A Nonparametric Approach to the Measurement of Markups and Productivity

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Abstract: The paper introduces a new nonparametric approach to the estimation of markups and the productivity of a production unit. The nonparametric approach to production theory originated with Farrell (1957) and it constructs estimates for production functions (or for production possibilities sets) and associated measures of efficiency using only observed quantity data (and price data if available) without any econometric estimation. This approach uses past vectors of inputs and outputs for the production unit under consideration to build up an approximation to the "true" underlying technology set at any point in time. The nonparametric approach proposed in the present paper is similar to the approach used by Diewert and Fox (2018) except our new approach allows for technology sets that can exhibit nonconstant returns to scale. An application to the Retail Trade Sector in Australia is presented.

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1. Introduction

The paper introduces a new nonparametric approach to the estimation of markups and the productivity of a production unit. The nonparametric approach to production theory originated with Farrell (1957) and it constructs estimates for production functions (or for production possibilities sets) and associated measures of efficiency using only observed quantity data (and price data if available) without any econometric estimation.[3] This approach uses past vectors of inputs and outputs for the production unit under consideration to build up an approximation to the "true" underlying technology set at any point in time. The nonparametric approach proposed in the present paper is similar to the approach used by Diewert and Fox (2018) except our new approach allows for technology sets that can exhibit nonconstant returns to scale. We also allow for the possibility that the production unit makes monopolistic profits; i.e. it can sell its outputs at prices that are greater than the cost of production.

We illustrate our new approach with an empirical example: the retail sector in Australia for the years 1994-2022. The subject of retail markups has assumed new importance in recent years due to the post covid spurt in inflation across many countries, including Australia. The question of the size of markups has also been of academic interest due to the recent paper by de Loecker, Eeckhout and Unger (2020), who found huge markups on prices over marginal costs in the US that were in the 20-60 percent range. Our costs are average costs (rather than marginal costs) which include returns to fixed factors, so our estimated markups are much smaller. Our markup growth estimates can be decomposed into contributions from output and input price changes, efficiency changes and the combined effects of technical progress and possible nonconstant returns to scale.

There are a number of limitations to our methodology:

- We do not obtain product by product markups; we only obtain an aggregate markup over all outputs in scope. In the case of a single output, our markup is a markup over average cost rather than marginal cost.
- Our methodology applies to a production unit (establishment, firm, sector) where we can observe output and input prices and quantities over time.[4]
- Our methodology combines the effects of nonconstant returns to scale with the effects of technical progress.
- Our measure of inefficiency is a measure of scale inefficiency and so some of our measured inefficiency could be due to decreasing returns to scale.
- Our Maximum Markup Function is a hypothetical construct; market conditions may prevent the production unit from attaining the maximum markup that is possible when we ignore the demand side of the market.
- The observed aggregate markup achieved by the production unit depends on exactly how the user costs for fixed assets was constructed: the choice of the unit's cost of capital and the treatment of expected capital gains (or losses) on assets is important.

[3] Early papers on the nonparametric approach to production theory include Farrell and Fieldhouse (1962), Afriat (1972), Hanoch and Rothschild (1972), Charnes, Cooper and Rhodes (1978), Färe and Lovell (1978), Diewert and Parkan (1983), Varian (1984), Charnes and Cooper (1985) and Färe, Grosskopf and Lovell (1985).
[4] It could be modified to apply in a cross sectional context but the production units need to produce more or less the same outputs and use the same inputs.

Section 2 explains our methodology for the case of a production unit that produces an arbitrary number of outputs and uses an arbitrary number of inputs over time. Section 3 illustrates the methodology for the case of one output and one input. Section 4 specializes the results in Section 2 to the case of one output and many inputs. Section 5 provides an empirical example of the Section 4 methodology to the Australian Retail Trade industry over the years 1994-2022. Section 6 concludes. Appendix A develops the necessary data for our empirical example. Appendix B goes back to the Diewert and Fox (2018a) nonparametric methodology and works out the implications of this approach (which assumed constant returns to scale in production) when there is only one output. We compare the new and old methodologies using our Australian data and find little difference in the results. Finally, Appendix C describes some alternative approaches to the measurement of markups and the problems associated with these approaches.

2. The Maximum Markup Function and its Decomposition into Explanatory Factors

The analysis in this section is analogous to the nonparametric decomposition of Total Factor Productivity (TFP) Growth obtained by Diewert and Fox (2018a). The main difference is that Diewert and Fox assumed that production was subject to constant returns to scale.

We relax the constant returns to scale assumption and at the same time, we allow the firm or production unit to set output prices such that aggregate revenues exceed aggregate costs; i.e., we allow the production unit to make pure profits. This means that we cannot apply the Jorgenson and Griliches (1967) approach to the measurement of Total Factor Productivity (TFP) which assumed a constant returns to scale technology set and competitive pricing of outputs and inputs so that the value of inputs used equaled the value of outputs produced during the accounting period. When the value of outputs does not equal the value of inputs, Schreyer (2010; 14) asked the following questions:

- How should TFP be conceptually defined, computed and interpreted?
- How should growth accounting exercises be carried out?
- How should measures of technical change be defined and evaluated?

These same questions are addressed in this paper.

For the most part, we follow Balk's (1998) (2001) (2003) (2010) methodological approach to the modelling of Total Factor Productivity (TFP) growth for a production unit when there are markups and his decomposition of TFP growth into explanatory factors.[5] In particular, we will make use of Balk's concept of *profitability* which we will interpret as the production unit's aggregate *markup factor*.

[5] Balk (2010) defined TFP Growth as an index number of the outputs produced by the production unit over the two periods being considered divided by an index number of the corresponding inputs used by the production unit and he also considered the dual version of this definition (a price index of inputs divided by a price index of outputs). These approaches to productivity measurement date back to Copeland (1937) (the primal version) and Copeland and Martin (1938) (for the dual version). The two approaches were implemented by Jorgenson and Griliches (1967) and Caves and Christensen (1969) for the case where the value of inputs equals the value of outputs in both periods. But what index number formula should be used to implement these approaches? Diewert (1992a) (1992b) suggested that the Fisher (1922) index be used because it satisfies more "reasonable" tests than competing indexes. Schreyer (2010; 28) ended up using the Törnqvist index in place of the Fisher index. Note that Diewert (1992a), Diewert and Nakamura (2003), Balk and Schreyer did not assume that the value of inputs equal the value of outputs in each period and this is what distinguished their approaches from the earlier approaches mentioned above.

What are the two most important factors that can explain TFP growth? Balk explains that these factors are technical change and changes in efficiency:

"In the old days one interpreted productivity change as completely due to and thus identical with technological change. Not so long ago one came to the insight that efficiency change is at least as important a factor. … The distinction between technological change and efficiency change can be made by conceiving the firm as operating in an exogenously determined environment, called the technology, which is the set of all at a given period feasible combinations of input and output quantities. A firm which operates on the boundary of this set is called technically efficient, whereas a firm which operates in the interior of this set is called technically inefficient. Technological change then means that the set of feasible combinations expands or contracts, while technical efficiency change means that the firm moves closer to or further away from the boundary. These two kinds of movement are clearly independent of each other: there can be technological change without efficiency change, and efficiency change without technological change. More usual, however, is a combination of both factors. But how to disentangle these factors in practice?" Bert Balk (2001; 160).

We will disentangle these two factors by making use of a production unit's *Empirical Maximum Markup Function*, which is defined below.

Suppose that in period t, a production unit produces combinations of J outputs, $y^t \equiv [y_1^t,...,y_J^t]$, using N inputs $x^t \equiv [x_1^t,...,x_N^t] > 0_N$, while facing the strictly positive vector of net output prices $p^t \equiv [p_1^t,...,p_J^t] >> 0_M$ and the strictly positive vector of primary input prices $w^t \equiv [w_1^t,...,w_N^t] >> 0_N$. The value of primary inputs used by the sector during period t is then $C^t \equiv w^t \cdot x^t \equiv \Sigma_{n=1}^N w_n^t x_n^t > 0$. Denote the *period t production possibilities set* for the production unit by S^t.[6] We assume that period t *value of gross output* or revenue $R^t \equiv p^t \cdot y^t \equiv \Sigma_{j=1}^J p_j^t y_j^t$ is positive. The production unit's *observed period t aggregate markup* m^t *and markup factor* M^t are defined as follows[7] for periods 1,2,…,T:

$$(1)\quad M^t \equiv (1+m^t) \equiv p^t \cdot y^t / w^t \cdot x^t ; \qquad t = 1,\ldots,T.$$

The period t observed *markup factor* M^t is equal to unity if period t gross output $p^t \cdot y^t$ equals period t primary input cost $w^t \cdot x^t$. The actual period t *markup* is m^t and it is interpreted as the aggregate percentage by which revenues exceed costs in period t. If $M^t = (1+m^t) > 1$, then the production unit is making pure profits at the rate m^t and if $M^t < 1$, then the unit is making a loss in period t; i.e., the unit is not making a high enough return on its fixed assets to cover the period t cost of capital r^t that is imbedded in the unit's user costs for fixed assets.

It should be emphasized that the markup m^t defined by (1) is an *aggregate markup* for the production unit over all outputs produced by the production unit in period t.[8] Thus it is different from an *individual product markup* that is the difference between the selling price of a product and the corresponding long run marginal cost.[9] When we define m^t by the equation $M^t \equiv (1 + m^t)$, then m^t is an aggregate counterpart to a weighted

[6] Our regularity conditions on the sets S^t are weak: we need only assume that S^t is such that the maximums in definition (2) below exist for observed period t output and input price vectors p^t and w^t in our sample of periods. Notation: $p \cdot y \equiv \Sigma_{j=1}^J p_j y_j$ is the inner product of the vectors p and y.

[7] Our definition of the *markup factor* is the same as Balk's (2003; 9-10) *profitability* definition.

[8] If a multiproduct production unit is known to provide some outputs at long run marginal costs, then these "competitive" outputs can be removed from the output aggregate and entered into the input aggregate (with negative signs for these competitive outputs).

[9] In order to estimate *individual* product markups, an econometric framework seems to be required. This entails making assumptions about the functional form for the production unit's joint cost function and on the output demand functions

sum of the individual product markups over their (imputed) average costs. It is difficult to measure long run marginal or average costs for *each* output but it is not so difficult to estimate the *aggregate* markup m^t.

Let S^t denote the production unit's production possibilities set for t = 1,…,T and let $p >> 0_M$ and $w >> 0_N$ denote generic positive output and input price vectors. Then the production unit's period t (*Maximum*) *Markup (Factor) Function*, $\mu^{t*}(p,w)$, is defined as follows:[10]

$$(2)\ \mu^{t*}(p,w) \equiv \max_{y,x} \{p\cdot y/w\cdot x : (y,x) \in S^t\}\ ; \qquad t = 1,\ldots,T.$$

For brevity, we will refer to the period t Maximum Markup Factor Function as the Markup function. Thus the Markup function is a function of the time period t (which indicates which production possibilities set is available) and the two price vectors p and w. Since the observed period t output and input vectors (y^t,x^t) belong to the period t production possibilities set, it will be the case that the observed period t markup factor M^t will satisfy the following inequality:

$$(3)\ M^t \equiv p^t\cdot y^t/w^t\cdot x^t \leq \mu^{t*}(p^t,w^t)\ ; \qquad t = 1,\ldots,T.$$

If it turns out that $M^t \equiv p^t\cdot y^t/w^t\cdot x^t = \mu^{t*}(p^t,w^t)$, then we say that the observed period t output-input vector (y^t,x^t) is *markup efficient* or *scale efficient*. In this case, the value of output $p^t\cdot y$ divided by the value of input used $w^t\cdot x$ to produce y is at a maximum over all feasible vectors (y,x) belonging to the period t production possibilities set S^t when we use the period t prices to value outputs and inputs. This concept of efficiency is not quite the same as being at a point of *maximum profits* in period t and it is not the same as being *technically efficient* (i.e., on the boundary of the period t production possibilities set). It is a generalization of Banker's (1984; 35) *scale efficiency* concept.

Our next assumption is that there is *no technological regress* in the technology of the production unit over time; i.e., we assume that:

$$(4)\ S^1 \subset S^2\ ;\ S^2 \subset S^3\ ;\ \ldots\ ;\ S^{T-1} \subset S^T\ .$$

Thus the production possibilities sets do not become smaller over time.[11]

We want to utilize the Markup function in the time series context without having to undertake econometric estimation of the production functions that could represent the various production possibilities sets S^t. Thus we will *approximate* the period t production possibilities set S^t by the union of the observed output and

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that the unit faces; see Appendix C. In addition, complicated stochastic assumptions have to be made. For examples of such studies, see Diewert and Fox (2008) and Schreyer and Zinni (2021). Our present approach is much simpler but it is an *aggregate* markup approach.

[10] Our assumptions on the S^t are not strong enough to ensure that $\mu^{t*}(p,w)$ is finite. However, our empirical markup function will always be well defined and finite. We note that in the case of one input and one output, our Maximum Markup function coincides with Banker's (1984; 35) *most productive scale size* concept. His Figure 1 on page 38 is related to our Figures in section 3 below.

[11] This is not an innocuous assumption over longer periods of time. Over time, the underlying technology can change dramatically: the introduction of new products and the disappearance of obsolete capital stocks means that it is not always possible to produce past vectors of outputs in the current period.

input vectors for periods 1,2,…,t: $(y^1,x^1), (y^2,x^2), \ldots, (y^t,x^t)$.[12] Using this very rough approximation to reality, the period t *Empirical Maximum Markup Factor Function* $\mu^t(p,w)$ becomes the following function:[13]

$$(5)\ \mu^t(p,w) \equiv \max_j \{p\cdot y^j/w\cdot x^j : j = 1,2,\ldots,t\}; \qquad t = 1,2,\ldots,T.$$

In order to save space, we will refer to $\mu^t(p,w)$ as the period t markup function. Definitions (5) imply that $\mu^1(p,w) = p\cdot y^1/w\cdot x^1$; $\mu^2(p,w) = \max\{p\cdot y^1/w\cdot x^1; p\cdot y^2/w\cdot x^2\}$; $\mu^3(p,w) = \max\{p\cdot y^1/w\cdot x^1; p\cdot y^2/w\cdot x^2; p\cdot y^3/w\cdot x^3\}$ and so on. Our observable markup functions $\mu^t(p,w)$ defined by (5) are well defined for each period t and for each possible positive output and input price vector (p,w).[14] Note that the function $\mu^t(p,w)$ defines a hypothetical markup that is based solely on the underlying technology; i.e., it is not necessarily the case that the production unit would be able to *sell* its outputs at the hypothetical prices p but it would be able to *produce* the vector of outputs y^j using the vector of inputs x^j that attains the maximum in definition (5).[15]

We will look at the rate of growth of the empirical markup functions, $\mu^t(p^t,w^t)$, over time. In particular, we want to decompose maximum markup factor growth, $\mu^t(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1})$, into the product of a term that represents changes in prices going from period t−1 to period t and a term that represents the combined effects of technical progress and changes in returns to scale. There are two decompositions of maximum markup factor growth that are equally valid and make use of the prices that the production unit faces during the two periods. The first decomposition is the following one:

$$(6)\ \mu^t(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1}) = [\mu^t(p^t,w^t)/\mu^{t-1}(p^t,w^t)][\mu^{t-1}(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1})] \qquad t = 2,\ldots,T$$
$$= \tau_P{}^t\alpha_L{}^t$$

where the *period t Paasche type index of the combined effects of technical progress and returns to scale on markup factor growth holding prices constant*, $\tau_P{}^t$, is defined as follows:[16]

$$(7)\ \tau_P{}^t \equiv \mu^t(p^t,w^t)/\mu^{t-1}(p^t,w^t) \geq 1\ ; \qquad t = 2,\ldots,T.$$

The $\tau_P{}^t$ measure of markup factor change holds prices constant at their period t levels. The numerator and denominator on the right hand side of (7) both calculate the maximum markup factor that is attainable by the production unit if it faces period t output and input prices but the numerator markup uses the period t technology while the denominator markup uses the technology of the previous period. We term (7) a Paasche type theoretical index of technical progress (and returns to scale) because we compare the maximum markup factor in period t to the maximum markup factor in period t−1 holding prices constant at the period t levels.

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[12] We assume free disposability of inputs and outputs in the charts in section 3. Thus in the one input, one output situation, we are assuming that the underlying (approximate) production function for each period is a step function.

[13] Note that $\mu^t(p,w)$ is a function of the price vectors p and w and t whereas $M^t \equiv (1+m^t) \equiv p^t\cdot y^t/w^t\cdot x^t$ is scalar number.

[14] Note that the functions $\mu^t(p,w)$ satisfy the following homogeneity properties: for any scalar $\lambda > 0$, we have $\mu^t(\lambda p,w) = \lambda\mu^t(p,w)$ and $\mu^t(p, \lambda w) = \lambda^{-1}\mu^t(p,w)$.

[15] Diewert and Fox (2018a; 637) defined their Empirical Cost Constrained Value Added Function using our present notation as follows: $R^t(p,w,x) \equiv w\cdot x \max_j \{p\cdot y^j/w\cdot x^j : j = 1,2,\ldots,t\} = w\cdot x\ \mu^t(p,w)$. Their Theoretical Cost Constrained Value Added Function was defined as $R^{t*}(p,w,x) \equiv \max_{y,z} \{p\cdot y : w\cdot z \leq w\cdot x\ ; (y,z)\in S^t\}$; see Appendix B for more details on the two approaches for the case of one output and many inputs. Thus our present nonparametric approach to production theory is related to our earlier approach, which assumed constant returns to scale in production.

[16] A feasibility argument shows that $\mu^t(p,w)/\mu^{t-1}(p,w) \geq 1$ for all p and w which establishes the inequalities in (7) and (10).

The *period t Laspeyres type measure of the effect of output and input price change on markup factor growth* going from period t−1 to period t is defined as:

$$(8)\ \alpha_L^t \equiv \mu^{t-1}(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1})\ ; \qquad t = 2,\ldots,T.$$

We term (8) a Laspeyres type theoretical index of price change because we compare the maximum markup factor in period t to the maximum markup factor in period t−1 holding the technology constant at the period t−1 levels.

The second decomposition is the following one:

$$(9)\ \mu^t(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1}) = [\mu^t(p^t,w^t)/\mu^t(p^{t-1},w^{t-1})][\mu^t(p^{t-1},w^{t-1})/\mu^{t-1}(p^{t-1},w^{t-1})] \qquad t = 2,\ldots,T$$
$$= \alpha_P^t\tau_L^t$$

where the *period t Laspeyres type index of the effects of technical progress and returns to scale on markup factor growth holding prices constant* τ_L^t is defined as:

$$(10)\ \tau_L^t \equiv \mu^t(p^{t-1},w^{t-1})/\mu^{t-1}(p^{t-1},w^{t-1}) \geq 1\ ; \qquad t = 2,\ldots,T.$$

The *period t Paasche type measure of the effect of price change on markup facto grow*th going from period t−1 to period t is defined as:

$$(11)\ \alpha_P^t \equiv \mu^t(p^t,w^t)/\mu^t(p^{t-1},w^{t-1})\ ; \qquad t = 2,\ldots,T.$$

We term (10) a Laspeyres type theoretical index of technical progress and returns to scale because we compare the maximum markup factor in period t to the maximum markup factor in period t−1 holding prices constant at the period t− 1 levels. Similarly, α_P^t defined by (11) is a Paasche type theoretical index of price change because we compare the maximum markup factor in period t to the maximum markup factor in period t−1 holding the technology constant at the period t levels.

If we ignore possible inefficiency, the indexes τ_L^t and τ_P^t could be regarded as indexes of *Total Factor Productivity growth*; they are generalizations of the one output, one input TFP growth indexes defined in the following section (assuming that production is efficient). However, in our present context, where we allow for inefficiency, we interpret the τ_L^t and τ_P^t as generalized measures of the combined effects of technical progress and of possible nonconstant returns to scale. Since τ_L^t and τ_P^t are equally relevant measures of generalized technical progress between periods t−1 and t, we use their geometric mean τ^t as our final estimate of *generalized technical progress* going from period t−1 to period t:

$$(12)\ \tau^t \equiv [\tau_L^t\tau_P^t]^{1/2}\ ; \qquad t = 2,\ldots,T.$$

Similarly, since α_L^t and α_P^t are equally relevant measures of the effects of output and input price change going from period t−1 to t, we take their geometric mean, α_L^t, as our final measure of *price change* going from period t−1 to t:

$$(13)\ \alpha^t \equiv [\alpha_L^t\alpha_P^t]^{1/2}\ ; \qquad t = 2,\ldots,T.$$

Using the above definitions, it can be seen that we have the following decomposition for the rate of growth of the Markup function going from period t−1 to period t:

(14) $\mu^t(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1}) = \alpha^t\tau^t$; $t = 2,\ldots,T.$

Following the example of Balk (1998; 143), we define the *scale efficiency* of the production unit during period t, e^t, as follows:

(15) $e^t \equiv [p^t\cdot y^t/w^t\cdot x^t]/M^t(p^t,w^t) \leq 1;$ $t = 1,2,\ldots,T$

where the inequality in (15) follows from definitions (5). Thus if $e^t = 1$, then production is scale efficient in period t and if $e^t < 1$, then production for the sector during period t is scale inefficient. Note that the above definition of scale efficiency is a markup function counterpart to Farrell's (1957; 255) cost based measure of *overall efficiency* which combined his measures of technical and (cost) allocative efficiency. As noted earlier, Farrell was an early pioneer on the nonparametric approach to production theory and the measurement of the efficiency of production by economists.[17]

In definitions (15), we defined the period t and period t−1 margin efficiencies as e^t and e^{t-1}, respectively. Now define an index of the *change in scale efficiency* ε^t for the production unit over the two periods as follows:

(16) $\varepsilon^t \equiv e^t/e^{t-1} = [(p^t\cdot y^t/w^t\cdot y^t)/\mu^t(p^t,w^t)]/[(p^{t-1}\cdot y^{t-1}/w^{t-1}\cdot x^{t-1})/\mu^{t-1}(p^{t-1},w^{t-1})]$; $t = 2,\ldots,T.$

Thus if $\varepsilon^t > 1$, then scale efficiency (or markup efficiency) has *improved* going from period t−1 to t whereas it has *fallen* if $\varepsilon^t < 1$.

Using the above definitions, it can be shown that the following decomposition of observed markup factors going from period t−1 to t is valid:

(17) $[R^t/C^t]/[R^{t-1}/C^{t-1}] = M^t/M^{t-1} = (1+m^t)/(1+m^{t-1}) = \varepsilon^t\alpha^t\tau^t$; $t = 2,3,\ldots,T.$

Since $R^t = p^t\cdot y^t$ and $C^t = w^t\cdot x^t$, equations (17) can be reorganized to give us the following *growth accounting decomposition* of nominal output growth, $p^t\cdot y^t/p^{t-1}\cdot y^{t-1}$, into explanatory factors:

(18) $p^t\cdot y^t/p^{t-1}\cdot y^{t-1} = [w^t\cdot x^t/w^{t-1}\cdot x^{t-1}]\varepsilon^t\alpha^t\tau^t$; $t = 2,3,\ldots,T.$

Define *Total Factor Productivity Growth* going from period t−1 to period t, π^t, as the product of efficiency growth ε^t times generalized technical progress τ^t:

(19) $\pi^t \equiv \varepsilon^t\tau^t$; $t = 2,3,\ldots,T.$

[17] Some studies of production inefficiency in a general equilibrium setting that preceded Farrell's 1957 study were by Allais (1943), Boiteux (1951) and Debreu (1951) (1954). See also Allais (1977) for a description in English of his earlier work in French.

In the following section, we will illustrate the decompositions given by (17) and (19) for the case of only one output, one input and two periods.

3. The One Output, One Input and Two Period Case

When there are only two periods and one output and one input, (17) becomes the following decomposition of markup factor growth:

(20) $M^2/M^1 = (1+m^2)/(1+m^1) = \varepsilon^2\alpha^2\tau^2$.

We will use the definitions in the previous section to evaluate the right hand side of (20) when p^t, y^t, w^t and x^t are scalars. Thus the observed markup factors, M^t, are defined as $M^t \equiv 1+m^t \equiv p^ty^t/w^tx^t = (p^t/w^t)(y^t/x^t)$ for $t = 1,2$. Using definitions (5) for our special case of two periods and one output and one input, we have the following equalities for the two empirical maximum markup factor functions, $\mu^1(p,w)$ and $\mu^2(p,w)$, for any scalars $p > 0$ and $w > 0$:

(21) $\mu^1(p,w) \equiv \max \{py^1/wx^1\} = (p/w)(y^1/x^1)$;
(22) $\mu^2(p,w) \equiv \max \{py^1/wx^1, py^2/wx^2\} = (p/w) \max\{y^1/x^1, y^2/x^2\}$.

We need to consider 9 cases assuming that x^1, x^2, y^1, y^2 are all positive numbers:

Case 1: $y^1/x^1 < y^2/x^2$; $x^1 < x^2$; **Case 2** : $y^1/x^1 < y^2/x^2$; $x^1 = x^2$; **Case 3** : $y^1/x^1 < y^2/x^2$; $x^1 > x^2$;
Case 4: $y^1/x^1 = y^2/x^2$; $x^1 < x^2$; **Case 5** : $y^1/x^1 = y^2/x^2$; $x^1 = x^2$; **Case 6** : $y^1/x^1 = y^2/x^2$; $x^1 > x^2$;
Case 7: $y^1/x^1 > y^2/x^2$; $x^1 < x^2$; **Case 8** : $y^1/x^1 > y^2/x^2$; $x^1 = x^2$; **Case 9** : $y^1/x^1 > y^2/x^2$; $x^1 > x^2$.

When there is only one output and one input, the markup factor growth rate, M^2/M^1, can be written as follows:

(23) $M^2/M^1 = [(p^2/p^1)/(w^2/w^1)][(y^2/x^2)/(y^1/x^1)]$.

Note that $(y^2/x^2)/(y^1/x^1) = (y^2/y^1)/(x^2/x^1)$ is the usual definition of Total Factor Productivity Growth when there is only one output and one input. Thus an increase in the output price ratio p^2/p^1 or an increase in TFP growth will increase markup growth and an increase in the input price ratio w^2/w^1 will decrease markup growth.

