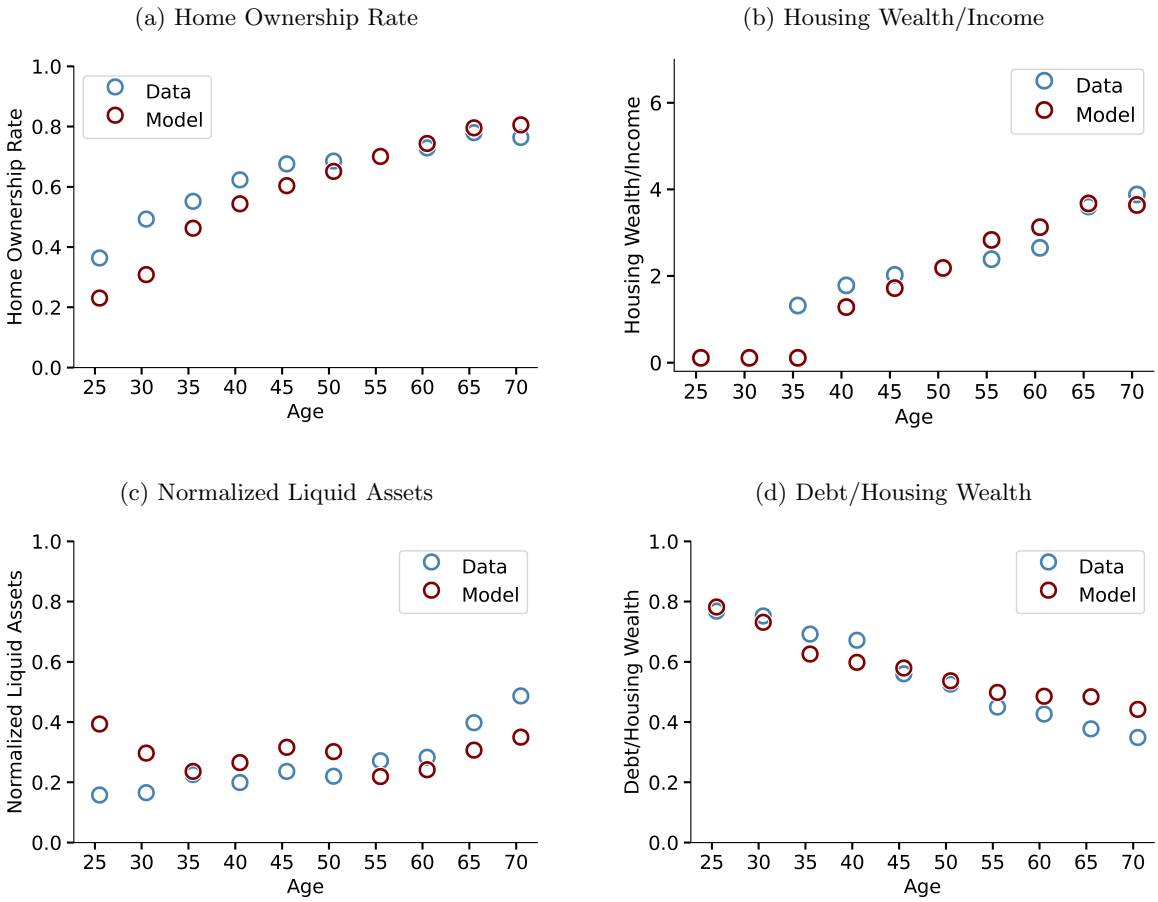


(b) By Age



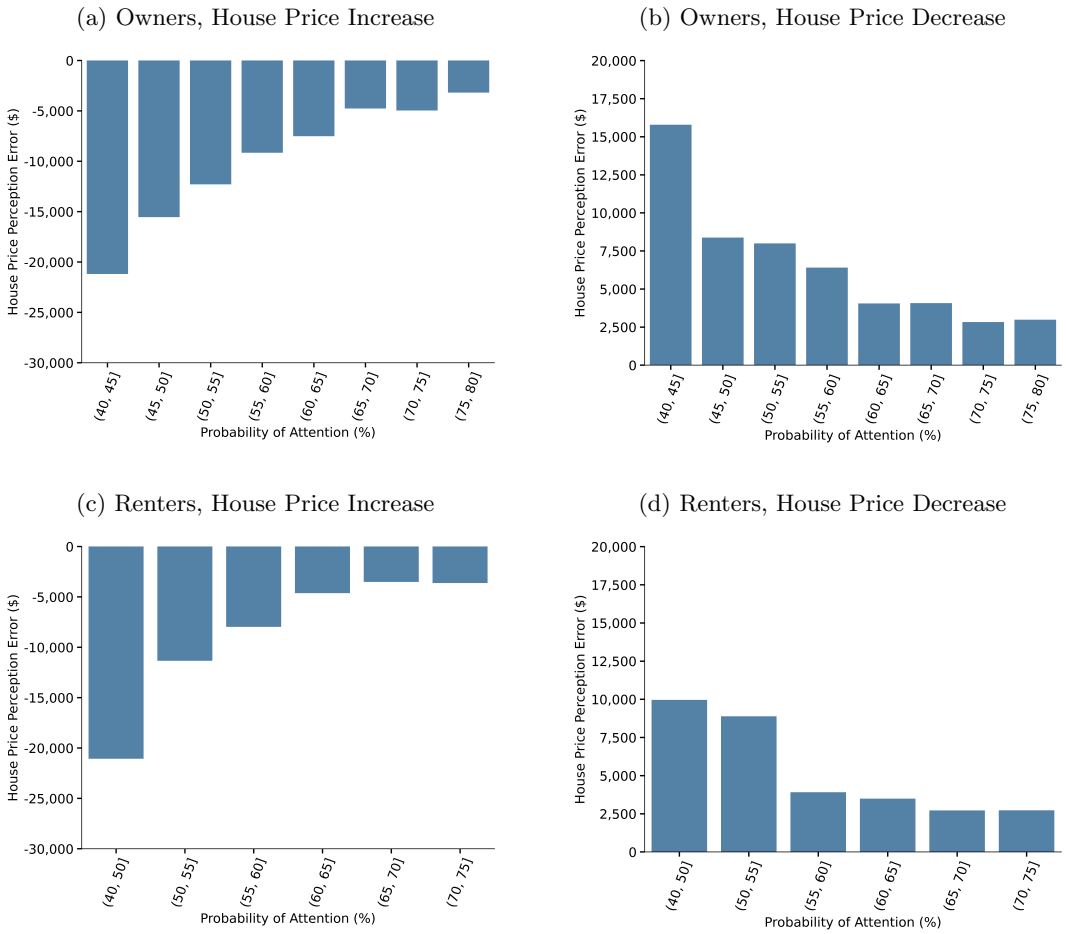
Note: The figure depicts renters' perception of house price growth over the last year, and their expected future house price growth in 1 and 5 years from the time of the survey, T. The figure plots estimated coefficients β_l of specification 22 for households of low, mid and high income and age. The data used for the estimation is SCE, including households' perception of the house price of a typical home in the zip code one year ago, at the time of the survey, as well as each household's expectation about the typical house price one year and five years in the future. Confidence intervals for 95% statistical significance.

Figure 9: Model vs. Data Moments



Note: The figure depicts the model-implied moments and their data moment counterparts used to calibrate the preference parameters of the model, specifically the coefficient of relative risk aversion in the per-period utility and the bequest utility, the non-housing share of consumption, the discount factor and the strength of the warm-glow bequest motive. The moments are the median housing wealth/income, the median normalized liquid wealth for owners, the median debt/housing wealth for owners and the home ownership rate by 5-year age group. The data used is the 2013-2019 Survey of Consumer Finances.

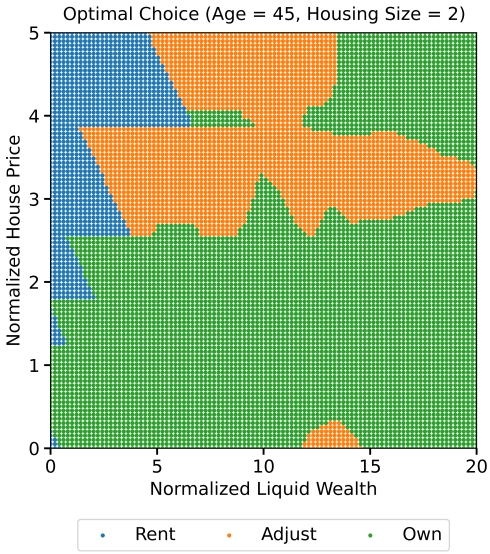
Figure 10: Attention and Belief Wedges



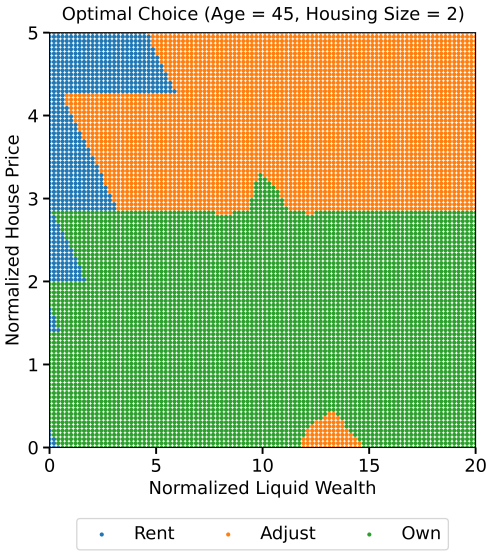
Note: The figure plots the average house price perception error by households' probability of being attentive to house price information. Each bar represents the mean of the household-specific difference between the house price, as perceived by households, minus the actual house price, for households with probability of attention within the 5% range specific to each bar (except for 40 to 50% for renters). The results are presented separately for owners and renters, and for cases when the actual house price has increased or decreased. The results use the baseline simulation of the model.

Figure 11: Attention and Impact on Housing Choice

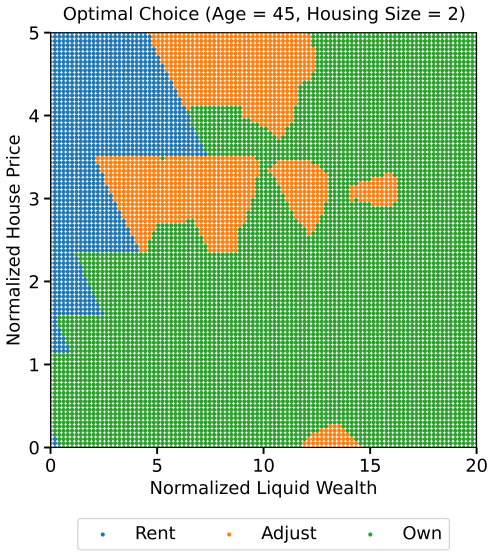
(a) Full Attention



(b) Perceived House Prices Lower

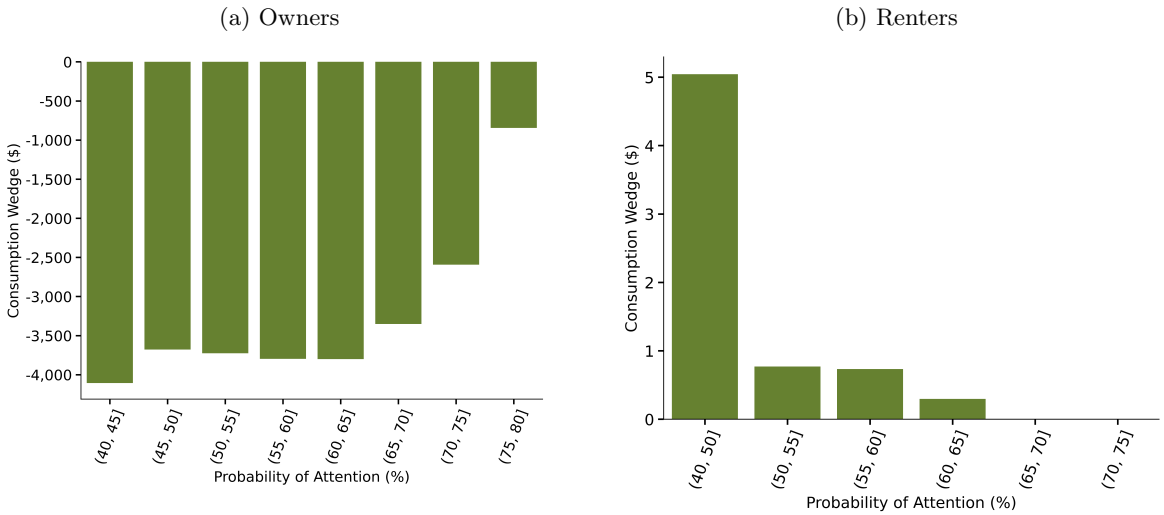


(c) Perceived House Prices Higher



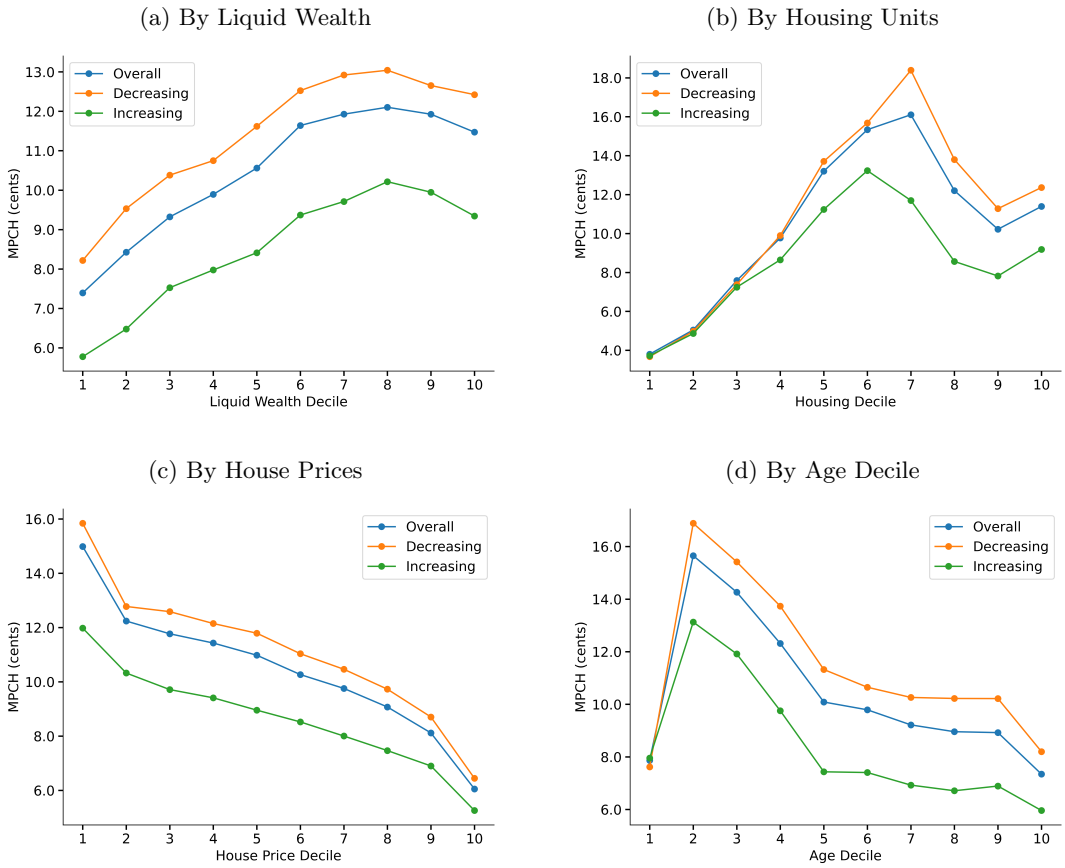
Note: The figure presents the impact of attention to house price information on the housing choice of households. Each panel presents the results for a separate simulation, where in Panel (a), households are attentive to house prices and their perceptions match actual prices, in Panel (b) households perceive house prices to be 10% lower than they actually are and in Panel (c) households perceive house prices to be 10% higher than they actually are. The results are shown for an example of a household that is 45 years old and the last time they observed house prices, they remained steady. For the choice to own, we take households that are considering a house of 2 housing units.

