Optimal Subsidies for Capital Replacement and the Green Transition*

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Abstract

We analyze optimal subsidies for the replacement of durable assets. We first develop a tractable model of capital replacement with heterogeneous producers, endogenous capital-embodied innovation, and environmental externalities that depend on capital vintages. We then characterize the constrained-efficient allocation that arises if a planner chooses capital replacement subject to the equilibrium evolution of innovation. Optimal subsidies to implement this allocation equal the sum of two terms: (i) the difference in present discounted value of damages associated with old capital vs. new capital and (ii) the social value of the marginal effect of capital replacement on innovation, net of the associated markup distortion. We generalize this formula to the case of heterogeneous technologies, such as internal combustion vs. electric vehicles. We then calibrate the model using empirical evidence on several types of capital, including aircraft and vehicles, and simulate the transitional dynamics induced by the optimal policy. Early in the transition, optimal subsidies are steeply increasing in the age of the replaced asset. In the long run, they are determined by the trade-off between innovation and markups.

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1 Introduction

Many governments are striving to design efficient policies to reduce the environmental impact of economic activity and address climate change. A core element of this transition to a cleaner economy is the replacement of old productive assets with new, cleaner capital.

As a consequence, subsidies for the replacement of durable assets are pervasive. Leading examples include: subsidies for corporate investment in energy-efficient structures and equipment; subsidies for the scrappage of old cars and their replacement with newer, energy-efficient vehicles, such as electric vehicles; subsidies for the purchase of energy-efficient household appliances.

These subsidies may increase aggregate welfare through two channels. First, by causing a change in the age composition of the capital stock, they improve the average quality of capital used in production, thereby reducing aggregate carbon emissions. In so doing, they address an *environmental externality*. Second, by sustaining demand for new technologies embodied in durable goods, they create incentives for capital-goods producers to accelerate the path of innovations, leading to an even higher quality of capital in the future. In so doing, they address an *innovation externality*.

The goal of this paper is to provide a theoretical and quantitative analysis of optimal capital replacement subsidies in the presence of environmental and innovation externalities. To accomplish this goal, our analysis proceeds in three main steps. First, we develop a tractable macroeconomic model of investment with heterogeneous producers, endogenous capital-embodied innovation, and environmental externalities.

In the model, heterogeneous firms choose the optimal timing to replace an indivisible asset subject to idiosyncratic shocks. Through a market-size effect, the fraction of firms replacing their capital generates incentives for capital-goods producers to invest and improve the quality of their products. In turn, this innovation process generates endogenous obsolescence of old capital, inducing its replacement with the new vintage. Critically, the quality of capital also affects the environmental damages induced by firm production, which firms do not internalize. Furthermore, the level of innovation in competitive equilibrium is inefficient, because capital-goods producers are only motivated by short-run profits, whereas improvements in capital quality have a long-lasting social value.

Second, we characterize three benchmark allocations: the decentralized equilibrium without policy interventions, the first-best allocation, and the *constrained-efficient allocation*, which is the main focus of this paper. This allocation arises when a benevolent

planner is restricted to distort capital-replacement decisions, but cannot directly choose innovation levels and prices for new goods.

This setup is relevant for our analysis because most governments around the world can subsidize domestic demand for durable goods, but have limited control over the supply side of this goods, which is highly concentrated geographically in few countries that specialize in the production of equipment. Moreover, even countries that have a large domestic industry producing durable goods, such as the US and China, leveraged significant replacement subsidies on the demand side to stimulate the transition to electric vehicles.

We formalize an equilibrium implementation of this constrained-efficient benchmark with subsidies for the replacement of old capital, which vary with capital age. We obtain a formula that expresses the optimal subsidy as the sum of two terms. The first term captures the internalization of the environmental externality taking as given the path of capital quality. This term equals the present discounted value of the difference between the environmental damages associated with continuing production with older capital and the environmental damages associated with production with the new vintage of capital. The second term captures the internalization of the social benefits of stimulating demand for capital-embodied innovation, net of the higher markup distortions required to induce innovation effort.

We show that this formal representation of optimal subsidies generalizes to the case of heterogeneous technologies, each embodied in durable assets. This extension is relevant, for instance, to analyze subsidies for the replacement of vehicles, in the presence of both electric vehicles and vehicles with internal-combustion engines.

Third, we combine several data sources on equipment goods, including aircraft and vehicles, to quantify the main margins in the model. We perform three separate calibrations, for aircraft, vehicles, and a generic capital good respectively. In each case, we calibrate the distribution of capital-replacement costs, the depreciation rate, and the speed of innovation to match the relevant empirical evidence. We leverage measures of technological improvements in fuel efficiency, which generate both internalized productivity gains and environmental externalities.

We compute optimal subsidies in the calibrated model and decompose the roles of environmental externalities and capital-embodied innovation using our analytical characterization. Starting from the calibrated competitive equilibrium, we simulate the effects of implementing the optimal subsidy scheme for capital replacement. We focus on both

short-run transitional dynamics and long-run outcomes as the policy change affects the evolution of the distribution of capital.

Along the decentralized-equilibrium balanced-growth path, capital is on average older than is desirable from the perspective of the planner. As a result, optimal subsidies are initially steeply increasing in capital age, which leads to a spike in capital replacement associated with scrappage of the oldest capital vintages. This finding reflects the important role of the first component of optimal subsidies, which addresses environmental externalities taking the evolution of technology as given.

Over time, however, the second component, which encodes the market-size effect of capital replacement on innovation and markups, becomes more important. This transition is reflected in the finding that optimal subsidies become flatter with respect to capital age.

We show that the long-run level of subsidies critically depends on the social value of capital-embodied innovation relative to the importance of markup distortions. When markups are high, such as in our vehicles calibration, the optimal policy may even call for a tax on replacement in the long run, although from a fist-best perspective it would be desirable to stimulate innovation. This finding highlights the importance of explicitly considering the trade-offs associated with second-best policies.

Overall, our analytical and quantitative findings provide a new framework to efficiently design a commonly used policy intervention to stimulate the green transition.

The rest of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents our model. Section 4 characterizes optimal policy in the model. Section 6 discusses the quantification of the model. Section 7 presents our quantitative results. Section 8 concludes.

2 Related Literature

This project contributes to three main strands of the academic literature. First, we contribute to the macroeconomic literature on heterogeneity and investment in durable assets (e.g., Cooper and Haltiwanger, 2006; Khan and Thomas, 2008; Lanteri, 2018; Baley and Blanco, 2021; Winberry, 2021) and on capital-embodied technological change (e.g., Greenwood, Hercowitz, and Krusell, 1997; Krusell, 1998; Boucekkine, del Río, and Licandro, 2005; Caunedo and Keller, 2021; Caunedo, Jaume, and Keller, 2023; Fiori and Scoccianti, 2025). Related to our focus on investment in the presence of environmental externalities

and endogenous innovation, Lanteri and Rampini (2025) analyze the role of financing constraints for investment in energy-efficient capital, whereas Ottonello and Winberry (2024) analyze the role of financing constraints for firm investment and innovation.[1] We provide a normative analysis of investment in the presence of both environmental and innovation externalities.

Our model builds on the theoretical framework of Bertolotti and Lanteri (2024), which combines a model of lumpy capital replacement (Cooper, Haltiwanger, and Power, 1999) with a theory of endogenous capital-embodied innovation. The aggregate volume of investment determines a market-size effect and creates incentives for innovators in the capital-goods sector to climb a quality ladder (Grossman and Helpman, 1991; Aghion and Howitt, 1992). In turn, as successive vintages of capital embody increasing levels of technology, innovation induces obsolescence of old capital, thereby stimulating capital replacement.

Relative to this framework, we introduce environmental externalities in production that depend on capital vintage. Furthermore, to analyze optimal policy, we introduce idiosyncratic adjustment costs, which allow for an analytical characterization of optimal replacement decisions. We use this formulation to characterize the planning problem building on the insights of Thomas (2002) and provide a decentralization of the planner's allocation with subsidies.

Second, we contribute to the literature that analyzes efficiency and policy in macroeconomic models that feature environmental externalities in production (e.g., Golosov, Hassler, Krusell, and Tsyvinski, 2014; Barrage, 2020) and endogenous innovation in clean technologies (e.g., Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Acemoglu, Akcigit, Hanley, and Kerr, 2016; Arkolakis and Walsh, 2023). Related to our focus on the role of durable assets, van der Ploeg and Rezai (2020) discusses the role of stranded assets in the green transition; Capelle, Kirti, Pierri, and Villegas-Bauer (2023) and Gilchrist, Martinez, and Rickard (2025) develop quantitative models that highlight the role of capital vintages for environmental externalities; and Caggese, Chiavari, Goraya, and Villegas-Sanchez (2024) analyze the role of climate for the allocation of factors across firms.[2] Relative to this literature, our contribution is to analyze optimal subsidies for capital replacement, which we first characterize formally and then simulate in several quantitative applications of our theory.

[1] In our model, we abstract from financial frictions. Our focus on constrained efficiency complements the contribution of Lanteri and Rampini (2023), which characterizes optimal investment subsidies to address externalities induced by financial frictions.

[2] Shin (2024) analyzes the role of imperfect capital and labor mobility for the transmission of climate policies.

Third, we contribute to the growing empirical and quantitative literature on the effects of subsidies for the replacement of durable goods (e.g., Adda and Cooper, 2000; Li, Linn, and Spiller, 2013; Gavazza and Lanteri, 2021; Allcott, Kane, Maydanchik, Shapiro, and Tintelnot, 2024; Brueckner, Kahn, and Nickelsburg, 2024; Buettner and Madzharova, 2024). Relatedly, Langer and Lemoine (2022) consider the efficient design of dynamic subsidies for technology adoption, with an application to solar panels. Relative to this literature, our main contribution is a normative theoretical and quantitative analysis of optimal subsidies for asset replacement to address environmental externalities and promote capital-embodied innovation. Our formula for optimal subsidy is related to and may provide a formalization for empirical calculations on the benefits of scrappage schemes.

3 Model of Capital Replacement, Innovation, and Environmental Externalities

We now describe our model of durable-asset replacement with innovation externalities and environmental externalities. The section is organized as follows. We begin by describing the problem of optimal capital replacement in decentralized equilibrium. We then characterize endogenous capital-embodied innovation. Finally, we introduce social preferences and analyze efficiency in the model.

3.1 Agents

Time is discrete and infinite, $t = 0, 1, ...$ The economy is populated by a representative household with linear preferences over streams of consumption and discount factor β. We focus on linear utility from consumption in the main text for simplicity of exposition; however, we also analyze optimal subsidies with a concave utility function in Appendix A.5. The household owns a continuum of final-good producing firms indexed by $i \in \mathcal{I} \equiv [0, 1]$ and a continuum of capital-good producing firms indexed by $j \in \mathcal{J} \equiv [0, 1]$. Final-good firms produce output using capital goods. Capital-good producers' innovations determine the path of technology ("quality") embodied in capital.

3.2 Capital Replacement

We begin by describing the capital-replacement problem of final-good producers. Each final-good firm produces a homogeneous output good with a linear technology that employs capital:

$$y_{i,t} = z\varepsilon_i k_{i,t-1}, \tag{1}$$

where z is an aggregate productivity parameter and ε_i is a constant idiosyncratic productivity type, distributed in the population with probability distribution $P(\varepsilon)$ on a discrete support $\mathcal{E}$.

We stress that for concreteness we refer to these agents as firms that use assets to produce a final good, but they could also be households deriving utility from a service flow (1) produced with a durable good, such as a vehicle.[3] In this case, we can interpret ε_i as an idiosyncratic valuation for the durable good.

Capital $k_{i,t-1}$ is subject to a one-period time to build and depreciates at rate δ. At each date t, firms face a discrete choice on their production capacity for the following date: They can either let their capital depreciate or they can replace it, in which case their new capacity is given by $\kappa_t \equiv \kappa q_t$, where $\kappa > 0$ is a parameter and q_t denotes the average quality of capital goods produced at date t. To focus on the extensive margin of firm investment and preserve the computational tractability of the model, we assume that the level of efficiency units of capital upon replacement is common across firms. The evolution of the capital stock is thus given by

$$k_{i,t} = \begin{cases} (1-\delta)k_{i,t-1} & \text{if } \; x_{i,t} = 0 \\ \kappa_t & \text{if } \; x_{i,t} = 1. \end{cases} \tag{2}$$

Capital replacement requires paying a cost $f_t(1+\phi_{i,t})$, where f_t is an aggregate term that we further specify below and $\phi_{i,t}$ is an idiosyncratic cost shock, independently distributed across firms and over time, with cumulative distribution function $G(\phi)$.[4]

[3] Gavazza and Lanteri (2021) develop a model of heterogeneous households that choose the replacement of durable goods of heterogeneous quality.

[4] Our assumption on idiosyncratic shocks to the cost of replacement implies a smooth replacement hazard as a function of capital age, consistent with the analysis of durable-goods replacement of Beraja and Zorzi (2024).

Firms solve the following dynamic capital-replacement problem:

$$\max_{\{x_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(z\varepsilon_i k_{i,t-1} - x_{i,t} f_t(1+\phi_{i,t}) \right), \tag{3}$$

subject to the capital accumulation equation (2), where $x_{i,t} \in \{0,1\}$ is an indicator variable that denotes the discrete capital replacement choice.

We now formulate this problem recursively. Let $a \in \mathcal{A} \equiv \{1,2,..,a_{max}\}$ denote capital age, or, equivalently, the number of periods elapsed since a firm's last capital replacement. Consider a firm with productivity ε that will enter period $t+1$ with capital of age a. We can express the value of this firm at the end of period t, $V_{\varepsilon,a,t}$, recursively as follows:

$$V_{\varepsilon,a,t} = \beta \left(z\varepsilon k_{a,t} + \max_{\overline{\phi} \in [\phi_{min}, \phi_{max}]} \left\{ -f_{t+1} \int_{\phi_{min}}^{\overline{\phi}} (1+\phi) dG(\phi) + G(\overline{\phi}) V_{\varepsilon,1,t+1} + (1-G(\overline{\phi})) V_{\varepsilon,a+1,t+1} \right\} \right). \tag{4}$$

Equation (4) reformulates the discrete choice of capital replacement at $t+1$ as a choice of a threshold $\overline{\phi}_{\varepsilon,a,t}$ for the idiosyncratic cost shock, such that replacement takes place if and only if the cost shock is lower than the threshold. Because the value of replacement is monotonically decreasing in the cost shock and the value of no replacement is independent of it, this threshold decision rule is equivalent to the discrete choice described above.

Firms replace their capital when their capital is sufficiently old and the idiosyncratic cost shock ϕ is sufficiently small. When replacement occurs, with probability $G(\overline{\phi}_{\varepsilon,a,t})$, the continuation value is the value of a firm with capital age $a = 1$; when no replacement occurs, the associated continuation value is the value of a firm with capital age $a+1$. At an interior solution, the first-order condition for optimal replacement is

$$f_t(1+\overline{\phi}_{\varepsilon,a,t}) = V_{\varepsilon,1,t} - V_{\varepsilon,a+1,t},$$

which equates the cost of replacement at the threshold and the benefit from replacement, expressed as the difference in value due to capital upgrade.

