Common Ownership and Strategic Investment Composition*

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Abstract

This paper studies how common ownership affects the allocation of strategic investment in a multi-market duopoly where firms face binding resource constraints. Two firms allocate effort across two innovation contests in which a single winner earns monopoly profit and simultaneous innovation yields lower duopoly profit. The model highlights a tension between technological concavity, which favors diversification, and a duplication penalty, which favors cross-specialization. Under separate ownership, firms diversify symmetrically in a unique interior equilibrium.

Common ownership, modeled as managers internalizing a fraction of rival profits, increases the effective duplication penalty. For low levels of common ownership, firms continue to diversify but shift investment toward lower-return, lower-dissipation markets. For high levels, the symmetric equilibrium becomes unstable and the industry moves to asymmetric cross-specialization, effectively implementing tacit market allocation.

Welfare effects are ambiguous. At high levels of common ownership, specialization reduces welfare when duplicate innovation benefits consumers, but increases welfare when duplication is wasteful. At low levels, reallocations may move the industry toward or away from the social optimum, depending on market appropriability.

Keywords: Common Ownership, Innovation Portfolio, Multi-market Competition

JEL Classification: L13, L41, O31, G34, D21

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1 Introduction

Common ownership—situations in which the same institutional investors hold stakes in multiple competing firms—has emerged as a defining feature of modern financial markets. By 2015, the largest institutional investors in the U.S. market—BlackRock, Vanguard, and State Street—collectively constituted the largest shareholder in 40% of all publicly listed American firms (Fichtner et al., 2017). Furthermore, Backus et al. (2021) find that their combined holdings in the average S&P 500 firm escalated from approximately 6% in 2000 to 21% by 2017. This ownership concentration matters because large diversified shareholders can influence corporate governance and executive incentives in ways that soften product-market competition (Schmalz, 2018). Specifically, Antón et al. (2023) demonstrate that common owners implement this influence by lowering the sensitivity of the CEOs' wealth to own-firm performance, thereby reducing the managerial incentive to compete aggressively. Consequently, this governance channel has been empirically linked to higher equilibrium prices and reduced competition in concentrated industries. Beyond the initial findings in airlines (Azar et al., 2018), recent studies have documented similar anticompetitive effects in banking (Azar et al., 2022), agricultural manufacturing (Torshizi and Clapp, 2021), and pharmaceuticals (Newham et al., 2025).

However, this extensive literature focuses almost entirely on the amount of effort managers exert: do they work harder or slack off? This perspective misses a crucial reality: managers have limited resources. Whether it is a fixed R&D budget or limited time, a manager cannot focus on everything at once; prioritizing one market often means neglecting another. Therefore, the relevant question is not just how much they invest, but where they invest. This paper examines a previously explored channel: how common ownership alters the way firms distribute their limited resources across competing markets.

To analyze this mechanism, we model two firms competing in two parallel R&D contests subject to a binding budget constraint—a structure formally equivalent to a continuous Colonel Blotto game (Roberson, 2006). We interpret this investment budget as a proxy for scarce managerial attention, consistent with the view that executive oversight is a binding and complementary input required for successful innovation (Gifford, 1992). Within each market, innovation follows a canonical stochastic patent race where investment exhibits diminishing returns (Loury, 1979). The critical feature of this environment is the payoff structure: if one firm succeeds, it captures monopoly rents; if both succeed, competition dissipates rents to a lower duopoly level. This creates a duplication penalty:

parallel success destroys industry value relative to the monopoly benchmark (Denicolò & Polo, 2018).

Common ownership is introduced through a standard governance channel: CEOs behave as if their objective internalizes a fraction of the rival's profit. Formally, each firm's manager maximizes their own expected profit plus a weight κ times the rival's expected profit (Backus et al., 2021). The parameter κ summarizes the effective strength of common-owner influence, acting as a reduced-form representation of the compensation distortions documented by Antón et al. (2023). This specification does not assume that common owners directly dictate product-market conduct in the second stage; rather, they dictate a first-stage reallocation of investment as the budget is fixed. Thus, common ownership in the baseline model does not reduce total investment; instead, it changes where investment is deployed.

The model highlights a tension between two forces. The first is technological concavity: because success probabilities are increasing and concave in own effort, a stand-alone firm would hedge by spreading investment across both markets to equalize marginal success probabilities. This is a *centripetal force* pushing toward interior diversification. The second is the duplication penalty: when both firms succeed in the same market, each firm's profit falls from monopoly to duopoly. This is a *centrifugal force* pushing toward asymmetric cross-specialization, so that firms avoid fighting head-to-head in the same market. Under separate ownership, the *centripetal force* dominates and the game admits a unique symmetric interior equilibrium in which both firms diversify, even though this generates duplication risk. As common ownership increases, it scales up the internalized cost of duplication penalty and the *centrifugal force* becomes stronger, eventually triggering a shift towards cross-specialization.

Our first set of results characterizes how common ownership reallocates investment while the equilibrium remains symmetric and interior. Under an exponential specification for success probabilities (a standard neutral benchmark in patent-race environments), we find that this reallocation is driven by the interplay between two distinct market characteristics. First, common ownership generates a *safe haven* effect: investment shifts toward the market where monopoly rents are more durable—specifically, where the ratio of rent dissipation to monopoly profit is lower. In these markets, parallel success destroys a smaller fraction of the total pie, making duplication less costly for the common owner. Second, this incentive is weighed against the intrinsic profitability of the innovation—defined as the monopoly prize scaled by the technological ease of innovation. Under separate ownership, firms aggressively target markets with high expected returns. Common own-

ership acts as a corrective device by reallocating investment towards markets with low expected returns. Consequently, we identify a sufficient condition for reallocation: common ownership shifts investment towards markets with low expected returns but serve as *safe havens*.

Our second set of results characterizes the breakdown of symmetry. A nontrivial stability threshold exists only if the industry exhibits the *efficiency effect*—that is, when joint monopoly profits exceed the sum of duopoly profits (Gilbert and Newbery, 1982). As common ownership increases beyond that threshold, the symmetric allocation becomes unstable, and stable outcomes feature cross-specialization. Depending on primitives, specialization may arise as interior asymmetric allocations, edge solutions where one firm is constrained at a boundary, or strict corner allocations in which each firm fully specializes in a different market. The core message is not that all economies necessarily pass through all regimes, but that common ownership strengthens the specialization motive.

These findings provide a rigorous micro-foundation for recent empirical evidence on non-price competition. By identifying strategic cross-specialization as the mechanism for relaxing rivalry, our model rationalizes the findings of Ruiz-Pérez (2024) and Koura (2023), who document that firms with higher common ownership actively increase product differentiation. Furthermore, our prediction that symmetry breaks precisely when the 'Efficiency Effect' is strong (i.e., when competition is most destructive) aligns with Geng and Pan (2023). They observe that common ownership drives differentiation most aggressively when firms initially exhibit high product similarity—consistent with our result that the symmetric 'head-to-head' equilibrium becomes dynamically unstable precisely when the profit gap between monopoly and duopoly is large.

Finally, we evaluate our outcomes using a consumer welfare standard. Our analysis reveals that the effect of common ownership on expected consumer surplus is fundamentally ambiguous: it hinges critically on the societal value of duplicate innovations.[1] When the value of duplicate innovation is high—typically because the resulting products are highly substitutable, leading to intense, price-lowering competition—elevated levels of common ownership harm consumers. In this regime, consumers maximize their surplus when firms compete head-to-head. However, common ownership incentivizes firms to specialize and retreat into separate markets to protect their margins, thereby stifling competition and reducing expected consumer surplus. Conversely, when the value of duplicate innovation is low—meaning the products exhibit low substitutability and a second success

[1]Unlike Denicolò and Polo (2018), who assume a winner-takes-all structure where simultaneous innovation is strictly redundant, our framework allows parallel successes to directly generate positive consumer surplus.

adds little competitive benefit—high levels of common ownership actually enhance consumer welfare. By encouraging firms to specialize, common ownership eliminates wasteful, redundant research and increases the overall probability that at least one successful innovation reaches the market.

From a policy perspective, antitrust authorities and regulators are primarily concerned with the competitive effects of high levels of common ownership, since those are the cases in which common ownership can materially soften rivalry and induce market separation. However, for the completeness of the theoretical framework, it is useful to characterize the reallocations that arise even at low levels of common ownership. At low levels of common ownership, the consumer-welfare effect is governed by the direction of marginal reallocation within the symmetric interior equilibrium. That is, firms continue to diversify across both markets, and common ownership only shifts investment at the margin rather than by inducing specialization. Consumer welfare therefore rises if this reallocation moves the industry closer to the socially optimal allocation, and falls if it pushes the market farther away. Thus, unlike the high-common-ownership case, the welfare effect at low levels of common ownership is inherently parameter-dependent: when the two markets differ only proportionally and therefore generate no wedge between private and social incentives, low common ownership reduces welfare; but when appropriability differs across markets, it can improve welfare by redirecting investment toward the underinvested market.

Literature Review. We place our work within the rapidly expanding literature on common ownership. The debate on the anticompetitive effects of common ownership was sparked by Azar et al. (2018), who documented that overlapping ownership by institutional investors increased prices in the US airline industry. Subsequent empirical work has identified similar anticompetitive price effects across diverse sectors, including banking (Azar et al., 2022), agricultural manufacturing (Torshizi Clapp, 2021), and pharmaceuticals (Newham et al., 2022).

Following these findings on price, a second strand of literature developed theoretical frameworks to analyze the impact on strategic investment and cost-reducing innovation. Lopez and Vives (2020) and Antón et al. (2023) are central to this development. Lopez and Vives (2020) examine the simultaneous choice of investment and output, showing that common ownership generally depresses both. Antón et al. (2023) provide the micro-foundation for this result, demonstrating that common ownership flattens the sensitivity of CEO wealth to own-firm performance. By dampening the managerial incentive to steal business from rival firms, common ownership reduces the aggregate

level of cost-reducing investment. Similarly, in a general equilibrium setting, Azar and Vives (2021) show that this softened competition suppresses the total demand for inputs (an "input markdown"). However, these theories largely focus on the level of investment rather than its composition. For instance, while Antón et al. (2023) consider multi-market firms, they model managerial effort as a non-rival input that scales uniformly, or assume costless transferability of innovation across markets. This abstracts from the "allocative margin" where managers must distribute scarce resources across competing projects.

Our paper aligns with a "third wave" of research that focuses on this allocative margin: specifically, how common ownership influences product differentiation and the direction of innovation. Recent empirical studies suggest that common ownership drives firms to avoid head-to-head overlap. Ruiz-Pérez (2024) and Koura (2023) find that firms with higher common ownership actively increase product differentiation to relax competitive pressure. Similarly, Geng and Pan (2023) show that common ownership promotes innovation most strongly when firms have high product similarity, but directs that innovation toward differentiation strategies to escape rivalry.

Our work provides the theoretical engine for these empirical patterns. By introducing a binding resource constraint, we show that the "input markdown" identified by Azar and Vives (2021) is not just a reduction in scale, but a driver of structural misallocation. We demonstrate that common ownership strengthens the motive to cross-specialize, providing a formal mechanism for the product differentiation observed by Ruiz-Pérez (2024) and others.

The rest of the paper is organized as follows. We introduce our general model and the separate ownership benchmark in Section 2. The market equilibrium under common ownership is analyzed in Section 3, where we characterize the investment reallocation effects and the structural transition to cross-specialization (symmetry breaking). We discuss the welfare implications of these distortions in Section 4. Finally, we discuss the conclusions in Section 6. The proofs of all propositions are assigned to the appendix.

2 Baseline Model

Consider a duopoly where two firms, i and j, compete simultaneously across two independent markets $\ell \in \{A, B\}$. In each market, firms invest in R&D to innovate. If only one firm succeeds in market ℓ, it acts as a monopolist earning a profit of π_ℓ^M. If both firms succeed, they compete in the product market earning the duopoly profit of $\pi_\ell^D < \pi_\ell^M$. If neither succeeds, profits are normalized

to zero. We define $\Delta_\ell = \pi_\ell^M - \pi_\ell^D$ as the cannibalization of innovation rent due to competition.

Firm i invests an amount $x_{\ell i} \in [0,1]$ in market ℓ. The probability of success is given by the production function $p_\ell(x_{\ell i}) : [0,1] \to [0,1)$. We assume $p_\ell(\cdot)$ is strictly increasing, twice differentiable, and strictly concave. Firms operate under a binding resource constraint, representing a fixed R&D budget or limited managerial attention. Each firm is endowed with a unit of resources to allocate across the two markets, such that $x_{Ai} + x_{Bi} \leqslant 1$. Since the marginal return to investment is positive, the constraint binds. We reduce the decision to a single strategic variable $x_{Ai} = x_i \in [0,1]$, which implies $x_{Bi} = 1 - x_i$.

