Trade, Research Productivity, and Growth: A Dynamic General Equilibrium Approach*

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Abstract

Exporting exposes firms to foreign buyers and rivals, providing knowledge that makes innovation more effective. How much does this mechanism raise innovation efficiency, and how does it affect long-run growth? I develop and estimate an endogenous growth model in which foreign-market exposure enhances firms' innovation efficiency, the exporting–innovation efficiency (EIE) channel, while broader product scope dilutes managerial attention and lowers marginal returns. The model predicts that innovation efficiency rises with export intensity but declines with product scope and firm size. Using firm-level data on Chinese manufacturers, I document patterns consistent with these predictions and estimate that export exposure increases the effective knowledge available for innovation by about 9% at the product level. Although firm-level gains are modest, the aggregate effect is economically meaningful: eliminating the EIE channel reduces the long-run growth rate by about 1.3 percentage points (around 10 percent of observed growth). With the EIE channel active, trade liberalization disproportionately benefits firms with broad export portfolios, boosting their innovation efficiency and growth and accelerating the exit of low-productivity firms. The result is a more right-skewed productivity distribution and greater market concentration.

Keywords— Export, Innovation Efficiency, Firm Dynamics, Endogenous Growth, General Equilibrium, Productivity Dispersion

JEL codes: C68, E25, F10, F41, F43, O24, O31, O41

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1 Introduction

Examples abound of manufacturers devising new products or improving processes once they begin exporting. Exposure to market-specific requirements, richer information flows, and disciplined feedback from foreign buyers helps R&D target implementable ideas. For instance, Haier entered the U.S. with compact refrigerators and, based on retailer and consumer feedback, broadened its line (e.g., wine refrigerators) as it built a U.S. presence.[1] In autos, BYD responded to Euro NCAP's 2024 "Not Recommended" grading of the ATTO 3's highway-assist by pushing a software update for existing models and launching a new system.[2] The medical device company Mindray upgraded patient monitors to meet U.S. FDA requirements—changes that forced better reliability, documentation, and features[3]. The same pattern shows up among exporters from advanced economies: Toyota created the Lexus LS400 specifically for its 1989 U.S. launch after months of on-the-ground consumer research into American luxury-car expectations[4]; German and Swedish suppliers engineered emissions technologies for the U.S. market—Mercedes-Benz's BlueTEC diesel after the tightening of U.S. Tier2 limits, and Bosch's lambda oxygen-sensor with a three-way catalyst that Volvo first launched on U.S. models in 1976—before these advances diffused globally; and Finland's Nokia engineered the 1100 for emerging-market conditions (dust-proof keypad, integrated flashlight) and then scaled it worldwide.[5][6]

Such episodes illustrate the paper's central premise: exporting does more than expand market size; it raises the efficiency of R&D, which is the expected flow of implementable ideas per unit of innovation input—by exposing firms to heterogeneous demand, rival practices, and production know-how. I refer to this mechanism as the *exporting–innovation-efficiency* (EIE) channel. Given export and innovation decisions are made jointly by forward-looking firms, assessing the macroeconomic importance of EIE requires a dynamic, general equilibrium framework in which innovation choices shape the allocation of resources over time. My model estimates the strength of EIE and quantifies its aggregate implications for growth, trade, and the distribution of firms.

To my knowledge, this paper is the first to develop and estimate a dynamic structural model in an open economy in which *export exposure directly raises the productivity of R&D* (the EIE channel) while *broader product scope dilutes marginal R&D returns*. The model provides clear microfoundations for export-driven knowledge diffusion: both market size and EIE effects shape firms' innovation and exporting choices, and these firm-level decisions aggregate to macroeconomic outcomes in general equilibrium. Embedding the EIE mechanism in an open-economy GE environment allows me to measure its magnitude at the firm level and quantify its long-run aggregate consequences.

In the model, forward-looking multiproduct firms allocate resources across production, ex-

[1] See Leinwand, Mainardi, and Kleiner (2015); Schuman (2014).
[2] See Euro NCAP (2025); Reuters (2024).
[3] See U.S. Food and Drug Administration (2008).
[4] See Toy (2025).
[5] See Die (2006); EPA (2016); Vol (2006); Bos (2016).
[6] See Virki (2007); Mob (2025).

porting, and R&D. Building on the multiproduct-firm literature (Bernard, Redding, and Schott, 2011), I treat product scope as a form of knowledge capital, following Klette and Kortum (2004) and Acemoglu et al. (2018). Exporting shifts the idea-production technology by exposing firms to foreign demand, rival practices, and production know-how, thereby raising the productivity of R&D—the EIE channel. At the same time, broader product scope stretches organizational and managerial bandwidth, reducing marginal R&D returns (scale-dilution), consistent with models of intangible, non-tradable capabilities (e.g., Crouzet et al., 2022; Nocke and Yeaple, 2014). I link a firm's intangible capacity to its *core productivity* in multiproduct models; innovation raises core productivity, which manifests as higher process efficiency and an expanded product set and ultimately in sales growth. A novel element is that firms choose their export portfolio by weighing both static profits *and* dynamic EIE benefits: a product is exported if the incremental value—current profit plus the expected return to EIE-enhanced innovation—exceeds its (line-specific) export fixed cost.

Entry, selection, and innovation jointly determine growth. Entrants pay a fixed cost and draw productivity; their initial draws co-move with the incumbent distribution through domestic knowledge spillovers, as in Sampson (2016). Incumbents invest in innovation to survive market selection. I convert the balanced-growth problem into a fixed-point in the *relative* productivity distribution, productivity normalized by the market cutoff, and compute the stationary density by solving a Kolmogorov forward (Fokker–Planck) equation with the associated infinitesimal generator. This adapts stationary-distribution methods in heterogeneous-firm models (Hopenhayn, 1992) to an environment with growth and spillover-induced co-movement between entrants and incumbents, in the spirit of balanced-growth approaches to firm dynamics.

The key market failure stems from the fact that new entrants learn from the technology of incumbent firms, while the latter innovate without taking this knowledge spillover into account. In this context, where trade affects innovation, both trade policy and innovation policy have the potential to achieve higher economic growth by stimulating the innovation of incumbents and reallocating resources to more productive firms.

The spillover block is not only economically motivated; it also delivers tractability. In models without growth, a stationary Markov process for firm productivity implies an invariant cross-sectional distribution, so the aggregate state can be characterized as a fixed point (Hopenhayn, 1992). With innovation-driven growth, aggregate (absolute) productivity drifts; by normalizing productivity by the cutoff and letting entrant draws co-move with the incumbent distribution via spillovers, the *relative* distribution becomes stationary, restoring a fixed-point characterization. I implement this by discretizing the Kolmogorov forward equation and constructing the infinitesimal generator matrix that encodes firms' innovation policy function and exit rule. The resulting sparse linear system yields the stationary density and thus the stationary relative equilibrium.

I discipline the model with Chinese manufacturing microdata for 2004–2007 by merging firm-destination-product customs records with the Industrial Enterprise Survey (production, financials, and R&D). These sources allow me to track entry and exit and to construct, at the firm level, export intensity and foreign-market scope (products and destinations), alongside

sales, value added, employment, and R&D. Moreover, 2004–2007 is a period of rapid integration of China into world markets. Over these four years, merchandise exports more than doubled—from about US$593 billion in 2004 to about US$1.22 trillion in 2007—while exports of goods and services reached roughly 35% of GDP in 2007 and real GDP grew at double-digit rates (about 10.1% in 2004, peaking near 14.2% in 2007). This window delivers sharp variation in foreign exposure against a backdrop of exceptionally fast growth, providing an informative environment to discipline the model's mechanisms.

This paper measures a firm's *innovation efficiency* as innovation output per unit of input: the growth of revenue-based total factor productivity (TFPR) per unit of R&D expenditure.[7] Two empirical patterns align with the model's predictions: *(i)* conditional on export exposure, innovation efficiency declines with firm scope/size (scale-dilution); and *(ii)* conditional on size, exporters exhibit higher innovation efficiency than non-exporters on the extensive margin, and innovation efficiency rises with export intensity on the intensive margin (EIE).

I estimate the model by a GMM procedure. Identification of the EIE parameter exploits cross and within firm covariation between innovation efficiency and export intensity (controlling for scale/scope), while the scale-dilution parameter is pinned down by the negative gradient of innovation efficiency with respect to product scale/scope conditional on foreign exposure. The moment set includes quantile statistics from the joint distribution of R&D shares, export intensity, sales levels, and sales growth. The estimates imply that adding one export line raises a firm's *effective knowledge* by about 9% of the marginal contribution of adding one *product* line: exporting materially enhances EIE, but the effect is smaller than that of expanding domestic scope. In counterfactuals, reducing the tariff reallocates activity toward more productive incumbents (raising dispersion). Ignoring EIE materially *understates* the gains from trade and can mis-rank policy packages that combine trade reforms with R&D incentives. Economic intuition suggests why: when exporting boosts innovation efficiency, firms may optimally retain marginal export lines because foreign exposure improves the payoff to R&D. Consequently, uniform trade policies can have unintended distributional effects; expanding foreign exposure tends to disproportionately benefit high-productivity incumbents with broad export portfolios, thickening the right tail of the firm distribution.

In summary, this paper's contributions are fourfold. *Mechanism:* It formalizes an *exporting–innovation efficiency* channel, distinct from market size, and articulate a countervailing *scale-dilution* force rooted in organizational constraints. *Theory/methodology:* I embed EIE in a dynamic open-economy GE model with domestic spillovers and selection, and solve for the relative stationary distribution via a Kolmogorov-Forward fixed point. *Evidence and measurement:* Using Chinese firm-level data, I document new facts on innovation efficiency vs. export intensity and scope. *Quantification and policy:* An additional export line raises product's effective knowledge for innovation by about 9%. ; the EIE effect contributes to the economy growth by about 1.5 pp and increases the inequality across firms; failing to model EIE materially understates the gains from trade and changes optimal policy rankings.

This paper connects to four broad literature: (i) trade, diffusion, and growth; (ii) learning-

[7]Following the TFPR/TFPQ distinction in Hsieh and Klenow (2009).

by-exporting; (iii) multiproduct dynamics; and (iv) innovation efficiency—and contributes by *endogenizing* firm-level innovation efficiency as a function of export exposure and product scope within a dynamic open-economy GE framework.

Classic endogenous-growth models emphasize market-size effects whereby openness raises the private return to innovation (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). A complementary literature highlights cross-border knowledge diffusion as a driver of aggregate productivity, with globalization relaxing selection, imitation, and adoption frictions (e.g., Coe and Helpman, 1995; Keller, 2004; Perla, Tonetti, and Waugh, 2021; Buera and Oberfield, 2020). Relative to these strands, I *endogenize the productivity of R&D* as a function of foreign exposure (the EIE channel) and embed it in a dynamic open-economy GE framework with selection and spillovers. This structure lets me quantify the consequences for the firm-size distribution, long-run growth, and the gains from trade, and to conduct trade-policy counterfactuals that explicitly account for export-mediated knowledge diffusion rather than treating diffusion as exogenous or uniform across firms.

A large empirical literature about learning-by-exporting (LBE) finds that exporting is associated with post-entry productivity and quality upgrading, with evidence ranging from reduced-form and structural estimations to randomized interventions (e.g., Clerides, Lach, and Tybout, 1998; Loecker, 2013; Aw, Roberts, and Xu, 2011; Atkin, Khandelwal, and Osman, 2017; Liang, 2024). Related work documents technology upgrading in response to export opportunities (Bustos, 2011; Lileeva and Trefler, 2010) and buyer-driven transfers of know-how and standards. My contribution is to provide a GE framework in which LBE operates *through* the EIE channel, foreign exposure raises the innovation efficiency, while a countervailing scale-dilution force tempers marginal innovation returns. The model nests the static market-size effect and thus separates informational gains from pure demand-scale forces.

Multiproduct models explain within-firm product churn, scope choices, and the role of "core" capabilities (Bernard, Redding, and Schott, 2011; Nocke and Yeaple, 2014). Work on intangible/organizational capital shows that non-tradable managerial resources shape firm productivity and reallocation (e.g., Crouzet et al., 2022). I build on these insights by (i) linking a firm's intangible capacity to a *core productivity* state that governs both production efficiency and the arrival of new product ideas; (ii) modeling *scale-dilution* as an equilibrium allocation of finite managerial attention across product lines; and (iii) showing how export-driven EIE and scope-driven dilution jointly generate a relationship between innovation efficiency and firm size.

This paper also enriches the literature of measurement and heterogeneity in innovation efficiency. Growth and firm-dynamics models often treat the idea-production technology as homogeneous across incumbents (Grossman and Helpman, 1991; Aghion and Howitt, 1992; Klette and Kortum, 2004; Lentz and Mortensen, 2008; Acemoglu et al., 2018). Recent work measures cross-sectional heterogeneity in innovation efficiency and knowledge production (e.g., Acemoglu et al., 2018; Fort et al., 2020; Sampson, 2023; Fan, 2025; Liu, 2024; Liang, 2024), typically taking it as exogenous. I instead *endogenize* firm-level innovation efficiency through export exposure (intensity and destination mix) and product scope, and I take the model to data to discipline these channels.

I build on stationary-distribution approaches to heterogeneous-firm dynamics (Hopenhayn, 1992), adapting the Kolmogorov forward equation for the distribution of *relative* productivity to accommodate balanced growth with spillover-induced co-movement. This formulation converts the aggregate problem—finding the stationary distribution of relative productivity—into a fixed-point problem. I model innovation in continuous time, which delivers a tractable infinitesimal-generator representation linking firm policies to the aggregate law of motion. This nonparametric solution strategy relaxes strong parametric restrictions on the productivity distribution, allowing flexible tail behavior and a transparent mapping from micro policies to the stationary density.

The remainder of this paper is organized as follows. Section 2 presents the model and solution method. Section 3 documents empirical patterns and measurement of innovation efficiency. Section 4 describes the estimation strategy and identification and reports quantitative results and policy counterfactuals. Section 5 concludes.

2 Model

The model has three tightly linked parts. (i) Static environment. We embed heterogeneous firms in a Melitz-style world with monopolistic competition and trade costs, augmented to allow multi-product firms in the spirit of Bernard–Redding–Schott; each firm manages a continuum of product lines, which we use to map product scope into an information/knowledge stock at the firm level. (ii) Innovation mechanism. Incumbents choose R&D and product scope (domestic lines and export lines) jointly. Export exposure raises innovation efficiency—capturing learning from foreign market penetration—while broader domestic scope dilutes innovative effort; firms trade off immediate operating profits against dynamic gains from higher innovation success, which endogenously pins down the attribute cutoffs (domestic/export) and the optimal success intensity. (iii) Entry, exit, and spillovers. New firms enter by paying a sunk cost and drawing core productivity; the incumbent cross-section evolves through selection (exit at a productivity cutoff), innovation jumps, and knowledge spillovers, yielding a Kolmogorov forward equation for the stationary relative distribution that is solved together with the firm HJB. In equilibrium, the aggregate state is obtained as a linear fixed-point problem in relative units, in the tradition of stationary industry models with entry and exit.

2.1 A Multi-Product Melitz Economy with a Continuum of Lines

Time is continuous, indexed by $t \in \mathbb{R}$. The world consists of two symmetric countries, $i \in \mathcal{C} = \{1, 2\}$. Each country hosts a continuum of symmetric products $j \in [0, 1]$.

2.1.1 Household

In country $n \in \mathcal{C}$, a representative household maximizes a constant relative risk aversion (CRRA) utility function:

$$U_t = \int_t^\infty e^{-\rho(s-t)} \frac{C_n(s)^{1-\eta} - 1}{1-\eta} \, ds, \tag{1}$$

where $\rho > 0$ is the subjective discount *rate* and $\eta > 0$ is the coefficient of relative risk aversion (the inverse of the intertemporal elasticity of substitution). The aggregate consumption index C_n is CES across products,

$$C_n = \left(\int_0^1 c_{nj}^{\frac{\theta-1}{\theta}} \, dj \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

with elasticity of substitution $\theta > 1$ across products.

Within each product j, there is a continuum of differentiated varieties ω offered by firms located in both countries. Let Q_{inj} denote the set of varieties of product j supplied to market n by firms headquartered in country i. Consumers in country n value variety ω in product j according to a *product attribute* (quality/taste shifter) $z_{nj}(\omega) > 0$.[8] The within-product aggregator is

$$c_{nj} = \left(\sum_{i \in \mathcal{C}} \int_{\omega \in Q_{inj}} \left[z_{nj}(\omega) \, c_{inj}(\omega) \right]^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $\sigma > 1$ is the elasticity of substitution across varieties within a product.

The associated price index of product j in country n is

$$p_{nj} = \left[\sum_{i \in \mathcal{C}} \int_{\omega \in Q_{inj}} \left(\frac{p_{inj}(\omega)}{z_{nj}(\omega)} \right)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \tag{4}$$

Let P_n denote the aggregate price index dual to Eq(2):

$$P_n = \left(\int_0^1 p_{nj}^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}}, \tag{5}$$

where θ represents the elasticity of substitution across products. We choose the final good as the numéraire, normalizing its price index to unity: $P_n(t) \equiv 1$ for all n and t.