Using (21) and (22) and our earlier definitions of ε^2, α^2 and τ^2, we can calculate ε^2, α^2 and τ^2 under the assumption that $y^1/x^1 < y^2/x^2$, which means that traditional TFP growth is greater than 1 (see Cases 1-3 above). It turns out that for Cases 1-6, $e^1 = e^2 = \varepsilon^2 = 1$, $\alpha_L{}^2 = \alpha_P{}^2 = \alpha^2 = (p^2/p^1)/(w^2/w^1)$ and $\tau_L{}^2 = \tau_P{}^2 = \tau^2 = (y^2/x^2)/(y^1/x^1)$. Thus for these Cases, we find that our measure of the combined effects of scale inefficiency, technical progress and returns to scale, $\pi^2 \equiv \varepsilon^2\tau^2$, is equal to a "traditional" measure of TFP growth.

For Cases 7-9, we have the assumption that $y^1/x^1 > y^2/x^2$, which means that traditional TFP growth is less than 1. For these cases, using (21) and (22), we find that $e^1 = 1$, $e^2 = [y^2/x^2]/[y^1/x^1] < 1$; $\varepsilon^2 = [y^2/x^2]/[y^1/x^1] < 1$; $\tau^2 = \tau_P{}^2 = \tau_L{}^2 = 1$ and $\alpha^2 = \alpha_P{}^2 = \alpha_L{}^2 = (p^2/p^1)/(w^2/w^1)$. Thus for these Cases, the decline in productivity $\pi^2 \equiv \varepsilon^2\tau^2$ is attributed to scale inefficiency.

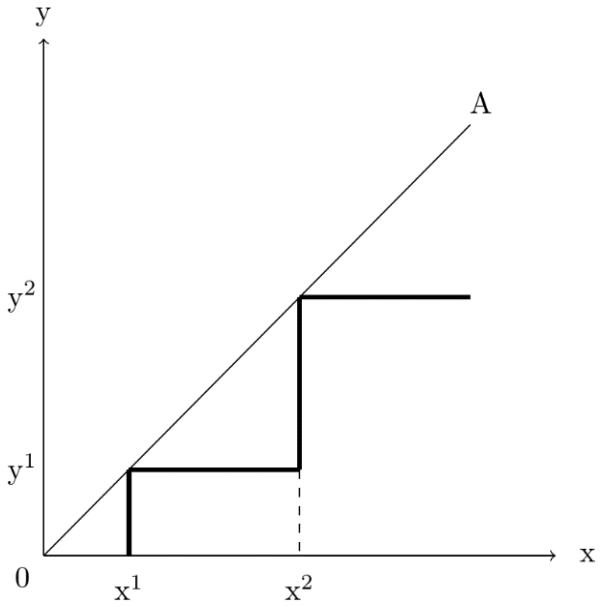
For all nine cases, we have π^2 equal to traditional TFP growth in the one output, one input case; i.e., we have:

(24) $\pi^2 \equiv \varepsilon^2\tau^2 = [y^2/x^2]/[y^1/x^1] = [y^2/y^1]/[x^2/x^1]$.

For Cases 1-6, $\varepsilon^2 = 1$ and $\tau^2 = [y^2/y^1]/[x^2/x^1]$. For Cases 7-9, we have $\varepsilon^2 = [y^2/y^1]/[x^2/x^1]$ and $\tau^2 = 1$. For all nine cases, we have $\alpha_L{}^2 = \alpha_P{}^2 = \alpha^2 = (p^2/p^1)/(w^2/w^1)$.

Below, we provide charts in input-output space to illustrate five of the above nine Cases.[18]

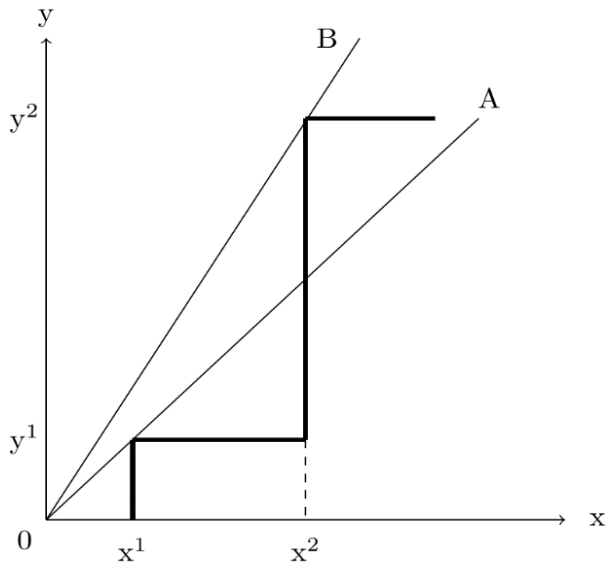
Figure 1: Case 4; Constant Total Factor Productivity with x^1 less than x^2



The slope of the ray OA is $y^1/x^1 = y^2/x^2$. There is no measured inefficiency for this case; the productivity levels are the same in periods 1 and 2.

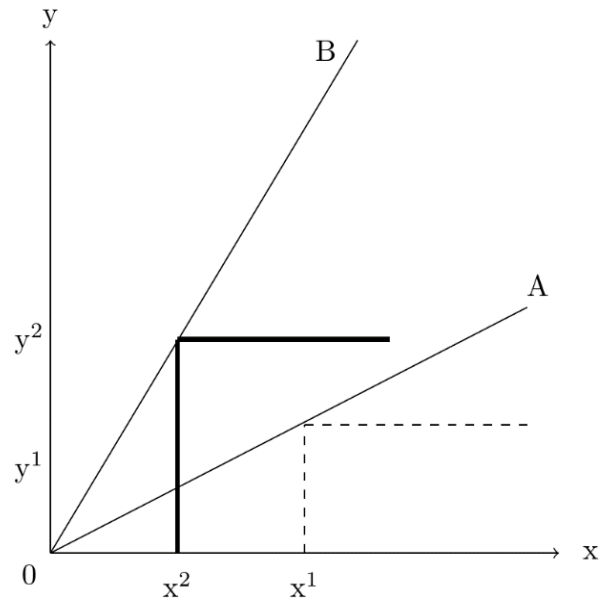
[18] It should be noted that our use of the free disposal hulls assumptions on production possibility sets was pioneered by Tulkens and his coauthors; see Deprins, Simar and Tulkens (1984), Tulkens (1993) and Tulkens and Eeckaut (1995). Diewert and Fox (2014) also developed a nonparametric approach to the measurement of TFP growth using free disposal hulls but they used a distance function approach to their analysis instead of the present profitability or markup approach that was pioneered by Balk.

Figure 2: Case 1; Increasing Total Factor Productivity with x^1 less than x^2



The ray OA has slope equal to y^1/x^1 and the ray OB has the steeper slope y^2/x^2. Thus the ratio of output to input has increased going from period 1 to period 2. In this case, there is no measured inefficiency so that $\varepsilon^2 = 1$. The measure of technical progress τ^2 is equal to y^2/x^2 divided by y^1/x^1. In this case, TFP growth π^2 is equal to τ^2. Again, the free disposal hull production function in each period is a step function.

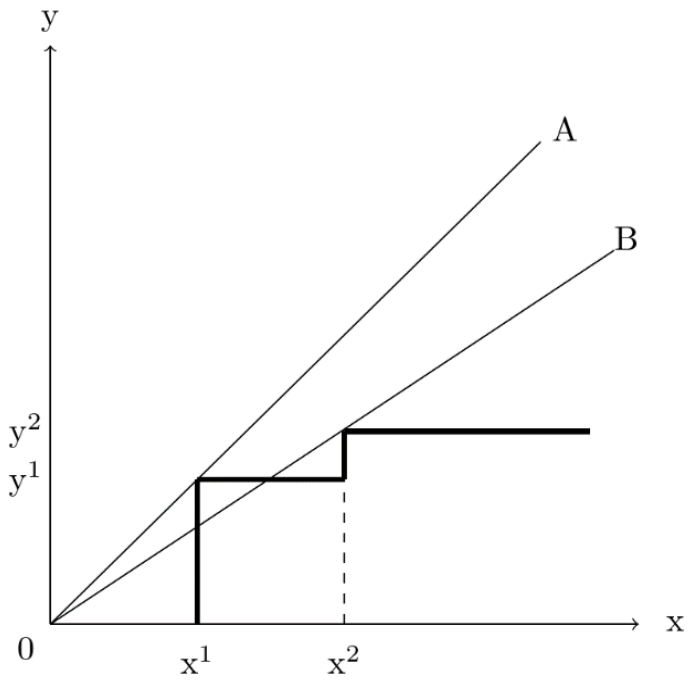
Figure 3: Case 3; Increasing Total Factor Productivity with x^2 less than x^1



This case is interesting because it shows that the period 1 efficient input output combination (x^1, y^1) is in the interior of the period 2 production possibilities set. But efficiency here is a relative concept: positive

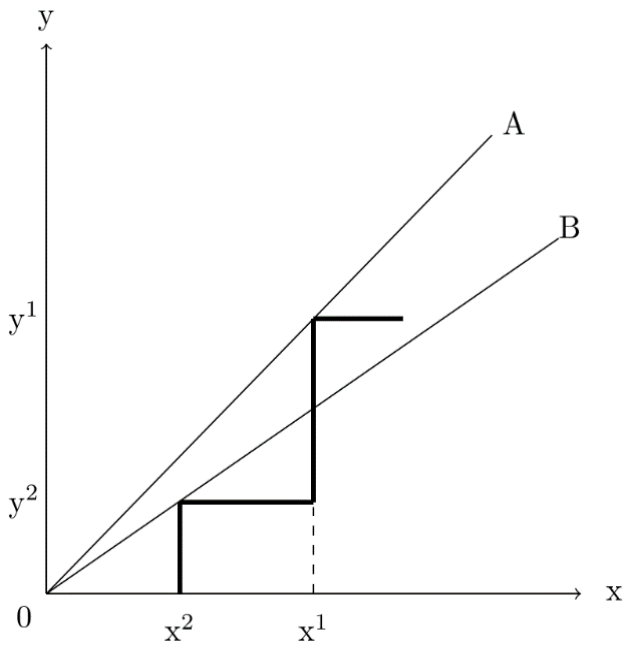
technical progress has made the observed efficient period 1 input output vector inefficient in period 2. The ray OA has slope equal to y^1/x^1 and the ray OB has the steeper slope y^2/x^2. Thus the ratio of output to input has increased going from period 1 to period 2. In this case as in the previous case, there is no measured inefficiency in either period so that $\varepsilon^2 = 1$. The measure of technical progress τ^2 is equal to y^2/x^2 divided by y^1/x^1. As in the previous case, TFP growth π^2 is then equal to τ^2.

Figure 4: Case 7; Decreasing Total Factor Productivity with x^1 less than x^2



In this case, the ray OA has a steeper slope than the ray OB and therefore, the point (x^1,y^1) has a higher output input coefficient than the period 2 point (x^2,y^2). In this case, it is plausible that there is no technical progress but there are decreasing returns to scale in the underlying technology. However, our measurement framework is such that our measures of technical progress and returns to scale are always equal to or greater than one. In this case, generalized technical progress τ^2 is set equal to 1 and efficiency change, $\varepsilon^2 = [(y^2/x^2)/(y^1/x^1)]$ is less than unity. Thus if there are decreasing returns to scale and x^2 is greater than x^1, our measure of efficiency change reflects the decreasing returns to scale and our measure of technical progress is set equal to one.

Figure 5: Case 9; Decreasing Total Factor Productivity with x^2 less than x^1



The period 1 ray OA has slope y^1/x^1 which is greater than the slope of the period 2 ray OB which is equal to y^2/x^2. We have a decline in input from the level x^1 to x^2 and a corresponding fall in output from y^1 to y^2 but the output quantity index declines more than the input quantity index; i.e., we have $y^2/y^1 < x^2/x^1$. This is the typical result when an economy goes into a *recession* in period 2. Total Factor Productivity growth going from period 1 to period 2 is $\pi^2 \equiv \varepsilon^2\tau^2 = [(y^2/x^2)/(y^1/x^1)][1] < 1$. In this Case, generalized technical progress τ^2 is set equal to 1 and efficiency change, $\varepsilon^2 = [(y^2/x^2)/(y^1/x^1)]$ is less than unity.

The growth accounting decomposition of nominal revenue growth R^2/R^1 defined by (18) becomes the following decomposition in the one output, one input, two period case:

(25) $R^2/R^1 = p^2y^2/p^1y^1 = [w^2x^2/w^1x^1]\varepsilon^2\alpha^2\tau^2 = [w^2x^2/w^1x^1]\alpha^2\pi^2 = [p^2/p^1][x^2/x^1]\pi^2$

where the final equality follows from the fact that $\alpha^2 = [p^2/p^1]/[w^2/w^1]$ for all nine Cases. Thus nominal output growth is equal to output price growth p^2/p^1 times real input quantity growth x^2/x^1 times TFP growth π^2. The equalities in (25) can be divided by p^2/p^1 to give us a decomposition of real output growth into real input growth times TFP growth.

4. Decomposing Price Change into Output and Input Price Change and the One Output Case

Recall the Laspeyres and Paasche measures of price change defined by (8) and (11), $\alpha_L^t \equiv \mu^{t-1}(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1})$ and $\alpha_P^t \equiv \mu^t(p^t,w^t)/\mu^t(p^{t-1},w^{t-1})$. It is useful to decompose the overall price change index $\alpha^t \equiv [\alpha_L^t\alpha_P^t]^{1/2}$ into separate indexes of output price change β^t and input price change γ^t. Thus α_L^t defined by (8) has the two exact decompositions defined by (26) and α_P^t defined by (11) has the two exact decompositions defined by (27):

(26) $\alpha_L{}^t = \beta_1{}^t/\gamma_1{}^t = \beta_2{}^t/\gamma_2{}^t$; $t = 2,3,\ldots,T$;
(27) $\alpha_P{}^t = \beta_3{}^t/\gamma_3{}^t = \beta_4{}^t/\gamma_4{}^t$; $t = 2,3,\ldots,T$.

The $\beta_i{}^t$ are defined by (28) and the $\gamma_i{}^t$ are defined by (29) for $t = 2,3,\ldots,T$:

(28) $\beta_1{}^t \equiv \mu^{t-1}(p^t,w^t)/\mu^{t-1}(p^{t-1},w^t)$; $\beta_2{}^t \equiv \mu^{t-1}(p^t,w^t)/\mu^{t-1}(p^{t-1},w^{t-1})$;
$\beta_3{}^t \equiv \mu^t(p^t,w^t)/\mu^t(p^{t-1},w^t)$; $\beta_4{}^t \equiv \mu^t(p^t,w^{t-1})/\mu^t(p^{t-1},w^{t-1})$;

(29) $\gamma_1{}^t \equiv \mu^{t-1}(p^{t-1},w^{t-1})/\mu^{t-1}(p^{t-1},w^t)$; $\gamma_2{}^t \equiv \mu^{t-1}(p^t,w^{t-1})/\mu^{t-1}(p^{t-1},w^t)$;
$\gamma_3{}^t \equiv \mu^t(p^t,w^{t-1})/\mu^t(p^{t-1},w^t)$; $\gamma_4{}^t \equiv \mu^t(p^t,w^{t-1})/\mu^t(p^t,w^{t-1})$.

Define β^t as the geometric mean of the $\beta_i{}^t$ and γ^t as the geometric mean of the $\gamma_i{}^t$:

(30) $\beta^t \equiv [\beta_1{}^t\beta_2{}^t\beta_3{}^t\beta_4{}^t]^{1/4}$; $\gamma^t \equiv [\gamma_1{}^t\gamma_2{}^t\gamma_3{}^t\gamma_4{}^t]^{1/4}$; $t = 2,3\ldots,T$.

It can be verified that $\alpha^t = \beta^t/\gamma^t$ for $t = 2,\ldots,T$ and substituting these equations into (18), we obtain the following alternative growth accounting decomposition of period t nominal output growth into explanatory factors:[19]

(31) $p^t\cdot y^t/p^{t-1}\cdot y^{t-1} = \varepsilon^t\beta^t\tau^t[w^t\cdot x^t/w^{t-1}\cdot x^{t-1}]/\gamma^t$; $t = 2,3,\ldots,T$.

The term $[w^t\cdot x^t/w^{t-1}\cdot x^{t-1}]/\gamma^t$ is nominal input growth divided by an input price index, γ^t. Thus nominal output growth is equal to the product of scale efficiency change ε^t times generalized technical progress τ^t times output price inflation β^t times nominal input growth $w^t\cdot x^t/w^{t-1}\cdot x^{t-1}$ divided by an index of input price inflation γ^t (which is real input quantity growth). This is similar to the Diewert and Morrison (1986) and Kohli (1990) decomposition of nominal GDP growth into explanatory factors that was based on the production unit's Revenue (or Value Added or GDP) Function.[20] The advantage of the present approach is that it is not necessary to make any assumptions about the functional form of the production unit's revenue function (or dual production function) or make assumptions about which index number formula is "best". Our present approach also allows for markups over average costs. A disadvantage of our approach is that the output and input price index functions which are implied by our approach look rather unfamiliar in the general case of many outputs and many inputs.[21]

Our nonparametric approach to productivity measurement turns out to be very similar to the nonparametric approach taken by Diewert and Fox (2018a). The main difference is that the Diewert and Fox approach assumed a constant returns to scale technology with no markups. In our present approach, we relax these assumptions.

Our empirical example has only one output (and eight inputs). It turns out that the formula for the Empirical Markup function $\mu^t(p,w)$ simplifies in the case of one output and this in turn simplifies the computation of the various values of $\mu^t(p^j,w^k)$. Assume that the number of outputs M = 1. In this case, the inner product $p\cdot y$

[19] The markup factor decomposition (17) becomes $M^t/M^{t-1} = (1+m^t)/(1+m^{t-1}) = \varepsilon^t\tau^t\beta^t/\gamma^t = \pi^t\beta^t/\gamma^t$.
[20] See Balk (1998) (2003) (2010) and Kohli (2025) for alternative growth decompositions.
[21] Below we provide alternative expressions for these price functions for the single output case.

just becomes the product of the scalar output price p and the scalar output quantity y. Using definition (5), the Empirical Markup function simplifies as follows for each period t:

$$\begin{aligned}(32)\ \mu^t(p,w) &\equiv \max_j \{py^j/w\cdot x^j : j = 1,2,\ldots,t\}\\ &= p \max_j \{y^j/w\cdot x^j : j = 1,2,\ldots,t\}\\ &= p \max_j \{1/w\cdot(x^j/y^j) : j = 1,2,\ldots,t\}\\ &= p/\min_j \{w\cdot(x^j/y^j) : j = 1,2,\ldots,t\}\\ &= p/c^t(w)\end{aligned}$$

where $c^t(w)$ is defined as follows:

$$(33)\ c^t(w) \equiv \min_j \{w\cdot(x^j/y^j) : j = 1,2,\ldots,t\}.$$

The function $c^t(w)$ is the period t *empirical minimum average cost function* for the production unit. Using assumption (4), it provides an approximation to the production unit's period t *true minimum average cost function* in the one output case. Define the production unit's period t *total cost function*, $C^t(y,w)$ for positive output level y and positive input price vector w as follows:

$$(34)\ C^t(y,w) \equiv \min_x \{w\cdot x : (y,x)\in S^t\}.$$

Define the production unit's period t *true minimum average cost function* as follows:[22]

$$(35)\ c^{t*}(w) \equiv \min_y \{C^t(y,w)/y : y > 0\}.$$

As mentioned above, the empirical minimum average cost function, $c^t(w)$ defined by (33) provides an approximation to the true minimum average cost function $c^{t*}(w)$ defined by (35).

For the one output case, the decomposition of period t nominal output growth defined by (31) above can be replaced by a simpler decomposition that is defined by using the empirical average cost function $c^t(w)$ defined by (33). Using the fact that $\mu^t(p,w)$ is equal to $p/c^t(w)$ in the one output case, we can establish the following relationships for t = 1,…,T:

$$(36)\ e^t \equiv [p^ty^t/w^t\cdot x^t]/\mu^t(p,w) = [p^ty^t/w^t\cdot x^t]/[p^t/c^t(w^t)] = c^t(w^t)/w^t\cdot(x^t/y^t).$$

Make the following definitions for t = 2,…,T:

$$(37)\ \varepsilon^t \equiv e^t/e^{t-1};$$
$$(38)\ \beta^t \equiv p^t/p^{t-1};$$
$$(39)\ \gamma_L{}^t \equiv c^{t-1}(w^t)/c^{t-1}(w^{t-1});\quad \gamma_P{}^t \equiv c^t(w^t)/c^t(w^{t-1});\quad \gamma^t \equiv [\gamma_L{}^t\gamma_P{}^t]^{1/2};$$
$$(40)\ \tau_L{}^t \equiv c^{t-1}(w^{t-1})/c^t(w^{t-1});\quad \tau_P{}^t \equiv c^{t-1}(w^t)/c^t(w^t);\quad \tau^t \equiv [\tau_L{}^t\tau_P{}^t]^{1/2}.$$

It can be seen that $\gamma_L{}^t = \mu^{t-1}(p^{t-1},w^t)/\mu^{t-1}(p^{t-1},w^{t-1})$ is a Laspeyres type theoretical producer input price index that is the ratio of the theoretical aggregate input price levels, $c^{t-1}(w^t)/c^{t-1}(w^{t-1})$ and $\gamma_P{}^t = \mu^t(p^t,w^t)/\mu^t(p^t,w^{t-1})$ is a Paasche type theoretical producer input price index that is the ratio of the theoretical aggregate input

[22] The minimum may not exist.

price levels, $c^t(w^t)/c^t(w^{t-1})$. Similarly, $\tau_L{}^t = \mu^{t-1}(p^{t-1},w^{t-1})/\mu^t(p^{t-1},w^{t-1}) = 1/[c^t(w^{t-1})/c^{t-1}(w^{t-1})]$ is a Laspeyres type measure of technical progress which is equal to the reciprocal of the amount of average cost reduction due to generalized technical progress holding constant the input prices of period $t-1$. It can be seen that $\tau_P{}^t = \mu^{t-1}(p^t,w^t)/\mu^t(p^t,w^t) = 1/[c^t(w^t)/c^{t-1}(w^t)]$ is a similar measure of technical progress holding constant the input prices of period t.[23]

Using the above definitions, it can be verified that the following identities hold for t = 2,…,T:

(41) $[p^ty^t/w^t\cdot x^t]/[p^{t-1}y^{t-1}/w^{t-1}\cdot x^{t-1}] = (1+m^t)/(1+m^{t-1}) = \beta^t\varepsilon^t\tau^t/\gamma^t$.

Equations (41) tell us that (one plus) the rate of markup growth is equal to an output price index β^t times an efficiency growth index ε^t times an index of generalized technical progress τ^t divided by an input price index γ^t. If we multiply both sides of (41) by $w^t\cdot x^t/w^{t-1}\cdot x^{t-1}$, we get the growth accounting identity (31) where the explanatory factors are now defined using $p/c^t(w)$ instead of the more general empirical markup function $\mu^t(p,w)$. As usual, our index of period t TFP growth is defined as $\pi^t \equiv \varepsilon^t\tau^t$.

It is useful to rearrange (41). Using the fact that β^t is equal to p^t/p^{t-1}, we can rewrite equations (41) as follows:

(42) $y^t/y^{t-1} = \varepsilon^t\tau^t[w^t\cdot x^t/w^{t-1}\cdot x^{t-1}]/\gamma^t$; $\qquad t = 2,\ldots,T$

where γ^t is the bilateral input price index defined by (39). Thus $[w^t\cdot x^t/w^{t-1}\cdot x^{t-1}]/\gamma^t = X^t/X^{t-1}$ is a bilateral input quantity index X^t/X^{t-1} where X^t is the period t aggregate input level. If we use this equation to eliminate γ^t from equations (42), we get the following growth decomposition:

(43) $y^t/y^{t-1} = (X^t/X^{t-1})\varepsilon^t\tau^t$; $\qquad t = 2,\ldots,T.$

For purposes of comparison with other measures of TFP growth, it is useful to convert the TFP growth equations (43) into TFP levels equations. Define aggregate input in period 1 as $X^1 \equiv w^1\cdot x^1$.[24] Define the X^t for subsequent periods $t \geq 2$ recursively using the equations $X^t \equiv X^{t-1}[w^t\cdot x^t/w^{t-1}\cdot x^{t-1}]/\gamma^t$ where γ^t is defined by (39). Define the corresponding aggregate input price levels by $W^t \equiv w^t\cdot x^t/X^t$. Define the period 1 level of technical progress $T^1 \equiv 1$ and define the technical progress levels for periods $t \geq 2$ recursively using the equations $T^t \equiv T^{t-1}\tau^t$ where τ^t is defined by (40). Define the period t Diewert Fox Nonparametric Level of Total Factor Productivity as Π^t:

(44) $\Pi^t \equiv [y^t/y^1]/[X^t/X^1] = e^t\,T^t$; $\qquad t = 1,\ldots,T.$

The last set of equalities in (44) follows from equations (43) and the above definitions.

5. Application to the Australian Retail Sector

In Appendix A, we explain how we used Australian Bureau of Statistics (ABS) data on Australia's Retail Trade sector for the 29 years 1994-2022 in order to construct estimates for the annual gross output of the

[23] Measuring technical progress by shifts in cost functions dates back to Farrell (1957) and Salter (1960).
[24] This definition assumes that the output price series p^t is normalized so that $p^1 = 1$. If not, define $X^1 \equiv w^1\cdot x^1/p^1$.

sector (year t price and quantity scalars are p^t and y^t respectively) and estimates for eight inputs (year t price and quantity vectors are $w^t \equiv [w_1^t,\ldots,w_8^t]$ and $x^t \equiv [x_1^t,\ldots,x_8^t]$ respectively). The nine price series are listed in Table A11 and the nine quantity series are listed in Table A12 of the Appendix. The eight inputs are: (1) quality adjusted labour input, (2) intermediate inputs, (3) non-dwelling construction services, (4) machinery and equipment, (5) research and development, (6) computer software, (7) inventory stocks and (8) land. The corresponding nine value series (in billions of Australian dollars) are listed in Table 1 below.