Upon replacement, capital is scrapped and the new capital is given by a bundle of investment goods $j \in \mathcal{J}$. The efficiency of this bundle depends on both the quantity $d_{i,j,t}$ and the *quality* $q_{j,t}$ of the capital goods purchased, according the following technology:

$$\kappa_t = \int_{j \in \mathcal{J}} q_{j,t}^{\gamma} d_{i,j,t}^{1-\gamma} dj, \tag{5}$$

with $\gamma \in (0,1)$. Hence, when firm i replaces its capital stock, it solves the following static expenditure minimization problem:

$$\min_{\{d_{i,j,t}\}_{j\in\mathcal{J}}} \int_{j\in\mathcal{J}} p_{j,t} d_{i,j,t} dj, \tag{6}$$

where $p_{j,t}$ denotes the price of investment good j, subject to the constraint (5).

Because all final-good firms face the same expenditure-minimization problem, the solution is independent of i and the resulting expenditure determines the capital-replacement cost: $f_t = \int_{j\in\mathcal{J}} p_{j,t} d_{j,t} dj$. We denote by λ_t the Lagrange multiplier on constraint (5), which is also common across replacing firms. The cost-minimizing quantity of variety j demanded by each replacing firm is given by

$$d_{j,t} = \lambda_t^{\frac{1}{\gamma}} (1-\gamma)^{\frac{1}{\gamma}} p_{j,t}^{-\frac{1}{\gamma}} q_{j,t}, \tag{7}$$

and aggregate demand for variety j is given by $M_t d_{j,t}$, where M_t is the mass of firms replacing their capital at date t.

Given the decision rules for optimal replacement, the distribution of firms over capital age, which we denote $m_{\varepsilon,a,t}$, evolves as follows:

$$m_{\varepsilon,1,t+1} = \sum_{a\in\mathcal{A}} m_{\varepsilon,a,t} G\left(\overline{\phi}_{\varepsilon,a,t}\right) \tag{8}$$

$$m_{\varepsilon,a+1,t+1} = m_{\varepsilon,a,t}\left(1 - G\left(\overline{\phi}_{\varepsilon,a,t}\right)\right). \tag{9}$$

and the aggregate fraction of firms replacing their capital is $M_t = \sum_{\varepsilon\in\mathcal{E}} m_{\varepsilon,1,t+1}$.

3.3 Capital-Embodied Innovation

We now describe the problem of capital-good producers. These firms invest to improve the quality of their products, which they sell to final-good producers. The success of their innovation effort is stochastic. Upon a successful innovation, the quality of variety j increases by an exogenous factor $\eta > 1$, $q_{j,t} = \eta q_{j,t-1}$, and capital-good producers enjoy profits from monopolistic competition for one period, before it becomes free to imitate the innovation. The technology requires c units of final output to produce one unit of capital,

giving the following profit-maximization problem:

$$\max_{p_{j,t},d_{j,t}} M_t d_{j,t} \left(p_{j,t} - c\right),$$

subject to the demand function given by (7). The optimality condition implies that capital-goods producers charge a constant markup, $p_{j,t} = \frac{c}{1-\gamma}$, and their optimal quantity produced satisfies

$$d_{j,t} = (1-\gamma)^{\frac{2}{\gamma}} c^{-\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} q_{j,t}. \tag{10}$$

Thus, profits for successful innovators are linear in quality $q_{j,t}$ and given by

$$\Pi_{j,t} = \gamma (1-\gamma)^{\frac{2-\gamma}{\gamma}} c^{\frac{\gamma-1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} M_t q_{j,t}.$$

In contrast, when innovation is not successful, the quality of variety j remains constant, $q_{j,t} = q_{j,t-1}$, and capital-goods producers sell their output at a price equal to their marginal cost c, because their predetermined quality level can be freely imitated by a competitive fringe.

To innovate, capital-goods producers invest $R_{j,t}$ units of the final good. Critically, this cost does not depend on the volume of sales of a successful product, which gives rise to a market-size effect on innovation: A large fraction of firms replacing their capital, M_t, induces a high innovation effort.

We define the innovation intensity of firm j as $\psi_{j,t} \equiv \frac{R_{j,t}}{\eta q_{j,t-1}}$. Innovation is successful with probability $\alpha(\psi_{j,t})$, with $\alpha(0) = 0$, $\alpha_\psi > 0$, $\alpha_{\psi\psi} < 0$, and $\lim_{\psi \to +\infty} \alpha(\psi) \leq 1$. The optimal expenditure on $R\&D$ for variety j solves the following maximization problem:

$$\max_{R_{j,t}} \alpha\left(\frac{R_{j,t}}{\eta q_{j,t-1}}\right) \Pi_{j,t} - R_{j,t}.$$

Optimal R&D intensity, which is constant across varieties, satisfies the following first-order condition:

$$\gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}} c \kappa^{\frac{1}{1-\gamma}} M_t \frac{\alpha_\psi(\psi_t)\left(1 + (\eta - 1)\alpha(\psi_t)\right)^{\frac{1}{1-\gamma}}}{\left(1 + (\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}} - 1)\alpha(\psi_t)\right)^{\frac{1}{1-\gamma}}} = 1, \tag{11}$$

Equation (11) defines an implicit function $\psi_t = \Psi(M_t)$: Market size determines the optimal innovation intensity because of its effect on expected profits.

As a result of this chosen innovation intensity, the average quality of new capital q_t, which accounts for both successful and unsuccessful innovations, evolves according to a law

of motion, $q_t = H(q_{t-1}, \psi_t)$, defined as follows:

$$\begin{aligned} q_t &= \int_{j \in \mathcal{J}} \left(\alpha(\psi_t)\eta q_{j,t-1} + (1 - \alpha(\psi_t))q_{j,t-1}\right) dj \\ &= q_{t-1} + \alpha(\psi_t)(\eta - 1)q_{t-1}. \end{aligned} \tag{12}$$

The equilibrium replacement cost, which also accounts for the fraction of successful innovation with are sold at a markup, can be expressed as follows:

$$f_t = F(q_{t-1}, \psi_t) \equiv c\kappa^{\frac{1}{1-\gamma}} q_{t-1} \left(1 + (\eta - 1)\alpha(\psi_t)\right) \mathcal{D}\left(\psi_t\right) = c\kappa^{\frac{1}{1-\gamma}} q_t \mathcal{D}\left(\psi_t\right). \tag{13}$$

where the term $\mathcal{D}\left(\psi_t\right) \equiv \frac{(1+(\eta-1)\alpha(\psi_t))^{\frac{\gamma}{1-\gamma}}}{\left(1+(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1)\alpha(\psi_t)\right)^{\frac{\gamma}{1-\gamma}}}$ represents the distortion induced by markups, which increases the cost of replacement.

Because of the assumption that the bundle of capital κ_t is proportional to capital quality q_t, the model admits a balanced-growth path (BGP) in which the fraction of firms replacing their capital is constant over time and capital quality as well as output grow at a constant rate. The BGP features an equilibrium feedback between capital replacement and innovation. The fraction of firms replacing their capital generate incentives for innovation. In turn, growth in the quality of capital renders old capital obsolete, and thus generates incentives for capital replacement.

3.4 Social Preferences and First Best

We now introduce social preferences and characterize the first-best allocation. The social welfare function is

$$\sum_{t=0}^{\infty} \beta^t (C_t - \Omega_t), \tag{14}$$

where C_t denotes aggregate consumption and Ω_t represents the environmental effects of production, which determine social preferences for the distribution of quality of capital used in production.

The aggregate resource constraint states that consumption equals aggregate output net of capital replacement and innovation expenditures:

$$C_t = \sum_{\varepsilon \in E} \sum_{a \in \mathcal{A}} m_{\varepsilon,a,t} \left(z\varepsilon k_{a,t-1} - c\kappa^{\frac{1}{1-\gamma}} q_t \left(G(\overline{\phi}_{\varepsilon,a,t}) + \Phi_{\varepsilon,a,t} \right) \right) - R_t, \tag{15}$$

where the term $c\kappa^{\frac{1}{1-\gamma}}q_t$ denotes the replacement cost in the absence of markup distortions and $\Phi_{\varepsilon,a,t} \equiv \int^{\overline{\phi}_{\varepsilon,a,t}} \phi dG(\phi)$ denotes the aggregate cost paid by firms with state variables ε, a at date t.

We define the environmental effect of production as follows:

$$\Omega_t \equiv \sum_{\varepsilon\in\mathcal{E}}\sum_{a\in\mathcal{A}} \omega_{a,t}(q_{t-a}) m_{\varepsilon,a,t} z\varepsilon k_{a,t-1}, \tag{16}$$

where $\omega_{a,t}(q_{t-a})$ denote social preference weights for different vintages of capital. We assume that these weights are such that capital with higher quality is preferred because it imposes a smaller environmental externality.

Notice that the model features two externalities. First, as equation (12) highlights, future innovations in capital quality build on current innovations, which induces a permanent component in the social value of capital quality. However, capital suppliers innovate only in response to short-run profits. Thus, capital-embodied innovation induces a positive externality increasing future capital quality. Second, as equation (16) highlights, changes in the quality of the capital stock affect welfare through an environmental externality. Overall, the quality of new capital affects both aggregate productivity and the environmental externality.

The first-best social planner fully internalizes these externalities: The planner chooses both capital replacement and innovation effort to maximize aggregate welfare (A16) subject to the resource constraint (15), the environmental effect of production (16), the evolution of the distribution of producers (8) with Lagrange multiplier $\theta_{\varepsilon,a,t}$, and the law of motion for capital quality (12) with multiplier ζ_t.

The planner's optimality conditions are as follows. The Lagrange multiplier on the evolution of the distribution of capital age satisfies the following Euler equation:

$$\begin{aligned}\theta_{\varepsilon,a,t} &= \beta\left((1-\omega_{a,t+1}(q_{t+1-a}))z\varepsilon k_{a,t} - c\kappa^{\frac{1}{1-\gamma}}q_{t+1}\left(G(\overline{\phi}_{\varepsilon,a,t+1}) + \Phi_{\varepsilon,a,t+1}\right)\right) \\ &\quad +\beta\left(G(\overline{\phi}_{\varepsilon,a,t+1})\theta_{\varepsilon,1,t+1} + (1-G(\overline{\phi}_{\varepsilon,a,t+1}))\theta_{\varepsilon,a+1,t+1}\right).\end{aligned} \tag{17}$$

Furthermore, optimal innovation and the evolution of capital quality satisfy:

$$\zeta_t = \eta q_{t-1}\left(H_\psi(q_{t-1},\psi_t)\right)^{-1}, \tag{18}$$

$$\zeta_t = \beta\zeta_{t+1}H_q(q_t,\psi_{t+1}) + \frac{\partial\left(\sum_{t=0}^{\infty}\beta^t(C_t-\Omega_t)\right)}{\partial q_t} \tag{19}$$

According to these optimality conditions, the social value of capital quality equals the sum of two terms: first, the effect of current quality on future quality through the law of motion (12); second, the effect of current quality on welfare. In turn, this welfare effect is given by the effect of quality on future output, as long as the current vintage of capital is used in production, net of its effect on the environmental externality and of its effect on the replacement cost.

3.5 Implementation of First-Best Allocation

To implement the first-best allocation in a decentralized equilibrium, three instruments are necessary. First, a Pigouvian tax on final-output production at rate $\tau^{\Omega}_{a,t}$ induces final-good firms to internalize the environmental externality in their capital-replacement decisions. Second, a production subsidy at rate s^{f}_{t} for capital-goods producers with a successful innovation allows to offset their markup distortion on the replacement cost. Third, an innovation subsidy at rate s^{ψ}_{t} induces capital-goods producers to internalize the social benefits of innovation, which do not coincide with their profits. These taxes and subsidies need are then financed with lump-sum taxes on households.

Consider the correction of the environmental externality. This can be achieved by taxing final-good production at a proportional rate as follows:

$$\tau^{\Omega}_{a,t} = \omega_{a,t}(q_{t-a}). \tag{20}$$

This Pigouvian tax, which critically depends on the capital vintage, aligns the private return from capital with its social return.

Next, consider the correction of the distortion induced by markups on successful innovation on the replacement cost f_t. This can be achieved by subsidizing the production of successful innovators at a proportional rate as follows:

$$s^{f}_{t} = \gamma. \tag{21}$$

Given this subsidy, capital-goods producers with a successful innovation perceive a marginal cost equal to $c(1-\gamma)$. Thus, their optimal price equals $\frac{c(1-\gamma)}{1-\gamma} = c$. As a result, they produce the same first-best quantity as competitive producers without a successful innovation.

Finally, consider the correction of innovation incentives. This can be achieved by sub-

sidizing innovation costs at a proportional rate as follows:

$$s_t^{\psi} = 1 - \frac{\Pi_t^{FB}}{(\eta - 1)\zeta_t q_{t-1}}, \tag{22}$$

where Π_t^{FB} denote profits upon a successful innovation in the presence of the production subsidy given by equation (21). To see this, notice that in a decentralized equilibrium with this production subsidy and with a generic innovation subsidy, optimal innovation solves

$$\alpha_{\psi}(\psi_t)\frac{\Pi_t^{FB}}{\eta q_{t-1}} = 1 - s_t^{\psi}. \tag{23}$$

In turn, the first-best optimality condition for innovation (18) implies

$$\left(\frac{\eta - 1}{\eta}\right)\alpha_{\psi}(\psi_t)\zeta_t = 1. \tag{24}$$

Thus, the subsidy given by equation (22) aligns private incentives for innovation with social incentives.

In practice, capital-goods producers are geographically concentrated in a small number of countries. Thus, for most governments, the possibility of interventions on the supply side is limited, whereas subsidy on the (domestic) demand side are a common solution. The complexity associated with the implementation of the first-best allocation, which requires three different distortions, and its requirement to distort the supply side of capital goods motivate us to analyze a constrained planning problem, in which only demand-side interventions on capital-replacement decisions are available.

4 Constrained Efficiency and Optimal Subsidies

We now analyze the constrained-efficient allocation that arises when a planner can only distort capital replacement, subject to the equilibrium dynamics of innovation and markups. We then provide an implementation of this allocation with subsidies for capital replacement.

4.1 Constrained-Efficient Capital Replacement

Whereas the first-best allocation represents a useful theoretical benchmark, we now focus on a second-best scenario, which we see as more relevant for real-world policy. Specifically, we assume that the planner can only distort the decision to replace assets, but cannot

directly intervene on innovation decisions or affect markups. Our goal is to explore the best way to use this limited set of policy tools.

The justification for analyzing this restricted policy problem is that governments can typically set taxes and subsidies on purchases of durable assets in the domestic economy. However, because capital goods are produced by a small number of countries and traded internationally, capital producers are often located in foreign economies. This fact implies that domestic policymakers can easily distort the demand for capital, but not the path of embodied innovation or the price of new capital goods.