The timing of the game is as follows: in Stage 1, firms simultaneously choose their investment allocations. In Stage 2, innovation outcomes realize, and market competition takes place.

2.1 The Separate Ownership Benchmark

We first characterize the baseline equilibrium under separate ownership where firm i chooses x_i to maximize its own expected profit

$$\pi_i(x_i, x_j) = p_A(x_i)\left[\pi_A^M - p_A(x_j)\Delta_A\right] + p_B(1-x_i)\left[\pi_B^M - p_B(1-x_j)\Delta_B\right]. \tag{1}$$

The presence of the terms, $-p_\ell(\cdot)\Delta_\ell$, ensures that the allocation game is one of *strategic substitutes*.

We rearrange the objective function (1) to highlight the tension between technological concavity (diminishing returns of the probability function) and strategic convexity (the duplication penalty):

$$\pi_i(x_i, x_j) = \underbrace{p_A(x_i)\pi_A^M + p_B(1-x_i)\pi_B^M}_{\text{Monopolist's Problem}} - \underbrace{\left(p_A(x_i)p_A(x_j)\Delta_A + p_B(1-x_i)p_B(1-x_j)\Delta_B\right)}_{\text{Duplication Penalties}}. \tag{2}$$

The first term represents the firm's incentive to allocate resources had it been a monopolist. Given the concavity of $p_\ell(\cdot)$, this term pushes the firm toward an interior "hedging" strategy to equalize marginal returns across markets. The second term represents the duplication penalty: the expected loss of monopoly rents due to the duplication of innovation. As this term is convex in the firms' joint probabilities, it pushes firms toward asymmetric specialization to avoid head-to-head competition.[2] The final equilibrium outcome depends on what strategic force dominates.

[2]Observe that the duplication penalty achieves a minimum of zero only when the firms choose asymmetric corner solutions; firm i chooses $x_i = 1$ while firm j chooses $x_j = 0$.

Proposition 1. *The investment allocation game under separate ownership admits a unique symmetric Nash equilibrium that is interior if and only if*

$$\frac{p'_B(1)}{p'_A(0)}\left(1-p_B(1)\frac{\Delta_B}{\pi_B^M}\right) < \frac{\pi_A^M}{\pi_B^M} < \frac{p'_B(0)}{p'_A(1)}\left(\frac{1}{1-p_A(1)\frac{\Delta_A}{\pi_A^M}}\right) \tag{3}$$

If condition (3) *holds, this equilibrium denoted by* $x_0^* \in (0,1)$ *is characterized by the equality of marginal returns:*

$$p'_A(x_0^*)\left[\pi_A^M - p_A(x_0^*)\Delta_A\right] = p'_B(1-x_0^*)\left[\pi_B^M - p_B(1-x_0^*)\Delta_B\right]. \tag{4}$$

This result shows that under separate ownership technological concavity always dominates the strategic convexity (discussed in (2)). Consequently, firms invest in both the markets while potentially duplicating innovations. The boundary conditions (3) ensure that neither firm finds it optimal to fully exit a market, even if the rival is fully invested in it.

3 Common Ownership

Following Antón et al. (2023), we assume that common owners do not dictate pricing or quantity choices to lower-level management; instead, they influence top-management by structuring executive compensation that lowers the sensitivity of CEO wealth to own-firm performance. Consequently, a rational CEO internalizes the externalities imposed on common shareholders to maximise the objective function $u_i(x_i, x_j, \kappa) = \pi_i(x_i, x_j) + \kappa\pi_j(x_j, x_i)$, where $\kappa \in [0,1]$ is the degree of common ownership that arises from the common owners' symmetric ownership stakes in the two firms.[3]

We revisit the mechanism driving investment choices discussed in the previous section following expression (2).

$$\begin{aligned} u_i(x_i, x_j; \kappa) = &\underbrace{[p_A(x_i) + \kappa p_A(x_j)]\pi_A^M + [p_B(1-x_i) + \kappa p_B(1-x_j)]\pi_B^M}_{\text{Internalized Benefit of Success}} \\ &- \underbrace{(1+\kappa)[p_A(x_i)p_A(x_j)\Delta_A + p_B(1-x_i)p_B(1-x_j)\Delta_B]}_{\text{Internalized Duplication Penalty}}. \end{aligned} \tag{5}$$

The first term now includes the rival's weighted profit, $\kappa\pi_\ell^M p_\ell(\cdot)$. However, because this term

[3] In the absence of a symmetric ownership structure, common ownership yields pair-specific, asymmetric weights ($\kappa_{ij} \neq \kappa_{ji}$) as formalized by Backus, Conlon, and Sinkinson (2021).

does not depend on firm i's investment, the marginal incentive to invest remains the same as the monopolists problem discussed earlier: $p'_A(x_i)\pi_A^M - p'_B(1-x_i)\pi_B^M$. Consequently, the *centripetal force* - driven by the diminishing returns of $p(\cdot)$ - pulls firms toward a symmetric, risk-hedging interior solution. The second term represents the *centrifugal force* that pushes firms toward asymmetric specialization. Under common ownership, this term is scaled by the factor $(1+\kappa)$. As κ rises, the cost of duplication increases and tends to push firms toward asymmetric specialization; i.e. firm i invests most of its budget in market A, while firm j invests most of its budget in market B.

We establish the conditions under which the symmetric equilibrium remains stable. We first state the general result applicable to any concave probability function $p_\ell(\cdot)$.

Lemma 1. *Given boundary conditions* (3)*, there exists a unique Nash equilibrium* x_κ^* *that is symmetric, interior, and stable if* $\kappa < \underline{\kappa} = \inf_{(x_i,x_j)\in[0,1]^2} T_1(x_i, x_j)$ *where* $T_1(\cdot)$ *is defined in the Appendix.*

This establishes that the symmetric interior equilibrium is stable only if the degree of common ownership remains below a critical threshold, $\underline{\kappa}$. As established in equation 2, an interior solution requires the *centripetal force* to dominate the *centrifugal force*. Economically, this means that if one firm slightly increases its investment in a market, the rival optimally responds by slightly reducing its own investment, reflecting standard strategic substitutability.

However, as the degree of common ownership, κ, increases, the *centrifugal force* intensifies. Once $\kappa > \underline{\kappa}$, this incentive to cut back increases and the symmetric equilibrium could degenerate into a saddle point that may triggering a symmetry breaking event towards asymmetric specialization.[4] Note that for $\kappa < \underline{\kappa}$, the firm's objective function (2) remains concave. This ensures that the sudden shift to specialization is a purely strategic phenomenon—driven by the tension between technological returns and common ownership—rather rather than a trivial consequence of convexity in the optimization problem.

While Lemma 1 establishes the existence of a stability threshold, determining how this threshold responds to market primitives is obscured by the curvature of $p_\ell(\cdot)$. In the general case, stability depends on the third derivative of the probability function. To isolate the strategic role of common ownership from varying technological returns to scale, the next section adopts a stylized functional form with constant absolute curvature as the benchmark.

[4]See Proposition 4 for the full characterization of asymmetric equilibria under Assumption 1 of exponential innovation technologies.

3.1 The Symmetric Equilibrium using the Exponential Benchmark

To derive sharp predictions regarding the impact of common ownership and market structure on the incentives to invest, we introduce the following benchmark assumption on the technological curvature $p_\ell(\cdot)$.

Assumption 1. *The probability of success follows an exponential distribution $p_\ell(z) = 1 - e^{-\lambda_\ell z}$ for both markets $\ell \in \{A, B\}$.*

We view this specific functional form as essential for isolating the true effects of common ownership.[5] With typical concave functions (like z^α), the rate of diminishing returns weakens as investment grows. We call this a "scale effect." If we used a standard concave function, equilibrium stability might change simply because a firm's investment reached a flatter part of the probability curve. These technological scale effects would obscure the strategic impact of common ownership.

The exponential form solves this problem. It ensures that the ratio of curvature to marginal return, $|p''_\ell(z)|/p'_\ell(z) = \lambda_\ell$, remains strictly constant regardless of the investment level, z. Because the relative strength of diminishing returns never changes, we can confidently attribute our results to strategic incentives rather than to probability function's changing curvature. Additionally, this setup aligns our static model with classic dynamic R&D models (e.g., Loury 1979), which treat innovation as a memoryless Poisson process. Therefore, we use the exponential specification for the remainder of this analysis.

Proposition 2. *If $\pi_\ell^M > 2\pi_\ell^D$ holds for both markets, then there exists a unique threshold $\bar{\kappa} < \max\{\pi_A^D/\Delta_A, \pi_B^D/\Delta_B\}$ such that:*

(A) For $\kappa < \underline{\kappa}$, there exists a unique symmetric equilibrium, x_κ^*, that is interior and stable.

(B) For $\underline{\kappa} < \kappa < \bar{\kappa}$, there exists a stable symmetric interior equilibrium, denoted by x_κ^*. This equilibrium may coexist with two stable asymmetric boundary equilibria, denoted by $(0, \bar{x}_\kappa^*)$ and $(\bar{x}_\kappa^*, 0)$, where $x_\kappa^* < \bar{x}_\kappa^*$.

(C) For $\kappa \geqslant \bar{\kappa}$, the symmetric equilibrium becomes unstable, and there exist exactly two stable asymmetric boundary equilibria, denoted by $(0, \bar{x}_\kappa^*)$ and $(\bar{x}_\kappa^*, 0)$, where $x_\kappa^* < \bar{x}_\kappa^*$.

[5] This approach is analogous to using Constant Absolute Risk Aversion (CARA) utility in decision theory. Just as CARA isolates risk preferences from wealth levels, the exponential probability function isolates strategic incentives from scale effects.

Conversely, if $\pi_\ell^M \leqslant 2\pi_\ell^D$ *holds for both markets, the symmetric interior equilibrium* x_κ^* *is unique and stable for all* $\kappa \in [0,1]$.

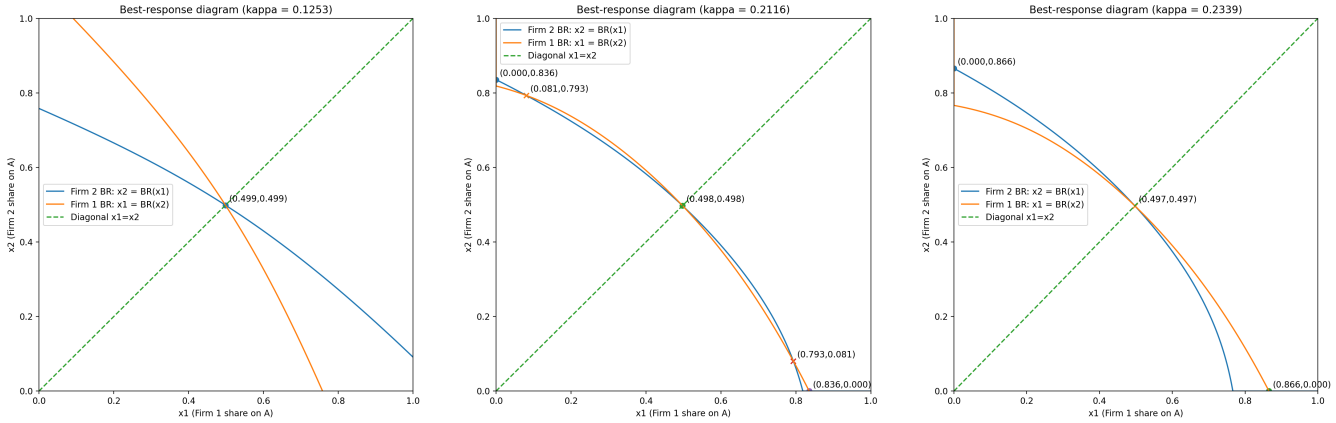
This proposition delineates three distinct strategic regimes determined by the extent of common ownership, culminating in a symmetry-breaking event. In the first regime (A), the unique equilibrium is interior and symmetric: firms commit the exact same level of R&D to a given market as their rivals, resulting in head-to-head competition on all fronts.[6] As common ownership rises, the industry enters a transitional, bistable regime where this symmetric equilibrium persists, but may coexist with asymmetric boundary solutions. Ultimately, once common ownership crosses the critical threshold, the symmetric state collapses entirely. The market bifurcates into a final regime characterized by strict asymmetric specialization, where firms divide the markets to minimize duplicative R&D and avoid direct product market competition.