Households supply labor inelastically, with total supply normalized to $L \equiv 1$. Each unit of labor earns a (real) wage w_t at time t. The household owns domestic firms. Let A_t denote the household's asset wealth, defined as the market value of claims on domestic firms. The flow budget constraint (HBC) is

$$\dot{A}_t \;=\; w_t \, L \;+\; r_t \, A_t \;-\; C_n(t), \tag{6}$$

where r_t is the real interest rate. The environment is deterministic. The economy satisfies the

[8] Following Bernard, Redding, and Schott (2011), $z_{nj}(\omega)$ can be read as a demand shifter (quality) that raises the effective units of consumption of ω; equivalently, it can be mapped to product-specific productivity inside the firm under a demand system with vertical differentiation.

standard no-Ponzi and transversality conditions. The no-Ponzi restriction (constant-r case) is

$$\lim_{T\to\infty} e^{-r(T-t)}A_T \geq 0, \tag{7}$$

and, more generally, with $R(t,T) \equiv \int_t^T r_s ds$, it is $\liminf_{T\to\infty} e^{-R(t,T)}A_T \geq 0$. The transversality condition is

$$\lim_{t\to\infty}\left\{A_t \exp\left[-\int_0^t r_s\, ds\right]\right\} = 0. \tag{8}$$

The household's first-order condition yields the standard Euler equation

$$\frac{\dot{C}_n(t)}{C_n(t)} = \frac{1}{\eta}\left(r_t - \rho\right). \tag{9}$$

Hence consumption grows (declines) whenever the real interest rate exceeds (falls short of) the subjective discount rate; η scales the sensitivity via the intertemporal elasticity $1/\eta$.

2.1.2 Firms

There is a continuum of potential entrants. To become active, a firm must pay a sunk entry cost $f^e > 0$ (in units of labor). Upon entry, the firm draws a *core productivity* q from a distribution with CDF $G_t(q)$. We interpret q as an ownership advantage or intangible capability in the spirit of Yeaple (2013) and Crouzet et al. (2022): a firm-wide efficiency that scales all product lines.

Incumbents' cross-sectional distribution of q at time t is denoted by Φ_t. Conditional on being active, each firm can operate multiple product lines. Within any product j, varieties produced by different firms are horizontally differentiated and compete under monopolistic competition.

For each potential line, the firm draws an idiosyncratic *attribute* (quality/taste shifter) $z \in [1,\infty)$ from a Pareto distribution

$$\Psi(z) = 1 - z^{-\iota} \quad \text{for } z \geq 1, \qquad \iota > 1.$$

Draws of q are independent across firms and independent of all z draws; within a firm, $\{z_j\}$ are i.i.d. across products. These orthogonality assumptions allow aggregation via the law of large numbers.

Incumbents face per-market, per-product fixed operating costs f_{in} (labor units), with $f_{in} > f_{ii}$ for all $i \neq n$. Each firm also pays a firm-level headquarters cost f_i^m every instant. Shipping a unit from i to n requires an iceberg trade cost $\tau_{in} \geq 1$ (with $\tau_{ii} = 1$). Wages are w_i.

Technology and pricing. A firm i with core productivity q uses a linear technology for any product line j,

$$y_j = q\,\ell_j,$$

so marginal cost is w_i/q. With CES demand within a product and monopolistic competition, the optimal consumer price in market n is a constant markup over delivered marginal cost:

$$p_{inj}(q, z_j) = \frac{\sigma}{\sigma - 1} \tau_{in} \frac{w_i}{q} \equiv p_{in}(q), \tag{10}$$

independent of z_j.

Given the demand system above, a variety with attribute z earns revenue

$$r_{inj}(q, z) = \left(\frac{p_{in}(q)}{z} \right)^{1-\sigma} P_n^{\sigma} C_n. \tag{11}$$

Under our numéraire $P_n \equiv 1$, this simplifies to $r_{inj}(q, z) = (p_{in}(q)/z)^{1-\sigma} C_n$.[9] Operating profits on a line equal a constant fraction of revenue, $\frac{1}{\sigma} r_{inj}(q, z)$, minus the per-line fixed cost:

$$\pi_{inj}(q, z) = \frac{1}{\sigma} r_{inj}(q, z) - w_i f_{in}. \tag{12}$$

Forward-looking firms choose innovation, production, and exporting jointly to maximize firm value, internalizing that strategic exporting can enhance innovation efficiency. They balance the short-run profit consequences of scope and exporting against the long-run benefits via innovation.

2.2 Innovation and Scope: Export-Induced Efficiency, Dilution, and R&D Choices

Firms allocate resources to research and development (R&D) to raise core productivity. This investment reflects the objective of generating new profitable knowledge based on available information. Knowledge improvements can take the form of more efficient production for existing lines, enhanced sales efficiency, or the development of new lines. (Innovation here is a general concept combining process and product innovation. Although the framework is multi-product, it differs from Klette and Kortum (2004): they model a discrete number of lines and a success rate that draws a new line with random productivity, yielding an innovation intensity independent of firm size. Acemoglu et al. (2018) extend Klette and Kortum (2004) by allowing an exogenous "innovation capacity" (our innovation efficiency) to differ by type and to switch exogenously over time. In contrast, here innovation efficiency is *endogenous* and shaped by the firm's export decision.)

We adopt a step-by-step quality-ladder structure: if a firm with productivity q innovates successfully, its q jumps to $(1+\lambda)q$, with step size $\lambda > 0$. Successes arrive according to a Poisson process with (endogenous) intensity $x(q) \geq 0$. Over an interval $[t, t + dt)$, a success occurs with probability $x(q)\, dt$; otherwise q is unchanged.

A key departure from the canonical growth model is that the efficiency of innovation depends on the firm's *scale/scope* and *export intensity* (foreign market penetration). For a firm with

[9] With symmetry across products j, p_{nj} is constant and equals P_n, so (11) is the standard nested-CES demand specialized to the symmetric case.

productivity q, let $m_{ii}(q)$ and $m_{in}(q)$ denote, respectively, the expected mass of active lines sold domestically and exported. With Pareto Ψ,

$$m_{ii}(q) = \int_{z^*_{ii}(q)}^{\infty} d\Psi(z) = \big(z^*_{ii}(q)\big)^{-\iota}, \qquad m_{in}(q) = \int_{z^*_{in}(q)}^{\infty} d\Psi(z) = \big(z^*_{in}(q)\big)^{-\iota},$$

where $z^*_{ii}(q)$ and $z^*_{in}(q)$ are the endogenous attribute cutoffs for operating a line in the domestic and foreign markets, determined by Eq(18)–Eq(19). Define export intensity

$$\nu(q) \equiv \frac{m_{in}(q)}{m_{ii}(q)} = \left(\frac{z^*_{ii}(q)}{z^*_{in}(q)}\right)^{\iota} \in [0,1].$$

Following Klette and Kortum (2004), the innovation success rate depends on both R&D investment and knowledge capital; we differ by incorporating export-generated knowledge. The innovation technology is

$$x(q) = c_x \frac{\overbrace{\big(m_{ii}(q) + \mu\, m_{in}(q)\big)^{\gamma}}^{\text{Knowledge}} \quad \overbrace{\big(h(q)\big)^{1-\gamma}}^{\text{Innovative effort}}}{\underbrace{m_{ii}(q)}_{\text{Scale dilution}}}, \qquad c_x > 0,\ \mu > 0,\ \gamma \in (0,1), \tag{13}$$

where $h(q) \geq 0$ is R&D labor. Knowledge and innovative effort enter in Cobb–Douglas form (unit elasticity of substitution); γ is the *share* on the knowledge term. The denominator $m_{ii}(q)$ captures scale/scope dilution: as a firm operates more domestic lines, knowledge and R&D effort are spread more thinly across lines, reducing marginal innovation efficiency. The component $\mu\, m_{in}(q)$ captures export-related knowledge—scaled by μ, which measures how strongly exporting boosts innovation efficiency (the export-induced efficiency, EIE, channel). Setting $\mu = 0$ and dropping per-line normalization collapses Eq(13) to the benchmark in which arrivals depend only on R&D input (or total scope times R&D), as in Klette and Kortum (2004); our formulation embeds both the EIE channel and scale-dilution. Rewriting Eq(13) as inputs *per line* gives

$$x(q) = c_x \left(\frac{m_{ii}(q) + \mu\, m_{in}(q)}{m_{ii}(q)}\right)^{\gamma} \left(\frac{h(q)}{m_{ii}(q)}\right)^{1-\gamma} = c_x \big(1 + \mu\, \nu(q)\big)^{\gamma} \left(\frac{h(q)}{m_{ii}(q)}\right)^{1-\gamma}. \tag{14}$$

As a suggestive empirical check of Eq(14), we take logs and regress a proxy for the innovation success rate (labor-productivity growth) on the logs of innovation intensity and export intensity, controlling for covariates. Both coefficients are positive and statistically significant (see Table **??** in the Appendix).

Inverting Eq(14) yields the R&D labor required to attain a given success rate x:

$$h(q,x) = \left(\frac{x}{c_x}\right)^{\frac{1}{1-\gamma}} \big(1 + \mu\, \nu(q)\big)^{-\frac{\gamma}{1-\gamma}}\, m_{ii}(q), \tag{15}$$

so the instantaneous R&D cost at wage w_i is $R(x,q) = w_i\, h(q,x)$.

Accordingly, define *innovation efficiency* as the average the success rate per unit of R&D

labor:

$$\frac{x(q)}{h(q)} = c_x^{\frac{-\gamma}{1-\gamma}} \Big(1 + \mu\,\nu(q)\Big)^{\gamma} \Big(\frac{h(q)}{m_{ii}(q)}\Big)^{-\gamma} \frac{1}{m_{ii}(q)}, \tag{16}$$

which exhibits diminishing returns ($\propto h(q)^{-\gamma}$) and increases with export intensity $\nu(q)$, holding $(h(q), m_{ii}(q))$ fixed.

2.2.1 Product-scope Decisions

Let $\Xi(q)$ denote the continuation value from R&D at core productivity q. For a chosen Poisson success intensity $x \geq 0$, the expected gain from R&D is $x\,\Delta V(q)$, where

$$\Delta V(q) \;\equiv\; \mathbb{E}\big[V\big((1+\lambda)q\big) - V(q)\big]$$

is the expected jump in firm value upon a successful innovation. Implementing intensity x requires $h(q,x)$ units of R&D labor, paid at wage w_i. Hence the R&D continuation value is

$$\Xi(q) \;\equiv\; \max_{x\geq 0} \Big\{\, x\,\Delta V(q) \;-\; w_i\, h(q,x) \,\Big\}. \tag{17}$$

Given q, a firm operates a line with draw z in market $n \in \{i, \text{foreign}\}$ if the instantaneous gain covers the per-line fixed cost. The marginal (cutoff) attributes $z_{ii}^*(q)$ and $z_{in}^*(q)$ satisfy

$$\frac{1}{\sigma}\, r_{iij}(q, z_{ii}^*(q)) \;+\; \frac{\partial \Xi(q)}{\partial m_{ii}} = w_i f_{ii}, \tag{18}$$

$$\frac{1}{\sigma}\, r_{inj}(q, z_{in}^*(q)) \;+\; \frac{\partial \Xi(q)}{\partial m_{in}} = w_i f_{in}. \tag{19}$$

Intuitively, a marginal line must cover operating cost *net* of the dynamic benefits/costs an extra line confers through innovation (via Ξ).

Profits earned by a firm i with q in market n are

$$\pi_{in}(q) = \int_{z_{in}^*(q)}^{\infty} \Big(\frac{1}{\sigma} r_{in}(q,z) - w_i f_{in}\Big)\, d\Psi(z), \qquad \pi_i(q) = \sum_n \pi_{in}(q). \tag{20}$$

2.2.2 Bellman Equation and Innovation Policy

Let $\Delta V(q) \equiv \mathbb{E}[V'((1+\lambda)q) - V'(q)]$. Given (m_{ii}, m_{in}), the incumbent chooses a success intensity x to maximize the present flow value:

$$r\,V(q) = \max\Big\{0,\; \pi_i(q) - w_i f_i^m \;+\; \underbrace{\max_{x\geq 0} \left[x\,\Delta V(q) - w_i\, h(q,x)\right]}_{\Xi(q)} \Big\}, \tag{21}$$

where $h(q,x)$ is given by Eq(15). The inner problem yields the R&D policy

$$x(q) = \max\left\{0,\; c_x^{1/\gamma} \Big(m_{ii}(q)\Big)^{\frac{\gamma-1}{\gamma}} \Big(1 + \mu\,\nu(q)\Big) \left[\frac{(1-\gamma)\,\Delta V(q)}{w_i}\right]^{\frac{1-\gamma}{\gamma}} \right\}, \tag{22}$$

and, using Eq(15),

$$h(q) = c_x^{1/\gamma} \left(m_{ii}(q)\right)^{\frac{\gamma-1}{\gamma}} \left(1+\mu\,\nu(q)\right) \left[\frac{(1-\gamma)\,\Delta V(q)}{w_i}\right]^{1/\gamma}. \tag{23}$$

For an interior solution, the associated R&D value is $\Xi(q) = \gamma\,\Delta V(q)\,x(q)$.

Comparative statics for $\Xi(q)$ with respect to scope. Using the closed-form continuation value $\Xi(q)$, for an interior R&D choice $x(q) > 0$ we obtain

$$\frac{\partial \Xi(q)}{\partial m_{ii}(q)} = -\frac{\Xi(q)}{m_{ii}(q)} \left[\frac{1-\gamma}{\gamma} + \frac{\mu\,\nu(q)}{1+\mu\,\nu(q)}\right], \tag{24}$$

$$\frac{\partial \Xi(q)}{\partial m_{in}(q)} = \frac{\mu}{1+\mu\,\nu(q)} \frac{\Xi(q)}{m_{ii}(q)}. \tag{25}$$

Proposition 1 (Dynamic marginal value of scope)**.** *Fix $\mu > 0$, $\gamma \in (0,1)$, and $m_{ii}(q) > 0$. If the R&D problem has an interior solution ($x(q) > 0$ so that $\Xi(q) > 0$), then*

$$\frac{\partial \Xi(q)}{\partial m_{ii}(q)} < 0 \qquad \textit{and} \qquad \frac{\partial \Xi(q)}{\partial m_{in}(q)} > 0.$$

Thus, (i) increasing domestic scope reduces the continuation value of R&D (a scale/scope-dilution effect), while (ii) increasing export scope raises it (an export-induced efficiency, EIE, effect).

Proof sketch. Equations Eq(24)–Eq(25) follow from the envelope theorem applied to $\Xi(q) = \max_{x \geq 0}\{x\,\Delta V(q) - w_i h(q,x)\}$ with h given by Eq(15) and $\nu(q) = m_{in}(q)/m_{ii}(q)$. For $\gamma \in (0,1)$ and $\mu \geq 0$, the bracket in Eq(24) is strictly positive, implying $\partial\Xi/\partial m_{ii} < 0$; Eq(25) is strictly positive since $\mu > 0$, $m_{ii} > 0$, and $\Xi > 0$. ■

Corollary 1 (Implications for static profits at the cutoffs)**.** *Using the cutoff conditions Eq(18)–Eq(19),*

$$\frac{1}{\sigma} r_{iij}(q, z_{ii}^*(q)) - w_i f_{ii} \;=\; -\frac{\partial \Xi(q)}{\partial m_{ii}(q)} \;>\; 0, \qquad \frac{1}{\sigma} r_{inj}(q, z_{in}^*(q)) - w_i f_{in} \;=\; -\frac{\partial \Xi(q)}{\partial m_{in}(q)} \;<\; 0,$$

whenever $x(q) > 0$. Hence the marginal domestic line breaks even with a positive *static profit, while the marginal export line breaks even with a* negative *static profit—capturing strategic exporting to raise future innovation efficiency.*

Remark 1 (Corner case)**.** *If $x(q) = 0$ (equivalently $\Delta V(q) \leq 0$), then $\Xi(q) = 0$ and, by the envelope theorem, $\partial\Xi(q)/\partial m_{ii} = \partial\Xi(q)/\partial m_{in} = 0$.*

Cutoff ratios. Combining Eq(18)–Eq(19) with Eq(11) yields the within-firm ratio of export to domestic attribute cutoffs:

$$\frac{z_{in}^*(q)}{z_{ii}^*(q)} = \frac{\tau_{in}}{\tau_{ii}} \left(\frac{w_i f_{in} - \partial\Xi(q)/\partial m_{in}}{w_i f_{ii} - \partial\Xi(q)/\partial m_{ii}} \cdot \frac{C_i}{C_n}\right)^{\frac{1}{\sigma-1}}. \tag{26}$$

Under country symmetry ($C_i = C_n$) and $P_n \equiv 1$, this simplifies to

$$\frac{z^*_{in}(q)}{z^*_{ii}(q)} = \frac{\tau_{in}}{\tau_{ii}} \left(\frac{w_i f_{in} - \partial \Xi(q)/\partial m_{in}}{w_i f_{ii} - \partial \Xi(q)/\partial m_{ii}} \right)^{\frac{1}{\sigma-1}}.$$

Since $f_{in} > f_{ii}$ and (for interior x) $\partial \Xi/\partial m_{in} > 0 > \partial \Xi/\partial m_{ii}$, the export cutoff exceeds the domestic cutoff: $z^*_{in}(q) > z^*_{ii}(q)$, all else equal.

Exit (core-productivity) cutoff. Let q^* be the marginal incumbent in country i. The zero-value condition is

$$\pi_i(q^*) + \Xi(q^*) = w_i f^m_i, \tag{27}$$

which pins down the market's core-productivity cutoff.

