Credit Market Imperfections in Production Networks

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Abstract

This paper studies how credit market imperfections shape interconnections among producers in economies with input–output linkages, and how they alter the propagation and aggregation of microeconomic shocks. I develop a dynamic, multi-sector model of production networks in which heterogeneous firms face collateral constraints to finance operations, in the spirit of Kiyotaki and Moore (1997). The model shows that when credit market imperfections restrict resource access, producers become additionally interconnected through their real credit conditions. Ex post, the sector-level profit-to-revenue ratio provides a sufficient statistic for tracking these conditions, consistent with recent evidence from Europe and the United States. Using this statistic, I establish an aggregation theorem that decomposes long-run aggregate TFP growth into micro-level sources and revises the notion of industry centrality for policy design: high profitability increases a sector's centrality for credit-market interventions (e.g., improving access to credit) while reducing its centrality for standard industrial policies (e.g., taxes or subsidies). A quantitative application to the U.S. economy from 1997–2016 shows that financial deepening accounts for nearly 76% of aggregate TFP growth. The *Housing* and *Chemical Products* sectors emerge as the most credit-central industries, while *Wholesale Trade*—ranked first by traditional production centrality—falls outside the top ten, underscoring how credit-based centrality highlights very different priorities for effective policy interventions.

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1 Introduction

Modern production features intricate input-output linkages among individual producers, as evidenced by the long and complex supply chains observed in many advanced economies. While these production networks enable the delivery of highly sophisticated products such as smartphones and automobiles, they, at the same time, increase the economy's sensitivity to micro-level shocks by making the performance of individual producers tightly depend on each other. That is, micro-level shocks originating from a small group of producers can propagate through input–output linkages, generate spillovers that affect others producers' performance, and ultimately induce large macroeconomic consequences—a fact vividly illustrated by recent supply-chain disruptions in semiconductors and food processing, highlighting the need to study the centrality of industries for micro-shock propagation and their implications for aggregate economic outcomes.

To date, the production-network literature has primarily focused on micro shocks to technology and goods-market imperfections. In reality, however, under credit market imperfections, access to credit is a key determinant of producers' performance. A deterioration in credit conditions can significantly disrupt firms' production decisions and, in turn, reduce their operational efficiency. Indeed, during the 2007–08 financial crisis, local credit shocks propagated across sectors through input–output linkages (Costello, 2020). How large are the resulting macroeconomic consequences—and can existing centrality measures, which capture the aggregate impact of micro shocks in production networks, account for them? Regrettably, the answer is no.

When credit-market imperfections restrict firms' access to production resources, the real credit condition of each producer emerges as an additional linkage that interacts with traditional input–output connections in the propagation of micro shocks. This financial linkage fundamentally alters the transmission of shocks across the network, rendering conventional measures of industry centrality—constructed solely from the input–output matrix—potentially misleading when financing frictions are present. These considerations motivate the development of a new framework for evaluating industry centrality under credit-market imperfections.

Formally, this paper asks the following question: How should we evaluate the macroeconomic consequences of micro-level shocks when producers are connected not only through input–output linkages within the production network but also through their credit conditions arising from credit-market imperfections? To address this question, I de-

velop a general yet tractable dynamic multi-sector heterogeneous-agent model of production networks in which credit-market imperfections constrain producers' access to production resources by their own wealth—a realistic feature supported by the widespread prevalence of asset-based lending worldwide. Within this framework, I derive three main results.

First, I show that the sector-level profit-over-revenue ratio serves as a nonparametric sufficient statistic for tracking each sector's real credit condition—a theoretical result consistent with recent empirical evidence from Europe and the United States. Second, using this sufficient statistic, I derive an aggregation theorem for production-network economies with credit-market imperfections, which decomposes aggregate TFP growth into distinct micro-level sources and fundamentally reshapes how we think about industry centrality in policy design. In particular, under credit frictions, higher sectoral profitability increases an industry's centrality for credit-market interventions (e.g., government-backed credit access) but decreases its centrality for traditional sectoral interventions (e.g., industrial taxation or subsidies). Finally, calibrating the model to the U.S. economy reveals that financial deepening accounts for roughly 76% of aggregate TFP growth between 1997 and 2016, with *Housing* and *Chemical Products* emerging as the most credit-central industries. By contrast, *Wholesale Trade*—the top-ranked sector by production centrality—falls outside the top ten, underscoring its suboptimality as a target for credit-market interventions. I now proceed to articulate these findings in detail.

I begin by analytically characterizing producers' optimal behavior under credit-market imperfections, leveraging the tractability of the model. This characterization makes explicit that individual producers are connected through two distinct channels: (i) production linkages, arising from production technology, and (ii) credit-condition linkages, stemming from credit-market imperfections—each governing a fundamentally different dimension of shock propagation. The production channel determines the optimal composition of inputs for cost-minimizing producers—that is, how a product is produced given input prices. By contrast, the credit-condition channel shapes the constrained optimal scale of production—that is, *how much* to produce given the minimized marginal cost of production. In environments with credit-market imperfections, the former (the minimized cost of production) is generally insufficient to infer the latter (the optimal production scale). This is the fundamental reason why the sufficient statistics developed in the existing production-network literature—constructed solely from the input–output matrix—fail to capture the full propagation of shocks when credit-market imperfections

are present. Moreover, by focusing exclusively on the input–output structure to assess industry centrality, these measures overlook the general-equilibrium interactions between production composition and credit conditions through prices and quantities, further undermining their validity in such environments.

Given this, a key theoretical contribution of the paper is to show that, ex post, the sector-level profit-over-revenue ratio serves as a nonparametric sufficient statistic for recovering the evolution of each sector's real credit condition—capturing the general-equilibrium interaction between optimal production composition and credit conditions at the producer level. The intuition is as follows: when credit-market imperfections constrain the optimal production scale of the most efficient producers within a sector, market equilibrium requires that total supply still meet total demand. To restore this balance, equilibrium prices adjust to incentivize less efficient producers to participate in production by offering them positive profits. This mechanism is one of the reasons why, in reality, we observe a continuum of heterogeneous firms within each industry actively producing and earning strictly positive profits—and the fundamental reason why, once properly processed, sector-level profitability can reveal its underlying real credit conditions, with low profitability reflecting favorable credit-market conditions and high profitability indicating tighter credit constraints.

More importantly, the above economic reasoning—and the implied mirror role of profitability—is empirically supported by recent firm-level evidence from both Europe and the United States. Amberg, Jacobson, and Qi (2024) exploit an exogenous innovation in supply-chain financing in Sweden and show that firms experiencing a positive credit shock significantly expand their operational scale, while their profitability slightly declines following the expansion. Likewise, Matray, Muller, Xu, and Kabir (2024) use the effective shutdown of the Export–Import Bank (EXIM) between 2015 and 2019 as a negative credit shock and find that affected firms contract their scale, while their profitability slightly increases. Together, these empirical findings lend strong support to the credibility of profitability as a sufficient statistic for tracing the evolution of real credit conditions.

Equipped with this sufficient statistic, I can now aggregate the disaggregated network economy to show how credit-market imperfections reshape the channels through which micro-level shocks manifest at the macro level. A key theoretical contribution lies in uncovering the overlooked interdependence between within-sector and across-sector misallocation in the economy. While both dimensions have been extensively studied in isolation, their joint analysis remains rare. Specifically, this paper shows that the interaction

between these two forms of misallocation fundamentally reshapes our understanding of industry centrality in enhancing aggregate efficiency through different types of microeconomic interventions.

I formalize this insight by deriving an aggregation theorem for micro shocks in production-network economies with credit-market imperfections. The theorem decomposes long-run aggregate TFP growth into distinct micro-level sources: pure technological change, changes in goods-market imperfections, changes in credit-market imperfections, and indirect efficiency gains through resource reallocation. It further reveals that, under credit-market imperfections, high sector-level profitability increases an industry's centrality for credit-market interventions (e.g., government-backed lending) while reducing its centrality for goods-market interventions (e.g., industrial taxation or subsidies).

The intuition behind this result is that when credit-market imperfections link a sector's profitability to its underlying real credit condition, the magnitude of profitability also reflects the extent of within-sector misallocation through firm dynamics. To see this, consider two sectors with similar production centrality—that is, sectors that occupy comparable positions in the production network and contribute similarly to the economy's across-sector misallocation. Suppose one sector faces severe credit constraints, while the other is only mildly constrained. In the more credit-constrained sector, limited access to credit restricts the production scale of its most productive firms, forcing the market to rely more heavily on less efficient producers, who are incentivized to operate through positive profit margins. In this context, higher sector-level profitability signals poorer within-sector allocation, reflected in the coexistence of highly productive and less efficient firms. This greater within-sector misallocation—manifested in high profitability—implies larger potential gains in aggregate efficiency from relaxing the sector's credit constraints, thereby increasing its centrality from the perspective of credit-market interventions.

A similar logic applies to goods-market interventions. When a sector exhibits high within-sector misallocation—again reflected in elevated profitability—a negative goods-market shock, such as an increase in industrial taxation, can help alleviate misallocation by forcing less efficient firms to exit the market. This mechanism explains why sector-level profitability is negatively correlated with the aggregate effects of goods-market interventions, thereby reducing the sector's centrality from this policy perspective.

In terms of methodological contribution, this paper illustrates how to bridge macroeconomic modeling with micro-level data in the study of macro-development. A long strand of literature in macro-finance seeks to understand whether financial deepening

truly contributes to enhancing one economy's aggregate efficiency by alleviating its resource misallocation. Equipped with this aggregation theorem, which explicitly decomposes long-run aggregate TFP growth into distinct micro-level sources, we can directly address this question by leveraging rich micro-level data—where financial deepening at the micro level can be measured using detailed firm-level financial datasets such as Compustat and so on.

Moreover, by taking a step further and imposing a set of commonly used assumptions on the primitives of the economy—specifically, Cobb–Douglas production technologies and a Pareto distribution of firm-level heterogeneity, the latter supported by empirical evidence on firm size distributions—the aggregation theorem does more than explain the sources of long-run growth ex post. It also provides ex ante policy guidance for micro-level credit-market interventions. The key insight is that, under these assumptions, both within-sector and across-sector misallocations are governed entirely by the structural primitives of the economy and remain invariant to the specific structure of exogenous credit shocks. As a result, the indirect efficiency effects are shut down, allowing the aggregation theorem to be used to design ex ante policy interventions targeting credit-market imperfections.

A quantitative application of the aggregation theorem to the United States indicates that financial deepening accounts for nearly 76% of aggregate TFP growth between 1997 and 2016. In terms of credit-market interventions, the *Housing* and *Chemical Products* sectors emerge as the two most credit-central industries in 2015. Both sectors exhibit severe credit-market imperfections, as reflected in their high sector-level profitability, and both occupy central positions in the production network. The housing sector is pivotal because of its dominant role in final household consumption, whereas the chemical products sector is crucial due to its upstream position—many downstream industries rely heavily on its outputs as intermediate inputs. By contrast, the *Wholesale Trade* sector, which ranks first in production centrality, falls outside the top ten in credit centrality owing to its low profitability, making it a suboptimal target for credit-market interventions.

Related Literature. This paper contributes to several strands of recent research. First, it relates to the growing literature on production networks and their macroeconomic implications in the presence of distortions; see Liu (2019), Baqaee and Farhi (2020), and Bigio and La'O (2020). These studies develop general frameworks to analyze how production networks shape the aggregate consequences of micro-level shocks under goods-market imperfections. The key contribution of this paper to this strand is the introduc-

tion of general and empirically grounded credit-market imperfections into the canonical framework. I show that when credit constraints limit the optimal production scale of individual producers, a novel credit-condition linkage emerges across firms, fundamentally altering how micro-level shocks propagate and aggregate in the economy. In doing so, the paper extends the existing analysis to a broader and more realistic setting in which credit-market imperfections play a central role.

Second, this paper contributes to the long strand of literature examining the role of financial market imperfections in economic development. More specifically, it relates to a series of recent studies linking financial frictions to aggregate productivity, including Moll (2014), Angeletos (2007), Buera, Kaboski, and Shin (2011), Buera and Shin (2013), Caselli and Gennaioli (2013), and Midrigan and Xu (2014), among others. This paper advances this strand of research by analyzing the implications of financial market imperfections for resource misallocation in a richer environment featuring extensive input–output linkages across sectors—a defining characteristic of modern advanced economies. It develops a highly tractable framework that highlights the interdependence between within-sector and across-sector misallocation in such settings, thereby reshaping our understanding of how microeconomic policy can address misallocation to foster long-run growth.

Third, this paper contributes to the growing use of the sufficient-statistics approach in studying misallocation, building on the influential work of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). It develops a general framework for applying this approach to a macro-finance setting with rich production linkages among heterogeneous producers. Moreover, it shows that, under standard macroeconomic assumptions—Cobb–Douglas production technologies and Pareto-distributed firm heterogeneity—the sufficient statistic measuring the direct effect of micro-level credit shocks on aggregate productivity, namely credit centrality, can also serve as an ex ante guide for credit-market interventions. This result represents a distinctive methodological contribution: it preserves the generality and tractability of the sufficient-statistics approach while incorporating the rich counterfactual analysis characteristic of structural methods, thereby enhancing its ability to inform effective policy design.

Finally, this paper contributes to the emerging literature linking markup dispersion to financial frictions. Specifically, Li, Lian, Ma, and Martell (2025) document that, in the United States, industries facing more severe asset-based lending constraints—as proxied by the low liquidation value of their assets—exhibit greater within-sector markup dispersion. By linking producers' real credit conditions to their realized profitability, this paper

develops a theoretical framework that rationalizes this empirical relationship.

Outline. The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 describes the network of interconnections among individual producers in the presence of credit-market imperfections. Section 4 illustrates how this network reshapes the propagation of microeconomic shocks. Section 5 theoretically examines the macroeconomic consequences of different types of microeconomic shocks under credit-market imperfections. Section 6 presents the quantitative results from calibrating the model to the U.S. economy. Section 7 concludes.