This symmetry-breaking event depends fundamentally on whether innovation dissipates total industry value. The condition $\pi_\ell^M > 2\pi_\ell^D$ identifies markets where aggregate monopoly profits strictly exceed duopoly profits, making symmetric rivalry inefficient from the perspective of common owners. Under this condition, the dynamic illustrated in Figure 1 takes hold.[7] For low levels of common ownership ($\kappa < \bar{\kappa}$), the *centripetal force* of diminishing returns dominates, investments act as standard strategic substitutes, and the symmetric equilibrium remains globally stable (Panel (a)).

[6]Note that *symmetry* in this context does not mean firms divide their budget evenly across different projects, but rather that their strategies perfectly mirror each other.

[7]The parameters for this numeric simulation are $\lambda_A = 3$, $\lambda_B = 4.5$, $\pi_A^M = 8.4$, $\pi_B^M = 11.5$, $\pi_A^D = 0.9$, $\pi_B^D = 2.7$. The assumptions are satisfied: $\pi_A^M > 2\pi_A^D$ since $8.4 > 1.8$, $\pi_B^M > 2\pi_B^D$ since $11.5 > 5.4$, and $\pi_A^D/\Delta_A = 0.12 < \pi_B^D/\Delta_B = 0.307$. Hence the admissible upper bound is $\max\{\pi_A^D/\Delta_A, \pi_B^D/\Delta_B\} = 0.307$. In addition, $\underline{\kappa} = 0.125$. The thresholds computed in Proposition 2 and Proposition 4 are

$$\underline{\kappa} = 0.125 < \bar{\kappa} = 0.223 < \hat{\kappa} = 0.289 < \max\{\pi_A^D/\Delta_A, \pi_B^D/\Delta_B\} = 0.307.$$



((a)) $\kappa = \underline{\kappa} < \bar{\kappa}$ ((b)) $\kappa = 0.95 \times \bar{\kappa}$ ((c)) Regime (iii): $\kappa = 1.05 \times \bar{\kappa}$

Figure 1: Transition of equilibria as common ownership (κ) increases

However, as κ rises, firms increasingly internalize their rival's potential profit losses. This internalization causes their best-response functions to steepen. Before the symmetric equilibrium completely breaks down, the system enters a bistable regime ($\underline{\kappa} < \kappa < \bar{\kappa}$), where unstable interior intersections create thresholds that can push the system toward asymmetric boundaries given a large enough shock (Panel (b)). Once κ surpasses the critical threshold $\bar{\kappa}$, the contraction property fails entirely. The symmetric equilibrium destabilizes, and the system structurally bifurcates toward cross-specialization (Panel (c)).[8] Conversely, if innovation is market-expanding ($\pi_\ell^M \leqslant 2\pi_\ell^D$), the reaction functions never violate the stability condition, and the symmetric equilibrium persists for all $\kappa \in [0, 1]$.

To ensure our model captures the full spectrum of equilibrium behavior within an economically meaningful range, we restrict our focus to the parameter space bounded by $\max\{\pi_A^D/\Delta_A, \pi_B^D/\Delta_B\}$. Crucially, this upper bound ensures that the firm's objective function remains strictly concave. This allows us to rule out a trivial mathematical artifact: the corner solutions in our model do not arise simply because the optimization problem becomes convex. Instead, the shift to these corners is a purely strategic phenomenon, driven entirely by the interplay between the centripetal and centrifugal forces discussed previously.

[8] While crossing $\bar{\kappa}$ guarantees local instability, Proposition 4 identifies the precise parametric conditions ensuring this specialization holds strictly for all $\kappa \geqslant \bar{\kappa}$.

3.2 Comparative Statics of the Symmetric Equilibrium

As Figure 1 illustrates, the symmetric interior equilibrium shifts as common ownership (κ) rises, even before the system bifurcates. While our primary focus is the collapse into asymmetric specialization, the marginal portfolio adjustments within the stable symmetric regime remain economically relevant. To analyze these comparative statics, we restrict our focus to the value-destroying environment ($\pi_\ell^M > 2\pi_\ell^D$ and $\kappa < \bar{\kappa}$), since the underlying mechanics naturally extend to the unconditionally stable market-expanding case ($\pi_\ell^M \leqslant 2\pi_\ell^D$). Within these bounds, firms continue competing head-to-head, but rising κ forces them to incrementally internalize their rival's cross-market externalities. This prompts a strategic rebalancing of their R&D portfolios that is formalized in the following result.

Proposition 3. *If $\pi_\ell^M > 2\pi_\ell^D$ holds for both markets, then any level of common ownership $\kappa < \bar{\kappa}$ induces a reallocation of investment towards Market A, $x_\kappa^* \geqslant x_0^*$ if and only if*

$$\frac{\lambda_A \pi_A^M}{\lambda_B \pi_B^M} \leqslant f(\bar{x}) = \frac{1 - p_B(1-\bar{x})}{1 - p_A(\bar{x})}, \tag{6}$$

where the critical value $\bar{x}$ is uniquely defined by the equality:

$$\frac{p_A(\bar{x})\Delta_A}{\pi_A^M} = \frac{p_B(1-\bar{x})\Delta_B}{\pi_B^M}. \tag{7}$$

To interpret this result, it is useful to categorise markets based on their *profitability*: the marginal return to innovation, $\lambda_\ell \pi_\ell^M$; and its *durability*: the resilience of rents against duplication, inversely proportional to Δ_ℓ / π_ℓ^M.

The inequality in (7) highlights that the direction of reallocation is determined by the interaction between the relative profitability ratio, $\lambda_A \pi_A^M / \lambda_B \pi_B^M$, and the durability correction factor, $f(\bar{x})$ that compares the durability of the two markets around a symmetric benchmark of $f(\bar{x}) = 1$. If Market A (resp. B) is more durable, $f(\bar{x}) >$ (resp. $<$) 1. Thus, if markets are equally profitable, $\lambda_A \pi_A^M = \lambda_B \pi_B^M$, common ownership reallocates investment toward the more durable market. On the other hand, if markets are equally durable, $f(\bar{x}) = 1$, common ownership reallocates investment toward the less profitable market. Consequently, common ownership introduces a preference for markets that are durable and less profitable.

If the less profitable market is also the durable one, these preferences align, and investment unambiguously shifts toward that market. However, if the less profitable market is not the durable

one, a trade-off emerges: common ownership shifts investment toward the less profitable market only if the gap in profitability is sufficiently large to compensate for its lack of durability determined by (6). Otherwise, the durability motive dominates, and investment shifts toward the more durable, albeit more profitable, market. Finally, the monotonic reallocation logic of Proposition 3 extends directly to the market-expanding regime ($\pi^M \leqslant 2\pi^D$), where the equilibrium remains interior for all κ.

3.3 Symmetric Breaking and Cross-Specialization

So far, our analysis has primarily dealt with the regime where the symmetric interior equilibrium is stable. Although Proposition 2 established the condition under which the symmetric equilibrium loses stability and firms transition toward asymmetry, it did not characterize these outcomes. In this section, we fully characterize the set of stable Nash equilibria for high levels of common ownership under Assumption 1.

Proposition 4. *Suppose $\pi_\ell^M > 2\pi_\ell^D$ holds for $\ell \in \{A, B\}$. Assume $\pi_A^D/\Delta_A > \pi_B^D/\Delta_B$ and condition (6) holds such that $dx_\kappa^*/d\kappa > 0$. There exist two critical thresholds, $\bar{\kappa}$ and $\hat{\kappa}$, with $\bar{\kappa} < \hat{\kappa}$, such that for all common ownership levels $\kappa \in [\bar{\kappa}, \max\{\pi_A^D/\Delta_A, \pi_B^D/\Delta_B\})$, the system is characterized by exactly two stable asymmetric equilibria as follows:*

(A) For $\kappa \in (\bar{\kappa}, \hat{\kappa})$, firm i, fully specializes in market B, and consequently exists market A: $x_i^* = 0$. On the other hand, the rival firm j maintains an interior allocation: $x_j^* = \tilde{x}_\kappa^* \in (0, 1)$. The exactly two stable equilibria are $(0, \tilde{x}_\kappa^*)$ and $(\tilde{x}_\kappa^*, 0)$ where $\tilde{x}_\kappa^* > x_\kappa^*$

(B) For $\kappa \in [\hat{\kappa}, \max\{\pi_A^D/\Delta_A, \pi_B^D/\Delta_B\})$, both firms fully specialize in opposite markets. The exactly two stable equilibria are the pure corner strategies $(1, 0)$ and $(0, 1)$.

Proposition 2 shows that common ownership above $\bar{\kappa}$ destabilizes the symmetric interior equilibrium. This boundary-seeking behavior unfolds in two distinct phases. Initially, in the intermediate range $\kappa \in (\bar{\kappa}, \hat{\kappa})$, the market enters a state of partial specialization. The positive feedback loop of strategic substitution pushes one firm (say, firm i) to entirely abandon market A, $x_i^* = 0$, leaving it fully specialized in market B. However, firm j does not immediately mirror this extreme behavior. Enjoying the newly uncontested environment in market A, firm j finds it optimal to maintain an interior allocation, $\bar{x}_\kappa^* \in (0, 1)$, securing the monopoly premium in market A while still deploying residual R&D capital into market B. As common ownership continues to rise and surpasses the

secondary threshold $\hat{\kappa}$, the firms transition to full cross-specialization. At this high level of common ownership, the internalized penalty for firm j to duplicate firm i's established efforts in Market B becomes prohibitively steep. Consequently, firm j entirely withdraws from Market B to allocate its full budget to Market A: $x_j^* = 1$. The result is perfect market segmentation: each firm acts as a pure monopolist in its respective domain, completely eliminating duplicative R&D waste to maximize aggregate shareholder value.

Note that we need condition (6) for a clean characterization of the resulting asymmetric equilibria. This condition ensures that the firms' best-response functions grow steeper as common ownership rises. Because these curves remain steep, firms cannot settle at an interior equilibrium. Slight perturbations would always push them toward corner solutions. We restrict our focus to this parameter space for both clarity and economic relevance. If condition (6) were to fail, the reaction curves could flatten at higher levels of κ. This would create complex cases where multiple interior equilibria appear and disappear. We intentionally abstract away from this scenario as they distract us from the core mechanism of joint-value maximization, and formally map a clean transition from symmetric equilibrium to cross-specialization.

4 The Social Value of Innovation

Having established how firms behave under both independent and common ownership, we analyze the impact on overall societal welfare. First, we identify natural market failures by comparing what independent firms choose to do versus what is best for society. Then, we examine whether common ownership pushes the market closer to, or further from, this socially optimal outcome.

We assume a social planner adopts a consumer surplus standard, maximizing expected surplus $S(x_i, x_j)$.[9] The planner's objective is to allocate resources to maximize:

$$S(x_i, x_j) = \underbrace{\sum_{\ell \in A,B} (p_\ell(x_{\ell i}) + p_\ell(x_{\ell j}))S_\ell^M}_{\text{Consumer's Benefit from Success}} + \underbrace{\sum_{\ell \in A,B} p_\ell(x_{\ell i})p_\ell(x_{\ell j})(S_\ell^D - 2S_\ell^M)}_{\text{Consumer's Duplication Value}} \tag{8}$$

Comparing the social planner's objective (8) with the private firm's payoff (2) reveals three main market distortions. The first is the *appropriability effect* (Arrow, 1962). This occurs because private

[9]While we focus on consumer surplus, the analysis extends to a total welfare standard by simply replacing the surplus terms S_ℓ with total welfare terms $W_\ell = \pi_\ell + S_\ell$ in the objective function.

firms only care about the profits they can capture (π_ℓ^M, π_ℓ^D), while a social planner cares about the consumer surplus (S_ℓ^M, S_ℓ^D). Because firms rarely capture the full value of their innovations, their private incentives will cause them to either under-invest or over-invest compared to the socially optimal level. The second distortion is the *business-stealing externality* (Mankiw & Whinston, 1986). This occurs when a firm invests primarily to capture existing profits from its rival. The third distortion is a novel effect we call the *consumer duplication value*. Denicolò and Polo (2018) identified a similar concept in the context of a full merger, assuming a winner-takes-all structure where simultaneous innovation is purely redundant. However, it is important to note that full common ownership ($\kappa = 1$) in our model is not equivalent to a full merger, because the firms continue to compete independently in prices and/or quantities. Furthermore, we depart from their framework by allowing simultaneous success to strictly increase consumer surplus ($S_\ell^D > S_\ell^M$).