Figure 12: Attention and Consumption Impact, Conditional on Housing Choice



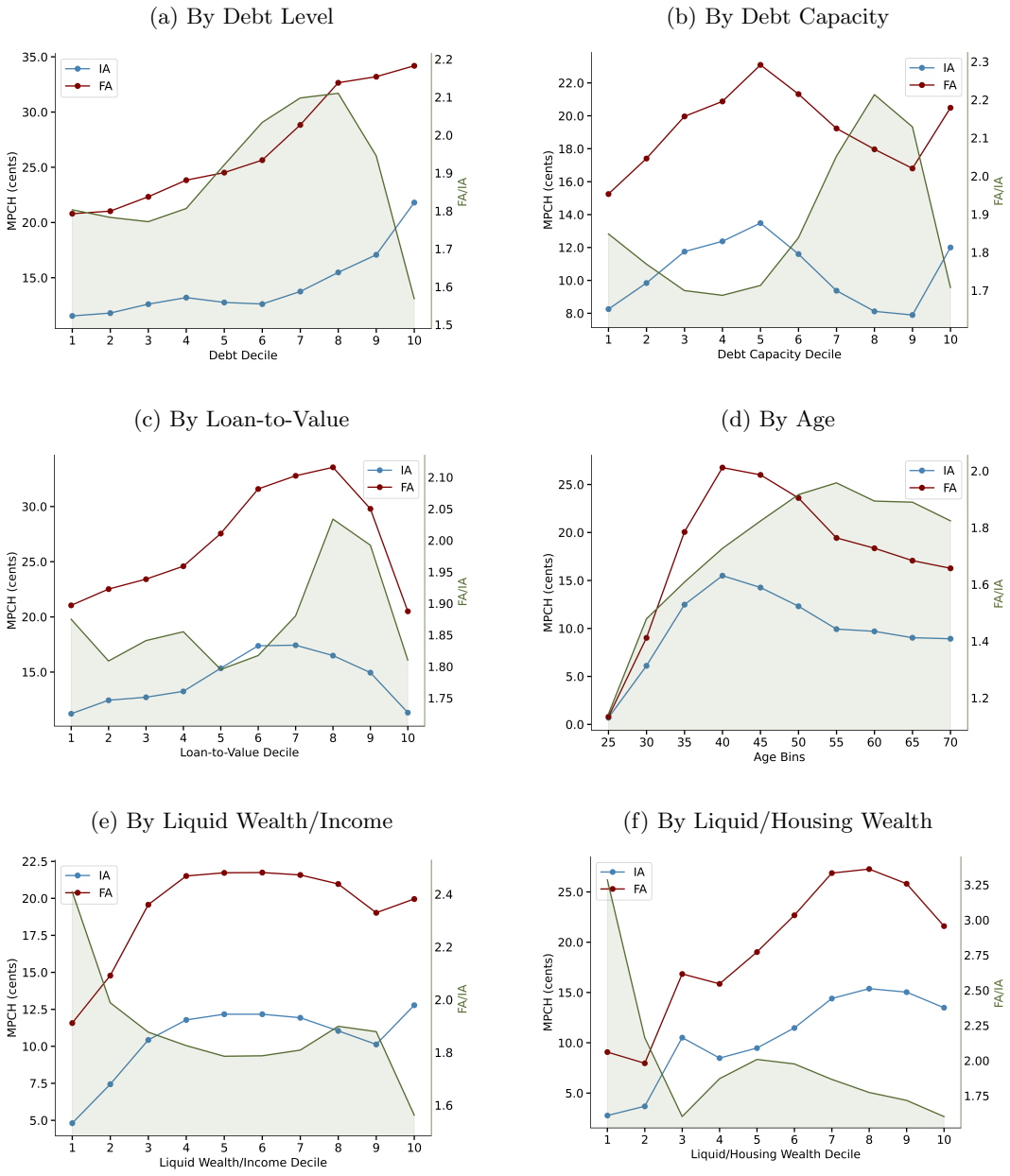
Note: The figure presents the impact of attention on consumption, conditional on households' housing choice. The bars represent the average difference in non-housing consumption over a year after a 10% house price shock, between the baseline simulation and the counterfactual where all households perceive the house price shock, after they have already decided their housing consumption. The average values are for households with probability of attention within the 5% range specific to each bar (except for 40 to 50% for renters). The results are shown separately for owners and renters.

Figure 13: MPCH By Household State Variable



Note: The figure presents the housing wealth effect along households' cross-section. The blue lines in the panels plot the mean MPCH by decile of each of the state variables of the households' problem, as annotated along the x-axis. Panel (a) presents the MPCH along the liquid wealth distribution, Panel (b) along the housing units distribution, Panel (c) along the house price distribution and Panel (d) along the age distribution. The orange and green lines show the average value of MPCH by decile of the state variable for the subset of households that expect local house prices to decrease or increase.

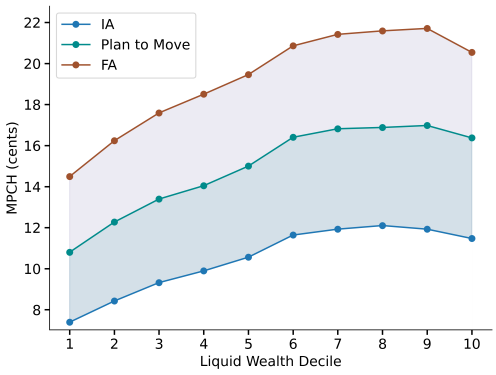
Figure 14: Attention Impact and Constraints



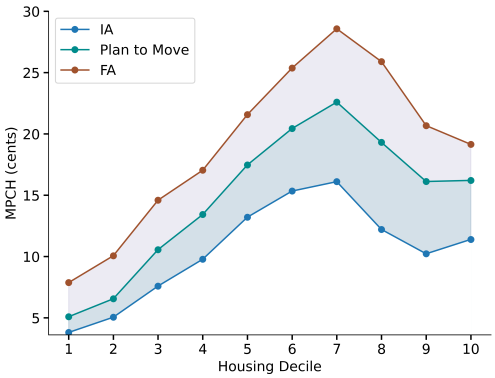
Note: The figure presents how the housing wealth effect interacts with household constraints, including borrowing constraints, constraints related to households' age and liquidity constraints. In the graphs, the blue lines show the average MPCH by decile in the baseline model, the red lines plot the average MPCH by decile in the full attention counterfactual, and the green shaded area that corresponds to the right vertical axis is the ratio of the value under FA over IA. The latter isolates the impact of inattention to house price information on household consumption out of a shock to house prices. Panel (a) presents the results along the debt level distribution, Panel (b) presents the results along the debt capacity distribution, Panel (c) along the LTV distribution, Panel (d) along different ages, Panel (e) along the liquid wealth/income distribution and Panel (f) along the liquid/housing wealth distribution.

Figure 15: Attention Impact Decomposition By Household State Variable

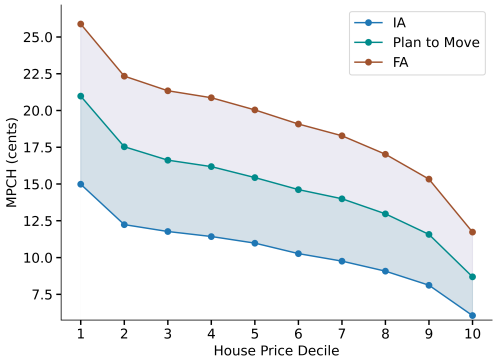
(a) By Liquid Wealth



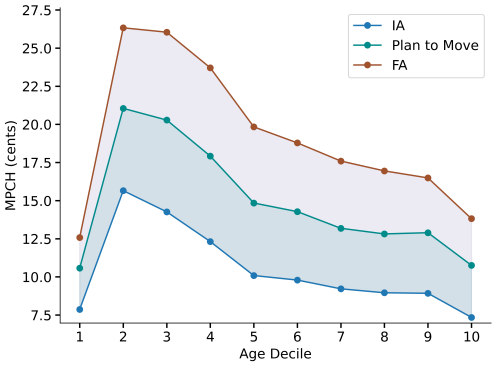
(b) By Housing Units



(c) By House Prices

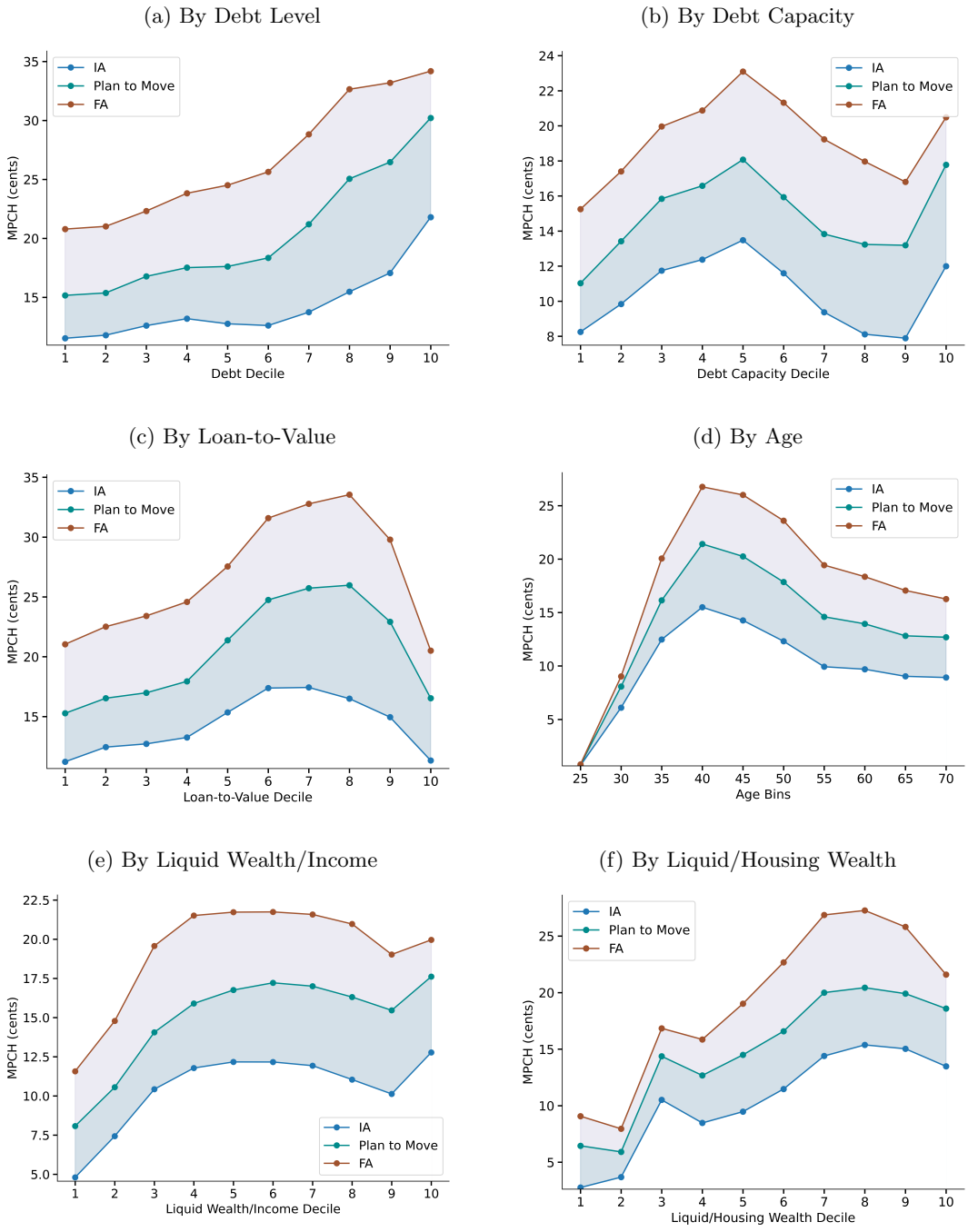


(d) By Age Decile



Note: The figure presents the results of the decomposition of attention consumption wedges, which result from households' inattention to house price information. To decompose the wedges, I run an additional counterfactual simulation, where the non-parametric function governing households' attention to house prices is calibrated using empirical predictions of households' probability of attention only for households that plan to move homes within the next 12 months. In the graphs, the blue line presents the results under the baseline IA simulation, the green line plots the results for the 'Plan to Move' counterfactual, and the red line shows the results for the full attention FA counterfactual. Panel (a) depicts the results along the liquid wealth distribution, Panel (b) along the housing distribution, Panel (c) along the house price distribution and Panel (d) along the age distribution.