A convenient restatement. Using Eq(22)–Eq(23) in Eq(21), one may equivalently write

$$rV(q) = \max\Big\{0,\ \pi_i(q) - w_i f^m_i + \Xi(q)\Big\}, \qquad \Xi(q) = \max_{x \geq 0} \big\{ x\, \Delta V(q) - w_i h(q,x) \big\}. \tag{28}$$

From Eq(15), $R(x,q) = w_i m_{ii}(q) \left(\frac{x}{c_x}\right)^{\frac{1}{1-\gamma}} (1 + \mu\nu(q))^{-\frac{\gamma}{1-\gamma}}$ and the (flow) innovation intensity can be defined as $R(q)/\pi_i(q)$.

2.3 Entry, Exit, and Spillovers: Cross-Sectional Dynamics, KFE, and Stationary Equilibrium

Entrants hire R&D labor to acquire an initial stock of knowledge and draw a core productivity. Paying f^e units of labor per draw (so cost $f^e w_t$), a mass R_t of draws requires $R_t f^e$ units of labor. Each draw yields a core productivity q.

Knowledge spillovers arise because process technologies are partially non-excludable: new firms can learn from incumbents' techniques (production, managerial practices, organization, input choices, etc.). Due to diffusion frictions (e.g., legal protection, imperfect observability), entrants do not simply copy the most productive firm; instead, their draw reflects the *entire* distribution of incumbent technologies, as in Sampson (2016).

We model entrants' productivities as

$$q = \bar{q}_t\, s, \tag{29}$$

where $\bar{q}_t$ is the average productivity of incumbents at time t and s is a draw from a time-invariant distribution $F(\cdot)$, which is assumed as a Pareto distribution following the literature. Spillovers operate through $\bar{q}_t$: if the incumbent distribution shifts by a factor k (so $\bar{q}_{t1} = k\bar{q}_{t0}$), the entrant distribution shifts proportionally. The induced entrant CDF is

$$G_t([0,q)) = F([0, q/\bar{q}_t))\,. \tag{30}$$

Free entry equates the expected cost of a draw to its expected value:

$$f^e w_t = \mathbb{E}_{G_t}[V_t(q)] \;=\; \int V_t(q)\, dG_t(q). \tag{31}$$

2.3.1 Relative Terms and Stationary Equilibrium

For computation, it is convenient to express objects in relative terms. In a stationary (balanced-growth) equilibrium, the cutoff productivity grows at a constant rate $g \equiv \dot{q}^*/q^*$. Define relative productivity by $\tilde{q} \equiv q/q_t^*$ and scale variables as

$$\tilde{V}_t(\tilde{q}) \equiv \frac{V_t(q)}{q_t^*}, \quad \tilde{\pi}_t(\tilde{q}) \equiv \frac{\pi_t(q)}{q_t^*}, \quad \tilde{w}_t \equiv \frac{w_t}{q_t^*}, \quad \tilde{\nu}_t(\tilde{q}) \equiv \nu_t(q), \quad \tilde{m}_{in,t}(\tilde{q}) \equiv m_{in,t}(q),$$

with $q = \tilde{q}\, q_t^*$. Then the Bellman equation in relative terms is

$$r\,\tilde{V}(\tilde{q}) \;=\; \max\Bigg\{0,\; \tilde{\pi}_i(\tilde{q}) - \tilde{w}_i f_i^m \;+\; \underbrace{\max_{\tilde{x}\geq 0}\Big[\tilde{x}\,\Delta\tilde{V}(\tilde{q}) - \tilde{w}_i\, h(\tilde{q},\tilde{x})\Big]}_{\tilde{\Xi}(\tilde{q})}\Bigg\}, \tag{32}$$

where

$$\Delta\tilde{V}(\tilde{q}) \equiv \tilde{V}\left(\frac{(1+\lambda)\tilde{q}}{1+g}\right) - \tilde{V}\left(\frac{\tilde{q}}{1+g}\right), \qquad h(\tilde{q},\tilde{x}) = \left(\frac{\tilde{x}}{c_x}\right)^{\frac{1}{1-\gamma}} \big(1+\mu\,\tilde{\nu}(\tilde{q})\big)^{-\frac{\gamma}{1-\gamma}}\, \tilde{m}_{ii}(\tilde{q}).$$

The associated policy in relative terms is

$$\tilde{x}(\tilde{q}) = \max\left\{0,\; c_x^{1/\gamma}\left(\tilde{m}_{ii}(\tilde{q})\,\tilde{w}_i\right)^{\frac{\gamma-1}{\gamma}}(1+\mu\tilde{\nu}(\tilde{q}))\left[(1-\gamma)\,\Delta\tilde{V}(\tilde{q})\right]^{\frac{1-\gamma}{\gamma}}\right\}. \tag{33}$$

Theorem 2 (Existence/uniqueness of firm value)**.** *Under standard regularity (e.g., $r > 0$ and bounded profits net of fixed costs), the Bellman operator in Eq(32) is a contraction on the space of bounded, continuous functions. Hence there exists a unique solution $\tilde{V}$, which is strictly increasing and strictly concave. See, e.g., Huggett (1993) and Aiyagari (1994) for analogous arguments establishing contraction and shape properties.*

2.3.2 Balanced Growth Path

In this section, we solve for a balanced growth path (BGP) equilibrium of the world economy. Along a BGP, the aggregate quantities

$$\{C_t, A_t, w_t, q_t^*, \bar{q}_t\}$$

grow at constant rates, the interest rate r_t and the masses (M_t, R_t) of incumbents and entrants are constant, and the distribution of *relative* productivities is stationary. Formally, writing $\tilde{q} := q/q_t^*$ and letting $\tilde{\Phi}_t$ be the cross-sectional distribution of $\tilde{q}$, stationarity means

$$\dot{\tilde{\Phi}}_t(\tilde{q}) = 0 \quad \text{for all } \tilde{q} \geq 1.$$

By focusing on the BGP we abstract from transitional dynamics that arise when the initial incumbent distribution differs from the stationary relative distribution sustained along the BGP.

We collect the time-invariant objects on the BGP in the tuple

$$\Big\{r,\ M,\ R,\ \tilde{h}(\tilde{q}),\ \tilde{m}_{ii}(\tilde{q}),\ \tilde{m}_{in}(\tilde{q}),\ \tilde{x}(\tilde{q})\Big\},$$

together with stationary functions $\tilde{V}(\tilde{q})$ and the stationary relative distributions of incumbents $\tilde{\Phi}$ and entrants $\tilde{G}$.

2.3.3 Stationary Equilibrium and the Law of Motion for Incumbents

Aggregate state. The measure of incumbents' core productivity distribution, denoted by Φ_t, serves as the aggregate state of the economy. It evolves over time according to the Markov jump process induced by firms' innovation and exit decisions under market selection. Current production and innovation depend on Φ_t, while its law of motion determines future economic dynamics. Unlike the stationary productivity distribution in Hopenhayn (1992), the present model features aggregate productivity that grows over time, so a stationary Φ_t does not exist. However, given the domestic spillover mechanism—which generates comovement between incumbents' productivity and entrants' draws—and the innovation-driven Markov process, we can normalize both incumbents' and entrants' productivity by the market cutoff q_t^*. This normalization yields a stationary relative productivity distribution, which in turn pins down a stationary equilibrium in relative terms. Accordingly, this paper focuses on the stationary equilibrium characterized by the relative productivity distribution of incumbents.

Let $\mathcal{Q} \subseteq \mathbb{R}_+$ be the support of productivity q. At date t, the cross-section of incumbents is a Borel probability measure Φ_t on $(\mathcal{Q}, \mathcal{B}(\mathcal{Q}))$: for any Borel set $B \subseteq \mathcal{Q}$, $\Phi_t(B)$ is the fraction of incumbents with $q \in B$. Let $M_t > 0$ denote the total mass of incumbents and $R_t \geq 0$ the mass inflow of entrants with entrant distribution G_t (a probability measure on $\mathcal{Q}$). The aggregate state is (Φ_t).

Law of motion in levels. Let q_t^* be the exit threshold; firms with $q < q_t^*$ exit immediately. Let $\Pi_t(\cdot \mid q)$ be the (one-period) transition kernel for a surviving incumbent with current productivity q. For any Borel set $B \subseteq \mathcal{Q}$ and small $\Delta > 0$,

$$M_{t+\Delta}\, \Phi_{t+\Delta}(B) = M_t \int_{q \geq q_t^*} \Pi_t(B \mid q)\, \Phi_t(dq) \ + \ R_{t+\Delta}\, G_{t+\Delta}(B). \tag{34}$$

BGP and relative variables. Along a BGP, q_t^* grows at the constant gross rate $1 + g$. Define the relative productivity $\tilde{q} := q/q_t^* \in [1, \infty)$ and the associated distribution $\tilde{\Phi}_t$ via $\tilde{\Phi}_t(B) = \Phi_t\big(\{q : q/q_t^* \in B\}\big)$. Let $\tilde{G}_t$ be the entrant distribution in relative units, which depends on the sampling distribution $F(\cdot)$ exogenously according to Eq(29). On the BGP, (M_t, R_t) are constant and all relative-objects are time-invariant. Suppressing time subscripts,

stationary in relative units is

$$M\,\tilde{\Phi}(B) = M\int_{\tilde{q}\geq 1}\tilde{\Pi}(B\mid\tilde{q})\,\tilde{\Phi}(d\tilde{q}) \;+\; R\,\tilde{G}(B), \quad \forall B\subseteq[1,\infty), \tag{35}$$

where $\tilde{\Pi}$ is the transition kernel in relative units. Equation (35) is the *law of motion* for incumbents at the relative level.

From law of motion to a fixed point. Discretize $[1,\infty)$ into N bins B_i with midpoints $\{\tilde{q}_i\}_{i=1}^N$, with $\tilde{q}_{\max}$ large enough that the tail mass above it is negligible. Let $\tilde{\boldsymbol{\phi}} = (\tilde{\Phi}(B_i))_{i=1}^N$ and $\tilde{\mathbf{g}} = (\tilde{G}(B_i))_{i=1}^N$. Denote by $\tilde{\boldsymbol{\mathcal{P}}}$ the $N\times N$ column-substochastic transition matrix implied by $\tilde{\Pi}$. Stationarity is then

$$\tilde{\boldsymbol{\phi}} = \tilde{\boldsymbol{\mathcal{P}}}\,\tilde{\boldsymbol{\phi}} \;+\; \frac{R}{M}\,\tilde{\mathbf{g}} \quad\Longleftrightarrow\quad (\mathbf{I}-\tilde{\boldsymbol{\mathcal{P}}})\,\tilde{\boldsymbol{\phi}} = \frac{R}{M}\,\tilde{\mathbf{g}}. \tag{36}$$

If the spectral radius $\rho(\tilde{\boldsymbol{\mathcal{P}}}) < 1$, then $(\mathbf{I}-\tilde{\boldsymbol{\mathcal{P}}})$ is invertible and a unique $\tilde{\boldsymbol{\phi}}$ exists (given a normalization such as $\mathbf{1}'\tilde{\boldsymbol{\phi}} = 1 = \mathbf{1}'\tilde{\mathbf{g}}$ and the entry ratio R/M implied by mass balance).[10] Here $\tilde{\boldsymbol{\mathcal{P}}}$ is the column-substochastic transition operator in relative units. $\tilde{\boldsymbol{\phi}}$ and $\tilde{\mathbf{g}}$ represent the relative productivity distribution density of incumbents and entrants, respectively. By the spillover mechanism in this economy, we have that $\tilde{\mathbf{g}}$ is exogenous and constant in this economy. $\frac{R}{M}$ is the inflow ratio of entrants on the BGP.

2.3.4 Kolmogorov Forward Equation (KFE) in Relative Units

To construct $\tilde{\boldsymbol{\mathcal{P}}}$ and to solve for the stationary density, it is convenient to use the continuous-time KFE associated with the relative-productivity process. Recall from Section 2 that innovations arrive as a Poisson process with *success intensity* $x(q)$; on success, q jumps to $(1+\lambda)q$, otherwise q is unchanged. In relative units $\tilde{q} = q/q_t^*$, the cutoff q_t^* grows at rate g, creating a *downward drift* in $\tilde{q}$ at speed $-g\tilde{q}$. Thus, in relative terms,

$$\text{drift: } \dot{\tilde{q}} = -g\,\tilde{q} \qquad \text{and} \qquad \text{jump (rate } \tilde{x}(\tilde{q})\text{): } \tilde{q}\mapsto(1+\lambda)\tilde{q}.$$

Let $\Phi(\tilde{q},t)$ be the density of $\tilde{\Phi}_t$ on $[1,\infty)$. The KFE (adjoint to the firm's HJB) is

$$\partial_t\tilde{\Phi}(\tilde{q},t) = \underbrace{g\,\partial_{\tilde{q}}\big[\tilde{q}\,\Phi(\tilde{q},t)\big]}_{\text{drift from growth}} + \underbrace{\frac{1}{1+\lambda}\,\tilde{x}\Big(\tfrac{\tilde{q}}{1+\lambda}\Big)\,\tilde{\Phi}\Big(\tfrac{\tilde{q}}{1+\lambda},t\Big) - \tilde{x}(\tilde{q})\,\Phi(\tilde{q},t)}_{\text{jump in/out}} + \underbrace{\frac{R}{M}\,\tilde{g}(\tilde{q})}_{\text{entry}}, \quad \tilde{q}\geq 1, \tag{37}$$

with boundary $\tilde{\Phi}(\tilde{q},t) = 0$ for $\tilde{q} < 1$ and normalization $\int_1^\infty \tilde{\Phi}(\tilde{q},t)\,d\tilde{q} = 1$. The first term transports mass *downward* in $\tilde{q}$ at speed g (since the frontier grows), the second term redistributes mass via Poisson successes (step size λ), and the last term injects entrants drawn from $\tilde{G}$ with density $\tilde{g}$. Because exit occurs only at the lower bound, no separate sink term is needed: a

[10] Equivalently, the Neumann series $\sum_{k\geq 0}\tilde{\boldsymbol{\mathcal{P}}}^{\,k}$ converges. The fixed-point structure is closely related to Hopenhayn (1992), here cast in relative variables to accommodate growth.

failure from $\tilde{q} \in [1, 1+g)$ exits the admissible set and is already accounted for by the generic outflow term $\tilde{\Phi}(\tilde{q}, t)$. In stationarity, $\partial_t \tilde{\Phi} \equiv 0$. [11]

Stationary mass balance. Integrating (37) over $\tilde{q} \in [1, \infty)$ yields the law of motion for the total incumbent mass:

$$\dot{M}_t = -\underbrace{M_t \int_1^{1+g} (1 - x(\tilde{q}))\, \tilde{\Phi}(\tilde{q}, t)\, d\tilde{q}}_{\text{exits}} + R_t.$$

In the stationary equilibrium, M_t is a constant, i.e., $\dot{M}_t = 0$. Therefore, on the BGP, we have the *entry-exit balance condition*:

$$\int_1^{1+g} (1 - x(\tilde{q}))\, \tilde{\Phi}(\tilde{q})\, d\tilde{q} = \frac{R}{M}, \tag{38}$$

i.e., in the stationary relative distribution, the inflow of entrants equals the outflow at the lower boundary driven by growth.

2.3.5 Stationary Density and an Infinitesimal Generator

Grid in logs. To implement the drift cleanly, work in logs $a := \log \tilde{q} \in [0, \log \tilde{q}_{\max}]$ with a uniform grid $a_i = a_1 + (i-1)\Delta a$, $a_1 = 0$, $a_N = \log \tilde{q}_{\max}$. Then $\dot{a} = -g$ (constant leftward drift), while a successful innovation shifts a up by $\log(1+\lambda)$.

Infinitesimal Generator. Let $\mathbf{p}(t) \in \mathbb{R}^N$ stack bin masses. The KFE discretization can be written as $\dot{\mathbf{p}}(t) = Q^\top \mathbf{p}(t) + \frac{R}{M}\, \tilde{\mathbf{g}}$ where the infinitesimal generator decomposes as $Q = A + J$:

- *Drift operator A:* with upwind advection, for interior nodes $i > 1$,

$$A(i-1, i) = \frac{g}{\Delta a}, \qquad A(i, i) = -\frac{g}{\Delta a},$$

  and no inflow for $i = 1$ (reflecting exit at the lower bound).

- *Jump operator J:* for each origin node i, let $k(i)$ be the nearest index to $a_i + \log(1+\lambda)$. Then

$$J(k(i), i) = \tilde{x}_i, \qquad J(i, i) = -\tilde{x}_i,$$

  where $\tilde{x}_i := \tilde{x}(\tilde{q}_i) \in [0, \infty)$ is the Poisson *rate* of successful innovations at node i.

The stationary density solves

$$Q^\top \tilde{\boldsymbol{\phi}} + \frac{R}{M}\, \tilde{\mathbf{g}} = 0, \qquad \mathbf{1}' \tilde{\boldsymbol{\phi}} = 1, \tag{39}$$

[11] For continuous-time Markov jump processes, the forward equation is $\dot{\mathbf{p}}(t) = Q^\top \mathbf{p}(t)$ for generator Q with nonnegative off-diagonals and diagonals equal to minus total outflow; the HJB operator is the adjoint of the KFE operator. See Achdou et al. (2022) and Whitt (2002).

with $\tilde{\mathbf{g}}$ the entrant density across bins (e.g., Pareto in relative units). Condition Eq(38) pins down R/M.