Nevertheless, policy interventions on the demand side affects the equilibrium path of innovation and the average markup through a market-size effect. A subsidy on capital replacement stimulates demand for new capital, and thus creates incentives for faster growth in the quality of new capital on the supply side. At the same time, faster embodied innovation comes with a higher average markup on new capital, inducing a policy trade-off in the domestic economy.

Formally, the constrained-efficient allocation maximizes social welfare (A16) subject to the resource constraint of the economy, the law of motion for the distribution of capital quality (8), the equilibrium evolution of innovation and capital quality that solves the innovators' problem of Section 3.3, as well as the equilibrium replacement cost (13). The assumption that the planner is subject to the equilibrium dynamics of the replacement cost implies that faster innovation and improvements in the quality of capital must come at the cost of higher markups in this second-best setup. We assume that the planner does not consider capital-goods producers' profits as part of aggregate consumption—that is, profits are distributed to other (foreign) agents that have no weight in the planner's objective function.

We can characterize the constrained-efficient replacement decision with the following recursive formulation of the Lagrange multiplier on the evolution of the distribution of capital age

$$\begin{aligned}\theta_{\varepsilon,a,t} &= \beta\left((1-\omega_{a,t+1}(q_{t+1-a}))z\varepsilon k_{a,t} - f_{t+1}\left(G(\overline{\phi}_{\varepsilon,a,t+1}) + \Phi_{\varepsilon,a,t+1}\right)\right) \\ &\quad +\beta\left(G(\overline{\phi}_{\varepsilon,a,t+1})\left(\theta_{\varepsilon,1,t+1} + \Delta^M_{t+1}\right) + (1-G(\overline{\phi}_{\varepsilon,a,t+1}))\theta_{\varepsilon,a+1,t+1}\right),\end{aligned} \tag{25}$$

and the optimal replacement threshold

$$f_t(1+\overline{\phi}_{\varepsilon,a,t}) = \theta_{\varepsilon,1,t} - \theta_{\varepsilon,a+1,t} + \Delta^M_t, \tag{26}$$

where Δ_t^M denotes the innovation externality induced by capital replacement. This externality is defined as follows:

$$\Delta_t^M \equiv \left(\zeta_t H_\psi(q_{t-1}, \psi_t) - \nu_t F_\psi(q_{t-1}, \psi_t)\right) \Psi_M(M_t), \tag{27}$$

where ζ_t and ν_t represent the Lagrange multipliers on the law of motion for capital quality (12) and the equilibrium price of new capital (13), respectively. Each additional sale of new capital increases M_t and thus profits, stimulating innovation effort and improving aggregate capital quality q_t, while at the same time increasing average markup and replacement cost F_t. The innovation externality is the net effect of higher capital quality and higher replacement cost.

The comparison of equation (A27) with the private valuation (4) reveals two differences. First, the planner internalizes the social value of capital used in production, encoded in the term $\omega_{a,t+1}(q_{t+1-a})$. Second, the planner internalizes the innovation externality of capital replacement Δ_{t+1}^M.

Finally, the shadow value of capital quality for the constrained planner satisfies a version of equation (A25), where the replacement cost that is relevant for the consumption effect of capital quality now includes markups, and the multiplier on equation (13) is given by $\nu_t = \sum_{\varepsilon\in\mathcal{E}} \sum_{a\in\mathcal{A}} m_{\varepsilon,a,t} \left(G(\overline{\phi}_{\varepsilon,a,t}) + \Phi_{\varepsilon,a,t}\right)$.

4.2 Implementation with Subsidies for Capital Replacement

We consider the following implementation of the constrained-efficient allocation. Upon replacement of an asset of age a with a new asset, firms receive a transfer $S_{\varepsilon,a,t}$ from the government, which is financed with lump-sum taxes on households. We now characterize the optimal subsidy scheme $S_{\varepsilon,a,t}$, that is, the subsidy scheme that supports the constrained-efficient allocation.

The competitive-equilibrium value function in the presence of replacement subsidies, which we denote $V^S_{\varepsilon,a,t}$, is defined recursively as follows:

$$\begin{aligned} V^S_{\varepsilon,a,t} \;=\; \beta \Bigg(z\varepsilon k_{a,t} + \max_{\overline{\phi}\in[\phi_{min},\phi_{max}]} \Bigg\{ -f_{t+1} \int_{\phi_{min}}^{\overline{\phi}} (1+\phi) dG(\phi) \\ + G(\overline{\phi}) \left(V^S_{\varepsilon,1,t+1} + S_{\varepsilon,a,t+1}\right) + (1 - G(\overline{\phi})) V^S_{\varepsilon,a+1,t+1} \Bigg\} \Bigg). \end{aligned} \tag{28}$$

Hence, the first-order condition for optimal replacement in the presence of subsidies is

$$f_t(1+\overline{\phi}_{\varepsilon,a,t}) = V^S_{\varepsilon,1,t} - V^S_{\varepsilon,a+1,t} + S_{\varepsilon,a,t}. \tag{29}$$

Upon replacement, the firm pays the replacement cost and obtains the difference in value between a new and old asset, plus the replacement subsidy. To implement the same replacement thresholds of the constrained planner given by equation (A22), we need

$$V^S_{\varepsilon,1,t+1} - V^S_{\varepsilon,a+1,t+1} + S_{\varepsilon,a,t+1} = \theta_{\varepsilon,1,t+1} - \theta_{\varepsilon,a+1,t+1} + \Delta^M_{t+1}.$$

Hence, we can formulate the optimal subsidy as follows:

$$\begin{aligned} S_{\varepsilon,a,t+1} &= \theta_{\varepsilon,1,t+1} - \theta_{\varepsilon,a+1,t+1} - \left(V^S_{\varepsilon,1,t+1} - V^S_{\varepsilon,a+1,t+1}\right) + \Delta^M_{t+1} \\ &= \theta_{\varepsilon,1,t+1} - V^S_{\varepsilon,1,t+1} - \left(\theta_{\varepsilon,a+1,t+1} - V^S_{\varepsilon,a+1,t+1}\right) + \Delta^M_{t+1}. \end{aligned} \tag{30}$$

The subsidy must reflect the difference in asset valuations between planner and private agents, as well as the social value of new-asset sales encoded in the innovation externality Δ^M_t.

Given this expression for the optimal subsidy and given a solution to the constrained-efficient allocation, we then obtain the private-sector value functions. Plugging in the subsidy (A28) in the value function (A18), we get

$$V^S_{\varepsilon,a,t} = \beta\left(z\varepsilon k_{a,t} - f_{t+1}\left(G\left(\overline{\phi}_{\varepsilon,a,t+1}\right) + \Phi_{\varepsilon,a,t+1}\right) + G\left(\overline{\phi}_{\varepsilon,a,t+1}\right)\left(\theta_{\varepsilon,1,t} - \theta_{\varepsilon,a+1,t+1} + \Delta^M_{t+1}\right) + V^S_{\varepsilon,a+1,t+1}\right).$$

We then add the environmental externality to both sides of the value function to write

$$\begin{aligned} V^S_{\varepsilon,a,t} - \beta\omega_{a,t+1}(q_{t+1-a})z\varepsilon k_{a,t} &= \beta\left(z\varepsilon k_{a,t}(1-\omega_{a,t+1}(q_{t+1-a})) - f_{t+1}\left(G\left(\overline{\phi}_{\varepsilon,a,t+1}\right) + \Phi_{\varepsilon,a,t+1}\right)\right. \\ &\quad \left. + G\left(\overline{\phi}_{\varepsilon,a,t+1}\right)\left(\theta_{\varepsilon,1,t} - \theta_{\varepsilon,a+1,t+1} + \Delta^M_{t+1}\right) + V^S_{\varepsilon,a+1,t+1}\right). \end{aligned} \tag{31}$$

Using equation (A42) and the planner's value (A27), we obtain the following recursion for the difference in value functions between planner and private agents,

$$\theta_{\varepsilon,a,t} - V^S_{\varepsilon,a,t} = -\beta\omega_{a,t+1}(q_{t+1-a})z\varepsilon k_{a,t} + \beta\left(\theta_{\varepsilon,a+1,t+1} - V^S_{\varepsilon,a+1,t+1}\right),$$

which we can solve to get the difference in value functions of equation (A28):

$$\theta_{\varepsilon,1,t}-V^S_{\varepsilon,1,t}-\left(\theta_{\varepsilon,a+1,t}-V^S_{\varepsilon,a+1,t}\right)=\sum_{\tau=0}^{\infty}\beta^{\tau+1}z\varepsilon\left(\omega_{a+1+\tau,t+1+\tau}(q_{t-a})k_{a+1+\tau,t+\tau}-\omega_{1+\tau,t+1+\tau}(q_t)k_{1+\tau,t+\tau}\right) \tag{32}$$

Thus, we obtain the following formal result.

Proposition 1: Optimal Subsidies

The capital-replacement subsidies that implement the constrained-efficient allocation are

$$S_{\varepsilon,a,t}=\Delta^{\Omega}_{\varepsilon,a,t}+\Delta^{M}_{t},$$

for all $t=0,1,..,\ \varepsilon\in\mathcal{E}$, and $a\in\mathcal{A}$, where

$$\begin{aligned}\Delta^{\Omega}_{\varepsilon,a,t} &= \sum_{\tau=0}^{\infty}\beta^{\tau+1}z\varepsilon\left(\omega_{a+1+\tau,t+1+\tau}(q_{t-a})k_{a+1+\tau,t+\tau}-\omega_{1+\tau,t+1+\tau}(q_t)k_{1+\tau,t+\tau}\right),\\ \Delta^{M}_{t} &= \left(\zeta_t H_{\psi}(q_{t-1},\psi_t)-\nu_t F_{\psi}(q_{t-1},\psi_t)\right)\Psi_M(M_t).\end{aligned}$$

Proposition 1 is our main formal result and expresses the optimal subsidy as the sum of two terms, which we now interpret. The first term, $\Delta^{\Omega}_{\varepsilon,a,t}$, is the difference in the present discounted value of environmental damages associated with continuing with an old asset of quality q_{t-a} instead of replacing it with the new vintage q_t, taking as given the path of capital quality.[5] As long as new capital is cleaner than old capital and capital quality increases over time, the term $\Delta^{\Omega}_{\varepsilon,a,t}$ is positive. Furthermore, it is increasing in capital age, which determines the slope of the optimal subsidy with respect to capital age. It is also increasing in productivity ε because higher output is associated with larger environmental externalities.

Notice that this present discounted value involves environmental externalities into the infinite future, although capital is replaced after a finite number of periods. Thus, the negative term inside the bracket defining Δ^{Ω}_{t} might suggest that when a firm invests in a new asset, it effectively pays for externalities that the firm will never induce, after scrapping the asset in the future. However, this is not the case, because when the firm then scraps the asset, the positive term inside the bracket rebates the tail of externalities that are not

[5] This term in our optimal subsidy formula is related to the empirical calculations of Li, Linn, and Spiller (2013) and Brueckner, Kahn, and Nickelsburg (2024) on the desirability of a scrappage subsidy for vehicles and aircraft respectively.

generated. Thus, overall the subsidy scheme ensures that firms internalize the environmental externality as if they were paying a Pigouvian tax in every period that they deploy the new capital.

The second term in the optimal subsidy, Δ_t^M, is the social value generated by an additional sale of new capital due to its market-size effect on innovation, net of its effect on markup distortions. This term highlights that the planner internalizes that the evolution of capital quality is endogenous with respect to the volume of replacement. Embodied innovation contributes both to privately valued capital quality (productivity) and to the reduction of environmental externalities. Overall, the term Δ_t^M may be positive or negative depending on whether the social value of innovation dominates the efficiency loss due to markups.

Moreover, this term is independent of capital age or productivity, because one additional sale of new capital has the same effect on innovation and markups, regardless of which firms is purchasing the additional unit. Thus, this terms represents the intercept of the optimal subsidy with respect to capital age.

Critically, the two components of the optimal subsidy interact over time along the equilibrium path of the economy. The first term focuses on substitution across vintages taking the equilibrium path of quality as given. However, the equilibrium path of quality is endogenously determined through innovation, as highlighted by the second term. Hence, endogenous changes over time in the intensity of innovation affect the relative importance of substitution across vintages, as we illustrate in our quantitative results.

To highlight the generality of our main insight, we also provide a version of Proposition 1 for two alternative modifications of our assumptions. In Appendix A.5 we consider a concave utility function from consumption. In this case, the discount factor becomes endogenous and we provide a generalization of our formula for optimal subsidies. In Appendix A.6 we obtain our formal result on optimal subsidies under a different set of assumptions on physical depreciation. Instead of geometric depreciation as in equation (2), there we assume that the asset does not lose efficiency over time until a stochastic shock destroys the asset entirely; this depreciation assumption is referred to as "one-hoss-shay depreciation."

5 Subsidies for Adoption of a New Technology

We now generalize our model to analyze the transition to a new technology embodied in capital. This extension is particularly relevant to the case of electric vehicles. We first describe the allocations in a decentralized equilibrium and then characterize the constrained-efficient allocation and the subsidies that implement it.

5.1 Heterogeneous Technologies in Decentralized Equilibrium

We assume that the initial condition of the economy is one with a single technology, which we refer to as "old" (superscript O) embodied in assets of heterogeneous vintage, as in the model of the previous sections.

A "new" technology (superscript N) becomes available and features a different level of quality and production costs. At each date, an exogenous fraction $\rho \in [0,1]$ of agents owning an asset with old technology have the opportunity the switch technology by purchasing a new asset emboding the new technology. With this assumption, the model nests the single-technology model of the previous section for $\rho = 0$.

The value function of an old-technology asset is

$$\begin{aligned} V^O_{\varepsilon,a,t} &= \beta\Big(z\varepsilon k^O_{a,t} \\ &\quad +(1-\rho) \max_{\overline{\phi}^O \in [\phi_{min},\phi_{max}]} \left\{ -f^O_{t+1} \int_{\phi_{min}}^{\overline{\phi}^O} (1+\phi) dG(\phi) + G(\overline{\phi}^O) V^O_{\varepsilon,1,t+1} + (1-G(\overline{\phi}^O)) V^O_{\varepsilon,a+1,t+1} \right\} \\ &\quad +\rho \max_{\overline{\phi}^{ON} \in [\phi_{min},\phi_{max}]} \left\{ -f^N_{t+1} \int_{\phi_{min}}^{\overline{\phi}^{ON}} (1+\phi) dG(\phi) + G(\overline{\phi}^{ON}) V^N_{\varepsilon,1,t+1} + (1-G(\overline{\phi}^{ON})) V^O_{\varepsilon,a+1,t+1} \right\} \Big), \end{aligned}$$

where we use the superscript ON to denote the variables associated with a switch from old to new technology. The value function after adoption of the new technology is

$$\begin{aligned} V^N_{\varepsilon,a,t} &= \beta\Big(z\varepsilon k^N_{a,t} \\ &\quad + \max_{\overline{\phi}^N \in [\phi_{min},\phi_{max}]} \left\{ -f^N_{t+1} \int_{\phi_{min}}^{\overline{\phi}^N} (1+\phi) dG(\phi) + G(\overline{\phi}^N) V^N_{\varepsilon,1,t+1} + (1-G(\overline{\phi}^N)) V^N_{\varepsilon,a+1,t+1} \right\} \Big). \end{aligned}$$

In the interest of tractability, we assume that adoption of the new technology is irreversible, that is, there is no opportunity to switch back to the old technology.

