Taxable and Tax-Deferred Investing: Implications for Target-Date Funds

By

Robert Dammon* and Chester Spatt**

March 16, 2026

*Robert Dammon is the Richard C. Green Professor of Financial Economics, Emeritus, and former Dean of the Tepper School of Business at Carnegie Mellon University.

**Chester Spatt is the Pamela and Kenneth Dunn Professor of Finance at the Tepper School of Business at Carnegie Mellon University.

Abstract

Target-Date Funds (TDFs) are diversified investment funds that offer investors a pre-determined path for asset allocations over their lifetimes. With TDFs, the asset allocations change over time (called the "glide path") to reduce portfolio risk as the investor approaches a "target retirement date". The rationale for declining portfolio risk is based upon the observation that the value of human capital is less risky than equity and declines as the investor ages. However, TDFs are designed for tax-deferred retirement accounts and are not appropriate for investors to hold in their taxable accounts due to the frequency of portfolio rebalancing and the realization of capital gains. The purpose of this paper is to investigate the optimal allocation of taxable and tax-deferred wealth during an individual's working years (i.e., prior to retirement) and identify the important factors that determine the optimal glide paths for taxable and tax-deferred accounts. We numerically solve a dynamic intertemporal model of asset allocation and use simulation methods to quantify the utility gains associated with following the optimal dynamic glide paths for the taxable and tax-deferred accounts relative to the standard "one size fits all" glide path for a typical TDF. The optimal glide paths for the taxable and tax-deferred accounts are shown to differ substantially from one another and from the glide path for a typical "one size fits all" TDF. The utility gains from following the optimal dynamic glide paths for the taxable and tax-deferred accounts are shown to be substantial.

1. Introduction

Target-Date Funds (TDFs) are diversified investment funds that offer investors a pre-determined path for asset allocations over their lifetimes. With TDFs, the asset allocations change over time (called the "glide path") to reduce portfolio risk as the investor approaches a "target date" (e.g., planned retirement date). The rationale for a downward-sloping glide path (i.e., a declining equity proportion as the investor ages) is based upon the observation that the value of human capital (i.e., the present value of future labor income) is less risky than equity and declines over time as the investor ages. The implication is that young investors should hold riskier financial portfolios (more equity) to balance off their relatively low risk, high value human capital.

Target-date funds became a central feature in many 401(k) and 403(b) tax-deferred retirement programs after the enactment of the Pension Protection Act (PPA) of 2006.[1] Prior to the passage of the PPA, plan sponsors typically assigned participants who did not make an investment election for their retirement plans into money market funds to avoid fiduciary liability risk. After passage of the PPA, target-date funds emerged as an important Qualified Default Investment Alternative (QDIA) for automatic enrollment without subjecting the sponsor to fiduciary liability. Since TDFs offer a higher expected rate of return, and manage the investor's risk exposure as they age, TDFs are viewed as a more appropriate default option than the traditional money market fund.

TDFs are typically designed for a specific target retirement date. Participants choose (or are assigned) to the TDF that most closely matches the participant's planned retirement date. The TDF glide path involves a reduction in the holdings of risky assets over time so that the TDF portfolio becomes less risky as it approaches its target date. One might view the TDF as intended for less financially sophisticated participants who are more comfortable delegating portfolio decisions to professional fund managers.[2] At the same time, this raises several issues with respect to the design of the TDF glide path. While participants in a specific TDF are likely to have similar target retirement dates (and ages), they may differ significantly

[1] Parker, Schoar, and Sun (2023) and Parker, Schoar, Cole and Simester (2025) emphasize the increased use of target-date funds since the enactment of the PPA and the latter also highlights the increase in equity ownership.
[2] The circumstances in which the participant selects the default portfolio rather than a customized portfolio is discussed in Spatt (2018).

along other important dimensions that impact portfolio choice. For example, investors with different degrees of risk aversion would optimally choose different portfolio holdings at different ages, resulting in different glide paths despite having identical target retirement dates. An important critique of the PPA framework is that it suggests that a "one glide path" approach meets the needs of all (or most) investors who utilize the default scenario. This can be a misleading and potentially harmful message for participants who have chosen to invest through standard TDFs.

The optimal glide path for an investor should be dependent upon both individual factors (e.g., risk aversion, wealth levels, future labor income, savings rates, tax rates, planned retirement date, etc.) and financial market factors (e.g., expected returns, and volatilities). Intertemporal models of consumption, savings, and investing that incorporate many of these factors have been developed by academics to study various important aspects of the problem.[3] The insights from these models have been used by investment management firms to develop proprietary models for glide path construction that create customized glide paths for their wealth management clients and standardized glide paths for their TDF offerings.[4]

One variable that is ignored by investment management firms in designing customized and standardized glide paths is the extent to which investors hold financial assets in both taxable and tax-deferred accounts. The implicit assumption being made by investment management firms is either that (1) investors only hold assets in tax-deferred retirement accounts, or (2) the optimal glide path is the same for both taxable and tax-deferred accounts. Although many investors do hold a significant portion of their savings in tax-

[3] Samuelson (1969) and Merton (1969, 1971) developed foundational models of lifetime consumption and investment in continuous time. Bodie, Merton, and Samuelson (1992) and Gomes, Kotlikoff, and Viceira (2008) incorporate an endogenous labor-leisure choice to show how human capital risk can affect life-time consumption and portfolio decisions. Heaton and Lucas (1996, 2000) and Constantinides and Duffie (1996) are part of a large literature that incorporates uninsurable labor income risk to study life-time savings and portfolio decisions. Dammon, Spatt, and Zhang (2004) examine the life-time consumption and investment decisions in an environment with both taxable and tax-deferred (retirement) savings accounts.

[4] While the proprietary models of glide path construction often incorporate many key factors that influence intertemporal investing (e.g., risk aversion, wealth levels, labor income, savings rates, retirement dates, replacement ratios (i.e., retirement consumption as a percentage of pre-retirement consumption), pension and social security benefits, mortality risk and bequest motives), they typically take consumption, savings, and replacement ratios to be exogenous. Aliaga-Diaz, Rayfield, Daga, and Dinerstein (2025) discuss Vanguard's proprietary life-cycle investing model.

deferred retirement accounts, it is obviously unrealistic to assume that non-retirement savings are zero. Dammon, Spatt and Zhang (2004) have shown that with taxable and tax-deferred accounts, investors optimally hold equity in the taxable account to the extent possible, with equity spilling over into the tax-deferred account only if the optimal holding of equity exceeds the amount of wealth in the taxable account.[5] This implies that (1) the optimal glide path (i.e., the allocation of wealth to risky assets) is likely to differ between the taxable and tax-deferred accounts and (2) these glide paths depend upon the split of wealth between the two accounts.

Another weakness of proprietary glide path construction, even when customized for a specific individual, is that the glide path is invariant to the realization of portfolio returns and the evolution of the investor's wealth levels. For example, over time the ratio of the investor's wealth in the taxable and tax-deferred accounts will change as investment returns are realized and new savings are invested. These stochastic changes in the investor's wealth levels will influence the optimal asset allocations in the taxable and tax-deferred accounts, making the optimal glide paths dynamic and state dependent. Hence, optimal glide path construction is not a one-time exercise, but requires dynamic adjustments through time.

The purpose of this paper is to investigate the optimal allocation of taxable and tax-deferred wealth during an individual's working years (i.e., prior to retirement) and identify the important factors that determine the optimal glide paths for taxable and tax-deferred accounts. We are also interested in quantifying the utility gains associated with following the optimal dynamic glide paths for the taxable and tax-deferred accounts relative to the standard "one size fits all" glide path for a typical TDF. The utility gains can be interpreted as the value of full customization of the TDF, where both asset allocation and asset location are dynamically optimized for the individual's specific circumstance.

[5] Equity is a desirable security for at least some of his funds in the taxable account because of its relatively low effective tax rate. Interest income on bonds and money market funds is taxed as ordinary income, while qualified dividends and capital gains are taxed at lower preferential rates (the effective taxation of capital gains may be even lower due to deferral of capital gains in practice). In practice, the investor possesses a valuable tax-timing option whereby capital losses can be realized when they occur (subject to loss limitations), while capital gains can be deferred indefinitely. See Constantinides (1983, 1984) and Dammon and Spatt (1996) for foundational work on tax-timing options.

2. The Model

Our main purpose is to investigate how taxable and tax-deferred savings and investing influence the optimal asset allocations over an individual's working years. To this end, we employ a simplified model of portfolio choice in which the individual maximizes the expected utility of terminal (retirement date) wealth. We incorporate labor income and savings exogenously into the model and, for simplicity, assume that future labor income is known with certainty.[6] Although we could allow savings rates to differ at various ages, we assume that the savings rate, and the proportion of savings that goes into the taxable and tax-deferred accounts, are held constant over the individual's working years.

Target-Date Funds (TDFs) typically invest in a host of asset classes, including domestic equity, international equity, domestic bonds, international bonds, inflation-indexed bonds, commodities, and, sometimes, alternative investments like private equity. We will simplify our analysis by assuming that there are only two asset classes: (1) a risky asset (which can broadly be thought of as a diversified portfolio of equity securities) and (2) a riskless asset (which can be broadly thought of as short-term government securities and money market funds). Interest rates, dividend yields, expected capital gain returns, and return volatilities are assumed to be constant over time.

Financial assets held in the taxable account are subject to taxation. Interest income from the riskless asset is taxed as ordinary income. In contrast, dividends and capital gains are taxed at preferential rates. Although in practice capital gains are taxed only when assets are sold, we assume that capital gains (and losses) are taxed on an accrual basis every year.[7]

---

[6] Exogenous labor income and savings rates imply that intertemporal consumption is also exogenous in our model. We could endogenize the consumption and savings decisions, as in Dammon, Spatt, and Zhang (2004), but this modification would not materially change our main conclusion regarding the need for customized glide paths for the taxable and retirement accounts. Likewise, as other authors have shown, labor income risk would produce a desire for precautionary savings and less equity holdings in the taxable account to reduce the overall risk of the investor's total wealth (including human capital). Since labor income risk would needlessly complicate our model without adding new insights, we have chosen to abstract from labor income risk in our model.

[7] This assumption greatly simplifies the model since it eliminates the tax basis as a state variable and allows us to make portfolio adjustments in the taxable account without the need to consider the tax consequences of doing so. However, it also results in an undervaluation of equity and can lead to a reduction in the optimal holding of equity in the taxable account.

At the end of each year t, the individual's financial wealth, $W_{F,t}$, is composed of the after-tax value of the taxable and retirement accounts. The wealth in these two accounts evolves through time as follows: [8]

$$W_{NR,t} = W_{NR,t-1}\{1 + x_{t-1}[d(1 - t_d) + g_t(1 - t_g)] + (1 - x_{t-1})r(1 - t_o)\} + S_{NR,t} \qquad (1a)$$

and

$$W_{R,t} = W_{R,t-1}\{1 + z_{t-1}(d + g_t) + (1 - z_{t-1})r\} + S_{R,t} \qquad (1b)$$

where $W_{NR,t}$ is taxable (non-retirement) wealth, $W_{R,t}$ is retirement wealth, x_t and z_t are the fractions of taxable and retirement wealth invested in the risky asset, d is the constant dividend yield on the risky asset, g_t is the capital gain (loss) on the risky asset, r is the risk-free interest rate, t_d is the tax rate on dividend income, t_g is the tax rate on capital gains (losses), t_o is the tax rate on interest income, and $S_{NR,t}$ and $S_{R,t}$ are the additional savings added to the taxable and retirement accounts, respectively.

The individual's total wealth, W_t, is composed of financial wealth plus the present value of future after-tax labor income, $PV_t(L)$:

$$W_t = W_{NR,t} + W_{R,t} + PV_t(L) \qquad (2)$$

where $PV_t(L)$ can be thought of as the value of the individual's human capital at date t.

The individual's optimization problem can now be expressed as follows:

$$\underset{x_t,\, z_t}{MAX} \quad E_t[U(W_T)], \qquad t = 0, 1, \ldots, T - 1 \qquad (3)$$

s.t.