4.1 Market Distortions with Separate Ownership

To understand how this impacts welfare, we can divide the social planner's objective (8) into two distinct components. The first component, proportional to $p_\ell(x_{\ell i}) + p_\ell(x_{\ell j})$, captures the baseline expected consumer surplus generated by individual firm successes. Because the probability of success features diminishing returns, this creates a strong pull (the centripetal force) toward a symmetric, risk-hedging interior optimum. The second component, $p_{\ell i}p_{\ell j}(S_\ell^D - 2S_\ell^M)$ captures the net duplication value—the incremental shift in consumer surplus—when both firms successfully innovate and compete in the same market. If simultaneous success is highly beneficial to consumers ($S_\ell^D \geqslant 2S_\ell^M$), it reinforces the pull toward symmetry. Conversely, if the second success adds less incremental value than the first ($S_\ell^D < 2S_\ell^M$), duplication becomes socially wasteful. In this scenario, the duplication term acts as a penalty, pushing the firms to segregate and specialize (the centrifugal force) in completely different markets.

Proposition 5. *If duplication is beneficial to consumers, $S_\ell^D \geqslant 2S_\ell^M$, the optimal allocation is the unique symmetric solution $x_W^* \in [0,1]$. If duplication is socially wasteful, $S_k^D < 2S_k^M$, the symmetric interior solution x_W^* is a stable maximum if and only if:*

$$\frac{\lambda_A + \lambda_B}{1 + \lambda_A + \lambda_B} > \left(\frac{2S_A^M - S_A^D}{S_A^M + S_B^M}\right) p_A'(x_W^*) + \left(\frac{2S_B^M - S_B^D}{S_A^M + S_B^M}\right) p_B'(1 - x_W^*). \tag{9}$$

Otherwise, the optimal allocation is full specialization; $x_W^* = 0$ or $x_W^* = 1$.

As the proposition demonstrates, if a second success creates sufficient value for consumers ($S_\ell^D \geqslant 2S_\ell^M$), the planner naturally prefers a symmetric, interior allocation where firms diversify across both markets. However, a fundamental trade-off emerges when duplicate innovations are socially wasteful ($S_\ell^D < 2S_\ell^M$). In this scenario, condition (9) dictates the socially optimal market structure. The left-hand side ($\lambda_A + \lambda_B$) captures the strength of the diminishing returns to research investment, which naturally encourages diversification. The right-hand side measures the marginal expected social cost of redundant parallel research. The planner will only maintain a symmetric interior allocation if the technological pull of diminishing returns strictly outweighs the expected waste of duplication. If the cost of redundant research is too high, the inequality flips, and the planner prefers to fully segregate the firms into separate markets.

4.2 Effect of Common Ownership on Welfare

To evaluate how common ownership impacts overall economic welfare, we must compare the market's private equilibrium against the socially optimal outcome.

First, we summarize the private market. Firms dictate the market equilibrium by weighing the returns of head-to-head competition against the benefits of specializing themselves into separate markets. As established in Proposition 2, the private market sustains a symmetric interior equilibrium (where firms compete head-to-head) under only two scenarios: either the standard efficiency effect fails ($\pi_\ell^M \leqslant 2\pi_\ell^D$), or the efficiency effect holds ($\pi_\ell^M > 2\pi_\ell^D$) but the level of common ownership remains low ($\kappa < \bar{\kappa}$). Conversely, firms will completely specialize to avoid competition only when the efficiency effect holds and common ownership crosses the critical threshold ($\kappa \geqslant \bar{\kappa}$).

Next, we summarize the social planner's objective. The planner must balance the gross consumer benefits of duplicate innovations against the social cost of redundant research. The socially optimal allocation is symmetric if duplicate innovations are highly beneficial to consumers ($S_\ell^D \geqslant 2S_\ell^M$). Even if duplication is socially wasteful ($S_\ell^D < 2S_\ell^M$), the planner will still prefer symmetry as long as the technological need to diversify satisfies the stability condition (9). The planner only prefers specialization when duplication is socially wasteful and that stability condition strictly fails.

By intersecting the two potential market outcomes with the two potential social optima, we map the full welfare space into four distinct economic regimes. These are summarized below in Table 1.

- **Regime 1:** Both the industry and the planner prefer a symmetric, competitive innovation race. However, the exact level of private investment may still differ from the social optimum

Table 1: Taxonomy of Welfare Regimes

Regime	Market Equilibrium	Social Optimum	Welfare Effect of $\kappa \uparrow$
1. Market Distortion	**Symmetric** ($\pi^M > \pi^D/2$ and $\kappa < \bar{\kappa}$; or $\pi^M \leqslant \pi^D/2$)	**Symmetric** ($S^D \leqslant 2S^M$ and (9) holds; or $S^D > 2S^M$)	**Ambiguous** Both seek competition in both markets, but distortion exists.
2. Excessive Entry	**Symmetric** ($\pi^M > \pi^D/2$ and $\kappa < \bar{\kappa}$; or $\pi^M \leqslant \pi^D/2$)	**Specialisation** ($S^D \geqslant 2S^M$ and (9) fails)	**Ambiguous** Firms continue to compete in both markets, but the planner wishes they wouldn't.
3. Insufficient Entry	**Specialisation** ($\pi^M > \pi^D/2$ and $\kappa \geqslant \bar{\kappa}$)	**Symmetric** ($S^D \leqslant 2S^M$ and (9) holds; or $S^D > 2S^M$)	**Negative** Firms avoid competing with each other, but the planner wants them to fight.
4. Natural Monopoly	**Specialisation** ($\pi^M > \pi^D/2$ and $\kappa \geqslant \bar{\kappa}$)	**Specialisation** ($S^M > S^D/2$ and (9) fails)	**Positive** Both agree one firm is enough in each market

due to the three market distortions discussed following equation (8).

- **Regime 2:** Firms engage in a socially wasteful business-stealing race. The planner prefers that firms specialize to avoid redundant duplication costs, but the private market fails to segregate.

- **Regime 3:** Firms retreat into specialization to protect their profit margins, avoiding head-to-head competition. Conversely, the planner prefers a symmetric race because the resulting consumer surplus outweighs the duplication costs.

- **Regime 4:** Both the private market and the social planner agree that a corner solution (full specialization) is optimal, typically driven by high research costs or low product differentiation.

The welfare implications of common ownership in Regimes 3 and 4 are straightforward. Because an increase in common ownership naturally shifts the market equilibrium away from a symmetric interior allocation and toward specialization (either full or partial), it actively harms welfare in Regime 3 while pushing the market toward the social optimum in Regime 4. From a policy perspective, antitrust authorities and regulators are primarily concerned with the competitive effects of

high levels of common ownership. At these levels (captured by Regimes 3 and 4 in our model), the conditions determining whether social welfare increases or decreases are relatively straightforward. We state this formally as follows:

Result: *High levels of common ownership drive the market away from symmetric interior allocations and toward specialization to reduce head-to-head competition. This shift lowers consumer surplus when the social value of duplication is high (Regime 3), but increases consumer surplus by aligning with the social optimum when the value of duplication is low (Regime 4).*

However, for the completeness of this theoretical framework, we must also analyze the reallocations that occur at low levels of common ownership (Regimes 1 and 2). We begin this analysis by focusing on Regime 1 in the following proposition.

Proposition 6. *In Regime 1, the market under-invests in Market A relative to the social optimum ($x_0^* < x_W^*$) if and only if $\bar{x}$:*

$$\lambda_A e^{-\lambda_A \hat{x}} \left(S_A^M + p_A(\hat{x})(S_A^D - 2S_A^M)\right) > \lambda_B e^{-\lambda_B(1-\hat{x})} \left(S_B^M + p_B(1-\hat{x})(S_B^D - 2S_B^M)\right) \tag{10}$$

where the threshold $\bar{x}$ is the unique allocation that equalizes the ratio of social to private marginal values across sectors:

$$\frac{S_A^M + p_A(\hat{x})(S_A^D - 2S_A^M)}{\pi_A^M - p_A(\hat{x})(\pi_A^M - \pi_A^D)} = \frac{S_B^M + p_B(1-\hat{x})(S_B^D - 2S_B^M)}{\pi_B^M - p_B(1-\hat{x})(\pi_B^M - \pi_B^D)}. \tag{11}$$

If $\pi_\ell^M \leqslant 2\pi_\ell^D$, then any for level of common ownership $\kappa \in [0,1]$ the industry under-invests in Market A relative to the social optimum if and only if (10) *is satisfied.*

Equation (11) defines a critical threshold allocation, $\hat{x}$. At this point, the ratio of marginal social value to marginal private value in Market A equals to that of Market B. To understand the wedge between the planner and the private market, we can break down the components of these ratios. The numerator of each side captures the marginal social value of investment. It is driven by the baseline consumer surplus (S_ℓ^M) and adjusted by the net social value of a duplicate innovation ($S_\ell^D - 2S_\ell^M$). The denominator captures the marginal private value. It is driven by monopoly profits (π_ℓ^M) but penalized by the "business-stealing" effect, which is the margin lost to competition ($\pi_\ell^M - \pi_\ell^D$).The tension between the social planner and the private market arises when these forces pull in opposite directions. For example, suppose Market A generates immense consumer surplus that firms cannot capture (a baseline appropriability failure), while duplicate innovations yield high social value but

severely cannibalize private margins (high business-stealing). In this scenario, the social planner aggressively pushes investment toward Market A to secure lower prices and product variety. The private firm, however, retreats from Market A to protect its margins and avoid cannibalization. Inequality (10) formalizes this dynamic, establishing the exact condition under which the private market's fear of business-stealing causes it to systematically under-invest in Market A relative to the social optimum.

While (10) establishes the condition under which the private market systematically under-invests in Market A, it leaves open a policy question: *if common ownership induces a reallocation of investment toward Market A, does that reallocation actually increase social welfare?* To understand whether common ownership closes the gap between private and social incentives, or instead exacerbates existing distortions, we evaluate both decision rules simultaneously. We do this by mapping both the private reallocation threshold and the social welfare threshold into a shared visual and analytical space: the duopoly profit space (π_A^D, π_B^D).

We first compute the critical allocation $\bar{x}$ by evaluating condition (6) with strict equality. We then substitute $\bar{x}$ into the threshold equality defined in (7). Because the profit differential is given by $\Delta_\ell = \pi_\ell^M - \pi_\ell^D$ for each market $\ell \in \{A, B\}$, we can rearrange (7) to express the reallocation indifference line as:

$$\pi_{B,\kappa}^D(\pi_A^D) = m_\kappa \pi_A^D + c_\kappa \quad \text{where} \quad m_\kappa = \frac{p_A(\bar{x})\pi_B^M}{p_B(1-\bar{x})\pi_A^M} \quad \text{and} \quad c_\kappa = \pi_B^M - m_\kappa \pi_A^M.$$

Along this threshold line—that is, for any pair $(\pi_A^D, \pi_{B,\kappa}^D(\pi_A^D))$—any level of common ownership $\kappa \in (0, \bar{\kappa})$ leaves the optimal allocation unchanged ($x_0^* = x_\kappa^*$). Furthermore, if the actual duopoly profit deviates from this threshold such that $\pi_{B,\kappa}^D(\pi_A^D) \gtrless \pi_B^D$, the firm's optimal allocation shifts, yielding $x_0^* \lesseqgtr x_\kappa^*$.

We apply an analogous procedure to derive the social welfare threshold. We compute the socially optimal threshold allocation $\hat{x}$ by setting the gradient condition (10) to equality, which identifies the point where the social marginal values across sectors are perfectly balanced. Substituting $\hat{x}$ back into the threshold definition (11) allows us to solve for π_B^D to generate the welfare contour $\pi_{B,W}^D = m_W \pi_A^D + c_W$. To simplify the notation, let $N_A(\hat{x})$ and $N_B(\hat{x})$ represent the numerators of (11), which capture the expected consumer surplus terms for markets A and B, respectively.

Rearranging (11) to isolate π_B^D gives the slope and intercept of the welfare indifference curve as:

$$m_W = \frac{N_B(\hat{x})p_A(\hat{x})}{N_A(\hat{x})p_B(1-\hat{x})} \quad \text{and} \quad c_W = \frac{N_B(\hat{x})(1-p_A(\hat{x}))\pi_A^M - N_A(\hat{x})(1-p_B(1-\hat{x}))\pi_B^M}{N_A(\hat{x})p_B(1-\hat{x})}$$

Along this threshold line—that is, for any pair $(\pi_A^D, \pi_{B,W}^D(\pi_A^D))$—the baseline private market naturally aligns with the social optimum ($x_0^* = x_W^*$). Furthermore, if the actual duopoly profit deviates from this threshold such that $\pi_{B,W}^D(\pi_A^D) \gtrless \pi_B^D$, the baseline market allocation diverges from the social optimum, yielding systematic under- or over-investment ($x_0^* \lesseqgtr x_W^*$).