Table 1: Retail Trade Output and Input Values 1994-2022 in Billions of Australian Dollars

Year	$p^t y^t$	$w^t \cdot x^t$	$w_1^t x_1^t$	$w_2^t x_2^t$	$w_3^t x_3^t$	$w_4^t x_4^t$	$w_5^t x_5^t$	$w_6^t x_6^t$	$w_7^t x_7^t$	$w_8^t x_8^t$
1994	54.51465	53.21520	17.96389	30.13065	1.00820	2.55199	0.01613	0.19078	1.14162	0.21194
1995	56.43017	54.68923	19.41232	29.88218	1.05342	2.63033	0.01878	0.19542	1.25842	0.23836
1996	57.45883	56.04662	21.66065	28.88581	1.09882	2.64533	0.02148	0.19351	1.27068	0.27033
1997	58.25967	57.51291	22.98743	28.77570	1.15036	2.71601	0.02309	0.20649	1.34954	0.30428
1998	61.25732	60.14167	23.83528	30.27129	1.22067	2.87245	0.02571	0.23162	1.33452	0.35014
1999	66.08819	65.34529	25.20477	33.63820	1.33160	2.97352	0.02828	0.25561	1.49033	0.42298
2000	69.85657	69.12622	26.08742	35.94156	1.42204	3.20021	0.03041	0.28723	1.62678	0.53057
2001	74.52927	73.38874	28.33932	37.36723	1.44795	3.35378	0.03361	0.30875	1.92940	0.60868
2002	73.41110	71.81859	29.75072	34.15810	1.47509	3.51994	0.03701	0.31470	1.91795	0.64509
2003	76.00042	73.14690	31.02585	34.02043	1.49818	3.55826	0.03879	0.32571	2.04693	0.63274
2004	83.30600	81.41777	33.82825	39.23200	1.55527	3.73728	0.04145	0.34990	2.05974	0.61388
2005	85.00775	82.92924	35.26910	38.73679	1.69808	4.05219	0.04642	0.38078	2.11561	0.63026
2006	87.99761	83.69183	37.17445	36.72062	2.00354	4.44224	0.05379	0.39892	2.17253	0.72573
2007	93.79305	89.55783	41.05374	37.86202	2.36281	4.74383	0.06391	0.41194	2.18263	0.87696
2008	98.66141	94.73029	42.65608	40.14644	2.71446	5.35503	0.07193	0.43800	2.32002	1.02833
2009	102.66752	98.85431	43.63373	42.74251	2.92790	5.66079	0.08063	0.44522	2.21047	1.15305
2010	108.83767	104.83948	47.01063	44.93665	3.11114	5.60204	0.09058	0.48820	2.35406	1.24617
2011	117.59393	110.95793	48.88091	48.88698	3.27777	5.44031	0.10132	0.53808	2.50648	1.32608
2012	118.93942	111.88444	49.65642	48.83339	3.33861	5.35687	0.11687	0.62875	2.57351	1.38002
2013	117.85113	111.72476	50.43980	47.46012	3.37516	5.44861	0.14005	0.74052	2.70237	1.41813
2014	117.69184	111.90363	51.31784	46.19479	3.47540	5.63420	0.16824	0.81271	2.80367	1.49677
2015	122.52411	120.79585	57.16943	48.64512	3.56814	5.90427	0.19170	0.83900	2.90556	1.57264
2016	126.12581	121.00165	55.61380	50.55383	3.62098	5.84364	0.20797	0.82543	2.76404	1.57196
2017	130.09973	125.09574	57.04895	53.19377	3.77155	5.84221	0.22394	0.79707	2.65320	1.56504
2018	136.91522	131.78633	59.06295	57.53418	3.99689	5.96665	0.24172	0.83396	2.59378	1.55620
2019	141.77827	132.79588	57.43527	59.58832	4.21036	6.24871	0.26191	0.89294	2.62253	1.53584
2020	156.00508	140.70145	59.65825	65.36909	4.33609	6.47190	0.28182	0.96999	2.14561	1.46871
2021	163.97504	152.41002	65.82741	69.68304	4.58466	7.05993	0.31126	1.22978	2.29638	1.41756
2022	180.63479	166.13330	73.74364	73.50779	5.01525	7.91561	0.35714	1.71601	2.49537	1.38251

Labour, intermediate inputs, structures and machinery and equipment (inputs 1,2,3 and 4) are the most important inputs while the remaining capital service flows (research and development 5, computer software 6, inventory stocks 7 and land 8) have relatively low service flows.[25]

In Appendix A, we explain in some detail the problems in constructing user costs for the 6 capital stocks. A user cost for an asset reflects the cost of purchasing an asset at the beginning of the accounting period

[25] The service flow from inventories and land is very low due to the fact that the depreciation rate for these assets is assumed to be equal to 0. But the value of these assets in the total value of assets in the Australian Retail Trade sector is not far from the value of structures and machinery and equipment; see Tables A4 and A5 in Appendix A. This means that the rapidly increasing price of land in Australia is crowding out the more productive investments in machinery and equipment and in R&D. This crowding out effect of increasing land prices can help to explain declining rates of TFP growth in many advanced countries. Note that this crowding out effect of high land prices applies to investments even though the user cost of land is low.

(including the interest foregone by tying up financial capital during the accounting period), using the asset and then (hypothetically) selling the depreciated asset at the end of the accounting period. The expected sale price at the end of the period depends on forecasts about depreciation rates and expected asset inflation rates. Thus there are three important ingredients in forming a period t user cost: (i) the opportunity cost of financial capital at the beginning of the period (the interest rate r^t), the asset depreciation rate δ^t and the expected asset inflation rate i^t. All three of these ingredients are subject to error.[26]

We use a variant of the user cost formula developed by Jorgenson and Griliches (1967) and Christensen and Jorgenson (1969). Jorgensonian user costs use ex post new asset inflation rates to determine the asset inflation rate i^t and r^t is the ex post rate of return earned by the production unit at the end of the accounting period. This ex post rate of return is determined by using ex post asset inflation rates in the user cost formula and solving for the Jorgenson and Griliches *balancing rate of return* rJ^t in each user cost that will make the sum of the user costs equal to the period t value of gross output less the values of labour and intermediate input services; see equation (A6) in the Appendix. Of course, if we followed this methodology, the value of outputs would equal the value of inputs in each period and markups m^t would be equal to 0. Thus the data in Table 1 above and in Tables A11 and A12 in Appendix A are based on user costs that smoothed the ex post asset inflation rates and we used an exogenous rate of return in our user costs. Our exogenous r^t was set equal to 7 per cent plus smoothed Consumer Price Index inflation going from the beginning of each June year to the following June year. Thus we are somewhat arbitrarily assuming that real rate of return to financial capital of 7 per cent is an appropriate exogenous rate of return for the Australian Retail Trade sector. If we add one per cent to the 7 per cent, markups decline by 0.01 times the value of the beginning of the year capital stock, which is listed in the Appendix. This hypothetical computation shows that it is important to include the value of inventories and land in the list of assets if we want to measure the aggregate markup over average cost correctly.[27]

Recall that the year t markup factor, M^t, is defined as $p^t y^t / w^t \cdot x^t$ which in turn is equal to the output price p^t divided by the year t average cost, $w^t \cdot x^t / y^t$. Our estimated markup factors, $M^t \equiv 1+m^t$, are listed in Table 2 and plotted in Chart 1. The average markup factor was 1.0407 but there is an increasing trend in the M^t, with a maximum of 10.9% in 2020. The average was 2.6% per year in the period 1994-2007 and 5.5% for the period 2008-2022.[28]

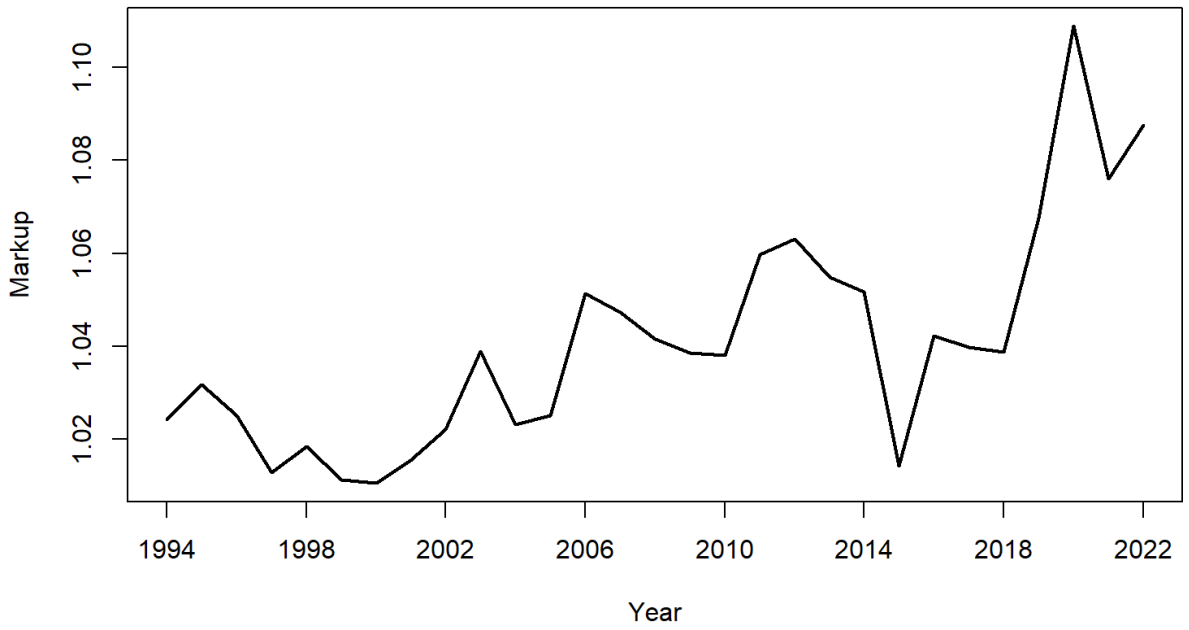
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[26] "In the previous section I may have left the impression that from a theoretical point of view constructing a capital aggregate is no more difficult than constructing a labor aggregate. In the present section, I will readjust this impression by cataloging some of the practical difficulties and sources of controversy that occur when researchers attempt to construct capital aggregates that are suitable for estimating production functions or for estimating total factor productivity." W.E. Diewert (1980; 475). The practical difficulties that Diewert discussed are precisely the difficulties in determining r^t, i^t and δ^t. Diewert has argued over the years that the Jorgenson approach is the "right" approach to determine ex post rates of return but that anticipated (or smoothed) asset inflation rates should be used when constructing TFP estimates or when using econometrics to estimate production functions or their dual representations.

[27] Another missing asset is monetary stocks held by the sector; see Diewert and Fox (2019) on the importance of this asset.

[28] However, our estimates for Retail Trade land values may be too small and our assumed real cost of financial capital (equal to 7%) may also be low.

Chart 1: Retail Sector Markups



The absolute amount of markup profit in year t is $p^t y^t - w^t \cdot x^t = m^t\, w^t \cdot x^t$. These estimated year t markup profits (m^t times the value of input costs $w^t \cdot x^t$) are listed in Table 2; they were very high in the last 3 years of our sample ($11.6 to $15.3 billion dollars). The absolute amount of markup profits has grown from 1.3 billion Australian dollars in 1994 to 14.5 billion dollars in 2022.

Recall the markup growth decomposition defined by (40) above: $M^t/M^{t-1} = \beta^t \varepsilon^t \tau^t / \gamma^t$ where the growth factors on the right hand side of this equation were defined by (36)-(39). The markup growth rates and the four explanatory growth factors are listed in Table 2 along with their sample averages. The growth factor $\beta^t \equiv p^t/p^{t-1}$ is a measure of output price change, γ^t is a measure of input price change, $\varepsilon^t \equiv e^t/e^{t-1}$ is an index of efficiency growth and τ^t is an index of generalized technical progress. Our index of Total Factor Productivity growth is $\pi^t \equiv \varepsilon^t \tau^t$ and this series is also listed in Table 2.

Table 2: Retail Trade Values of Markups over Average Cost, Markup Factors and MF Growth, Inefficiency and Inefficiency Growth, Output Price Growth, Input Price Growth and TFP Growth

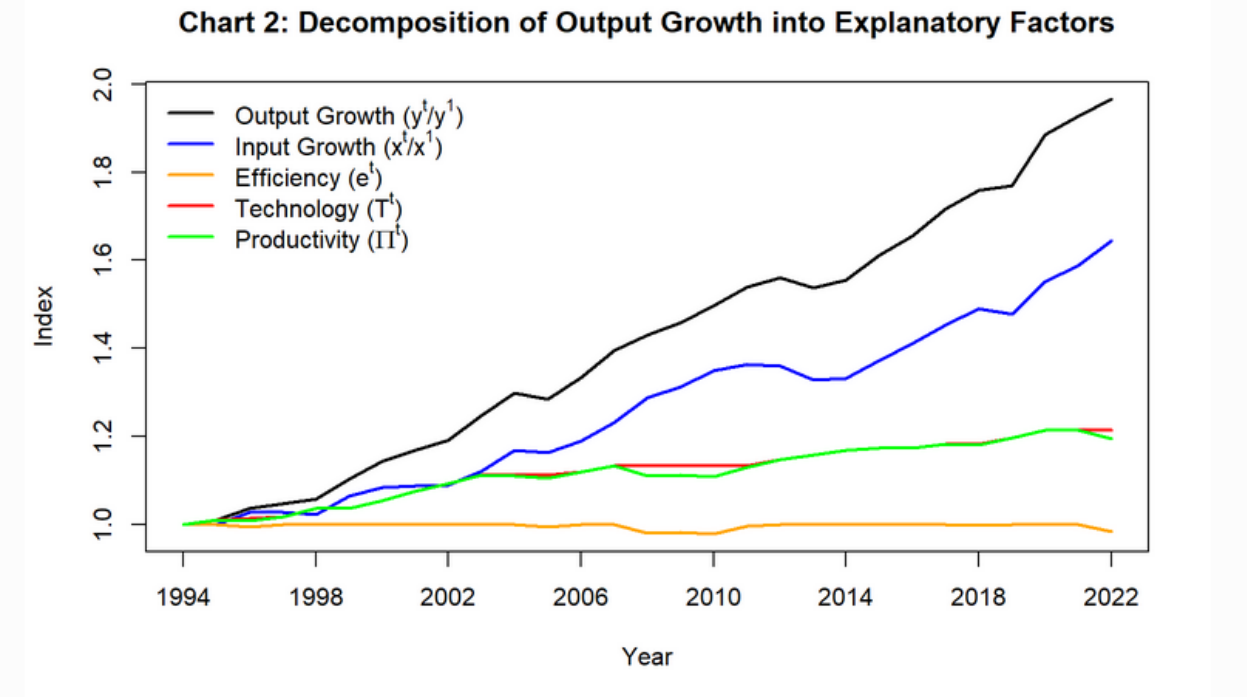
Year	$m^t w^t \cdot x^t$	M^t	M^t/M^{t-1}	e^t	$\varepsilon^t = e^t/e^{t-1}$	$\beta^t = p^t/p^{t-1}$	γ^t	τ^t	$\pi^t = \varepsilon^t\tau^t$
1994	1.29430	1.02432		1.00000					
1995	1.73736	1.03177	1.00727	1.00000	1.00000	1.02526	1.02746	1.00943	1.00943
1996	1.41045	1.02516	0.99360	0.99504	0.99504	0.99177	0.99660	1.00341	0.99844
1997	0.74694	1.01299	0.98812	1.00000	1.00498	1.00422	1.02618	1.00472	1.00972
1998	1.11761	1.01858	1.00552	1.00000	1.00000	1.03974	1.05256	1.01793	1.01793
1999	0.74622	1.01142	0.99297	0.99979	0.99979	1.03540	1.04251	1.00000	0.99979
2000	0.73334	1.01061	0.99920	1.00000	1.00021	1.01976	1.03837	1.01722	1.01744
2001	1.14185	1.01556	1.00490	1.00000	1.00000	1.04303	1.05972	1.02098	1.02098
2002	1.59100	1.02215	1.00649	1.00000	1.00000	0.96727	0.97600	1.01557	1.01557
2003	2.85150	1.03898	1.01646	1.00000	1.00000	0.98859	0.98985	1.01776	1.01776
2004	1.88830	1.02319	0.98480	0.99888	0.99888	1.05296	1.06802	1.00000	0.99888
2005	2.07919	1.02507	1.00184	0.99374	0.99486	1.03105	1.02386	1.00000	0.99486
2006	4.30659	1.05146	1.02574	1.00000	1.00629	0.99776	0.98575	1.00705	1.01339
2007	4.24037	1.04735	0.99609	1.00000	1.00000	1.01811	1.03457	1.01220	1.01220
2008	3.93770	1.04157	0.99448	0.98001	0.98001	1.02617	1.01124	1.00000	0.98001
2009	3.81648	1.03861	0.99716	0.98125	1.00126	1.02000	1.02420	1.00000	1.00126
2010	3.99927	1.03815	0.99956	0.97819	0.99688	1.03337	1.03060	1.00000	0.99688
2011	6.63737	1.05982	1.02088	0.99688	1.01911	1.05038	1.04856	1.00000	1.01911
2012	7.05703	1.06308	1.00307	1.00000	1.00313	0.99844	1.00990	1.01142	1.01458
2013	6.12708	1.05484	0.99225	1.00000	1.00000	1.00491	1.02312	1.01023	1.01023
2014	5.78574	1.05170	0.99702	1.00000	1.00000	0.98779	0.99877	1.00811	1.00811
2015	1.72395	1.01427	0.96441	1.00000	1.00000	1.00436	1.04741	1.00575	1.00575
2016	5.11699	1.04229	1.02762	0.99905	0.99905	1.00214	0.97427	1.00000	0.99905
2017	4.99170	1.03990	0.99771	1.00000	1.00096	0.99512	1.00463	1.00629	1.00725
2018	5.12336	1.03887	0.99901	0.99851	0.99851	1.02663	1.02611	1.00000	0.99851
2019	8.98332	1.06765	1.02770	1.00000	1.00150	1.02988	1.01646	1.01279	1.01431
2020	15.31171	1.10883	1.03857	1.00000	1.00000	1.03242	1.00880	1.01482	1.01482
2021	11.58246	1.07600	0.97040	0.99962	0.99962	1.02762	1.05857	1.00000	0.99962
2022	14.53185	1.08749	1.01067	0.98370	0.98408	1.08056	1.05212	1.00000	0.98408
Mean	4.5038	1.0407	1.0023	0.99671	0.99671	0.99943	1.0191	1.0070	1.0064

The sample average TFP growth rate was 0.64 percentage points per year.[29] Over the years 1994-2007, the average TFP growth rate was 0.97 pp but this average fell to 0.36 percentage points over the years 2008-2022. Thus there was a tendency for monopoly profits to go up and TFP growth to go down over the sample period. There were 12 years where efficiency dipped below 1. The effects of this inefficiency were small in the first part of the sample period but averaged – 0.1 percentage points per year over the years 2008-2022.

Using definitions (41)-(44), we can compute our period t aggregate input price levels, W^t, and the corresponding input quantity levels, X^t. These series are listed in Table 3 along with the period t output and input levels relative to their period 1 levels, y^t/y^1 and X^t/X^1. The period t efficiency levels, e^t defined by (36), are also listed in Table 3. The period t technology levels, T^t defined at the end of section 4, are listed

[29] Using Jorgensonian user costs and the Jorgenson-Griliches balancing rate of return methodology which sets the value of inputs equal to the value of outputs, we found that the average TFP growth rate was 0.59 percentage points per year. The ABS gross output TFP average growth rate was only 0.47 percentage points per year. However, the ABS estimates did not include inventories or land and because these inputs grew more slowly than the included inputs, TFP growth will tend to increase if these assets are included. See Table A7 in Appendix A.

in Table 3 along with the nonparametric period t productivity levels $\Pi^t = (y^t/X^t)/(y^1/X^1) = (y^t/y^1)/(X^t/X^1) = e^t T^t$. These five series are plotted on Chart 2.[30]



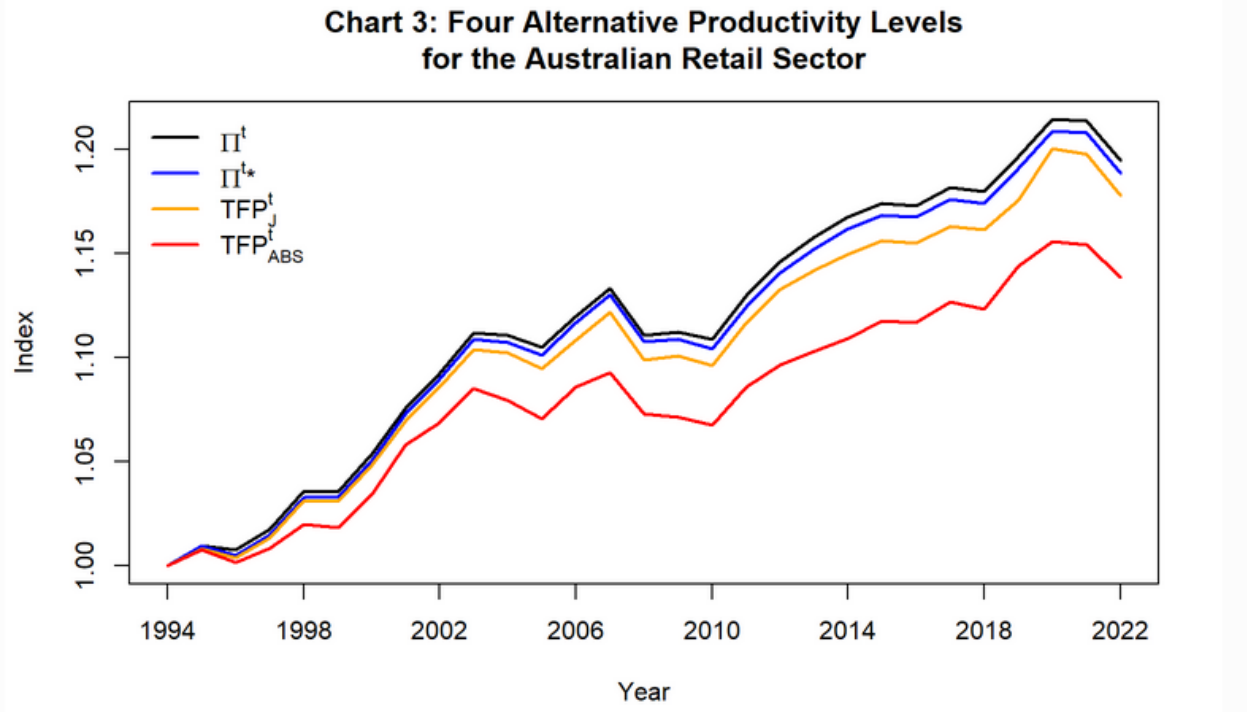
Our new nonparametric TFP levels, Π^t, are compared with our 2018 nonparametric productivity levels, Π^{t*}, in Table 3. Our estimates Π^{t*} based on the 2018 methodology used the same data as were used to compute the new estimates Π^t. Our previous methodology is explained in Appendix B for the case of one output and it assumes constant returns to scale in production. Using the new methodology, the chain link in the aggregate input index is $X^t/X^{t-1} \equiv [w^t \cdot x^t / w^{t-1} \cdot x^{t-1}]/\gamma^t$; using the old methodology, the chain link is the Fisher (1922) input quantity index linking period t−1 input to period t inputs, defined as $X^t/X^{t-1} \equiv \{[w^{t-1} \cdot x^t / w^{t-1} \cdot x^{t-1}][w^t \cdot x^t / w^t \cdot x^{t-1}]\}^{1/2}$. As noted in Appendix B, the old methodology has an advantage over the new methodology in that period t TFP growth can be computed using only the price and quantity data pertaining to periods t−1 and t whereas computing the new TFP growth requires price and quantity information back to the initial period 1. In addition to listing the new and old Diewert Fox TFP levels Π^t and Π^{t*}, Table 3 lists the TFP levels TFP_J^t and TFP_{ABS}^t. TFP_J^t are the TFP levels that correspond to the Jorgenson and Griliches (1967) methodology where user costs are constructed using a balancing rate of return and the capital gains terms in the asset user costs are not smoothed.[31] TFP_{ABS}^t are the TFP levels that were constructed by the Australian Bureau of Statistics; see Appendix A for the details. The ABS also uses a balancing rate of return methodology (so there are no markups).

[30] Note that this equation gives us the following long term growth decompositions: $y^t/y^1 = (X^t/X^1)\Pi^t = (X^t/X^1)e^t T^t$.
[31] Our exogenous rates of return were smaller than the balancing rates of return and our user costs used smoothed capital gains in the user cost formula. See Appendix A for the details.

Table 3: Input Price and Quantity Indexes, Long Term Output and Input Growth, Inefficiency and Technology Levels and Four Alternative Total Factor Productivity Levels.

Year	W^t	X^t	y^t/y^1	X^t/X^1	e^t	T^t	Π^t	Π^{t*}	TFP_J^t	TFP_{ABS}^t
1994	1.00000	53.22035	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1995	1.02746	53.23097	1.00963	1.00020	1.00000	1.00943	1.00943	1.00943	1.00826	1.00787
1996	1.02397	54.73643	1.03656	1.02849	0.99504	1.01287	1.00785	1.00492	1.00405	1.00152
1997	1.05078	54.73362	1.04659	1.02843	1.00000	1.01765	1.01765	1.01488	1.01338	1.00841
1998	1.10601	54.37559	1.05838	1.02171	1.00000	1.03590	1.03590	1.03307	1.03129	1.01985
1999	1.15302	56.67024	1.10281	1.06482	0.99979	1.03590	1.03568	1.03288	1.03086	1.01838
2000	1.19726	57.73447	1.14311	1.08482	1.00000	1.05373	1.05373	1.05088	1.04857	1.03459
2001	1.26877	57.84158	1.16925	1.08683	1.00000	1.07584	1.07584	1.07292	1.06974	1.05829
2002	1.23831	57.99849	1.19068	1.08978	1.00000	1.09259	1.09259	1.08963	1.08607	1.06892
2003	1.22574	59.67727	1.24690	1.12132	1.00000	1.11199	1.11199	1.10898	1.10405	1.08529
2004	1.30911	62.19306	1.29801	1.16860	0.99888	1.11199	1.11075	1.10731	1.10222	1.07942
2005	1.34035	61.87071	1.28465	1.16254	0.99374	1.11199	1.10504	1.10124	1.09469	1.07063
2006	1.32125	63.34233	1.33281	1.19019	1.00000	1.11983	1.11983	1.11675	1.10863	1.08597
2007	1.36692	65.51404	1.39532	1.23100	1.00000	1.13349	1.13349	1.13038	1.12206	1.09284
2008	1.38229	68.52647	1.43031	1.28760	0.98001	1.13349	1.11084	1.10761	1.09910	1.07292
2009	1.41574	69.82291	1.45921	1.31196	0.98125	1.13349	1.11224	1.10890	1.10072	1.07162
2010	1.45907	71.85302	1.49695	1.35010	0.97819	1.13349	1.10877	1.10419	1.09633	1.06763
2011	1.52992	72.52428	1.53981	1.36272	0.99688	1.13349	1.12995	1.12470	1.11674	1.08598
2012	1.54507	72.41265	1.55986	1.36062	1.00000	1.14643	1.14643	1.14087	1.13288	1.09670
2013	1.58078	70.67635	1.53804	1.32800	1.00000	1.15817	1.15817	1.15254	1.14204	1.10311
2014	1.57885	70.87844	1.55494	1.33179	1.00000	1.16756	1.16756	1.16189	1.14988	1.10921
2015	1.65370	73.04827	1.61176	1.37256	1.00000	1.17427	1.17427	1.16857	1.15610	1.11765
2016	1.61116	75.10676	1.65559	1.41124	0.99905	1.17427	1.17315	1.16759	1.15528	1.11703
2017	1.61862	77.29317	1.71614	1.45232	1.00000	1.18165	1.18165	1.17600	1.16329	1.12693
2018	1.66088	79.35067	1.75919	1.49098	0.99851	1.18165	1.17989	1.17427	1.16175	1.12342
2019	1.68822	78.65974	1.76882	1.47800	1.00000	1.19677	1.19677	1.19112	1.17568	1.14399
2020	1.70308	82.61093	1.88520	1.55224	1.00000	1.21450	1.21450	1.20877	1.20041	1.15574
2021	1.80283	84.52969	1.92825	1.58830	0.99962	1.21450	1.21404	1.20828	1.19775	1.15430
2022	1.89680	87.57002	1.96580	1.64542	0.98370	1.21450	1.19470	1.18878	1.17790	1.13875

There is not a lot of difference between our two sets of nonparametric estimates for the Total Factor Productivity levels of the Australian Retail Trade sector. Our new methodology estimates Π^t are a bit higher than the corresponding old constant returns to scale methodology estimates Π^{t*}. The Jorgenson estimates are about 2 percentage points lower than the preferred Π^t estimates at the end of the sample period and the ABS estimates are about 5 percentage points below the Π^t estimates. Chart 3 plots the four TFP series: Π^t, Π^{t*}, TFP_J^t and TFP_{ABS}^t.