[8] Wealth is measured on an after-tax basis. For retirement accounts in which the individual invests after-tax dollars (e.g., Roth IRAs) the after-tax value is simply the year-end balance of the account. For traditional pre-tax retirement accounts, the after-tax value is equal to the year-end balance multiplied by $(1-t_o)$, where t_o is the individual's ordinary tax rate. In essence, for pre-tax retirement accounts the government is effectively a co-investor owning the fraction t_o of the assets held in the account with the individual owning the remainder.

$$0 \le x_t \le 1 \text{ and } 0 \le z_t \le 1, \qquad t = 0, 1, \ldots, T-1 \tag{4a}$$

$$W_t = W_{F,t} + PV_t(L) \tag{4b}$$

$$W_{F,t} = W_{NR,t} + W_{R,t} \tag{4c}$$

$$PV_t(L) = \sum_{j=t+1}^{T} \frac{L_1(1+i)^{j-1}}{[1+r(1-t_o)]^{j-t}} = L_t(1+i)A_t \tag{4d}$$

$$W_{NR,t+1} = W_{NR,t}\{1 + x_t[d(1-t_d) + g_{t+1}(1-t_g)] + (1-x_t)r(1-t_o)\} + S_{NR,t+1} \tag{4e}$$

$$W_{R,t+1} = W_{R,t}\{1 + z_t(d + g_{t+1}) + (1-z_t)r\} + S_{R,t+1} \tag{4f}$$

$$S_{NR,t+1} = y_{NR}L_t(1+i) \tag{4g}$$

$$S_{R,t+1} = y_R L_t(1+i) \tag{4h}$$

where $L_t = L_1(1+i)^{t-1}$ is labor income at date t, i is the annual growth rate in labor income, y_{NR} is the savings rate for the taxable (non-retirement) account, y_R is the savings rate for the retirement account, and A_t is the present value factor for a (T-t)-period growing annuity:

$$A_t = \left[\frac{1}{r(1-t_o) - i}\right]\left[1 - \frac{(1+i)^{T-t}}{[1+r(1-t_o)]^{T-t}}\right]$$

The choice variables for the optimization problem are the fraction of wealth invested in equity in the taxable account, x_t, and the fraction of wealth invested in equity in the

retirement account, z_t.[9] The relevant state variables are: $H_t = \{W_{F,t}, W_{NR,t}, L_t\}$.[10] As a quick reference, **Table 1** provides a list of the variables and notation used in our model.

Bellman's (1952) optimality condition allows us to define an intertemporal value function for the above optimization problem:

$$V(W_t, H_t) = \underset{x_t,\, z_t}{MAX} \quad E_t[V(W_{t+1}, H_{t+1})], \qquad t = 0, 1, \ldots, T-1 \tag{5}$$

subject to the constraints in (4a)-(4h). At date T, the value function is $V(W_T, H_T) = U(W_T)$.

Define $w_{F,t} = W_{F,t}/W_t$ as the ratio of financial wealth to total wealth, $w_{NR,t} = W_{NR,t}/W_{F,t}$ as the fraction of financial wealth invested in the taxable (non-retirement) account, $w_{R,t} = W_{R,t}/W_{F,t} = (1\text{-}w_{NR,t})$ as the fraction of financial wealth invested the retirement account, and $l_t = L_t/W_t$ as the labor income-to-total wealth ratio. Then, the dynamic wealth equation can be written as follows:

$$W_{t+1} = W_{NR,t+1} + W_{R,t+1} + PV_{t+1}(L) \tag{6}$$

$$= W_t\{[w_{F,t}w_{NR,t}\rho_{NR,t+1} + y_{NR}l_t(1+i)] + [w_{F,t}(1-w_{NR,t})\rho_{R,t+1} + y_R l_t(1+i)]$$

$$+ l_t(1+i)[A_t[1 + r(1-t_o)] - 1]\}$$

where $\rho_{NR,t+1}$ is one plus the after-tax return on taxable wealth at date t+1 and $\rho_{R,t+1}$ is one plus the return on retirement wealth at date t+1.

$$\rho_{NR,t+1} = \{1 + x_t[d(1-t_d) + g_{t+1}(1-t_g)] + (1-x_t)r(1-t_o)\} \tag{7a}$$

[9] Note that the individual's consumption at date t is exogenous and equal to $C_t = L_t - S_{NR,t} - S_{R,t} = L_1(1+i)^{t-1}[1\text{-}y_{NR} - y_R]$. The exogeneity of consumption is due to the assumption that future labor income and savings rates are exogenous, which is the standard approach taken by financial firms in designing portfolio allocations for their clients. Dammon, Spatt, and Zhang (2004) endogenize both the consumption and portfolio allocation decisions in an environment with both a taxable and retirement account. Interestingly, the intertemporal asset allocations that arise in the current model are consistent with those derived in Dammon, Spatt, and Zhang (2004) even though we do not solve endogenously for optimal intertemporal consumption.

[10] There are alternative state variables that provide equivalent information. Any set of state variables that allows us to determine W_t, $W_{NR,t}$, $W_{R,t}$, $S_{R,t}$, $S_{NR,t}$, and $PV_t(L)$ is acceptable.

and

$$\rho_{R,t+1} = \{1 + z_t(d + g_{t+1}) + (1 - z_t)r\} \qquad (7b)$$

Dividing the left- and right-hand sides of Equation (6) by total wealth, W_t, produces the following expression:

$$\rho_{t+1} = w_{F,t}\{w_{NR,t}\rho_{NR,t+1} + (1 - w_{NR,t})\rho_{R,t+1}\} \qquad (8)$$

$$+l_t(1 + i)\{y_{NR} + y_R + A_t[1 + r(1 - t_o)] - 1\}$$

where $\rho_{t+1} = W_{t+1}/W_t$ is one plus the growth rate in total wealth.

Substituting Equation (8) into Equation (5) allows us to write the value function as follows:

$$V(W_t, H_t) = \underset{x_t,\, z_t}{MAX}\ E_t[V(W_t\rho_{t+1}, H_{t+1})], \qquad t = 0, 1, \ldots, T - 1 \qquad (9)$$

Next, we augment the above maximization problem by introducing Lagrange multipliers into Equation (9) to account for the upper and lower bounds for x_t and z_t in Equation (4a).

$$V(W_t, H_t) = \underset{x_t,\, z_t}{MAX}\ E_t[V(W_t\rho_{t+1}, H_{t+1})] + W_{NR,t}[\lambda_{1,t}x_t + \lambda_{2,t}(1 - x_t)] \qquad (10)$$

$$+W_{R,t}[\lambda_{3,t}z_t + \lambda_{4,t}(1 - z_t)]$$

where the second term in Equation (10) deals with the restriction that the fraction of equity in the taxable account satisfies $0 \le x_t \le 1$ and the third term deals with the restriction that the fraction of equity in the retirement account satisfies $0 \le z_t \le 1$. In particular, $\lambda_{1,t} \ge 0$ if $x_t^* = 0$, $\lambda_{2,t} \ge 0$ if $x_t^* = 1$, $\lambda_{3,t} \ge 0$ if $z_t^* = 0$, and $\lambda_{4,t} \ge 0$ if $z_t^* = 1$.

The first-order condition for x_t is:

$$\frac{\partial V(W_t, H_t)}{\partial x_t} = W_t E_t\left\{V'_W\left(\frac{\partial \rho_{t+1}}{\partial x_t}\right)\right\} + W_{NR,t}(\lambda_{1,t} - \lambda_{2,t}) = 0 \qquad (11)$$

where $V'_W = \partial V\left(W_{t+1,}H_{t+1}\right)/\partial W_{t+1}$. But from the definition of ρ_{t+1} in Equation (8) we have:

$$\frac{\partial \rho_{t+1}}{\partial x_t} = w_{\mathrm{F,t}} w_{\mathrm{NR,t}}\left[\mathrm{d}(1-\mathrm{t_d}) + \mathrm{g_{t+1}}\left(1-\mathrm{t_g}\right) - \mathrm{r}(1-\mathrm{t_o})\right] \tag{12}$$

Substituting Equation (12) into Equation (11), recognizing that $w_{F,t}w_{NR,t} = W_{NR,t}/W_t$, we have the following first-order condition for x_t:

$$\mathrm{E_t}\left\{\mathrm{V'_W}\left[\mathrm{d}(1-\mathrm{t_d}) + \mathrm{g_{t+1}}\left(1-\mathrm{t_g}\right) - \mathrm{r}(1-\mathrm{t_o})\right]\right\} + \lambda_{1,\mathrm{t}} - \lambda_{2,\mathrm{t}} = 0 \tag{13a}$$

Similarly, the first-order condition for z_t is:

$$\mathrm{E_t}\{\mathrm{V'_W}[\mathrm{d} + \mathrm{g_{t+1}} - \mathrm{r}]\} + \lambda_{3,\mathrm{t}} - \lambda_{4,\mathrm{t}} = 0 \tag{13b}$$

The optimality conditions in Equations (13a) and (13b) make clear that the optimal holding of equity in the taxable and retirement accounts cannot both be positive unless $x_t^* = 1$ (i. e. , $\lambda_{2,\mathrm{t}} > 0$) or all tax rates are equal (i.e., $t_d = t_g = t_o$). To see this, let $r > d$ and $t_g \leq t_d < t_o$. Then, if the optimal holding of equity in the retirement account satisfies $0 < z_t < 1$, the Lagrange multipliers in Equation (13b) are zero and the optimality condition becomes:

$$\mathrm{E_t}\{\mathrm{V'_W}[\mathrm{d} + \mathrm{g_{t+1}} - \mathrm{r}]\} = 0 \tag{14}$$

Substituting Equation (14) into Equation (13a) and simplifying terms yields:

$$\mathrm{E_t}[\mathrm{V'_W}][\mathrm{r}\left(\mathrm{t_0} - \mathrm{t_g}\right) - \mathrm{d}(\mathrm{t_d} - \mathrm{t_g})] + \lambda_{1,\mathrm{t}} - \lambda_{2,\mathrm{t}} = 0 \tag{15}$$

Since $\mathrm{r}\left(\mathrm{t_0} - \mathrm{t_g}\right) > \mathrm{d}(\mathrm{t_d} - \mathrm{t_g})$, Equation (15) requires $\lambda_{2,\mathrm{t}} > 0$. This implies that the optimal equity holding in the taxable account is at a corner solution in which $x_t = 1$. This leads to the following ***optimal asset location policy***: [11]

> The optimal asset location policy involves holding equity in the taxable account to the extent possible, with equity spilling over into the retirement

[11] This optimal asset location policy was derived previously by Dammon, Spatt, and Zhang (2004).

account only if the optimal overall holding of equity exceeds the individual's wealth in the taxable account. In other words, $z_t^* > 0\%$ only if $x_t^* = 100\%$.

In what follows, we assume that the individual has constant relative risk averse (CRRA) preferences, which ensures that our model has the property that the asset allocation decision rules $\{x_t, z_t\}$ are independent of total wealth, W_t. With CRRA preferences, the utility of terminal wealth is:

$$U(W_T) = \left(\frac{W_T^{1-\gamma}}{1-\gamma}\right) = \left(\frac{W_t^{1-\gamma}}{1-\gamma}\right)\left[\prod_{j=t+1}^{T} \rho_j^{1-\gamma}\right] \tag{16}$$

Dividing both sides of Equation (16) by $(W_t)^{1-\gamma}$ yields the following normalized value function at date t:

$$v(h_t) = \underset{x_t,\, z_t}{\text{MAX}}\ E_t[v(h_{t+1})] + [\lambda_{1,t}x_t + \lambda_{2,t}(1-x_t)] + [\lambda_{3,t}z_t + \lambda_{4,t}(1-z_t)] \tag{17}$$

where

$$v(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\phi_{t+1}(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\prod_{j=t+2}^{T}\left(\rho_j^*\right)^{1-\gamma} \tag{18}$$

$\phi_{t+1}(h_{t+1})$ is a time-dependent function of future one-period gross returns under the optimal investment policies $\{x_j^*, z_j^*\}$, $j = t+1,\ldots,T-1$, $h_t = \{w_{F,t}, w_{NR,t}\}$ is the vector of state variables at date t, and $v(h_T) = \left(\rho_T^{1-\gamma}\right)/(1-\gamma)$ at the terminal date.

It is worth noting that if ρ_{t+1} and $\phi_{t+1}(h_{t+1})$ are independent at time t, then

$$E_t\left[\left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\phi_{t+1}(h_{t+1})\right] = E_t\left[\left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\right]E_t[\phi_{t+1}(h_{t+1})] \tag{19}$$

This implies that the optimal asset allocations, $\{x_t, z_t\}$, can be found by solving a one-period optimization problem of the form:

$$\underset{x_t,\, z_t}{\text{MAX}} \quad \mathrm{E_t}\left[\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right] + [\lambda_{1,t}x_t + \lambda_{2,t}(1-x_t)] + [\lambda_{3,t}z_t + \lambda_{4,t}(1-z_t)] \qquad (20)$$

In this case, the optimal investment policy is myopic.