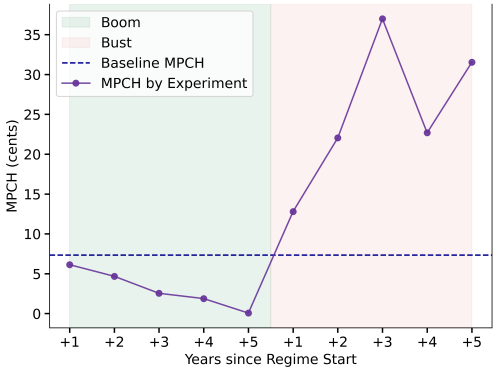
Figure 16: Attention Impact Decomposition and Constraints



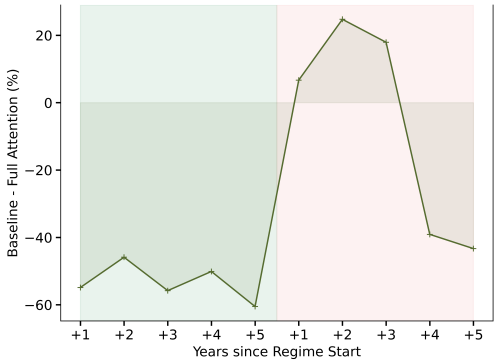
Note: Similarly to Figure 15, this figure presents the results of the decomposition of attention consumption wedges, along the distribution of variables that capture different constraints households are exposed to. Panel (a) depicts the results along the debt distribution, Panel (b) along the debt capacity distribution, Panel (c) along the loan-to-value distribution, Panel (d) along different ages, Panel (e) along the liquid wealth/income distribution and Panel (f) along the liquid/housing wealth distribution.

Figure 17: Cyclical MPCH and Impact of Attention

(a) MPCH



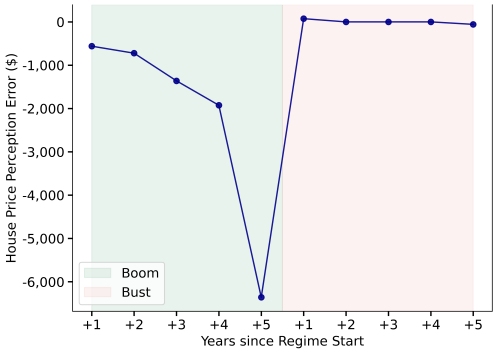
(b) Attention Consumption Wedge



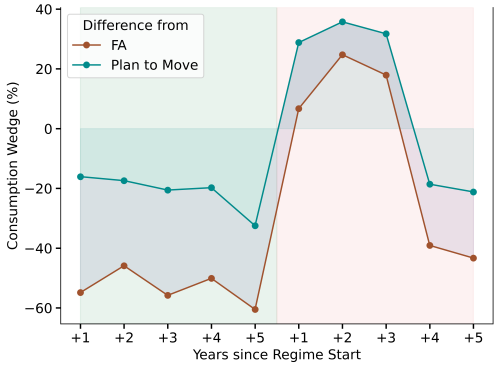
Note: The figure presents the cyclical effect of attention on the average MPCH among households, as predicted by the model. In Panel (a), the solid line shows the average value of MPCH separately for each experiment ran, and the dashed line shows the baseline estimate for owners before the boom and bust periods. The green areas in Panel (b) present the attention consumption wedges, that are calculated by subtracting from the average MPCH value of each experiment the corresponding averages produced by running a counterfactual specification of the model, where all households are fully attentive to house prices and house price changes.

Figure 18: Decomposition Under Different Economic Conditions

(a) House Price Perception Error



(b) Attention Consumption Wedge Decomposition



Note: The figure examines the attention consumption wedge in further detail, in order to determine what drives the change in the average MPCH estimated for each of the experiments ran. Panel (a) plots the average house price perception error by experiment, and therefore summarizes the variation of attention over time. Panel (b) decomposes the attention consumption wedge by running a counterfactual simulation, where the non-parametric function governing households' attention to house prices is calibrated using empirical predictions of households' probability of attention only for households that plan to move homes within the next 12 months.

Internet Appendix:

Household Consumption and Inattention to House Prices

A Additional Data Information

A.1 American Community Survey

The American Community Survey (ACS) sample micro data is a 1-in-100 national random sample of the population, that contains information for at least 2,800,000 survey respondents. For the analysis in the paper, I weigh the data by the field HHWT, which is the household weight indicating how many households in the U.S. population are represented by each data entry, such as to obtain nationally representative results. The main field used is VALUEH, which provides the continuous value of housing units in nominal dollars for the year of the survey. Specifically, the values in the data are the respondents' answer to the following question: "About how much do you think this house and lot, apartment, or mobile home (and lot, if owned) would sell for if it were for sale?". Therefore, this value captures households' perceived value of their own home, and is available at an annual frequency. In the paper, I take the weighted average perceived house price by county for survey data spanning 2005 to 2022. For the perceived house price growth, I use the annual change of the county-weighted average of the perceived house price level from the survey. The VALUEH field in the survey is specific to owners, therefore this perception data refers only to owners and their valuation for the value of their own house.

A.2 Federal Housing Finance Agency

The Federal Housing Finance Agency House Price Index used in this paper is the all-transactions index, which is estimated with prices from sales transactions of mortgage data, as well as prices from appraisal data, both obtained from Fannie Mae or Freddie Mac due to their purchase of the

mortgages collateralized by the valued properties. It is a repeat-sales index measuring changes in single-family home values from all states and more than 400 cities in the U.S.. The FHFA HPI gives the average price changes in repeat sales, refinancings or appraisals of the same properties over time. In the empirical analysis of the paper, I use the county-level HPI as the actual house price changes by county. The FHFA data is particularly suited to this analysis, as the breadth of the housing sample used and the incorporation of transaction data provides the best counterpart to the ACS house value perception data.

A.3 Survey of Consumer Finances

The Survey of Consumer Finances is a triennial cross-sectional survey of households in the U.S., including data on household balance sheets, pensions, income, and demographic characteristics. I use the SCF waves between 2013 and 2019 to calculate the empirical moments used in the estimation of the model parameters, as outlined in Section 3 of the paper. For households' wage income, I add fields X5702, X5704, X5716, X5718, X5720 and X5722, which correspond to the total income from wages and salaries, from a professional practice, business, or farm, from unemployment or worker's compensation, from child support or alimony, from ADC, AFDC, food stamps, or other forms of welfare or assistance such as SSI, or from Social Security or other pensions, annuities, or other disability or retirement programs, all before deductions for taxes and anything else. The value of income then is normalized by the median wage income in the data, calculated by taking into account the survey weights. The liquid wealth before debt deductions is the sum of fields NMMF, the total value of directly held pooled investment funds held by household, STOCKS, the total value of directly held stocks held by household, BOND, the total value of directly held bonds held by household. The total of all types of transactions accounts is adjusted upwards by 5% to account for any cash. The total value of debt is given by field MRTHEL and it the total value of debt secured by the primary residence held by the household, including mortgages, home equity loans and HELOCs secured by the primary residence. The liquid wealth of households is the sum of

liquid wealth, plus the total value of quasi-liquid retirement wealth held by household over 68 using field RETQLIQ. The home value is the sum of the value of real estate owned by the respondent, or the sum of fields HOUSES, the total value of primary residence of household and ORESRE, the total value of other residential real estate held by household. Finally, households are classified as owners if field X701 has value 1, 3, 4, or 5, so if respondents say they own their residence, condo, co-op or townhouse association or are buying/land contract.

A.4 New York Fed Survey of Consumer Expectations

The New York Fed Survey of Consumer Expectations is a survey of U.S. households, including data on consumer expectations and decisions related to inflation, household finance, the labor market, the housing market and other topics. For the analysis in this paper, I leverage the core survey, the housing module and the household finance module. From the core survey, I retain household characteristics on their credit score, gender, ethnicity, race, health state, marital status, risk aversion, employment status, as well as the commuting zone in which they live. The core questionnaire runs monthly, but I keep a single data point for each household and the aforementioned fields. From the housing module that is run annually, one wave is available by household. The information useful to this paper is on households' frequency of consultation of sources of information on the estimated current value of properties in area, their perception of house prices a year ago, five years ago and at the time of the survey, their expectation for future house prices in a year and 5 years from the time of the survey, as well as their confidence level with respect to their answers to what they believe the value of a typical home in their zip code was one and five years ago. In addition, from the housing module I also use data on the respondents' housing, the household's primary residency type and location description. From the household finance module I use data on households' liquid wealth and debt.

A.5 Health and Retirement Study

The Health and Retirement Study is longitudinal survey of 7,700 households in the U.S. headed by older individuals, aiming to collect data on retirement behavior, decisions related to Social Security, as well as the savings and health status of its respondents. The survey includes data on households' perception about how much their house is worth and the actual price at which they sell the home, if and when they do so. As mentioned in the main text of this paper, since survey respondents will report the price at which their home was sold at the next survey wave after the sale, there can be up to a 2-year gap between the perceived and actual price to be sued in the empirical analysis. Therefore to account for this possibility, I regress the number of months between the survey date for the perceived price and the date for the sale, and use the residual in the remaining analysis. The HRS is sponsored by the National Institute on Aging (grant number NIA U01AG009740) and is conducted by the University of Michigan.

B Detailed Proofs

Proof for Lemma 1:

To determine the range of inaction, I reduce the dimensionality of the problem. For owners, the value function can be written in its intensive form, defining a, y and c with respect to the housing choice h, in a similar vein to Stokey (2009). Therefore, $\boldsymbol{a}_{i,t} = \frac{a_{i,t}}{h_{i,t}}$, $\boldsymbol{c}_{i,t} = \frac{c_{i,t}}{h_{i,t}}$, $\boldsymbol{y}_{i,t} = \frac{y_{i,t}}{h_{i,t}}$, $\bar{u}(\boldsymbol{c}_{i,t})$ where $h_{i,t} = 1$ and $v(\boldsymbol{a'}, \boldsymbol{y'}, P_j^{H'}, \phi_j^{H'})$ where $h = 1$. At adjustment, households incur the sum of the transaction and observation costs. The intensive form of the continuation value is:

$$\begin{aligned} V(a', y', h', \tilde{P}_{i,j}^{H'}, \phi_j^H) =& (h')^{1-\rho} V(\boldsymbol{a'}, \boldsymbol{y'}, 1, \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'}) \\ =& \left(\frac{\boldsymbol{a}(\tau') - (\kappa^O + \kappa^T) P_j^H}{\boldsymbol{a'}} \right)^{1-\rho} v(\boldsymbol{a'}, \boldsymbol{y'}, \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'}) \end{aligned}$$

where ρ is the coefficient of relative risk aversion in the per-period utility. Using these definitions, the owner's problem when they decide to adjust their housing can be written as follows:

$$V^{adj|own}(\boldsymbol{a}, \boldsymbol{y}, P_j^H, \phi_j^H) = \max_{\{\boldsymbol{c}_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s \bar{u}(\boldsymbol{c}_{i,s}) + \beta^{\tau'} \left(\frac{\boldsymbol{a}(\tau') - (\kappa^O + \kappa^T) P_j^H}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \boldsymbol{y}', \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'}) \right]$$

$$\begin{aligned}
\text{subject to:} \quad \boldsymbol{a}_{i,t+1} =& (1+r)\boldsymbol{a}_{i,t} + \boldsymbol{y}_{i,t+1} - \boldsymbol{c}_{i,t} - \Xi_{i,t}^H \\
\tilde{P}_{i,j,s+1}^H =& \mathbb{E}[P_{j,s+1}^H | P_j^H, \phi_j^H] = (\phi_j^H)^s P_j^H \Psi_{s+1}^H \quad \text{for } 0 \leq s \leq \tau' - 1 \\
\tilde{\phi}_{i,j}^{H'} =& \phi_j^H \\
\boldsymbol{a}_{i,t} \geq& -(1-\theta)\tilde{P}_{i,j,t}^H
\end{aligned}$$

To continue, we consider the continuous-time equivalent problem:

$$V^{adj|own}(\boldsymbol{a}, P_j^H, \phi_j^H) = \max_{\{\boldsymbol{c}_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\int_{s=0}^{\tau'} e^{-\delta t} \bar{u}(\boldsymbol{c}_{i,t}) \mathrm{d}t + e^{-\delta \tau'} \left(\frac{\boldsymbol{a}(\tau') - (\kappa^O + \kappa^T) P_j^H}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'}) \right]$$

$$\begin{aligned}
\text{subject to:} \quad \mathrm{d}\boldsymbol{a}_{i,t} =& [r\boldsymbol{a}_{i,t} - \boldsymbol{c}_{i,t} - \Xi_{i,t}^H]\mathrm{d}t + \frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T \\
d\tilde{P}_{i,j,t+1}^H =& \tilde{P}_{i,j,t}^H [\ln \phi_j^H \mathrm{d}t + \sigma^{P^H} \mathrm{d}z_t^{P^H}] \\
\tilde{\phi}_{i,j}^{H'} =& \phi_j^H \\
\boldsymbol{a}_{i,t} \geq& -(1-\theta)\tilde{P}_{i,j,t}^H
\end{aligned}$$

As we see above, $\boldsymbol{a}$ is a sufficient statistic to capture the evolution of liquid wealth in continuous time, so there is no need to keep $\boldsymbol{y}$ as a state variable while the proof continues. For the law of motion of $\boldsymbol{a}$, the present value of expected future income y includes the transitory income component, that follows a drift-less Brownian motion with standard deviation σ^P as well as the permanent income component, that follows a continuous time AR(1), or an Orstein–Uhlenbeck process, with mean reversion parameter ψ and standard deviation σ^T.