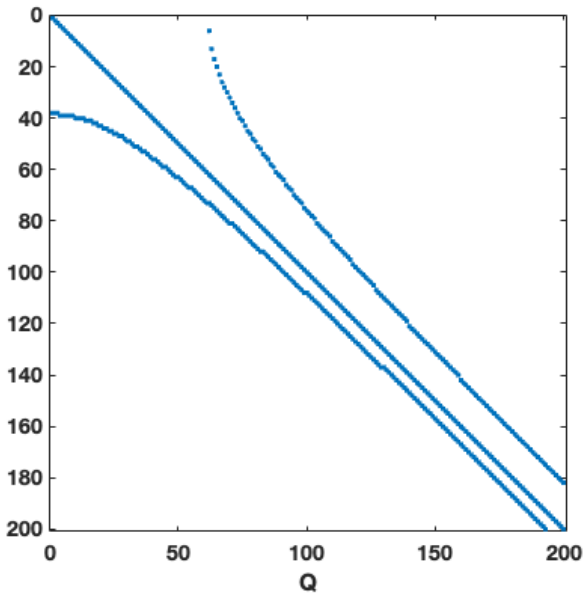


Figure 1: Sparsity of the infinitesimal generator, spy(Q)

Figure 1 displays the sparsity pattern of the estimated infinitesimal generator Q using MATLAB's `spy`, which marks nonzeros and leaves zeros blank. Dots therefore indicate the locations of transitions in the discretized KFE. The narrow subdiagonal band arises from the upwind drift operator A, which moves mass from bin i to $i-1$ at rate $g/\Delta a$. The diagonal collects the negative total hazards $-g/\Delta a - \tilde{x}_i$, while the offset band at distance $k(i)-i$ reflects jump transitions $i \to k(i)$ occurring at rate $\tilde{x}_i$. As required for a CTMC generator, each row of Q sums to zero (equivalently, the columns of $Q^\top$ sum to zero), which ensures mass conservation in the Kolmogorov forward equation $\dot{\mathbf{p}}(t) = Q^\top \mathbf{p}(t)$.

2.3.6 Stationary Equilibrium

A balanced-growth-path (BGP) equilibrium is a collection of constants (r, M, R, g); time-invariant functions $\{\tilde{V}(\cdot), \tilde{x}(\cdot), \tilde{h}(\cdot), \tilde{m}_{ii}(\cdot), \tilde{m}_{in}(\cdot)\}$; and a stationary relative distribution $\tilde{\Phi}$ such that:

1. **Household optimality:** The representative household solves (1)–(9) subject to (6), the Euler equation and budget constraint hold with C_t, A_t, w_t growing at rate g, and the no-Ponzi and transversality conditions (7)–(8) satisfied.

2. **Firms' production choices:** For each core productivity q, the domestic and export cutoffs $z^*_{ii}(q)$ and $z^*_{in}(q)$ satisfy the marginal conditions of production (18)–(19).

3. **Firms' innovation choice:** For each q, the Bellman equation (21) holds and the policy functions (22)–(23) solve the inner innovation problem.

4. **Free entry:** Entrants draw q from G_t induced by (29) and free entry holds: (31).

5. **Incumbent dynamics:** The stationary distribution $\tilde{\Phi}$ solves (39) (equivalently (36)), with entry–exit balance (38).

6. **Market clearing.** Goods, labor, and asset markets clear. In particular,

   - *Labor:* with total supply normalized to $L \equiv 1$, aggregate labor demand equals one:

$$\underbrace{M \int_{q \geq q^*} \Big\{ L_{\text{prod}}(q) + \underbrace{h(q)}_{\text{R\&D}} + \underbrace{f_i^m}_{\text{HQ}} + \underbrace{f_{ii}\, m_{ii}(q) + f_{in}\, m_{in}(q)}_{\text{per-line operating}} \Big\} \Phi(dq)}_{\text{labor used by incumbents}} + \underbrace{R\, f^e}_{\text{Entry R\&D}} = 1. \tag{40}$$

     where $L_{\text{prod}}(q)$ is variable production labor implied by the demand system and technology (any equivalent consistent aggregation is admissible).[12] $L_{\text{prod}}(q)$ is integrated across the active product lines for domestic and foreign sales which are determined by the attribute cutoff of firms, $z_{ii}^*(q)$ and $z_{in}^*(q)$ (cf(Appendix)).

   - *Assets:* the asset market clears—aggregate household assets equal the value of firms (the present value of profits net of costs), consistent with Eq(6).

   - *Goods:* with the normalization $P_n \equiv 1$, goods market clearing is implied by labor market clearing and the household budget constraint.

2.3.7 Stationary Growth

From the household budget constraint and the numéraire normalization, assets, wages, and consumption grow at the same rate as the productivity cutoff:

$$\frac{\dot{A}_t}{A_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{C}_t}{C_t} = \frac{\dot{q}_t^*}{q_t^*} = g.$$

Because entry, headquarter management and operation costs are denominated in labor units, they scale with the wage; hence on the BGP they grow at rate g (consistent with the idea that entry costs rise with growth, e.g., Klenow and Li (2025)). The Euler equation implies

$$g = \frac{r - \rho}{\eta}, \tag{41}$$

and the transversality/no-Ponzi restriction requires $r > g$, i.e.,

$$r > g \quad \Longleftrightarrow \quad \rho + (\eta - 1)g > 0. \tag{42}$$

In particular, $r > g$ implies $\rho > (1 - \eta)g$, which is sufficient for the utility integral to converge. Along the BGP, the masses L_t, M_t, and R_t are constant:

$$\dot{L}_t / L_t = \dot{M}_t / M_t = \dot{R}_t / R_t = 0,$$

[12] All fixed/operating costs are measured in labor units; the wage w_t converts these flows into the numéraire good by the normalization $P_n \equiv 1$.

with R/M pinned down by Eq(38). See Appendix 5 for the numerical scheme used to compute the stationary equilibrium.

2.4 A Firm's Life Cycle

We summarize the within-cohort dynamics implied by the model—entry, survival and scope expansion, export adoption, maturity, and exit—under the BGP as follows. The evolution of an individual firm is entirely state-dependent, with its core productivity q (or relative productivity $\tilde{q} = q/q_t^*$), innovation policy $x(q)$, and product scopes $(m_{ii}(q), m_{in}(q))$ jointly determining outcomes.

Entry. A potential entrant pays $f^e w_t$ to draw a core productivity q_0 from G_t; with knowledge spillovers, G_t scales with the incumbent frontier (cf. Eq(29)). Free entry Eq(31) equates the expected value of a draw to the entry cost. An entrant operates only if its draw lies above the country cutoff q_t^*; otherwise it exits immediately. Given its draw q_0, the firm chooses domestic and export attribute cutoffs $z_{ii}^*(q_0)$ and $z_{in}^*(q_0)$ from Eq(18)–Eq(19).

Innovation and state evolution. Innovation successes arrive as a Poisson process at rate $x(q)$, raising q to $(1+\lambda)q$; otherwise q is unchanged. In relative units, the threshold grows at rate g, so $\tilde{q}$ drifts downward deterministically between successes. Let $a := \log \tilde{q}$. Conditional on survival, the generator of the relative process implies the expected instantaneous drift

$$\frac{d}{dt}\,\mathbb{E}[a_t \mid q_t = q] \;=\; -\,g \;+\; x(q)\,\log(1+\lambda). \tag{43}$$

Hence, a firm's relative position improves on average iff $x(q) \geq \bar{x} := g/\log(1+\lambda)$ and deteriorates otherwise. Because $x(q)$ rises with export intensity $\nu(q)$ and falls with domestic scope $m_{ii}(q)$ (via Eq(22)), export penetration is a key margin for survival and growth.

Scope management and export adoption. Following a relative position improves, variable profits rise and cutoffs fall; both domestic and export scopes expand (z^* decrease, m increase). Two forces discipline scope: (i) the scale/scope dilution term $\tilde{m}_{ii}(\tilde{q})$ in Eq(14) lowers innovation efficiency as domestic scope grows; and (ii) the export-induced efficiency (EIE) term $1 + \mu\tilde{\nu}(\tilde{q})$ raises it as the firm opens export lines. At the margin, the dynamic value of an extra domestic line is negative, while that of an export line is positive (Proposition 1). This rationalizes *strategic exporting*: firms may accept static losses on marginal export lines to raise $\tilde{\Xi}(\tilde{q})$ (the continuation value of R&D) and thereby $\tilde{x}(\tilde{q})$.

Maturity. As a firm ages and (stochastically) accumulates successes and failures. The innovation policy Eq(22) implies

$$x(q) \;\propto\; \Big(m_{ii}(q)\Big)^{\frac{\gamma-1}{\gamma}}\,(1+\mu\nu(q))\,[\Delta V(q)]^{\frac{1-\gamma}{\gamma}}.$$

Thus, the net effect of growth in q on $x(q)$ is ambiguous ex ante: the expansion of domestic scope (dilution) pushes x down, while export penetration and larger continuation values push x up. Depending on parameters, $x(q)$ can be monotone or hump-shaped over the life cycle; in either case, firms that sustain $x(q) \geq \bar{x}$ on average keep up with the frontier, while those that fall short gradually slide toward the exit region.

Exit and pruning. Two (equivalent) exit rules operate: (i) in levels, the firm exits when $\pi_i(q) + \Xi(q) < w_i f_i^m$, i.e., $q < q^*$ (see Eq(27)); and (ii) in relative units, the state $a = \log \tilde{q}$ drifts left at speed g between successes and hits the market selection boundary $\tilde{q} = 1$. Before exit, scope is gradually *pruned* as cutoffs rise back toward the support lower bound; export lines (the higher cutoff set) are dropped first, then domestic lines.

In summary, an incumbent's life-cycle is as follows: At birth, a potential entrant draws an initial core productivity q_0. In the early growth/selection phase, the firm invests in R&D; its survival and growth hinge on whether the innovation success intensity exceeds the keep-up threshold, $x(q_0) \gtrless \bar{x}$. As profitability improves and the export cutoff $z_{in}^*(q)$ falls, the firm adopts exporting, which raises export intensity $\nu(q)$ and—through the export-induced efficiency channel—increases $x(q)$, supporting further innovation. In maturity, the state alternates between successful innovations and downward drift from frontier growth; the firm manages scope by trading off dilution (via m_{ii}) against export learning (via ν). Ultimately, firms with persistently low $x(q)$ prune lines and hit the exit threshold q^* (equivalently $\tilde{q} = 1$), whereas firms that sustain $x(q) \geq \bar{x}$ keep pace with the frontier and remain in the market.

2.5 Comparative Statics

This subsection collects transparent, testable comparative-statics results for innovation intensity and exporting. Throughout we work with the stationary (time-invariant) policy

$$\tilde{x}(\tilde{q}) = c_x^{1/\gamma} \left(\tilde{m}_{ii}(\tilde{q})\right)^{\frac{\gamma-1}{\gamma}} \left(1 + \mu\, \tilde{\nu}(\tilde{q})\right) \left[\frac{(1-\gamma)}{\tilde{w}_i}\, \Delta\tilde{V}(\tilde{q})\right]^{\frac{1-\gamma}{\gamma}}, \qquad \gamma \in (0,1),\ \mu \geq 0, \tag{44}$$

where $\tilde{\nu}(\tilde{q}) = \tilde{m}_{in}(\tilde{q})/\tilde{m}_{ii}(\tilde{q})$ and $\Delta\tilde{V}(\tilde{q}) := \mathbb{E}\Big[\tilde{V}\Big(\frac{(1+\lambda)\tilde{q}}{1+g}\Big) - \tilde{V}\Big(\frac{\tilde{q}}{1+g}\Big)\Big]$. Stationarity of the BGP *does not* require $\tilde{x}(\tilde{q})$ to be constant across states; it only requires $\tilde{x}(\cdot)$ to be time-invariant.

Export intensity via cutoff ratios. Under symmetry and $P_n \equiv 1$, the within-firm cutoff ratio is (see (26))

$$\frac{z_{in}^*(q)}{z_{ii}^*(q)} = \tau_{in} \left(\frac{w_i f_{in} - \partial\Xi(q)/\partial m_{in}}{w_i f_{ii} - \partial\Xi(q)/\partial m_{ii}}\right)^{\frac{1}{\sigma-1}}, \qquad \tilde{\nu}(q) = \left(\frac{z_{ii}^*(q)}{z_{in}^*(q)}\right)^{\iota}.$$

Proposition 2 (Determinants of export intensity)**.** *Holding other arguments fixed and for*

$x(q) > 0$,

$$\frac{\partial \ln \tilde{\nu}}{\partial \ln \tau_{in}} = -\frac{\iota}{\sigma - 1} < 0, \qquad \frac{\partial \ln \tilde{\nu}}{\partial f_{in}} < 0, \qquad \frac{\partial \ln \tilde{\nu}}{\partial f_{ii}} > 0, \qquad \frac{\partial \ln \tilde{\nu}}{\partial \mu} > 0.$$

Sketch. *Differentiate* $\ln \tilde{\nu} = -\iota \left[\ln \tau_{in} + \frac{1}{\sigma-1}\{\ln(\cdot_{num}) - \ln(\cdot_{den})\}\right]$. *Trade costs and export fixed costs raise the numerator; a higher domestic fixed cost raises the denominator. Since* $\partial \Xi / \partial m_{in}$ *increases with* μ*, the numerator falls with* μ*, so* $\tilde{\nu}$ *rises. Therefore, a stronger EIE effect encourages firms to choose a higher export intensity.*

Corollary 2 (International trade to innovation)**.** *Both trade liberalization and exporting–innovation efficiency have the effect on boosting a country's innovation. Via* (44)*, the signs in Proposition 2 imply*

$$\tau_{in}\downarrow \Rightarrow \tilde{\nu}\uparrow \Rightarrow \tilde{x}\uparrow,$$

$$\mu\uparrow \Rightarrow \tilde{\nu}\uparrow \Rightarrow \tilde{x}\uparrow .$$

Both trade cost and EIE effect have impact on firm's decisions on its innovation production. Government can boost economy growth via innovation by being more opening to trade (subsidizing export) or encouraging exporters to learn more diligently in the foreign market (stronger EIE).

3 Data

3.1 Data Description

Firstly, we consider firm-level data for China and examine the relationship between a firm's innovation activity and export product scope. Our main sample contains only "innovators" in the manufacturing sector which reported positive R&D expenditure at least once from 2005-2007. Overall, in 2007, the innovators accounted for 17.6 percent of firms, 52.9 percent of sales, 54.3 percent of export value, and 39.3 percent of employment.

To begin with, Table 1 presents summary statistics of the innovation activity for firms with different numbers of exported products in 2007. Firms with zero exported products are simply non-exporters. It is evident that as the number of exported products increases, firms have more current patent applications. However, the gap between exporters and non-exporters is not significant until the export product scope is large enough. The innovation premium of firms with more exported products manifests in the near future. In 2009, non-exporters, on average, applied 0.16 patents more than in 2007, while the number of patent applications for exporters increased by over 0.4 from 2007 to 2009.

While recent studies have primarily examined how the extensive-margin export decision (i.e., whether to export or not) influence firm innovation, they often overlook the nuanced effects stemming from the intensity of export activities (i.e., the volume of firm export, export product counts, and export intensity).[13] This oversight is significant given that the largest exporters

[13]See Melitz and Redding (2021) for a comprehensive review of the literature.

Table 1: Number of Exported Products and Innovation: Summary Statistics

# of Exported Products	0	1	2	3	4	[5,10)	[10,20)	[20,30)	30+
R&D Dummy	0.60	0.62	0.62	0.63	0.64	0.65	0.67	0.68	0.70
log Sales	6.20	6.55	6.63	6.79	6.85	6.99	7.42	7.94	8.29
log Employment	5.03	5.37	5.46	5.59	5.65	5.80	6.12	6.56	6.73
log Value Added per Worker	1.22	1.23	1.21	1.23	1.23	1.22	1.31	1.40	1.57
log Export Value	-	2.24	3.40	4.06	4.36	4.97	5.77	6.55	6.99
Number of Export Destinations	-	3.43	5.95	8.51	9.98	13.63	18.71	25.37	30.96
Number of Firms	36,315	3,471	2,379	1,695	1,132	3,008	1,089	318	265

Note: This table presents several descriptive statistics across groups of firms with different numbers of exported products. The exported products are encoded at the Harmonized-System 6-digital level. The sample period is 2007, and we restrict attention to firms who reported positive R&D expenditure during 2005-2007. All the statistics except the last row are sample averages across firms within the same group. Current patent applications refer to the firm's application for invention patents in 2007, while future patent applications refer to the firm's application for invention patents in 2009. R&D dummy equals one if a firm reported positive R&D expenditure in 2007, and the statistics in this row stand for a fraction of firms with positive R&D expenditure in each group.

are usually those who spend a substantial amount in R&D. Figure2 presents this pattern for Chinese firms with positive R&D expenditure in 2007.