In our model, however, ρ_{t+1} and $\phi_{t+1}(h_{t+1})$ are not independent since the realization of ρ_{t+1} will influence the vector of state variables at future dates, $h_j = \{w_{F,j}, w_{NR,j}\}$, j= t+1, … , T-1. In this case, solving for the optimal holding of equity in the taxable and retirement accounts for each date t involves the use of dynamic programming. Because analytical solutions are not available, it is necessary to use numerical methods to derive solutions to the intertemporal optimization problem. Appendix A provides the details of the numerical solution methodology we employ.

3. Numerical Solutions

We begin by presenting the solution to the intertemporal investment problem for the base-case parameters in **Table 1**. **Figure 1** is a 3D representation of the optimal equity allocations in the taxable and tax-deferred accounts for an individual at age 30 (T = 35 years until retirement). The optimal equity allocations are shown as a function of the value of future labor income (normalized by total wealth) and the value of taxable wealth (normalized by the value of financial wealth). [12] While the 3D surface in **Figure 1** illustrates how the optimal holdings of equity vary across the entire state space, a cross-section of **Figure 1** shown in 2D can be easier to interpret.

Figure 2 shows the optimal holdings of equity in the taxable and tax-deferred accounts as a function of the value of future labor income (normalized by total wealth) for three different levels of taxable wealth (normalized by the value of financial wealth). There are several interesting observations that can be made about the equity allocations shown in **Figure 2**. First, note that the optimal equity holdings are very different in the taxable (**Figure 2A**) and retirement (**Figure 2B**) accounts. As shown earlier, the optimal asset location policy is to hold equity in the taxable account to the extent possible, with equity spilling over into the

[12] Recall that financial wealth is the wealth held in the taxable and retirement accounts combined. Total wealth is equal to financial wealth plus the present value of future labor income.

retirement account only if the taxable account is invested entirely in equity. Second, the optimal overall holding of equity (**Figure 2C**) is higher for individuals with higher levels of labor income (as percentage of total wealth). This is because labor income is riskless in our model (although generally less risky than equity), giving the individual an incentive to invest a higher percentage of financial wealth in equity to achieve the desired overall risk exposure. Finally, the optimal overall holding of equity is also increasing with the level of taxable wealth (as a percentage of financial wealth) as shown in **Figure 2C**. Since the taxable account is the preferred location for equity, the higher the percentage of financial wealth in the taxable account the higher the optimal overall holding of equity.

The optimal equity allocations illustrated in **Figure 2** have important implications for target-date funds. Target-date funds are typically designed for tax-deferred retirement accounts and for individuals with similar retirement dates. But as **Figure 2** illustrates, the optimal asset allocations depend upon much more than just the planned retirement date.

Take, for example, two individuals with the same 35-year target retirement date, identical risk aversion ($\gamma = 3$), and identical savings rates ($y_{NR} = 4\%$ and $y_R = 16\%$). Investor A has 20% of his financial wealth invested in a taxable account, while investor B has 80% of her financial wealth invested in a taxable account. Both investors have human capital (present value of future labor income) that represents 50% of total wealth. The optimal equity allocations for these two individuals are as follows:

	Individual A ($w_{NR} = 20\%$)	**Individual B ($w_{NR} = 80\%$)**
Taxable Account Equity	100.0%	100.0%
Retirement Account Equity	48.5%	0.0%
Overall Equity	58.8%	80.0%

Both investors allocate 100% of their taxable wealth to equity. However, the optimal holding of equity in the retirement account, and the overall equity allocations, are quite different for these two individuals. Target-date funds that invest identically for all investors based solely on the target retirement date will likely be suboptimal for at least some, and perhaps even most, investors. (Utility losses associated with following the suboptimal investment policy advocated by a typical target-date fund will be investigated in a later section.)

Figure 2 highlights the importance of the split of wealth between the taxable and retirement accounts and the value of human capital for optimal asset allocation. A much more obvious difference across investors that clearly has an important bearing on optimal asset allocation is risk aversion. **Figure 3** replicates **Figure 2** for a risk aversion parameter of $\gamma = 5$. All other variables take on the same base-case parameter values as in **Figure 2**.

From **Figure 3**, risk aversion plays an important role in the determination of the optimal equity holdings for both the taxable and retirement accounts, as well as for the overall portfolio. Not surprisingly, higher risk aversion reduces the optimal holding of equity in both the taxable and retirement accounts, except possibly when the optimal holding of equity hits a boundary constraint of either 0% or 100%.

The magnitude of the effect of risk aversion on the optimal equity allocations depends upon the split of wealth between the taxable and retirement accounts and the value of future labor income (human capital). The table below shows the interaction effects. Clearly, risk aversion, the split of wealth between the taxable and retirement accounts, and the value of future labor income (human capital) are three important variables to consider when designing optimal portfolio allocations for individuals.

	Risk Aversion: $\gamma = 3$		**Risk Aversion: $\gamma = 5$**	
Panel A: Labor Income = 20%	**$w_{NR} = 20\%$**	**$w_{NR} = 80\%$**	**$w_{NR} = 20\%$**	**$w_{NR} = 80\%$**
Equity in Taxable Account	100.0%	90.2%	100.0%	53.4%
Equity in Retirement Account	41.6%	0.0%	17.8%	0.0%
Overall Equity Proportion	53.3%	72.1%	34.3%	42.7%
Panel B: Labor Income = 80%	**$w_{NR} = 20\%$**	**$w_{NR} = 80\%$**	**$w_{NR} = 20\%$**	**$w_{NR} = 80\%$**
Equity in Taxable Account	100.0%	100.0%	100.0%	71.2%
Equity in Retirement Account	67.5%	59.7%	27.9%	0.0%
Overall Equity Proportion	74.0%	91.9%	42.3%	56.9%

Next, consider the optimal asset allocations for individuals with different target retirement dates. **Figure 4** illustrates the optimal equity holdings for three target retirement dates (T = 5, 20, 35). **Figure 4** is drawn assuming that wealth in the taxable account is 50% of financial

wealth. All other variables are set at their base-case parameter values. It is surprising to note about **Figure 4** is how similar the optimal equity allocations are across all three target retirement dates. Although younger investors, with more distant target retirement dates, will likely have more valuable human capital and higher equity holdings, **Figure 4** suggests that the target retirement date itself is somewhat irrelevant.

To see what we mean, consider three individuals with the following attributes and optimal equity allocations:

	Investor A	**Investor B**	**Investor C**
Target Date (Years)	5	20	35
Value of Human Capital (Pct. of Total Wealth)	10%	45%	80%
Equity in Taxable Account	100%	100%	100%
Equity in Retirement Account	16.8%	30.4%	66.3%
Overall Equity Proportion	58.4%	65.2%	83.2%

The above table makes clear that individuals with more distant target retirement dates hold more equity, which is the primary thesis of target-date funds. However, we observe that the higher equity holdings for individuals with more distant retirement dates (or younger age) is simply a consequence of the target date being highly correlated with the value of human capital. As **Figure 4** illustrates, what really matters is not so much the target retirement date, but the value of the investor's human capital relative to total wealth. Once the value of human capital is accounted for, the planned retirement date has less importance.[13]

With both taxable and retirement savings, there is a need to take an integrated approach to the design of optimal investment portfolios. Optimal asset allocations should utilize the retirement account to hold highly-taxed assets (e.g., taxable fixed income securities), while the taxable account is more appropriate for tax-advantaged securities (e.g., equities and tax-

[13] The target retirement date (and indirectly the investor's age) still has some relevance for the investor because it determines the rate at which the value of human capital declines over the remaining years prior to retirement. For example, two individuals with identical values for future labor income, but with different target retirement dates, will experience different rates of decline in the value of their human capital over time, which will influence their current and future portfolio choices.

exempt bonds). Moreover, the optimal holdings in these two accounts will depend primarily upon four individual-specific factors: risk aversion, the split of wealth between the taxable and retirement accounts, the present value of future labor income (human capital), and the future savings rates for the taxable and retirement accounts.[14] The target retirement date (or the investor's age) is of secondary importance, despite it being the primary focus of most target-date funds.

4. Simulation and Welfare Analysis

In this section, we conduct a simulation analysis to examine how the investor's optimal portfolio allocations in the taxable and retirement accounts evolve over time and compare the utility gains (losses) against a typical target-date fund investment policy. We consider an individual at age 20 who is planning to retire at age 65 (T = 45 years). Our simulations are conducted using the base-case parameter values in Table 1 and assume that equity returns follow a lognormal distribution.

We simulate returns, wealth accumulation, and utility under three different investment policies for the taxable and retirement accounts: (1) a target-date fund policy, (2) a policy with the same year-by-year portfolio risk as the target-date fund, but with tax-efficient asset location across taxable and retirement accounts, and (3) an optimal dynamic asset allocation and location policy for the taxable and retirement accounts. A more detailed description of each of these policies is provided below:

4.1. Target-Date Fund Policy

We assume a typical target-date fund equity allocation that declines as a function of the investor's age as shown in **Figure 5**. The equity allocation remains constant at 95% from age 20 to age 30, gradually declines to 60% by age 60, and then declines further to 50% by age 65. Because target-date funds are designed primarily for retirement accounts, they implicitly assume that either (1) investors do not have, and will never have, wealth invested in taxable accounts or (2) the equity allocations for both taxable and retirement accounts are the

[14] The value of future labor income, combined with the future savings rates, determine the value of future savings in both the taxable and tax-deferred accounts. It is the value of future savings in these two accounts that is relevant for the design of the investor's portfolio.

same. Since the first of these assumptions seems unrealistic, we impose the same equity allocation in the taxable account as in the retirement account when evaluating the performance of the target-date fund strategy.

4.2. Target-Date Fund Risk Profile with Optimal Asset Location

As discussed earlier, maintaining identical asset allocations in both taxable and retirement accounts is tax inefficient. The optimal asset location policy is to hold equity in the taxable account to the extent possible, with equity spilling over into the retirement account only if the taxable account is invested entirely in equity. This leads us to an alternative policy in which the risk profile of the target-date fund is maintained over time, but where assets are held in the most tax efficient location.

The risk of the investor's financial wealth (taxable plus retirement wealth) under the target-date fund strategy changes over time for two reasons: (1) the split of wealth between the taxable and retirement accounts evolves based upon the realization of asset returns and (2) the allocation to risky equity under the target-date fund policy is declining over time according to the pre-specified glide path. The alternative investment policy we have in mind allocates wealth in the taxable and retirement accounts to achieve two objectives: (1) match the year-by-year risk profile (standard deviation of after-tax returns) of the target-date fund strategy and (2) hold assets in the taxable and retirement accounts to maximize tax efficiency. Notice that this alternative investment policy, while tax efficient in terms of asset location, may not necessarily be optimal in terms of its overall risk exposure.

4.3. Optimal Dynamic Asset Allocation and Location

The optimal dynamic asset allocation and location policy maximizes the expected utility of the investor's terminal (target date) after-tax wealth. It is the solution to the dynamic intertemporal investment problem presented in **Section 2**. Under this policy, the investor's optimal asset allocations in the taxable and retirement accounts will evolve over time as a function of (1) the split of wealth between the taxable and retirement accounts, (2) the present value of future labor income (human capital), and (3) the remaining time until retirement. Assets are held in the taxable and retirement accounts to maximize tax efficiency.

4.4. Simulation and Welfare Results

We run 5,000 simulations over a 45-year period. The initial conditions (at age 20) are that the fraction of financial wealth held in the taxable account is 50% and the present value of labor income as a percentage of total wealth is 90%. We assume constant relative risk averse (CRRA) preferences and set all other parameters of our model to their base-case values shown in **Table 1**.

As is the nature of target-date funds, the asset allocations over time follow a fixed, pre-determined glide path as shown in **Figure 5**. When asset returns push the portfolio allocations away from the glide path, the target-date funds rebalance the portfolio back to the glide path. Once set, the glide path is rarely, if ever, changed.

In contrast, the glide paths (optimal equity allocations) change dynamically over time under both alternative investment policies discussed earlier. **Figures 6** shows the glide paths for the two alternative investment policies for one unique realization of equity returns over the 45-year period. Notice that the glide paths are dynamic and depend upon the specific pattern of realized returns. This is particularly true for the optimal dynamic policy shown in **Figure 6B**, where the optimal equity allocations depend upon both (1) the dynamics of the investor's wealth in the taxable and retirement accounts and (2) the value of human capital as a percentage of total wealth.

The optimal equity holdings shown in **Figure 6** are for a simulation in which equity returns are above average (average annual realized capital gain return of 13%). Notice that in this specific simulation the taxable account is optimally invested entirely in equity over most of the 45-year period, with a slight drop below 100% for a couple of years when the investor is in his mid-40s. The optimal equity allocation in the retirement account declines rapidly from 100% in the early-20s to 0% by age 45 and then remains near 0% through retirement. Finally, the overall equity allocation declines from 100% in the early-20s to about 65% in the mid-30s, before rising again to approximately 70% from age 45 through retirement. These optimal equity holdings show a very different pattern from the glide path shown in **Figure 5** for the target-date fund.