Similarly to the owner case, the renters' value function can be written in its intensive form,

defining a, y and c with respect to the housing choice h, as follows:

$$\begin{aligned} V(a', y', h', \tilde{P}^{H'}_{i,j}, \tilde{\phi}^{H'}_{i,j}) =& (h')^{1-\rho} V(\boldsymbol{a}', \boldsymbol{y}', 1, \tilde{P}^{H'}_{i,j}, \tilde{\phi}^{H'}_{i,j}) \\ =& \left(\frac{\boldsymbol{a}(\tau') - (\kappa^O + \kappa^T) P^H_j}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \boldsymbol{y}', \tilde{P}^{H'}_{i,j}, \tilde{\phi}^{H'}_{i,j}) \end{aligned}$$

When renters decide to adjust their housing:

$$V^{adj|rent}(\boldsymbol{a}, \boldsymbol{y}, P^H_j, \phi^H_j) = \max_{\{c_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s \bar{u}(\boldsymbol{c}_{i,s}) + \beta^{\tau'} \left(\frac{\boldsymbol{a}(\tau') - (\kappa^O + \kappa^T) P^H_j}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \boldsymbol{y}', \tilde{P}^{H'}_{i,j}, \tilde{\phi}^{H'}_{i,j}) \right]$$

$$\begin{aligned} \text{subject to:} \quad \boldsymbol{a}_{i,t+1} =& (1+r)\boldsymbol{a}_{i,t} - \boldsymbol{c}_{i,t} - R^H_{i,t} \\ \tilde{P}^H_{i,j,s+1} =& \mathbb{E}[P^H_{j,s+1} | P^H_j, \phi^H_j] = (\phi^H_j)^s P^H_j \Psi^H_{s+1} \quad \text{for } 0 \le s \le \tau' - 1 \\ \tilde{\phi}^{H'}_{i,j} =& \phi^H_j \\ \boldsymbol{a}_{i,t} \ge& -\sigma \boldsymbol{y}_{i,t} \end{aligned}$$

For the continuous-time equivalent problem:

$$\begin{aligned} & V^{adj|rent}(\boldsymbol{a}, \boldsymbol{y}, P^H_j, \phi^H_j) = \\ & \quad \max_{\{c_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\int_{s=0}^{\tau'} e^{-\delta t} \bar{u}(\boldsymbol{c}_{i,t}) \mathrm{d}t + e^{-\delta \tau'} \left(\frac{\boldsymbol{a}(\tau') - (\kappa^O + \kappa^T) P^H_j}{\boldsymbol{a}'} \right)^{1-\rho} V(\boldsymbol{a}', \boldsymbol{y}', \tilde{P}^{H'}_{i,j}, \tilde{\phi}^{H'}_{i,j}) \right] \\ & \text{subject to:} \quad \mathrm{d}\boldsymbol{a}_{i,t} = [r\boldsymbol{a}_{i,t} - \boldsymbol{c}_{i,t} - R^H_{i,t}]\mathrm{d}t + \frac{\sigma^P}{r} \mathrm{d}z^P_t + \frac{\sigma^T}{r+\psi} \mathrm{d}z^T_t \\ & d\tilde{P}^H_{i,j,t+1} = \tilde{P}^H_{i,j,t} [\ln \phi^H_j \mathrm{d}t + \sigma^{P^H} \mathrm{d}z^{P^H}_t] \\ & \tilde{\phi}^{H'}_{i,j} = \phi^H_j \\ & \boldsymbol{a}_{i,t} \ge -\sigma \boldsymbol{y}_{i,t} \end{aligned}$$

From the above Bellman equations of owners and renters, we see that, in the case of the owner, for some current house price level and growth, households will optimally choose a lower and upper threshold b and B. There is no upper bound for the thresholds, but there is a lower bound based on the borrowing constraint $\boldsymbol{a}_{i,t} \ge -(1-\theta)\tilde{P}^H_{i,j,t}$. This implies that the optimal thresholds will be

chosen such that $-(1-\theta)\tilde{P}^H_{i,j,t} \leq b < B < +\infty$. Therefore, the state variable $\boldsymbol{a}$ has an inaction region which is the open interval (b, B). The renter also does not face an upper bound, but the lower bound is again derived from the borrowing constraint, such that $-\sigma \boldsymbol{y}_{i,t} \leq b < B < +\infty$.

Proof for Corollary 1:

As follows directly from Lemma 1, the household's choice of the optimal period until the next observation and adjustment will be such that:

$$\tau'(\boldsymbol{a}, \tilde{P}^H_{i,j}, \tilde{\phi}^H_{i,j}) = \tau'(b, \tilde{P}^H_{i,j}, \tilde{\phi}^H_{i,j}) \wedge \tau'(B, \tilde{P}^H_{i,j}, \tilde{\phi}^H_{i,j}) \tag{33}$$

In other words, the inaction region of state variable $\boldsymbol{a}$ which is the open interval (b, B) can be expressed in terms of time as shown.

Proof for Proposition 1:

As consumption is continuously adjusted within the τ' period, there exists a maximization inside the HJB equation. Define:

$$M^\star = \max_{\boldsymbol{a}'} \frac{v(\boldsymbol{a}', \tilde{P}^{H'}_{i,j}, \tilde{\phi}^{H'}_{i,j})}{\boldsymbol{a}'^{1-\rho}} \tag{34}$$

and let S be the return point, the maximizing value of $\boldsymbol{a}'$. In order to express $M^\star$ with respect to the return point S, I use the value function $v(\cdot)$ for a household with net wealth equal to 1. Then the following holds:

$$v(1, \tilde{P}^H_{i,j}, \tilde{\phi}^H_{i,j}) = S^{-(1-\rho)} v(S, \tilde{P}^H_{i,j}, \tilde{\phi}^H_{i,j})$$

Therefore, $M^\star$ can be expressed with respect to the return point S:

$$M^\star = S^{-(1-\rho)} v(S, \tilde{P}^H_{i,j}, \tilde{\phi}^H_{i,j}) \tag{35}$$

The above expression is the optimized value for this household, after paying the observation and transaction costs and choosing S as its non-housing wealth over housing ratio.

The household immediately observes house price data and adjusts its housing once outside the inaction region, and the value function outside (b, B) has the form:

$$v(\boldsymbol{a}, P_j^H, \phi_j^H) = [\boldsymbol{a} - (\kappa^O + \kappa^T)P_j^H]^{1-\rho} M \text{ where } w \notin (b, B) \tag{36}$$

To characterize the value function v, the policy function c, and the critical points b, S, B, I can leverage the fact that the value function satisfies the following Hamilton-Jacobi-Bellman equation in the inaction region:

$$\begin{aligned}
&\delta v(\boldsymbol{a},P_j^H, \phi_j^H) = \qquad (37)\\
&\max_{c\geq 0, \boldsymbol{a}' \geq -(1-\theta)P_j^H} \frac{\bar{u}(\boldsymbol{c})^{1-\rho}}{1-\rho} + m(\boldsymbol{a}) v_{\boldsymbol{a}}(\boldsymbol{a}, P_j^H, \phi_j^H) + m(P_j^H) v_{P^H}(\boldsymbol{a}, P_j^H, \phi_j^H) + m(\phi_j^H) v_{\phi^H}(\boldsymbol{a}, P_j^H, \phi_j^H)\\
&+ \frac{1}{2} s^2(\boldsymbol{a}) v_{\boldsymbol{aa}}(\boldsymbol{a}, P_j^H, \phi_j^H) + \frac{1}{2} s^2(P_j^H) v_{P^H P^H}(\boldsymbol{a}, P_j^H, \phi_j^H) + \frac{1}{2} s(\boldsymbol{a}) s(P_j^H) v_{\boldsymbol{a} P^H}(\boldsymbol{a}, P_j^H, \phi_j^H)\\
&+ \frac{1}{2} s^2(\phi_j^H) v_{\phi_j^H \phi_j^H}(\boldsymbol{a}, P_j^H, \phi_j^H) + \frac{1}{2} s(\boldsymbol{a}) s(\phi_j^H) v_{\boldsymbol{a}\phi_j^H}(\boldsymbol{a}, P_j^H, \phi_j^H) + \frac{1}{2} s(\phi_j^H) s(P_j^H) v_{\phi_j^H P^H}(\boldsymbol{a}, P_j^H, \phi_j^H)
\end{aligned}$$

where:

$$m(\boldsymbol{a}) = r\boldsymbol{a} - \boldsymbol{c} \tag{38}$$

$$s^2(\boldsymbol{a}) = \left(\frac{\sigma^P}{r}\right)^2 + \left(\frac{\sigma^T}{r+\psi}\right)^2 \tag{39}$$

$$m(P_j^H) = ln\phi_{j,t}^H \tag{40}$$

$$s^2(P_j^H) = \sigma^{P^H 2} \tag{41}$$

The condition for non-durable consumption $c(\boldsymbol{a}, P_j^H, \phi_j^H)$ is $\bar{u}(c)^{-\rho}\bar{u}'(c) = v'$, therefore the marginal utility of non-durable consumption keeping housing fixed must equal the marginal value of wealth. The optimal thresholds b and B satisfy the value matching and smooth pasting conditions. In other words, both v and v' should be continuous at the thresholds b and B.

$$\lim_{\boldsymbol{a}\downarrow b} v(\boldsymbol{a}, P_j^H, \phi_j^H) = [b - (\kappa^O + \kappa^T)P_j^H]^{1-\rho} M \tag{42}$$

$$\lim_{\boldsymbol{a}\uparrow B} v(\boldsymbol{a}, P_j^H, \phi_j^H) = [B - (\kappa^O + \kappa^T)P_j^H]^{1-\rho} M \tag{43}$$

$$\lim_{\boldsymbol{a}\downarrow b} v'(\boldsymbol{a}, P_j^H, \phi_j^H) = (1-\rho)[b - (\kappa^O + \kappa^T)P_j^H]^{-\rho} M \tag{44}$$

$$\lim_{\boldsymbol{a}\uparrow B} v'(\boldsymbol{a}, P_j^H, \phi_j^H) = (1-\rho)[B - (\kappa^O + \kappa^T)P_j^H]^{-\rho} M \tag{45}$$

Using the above properties of the HJB, the numerical solution in the inaction region provides the properties of the function $\tau(\boldsymbol{a}, P_j^H, \phi_j^H)$. Since τ gives the optimal number of years until the next observation of house prices, the inverse of the value will provide the probability that households pay attention to house price information in a year. Outside the inaction region defined by the optimally chosen boundaries b and B, households pay attention and adjust their portfolio. Within the inaction region, the probability of attention is convex, where $\boldsymbol{a} \downarrow b$ or $\boldsymbol{a} \uparrow B$, the probability of adjustment increases, and so does the probability of attention.

Proof for Proposition 2:

Immediately after adjustment, when a household will plan to remain an owner, they will again have to choose τ', which now represents the interval until the next observation of house prices, without housing adjustment. The household's problem is then:

$$\begin{aligned}
V^{own}(a, h, P_j^H, \phi_j^H) = &\max_{\{c_{i,t}\},\tau'} \mathbb{E}_t \left[\int_{s=0}^{\tau'} e^{-\delta t} u(c_{i,t}, h) \mathrm{d}t + e^{-\delta \tau'} V(a', h, \tilde{P}_{i,j}^{H'}, \tilde{\phi}_{i,j}^{H'}) \right] \\
\text{subject to:} \quad \mathrm{d}a_{i,t} =& [r a_{i,t} - c_{i,t} - \Xi_{i,t}^H]\mathrm{d}t + \frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T \\
d\tilde{P}_{i,j,t+1}^H =& \tilde{P}_{i,j,t}^H [\ln \phi_j^H \mathrm{d}t + \sigma^{P^H} \mathrm{d}z_t^{P^H}] \\
\tilde{\phi}_{i,j}^{H'} =& \phi_j^H \\
a_{i,t} \geq& -(1-\theta)\tilde{P}_{i,j,t}^H \\
a' =& a(\tau') - \kappa^O P_j^H h
\end{aligned}$$

Wealth after the next observation is now equal to wealth just before observation $a(\tau')$, minus the observation cost $\kappa^O P_j^H h$. For the value of wealth just before observation, the dynamics of

wealth can be integrated for the period t between observation dates of duration τ'.

$$a(\tau') = e^{r\tau'}\left[a - \int_0^{\tau'} e^{-rt}(c_{i,t} + \Xi_{i,t}^H)\mathrm{d}t + \int_0^{\tau'} e^{-rt}\left(\frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T\right)\right] \tag{46}$$

The post-observation wealth that captures the owner's budget constraint is:

$$a' = e^{r\tau'}\left[a - \int_0^{\tau'} e^{-rt}(c_{i,t} + \Xi_{i,t}^H)\mathrm{d}t + \int_0^{\tau'} e^{-rt}\left(\frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T\right)\right] - \kappa^O P_j^H h \tag{47}$$

Define as $C(0,t)$ the present value of the flow of consumption over the time interval between 0 and some period t:

$$C(0,t) = \int_0^t e^{-rs} c_{i,s}\mathrm{d}s \tag{48}$$

where $c_{i,s}$ is the path of consumption chosen between $0 \leq s \leq t$ that is chosen to maximize the utility over the same period. Optimal path $c_{i,s}$ therefore solves the following:

$$U(C(0,t),h) = \max_{\{c_{i,t}\}} \int_0^t e^{-\delta s}\frac{1}{1-\rho}(c_{i,s}^{\alpha} h^{1-\alpha})^{1-\rho}\mathrm{d}s \tag{49}$$

Taking the derivative at each of the integral limits:

$$\frac{\partial u(c,h)}{\partial c} = \alpha c^{\alpha(1-\rho)-1} h^{(1-\alpha)(1-\rho)} \tag{50}$$

$$\frac{\partial u(c_{i,t},h)}{\partial c_{i,t}} = \alpha e^{-\delta t} c_{i,t}^{\alpha(1-\rho)-1} h^{(1-\alpha)(1-\rho)} \tag{51}$$

Dividing $\frac{\partial u(c_{i,t},h)}{\partial c_{i,t}}$ by $\frac{\partial u(c,h)}{\partial c}$ gives:

$$\frac{\frac{\partial u(c_{i,t},h)}{\partial c_{i,t}}}{\frac{\partial u(c,h)}{\partial c}} = e^{-\delta t}\left(\frac{c_{i,t}}{c}\right)^{\alpha(1-\rho)-1} \tag{52}$$

Because the product of the marginal rate of substitution and the gross rate of return equals 1, the following holds:

$$c_{i,t} = e^{\frac{\delta - r}{\alpha(1-\rho)-1}t} c \tag{53}$$

Substituting this expression in the budget constraint then gives:

$$a' = e^{r\tau'}\left[a - \int_0^{\tau'} e^{-rt}\left((e^{\frac{\delta-r}{\alpha(1-\rho)-1}t}c + +\Xi_{i,t}^H)\mathrm{d}t + \int_0^{\tau'} e^{-rt}\left(\frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T\right)\right)\right] - \kappa^O P_j^H h \tag{54}$$

Simplify the integral $\int_0^{\tau'} e^{-rt}e^{\frac{\delta-r}{\alpha(1-\rho)-1}t}c\mathrm{d}t$ and define the parameter γ for convenience:

$$\gamma = \frac{\delta - r}{\alpha(1-\rho) - 1} \tag{55}$$

$$\int_0^{\tau'} e^{-rt}e^{\frac{\delta-r}{\alpha(1-\rho)-1}t}c\mathrm{d}t = \int_0^{\tau'} e^{-rt}e^{\gamma t}c\mathrm{d}t = c\int_0^{\tau'} e^{(\gamma-r)t}\mathrm{d}t = c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}$$