The first-order conditions with respect to the replacement-cost thresholds are as follows:

$$\begin{aligned} f_t^O(1+\overline{\phi}_{\varepsilon,a,t}^O) &= V_{\varepsilon,1,t}^O - V_{\varepsilon,a+1,t}^O \\ f_t^N(1+\overline{\phi}_{\varepsilon,a,t}^{ON}) &= V_{\varepsilon,1,t}^N - V_{\varepsilon,a+1,t}^O \\ f_t^N(1+\overline{\phi}_{\varepsilon,a,t}^N) &= V_{\varepsilon,1,t}^N - V_{\varepsilon,a+1,t}^N, \end{aligned}$$

where the first equation refers to replacement within the old technology, the second equation refers to a switch from old to new technology, and the third equation refers to a replacement within the new technology. Accordingly, the law of motion of the distribution of capital of old and new technology is

$$\begin{aligned} m_{\varepsilon,1,t+1}^O &= (1-\rho)\sum_{a\in A} m_{\varepsilon,a,t}^O G(\overline{\phi}_{\varepsilon,a,t}^O) \\ m_{\varepsilon,a+1,t+1}^O &= (1-\rho)m_{\varepsilon,a,t}^O(1-G(\overline{\phi}_{\varepsilon,a,t}^O)) + \rho m_{\varepsilon,a,t}^O(1-G(\overline{\phi}_{\varepsilon,a,t}^{ON})) \\ m_{\varepsilon,1,t+1}^N &= \rho\sum_{a\in A} m_{\varepsilon,a,t}^O G(\overline{\phi}_{\varepsilon,a,t}^{ON}) + \sum_{a\in A} m_{\varepsilon,a,t}^N G(\overline{\phi}_{\varepsilon,a,t}^N) \\ m_{\varepsilon,a+1,t+1}^N &= m_{\varepsilon,a,t}^N(1-G(\overline{\phi}_{\varepsilon,a,t}^N)). \end{aligned}$$

5.2 Constrained Efficiency and Optimal Subsidies

We now analyze constrained efficiency in this version of the model. We focus on the main result in the main text and report all other equations in Appendix A.7.

As in Section 4, we assume that a constrained planner chooses capital replacement subject to the resource constraints of the economy and the endogenous evolution of innovation. The Lagrange multipliers on the distribution of old-technology capital satisfy the following recursive formulation:

$$
\begin{aligned}
\theta^O_{\varepsilon,a,t} \;=\; & \beta\Big(z\varepsilon k^O_{a,t}\left(1-\omega^O_{a,t+1}\left(q^O_{t-a+1}\right)\right) \\
& +(1-\rho)\Big(-f^O_{t+1}\left(G\left(\overline{\phi}^O_{\varepsilon,a,t+1}\right)+\Phi^O_{\varepsilon,a,t+1}\right)+G\left(\overline{\phi}^O_{\varepsilon,a,t+1}\right)\left(\theta^O_{\varepsilon,1,t+1}+\Delta^{M,O}_{t+1}\right)\Big) \\
& +(1-\rho)\left(1-G\left(\overline{\phi}^O_{\varepsilon,a,t+1}\right)\right)\theta^O_{\varepsilon,a+1,t+1} \\
& +\rho\left(-f^N_{t+1}\left(G\left(\overline{\phi}^{ON}_{\varepsilon,a,t+1}\right)+\Phi^{ON}_{\varepsilon,a,t+1}\right)+G\left(\overline{\phi}^{ON}_{\varepsilon,a,t+1}\right)\left(\theta^N_{\varepsilon,1,t+1}+\Delta^{M,N}_{t+1}\right)\right) \\
& +\rho\left(1-G\left(\overline{\phi}^{ON}_{\varepsilon,a,t+1}\right)\right)\theta^O_{\varepsilon,a+1,t+1}\Big)
\end{aligned}
$$

The Lagrange multipliers on the distribution of new-technology capital satisfy the following recursive formulation:

$$
\begin{aligned}
\theta^N_{\varepsilon,a,t} \;=\; & \beta\Big(z\varepsilon k^N_{a,t}\left(1-\omega^N_{a,t+1}\left(q^N_{t-a+1}\right)\right)-f^N_{t+1}\left(G\left(\overline{\phi}^N_{\varepsilon,a,t+1}\right)+\Phi^N_{\varepsilon,a,t+1}\right) \\
& +G\left(\overline{\phi}^N_{\varepsilon,a,t+1}\right)\left(\theta^N_{\varepsilon,1,t+1}+\Delta^{M,N}_{t+1}\right)+\left(1-G\left(\overline{\phi}^N_{\varepsilon,a,t+1}\right)\right)\theta^N_{\varepsilon,a+1,t+1}\Big).
\end{aligned}
$$

These multipliers differ from the firm value functions in competitive equilibrium because of the presence of the environmental externality in production as well as the innovation externalities associated with capital replacement.

The replacement thresholds satisfy the following first-order conditions:

$$
\begin{aligned}
f^O_t(1+\overline{\phi}^O_{\varepsilon,a,t}) &= \theta^O_{\varepsilon,1,t}-\theta^O_{\varepsilon,a+1,t}+\Delta^{M,O}_t \\
f^N_t(1+\overline{\phi}^{ON}_{\varepsilon,a,t}) &= \theta^N_{\varepsilon,1,t}-\theta^O_{\varepsilon,a+1,t}+\Delta^{M,N}_t \\
f^N_t(1+\overline{\phi}^N_{\varepsilon,a,t}) &= \theta^N_{\varepsilon,1,t}-\theta^N_{\varepsilon,a+1,t}+\Delta^{M,N}_t
\end{aligned}
$$

We formalize an implementation of the constrained-efficient allocation with subsidies as in Section 4.2. Upon replacement of an old-technology asset of age a with an old-technology asset of the new vintage, firms receive a transfer $S^O_{\varepsilon,a,t}$; upon replacement of an old-technology asset of age a with a new-technology asset of the new vintage, firms receive a transfer $S^{ON}_{\varepsilon,a,t}$; upon replacement of a new-technology asset of age a with a new-technology asset of the new vintage, firms receive a transfer $S^N_{\varepsilon,a,t}$.

Following the same steps as in Section 4.2 to match the optimality conditions across

decentralized equilibrium with subsidies and constrained-efficient allocation, we obtain the following result, which generalizes Proposition 1 to the case of heterogeneous technologies:

Proposition 2: Optimal Subsidies with Heterogeneous Technologies

The capital-replacement subsidies that implement the constrained-efficient allocation with heterogeneous technologies are

$$\begin{aligned} S^{O}_{\varepsilon,a,t} &= \Delta^{\Omega,O}_{\varepsilon,a,t} + \Delta^{M,O}_{t} \\ S^{ON}_{\varepsilon,a,t} &= \Delta^{\Omega,ON}_{\varepsilon,a,t} + \Delta^{M,N}_{t} \\ S^{N}_{\varepsilon,a,t} &= \Delta^{\Omega,N}_{\varepsilon,a,t} + \Delta^{M,N}_{t}, \end{aligned}$$

for all $t = 0, 1, .., \varepsilon \in \mathcal{E}$, and $a \in \mathcal{A}$, where

$$\begin{aligned} \Delta^{\Omega,O}_{\varepsilon,a,t} &\equiv \sum_{\tau=0}^{\infty} \beta^{\tau+1} z\varepsilon \left(\omega^{O}_{a+1+\tau,t+1+\tau}(q^{O}_{t-a}) k^{O}_{a+1+\tau,t+\tau} - \omega^{O}_{1+\tau,t+1+\tau}(q^{O}_{t}) k^{O}_{1+\tau,t+\tau} \right) \\ \Delta^{\Omega,ON}_{\varepsilon,a,t} &\equiv \sum_{\tau=0}^{\infty} \beta^{\tau+1} z\varepsilon \left(\omega^{O}_{a+1+\tau,t+1+\tau}(q^{O}_{t-a}) k^{O}_{a+1+\tau,t+\tau} - \omega^{N}_{1+\tau,t+1+\tau}(q^{N}_{t}) k^{N}_{1+\tau,t+\tau} \right) \\ \Delta^{\Omega,N}_{\varepsilon,a,t} &\equiv \sum_{\tau=0}^{\infty} \beta^{\tau+1} z\varepsilon \left(\omega^{N}_{a+1+\tau,t+1+\tau}(q^{N}_{t-a}) k^{N}_{a+1+\tau,t+\tau} - \omega^{N}_{1+\tau,t+1+\tau}(q^{N}_{t}) k^{N}_{1+\tau,t+\tau} \right) \end{aligned}$$

and

$$\begin{aligned} \Delta^{M,O}_{t} &\equiv \left(\zeta^{O}_{t} H^{O}_{\psi}(q^{O}_{t-1}, \psi^{O}_{t}) - \nu^{O}_{t} F^{O}_{\psi}(q^{O}_{t-1}, \psi^{O}_{t}) \right) \Psi^{O}_{M}(M^{O}_{t}) \\ \Delta^{M,N}_{t} &\equiv \left(\zeta^{N}_{t} H^{N}_{\psi}(q^{N}_{t-1}, \psi^{N}_{t}) - \nu^{N}_{t} F^{N}_{\psi}(q^{N}_{t-1}, \psi^{N}_{t}) \right) \Psi^{N}_{M}(M^{N}_{t}). \end{aligned}$$

These optimal subsidies depend on the technology of the asset being scrapped as well as the technology of the new asset that is purchased. However, as in the case of a single technology, the capital replacement subsidy can be expressed as the sum of a two terms, namely the difference in present discounted value of environmental externalities and the effect of an additional new-capital sale on aggregate welfare through its effect on capital quality net of its effect on the replacement cost. Thus, the main insight of Section 4 generalizes to the case of heterogeneous technologies.

Table 1: Parameters Values

	Parameter	Symbol	Aircraft	Vehicles	Macro
Replacement	Discount Factor	β	0.96	0.96	0.96
	Physical Depreciation	δ	0.06	0.114	0.08
	Min. Replacement Cost	ϕ_{min}	-0.4	-0.55	-0.39
	Max. Replacement Cost	ϕ_{max}	0.4	0.55	0.39
	Investment Scale	κ	33.35	6.25	0.93
	Maximum Age	a_{max}	69	21	
Innovation	Max. Innovation Prob.	α_0	0.197	0.24	0.22
	Curvature of Innov. Prob.	α_1	7.45	11.7	284
	Innovation Step Size	η	1.046	1.045	1.08
	Quality Weight	γ	0.365	0.532	0.5
	Marginal Cost	c	1	1	1
Externality	Scale	ω_0	0.028	0.099	0.019
	Curvature	ω_1	6.07	4.93	3.92

Notes: The table reports the parameter values used in the analysis.

6 Calibration

We perform three separate calibrations of our model. We first focus on two important types of equipment: vehicles and aircraft. For both types of asset, our quantitative results contribute to an active policy debate on incentives for innovation and decarbonization. We then perform a macro calibration that targets a generic—i.e., average—type of capital, following the approach of Bertolotti and Lanteri (2024).

In this section we describe our choices of functional forms and parameter values, which we report in Table 1. At this stage, we abstract from heterogeneity in productivity ε; hence, firms are heterogeneous in terms of capital age and the realization of the replacement cost ϕ. We first consider a single technology for each asset, and extend the vehicles calibration to the model with two technologies in Section 7.2. We will analyze the quantitative roles of heterogeneity in ε in future versions of the paper.

6.1 Functional Forms

We specify the following functional forms, which we maintain across all calibrations. We assume that the innovation probability function takes the form

$$\alpha(\psi) \equiv \alpha_0 \left(1 - e^{-\alpha_1 \psi}\right), \tag{33}$$

with $\alpha_0 \in [0, 1]$ denoting the upper bound on the innovation probability and $\alpha_1 > 0$ governing the curvature of the function.

We then assume that the idiosyncratic replacement cost is independent and identically distributed according to a uniform distribution on the support $[\phi_{min}, \phi_{max}]$. We set the bounds of this support to be symmetric around zero.

We further assume that the environmental externality takes the functional form

$$\omega_{a,t}(q_{t-a}) = \omega_0 \left(q_{t-a}\right)^{-\omega_1} (1 - \delta)^{-a+1}, \tag{34}$$

with $\omega_0 > 0$ representing the scale of the externality (expressed in percent of vintage-specific output value gross of asset depreciation) and $\omega_1 > 0$ governing the curvature of the social preferences for capital of different ages. To calibrate the parameters that govern the environmental externality, we consider the value of this functional form across two subsequent periods along the competitive balanced growth path.

In choosing this functional form, we assume that physical depreciation reduces output as capital ages, but does not reduce the environmental damage associated with the use of the asset.[6] The term $(1 - \delta)^{-a+1}$ reflects this assumption and implies that the total environmental cost of an asset with age a operated by a firm with productivity ε equals $\omega_0 z \varepsilon q_{t-a}^{\omega_1 - 1}$.

6.2 Aircraft Calibration

We begin by discussing our choice of parameter values for the aircraft calibration. We calibrate the replacement problem to match several moments of the empirical distribution of aircraft age. We obtain detailed data on retirement and scrappage events of individual commercial aircraft from online public sources (https://planelist.net/). We compute vintage-specific capital-replacement rates and key moments of the capital-age distribution,

[6] Physical depreciation can be interpreted as a maintenance cost in units of output that increases as the asset ages.

which inform the distribution of replacement costs and the scale of new-capital investment. Specifically, we jointly set the symmetric bounds of the uniform replacement cost distribution and investment scale to match two empirical moments: an aggregate replacement rate equal to 4.3% and an average age of aircraft equal to 12.8 years. We fix maximum aircraft age–after which we impose replacement–to the upper bound of the empirical distribution (69 years). We calibrate the rate of physical depreciation as the difference between BEA estimates of aircraft economic depreciation and estimates of aircraft quality growth discussed below.

We exploit the estimates of technological progress in new aircraft efficiency in Bertolotti and Lanteri (2024) to calibrate the innovation side of the model. We set the relative fuel efficiency of new and continuing aircraft models to match a quality difference equal to 4.6%. We then calibrate the remaining three parameters of the innovation problem, namely α_0, α_1, and γ to jointly match three empirical targets. Specifically, we target: a quality growth rate equal to 0.6%; an elasticity of R&D with respect to its user cost equal to 1, which is in the range of estimates in the innovation economics literature (Becker, 2015; Agrawal, Rosell, and Simcoe, 2020; Dechezleprêtre, Einiö, Martin, Nguyen, and Van Reenen, 2023); an average markup of 7.5% consistent with empirical markup estimates in the commercial aircraft manufacturing industry in Irwin and Pavcnik (2004).