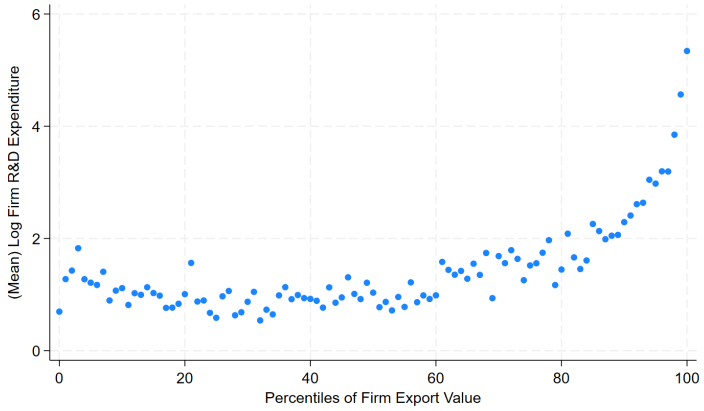


Figure 2: Firm Export Value and R&D Expenditure

Note: This figure plots the mean level of log firm R&D expenditure within percentiles of Chinese firms in year 2007. We categorize firms into percentiles based on their export revenue. Non-exporters belong to the zero percentile. Here we limit attention to firms with positive R&D expenditure.

4 Calibration and Quantitative Analysis

4.1 Calibration

The model parameters will be grouped into two parts. The first part consists of parameters drawn from the literature. The second part will be a vector of parameters,

$$\Theta \equiv \{f^e, f^m, f_{ii}, f_{in}, \gamma, \mu, c_x\},$$

estimated internally, chosen to minimize the distance between a large number of model moments and their counterparts in the Chinese data.

4.1.1 Externally Calibrated Parameters

Parameter	Value	Meaning	Reference
ρ	0.04	Discount Rate	Garcia-Penalosa and Turnovsky (2006)
η	2	Intertemporal elasticity of substitution	Acemoglu et al. (2018)
θ	5	Elasticity of substitution across products	Atkeson and Burstein (2010)
σ	5	Elasticity of substitution across varieties within a product	Heo and Kim (2025)
ι	5.78	Pareto shape parameter of product attributes	Heo and Kim (2025)
a_F	1.055	Pareto shape parameter of entrant distribution	Brandt, Kambourov, and Storesletten (2025)

Table 2: Externally Calibrated Parameters

Table 2 summarizes the parameters calibrated from prior studies. The discount rate is set to $\rho = 0.04$ Garcia-Penalosa and Turnovsky (2006); the intertemporal elasticity of substitution to $\eta = 2$ Acemoglu et al. (2018); the elasticity of substitution across products to $\theta = 5$ Atkeson and Burstein (2010); the elasticity of substitution within product varieties to $\sigma = 5$ Heo and Kim (2025); the tail parameter of product attributes to $\iota = 5.78$ Heo and Kim (2025); and the entry distribution parameter to $a_F = 1.055$ Brandt, Kambourov, and Storesletten (2025). The choices of ρ, η, θ, and a_F are consistent with standard macroeconomic calibrations. The parameters σ and ι are adopted from Heo and Kim (2025), which also models multiproduct firms with a continuum of product attributes following a Pareto distribution. Their calibration of ι relies on the same Chinese firm-level data sources used in this paper.

4.2 Estimation Procedure

We estimate Θ by GMM. Let $m(\Theta)$ denote the vector of simulated model moments. We choose $\hat{\Theta}$ to minimize

$$\widehat{Q}(\Theta) = \left[m(\Theta) - m^{\text{data}} \right]' \widehat{W} \left[m(\Theta) - m^{\text{data}} \right], \tag{45}$$

where $\widehat{W}$ is a positive definite weighting matrix. We initialize with a diagonal $\widehat{W}$ inversely proportional to the empirical variances of the data moments, then iterate once to the optimal

weighting matrix using the sample covariance of the moment conditions at the first–stage estimate (cf. Hansen (1982)). Standard errors for $\hat{\Theta}$ obtain from the usual GMM sandwich formula with numerical Jacobians of the moments.

Targeted moments. Let m^{data} collect the seven aggregated-share and growth moments constructed above: (i) the *entry rate* (entrants over incumbents), (ii)–(iii) *innovation intensity* for the full economy and the top 50% (R&D wage bill over profits, aggregated within each bin), (iv)–(v) *export intensity* for the full economy and the top 10% (exports over sales, aggregated), and (vi)–(vii) *sales growth* for all firms and for the top 50%. As emphasized, we compute all share-type moments as ratios of totals (not averages of firm-level ratios), using the same bin definitions in the model and the data. We estimate Θ by matching the seven moments above.

Identification. We discipline parameters by choosing moments that are most sensitive to the underlying mechanisms they control. *Entry cost* f^e is pinned down by the equilibrium entry rate, while *overhead* f^m shapes the fixed–to–variable cost ratio and thereby size-conditional profitability; together, these moments trace the stationary entry–exit condition and profit levels in industry dynamics. The *domestic per-product operating cost* f_{ii} governs firms' domestic product scope and thus the share of revenue generated at home, tightening the fit of the domestic sales margin. The *foreign per-product operating cost* f_{in} is identified by export participation and the export revenue share—precisely the margins most responsive to fixed costs of exporting in heterogeneous-firm trade models. The *export-induced innovation elasticity* μ is disciplined by cross-sectional covariation between export intensity, innovation effort, and subsequent profit/productivity growth, in line with evidence that exporting and R&D jointly raise future efficiency. (Aw, Roberts, and Xu, 2011) Finally, the *scale* of the innovation technology c_x is set by the level of economy-wide innovation intensity, consistent with Poisson-arrival models where R&D outlays scale the innovation rate. (Klette and Kortum, 2004)

Parameter	Value	Description
μ	0.09	EIE scaling paprameter
γ	0.60	Cobb-Douglas share on knowledge term
f_{ii}	0.001	Domestic operation cost per product
c_x	0.20	Scaling parameter of innovation technology
f_i^e	0.001	Entry cost
f_i^m	0.001	Headquarter management cost
f_{in}	0.011	Foreign operation cost per product

Table 3: GMM Calibration

Results. Table 3 reports the estimated parameter vector and (robust) standard errors. Three qualitative features are worth highlighting: *Fixed costs at home vs. abroad.* The foreign per-product operating cost f_{in} exceeds its domestic counterpart f_{ii}, consistent with higher organizational and market-entry hurdles in export markets. This gap is crucial for matching the aggregate export share and the concentration of exports among very large firms (Top 10%).

Trade–innovation complementarity. The export-induced innovation elasticity μ is positive and economically meaningful, allowing the model to reproduce the lower innovation intensity among large, export-active firms (Top 50%) while preserving the aggregate level. This aligns with the regression-based reduced-form evidence in Table 5. *Scale of innovation.* The innovation scale parameter c_x brings the level of economy-wide innovation intensity to the data, while leaving the cross-bin dispersion to be governed by μ and γ.

The model reproduces (by construction) the targeted moments—entry, export shares, innovation intensity levels across bins, and sales growth patterns—and it also matches non-targeted distributional features reasonably well (e.g., the right tail of exporter sales), as we document in the appendix (results coming soon).

Validation. As an external check, the reduced-form ratio $\widehat{\gamma} \simeq \hat{\beta}_1/(\hat{\beta}_1 + \hat{\beta}_2)$ from Table 5 is close to the structural value when γ is estimated jointly, lending credibility to the model's innovation mechanism.

4.3 Quantitative Results

Table 3 reports our preferred parameter vector. Three features are worth highlighting. (i) The estimated export–induced externality (EIE) parameter, $\mu = 0.09$, implies a moderate but economically meaningful responsiveness of innovation intensity to export market access, consistent with observed exporter–non-exporter innovation gaps. (ii) The scaling parameter $c_x = 0.20$ pins down the level and dispersion of patents per revenue/worker. (iii) Although the fixed costs $\{f^e, f^m, f_{ii}, f_{in}\}$ are small in wage units, they are essential to matching entry and exit rates as well as the quartiles of sales growth, export intensity, and innovation intensity.

Counterfactuals. We study three experiments with the estimated model: (1) *Shut down* the EIE channel by setting $\mu = 0$ to quantify its contribution to long-run growth; (2) *Strengthen* EIE (vary μ) to trace out distributional effects; (3) Evaluate the general-equilibrium impact of *trade liberalization* (vary iceberg trade costs τ) when EIE operates.

Counterfactual 1: Removing EIE ($\mu = 0$). Setting $\mu = 0$ lowers the stationary growth rate g by *1.30 percentage points* (from 10.29% to 8.99%) and reduces firm mass M by about *1.8%* (from 0.111 to 0.109).[14] *Mechanism.* With EIE switched off, exporting no longer augments the knowledge stock that feeds innovation. This lowers the private return to R&D (via Eq. (13)), reduces the intensity of upward productivity "jumps," and weakens selection. The economy therefore features fewer highly productive incumbents and slightly lower net entry, so both growth and the stationary firm mass fall. Consistent with this, the cross-sectional variance of relative productivity, $\mathrm{Var}(\tilde{q})$, declines slightly from 0.0846 to 0.0842. We return to distributional implications in Counterfactual 2.

[14] The decline in g is $10.29 - 8.99 = 1.30$ percentage points; the decline in M is $(0.111 - 0.109)/0.111 \approx 1.8\%$.

Counterfactual 2: Stronger EIE ($\mu \in \{0, 0.01, 0.02\}$, other parameters fixed). Figure 3 plots the stationary density $\tilde{\phi}(\tilde{q})$. As μ rises, the distribution becomes more dispersed: the mode shifts down and the right tail thickens. *Mechanism.* A higher μ raises the innovation premium associated with exporting; firms with higher relative productivity $\tilde{q}$—and thus higher export intensity ν—choose a larger innovation success rate x (Eq. (13)). This increases the Poisson arrival rate of productivity improvements for these firms and reallocates market share toward the right tail, amplifying between-firm inequality in productivity and size.

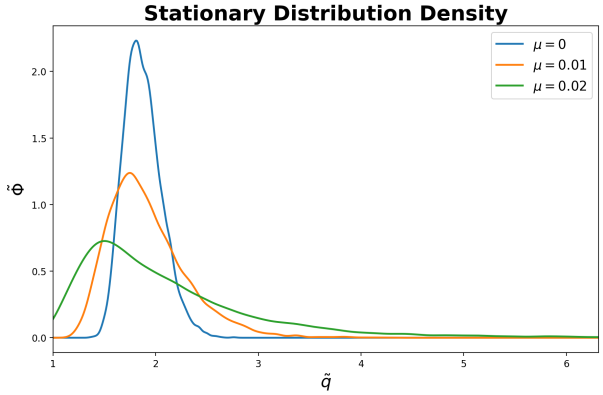


Figure 3: Stationary density $\tilde{\phi}(\tilde{q})$ as μ varies.

Counterfactual 3: Trade liberalization with EIE (varying τ). We vary iceberg trade costs over $\tau \in \{1.30, 1.25, 1.20\}$ (lower τ means greater liberalization). Figure 4 shows that liberalization shifts mass to the right tail and lowers the peak—echoing Counterfactual 2. *Mechanism.* Cheaper exporting raises profits disproportionately for more productive firms (the standard Melitz selection-cum-reallocation effect), which further strengthens innovation incentives when EIE is active. The joint effect is a thicker right tail in $\tilde{\phi}(\tilde{q})$ and greater between-firm dispersion.

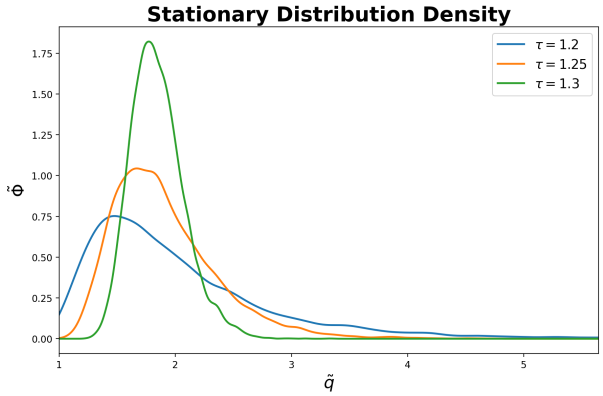


Figure 4: Stationary density $\tilde{\phi}(\tilde{q})$ as iceberg costs τ vary (lower τ = more liberalization).

The drop in g when μ is shut down indicates that export-mediated knowledge flows are a first-order growth mechanism in this environment. Increasing μ or lowering τ both reallocate resources toward high-$\tilde{q}$ firms, but through distinct channels: μ operates through *innovation*

productivity (exporting makes R&D more effective), while τ operates through *market access and selection*. Their effects are complementary when present together.

Our reallocation and dispersion results under trade liberalization mirror canonical heterogeneous-firm models and evidence: trade exposure induces within-industry selection and shifts resources to more productive firms Melitz (2003); exporters are systematically larger and more productive Bernard et al. (2007); and liberalization raises productivity via selection and technology upgrading Lileeva and Trefler (2010); Bustos (2011). The novel contribution here is to show that, in the presence of EIE, liberalization not only reallocates market share toward high-productivity incumbents, but also increases the mass of high-$\tilde{q}$ firms by boosting their innovation incentives, thereby altering the *distribution* of firm productivity itself.

5 Conclusions

This paper identifies and quantifies a new growth margin—the **Exporting–Innovation Efficiency (EIE)** channel—in China. We show that access to foreign markets does more than reallocate activity or scale quantities: it *raises the efficiency with which firms convert innovative effort into productivity gains*. We embed exporting-induced knowledge diffusion in a dynamic endogenous growth model, which brings the mechanism that raises firms' innovation efficiency into focus and allows us to quantify its aggregate contribution to growth. In our baseline quantification, the EIE channel accounts for about 1.3 percentage points of China's average annual GDP growth during 2004–2007, a period when the economy expanded at roughly 12.1% per year. This magnitude implies that dynamic innovation responses to exporting explain a meaningful share of realized growth rather than a second-order nuance around static reallocation.

The mechanism has nontrivial distributional consequences. Because the efficiency gain from exporting is heterogeneous—stronger for firms with complementary capabilities and access to intangible inputs—EIE *amplifies productivity dispersion* and *increases firm-level inequality* during trade liberalization, even as aggregate efficiency rises. The pattern is consistent with tougher competition and larger market size shifting market share toward firms best able to translate innovation into performance.

Our results also speak to how we measure the gains from trade. Canonical, largely static approaches understate the welfare effects of openness when they ignore persistent changes in domestic productivity that operate through innovation. By explicitly modeling and quantifying EIE, we show that *omitting this channel leads to an underestimation of the long-run gains from trade*. Innovation-efficiency improvements raise welfare directly (via higher productivity) and indirectly (through dynamic reallocation and quality upgrading), pushing estimated gains beyond what static sufficient-statistics formulas imply.

Trade and innovation policy are complements. Market-access initiatives and export promotion yield larger and more durable payoffs when paired with instruments that strengthen firms' innovative capacity (e.g., R&D tax incentives, diffusion and supplier-development programs, managerial training, and robust IP enforcement). Because EIE can widen dispersion, complementary policies that expand access to intangible capital and absorptive capacity for lagging

firms can preserve aggregate efficiency while tempering distributional costs. Sequencing also matters: lowering frictions to high-quality imported inputs and standards that facilitate quality upgrading magnify the dynamic returns to exporting.

Our estimates are anchored in a high-growth window and a specific liberalization episode; extrapolations to slower-growth periods or to environments with tighter trade restrictions should be cautious. Nonetheless, the mechanism we uncover is general: exporting can raise not only how much firms innovate but *how productively* they turn innovation into performance.

In sum, trade can be a catalyst not only for "more" innovation but for *smarter* innovation. Accounting for the EIE channel clarifies why productivity dispersion may initially widen, raises the estimated gains from openness, and points toward trade-cum-innovation policies that are both growth-enhancing and distribution-aware.

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Appendix

Appendix.A

Appendix.A.1 Derivation of Proposition 1

Envelope derivatives for the inner R&D problem (interior case). Consider the interior case $x(q) > 0$ for the inner R&D problem

$$\Xi(q) = \max_{x \geq 0} \Big\{ x\,\Delta V(q) \;-\; w_i\, m_{ii}(q) \Big[\tfrac{x}{c_x(1+\mu\nu(q))^\gamma} \Big]^{\frac{1}{1-\gamma}} \Big\}, \qquad \Delta V(q) := \mathbb{E}\big[V'((1+\lambda)q) - V'(q)\big], \tag{46}$$

where $\nu(q) = m_{in}(q)/m_{ii}(q)$ and $\gamma \in (0,1)$. Let $A(q) := c_x(1+\mu\nu(q))^\gamma$ and $B(q,x) := [x/A(q)]^{1/(1-\gamma)}$ for brevity. At the optimum $x = x(q)$, the envelope theorem gives (holding x fixed when differentiating w.r.t. scope variables), but we must account for the dependence $\nu = m_{in}/m_{ii}$ via the chain rule.