For the house value depreciation term, since the household does not observe house price information between observation dates, it assumes that house price growth remains constant between observation dates. The household's perception for the dynamics of local house prices between observations dates are then:

$$P_{j,t}^H = (\phi_j^H)^t \Psi_t^H P_j^H \tag{56}$$

Defining the per-period depreciation cost as $\Xi_{i,t}^H = \xi P_{j,t}^H h$, then the total depreciation cost sums to:

$$P_{j,t}^H = (\phi_j^H)^t \Psi_t^H P_j^H \tag{57}$$

$$\begin{aligned}&\Rightarrow \int_0^{\tau'} e^{-rt}\xi P_{j,t}^H h\mathrm{d}t = \int_0^{\tau'} e^{-rt}\xi(\phi_j^H)^t\Psi_t^H P_j^H h\mathrm{d}t = \xi P_j^H h\int_0^{\tau'} e^{-rt}(\phi_j^H)^t\Psi_t^H\mathrm{d}t \\ &= \xi P_j^H h\int_0^{\tau'} e^{t[\ln(\phi_j^H)-r]}\Psi_t^H\mathrm{d}t = \xi P_j^H h\frac{e^{\tau'[\ln(\phi_j^H)-r]}-1}{\ln(\phi_j^H)-r} = \xi P_j^H h\frac{(\phi_j^H)^{\tau'}e^{-r\tau'}-1}{\ln(\phi_j^H)-r}\end{aligned} \tag{58}$$

Substituting back into the budget constraint:

$$\begin{aligned}a' =& e^{r\tau'}\left[a - c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} - \xi P_j^H h\frac{(\phi_j^H)^{\tau'}e^{-r\tau'}-1}{\ln(\phi_j^H)-r}\right] - \kappa^O P_j^H h \\ &+ e^{r\tau'}\int_0^{\tau'} e^{-rt}\left(\frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T\right)\end{aligned} \tag{59}$$

Based on these dynamics, the mean and variance of a' are:

$$\mathbb{E}[a'] = e^{r\tau'}\left[a - c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} - \xi P_j^H h\frac{(\phi_j^H)^{\tau'}e^{-r\tau'}-1}{\ln(\phi_j^H)-r}\right] - \kappa^O P_j^H h \tag{60}$$

$$\begin{aligned}\mathrm{Var}[a'] =& \int_0^{\tau'}\left[e^{r\tau'}e^{-rt}\left(\frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T\right)\right]^2 \\ =& e^{2r\tau'}\int_0^{\tau'} e^{-2rt}\left(\frac{\sigma^P}{r}\mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T\right)^2 \\ =& \frac{\sigma^2}{2r}(e^{2r\tau'}-1)\end{aligned} \tag{61}$$

where $\sigma^2 \equiv (\frac{\sigma^P}{r}\mathrm{d}z_t^P)^2 + (\frac{\sigma^T}{r+\psi}\mathrm{d}z_t^T)^2$. I make an informed guess for the form of the value function:

$$V^{own}(a, h, P_j^H, \phi_j^H) = \frac{1}{1-\rho}Aa^{\alpha(1-\rho)} \tag{62}$$

where A is a positive constant to be determined. According to the envelope theorem:

$$\begin{aligned}\frac{1}{1-\rho}A\alpha(1-\rho)a^{\alpha(1-\rho)-1} =& e^{-\delta\tau'}\,\mathbb{E}\left[\frac{1}{1-\rho}A\alpha(1-\rho)a'^{\alpha(1-\rho)-1}\frac{\partial a'}{\partial a}\right] \\ a^{\alpha(1-\rho)-1} =& e^{-\delta\tau'}\,\mathbb{E}\left[a'^{\alpha(1-\rho)-1}\frac{\partial a'}{\partial a}\right] \\ a^{\alpha(1-\rho)-1} =& e^{-\delta\tau'}\,\mathbb{E}\left[a'^{\alpha(1-\rho)-1}e^{r\tau'}\right] \\ a^{\alpha(1-\rho)-1} =& e^{(r-\delta)\tau'}\,\mathbb{E}\left[a'^{\alpha(1-\rho)-1}\right] \\ (\alpha(1-\rho)-1)\ln(a) =& (r-\delta)\tau' + \ln(\mathbb{E}[a'^{\alpha(1-\rho)-1}])\end{aligned} \tag{63}$$

recalling that a' is log-normally distributed and assuming that $\alpha(1-\rho)-1<0$ as is typical for a CRRA utility function, we can express the term $\mathbb{E}[a'^{\alpha(1-\rho)-1}]$ as:

$$\mathbb{E}[a'^{\alpha(1-\rho)-1}] = e^{(\alpha(1-\rho)-1)\,\mathbb{E}[\ln(a')] + \frac{(\alpha(1-\rho))^2}{2}\mathrm{Var}[\ln(a')]} \tag{64}$$

The moments in the expression can be approximated for analytical tractability:

$$\mathbb{E}[\ln(a')] \approx \ln(\mathbb{E}[a']) - \frac{\mathrm{Var}[a']}{2(\mathbb{E}[a'])^2} \tag{65}$$

$$\text{Var}[\ln(a')] \approx \frac{\text{Var}[a']}{(\mathbb{E}[a'])^2} \tag{66}$$

Substituting in these approximations then gives:

$$\begin{aligned}
\mathbb{E}[a'^{\alpha(1-\rho)-1}] \approx & e^{(\alpha(1-\rho)-1)\left[\ln(\mathbb{E}[a'])-\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2}\right]+\frac{(\alpha(1-\rho))^2}{2}\left[\frac{\text{Var}[a']}{(\mathbb{E}[a'])^2}\right]} \\
= & e^{\ln(\mathbb{E}[a'])^{\alpha(1-\rho)-1}+(\alpha(1-\rho)-1)(\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2}} \\
= & \mathbb{E}[a']^{\alpha(1-\rho)-1} e^{(\alpha(1-\rho)-1)(\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2}}
\end{aligned} \tag{67}$$

Substituting 67 in the equation derived from the envelope theorem 63:

$$\begin{aligned}
(\alpha(1-\rho)-1)\ln(a) = & (r-\delta)\tau' + \ln(\mathbb{E}[a'^{\alpha(1-\rho)-1}]) \\
= & (r-\delta)\tau' + (\alpha(1-\rho)-1)\ln(\mathbb{E}[a']) + (\alpha(1-\rho)-1)(\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2} \\
\Rightarrow \ln(a) = & \frac{(r-\delta)\tau'}{\alpha(1-\rho)-1} + \ln(\mathbb{E}[a']) + (\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2} \\
a = & \mathbb{E}[a'] e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}} e^{(\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2}}
\end{aligned} \tag{68}$$

Define the following parameters for tractability as we proceed with the derivation steps:

$$B = e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}} \tag{69}$$

$$G = (\alpha(1-\rho)-2)\text{Var}[a'] \tag{70}$$

$$\Xi = \xi P_j^H h \frac{(\phi_j^H)^{\tau'} e^{-r\tau'} - 1}{\ln(\phi_j^H) - r} \tag{71}$$

$$K = \kappa^O P_j^H h \tag{72}$$

Substituting the above in expression 68:

$$a = \left[e^{r\tau'} \left(a - c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} - \Xi \right) - K \right] B e^{\frac{G}{2(E[a'])^2}}$$

$$a = e^{r\tau'} a B e^{\frac{G}{2(E[a'])^2}} - e^{r\tau'} c \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} B e^{\frac{G}{2(E[a'])^2}} - e^{r\tau'} \Xi B e^{\frac{G}{2(E[a'])^2}} - K B e^{\frac{G}{2(E[a'])^2}}$$

$$a + e^{r\tau'} c \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} B e^{\frac{G}{2(E[a'])^2}} + e^{r\tau'} \Xi B e^{\frac{G}{2(E[a'])^2}} + K B e^{\frac{G}{2(E[a'])^2}} = e^{r\tau'} a B e^{\frac{G}{2(E[a'])^2}}$$

$$e^{r\tau'} c \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} B e^{\frac{G}{2(E[a'])^2}} = e^{r\tau'} a B e^{\frac{G}{2(E[a'])^2}} - a - e^{r\tau'} \Xi B e^{\frac{G}{2(E[a'])^2}} - K B e^{\frac{G}{2(E[a'])^2}}$$

$$c = \frac{e^{r\tau'} a B e^{\frac{G}{2(E[a'])^2}} - a - e^{r\tau'} \Xi B e^{\frac{G}{2(E[a'])^2}} - K B e^{\frac{G}{2(E[a'])^2}}}{e^{r\tau'} \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} B e^{\frac{G}{2(E[a'])^2}}}$$

$$c = \frac{a(e^{r\tau'} B e^{\frac{G}{2(E[a'])^2}} - 1) - e^{r\tau'} \Xi B e^{\frac{G}{2(E[a'])^2}} - K B e^{\frac{G}{2(E[a'])^2}}}{e^{r\tau'} \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} B e^{\frac{G}{2(E[a'])^2}}}$$

$$c = a \frac{1 - \frac{1}{e^{r\tau'} B e^{\frac{G}{2(E[a'])^2}}}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'} \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \tag{73}$$

Applying the Taylor rule expansion for the exponential term:

$$e^{\frac{G}{2(E[a'])^2}} \approx 1 + \frac{G}{2(E[a'])^2} \tag{74}$$

Substituting back, the equation for initial consumption c becomes:

$$c \approx a \frac{1 - \frac{1}{e^{r\tau'} B \left[1 + \frac{G}{2(E[a'])^2}\right]}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'} \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \tag{75}$$

Further simplifying using the approximation $\frac{1}{1+x} \approx 1 - x$ for small values of x:

$$\begin{aligned} c \approx & a \frac{1 - \frac{1}{e^{r\tau'} B} \left(1 - \frac{G}{2(E[a'])^2}\right)}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'} \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ = & a \frac{1 - \frac{1}{e^{r\tau'} B} + \frac{1}{e^{r\tau'} B} \frac{G}{2(E[a'])^2}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'} \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \end{aligned}$$

Substituting the expression for $(E[a'])^2$ in the above:

$$\begin{aligned} c = & a \frac{1 - \frac{1}{e^{r\tau'} B} + \frac{1}{e^{r\tau'} B} \frac{G}{2} \left[e^{2r\tau'} \left((a-\Xi)^2 - 2(a-\Xi) c \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + c^2 \left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} \right)^2 \right) \right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ & - a \frac{\frac{1}{e^{r\tau'} B} \frac{G}{2} \left[2K e^{r\tau'} \left[(a-\Xi) - c \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} \right] + K^2 \right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'} \frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \end{aligned}$$

$$c = a\frac{1-\frac{1}{e^{r\tau'}B}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} + a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-\Xi)^2 - 2e^{2r\tau'}(a-\Xi)c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + e^{2r\tau'}c^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$- a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-\Xi) - 2Ke^{r\tau'}c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

The above expression can be written in the standard quadratic form $\alpha_1 c^2 + \alpha_2 c + \alpha_3 = 0$, and solved as a quadratic function, where:

$$\alpha_1 = - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}e^{2r\tau'}\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$\alpha_2 = 1 + 2a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}e^{2r\tau'}(a-\Xi)\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}2Ke^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$\alpha_3 = - a\frac{1-\frac{1}{e^{r\tau'}B}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}[e^{2r\tau'}(a-\Xi)^2 - 2Ke^{r\tau'}(a-\Xi) + K^2]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} + \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} + \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

Note that $G = (\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)$. Substituting back in for α_1 and α_2:

$$\alpha_1 = - a\frac{\frac{1}{e^{r\tau'}B}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}e^{2r\tau'}\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$\alpha_2 = 1 + 2a\frac{\frac{1}{e^{r\tau'}B}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}e^{2r\tau'}(a-\Xi)\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - a\frac{\frac{1}{e^{r\tau'}B}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}2Ke^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

Given that the quadratic formula is $c = \frac{-\alpha_2\sqrt{\alpha_2^2-4\alpha_1\alpha_3}}{2\alpha_1}$, we can use the perturbation method and write the solution of c as a power series: $c = c^0 + \sigma^2 c^1 + O(\sigma^4)$. The deterministic solution c^0 is the solution when $\sigma = 0$ and c^1 is the first-order risk adjustment. From the expression for G, we have $\sigma = 0 \Rightarrow G = 0$. Then:

$$\begin{aligned} c^0 =& a\frac{1-\frac{1}{e^{r\tau'}e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}}}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{\Xi}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ =& a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} - \Xi\frac{e^{r\tau'}(r-\gamma)}{e^{r\tau'}-1} - K\frac{r-\gamma}{e^{r\tau'}-1} \end{aligned} \tag{76}$$

For c^1, we start by isolating all terms of c that contain σ^2:

$$c|\sigma^2 = a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-\Xi)^2 - 2e^{2r\tau'}(a-\Xi)c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + e^{2r\tau'}c^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$- a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-\Xi) - 2Ke^{r\tau'}c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

For the first-order approximation, substituting c for c^0:

$$c|\sigma^2 = a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-\Xi)^2 - 2e^{2r\tau'}(a-\Xi)c^0\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + e^{2r\tau'}(c^0)^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$- a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-\Xi) - 2Ke^{r\tau'}c^0\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

Substitute the approximation for $c^0 = a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1}$, based on 76:

$$c|\sigma^2 = a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-\Xi)^2 - 2e^{2r\tau'}(a-\Xi)a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + e^{2r\tau'}(a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1})^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$- a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-\Xi) - 2Ke^{r\tau'}a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

Simplify the leading coefficient of the expression:

$$a\frac{\frac{1}{e^{r\tau'}}\frac{G}{2}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} = a\frac{\frac{1}{e^{r\tau'}e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}}}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} = \frac{a(\alpha(1-\rho)-2)\sigma^2}{4r}(e^{2r\tau'}-1)(\gamma-r)\frac{e^{(\gamma-r)\tau'}}{e^{(\gamma-r)\tau'}-1}$$

Therefore, c^1 is the following:

$$c^1 = \frac{(\alpha(1-\rho)-2)}{4r}(e^{2r\tau'}-1)(\gamma-r)\frac{e^{(\gamma-r)\tau'}}{e^{(\gamma-r)\tau'}-1}a^{-1}\left(-\frac{1}{2}\right) \tag{77}$$

Using that $c = c^0 + \sigma^2 c^1 + O(\sigma^4)$:

$$c = a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} - \Xi\frac{e^{r\tau'}(r-\gamma)}{e^{r\tau'}-1} - K\frac{r-\gamma}{e^{r\tau'}-1} - \frac{\sigma^2}{4a}\frac{(\alpha(1-\rho)-2)}{r}(e^{r\tau'}+1)\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} \tag{78}$$

I define k, m, n, functions of the parameters r, δ, α, ρ and τ':

$$k = \frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} \tag{79}$$

$$m = \frac{r-\gamma}{e^{r\tau'}-1} \tag{80}$$

$$n = \frac{(\alpha(1-\rho)-2)}{r}\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} \tag{81}$$

The optimal initial consumption can be written as:

$$c = ka - e^{r\tau'}m\Xi - mK - n\frac{\sigma^2}{4}(e^{r\tau'}+1) \tag{82}$$

As we can see from this expression, k captures the marginal propensity to consume out of wealth, m is the effect of the costs involved in planning ahead consumption that include depreciation costs and observation costs, and n is the risk adjustment factor, appropriately scaled for this context.