2 Model

2.1 Environment

I consider a dynamic, discrete-time economy endowed with a fixed unit of a nonperishable factor $K \equiv 1$, referred to as *capital*. Owing to its nonperishable nature, capital serves two roles in the economy: (i) as a productive input and (ii) as a store of value for saving over time. When capital is employed as a production input, I denote it by k; when it functions as an asset, I denote it by a. Both k and a are individual-level variables.

There are N production sectors, indexed by $i \in I = \{1, ..., N\}$. Each sector produces a perishable, homogeneous good that can be consumed directly or used as an intermediate input in the production of other sectoral goods. Each sector is populated by a unit mass of entrepreneurs, heterogeneous in their idiosyncratic efficiency z and asset holdings a. The efficiency z is drawn i.i.d. at the beginning of each period from an exogenous distribution with probability density function (pdf) $\psi(z)$, identical across sectors. Asset holdings a are endogenous, determined by entrepreneurs' optimal consumption–saving decisions, with the marginal pdf for sector i at time t denoted by $\xi_{it}(a)$. The state of sector i at time t is therefore described by the following joint pdf $g_{it}(a, z) \equiv \xi_{it}(a) \cdot \psi_(z)$, reflecting the independence between a and z at the individual level.

There is no aggregate uncertainty in the model economy, and entrepreneurs are the only economic agents at this stage. Accordingly, for the remainder of the paper, I use the terms *entrepreneurs* and *individuals (or individual producers)* interchangeably.

2.1.1 Preference

Entrepreneurs share the following lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \, ln(c_t) \tag{1}$$

where c_t denotes final consumption at time t.

The final consumption good c_t, which serves as the numéraire of the economy, is produced from sectoral goods using a constant-returns-to-scale aggregator

$$c_t = Y(c_{1t}, ..., c_{Nt}), \tag{2}$$

where c_{it} is the individual's consumption of sectoral good i at time t

2.1.2 Technology

The production technology available to an individual entrepreneur in sector i with efficiency z is

$$y_i(z) = A_i \cdot z \cdot F_i(k_i, \{m_{ij}\}), \tag{3}$$

where k_i denotes her demand for the primary factor (*capital*), m_{ij} her demand for intermediate inputs from sector j, and z her idiosyncratic production efficiency. The function $F_i(\cdot)$ represents the production technology for good i. It is identical across entrepreneurs within the same sector but heterogeneous across sectors. I assume all technologies $F_i(\cdot)$ are homogeneous of degree one (exhibit constant returns to scale), twice continuously differentiable in all arguments, and exhibit positive and diminishing marginal products in all arguments. A_i is a Hicks–neutral, sector-specific productivity shifter through which sector-level technological shocks enter.

Given the dynamic nature of the model, a few clarifications are in order. All sector-level primitives, including A_i and $F_i(\cdot)$, are time-invariant. Even when shocks to sector-level primitives are introduced—such as $\Delta \, log \, A_i$—they should be interpreted as permanent structural changes in the economy rather than as transitory volatility shocks. In contrast, the individual-level variables—namely, the idiosyncratic efficiency z and the input demands $(k_i, \{m_{ij}\})$—are stochastic and time-dependent: z evolves exogenously over time due to its random-draw nature, while input demands evolve endogenously with the individual's state (a, z) and the prevailing economic conditions in each period. Since this subsection focuses on the technological primitives of the model, I omit time subscripts here. These subscripts will be reinstated in *Section 3.1* when discussing individuals' opti-

mal production decisions.

2.1.3 Market Structure: Competitive Markets with Goods-Market Imperfections

The model economy relies on a competitive market structure to guide resource allocation, implying that all individuals are price takers. Specifically, in each period t, there are: i) N sectoral good markets, where individuals freely trade sectoral goods at prices $\{p_{it}\}$; ii) a rental (factor) market, where individuals can rent or lease out their capital assets for production at a rental rate r_t; and iii) an asset market, where individuals can freely trade capital as a store of value at a price q_t for saving.

Importantly, I introduce **goods-market imperfections** into the system. These imperfections represent inefficiencies in the economy that distort its **across-sectors** resource allocation. They may arise from policy interventions—such as industrial taxation—or from intrinsic market frictions, such as contracting difficulties or transaction costs. In the model, these imperfections are represented by sector-specific reduced-form wedges $\mu^\tau \equiv \{\mu_i^\tau\}$, which introduce shocks to goods-market conditions and satisfy two properties: i) they increase individual entrepreneurs' cost of production, and ii) all additional payments induced by these imperfections are fully rebated to individuals after production. For the remainder of the paper, I use industrial taxation as the running narrative for μ^τ, while acknowledging that they can also be interpreted as arising from other micro-level physical market imperfections.

So far, the setup presented here is a relatively standard, multi-sector framework of intermediate goods trade with generic constant-returns-to-scale technologies. Moreover, taxation is a standard accounting device for goods-market imperfections. The novel aspect is that I will now introduce credit market imperfections into this framework and examine how their interaction with the economy's input–output linkages fundamentally reshapes the network of interconnections between producers—transforming the very channels through which micro-level shocks propagate and aggregate at the macroeconomic level.

2.1.4 Credit Market Imperfections

Since sectoral goods are perishable, entrepreneurs enter each period holding only a stock of assets a. To engage in production, they must obtain credit to finance their working capital. In the presence of credit-market imperfections—where repayment is unenforce-

able unless backed by the borrower's own wealth—the amount of credit available must be collateralized by the entrepreneur's assets.

A Collateral Constraint on Active Entrepreneurs' Working Capital. Formally, an entrepreneur in sector i has to respect the following within-period collateral constraint on working capital:

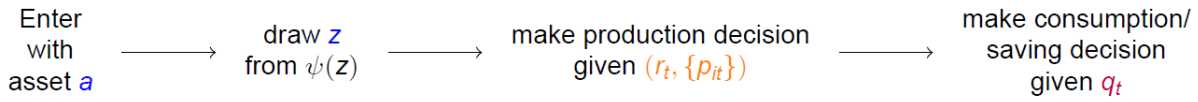
$$r_t \cdot k_{it} + \sum_j p_{jt} \cdot m_{ijt} \leq \phi_i \cdot q_t \cdot a \tag{4}$$

where ϕ_i is a primitive of the economy capturing the severity of credit market imperfections faced by sector i, through which sector-level credit shocks are introduced, and $q_t \cdot a$ denotes the value of the entrepreneur's asset holdings.

These credit-market imperfections mirror the widespread use of asset-based lending worldwide and the financing constraints that entrepreneurs face in practice. By explicitly incorporating these frictions, the framework shows how such imperfections restrict entrepreneurs' access to production resources, introduce microeconomic shocks to credit availability, and provide a foundation for analyzing how imperfections in the credit market would shape the propagation and aggregation of microeconomic shocks in production network economies.

2.2 Within-period Timeline

Within-period Timeline. This subsection outlines the within-period timeline that agents in the model economy follow—a sequence that is central to the subsequent analysis. The within-period timeline for an entrepreneur in sector i is as follows:



- At the start of period t, she enters with an asset holding a, an individual-level state variable determined at the end of the previous period.
- Immediately upon entering the new period, she learns her idiosyncratic efficiency z for the period. This is drawn from a general probability density function $\psi(z)$,

assumed i.i.d. across individuals and over time, and identical across sectors for simplicity.

- Given the state (a, z) and prices $P_t \equiv \{r_t, \{p_{jt}\}\}$, she makes her production decision to maximize within-period income, $\Pi_{it}(a, z)$, subject to the collateral constraint (4). Specifically, given her idiosyncratic productivity z, she chooses whether to be active in production. Importantly, regardless of her production status, she always rents out her personal asset holdings and earns $r_t \cdot a$ on them.[1] Hence, the maximized within-period income $\Pi_{it}(a, z)$ satisfies

$$\Pi_{it}(a,z) = \begin{cases} \underbrace{\pi_{it}(a,z)}_{\text{profit from production}} + \underbrace{r_t \cdot a}_{\text{rental income}}, & \text{if active,} \\ \underbrace{r_t \cdot a}_{\text{rental income}}, & \text{if inactive.} \end{cases}$$

- Then, given the resulting maximized income, $\Pi_{it}(a, z)$, and the asset price q_t, she decides how much to consume $c(a, z)$ and how much to save $a'(a, z)$, to maximize her lifetime utility (1) subject to the standard intertemporal budget constraint:

$$c(a,z) + q_t \cdot a'(a,z) = \Pi_{it}(a,z) + q_t \cdot a \tag{5}$$

The within-period timeline above reveals a clear and natural separation between individuals' production decisions and their consumption–saving decisions: the former is essentially an intra-period problem, while the latter is inter-period. This convenient separation makes it possible—given the presence of credit-market imperfections—to examine their partial-equilibrium implications for each decision in isolation and, in turn, to characterize their general-equilibrium implication in a more organized way.

2.3 Individual Behavior and Equilibrium

Individual Behavior. Given the model environment, the optimal behavior of individual entrepreneurs follows standard principles. They maximize the present-discounted value of utility from consumption (1) by sequentially choosing their within-period production decisions and intertemporal consumption–saving decisions, subject to the stan-

[1] The reason is that renting out an individual's capital does not undermine ownership rights, which are essential for its use as collateral.

dard intertemporal budget constraint (5). The only novel feature is that entrepreneurs must now make their within-period production decisions subject to the collateral constraint (4) arising from credit-market imperfections.

Equilibrium. Given the model environment and the corresponding optimal behavior of agents, the relevant notion of equilibrium is that of a standard competitive equilibrium, defined as follows.

> ***Definition 1 (Competitive Equilibrium).*** *An equilibrium of the model economy is a set of time paths for prices (p_{it}, r_t, q_t) and corresponding quantities such that (i) individuals maximize their lifetime utility (1) subject to the collateral constraint (4) and the standard intertemporal budget constraint (5), taking as given equilibrium prices, and (ii) all markets clear at each point in time.*

Having outlined agents' optimal behavior and established the equilibrium of the model economy at a conceptual level, the next section formally characterizes individual entrepreneurs' optimal production and consumption–saving decisions in the presence of credit-market imperfections. This characterization clarifies how such imperfections interact with agents' positions within the production network to shape the pattern of interconnections among them and, in turn, transform the channels through which microeconomic shocks propagate and aggregate at the macroeconomic level.

3 Network of Interconnections under Credit Frictions

Given the two-layer structure of individual entrepreneurs' optimization problem—namely, a first layer consisting of within-period production decisions to maximize current income and a second layer consisting of intertemporal consumption–saving decisions to maximize lifetime utility, as revealed by the within-period timeline—credit-market imperfections that make access to production resources contingent on entrepreneurs' wealth naturally generate two types of financial frictions, each corresponding to one layer of the optimization.

The first is captured by equation (4): a collateral constraint on entrepreneurs' working capital that directly affects their optimal production decisions. The second is more subtle: an incomplete-market friction that limits entrepreneurs' ability to insure against idiosyncratic shocks, which arises naturally from the fact that such shocks cannot be fully hedged

when the only asset available for intertemporal trade is physical capital. Returning to the natural separation between individuals' optimal production and consumption–saving decisions, an important feature of the economy is that the first type of financial friction affects only production decisions, while the second affects only consumption–saving decisions. This separation allows each decision to be analyzed in isolation, thereby enabling a clear analytical characterization of individuals' optimal behavior.

In what follows, I elaborate on these two types of financial frictions and characterize the corresponding optimal behavior of individuals, establishing how credit market imperfections reshape the interconnections between them.

3.1 Collateral Constraint and Optimal Production Decisions

Individuals' Optimal Production Decisions. With the amount of credit available constrained by the collateral condition (4), individuals' optimal production decisions consist of two components:

1. Cost-Minimization. Given $P_t \equiv (r_t, \{p_{it}\})$, individuals in each sector first choose their production plan to minimize the marginal cost of production:

$$\begin{aligned} C_i(P_t) = \min_{k_{it}, \{m_{ijt}\}} & \; r_t \cdot k_{it} + \sum_j p_{jt} \cdot m_{ijt} \\ s.t. \;\; & F_i(k_{it}, \{m_{ijt}\}) \geq 1. \end{aligned} \tag{6}$$

The I.O. linkages: Through this process, input–output linkages among sectors emerge, creating a potentially powerful transmission channel through which micro-level shocks propagate throughout the economy.

2. Participation. With the minimized production cost $C_i(P_t)$ and the tax rate μ_i^τ imposed by the government, individuals in sector i now decide whether to engage in production based on their idiosyncratic efficiency z. An individual in sector i will participate if and only if her idiosyncratic efficiency z allows her to "beat the market"—that is, if her private after-tax marginal cost of production is no greater than the market price:

$$\underbrace{\frac{\mu_i^\tau}{A_i \cdot z} \cdot C_i(P_t)}_{mc_i(z)} \leq p_{it}. \tag{7}$$

This means that, in equilibrium, for each sector i, there exists an endogenous productivity cutoff $z_{it} \equiv z_i(P_t)$ for its entrepreneurs to be active:

$$z \geq z_i(P_t) \equiv \frac{\mu_i^\tau}{A_i \cdot p_{it}} \cdot C_i(P_t). \tag{8}$$

The evolution of $z_{it} \equiv z_i(P_t)$ thus constitutes a natural channel through which a sector's real credit conditions shape its within-sector allocation of resources between high- and low-efficiency entrepreneurs.

Moreover, for those who choose to engage in production, the strictly positive marginal profits together with the *constant-returns-to-scale* nature of their technology induce them to produce as much as possible until they hit the limit imposed by their credit constraint. Hence, in equilibrium, the collateral constraint is always binding for active entrepreneurs, with their constrained optimal production scale given by

$$y_{it}(a, z) = A_i \cdot z \cdot \frac{\phi_i \cdot q_t \cdot a}{C_i(P_t)}. \tag{9}$$

The Credit-Condition linkage: With credit-market imperfections, the real credit condition of a sector—captured by the constrained optimal production scale of its active entrepreneurs (Equation 9)—emerges as an additional linkage that interacts with I.O. linkages in propagating micro-level shocks. Specifically, when a negative shock raises the price of a sector's output, the performance of its trading partners—who rely on that output as an input—is affected not only through higher production costs but also through a direct deterioration in their real credit conditions: given the same liquidity available from the financial sector, they can now produce less than before. Moreover, the inclusion of the asset price q_t in the credit condition (Equation 9) introduces an additional layer of interdependence between the economy's micro- and macro-level performance in general equilibrium, rendering the aggregate consequences of micro-level shocks more complex to characterize. These additional propagation channels must therefore be accounted for to properly assess the macroeconomic consequences of micro-level shocks under credit-market imperfections.