Following the example of Schreyer (2010), we conclude this section with a brief discussion on how markups fit into the production accounts of the international System of National Accounts. Schreyer (2010; 15) explained that the value of markups in period t, $m^t w^t \cdot x^t \equiv$ Value of Markups $\equiv v_M^t$, is simply a part of what is called Gross Operating Surplus in the System of National Accounts. GOS is defined as the value of output less the value of labour less the value of intermediate inputs which in turn is equal to the value of capital services plus the value of markup pure profits. The accounting identities for GOS^t using our notation for year t are as follows:

(45) $GOS^t \equiv p^t y^t - w_1^t x_1^t - w_2^t x_2^t = \Sigma_{n=3}^{8} w_n^t x_n^t + v_M^t$.

Barkai (2020) constructed user costs for many US companies using exogenous rates of return and used a rearrangement of the above equation to decompose the value added[32] generated from production into the share of labour, capital services (evaluated at exogenous costs of capital) plus the share of markup profits. In the following Table, we provide a similar decomposition of Retail Trade value added into the shares of Labour s_2^t, the shares of capital services, $s_2^t,\ldots,s_8^t$ and the share of markup profits s_M^t. These shares are defined as follows:

(46) $s_n^t \equiv w_n^t x_n^t / [p^t y^t - w_1^t x_1^t]$; $\quad n = 2,3,\ldots,8$; $\quad t = 1,\ldots,T$;
(47) $s_M^t \equiv v_M^t / [p^t y^t - w_1^t x_1^t]$; $\quad t = 1,\ldots,T$.

Table 4 lists these shares and Chart 4 plots them.

[32] Period t value added is defined as $va^t \equiv p^t y^t - w_1^t x_1^t$, which is the value of output less the value of intermediate inputs.

Table 4: Retail Trade Shares of Labour, Capital Services and Markup Profits in Value Added

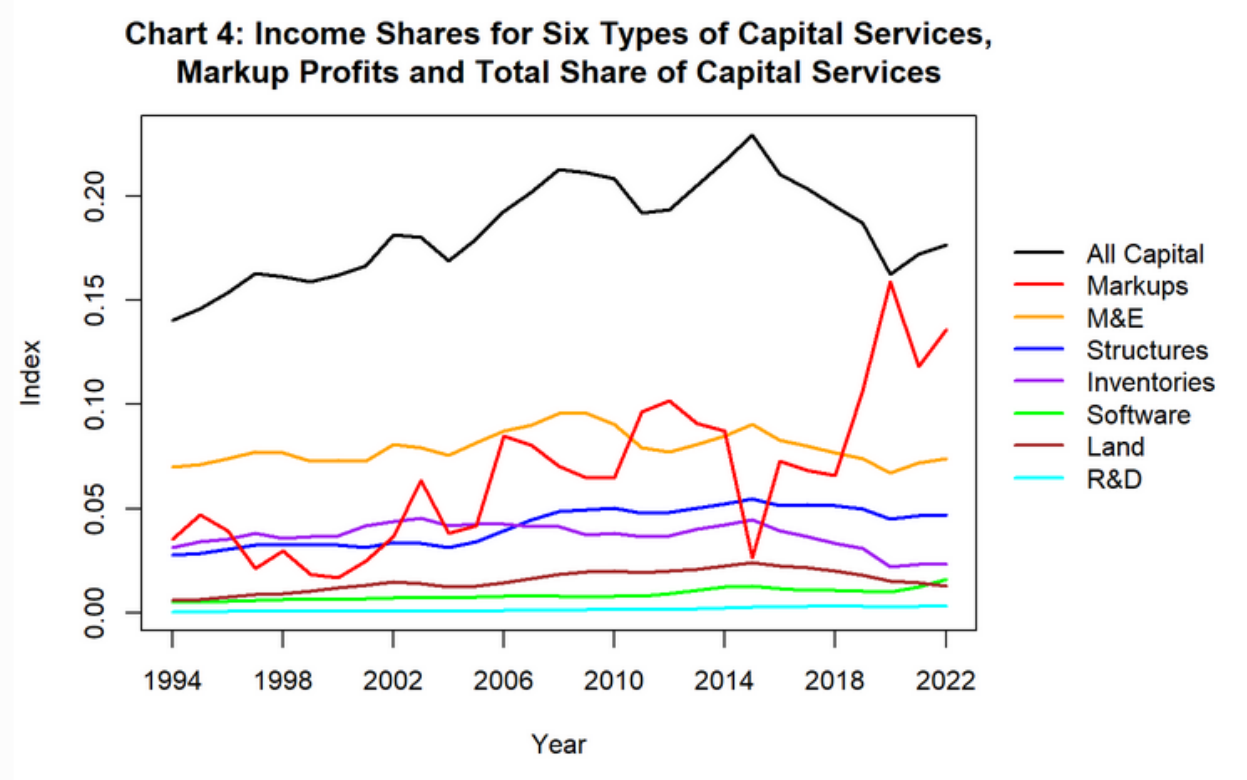
Year	va^t	s_2^t	s_3^t	s_4^t	s_5^t	s_6^t	s_7^t	s_8^t	s_M^t	s_K^t
1994	36.55076	0.82435	0.02762	0.06986	0.00044	0.00522	0.03128	0.00582	0.03541	0.14024
1995	37.01785	0.80724	0.02848	0.07108	0.00051	0.00528	0.03403	0.00645	0.04693	0.14583
1996	35.79818	0.80691	0.03071	0.07391	0.00060	0.00541	0.03551	0.00756	0.03940	0.15369
1997	35.27224	0.81582	0.03261	0.07700	0.00065	0.00585	0.03826	0.00863	0.02118	0.16301
1998	37.42204	0.80892	0.03261	0.07675	0.00069	0.00619	0.03564	0.00935	0.02987	0.16122
1999	40.88342	0.82278	0.03255	0.07271	0.00069	0.00625	0.03642	0.01033	0.01825	0.15896
2000	43.76914	0.82116	0.03247	0.07310	0.00069	0.00656	0.03714	0.01211	0.01675	0.16208
2001	46.18995	0.80899	0.03134	0.07260	0.00073	0.00668	0.04176	0.01317	0.02472	0.16629
2002	43.66038	0.78236	0.03379	0.08063	0.00085	0.00721	0.04394	0.01478	0.03644	0.18120
2003	44.97457	0.75644	0.03332	0.07913	0.00086	0.00724	0.04553	0.01408	0.06340	0.18016
2004	49.47775	0.79292	0.03143	0.07553	0.00084	0.00707	0.04163	0.01241	0.03816	0.16891
2005	49.73865	0.77881	0.03414	0.08147	0.00093	0.00766	0.04253	0.01267	0.04180	0.17939
2006	50.82316	0.72252	0.03942	0.08740	0.00106	0.00785	0.04274	0.01428	0.08474	0.19275
2007	52.73931	0.71791	0.04477	0.08993	0.00121	0.00781	0.04135	0.01661	0.08040	0.20169
2008	56.00533	0.71683	0.04844	0.09559	0.00128	0.00782	0.04139	0.01834	0.07031	0.21286
2009	59.03379	0.72403	0.04958	0.09588	0.00137	0.00754	0.03743	0.01952	0.06465	0.21132
2010	61.82704	0.72681	0.05032	0.09060	0.00146	0.00790	0.03807	0.02015	0.06468	0.20850
2011	68.71302	0.71147	0.04770	0.07917	0.00147	0.00783	0.03647	0.01930	0.09660	0.19194
2012	69.28300	0.70484	0.04818	0.07731	0.00169	0.00907	0.03714	0.01991	0.10186	0.19330
2013	67.41133	0.70404	0.05007	0.08082	0.00208	0.01098	0.04008	0.02104	0.09089	0.20507
2014	66.37400	0.69598	0.05237	0.08489	0.00253	0.01224	0.04225	0.02256	0.08717	0.21685
2015	65.35469	0.74432	0.05461	0.09036	0.00293	0.01284	0.04448	0.02408	0.02638	0.22930
2016	70.51200	0.71695	0.05138	0.08290	0.00295	0.01171	0.03923	0.02231	0.07257	0.21048
2017	73.05078	0.72818	0.05168	0.08001	0.00307	0.01091	0.03637	0.02146	0.06833	0.20349
2018	77.85227	0.73902	0.05136	0.07666	0.00311	0.01071	0.03334	0.02000	0.06581	0.19517
2019	84.34299	0.70650	0.04992	0.07408	0.00311	0.01059	0.03109	0.01821	0.10651	0.18699
2020	96.34683	0.67848	0.04498	0.06715	0.00292	0.01007	0.02225	0.01523	0.15892	0.16260
2021	98.14762	0.70998	0.04666	0.07189	0.00317	0.01253	0.02335	0.01441	0.11801	0.17201
2022	106.8911	0.68769	0.04684	0.07399	0.00334	0.01605	0.02328	0.01287	0.13595	0.17636
Mean	59.4990	0.75042	0.04170	0.07939	0.00163	0.00866	0.03703	0.01543	0.06573	0.18385

Below is a chart of the shares s_3^t (Structures), s_4^t (Machinery and Equipment), s_5^t (Research and Development), s_6^t (Software), s_7^t (Inventory Stocks), s_8^t (Land), s_M^t (Markup Profits) and s_K^t (Aggregate Capital Services). We did not include s_2^t (Labour Services) because it is much larger than the other income components.

The top line is the income share of Aggregate Capital Services, followed by the very volatile Markups share and followed by the fairly smooth Machinery and Equipment share. In order of their importance, the next three shares are Structures, Inventory Stocks and Land. The income shares of Software and R&D are very low.

The increase in the Markup share over the sample period is a cause for concern. Markups act in much the same way as taxes on outputs and inputs and these tax and markup wedges lead to deadweight losses. [33]

[33]See Allais (1943) (1977), Boiteux (1951), Debreu (1951) (1954) and Diewert (1983) on measuring deadweight loss in the production sector of an economy.



6. Conclusion

We have suggested a very obvious method for measuring the aggregate markup of revenues over costs. This method is reasonably robust but it does depend on what is assumed about the opportunity cost of financial capital. A higher opportunity cost will lead to a lower aggregate markup. Thus attention shifts to the construction of user costs including the treatment of asset price changes in the user cost formula. Including more assets in scope will also decrease the aggregate markup. For our particular example, including land and inventories as in scope assets led to higher estimates of TFP growth.

Our methodology requires minimal assumptions on producer behavior. The main assumptions are:

- Past period input and output quantity vectors can be meaningfully compared with current period input and output quantity vectors.[34]
- The price vectors are the "correct" prices to use in our comparisons.[35]

We also note that there is some arbitrariness in our measures of price change and technology change. In the case of a single output, our measures of TFP growth turn out to be quite conventional but our new measures of price change have not been tested in the case of multiple outputs. Finally, our decomposition of TFP growth does not have a *separate* component that reflects possible increasing or decreasing returns to scale. In view of the prospect of population decline for many countries and hence of labour input decline, it may be important to be able to estimate returns to scale.

[34] Over time, there can be a lack of matching problem; i.e., the introduction of new products and the disappearance of obsolete products can lead to intertemporal comparisons of output and input that are meaningless.
[35] In Appendix A, it will be seen that many assumptions have to be made to construct user costs.

On a positive note, our methodology can deal with both micro and macro data; i.e., our methodology is not overly complicated and in principle, can deal with many outputs and inputs.[36]

How does our new methodology fit into the System of National Accounts? This issue was discussed by Balk (2010) and Schreyer (2010) and the answer is simple: we just need to add an additional category to Gross Operating Surplus that corresponds to pure profits or the value of revenues over costs. However, many countries and organizations that produce country estimates of Multifactor or Total Factor Productivity use the Jorgenson-Griliches framework where the value of inputs is forced to equal the value of outputs and there is no category for pure profits. Our empirical example indicates that pure profits may not be negligible and that markups tend to be increasing over time.[37] This means that National Statistical Offices that produce TFP estimates may have to adjust their measurement methodologies to deal with the existence of substantial markups.

[36] An open issue is: how many periods of data do we need in order to implement the methodology in a meaningful way? When can we start dropping the data of the first period as we add the data for a new period?
[37] This agrees with the findings of Barkai (2020) and others that markups appear to be increasing over time.

Appendix A: Data Construction

Constructing the price and quantity data for the Australian Retail Trade sector was not a straightforward task.

From the Australian Bureau of Statistics (ABS) (2025a), we obtained information on the *value added* produced by the Australian Retail Trade sector for the split years 1994-2022 both in current prices and in chained volume measures. The first split year runs from July1, 1994 to June 30, 1995. We will label this split year as year 1994, the second split year as 1995 and so on. Denote the year t nominal value added produced by the Retail sector as VY^t and the corresponding volume measure as the split year t quantity QY^t for t = 1994,…,2022. Define the year t price of value added as $PY^t \equiv VY^t/QY^t$ for t = 1994,…,2022. We normalized the prices so that $PY^{1994} = 1$ and $QY^{1994} = VY^{1994}$. The units of measurement for the VY^t are in billions of Australian dollars. The series VY^t, QY^t and PY^t are listed in Table A1 below.

In ABS (2025b), we can find estimates for the Multifactor Value Added Productivity of the Retail sector in Australia for the split years 1994-2023. In Tables 9 and 10 of this publication, we obtained indexes for quality adjusted labour input QX_1^t and for real capital services input QX_2^t. In Table 14 of this publication, we obtained labour and capital income shares for the Retail Trade sector, SX_1^t and SX_2^t. Since the Australian Bureau of Statistics uses a methodology that sets the value of sectoral inputs equal to the value of sectoral outputs, we can use these shares along with the ABS estimates for the value of output in year t, VY^t. to obtain estimates for the value of labour and capital services input in year t, $VX_1^t \equiv VY^t \times SX_1^t$ and $VX_2^t \equiv VY^t \times SX_2^t$ for t = 1994,…,2022. We normalized the input prices so that $PX_1^{1994} = 1$, $QX_1^{1994} = VX_1^{1994}$ and $PX_2^{1994} = 1$, $QX_2^{1994} = VX_2^{1994}$. The units of measurement for VX_1^t and VX_2^t are in billions of Australian dollars. The series PX_1^t, PX_2^t, QX_1^t and QX_2^t are listed in Table A1 below.

We use chained Törnqvist input quantity indexes[38] to aggregate the labour and capital input quantities into aggregate input QX^t with corresponding aggregate input price index, PX^t with $VX^t = PX^t \times QX^t = VX_1^t + VX_2^t = VY^t$. Diewert and Morrison (1986) showed that the use of this index number formula was consistent with the functional form for the Retail sector Value Added function being a translog function, which is a flexible functional form. The year t aggregate input quantity and price, QX^t and PX^t, are also listed in Table A1. The aggregate input value series, VX^t, is not listed because $VX^t = VY^t$ for all years t.

With estimates for real output and input in hand, the year t level of Total Factor Productivity[39] for the Retail sector is defined as $TFP^t \equiv QY^t/QX^t$ and TFP growth for year t = 1994,…,2022 is defined as $\Delta TFP^t \equiv TFP^t/TFP^{t-1} = [QY^t/QY^{t-1}]/[QX^t/QX^{t-1}]$. TFP^t and ΔTFP^t are listed in Table A1.

Table A1: Inputs, Outputs and Productivity Levels for Retail Sector Value Added

Year t	PY^t	PX_1^t	PX_2^t	PX^t	VY^t	QY^t	QX^t	QX_1^t	QX_2^t	TFP^t	$\Delta TFPG^t$
1994	1.0000	1.0000	1.0000	1.0000	24.3840	24.3840	24.3840	17.9639	6.4201	1.0000	

[38] Let $p^t \equiv [p_1^t,\ldots,p_N^t]$ and $q^t \equiv [q_1^t,\ldots,q_N^t]$ be positive price and quantity vectors for t = 1,2,… . The bilateral Törnqvist quantity index for period t relative to period t−1 is defined as $Q_T(p^{t-1},p^t,q^{t-1},q^t) \equiv \exp[\Sigma_{n=1}^N (1/2)(s_n^{t-1} + s_n^t)\ln(q_n^t/q_n^{t-1})]$ where $s_n^t \equiv p_n^t q_n^t/p^t \cdot q^t$ is the expenditure share for product n in period t. The sequence of chained Törnqvist quantity indexes is defined as follows: for period 1, $Q^1 \equiv p^1 \cdot q^1$, the value aggregate for period 1. For subsequent periods, $Q^t \equiv Q^{t-1} \times Q_T(p^{t-1},p^t,q^{t-1},q^t)$ for t = 2,3,… . The ABS constructs chained Törnqvist price indexes first and then constructs the corresponding quantity indexes for capital by deflating the value series by the price index series.

[39] This terminology follows that of Jorgenson and Griliches (1967). It is the same concept as Multifactor Productivity.

1995	**1.0423**	**1.0608**	**1.0634**	**1.0615**	**26.5480**	**25.4718**	**25.0105**	**18.2997**	**6.7106**	**1.0184**	**1.0184**
1996	**1.0666**	**1.1069**	**0.9811**	**1.0733**	**28.5730**	**26.7897**	**26.6207**	**19.5694**	**7.0457**	**1.0064**	**0.9881**
1997	**1.0605**	**1.1576**	**0.8740**	**1.0817**	**29.4840**	**27.8028**	**27.2572**	**19.8581**	**7.4330**	**1.0200**	**1.0136**
1998	**1.0572**	**1.1777**	**0.8860**	**1.0996**	**30.9860**	**29.3086**	**28.1789**	**20.2381**	**8.0711**	**1.0401**	**1.0197**
1999	**1.0665**	**1.2058**	**0.8405**	**1.1065**	**32.4500**	**30.4255**	**29.3274**	**20.9039**	**8.6197**	**1.0374**	**0.9975**
2000	**1.0717**	**1.2604**	**0.8524**	**1.1487**	**33.9150**	**31.6473**	**29.5251**	**20.6977**	**9.1833**	**1.0719**	**1.0332**
2001	**1.1049**	**1.3567**	**0.9247**	**1.2387**	**37.1620**	**33.6338**	**30.0011**	**20.8891**	**9.5408**	**1.1211**	**1.0459**
2002	**1.1123**	**1.4064**	**0.9263**	**1.2736**	**39.2530**	**35.2907**	**30.8213**	**21.1543**	**10.2583**	**1.1450**	**1.0213**
2003	**1.1247**	**1.4533**	**0.9845**	**1.3254**	**41.9800**	**37.3245**	**31.6742**	**21.3487**	**11.1272**	**1.1784**	**1.0292**
2004	**1.1237**	**1.5054**	**0.8424**	**1.3097**	**44.0740**	**39.2210**	**33.6512**	**22.4711**	**12.1625**	**1.1655**	**0.9891**
2005	**1.1637**	**1.5510**	**0.8327**	**1.3363**	**46.2710**	**39.7610**	**34.6255**	**22.7391**	**13.2126**	**1.1483**	**0.9852**
2006	**1.2083**	**1.5873**	**1.0015**	**1.4253**	**51.2770**	**42.4387**	**35.9758**	**23.4196**	**14.0816**	**1.1796**	**1.0273**
2007	**1.2371**	**1.6829**	**0.9701**	**1.4747**	**55.9310**	**45.2119**	**37.9275**	**24.3947**	**15.3353**	**1.1921**	**1.0105**
2008	**1.2963**	**1.7108**	**0.9816**	**1.4972**	**58.5150**	**45.1405**	**39.0828**	**24.9338**	**16.1570**	**1.1550**	**0.9689**
2009	**1.3053**	**1.7340**	**0.9633**	**1.5043**	**59.9250**	**45.9080**	**39.8355**	**25.1636**	**16.9118**	**1.1524**	**0.9978**
2010	**1.3619**	**1.8167**	**0.9693**	**1.5591**	**63.9010**	**46.9216**	**40.9860**	**25.8765**	**17.4257**	**1.1448**	**0.9934**
2011	**1.4156**	**1.8938**	**1.1118**	**1.6687**	**68.7070**	**48.5373**	**41.1737**	**25.8117**	**17.8328**	**1.1788**	**1.0297**
2012	**1.4002**	**1.9114**	**1.1107**	**1.6792**	**70.1060**	**50.0689**	**41.7486**	**25.9796**	**18.4113**	**1.1993**	**1.0174**
2013	**1.3884**	**1.9602**	**1.0465**	**1.6807**	**70.3910**	**50.7002**	**41.8825**	**25.7321**	**19.0642**	**1.2105**	**1.0094**
2014	**1.3618**	**1.9540**	**1.0153**	**1.6626**	**71.4970**	**52.5032**	**43.0039**	**26.2624**	**19.8760**	**1.2209**	**1.0086**
2015	**1.3623**	**2.1373**	**0.8195**	**1.6837**	**73.8790**	**54.2293**	**43.8799**	**26.7485**	**20.3899**	**1.2359**	**1.0123**
2016	**1.3753**	**2.0558**	**0.9573**	**1.6983**	**75.5720**	**54.9479**	**44.4975**	**27.0519**	**20.8492**	**1.2349**	**0.9992**
2017	**1.3635**	**2.0898**	**0.9349**	**1.7085**	**76.9060**	**56.4032**	**45.0151**	**27.2993**	**21.2390**	**1.2530**	**1.0147**
2018	**1.3949**	**2.1485**	**0.9242**	**1.7388**	**79.3810**	**56.9075**	**45.6521**	**27.4908**	**21.9838**	**1.2466**	**0.9949**
2019	**1.4430**	**2.1787**	**1.1269**	**1.8560**	**82.1900**	**56.9597**	**44.2828**	**26.3626**	**21.9664**	**1.2863**	**1.0319**
2020	**1.5128**	**2.1932**	**1.3519**	**1.9766**	**90.6360**	**59.9137**	**45.8538**	**27.2021**	**22.9148**	**1.3066**	**1.0158**
2021	**1.5311**	**2.3624**	**1.1896**	**1.9951**	**94.2920**	**61.5855**	**47.2619**	**27.8649**	**23.9277**	**1.3031**	**0.9973**
2022	**1.6897**	**2.5033**	**1.3447**	**2.1564**	**107.1270**	**63.4001**	**49.6785**	**29.4587**	**24.8264**	**1.2762**	**0.9794**

Over the 28 years 1995-2022, Value Added TFP growth (ΔTFP^t) averaged 0.89 percentage points per year.[40] However, TFP growth slowed down after 2007: over the 13 years 1995-2007, TFP growth averaged 1.38 percentage points per year while over the 15 years, 2008-2022, TFP growth averaged only 0.47 percentage points per year. Note also the spike in output price inflation in 2022 relative to 2021: a 10.4 percent increase.

The above data is not suitable for our particular application: we require data on the *gross output* produced by the Retail Trade sector along with data on the *intermediate inputs* used during each split year. The same publication that had data on value added productivity also has some information on the gross output productivity of the Australian Retail sector for the split years 1994-2022. Thus we obtained indexes of Retail sector real gross output from Table 16 in ABS (2025b) which we denote by QYG^t for split year t. From Table 18 in ABS (2025b), we obtained indexes of real intermediate input which we denote by QX_3^t for split year t. Table 19 in ABS (2025b) has estimates for the gross input cost shares for labour, capital services and intermediate inputs which we denote by SXG_1^t, SXG_2^t and SXG^t. However, this publication did not list the value of gross output for each year, VYG^t, or the values of Labour, Capital services and Intermediates (VX_1^t, VX_2^t and VX_3^t).[41]

Table A1 allows us to determine the values for the first two inputs (quality adjusted labour and capital services), VX_1^t and VX_2^t. We also know that $SXG_1^t = VX_1^t/VYG^t$ and $SXG_2^t = VX_2^t/VYG^t$ since the ABS sets the value of gross output VYG^t equal to the value of gross input VXG^t for each year t. Thus it should be the case that $VYG^t = VX_1^t/SXG_1^t$ and $VYG^t = VX_2^t/SXG_2^t$ for each year t. We calculated these two

[40] Our estimates of value added TFP coincide with the ABS estimates of MFP to two decimal places.
[41] ABS (2025b) does list estimates for gross output TFP which are listed as TFP_{ABS}^t in Table A7 below. Their estimates are very close to our estimates $TFPG^t$ listed in Table A2.

estimates for the value of gross output in year t and found that they coincided to two decimal places. We took the arithmetic average of the two estimates for our final estimate for the value of gross output, which is denoted by VYG^t in Table A2. We set the aggregate value of inputs (including the value of intermediate inputs) VXG^t equal to the value of gross output, VYG^t. Now the year t value of each input can be determined by multiplying the relevant cost share by the value of gross output. Thus, $VX_3^t = VYG^t \times SXG_3^t$. Recall that QX_1^t and QX_2^t (the quantities of labour and capital input) are listed in Table A1 along with the corresponding prices, PX_1^t and PX_2^t. Define $PX_3^t \equiv VX_3^t/QX_3^t$. As usual, we normalized the intermediate input prices so that $PX_3^{1994} = 1$ and $QX_3^{1994} = VX_3^{1994}$. The normalized series PX_3^t and QX_3^t are listed in Table A2 below. Above, we explained how the nominal and real gross output series, VYG^t and QYG^t, were constructed. Define the corresponding year t price level for gross output as $PYG^t \equiv VYG^t/QYG^t$ for years t = 1994,…,2022. Normalize the price and volume series for gross output so that $PYG^{1994} = 1$ and $QYG^{1994} = VYG^{1994}$. The normalized series PYG^t and QYG^t are also listed in Table A2 below.