We set ω_1 to match a yearly decline in emission intensity—defined as the volume of CO_2 emissions per dollar of output—in the U.S. air transportation industry between 1995 and 2018 equal to -3.6%. To this end, we leverage data on emission intensity by the International Monetary Fund (IMF) based on OECD emissions data (Yamano, Lioussis, and Cimper, 2024). We then set the scale of the externality to match the ratio of the social cost of total CO_2 emissions to GDP in the same industry. We experiment with a range of alternative social cost of carbon estimates between \$51 and \$337 per metric tonne of CO_2, with a central value of \$193.

6.3 Vehicles Calibration

Next, we discuss our vehicles calibration. We obtain data on scrappage rates for vehicles of different age from Jacobsen and Van Benthem (2015). We fix maximum vehicle age–after which we impose replacement–to the upper bound of the empirical distribution (21 years). We then use scrappage rates to compute the aggregate replacement rate (6.5%) and the stationary distribution of vehicles over age. The average vehicle is approximately 9 years

old. We set the symmetric bounds of the uniform distribution of replacement costs and the scale of new-capital investment to match these targets. We calibrate the rate of physical depreciation as the difference between estimates of the rate of economic depreciation from the Bureau of Labor Statistics and estimates of the quality growth rate for cars discussed below.

We exploit the estimates of technological progress in new-vehicle quality of Bertolotti, Gavazza, and Lanteri (2023) to calibrate the innovation side of the model. We set the relative fuel efficiency of new and continuing car models to match a quality difference approximately equal to 4.5%. We then calibrate the remaining three parameters of the innovation problem, namely α_0, α_1, and γ to jointly match three empirical targets: a yearly quality growth rate equal to 0.7%; an elasticity of R&D with respect to its user cost equal to 1, which is in the range of estimates in the innovation economics literature (Becker, 2015; Agrawal, Rosell, and Simcoe, 2020; Dechezleprêtre, Einiö, Martin, Nguyen, and Van Reenen, 2023); an average markup of 18% consistent with empirical markup estimates in the car manufacturing industry in Grieco, Murry, and Yurukoglu (2023).

Because time-series measures of emission intensity are not readily available, we set ω_1 to match a decline in *aggregate* emissions equal to -1.7% per year, as observed in EPA data on U.S. passenger vehicles emissions between 1990 and 2022. In so doing, we assume that vehicles' output and aggregate emissions not only increase with quality growth but also with population growth, which we set to one percent. We then set the scale of the externality to match the ratio of the social cost of CO_2 emissions by an average vehicle over its service life to the private value of the asset, which we proxy by the average vehicle price in the secondary market. We obtain CO_2 statistics for the average emissions of a car from the EPA and we employ a social cost of carbon equal to \$193 per metric tonne, but experiment with a range of alternative estimates.

6.4 Macro Calibration: Aggregate Capital

We now describe our macro calibration. We calibrate the capital-replacement problem using a standard strategy and parameter values—e.g., for discount factor and depreciation rate—commonly used in the quantitative literature on investment dynamics with heterogeneous firms. We set the symmetric bounds of idiosyncratic replacement cost distribution such that the constant fraction of final-good producers with investment rate larger than 20% in the competitive-equilibrium balanced-growth path equals 0.2, consistent with the micro

empirical evidence of Cooper and Haltiwanger (2006) on the frequency of large investment episodes (spikes). We let the maximum capital age be endogenously determined by the model.

We set the scale of new capital purchases κ to obtain that the ratio of aggregate investment expenditures to aggregate output is approximately equal to 20% on the competitive balanced-growth path, consistent with the average ratio of gross private investment to GDP for the U.S.[7] We set the marginal cost of producing new capital $c = 1$ consistent with the standard assumption that one unit of output is needed to produce one unit of new investment goods.

We then set the four parameters of the innovation problem, namely α_0, α_1, η, and γ to jointly match four empirical targets. Specifically, we target: a growth rate equal to 1.2%, consistent with the contribution of investment-specific technical change to aggregate productivity growth (Greenwood, Hercowitz, and Krusell, 1997); an elasticity of R&D with respect to its user cost equal to 1, which is in the range of estimates in the innovation economics literature (Becker, 2015; Agrawal, Rosell, and Simcoe, 2020; Dechezleprêtre, Einiö, Martin, Nguyen, and Van Reenen, 2023); an average share of new products approximately equal to 15%, consistent with evidence from Bertolotti and Lanteri (2024); an aggregate markup rate equal to 15%, consistent with Edmond, Midrigan, and Xu (2015, 2023).

We set ω_1 to match a yearly growth rate of emission intensity equal to -4.6%, consistent with the observed decline in the ratio of the volume of CO_2 emissions (Environmental Protection Agency, EPA) to GDP in the U.S. between 1997 and 2021.[8] We then set the scale of the externality to match the ratio of the social cost of total CO_2 emissions to GDP. We consider a baseline value of the social cost of carbon equal to \$193 per metric tonne of CO_2, consistent with Hahn, Hendren, Metcalfe, and Sprung-Keyser (2024) and with values recently proposed by EPA. We experiment with a range of alternative estimates between \$51, in line with values used by the US government, and \$337, the upper bound considered by Hahn, Hendren, Metcalfe, and Sprung-Keyser (2024).

[7] We consider aggregate investment expenditures including wedges, i.e., $M_t f_t + \left(\sum_{\varepsilon\in\mathcal{E}}\sum_{a\in\mathcal{A}} m_{\varepsilon,a,t} G(\bar{\phi}_{\varepsilon,a,t})\Phi_{\varepsilon,a,t}\right) f_t$.

[8] We compute emission intensity for the U.S. as the ratio of CO_2 emissions published by the Environmental Protection Agency to U.S. GDP estimated by the Bureau of Economic Analysis.

7 Quantitative Results

We now present our quantitative results. For each calibration, we perform the following analysis. We initialize the economy in the competitive-equilibrium BGP and assume that from $t = 0$ onward the optimal policy to sustain the constrained-efficient allocation is implemented. We then solve for the entire transition path until the economy converges to the constrained-efficient BGP. To benchmark our results, we compare this baseline scenario with a counterfactual economy that remains in the decentralized equilibrium as well as an economy where the first-best allocation is implemented.

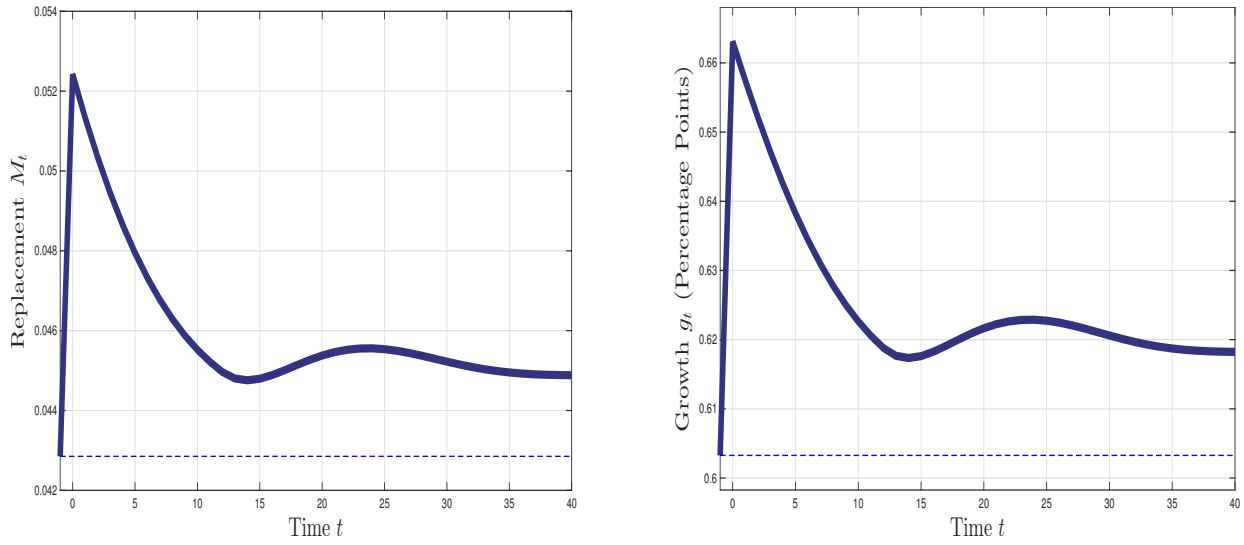
7.1 Aircraft

We first focus on the aircraft calibration. Figure 1 displays the dynamics of the replacement rate M_t (left panel) and of the growth rate of capital quality $g_t \equiv \frac{q_t}{q_{t-1}} - 1$, a measure of capital-embodied innovation (right panel). Upon implementation of the optimal policy at $t = 0$, the economy experiences a boom in replacement and innovation. On impact, the fraction of firms that invest in the new vintage of aircraft increases substantially, from 4.3% to 5.2%. Because of the market-size effect on innovation, this replacement boom implies that the growth rate of capital quality jumps from 0.6% to 0.66%. Then, over time, the volume of replacement activity gradually decreases and eventually converges to a higher level than the initial one, approximately 4.5%; The growth rate settles to 0.62%.

The reason for the overshooting of capital replacement is that in the initial BGP features relatively old capital, which the planner wants to scrap quickly. Consistent with this explanation, we analyze the subsidies that implement the optimal allocation, which Figure 2 illustrates. The three panels display subsidies as a function of capital age at $t = 0$, $t = 10$, and $t = 30$ respectively. Furthermore, the overall subsidy $S_{a,t}$ (solid blue line) is decomposed into Δ_t^M (dashed grey line), which is constant with respect to age, and $\Delta_{a,t}^{\Omega}$ (dashed-dotted light blue line), which determines the slope of subsidies with respect to capital age. To interpret the magnitudes and facilitate comparison across dates, we express all subsidies and their components as fractions of the replacement cost f_t evaluated at its BGP value for the corresponding date.

At the beginning of the transition, subsidies are steep in capital age: 10-year-old aircraft is associated with a subsidy that is approximately equal to 5% of the replacement cost, whereas 40-year-old aircraft receives a subsidy that exceeds 20% of the replacement cost. As

Figure 1: Replacement and Innovation (Aircraft)



(a) Replacement (b) Quality Growth

Notes: The figure displays the fraction of firms replacing their capital M_t (left panel) and the growth rate of new capital quality g_t (right panel) after implementing the constrained-efficient optimal policy in our calibration of the model to aircraft. The economy is assumed to be on the decentralized-equilibrium BGP at $t = -1$ (represented by the horizontal dashed blue line) and the optimal policy is implemented at $t = 0$.

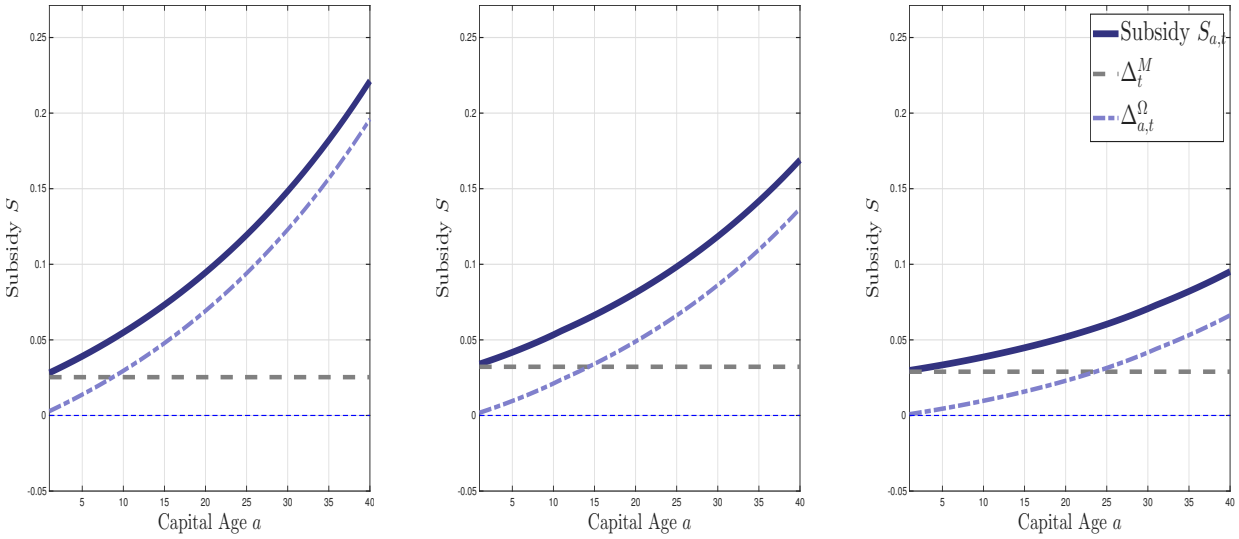
the transition evolves and the oldest, lowest-quality assets are scrapped, subsidies become significantly flatter. After 30 years, even the oldest aicraft is associated with subsidies below 10% of the replacement cost.

These dynamics also imply that the relative importance of $\Delta^{\Omega}_{a,t}$ and Δ^{M}_{t} changes significantly along the transition. Early in the transition, subsidies are predominantly dictated by the term $\Delta^{\Omega}_{a,t}$. Over time, the term Δ^{M}_{t} becomes increasingly important and in the long run it is the only determinant of subsidies, after the environmental externality becomes negligible.

In Figure 3 we analyze the effects of these subsidies on the aggregate environmental externality Ω_t. We compare the constrained-efficient allocation (solid blue line) with the decentralized equilibrium (dashed red line) and the first-best allocation (dashed-dotted light blue line); each transition begins at the same initial condition, namely the decentralized-equilibrium BGP, and we normalize $\Omega_0 = 1$ to display the changes over time in this variable.

While all three allocations feature a reduction in Ω_t over time due to technological progress increasing capital quality, this reduction is slowest in the decentralized equilibrium and fastest in the first-best allocation. The constrained-efficient allocation induces a rapid improvement in the environmental externality early in the transition, because it leads to

Figure 2: Optimal Subsidies (Aircraft)



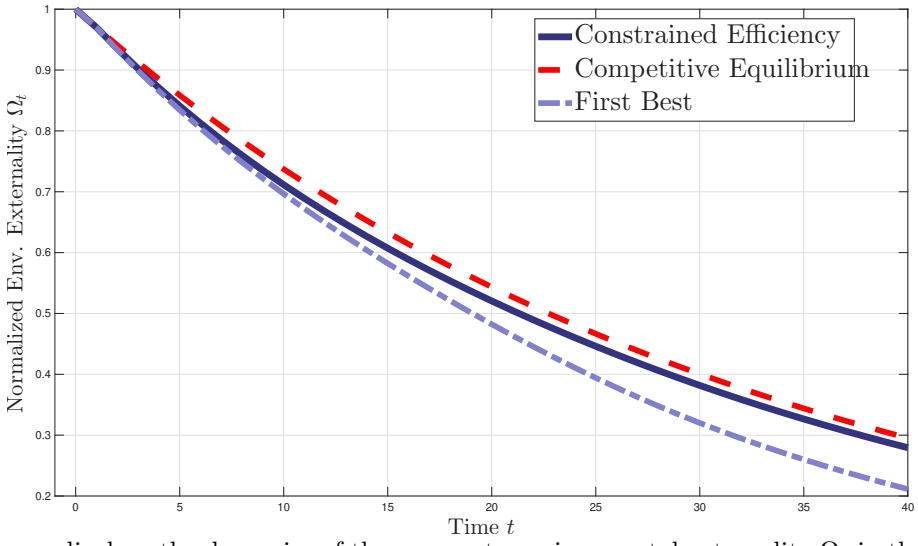
(a) Year $t = 0$ (b) Year $t = 10$ (c) Year $t = 30$

Notes: The figure displays the age profile of the constrained-efficient optimal replacement subsidies in the aircraft calibration of the model, at different points of the transition. The subsidies are represented as a function of the age of the asset (x-axis) and expressed as a fraction of the replacement cost in the competitive BGP (y-axis). Panel (a) reports optimal subsidies at $t = 0$; panel (b) at $t = 10$; and panel (c) at $t = 30$. In all panels, the solid blue line represents the overall optimal subsidy $S_{a,t}$ defined in Proposition 1; the dashed-dotted light-blue line refers to the $\Delta_{a,t}^{\Omega}$ component; and the gray dashed line shows the Δ_t^M component.

the scrappage of the oldest, lowest-quality capital in the first few years. As a result, the constrained-efficient line is closer to the first-best line for the first 10 years. Over time, however, the gap with first best increases because of the cumulative effect of faster quality growth in the first-best allocation.