Individuals' Maximized Profit. Having characterized individuals' optimal production decisions under credit market imperfections, we are now in a position to derive the resulting maximized within-period profit. This result, presented in Lemma 1, will later serve as the foundation for identifying the evolution of each sector's real credit condition.

LEMMA 1 (Individuals' Maximized Profit). *Given prices $P_t \equiv (r_t, \{p_{it}\})$—where r_t is the rental rate of capital, p_{it} are sectoral output prices, and q_t, the price of the asset—together with the primitives of the economy, individuals' optimal production decisions imply, in equilibrium,*

$$p_{it} = \frac{\mu_i^\tau}{A_i \cdot z_{it}} \cdot C_i(P_t), \quad \forall i. \tag{10}$$

Each active entrepreneur (a, z) in sector i therefore earns a positive profit:

$$\underbrace{\pi_{it}(a,z)}_{\text{maxi. profit}} = \mu_i^\tau \cdot (\frac{z}{z_{it}} - 1) \cdot \phi_i \cdot q_t \cdot a, \quad \text{for } z \geq z_{it} \tag{11}$$

Equation (10) in Lemma 1 implies that, for individual entrepreneurs at the productivity cutoff $z_{it} \equiv z_i(P_t)$, marginal revenue equals the after-tax marginal cost, yielding zero profit. For entrepreneurs with strictly higher efficiency $(z > z_{it})$, an endogenous profit wedge between their marginal cost and output price arises:

$$\mu_i^\pi(z; P_t) = \underbrace{\frac{\mu_i^\tau}{A_i \cdot z_{it}} \cdot C_i(P_t)}_{P_{it}} - \underbrace{\frac{\mu_i^\tau}{A_i \cdot z} \cdot C_i(P_t)}_{MC_i(z;P_t)} \tag{12}$$

which generates strictly positive profits at the constrained optimal production scale (9) imposed by the collateral constraint (4), explicitly characterized by the maximized profit function in Equation (11).

Sectoral Aggregation Lemma 1 gives us a closed-form expression for individuals' maximized profits under the collateral constraint. This expression directly links credit conditions—captured by the constrained optimal production scale—to maximized profit, and constitutes a key building block for the subsequent analysis of credit market imperfections in a production-networked economy. Moreover, the always-binding collateral constraint induces a convenient linear relationship between individuals' maximized profit and their current asset holdings, which facilitates sectoral aggregation.

LEMMA 2 (Sector-level Accounting Items). *Sectoral aggregation yields the following sector-level accounting items:*

$$\underbrace{\pi_{it}}_{\textit{sectoal profits}} = \int\int \pi_{it}(a,z) \cdot g_{it}(a,z)\, dadz \tag{13}$$

$$= \mu_i^{\tau} \cdot (\mu_{it}^{\pi} - 1) \cdot \underbrace{\phi_i \cdot q_t \cdot a_{it} \cdot [1 - \Psi(z_{it})]}_{\textit{sector-level production cost}} \tag{14}$$

$$\underbrace{T_{it}}_{\textit{sectoral taxes}} = (\mu_i^{\tau} - 1) \cdot \underbrace{\phi_i \cdot q_t \cdot a_{it} \cdot [1 - \Psi(z_{it})]}_{\textit{sector-level production cost}} \tag{15}$$

$$\underbrace{R_{it}}_{\textit{sectoral sales}} = \mu_i^{\tau} \cdot \mu_{it}^{\pi} \cdot \underbrace{\phi_i \cdot q_t \cdot a_{it} \cdot [1 - \Psi(z_{it})]}_{\textit{sector-level production cost}} \tag{16}$$

with the state of each sector characterized by

$$\underbrace{a_{it}}_{\textit{sectoral assets}} = \int a \cdot \xi_{it}(a)\, da \tag{17}$$

$$\underbrace{1 - \Psi(z_{it})}_{\substack{\textit{the share of active} \\ \textit{entrepreneurs}}} = \int_{z_{it}}^{\overline{z}} \psi(z)\, dz \tag{18}$$

$$\underbrace{\mu_{it}^{\pi}}_{\substack{\textit{an endogenous} \\ \textit{profit wedge}}} = \frac{\int_{z_{it}}^{\overline{z}} z\, d\Psi(z)}{z_{it} \cdot [1 - \Psi(z_{it})]} \tag{19}$$

Lemma 2 characterizes the sector-level accounting variables by aggregating over the actively producing individuals in each sector. The aggregate state of sector i at time t is thereby summarized by three variables: (i) the stock of sectoral assets a_{it}; (ii) the share of active entrepreneurs $1 - \Psi(z_{it})$; and (iii) a novel endogenous term, the sector-level profit wedge μ_{it}^{π}.

This novel endogenous profit wedge arises because, when the optimal production scale of a sector's most efficient entrepreneurs is constrained by the exogenous collateral

constraint, equilibrium prices must generate sufficient profitability for less efficient entrepreneurs to also engage in production, thereby ensuring that the sector's aggregate supply meets total demand. And, as reflected in Equation (16), this endogenous profit wedge, μ_{it}^{π}, compounds with the sector's exogenous tax wedge, μ_i^{τ}, further distorting the economy's cross-sector resource allocation.

Crucially, the profit wedge μ_{it}^{π} can be fully recovered from the sector-level profit-to-revenue ratio:

$$\frac{\pi_{it}}{R_{it}} = \frac{\mu_{it}^{\pi} - 1}{\mu_{it}^{\pi}}. \tag{20}$$

This relationship allows us to disentangle the exogenous component of sectoral wedges —arising from goods-market imperfections—from its endogenous counterpart, which stems from within-sector misallocation. In doing so, it provides a sufficient-statistics foundation for identifying and quantifying each wedge, even when goods-market imperfections are not directly observable in the data (e.g., contracting frictions, transaction costs, and so forth). Ultimately, this expression underpins our ability to trace the evolution of each sector's real credit conditions over time.

Income Accounting Identify. With the sector-level accounting variables characterized in Lemma 2, we can now state the income accounting identity of the economy, which articulates how the resources produced within a period—measured as GDP—are allocated across different income categories.

> ***Definition 2 (Income Accounting Identity).*** *The income accounting identity of the model economy is*

$$\underbrace{Y_t}_{\text{total output}} = \underbrace{r_t \cdot K}_{\text{factor income}} + \sum_i \underbrace{\pi_{it}}_{\text{sectoral profits}} + \sum_i \underbrace{T_{it}}_{\text{sectoral tax revenue}} \tag{21}$$

The income accounting identity decomposes aggregate GDP into three components: factor income $r_t \cdot K$, tax revenue from each sector through industrial taxation T_{it}, and profit income earned by each sector's active entrepreneurs π_{it}.

Aggregation in Partial Equilibrium. At this stage, equipped with the economy's income accounting identity, we can now aggregate the disaggregated networked economy

within each period to see what determines the model economy's aggregate efficiency. To keep the exposition clear, it is useful to first define the profitability of assets in each sector—a measure capturing the return from holding one unit of assets in that sector, net of any tax rebates.

Definition 3 (Profitability of Assets). *The profitability of assets, net of tax rebates, for an individual entrepreneur with efficiency z in sector i is given by*

$$\Theta_{it}(z) = \begin{cases} r_t + \tau_t + \mu_i^{\tau} \cdot (\dfrac{z}{z_{it}} - 1) \cdot \phi_i \cdot q_t, & \text{for } \ z \geq z_{it} \\ r_t + \tau_t, & \text{for } \ z < z_{it} \end{cases} \tag{22}$$

where τ_t denotes the tax rebate rate in period t, defined as

$$\tau_t = \frac{\sum_i T_{it}}{K}. \tag{23}$$

Corresponding, the profitability of assets at the sector level is defined as

$$\Theta_{it} \equiv \int_z \Theta_{it}(z) \cdot \psi(z)\, dz \tag{24}$$

The profitability of assets at the individual level (22) in Definition 3 articulates the sources of income from holding assets for different entrepreneurs. For low-efficiency entrepreneurs, the income for holding assets consists solely of two components: rental income and tax rebates. For high-efficiency entrepreneurs, there is an additional source of income—the ability to use assets as collateral to undertake profit-generating production.

Building on the characterization of asset profitability, the income accounting identity (21) implies that, in equilibrium,

$$TFP_t \equiv \frac{Y_t}{K} = \sum_i a_{it} \cdot \Theta_{it}. \tag{25}$$

which says the endogenous aggregate TFP of the economy is just the asset-weighted average of each sector's sector-level profitability of assets.

While Equation (25) neatly links sectoral profitability to aggregate efficiency, it is inherently partial-equilibrium, as it takes the asset price q_t as given. To fully understand the model, we must examine how the economy evolves over time and how q_t is determined in equilibrium. Hence, we now turn to the second type of financial friction in the

economy, which shapes its dynamic evolution through individuals' consumption–saving decisions.

3.2 Incomplete Markets and Optimal Saving Decisions

Individuals' intertemporal problem. After maximizing their within-period income, individuals in the economy now solve the following intertemporal optimization problem to maximize their lifetime utility:

$$\begin{aligned} V_{it}(a,z) &= \max_{c,a'} ln(c) + \beta \cdot E_t\, V_{i,t+1}(a',z') \\ s.t.\;\; & c + q_t \cdot a' = [\Theta_{it}(z) + q_t] \cdot a \end{aligned} \tag{26}$$

From Problem (26), the linearity of income in asset holdings captures much of the tractability of the model, yielding a law of motion for assets that is linear in a:

$$a'(a,z) = (\frac{\Theta_{it}(z)}{q_t} + 1) \cdot a - \frac{c(a,z)}{q_t}, \tag{27}$$

which, together with log flow utility, admits a closed-form solution for the individual's optimal saving policy.

> ***LEMME 3 (Individuals' Optimal Saving Behavior).*** *Individuals' optimal saving policy is linear in their asset holdings:*
>
> $$a'(a,z) = \beta \cdot (\frac{\Theta_{it}(z)}{q_t} + 1) \cdot a, \tag{28}$$
>
> *where the constant saving rate is determined by the time discount factor β.*

From Lemma 2, the constant saving rate β follows directly from log utility and linear wealth dynamics. Moreover, this linearity ensures that optimal asset holdings never become negative ($a \geq 0$), even without this constraint being imposed.

Given individuals' optimal saving policy, the asset market–clearing condition

$$\sum_i \int \int a'(a,z) \cdot g_{it}(a,z)\, dadz = K \tag{29}$$

delivers

$$q_t = \frac{\beta}{1-\beta} \cdot TFP_t = \frac{\beta}{1-\beta} \cdot \sum_i a_{it} \cdot \Theta_{it} \tag{30}$$

which implies that, in equilibrium, the asset price q_t serves as a perfect mirror of the economy's aggregate performance. Furthermore, by embedding sector-specific credit conditions, q_t establishes an interdependence between micro-level resource allocation and macro-level outcomes.

Aggregating individuals' optimal saving behavior across entrepreneurs within each sector yields the law of motion for sector-level asset holdings:

$$a_{i,t+1} = \beta \cdot [\frac{\Theta_{it}}{q_t} + 1] \cdot a_{it} \tag{31}$$

which governs the dynamic evolution of sectoral asset holdings and facilitates the characterization of the steady state of the model economy.

3.3 Steady State

From the sector-level law of motion for assets (Equation 31), assets reallocate each period toward sectors with higher asset profitability and away from those with lower profitability. In the long run, this process equalizes the profitability of assets across sectors:

$$\Theta_i^{ss} = \Theta_j^{ss} = TFP^{ss}, \ \ \forall i, j \tag{32}$$

which leads to the first proposition of this paper.

> ***PROPOSITION 1.*** *In the steady state, the aggregate performance of the disaggregated network economy is isomorphic to that of a single-sector framework:*
>
> $$Y^{ss} = TFP^{ss} \cdot K \tag{33}$$

Proposition 1 implies that, despite the model's rich micro linkages and substantial heterogeneity across many economically relevant dimensions—such as technologies, goods-market distortions, and credit-market imperfections—in the long-run, the reallocation of assets will fully eliminate these differences. As a result, the aggregate performance of the disaggregated network economy coincides with that of a simple representative-sector economy. This finding provides theoretical justification for the widespread use

of single-sector frameworks in macroeconomic analysis: although seemingly abstract, they may properly approximate the economy's aggregate outcomes, and therefore the insights derived from them may remain valid for understanding our disaggregated network economies.

Although the aggregate performance of the disaggregated economy is isomorphic to that of a single-sector model, the presence of rich micro-level linkages among individual producers creates channels via which microeconomic shocks will propagate to other sectors of the economy and generate substantial macroeconomic consequences. Accordingly, the analysis henceforth focuses on the economy's steady state. The objective is to derive a set of empirically measurable sufficient statistics that capture how changes in sectoral primitives $\{A_i, \mu_i, \phi_i\}$ affect the economy's long-run aggregate efficiency—namely, the centrality of industries for long-run growth—and to use this framework to guide the design of microeconomic interventions in support of macroeconomic development.

4 Propagation with Credit Market Imperfections

4.1 Notations and Definitions

Before proceeding, I first introduce the notations and definitions that characterize local features of the initial steady state, which will be central to our analysis. All objects are equilibrium concepts defined at the initial steady state.

Final Expenditure Shares

Let $b \equiv [b_i]$ be the $N \times 1$ vector of final consumption shares, where the i^{th} element is defined as the share of good i in the budget for final consumption:

$$b_i \equiv \frac{p_i \cdot c_i}{\sum_j p_j \cdot c_j} = \frac{\partial \, log \, C_Y(P)}{\partial \, log \, p_i} \tag{34}$$

This measure captures how important good i is for final consumption in the initial steady state.