As a comparison, **Figure 7** replicates **Figure 6** for a different sequence of realized equity returns over a 45-year period. The optimal equity holdings shown in **Figure 7** are for a simulation in which equity returns are below average (average annual realized capital gain return of 4%). For this sequence of realized equity returns, the optimal dynamic policy shown in **Figure 7B** allocates 100% of the wealth in the taxable account to equity throughout the 45-year period. Unlike **Figure 6B**, the optimal holding of equity in the retirement account remains at 100% for a much longer period of time (through the early-30s). From there, the optimal equity holding in the retirement account gradually declines to about 40% by the target retirement date but never comes close to hitting 0% over the entire 45-year period. The optimal overall holding of equity also remains at 100% through the early-30s, before declining to about 55% by the target retirement date. Once again, the patterns shown in **Figures 6 and 7** illustrate that the optimal holdings of equity (i.e., the glide path) is different for taxable and retirement accounts and evolve dynamically over time in response to changes in the split of wealth between the taxable and retirement accounts and changes in the present value of future labor income.

Next, consider the differences in expected utility for the three investment strategies. **Table 2** summarizes the results over the 5,000 simulations. In **Table 2**, we report the average terminal wealth relative (i.e., ratio of after-tax terminal wealth to initial wealth), the average utility of terminal wealth, and the certainty equivalent of terminal wealth.[15] The results are reported for three different risk aversion parameters (γ = 1, 3, 5).[16] Panel A reports the results for the case in which the value of future labor income represents 90% of the investor's total wealth at age 20. Panels B and C report similar results when the value of future labor income as a percentage of total wealth at age 20 is 70% and 50%, respectively.

Consider first the comparison of the target-date fund policy (first column in **Table 2**) with the alternative policy that maintains the same portfolio risk as the target-date fund but does so by locating assets in a tax efficient manner (second column in **Table 2**). Surprisingly, the alternative policy performs only slightly better than the target-date fund. The certainty equivalent of terminal wealth for the alternative policy is only 0.6% to 2.5% higher than the

[15] The certainty equivalent is the after-tax wealth the investor would be willing to accept with certainty at the terminal date instead of the risky after-tax wealth generated by his portfolio. The utility of the certainty equivalent is equal to the expected utility of risky terminal wealth for the investor.
[16] Note that γ = 1 corresponds to log utility.

certainty equivalent for the target-date fund across all three panels of **Table 2**. It is tempting to interpret this result as suggesting that the glide paths for target-date funds, if used for both taxable and retirement accounts alike, do not cost investors much in terms of utility losses. However, it is worth noting that in our model capital gains and losses are taxed on an accrual basis, eliminating the possibility of deferring the realization of capital gains. With the possibility of capital gain deferral, the optimal asset location policy becomes more valuable to investors.[17]

The optimal dynamic investment policy (last column in **Table 2**) generates the highest utility gains for investors. For investors with moderate risk aversion ($\gamma = 3$), the gains relative to the target-date fund are modest, ranging from 1.9% to 9.2% across the three panels of **Table 2**. This suggests that the glide path we assumed for the target-date fund (see **Figure 5**) is reasonable for investors with moderate risk. However, for investors who are more or less risk averse, the utility gains are significantly higher. The glide path for the target-date fund is suboptimal for these investors, investing too much wealth in equity for investors with high risk aversion ($\gamma = 5$) and too little wealth in equity for investors with low risk aversion ($\gamma = 1$). The utility gains for the optimal dynamic investment policy are around 24%-25% for investors with $\gamma = 1$ and between 12.9% and 53.3% for investors with $\gamma = 5$.

One way to interpret these utility gains is that they reflect the value of customization in portfolio construction and management over an investor's lifetime. Customization involves the dynamic optimization of portfolio allocations considering the evolution of the investor's taxable and retirement wealth, value of human capital, risk aversion, and savings rates. As we have seen, the utility gains from this customization can be significant and more than outweigh any costs of providing a customized approach for investors.

[17] Dammon, Spatt and Zhang (2001) characterize the optimal dynamic consumption and portfolio decisions in the presence of capital gains taxes. Their results capture the tradeoff between the diversification benefits and tax costs of portfolio rebalancing over an investor's lifetime. They show that the investor's incentive to rebalance is inversely related to the size of the embedded capital gain and the investor's age. Optimal equity holdings are shown to increase for elderly investors due to the forgiveness of capital gains taxes at death.

5. Concluding Comments

Standard TDFs are designed for a specific target retirement date and assume that either (1) all wealth is held in the tax-deferred account or (2) the optimal asset allocations are the same for the taxable and tax-deferred accounts. Furthermore, TDF glide paths are designed for the "average" investor, assuming population averages for risk aversion, value of human capital, and savings rates. This makes investment in a TDF most appropriate for the "average" investor, with most (or all) of their current wealth and future savings in a tax-deferred account. For investors with both taxable and tax-deferred wealth and future savings, or whose risk aversion, value of human capital, or savings rates differ from the "average" investor, an investment in a TDF may not be utility maximizing.

In this paper, we utilize dynamic programming methods to derive the optimal glide paths for both the taxable and tax-deferred accounts as a function of the investor's age, split of financial wealth between the taxable and tax-deferred accounts, and the value of human capital. We show that the utility gains from following the optimal dynamic glide paths are substantial in comparison to the TDF glide path, especially for investors with either higher or lower risk aversion than the "average" investor. These utility gains can be interpreted as the gains to customization in the design of the optimal glide paths.

Under the Pension Protection Act (PPA), the plan sponsor can automatically enroll the plan participant into the plan's default option. In the case of a TDF, this would specify the investor's asset allocation based solely on the participant's age (or years until retirement). While a participant's optimal asset allocation depends upon much more than age, it is difficult to incorporate these other factors (e.g., risk aversion, split of wealth between taxable and tax-deferred accounts, value of human capital, and savings rates) into the design of a default TDF that meets the requirements of the PPA. The inherent lack of knowledge of the participant's risk aversion and other relevant state variables, and the desire to meet the needs of many plan participants, limits the design of a single default TDF. Nevertheless, as we have shown, there is considerable value in following a dynamic strategy based upon the evolution of the relevant state variables.

In a future draft of the paper we plan to investigate how investors can design a "do-it-yourself" TDF using a simplified approach that can be easily implemented in practice.

References

Aliaga-Díaz, R. A., Rayfield, Z., Daga, A., & Dinerstein, M. (2025). Vanguard's Life-Cycle Investing Model (VLCM): A general portfolio framework for goals-based investing. Vanguard Research.

Bodie, Z., Merton, R. C., & Samuelson, W. F. (1992). Labor supply flexibility and portfolio choice in a life cycle model. *Journal of Economic Dynamics and Control, 16*(3–4), 427–449. https://doi.org/10.1016/0165-1889(92)90044-F

Constantinides, G. (1983). Capital market equilibrium with personal tax. *Econometrica*, 51, 611-636.

Constantinides, G. (1984). Optimal stock trading with personal taxes: Implications for prices and the abnormal January returns. *Journal of Financial Economics*,13, 65-89.

Constantinides, G. M., & Duffie, D. (1996). Asset pricing with heterogeneous consumers. *Journal of Political Economy*, *104*(2), 219–240. https://doi.org/10.1086/262023

Dammon, R. M., & Spatt, C. S. (1996). The optimal trading and pricing of securities with asymmetric capital gains taxes and transaction costs. *Review of Financial Studies, 9*(3), 921–952. https://doi.org/10.1093/rfs/9.3.921

Dammon, R. M., Spatt, C. S., & Zhang, H. H. (2001). Optimal consumption and investment with capital gains taxes. *Review of Financial Studies, 14*(3), 583–616.

Dammon, R. M., Spatt, C. S., & Zhang, H. H. (2004). Optimal asset location and allocation with taxable and tax-deferred investing. *Journal of Finance, 59*(3), 999–1037. https://doi.org/10.1111/j.1540-6261.2004.00655.x

Gomes, F. J., Kotlikoff, L. J., & Viceira, L. M. (2008). Optimal life-cycle investing with flexible labor supply: A welfare analysis of life-cycle funds. *American Economic Review: Papers & Proceedings, 98*(2), 297–303. https://doi.org/10.1257/aer.98.2.297

Heaton, J., & Lucas, D. (1996). Evaluating the effects of incomplete markets on risk sharing and asset pricing. *Journal of Political Economy, 104*(3), 443–487. https://doi.org/10.1086/262030

Heaton, J., & Lucas, D. (2000). Portfolio choice and asset prices: The importance of entrepreneurial risk. *Journal of Finance, 55*(3), 1163–1198. https://doi.org/10.1111/0022-1082.00244

Merton, R. C. (1969). Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case. *Review of Economics and Statistics,* 51(3), 247–257.

Merton, R, C. (1971). Optimum Consumption and Portfolio Rules in a Continuous-Time Model. *Journal of Economic Theory,* 3(4), 373–413.

Parker, J. A., Schoar, A., Cole, A., & Simester, D. (2025). Household portfolios and retirement saving over the life cycle. *Journal of Finance*, 80(5), 2739–2787. https://doi.org/10.1111/jofi.13473

Parker, J. A., Schoar, A., & Sun, Y. (2023). Retail financial innovation and stock market dynamics: The case of target date funds. *Journal of Finance*, 78(5), 2673–2723. https://doi.org/10.1111/jofi.13258

Samuelson, P. A. (1969). Lifetime portfolio selection by dynamic stochastic programming. *Review of Economics and Statistics,* 51(3), 239–246. https://doi.org/10.2307/1926559

Spatt, C. S. (2018). The role of the employer default allocation in defined-contribution retirement plan design. (Research Dialogue No. 149). TIAA Institute.

Table 1: Model Variables and Notation

Variable	Notation	Base-Case Values
Return Variables:		
Risk-free interest rate	R	3.0%
Dividend yield on risky asset	D	1.0%
Capital gain return on risky asset	g	7.0%
Standard deviation of returns on risky asset	σ	20.0%
Tax Rates:		
Ordinary income tax rate	t_o	36.0%
Dividend tax rate	t_d	20.0%
Capital gains tax rate	t_g	20.0%
Utility and Value Function:		
Utility of terminal wealth	$U(W_T)$	
Value function at date t	$V(H_t)$	
Vector of state variables at date t	H_t	
Constant of relative risk aversion	γ	3.0
Target (retirement) date	T	35 years
Labor Income and Savings Rate Variables:		
After-tax labor income at date t	L_t	
Annual growth rate in labor income	i	2.0%
Savings rate (as percent of labor income) in the retirement account	y_R	16%
Savings rate (as percent of labor income) in the taxable account	y_{NR}	4%
Wealth Variables:		
Total wealth at date t	W_t	
Financial wealth at date t	$W_{F,t}$	
Wealth held in the retirement account at date t	$W_{R,t}$	
Wealth held in the taxable (non-retirement) account at date t	$W_{NR,t}$	
Present value of future labor income at date t	$PV_t(L)$	
After-tax growth rate (gross) in total wealth at date t	ρ_t	
After-tax growth rate (gross) in retirement wealth at date t	$\rho_{R,t}$	
After-tax growth rate (gross) in taxable wealth at date t	$\rho_{NR,t}$	
Choice Variables:		
Proportion of taxable account wealth invested in equity at date t	x_t	
Proportion of retirement account wealth invested in equity at date t	z_t	
Normalized Variables:		
Labor income-to-total wealth at date t	l_t	
Financial wealth as fraction of total wealth at date t	$w_{F,t}$	
Retirement wealth as fraction of financial wealth at date t	$w_{R,t}$	
Taxable wealth as fraction of financial wealth at date t	$w_{NR,t}$	
Value function normalized by total wealth at date t	$v(h_t)$	
Vector of normalized state variables at date t	h_t	

Table 2: Welfare Analysis

Panel A:

Initial Conditions (at age 20)				
Taxable Wealth/Financial Wealth	50.00%			
PV of Labor Income/Total Wealth	90.00%			
Risk Aversion Parameter (CRRA)	1		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$2.74	$2.74	$4.30
Expected Utility of Terminal Wealth		0.7469	0.7527	0.9635
Certainty Equivalent of Terminal Wealth		$2.11	$2.12	$2.62
Pct. Improvement Over TDF Policy			**0.58%**	**24.18%**
Risk Aversion Parameter (CRRA)	3		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$2.73	$2.74	$2.34
Expected Utility of Terminal Wealth		-0.2610	-0.2558	-0.2515
Certainty Equivalent of Terminal Wealth		$1.38	$1.40	$1.41
Pct. Improvement Over TDF Policy			**1.01%**	**1.87%**
Risk Aversion Parameter (CRRA)	5		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$2.67	$2.68	$1.64
Expected Utility of Terminal Wealth		-0.2243	-0.2140	-0.1382
Certainty Equivalent of Terminal Wealth		$1.03	$1.04	$1.16
Pct. Improvement Over TDF Policy			**1.18%**	**12.87%**

Table 2: Welfare Analysis

Panel B:

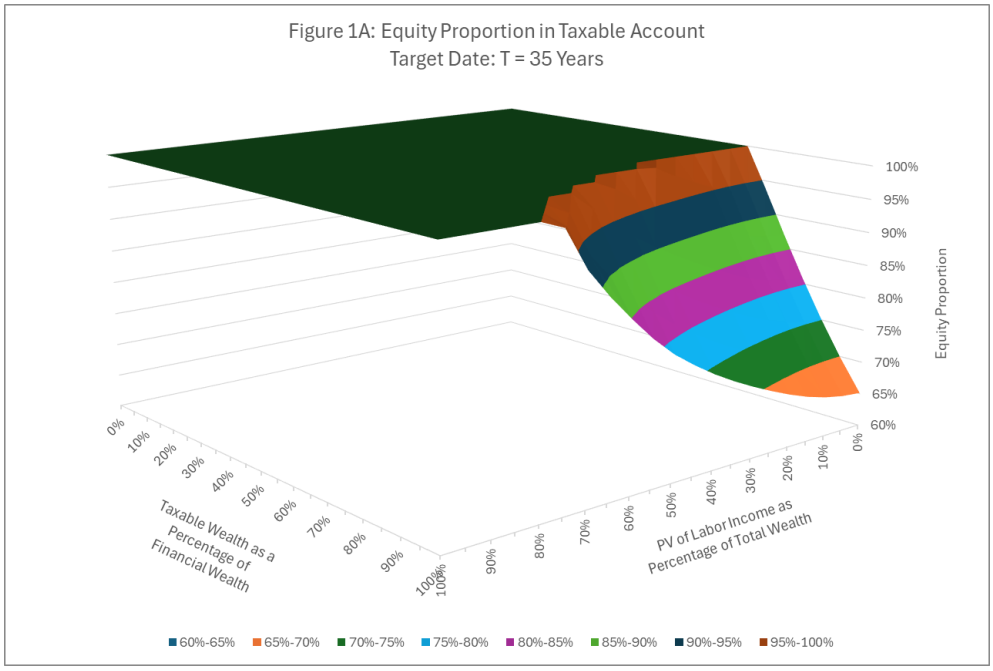
Initial Conditions (at age 20)				
Taxable Wealth/Financial Wealth	50.00%			
PV of Labor Income/Total Wealth	70.00%			
Risk Aversion Parameter (CRRA)	1		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$5.49	$5.51	$8.64
Expected Utility of Terminal Wealth		1.3764	1.3843	1.5967
Certainty Equivalent of Terminal Wealth		$3.96	$3.99	$4.94
Pct. Improvement Over TDF Policy			**0.79%**	**24.65%**
Risk Aversion Parameter (CRRA)	3		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$5.46	$5.47	$4.07
Expected Utility of Terminal Wealth		-0.0999	-0.0969	-0.0909
Certainty Equivalent of Terminal Wealth		$2.24	$2.27	$2.35
Pct. Improvement Over TDF Policy			**1.54%**	**4.83%**
Risk Aversion Parameter (CRRA)	5		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$5.53	$5.55	$2.79
Expected Utility of Terminal Wealth		-0.0705	-0.0652	-0.0198
Certainty Equivalent of Terminal Wealth		$1.37	$1.40	$1.89
Pct. Improvement Over TDF Policy			**1.97%**	**37.37%**

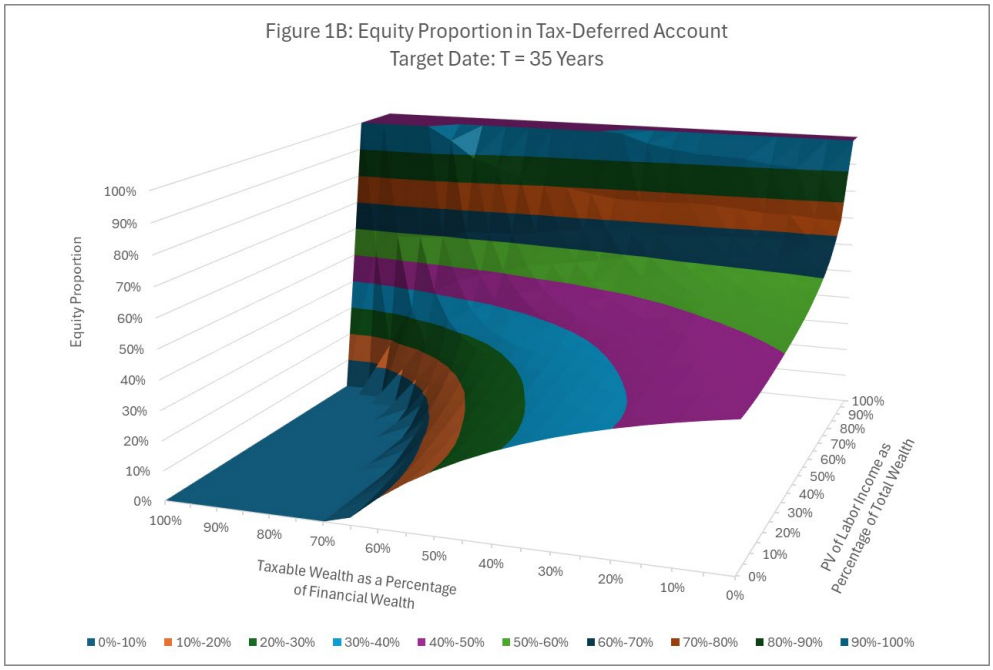
Table 2: Welfare Analysis

Panel C:

Initial Conditions (at age 20)				
Taxable Wealth/Financial Wealth	50.00%			
PV of Labor Income/Total Wealth	50.00%			
Risk Aversion Parameter (CRRA)	1		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$8.11	$8.13	$12.86
Expected Utility of Terminal Wealth		1.7272	1.7362	1.9443
Certainty Equivalent of Terminal Wealth		$5.62	$5.68	$6.99
Pct. Improvement Over TDF Policy			**0.90%**	**24.25%**
Risk Aversion Parameter (CRRA)	3		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$8.22	$8.24	$5.72
Expected Utility of Terminal Wealth		-0.0575	-0.0554	-0.0482
Certainty Equivalent of Terminal Wealth		$2.95	$3.00	$3.22
Pct. Improvement Over TDF Policy			**1.88%**	**9.22%**
Risk Aversion Parameter (CRRA)	5		**TDF with**	**Optimal Dynamic**
		TDF	**Optimal Location**	**Policy**
Expected Wealth Relative $E(W_T)/W_0$		$8.31	$8.34	$3.90
Expected Utility of Terminal Wealth		-0.0304	-0.0276	-0.0055
Certainty Equivalent of Terminal Wealth		$1.69	$1.73	$2.60
Pct. Improvement Over TDF Policy			**2.45%**	**53.33%**

Figure 1: 3D Surface of Optimal Equity Allocations (Using Base-Case Parameter Values)





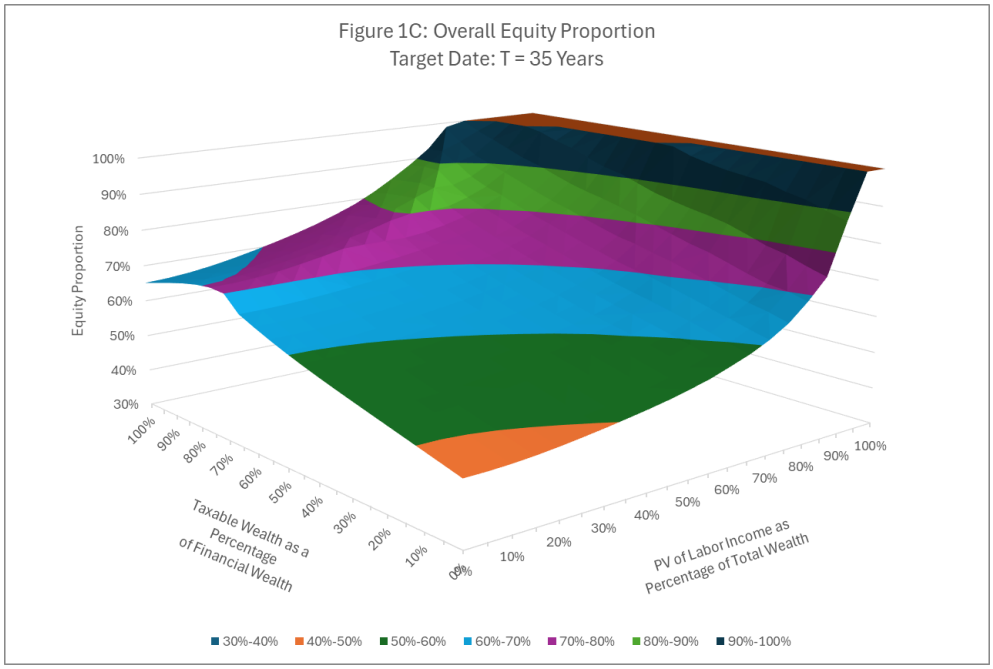
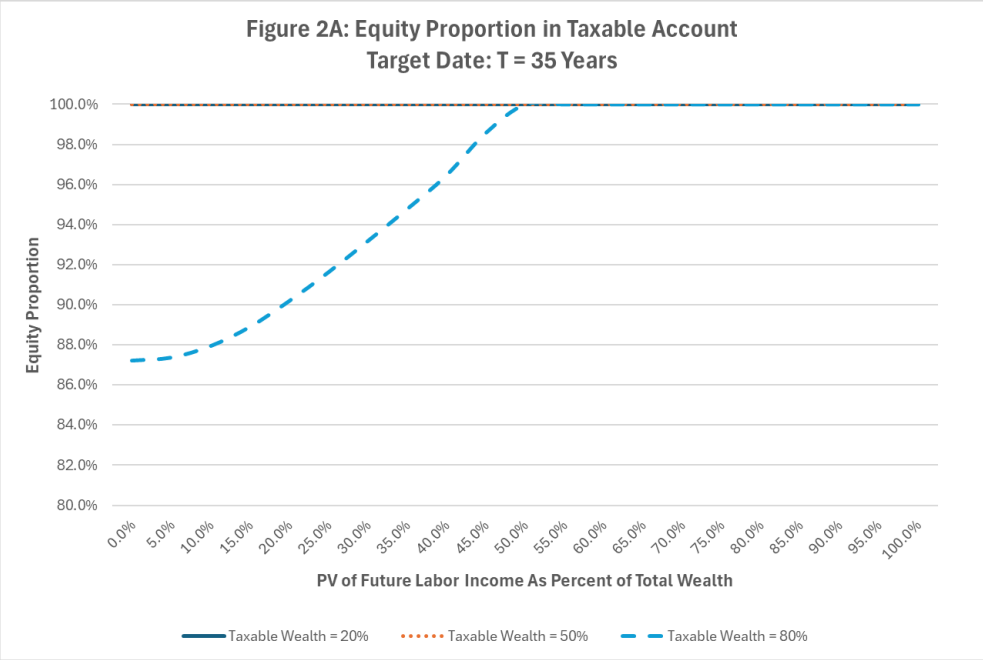
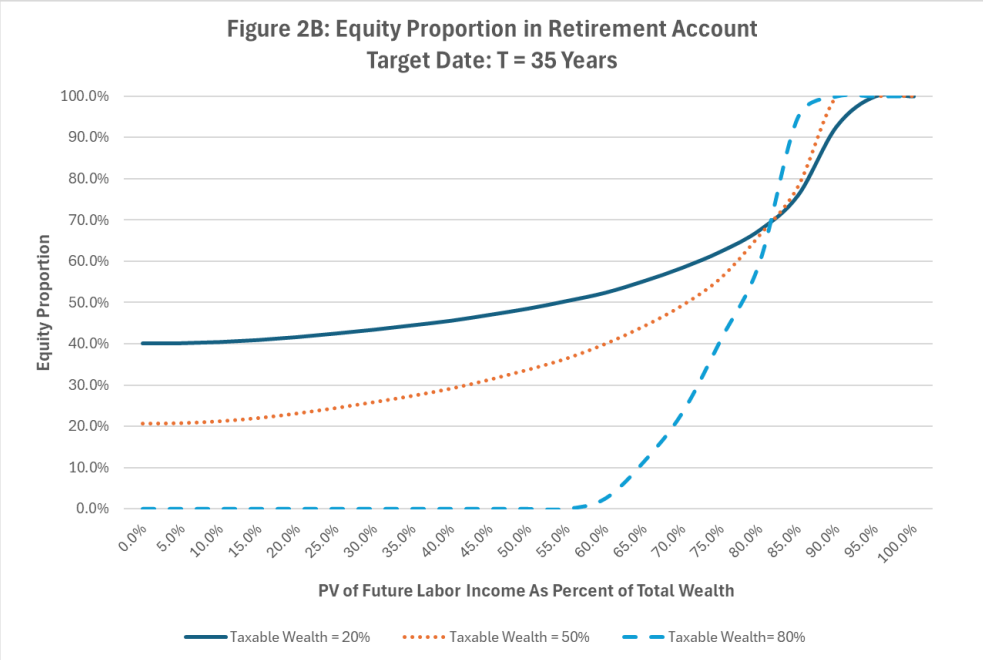