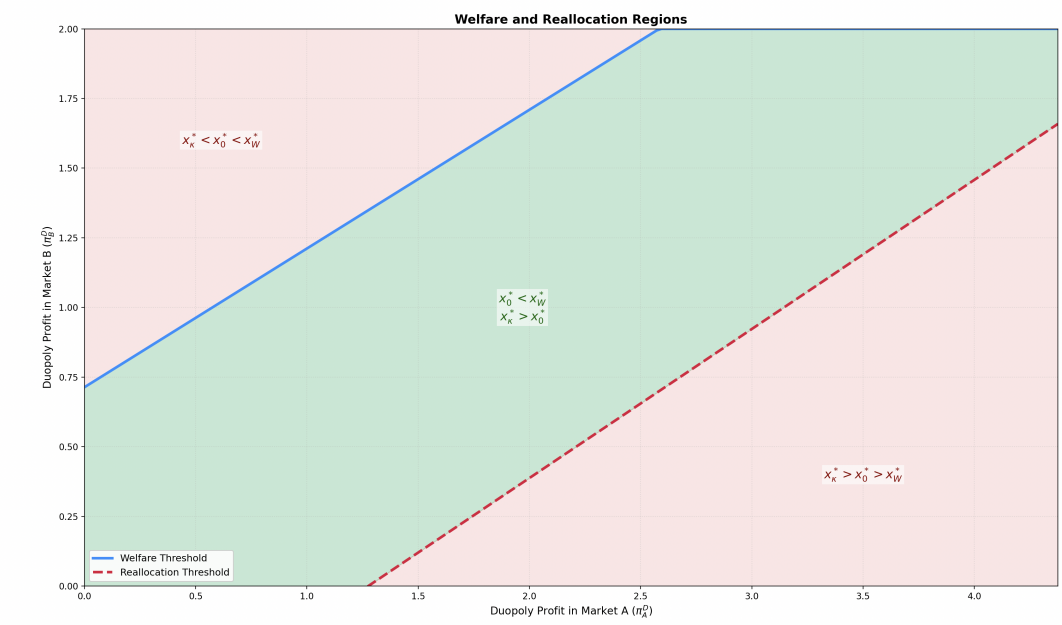


Figure 2: Welfare and Reallocation Regions.

Figure 2 visually partitions the parameter space into three distinct regions. The shaded areas are defined by the intersection of the private reallocation threshold and the social welfare threshold. In the red regions, the strategic incentives introduced by common ownership strictly conflict with the social planner's objectives. In the upper-left region, the baseline market already under-invests in Market A relative to the social optimum ($x_0^* < x_W^*$). Common ownership exacerbates this distortion by further reducing investment in Market A ($x_\kappa^* < x_0^*$). Conversely, in the lower-right region, the baseline market over-invests in Market A ($x_0^* > x_W^*$), and common ownership pushes even more resources into Market A ($x_\kappa^* > x_0^*$). Thus, in both red regions, the reallocation induced by common ownership strictly moves the interior allocation further away from the socially optimal level, unambiguously reducing social welfare.

The green region between the two thresholds represents the domain where common ownership introduces the potential for a welfare improvement. In this space, the baseline private market underinvests in Market A ($x_0^* < x_W^*$), but common ownership induces a corrective reallocation toward Market A ($x_\kappa^* > x_0^*$). Because the investment moves in the direction of the social optimum, welfare initially increases. However, it is important to note that this alignment is directional. Depending on the magnitude of the common ownership parameter κ, it is theoretically possible for the reallocation to overshoot the social optimum ($x_\kappa^* > x_W^*$). If this overshoot is sufficiently large, the net effect on welfare could eventually become negative. While we note this possibility, a full mathematical characterization of the exact overshoot boundary is outside the scope of this baseline analysis; our primary focus is establishing the structural conditions under which private and social directional incentives align.

To generate the exact thresholds depicted in Figure 3, we set the baseline market parameters to $\lambda_A = 0.7$, $\lambda_B = 1.4$, $S_B^M = 5.0$, $S_B^D = 9.0$, and $\pi_B^M = 4.0$. For Market A, we introduce a slight asymmetry between private appropriability and consumer surplus by setting $\pi_A^M = 8.75$, $S_A^M = 9.75$, and $S_A^D = 18.0$. This specific wedge between private profits and social surplus creates the central green region where common ownership can directionally align with the social optimum.

However, it is entirely possible for this green region of potential welfare improvement to vanish completely (see figure 3). This collapse occurs when the markets are strictly proportional—for example, in standard frameworks with symmetric, linear demands, such as the Singh-Vives model of differentiated products under either Cournot or Bertrand competition. To illustrate this proportionality, suppose we adjust Market A's parameters to $S_A^M = 10$ and $\pi_A^M = 8$. In this scenario, the ratio of monopoly consumer surplus to monopoly profit in Market A ($10/8 = 1.25$) perfectly matches that of Market B ($5/4 = 1.25$). Because the relative private and social incentives are identical across sectors, the reallocation and welfare thresholds perfectly overlap, eliminating the green region entirely. In this purely proportional benchmark, any reallocation induced by common ownership unambiguously decreases welfare.

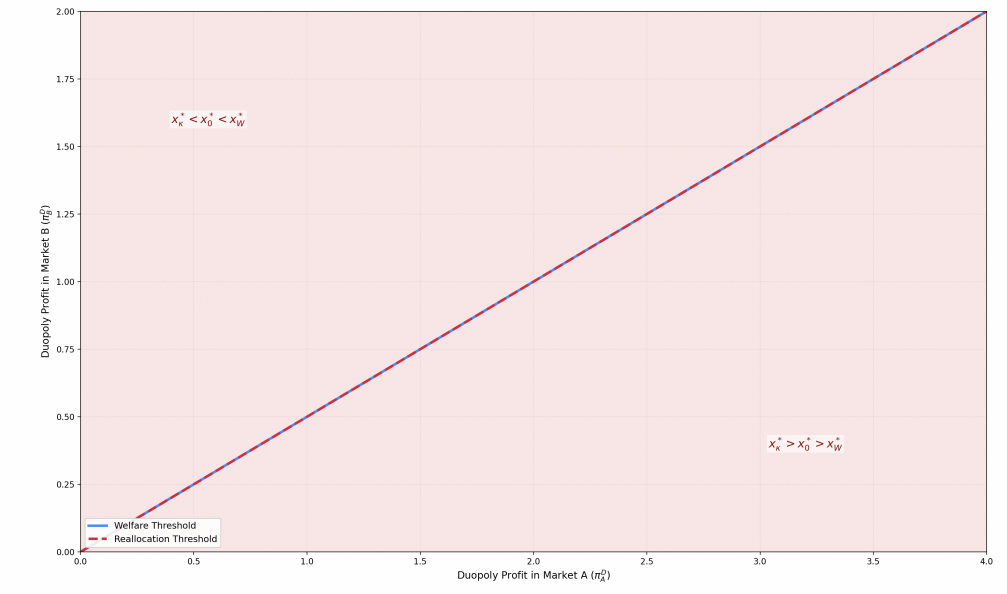


Figure 3: Welfare Graph for Proportional Markets

As we move away from proportionality, the green region emerges and grows. If we increase Market A's profitability to $\pi_A^M = 8.75$ but keep consumer surplus relatively high at $S_A^M = 9.75$, the lines separate to form the moderate green region seen in Figure 2. On the other hand, if we hold $\pi_A^M = 8.75$ but apply a sharper decrease to baseline consumer surplus, such that $S_A^M = 7.5$, the wedge between the planner's objectives and the firm's incentives widens significantly. Consequently, the green area expands further, as illustrated in Figure 4, reflecting a broader parameter space where common ownership corrects severe baseline misallocations.

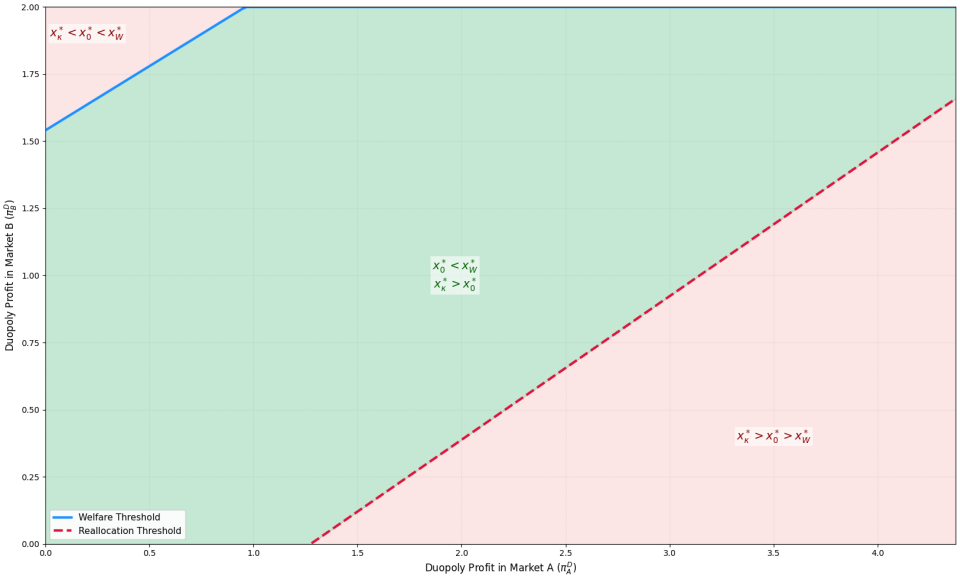


Figure 4

Having fully characterized the welfare dynamics of Regime 1—where both the private firm and the social planner prefer interior allocations—we briefly note the mechanics of Regime 2 to complete our taxonomy of low common ownership levels ($\kappa < \bar{\kappa}$). In Regime 2, the baseline private market settles at an interior equilibrium, but this allocation is socially wasteful; the planner strictly prefers specialization to avoid redundant duplication costs. Because common ownership remains low, it induces only a marginal shift in the interior allocation rather than a jump to full specialization. Consequently, the welfare effect is mechanically straightforward: common ownership strictly increases welfare if and only if it pushes investment in the same direction as the planner's preferred boundary. Mathematically, $W(x_\kappa^*) > W(x_0^*)$ if and only if $\Psi(x_0^*) \cdot (x_\kappa^* - x_0^*) > 0$, where $\Psi(x_0^*)$ is the marginal social gradient evaluated at the baseline equilibrium. Because this directional alignment relies on the same fundamental parameter wedges explored in Regime 1, we do not belabor the formal proofs here, as the primary economic tension resides in the interior trade-offs analyzed above.

5 Extensions

The baseline model relies on a stylized symmetric duopoly with independent markets and fixed R&D budgets to cleanly isolate the novel allocative margin of common ownership. To ensure the theoretical robustness of our core symmetry-breaking mechanism, we discuss four natural extensions. These extensions explore how endogenous investment levels, ex-ante asymmetries, R&D spillovers, and multi-firm market structures interact with the fundamental tension between technological concavity and the duplication penalty. While the formal mathematical characterizations of these generalized environments are part of ongoing research and will be provided in detail within the Online Appendix, we briefly outline the economic intuition behind these extensions here.

5.1 Endogenous Total Investment

In our baseline model, we impose a strictly binding budget constraint to cleanly isolate the directional allocation of R&D from the overall scale of investment. A natural question is whether our symmetry-breaking results hold if firms can endogenously choose their total investment level subject to a convex joint cost function, $c(x_{iA} + x_{iB})$. In the Online Appendix, we formally extend the model to allow for flexible budgets. We show that as long as the cost function is sufficiently

convex—capturing the inherent limits of managerial bandwidth and organizational capacity—the fundamental strategic trade-off across markets persists. In this generalized setting, common ownership simultaneously depresses the aggregate level of investment (consistent with the scale effects documented by Antón et al., 2023) and drives firms toward asymmetric cross-specialization to mitigate duplication penalties. By abstracting from the scale effect in the main text, our fixed-budget benchmark provides a much more tractable and transparent characterization of this novel allocative margin.