Cost-based Input-output Matrix

Let $\tilde{\Omega}$ be the $N \times N$ cost-based input–output matrix, where ij^{th} element is defined as the expenditure share of good j in the production cost of good i:

$$\tilde{\Omega}_{ij} \equiv \frac{p_j \cdot m_{ij}}{C_i(P)} = \frac{\partial \, log \, C_i(P)}{\partial \, log \, p_j} \tag{35}$$

This measure captures how important good j is for producing good i in the initial steady state.

Cost-based Domar Weight

The cost-based Domar weight is defined as:

$$\tilde{\lambda} \equiv b' \cdot (I - \tilde{\Omega})^{-1} \tag{36}$$

with its i^{th} element representing an elasticity-based centrality measure of the importance of good i for final consumption, accounting for all higher-order network effects. In essence, $\tilde{\lambda}_i$ measures—given the distribution of resources in the initial steady state—how much additional final consumption could be obtained in equilibrium if the economy were endowed with one extra unit of good i, taking into account the infinite sequence of production linkages implied by the production technology.

Factor Share

Let Λ^{ss} denote the steady-state factor share:

$$\Lambda^{ss} = \frac{r^{ss} \cdot K}{Y^{ss}} \tag{37}$$

Profit Share

Let Λ_{π}^{ss} denote the steady-state aggregate profit share:

$$\Lambda_{\pi}^{ss} = \frac{\sum_i \pi_i^{ss}}{Y^{ss}} \tag{38}$$

Equipped with these objects that capture the local features of the model economy in steady state, we are now ready to examine the fundamental difference between input–output linkages and credit-condition linkages in how they propagate microeconomic shocks, and how the presence of credit market imperfections fundamentally reshapes the propagation mechanism in production-networked economies.

4.2 Distinction between the I.O. and Credit-Condition Linkages

Distinction between the Two Linkages for Micro-shock Propagation. From the decomposition of individuals' optimal production decisions under credit market imperfections—that is, the two-stage structure comprising the cost-minimization and participation decisions—we can clearly see the fundamental difference in how input–output linkages and credit-condition linkages operate in propagating microeconomic shocks.

Specifically, the input–output linkages operate through the minimized marginal cost of production $C_i(P)$ by shaping the expenditure shares of different inputs in the minimized cost of production:

$$\tilde{\Omega}_{ij} \equiv \frac{\partial \, log \, C_i(P)}{\partial \, log \, p_j} = \frac{p_j \cdot m_{ij}}{C_i(P)} \tag{35}$$

That is, the expenditure share of sector i on the output of sector j—which measures the extent to which one sector relies on another's production—determines its direct exposure to shocks originating from that sector, as well as those transmitted through it. This input–output interdependence is fully recoverable from the economy's cost-based input–output matrix $\tilde{\Omega}$. Accordingly, through the cost-based Domar weight $\tilde{\lambda}$, derived from the Leontief inverse of $\tilde{\Omega}$, we obtain a comprehensive measure of an industry's production centrality that accounts for the entire infinite sequence of production linkages in the production-networked economy.

In contrast, the credit-condition linkage is more subtle, operating through individuals' constrained optimal production scale:

$$\underbrace{y_{it}(a,z)}_{\textit{Constrained Production}} = A_i \cdot z \cdot \frac{\phi_i \cdot q_t \cdot a}{C_i(P_t)} \tag{9}$$

taking the minimized marginal cost of production, $C_i(P_t)$, determined in the preceding cost-minimization process as given.

That is, in settings with credit market imperfections, beyond individuals' minimized cost of production $C_i(P_t)$, the severity of each sector's credit-market imperfection, ϕ_i, exogenously shifts the optimal production scale at the individual level. The exogeneity of this shift implies that the cost-based input–output matrix $\tilde{\Omega}$, while fully capturing the interdependence arising from the cost-minimization process, becomes insufficient to infer the corresponding constrained optimal production scale in such environments. This is the fundamental reason why existing sufficient statistics—constructed solely from the

economy's input–output structure—generally fail when extended to settings with credit market imperfections.

Recognizing the general failure of existing industry centrality measures in the presence of credit market imperfections, one key theoretical contribution of this paper is to derive a sufficient statistic that identifies the evolution of each sector's real credit conditions, thereby allowing us to characterize how the presence of credit-market imperfections fundamentally reshapes the propagation and aggregation of microeconomic shocks in production-networked economies.

4.3 Sufficient Statistics for One Sector's Real Credit Condition

This subsection turns to investigating how a sector's real credit condition manifests itself in the data. Specifically, the analysis proceeds by comparing the economy's income accounting identity with its counterpart in the absence of credit market imperfections, thereby making explicit the channel through which a sector's real credit condition manifests itself in equilibrium.

With and W.O. Credit Market Imperfections. Recall that the economy's income identity is expressed as

$$\underbrace{Y_t}_{\textit{total output}} = \underbrace{r_t \cdot K}_{\textit{factor income}} + \sum_i \underbrace{\pi_{it}}_{\textit{sectoral profits}} + \sum_i \underbrace{T_{it}}_{\textit{sectoral tax revenue}} \tag{21}$$

which decomposes the total output of the economy into factor incomes, sectoral profits, and sectoral tax revenues.

When there are no credit market imperfections constraining individual entrepreneurs' production scale, the fierce competition among them within a sector will ensure that, in equilibrium, only the most efficient entrepreneurs engage in production, and they will earn zero profit. That is,

$$\pi_{it} = 0, \ \ \textit{for } \forall i, t \tag{39}$$

However, when credit markets are imperfect and the optimal production scale of the most efficient entrepreneurs is constrained, the equilibrium prices must adjust to incentivize less efficient producers to participate in production through positive profits, thereby ensuring that the sector's total supply meets its total demand. Consequently,

in the presence of credit market imperfections, equilibrium features a continuum of heterogeneous firms within each industry actively producing and earning strictly positive profits:

$$\pi_{it} > 0, \ \ for \ \forall i, t \tag{40}$$

The above comparison reveals that the equilibrium profitability of a sector's active entrepreneurs, once properly processed, can mirror its underlying real credit condition. Specifically, guided by this economic reasoning, I construct the following sector-level equilibrium object, which I term the profitability of a sector's active entrepreneurs.

Definition 4 (Profitability of a Sector's Active Entrepreneurs). *In period t, the profitability of a sector's active entrepreneurs is defined as*

$$H_{it} \equiv \frac{\pi_{it}}{q_t \cdot a_{it}} = \mu_i^{\tau} \cdot H(z_{it}) \cdot \phi_i, \ \ for \ \forall i, t \tag{41}$$

where $H(\cdot)$ is the sector-level profitability function, given by

$$H(z_{it}) \equiv (\mu_{it}^{\pi} - 1) \cdot [1 - \Psi(z_{it})], \tag{42}$$

which evolves endogenously with the sector-level productivity cutoff z_{it}.

Equation (41) in *Definition 4* reveals that the profitability of a sector's active entrepreneurs, H_{it}, is jointly determined by the sector's goods-market imperfection, μ_i^{τ}, and credit-market imperfection, ϕ_i. Moreover, from the profitability function $H(z_{it})$, we obtain a direct mapping from the sector's real credit condition to its within-sector allocation of resources between high- and low-efficiency entrepreneurs, thereby allowing us to trace how a sector's real credit condition ultimately translates into its within-sector misallocation.

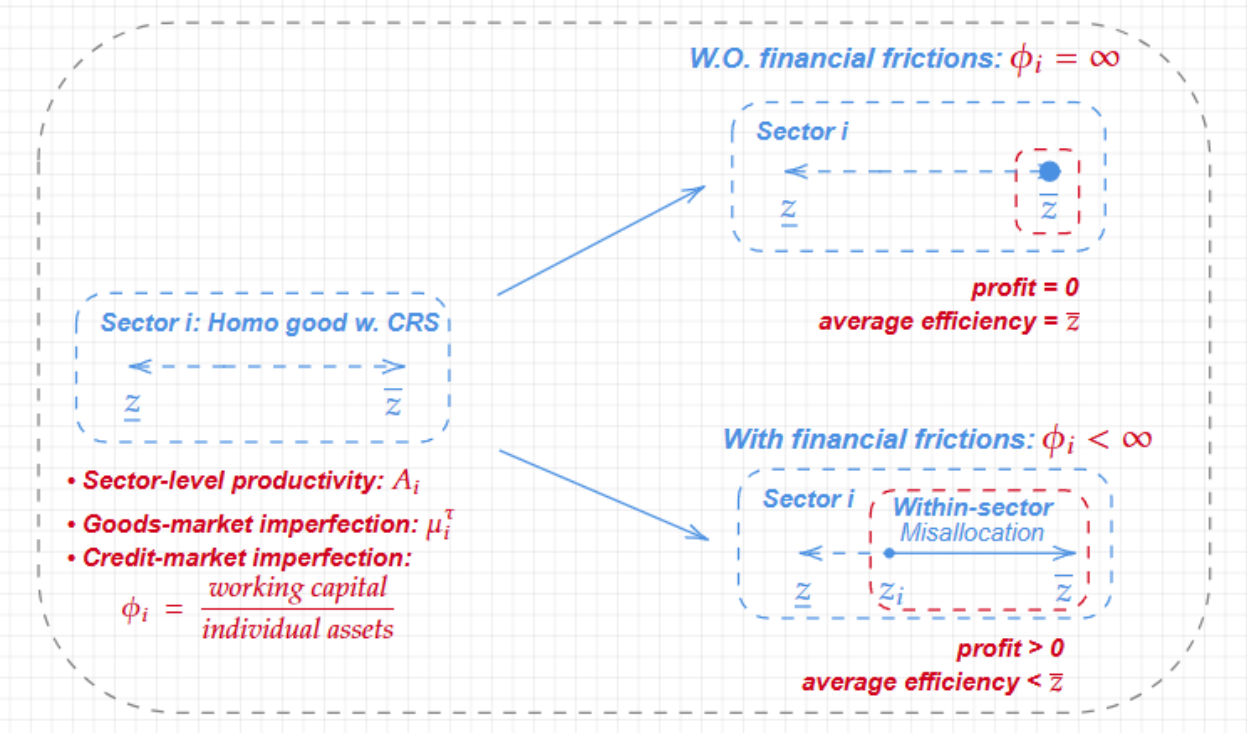


Figure 1: Within-period Misallocation with v.s. w.o. Credit Market Imperfections

Figure 1 visually illustrates the distinct features of a sector's equilibrium behavior when it is subject to, versus exempt from, credit market imperfections. As the figure shows, when credit market imperfections constrain the optimal production scale of the most efficient entrepreneurs, a sector's real credit condition effectively introduces a within-sector dimension of resource misallocation between high- and low-efficiency entrepreneurs in the model economy. Embedding this within-sector block into the cross-sector production network then makes it conceptually clear why the presence of credit market imperfections fundamentally reshapes the propagation mechanism of microeconomic shocks in production-networked economies, thereby motivating the following subsection.

4.4 Propagation with Credit Market Imperfections

This subsection visually illustrates how the presence of credit market imperfections fundamentally reshapes the propagation of microeconomic shocks in production-networked economies, thereby establishing the conceptual foundation for the rigorous aggregation developed in Section 5. In particular, the analysis proceeds by comparing the propagation of microeconomic shocks in economies with and without credit market imperfections.

4.4.1 Propagation without Credit Market Imperfections: An Illustration

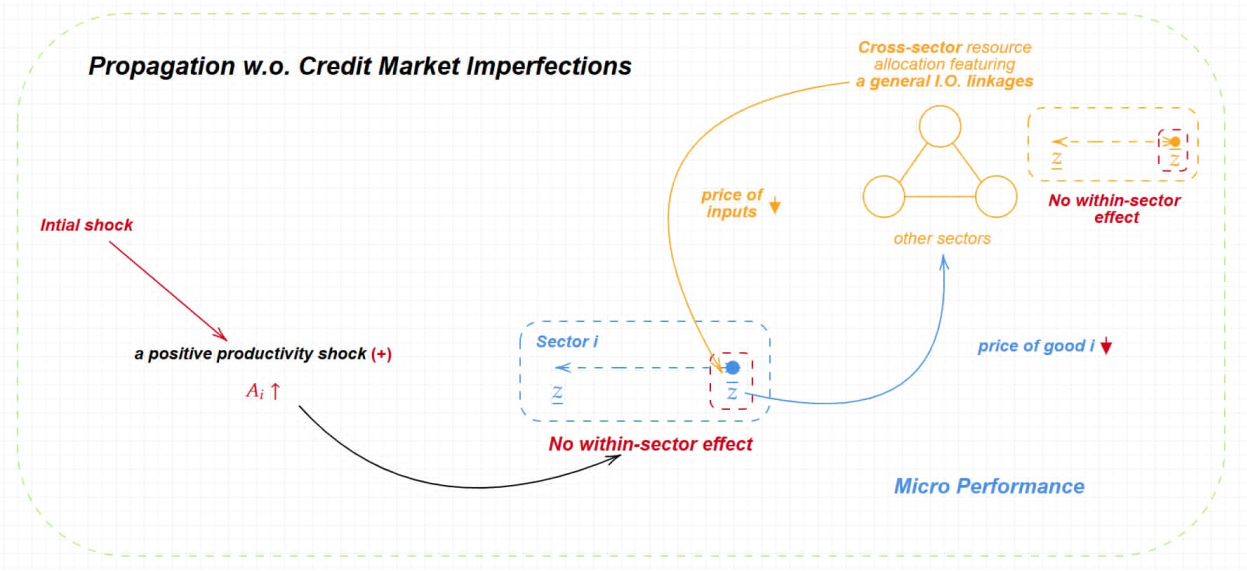


Figure 2: Propagation of Technological Shock w.o. Credit Market Imperfection

Figure 2 illustrates the canonical propagation of a microeconomic shock in a production-networked economy absent credit market imperfections, using a technological shock as an example. A positive technology shock to sector i directly benefits its trading partners by lowering the price of good i as an input (as indicated by the downward red arrow in Figure 2), thereby reducing the marginal cost of production for sectors that employ good i in their production processes. This positive shock then propagates further to the trading partners of sector i's trading partners: as their input costs decline, lower marginal costs translate into lower equilibrium output prices, which in turn benefit the downstream sectors that rely on their goods as inputs.