Figure 1C: Overall Equity Proportion
Target Date: T = 35 Years
Equity Proportion
100%
90%
80%
70%
60%
50%
40%
30%
Taxable Wealth as a Percentage of Financial Wealth
PV of Labor Income as Percentage of Total Wealth
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
30%-40%
40%-50%
50%-60%
60%-70%
70%-80%
80%-90%
90%-100%

Figure 2: Cross-Section of Optimal Equity Allocations (Using Base-Case Parameter Values)





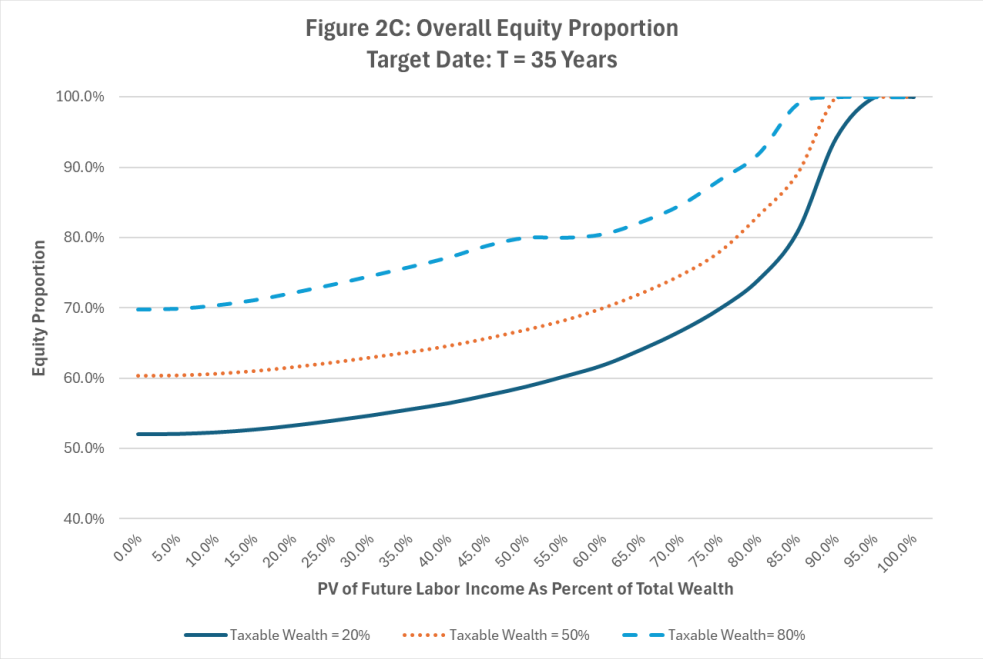
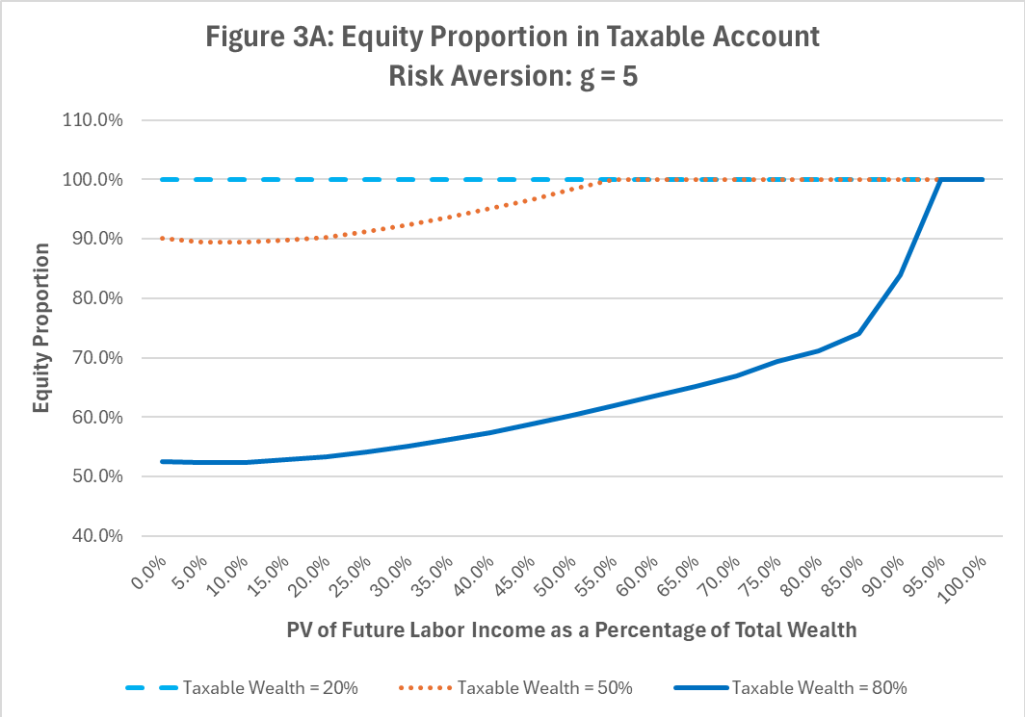
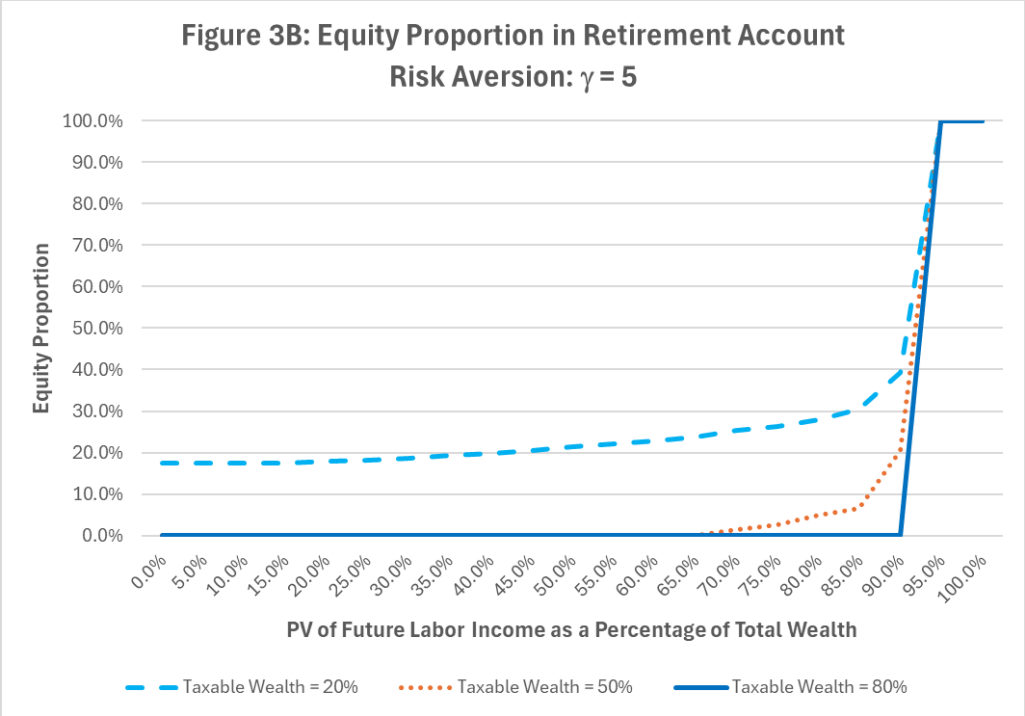

Figure 2C: Overall Equity Proportion
Target Date: T = 35 Years
Equity Proportion
100.0%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
0.0% 5.0% 10.0% 15.0% 20.0% 25.0% 30.0% 35.0% 40.0% 45.0% 50.0% 55.0% 60.0% 65.0% 70.0% 75.0% 80.0% 85.0% 90.0% 95.0% 100.0%
PV of Future Labor Income As Percent of Total Wealth
Taxable Wealth = 20%
Taxable Wealth = 50%
Taxable Wealth= 80%

Figure 3: Cross-Section of Optimal Equity Allocations
Risk Aversion: $\gamma = 5$