5.2 Asymmetric Firms and Capability Differences

Our baseline framework assumes identical R&D budgets and innovation capabilities, implying that the shift to cross-specialization arises from spontaneous symmetry breaking. If we introduce ex-ante heterogeneity—such as Firm i possessing a strictly larger budget or a technological cost advantage—the fundamental centrifugal force remains, but the resulting market structure becomes deterministic. The highly capable firm is incentivized to secure the more lucrative market, while the constrained firm strategically retreats into the less contested "safe haven" market to avoid the duplication penalty. Consequently, while ex-ante asymmetry eliminates the multiplicity of the stable corner equilibria, high levels of common ownership continue to act as a catalyst for product-market differentiation, dictating that the smaller firm yields rather than competes head-to-head.

5.3 Technological R&D Spillovers

Following the literature on overlapping ownership and innovation (e.g., Lopez and Vives, 2019), it is natural to consider environments where R&D exhibits positive technological spillovers. Introducing a spillover parameter implies that Firm i's investment in Market A positively impacts Firm j's probability of success. This creates a countervailing strategic force: while common ownership penalizes simultaneous success, spillovers allow firms to free-ride on rival investments. If spillovers are exceptionally high, this free-riding effect can dominate, potentially restoring the stability of a symmetric interior equilibrium where firms passively co-invest. However, for moderate and empirically plausible levels of spillovers, the duplication penalty remains the dominant force. The primary effect of spillovers in our framework is to shift the critical threshold $\bar{\kappa}$ rather than eliminate the eventual bifurcation into cross-specialization.

5.4 Multi-Firm and Multi-Market Competition

The baseline model restricts the analysis to two firms and two markets, allowing for perfect cross-specialization. Extending the framework to N firms and M markets introduces the "pigeonhole problem": if $N > M$, perfect monopoly segmentation is mathematically impossible, and at least some firms must overlap. In this generalized setting, the strategic tension shifts from entirely avoiding duplication to minimizing the density of overlap. As common ownership increases, the internalized duplication penalty still drives firms away from the fully symmetric equilibrium (where all N firms invest proportionally in all M markets). Instead, the industry partitions into localized, specialized clusters. Firms distribute themselves as sparsely as possible across the available markets, confirming that the core mechanism—differentiation to preserve joint industry surplus—scales robustly to more complex oligopoly structures.

6 Conclusion

In this paper, we advance the literature on common ownership by shifting the analytical focus from the overall scale of managerial effort to the strategic allocation of limited R&D resources across competing markets. By modeling multi-market patent races under a binding budget constraint, we demonstrate that overlapping ownership fundamentally alters the direction of innovation. As institutional investors increase their stakes, the internalized penalty of rent-dissipating parallel success overcomes the natural technological drive for firms to diversify. Consequently, firms systematically reallocate their investments away from highly contested markets, seeking instead "safe havens" where monopoly rents are more resilient to duplication. When the cost of head-to-head competition is sufficiently high, this dynamic destabilizes symmetric market structures entirely, driving competing firms toward strategic cross-specialization—a theoretical result that perfectly rationalizes recent empirical evidence on how common ownership increases product differentiation.

Crucially, our framework reveals that the welfare implications of this strategic segregation are not unilaterally detrimental; rather, they depend heavily on the underlying societal value of duplicate innovation. If overlapping R&D yields highly substitutable products that drive down prices, the cross-specialization induced by common ownership stifles necessary competition and strictly harms consumer surplus. However, in industries where the consumer benefits of a second, parallel success are minimal, this exact same mechanism acts as a highly efficient coordinating device. By

encouraging firms to specialize, common ownership prevents the deadweight loss of redundant R&D and increases the aggregate probability that at least one new innovation reaches the market. Ultimately, antitrust and regulatory scrutiny of common ownership should eschew blanket assumptions, recognizing that its effect on consumer welfare is inextricably tied to the specific technological and competitive parameters of the markets in question.

7 Appendix

Proof of Proposition 1

Proof. The first-order condition for firm i maximizing (1) with respect to x_i is given by

$$\frac{\partial \pi(x_i, x_j)}{\partial x_i} = p'_A(x_i)\left[\pi_A^M - p_A(x_j)\Delta_A\right] - p'_B(1-x_i)\left[\pi_B^M - p_B(1-x_j)\Delta_B\right] = 0. \tag{12}$$

As $p_\ell(\cdot)$ is strictly concave by assumption, the second order condition is always negative:

$$\frac{\partial^2 \pi(x_i, x_j)}{\partial x_i^2} = p''_A(x_i)\left[\pi_A^M - p_A(x_j)\Delta_A\right] + p''_B(1-x_i)\left[\pi_B^M - p_B(1-x_j)\Delta_B\right] < 0. \tag{13}$$

As the firms are symmetric, there exists a unique symmetric equilibrium denoted by $x_i^* = x_j^* = x^*$. Define the function $\Phi(z)$ as the FOC (12) evaluated at the symmetric allocation:

$$\Phi(x^*) \equiv p'_A(x^*)\left[\pi_A^M - p_A(x^*)\Delta_A\right] - p'_B(1-x^*)\left[\pi_B^M - p_B(1-x^*)\Delta_B\right].$$

This symmetric equilibrium is interior if and only if the boundary conditions (3) hold:

$$\Phi(0) = p'_A(0)\pi_A^M - p'_B(1)\left[\pi_B^M - p_B(1)\Delta_B\right] > 0$$

$$\Phi(1) = p'_A(1)\left[\pi_A^M - p_A(1)\Delta_A\right] - p'_B(0)\pi_B^M < 0.$$

If $\Phi(0) \leqslant 0$, the unique solution is the corner $x^* = 0$. If $\Phi(1) \geqslant 0$, the unique solution is $x^* = 1$.

We next verify that x^* is the unique Nash equilibrium, not just the unique symmetric candidate. For that purpose, we first determine the nature of strategic interaction, we compute the cross-partial derivative:

$$\frac{\partial^2 \pi(x_i, x_j)}{\partial x_i \partial x_j} = -p'_A(x_i)p'_A(x_j)\Delta_A - p'_B(1-x_i)p'_B(1-x_j)\Delta_B.$$

Since $p'_\ell(\cdot) > 0$ and $\Delta_\ell > 0$, we have $\partial^2\pi(x_i, x_j)/\partial x_i \partial x_j < 0$. This confirms that the game exhibits strategic substitutes (decreasing best replies). With strictly concave payoff functions and strictly decreasing best replies, the reaction functions intersect at most once. Since we have already established the existence of a unique symmetric intersection x^* via $\Phi(x) = 0$, this x^* is the unique Nash equilibrium of the game (Vives, 1999).■

Proof of Lemma 1

Proof. The first-order condition (FOC) for firm i is given by:

$$\Psi(x_i, x_j; \kappa) = p'_A(x_i)[\pi_A^M - (1+\kappa)p_A(x_j)\Delta_A] - p'_B(1-x_i)[\pi_B^M - (1+\kappa)p_B(1-x_j)\Delta_B] = 0. \quad (14)$$

Consider symmetric actions by the firms where $x_i = x_j = x$. The boundary conditions (3) imply that $\Psi(0, 0; \kappa) > 0$ and $\Psi(1, 1; \kappa) < 0$. Since the function $\Psi(\cdot)$ is continuous in x, the Intermediate Value Theorem guarantees the existence of at least one interior symmetric critical point $x^*_\kappa \in (0, 1)$ such that $\Psi(x^*_\kappa, x^*_\kappa; \kappa) = 0$.

Following Rosen (1965), a sufficient condition for the Nash equilibrium to be unique is that for all strategy profiles $(x_i, x_j) \in [0, 1]^2$, the Jacobian of the system satisfies strict diagonal dominance:

$$\frac{\partial^2 u_i(x_i, x_j; \kappa)}{\partial x_i^2} + \left| \frac{\partial^2 u_i(x_i, x_j; \kappa)}{\partial x_i \partial x_j} \right| < 0. \quad (15)$$

First, the cross-partial derivative is strictly negative:

$$\frac{\partial^2 u_i(x_i, x_j; \kappa)}{\partial x_i \partial x_j} = -(1+\kappa)\left[p'_A(x_i)p'_A(x_j)\Delta_A + p'_B(1-x_i)p'_B(1-x_j)\Delta_B\right] < 0. \quad (16)$$

Hence, the reaction functions exhibit strategic substitutability. Next, we compute the second-order derivative:

$$\frac{\partial^2 u_i(x_i, x_j; \kappa)}{\partial x_i^2} \equiv p''_A(x_i)[\pi_A^M - (1+\kappa)p_A(x_j)\Delta_A] + p''_B(1-x_i)[\pi_B^M - (1+\kappa)p_B(1-x_j)\Delta_B]. \quad (17)$$

This derivate if negative for all strategy profiles $(x_i, x_j) \in [0, 1]^2$ if $\kappa < T_0(x_i, x_j)$ where

$$T_0(x_i, x_j) = \frac{\sum_{\ell \in \{A,B\}} |p''_\ell(x_{\ell i})|\pi_\ell^M}{\sum_{\ell \in \{A,B\}} |p''_\ell(x_{\ell i})|p_\ell(x_{\ell j})\Delta_\ell} - 1, \quad x_{Ai} = x_i \text{ and } x_{Bi} = 1 - x_i.$$

Thus, $\kappa < \kappa_{max}$ is a sufficient condition for concavity where

$$\kappa_{max} = \inf_{x_i, x_j \in [0,1]^2} T_0(x_i, x_j). \tag{18}$$

Assuming $\kappa < \kappa_{max}$, we substitute these derivatives into the diagonal dominance condition (15) and rearranging terms to isolate κ, we find that diagonal dominance holds if $\kappa < T_1(x_i, x_j)$ where

$$T_1(x_i, x_j) = \frac{\sum_{\ell \in \{A,B\}} |p''_\ell(x_{\ell i})| \pi^M_\ell}{\sum_{\ell \in \{A,B\}} |p''_\ell(x_{\ell i})| p_\ell(x_{\ell j}) \Delta_\ell + \sum_{\ell \in \{A,B\}} p'_\ell(x_{\ell i}) p'_\ell(x_{\ell j}) \Delta_\ell} - 1.$$

Thus, the condition $\kappa < \bar{\kappa}$ where

$$\bar{\kappa} = \inf_{x_i, x_j \in [0,1]^2} T(x_i, x_j),$$

ensures the Jacobian is diagonally dominant everywhere to ensure uniqueness and stability of the equilibrium. Since $p'_\ell(x_{\ell i}) p'_\ell(x_{\ell j}) \Delta_\ell > 0$, the denominator of $T_1(x_i, x_j)$ is strictly larger than that of $T_0(x_i, x_j)$ for any pair (x_i, x_j). Given the compactness of the strategy space $[0, 1]^2$ and the continuity of $p_\ell(\cdot)$, by the Extreme Value Theorem, the minimum of the difference, $\min T_0(x_i, x_j) < \min T_1(x_i, x_j)$ ensuring that $\bar{\kappa} < \kappa_{max}$.

Finally, since the game is symmetric and the equilibrium is unique, guaranteed by $\kappa < \bar{\kappa}$, the unique equilibrium must be the symmetric allocation x^*_κ computed in Step 1. Thus, $\kappa < \bar{\kappa}$ is sufficient for existence, uniqueness, and stability. ■

Proof of Proposition 2

Proof. Under Assumption 1, the stability threshold derived in Lemma 1 takes the form:

$$T_1(x_i, x_j) = \tau(x_i) = \frac{\lambda_A^2 e^{-\lambda_A x_i} \pi^D_A + \lambda_B^2 e^{-\lambda_B (1-x_i)} \pi^D_B}{\lambda_A^2 e^{-\lambda_A x_i} \Delta_A + \lambda_B^2 e^{-\lambda_B (1-x_i)} \Delta_B}. \tag{19}$$

As $T_1(x_i, x_j)$ is independent of x_j under Assumption 1, we denote the threshold by $\tau(x_i)$. If $\kappa < \underline{\kappa} = \min_{x_i} \tau(x_i)$, then by Lemma 1 there exists a unique Nash equilibrium that satisfies the first-order condition (14).