As Figure 2 shows, this initial technological improvement in sector i ripples through the economy's input–output linkages via adjustments in marginal production costs, inducing resource reallocation across sectors along the way. In a general production network, the ensuing sequence of cost and price adjustments eventually feeds back to the original sector i, which benefits from lower input prices (as shown by the downward yellow arrow in Figure 2). This iterative process generates an infinite sequence of price feedback loops, with the overall general-equilibrium effect of the original local technology shock ultimately pinned down by the fixed point of this propagation mechanism.

The key feature of the above propagation mechanism is that, in the absence of credit

market imperfections, only the most efficient entrepreneurs in each sector engage in production. Consequently, although the infinite sequence of price feedback loops alters the economy's across-sector resource allocation, it has no effect on within-sector efficiency, since there is no misallocation within sectors to begin with. This stands in sharp contrast to the case with credit market imperfections, which the following subsection examines in detail.

4.4.2 Propagation with Credit Market Imperfections

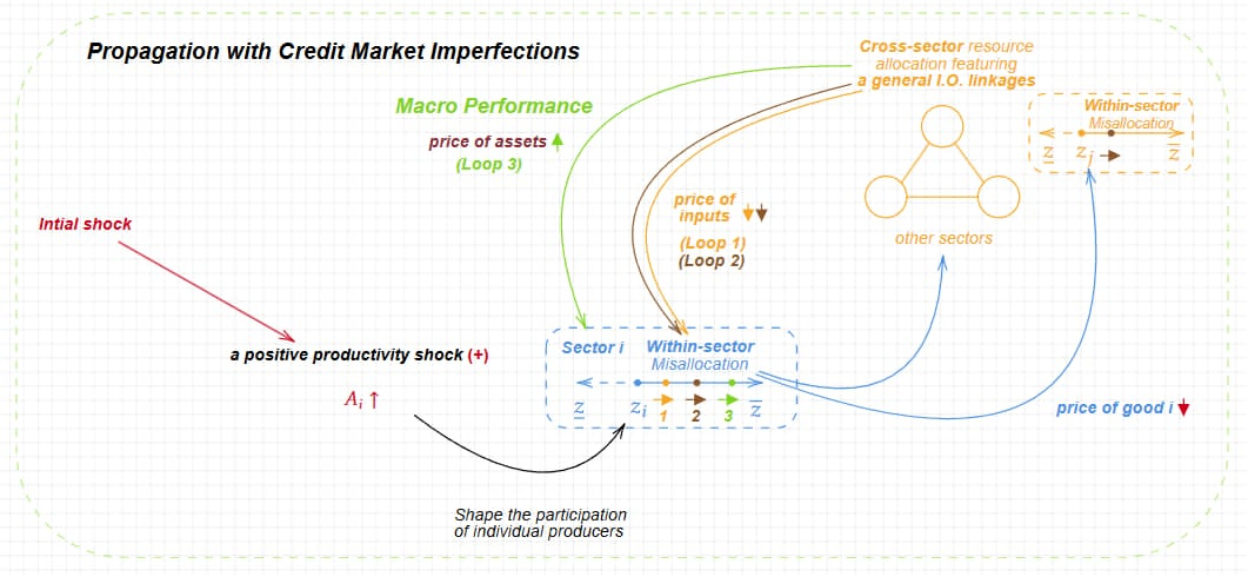


Figure 3: Propagation of a Technological Shock with Credit Market Imperfection

Sector-Level Shock to Technology. Figure 3 illustrates the propagation of a sector-level technological shock in the presence of credit market imperfections. Following a positive technological shock to a particular sector, the dense web of input–output linkages across sectors and industries generates the same infinite sequence of price feedback loops as described in the preceding subsection. Along these loops, producers adjust their optimal input expenditure shares to minimize marginal production costs, thereby inducing systematic cross-sector reallocation of resources throughout the economy, as illustrated by the inner orange loop labeled ***Loop 1*** in Figure 3.

What fundamentally reconfigures the propagation mechanism of a local technological shock is the presence of credit market imperfections that constrain the optimal production scale of the most efficient entrepreneurs. In equilibrium, a continuum of entrepreneurs,

heterogeneous in their idiosyncratic efficiencies, must actively engage in production to ensure that the sector's aggregate supply meets aggregate demand. Because the marginal benefit of production remains strictly positive, each active entrepreneur produces up to the limit imposed by their credit constraint. This equilibrium configuration introduces a dimension of within-sector resource misallocation—where less efficient producers remain active alongside constrained efficient ones—which, in turn, reshapes how sectors transmit technological shocks through the production network, to which we now turn.

First, since the collateral constraints of all active entrepreneurs in each sector bind, a positive sector-level technological shock can substantially ease the credit tension faced by entrepreneurs in downstream sectors by lowering their input prices. Given the same amount of liquidity obtained from the financial sector, these downstream producers can now afford to purchase a larger quantity of inputs. This effectively relaxes their production constraints and improves within-sector efficiency by shifting the productivity cutoff for active production rightward. In other words, even though the exogenous credit-market imperfection parameter, ϕ_j, remains unchanged, the lower input price allows more efficient entrepreneurs to expand production. The resulting decline in the sector's equilibrium output price then drives marginal producers—those near the initial productivity cutoff—out of the market. This improvement in the downstream sector's within-sector efficiency further reduces its output price, reinforcing the initial decline induced by the lower input price of good i and thereby triggering a second layer of price feedback loops, as illustrated by the brown loop labeled ***Loop 2*** in Figure 3.

Furthermore, this improvement in overall efficiency raises the aggregate performance of the economy, thereby increasing the price of capital and, in turn, the value of each entrepreneur's collateral. This mechanism triggers a third layer of feedback loops—operating through the macroeconomic performance and asset prices of the model economy—as illustrated by the green loop labeled ***Loop 3*** in Figure 3.

Finally, as illustrated in Figure 3, in an economy with a general production network, these beneficial efficiency effects eventually feed back to the originating sector i through lower input prices and higher collateral values, thereby further enhancing its own within-sector efficiency. The key to evaluating the aggregate macroeconomic consequences of such local technological shocks under credit market imperfections is therefore to identify an object that enables us to track each sector's real credit condition and, in turn, its underlying within-sector misallocation—an object that we formally derive in Section 5.

Sectoral Shock to Goods-Market and Credit-Market Imperfections

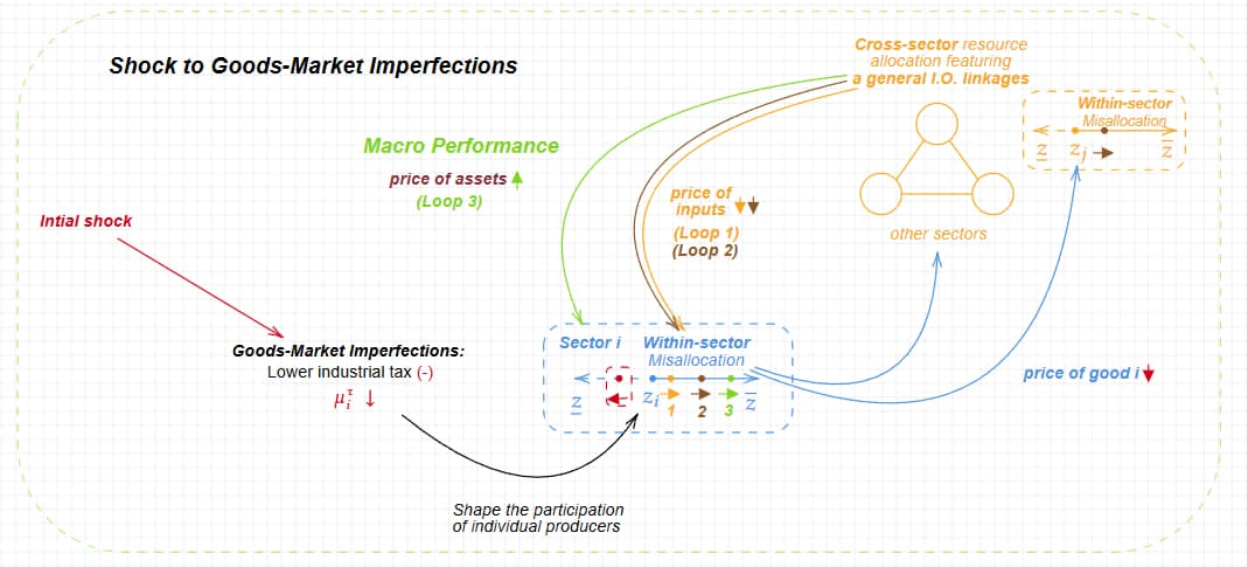


Figure 4: Propagation of an Industrial Tax Shock with Credit Market Imperfection

Sectoral Shock to Goods-Market Imperfections. Figure 4 illustrates the propagation of a sector-level shock to goods-market imperfections in the presence of credit market imperfections, using a negative shock to one sector's industrial tax rate as an example to convey the intuition. By lowering the output price of the originating sector, a negative tax shock triggers the same infinite sequence of price feedback loops as a positive sectoral technological shock—that is, ***Loops 1, 2, and 3***.

The key distinction between a sector-level shock to technology and one to the tax rate is that, unlike a technological shock, a reduction in the tax rate directly alters the within-sector allocation of resources between high- and low-efficiency entrepreneurs. A lower tax rate improves the transaction environment for the affected sector but simultaneously generates an adverse self-selection effect by shifting the productivity cutoff leftward, as some less efficient entrepreneurs now find it profitable to enter production. Consequently, to properly evaluate the macroeconomic implications of a local tax shock, it is necessary to identify sufficient statistics that can capture this direct deterioration in within-sector efficiency, which we will turn to in Section 5.

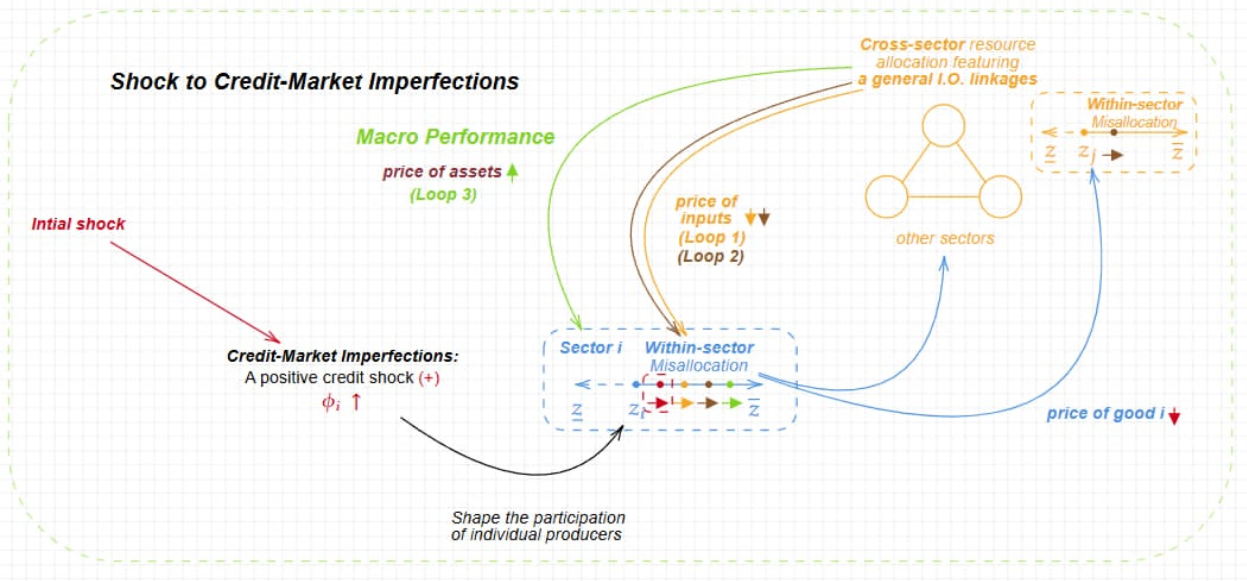


Figure 5: Propagation of a Credit Shock with Credit Market Imperfection

Sectoral Shock to Credit-Market Imperfections. Figure 5 illustrates the propagation of a sector-level shock to credit using a positive one as an example to convey the intuition. By alleviating the original sector's within-sector misallocation and thereby lowering its output price, a positive credit shock triggers the same infinite sequence of price feedback loops as a positive sectoral technological shock—that is, ***Loops 1, 2, and 3***.

The only difference is that the lower sectoral price here arises from a direct alleviation of the sector's within-sector misallocation—specifically, the rightward shift of the productivity cutoff. Therefore, to properly evaluate the macroeconomic consequences of a sector-level credit shock, it is essential to identify sufficient statistics that map credit conditions to within-sector misallocation. The formal derivation of these sufficient statistics is presented in Section 5, to which we now turn.

5 Aggregation: Macro Effects of Micro Shocks

In Section 4, we have seen that the presence of credit market imperfections—by opening up the within-sector misallocation margin—fundamentally reconfigures the propagation mechanism of micro-level shocks. This section now turns to aggregate these effects, examining how the altered propagation channels ultimately shape the way in which microeconomic shocks manifest at the macro level.

5.1 I.O. Linkages: the Across-sector Misallocation

PROPOSITION 2. *Following Baqaee and Farhi (2020),*

$$d\log TFP^{ss} = \underbrace{\tilde{\lambda}' \cdot (d\log A - d\log \mu^{\tau} + \overbrace{d\log z^{ss}}^{\text{a novel self-selection effect}})}_{\text{direct effect holding the distribution of resource across sectors constant}} \quad \underbrace{-d\log \Lambda^{ss}}_{\text{indirect effect via resource reallocation}} \tag{43}$$

Proposition 2 states that, in general, the response of an economy's aggregate efficiency to micro-level shocks can be decomposed into two components: a **direct effect**—the impact of the shock while holding the resource distribution across sectors fixed—and an **indirect effect** arising from the reallocation of resources across sectors.