We know that the inverse of the optimal attention interval τ gives the probability that households pay attention to house price information in a year. Therefore, I determine the optimal attention interval τ'^* by deriving the relevant optimality condition: the first-order derivative of the value function V^{own} with respect to τ'. Taking the first-order condition with respect to the period until the next observation, and substituting in $u(c', h)$ and $V^{own}(a', h, P_j^{H'}, \phi_j^{H'})$:

$$\begin{aligned}
&e^{-\delta\tau'}u(c',h) - \delta e^{-\delta\tau'}\,\mathbb{E}[V^{own}(a',h,P_j^{H'},\phi_j^{H'})] + e^{-\delta\tau'}\frac{\partial\,\mathbb{E}\left[V^{own}(a',h,P_j^{H'},\phi_j^{H'})\right]}{\partial\tau'} = 0\\
&e^{-\delta\tau'}\left[\frac{1}{1-\rho}(c'^{\alpha}h^{1-\alpha})^{1-\rho}\right] - \delta e^{-\delta\tau'}\,\mathbb{E}\left[\frac{1}{1-\rho}Aa'^{\alpha(1-\rho)}\right] + e^{-\delta\tau'}\frac{\partial\,\mathbb{E}\left[\frac{1}{1-\rho}Aa'^{\alpha(1-\rho)}\right]}{\partial\tau'} = 0\\
&e^{-\delta\tau'}\left[\frac{1}{1-\rho}(c'^{\alpha}h^{1-\alpha})^{1-\rho}\right] - \delta e^{-\delta\tau'}\frac{1}{1-\rho}A\,\mathbb{E}\left[a'^{\alpha(1-\rho)}\right] + e^{-\delta\tau'}\frac{A}{1-\rho}\frac{\partial\,\mathbb{E}[a'^{\alpha(1-\rho)}]}{\partial\tau'} = 0
\end{aligned} \tag{83}$$

For the expectation $\mathbb{E}[a'^{\alpha(1-\rho)}]$, I apply Ito's lemma:

$$\begin{aligned}
\mathbb{E}[a'^{\alpha(1-\rho)}] &= \mathbb{E}[a']^{\alpha(1-\rho)}\left(1 + (\alpha(1-\rho)-1)\alpha(1-\rho)\frac{\mathrm{Var}[a']}{2\,\mathbb{E}[a']^2} + \text{higher order terms}\right)\\
\Rightarrow \frac{\partial\,\mathbb{E}[a'^{\alpha(1-\rho)}]}{\partial\tau'} &= \frac{A}{1-\rho}\alpha(1-\rho)\,\mathbb{E}[a']^{\alpha(1-\rho)-1}\frac{\partial\,\mathbb{E}[a']}{\partial\tau'} + \frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)}\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2\,\mathbb{E}[a']^2}\frac{\partial\mathrm{Var}[a']}{\partial\tau'}
\end{aligned}$$

I can also determine the following derivatives:

$$\frac{\partial \mathbb{E}[a']}{\partial \tau'} = re^{r\tau'}\left(a - c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} - \Xi\right) - e^{r\tau'}c'$$
$$\frac{\partial \text{Var}[a']}{\partial \tau'} = \sigma^2 e^{2r\tau'}$$

Applying the envelope condition, I note down the relationship below:

$$\frac{\partial V^{own}(a,h,P_j^H,\phi_j^H)}{\partial a} = A\alpha(1-\rho)a^{\alpha(1-\rho)-1} = e^{-\delta\tau'+r\tau'}A\alpha(1-\rho)\,\mathbb{E}[a']^{\alpha(1-\rho)-1} \tag{84}$$

Returning to 83 and substituting the aforementioned simplified terms, I arrive at:

$$e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)-1}\left(\Xi + rK - e^{r\tau'}c'\right) + e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)}\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2\,\mathbb{E}[a']^2}\sigma^2 e^{2r\tau'} = 0$$
$$(\Xi + rK)e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)-1}$$
$$= e^{r\tau'}c'e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\theta)-1} - e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\theta)}\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2\,\mathbb{E}[a']^2}\sigma^2 e^{2r\tau}$$
$$\Xi + rK = e^{r\tau'}c' - \mathbb{E}[a']\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2}\frac{\sigma^2 e^{2r\tau'}}{\mathbb{E}[a']}$$
$$\Xi + rK = e^{r\tau'}c' - \frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2}\sigma^2 e^{2r\tau}$$
$$(\Xi + rK)^2 = \frac{\alpha^2(1-\rho)^2\sigma^4 e^{4r\tau'}}{4}\left[1 + \frac{(\delta-r)^2}{(\alpha(1-\rho)-1)^2 r} + \text{higher order terms}\right]$$
$$(e^{r\tau'}-1)^2 = \frac{4(\Xi+rK)}{\alpha\sigma^2}\left(1 + f(\alpha,\rho,\delta,r)\right) \tag{85}$$

where:

$$f(\alpha,\rho,\delta,r) \approx \frac{(\delta-r)^2}{r(\alpha(1-\rho)-1)^2} \tag{86}$$

So the optimal attention interval τ', or more generally τ, can be solved in closed form:

$$\tau = \frac{1}{r}\ln\left(a + \sqrt{\frac{4(\Xi+rK)}{\alpha\sigma^2}(1+f(\alpha,\rho,\delta,r))}\right) \tag{87}$$

Taking the inverse of τ, and using the approximation $\ln(1+x) \approx x$ for depreciation and observation costs that are small relative to the sum of the variance of income and house prices scaled by α, the

probability that households pay attention to house price information in a year is approximated to be:

$$P^{att} \approx \frac{r\sigma\sqrt{\alpha}}{2\sqrt{\Xi + r\bar{K}}\sqrt{1 + f(\alpha, \rho, \delta, r)}} \tag{88}$$

This closed-form solution informs the properties of the optimal probability that households pay attention to house prices when they have chosen to remain owners, as outlined in Proposition 2.

For households that plan to remain renters, immediately after adjustment, they will again have to choose τ', which now represents the interval until the next observation of house prices, without housing adjustment. The household's problem is then:

$$\begin{aligned} V^{rent}(a, \bar{h}, P_j^H, \phi_j^H) = &\max_{\{c_{i,t}\}, \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s u(c_{i,s}, \bar{h}) + \beta^{\tau'} V(a', \bar{h}, \tilde{P}_{i,j}^{H'}, \phi_j^H) \right] \\ \text{subject to: } \tilde{P}_{i,j,s+1}^H = &\mathbb{E}[P_{j,s+1}^H | P_j^H, \phi_j^H] = (\phi_j^H)^s P_j^H \Psi_{s+1}^H \quad \text{for } 0 \leq s \leq \tau' - 1 \\ a' = &a(\tau') - \kappa^O P_j^H \bar{h} \end{aligned}$$

Wealth after the next observation is now equal to wealth just before observation $a(\tau')$, minus the observation cost $\kappa^O P_j^H h$. For the value of wealth just before observation, the dynamics of wealth can be integrated for the period t between observation dates of duration τ'.

$$a(\tau') = e^{r\tau'} \left[a - \int_0^{\tau'} e^{-rt} (c_{i,t} + R_{i,t}^H) \mathrm{d}t + \int_0^{\tau'} e^{-rt} \left(\frac{\sigma^P}{r} \mathrm{d}z_t^P + \frac{\sigma^T}{r + \psi} \mathrm{d}z_t^T \right) \right] \tag{89}$$

The post-observation wealth that captures the owner's budget constraint is:

$$a' = e^{r\tau'} \left[a - \int_0^{\tau'} e^{-rt} (c_{i,t} + R_{i,t}^H h) \mathrm{d}t + \int_0^{\tau'} e^{-rt} \left(\frac{\sigma^P}{r} \mathrm{d}z_t^P + \frac{\sigma^T}{r + \psi} \mathrm{d}z_t^T \right) \right] - \kappa^O P_j^H \bar{h} \tag{90}$$

The definition of $C(0, t)$ and the optimal path $c_{i,s}$ are the same as in the owner case, so the budget constraint now becomes:

$$a' = e^{r\tau'} \left[a - \int_0^{\tau'} e^{-rt} \left((e^{\frac{\delta - r}{\alpha(1-\rho)-1} t} c + R_{i,t}^H) \mathrm{d}t + \int_0^{\tau'} e^{-rt} \left(\frac{\sigma^P}{r} \mathrm{d}z_t^P + \frac{\sigma^T}{r + \psi} \mathrm{d}z_t^T \right) \right) \right] - \kappa^O P_j^H \bar{h}$$

$$=e^{r\tau'}\left[a-c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}-R^H_{i,t}\right]-\kappa^O P^H_j\bar{h}+e^{r\tau'}\int_0^{\tau'}e^{-rt}\left(\frac{\sigma^P}{r}\mathrm{d}z^P_t+\frac{\sigma^T}{r+\psi}\mathrm{d}z^T_t\right) \tag{91}$$

Based on these dynamics, the mean and variance of a' are:

$$\mathbb{E}[a']=e^{r\tau'}\left[a-c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}-R^H_{i,t}\right]-\kappa^O P^H_j\bar{h} \tag{92}$$

$$\begin{aligned}\mathrm{Var}[a']=&\int_0^{\tau'}\left[e^{r\tau'}e^{-rt}\left(\frac{\sigma^P}{r}\mathrm{d}z^P_t+\frac{\sigma^T}{r+\psi}\mathrm{d}z^T_t\right)\right]^2\\ =&\frac{\sigma^2}{2r}(e^{2r\tau'}-1)\end{aligned} \tag{93}$$

where $\sigma^2\equiv(\frac{\sigma^P}{r}\mathrm{d}z^P_t)^2+(\frac{\sigma^T}{r+\psi}\mathrm{d}z^T_t)^2$. Making the same informed guess as in the owner case:

$$V^{rent}(a,\bar{h},P^H_j,\phi^H_j)=\frac{1}{1-\rho}Aa^{\alpha(1-\rho)} \tag{94}$$

where A is a positive constant to be determined. According to the envelope theorem:

$$\begin{aligned}\frac{1}{1-\rho}A\alpha(1-\rho)a^{\alpha(1-\rho)-1}=&e^{-\delta\tau'}\,\mathbb{E}\left[\frac{1}{1-\rho}A\alpha(1-\rho)a'^{\alpha(1-\rho)-1}\frac{\partial a'}{\partial a}\right]\\ (\alpha(1-\rho)-1)\ln(a)=&(r-\delta)\tau'+\ln\left(\mathbb{E}[a'^{\alpha(1-\rho)-1}]\right)\end{aligned} \tag{95}$$

recalling that a' is log-normally distributed and assuming that $\alpha(1-\rho)-1<0$ as is typical for a CRRA utility function, we can express the term $\mathbb{E}[a'^{\alpha(1-\rho)-1}]$ as:

$$\begin{aligned}\mathbb{E}[a'^{\alpha(1-\rho)-1}]=&e^{(\alpha(1-\rho)-1)\,\mathbb{E}[\ln(a')]+\frac{(\alpha(1-\rho))^2}{2}\mathrm{Var}[\ln(a')]}\\ \approx&e^{(\alpha(1-\rho)-1)\left[\ln(\mathbb{E}[a'])-\frac{\mathrm{Var}[a']}{2(\mathbb{E}[a'])^2}\right]+\frac{(\alpha(1-\rho))^2}{2}\left[\frac{\mathrm{Var}[a']}{(\mathbb{E}[a'])^2}\right]}\\ =&e^{\ln(\mathbb{E}[a'])^{\alpha(1-\rho)-1}+(\alpha(1-\rho)-1)(\alpha(1-\rho)-2)\frac{\mathrm{Var}[a']}{2(\mathbb{E}[a'])^2}}\\ =&\mathbb{E}[a']^{\alpha(1-\rho)-1}e^{(\alpha(1-\rho)-1)(\alpha(1-\rho)-2)\frac{\mathrm{Var}[a']}{2(\mathbb{E}[a'])^2}}\end{aligned} \tag{96}$$

Substituting 96 in the equation derived from the envelope theorem 95:

$$(\alpha(1-\rho)-1)\ln(a)=(r-\delta)\tau'+\ln\left(\mathbb{E}[a'^{\alpha(1-\rho)-1}]\right)$$

$$
\begin{aligned}
&= (r-\delta)\tau' + (\alpha(1-\rho)-1)\ln(\mathbb{E}[a']) + (\alpha(1-\rho)-1)(\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2} \\
\Rightarrow \ln(a) &= \frac{(r-\delta)\tau'}{\alpha(1-\rho)-1} + \ln(\mathbb{E}[a']) + (\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2} \\
a &= \mathbb{E}[a']e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}} e^{(\alpha(1-\rho)-2)\frac{\text{Var}[a']}{2(\mathbb{E}[a'])^2}} \qquad (97)
\end{aligned}
$$