Step 1: Direct derivatives. With x fixed and m_{in} held fixed,

$$\frac{\partial \Xi(q)}{\partial m_{ii}} = -\,w_i \Big\{ B(q,x) \;+\; m_{ii}\,\frac{\partial B(q,x)}{\partial \nu}\,\frac{\partial \nu}{\partial m_{ii}} \Big\}, \tag{47}$$

$$\frac{\partial \Xi(q)}{\partial m_{in}} = -\,w_i\, m_{ii}\,\frac{\partial B(q,x)}{\partial \nu}\,\frac{\partial \nu}{\partial m_{in}}. \tag{48}$$

Since $A = c_x(1+\mu\nu)^\gamma$,

$$\frac{\partial B}{\partial \nu} = -\,\frac{\gamma\mu}{1-\gamma}\,\frac{B}{1+\mu\nu}, \qquad \frac{\partial \nu}{\partial m_{ii}} = -\,\frac{\nu}{m_{ii}}, \qquad \frac{\partial \nu}{\partial m_{in}} = \frac{1}{m_{ii}}.$$

Hence,

$$\frac{\partial \Xi(q)}{\partial \nu} = -w_i m_{ii}\,\frac{\partial B}{\partial \nu} = \frac{\mu}{1+\mu\nu}\,\Xi(q), \tag{49}$$

where in the last equality we used the optimality identity (see below). Plugging (49) into (47) yields

$$\frac{\partial \Xi(q)}{\partial m_{ii}} = -\,w_i\,B \;-\; \frac{\mu\nu}{1+\mu\nu}\,\frac{\Xi(q)}{m_{ii}}, \qquad \frac{\partial \Xi(q)}{\partial m_{in}} = \frac{1}{m_{ii}}\cdot\frac{\partial \Xi(q)}{\partial \nu} = \frac{\mu}{1+\mu\nu}\,\frac{\Xi(q)}{m_{ii}}. \tag{50}$$

Step 2: Optimality identities. The FOC for (46) implies the familiar identities

$$\Xi(q) = \gamma\,\Delta V(q)\,x(q) = \frac{\gamma}{1-\gamma}\,w_i m_{ii}(q)\,B\big(q, x(q)\big). \tag{51}$$

Using (51) to eliminate $w_i B$ in (50) gives

$$\boxed{\frac{\partial \Xi(q)}{\partial m_{ii}} = -\frac{\Xi(q)}{m_{ii}(q)}\left[\frac{1-\gamma}{\gamma} + \frac{\mu\,\nu(q)}{1+\mu\,\nu(q)}\right], \qquad \frac{\partial \Xi(q)}{\partial m_{in}} = \boxed{\frac{\mu}{1+\mu\,\nu(q)}\,\frac{\Xi(q)}{m_{ii}(q)}}}. \tag{52}$$

Thus, the dynamic marginal value of an additional *domestic* line is strictly negative, while that of an additional *export* line is strictly positive, whenever $x(q) > 0$. The first term in $\partial\Xi/\partial m_{ii}$

$(-(1-\gamma)/\gamma \cdot \Xi/m_{ii})$ is the *direct* scale-dilution effect; the second term $(-\mu\nu/(1+\mu\nu)\cdot\Xi/m_{ii})$ captures the induced decline in export intensity ν when m_{ii} rises (holding m_{in} fixed).

Closed form for $\Xi(q)$. Substituting the policy function $x(q)$ from (**??**) into $\Xi(q) = \gamma\,\Delta V(q)\,x(q)$ yields

$$\Xi(q) = c_x^{\frac{1}{\gamma}} \underbrace{\left[(1-\gamma)^{\frac{1}{\gamma}}\frac{\gamma}{1-\gamma}\right]}_{\mathcal{H}} \left(w_i m_{ii}(q)\right)^{\frac{\gamma-1}{\gamma}} \left(1+\mu\,\nu(q)\right)\left(\Delta V(q)\right)^{\frac{1}{\gamma}}, \qquad \mathcal{H} \equiv (1-\gamma)^{\frac{1}{\gamma}}\frac{\gamma}{1-\gamma}. \tag{53}$$

Plugging Eq(53) into Eq(52) reproduces the compact derivatives in Eq(52).

Appendix.A.2 Cutoff conditions for z^* and q^*

Plugging the partial derivatives, $\frac{\partial\Xi(q)}{\partial m_{ii}}$ and $\frac{\partial\Xi(q)}{\partial m_{in}}$, in eq(52) into the cutoff conditions, eq(18) and eq(19), we can obtain

$$\mathcal{J}\left(\frac{q z_{ii}^*}{\tau_{ii} w_i}\right)^{\sigma-1} C_i + m_{ii}^{-\frac{1}{\gamma}}(1-\frac{1}{\gamma}\frac{m_{ii}+m_{in}}{m_{ii}})\mathbb{E}\left[V'((1+\lambda)q) - V'(q)\right]^{\frac{1}{\gamma}}(w_i)^{\frac{\gamma-1}{\gamma}}\mathcal{H} = w_i f_{ii}, \tag{54}$$

$$\mathcal{J}\left(\frac{q z_{in}^*}{\tau_{in} w_i}\right)^{\sigma-1} C_n + \mu m_{ii}^{-\frac{1}{\gamma}}\mathbb{E}\left[V'((1+\lambda)q) - V'(q)\right]^{\frac{1}{\gamma}}(w_i)^{\frac{\gamma-1}{\gamma}}\mathcal{H} = w_i f_{in}. \tag{55}$$

where $\mathcal{J} \equiv \sigma^{-\sigma}(\sigma-1)^{\sigma-1}$ and $\mathcal{H} \equiv (1-\gamma)^{\frac{1}{\gamma}}\frac{\gamma}{1-\gamma}$.

Combine , Rewrite the cutoff conditions with relative terms,

$$m_{ii}^{-\frac{1}{\gamma}}(1-\frac{1}{\gamma}\frac{m_{ii}+m_{in}}{m_{ii}})\left[\tilde{V}((1+\lambda)\tilde{q}/(1+g)) - \tilde{V}(\tilde{q}/(1+g))\right]^{\frac{1}{\gamma}}(\tilde{w}_i)^{\frac{\gamma-1}{\gamma}}\mathcal{H} + \mathcal{J}\left(\frac{\tilde{q} z_{ii}^*}{\tau_{ii}\tilde{w}_i}\right)^{\sigma-1}\tilde{C}_i - \tilde{w}_i f_{ii} = 0, \tag{56}$$

$$\mu m_{ii}^{-\frac{1}{\gamma}}\left[\tilde{V}((1+\lambda)\tilde{q}/(1+g)) - \tilde{V}(\tilde{q}/(1+g))\right]^{\frac{1}{\gamma}}(\tilde{w}_i)^{\frac{\gamma-1}{\gamma}}\mathcal{H} + \mathcal{J}\left(\frac{\tilde{q} z_{in}^*}{\tau_{in}\tilde{w}_i}\right)^{\sigma-1}\tilde{C}_n - \tilde{w}_i f_{in} = 0. \tag{57}$$

Plugging $m_{ii} = (z_{ii}^*)^{-\iota}$ and $m_{in} = (z_{in}^*)^{-\iota}$ into eq(56) and eq(57), we obtain $z_{ii}^*(q)$ and $z_{in}^*(q)$.

Appendix A.3 Cutoff condition of core productivity

Using (20) and (12) with $P_n \equiv 1$, one can write

$$\underbrace{\sum_n \int_{z_{in}^*(q^*)}^{\infty}\left[\mathcal{J}\left(\frac{q^* z}{\tau_{in} w_i}\right)^{\sigma-1} C_n - w_i f_{in}\right] d\Psi(z)}_{\pi_i(q^*)} + \Xi(q^*) = w_i f_i^m, \qquad \mathcal{J} \equiv \sigma^{-\sigma}(\sigma-1)^{\sigma-1}. \tag{58}$$

Rewrite the productivity cutoff condition in eq(58) by plugging eq(17) into it. Then we have

$$w_i f_i^m = \sum_n \int_{z_{in}^*(q^*)}^{\infty} \left[\mathcal{J} \left(\frac{q^* z}{\tau_{in} w_i} \right)^{\sigma-1} C_n - w_i f_{in} \right] d\Psi(z) \tag{59}$$

$$+ \mathcal{H}\mathbb{E}\left[V((1+\lambda)q^*) - V(q^*) \right]^{\frac{1}{\gamma}} \left(m_{ii}(q^*) + \mu m_{in}(q^*) \right)^{\frac{\gamma-1}{\gamma}} (w_i)^{\frac{\gamma-1}{\gamma}}. \tag{60}$$

Plugging $m_{ii} = (z_{ii})^{-\iota}$ and $m_{in} = (z_{in})^{-\iota}$ into the above equation, we obtain q^*.

Appendix.A.4 Functions in relative terms

$$r\tilde{V}_t(\tilde{q}_t) = \max\Big\{ 0, \tilde{\pi}_i(q) - \tilde{w}_i f_i^m$$

$$+\mathcal{H}(\tilde{w}_i \tilde{m}_{ii}(\tilde{q}_t))^{\frac{\gamma-1}{\gamma}} (1+\mu\tilde{\nu}(\tilde{q}_t)) \left[\tilde{V}((1+\lambda)\tilde{q}/(1+g)) - \tilde{V}(\tilde{q}/(1+g)) \right]^{\frac{1}{\gamma}} \Big\}. \tag{61}$$

$$\tilde{x}(\tilde{q}) = \max\left\{ 0, (\tilde{m}_{ii}(\tilde{q})\tilde{w}_i)^{\frac{\gamma-1}{\gamma}} (1+\mu\tilde{\nu}(\tilde{q})) \left[(1-\gamma) \left[\tilde{V}((1+\lambda)\tilde{q}/(1+g)) - \tilde{V}(\tilde{q}/(1+g)) \right] \right]^{\frac{1-\gamma}{\gamma}} \right\}. \tag{62}$$

$\Rightarrow$

$$r\tilde{V}_t(\tilde{q}_t) = \max\left\{ 0, \tilde{\pi}_i(q) - \tilde{w}_i f_i^m + \gamma \tilde{x}(\tilde{q}) \left[\tilde{V}((1+\lambda)\tilde{q}/(1+g)) - \tilde{V}(\tilde{q}/(1+g)) \right] \right\}. \tag{63}$$

$\Rightarrow$

$$\tilde{x}(\tilde{q}) = \frac{r\tilde{V}_t(\tilde{q}_t) - \tilde{\pi}_i(\tilde{q}) + \tilde{w}_i f_i^m}{\gamma \left[\tilde{V}((1+\lambda)\tilde{q}/(1+g)) - \tilde{V}(\tilde{q}/(1+g)) \right]}, \tag{64}$$

where

$$\pi_i(q) = \sum_n \int_{z_{in}^*(q)}^{\infty} \left(\frac{r_{in}(q,z)}{\sigma} - w_i f_{in} \right) d\Psi(z),$$

$$r_{in}(q,z) = \left(\frac{\frac{\sigma}{\sigma-1} \tau_{in} \frac{w_i}{q}}{z} \right)^{1-\sigma} C_n. \tag{65}$$

$$\tilde{\pi}_i(\tilde{q}) = \frac{\pi_i(q)}{q_t^*} \tag{66}$$

$$= \sum_n \int_{z_{in}^*(q)}^{\infty} \left(\frac{r_{in}(q,z)}{\sigma q_t^*} - \tilde{w}_i f_{in} \right) d\Psi(z) \tag{67}$$

$$= \sum_n \int_{z_{in}^*(q)}^{\infty} \left(\mathcal{J} \left(\frac{\tau_{in}\tilde{w}_i}{z\tilde{q}} \right)^{1-\sigma} \tilde{C}_n - \tilde{w}_i f_{in} \right) d\Psi(z), \tag{68}$$

$$= \sum_n \left(\mathcal{J} \left(\frac{\tau_{in}\tilde{w}_i}{\tilde{q}} \right)^{1-\sigma} \tilde{C}_n \int_{z_{in}^*(q)}^{\infty} z^{1-\sigma} d\Psi(z) - \tilde{w}_i f_{in} \int_{z_{in}^*(q)}^{\infty} d\Psi(z) \right), \tag{69}$$

$$= \sum_n \left(\mathcal{J} \left(\frac{\tau_{in}\tilde{w}_i}{\tilde{q}} \right)^{1-\sigma} \tilde{C}_n \frac{\iota}{\sigma + \iota - 1} (z_{in}^*(q))^{-\sigma-\iota+1} - \tilde{w}_i f_{in} (z_{in}^*(q))^{-\iota} \right), \tag{70}$$

where $\tilde{C} = \frac{C}{q_t^*}$ and $\mathcal{J} \equiv \sigma^{-\sigma}(\sigma-1)^{\sigma-1}$.

Appendix B.5 Aggregate Labor Demand

Labor market clearing. With total labor supply L (normalized to $L = 1$ in the main text), market clearing requires

$$L = M_t \int_{\tilde{q} \geq 1} \ell(\tilde{q})\, d\tilde{\Phi}_t(\tilde{q}) \;+\; R_t\, f^e, \tag{71}$$

where $\ell(\tilde{q})$ is the total labor used by an incumbent with relative productivity $\tilde{q} := q/q_t^*$, and $R_t f^e$ is labor used by entrants (entry R&D), with f^e measured in labor units.

Incumbent-level labor demand. For a firm with relative productivity $\tilde{q}$,

$$\ell(\tilde{q}) = \underbrace{h(\tilde{q})}_{\text{R\&D}} + \underbrace{f_i^m}_{\text{HQ}} + \sum_{n \in \mathcal{C}} \Big\{ \underbrace{f_{in}\, m_{in}(\tilde{q})}_{\text{per-line fixed operating}} + \underbrace{L_{in}^{\text{var}}(\tilde{q})}_{\text{variable production}} \Big\}. \tag{72}$$

Here $m_{in}(\tilde{q})$ is the expected mass of active lines sold by a firm headquartered in i to market n, and

$$m_{in}(\tilde{q}) = \int_{z \geq z_{in}^*(\tilde{q})} d\Psi(z) = \big(z_{in}^*(\tilde{q})\big)^{-\iota},$$

with $z_{in}^*(\tilde{q})$ the attribute cutoff defined in the main text.

Variable production labor. Under $y_j = q\,\ell_j$ and the constant-markup price $p_{in}(q) = \frac{\sigma}{\sigma-1}\,\tau_{in}\,\frac{w_i}{q}$, a variety with attribute z in market n produces

$$y_{inj}(q,z) = \frac{r_{inj}(q,z)}{p_{in}(q)} = \left(\frac{p_{in}(q)}{z} \right)^{-\sigma} \frac{C_n}{p_{in}(q)} = \left(\frac{\sigma-1}{\sigma} \right)^{\sigma} \frac{q^\sigma}{w_i^\sigma\, \tau_{in}^\sigma}\, z^{\sigma-1}\, C_n,$$

where we used $P_n \equiv 1$ so $r_{inj}(q,z) = (p_{in}(q)/z)^{1-\sigma} C_n$. Hence the variable *production labor* for that line is $y_{inj}(q,z)/q$, and integrating over active lines yields

$$L_{in}^{\text{var}}(q) = \int_{z \geq z_{in}^*(q)} \frac{y_{inj}(q,z)}{q}\, d\Psi(z) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \frac{q^{\sigma-1}}{w_i^{\sigma}\, \tau_{in}^{\sigma}}\, C_n \int_{z \geq z_{in}^*(q)} z^{\sigma-1}\, d\Psi(z). \tag{73}$$

In relative units (using $\tilde{w}_i := w_i/q_t^*$ and $q = \tilde{q}\, q_t^*$, which cancels out in p_{in}), this becomes

$$L_{in}^{\text{var}}(\tilde{q}) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \frac{\tilde{q}^{\sigma-1}}{\tilde{w}_i^{\sigma}\, \tau_{in}^{\sigma}}\, C_n \int_{z \geq z_{in}^*(\tilde{q})} z^{\sigma-1}\, d\Psi(z). \tag{74}$$

Optional closed form under Pareto: if $d\Psi(z) = \iota z^{-\iota-1}\, dz$ for $z \geq 1$ and $\iota > \sigma - 1$, then

$$\int_{z \geq z_{in}^*(\tilde{q})} z^{\sigma-1} d\Psi(z) = \frac{\iota}{\iota - \sigma + 1} \left(z_{in}^*(\tilde{q})\right)^{\sigma-\iota-1}.$$

Incumbents' R&D labor. Consistent with the innovation policy in the main text, the firm's total R&D labor in relative units is

$$h(\tilde{q}) = c_x^{1/\gamma} \left(\tilde{m}_{ii}(\tilde{q})\right)^{\frac{\gamma-1}{\gamma}} \left(1 + \mu\, \tilde{\nu}(\tilde{q})\right) \left[\frac{(1-\gamma)}{\tilde{w}_i} \left(\tilde{V}\Big(\tfrac{(1+\lambda)\tilde{q}}{1+g}\Big) - \tilde{V}\Big(\tfrac{\tilde{q}}{1+g}\Big)\right)\right]^{1/\gamma}, \tag{75}$$

where $\tilde{\nu}(\tilde{q}) = \tilde{m}_{in}(\tilde{q})/\tilde{m}_{ii}(\tilde{q})$ and $\tilde{m}_{in}, \tilde{m}_{ii}$ are the relative masses of export and domestic lines, respectively.[15]

Putting it together. Equations (71)–(75) fully characterize labor demand on the BGP. In symmetric equilibria, $n \in \{i, \text{foreign}\}$ with $\tau_{ii} = 1$ and $f_{in} > f_{ii}$. Integrability of the production labor integral requires $\iota > \sigma - 1$, which we assume throughout the paper.

[15] This corrects a common mis-specification: $h(\tilde{q})$ is the firm's *total* R&D labor; it is *not* multiplied by $m_{ii} + m_{in}$. The dependence on scope enters through $\tilde{m}_{ii}$ and $\tilde{\nu}$ in (75).

Appendix.B Numerical Scheme

State and grids. The single continuous state is a firm's relative productivity $q \in [1, q_{\max}]$, discretized on a monotone, lower-bound–dense grid

$$q_i \;=\; 1 + (q_{\max} - 1)\left(\frac{i-1}{n-1}\right)^2, \qquad i = 1, \ldots, n,$$

with node number, n, and upper bound of grids, $q_{\max}$. Entrants draw q from a Pareto distribution with shape a_F; on the grid we set $G_g(i) \propto a_F\, q_i^{-a_F-1}$ and normalize to $\sum_i G_g(i) = 1$.