Given the definition of the cost-based Domar weights, $\tilde{\lambda}$ (equation 36), the direct effect is immediate. Conditional on the initial allocation of resources across sectors, $\tilde{\lambda}_i$ quantifies the equilibrium increase in final consumption that would arise if the economy were endowed with one additional unit of good i—for instance, through an exogenous sectoral technology shock $d\log A_i$. This gain in final consumption translates one-for-one into an improvement in aggregate productivity, $d\log TFP^{ss}$, since the total endowment of primary factors remains fixed.

The same logic applies to industrial tax shocks, μ^{τ}. An increase in the tax wedge, $d\log \mu^{\tau}$, is perceived by the economy as equivalent to a reduction in sectoral productivity, as it effectively withdraws resources from the system. This equivalence explains the negative sign in front of $d\log \mu^{\tau}$ in the aggregation theorem: higher tax rates elevate production costs and thereby directly reduce aggregate efficiency.

Beyond the direct effect, the distribution of resources across sectors is itself an equilibrium outcome that adjusts endogenously in response to the shock, reflecting the reallocation of resources through the infinite sequence of price feedback loops induced by producers' cost-minimizing behavior. When the initial allocation of resources is inefficient, this reallocation generates a non-zero first-order effect on aggregate efficiency. Moreover, when the shock pertains to tax wedges, any resources artificially withdrawn from the system must be rebated, further modifying the post-shock resource allocation across sectors and, consequently, the aggregate efficiency response. Although these indirect efficiency effects operating through cross-sectoral reallocation in inefficient economies may appear complex, Baqaee and Farhi (2020) demonstrate that, ex post, they can be summarized by

a single sufficient statistic—the change in the economy's factor income share, $d \log \Lambda^{ss}$—a result that marks a significant advance in the study of misallocation.

The key novelty under credit-market imperfections lies in the emergence of a self-selection effect in production participation within each sector, driven by the endogenous evolution of sectoral credit conditions following a shock. This mechanism induces a systematic reallocation of resources between high- and low-efficiency entrepreneurs and thus constitutes the theoretical counterpart capturing the within-sector dimension of micro-shock propagation in the presence of credit frictions. The remainder of this section focuses on this novel component and proceeds as follows. I first characterize the sufficient statistic that captures the endogenous evolution of each sector's real credit condition, and then derive the sufficient statistic that maps this credit evolution into within-sector resource reallocation between high- and low-efficiency entrepreneurs, thereby tracing the evolution of the self-selection effect in production participation.

5.2 Evolution of Each Sector's Real Credit Condition

Recall that, in the long run, the flow of assets across sectors perfectly equalizes the return on assets in each sector (equation 32). This implies that, in the steady state, the profitability of each sector's active entrepreneurs, H_i^{ss}, is perfectly equalized.

LEMMA 4. *In the steady state, the profitability of each sector's active entrepreneurs satisfies*

$$H_i^{ss} \equiv \frac{\pi_i^{ss}}{q_t \cdot a_i^{ss}} = \mu_i^{\tau} \cdot \mathcal{H}(z_i^{ss}) \cdot \phi_i = \frac{\beta}{1-\beta} \cdot \Lambda_{\pi}^{ss}, \quad for\ \forall i \tag{44}$$

which implies

$$d\,log\, H_i^{ss} = d\,log\, H_j^{ss} = d\,log\, \Lambda_{\pi}^{ss}, \quad for\ \forall i, j \tag{45}$$

Lemma 4 establishes that, *ex post*, the evolution of the economy's aggregate profit share, $d\,log\,\Lambda_{\pi}^{ss}$, serves as a nonparametric sufficient statistic for the evolution of each sector's real credit condition in the steady state. The last equality in equation (44) follows from the proportional relationship between the equilibrium asset price q_t and aggregate productivity TFP_t in each period, as characterized by equation (30) through the asset-market-clearing condition.

Equipped with this result, which identifies the sufficient statistic for the steady-state evolution of each sector's real credit condition following a microeconomic shock, we now proceed to characterize how this credit-condition evolution translates into within-sector resource reallocation between high- and low-efficiency entrepreneurs.

5.3 Credit-Condition Linkage: the Within-sector Misallocation

By examining the composition of a sector's profitability of active entrepreneurs (equation 41), we can trace how changes in its credit condition translate into within-sector misallocation through the profitability function $\mathcal{H}(\cdot)$.

PROPOSITION 3. *In the steady state, the evolution of sector i's self-selection effect satisfies*

$$d\,log\,z_i^{ss} = h_i \cdot (d\,log\,\mu_i^{\tau} + d\,log\,\phi_i - d\,log\,\Lambda_{\pi}^{ss}), \tag{46}$$

where h_i denotes the elasticity of the sector-specific productivity cutoff z_i^{ss} with respect to the profitability function $H(z_i^{ss})$, and is given by

$$h_i \equiv -\frac{d\,log\,z_i^{ss}}{d\,log\,\mathcal{H}(z_i^{ss})} = \frac{\pi_i^{ss}}{R_i^{ss}} \tag{47}$$

which can be fully recovered from the sector-level profit-to-revenue ratio in the initial steady state.

Proposition 3 establishes that, irrespective of the distribution of each sector's within-sector heterogeneity $\psi(z)$, the sector-level profit-to-revenue ratio in the initial steady state serves as a nonparametric sufficient statistic for the elasticity of the sector-specific productivity cutoff z_i^{ss} with respect to the sector's real credit condition. This distributional irrelevance arises because the characterization of the elasticity h_i depends only on two moments of $\psi(z)$—the cutoff z_i^{ss} and the average productivity among active entrepreneurs, $E[z|z \geq z_i^{ss}]$—and can be expressed as

$$h_i = 1 - \frac{z_i^{ss}}{E[z|z \geq z_i^{ss}]}, \tag{48}$$

a ratio of the cutoff to the tail mean, which is independent of the specific shape of $\psi(z)$.

Equation (46) demonstrates that both sectoral shocks to goods-market imperfections, $d\log\mu^{\tau}$, and shocks to credit-market imperfections, $d\log\phi_i$, exert a direct impact on the originating sector's within-sector resource misallocation. The sector-level profit-to-revenue ratio in the initial steady state—reflecting the extent of the sector's initial within-sector misallocation—fully captures the direct effects of these exogenous shocks on the sector's internal allocation efficiency. Sectors with a higher initial degree of within-sector misallocation experience a correspondingly larger first-order efficiency gain in response to such shocks. Therefore, as shown by equation (46) in Proposition 3, the sector-level profit-to-revenue ratio serves as a nonparametric sufficient statistic that captures the direct efficiency effects of sectoral shocks to market imperfections on the sector's internal allocation—corresponding to the red leftward arrow in Figure 4 and the red rightward arrow in Figure 5.

We have now fully characterized the sufficient statistics that capture all propagation channels of microeconomic shocks in a production-networked economy with credit-market imperfections. The next subsection examines how these sufficient statistics determine the channels through which microeconomic shocks manifest at the macro level—that is, in the economy's aggregate efficiency.

5.4 Aggregation Theorem with Credit Market Imperfections

Accounting Items for the Credit-Condition Linkage. Before proceeding to the aggregation theorem, I first introduce the credit-condition accounting items, guided by *Proposition 3*, using sector-level profit-to-revenue data from the initial steady state:

$$\mathcal{H} \equiv \begin{bmatrix} h_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & h_N \end{bmatrix}, \quad \eta \equiv (h_1, \dots, h_N)'$$

where $\mathcal{H}$ and η denote, respectively, the diagonal matrix and the vector of the economy's sector-level profit-over-revenue ratio.

Combining Propositions 2 and 3 then yields the aggregation theorem for the model economy, which formalized how micro-level changes—stemming from technology, goods-market, and credit-market imperfections—jointly determine the long-run evolution of aggregate productivity.

THEOREM 1 (Aggregation Theorem). *Let* $(d\log A,\ d\log\mu^{\tau},\ d\log\phi)$ *denote the vectors*

of sector-level shocks to technology, goods-market imperfections, and credit-market imperfections over the period under consideration, defined as

$$d\log A \equiv (d\log A_1, ..., d\log A_N)'$$
$$d\log \mu^\tau \equiv (d\log \mu_1^\tau, ..., d\log \mu_N^\tau)'$$
$$d\log \phi \equiv (d\log \phi_1, ..., d\log \phi_N)'$$

Then, in the steady state, the response of an economy's aggregate productivity to micro-level shocks is given by

$$d\log TFP^{ss} = \underbrace{\tilde{\lambda}' \cdot d\log A}_{\Delta Technology} \underbrace{-\tilde{\lambda}' \cdot [I - \mathcal{H}] \cdot d\log \mu^\tau}_{\substack{\Delta Goods\text{-}Market \\ Imperfections}} + \underbrace{\tilde{\lambda}' \cdot \mathcal{H} \cdot d\log \phi}_{\substack{\Delta Credit\text{-}Market \\ Imperfections}} \underbrace{-\tilde{\lambda}' \cdot \eta \cdot d\log \Lambda_\pi^{ss} - d\log \Lambda^{ss}}_{\Delta Resource\ Reallocation}$$

where $\tilde{\lambda}' = (\tilde{\lambda}_1, ..., \tilde{\lambda}_N)$ denotes the vector of each sector's cost-based Domar Weights in the initial steady state.

Theorem 1 establishes that the evolution of an economy's aggregate productivity can always be decomposed into four components: pure technological changes, changes in goods-market imperfections, changes in credit-market imperfections, and the indirect efficiency effect operating through resource reallocation, with the first term of the indirect effect capturing the within-sector dimension of reallocation and the second capturing the across-sector dimension. For sector-level technological shocks, $d\log A$, the cost-based Domar weights $\tilde{\lambda}$ remain the nonparametric sufficient statistics summarizing the direct contribution of these shocks to aggregate efficiency. By contrast, for micro-level shocks to goods-market or credit-market imperfections, the direct effects differ because such shocks also alter the originating sector's within-sector misallocation. Specifically, Theorem 1 implies that higher sectoral profitability increases a sector's centrality—that is, its macroeconomic influence—with respect to credit shocks but decreases its centrality with respect to conventional industrial-tax shocks. The intuition is that high profitability reflects poor within-sector resource allocation in the initial steady state, indicating greater potential gains from relaxing credit constraints in that sector. Conversely, a reduction in the sector's tax rate would further distort its internal allocation, thereby attenuating the aggregate benefit of what would otherwise appear to be a positive goods-market intervention.

More importantly, this decomposition is not only theoretically elegant but also empirically powerful, as all the sufficient statistics on which it relies are directly observable in microeconomic data. Consequently, the theorem offers a transparent framework for attributing long-run aggregate productivity growth to its underlying microeconomic sources. In particular, once the evolution of an economy's micro-level fundamentals —technology, goods-market imperfections, and credit-market imperfections—can be accurately measured, the aggregation theorem enables a quantitative assessment of their respective contributions to the long-run productivity growth observed in the data.

This framework is particularly well suited to addressing a central question in the macro-finance literature: to what extent does financial deepening enhance aggregate efficiency by alleviating resource misallocation? Equipped with the aggregation theorem and today's rich firm-level financial datasets—such as Compustat—we can quantify the contribution of financial deepening by directly measuring the evolution of sectoral credit-market imperfections, captured by $d\log\phi$.

Moreover, if we take a step further and impose a set of commonly used assumptions on the economy's primitives—specifically, Cobb–Douglas production technologies and a Pareto distribution of firm-level heterogeneity (that is, $\psi(z)$ following a Pareto distribution, as supported by empirical evidence on firm size distributions)—then the aggregation theorem does more than merely account for the sources of long-run growth *ex post*. It also provides *ex ante* policy guidance for credit-market interventions at the micro level. The key insight is that, under these assumptions, both within-sector and across-sector misallocations are determined entirely by the structural primitives of the economy and remain invariant to the specific structure of exogenous credit shocks. As a result, the indirect efficiency effects vanish, allowing the aggregation theorem to serve as a foundation for designing ex ante policy interventions targeting credit-market imperfections.

Corollary 1 (Credit-Market Intervention Centrality). *Under the Cobb-Douglas and Pareto-distributed assumptions stated above, the centrality of sector i with respect to credit-market interventions is given by:*

$$\underbrace{\frac{d\log TFP^{ss}}{d\log\phi_i}}_{\substack{\text{Credit Centrality}\\ \text{for sector } i}} = \underbrace{\tilde{\lambda}_i}_{\substack{\text{Centrality of}\\ \text{Production for } i}} \cdot \underbrace{h_i}_{\substack{\text{profitability}\\ \text{of sector } i}} \tag{49}$$

where

- *$\tilde{\lambda}_i$ is the sector's cost-based Domar weight, measuring its production centrality*
- *h_i is its profitability, measured by the sector-level profit-over-revenue ratio*

This expression shows that, under Cobb–Douglas technology and Pareto-distributed within-sector heterogeneity, the efficiency gain from a sector-level credit shock equals the product of the sector's production centrality and its profitability. Hence, a sector that is highly central in production is not necessarily the optimal target for credit-market interventions; it must also exhibit substantial within-sector misallocation, as reflected in high profitability.

With the aggregation theorem in hand, we now turn to a quantitative application to the U.S. economy to address two closely related questions: To what extent has financial deepening contributed to improvements in the economy's aggregate efficiency? And do industries that are central in production also occupy a similarly prominent position when it comes to credit-market interventions?