We are now in a position to calculate Total Factor Productivity Indexes using Retail sector gross output as our output concept. There is only one output (QYG^t) so the new output growth index is QYG^t/QYG^{t-1}. Again, we chose to use chained Törnqvist input quantity indexes to aggregate the labour, capital and intermediate input quantities into an aggregate input QXG^t with corresponding aggregate input price index, PXG^t with $VXG^t = PXG^t \times QXG^t = VX_1^t + VX_2^t + VX_3^t = VYG^t$. The year t gross output level of Total Factor Productivity[42] for the Retail sector is defined as $TFPG^t \equiv QYG^t/QXG^t$ and TFP growth for year t = 1994,…,2022 is defined as $\Delta TFPG^t \equiv TFPG^t/TFPG^{t-1} = [QYG^t/QYG^{t-1}]/[QXG^t/QXG^{t-1}]$. $TFPG^t$ and $\Delta TFPG^t$ are listed in Table A2.

Table A2: Inputs, Outputs and Productivity Levels for Retail Sector Gross Output

Year t	PYG^t	PXG^t	PX_3^t	VYG^t	QYG^t	QXG^t	QX_3^t	$TFPG^t$	$\Delta TFPG^t$
1994	1.00000	1.00000	1.00000	54.51465	54.51465	54.51465	30.13065	1.00000	
1995	1.02526	1.03333	1.01005	56.43018	55.03976	54.61005	29.58482	1.00787	1.00787
1996	1.01683	1.01837	0.97175	57.45882	56.50791	56.42230	29.72568	1.00152	0.99370
1997	1.02112	1.02971	0.98585	58.25969	57.05445	56.57862	29.18866	1.00841	1.00688
1998	1.06170	1.08277	1.07318	61.25730	57.69744	56.57439	28.20704	1.01985	1.01135
1999	1.09928	1.11949	1.13989	66.08822	60.11936	59.03433	29.50999	1.01838	0.99856
2000	1.12100	1.15978	1.17857	69.85656	62.31624	60.23279	30.49601	1.03459	1.01592
2001	1.16924	1.23740	1.24454	74.52924	63.74153	60.23064	30.02501	1.05829	1.02291
2002	1.13097	1.20892	1.15133	73.41109	64.90962	60.72437	29.66846	1.06892	1.01005
2003	1.11807	1.21343	1.10694	76.00043	67.97453	62.63287	30.73371	1.08529	1.01531
2004	1.17729	1.27080	1.24131	83.30600	70.76081	65.55420	31.60527	1.07942	0.99460
2005	1.21384	1.29957	1.27280	85.00777	70.03209	65.41207	30.43438	1.07063	0.99185
2006	1.21113	1.31525	1.20377	87.99763	72.65763	66.90586	30.50481	1.08597	1.01433
2007	1.23306	1.34754	1.21609	93.79302	76.06546	69.60337	31.13427	1.09284	1.00633
2008	1.26533	1.35760	1.21136	98.66145	77.97299	72.67364	33.14152	1.07292	0.98177
2009	1.29063	1.38306	1.25877	102.66752	79.54831	74.23192	33.95586	1.07162	0.99879
2010	1.33370	1.42390	1.28378	108.83764	81.60587	76.43650	35.00350	1.06763	0.99628
2011	1.40089	1.52134	1.36824	117.59397	83.94206	77.29624	35.72980	1.08598	1.01719
2012	1.39871	1.53396	1.38344	118.93941	85.03514	77.53763	35.29842	1.09670	1.00987
2013	1.40557	1.55051	1.41867	117.85111	83.84561	76.00814	33.45405	1.10311	1.00585
2014	1.38841	1.54004	1.41778	117.69180	84.76723	76.42129	32.58248	1.10921	1.00553
2015	1.39447	1.55854	1.43334	122.52413	87.86429	78.61487	33.93825	1.11765	1.00761
2016	1.39745	1.56100	1.42032	126.12582	90.25406	80.79828	35.59335	1.11703	0.99944
2017	1.39063	1.56713	1.42170	130.09975	93.55473	83.01766	37.41571	1.12693	1.00886
2018	1.42766	1.60386	1.46645	136.91517	95.90164	85.36610	39.23367	1.12342	0.99689

[42] This terminology follows that of Jorgenson and Griliches (1967). It is the same concept as Multifactor Productivity.

2019	**1.47032**	**1.68203**	**1.50095**	**141.77831**	**96.42674**	**84.28991**	**39.70027**	**1.14399**	**1.01831**
2020	**1.51799**	**1.75441**	**1.52109**	**156.00509**	**102.77089**	**88.92184**	**42.97525**	**1.15574**	**1.01028**
2021	**1.55992**	**1.80061**	**1.59726**	**163.97504**	**105.11780**	**91.06660**	**43.62672**	**1.15430**	**0.99875**
2022	**1.68558**	**1.91946**	**1.66993**	**180.63480**	**107.16464**	**94.10715**	**44.01848**	**1.13875**	**0.98653**

Over the 28 years 1995-2022, Gross Output TFP growth ($\Delta TFPG^t$) averaged 0.47 percentage points per year.[43] However, TFP growth slowed down after 2007: over the 13 years 1995-2007, TFP growth averaged 0.69 percentage points per year while over the 15 years, 2008-2022, Gross Output TFP growth averaged only 0.28 percentage points per year. These average growth rates are approximately one half of the corresponding growth rates for Value Added TFP growth. This drop is due to the fact that the gross output value of inputs (which includes intermediate inputs) is about twice as big as the value of inputs excluding intermediates so the absolute amount of extra output produced due to technical progress is measured relative to an input base that is twice as big as the input base for value added productivity growth.

Looking ahead, we will use the price series for gross output PYG^t to value inventory stocks at the beginning and end of each year. To obtain estimates for beginning and end of year prices for inventory stocks, it proved to be useful to extend the series PYG^t back to 1993 and forward to 2023. We used the movements in the price series for value added output, PY^t for t =1993, 1994, 2022, 2023, in order to do these extensions. The resulting estimated prices end up as follows: $PYG^{1993} = 0.97227$ and $PYG^{2023} = 1.81582$. We take the average of PYG^{t-1} and PYG^t to form the price series PK_5^t which is an approximation to the price of inventories at the beginning of split year t. The series PK_5^t was normalized so that $PK_5^{1994} =1$ and it is listed in Table A5 below.

The data contained in Tables A1 and A2 are not suitable for estimating aggregate markups in the Australian Retail Trade sector: the ABS data were constructed so that the value of inputs equals the value of outputs for each year and thus markups m^t are equal to 0 for each year t. Thus we need to construct user costs for the various types of capital that the Retail Trade sector uses that are based on exogenous rates of return that reflect the opportunity cost of financial capital. Fortunately, the ABS does provide information on capital stocks used by the Retail sector and it also provides information on depreciation rates that are ingredients used to construct user costs.

From ABS (2025c), we obtained beginning of the split year t estimates for the current value of the net capital stock along for four types of capital for the Retail Trade sector for t = 1994,…,2025:

VK_1^t = Value of non-dwelling construction;
VK_2^t = Value of machinery and equipment;
VK_3^t = Value of Research and Development (R&D);
VK_4^t = Value of computer software.

ABS(2025c) also provided the corresponding chain volume estimates for these four capital stock components, QK_j^t for j = 1,…,4. Define the corresponding beginning of the year prices, $PK_j^t \equiv VK_j^t/QK_j^t$ for j = 1,…,4. As usual, we normalized the price and quantity series so that $PK_j^{1994} = 1$ and $QK_j^{1994} = VK_j^{1994}$ for j = 1,2,3,4. These price and quantity series are listed in Table A3. The current dollar units are in billions of Australian dollars.

[43] Our estimates of value added TFP coincide with the ABS (2025b) estimates of MFP to two decimal places.

Table A3: Retail Trade Capital Stock Prices and Quantities for Four Assets

Year t	PK_1^t	PK_2^t	PK_3^t	PK_4^t	QK_1^t	QK_2^t	QK_3^t	QK_4^t
1994	1.00000	1.00000	1.00000	1.00000	10.30200	10.57400	0.06400	0.34400
1995	1.03622	0.99321	1.05785	0.93615	10.33564	10.91010	0.06995	0.37921
1996	1.06704	0.96357	1.07842	0.87557	10.36889	11.59855	0.07789	0.40088
1997	1.09579	0.93519	1.09726	0.81575	10.42898	11.92272	0.08384	0.45235
1998	1.12840	0.92949	1.13379	0.79111	10.68862	12.61876	0.08732	0.53090
1999	1.16746	0.91282	1.17578	0.78623	11.21924	13.31589	0.08930	0.58507
2000	1.20521	0.90696	1.25417	0.79134	11.82695	14.35020	0.08930	0.64195
2001	1.22777	0.86453	1.29263	0.77955	12.17162	15.20128	0.09129	0.70425
2002	1.25850	0.85583	1.32661	0.73049	12.69127	16.15969	0.09724	0.76113
2003	1.32045	0.82516	1.33089	0.65608	13.33338	17.36747	0.10369	0.89928
2004	1.40783	0.79172	1.32607	0.60256	14.00307	19.14444	0.11312	1.04554
2005	1.49622	0.77769	1.34910	0.58405	14.50988	21.62179	0.12453	1.17285
2006	1.59137	0.77076	1.39770	0.57244	14.97511	24.44716	0.13594	1.25953
2007	1.69037	0.75783	1.44727	0.54935	15.26233	26.83452	0.17067	1.35433
2008	1.78104	0.75755	1.50064	0.53868	15.50383	30.11416	0.18059	1.45726
2009	1.81700	0.76387	1.55447	0.52070	15.73475	32.74113	0.19299	1.51143
2010	1.82697	0.74105	1.57543	0.48916	15.84095	34.26332	0.21581	1.83376
2011	1.86745	0.71575	1.59243	0.46847	16.08131	34.96044	0.22921	2.07483
2012	1.90301	0.70405	1.62262	0.45734	16.17126	35.45483	0.25699	2.47301
2013	1.94103	0.70975	1.63246	0.45145	16.25062	36.34819	0.31057	3.03912
2014	1.97994	0.72397	1.64679	0.44515	16.64594	37.67956	0.38499	3.56731
2015	2.00668	0.74274	1.66419	0.44060	17.14178	39.28415	0.44105	3.54293
2016	2.02609	0.74576	1.67378	0.43153	17.58018	40.00730	0.47975	3.67294
2017	2.05940	0.73446	1.67936	0.41498	18.37724	40.80525	0.51150	3.55918
2018	2.11423	0.73850	1.69365	0.40366	19.11949	41.28771	0.53730	3.77046
2019	2.16257	0.75572	1.72157	0.39847	19.98684	42.53669	0.56112	4.06299
2020	2.18862	0.77306	1.75504	0.38703	20.53484	43.11672	0.57947	4.48283
2021	2.26039	0.80295	1.79311	0.37548	20.97248	45.09969	0.61569	5.39836
2022	2.42345	0.86442	1.87532	0.37094	21.38027	46.65440	0.66282	7.96346
2023	2.58526	0.91210	1.97392	0.36937	21.60211	48.38150	0.71938	10.85361
2024	2.69676	0.92518	2.05749	0.37091	21.80431	49.58168	0.78834	12.21065
2025	2.78019	0.93976	2.14247	0.36122	22.12328	50.51950	0.86722	13.43225

The corresponding values for the four capital stock components are listed in Table A4 below as VK_1^t-VK_4^t along with the corresponding ABS total capital stock, VK_{ABS}^t.

From ABS (2025d), information on depreciation in current dollars for the four assets in the Retail Trade sector can be obtained. These depreciation estimates are divided by the corresponding beginning of the period value of the capital stock to give us depreciation rates d_j^t by year t and asset j. These depreciation rates are listed in Table A4.

Table A4: Beginning of the Year Capital Stock Values and Depreciation Rates for Four Assets

Year t	VK_1^t	VK_2^t	VK_3^t	VK_4^t	VK_{ABS}^t	d_1^t	d_2^t	d_3^t	d_4^t
1994	10.30200	10.57400	0.06400	0.34400	21.28400	0.03427	0.13268	0.18750	0.42442
1995	10.71000	10.83600	0.07400	0.35500	21.97400	0.03455	0.13372	0.18919	0.42535
1996	11.06400	11.17600	0.08400	0.35100	22.67500	0.03480	0.12643	0.19048	0.43020
1997	11.42800	11.15000	0.09200	0.36900	23.03800	0.03535	0.13193	0.18478	0.44173
1998	12.06100	11.72900	0.09900	0.42000	24.30900	0.03549	0.13198	0.19192	0.43571
1999	13.09800	12.15500	0.10500	0.46000	25.81800	0.03512	0.13015	0.20000	0.44130
2000	14.25400	13.01500	0.11200	0.50800	27.88800	0.03410	0.12647	0.19643	0.44488
2001	14.94400	13.14200	0.11800	0.54900	28.75400	0.03379	0.13149	0.20339	0.43534
2002	15.97200	13.83000	0.12900	0.55600	30.48700	0.03393	0.12979	0.20155	0.43525

2003	**17.60600**	**14.33100**	**0.13800**	**0.59000**	**32.66500**	**0.03374**	**0.12623**	**0.19565**	**0.42034**
2004	**19.71400**	**15.15700**	**0.15000**	**0.63000**	**35.65100**	**0.03333**	**0.12918**	**0.19333**	**0.42698**
2005	**21.71000**	**16.81500**	**0.16800**	**0.68500**	**39.37800**	**0.03275**	**0.12774**	**0.19643**	**0.43212**
2006	**23.83100**	**18.84300**	**0.19000**	**0.72100**	**43.58400**	**0.03277**	**0.12498**	**0.20526**	**0.43273**
2007	**25.79900**	**20.33600**	**0.24700**	**0.74400**	**47.12600**	**0.03244**	**0.12284**	**0.18219**	**0.43414**
2008	**27.61300**	**22.81300**	**0.27100**	**0.78500**	**51.48300**	**0.03256**	**0.12462**	**0.18819**	**0.43949**
2009	**28.59000**	**25.01000**	**0.30000**	**0.78700**	**54.68700**	**0.03151**	**0.11847**	**0.19000**	**0.44981**
2010	**28.94100**	**25.39100**	**0.34000**	**0.89700**	**55.56900**	**0.03234**	**0.11642**	**0.18529**	**0.43032**
2011	**30.03100**	**25.02300**	**0.36500**	**0.97200**	**56.39100**	**0.03230**	**0.11805**	**0.19452**	**0.44342**
2012	**30.77400**	**24.96200**	**0.41700**	**1.13100**	**57.28400**	**0.03246**	**0.12078**	**0.19664**	**0.45004**
2013	**31.54300**	**25.79800**	**0.50700**	**1.37200**	**59.22000**	**0.03291**	**0.12315**	**0.19329**	**0.43732**
2014	**32.95800**	**27.27900**	**0.63400**	**1.58800**	**62.45900**	**0.03280**	**0.12288**	**0.18297**	**0.41058**
2015	**34.39800**	**29.17800**	**0.73400**	**1.56100**	**65.87100**	**0.03244**	**0.12191**	**0.17984**	**0.43754**
2016	**35.61900**	**29.83600**	**0.80300**	**1.58500**	**67.84300**	**0.03254**	**0.11872**	**0.18057**	**0.42145**
2017	**37.84600**	**29.97000**	**0.85900**	**1.47700**	**70.15200**	**0.03232**	**0.12009**	**0.18393**	**0.43940**
2018	**40.42300**	**30.49100**	**0.91000**	**1.52200**	**73.34600**	**0.03218**	**0.12236**	**0.18681**	**0.44152**
2019	**43.22300**	**32.14600**	**0.96600**	**1.61900**	**77.95400**	**0.03149**	**0.12225**	**0.19048**	**0.43916**
2020	**44.94300**	**33.33200**	**1.01700**	**1.73500**	**81.02700**	**0.03128**	**0.12345**	**0.19469**	**0.44092**
2021	**47.40600**	**36.21300**	**1.10400**	**2.02700**	**86.75000**	**0.03202**	**0.12554**	**0.19746**	**0.48298**
2022	**51.81400**	**40.32900**	**1.24300**	**2.95400**	**96.34000**	**0.03248**	**0.12782**	**0.20032**	**0.44990**
2023	**55.84700**	**44.12900**	**1.42000**	**4.00900**	**105.40500**	**0.03212**	**0.12318**	**0.19789**	**0.41257**
2024	**58.80100**	**45.87200**	**1.62200**	**4.52900**	**110.82200**	**0.03224**	**0.12195**	**0.19605**	**0.42349**

The problem with the above ABS capital stock data is that two important capital stocks for the Retail Trade sector are missing. These two assets are inventory stocks and the stock of land that retail structures sit on.

From ABS (2025e), we can obtain quarterly estimates for the value of inventories in the Retail Trade sector in current dollars by quarter in billions of dollars. We used this series to form the beginning of split year t series VK_5^t for the value of inventories held by the Retail Trade sector. Recall that we defined a beginning of split year t price of inventories series PK_5^t earlier. Define $QK_5^t \equiv VK_5^t/PK_5^t$ for t = 1994,…,2022. The series VK_5^t, PK_5^t and QK_5^t are listed in Table A5.

Obtaining official estimates for the price and value of Retail Trade land proved to be difficult. We were able to obtain an (unpublished) Retail Land price series from the ABS and we list this series as PK_6^t in Table A5. We were not able to obtain value or quantity series for Retail Trade land in Australia so we assumed that the quantity of land was proportional to the quantity of Retail Trade structures. Finally, we assumed that the value of Retail Trade land at the beginning of split year 1994 was equal to one half of the value of retail structures. The resulting very rough estimates for beginning of split year t Retail trade price, quantity and value of land are listed in Table A5 as PK_6^t, PQ_6^t and VK_6^t. The depreciation rates for assets 5 and 6 are set equal to 0; i.e., $d_5^t \equiv d_6^t \equiv 0$ for all t.

Table A5: Retail Trade Prices, Quantities and Values for Inventories and Land

Year t	PK_5^t	PK_6^t	QK_5^t	VK_5^t	QK_6^t	VK_6^t
1994	**1.000000**	**1.000000**	**14.610000**	**14.610000**	**5.151000**	**5.151000**
1995	**1.026869**	**1.069293**	**15.982567**	**16.412000**	**5.167818**	**5.525912**
1996	**1.035401**	**1.136214**	**16.116461**	**16.687000**	**5.184447**	**5.890643**
1997	**1.033303**	**1.184255**	**17.100501**	**17.670000**	**5.214492**	**6.175289**
1998	**1.056054**	**1.237186**	**16.452758**	**17.375000**	**5.344311**	**6.611905**
1999	**1.095683**	**1.318552**	**17.573514**	**19.255000**	**5.609618**	**7.396573**
2000	**1.125751**	**1.414396**	**17.677977**	**19.901000**	**5.913474**	**8.363992**
2001	**1.161222**	**1.487705**	**19.538902**	**22.689000**	**6.085810**	**9.053890**
2002	**1.166278**	**1.522504**	**18.873709**	**22.012000**	**6.345637**	**9.661259**
2003	**1.140334**	**1.547016**	**20.580812**	**23.469000**	**6.666689**	**10.313477**

2004	**1.163818**	**1.604183**	**20.771294**	**24.174000**	**7.001535**	**11.231744**
2005	**1.212375**	**1.701877**	**21.407566**	**25.954000**	**7.254938**	**12.347009**
2006	**1.229532**	**1.852817**	**22.555746**	**27.733000**	**7.487554**	**13.873065**
2007	**1.239275**	**2.018263**	**22.928732**	**28.415000**	**7.631167**	**15.401699**
2008	**1.266756**	**2.134524**	**24.336959**	**30.829000**	**7.751916**	**16.546651**
2009	**1.295948**	**2.184333**	**23.097370**	**29.933000**	**7.867373**	**17.184965**
2010	**1.330614**	**2.210173**	**23.626684**	**31.438000**	**7.920473**	**17.505619**
2011	**1.386521**	**2.278352**	**23.323123**	**32.338000**	**8.040654**	**18.319438**
2012	**1.419483**	**2.393868**	**22.503261**	**31.943000**	**8.085628**	**19.355923**
2013	**1.421855**	**2.495808**	**22.728053**	**32.316000**	**8.125311**	**20.279217**
2014	**1.416634**	**2.567818**	**23.088531**	**32.708000**	**8.322968**	**21.371866**
2015	**1.411005**	**2.617641**	**24.521533**	**34.600000**	**8.570891**	**22.435516**
2016	**1.415589**	**2.662514**	**25.072246**	**35.492000**	**8.790090**	**23.403742**
2017	**1.413641**	**2.728438**	**25.984680**	**36.733000**	**9.188618**	**25.070571**
2018	**1.428958**	**2.826362**	**25.946184**	**37.076000**	**9.559745**	**27.019302**
2019	**1.469365**	**2.936665**	**26.733995**	**39.282000**	**9.993420**	**29.347330**
2020	**1.515163**	**3.047451**	**22.335545**	**33.842000**	**10.267419**	**31.289461**
2021	**1.560591**	**3.247877**	**24.361923**	**38.019000**	**10.486241**	**34.058025**
2022	**1.645566**	**3.550947**	**26.221984**	**43.150000**	**10.690134**	**37.960103**
2023	**1.775314**	**3.931967**	**25.320025**	**44.951000**	**10.801057**	**42.469403**

With estimates for the 6 capital stock components in hand, we can construct a Retail Trade capital stock aggregate for year t, $VK^t = \Sigma_{n=1}^{6} VK_n^t$ and compare it to the corresponding ABS year t reproducible capital stock aggregate VK_A^t from ABS (2025c) which does not include inventories or land. VK^t and VK_A^t are listed in Table A6 for the split years 1994-2022; the units are in billions of Australian Dollars. ABS (2025c) also has estimates for the real reproducible capital stock, QK_A^t, which are also listed in Table A6. We constructed a chained Tornqvist price series for the total capital stock, PK^t, and the corresponding capital stock volume index, $QK^t \equiv VK^t/PK^t$ for t = 1994,…,2022. From Table A1, we can find year t estimates for Retail Trade nominal and real output, VY^t and QY^t, which are also listed in Table A6. The nominal capital output ratio using our 6 asset value of capital stocks for year t is VK^t/VY^t and the nominal capital output ratio using the ABS 4 asset value of capital stocks for year t is VK_A^t/VY^t. The corresponding real capital output ratios for year t are QK^t/QY^t and QK_A^t/QY^t. These series are also listed in Table A6. Note that QY^t is the volume of value added output in year t; i.e., it is not the volume of gross output.