To compute $\underline{\kappa}$ for the stylized exponential functional form (Assumption 1), we differentiate $\tau(\cdot)$ with respect to x_i to obtain:

$$\tau'(x_i) = \frac{(\lambda_A + \lambda_B) \lambda_A^2 \lambda_B^2 e^{-[\lambda_A x_i + \lambda_B (1-x_i)]} (\pi^D_B \Delta_A - \pi^D_A \Delta_B)}{\left(\lambda_A^2 e^{-\lambda_A x_i} \Delta_A + \lambda_B^2 e^{-\lambda_B (1-x_i)} \Delta_B\right)^2}.$$

Without loss of generality, let $\pi_A^D/\Delta_A < \pi_B^D/\Delta_B$. Thus, $\tau(\cdot)$ is increasing for all x_i and achieves a minimum at $x_i = 0$.

$$\underline{\kappa} = \tau(0) = \frac{\lambda_A^2 \pi_A^D + \lambda_B^2 e^{-\lambda_B} \pi_B^D}{\lambda_A^2 \Delta_A + \lambda_B^2 e^{-\lambda_B} \Delta_B} \tag{20}$$

Thus, for all $\kappa < \underline{\kappa}$, the dominant diagonal condition is satisfied for all $x_i \in [0, 1]$. Hence, there exists a unique equilibrium that is symmetric and stable (Rosen, 1965). Observe that $\underline{\kappa}$ is a convex combination of the weights $\omega_A = \lambda_A^2 \Delta_A$ and $\omega_B = \lambda_B^2 e^{-\lambda_B} \Delta_B$:

$$\underline{\kappa} = \frac{\omega_A(\pi_A^D/\Delta_A) + \omega_B(\pi_B^D/\Delta_B)}{\omega_A + \omega_B}$$

Hence, the threshold must lie in the following range: $\pi_A^D/\Delta_A < \underline{\kappa} < \pi_B^D/\Delta_B$. We denote this unique Nash equilibrium that is symmetric and stable by $x_i = x_j = x_\kappa^*$ that satisfies the first-order condition (14).

However, the dominant diagonal condition is too strict, and the equilibrium x_κ^* exists for common ownership levels $\kappa \geqslant \underline{\kappa}$.

Lemma 2. *There exists a threshold $\bar{\kappa} \in [\underline{\kappa}, \pi_B^D/\Delta_B]$, such that there exists a unique symmetric equilibrium x_κ^* that is stable only if $\kappa < \bar{\kappa} \in [\underline{\kappa}, \bar{\kappa})$.*

Proof. Given Assumption 1, the reaction function for firm i (derived from FOC (14)) is denoted by

$$R(x_j) = \frac{\ln(\lambda_A/\lambda_B) + \lambda_B}{\lambda_A + \lambda_B} + \frac{1}{\lambda_A + \lambda_B} \ln \left(\underbrace{\frac{\pi_A^M - (1+\kappa)\big(1 - e^{-\lambda_A x_j}\big)\Delta_A}{\pi_B^M - (1+\kappa)\big(1 - e^{-\lambda_B(1-x_j)}\big)\Delta_B}}_{N(x_j)/D(x_j)} \right) \tag{21}$$

In order that this best response is well-defined, the ration inside the logarithm, $N(x_j)/D(x_j)$, must be positive and $R(x_j) \geqslant 0$. The denominator $D(x_j)$ is always strictly positive for $\kappa \leqslant \pi_B^D/\Delta_B$. The numerator term remains positive if and only if $x_j < \bar{x}$ where $\bar{x} > 0$ satisfies $\pi_A^M - (1+\kappa)\big(1 - e^{-\lambda_A \bar{x}}\big)\Delta_A = 0$. The reaction function is strictly decreasing on the interval $x_j \in [0, \min\{\bar{x}, 1\}]$:

$$R'(x_j) = -\frac{1+\kappa}{\lambda_A + \lambda_B} \left(\frac{\lambda_A e^{-\lambda_A x_j} \Delta_A}{\pi_A^M - (1+\kappa)\big(1 - e^{-\lambda_A x_j}\big)\Delta_A} + \frac{\lambda_B e^{-\lambda_B(1-x_j)} \Delta_B}{\pi_B^M - (1+\kappa)\big(1 - e^{-\lambda_B(1-x_j)}\big)\Delta_B} \right) < 0,$$

and satisfies the boundary condition $R(0) > 0$. If $R(\min\{\bar{x}, 1\}) < 0$ there exists a unique intercept

$\hat{x} < \bar{x}$ such that

$$R(\hat{x}) = 0 \iff \frac{\pi_A^M - (1+\kappa)\big(1 - e^{-\lambda_A \hat{x}}\big)\Delta_A}{\pi_B^M - (1+\kappa)\big(1 - e^{-\lambda_B(1-\hat{x})}\big)\Delta_B} = \frac{\lambda_B}{\lambda_A} e^{-\lambda_B}. \tag{22}$$

Note that as $x_j \to \bar{x}^-$ the ratio term inside the logarithm in (21) tends to zero: $\lim_{x_j \to \bar{x}^-} N(x_j)/D(x_j) = 0$. This implies $\lim_{x_j \to \bar{x}^-} R(x_j) = -\infty$. Since $R(x_j)$ is strictly decreasing, $R(\hat{x}) = 0$, and $\lim_{x_j \to \bar{x}^-} R(x_j) = -\infty$, it strictly follows that $\hat{x} < \bar{x}$. Thus, the best response function is:

$$R(x_j) = \begin{cases} \frac{\ln(\lambda_A) - \ln(\lambda_B) + \lambda_B}{\lambda_A + \lambda_B} + \frac{1}{\lambda_A + \lambda_B} \ln\left(\frac{\pi_A^M - (1+\kappa)\left(1 - e^{-\lambda_A x_j}\right)\Delta_A}{\pi_B^M - (1+\kappa)\left(1 - e^{-\lambda_B(1-x_j)}\right)\Delta_B} \right) & \text{if } 0 \leqslant x_j < \min\{\hat{x}, 1\}, \\ 0 & \text{if } x_j \geqslant \hat{x}, \end{cases} \tag{23}$$

Taking the second order derivative of the reaction function yields

$$R''(x_j) = \frac{1+\kappa}{\lambda_A + \lambda_B} \left(\frac{\lambda_B^2 \Delta_B \big(\pi_B^D - \kappa \Delta_B\big) e^{-\lambda_B(1-x_j)}}{\big(\pi_B^M - (1+\kappa)(1 - e^{-\lambda_B(1-x_j)})\Delta_B\big)^2} - \frac{\lambda_A^2 \Delta_A \big(\pi_A^D - \kappa \Delta_A\big) e^{-\lambda_A x_j}}{\big(\pi_A^M - (1+\kappa)(1 - e^{-\lambda_A x_j})\Delta_A\big)^2} \right).$$

Since $\pi_A^D/\Delta_A < \underline{\kappa} < \kappa < \pi_B^D/\Delta_B$, $R''(x_j) > 0$ for all $x_j \in [0, \min\{\hat{x}, 1\})$.

As the reaction curve $R(\cdot)$ is strictly convex, there exists a unique fixed point x_κ^* such that $x_\kappa^* = R(x_\kappa^*)$ on the interval $[0, \hat{x})$. To see this, let $f(x_j) = R(x_j) - x_j$. At the lower bound $x_j = 0$, we have $R(0) > 0$. Thus, $f(0) = R(0) - 0 > 0$.

At the upper bound $x_j = \min\{\hat{x}, 1\}$, we evaluate $f(x_j)$. If $\hat{x} \leqslant 1$, then $R(\hat{x}) = 0$, which trivially yields $f(\hat{x}) = -\hat{x} < 0$. If $\hat{x} > 1$, we evaluate the boundary at $x_j = 1$:

$$R(1) = \frac{\ln(\lambda_A/\lambda_B) + \lambda_B + \ln\left(\frac{\pi_A^M - (1+\kappa)(1 - e^{-\lambda_A})\Delta_A}{\pi_B^M}\right)}{\lambda_A + \lambda_B}$$

The boundary condition (3) implies

$$\lambda_A e^{-\lambda_A} [\pi_A^M - (1 - e^{-\lambda_A})\Delta_A] < \lambda_B \pi_B^M$$

Multiplying by e^{λ_A}, isolating the parameter terms, and taking the natural logarithm yields:

$$\ln(\lambda_A) - \ln(\lambda_B) + \ln\left(\frac{\pi_A^M - (1 - e^{-\lambda_A})\Delta_A}{\pi_B^M}\right) < \lambda_A$$

Because common ownership weakly increases the internalized cannibalization penalty ($\kappa \geqslant 0$), the

numerator inside the logarithm of $R(1)$ is bounded above by the independent firm case: $\pi_A^M - (1+\kappa)(1-e^{-\lambda_A})\Delta_A \leqslant \pi_A^M - (1-e^{-\lambda_A})\Delta_A$. Adding λ_B to both sides and substituting this upper bound directly into the closed-form expression for $R(1)$ gives:

$$R(1) = \frac{\ln(\lambda_A/\lambda_B) + \lambda_B + \ln\left(\frac{\pi_A^M - (1+\kappa)(1-e^{-\lambda_A})\Delta_A}{\pi_B^M}\right)}{\lambda_A + \lambda_B} < \frac{\lambda_A + \lambda_B}{\lambda_A + \lambda_B} = 1$$

Thus, $R(1) < 1$, yielding $f(1) = R(1) - 1 < 0$. Consequently, $f(\min\{\hat{x}, 1\}) < 0$ holds unconditionally. Since $R(\cdot)$ is continuous, $f(x)$ is continuous. By the Intermediate Value Theorem, there exists at least one $x_\kappa^* \in (0, \hat{x})$ such that $f(x_\kappa^*) = 0$.

Next, we examine the curvature of $f(\cdot)$ to prove the uniqueness of this equilibrium. As the reaction curve is strictly convex, the function $f(x)$ is also a strictly convex function; i.e. $f''(x) = R''(x) > 0$ for all $x \in [0, \hat{x})$. A continuous, strictly convex function $f(x)$ intersecting the y-axis at $f(0) > 0$ can cross the x-axis at most once before turning upward, ensuring the root x_κ^* is strictly unique in the relevant interval.

Lastly, we now evaluate this equilibrium's stability. Let $\Phi(\kappa) \equiv \tau(x_\kappa^*) - \kappa$ represent the stability condition where $\tau(\cdot)$ is defined in (19).

Case 1: $\pi_\ell^M > 2\pi_\ell^D \iff \pi_\ell^D/\Delta_\ell < 1$.

If this condition holds for both markets, then $\max_{\ell \in \{A,B\}}\{\pi_\ell^D/\Delta_\ell\} < 1$. By the boundedness property (3), it follows that $\tau(x) < 1$ for all feasible x. Therefore, at the upper bound, $\kappa = 1$, $\Phi(1) = \tau(x_1^*) - 1 < 0$. At the lower bound, $\kappa = 0$, $\tau(x_0^*) > 0$, which implies $\Phi(0) = \tau(x_0^*) - 0 > 0$. Since $\Phi(\kappa)$ is continuous, $\Phi(0) > 0$, and $\Phi(1) < 0$, the Intermediate Value Theorem implies there exists at least one root in $(0, 1)$ such that $\Phi(\kappa) = 0$. We define $\bar{\kappa} = \inf\{\kappa \in (0, 1] \mid \Phi(\kappa) = 0\}$ as the infimum of these roots. This definition mathematically guarantees that $\Phi(\kappa) \geqslant 0$, and thus the interior equilibrium x_κ^* remains stable, for all $\kappa \leqslant \bar{\kappa}$. Lastly, as $\Phi(\kappa) < 0$ in the strictly positive neighborhood of $\bar{\kappa}^+$, there must exist some values of $\kappa > \bar{\kappa}$ for which the interior equilibrium is unstable.

Case 2: $\pi_\ell^M \leqslant 2\pi_\ell^D \iff \pi_\ell^D/\Delta_\ell \geqslant 1$.If $\pi_\ell^M \leqslant 2\pi_\ell^D$, then $\Delta_\ell \leqslant \pi_\ell^D$, which implies $\pi_\ell^D/\Delta_\ell \geqslant 1$. By (3), $\tau(x) \geqslant 1$ for all x, and the formal stability threshold satisfies $\bar{\kappa} \geqslant 1$. Because $\Phi(\kappa) = \tau(x_\kappa^*) - \kappa \geqslant 1 - \kappa \geqslant 0$ for all $\kappa \in [0, 1]$, the stability condition is never violated. Thus, the symmetric equilibrium is globally stable for all feasible levels of common ownership.