6 Quantitative Application to the U.S.

In this section, I present a quantitative application of the aggregation theorem to the U.S. economy to illustrate its usefulness in quantifying the contribution of financial deepening to the economy's aggregate TFP growth and in assessing the centrality of industries with respect to credit-market interventions. Specifically, I assume that production technologies are Cobb–Douglas and that within-sector firm-level heterogeneity, $\psi(z)$, follows a Pareto distribution—an assumption supported by empirical evidence on firm size distributions. I also introduce hand-to-mouth workers into the framework to account for the large share of labor income, with its evolution over time captured by $d\,log\,L_t$. Since these hand-to-mouth workers consume their earnings immediately and do not participate in asset markets, their presence leaves entrepreneurs' optimal behavior unchanged, and therefore all results derived thus far continue to hold. Moreover, I allow the total capital endowment to evolve over time, denoted by $d\,log\,K_t$. Given that the main objective of this study is to examine how, for a fixed unit of capital, credit-market imperfections affect resource allocation—both across sectors and within sectors among heterogeneous producers in production-networked economies—allowing the aggregate capital endowment to vary exogenously does not alter any of the theoretical results established earlier.

6.1 Data and Calibration

The analysis draws on three main sources of data. First, I use operational and financial information from publicly listed firms in the Compustat database and inflation-adjusted annual risk free rates from Federal Reserve Economic Data (FRED) to construct industry-level measures of profitability and financial deepening. Each firm is assigned to a North American Industry Classification System (NAICS) industry at the threee-digit level, from which I compute the sector-level profit-to-revenue ratio h_{it} and the financial deepening parameter ϕ_{it} for each year.

Specifically, I calibrate these measures as follows:

i). User cost of Capital — For each active firm f in year t,

$$UCOC_{ft} = r_t \cdot k_{ft}, \tag{50}$$

where r_t is the inflation-adjusted risk-free rate in year t and k_{ft} is firm f's capital input measured by its net value of tangible fixed assets (PPENT)

ii). Profit-to-revenue ratio — For each industry i in each year t,

$$h_{it} = \frac{\sum_{f \in \mathcal{F}_{it}} Operational\ Profit_{ft}}{\sum_{f \in \mathcal{F}_{it}} Revenue_{ft}}, \tag{51}$$

where $\mathcal{F}_{it}$ denotes the set of active firms in industry i in year t. Operational profit of firm f is measured by its operating income after depreciation (OIADP) minus the user cost of capital $UCOC_{ft}$, and revenue is measured by total sales (SALE).

iii). Financial deepening parameter — For each industry i in year t, the amount of external liabilities accessible relative to the value of collateral is given by:

$$\phi_{it} = \frac{\sum_{f \in \mathcal{F}_{it}} External\ Debt_{ft} + Accounts\ Payable_{ft}}{\sum_{f \in \mathcal{F}_{it}} Value\ of\ Collateral_{ft}} \tag{52}$$

where a firm's external liabilities are the sum of its external debt and accounts payable, and collateral value is proxied by the net value of tangible fixed assets (PPENT).

After standard data cleaning and ensuring balanced panel coverage, the final sample consists of 52 industries with complete annual observations from 1997 to 2015. I assume that, in each year, the observed endogenous variables correspond to a steady-state equilibrium of the economy. This assumption applies uniformly across all three datasets used in the analysis.

Second, I use the U.S. input–output tables from the Bureau of Economic Analysis (BEA) to calibrate a Cobb–Douglas production network. These tables are produced annually beginning in 1997 at the three-digit NAICS level. After excluding government, noncomparable imports, and second-hand scrap industries, the dataset reports annual industrial output and intermediate input flows for 66 industries between 1997 and 2015. For industries that match the 52 sectors with continuous Compustat coverage, I assign the corresponding annual profitability measures from Compustat. For the remaining 14 industries—where Compustat data are insufficient—I set annual profitability to zero throughout the analysis period. This approach allows for a complete calibration of the production network while excluding industries without adequate operational data from the financial deepening analysis.

Specifically, for each sector i in year t, we observe: (i) total sales, the empirical analogue of$p_{it} \cdot y_{it}$; (ii) expenditures on intermediate goods produced by sector j, the empirical analogue of $p_{jt} \cdot m_{ij,t}$; (iii) value-added components, including labor compensation—the empirical analogue of $w_{it} \cdot L_{it}$—and gross operating surplus (OS)—the empirical analogue of $\pi_{it} + r_t \cdot K_{it}$; and (iv) the final use of the sector's output, the empirical analogue of $p_{it}\, c_{it}$.

Equipped with the sector-level profitability h_{it}, we can back out each sector's user cost of capital in year t as :

$$r_t \cdot K_{it} = OS_{it} - h_{it} \cdot SALE_{it} \tag{53}$$

where OS_{it} denotes gross operating surplus and $SALE_{it}$ total sales.

We can now construct each sector's annual expenditure shares for intermediate goods, labor, and capital inputs, as well as its final-use (consumption) share. Specifically, for each industry i in year t:

- **Expenditure share of good j in the production cost of good i:**

$$\tilde{\Omega}_{ij,t} = \frac{p_{jt} \cdot m_{ij,t}}{w_t \cdot L_{it} + r_t \cdot K_{it} + \sum_j p_{jt} \cdot m_{ij,t}} \tag{54}$$

- **Expenditure share of labor input:**

$$\tilde{\Omega}_{iL,t} = \frac{w_t \cdot L_{it}}{w_t \cdot L_{it} + r_t \cdot K_{it} + \sum_j p_{jt} \cdot m_{ij,t}} \tag{55}$$

- **Expenditure share of capital input:**

$$\tilde{\Omega}_{iK,t} = \frac{r_t \cdot K_{it}}{w_t \cdot L_{it} + r_t \cdot K_{it} + \sum_j p_{jt} \cdot m_{ij,t}} \tag{56}$$

- **Final consumption share:**

$$b_{it} = \frac{p_{it} \cdot c_{it}}{\sum_j p_{jt} \cdot c_{jt}} \tag{57}$$

From these expenditure shares, we derive the corresponding cost-based Domar weights for the economy as follows:

- **Cost-based I.O. matrix** of the economy in year t:

$$\tilde{\Omega}_t = \left[\begin{array}{cccc|cc} \tilde{\Omega}_{11,t} & \tilde{\Omega}_{12,t} & \cdots & \tilde{\Omega}_{1N,t} & \tilde{\Omega}_{1L,t} & \tilde{\Omega}_{1K,t} \\ \tilde{\Omega}_{21,t} & \tilde{\Omega}_{22,t} & \cdots & \tilde{\Omega}_{2N,t} & \tilde{\Omega}_{2L,t} & \tilde{\Omega}_{2K,t} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \underbrace{\tilde{\Omega}_{N1,t}}_{\tilde{\Omega}_t^p} & \tilde{\Omega}_{N2,t} & \cdots & \tilde{\Omega}_{NN,t} & \underbrace{\tilde{\Omega}NL,t}_{\tilde{\Omega}_t^f} & \tilde{\Omega}_{NK,t} \end{array}\right] \tag{58}$$

- **Cost-based Domar weights for industries** are given by:

$$\tilde{\lambda}_t' = b_t' \cdot (1 - \tilde{\Omega}_t^p)^{-1} \tag{59}$$

- the corresponding **cost-based factor shares** are

$$(\tilde{\Lambda}_{L,t}, \tilde{\Lambda}_{K,t}) = b_t' \cdot (1 - \tilde{\Omega}_t^p)^{-1} \cdot \tilde{\Omega}_t^f \tag{60}$$

Finally, I use macroeconomic aggregates from the Federal Reserve Bank of San Francisco, which provide real GDP growth and growth rates of labor and capital inputs. These series are employed to compute annual aggregate TFP growth (the distortion-adjusted Solow residual) following Baqaee and Farhi (2020) for the period 1997–2015. Specifically, aggregate TFP growth in year t is is computed as

$$\Delta\, Aggregate\ TFP_t = \underbrace{\Delta \log Y_t - \tilde{\Lambda}_{K,t-1} \cdot \Delta \log K_t - \tilde{\Lambda}_{L,t-1} \cdot \Delta \log L_t}_{\text{the distortion-adjusted Solow residual}} \tag{61}$$

where the factor shares $\tilde{\Lambda}_{L,t-1}$ and $\tilde{\Lambda}_{K,t-1}$ are cost-based shares from the previous year, rather than the standard revenue-based factor shares. The use of cost-based shares is crucial when the economy is subject to inefficiencies, as revenue-based measures rely on the macro–envelope condition implied by the first welfare theorem, which fails under resource misallocation.

6.2 Contribution of Financing Deepening

We now turn to examining the contribution of financial deepening to aggregate TFP growth in the U.S. economy.

Figure 6 plots the cumulative contribution of financial deepening to aggregate TFP growth in the United States over the period 1997–2016. It shows that approximately 76.6% of the observed increase in aggregate efficiency can be attributed to improvements in financial deepening benefiting the corporate sector.

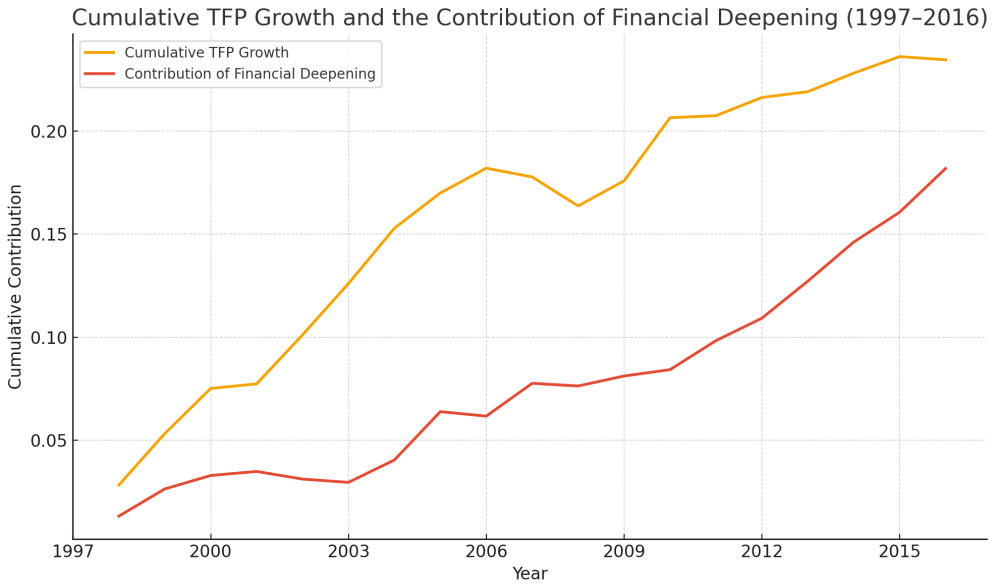


Figure 6: The Cumulative Contribution of Financing Deepening (FD) between 1997–2016

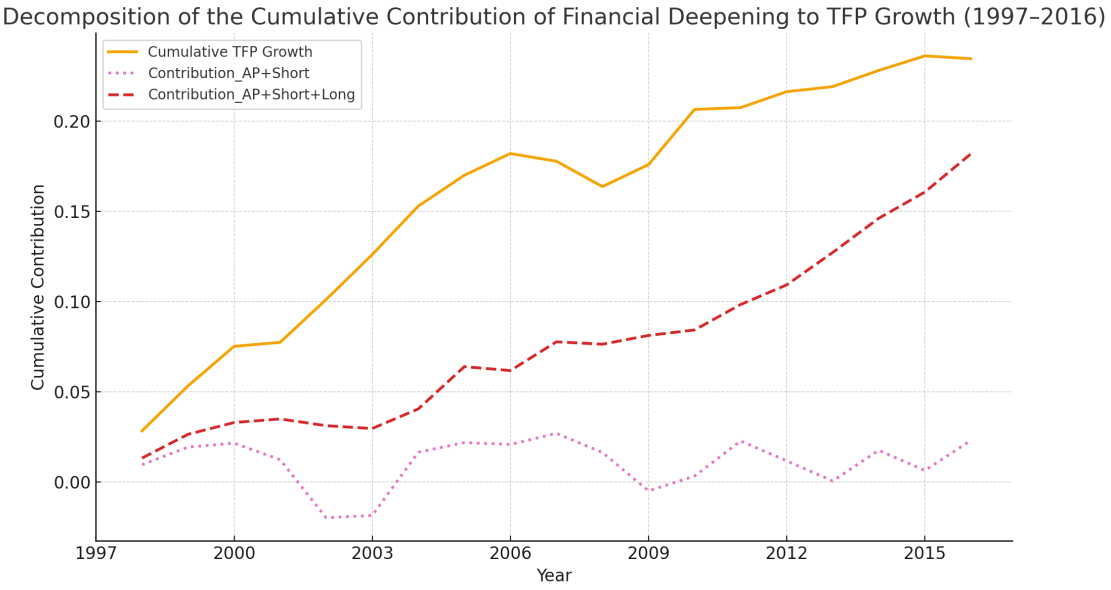


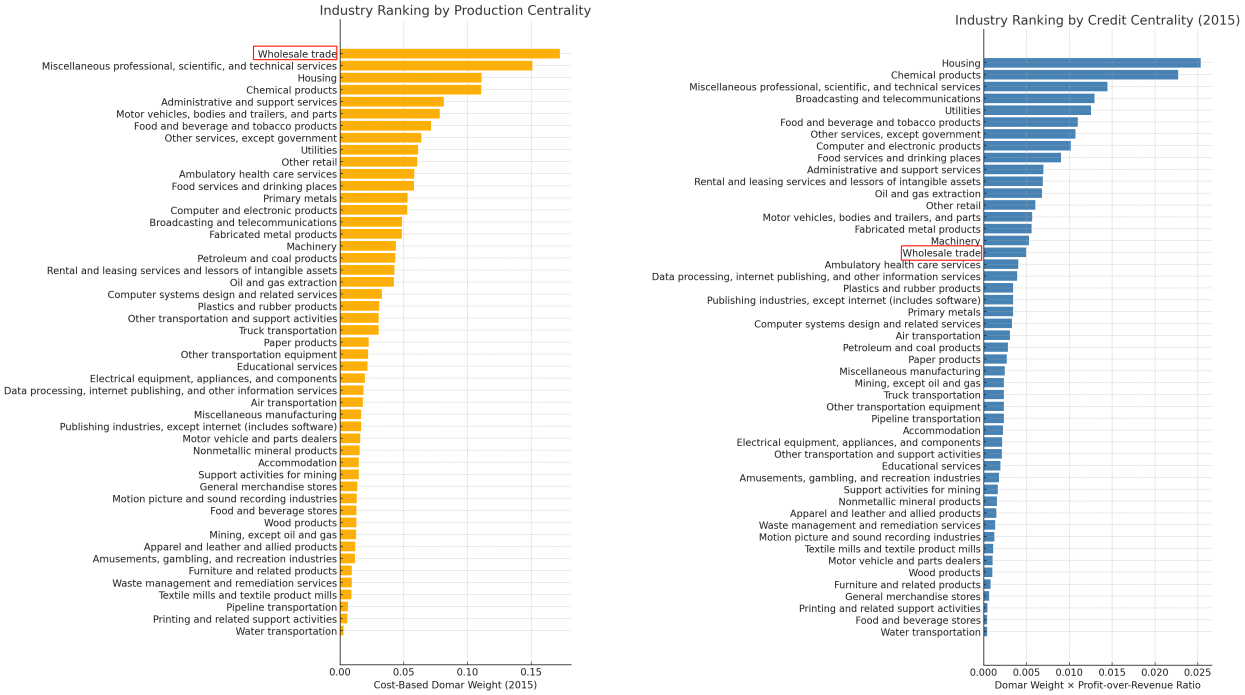
Figure 7: Decomposition of the FD Contribution

Figure 7 further decomposes the contribution of financial deepening into long-term debt—perceived as primarily enhancing the efficient allocation of capital across different production units—and short-term debt plus trade credit between trading partners, which constitutes the main component of the economy's short-term borrowing. The results suggest that developments in long-term financial deepening, accounting for approximately 67.5% of aggregate TFP growth, are the primary driver of improvements in U.S. aggregate efficiency, whereas changes in short-term borrowing contribute more to business cycle fluctuations.