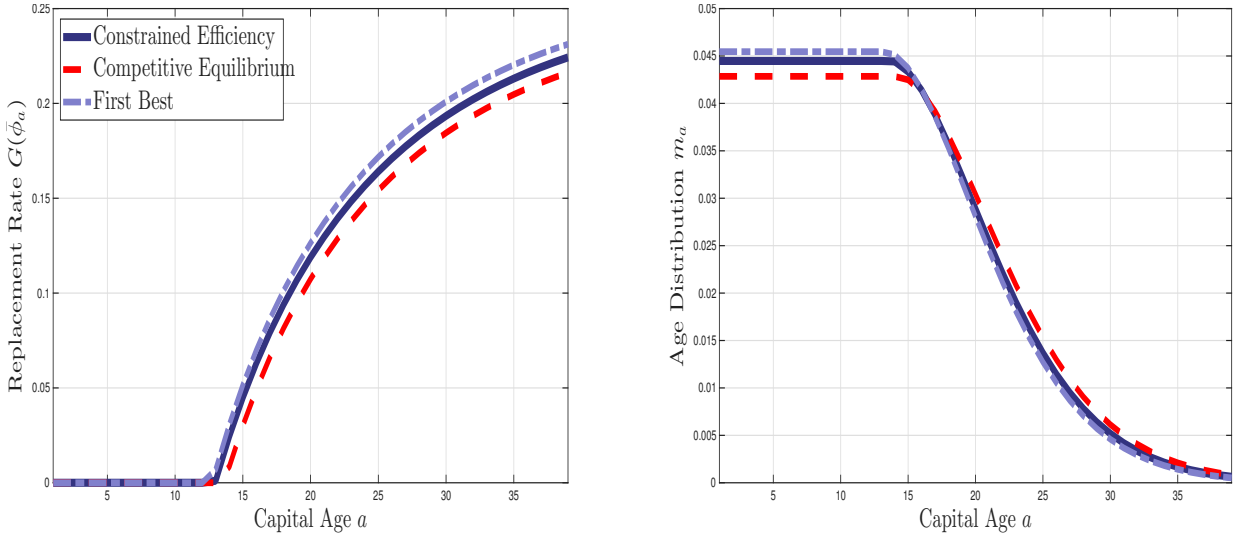
The implementation of the optimal policy leads to sizeable changes in the long-run distribution of capital age, which we illustrate in Figure 4, again comparing the same three allocations, each considered in its BGP. The left panel shows the replacement rate as a function of capital age, $G(\overline{\phi}_{a,t})$ under the three allocations. No aircraft is replaced before 10 years. In the decentralized equilibrium, the replacement rate remains relatively low for older aircraft. Replacement is fastest in the first-best allocation and the constrained-efficient allocation represents an intermediate case. These patterns manifest in the stationary distribution of capital age, shown in the right panel. The figure shows a higher mass for young capital under the two planning allocations, relative to the decentralized equilibrium.

Figure 3: Environmental Externality (Aircraft)



Notes: The figure displays the dynamics of the aggregate environmental externality Ω_t in the constrained-efficient equilibrium (solid blue line), in the competitive equilibrium (red dashed line), and in the first best (dash-dotted light-blue line) for the aircraft calibration of the model. For all the allocations, we normalize the aggregate externality Ω_t to 1 in the initial period $t = -1$, when we assume that the economy is in the competitive-equilibrium BGP. For the constrained-efficient and first-best allocations, we assume that the optimal policies are implemented at $t = 0$.

Figure 4: BGP Distribution (Aircraft)



(a) Replacement Rate (b) Quality Growth

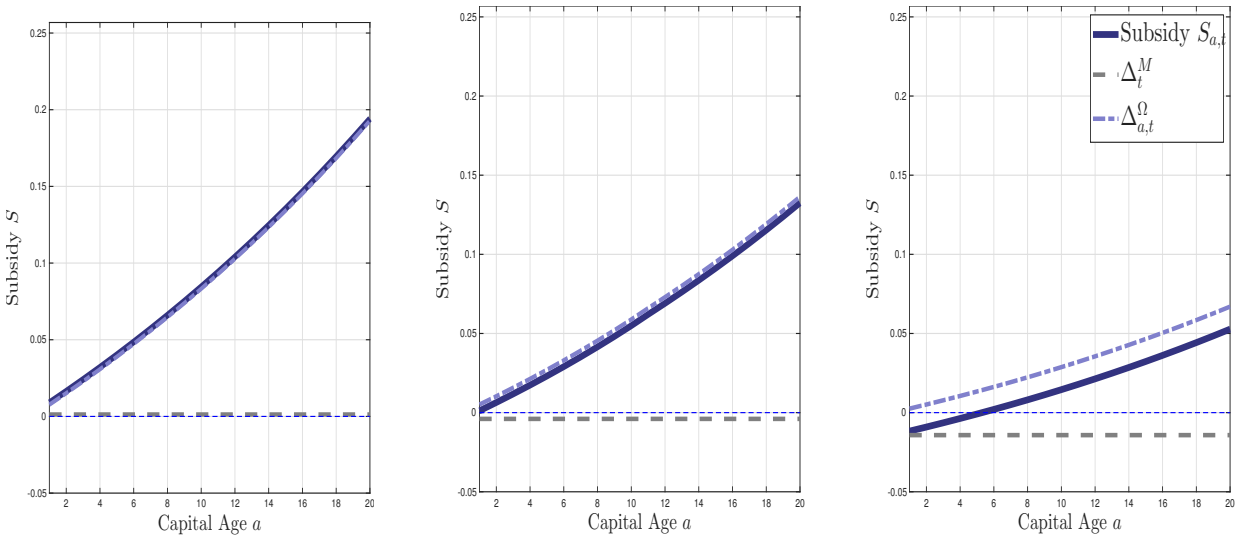
Notes: The figure represents the age-profile of the replacement rate $G(\bar{\phi}_a)$ (panel a) and of the firm distribution m_a (panel b) in the BGP of the constrained-efficient optimal allocation (solid blue line), of the competitive equilibrium (red dashed line), and of the first-best allocation (light blue dashed-dotted line), for the aircraft calibration of the model.

7.2 Vehicles

Next, we analyze optimal subsidies in the vehicles calibration. The three panels of Figure 5 display the subsidies as a function of capital age at $t = 0$, $t = 10$, and $t = 30$ respectively. Consistent with our findings in the aircraft calibration, the subsidies are initially steep in capital age, reflecting a large role for $\Delta^{\Omega}_{a,t}$ early in the transition, and then become flatter.

However, we also uncover a significant difference with respect to the aircraft calibration. Namely, the innovation externality term Δ^{M}_{t} is smaller and, in the long run, it becomes negative for vehicles. This result suggests that after the environmental damage from vehicles emission is addressed, the social cost of markups dominates the benefits from additional innovation when the planner is constrained to only subsidize replacement. Overall, the magnitude of the optimal subsidies (as a fraction of the corresponding replacement cost) is thus lower than for aircraft.

Figure 5: Optimal Subsidies (Vehicles)



(a) Year $t = 0$ (b) Year $t = 10$ (c) Year $t = 30$

Notes: The figure displays the age profile of the constrained-efficient optimal replacement subsidies in the single-technology vehicle calibration of the model, at different points of the transition. The subsidies are represented as a function of the age of the asset (x-axis) and expressed as a fraction of the replacement cost in the competitive BGP (y-axis). Panel (a) reports optimal subsidies at $t = 0$; panel (b) at $t = 10$; and panel (c) at $t = 30$. In all panels, the solid blue line represents the overall optimal subsidy $S_{a,t}$ defined in Proposition 1; the dashed-dotted light-blue line refers to the $\Delta^{\Omega}_{a,t}$ component; and the gray dashed line shows the Δ^{M}_{t} component.

Consistent with these findings, in Appendix B we show that the transitional dynamics for vehicles feature an initial overshoot in replacement. In the long run, however, the opti-

mal replacement rate is lower than in the decentralized equilibrium, reflecting the planner's internalization of significant markup distortions.

The transition to a new technology, such as electric motors, is of particular interest for the case of vehicle. We thus perform a quantitative experiment that leverages Proposition 2 in the vehicles calibration.

We calibrate this extension of the model according to the parameter values reported in Table 1 for both technologies. However, we set the scale of the environmental externality equal to zero for the new technology, because operating electric vehicles does not generate any (direct) carbon emissions. Moreover, we assume that a fraction $\rho = 0.35$ of firms using the old technology are given the opportunity to switch to the new technology each period. This parameter value implies that the new technology accounts for approximately 90% of total replacement 40 years since the start of the transition, consistent with estimates from the engineering literature (Diouf, 2024). Furthermore, the new technology is assumed to have 50% higher quality compared to the old technology to match the average price premium of EVs over combustion-engine cars in the U.S. (IEA, 2025).

Figure 6 shows the transition of the replacement share M_t for vehicle when both technologies are present, comparing the optimal policy with the decentralized equilibrium. The figure illustrates how the optimal replacement policy accelerates the transition, resulting in a faster convergence to the BGP with full adoption of the new technology.

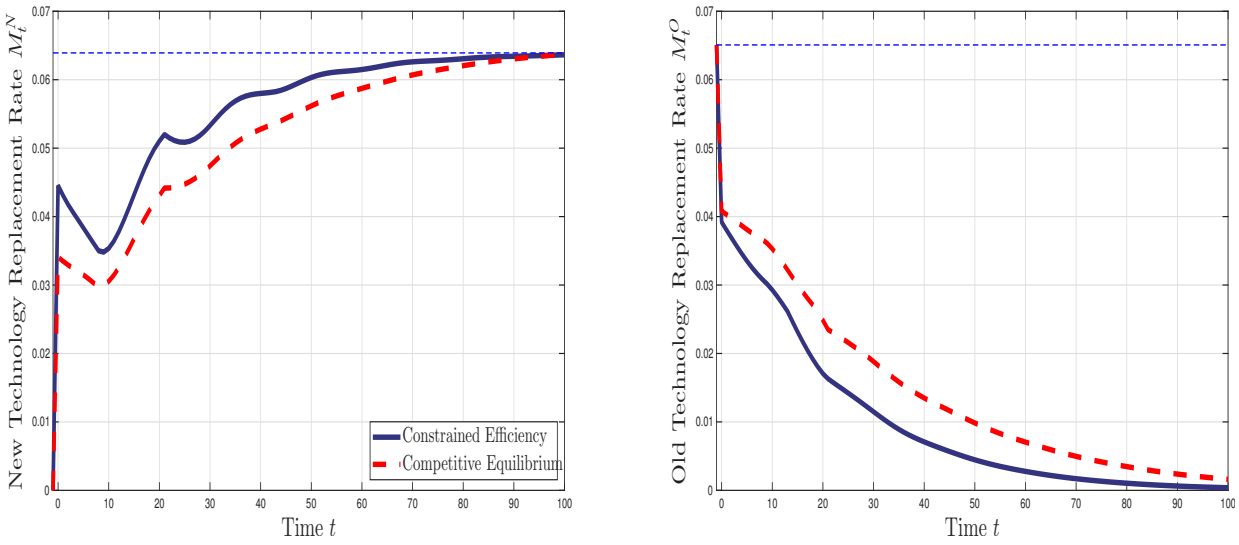
7.3 Aggregate Capital

Finally, we consider the macro calibration, interpreting the asset in the model as a generic type of capital. Figure 7 displays the optimal subsidies. Similar to the other two calibrations, along the transition path we obtain a slope of the subsidies with respect to capital age that is initially high and then decreases.

In this calibration, which targets a higher growth rate of capital quality, the innovation externality is significantly more powerful, leading to a large term Δ_t^M and thus an overall higher level for the subsidies. Even in the long run these subsidies exceed 30% of the replacement cost, reflecting a high social value of innovations in capital productivity, net of markup distortions, also beyond the environmental concerns.

In Appendix B we show that the planner optimally induces fast capital replacement in the long run, leading to a distribution of capital that features even younger capital than in the first-best allocation. In so doing, the planner provides large incentives for

Figure 6: Constrained-Efficient Replacement with Two Technologies (Vehicles)



(a) New Technology (b) Old Technology

Notes: The figure displays the fraction of firms replacing their asset for the case of the vehicle calibration with two technologies. We assume that at time $t = -1$ the economy on the competitive-equilibrium BGP with only the old technology. At $t = 0$ the new technology becomes available and it is assumed to have initial quality 50% higher than the old technology. We further assume that $\rho = 0.35$. We consider the transition of the economy to the new equilibrium under the constrained-efficient allocation (solid lines) and in the decentralized equilibrium (dashed lines). The left (right) panel shows replacement dynamics for the new (old) technology. The horizontal blue dashed line in the left (right) panel denotes the equilibrium replacement rate for the new (old) technology in the constrained-efficient BGP (initial BGP).