■ ■

Proof of Proposition 3

Proof. Define the function $F(x, \kappa)$ as any firm's marginal profit for symmetric strategies:

$$F(x,\kappa) = \lambda_A e^{-\lambda_A x}\left(\pi_A^M - (1+\kappa)p_A(x)\Delta_A\right) - \lambda_B e^{-\lambda_B(1-x)}\left(\pi_B^M - (1+\kappa)p_B(x)\Delta_B\right). \tag{24}$$

The symmetric interior equilibrium x_κ^* is defined by the first-order condition $F(x_\kappa^*, \kappa) = 0$. By the Implicit Function Theorem, the sensitivity of the equilibrium investment to common ownership is:

$$\frac{dx_\kappa^*}{d\kappa} = -\frac{\partial F(x_\kappa^*,\kappa)/\partial\kappa}{\partial F(x_\kappa^*,\kappa)/\partial x_\kappa^*}.$$

As $\kappa < \bar{\kappa}$, the second-order condition is satisfied by Lemma 1, implying $\partial F/\partial x_\kappa^* < 0$.
Thus, the sign of the total derivative is determined solely by the numerator:

$$\frac{\partial F(x_\kappa^*,\kappa)}{\partial\kappa} = \lambda_B e^{-\lambda_B(1-x_\kappa^*)}p_B(x_\kappa^*)\Delta_B - \lambda_A e^{-\lambda_A x_\kappa^*}p_A(x_\kappa^*)\Delta_A.$$

Substituting the equilibrium relationship from (24) into the inequality $\partial F/\partial\kappa > 0$ yields:

$$\frac{p_A(x_\kappa^*)\Delta_A}{\pi_A^M - (1+\kappa)p_A(x_\kappa^*)\Delta_A} < \frac{p_B(x_\kappa^*)\Delta_B}{\pi_B^M - (1+\kappa)p_B(x_\kappa^*)\Delta_B} \iff p_B(x_\kappa^*)\frac{\Delta_B}{\pi_B^M} > p_A(x_\kappa^*)\frac{\Delta_A}{\pi_A^M}. \tag{25}$$

We define the cannibalization risk differential as $\Delta\theta(x) \equiv p_B(x)(\Delta_B/\pi_B^M) - p_A(x)(\Delta_A/\pi_A^M)$. Note that $\Delta\theta(x)$ is strictly monotonically decreasing because $p_B(x)$ is strictly decreasing while $p_A(x)$ is strictly increasing in x. Consequently, investment x_κ^* increases monotonically in κ if and only if x_κ^* remains below the critical value $\bar{x}$ where cannibalization risks equalize: $\Delta\theta(\bar{x}) = 0$. Since $F(x, \kappa)$ is decreasing in x, the condition $x_\kappa^* < \bar{x}$ is equivalent to $F(\bar{x}, \kappa) < 0$. Substituting the condition $\Delta\theta(\bar{x}) = 0$ into $F(\bar{x}, \kappa) < 0$ yields condition (3) in the Proposition. ■

Proof of Proposition 4

Proof. First, we establish that $\bar{\kappa}$ is a unique root of the expression $\bar{\kappa} = \tau(x_{\bar{\kappa}}^*)$ under the stated assumptions where $\tau(\cdot)$ is defined by (19). For that purpose, consider the derivate

$$\frac{d\tau(x_\kappa^*)}{d\kappa} = \tau'(x_\kappa^*) \times \frac{dx_\kappa^*}{d\kappa} = \underbrace{\frac{\lambda_A^2\lambda_B^2 e^{-(\lambda_B+(\lambda_A-\lambda_B)x)}(\lambda_A+\lambda_B)}{\left(\lambda_A^2 e^{-\lambda_A x}\Delta_A + \lambda_B^2 e^{-\lambda_B(1-x)}\Delta_B\right)^2}}_{\text{Positive}} \times \underbrace{\left(\Delta_A\pi_B^D - \Delta_B\pi_A^D\right)}_{\text{Negative}} \times \underbrace{\frac{dx_\kappa^*}{d\kappa}}_{\text{Positive}}.$$

If $\Delta_A/\pi_A^M \leqslant \Delta_B/\pi_B^M$ and condition (6) holds (that implies $dx_\kappa^*/d\kappa > 0$), we get $\tau(x_\kappa^*)$ strictly decreases with κ. Given $\pi_\ell^M > 2\pi_\ell^D$, at the upper bound, $\kappa = 0$, $t(x_1^*) < 1$. At the lower bound, $\kappa = 0$, $t(x_0^*) > 0$. Hence, by the Intermediate Value Theorem, there exists a unique fixed point $\bar{\kappa} = t(x_{\bar{\kappa}}^*)$ for which $\kappa <$ (resp. $>$) $\tau(x_\kappa^*)$ for all $\kappa <$ (resp. $>$) $\bar{\kappa}$. Hence, the interior solution x_κ^* is an equilibrium, but is unstable for all $\kappa > \bar{\kappa}$.

Given the compact strategy space $[0,1]^2$ and continuous payoffs, the instability of the unique symmetric interior fixed point implies the existence of exactly two stable asymmetric equilibria (see Proposition 2). To determine the location of these two stable equilibria, we analyze the boundary conditions. First, we derive the condition for the stability of the equilibrium $(1,0)$. For Firm 1 to maintain full investment in market A, we require $FOC(1,0) > 0$. Substituting the corner values into the first-order condition yields:

$$\lambda_A e^{-\lambda_A}\pi_A^M - \lambda_B\left[\pi_B^M - (1+\kappa)(1-e^{-\lambda_B})\Delta_B\right] > 0.$$

Rearranging to isolate κ, we find that the incumbency at $(1,0)$ is stable if and only if κ exceeds the threshold:

$$\kappa > \frac{\lambda_B\pi_B^M - \lambda_A e^{-\lambda_A}\pi_A^M}{\lambda_B(1-e^{-\lambda_B})\Delta_B} - 1.$$

Conversely, we derive the condition for the stability of the mirrored equilibrium, or specifically, the entry deterrence condition at $x = 0$. For a firm to remain at $x = 0$ (e.g., Firm 1 in the profile $(0,1)$), the marginal profit of entering must be negative: $FOC(0,1) < 0$. Expanding this condition:

$$\lambda_A\left[\pi_A^M - (1+\kappa)(1-e^{-\lambda_A})\Delta_A\right] - \lambda_B e^{-\lambda_B}\pi_B^M < 0.$$

Solving for κ, we obtain the threshold:

$$\kappa > \frac{\lambda_A\pi_A^M - \lambda_B e^{-\lambda_B}\pi_B^M}{\lambda_A(1-e^{-\lambda_A})\Delta_A} - 1.$$

Let

$$\hat{\kappa} = \min\left\{\frac{\lambda_B\pi_B^M - \lambda_A e^{-\lambda_A}\pi_A^M}{\lambda_B(1-e^{-\lambda_B})\Delta_B} - 1, \frac{\lambda_A\pi_A^M - \lambda_B e^{-\lambda_B}\pi_B^M}{\lambda_A(1-e^{-\lambda_A})\Delta_A} - 1\right\}$$

Thus, if $\bar{\kappa} < \kappa < \hat{\kappa}$ the equilibrium must be $(0, \bar{x}_\kappa^*)$ and $(\bar{x}_\kappa^*, 0)$. If $\kappa \geqslant \hat{\kappa}$, then the equilibrium must

be $(0,1)$ and $(1,0)$. ■

Proof of Proposition 5

Proof. The planner's first order condition (FOC) is computed by taking the first order derivative of (8) with respect to one of its arguments, say x_i, and setting it to zero: $\partial S(x_i, x_j)/\partial x_i = 0$. For the existence of a unique equilibrium, the dominant diagonal condition must be satisfied:

$$\frac{\partial^2 S(x_i, x_j)}{\partial x_i^2} > \frac{\partial^2 S(x_i, x_j)}{\partial x_i \partial x_j}. \tag{26}$$

Using the property of exponential functions $p''_\ell(x_{\ell i}) = -\lambda_\ell p'_\ell(x_{\ell i})$ where $\ell \in \{A, B\}$, we get

$$\begin{aligned}\frac{\partial^2 S(x_i, x_j)}{\partial x_i^2} =& -\lambda_A p'_A(x_i)[S_A^M - (2S_A^M - S_A^D)p_A(x_i)] \\ & -\lambda_B p'_B(1-x_i)[S_B^M - (2S_B^M - S_B^D)p_B(1-x_i)] \qquad (27)\\ \frac{\partial^2 S(x_i, x_j)}{\partial x_i \partial x_j} =& -\left[(2S_A^M - S_A^D)(p'_A)^2 + (2S_B^M - S_B^D)(p'_B)^2\right] \qquad (28)\end{aligned}$$

If a symmetric equilibrium x_W^* exists, it satisfies the planner's FOC where the Marginal Social Value M is equal across markets.

$$p'_A(x_W^*)\left[S_A^M - (2S_A^M - S_A^D)p_A(x_W^*)\right] = p'_B(1-x_W^*)\left[S_B^M - (2S_B^M - S_B^D)p_B(1-x_W^*)\right] = M. \tag{29}$$

We evaluate (27) and (28) at the social equilibrium x_W^* and subsitute them into the inequality (26) along with (29) to get

$$M(\lambda_A + \lambda_B) > (2S_A^M - S_A^D)(p'_A)^2 + (2S_B^M - S_B^D)(p'_B)^2$$

This can be rewritten to obtain (9).

■

Proof of Proposition 6

Proof. Let $\Psi(x)$ denote the social planner's marginal gradient evaluated at a symmetric allocation

$x_i = x_j = x$, derived from the social welfare First-Order Condition:

$$\Psi(x) \equiv \lambda_A e^{-\lambda_A x}\left[S_A^M + p_A(x)(S_A^D - 2S_A^M)\right] - \lambda_B e^{-\lambda_B(1-x)}\left[S_B^M + p_B(1-x)(S_B^D - 2S_B^M)\right]$$

The socially optimal allocation x_W^* is an interior solution satisfying $\Psi(x_W^*) = 0$. From Proposition 5, we know $\Psi(\cdot)$ is a strictly decreasing function because the second-order condition is always negative. Therefore, the social planner desires more investment in Market A than the private market provides ($x_W^* > x_\kappa^*$) if and only if $\Psi(x_\kappa^*) > 0$.

Recall that the market equilibrium x_κ^* is defined by the point where the firms' marginal returns in Market A and Market B are equal:

$$\lambda_A e^{-\lambda_A x_0^*}\left[\pi_A^M - p_A(x_0^*)\Delta_A\right] = \lambda_B e^{-\lambda_B(1-x_0^*)}\left[\pi_B^M - p_B(1-x_0^*)\Delta_B\right]$$

As the private marginal returns are equal and positive at x_κ^*, we can divide the social gradient $\Psi(x_\kappa^*) > 0$ by these private returns without changing the inequality. This cleanly eliminates the exponential terms and yields a new ratio function, $\xi(x)$, representing the difference between the social-to-private value ratios of the two markets:

$$\xi(x_\kappa^*) \equiv \frac{S_A^M - p_A(x_\kappa^*)(2S_A^M - S_A^D)}{\pi_A^M - p_A(x_\kappa^*)\Delta_A} - \frac{S_B^M - p_B(1-x_\kappa^*)(2S_B^M - S_B^D)}{\pi_B^M - p_B(1-x_\kappa^*)\Delta_B} > 0 \tag{30}$$

To determine the behavior of $\xi(\cdot)$, we define the standalone ratio function for a generic market ℓ as

$$R_\ell(p_\ell) = \frac{S_\ell^M - p_\ell(2S_\ell^M - S_\ell^D)}{\pi_\ell^M - p\Delta_\ell} \quad \text{where} \quad p_A = p_A(x_\kappa^*) \quad \text{and} \quad p_B = p_B(1-x_\kappa^*).$$

Differentiating this with respect to the probability of success p yields:

$$R'_\ell(p_\ell) = \frac{\Delta_\ell S_\ell^M - (2S_\ell^M - S_\ell^D)\pi_\ell^M}{(\pi_\ell^M - p\Delta_\ell)^2}$$

If $\Delta_\ell/\pi_\ell^M > (2S_\ell^M - S_\ell^D)/S_\ell^M$, we have $R'_\ell(p_\ell) > 0$. For Market A, $p_A = 1 - e^{-\lambda_A x}$ is strictly increasing in x, so $R_A(p_A)$ is strictly increasing in x. For Market B, $p_B = 1 - e^{-\lambda_B(1-x)}$ is strictly decreasing in x, so $R_B(p_B)$ is strictly decreasing in x. Now,

$$\xi(x_\kappa^*) > 0 \iff R_A(p_A) > R_B(p_B)$$

As $R_A(p_A)$ is continuous and increasing, and $R_B(p_B)$ is continuous and decreasing, the Intermediate Value Theorem guarantees a unique intersection point, defined as $\hat{x}$, where $\xi(\hat{x}) = 0$. Consequently, the inequality $\xi(x_\kappa^*) > 0$ holds if and only if $x_\kappa^* > \hat{x}$. Therefore, the industry under-invests in Market A relative to the social optimum if and only if (10) is satisfied. ■

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