Table A6: Retail Trade Total and Reproducible Capital Stocks and Real and Nominal Capital Output Ratios 1994-2022

Year t	VY^t	QY^t	VK^t	QK^t	VK_A^t	QK_A^t	VK^t/VY^t	QK^t/QY^t	VK_A^t/VY^t	QK_A^t/QY^t
1994	**54.51465**	**54.51465**	**41.04500**	**41.04500**	**21.28400**	**21.28400**	**0.75292**	**0.75292**	**0.39043**	**0.39043**
1995	**56.43017**	**55.03976**	**43.91292**	**42.84041**	**21.97399**	**21.69943**	**0.77818**	**0.77835**	**0.38940**	**0.39425**
1996	**57.45883**	**56.50791**	**45.25264**	**43.70711**	**22.67500**	**22.44490**	**0.78757**	**0.77347**	**0.39463**	**0.39720**
1997	**58.25967**	**57.05445**	**46.88430**	**45.12971**	**23.03800**	**22.87382**	**0.80475**	**0.79099**	**0.39544**	**0.40091**
1998	**61.25732**	**57.69744**	**48.29592**	**45.59208**	**24.30901**	**23.87419**	**0.78841**	**0.79019**	**0.39683**	**0.41378**
1999	**66.08819**	**60.11936**	**52.46959**	**48.23872**	**25.81801**	**25.14425**	**0.79393**	**0.80238**	**0.39066**	**0.41824**
2000	**69.85657**	**62.31624**	**56.15397**	**50.26880**	**27.88801**	**26.80341**	**0.80385**	**0.80667**	**0.39922**	**0.43012**
2001	**74.52927**	**63.74153**	**60.49591**	**53.47369**	**28.75400**	**27.99320**	**0.81171**	**0.83891**	**0.38581**	**0.43917**
2002	**73.41110**	**64.90962**	**62.16027**	**54.47528**	**30.48699**	**29.47386**	**0.84674**	**0.83925**	**0.41529**	**0.45408**
2003	**76.00042**	**67.97453**	**66.44748**	**58.34223**	**32.66501**	**31.36674**	**0.87430**	**0.85830**	**0.42980**	**0.46145**
2004	**83.30600**	**70.76081**	**71.05675**	**61.12372**	**35.65101**	**33.75403**	**0.85296**	**0.86381**	**0.42795**	**0.47702**
2005	**85.00775**	**70.03209**	**77.67902**	**64.45579**	**39.37800**	**36.40650**	**0.91379**	**0.92038**	**0.46323**	**0.51985**
2006	**87.99761**	**72.65763**	**85.19104**	**68.36065**	**43.58401**	**39.17263**	**0.96811**	**0.94086**	**0.49529**	**0.53914**
2007	**93.79305**	**76.06546**	**90.94266**	**70.84009**	**47.12599**	**41.35702**	**0.96961**	**0.93130**	**0.50245**	**0.54370**
2008	**98.66141**	**77.97299**	**98.85766**	**74.66670**	**51.48302**	**44.00243**	**1.00199**	**0.95760**	**0.52182**	**0.56433**
2009	**102.66752**	**79.54831**	**101.80497**	**75.50673**	**54.68698**	**46.10271**	**0.99160**	**0.94919**	**0.53266**	**0.57956**
2010	**108.83767**	**81.60587**	**104.51263**	**77.24573**	**55.56901**	**47.42092**	**0.96026**	**0.94657**	**0.51057**	**0.58110**
2011	**117.59393**	**83.94206**	**107.04845**	**77.93958**	**56.39101**	**48.35901**	**0.91032**	**0.92849**	**0.47954**	**0.57610**
2012	**118.93942**	**85.03514**	**108.58291**	**77.72606**	**57.28402**	**49.01009**	**0.91293**	**0.91405**	**0.48162**	**0.57635**

2013	**117.85113**	**83.84561**	**111.81521**	**78.82343**	**59.21999**	**49.97708**	**0.94878**	**0.94010**	**0.50250**	**0.59606**
2014	**117.69184**	**84.76723**	**116.53889**	**80.99340**	**62.45899**	**51.72998**	**0.99020**	**0.95548**	**0.53070**	**0.61026**
2015	**122.52411**	**87.86429**	**122.90652**	**84.38913**	**65.87098**	**53.57150**	**1.00312**	**0.96045**	**0.53762**	**0.60971**
2016	**126.12581**	**90.25406**	**126.73878**	**86.37321**	**67.84302**	**54.82357**	**1.00486**	**0.95700**	**0.53790**	**0.60744**
2017	**130.09973**	**93.55473**	**131.95558**	**89.49308**	**70.15201**	**56.60666**	**1.01426**	**0.95659**	**0.53922**	**0.60506**
2018	**136.91522**	**95.90164**	**137.44127**	**91.51592**	**73.34601**	**58.22665**	**1.00384**	**0.95427**	**0.53570**	**0.60715**
2019	**141.77827**	**96.42674**	**146.58331**	**95.02206**	**77.95400**	**60.53946**	**1.03389**	**0.98543**	**0.54983**	**0.62783**
2020	**156.00508**	**102.77089**	**146.15847**	**92.51702**	**81.02700**	**61.95848**	**0.93688**	**0.90023**	**0.51939**	**0.60288**
2021	**163.97504**	**105.11780**	**158.82701**	**96.70946**	**86.75002**	**64.18781**	**0.96860**	**0.92001**	**0.52904**	**0.61063**
2022	**180.63479**	**107.16464**	**177.45012**	**100.80589**	**96.34002**	**66.59244**	**0.98237**	**0.94066**	**0.53334**	**0.62140**

It can be seen that there are substantial differences in the ABS capital output ratios (which include only the 4 reproducible capital stock assets) and our capital output ratios (which include all six types of capital). Our starting capital output ratios are 92.8% higher than the corresponding ABS capital output ratios. In 2022, our nominal capital output ratio, 0.98247, is 84.2% higher than the ABS nominal capital output ratio (0.53334). In 2022, our real capital output ratio, 0.94066, is 51.4% higher than the ABS real capital output ratio (0.62140). All of these capital output ratios indicate that the Retail Trade sector is not very capital intensive. The nominal capital output ratio is important because it tells us (on average) how much financial capital is required to produce one current dollar of output. A higher nominal capital output ratio means that it is more expensive for new firms to enter the industry. A high capital output ratio acts as a barrier to entry.[44]

We turn our attention to the problems associated with the construction of prices for capital services. By definition, a capital input or durable input is an input whose services can be used for multiple accounting periods. Thus it does not make sense to charge the entire cost of the durable good to the period of purchase: the initial purchase price must be allocated or distributed over the useful lifetime of the good. This is *the fundamental problem of accounting*.[45] In order to solve this intertemporal cost allocation problem, economists have measured the services of a durable input over an accounting period by its *user cost*.

A simple way to explain the user cost concept proceeds as follows. Suppose a producer purchases a unit of a durable input at the beginning of period t at the price P^t. In order to finance this purchase, the producer borrows the amount P^t at the interest rate r^t. At the end of the accounting period, the producer (hypothetically) sells the depreciated asset at the price P^{t+1*}. Thus the period t user cost for the asset, u^t, is the cost of the purchase of the asset at the beginning of the period less the imputed value of the asset at the end of the period; i.e., we have:

(A1) $u^t \equiv P^t(1+r^t) - P^{t+1*}$.

We want to relate the end of period value of the asset, P^{t+1*}, to the end of period price of a "new" asset of the same quality as the asset that was purchased at the beginning of period t. Call this price P^{t+1}. Define the "new" asset *period t inflation rate* i^t by the following equation:

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[44] These considerations indicate the importance of including land and inventories as important assets that make up the productive capital stock. There is another important asset which probably should be included in the list of productive assets and that is monetary holdings.

[45] "Before we begin with our algebra, it seems appropriate to explain why accounting for the contribution of capital to production is more difficult than accounting for the contributions of labour or materials. The main problem is that when a reproducible capital input is purchased for use by a production unit at the beginning of an accounting period, we cannot simply charge the entire purchase cost to the period of purchase. Since the benefits of using the capital asset extend over more than one period, the initial purchase cost must be distributed somehow over the useful life of the asset. This is the fundamental problem of accounting." Diewert (2005; 481).

(A2) $1+i^t = P^{t+1}/P^t$.

We can relate the end of period value P^{t+1*} to the end of period t price of a "new" asset P^{t+1} by assuming that the asset *depreciates* or deteriorates at the rate δ^t over period t so that the following equation holds:

(A3) $(1-\delta^t) = P^{t+1*}/P^{t+1}$.

Using (A2) and (A3), we can deduce that $P^{t+1*} = (1+i^t)(1-\delta^t)P^t$. Substituting this equation into definition (A1) gives us the following formula for the period t user cost for the asset:[46]

(A4) $u^t = P^t[(1+r^t) - (1+i^t)(1-\delta^t)] = [r^t - i^t + (1+i^t)\delta^t]P^t$.

If the asset inflation rate i^t is equal to 0, then the user cost formula simplifies to $u^t = (r^t+\delta^t)P^t$ which is a user cost formula that dates back to Walras (1954; 269).[47]

We note that the above justification for the user cost formula (A4) does not apply to a sunk cost asset like Software and R&D. We agree with Pitzer (2004) that R&D investments are fundamentally different from investments in reproducible capital. Pitzer (2004; 2) regarded R&D investments as producing "recipes" but a recipe is not the usual type of "productive" input like machine services or structure services. The value of a new recipe is the discounted stream of incremental profits that the recipe is expected to produce before it becomes obsolete. The period t cost of a new R&D project needs to be distributed (or amortized) over the useful life of the project. Period by period depreciation charges are replaced by period by period amortization charges. The industrial engineer Church (1917; 193) and the accountants Hatfield (1927; 18) and Paton and Littleton (1940; 123) argued that the amortization charges should be *matched* (i.e., made proportional) to the incremental profits. Thus the depreciation rates δ^t in the user cost formula (A4) are to be interpreted as amortization rates. In general, accountants will not have accurate information on incremental profits so more or less arbitrary assumptions are made about these amortization rates. We simply use the ABS depreciation rates as amortization rates for Software and R&D stocks.[48]

We use a slight modification of the user cost formula (A4) to calculate user costs for our 6 capital stock assets: we add an asset specific tax rate τ_n^t as an additional cost of using asset n for our list of 6 assets. Thus our user cost formula for asset n in period t has the following form:

(A5) $u_n^t = PK_n^t[(1+r^t+\tau_n^t) - (1+i^t)(1-\delta_n^t)] = [r^t + \tau_n^t - i_n^t + (1+i_n^t)\delta_n^t]\, PK_n^t$; n = 1,…,6 ; t = 1994,…,2022.

We assume that $\tau_n^t = 0$ for assets n = 1,…,5 for all t and we make a very rough estimate that the land tax rate on Retail Trade establishments is equal to one percent so that $\tau_6^t = 0.01$ for all years t. With these assumptions, all of the variables in the user costs u_n^t are known, with the exception of the cost of financial capital r^t. The strategy used by Jorgenson and Griliches (1967) was to determine the r^t in their particular context by setting the value of capital services equal to the production unit's gross operating surplus which

[46] The technique of specializing Hicks' (1946; 193-194]) general intertemporal production framework in order to derive this user cost formula was used repeatedly by Diewert (1974; 503-504), (1980; 470-475) (2014).
[47] For alternative derivations of a user cost formula, see Jorgenson (1963) (1989), Hall and Jorgenson (1967), Jorgenson and Griliches (1967) and Christensen and Jorgenson (1969).
[48] There are other complications associated with forming real investments in R&D; see Diewert and Huang (2011a) (2011b).

in our context is the value of gross output less the value of labour and intermediate input services. Thus for each year t, the Jorgenson and Griliches *ex post balancing rate of return on assets*, r_J^t, can be determined by solving the following equation:

(A6) $VYG^t - VX_1^t - VX_3^t = \Sigma_{n=1}^6 [r_J^t + \tau_n^t - i_n^t + (1+i_n^t)\delta_n^t] PK_n^t QK_n^t$; t = 1994,…,2022.

These Jorgenson and Griliches balancing rates of return r_J^t are listed in Table A7 below. Once the r_J^t are known, the Jorgensonian user costs[49] u_{Jn}^t can be defined using equations (A5).

There is only one output (gross output QYG^t listed in Table A2) so the new output growth index is QYG^t/QYG^{t-1}. Again, in order to be consistent with the Diewert and Morrison (1986) translog methodology, we chose to use chained Törnqvist input quantity indexes to aggregate the labour, 6 types of capital services and intermediate input quantities into an aggregate input QX_J^t, which is listed in Table A7. The year t gross output level of Jorgensonian Total Factor Productivity for the Retail sector is defined as $TFP_J^t \equiv QYG^t/QX_J^t$ and Jorgensonian TFP growth for year t = 1994,…,2022 is defined as $\Delta TFP_J^t \equiv TFP_J^t/TFP_J^{t-1}$. TFP_J^t (an aggregate of 8 inputs), is compared to the ABS (2025b) estimates for Multifactor Productivity, TFP_{ABS}^t (an aggregate of 6 inputs) in Table A7. The Jorgensonian user costs u_{Jn}^t were normalized to equal 1 in the split year 1994 and they are also listed in Table A7.

Table A7: Jorgensonian User Costs, Rates of Return and Gross Output TFP Levels; ABS TFP Levels

Year t	r_J^t	u_{J1}^t	u_{J2}^t	u_{J3}^t	u_{J4}^t	u_{J5}^t	u_{J6}^t	TFP_J^t	TFP_{ABS}^t	QX_J^t
1994	0.1338	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	54.5147
1995	0.1274	1.0371	1.0463	1.1600	0.9282	1.1433	1.0733	1.0083	1.0079	55.0398
1996	0.1125	0.9724	0.9363	1.1356	0.8559	1.1089	1.2232	1.0040	1.0015	56.5079
1997	0.1133	0.9871	0.8600	1.0836	0.7840	0.8816	1.2485	1.0133	1.0084	57.0545
1998	0.1287	1.1087	0.9426	1.2014	0.7551	0.9002	1.2106	1.0312	1.0199	57.6974
1999	0.1204	1.0907	0.8585	1.1449	0.7375	0.9528	1.0218	1.0308	1.0184	60.1194
2000	0.1095	1.1365	0.9218	1.2860	0.7483	0.8210	1.2843	1.0483	1.0346	62.3162
2001	0.1120	1.1218	0.8006	1.3875	0.7637	1.1689	1.9687	1.0696	1.0583	63.7415
2002	0.1088	0.8999	0.8476	1.4884	0.7386	1.4291	2.0982	1.0863	1.0689	64.9096
2003	0.1449	1.1376	0.9286	1.6663	0.6754	1.3253	2.4482	1.1042	1.0853	67.9745
2004	0.1389	1.1792	0.8239	1.5381	0.5909	1.0570	1.8933	1.1021	1.0794	70.7608
2005	0.1333	1.1751	0.7675	1.4793	0.5661	1.3507	1.2471	1.0944	1.0706	70.0321
2006	0.1536	1.5083	0.8297	1.6846	0.5861	1.6743	1.8463	1.1082	1.0860	72.6576
2007	0.1525	1.6896	0.7670	1.6067	0.5518	1.5102	2.8409	1.1216	1.0928	76.0655
2008	0.1351	1.9814	0.7019	1.6090	0.5371	1.3267	3.4863	1.0990	1.0729	77.9730
2009	0.1210	2.0094	0.7454	1.7005	0.5287	1.1422	3.4932	1.1005	1.0716	79.5483
2010	0.1346	1.9976	0.7651	1.7869	0.4843	1.1523	3.3746	1.0961	1.0676	81.6059
2011	0.1597	2.4357	0.7678	1.9679	0.4853	1.7627	3.6392	1.1164	1.0860	83.9421
2012	0.1593	2.4649	0.7055	2.0765	0.4738	2.0919	4.0698	1.1326	1.0967	85.0351
2013	0.1463	2.3308	0.6563	1.9788	0.4488	1.9940	4.2685	1.1418	1.1031	83.8456
2014	0.1381	2.3481	0.6333	1.8757	0.4150	1.8821	4.4349	1.1497	1.1092	84.7672
2015	0.0969	1.8083	0.5868	1.6500	0.4043	1.2352	3.1516	1.1559	1.1177	87.8643
2016	0.1174	2.0398	0.6830	1.8013	0.4069	1.5718	3.6660	1.1551	1.1170	90.2541
2017	0.1241	2.0218	0.6455	1.8434	0.4037	1.4974	3.5963	1.1632	1.1269	93.5547
2018	0.1295	2.2164	0.6273	1.8701	0.3923	1.3523	3.8101	1.1617	1.1234	95.9016
2019	0.1484	2.7325	0.6949	2.0277	0.4043	1.6101	4.7538	1.1755	1.1440	96.4267
2020	0.2039	3.3444	0.8328	2.4383	0.4303	2.4638	6.0569	1.2000	1.1557	102.7709
2021	0.2007	2.7659	0.7643	2.3620	0.4354	2.1338	5.1150	1.1972	1.1543	105.1178

[49] We attribute these user costs to Jorgenson because he firmly believed that the use of ex post inflation rates in the user cost formula was the best way to define user costs; see Jorgenson (1989) and Christensen and Jorgenson (1969).

2022	**0.2107**	**3.2522**	**0.9217**	**2.5227**	**0.4133**	**2.0294**	**5.4053**	**1.1773**	**1.1388**	**107.1646**
Mean	**0.1384**	**1.7860**	**0.7952**	**1.6680**	**0.5843**	**1.4334**	**2.9216**	**1.1050**	**1.0799**	**69.5390**

The balancing rate of return on Retail Trade assets, r_J^t, varied from a low of 9.7% in 2015 to a high of 21.1% in 2022. The average rate was 13.8%. Jorgensonian gross output TFP ended up growing 17.7% over the sample period while the ABS estimated gross output TFP ended up growing 13.9%. Thus adding inventory stocks and land to the list of assets did lead to a somewhat higher rate of TFP growth. The average rate of TFP growth over the 28 years 1995-2022 was 0.59 percentage points per year for ΔTFP_J^t and 0.47 for ΔTFP_{ABS}^t. However, there was a significant productivity slowdown over the sample period. Over the 13 years, 1995-2007, average Jorgensonian TFP growth was 0.89 percentage points per year while ABS average growth was 0.69. Over the 15 years 2008-2022, average Jorgensonian TFP growth was 0.33 percentage points per year while ABS average growth was 0.28 percentage points per year.

The user costs for assets 1, 5 and 6 (structures, inventories and land) were very volatile; see Table A7. This is due to the fact that depreciation rates for these assets are close to zero so the volatile ex post asset inflation rates i_1^t, i_5^t and i_6^t cause the corresponding Jorgensonian user costs u_{J1}^t, u_{j5}^t and u_{J6}^t to fluctuate. We would like our user costs to approximate annual rental prices for these assets. But rental prices for assets do not (in general) fluctuate wildly. In order to obtain user costs that better approximate rental prices, we smoothed the asset inflation rates i_n^t [50] and inserted these smoothed rates (i_n^{t*}) into the relevant user cost formula (A5) in order to obtain new smoothed user costs, u_{Sn}^t. These smoothed asset inflation rates i_n^{t*} are listed in Table A8.

Table A8: Original and Smoothed Asset Inflation Rates

Year	i_1^t	i_1^{t*}	i_2^t	i_2^{t*}	i_3^t	i_3^{t*}	i_4^t	i_4^{t*}	i_5^t	i_5^{t*}	i_6^t	i_6^{t*}
1994	**0.0362**	**0.0321**	**-0.0068**	**-0.0163**	**0.0578**	**0.0369**	**-0.0638**	**-0.0619**	**0.0269**	**0.0164**	**0.0693**	**0.0634**
1995	**0.0297**	**0.0315**	**-0.0298**	**-0.0171**	**0.0195**	**0.0366**	**-0.0647**	**-0.0538**	**0.0083**	**0.0175**	**0.0626**	**0.0611**
1996	**0.0269**	**0.0310**	**-0.0295**	**-0.0181**	**0.0175**	**0.0361**	**-0.0683**	**-0.0469**	**-0.0020**	**0.0183**	**0.0423**	**0.0585**
1997	**0.0298**	**0.0305**	**-0.0061**	**-0.0195**	**0.0333**	**0.0350**	**-0.0302**	**-0.0414**	**0.0220**	**0.0184**	**0.0447**	**0.0555**
1998	**0.0346**	**0.0303**	**-0.0179**	**-0.0207**	**0.0370**	**0.0335**	**-0.0062**	**-0.0370**	**0.0375**	**0.0181**	**0.0658**	**0.0520**
1999	**0.0323**	**0.0306**	**-0.0064**	**-0.0212**	**0.0667**	**0.0334**	**0.0065**	**-0.0327**	**0.0274**	**0.0187**	**0.0727**	**0.0489**
2000	**0.0187**	**0.0346**	**-0.0468**	**-0.0233**	**0.0307**	**0.0298**	**-0.0149**	**-0.0387**	**0.0315**	**0.0173**	**0.0518**	**0.0456**
2001	**0.0250**	**0.0397**	**-0.0101**	**-0.0256**	**0.0263**	**0.0251**	**-0.0629**	**-0.0453**	**0.0044**	**0.0164**	**0.0234**	**0.0443**
2002	**0.0492**	**0.0454**	**-0.0358**	**-0.0258**	**0.0032**	**0.0212**	**-0.1019**	**-0.0505**	**-0.0222**	**0.0151**	**0.0161**	**0.0455**
2003	**0.0662**	**0.0513**	**-0.0405**	**-0.0241**	**-0.0036**	**0.0193**	**-0.0816**	**-0.0530**	**0.0206**	**0.0137**	**0.0370**	**0.0496**
2004	**0.0628**	**0.0558**	**-0.0177**	**-0.0205**	**0.0174**	**0.0205**	**-0.0307**	**-0.0504**	**0.0417**	**0.0143**	**0.0609**	**0.0549**
2005	**0.0636**	**0.0554**	**-0.0089**	**-0.0162**	**0.0360**	**0.0239**	**-0.0199**	**-0.0435**	**0.0142**	**0.0176**	**0.0887**	**0.0580**
2006	**0.0622**	**0.0497**	**-0.0168**	**-0.0131**	**0.0355**	**0.0271**	**-0.0403**	**-0.0374**	**0.0079**	**0.0210**	**0.0893**	**0.0570**
2007	**0.0536**	**0.0417**	**-0.0004**	**-0.0124**	**0.0369**	**0.0281**	**-0.0194**	**-0.0354**	**0.0222**	**0.0227**	**0.0576**	**0.0526**
2008	**0.0202**	**0.0340**	**0.0083**	**-0.0132**	**0.0359**	**0.0263**	**-0.0334**	**-0.0355**	**0.0230**	**0.0233**	**0.0233**	**0.0464**
2009	**0.0055**	**0.0269**	**-0.0299**	**-0.0124**	**0.0135**	**0.0225**	**-0.0606**	**-0.0345**	**0.0267**	**0.0231**	**0.0118**	**0.0399**
2010	**0.0222**	**0.0213**	**-0.0341**	**-0.0095**	**0.0108**	**0.0180**	**-0.0423**	**-0.0319**	**0.0420**	**0.0209**	**0.0308**	**0.0346**
2011	**0.0190**	**0.0181**	**-0.0163**	**-0.0057**	**0.0190**	**0.0140**	**-0.0238**	**-0.0284**	**0.0238**	**0.0168**	**0.0507**	**0.0320**
2012	**0.0200**	**0.0171**	**0.0081**	**-0.0014**	**0.0061**	**0.0111**	**-0.0129**	**-0.0242**	**0.0017**	**0.0120**	**0.0426**	**0.0313**
2013	**0.0200**	**0.0170**	**0.0200**	**0.0029**	**0.0088**	**0.0094**	**-0.0139**	**-0.0211**	**-0.0037**	**0.0069**	**0.0289**	**0.0306**
2014	**0.0135**	**0.0172**	**0.0259**	**0.0064**	**0.0106**	**0.0084**	**-0.0102**	**-0.0202**	**-0.0040**	**0.0036**	**0.0194**	**0.0293**
2015	**0.0097**	**0.0175**	**0.0041**	**0.0089**	**0.0058**	**0.0084**	**-0.0206**	**-0.0208**	**0.0032**	**0.0043**	**0.0171**	**0.0281**
2016	**0.0164**	**0.0178**	**-0.0152**	**0.0105**	**0.0033**	**0.0097**	**-0.0384**	**-0.0224**	**-0.0014**	**0.0085**	**0.0248**	**0.0292**
2017	**0.0266**	**0.0200**	**0.0055**	**0.0135**	**0.0085**	**0.0122**	**-0.0273**	**-0.0242**	**0.0108**	**0.0145**	**0.0359**	**0.0343**
2018	**0.0229**	**0.0279**	**0.0233**	**0.0232**	**0.0165**	**0.0183**	**-0.0129**	**-0.0228**	**0.0283**	**0.0237**	**0.0390**	**0.0461**
2019	**0.0120**	**0.0361**	**0.0229**	**0.0328**	**0.0194**	**0.0250**	**-0.0287**	**-0.0204**	**0.0312**	**0.0341**	**0.0377**	**0.0586**
2020	**0.0328**	**0.0443**	**0.0387**	**0.0427**	**0.0217**	**0.0319**	**-0.0298**	**-0.0180**	**0.0300**	**0.0447**	**0.0658**	**0.0712**
2021	**0.0721**	**0.0528**	**0.0765**	**0.0530**	**0.0458**	**0.0390**	**-0.0121**	**-0.0154**	**0.0545**	**0.0554**	**0.0933**	**0.0842**

[50] We used the nonparametric Lowess smoothing method in Shazam with the smoothing parameter set equal to 0.4; see White (2004). Alternative smoothing techniques were used in Diewert (2005) and Diewert and Fox (2018b).

2022	0.0668	0.0618	0.0552	0.0638	0.0526	0.0464	-0.0042	-0.0125	0.0788	0.0663	0.1073	0.0977
Mean	0.0335	0.0341	-0.0028	-0.0020	0.0239	0.0244	-0.0334	-0.0338	0.0202	0.0208	0.0486	0.0497

It can be seen that the smoothed asset inflation rates i_n^{t*} are very smooth compared to the true ex post inflation rates i_n^t and that the average smoothed rate is reasonably close to the true ex post average rate. We used these smoothed asset inflation rates i_n^{t*} in place of the i_n^t in definitions (A5) in order to construct the new smoothed user costs u_n^{t*}. The new period t smoothed balancing rate of return rs^t was determined by solving the following counterpart to equations (A6):

(A7) $VYG^t - VX_1^t - VX_3^t = \Sigma_{n=1}^6 [rs^t + \tau_n^t - i_n^{t*} + (1+i_n^{t*})\delta_n^t] PK_n^t QK_n^t$; t = 1994,…,2022.

Once the smoothed rates of return rs^t have been determined, the new smoothed user costs are defined by the following counterparts to equations (A5):

(A8) $us_n^t = [rs^t + \tau_n^t - i_n^{t*} + (1+i_n^{t*})\delta_n^t] PK_n^t$; n = 1,…,6 ; t = 1994,…,2022.

The new output growth index is still QYG^t/QYG^{t-1}. Again, in order to be consistent with the Diewert and Morrison (1986) translog methodology, we chose to use chained Törnqvist input quantity indexes to aggregate the labour, intermediate inputs and 6 types of capital services using smoothed user costs into an aggregate input QXS^t, which is listed in Table A9. The year t gross output level of Smoothed Total Factor Productivity for the Retail sector is defined as $TFPS^t \equiv QYG^t/QXS^t$ and Smoothed TFP growth for year t = 1994,…,2022 is defined as $\Delta TFPS^t \equiv TFPS^t/TFPS^{t-1}$. $TFPS^t$ is compared to the Jorgenson type estimates $TFPJ^t$ in Table A9. The Smoothed user costs us_n^t were normalized to equal 1 in the split year 1994 and the normalized smoothed user costs us_n^{t*} are also listed in Table A9.