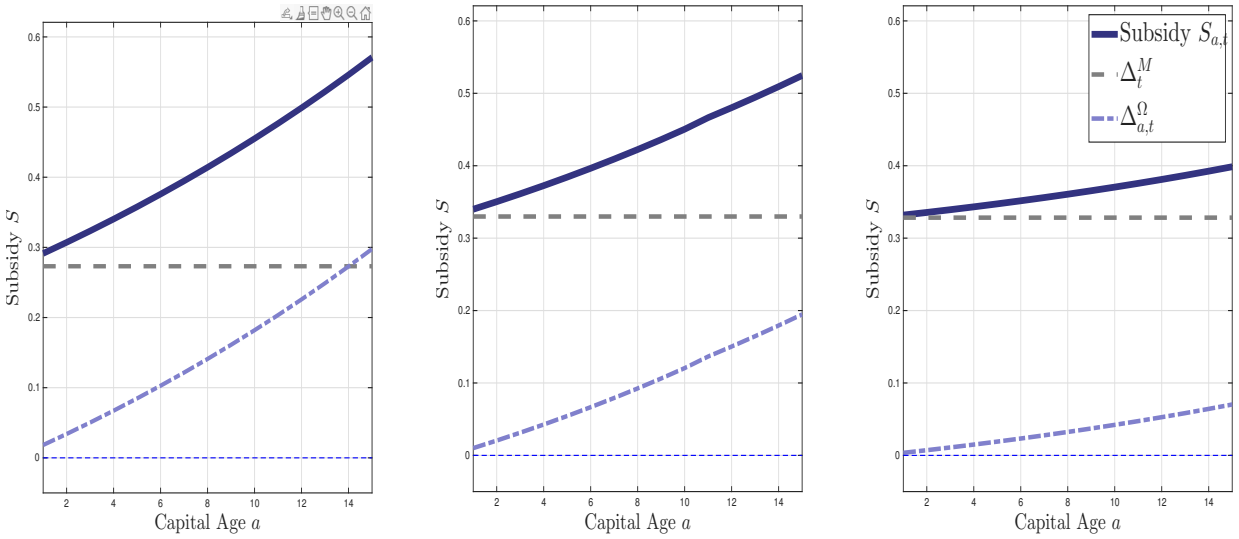
capital-embodied innovations.

8 Conclusions

This paper provides theoretical and quantitative guidance on a common policy intervention to stimulate the green transition: subsidies to scrap and replace durable assets. We develop a tractable model of capital replacement with capital-embodied innovation and environmental externalities. We use the model to characterize optimal subsidies for capital replacement when a planner internalizes environmental and innovation externalities.

We obtain a formal representation of optimal subsidies as the sum of two terms. The first term is the present discounted value of the difference in environmental damages between the asset being scrapped and a new asset. This term corrects environmental externalities taking the evolution of technology as given. The second term encodes the equilibrium effect of an additional sale of new capital on innovation, net of its effect on markup distortions.

Figure 7: Optimal Subsidies (Aggregate Capital)



(a) Year $t = 0$ (b) Year $t = 10$ (c) Year $t = 30$

Notes: The figure displays the age profile of the constrained-efficient optimal replacement subsidies in the calibration of the model to aggregate capital, at different points of the transition. The subsidies are represented as a function of the age of the asset (x-axis) and expressed as a fraction of the replacement cost in the competitive BGP (y-axis). Panel (a) reports optimal subsidies at $t = 0$; panel (b) at $t = 10$; and panel (c) at $t = 30$. In all panels, the solid blue line represents the overall optimal subsidy $S_{a,t}$ defined in Proposition 1; the dashed-dotted light-blue line refers to the $\Delta_{a,t}^{\Omega}$ component; and the gray dashed line shows the Δ_t^M component.

This term addresses the innovation externality and, importantly, contributes to correcting future environmental externalities as well.

We quantify the model in three different ways, targeting empirical features of specific assets—aircraft and vehicles—as well as targeting macroeconomic moments. For each calibration, we simulate the transitional dynamics that arise when the optimal policy is implemented starting from the decentralized equilibrium. We find that optimal subsidies are initially large and steeply increasing in capital age. This feature induces an initial spike in capital replacement that reduces significantly capital age. Then, over time, subsidies become flatter in capital age and their long-run level depends critically on the importance of the innovation externality relative to markup distortions.

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APPENDIX

A Model Appendix

A.1 Lagrangian for First Best

$$\mathcal{L}^{FB} \equiv \sum_{t=0}^{\infty} \beta^t \Big\{ (C_t - \Omega_t)$$

$$-\mu_t^C \left(C_t - \left(\sum_{\varepsilon \in E} \sum_{a \in A} m_{\varepsilon,a,t} \left(z\varepsilon\kappa(1-\delta)^{a-1} q_{t-a} - c\kappa^{\frac{1}{1-\gamma}} q_t \left(G(\overline{\phi}_{\varepsilon,a,t}) + \Phi_{\varepsilon,a,t} \right) \right) - \eta q_{t-1} \psi_t \right) \right)$$

$$+\mu_t^{\Omega} \left(\Omega_t - \sum_{\varepsilon \in E} \sum_{a \in A} \omega_{a,t}(q_{t-a}) m_{\varepsilon,a,t} z\varepsilon\kappa(1-\delta)^{a-1} q_{t-a} \right)$$

$$- \sum_{\varepsilon \in E} \theta_{\varepsilon,1,t} \left(m_{\varepsilon,1,t+1} - \sum_{a \in A} m_{\varepsilon,a,t} G(\overline{\phi}_{\varepsilon,a,t}) \right)$$

$$- \sum_{\varepsilon \in E} \sum_{a \in A} \theta_{\varepsilon,a+1,t} \left(m_{\varepsilon,a+1,t+1} - m_{\varepsilon,a,t}(1 - G(\overline{\phi}_{\varepsilon,a,t})) \right)$$

$$-\zeta_t \left(q_t - H(q_{t-1}, \psi_t) \right) \Big\}$$

A.2 Lagrangian for Constrained Efficiency

$$
\begin{aligned}
\mathcal{L} \equiv \sum_{t=0}^{\infty} \beta^t \Big\{ & (C_t - \Omega_t) \\
& -\mu_t^C \left(C_t - \left(\sum_{\varepsilon \in E} \sum_{a \in A} m_{\varepsilon,a,t} \left(z\varepsilon\kappa(1-\delta)^{a-1} q_{t-a} - f_t \left(G(\overline{\phi}_{\varepsilon,a,t}) + \Phi_{\varepsilon,a,t} \right) \right) \right) \right) \\
& +\mu_t^{\Omega} \left(\Omega_t - \sum_{\varepsilon \in E} \sum_{a \in A} \omega_{a,t}(q_{t-a}) m_{\varepsilon,a,t} z\varepsilon\kappa(1-\delta)^{a-1} q_{t-a} \right) \\
& -\sum_{\varepsilon \in E} \theta_{\varepsilon,1,t} \left(m_{\varepsilon,1,t+1} - \sum_{a \in A} m_{\varepsilon,a,t} G(\overline{\phi}_{\varepsilon,a,t}) \right) \\
& -\sum_{\varepsilon \in E} \sum_{a \in A} \theta_{\varepsilon,a+1,t} \left(m_{\varepsilon,a+1,t+1} - m_{\varepsilon,a,t}(1 - G(\overline{\phi}_{\varepsilon,a,t})) \right) \\
& -\zeta_t \left(q_t - H(q_{t-1}, \psi_t) \right) \\
& +\nu_t \left(f_t - F(q_{t-1}, \psi_t) \right) \\
& -\xi_t \left(\psi_t - \Psi \left(\sum_{\varepsilon \in E} \sum_{a \in A} m_{\varepsilon,a,t} G\left(\overline{\phi}_{\varepsilon,a,t} \right) \right) \right) \Big\}
\end{aligned}
$$

A.3 Additional Derivations and Optimality Conditions

A.3.1 R&D Optimality Conditions and Replacement Cost

In this subsection we present additional model derivations for the innovation block of the model. First, we solve for the Lagrange multiplier λ_t of the constraint on the expenditure minimization problem of final good producers that choose to replace their capital. Second, we solve for the replacement cost f_t as a function of R&D intensity, past quality, and parameters.

Lagrange Multiplier. As a preliminary step, we derive quantity of $d_{j,t}$ of each capital variety j optimally used in the capital bundle. For monopolistic varieties with price $p_{j,t} = \frac{c}{1-\gamma}$, the demand function (7) implies

$$d_{j,t} = \lambda_t^{\frac{1}{\gamma}}(1-\gamma)^{\frac{1}{\gamma}}\left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}} q_{j,t} \tag{A1}$$

where $q_{j,t} = \eta q_{j,t-1}$ due to successful innovation. In contrast, for competitive varieties, for which the price is driven to the marginal cost $p_{j,t} = c$, the quantity is

$$d_{j,t} = c^{-\frac{1}{\gamma}}(1-\gamma)^{\frac{1}{\gamma}}\lambda_t^{\frac{1}{\gamma}} q_{j,t} \tag{A2}$$

where $q_{j,t} = q_{j,t-1}$ because innovation was not successful.

Because varieties become competitive after one period and R&D intensity is symmetric across them, in equilibrium each variety j faces an independent and identical probability $\alpha(\psi_t)$ of being monopolistic. Therefore, in each period a fraction $\alpha(\psi_t)$ is monopolistic and a fraction $(1-\alpha(\psi_t))$ is competitive. Replacing prices and quantities of monopolistic and competitive varieties in equation (5), we can derive

$$
\begin{aligned}
\kappa q_t = &\int_0^1 q_{j,t}^{\gamma} v_{j,t}^{1-\gamma} dj = \\
&= \int_{j\in\mathcal{J}^{inn}} \underbrace{\eta^{\gamma} q_{j,t-1}^{\gamma}}_{q_{j,t}^{\gamma}} \underbrace{\Big((1-\gamma)^{\frac{2}{\gamma}} c^{-\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} \eta q_{j,t-1}\Big)^{1-\gamma}}_{d_{j,t}^{1-\gamma}} dj + \\
&\int_{j\in\mathcal{J}^{no}} \underbrace{q_{j,t-1}^{\gamma}}_{q_{j,t}^{\gamma}} \underbrace{\Big((1-\gamma)^{\frac{1}{\gamma}} c^{-\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} q_{j,t-1}\Big)^{1-\gamma}}_{d_{j,t}^{1-\gamma}} dj = \\
&= \lambda_t^{\frac{1-\gamma}{\gamma}} \Big(\frac{1-\gamma}{c}\Big)^{\frac{1-\gamma}{\gamma}} \Big[\alpha(\psi_t)\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}} + (1-\alpha(\psi_t))\Big] q_{t-1}
\end{aligned}
\tag{A3}
$$

where from the first to the second and third lines we distinguish varieties j's based on their innovation outcome ($\mathcal{J}^{inn}$ denotes the set of varieties with a successful innovation and $\mathcal{J}^{no}$ the set of competitive varieties). We further replace equilibrium quality and quantity and finally we leverage the independence of innovation probability with respect to past quality $q_{j,t-1}$ across varieties.

We substitute the law of motion for quality q_t (12) on the left hand side of (A3) and after re-arranging we obtain

$$
\lambda_t^{\frac{1}{\gamma}} = \frac{\kappa^{\frac{1}{1-\gamma}} \Big[1 + (\eta-1)\alpha(\psi_t)\Big]^{\frac{1}{1-\gamma}}}{\Big(\frac{1-\gamma}{c}\Big)^{\frac{1}{\gamma}} \Big[1 + \Big(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}} - 1\Big)\alpha(\psi_t)\Big]^{\frac{1}{1-\gamma}}}
\tag{A4}
$$

Replacement Cost. Substituting equilibrium prices and quantities in the objective function of the expenditure minimization (6), we obtain

$$
\begin{aligned}
f_t \equiv &\int_0^1 p_{j,t} d_{j,t} dj = \\
&= \int_{j\in\mathcal{J}^{inn}} \underbrace{\Big(\frac{c}{1-\gamma}\Big)}_{p_{j,t}} \underbrace{\frac{(1-\gamma)^{\frac{2}{\gamma}}}{c^{\frac{1}{\gamma}}} \lambda_t^{\frac{1}{\gamma}} \eta q_{j,t-1}}_{d_{j,t}} dj + \int_{j\in\mathcal{J}^{no}} \underbrace{c}_{p_{j,t}} \underbrace{\Big(\frac{1-\gamma}{c}\Big)^{\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} q_{j,t-1}}_{d_{j,t}} dj \\
&= c^{1-\frac{1}{\gamma}} (1-\gamma)^{\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} \Big[1 + \Big((1-\gamma)^{\frac{1}{\gamma}-1}\eta - 1\Big)\alpha(\psi_t)\Big] q_{t-1} \\
&= c\kappa^{\frac{1}{1-\gamma}} \underbrace{\Big[1 + (\eta-1)\phi(\psi_t)\Big]^{\frac{\gamma}{1-\gamma}} \Big[1 + \Big((1-\gamma)^{\frac{1}{\gamma}-1}\eta - 1\Big)\phi(\psi_t)\Big]^{-\frac{\gamma}{1-\gamma}}}_{\equiv \mathcal{D}(\psi_t)} \Big[1 + (\eta-1)\phi(\psi_t)\Big] q_{t-1} \\
&= c\kappa^{\frac{1}{1-\gamma}} \mathcal{D}(\psi_t) q_t
\end{aligned}
\tag{A5}
$$

where from the first to second line we distinguish between monopolistic and competitive varieties and substitute for equilibrium prices and quantities, from the second to the third line we leverage the independence of innovation probability with respect to past quality $q_{j,t-1}$, and from the third to the fourth line we substitute the expression of $\lambda_t^{\frac{1}{\gamma}}$ from equation (A4) and define the distortion term $\mathcal{D}(\psi_t)$.

A.3.2 Derivations for the First Best

We now derive the partial derivative $\frac{\partial\left(\sum_{t=0}^{\infty}\beta^t(C_t-\Omega_t)\right)}{\partial q_t}$ from equation (A25) for the first best equilibrium.

Using the expressions for C_t and Ω_t determined by the first and second constraints of the first-best Lagrangian (Section A.1), we derive:

$$\begin{aligned}\frac{\partial\left(\sum_{t=0}^{\infty}\beta^t(C_t-\Omega_t)\right)}{\partial q_t} &= \sum_{\varepsilon\in E}\sum_{a\in A}\beta^a m_{\varepsilon,a,t+a} z\varepsilon\kappa(1-\delta)^{a-1} \\ &- c\kappa^{\frac{1}{1-\gamma}}\sum_{\varepsilon\in E}\sum_{a\in A} m_{\varepsilon,a,t}\left(G(\bar{\phi}_{\varepsilon,a,t})+\Phi_{\varepsilon,a,t}\right)-\beta\eta\psi_{t+1} \\ &- \sum_{\varepsilon\in E}\sum_{a\in A}\beta^a m_{\varepsilon,a,t+a} z\varepsilon\kappa(1-\delta)^{a-1}\left(\omega_{a,t+a}(q_t)+\frac{\partial\omega_{a,t+a}(q_t)}{\partial q_t}q_t\right)\end{aligned} \tag{A6}$$

where $\frac{\partial\omega_{a,t+a}(q_t)}{\partial q_t} = -\omega_0\omega_1 q_t^{-\omega_1-1}$. If the planner does not internalize the effect of quality on the R&D cost of foreign producers, the term $-\beta\eta\psi_{t+1}$ would be absent from the second line.

A.3.3 Derivations for Constrained Efficiency

We first derive the partial derivative $\frac{\partial\left(\sum_{t=0}^{\infty}\beta^t(C_t-\Omega_t)\right)}{\partial q_t}$ from equation (A25) for the constrained-efficient equilibrium.