Computation at a glance. We solve a fixed point in the stationary distribution Φ as (i) the equilibrium vector $EQ = (g, r, A, R, C, M)$, (ii) policies (domestic and export attribute cutoffs, z_d^* and z_e^*, and innovation x), and (iii) firm values V are functions of Φ. The iteration is organized into an *outer loop* (fixed point on V and Φ) with three nested blocks:

1. **Equilibrium block (given $V^{(t)}$ and $\Phi^{(t)}$).** We choose EQ to minimize the squared norm of six equilibrium conditions' residuals. For any candidate EQ, this block updates the current policies internally (attribute cutoffs and innovation) to compute the residuals:

$$\begin{aligned}
\text{(i) Labor market:} \quad & y_1 = L - L^d(EQ; z_d, z_e, x, \Phi^{(t)}),\\
\text{(ii) HBC:} \quad & y_2 = C - wL - (r-g)A,\\
\text{(iii) Euler:} \quad & y_3 = g - \frac{r-\rho}{\eta},\\
\text{(iv) Asset market:} \quad & y_4 = A - M\,\overline{V}, \qquad \overline{V} = \sum_i V^{(t)}(q_i)\,\Phi^{(t)}(q_i),\\
\text{(v) Free entry:} \quad & y_5 = w f_e - \sum_i V^{(t)}(q_i)\,G_g(q_i),\\
\text{(vi) Mass invariance:} \quad & y_6 = \int_{q \le 1+g} [1 - x(q)]\,\Phi^{(t)}(q)\,dq \;-\; \frac{R}{M},
\end{aligned}$$

   with the last integral evaluated on the grid by the trapezoidal rule.

2. **Policy/Value block (given $EQ^{(t)}$).** We update future images of the value function under innovation and growth,

$$q^{\text{succ}} = \tfrac{1+\lambda}{1+g}\,q, \qquad q^{\text{fail}} = \tfrac{1}{1+g}\,q, \qquad V_s(q) = V^{(t)}(q^{\text{succ}}), \quad V_f(q) = V^{(t)}(q^{\text{fail}}),$$

   by monotone cubic interpolation; if an image exits the grid on the left, the value is set to 0. At each grid point we then solve a two-equation zero-profit system for the domestic/export attribute cutoffs $(z_d(q), z_e(q)) \ge 1$. Given cutoffs, innovation success $x(q) \in [0, 1)$ is computed from the first-order condition in closed form,

$$x(q) \;=\; \max\!\big[0,\; c_x^{1/\gamma}\,\big(\tfrac{(1-\gamma)\,\Delta V(q)}{w}\big)^{1/\gamma-1}\,(1 + \tfrac{m_e(q)}{m_d(q)})\,m_d(q)^{\,1-1/\gamma}\big],$$

   where $\Delta V(q) = \max\{V_s(q) - V_f(q), 0\}$, $m_d(q) = z_d(q)^{-\iota}$, $m_e(q) = z_e(q)^{-\iota}$. Flow profits

combine domestic and export components. Then, we update the innovation expected payoff continuation/cost term and value update are

$$\Xi(q) \;=\; x(q)\,[V_s(q) - V_f(q)] - w\,h(q), \qquad h(q) \;=\; \Big(\tfrac{x(q)}{c_x}\Big)^{\frac{1}{1-\gamma}} \Big(1 + \mu\,\tfrac{m_e(q)}{m_d(q)}\Big)^{-\frac{\gamma}{1-\gamma}}\, m_d(q).$$

By Bellman equation, update the value function series as $V^{(t+1)}(q) \;=\; \frac{\Pi(q)+\Xi(q)}{r}$, re-anchored each iteration to enforce $V^{(t+1)}(1) = 0$.

3. **Distribution block (given $x^{(t)}$ and $EQ^{(t)}$).** Innovation induces multiplicative jumps in q, yielding an infinitesimal generator $Q \in \mathbb{R}^{n\times n}$: each column i sends mass to the nearest up/down images $j^{\uparrow}(i)$ and $j^{\downarrow}(i)$ with rates

$$Q_{\,j^{\uparrow}(i),\,i} = x(q_i) \quad \text{if } q_i \le \tfrac{q_{\max}(1+g)}{1+\lambda}, \qquad Q_{\,j^{\downarrow}(i),\,i} = 1 - x(q_i) \quad \text{if } q_i \ge 1+g,$$

and diagonal entries chosen so columns sum to zero. The stationary cross-section solves the entry-augmented system

$$Q'\Phi \;=\; -\frac{R}{M}\,G_g, \qquad \mathbf{1}'\Phi = 1,$$

implemented by augmenting with the normalization row and solving a sparse linear system. Tiny negative entries (from interpolation/discretization) are truncated and Φ is renormalized.

Outer loop, initialization, and stopping rule. We initialize

$$\{g^{(0)}, \quad r^{(0)}, \quad A^{(0)}, \quad R^{(0)}, \quad C^{(0)}, \quad M^{(0)}\},$$

set $V^{(0)}(q) = \log q$ (anchored to $V^{(0)}(1) = 0$), uniform cutoffs $(z_d^{(0)}, z_e^{(0)})$, and $\Phi^{(0)}$.

The outer loop stops when the value function stabilizes,

$$\frac{\|V^{(t+1)} - V^{(t)}\|_2}{\|V^{(t+1)}\|_2} \;<\; 10^{-4}.$$

Labor aggregation (used in y_1). Total labor demand combines incumbents (production plus innovation) and entrants:

$$L^d \;=\; M\,\Phi'\big[\ell_p(q; z_d, z_e, C, w) + h(q)\big] \;+\; Rf_e,$$

where $\ell_p(\cdot)$ is the implied production labor from the CES technology and the Pareto attribute distribution.

Numerical safeguards. Double precision throughout; clip $x \in [0, 0.999]$, $z_d, z_e \ge 1$; enforce column sums of Q to zero and guard off-grid jumps; truncate negative components of Φ and

renormalize. The policy (cutoff) solves use tight tolerances (10^{-6}), while the equilibrium solve uses 10^{-4} tolerances; finite differences are central.

Pseudocode (outer–inner fixed point).

Initialize $(EQ^{(0)}, V^{(0)}, \Phi^{(0)}, z_d^{(0)}, z_e^{(0)})$
Repeat for $t = 0, 1, 2, \ldots$
  [Equilibrium] $EQ^{(t)} \leftarrow \arg\min_{EQ} \|(y_1, \ldots, y_6)\|_2$ (policies updated inside)
  [Policy/Value] $V_s, V_f \leftarrow \text{interp}(V^{(t)}; g^{(t)})$; $(z_d, z_e) \leftarrow$ solve cutoffs; $x \leftarrow$ FOC closed form
    $V^{(t+1)} \leftarrow (\Pi + x(V_s - V_f) - wh)/r$, then re-anchor
  [Distribution] $Q \leftarrow$ build from $(x, g^{(t)})$; $\Phi^{(t+1)} \leftarrow$ solve $[Q'\Phi = -(R/M)G_g,\ \mathbf{1}'\Phi = 1]$
  [Stop] $\|V^{(t+1)} - V^{(t)}\|_2 / \|V^{(t+1)}\|_2 < 10^{-4}$
End

Appendix.C Reduced-form Tests of the Innovation Mechanism

Propensity to Invest in R&D

Table 4 reports Tobit estimates of a firm's R&D investment: the dependent variable is current R&D expenditure. Regressors include lagged R&D (both a dummy and log level), measures of foreign–market exposure—either the (log) export value or the (log) number of exported products—and their interactions with lagged R&D. We also control for contemporaneous value–added per worker and capital, and include industry–year, province, and age fixed effects; standard errors are clustered at the firm level. The estimation sample contains 75,003 firm–year observations.

The coefficients on lagged R&D are large and tightly estimated: the *R&D dummy* ranges from 0.375 to 0.478 (cols. 1, 3, 5) and the *log R&D* coefficient is about 0.44–0.45 (cols. 2–5). This is consistent with adjustment costs and/or knowledge capital that make R&D highly state–dependent. The results also indicate that export exposure raises the propensity to invest in R&D. Both intensive export measures enter positively: the coefficient on *log export value* is 0.005^{***} in col. (2) and 0.018^{***} in col. (3), while the coefficient on *log # exported products* is 0.034^{***} in col. (4) and 0.033^{***} in col. (5). Hence, conditional on fundamentals and fixed effects, more export scale/scope is associated with higher current R&D outlays. The key result in the table is the complementarity between R&D and exporting. The interactions between lagged R&D and export exposure are uniformly positive and precisely estimated when we measure both on the intensive margin: *Export scale (cols. 2–3):* $\log \text{Export Value}_{t-1} \times \log \text{R\&D}_{t-1}$ has a coefficient of 0.006^{***} in both specifications. *Export scope (cols. 4–5):* $\log \#\,\text{Products}_{t-1} \times \log \text{R\&D}_{t-1}$ loads at 0.024^{***} and 0.025^{***}. These estimates imply that the marginal effect of past R&D on the (latent) current R&D decision is *increasing* in a firm's export scale/scope.

When only dummies are used, the *Export Dummy*$\times$*R&D Dummy* is positive (0.199^{***}, col. 1), indicating that past exporters who were already doing R&D are particularly likely to keep investing. Once we add intensive–margin measures for both activities, this residual extensive–margin interaction turns negative (cols. 3 and 5), which is natural: the continuous interactions absorb the economically relevant complementarity, while the remaining dummy interaction mostly captures selection at the corner.[16]

Prior work emphasizes productivity gains from starting to export (the extensive margin, "learning by exporting") and from the joint choice of R&D and exporting (Aw, Roberts, and Xu, 2008, 2011). Our results complement that evidence by showing *intensive–margin* complementarities: greater export *scale* (value) and *scope* (number of products) make R&D more attractive at the margin, consistent with market–size mechanisms in which foreign demand raises the payoff to innovation and technology upgrading (Lileeva and Trefler, 2010; Bustos, 2011). The positive interaction terms indicate that a given unit of prior R&D becomes *more effective for sustaining current R&D investment* when firms are more deeply engaged in export markets.

[16] This pattern is common when moving from extensive to intensive measures for choices that are highly persistent and skewed.

Table 4: Estimating R&D Expenditure: Tobit Model

	(1)	(2)	(3)	(4)	(5)
Value-Added per Worker$_{ft}$	0.273***	0.163***	0.165***	0.163***	0.166***
	(0.017)	(0.010)	(0.010)	(0.010)	(0.010)
Capital$_{ft}$	0.380***	0.227***	0.222***	0.226***	0.221***
	(0.013)	(0.006)	(0.006)	(0.006)	(0.006)
SOE$_{ft}$	0.097**	0.050	0.034	0.049	0.034
	(0.049)	(0.033)	(0.033)	(0.033)	(0.033)
R&D Dummy$_{ft-1}$	0.478***		0.378***		0.375***
	(0.020)		(0.018)		(0.018)
Export Dummy$_{ft-1}$	0.065***		-0.021		0.146***
	(0.022)		(0.072)		(0.028)
.× R&D Dummy$_{ft-1}$	0.199***		-0.263***		-0.246***
	(0.032)		(0.028)		(0.027)
log R&D Expenditure$_{ft-1}$		0.448***	0.437***	0.454***	0.443***
		(0.010)	(0.008)	(0.010)	(0.008)
log Export Value$_{ft-1}$		0.005***	0.018***		
		(0.001)	(0.005)		
.×log R&D Expenditure$_{ft-1}$		0.006***	0.006***		
		(0.001)	(0.001)		
log # of Exported Products$_{ft-1}$				0.034***	0.033***
				(0.006)	(0.008)
.×log R&D Expenditure$_{ft-1}$				0.024***	0.025***
				(0.003)	(0.003)
Constant	-2.255***	-1.072***	-1.236***	-1.070***	-1.234***
	(0.184)	(0.156)	(0.152)	(0.157)	(0.153)
Obs.	75,003	75,003	75,003	75,003	75,003

Notes: The dependent variable is current R&D expenditure; observations with zero R&D are left–censored. Regressors include lagged R&D (dummy and log), export exposure (log export value or log number of exported products), and their interactions, plus value–added per worker and capital. All columns include industry–year, province, and age fixed effects. Standard errors (in parentheses) are clustered at the firm level. Asterisks denote significance at 1% (***), 5% (**), and 10% (*).

From the Policy Function in the Model to an Estimable Equation

The model delivers the innovation technology

$$\log x(q) \;=\; \log c_x \;+\; \gamma \,\log\big(1+\nu(q)\big) \;+\; \big(1-\gamma\big)\,\log\!\left(\frac{h(q)}{m_{ii}(q)}\right), \tag{76}$$

where $x(q)$ is the *innovation success rate for productivity growth*, $\nu(q)$ captures foreign market access (export scale/scope), and $h(q)/m_{ii}(q)$ is the *innovation intensity* (innovation effort per product). This structure follows models in which R&D effort raises a Poisson-type arrival rate of successful innovations and market size magnifies the return to that effort.

Because $x(q)$ is unobserved, we test (76) using the following proxies: (i) $\nu(q)$ with firm export intensity, $\text{ExpInt}_{i,t} \equiv X_{i,t}/S_{i,t}$; (ii) $h(q)/m_{ii}(q)$ with innovation intensity constructed from the data, $\text{RDInt}_{i,t}$ (baseline: R&D-to-sales; robustness: R&D per product or R&D-to-profits); and

(iii) the success rate $x(q)$ with next period's improvement in labor productivity, either the continuous growth of $\log(\mathrm{VA}/L)$ or an indicator for positive growth. With firm fixed effects absorbing $\log c_x$, our baseline panel equation is

$$\Delta \log\left(\frac{\mathrm{VA}}{L}\right)_{i,t+1} = \beta_1 \log\bigl(1+\mathrm{ExpInt}_{i,t}\bigr) + \beta_2 \log \mathrm{RDInt}_{i,t} + \mathbf{X}'_{i,t}\kappa + \zeta_i + \chi_{s\times t} + u_{i,t+1}, \quad (77)$$

and we also estimate a binary specification where the dependent variable equals one if $\Delta \log(\mathrm{VA}/L)_{i,t+1} > 0$. If expected productivity growth is approximately proportional to the (log) success rate, then $\beta_1 \propto \gamma$ and $\beta_2 \propto (1-\gamma)$, yielding a reduced-form counterpart to γ,

$$\widehat{\gamma} \approx \frac{\hat{\beta}_1}{\hat{\beta}_1 + \hat{\beta}_2},$$

up to a common scale factor.[17]

Table 5 reports estimates of (77). Columns (1)–(3) use the *continuous* growth rate of labor productivity from t to $t{+}1$ as the dependent variable; columns (4)–(6) use a *growth dummy* that equals one if the firm's labor productivity rises between t and $t{+}1$. All specifications include firm and industry–year fixed effects, with standard errors clustered at the firm level.

Two patterns line up directly with (76): *Export access (ν) raises the success-rate proxy.* Across specifications, the coefficient on $\log\bigl(1+\mathrm{ExpInt}_{i,t}\bigr)$ is positive and statistically significant in the continuous-growth regressions and remains positive in the growth-dummy models. This is consistent with market-size mechanisms: better foreign access raises the payoff to innovation and the likelihood that innovation translates into productivity gains. *Innovation intensity (h/m_{ii}) raises the success-rate proxy.* The coefficient on $\log \mathrm{RDInt}_{i,t}$ is likewise positive and significant, indicating that firms with higher innovation effort per product are more likely to realize subsequent productivity improvements.

When both regressors are included, each remains economically and statistically relevant, implying that *export access and innovation intensity contribute independently* to subsequent productivity growth, as in dynamic models where exporting and R&D jointly push future efficiency.

Remark (link to structural γ). *From column (1), $\widehat{\gamma} \approx \hat{\beta}_1/(\hat{\beta}_1+\hat{\beta}_2) \approx 0.60$; from column (2), $\widehat{\gamma} \approx 0.64$. These values are in line with the GMM estimate from the structural model, $\hat{\gamma} = 0.60$, providing an external validation of the structural calibration.*

The earlier Tobit results showed that exporting amplifies the *propensity to invest in R&D* via positive interactions between export exposure and past R&D. Here we show that exporting and innovation effort per product are also associated with higher *realized* productivity improve-

[17] This mapping is exact if $\mathbb{E}[\Delta \log(\mathrm{VA}/L)_{i,t+1} \mid \cdot] = \kappa \log x_{i,t}$ and the proxies are measured without error. A delta-method standard error for $\widehat{\gamma} = g(\hat{\beta}_1, \hat{\beta}_2)$ with $g(b_1,b_2) = b_1/(b_1+b_2)$ uses $\nabla g(b_1,b_2) = \bigl(\frac{b_2}{(b_1+b_2)^2}, -\frac{b_1}{(b_1+b_2)^2}\bigr)'$.