Define the following parameters for tractability as we proceed with the derivation steps:

$$B = e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}} \qquad (98)$$

$$G = (\alpha(1-\rho)-2)\text{Var}[a'] \qquad (99)$$

$$K = \kappa^O P_j^H \bar{h} \qquad (100)$$

Substituting the above in expression 97:

$$
\begin{aligned}
a &= \left[e^{r\tau'}\left(a - c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} - R_{i,t}^H\right) - K\right] Be^{\frac{G}{2(E[a'])^2}} \\
a &= e^{r\tau'}aBe^{\frac{G}{2(E[a'])^2}} - e^{r\tau'}c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}Be^{\frac{G}{2(E[a'])^2}} - e^{r\tau'}R_{i,t}^H Be^{\frac{G}{2(E[a'])^2}} - KBe^{\frac{G}{2(E[a'])^2}} \\
a &+ e^{r\tau'}c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}Be^{\frac{G}{2(E[a'])^2}} + e^{r\tau'}R_{i,t}^H Be^{\frac{G}{2(E[a'])^2}} + KBe^{\frac{G}{2(E[a'])^2}} = e^{r\tau'}aBe^{\frac{G}{2(E[a'])^2}} \\
e^{r\tau'}c&\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}Be^{\frac{G}{2(E[a'])^2}} = e^{r\tau'}aBe^{\frac{G}{2(E[a'])^2}} - a - e^{r\tau'}R_{i,t}^H Be^{\frac{G}{2(E[a'])^2}} - KBe^{\frac{G}{2(E[a'])^2}} \\
c &= \frac{e^{r\tau'}aBe^{\frac{G}{2(E[a'])^2}} - a - e^{r\tau'}R_{i,t}^H Be^{\frac{G}{2(E[a'])^2}} - KBe^{\frac{G}{2(E[a'])^2}}}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}Be^{\frac{G}{2(E[a'])^2}}} \\
c &= \frac{a(e^{r\tau'}Be^{\frac{G}{2(E[a'])^2}} - 1) - e^{r\tau'}R_{i,t}^H Be^{\frac{G}{2(E[a'])^2}} - KBe^{\frac{G}{2(E[a'])^2}}}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}Be^{\frac{G}{2(E[a'])^2}}} \\
c &= a\frac{1 - \frac{1}{e^{r\tau'}Be^{\frac{G}{2(E[a'])^2}}}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{R_{i,t}^H}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \qquad (101)
\end{aligned}
$$

Applying the Taylor rule expansion for the exponential term:

$$e^{\frac{G}{2(E[a'])^2}} \approx 1 + \frac{G}{2(E[a'])^2} \qquad (102)$$

Substituting back, the equation for initial consumption c becomes:

$$c \approx a\frac{1-\frac{1}{e^{r\tau'}B\left[1+\frac{G}{2(E[a'])^2}\right]}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{R^H_{i,t}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \tag{103}$$

Further simplifying using the approximation $\frac{1}{1+x} \approx 1-x$ for small values of x:

$$\begin{aligned} c \approx & a\frac{1-\frac{1}{e^{r\tau'}B}\left(1-\frac{G}{2(E[a'])^2}\right)}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{R^H_{i,t}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ = & a\frac{1-\frac{1}{e^{r\tau'}B}+\frac{1}{e^{r\tau'}B}\frac{G}{2(E[a'])^2}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{R^H_{i,t}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \end{aligned}$$

Substituting expression for $(E[a'])^2$ in the expression:

$$\begin{aligned} c = & a\frac{1-\frac{1}{e^{r\tau'}B}+\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}\left((a-R^H_{i,t})^2-2(a-R^H_{i,t})c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}+c^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right)\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ & - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}\left[(a-R^H_{i,t})-c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right]+K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{R^H_{i,t}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ c = & a\frac{1-\frac{1}{e^{r\tau'}B}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} + a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-R^H_{i,t})^2-2e^{2r\tau'}(a-R^H_{i,t})c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}+e^{2r\tau'}c^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ & - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-R^H_{i,t})-2Ke^{r\tau'}c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}+K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{R^H_{i,t}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \end{aligned}$$

The above expression can be written in the standard quadratic form $\alpha_1 c^2 + \alpha_2 c + \alpha_3 = 0$, and solved as a quadratic function, where:

$$\begin{aligned} \alpha_1 = & -a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}e^{2r\tau'}\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ \alpha_2 = & 1+2a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}e^{2r\tau'}(a-R^H_{i,t})\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}2Ke^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \end{aligned}$$

$$\alpha_3 = -a\frac{1-\frac{1}{e^{r\tau'}B}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}[e^{2r\tau'}(a-R^H_{i,t})^2 - 2Ke^{r\tau'}(a-R^H_{i,t})+K^2]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} + \frac{R^H_{i,t}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} + \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

Note that $G = (\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)$. Substituting back in for α_1 and α_2:

$$\alpha_1 = -a\frac{\frac{1}{e^{r\tau'}B}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}e^{2r\tau'}\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

$$\alpha_2 = 1 + 2a\frac{\frac{1}{e^{r\tau'}B}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}e^{2r\tau'}(a-R^H_{i,t})\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - a\frac{\frac{1}{e^{r\tau'}B}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}2Ke^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

Given that the quadratic formula is $c = \frac{-\alpha_2\sqrt{\alpha_2^2-4\alpha_1\alpha_3}}{2\alpha_1}$, we can use the perturbation method and write the solution of c as a power series: $c = c^0 + \sigma^2 c^1 + O(\sigma^4)$. The deterministic solution c^0 is the solution when $\sigma = 0$ and c^1 is the first-order risk adjustment. For $\sigma = 0 \Rightarrow G = 0$. Then:

$$\begin{aligned} c^0 =& a\frac{1-\frac{1}{e^{r\tau'}e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}}}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{R^H_{i,t}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} - \frac{K}{e^{r\tau'}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ =& a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} - R^H_{i,t}\frac{e^{r\tau'}(r-\gamma)}{e^{r\tau'}-1} - K\frac{r-\gamma}{e^{r\tau'}-1} \end{aligned} \tag{104}$$

For c^1, all terms of c that contain σ^2 are isolated:

$$\begin{aligned} c|\sigma^2 =& a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-R^H_{i,t})^2 - 2e^{2r\tau'}(a-R^H_{i,t})c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + e^{2r\tau'}c^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ & - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-R^H_{i,t}) - 2Ke^{r\tau'}c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \end{aligned}$$

For the first-order approximation, substituting c for c^0:

$$\begin{aligned} c|\sigma^2 =& a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-R^H_{i,t})^2 - 2e^{2r\tau'}(a-R^H_{i,t})c^0\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + e^{2r\tau'}(c^0)^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \\ & - a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-R^H_{i,t}) - 2Ke^{r\tau'}c^0\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} \end{aligned}$$

Substituting an approximation for $c^0 = a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1}$:

$$c|\sigma^2 = a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[e^{2r\tau'}(a-R^H_{i,t})^2 - 2e^{2r\tau'}(a-R^H_{i,t})a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + e^{2r\tau'}(a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1})^2\left(\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}\right)^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$
$$- a\frac{\frac{1}{e^{r\tau'}B}\frac{G}{2}\left[2Ke^{r\tau'}(a-R^H_{i,t}) - 2Ke^{r\tau'}a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1}\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} + K^2\right]}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}}$$

Simplifying the leading coefficient of the expression:

$$a\frac{\frac{1}{e^{r\tau'}}\frac{G}{2}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} = a\frac{\frac{1}{e^{r\tau'}e^{\frac{(r-\delta)\tau'}{\alpha(1-\rho)-1}}}\frac{(\alpha(1-\rho)-2)\frac{\sigma^2}{2r}(e^{2r\tau'}-1)}{2}}{\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r}} = \frac{a(\alpha(1-\rho)-2)\sigma^2}{4r}(e^{2r\tau'}-1)(\gamma-r)\frac{e^{(\gamma-r)\tau'}}{e^{(\gamma-r)\tau'}-1}$$

Therefore, c^1 is the following:

$$c^1 = \frac{(\alpha(1-\rho)-2)}{4r}(e^{2r\tau'}-1)(\gamma-r)\frac{e^{(\gamma-r)\tau'}}{e^{(\gamma-r)\tau'}-1}a^{-1}\left(-\frac{1}{2}\right) \tag{105}$$

Using that $c = c^0 + \sigma^2 c^1 + O(\sigma^4)$:

$$c = a\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} - R^H_{i,t}\frac{e^{r\tau'}(r-\gamma)}{e^{r\tau'}-1} - K\frac{r-\gamma}{e^{r\tau'}-1} - \frac{\sigma^2}{4a}\frac{(\alpha(1-\rho)-2)}{r}(e^{r\tau'}+1)\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} \tag{106}$$

I define k, m, n, functions of the parameters r, δ, α, ρ and τ':

$$k = \frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} \tag{107}$$
$$m = \frac{r-\gamma}{e^{r\tau'}-1} \tag{108}$$
$$n = \frac{(\alpha(1-\rho)-2)}{r}\frac{r-\gamma}{e^{(\gamma-r)\tau'}-1} \tag{109}$$

The optimal initial consumption can be written as:

$$c = ka - e^{r\tau'}mR^H_{i,t} - mK - n\frac{\sigma^2}{4}(e^{r\tau'}+1) \tag{110}$$

As we can see from this expression, k captures the marginal propensity to consume out of wealth, m is the effect of the costs involved in planning ahead consumption that include rental costs and

observation costs, and n is the risk adjustment factor, appropriately scaled for this context.

The optimal attention interval τ'^* is determined by deriving the relevant optimality condition: the first-order derivative of the value function V^{rent} with respect to τ'. Taking the first-order condition with respect to the period until the next observation, and substitute in $u(c', \bar{h})$ and $V^{rent}(a', \bar{h}, P_j^{H'}, \phi_j^{H'})$:

$$
\begin{aligned}
&e^{-\delta\tau'} u(c', \bar{h}) - \delta e^{-\delta\tau'} \mathbb{E}[V^{rent}(a', \bar{h}, P_j^{H'}, \phi_j^{H'})] + e^{-\delta\tau'} \frac{\partial \mathbb{E}\left[V^{rent}(a', \bar{h}, P_j^{H'}, \phi_j^{H'})\right]}{\partial \tau'} = 0 \\
&e^{-\delta\tau'} \left[\frac{1}{1-\rho}(c'^{\alpha}\bar{h}^{1-\alpha})^{1-\rho}\right] - \delta e^{-\delta\tau'} \mathbb{E}\left[\frac{1}{1-\rho} A a'^{\alpha(1-\rho)}\right] + e^{-\delta\tau'} \frac{\partial \mathbb{E}\left[\frac{1}{1-\rho} A a'^{\alpha(1-\rho)}\right]}{\partial \tau'} = 0 \\
&e^{-\delta\tau'} \left[\frac{1}{1-\rho}(c'^{\alpha}\bar{h}^{1-\alpha})^{1-\rho}\right] - \delta e^{-\delta\tau'} \frac{1}{1-\rho} A \mathbb{E}\left[a'^{\alpha(1-\rho)}\right] + e^{-\delta\tau'} \frac{A}{1-\rho} \frac{\partial \mathbb{E}[a'^{\alpha(1-\rho)}]}{\partial \tau'} = 0 \qquad (111)
\end{aligned}
$$

For the expectation $\mathbb{E}[a'^{\alpha(1-\rho)}]$, I apply Ito's lemma:

$$
\begin{aligned}
\mathbb{E}[a'^{\alpha(1-\rho)}] =& \mathbb{E}[a']^{\alpha(1-\rho)} \left(1 + (\alpha(1-\rho)-1)\alpha(1-\rho)\frac{\mathrm{Var}[a']}{2\,\mathbb{E}[a']^2} + \text{higher order terms}\right) \\
\Rightarrow \frac{\partial \mathbb{E}[a'^{\alpha(1-\rho)}]}{\partial \tau'} =& \frac{A}{1-\rho}\alpha(1-\rho)\,\mathbb{E}[a']^{\alpha(1-\rho)-1}\frac{\partial \mathbb{E}[a']}{\partial \tau'} + \frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)}\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2\,\mathbb{E}[a']^2}\frac{\partial \mathrm{Var}[a']}{\partial \tau'}
\end{aligned}
$$

I can also determine the following derivatives:

$$
\begin{aligned}
\frac{\partial \mathbb{E}[a']}{\partial \tau'} =& r e^{r\tau'}\left(a - c\frac{e^{(\gamma-r)\tau'}-1}{\gamma-r} - R_{i,t}^H\right) - e^{r\tau'}c' \\
\frac{\partial \mathrm{Var}[a']}{\partial \tau'} =& \sigma^2 e^{2r\tau'}
\end{aligned}
$$

Applying the envelope condition, I note down the relationship below:

$$
\frac{\partial V^{rent}(a, \bar{h}, P_j^H, \phi_j^H)}{\partial a} = A\alpha(1-\rho)a^{\alpha(1-\rho)-1} = e^{-\delta\tau'+r\tau'} A\alpha(1-\rho)\,\mathbb{E}[a']^{\alpha(1-\rho)-1} \qquad (112)
$$

Returning to 111 and substituting the aforementioned simplified terms, I arrive at:

$$
\begin{aligned}
&e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)-1}\left(R_{i,t}^H + rK - e^{r\tau'}c'\right) + e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)}\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2\,\mathbb{E}[a']^2}\sigma^2 e^{2r\tau'} = 0 \\
&(R_{i,t}^H + rK)e^{-\delta\tau'}\frac{A}{1-\rho}\,\mathbb{E}[a']^{\alpha(1-\rho)-1}
\end{aligned}
$$

$$= e^{r\tau'}c'e^{-\delta\tau'}\frac{A}{1-\rho}\mathbb{E}[a']^{\alpha(1-\theta)-1} - e^{-\delta\tau'}\frac{A}{1-\rho}\mathbb{E}[a']^{\alpha(1-\rho)}\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2\,\mathbb{E}[a']^2}\sigma^2 e^{2r\tau}$$

$$R^H_{i,t} + rK = e^{r\tau'}c' - \mathbb{E}[a']\frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2}\frac{\sigma^2 e^{2r\tau'}}{\mathbb{E}[a']}$$

$$R^H_{i,t} + rK = e^{r\tau'}c' - \frac{(\alpha(1-\rho)-1)\alpha(1-\rho)}{2}\sigma^2 e^{2r\tau}$$

$$(R^H_{i,t} + rK)^2 = \frac{\alpha^2(1-\rho)^2\sigma^4 e^{4r\tau'}}{4}\left[1 + \frac{(\delta-r)^2}{(\alpha(1-\rho)-1)^2 r} + \text{higher order terms}\right]$$

$$(e^{r\tau'}-1)^2 = \frac{4(R^H_{i,t}+rK)}{\alpha\sigma^2}\left(1 + f(\alpha,\rho,\delta,r)\right) \tag{113}$$

where:

$$f(\alpha,\rho,\delta,r) \approx \frac{(\delta-r)^2}{r(\alpha(1-\rho)-1)^2} \tag{114}$$

So the optimal attention interval τ can be solved in closed form:

$$\tau = \frac{1}{r}\ln\left(a + \sqrt{\frac{4(R^H_{i,t}+rK)}{\alpha\sigma^2}(1+f(\alpha,\rho,\delta,r))}\right) \tag{115}$$

Taking the inverse of τ, and using the approximation $\ln(1+x) \approx x$ for rental and observation costs that are small relative to the sum of the variance of income and house prices scaled by α, the probability that households pay attention to house price information in a year is approximated to be:

$$P^{att} \approx \frac{r\sigma\sqrt{\alpha}}{2\sqrt{R^H_{i,t}+rK}\sqrt{1+f(\alpha,\rho,\delta,r)}} \tag{116}$$

This closed-form solution informs the properties of the optimal probability that households pay attention to house prices when they have chosen to remain renters, as outlined in Proposition 2.