Table A9: Smoothed User Costs, Rates of Return and Smoothed Gross Output TFP Levels

Year t	r_S^t	u_{S1}^{t*}	u_{S2}^{t*}	u_{S3}^{t*}	u_{S4}^{t*}	u_{S5}^{t*}	u_{S6}^{t*}	TFP_S^t	TFP_J^t	QX_S^t
1994	0.12622	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	54.51465
1995	0.13385	1.10406	1.02733	1.09372	0.94232	1.08788	1.21583	1.00826	1.00825	54.58886
1996	0.12562	1.07526	0.94557	1.09038	0.87000	1.01236	1.20325	1.00405	1.00402	56.28014
1997	0.11070	0.98637	0.88898	1.03221	0.80080	0.86865	1.06060	1.01338	1.01334	56.30089
1998	0.11801	1.08296	0.91245	1.13010	0.77536	0.96091	1.29244	1.03129	1.03116	55.94713
1999	0.11024	1.04397	0.86530	1.17478	0.76419	0.91366	1.29212	1.03086	1.03076	58.31935
2000	0.11206	1.04933	0.86007	1.25754	0.78076	0.97145	1.48506	1.04857	1.04832	59.42998
2001	0.12033	1.09717	0.86784	1.38362	0.77288	1.09873	1.75903	1.06974	1.06964	59.58627
2002	0.12786	1.14628	0.87818	1.46103	0.73718	1.19764	1.93210	1.08607	1.08630	59.76548
2003	0.14391	1.30531	0.88026	1.52000	0.66586	1.35183	2.21617	1.10405	1.10418	61.56833
2004	0.12612	1.14634	0.79212	1.41550	0.59816	1.18480	1.78983	1.10222	1.10208	64.19828
2005	0.12584	1.21264	0.76270	1.44096	0.58045	1.19551	1.81875	1.09469	1.09444	63.97425
2006	0.14989	1.65394	0.80828	1.64346	0.58961	1.44320	2.61760	1.10863	1.10821	65.53806
2007	0.14609	1.80303	0.77681	1.55735	0.56246	1.39259	2.86949	1.12206	1.12161	67.79101
2008	0.13835	1.89855	0.76170	1.61412	0.54922	1.32699	2.98797	1.09910	1.09897	70.94228
2009	0.13441	1.96209	0.73811	1.67757	0.53573	1.31371	3.13689	1.10072	1.10054	72.26961
2010	0.13403	2.05592	0.70273	1.69154	0.48604	1.37105	3.32245	1.09633	1.09612	74.43570
2011	0.15634	2.46742	0.73253	1.90583	0.49190	1.76158	4.20525	1.11674	1.11639	75.16712
2012	0.15754	2.54855	0.72099	1.97421	0.48437	1.88153	4.48071	1.13288	1.13257	75.06102
2013	0.14536	2.42461	0.69152	1.90450	0.45782	1.79240	4.27558	1.14204	1.14176	73.41772
2014	0.13896	2.37117	0.67943	1.82837	0.42632	1.74676	4.22194	1.14988	1.14969	73.71815
2015	0.10231	1.82493	0.58878	1.61423	0.41450	1.25986	3.02586	1.15610	1.15593	76.00073
2016	0.12681	2.22265	0.64547	1.76622	0.41310	1.52530	3.93496	1.15528	1.15512	78.12320
2017	0.12462	2.18745	0.62645	1.76741	0.40884	1.41818	3.76067	1.16329	1.16320	80.42275
2018	0.13101	2.22311	0.63029	1.80823	0.40298	1.39612	3.68466	1.16175	1.16166	82.54948

2019	**0.16219**	**2.64952**	**0.70772**	**2.01683**	**0.41653**	**1.71347**	**4.58272**	**1.17568**	**1.17554**	**82.01811**
2020	**0.21282**	**3.39953**	**0.84631**	**2.36164**	**0.43821**	**2.31977**	**6.34762**	**1.20041**	**1.20001**	**85.61285**
2021	**0.18860**	**2.95849**	**0.78756**	**2.24185**	**0.43522**	**1.89342**	**5.10520**	**1.19775**	**1.19721**	**87.76261**
2022	**0.20585**	**3.34010**	**0.88023**	**2.43902**	**0.41927**	**2.09147**	**5.76225**	**1.17790**	**1.17729**	**90.97918**
Mean	**0.13917**	**1.83590**	**0.79330**	**1.61420**	**0.59380**	**1.39620**	**2.97890**	**1.10520**	**1.10500**	**69.52700**

From Table A9, it can be seen that the new user costs u_{Sn}^t for the low depreciation assets are smoother than the Jorgensonian user costs u_J^t but the low depreciation assets (1, 5 and 6) still have substantial volatility due to the volatility of the balancing rates of return, r_S^t. The average of the r_S^t is 13.92% which is close to the average of the Jorgensonian balancing rates of return which was 13.84%. Note also that Jorgensonian TFP_J^t levels are very close to their smoothed counterparts, TFP_S^t. Thus the average rate of smoothed TFP growth over the sample period and over the two subperiods 1995-2007 and 2008-2022 was the same as the Jorgensonian average rates over these time periods.

We turn now to the construction of user costs that use an exogenous cost of capital.[51] The problem with the use of an exogenous rate of return on assets is that there is uncertainty as to what exactly this rate should be. The finance literature advocates a weighted cost of capital approach that is an average of debt and equity rates but it is difficult to pin down precisely what these rates should be. Our approach is a very simple one: we chose the year t exogenous rate, r_E^t, to be 7 percent[52] plus a (smoothed) Consumer Price inflation rate, i^t:

(A9) $r_E^t \equiv 0.07 + i^t$; t = 1994,…,2022.

From ABS (2025f), we used the June quarter All Groups CPI indexes for the 30 years 1994-2023. This series is listed as P_{CPI}^t for year t in Table 10. Define the raw year over year CPI inflation rates as $i_{CPI}^t \equiv (P_{CPI}^{t+1}/P_{CPI}^t) - 1$. We smoothed the raw rates i_{CPI}^t using the nonparametric Lowess smoothing method in Shazam with the smoothing parameter set equal to 0.4. The resulting smoothed CPI inflation rates, i^t, are listed in Table A10 along with the series P_{CPI}^t and i_{CPI}^t. The exogenous rates of return r_E^t along with the corresponding Jorgenson and Smoothed Jorgenson rates, r_J^t and r_S^t, are also listed in Table A10.

Table A10: CPI, CPI Raw and Smoothed Inflation Rates and Exogenous, Smoothed and Jorgensonian Interest Rates

Year t	P_{CPI}^t	i_{CPI}^t	i^t	r_E^t	r_S^t	r_J^t
1994	**42.99**	**0.04489**	**0.02456**	**0.09456**	**0.12622**	**0.13382**
1995	**44.92**	**0.03094**	**0.02421**	**0.09421**	**0.13385**	**0.12739**
1996	**46.31**	**0.00345**	**0.02441**	**0.09441**	**0.12562**	**0.11251**
1997	**46.47**	**0.00667**	**0.02477**	**0.09477**	**0.11070**	**0.11326**
1998	**46.78**	**0.01090**	**0.02491**	**0.09491**	**0.11801**	**0.12869**
1999	**47.29**	**0.03130**	**0.02608**	**0.09608**	**0.11024**	**0.12044**
2000	**48.77**	**0.06069**	**0.02906**	**0.09906**	**0.11206**	**0.10950**

[51] Previous productivity studies that used exogenous rates of return include Schreyer (2010) and Diewert and Fox (2018b) and Barkai (2020). Our exogenous real rate needs to include the equity premium and an expected income tax rate (less the income subsidy rate if applicable). Following Jorgenson and Griliches (1972) and Schreyer (2010), output prices should exclude taxes (less specific subsidies) on outputs and include indirect taxes on inputs. In the System of National Accounts, these tax adjusted prices are called basic prices.

[52] Barkai (2020; 2431) chose his exogenous r as a weighted cost of equity and debt. His equity cost of capital was defined as the sum of the yield on a 10-year U.S. treasury bond and a constant 5% equity risk premium. Some of his user costs of capital turned out to be negative and he set those negative prices equal to 0. Our methodology can accommodate negative user costs on land because the short term costs of buying and selling land can be large.

2001	**51.73**	**0.02842**	**0.03148**	**0.10148**	**0.12033**	**0.11201**
2002	**53.20**	**0.02669**	**0.03225**	**0.10225**	**0.12786**	**0.10880**
2003	**54.62**	**0.02472**	**0.03096**	**0.10096**	**0.14391**	**0.14489**
2004	**55.97**	**0.02466**	**0.02955**	**0.09955**	**0.12612**	**0.13885**
2005	**57.35**	**0.04010**	**0.02908**	**0.09908**	**0.12584**	**0.13330**
2006	**59.65**	**0.02096**	**0.02935**	**0.09935**	**0.14989**	**0.15355**
2007	**60.90**	**0.04483**	**0.02952**	**0.09952**	**0.14609**	**0.15251**
2008	**63.63**	**0.01446**	**0.02859**	**0.09859**	**0.13835**	**0.13505**
2009	**64.55**	**0.03052**	**0.02696**	**0.09696**	**0.13441**	**0.12101**
2010	**66.52**	**0.03623**	**0.02578**	**0.09578**	**0.13403**	**0.13463**
2011	**68.93**	**0.01175**	**0.02435**	**0.09435**	**0.15634**	**0.15974**
2012	**69.74**	**0.02352**	**0.02257**	**0.09257**	**0.15754**	**0.15928**
2013	**71.38**	**0.03068**	**0.02057**	**0.09057**	**0.14536**	**0.14631**
2014	**73.57**	**0.01522**	**0.01929**	**0.08929**	**0.13896**	**0.13811**
2015	**74.69**	**0.01018**	**0.01825**	**0.08825**	**0.10231**	**0.09687**
2016	**75.45**	**0.01895**	**0.01638**	**0.08638**	**0.12681**	**0.11737**
2017	**76.88**	**0.02094**	**0.01670**	**0.08670**	**0.12462**	**0.12412**
2018	**78.49**	**0.01542**	**0.02369**	**0.09369**	**0.13101**	**0.12949**
2019	**79.70**	**-0.00326**	**0.03091**	**0.10091**	**0.16219**	**0.14836**
2020	**79.44**	**0.03814**	**0.03811**	**0.10811**	**0.21282**	**0.20389**
2021	**82.47**	**0.06208**	**0.04578**	**0.11578**	**0.18860**	**0.20068**
2022	**87.59**	**0.06028**	**0.05413**	**0.12413**	**0.20585**	**0.21074**
Mean	**63.103**	**0.02705**	**0.02766**	**0.09766**	**0.13917**	**0.13845**

The average raw CPI inflation rate i_{CPI}^t over the sample period 2.705% per year which is close to the average of the smoothed CPI inflation rates, 2.766% per year. Adding the smoothed CPI inflation rates i^t to the constant (assumed) real rate of 7.0% per year gives us the exogenous cost of capital r_E^t, which averaged 9.766% per year. This is well below the average of the Jorgensonian ex post rates of return r_J^t which was 13.845% per year. The average of the smoothed Jorgensonian rates of return r_S^t was 13.917% per year. The standard deviations of the r_E^t, r_S^t and r_J^t were 0.008. 0.026 and 0.028 respectively. Thus the volatility of the exogenous cost of financial capital was (not surprisingly) well below the volatility of the Jorgensonian endogenous rates of return.

With the exogenous rates r_E^t determined, we used equations (A8) with r_S^t replaced by r_E^t to define new exogenous user costs, u_{En}^t for t = 1994,…,2022. These u_{En}^t and the corresponding QK_n^t were normalized so that $u_{En}^{1*} = 1$ and $QK_n^{1*} = u_{En}^1 QK_n^1$ for n = 1,…,6. The normalized u_{En}^{t*} along with PYG^t, PX_1^t and PX_3^t are listed in Table A11 and the normalized QK_n^{1*} along with QYG^t, QX_1^t and QX_3^t are listed in Table A12.

Table A11: Gross Output, Labour and Intermediate Input Price Indexes and Exogenous User Costs

Year t	PYG^t	PX_1^t	PX_3^t	u_{E1}^{t*}	u_{E2}^{t*}	u_{E3}^{t*}	u_{E4}^{t*}	u_{E5}^{t*}	u_{E6}^{t*}
1994	**1.000000**	**1.000000**	**1.000000**	**1.000000**	**1.000000**	**1.000000**	**1.000000**	**1.000000**	**1.000000**
1995	**1.025262**	**1.060799**	**1.010051**	**1.041450**	**0.998945**	**1.064710**	**0.929192**	**1.007644**	**1.121032**
1996	**1.016828**	**1.106864**	**0.971746**	**1.082852**	**0.945010**	**1.093819**	**0.870399**	**1.009009**	**1.267309**
1997	**1.021124**	**1.157586**	**0.985852**	**1.127116**	**0.943879**	**1.092513**	**0.823089**	**1.009962**	**1.418244**
1998	**1.061699**	**1.177743**	**1.073182**	**1.166947**	**0.943181**	**1.168129**	**0.786670**	**1.038042**	**1.592325**
1999	**1.099283**	**1.205747**	**1.139892**	**1.212793**	**0.925251**	**1.256279**	**0.787746**	**1.085307**	**1.832607**
2000	**1.121001**	**1.260405**	**1.178566**	**1.228611**	**0.924019**	**1.350767**	**0.806774**	**1.177670**	**2.180654**
2001	**1.169242**	**1.356654**	**1.244537**	**1.215575**	**0.914144**	**1.460717**	**0.790501**	**1.263722**	**2.430837**
2002	**1.130974**	**1.406370**	**1.151327**	**1.187649**	**0.902532**	**1.509694**	**0.745516**	**1.300496**	**2.470769**
2003	**1.118072**	**1.453291**	**1.106942**	**1.148152**	**0.848908**	**1.484173**	**0.653070**	**1.272827**	**2.306749**
2004	**1.177290**	**1.505414**	**1.241312**	**1.134902**	**0.808859**	**1.453479**	**0.603432**	**1.269044**	**2.130965**
2005	**1.213840**	**1.551031**	**1.272797**	**1.195833**	**0.776530**	**1.478726**	**0.585404**	**1.264727**	**2.111407**
2006	**1.211127**	**1.587320**	**1.203765**	**1.367113**	**0.752893**	**1.569624**	**0.571095**	**1.232642**	**2.355715**
2007	**1.233057**	**1.682895**	**1.216088**	**1.581916**	**0.732479**	**1.485570**	**0.548443**	**1.218225**	**2.793034**

2008	**1.265328**	**1.710773**	**1.211364**	**1.789039**	**0.736802**	**1.580112**	**0.541952**	**1.219984**	**3.224117**
2009	**1.290631**	**1.734003**	**1.258767**	**1.901395**	**0.716379**	**1.657290**	**0.531147**	**1.224761**	**3.562087**
2010	**1.333699**	**1.816732**	**1.283776**	**2.006843**	**0.677449**	**1.664902**	**0.480041**	**1.275098**	**3.823956**
2011	**1.400894**	**1.893752**	**1.368241**	**2.082728**	**0.644772**	**1.753590**	**0.467612**	**1.375328**	**4.008344**
2012	**1.398709**	**1.911363**	**1.383444**	**2.109587**	**0.626031**	**1.803933**	**0.458436**	**1.463551**	**4.148191**
2013	**1.405573**	**1.960187**	**1.418666**	**2.122264**	**0.621102**	**1.788875**	**0.439355**	**1.521635**	**4.241932**
2014	**1.388412**	**1.954043**	**1.417780**	**2.133401**	**0.619564**	**1.733534**	**0.410790**	**1.554030**	**4.370828**
2015	**1.394470**	**2.137298**	**1.433342**	**2.126968**	**0.622743**	**1.724132**	**0.426996**	**1.516388**	**4.459535**
2016	**1.397453**	**2.055820**	**1.420317**	**2.104644**	**0.605207**	**1.719593**	**0.405221**	**1.410844**	**4.346439**
2017	**1.390627**	**2.089756**	**1.421696**	**2.097082**	**0.593227**	**1.736744**	**0.403807**	**1.306715**	**4.139628**
2018	**1.427663**	**2.148461**	**1.466449**	**2.136098**	**0.598783**	**1.784611**	**0.398820**	**1.279344**	**3.956435**
2019	**1.470321**	**2.178669**	**1.500955**	**2.152536**	**0.608676**	**1.851632**	**0.396277**	**1.255409**	**3.735244**
2020	**1.517989**	**2.193147**	**1.521087**	**2.157656**	**0.621936**	**1.929262**	**0.390159**	**1.229367**	**3.476642**
2021	**1.559917**	**2.362374**	**1.597256**	**2.233739**	**0.648614**	**2.005462**	**0.410763**	**1.206313**	**3.285554**
2022	**1.685582**	**2.503293**	**1.669930**	**2.396926**	**0.702994**	**2.137395**	**0.388545**	**1.217859**	**3.143190**

It can be seen that the exogenous user costs u_E^t are much less volatile than the previous user costs. Thus the exogenous user costs may be closer to rental prices for the capital stock assets used in production than the Jorgensonian user costs. We relabel the above series when they are used in the main text: PYG^t becomes the single year t output price p^t and the 8 prices which follow PYG^t in the above Table become the vector $w^t \equiv [w_1^t,\ldots,w_8^t] \equiv [PX_1^t,PX_3^t,u_{E1}^{t*},u_{E2}^{t*},u_{E3}^{t*},u_{E4}^{t*},u_{E5}^{t*},u_{E6}^{t*}]$ for t = 1994,…,2022.

Table A12: Gross Output, Labour and Intermediate Input Quantity Indexes and Normalized Capital Stocks

Year t	QYG^t	QX_1^t	QX_3^t	QK_1^{t*}	QK_2^{t*}	QK_3^{t*}	QK_4^{t*}	QK_5^{t*}	QK_6^{t*}
1994	**54.51465**	**17.96389**	**30.13065**	**1.00820**	**2.55199**	**0.01613**	**0.19078**	**1.14162**	**0.21194**
1995	**55.03976**	**18.29972**	**29.58482**	**1.01149**	**2.63311**	**0.01763**	**0.21031**	**1.24887**	**0.21263**
1996	**56.50792**	**19.56939**	**29.72568**	**1.01475**	**2.79926**	**0.01964**	**0.22233**	**1.25934**	**0.21331**
1997	**57.05445**	**19.85808**	**29.18866**	**1.02063**	**2.87750**	**0.02114**	**0.25087**	**1.33623**	**0.21455**
1998	**57.69744**	**20.23810**	**28.20705**	**1.04604**	**3.04549**	**0.02201**	**0.29443**	**1.28561**	**0.21989**
1999	**60.11936**	**20.90386**	**29.50999**	**1.09796**	**3.21374**	**0.02251**	**0.32448**	**1.37319**	**0.23081**
2000	**62.31624**	**20.69765**	**30.49601**	**1.15744**	**3.46336**	**0.02251**	**0.35602**	**1.38135**	**0.24331**
2001	**63.74153**	**20.88913**	**30.02501**	**1.19117**	**3.66877**	**0.02301**	**0.39057**	**1.52676**	**0.25040**
2002	**64.90962**	**21.15426**	**29.66846**	**1.24202**	**3.90008**	**0.02451**	**0.42212**	**1.47479**	**0.26109**
2003	**67.97453**	**21.34869**	**30.73371**	**1.30486**	**4.19157**	**0.02614**	**0.49873**	**1.60818**	**0.27430**
2004	**70.76081**	**22.47106**	**31.60527**	**1.37040**	**4.62043**	**0.02852**	**0.57985**	**1.62306**	**0.28808**
2005	**70.03209**	**22.73914**	**30.43438**	**1.42000**	**5.21833**	**0.03139**	**0.65046**	**1.67278**	**0.29850**
2006	**72.65763**	**23.41963**	**30.50481**	**1.46553**	**5.90023**	**0.03427**	**0.69853**	**1.76250**	**0.30807**
2007	**76.06546**	**24.39471**	**31.13427**	**1.49364**	**6.47641**	**0.04302**	**0.75110**	**1.79164**	**0.31398**
2008	**77.97299**	**24.93381**	**33.14152**	**1.51727**	**7.26793**	**0.04552**	**0.80819**	**1.90168**	**0.31895**
2009	**79.54831**	**25.16358**	**33.95586**	**1.53987**	**7.90194**	**0.04865**	**0.83823**	**1.80482**	**0.32370**
2010	**81.60587**	**25.87648**	**35.00350**	**1.55026**	**8.26932**	**0.05440**	**1.01699**	**1.84618**	**0.32589**
2011	**83.94206**	**25.81167**	**35.72980**	**1.57379**	**8.43757**	**0.05778**	**1.15069**	**1.82246**	**0.33083**
2012	**85.03514**	**25.97959**	**35.29842**	**1.58259**	**8.55688**	**0.06478**	**1.37152**	**1.75840**	**0.33268**
2013	**83.84561**	**25.73214**	**33.45405**	**1.59036**	**8.77249**	**0.07829**	**1.68548**	**1.77596**	**0.33431**
2014	**84.76723**	**26.26239**	**32.58248**	**1.62904**	**9.09381**	**0.09705**	**1.97841**	**1.80413**	**0.34245**
2015	**87.86429**	**26.74846**	**33.93825**	**1.67757**	**9.48107**	**0.11118**	**1.96489**	**1.91611**	**0.35265**
2016	**90.25406**	**27.05188**	**35.59335**	**1.72047**	**9.65560**	**0.12094**	**2.03699**	**1.95914**	**0.36167**
2017	**93.55473**	**27.29934**	**37.41571**	**1.79848**	**9.84819**	**0.12894**	**1.97390**	**2.03044**	**0.37806**
2018	**95.90164**	**27.49082**	**39.23368**	**1.87112**	**9.96463**	**0.13545**	**2.09107**	**2.02743**	**0.39333**
2019	**96.42674**	**26.36255**	**39.70027**	**1.95600**	**10.26606**	**0.14145**	**2.25331**	**2.08899**	**0.41118**
2020	**102.77089**	**27.20212**	**42.97525**	**2.00963**	**10.40605**	**0.14608**	**2.48615**	**1.74529**	**0.42245**
2021	**105.11780**	**27.86494**	**43.62672**	**2.05246**	**10.88463**	**0.15521**	**2.99390**	**1.90363**	**0.43145**
2022	**107.16464**	**29.45866**	**44.01848**	**2.09237**	**11.25986**	**0.16709**	**4.41649**	**2.04898**	**0.43984**

Because the depreciation rates for inventories and land (assets 5 and 6) are assumed to be equal to zero, it can be seen that the value of inventory and land services is relatively small compared to value of intermediate, labour, and machinery and equipment (asset 2) inputs. This may explain why the ABS did not include these assets in their Retail Trade capital stocks.

We relabel the above series when they are used in the main text: QYG^t becomes the single year t gross output quantity y^t and the 8 quantities which follow QYG^t in the above Table become the input vector $x^t \equiv [x_1^t,\ldots,x_8^t] \equiv [QX_1^t,QX_3^t,QK_1^{t*},QK_2^{t*},QK_3^{t*},QK_4^{t*},QK_5^{t*},QK_6^{t*}]$ for t = 1994,…,2022.

Recall that the eight inputs are: (1) quality adjusted labour input, (2) intermediate inputs, (3) non-dwelling construction services, (4) machinery and equipment, (5) research and development, (6) computer software, (7) inventory stocks and (8) land.

Appendix B: The Relationship of the Present Methodology to the Earlier Diewert-Fox Methodology

We use the same notation that was used in section 4. Recall that the period t *empirical minimum average cost function* $c^t(w)$ waw defined by equation (33). We repeat this definition below for t = 1,…,T:

(B1) $c^t(w) \equiv \min_j \{w\cdot(x^j/y^j) : j = 1,2,\ldots,t\}$.

Diewert and Fox (2018a) developed their nonparametric approach to the measurement of TFP using the cost constrained value added function. In our present context where we have one output y and many inputs x, we define the production unit's period t *cost constrained revenue function* for positive output price p, positive input price vector w and positive input quantity vector x, $R^{t*}(p,w,x)$, as follows:

(B2) $R^{t*}(p,w,x) \equiv \max_{y,z} \{py : (y,z)\in S^t ; w\cdot z \leq w\cdot x\}$; t = 1,…,T.

Diewert and Fox(2018a) did not spell out the implications of their methodology if the number of outputs is equal to 1.[53] We undertake this task in this appendix.

Diewert and Fox assumed constant returns to scale in production. Thus if an observed output input vector (y^t,x^t) belonged to the period t production possibilities set S^t, then they assumed that $(\lambda y^t,\lambda x^t)$ also belonged to S^t for any $\lambda > 0$. As in the present paper, they assumed that the sets S^t did not become smaller over time.

Let (y^t,x^t) be the observed output-input vector for period t for t = 1,…,T. Consider the period t cost constrained revenue maximization problem defined by (B2). If $j \leq t$, then $(\lambda y^j,\lambda x^j)\in S^t$. Choose the number λ_j so that $w\cdot\lambda_j x^j = w\cdot x$. Hence, choose $\lambda_j = w\cdot x/w\cdot x^j$. Thus $(\lambda_j y^j, \lambda_j x^j)$ is a feasible solution for the maximization problem defined by (B2) and $R^{t*}(p,w,x) \geq p(\lambda_j y^j) = (w\cdot x/w\cdot x^j)py^j = w\cdot xpy^j/w\cdot x^j$. This algebra holds for any $j \leq t$. Thus for t = 1,…,T, we have the following inequalities and definitions for the *empirical cost constrained revenue functions*, $R^t(p,w,x)$:

(B3) $R^{t*}(p,w,x) \geq \max_j \{w\cdot xpy^j/w\cdot x^j : j = 1,\ldots,t) \equiv R^t(p,w,x)$.

[53] They also failed to mention that their methodology does not require that the value of inputs be equal to the value of outputs in each period; i.e., their algebra is valid even if there are markups over average costs. Thus the only difference between the 2018 methodology and the present methodology is that the earlier methodology assumed constant returns to scale in production.

As in section 4, we approximate the unobserved true cost constrained revenue function, $R^{t*}(p,w,x)$, by the observable cost constrained revenue function, $R^t(p,w,x)$. It turns out that there is a relationship between the $R^t(p,w,x)$ functions and the empirical average cost functions $c^t(w)$ defined above by definitions (B1). Using definitions (B3), we have for t = 1,…,T:

$$\begin{aligned}(B4)\ R^t(p,w,x) &\equiv \max_j \{w\cdot xpy^j/w\cdot x^j : j = 1,\ldots,t\}\\ &= p\ w\cdot x \max_j \{y^j/w\cdot x^j : j = 1,\ldots,t\}\\ &= p\ w\cdot x \max_j \{1/w\cdot(x^j/y^j) : j = 1,\ldots,t\}\\ &= p\ w\cdot x / \min_j \{w\cdot(x^j/y^j) : j = 1,\ldots,t)\\ &= p\ w\cdot x / c^t(w)\end{aligned}$$

where $c^t(w)$ is defined by (B1). Below, we will use the fact that $R^t(p,w,x)$ equals $pw\cdot x/c^t(w)$ in order to decompose the revenue ratio, $p^ty^t/p^{t-1}y^{t-1}$, into a product of explanatory factors that are similar to the decomposition given by (41) in the main text.

For t = 1,…,T, define the *revenue efficiency factors* e^{t*} as follows:

$$(B5)\ e^{t*} \equiv p^ty^t/R^t(p,w,x) = c^t(w^t)/[w^t\cdot(x^t/y^t)].$$

Note that the last equality in (B5) says that period t revenue efficiency e^{t*} is equal to "optimal" average cost $c^t(w^t)$ divided by observed average cost, $w^t\cdot x^t/y^t$. Note also that e^{t*} equals our new efficiency factor in the main text, e^t defined by (36).