This accounting result underscores the critical role of credit market interventions in enhancing aggregate productivity in the U.S. economy. If the U.S. government's credibility is firmly established in the eyes of the financial sector, policymakers may possess limited, yet valuable, capacity to mitigate credit market imperfections by serving as the ultimate provider of guarantees or insurance. A natural question then arises: to which sectors should such scarce credibility be directed in order to maximize its impact on economic performance? Addressing this question motivates the second application of the aggregation theorem, which evaluates the centrality of industries with respect to credit-market interventions, thereby offering ex-ante policy guidance.

6.3 Industry Centrality for Credit Market Interventions

Recall that Corollary 1 establishes that, under the Cobb–Douglas and Pareto-distribution assumptions, an industry's credit centrality—measured by the efficiency gain from a sector-level credit shock—is equal to the product of its production centrality and its profitability.



(a) Ranking by Production Centrality

(b) Ranking by Credit Centrality

Figure 8: Industry Ranking: Production v.s. Credit Centrality

Figure 8 compares the 2015 ranking of U.S. industries by production centrality and credit centrality. In terms of credit centrality, the *Housing* and *Chemical Products* sectors emerge as the top candidates for credit-market interventions. Both exhibit severe within-sector misallocation, as indicated by high sector-level profitability, suggesting substantial potential efficiency gains from easing credit constraints. Moreover, both hold central positions in the production network: *Housing* due to its dominant share in household final consumption, and *Chemical Products* owing to its upstream role as a critical supplier to numerous downstream industries.

Furthermore, the Wholesale Trade sector—ranked first in production centrality—falls outside the top ten in credit centrality due to its low profitability, rendering it a clearly suboptimal target for credit-market interventions. This stark divergence highlights how

credit market imperfections fundamentally reshape sectoral centrality for different types of microeconomic interventions, thereby further underscoring the conceptual contribution of this paper.

7 Conclusion

This paper develops a general yet highly tractable dynamic, multi-sector, heterogeneous-agents model of production networks with collateral-constrained producers, in the spirit of Kiyotaki and Moore (1997), to examine how credit-market imperfections shape the propagation and aggregation of microeconomic shocks in economies with complex production networks.

The analysis shows that when credit-market imperfections restrict producers' access to production resources, an additional channel of interdependence emerges—the producers' real credit condition—which interacts with existing production linkages to amplify the propagation of shocks. The model demonstrates that, ex post, the sector-level profit-over-revenue ratio serves as a non-parametric sufficient statistic for tracking the evolution of each sector's real credit condition, a finding supported by recent empirical evidence from both Europe and the United States.

Using this sufficient statistic, I characterize an aggregation theorem for micro shocks in production-network economies with credit-market imperfections. This theorem decomposes long-run aggregate TFP growth into distinct micro-level sources and substantially reshapes our understanding of industry centrality in improving aggregate efficiency through different types of microeconomic interventions. In particular, high sector-level profitability increases an industry's centrality for credit-market interventions, while reducing its centrality for standard industrial-policy interventions (e.g., taxes).

A quantitative application of the model to the U.S. economy indicates that financial deepening accounts for nearly 76% of aggregate TFP growth over the period 1997–2016. The *Housing* and *Chemical Products* sectors emerge as the top two industries in terms of credit centrality, both exhibiting high profitability and occupying central positions in the production network. In contrast, the *Wholesale Trade* sector—ranked first in production centrality by cost-based Domar weights—falls outside the top ten in credit centrality, making it a clearly suboptimal target for credit-market interventions.

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A Appendix

A.1 Derive the shadow credit linkage accounting item h

$$
\begin{aligned}
H(z_i^{ss}) &= (\mu_i^{\pi,ss} - 1) \cdot [1 - \Phi(z_i^{ss})] \\
&= \frac{\int_{z_i^{ss}}^{\overline{z}} z d\,\Phi(z)}{z_i^{ss}} - [1 - \Phi(z_i^{ss})]
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\frac{d\,H(z_i^{ss})}{d\,z_i^{ss}} &= \frac{-z_i^{ss} \cdot \phi(z_i^{ss}) \cdot z_i^{ss} - \int_{z_i^{ss}}^{\overline{z}} z d\,\Phi(z)}{(z_i^{ss})^2} + \phi(z_i^{ss}) \\
&= -\frac{\int_{z_i^{ss}}^{\overline{z}} z d\,\Phi(z)}{(z_i^{ss})^2}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
h_i &\equiv -\frac{d\,log\,z_i^{ss}}{d\,log\,H(z_i^{ss})} = -\frac{d\,z_i^{ss}}{d\,H(z_i^{ss})} \cdot \frac{H(z_i^{ss})}{z_i^{ss}} = \frac{(z_i^{ss})^2}{\int_{z_i^{ss}}^{\overline{z}} z d\,\Phi(z)} \cdot \frac{H(z_i^{ss})}{z_i^{ss}} \\
&= \frac{z_i^{ss}}{\int_{z_i^{ss}}^{\overline{z}} z d\,\Phi(z)} \cdot \underbrace{(\frac{\int_{z_i^{ss}}^{\overline{z}} z d\,\Phi(z)}{z_i^{ss}} - [1 - \Phi(z_i^{ss})])}_{H(z_i^{ss})} \\
&= 1 - \underbrace{\frac{z_i^{ss} \cdot [1 - \Phi(z_i^{ss})]}{\int_{z_i^{ss}}^{\overline{z}} z d\,\Phi(z)}}_{\frac{1}{\mu_i^{\pi,ss}}} \\
&= \frac{\mu_i^{\pi,ss} - 1}{\mu_i^{\pi,ss}} \in (0, 1)
\end{aligned}
$$

A.2 The sectoral profit-revenue ratio

Given price $P_t = \{r_t, \{p_{it}\}\}$, the net profit of an individual entrepreneur, (a, z), in sector i is

$$\begin{aligned}\pi_{it}(a,z) &= (p_{it} - mc_{it}(z)) \cdot \underbrace{A_i \cdot z \cdot \frac{\phi_i \cdot q_t \cdot a}{C_i(P_t)}}_{y_{it}(a,z)} \\ &= \underbrace{(\frac{\mu_i}{A_i \cdot z_{it}} - \frac{\mu_i}{A_i \cdot z}) \cdot C_i(P_t)}_{p_{it} - mc_{it}(z)} \cdot \underbrace{A_i \cdot z \cdot \frac{\phi_i \cdot q_t \cdot a}{C_i(P_t)}}_{y_{it}(a,z)} \\ &= \mu_i \cdot (\frac{z}{z_{it}} - 1) \cdot \underbrace{\phi_i \cdot q_t \cdot a}_{\equiv L_{it}(a,z)}\end{aligned}$$

Aggregating at the sector level gives the total net profits of active entrepreneurs in sector i, which is

$$\pi_{it} = \mu_i^\tau \cdot (\mu_{it}^\pi - 1) \cdot \underbrace{\phi_i \cdot q_t \cdot a_{it} \cdot [1 - \Phi(z_{it})]}_{\equiv L_{it}}$$

The corresponding sectoral revenue is

$$R_{it} = \mu_i^\tau \cdot \mu_{it}^\pi \cdot L_{it}$$

In the equilibrium, the total liquidity utilized by active entrepreneurs in the sector i, L_{it}, is equivalent to the total cost of production of that sector. And, therefore, the total revenue earned in that sector is just the markup, both the exogenous and endogenous ones, times the cost of production.

Therefore, in the equilibrium, we have

$$\frac{\pi_{it}}{R_{it}} = \frac{\mu_{it}^\pi - 1}{\mu_{it}^\pi}$$

and, in the steady state, we have

$$\frac{\pi_i^{ss}}{R_i^{ss}} = \frac{\mu_i^{\pi,ss} - 1}{\mu_i^{\pi,ss}} = h_i$$

A.3 Deriving Proposition 2

Given prices $P^{ss} = \{r^{ss}, \{p_i^{ss}\}\}$ in the steady state, individuals' cost-minimizing behavior delivers

$$p_i^{ss} = \frac{\mu_i^{\tau}}{A_i \cdot z_i^{ss}} \cdot C_i(P^{ss})$$

Log-linearizing the above equation gives us

$$d\,log\, p_i^{ss} = -d\,log\, A_i + d\,log\, \mu_i^{\tau} - d\,log\, z_i^{ss} + d\,log\, C_i(P^{ss})$$

with

$$d\,log\, C_i(P^{ss}) = \sum_j \underbrace{\frac{p_j^{ss} \cdot M_{ij}(P^{ss})}{C_i(P^{ss})}}_{=\tilde{\Omega}_{ij}} \cdot d\,log\, p_j^{ss} + \underbrace{\frac{r^{ss} \cdot K_i(P^{ss})}{C_i(P^{ss})}}_{=\tilde{\Omega}_{ik}} \cdot d\,log\, r^{ss}$$

by applying the envelope theorem on individuals' cost-minimizing behavior.

Rewrite the above system in a matrix, we have

$$[d\,log\, p_j^{ss}] = -\underbrace{[d\,log\, A_j]}_{\equiv d\,log\, A} + \underbrace{[d\,log\, \mu_j^{\tau}]}_{\equiv d\,log\, \mu^{\tau}} - \underbrace{[d\,log\, z_j^{ss}]}_{\equiv d\,log\, z^{ss}} + \underbrace{\tilde{\mathbf{\Omega}}}_{\equiv[\tilde{\Omega}_{ij}]} \cdot [d\,log\, p_j^{ss}] + [\tilde{\Omega}_{jk}] \cdot d\,log\, r^{ss}$$

Isolating $[d\,log\, p_j^{ss}]$ delivers

$$[d\,log\, p_j^{ss}] = [I - \tilde{\mathbf{\Omega}}]^{-1} \cdot \{-d\,log\, A + d\,log\, \mu^{\tau} - d\,log\, z^{ss}\} + [I - \tilde{\mathbf{\Omega}}]^{-1} \cdot [\tilde{\Omega}_{jk}] \cdot d\,log\, r^{ss}$$

Applying the envelope theorem on the price normalization delivers

$$b' \cdot [d\,log\, p_j^{ss}] = 0 \tag{62}$$

Note: (62) holds because I choose the final consumption goods as the numeraire of the economy

Combining the above two equations gives us

$$b' \cdot [I - \tilde{\mathbf{\Omega}}]^{-1} \cdot [\tilde{\Omega}_{ik}] \cdot d\,log\, r^{ss} = b' \cdot [I - \tilde{\mathbf{\Omega}}]^{-1} \cdot \{d\,log\, A - d\,log\, \mu^{\tau} + d\,log\, z^{ss}\} \tag{63}$$

By definition,

$$\tilde{\lambda}' \equiv b' \cdot [I - \tilde{\mathbf{\Omega}}]^{-1}$$

And, CRS technologies deliver

$$\tilde{\Omega} \cdot \mathbf{1} + [\tilde{\Omega}_{ik}] = \mathbf{1}$$

$$\Downarrow$$

$$[I - \tilde{\mathbf{\Omega}}]^{-1} \cdot [\tilde{\Omega}_{ik}] = \mathbf{1}$$

$$\Downarrow$$

$$b' \cdot [I - \tilde{\mathbf{\Omega}}]^{-1} \cdot [\tilde{\Omega}_{ik}] = b' \cdot \mathbf{1} = 1$$

Therefore, equation (63) can be further simplified as

$$d\,log\, r^{ss} = \tilde{\lambda}' \cdot (d\,log\, A - d\,log\, \mu^{\tau} + d\,log\, z^{ss})$$

To find the mapping from $d\,log\, r^{ss}$ to $d\,log\, TFP^{ss}$, we rely on the following accounting definition

$$\Lambda^{ss} = \frac{r^{ss}}{TFP^{ss}}$$

which delivers

$$d\,log\,TFP^{ss} = d\,log\,r^{ss} - d\,log\,\Lambda^{ss}$$

Therefore, we have

$$d\,log\,TFP^{ss} = \tilde{\lambda}' \cdot (d\,log\,A - d\,log\,\mu^{\tau} + d\,log\,z^{ss}) - d\,log\,\Lambda^{ss} \tag{64}$$