Table 5: Labor Productivity Growth, Export Intensity and Innovation Intensity

	(1)	(2)	(3)	(4)	(5)	(6)
Dep. Var.:	Growth Rate of Labor Productivity$_{ft}$			Dummy of Positive Growth$_{ft}$		
log Export Int.$_{ft}$	0.0141***	0.0244***	0.0270***	0.00276***	0.00406***	0.00410***
	(0.00361)	(0.00350)	(0.00378)	(0.000568)	(0.000557)	(0.000601)
log R&D Int.$_{ft}$	0.00958***	0.0138***	0.0155***	0.00308***	0.00424***	0.00427***
	(0.00237)	(0.00226)	(0.00239)	(0.000458)	(0.000448)	(0.000472)
.× log Export Int.$_{ft}$			0.00233**			3.97e-05
			(0.00105)			(0.000210)
log Capital$_{ft}$		-0.133***	-0.133***		-0.0410***	-0.0410***
		(0.00752)	(0.00752)		(0.00115)	(0.00115)
log Employment$_{ft}$		1.488***	1.488***		0.294***	0.294***
		(0.0145)	(0.0145)		(0.00192)	(0.00192)
SOE$_{ft}$		0.0315	0.0310		-0.0201	-0.0201
		(0.0773)	(0.0773)		(0.0133)	(0.0133)
Constant	0.487***	-6.149***	-6.149***	0.767***	-0.463***	-0.463***
	(0.00205)	(0.0729)	(0.0729)	(0.000353)	(0.00962)	(0.00962)
Industry-Year FE	YES	YES	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES	YES	YES
Obs.	361,968	361,968	361,968	677,464	677,464	677,464
R^2	0.424	0.482	0.482	0.430	0.457	0.457

Notes: This table shows the correlation between future growth of firms' value-added per worker (i.e., labor productivity) and their current (log) export intensity as well as (log) R&D intensity. Columns (1) to (3) use the growth rate of labor productivity from year t to $t+1$ as the dependent variable, while columns (4) to (6) employ a dummy which equals one if the firm experiences a positive productivity growth from t to $t+1$. All exercises include industry-year and firm fixed effects. Standard errors are in parentheses and clustered at the firm level. The number of asterisks indicates significance at 1%(***), 5%(**), and 10%(*) level.

ments one year ahead. The two findings are complementary: the policy regression captures how exporting raises the incentive to invest, while Table 5 indicates that exporting and innovation intensity jointly raise the success rate with which those investments translate into productivity growth.

Innovation Efficiency with Firm Scale and Export Intensity

The following documents a set of data patterns that are directly implied by the two mechanisms in the model: (i) the *Exporting–Innovation Efficiency* (EIE) effect—greater export exposure raises the payoff to innovative effort; and (ii) the *scale–dilution* effect—holding technology fixed, a larger production scale lowers the return to an additional unit of R&D. The goal here is not to make causal claims. Establishing causality would require valid instruments for exporting and R&D to deal with the well-known endogeneity between these choices. Instead, we show that reduced-form correlations in the data line up with the model's comparative statics.

Constructing innovation efficiency. We define a firm's innovation efficiency, IE_{it}, as the *return per unit of R&D expenditure*. Operationally, we compute the one-year change in productivity and scale it by contemporaneous R&D intensity:

$$IE_{it} \equiv \frac{\Delta \ln \text{TFPR}_{i,t+1}}{\text{R\&D}_{it}},$$

where $R\&D_{it}$ is the firm's R&D spending in year t, and $\Delta \ln \text{TFPR}_{i,t+1}$ is the growth in productivity between t and $t+1$. Using the lead of productivity growth accommodates the implementation lag of R&D and avoids a mechanical contemporaneous correlation between output and inputs.

Why TFPR? We measure productivity as revenue productivity (TFPR) following De Loecker (2007). There are three reasons for this choice. *Object of interest.* The model maps innovation into profits through both cost and demand channels. TFPR loads on technological efficiency and on price/quality premia, so it is the relevant summary statistic for profitability. In contrast, physical productivity (TFPQ) would intentionally strip out demand-side gains that product innovation is designed to create. *Price heterogeneity.* Exporters and non-exporters face different demand conditions and charge different prices. With firm-level output prices unavailable, deflating revenues with common industry deflators can bias TFPQ. The De Loecker (2007) approach explicitly allows for firm-specific prices and for selection into exporting, yielding a revenue-based productivity measure that is comparable across exporters and non-exporters. *Precedent in trade and innovation.* TFPR estimated with a control-function production framework (as in De Loecker, 2007) is standard in the literature studying how exporting and innovation interact, and it uses exactly the information available in our data (revenues and input expenditures).

Figure 5 plots a binned scatter of IE_{it} against firm scale, measured by log(Value Added). Blue markers trace non-exporters; red markers trace exporters. Two patterns are immediate. First, within both groups there is a strong negative slope: larger firms exhibit lower innovation efficiency, consistent with scale–dilution in the model. Second, at a given scale, the exporter

curve lies above the non-exporter curve, indicating a higher return to R&D for exporters along the extensive margin. The corresponding regression in Table 6 confirms these patterns: IE_{it} is significantly negatively correlated with log(VA) and positively correlated with an exporter indicator.[18]

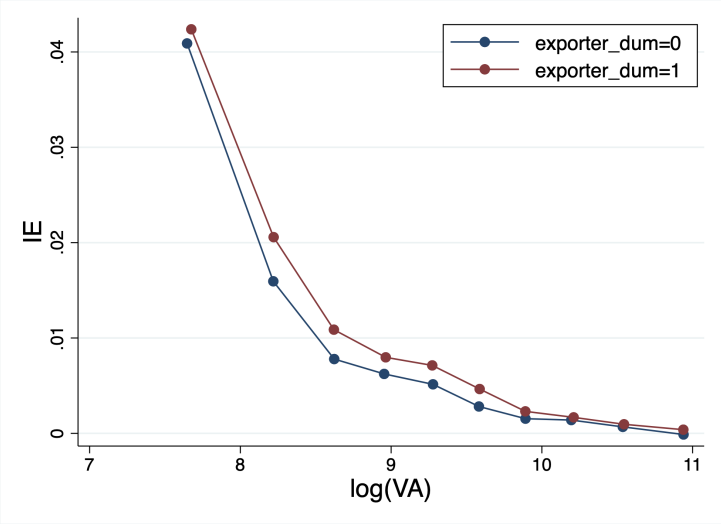


Figure 5: Innovation efficiency vs. scale, by export status.

Export exposure on the intensive margin. We next restrict attention to exporters and relate IE_{it} to *export intensity*, defined as the ratio of export sales to total sales in year t. Figure 6 shows a clear positive relationship: firms that sell a larger share of their output abroad obtain a higher productivity payoff per unit of R&D. Table 7 provides the corresponding regression evidence and finds a statistically significant positive coefficient on export intensity, in line with the EIE mechanism of the model.

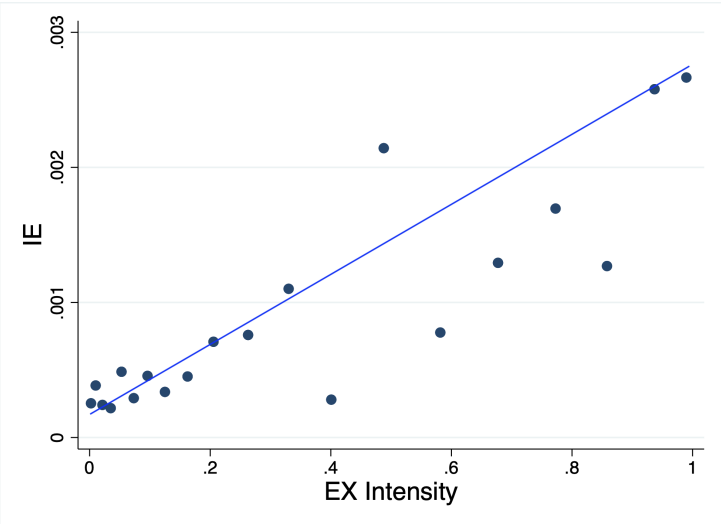


Figure 6: Innovation efficiency vs. export intensity (exporters only).

[18]Specifications include firm-level controls and fixed effects as reported in the table; standard errors are clustered at the firm level.

Across both the extensive and intensive margins of trade, the data display the monotonic relationships predicted by the model: (i) innovation efficiency falls with scale (scale–dilution), and (ii) increases with export exposure (EIE). While these are not causal estimates, the alignment between theory and the descriptive evidence provides external validation of the mechanisms embedded in the model.

	Innovation Efficiency	
	(1)	(2)
log(VA)	-0.0354***	-0.1690***
	(0.00153)	(0.00698)
Exporter	0.0111***	0.0015
	(0.00297)	(0.0105)
Const.	0.3630***	1.6210***
	(0.0147)	(0.0663)
Ind. × Year FE	Yes	Yes
Firm FE		Yes
*** p<0.01, ** p<0.05, * p<0.1		

Table 6: IE vs. Scale and Export Extensive Margin

	Innovation Efficiency	
	(1)	(2)
Exp. Int.	.01480***	
log(EX)		.0050***
log(VA)		-0.0339***
Const.	.017423 ***	.3122***
Ind. × Year FE	Yes	Yes
Obs.	14,30428,269	10,35914,776
*** p<0.01, ** p<0.05, * p<0.1		

Table 7: IE vs. Export Intensive Margin

We further validate this point with an empirical model. Table 9 regresses the (log) number of future patent applications on the (log) number of current exported products.

	(1)	(2)	(3)	(4)	(5)	(6)
Dep. Var.:	Growth Rate of Labor Productivity$_{ft}$			Dummy of Positive Growth$_{ft}$		
log Export Int.$_{ft}$	0.0141***	0.0244***	0.0270***	0.00276***	0.00406***	0.00410***
	(0.00361)	(0.00350)	(0.00378)	(0.000568)	(0.000557)	(0.000601)
log R&D Int.$_{ft}$	0.00958***	0.0138***	0.0155***	0.00308***	0.00424***	0.00427***
	(0.00237)	(0.00226)	(0.00239)	(0.000458)	(0.000448)	(0.000472)
.× log Export Int.$_{ft}$			0.00233**			3.97e-05
			(0.00105)			(0.000210)
log Capital$_{ft}$		-0.133***	-0.133***		-0.0410***	-0.0410***
		(0.00752)	(0.00752)		(0.00115)	(0.00115)
log Employment$_{ft}$		1.488***	1.488***		0.294***	0.294***
		(0.0145)	(0.0145)		(0.00192)	(0.00192)
SOE$_{ft}$		0.0315	0.0310		-0.0201	-0.0201
		(0.0773)	(0.0773)		(0.0133)	(0.0133)
Constant	0.487***	-6.149***	-6.149***	0.767***	-0.463***	-0.463***
	(0.00205)	(0.0729)	(0.0729)	(0.000353)	(0.00962)	(0.00962)
Industry-Year FE	YES	YES	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES	YES	YES
Obs.	361,968	361,968	361,968	677,464	677,464	677,464
R^2	0.424	0.482	0.482	0.430	0.457	0.457

Table 8: Data consistency with innovation policy function

Table 9: Number of Exported Products and Innovation: Reduced-Form Evidence

	Dependent Variable: log (1+Future Patent App.)$_{ft}$					
	(1)	(2)	(3)	(4)	(5)	(6)
log (1+ Exported Products)$_{ft}$	0.121***	0.175***	0.169***	0.100**	0.0500**	
	(0.0291)	(0.0430)	(0.0425)	(0.0446)	(0.0209)	
log (1+ Export Destinations)$_{ft}$				0.0693***	0.0432***	
				(0.00996)	(0.00917)	
log (1+ Exported Varieties)$_{ft}$						0.0672***
						(0.0128)
log Value Added per Worker$_{ft}$	0.0947***	0.0946***	0.0915***	0.0904***	0.0384***	0.0383***
	(0.0137)	(0.0126)	(0.0128)	(0.0130)	(0.00445)	(0.00447)
SOE$_{ft}$		0.200***	0.127***	0.130***	0.0253	0.0248
		(0.0598)	(0.0454)	(0.0457)	(0.0277)	(0.0278)
Export Intensity$_{ft}$		-0.486***	-0.470***	-0.544***	-0.284***	-0.294***
		(0.133)	(0.132)	(0.131)	(0.0553)	(0.0596)
log (1+Current Patent App.)$_{ft}$					0.613***	0.614***
					(0.0555)	(0.0557)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Age FE	No	No	Yes	Yes	Yes	Yes
Province FE	No	No	Yes	Yes	Yes	Yes
Obs.	49,672	49,550	49,461	49,461	49,461	49,461
Adj. R^2	0.367	0.367	0.367	0.367	0.367	0.367

Note: This table shows the relationship between the log number of firm future patent applications in 2009 and the number of firm current exported products in 2007. The exported products are encoded at the Harmonized System 6-digit level. The sample period is 2007, and we restrict attention to firms that applied for at least one invention patent in China during the period 1998-2009 and those that reported positive R&D expenditure during 2005-2007. Standard errors are in the parentheses and clustered at industry-level. Observations are weighted by employment. The number of asterisks indicates significance at 1%(***), 5%(**), and 10%(*) levels.

	Dependent Variable: log Sales Income$_{ft}$			
	(1)	(2)	(3)	(4)
log Employment	0.563***	0.562***	0.562***	0.333***
	(0.00384)	(0.00383)	(0.00383)	(0.00388)
Labor Productivity	0.396***	0.395***	0.395***	0.271***
	(0.00280)	(0.00280)	(0.00280)	(0.00275)
R&D Dummy	0.0267***	0.0269***	0.0269***	0.00686***
	(0.00321)	(0.00320)	(0.00320)	(0.00229)
log R&D Expenditure		0.0121***	0.0121***	0.00553***
		(0.00131)	(0.00131)	(0.000979)
Export Dummy	0.0418***	-0.144***	-0.144***	-0.0650***
	(0.00361)	(0.0145)	(0.0145)	(0.01000)
log Export Revenue		0.0154***	0.0154***	0.00690***
		(0.00116)	(0.00116)	(0.000790)
Import Dummy				0.00457*
				(0.00263)
Entrant Dummy	-0.164***	-0.164***	-0.165***	-0.0567***
	(0.00294)	(0.00294)	(0.00294)	(0.00214)
Exiter Dummy	-0.134***	-0.134***	-0.135***	-0.0478***
	(0.00366)	(0.00366)	(0.00371)	(0.00274)
.× Entrant Dummy			0.0208	-0.00339
			(0.0224)	(0.0143)
Industry-Year FE	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES
Additional Controls	NO	NO	NO	YES
Obs.	404,528	404,528	404,528	404,528
Adj. R^2	0.974	0.974	0.974	0.987

Dependent Var.:	Incumbent Dummy		Entrant Dummy		Exiter Dummy	
	(1)	(2)	(3)	(4)	(5)	(6)
log Employment	0.183***	0.119***	-0.0932***	-0.0504***	-0.0917***	-0.0693***
	(0.00309)	(0.00322)	(0.00254)	(0.00262)	(0.00231)	(0.00237)
Labor Productivity	0.107***	0.0724***	-0.0742***	-0.0512***	-0.0345***	-0.0226***
	(0.00172)	(0.00176)	(0.00140)	(0.00139)	(0.00124)	(0.00128)
Export Dummy	0.0287**	0.0487***	-0.0540***	-0.0676***	0.0248**	0.0171*
	(0.0141)	(0.0138)	(0.0119)	(0.0118)	(0.00985)	(0.00982)
log Export Revenue	0.00398***	0.00142	-0.000185	0.00155	-0.00383***	-0.00299***
	(0.00114)	(0.00112)	(0.000974)	(0.000968)	(0.000799)	(0.000801)
R&D Dummy	0.0125***	0.00668*	0.00352	0.00727**	-0.0163***	-0.0160***
	(0.00395)	(0.00392)	(0.00339)	(0.00337)	(0.00244)	(0.00278)
log R&D Expenditure	0.00149	-0.000542	-0.000346	0.001000	-0.00103	-0.000312
	(0.00159)	(0.00158)	(0.00136)	(0.00136)	(0.000970)	(0.00114)
Industry-Year FE	YES	YES	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES	YES	YES
Additional Controls	NO	YES	NO	YES	NO	YES
Obs.	404,528	404,528	404,528	404,528	404,528	404,528
Adj. R^2	0.561	0.568	0.530	0.533	0.575	0.586

	Dependent Variable: Δ R&D Expenditure$_{ft}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δ Unadj. Demand$_{ft}$	0.120	-0.132	-0.0726	-1.775**	-1.603**	-1.806
	(0.175)	(0.182)	(0.176)	(0.693)	(0.709)	(2.628)
Δ Adj. Demand$_{ft}$		0.671**	0.727***			
		(0.273)	(0.261)			
Δ Adj. Demand (Patent App. Non-resident)$_{ft}$				0.200**		
				(0.0786)		
Δ Adj. Demand (Patent App. Resident)$_{ft}$					0.176**	
					(0.0751)	
Δ Adj. Demand (Researcher per Million)$_{ft}$						0.270
						(0.338)
Skill Intensity$_{ft_0}$			0.141***	0.140***	0.141***	0.141***
			(0.0445)	(0.0447)	(0.0446)	(0.0449)
R&D Intensity$_{ft_0}$			-5.454***	-5.451***	-5.452***	-5.448***
			(1.518)	(1.523)	(1.521)	(1.523)
Export Revenue$_{ft_0}$			0.00255**	0.00257**	0.00258**	0.00258**
			(0.00116)	(0.00115)	(0.00115)	(0.00115)
Export Intensity$_{ft_0}$			-0.170	-0.186*	-0.176*	-0.201*
			(0.108)	(0.104)	(0.103)	(0.113)
Industry-Year FE	YES	YES	YES	YES	YES	YES
Additional Controls	YES	YES	YES	YES	YES	YES
Obs.	58,422	58,422	58,422	58,422	58,422	58,422
Adj. R^2	0.003	0.003	0.015	0.015	0.015	0.015