For t = 2,…,T, we define a long list of explanatory growth factors below:

$$(B6)\ \varepsilon^{t*} \equiv e^{t*}/e^{t-1*};$$
$$(B7)\ \beta^{t*} \equiv R^s(p^t,w,x)/R^s(p^{t-1},w,x) = p^t/p^{t-1};$$
$$(B8)\ \gamma_L{}^{t*} \equiv R^{t-1}(p^{t-1},w^t,x^{t-1})/R^{t-1}(p^{t-1},w^{t-1},x^{t-1}) = [w^t\cdot x^{t-1}/c^{t-1}(w^t)]/[w^{t-1}\cdot x^{t-1}/c^{t-1}(w^{t-1})];$$
$$(B9)\ \gamma_P{}^{t*} \equiv R^t(p^t,w^t,x^t)/R^t(p^t,w^{t-1},x^t) = [w^t\cdot x^t/c^t(w^t)]/[w^{t-1}\cdot x^t/c^t(w^{t-1})];$$
$$(B10)\ \gamma^{t*} \equiv [\gamma_L{}^{t*}\gamma_P{}^{t*}]^{1/2} = [\{w^t\cdot x^{t-1}/w^{t-1}\cdot x^{t-1}\}\{w^t\cdot x^t/w^{t-1}\cdot x^t\}]^{1/2}/[\{c^{t-1}(w^t)/c^{t-1}(w^{t-1})\}\{c^t(w^t)/c^t(w^{t-1})\}]^{1/2}$$
$$(B11)\ \tau_L{}^{t*} \equiv R^t(p^{t-1},w^{t-1},x^{t-1})/R^{t-1}(p^{t-1},w^{t-1},x^{t-1}) = c^{t-1}(w^{t-1})/c^t(w^{t-1});$$
$$(B12)\ \tau_P{}^{t*} \equiv R^t(p^t,w^t,x^t)/R^{t-1}(p^t,w^t,x^t) = c^{t-1}(w^t)/c^t(w^t);$$
$$(B13)\ \tau^{t*} \equiv [\tau_L{}^{t*}\tau_P{}^{t*}]^{1/2};$$
$$(B14)\ \delta_L{}^{t*} \equiv R^{t-1}(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^{t-1}) = w^{t-1}\cdot x^t/w^{t-1}\cdot x^{t-1};$$
$$(B15)\ \delta_P{}^{t*} \equiv R^t(p^t,w^t,x^t)/R^t(p^t,w^t,x^{t-1}) = w^t\cdot x^t/w^t\cdot x^{t-1};$$
$$(B16)\ \delta^{t*} \equiv [\delta_L{}^{t*}\delta_P{}^{t*}]^{1/2} = [\{w^{t-1}\cdot x^t/w^{t-1}\cdot x^{t-1}\}\{w^t\cdot x^t/w^t\cdot x^{t-1}\}]^{1/2} \equiv Q_F(w^{t-1},w^t,x^{t-1},x^t).$$

Note that the 2018 Diewert and Fox output price index β^{t*} defined by (B7) is equal to the new methodology output price index $\beta^t \equiv p^t/p^{t-1}$ defined by (38) in the main text. The new input methodology price index defined by (39) in the main text is $\gamma^t \equiv [\{c^{t-1}(w^t)/c^{t-1}(w^{t-1})\}\{c^t(w^t)/c^t(w^{t-1})\}]^{1/2}$. The Fisher (1922) period t chain link price index is $P_F(w^{t-1},w^t,x^{t-1},x^t) \equiv [\{w^t\cdot x^{t-1}/w^{t-1}\cdot x^{t-1}\}\{w^t\cdot x^t/w^{t-1}\cdot x^t\}]^{1/2}$. The 2018 Diewert Fox period t chain link input price index γ^{t*} defined by (B10) is equal to $P_F(w^{t-1},w^t,x^{t-1},x^t)/\gamma^t$ where γ^t is defined by (39). It turns out that the input price index γ^t is either equal to the corresponding Fisher input price index

$P_F(w^{t-1},w^t,x^{t-1},x^t)$ or is close to it and so typically, γ^{t*} defined by (B10) will be close to unity.[54] The technical progress index τ^{t*} defined by (B13) is equal to the Salter type technical progress index τ^t defined by (40) in the main text. The period t Laspeyres and Paasche indexes of input quantity change defined by (B14) and (B15) are equal to the corresponding ordinary Laspeyres and Paasche input quantity indexes that compare the level of aggregate input in period t to period t−1, Thus δ^{t*} defined by (B16) is equal to the Fisher input quantity index, $Q_F(w^{t-1},w^t,x^{t-1},x^t)$, that compares the aggregate input level in period t to the corresponding input level in period t−1. .

Using the above definitions, it can be shown that we have the following decomposition of revenue growth into explanatory factors going from period t−1 to period t:

(B17) $p^t y^t/p^{t-1}y^{t-1} = \varepsilon^{t*}\gamma^{t*}\tau^{t*}\delta^{t*}\beta^{t*}$; $t = 2,3,\ldots,T.$

Diewert-Fox 2018 TFP growth going from period t−1 to period t, π^{t*}, is defined as follows:

(B18) $\pi^{t*} \equiv \varepsilon^{t*}\gamma^{t*}\tau^{t*}$; $t = 2,3,\ldots,T$
$= [y^t/y^{t-1}]/Q_F(w^{t-1},w^t,x^{t-1},x^t)$ using (B7), (B16) and (B17).

Thus 2018 Diewert-Fox period t nonparametric TFP growth in the case of one output is equal to y^t/y^{t-1} (which is equal to the period t Fisher output quantity index in the case of only one output) divided by the period t Fisher input quantity index, $Q_F(w^{t-1},w^t,x^{t-1},x^t)$. This is a very interesting result: if we have only one output and we are not interested in decomposing TFP growth into technical progress and inefficiency terms, then we can measure TFP growth just using the Balk bilateral index number approach to measuring TFP. The last equation in (B18) involves only price and quantity data pertaining to periods t−1 and t. Using (B7), we see that (B17) is equivalent to $y^t/y^{t-1} = \varepsilon^{t*}\gamma^{t*}\tau^{t*}\delta^{t*}$ for t = 2,3,…,T.

For some purposes, it is convenient to convert the short term growth factor identities (B17) and (B18) into longer term levels identities as was done at the end of section 4. Define $\Gamma^{1*} \equiv 1$, $T^{1*} \equiv 1$, $X^{1*} = w^1 \cdot x^1$ and $\Pi^{1*} = 1$. For t = 2,…,T, define $\Gamma^{t*} \equiv \Gamma^{t-1*}\gamma^{t*}$, $T^{t*} \equiv T^{t-1*}\tau^{t*}$, $X^{t*} \equiv X^{t-1}\delta^{t*}$ and $\Pi^{t*} \equiv \Pi^{t-1*}\pi^{t*}$ where γ^{t*}, τ^{t*}, δ^{t*} and π^{t*} are defined by (B10), (B13), (B16) and (B18) respectively. Normalize the output prices so that $p^1 = 1$. Recall that e^{t*} was defined by (B5) above. Using these definitions, the long term counterparts to (B17) and (B18) are (B19) and (B20) below:

(B19) $y^t/y^1 = (X^{t*}/X^{1*})e^{t*}\Gamma^{t*}T^{t*}$; $t = 1,\ldots,T;$
(B20) $\Pi^{t*} \equiv (y^t/y^1)/(X^{t*}/X^{1*}) = e^{t*}\Gamma^{t*}T^{t*}$; $t = 1,\ldots,T.$

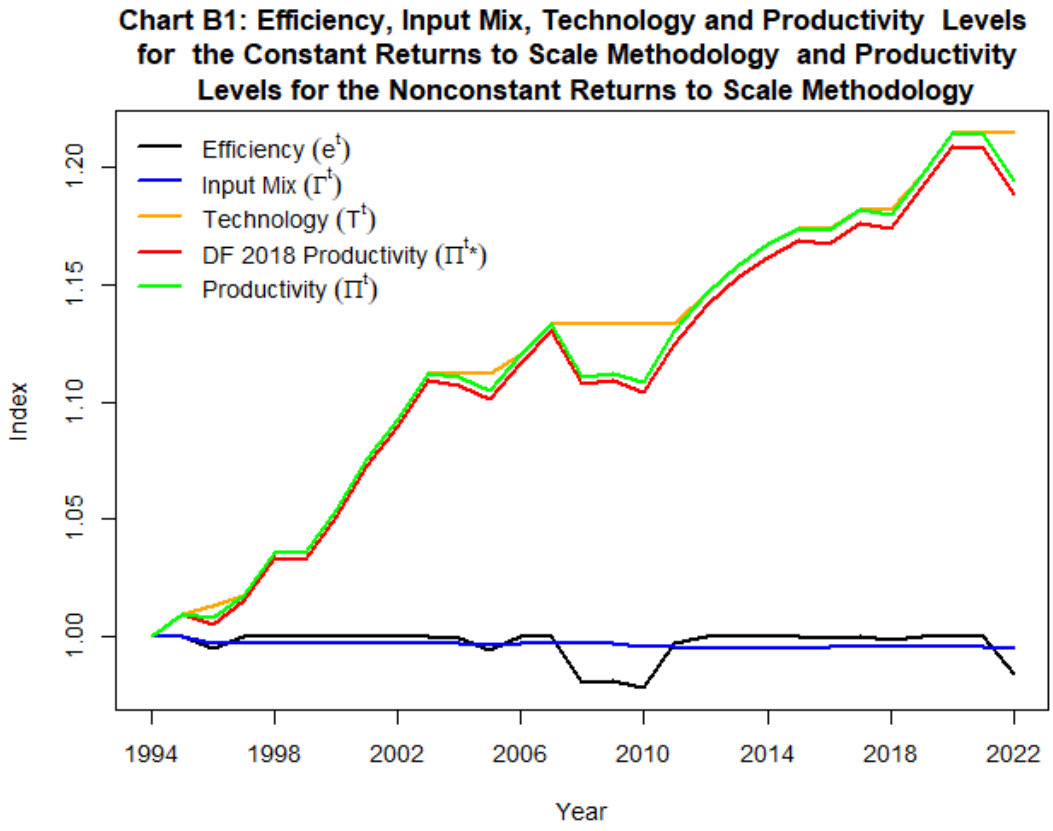
Note that X^{t*} is equal to the Fisher chained input quantity index and the period t technical progress level T^{t*} is equal to the new Diewert Fox period t technical progress level T^t defined at the end of section 4. The six indexes y^t/y^1, X^{t*}/X^{1*}, e^{t*}, Γ^{t*}, T^{t*} and Π^{t*} are listed in Table B1 and plotted on Chart B1 below. For comparison purposes, our new nonconstant returns to scale TFP levels, Π^t, are also listed below.

[54] Diewert and Fox (2018a; 631) called γ^{t*} an *input mix index* and noted that it was typically close to one.

Table B1: Diewert Fox Output, Input, Efficiency, Technical Progress and TFP Indexes

Year t	y^t/y^1	X^{t*}/X^{1*}	e^t	Γ^{t*}	T^{t*}	Π^{t*}	Π^t
1994	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1995	1.00963	1.00020	1.00000	1.00000	1.00943	1.00943	1.00943
1996	1.03656	1.03149	0.99504	0.99709	1.01287	1.00492	1.00785
1997	1.04659	1.03125	1.00000	0.99727	1.01765	1.01488	1.01765
1998	1.05838	1.02450	1.00000	0.99727	1.03590	1.03307	1.03590
1999	1.10281	1.06770	0.99979	0.99730	1.03590	1.03288	1.03568
2000	1.14311	1.08777	1.00000	0.99729	1.05373	1.05088	1.05373
2001	1.16925	1.08979	1.00000	0.99729	1.07584	1.07292	1.07584
2002	1.19068	1.09274	1.00000	0.99729	1.09259	1.08963	1.09259
2003	1.24690	1.12437	1.00000	0.99729	1.11199	1.10898	1.11199
2004	1.29801	1.17222	0.99888	0.99690	1.11199	1.10731	1.11075
2005	1.28465	1.16655	0.99374	0.99656	1.11199	1.10124	1.10504
2006	1.33281	1.19347	1.00000	0.99725	1.11983	1.11675	1.11983
2007	1.39532	1.23439	1.00000	0.99725	1.13349	1.13038	1.13349
2008	1.43031	1.29135	0.98001	0.99709	1.13349	1.10761	1.11084
2009	1.45921	1.31591	0.98125	0.99700	1.13349	1.10890	1.11224
2010	1.49695	1.35570	0.97819	0.99587	1.13349	1.10419	1.10877
2011	1.53981	1.36908	0.99688	0.99535	1.13349	1.12470	1.12995
2012	1.55986	1.36726	1.00000	0.99515	1.14643	1.14087	1.14643
2013	1.53804	1.33447	1.00000	0.99515	1.15817	1.15254	1.15817
2014	1.55494	1.33829	1.00000	0.99515	1.16756	1.16189	1.16756
2015	1.61176	1.37926	1.00000	0.99515	1.17427	1.16857	1.17427
2016	1.65559	1.41795	0.99905	0.99527	1.17427	1.16759	1.17315
2017	1.71614	1.45930	1.00000	0.99522	1.18165	1.17600	1.18165
2018	1.75919	1.49811	0.99851	0.99524	1.18165	1.17427	1.17989
2019	1.76882	1.48500	1.00000	0.99528	1.19677	1.19112	1.19677
2020	1.88520	1.55960	1.00000	0.99528	1.21450	1.20877	1.21450
2021	1.92825	1.59587	0.99962	0.99526	1.21450	1.20828	1.21404
2022	1.96580	1.65363	0.98370	0.99504	1.21450	1.18878	1.19470

Below is Chart B1 that plots the inefficiency level e^{t*}, the input mix level Γ^{t*}, the technology level T^{t*} and the Diewert Fox productivity level Π^{t*} for the Diewert and Fox (2018a) nonparametric methodology. For comparison purposes, this chart also plots the new productivity levels Π^t for year t that were defined in the main text. It can be seen that the new methodology productivity estimates are slightly above the old constant returns to scale methodology estimates but the differences are not large. The input mix levels, Γ^{t*}, are all equal to 1 or slightly below the level 1. When the efficiency level e^{t*} drops below one, the technology level T^{t*} remains constant.



Appendix C: Production Function Approaches to Measuring Markups

We explain single output production function approaches in the literature to the measurement of markups in this Appendix and relate them to our approach. We use the notation that was used in Appendix B where we also assumed that there was only a single output. Denote the observed period t price and quantity data as p^t, y^t, $w^t \equiv [w_1^t,\ldots,w_N^t]$ and $x^t \equiv [x_1^t,\ldots,x_N^t]$. We assume that all prices and quantities are positive.

Assume that there is a differentiable period t production function $F^t(x)$ where $y^t = F^t(x^t)$. We assume each partial derivative of F(x) with respect to an input n evaluated at x^t is positive; i.e., $\partial F^t(x^t)/\partial x_n > 0$ for n = 1,…,N. Finally, we assume that the observed x^t is a solution to the following period t cost minimization problem:

(C1) $\min_x \{w^t \cdot x : F^t(x) = y^t ; x \geq 0_N\} \equiv C^t(y^t,w^t)$

where $C^t(y,w) \equiv \min_x \{w \cdot x : F^t(x) = y ; x \geq 0_N\}$ is the period t cost function that is dual to $F^t(x)$. Using the assumption that x^t solves the cost minimization problem defined by (C1)and using the differentiability of F(x) at $x = x^t$, the following first order Lagrangian conditions must be satisfied:

(C2) $w_n^t = \lambda^t \partial F^t(x^t)/\partial x_n$; n = 1,…,N ;
(C3) $y^t = F(x^t)$.

The variable λ^t is the optimal Lagrange multiplier for the minimization problem defined by (C1) and it can be interpreted as the *period t marginal cost*; i.e., $\lambda^t = \partial C^t(y^t,w^t)/\partial y$.[55] Define the period t *elasticity of output with respect to input n*, $\theta_n(x^t,t)$, as follows:

(C4) $\theta_n(x^t,t) \equiv x_n^t [\partial F^t(x^t)/\partial x_n]/F^t(x^t)$; n = 1,...,N.

Multiply both sides of equation n in equations (C2) by $x_n^t/p^t y^t$ for n = 1,…,N. Using (C3) and (C4), the resulting equations become the following system of equations:

(C5) $w_n^t x_n^t/p^t y^t = (\lambda^t/p^t)\theta_n(x^t,t)$; n = 1,…,N.

A measure of monopoly power that is frequently used in the literature on markups[56] is the ratio of price to marginal cost which is p^t/λ^t. Rearranging equations (C5) gives us the following N estimating equations for the *price to marginal cost markup ratio*:

(C6) $p^t/\lambda^t = \theta_n(x^t,t)/[w_n^t x_n^t/p^t y^t]$; n = 1,…,N.

If we have time series data on a production unit for T periods, then (C1)-(C6) hold for t = 1,…,T. Hall (1988; 925) derived equations (C6) but De Loecker and Warzynski (2012; 2443) gave the more complete exposition defined by equations (C1)-(C6) above in their paper. De Loecker, Eeckhout and Unger (2020) were the first to realize the empirical importance of equations (C6): information on expenditures on a single variable input n, $v_n^t \equiv w_n^t x_n^t$, information on the value of output $p^t y^t$ and a knowledge of the output elasticity $\theta_n(x^t,t)$ enables one to estimate the markup ratio, p^t/λ^t (under the assumption of cost minimizing behavior on the part of the production unit).[57] Thus this approach to the measurement of markups is particularly well suited to working with firm level data because firm level data provides (for the most part) only information on output and input *values*; information on output and input *quantities* or *prices* is usually not available.

Of course, there are two major problems with the De Loecker, Eeckhout and Unger methodology:[58]

- It is not straightforward to estimate the output elasticity $\theta_n(x^t,t) \equiv x_n^t [\partial F^t(x^t)/\partial x_n]/F^t(x^t)$;
- Why look at just one of the N equations in equations (C6)? Each equation is a valid estimator of p^t/λ^t.

We address the second objection first. Obviously, the separate possible estimators for p^t/λ^t should be combined somehow. One way of proceeding is to derive a *system* of estimating equations.

Option 1 just uses the N+1 equations (C2) and (C3) for periods t = 1,…,T as estimating equations. The T Lagrange multipliers λ^t are added to the unknown parameters that are imbedded in the production functions $F^1(x)$, ….,$F^T(x)$.

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[55] This follows using the Envelop Theorem under some regularity conditions; see Samuelson (1947; 34-35).
[56] For excellent summaries of the recent markup literature, see Basu (2019) and Syverson (2019).
[57] There is also the assumption that there is only a single output or that outputs can be aggregated by a linearly homogeneous aggregator function g(y). In this case, aggregate output in period t, y^t, is equal to $g(y_1^t,\ldots,y_M^t)$. Using the linear homogeneity restriction on $g(y_1,\ldots,y_M)$ leads to a dual unit revenue function $r(p_1^t,\ldots,p_M^t)$ that can act as the period t price aggregate p^t. Note that our methodology can deal with the many outputs, many inputs situation.
[58] See Basu (2019) and Syverson (2019) for more extended discussions of the methodological problems.

Option 2 eliminates the Lagrange multipliers from the estimating equations. One useful way of doing this is to multiply both sides of equation n in (C2) by x_n^t and to sum the N resulting equations. This summed equation can be solved for λ^t:

(C7) $\lambda^t = w^t \cdot x^t / x^t \cdot \nabla F^t(x^t)$; $t = 1,\ldots,T.$

$\nabla F^t(x^t)$ is the vector of first order partial derivatives of the period t production function $F^t(x)$ evaluated at $x = x^t$. Substitute λ^t defined by (C7) back into equations (C2) and after a bit of rearrangement, we obtain the following system of *inverse demand input share equations*:

(C8) $s_n^t \equiv w_n^t x_n^t / w^t \cdot x^t = x_n^t [\partial F^t(x^t)/\partial x_n] / x^t \cdot \nabla F^t(x^t)$; $n = 1,\ldots,N$; $t = 1,\ldots,T.$

Not all of the parameters of the production functions $F^t(x)$ can be identified using equations (C8): we need to add equations (C3) for $t = 1,\ldots,T$ in order to identify all of the parameters imbedded in the $F^t(x)$. It can be seen that the N share equations that pertain to period t are not independent since $\Sigma_{n=1}^N s_n^t = 1$ for $t = 1,\ldots,T$. Equations (C8) and (C3) form our Option 2 system of estimating equations.

Option 3 estimates a system of input demand functions based on the estimation of the dual cost functions, $C^t(y,w)$ defined by (C1) for $t = 1,\ldots,T$. Choose differentiable functional forms for the $C^t(y,w)$ that are linearly homogeneous and concave in the components of w. Using Shephard's Lemma, the system of period t input demand functions is given by equations (C9) below:

(C9) $x_n^t = \partial C^t(y^t,w^t)/\partial w_n$; $n = 1,\ldots,N$; $t = 1,\ldots,T.$

Once the unknown parameters in the functional form for $C^t(y,w)$ have been estimated, marginal cost for period t is obtained by differentiating the cost function with respect to the output level y:[59]

(C10) $\lambda^t \equiv \partial C^t(y^t,w^t)/\partial y$; $t = 1,\ldots,T.$

It can be seen that it is an extremely difficult task to estimate marginal costs using econometric techniques. Obviously, completely independent period t production functions $F^t(x)$ or cost functions $C^t(y,w)$ cannot be estimated: most of the parameters in these functions cannot depend on time t. To provide second order approximations to an $F^t(x)$ or $C^t(y,w)$ that are independent of time requires $N(N+1)/2$ independent parameters if production is subject to constant returns to scale. Allowing for nonconstant returns to scale will require at least N additional parameters. Allowing for biased technical progress will again require at least N additional parameters and since technical progress is not easily explained by sample wide linear or log linear trends, multiples of N extra parameters will usually be required. Attempting to estimate a functional form that allows for non constant returns to scale as well as technical progress will typically lead to multicollinearity problems. On top of all of these problems, the assumption of cost minimizing behavior on the part of the production unit places curvature restrictions on the $F^t(x)$ and $C^t(y,w)$: the $F^t(x)$ need to be quasiconcave in x and the $C^t(y,w)$ need to be linearly homogeneous and concave in w. If one uses translog

[59] This cost function methodology can easily extended to the case of many outputs. Equations (C9) are still the input demand estimating equations with $y^t \equiv [y_1^t,\ldots,y_J^t]$. Once $C^t(y^t,w^t)$ has been estimated, marginal cost j can be calculated as $\lambda_j^t \equiv \partial C^t(y^t,w^t)/\partial y_j$ for $j = 1,\ldots,J$ and $t = 1,\ldots,T$.

functional forms and there are more than three inputs, typically curvature conditions are not satisfied.[60] All of these problems with the systems approaches to measuring marginal cost carry over to the problem of measuring a single output elasticity $\theta_n(x^t,t)$ defined by (C4) or a single partial derivative of the period t production function, $\partial F^t(x^t)/\partial x_n$. Our accounting approach to the measurement of aggregate markups avoids all of these econometric problems.

We conclude this appendix by noting another approach to measuring marginal cost that was introduced by Basu and Fernald (1997; 253). First, they noted that in the case of a differentiable production function $F^t(x)$, a measure of period t local returns to scale, $\rho^t(x^t)$, is defined as follows:

(C11) $\rho^t(x^t) \equiv x^t \cdot \nabla F^t(x^t)/F^t(x^t)$.

Using (C7), we have the following equation:

(C12) $\lambda^t = w^t \cdot x^t / x^t \cdot \nabla F^t(x^t)$
$= [w^t \cdot x^t/y^t]/[x^t \cdot \nabla F^t(x^t)/y^t]$
$= [w^t \cdot x^t/y^t]/\rho^t(x^t)$

where the last equality follows using $y^t = F^t(x^t)$ and definition (C11). Divide both sides of (C12) by p^t and rearrange terms to obtain the following expression for the markup:

(C13) $p^t/\lambda^t = [p^t/(w^t \cdot x^t/y^t)]\rho^t(x^t)$.

Thus price over marginal cost, p^t/λ^t, is equal to price over average cost, $p^t/(w^t \cdot x^t/y^t)$, times returns to scale, $\rho^t(x^t)$.[61] Thus since average costs can be measured, if one has econometric estimates for returns to scale or makes reasonable guesses about returns to scale, then bounds on the markup ratio can be determined using (C13).[62]

Note that our general markup factor for period t is $M^t \equiv p^t \cdot y^t / w^t \cdot x^t$. In the case of one output, M^t simplifies to the following expression:

(C14) $M^t \equiv p^t y^t / w^t \cdot x^t = p^t/[w^t \cdot x^t/y^t]$.

Thus in the case of one output, our markup concept M^t is equal to output price p^t divided by average cost, $w^t \cdot x^t/y^t$. Our markup can be defined using accounting data; there is no need to estimate returns to scale using econometrics to determine price over average cost.

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[60] For an attempt to address many of these problems when attempting to estimate a joint cost function, see Diewert, Nomura and Shimizu (2025). They did not address all of the problems because they assumed constant returns to scale technologies and no markups. However, they did address the curvature problem and the modeling of technical progress in a more flexible manner.

[61] Basu (2019) and Syverson (2019) use this equation to suggest that the very high markups obtained by De Loecker, Eeckhout and Unger (2020) are implausible.

[62] Of course, if one assumes constant returns to scale, then $x^t \cdot \nabla F^t(x^t) = F^t(x^t)$ and $\rho^t(x^t) = 1$. Thus marginal cost λ^t equals average cost, $w^t \cdot x^t/y^t$.

Historically, the markup literature often uses price over marginal cost as a welfare indicator of the costs of monopoly.[63] However, it was soon realized that pricing outputs at marginal costs for firms that exhibited increasing returns to scale would require government subsidies to enable them to pay their input costs. If governments could not raise enough revenue to cover the subsidies by taxing fixed factors, then taxing variable factors of production would create distortions and deadweight losses in the rest of the economy.[64] Using price over average cost (instead of using price over marginal cost) seems to us to be a reasonable alternative measure for the costs of monopoly that avoids the subsidy problem for production units that exhibit increasing returns to scale. Our general methodology that allows for many outputs is a generalization of the price over average cost approach to measuring the costs of monopoly.

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[63] See Basu (2019) and Syverson (2019). This preference for measuring the welfare cost of a markup dates back to the early literature on the regulation of monopolies. This early literature thought that forcing a monopolist to sell each product at a price equal to the marginal cost of producing it would suffice to bring about an ideal or optimal allocation of resources; see Walras (1980; 83), Pigou (1920; 278), Hotelling (1938; 256) and Lerner (1944; 182).
[64] See Hotelling (1939; 155) and Coase (1946; 179).

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