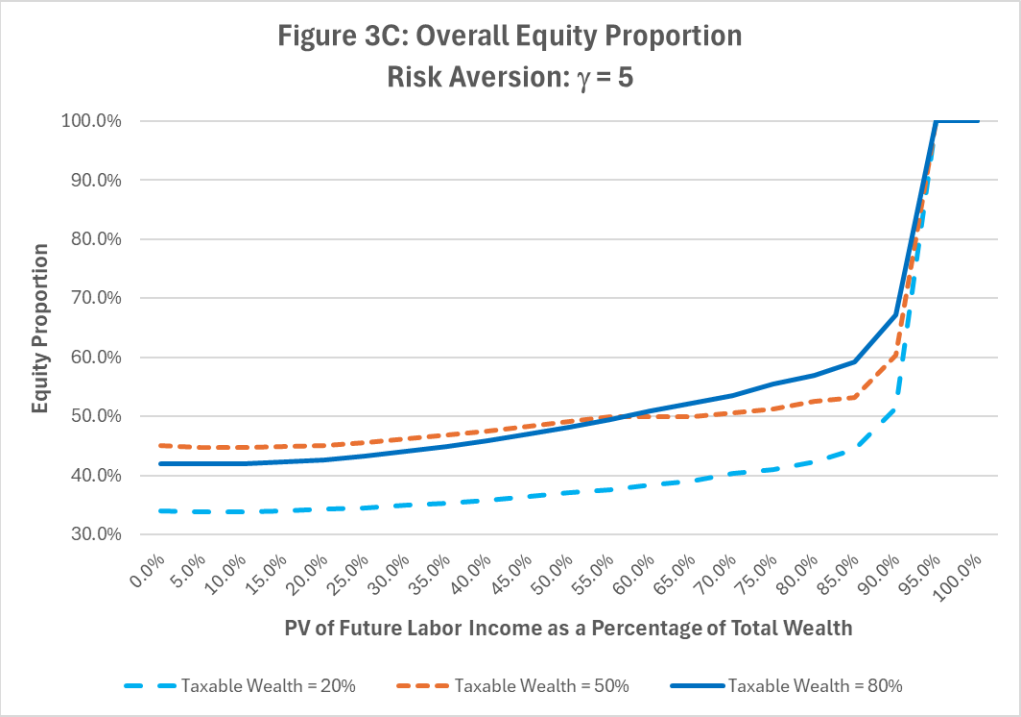
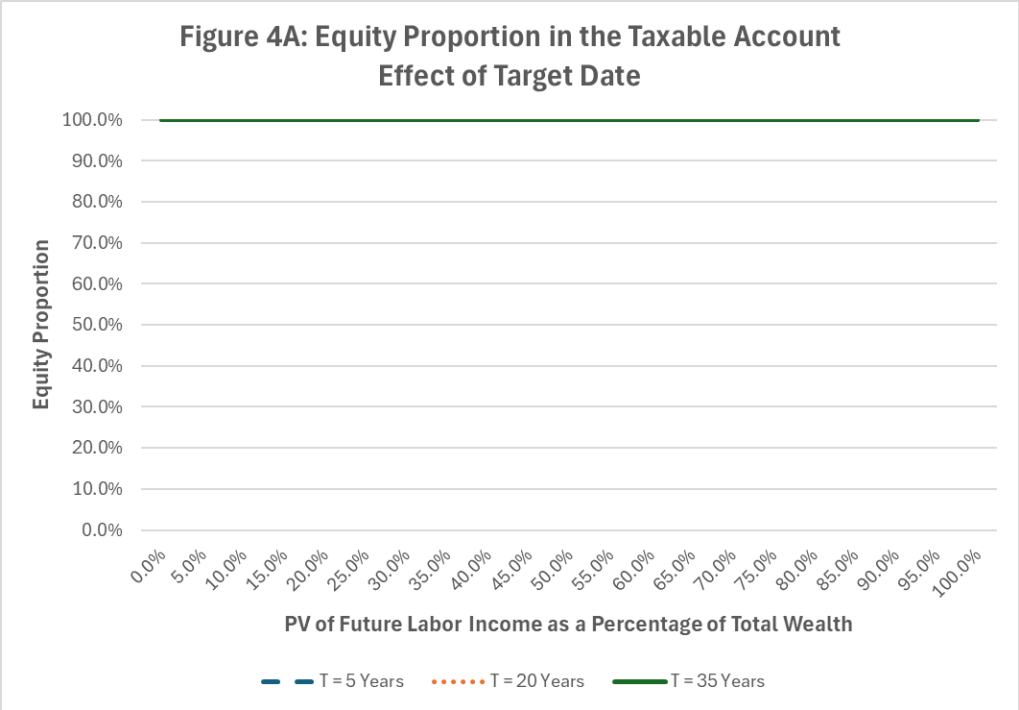
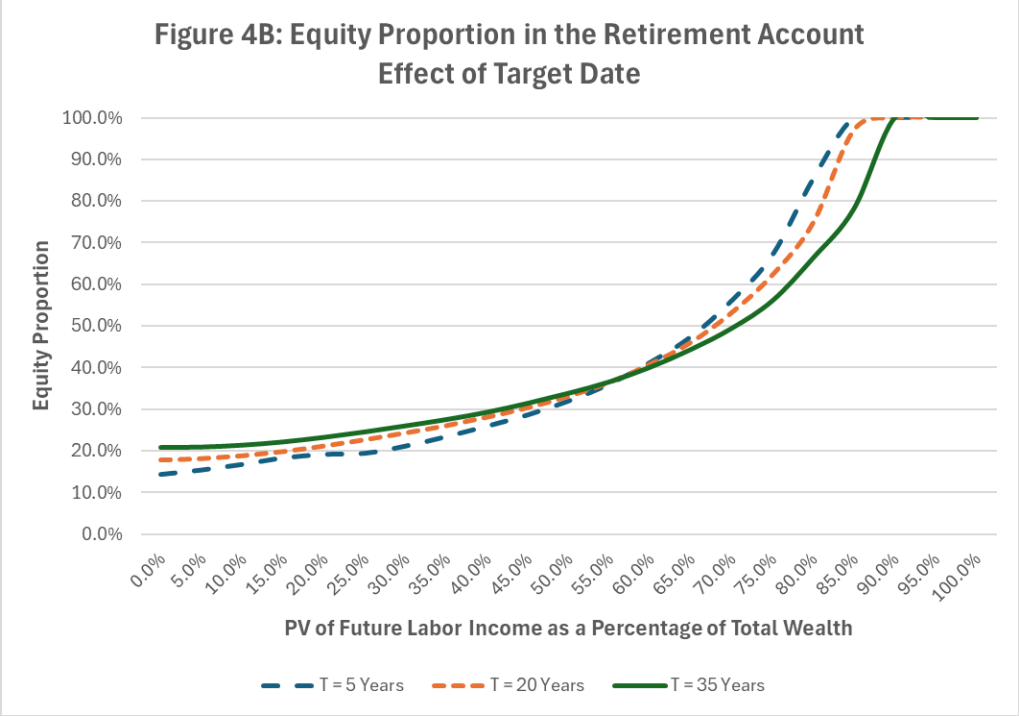

Figure 3C: Overall Equity Proportion
Risk Aversion: γ = 5
Equity Proportion
100.0%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
0.0% 5.0% 10.0% 15.0% 20.0% 25.0% 30.0% 35.0% 40.0% 45.0% 50.0% 55.0% 60.0% 65.0% 70.0% 75.0% 80.0% 85.0% 90.0% 95.0% 100.0%
PV of Future Labor Income as a Percentage of Total Wealth
Taxable Wealth = 20%
Taxable Wealth = 50%
Taxable Wealth = 80%

**Figure 4: Effect of Age (Target Retirement Date)
on Optimal Equity Allocations**





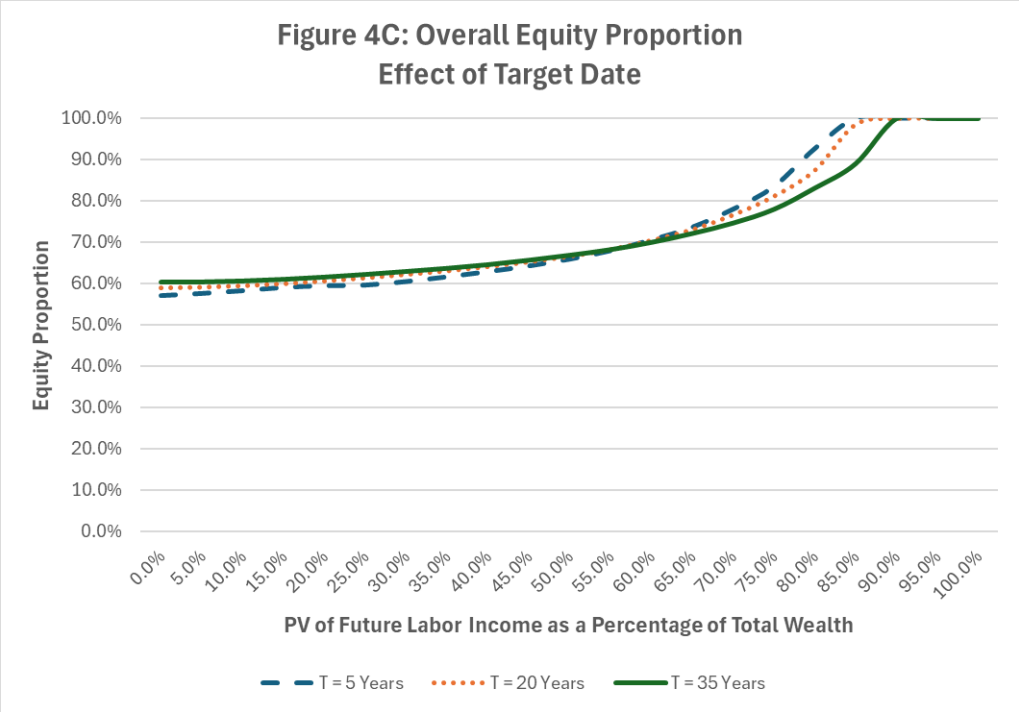

Figure 4C: Overall Equity Proportion
Effect of Target Date
Equity Proportion
100.0%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
0.0% 5.0% 10.0% 15.0% 20.0% 25.0% 30.0% 35.0% 40.0% 45.0% 50.0% 55.0% 60.0% 65.0% 70.0% 75.0% 80.0% 85.0% 90.0% 95.0% 100.0%
PV of Future Labor Income as a Percentage of Total Wealth
T = 5 Years
T = 20 Years
T = 35 Years

Figure 5: Target-Date Fund Glide Path

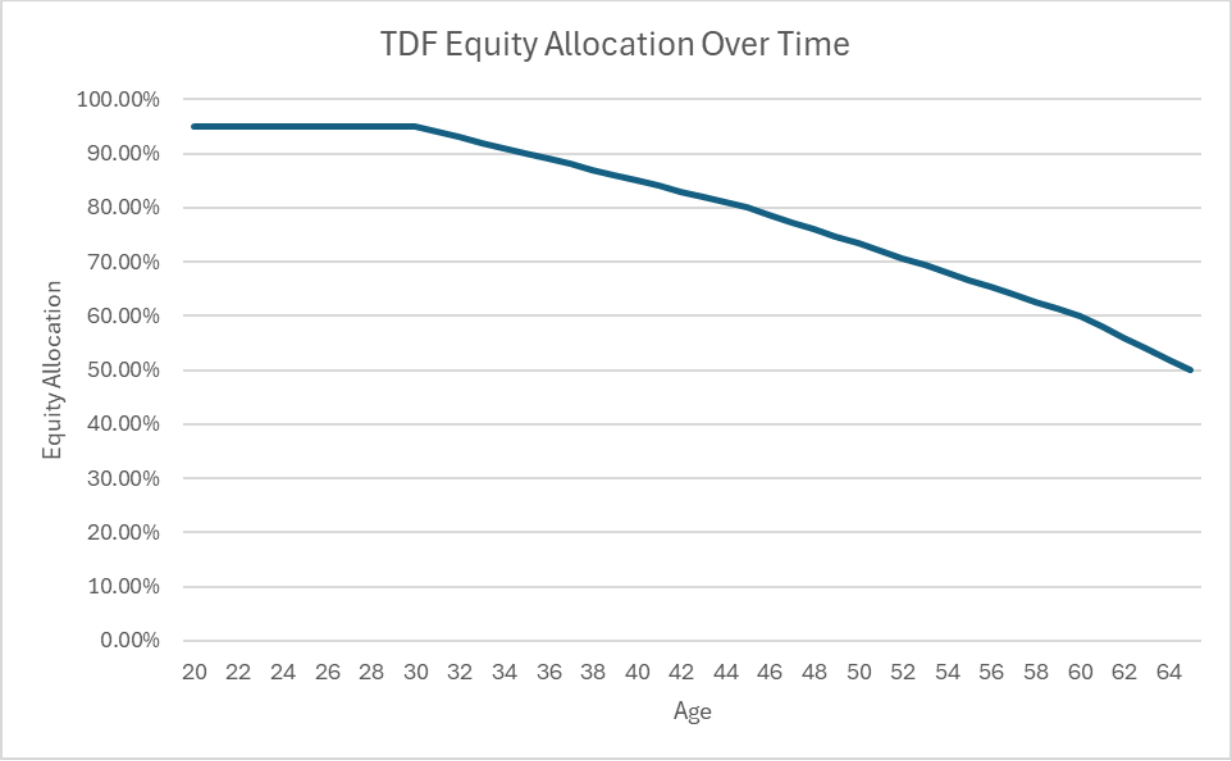
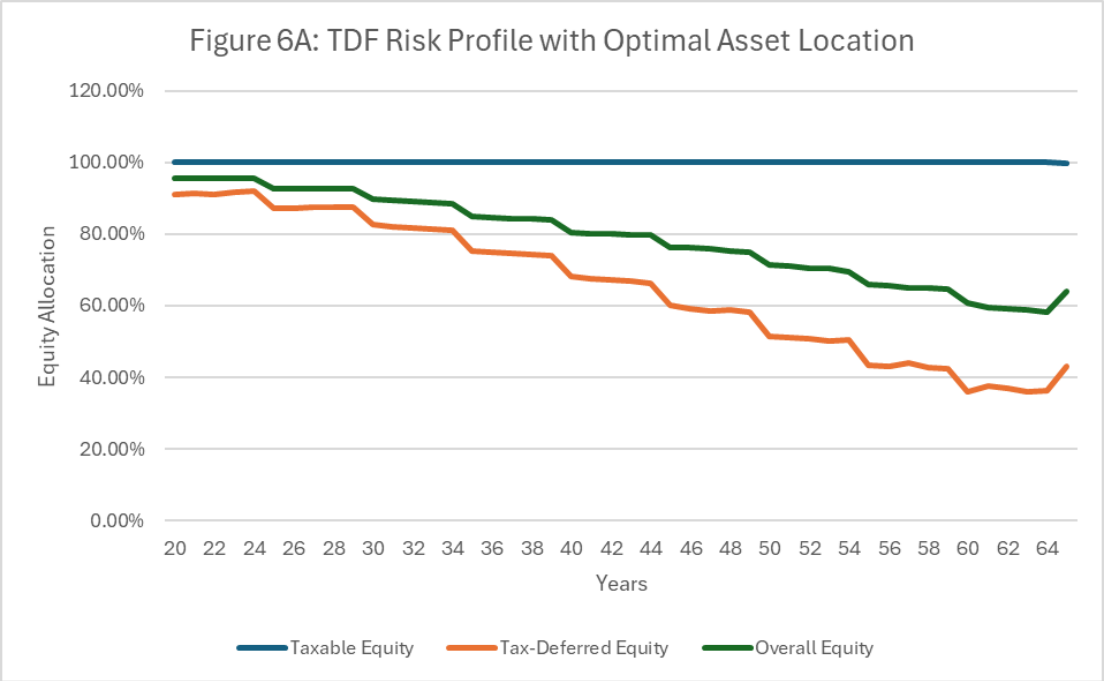