Proof for Lemma 2:

This Lemma is equivalent to Lemma 1, but this time considering households that are constantly and fully attentive to house price information. Starting with the owners, to reduce the dimensionality of their problem, the value function can be written in its intensive form, defining a, y and c with

respect to the housing choice h, in a similar vein to Stokey (2009). Therefore, $\boldsymbol{a}_{i,t} = \frac{a_{i,t}}{h_{i,t}}$, $\boldsymbol{y}_{i,t} = \frac{y_{i,t}}{h_{i,t}}$, $\boldsymbol{c}_{i,t} = \frac{c_{i,t}}{h_{i,t}}$, $\bar{u}(\boldsymbol{c}_{i,t})$ where $h_{i,t} = 1$ and $v(\boldsymbol{a}', P_j^{H'}, \phi_j^{H'})$ where $h = 1$. The intensive form of the continuation value is then:

$$\begin{aligned} V(a', y', h', P_j^{H'}, \phi_j^{H'}) =& (h')^{1-\rho} V(\boldsymbol{a}', \boldsymbol{y}', 1, P_j^{H'}, \phi_j^{H'}) \\ =& \left(\frac{\boldsymbol{a}(\tau') - \kappa^T P_j^H}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \boldsymbol{y}', P_j^{H'}, \phi_j^{H'}) \end{aligned}$$

Using these definitions, the owner's problem under full attention, when they decide to adjust their housing, can be written as follows:

$$V^{adj|own}(\boldsymbol{a}, \boldsymbol{y}, P_j^H, \phi_j^H) = \max_{\{c_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s \bar{u}(\boldsymbol{c}_{i,s}) + \beta^{\tau'} \left(\frac{\boldsymbol{a}_i(\tau') - \kappa^T P_j^H}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \boldsymbol{y}', P_j^{H'}, \phi_j^{H'}) \right]$$

$$\begin{aligned} \text{subject to:} \quad \boldsymbol{a}_{i,t+1} =& (1+r)\boldsymbol{a}_{i,t} + \boldsymbol{y}_{i,t+1} - \boldsymbol{c}_{i,t} - \boldsymbol{\Xi}_{i,t}^H \\ P_{j,t+1}^H =& \phi_{j,t}^H P_{j,t}^H \Psi_{s+1}^H \\ \boldsymbol{a}_{i,t} \geq& -(1-\theta) P_{j,t}^H \end{aligned}$$

To continue, we consider the continuous-time equivalent problem:

$$V^{adj|own}(\boldsymbol{a}, P_j^H, \phi_j^H) = \max_{\{c_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\int_{s=0}^{\tau'} e^{-\delta t} \bar{u}(\boldsymbol{c}_{i,t}) \mathrm{d}t + e^{-\delta \tau'} \left(\frac{\boldsymbol{a}_i(\tau') - \kappa^T P_j^H}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', P_j^{H'}, \phi_j^{H'}) \right]$$

$$\begin{aligned} \text{subject to:} \quad \mathrm{d}\boldsymbol{a}_{i,t} =& [r\boldsymbol{a}_{i,t} - \boldsymbol{c}_{i,t} - \boldsymbol{\Xi}_{i,t}^H]\mathrm{d}t + \frac{\sigma^P}{r} \mathrm{d}z_t^P + \frac{\sigma^T}{r+\psi} \mathrm{d}z_t^T \\ dP_{j,t+1}^H =& P_{j,t}^H [\ln \phi_{j,t}^H \mathrm{d}t + \sigma^{P^H} \mathrm{d}z_t^{P^H}] \\ \boldsymbol{a}_{i,t} \geq& -(1-\theta) P_{j,t}^H \end{aligned}$$

Similarly to the owner case, the renters' value function can be written in its intensive form, defining a, y and c with respect to the housing choice h, as follows:

$$V^{adj|rent}(\boldsymbol{a}, \boldsymbol{y}, P_j^H, \phi_j^H) = \max_{\{c_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s \bar{u}(\boldsymbol{c}_{i,s}) + \beta^{\tau'} \left(\frac{\boldsymbol{a}_i(\tau') - \kappa^T P_j^H}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \boldsymbol{y}', P_j^{H'}, \phi_j^{H'}) \right]$$

$$
\begin{aligned}
\text{subject to:} \quad \boldsymbol{a}_{i,t+1} =& (1+r)\boldsymbol{a}_{i,t} - \boldsymbol{c}_{i,t} - R^H_{i,t} \\
P^H_{j,t+1} =& \phi^H_{j,t} P^H_{j,t} \Psi^H_{t+1} \\
\boldsymbol{a}_{i,t} \geq& -\sigma \boldsymbol{y}_{i,t}
\end{aligned}
$$

We consider the continuous-time equivalent problem:

$$
V^{adj|rent}(\boldsymbol{a}, \boldsymbol{y}, P^H_j, \phi^H_j) = \max_{\{c_{i,t}\}, \boldsymbol{a}', \tau'} \mathbb{E}_t \left[\int_{s=0}^{\tau'} e^{-\delta t} \bar{u}(\boldsymbol{c}_{i,t}) \mathrm{d}t + e^{-\delta \tau'} \left(\frac{\boldsymbol{a}_i(\tau') - \kappa^T P^H_j}{\boldsymbol{a}'} \right)^{1-\rho} v(\boldsymbol{a}', \boldsymbol{y}', P^{H'}_j, \phi^{H'}_j) \right]
$$

$$
\begin{aligned}
\text{subject to:} \quad \mathrm{d}\boldsymbol{a}_{i,t} =& [r\boldsymbol{a}_{i,t} - \boldsymbol{c}_{i,t} - R^H_{i,t}]\mathrm{d}t + \frac{\sigma^P}{r}\mathrm{d}z^P_t + \frac{\sigma^T}{r+\psi}\mathrm{d}z^T_t \\
dP^H_{j,t+1} =& P^H_{j,t}[\ln \phi^H_{j,t} \mathrm{d}t + \sigma^{P^H} \mathrm{d}z^{P^H}_t] \\
\boldsymbol{a}_{i,t} \geq& -\sigma \boldsymbol{y}_{i,t}
\end{aligned}
$$

Similarly to the discussion related to Lemma 1, the lower bound that defines households' inaction region is determined by their borrowing constraints. For owners, the optimal thresholds will be such that $-(1-\theta)P^H_{j,t} \leq b < B < +\infty$, and for renters, the range is $-\sigma \boldsymbol{y}_{i,t} \leq b < B < +\infty$.

Proof for Corollary 2:

Corollary 2 follows directly from Lemma 2, such that we define the optimal time period that corresponds to the inaction region of the state variable $\boldsymbol{a}$. Therefore, for both owners and renters, the household's choice of the optimal period until the next observation and adjustment will be such that:

$$
\tau'(\boldsymbol{a}, P^H_j, \phi^H_j) = \tau'(b, P^H_j, \phi^H_j) \wedge \tau'(B, P^H_j, \phi^H_j) \tag{117}
$$

where thresholds b and B are optimally chosen by the owner or renter.

C Stylized Model with Fully Attentive Households

This appendix considers the conditional value functions, the state variable dynamics and the optimal policy functions describing the control variables that correspond to the available housing choices, for households that pay full attention to house price information.

Owner with Full Attention to House Price Information: For a household that pays full attention to house price information, their perception of current house prices and house price growth is updated in every period, and matches the actual data. For these households, we assume that the observation cost for observing house price information is 0. The value function of the household is then:

$$
\begin{aligned}
V^{adj|own}(a, y, h, P_j^H, \phi_j^H) = & \max_{\{c_{i,t}\}, h', \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s u(c_{i,s}, h) + \beta^{\tau'} V(a', y', h', P_j^{H'}, \phi_j^{H'}) \right] \\
\text{subject to:} \quad a_{i,t+1} = & (1+r) a_{i,t} + y_{i,t+1} - c_{i,t} - \Xi_{i,t}^H \\
P_{j,t+1}^H = & \phi_{j,t}^H P_{j,t}^H \Psi_{t+1}^H \\
a_{i,t+1} \geq & -(1-\theta) P_{j,t}^H h \\
a' = & a(\tau') - \kappa^T P_{j,t}^H \bar{h}
\end{aligned}
$$

Note that during the inaction interval τ', a grows as determined by the dynamics of the household's income process, choice of consumption in each period and housing depreciation cost. At dates when an adjustment is made, $a(\tau')$ falls by the amount of the transaction cost $\kappa^T P_{j,t}^H h$.

Renter with Full Attention to House Price Information: The equivalent value function of a household that is currently a renter is the following:

$$
\begin{aligned}
V^{adj|rent}(a, y, \bar{h}, P_j^H, \phi_j^H) = & \max_{\{c_{i,t}\}, h', \tau'} \mathbb{E}_t \left[\sum_{s=0}^{\tau'} \beta^s u(c_{i,s}, \bar{h}) + \beta^{\tau'} V(a', y', h', P_j^{H'}, \phi_j^{H'}) \right] \\
\text{subject to:} \quad a_{i,t+1} = & (1+r) a_{i,t} - c_{i,t} - R_{i,t}^H \\
P_{j,t+1}^H = & \phi_{j,t}^H P_{j,t}^H \Psi_{t+1}^H
\end{aligned}
$$

$$a_{i,t+1} \geq -\sigma y_{i,t}$$
$$a' = a(\tau') - \kappa^T P^H_{j,t} \bar{h}$$

Lemma 2: There exists a range of values for the wealth/housing ratio where, at a perceived current house price level and perceived current house price growth, households will choose to remain inactive and not adjust their housing. The optimal thresholds that defines this inaction region are as follows:

$$-(1-\theta)P^H_{j,t} \leq b < B < +\infty \quad \text{for owners}$$

$$-\sigma y_{i,t} \leq b < B < +\infty \quad \text{for renters}$$

These thresholds b and B are optimally chosen by the household. There is no upper bound for the thresholds, but there is a lower bound based on the borrowing constraint $a_{i,t} \geq -(1-\theta)P^H_{j,t}h$ for owners and $a_{i,t} \geq -\sigma y_{i,t}$ for renters respectively. Therefore, the ratio $a_{i,t}/h_{i,t}$ has an inaction region which is the open interval (b, B). *See Proof in Appendix B.*

Corollary 2: The inaction region defined for a range of values for the wealth/housing ratio can be expressed in terms of time and defines the optimal period until the next housing adjustment τ'. The household's choice of the optimal period until the next adjustment will be such that:

$$\tau'(a/h, P^{H'}_j, \phi^{H'}_j) = \tau'(b, P^{H'}_j, \phi^{H'}_j) \wedge \tau'(B, P^{H'}_j, \phi^{H'}_j)$$

See Proof in Appendix B.

The optimal period until the next housing adjustment τ' therefore drives when households will choose to adjust their housing. Conditional on this decision, households can determine their remaining 2 control variables, non-housing and housing consumption, $c_{i,t}$ and h', or $h_{i,t}$ more generally. Thus, there exists optimal policy functions for these two variables, $c^\star$ and $h^\star$ that are determined jointly, conditional on $V(a, y, h, P^H, \phi^H) = V^{adj}(a, y, h, P^H, \phi^H)$:

$$c^*_{i,t} = c(a_{i,t}, y_{i,t}, h_{i,t}, P^H_{j,t}, \phi^H_{j,t})$$

$$h^*_{i,t} = h(a_{i,t}, y_{i,t}, c_{i,t}, P^H_{j,t}, \phi^H_{j,t})$$

D Model Solution and Computational Procedure

Households in the model are faced with an endogenous liquidity constraint, where owners can borrow up to some proportion of their housing value, and renters can borrow up to some proportion of their permanent income. To simplify the computational procedure, I follow the approach of Díaz and Luengo-Prado (2010). Based on this approach, households' liquid wealth state variable $a_{i,t}$ is transformed to a voluntary equity state variable $q_{i,t}$, where:

$$q_{i,t} = \begin{cases} a_{i,t} + (1-\theta)P^H_{j,t}h_{i,t} & \text{for owners} \\ a_{i,t} + \sigma p_{i,t} & \text{for renters} \end{cases} \tag{118}$$

Following the procedure outlined in Díaz and Luengo-Prado (2010) by substituting 118 in the budget constraint for each of the conditional value functions, the households' problem can be rewritten with respect to non-negative state variables $q_{i,t}$, $y_{i,t}$, $h_{i,t}$, $age_{i,t}$, $P^H_{j,t}$, $\phi^H_{j,t}$ and time t.

After applying these transformations, I solve the model by backward induction, assuming that the terminal utility of the household is $u(W_{i,T}) = B_0 \frac{1}{1-\rho^B} W_{i,T}^{1-\rho^B}$, where $W_{i,T}$ is the sum of liquid and housing wealth of the household in the terminal period, as defined in the paper with $W_{i,T} = (1+r_T)a_{i,T-1} + w^H_{i,T}$. This model's computational package has made use of some of the methods available in the HARK package (Carroll et al., 2018). Unlike models of consumption and saving, the complication in solving this paper's model is the discrete housing choice that is made before the continuous housing and non-housing consumption choices. To improve the efficiency of the model solution, I apply the nested endogenous grid method in combination with loop reordering developed by Druedahl (2021). For the upper envelope, the optimal housing consumption in the adjusting conditional value function is computed along the state variables using Nelder-Meade optimization. The value of adjusting, owning and renting are then compared to generate the

overall policy function. The process can be iterated backward until reaching the age of birth of households in the model.