Figure 6: Dynamic Glide Paths for Alternative Investment Policies (Realized Equity Returns Above Average)



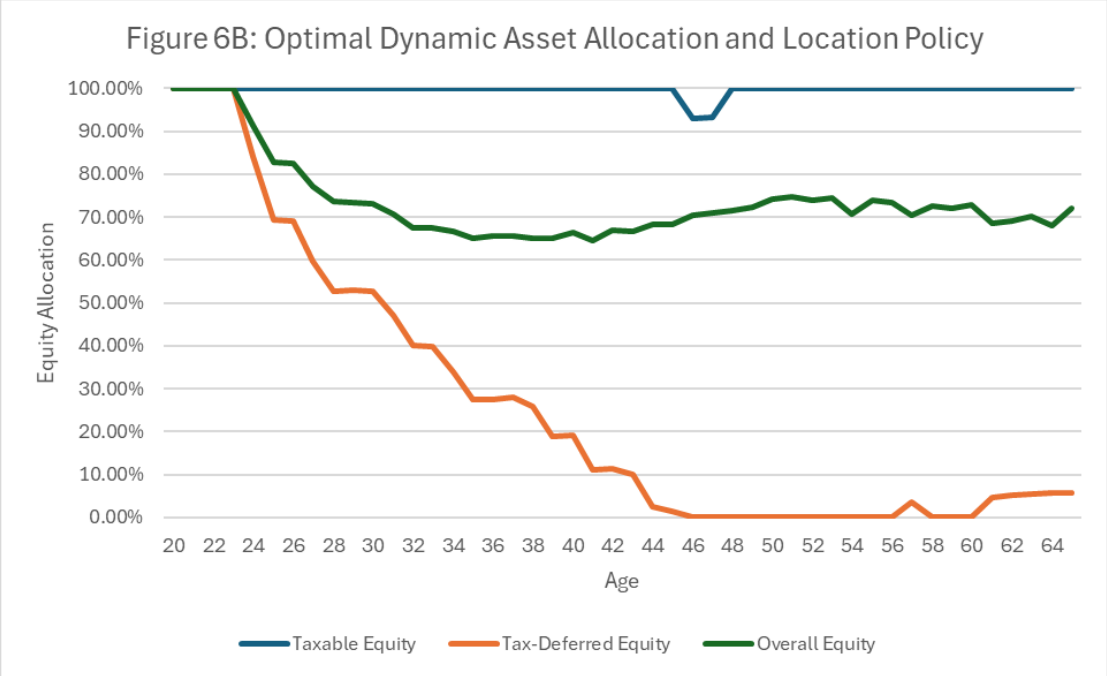
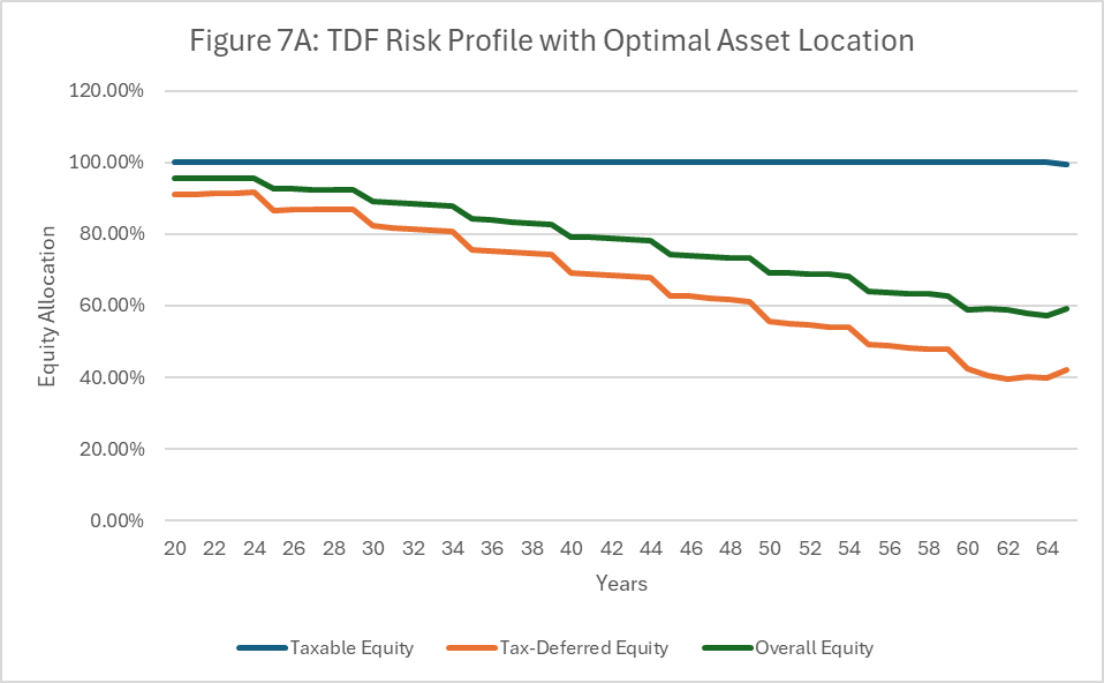
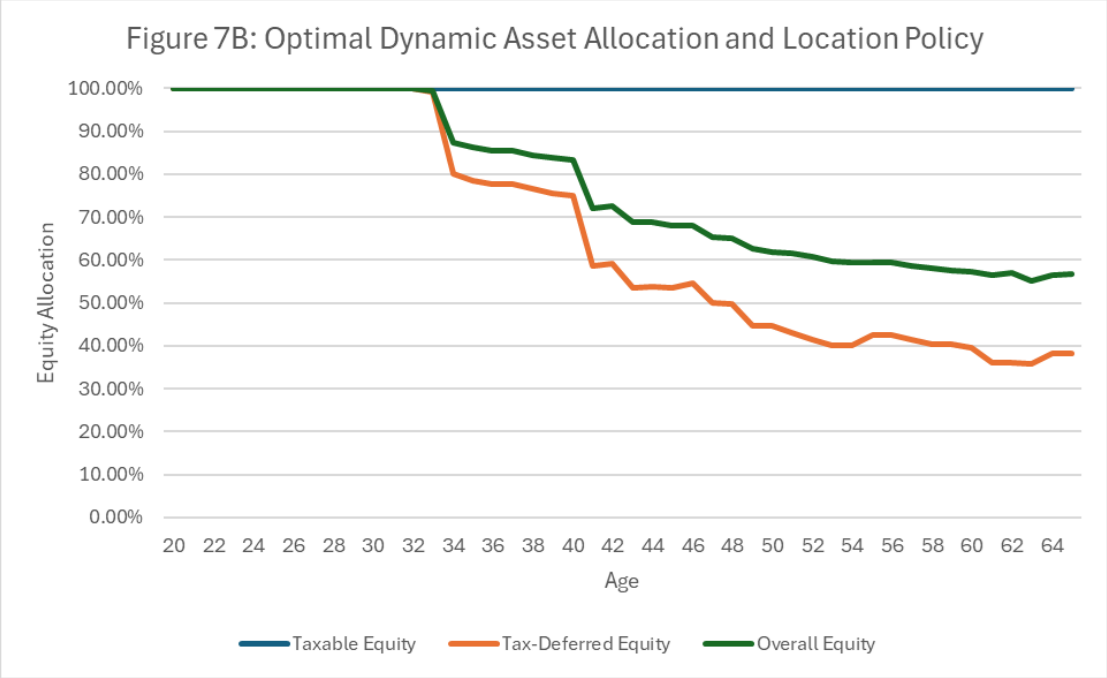


Figure 7: Dynamic Glide Paths for Alternative Investment Policies (Realized Equity Returns Below Average)





Appendix A

Our objective is to solve the intertemporal investment problem stated below:

$$v(\mathrm{h_t}) = \underset{\mathrm{x_t},\, \mathrm{z_t}}{\mathrm{MAX}}\ \mathrm{E_t}[v(h_{t+1})] + [\lambda_{1,\mathrm{t}}\mathrm{x_t} + \lambda_{2,\mathrm{t}}(1-\mathrm{x_t})] + [\lambda_{3,\mathrm{t}}\mathrm{z_t} + \lambda_{2,\mathrm{t}}(1-z_t)] \qquad (\mathrm{A1})$$

for all t = 0, … , T-1, where

$$v(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\phi_{t+1}(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\prod_{j=t+2}^{T}\left(\rho_j^*\right)^{1-\gamma} \qquad (\mathrm{A2})$$

is the date-t+1 value function,

$$\rho_{t+1} = w_{F,t}\{w_{NR,t}\rho_{NR,t+1} + (1-w_{NR,t})\rho_{R,t+1}\} \qquad (\mathrm{A3})$$

$$+l_t(1+i)\{y_{NR} + y_R + A_t[1+r(1-t_o)] - 1\}$$

is the gross after-tax growth rate in total wealth between dates t and t+1, and

$$\rho_{NR,t+1} = \left\{1 + \mathrm{x_t}\left[\mathrm{d}(1-\mathrm{t_d}) + \mathrm{g_{t+1}}\left(1-\mathrm{t_g}\right)\right] + (1-\mathrm{x_t})\mathrm{r}(1-\mathrm{t_o})\right\} \qquad (A4)$$

$$\rho_{R,t+1} = \{1 + \mathrm{z_t}(\mathrm{d} + \mathrm{g_{t+1}}) + (1-\mathrm{z_t})\mathrm{r}\} \qquad (A5)$$

are the gross after-tax growth rates in taxable (non-retirement) wealth and retirement wealth, respectively, between dates t and t+1.

The control (choice) variables are the fraction of taxable wealth and retirement wealth invested in equity at each date: $\{x_t, z_t\}$, t = 1, … , T-1. The state variables that are relevant for solving this intertemporal investment problem are not unique. There are a variety of state variable combinations that provide the same relevant information. We have chosen to use the fraction of financial wealth invested in the taxable account, $w_{NR,t}$, and the present value

of future labor income as a fraction of total wealth, $l_t(1+i)A_t$, as our state variables.[18] Of course, the number of years prior to the retirement date, T-t, is also a relevant state variable for the intertemporal investment problem.

The objective is to find the optimal holdings of equity in both the taxable and retirement accounts at each date t = 1, …, T-1. Since the investment choices at date t affect the future state variables, it is necessary to use dynamic programming to solve the intertemporal investment problem. However, because we do not have an analytical solution, we must use numerical methods. To facilitate the implementation, we assume that equity returns follow a lognormal distribution and that the individual has constant relative risk averse preferences. The state variables, $w_{NR,t}$ and $l_t(1+i)A_t$, have natural upper and lower bounds of 0% and 100%.

We discretize the state variables by assuming 5% increments between 0% and 95%. Because there is more variation at the boundaries of the grid, we increment the state variables by 1% for the state variables between 95% and 100%. This, then, gives us a 25 x 25 state variable grid. We begin by solving the optimization problem at date T-1 and working backward period-by-period until we have solved for the optimal equity holdings and value functions for each date t = 0, …, T-1. The dynamic programming approach accounts for the fact that current portfolio decisions affect future state variables. However, because there is no guarantee that the realization of returns in the current period will produce future state variables that lie on the discretized grid, we also need to utilize interpolation methods to find solutions that lie between grid points.

We formulate the intertemporal investment problem in Excel and utilize the VBA programming feature to design a Macro that finds the optimal equity allocations in the taxable and retirement accounts, as well as the resulting optimized value function, for each point on the 25x25 state-variable grid and for each date t = 0, …, T. We use a simple bi-linear interpolation method to find optimal equity allocations and value functions for state variables that lie between grid points. Base-case parameter values are provided in Table 1.

[18] It may seem simpler to use the current labor income-to-total wealth ratio, l_t, instead of $l_t(1+i)A_t$ as the second state variable, but the advantage of $l_t(1+i)A_t$ is that it has natural lower and upper bounds of 0% and 100%, which facilitates the implementation of our numerical methodology.

There are points on the boundary of the 25x25 state variable grid where the optimal equity holdings for the taxable and retirement accounts are indeterminate. In particular, the optimal equity holdings in the taxable and retirement accounts are undefined when the present value of future labor income represents 100% of total wealth. Likewise, the optimal equity allocations in the retirement (taxable) account are undefined when the fraction of financial wealth held in the taxable account is 100% (0%). However, the optimal equity allocations at the boundary of the state variable grid are still important because they are used for interpolation purposes. In assigning values for the optimal equity allocations at the boundary of the grid, we rely on the solutions at nearby grid points inside the boundary to ensure that there is a smooth transition to the boundary.