Using the expressions for C_t and Ω_t determined by the first and second constraints of the first-best Lagrangian (Section A.2) together with the equilibrium value of capital price (13), we derive:

$$\frac{\partial\left(\sum_{t=0}^{\infty}\beta^t(C_t-\Omega_t)\right)}{\partial q_t}=\sum_{\varepsilon\in E}\sum_{a\in A}\beta^a m_{\varepsilon,a,t+a}z\varepsilon\kappa(1-\delta)^{a-1}$$
$$-\beta c\kappa^{\frac{1}{1-\gamma}}\mathcal{D}(\psi_{t+1})(1+(\eta-1)\alpha(\psi_{t+1}))\sum_{\varepsilon\in E}\sum_{a\in A}m_{\varepsilon,a,t+1}\left(G(\bar{\phi}_{\varepsilon,a,t+1})+\Phi_{\varepsilon,a,t+1}\right)$$
$$-\sum_{\varepsilon\in E}\sum_{a\in A}\beta^a m_{\varepsilon,a,t+a}z\varepsilon\kappa(1-\delta)^{a-1}\left(\omega_{a,t+a}(q_t)+\frac{\partial\omega_{a,t+a}(q_t)}{\partial q_t}q_t\right) \tag{A7}$$

where $\frac{\partial F(q_t,\psi_{t+1})}{\partial q_t}=c\kappa^{\frac{1}{1-\gamma}}\mathcal{D}(\psi_{t+1})(1+(\eta-1)\alpha(\psi_{t+1}))$ and we assume that the constrained-efficient planner does not internalize the effect of quality on foreign consumption. In the alternative case in which the planner does consider the impact of quality on $\tilde{C}_t=\Pi_t-\eta q_{t-1}\psi_t$, equation (A7) would include the additional terms $\beta\frac{\partial\Pi_{t+1}}{q_t}-\beta\eta\psi_{t+1}$ on the right-hand side, where:

$$\beta\frac{\partial\Pi_{t+1}}{\partial q_t}=\beta\gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}}\frac{c\kappa^{\frac{1}{1-\gamma}}\left[1+(\eta-1)\alpha(\psi_{t+1})\right]^{\frac{1}{1-\gamma}}}{\left[1+\left(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\right)\alpha(\psi_{t+1})\right]^{\frac{1}{1-\gamma}}}M_{t+1}\eta\alpha(\psi_{t+1}).$$

Moreover, the constrained-efficient allocation involves the partial derivatives $H_\psi(q_{t-1},\psi_t)$, $F_\psi(q_{t-1},\psi_t)$, $\Psi_M(M_t)$, which we derive next.

We obtain $\Psi_M(M_t)$ by differentiating the optimality condition (11) with respect to M_t:

$$\underbrace{\alpha_{\psi\psi}(\psi_t)\gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}}\frac{c\kappa^{\frac{1}{1-\gamma}}\left[1+(\eta-1)\alpha(\psi)_t\right]^{\frac{1}{1-\gamma}}}{\left[1+(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1)\alpha(\psi_t)\right]^{\frac{1}{1-\gamma}}}M_t\,\Psi_{M,t}(M_t)+}_{\equiv A_{\Psi_M,t}}$$

$$+\underbrace{\alpha_{\psi}(\psi_t)\gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}-1}\frac{c\kappa^{\frac{1}{1-\gamma}}\left[1+(\eta-1)\alpha(\psi_t)\right]^{\frac{1}{1-\gamma}-1}}{\left[1+(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1)\alpha(\psi_t)\right]^{\frac{1}{1-\gamma}}}M_t(\eta-1)\alpha_{\psi}(\psi_t)\,\Psi_{M,t}(M_t)+}_{\equiv B_{\Psi_M,t}}$$

$$-\underbrace{\alpha_{\psi}(\psi_t)\gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}-1}\frac{c\kappa^{\frac{1}{1-\gamma}}\left[1+(\eta-1)\alpha(\psi_t)\right]^{\frac{1}{1-\gamma}}}{\left[1+(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1)\alpha(\psi_t)\right]^{\frac{1}{1-\gamma}+1}}M_t\left(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\right)\alpha_{\psi}(\psi_t)\,\Psi_{M,t}(M_t)+}_{\equiv C_{\Psi_M,t}}$$

$$+\underbrace{\alpha_{\psi}(\psi_t)\gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}}\frac{c\kappa^{\frac{1}{1-\gamma}}\left[1+(\eta-1)\alpha(\psi_t)\right]^{\frac{1}{1-\gamma}}}{\left[1+(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1)\alpha(\psi_t)\right]^{\frac{1}{1-\gamma}}}}_{\equiv D_{\Psi_M,t}}=0 \tag{A8}$$

Therefore, we can express $\Psi_{M,t}$ as:

$$\Psi_M(M_t)=\frac{D_{\Psi_M,t}}{C_{\Psi_M,t}-A_{\Psi_M,t}-B_{\Psi_M,t}}. \tag{A9}$$

Furthermore, we derive $F_{\psi}(q_{t-1},\psi_t)$ by differentiating (13) for ψ_t:

$$\begin{aligned}F_{\psi}(q_{t-1},\psi_t)=&\frac{c\kappa^{\frac{1}{1-\gamma}}}{1-\gamma}\mathcal{D}(\psi_t)\left(1+(\eta-1)\alpha(\psi_t)\right)^{-1}(\eta-1)\alpha_{\psi}(\psi_t)q_{t-1}+\\&-\frac{\gamma c\kappa^{\frac{1}{1-\gamma}}}{1-\gamma}\mathcal{D}(\psi_t)\frac{\left(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\right)}{\left(1+(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1)\alpha(\psi_t)\right)}\alpha_{\psi}(\psi_t)q_{t-1}\end{aligned} \tag{A10}$$

Finally, from the law of motion of quality (12) we obtain:

$$H_{\psi}(q_{t-1},\psi_t)=(\eta-1)\alpha_{\psi}(\psi_t)q_{t-1} \tag{A11}$$

A.4 Decomposition of Δ^M

To decompose the shadow value of capital quality into its resource component and its environmental component, consider the two recursive definitions:

$$\zeta_t^C \equiv \beta\zeta_{t+1}^C H_q(q_t,\psi_{t+1}) + \frac{\partial\left(\sum_{t=0}^{\infty}\beta^t C_t\right)}{\partial q_t} \tag{A12}$$

$$\zeta_t^\Omega \equiv \beta\zeta_{t+1}^\Omega H_q(q_t,\psi_{t+1}) - \frac{\partial\left(\sum_{t=0}^{\infty}\beta^t \Omega_t\right)}{\partial q_t}. \tag{A13}$$

Adding up both sides of these two equations and using equation (A25) we get $\zeta_t = \zeta_t^C + \zeta_t^\Omega$. We can then decompose the innovation externality as follows:

$$\Delta_t^{M,C} \equiv \left(\zeta_t^C H_\psi(q_{t-1},\psi_t) - \nu_t F_\psi(q_{t-1},\psi_t)\right)\Psi_M(M_t) \tag{A14}$$

$$\Delta_t^{M,\Omega} \equiv \zeta_t^\Omega H_\psi(q_{t-1},\psi_t)\Psi_M(M_t), \tag{A15}$$

which implies $\Delta_t^M = \Delta_t^{M,C} + \Delta_t^{M,\Omega}$.

A.5 Concave Utility and Endogenous Interest Rate

We now assume the utility function is

$$\sum_{t=0}^{\infty}\beta^t(u(C_t) - \Omega_t), \tag{A16}$$

with $u_c > 0$ and $u_{cc} < 0$. The representative household can trade risk-free bonds in zero net supply that pay an interest rate r_t. In equilibrium, firms discount future profits using the interest rate

$$1 + r_t = \frac{u_C(C_t)}{\beta u_C(C_{t+1})}. \tag{A17}$$

In a decentralized equilibrium with generic replacement subsidies, the firm value function becomes

$$\begin{aligned} V_{\varepsilon,a,t}^S = (1+r_t)^{-1}\Bigg(z\varepsilon k_{a,t} + \max_{\overline{\phi}\in[\phi_{min},\phi_{max}]}\Bigg\{-f_{t+1}\int_{\phi_{min}}^{\overline{\phi}}(1+\phi)dG(\phi) \\ +G(\overline{\phi})\left(V_{\varepsilon,1,t+1}^S + S_{\varepsilon,a,t+1}\right) + (1-G(\overline{\phi}))V_{\varepsilon,a+1,t+1}^S\Bigg\}\Bigg), \end{aligned} \tag{A18}$$

and the first-order condition is again

$$f_t(1+\overline{\phi}_{\varepsilon,a,t}) = V^S_{\varepsilon,1,t} - V^S_{\varepsilon,a+1,t} + S_{\varepsilon,a,t}. \tag{A19}$$

The optimality conditions for the constrained-efficient allocation are as follows:

$$\mu^C_t = u_C(C_t) \tag{A20}$$

$$\begin{aligned}\theta_{\varepsilon,a,t} &= \beta\mu^C_{t+1}\left((1-\omega_{a,t+1}(q_{t+1-a}))z\varepsilon k_{a,t} - f_{t+1}\left(G(\overline{\phi}_{\varepsilon,a,t+1}) + \Phi_{\varepsilon,a,t+1}\right)\right) \\ &\quad +\beta\left(G(\overline{\phi}_{\varepsilon,a,t+1})\left(\theta_{\varepsilon,1,t+1} + \Delta^M_{t+1}\mu^C_{t+1}\right) + (1-G(\overline{\phi}_{\varepsilon,a,t+1}))\theta_{\varepsilon,a+1,t+1}\right)\end{aligned} \tag{A21}$$

$$f_t(1+\overline{\phi}_{\varepsilon,a,t})\mu^C_t = \theta_{\varepsilon,1,t} - \theta_{\varepsilon,a+1,t} + \Delta^M_t\mu^C_t, \tag{A22}$$

where we define the externality on capital quality in units of consumption,

$$\Delta^M_t \equiv \left(\mu^C_t\right)^{-1}\left(\zeta_t H_\psi(q_{t-1},\psi_t) - \nu_t F_\psi(q_{t-1},\psi_t)\right)\Psi_M(M_t), \tag{A23}$$

$$\nu_t = \mu^C_t \sum_{\varepsilon\in\mathcal{E}}\sum_{a\in\mathcal{A}} m_{\varepsilon,a,t}\left(G(\overline{\phi}_{\varepsilon,a,t}) + \Phi_{\varepsilon,a,t}\right) \tag{A24}$$

$$\zeta_t = \beta\zeta_{t+1}H_q(q_t,\psi_{t+1}) + \frac{\partial\left(\sum_{t=0}^{\infty}\beta^t(u(C_t)-\Omega_t)\right)}{\partial q_t} \tag{A25}$$

To analyze the implementation, first notice that a sequence of subsidies for capital replacement determines the sequence of replacement thresholds, which in turn pin down aggregate investment, innovation effort, and, through the resource constraint, aggregate consumption at all dates. To characterize optimal subsidies, first define the shadow value of the distribution, expressed in units of consumption:

$$\tilde{\theta}_{\varepsilon,a,t} \equiv \frac{\theta_{\varepsilon,a,t}}{\mu^C_t}, \tag{A26}$$

which we can write as follows

$$\begin{aligned}\tilde{\theta}_{\varepsilon,a,t} &= \beta\frac{\mu^C_{t+1}}{\mu^C_t}\left((1-\omega_{a,t+1}(q_{t+1-a}))z\varepsilon k_{a,t} - f_{t+1}\left(G(\overline{\phi}_{\varepsilon,a,t+1}) + \Phi_{\varepsilon,a,t+1}\right)\right) \qquad \text{(A27)}\\ &\quad +\beta\frac{\mu^C_{t+1}}{\mu^C_t}\left(G(\overline{\phi}_{\varepsilon,a,t+1})\left(\tilde{\theta}_{\varepsilon,1,t+1}+\Delta^M_{t+1}\right) + (1-G(\overline{\phi}_{\varepsilon,a,t+1}))\tilde{\theta}_{\varepsilon,a+1,t+1}\right).\end{aligned}$$

Comparing the shadow value with the decentralized-equilibrium value funcion, we find that the optimal subsidies to implement the replacement thresholds of the constrained efficient allocation are:

$$\begin{aligned}S_{\varepsilon,a,t+1} &= \tilde{\theta}_{\varepsilon,1,t+1} - \tilde{\theta}_{\varepsilon,a+1,t+1} - \left(V^S_{\varepsilon,1,t+1} - V^S_{\varepsilon,a+1,t+1}\right) + \Delta^M_{t+1} \qquad \text{(A28)}\\ &= \tilde{\theta}_{\varepsilon,1,t+1} - V^S_{\varepsilon,1,t+1} - \left(\tilde{\theta}_{\varepsilon,a+1,t+1} - V^S_{\varepsilon,a+1,t+1}\right) + \Delta^M_{t+1}.\end{aligned}$$

We then define the term $\Delta^{\Omega}_{\varepsilon,a,t}$ accounting for the endogenous interest rate to discount future damages:

$$\Delta^{\Omega}_{\varepsilon,a,t} \equiv \tilde{\theta}_{\varepsilon,1,t} - V^S_{\varepsilon,1,t} - \left(\tilde{\theta}_{\varepsilon,a+1,t} - V^S_{\varepsilon,a+1,t}\right) = \qquad \text{(A29)}$$

$$\sum_{\tau=0}^{\infty}\beta^{\tau+1}\frac{u_C(C_{t+1+\tau})}{u_C(C_t)}z\varepsilon\left(\omega_{a+1+\tau,t+1+\tau}(q_{t-a})k_{a+1+\tau,t+\tau} - \omega_{1+\tau,t+1+\tau}(q_t)k_{1+\tau,t+\tau}\right). \qquad \text{(A30)}$$

Thus, with these expressions for $\Delta^{\Omega}_{\varepsilon,a,t}$ and Δ^M_t, we obtain again the following formula for optimal subsidies, which is a generalization of Proposition 1 with endogenous interest rate:

$$S_{\varepsilon,a,t} = \Delta^{\Omega}_{\varepsilon,a,t} + \Delta^M_t. \qquad \text{(A31)}$$

A.6 One-Hoss-Shay Depreciation

Unless a depreciation shock takes place,

$$k_{i,t} = \begin{cases} k_{i,t-1} & \text{if } \ x_{i,t} = 0 \\ \kappa_t & \text{if } \ x_{i,t} = 1. \end{cases} \qquad \text{(A32)}$$

If a depreciation shock takes place (with probability δ), then $k_{i,t} = \kappa_t$.

The firm value function in the absence of subsidies is

$$V_{\varepsilon,a,t} = \beta\Bigg(z\varepsilon k_{a,t} + \delta\left(V_{\varepsilon,1,t+1} - f_{t+1}(1+\Phi)\right) \tag{A33}$$

$$+(1-\delta)\max_{\overline{\phi}\in[\phi_{min},\phi_{max}]}\left\{-f_{t+1}\int_{\phi_{min}}^{\overline{\phi}}(1+\phi)dG(\phi) + G(\overline{\phi})V_{\varepsilon,1,t+1} + (1-G(\overline{\phi}))V_{\varepsilon,a+1,t+1}\right\}\Bigg),$$

where $\Phi \equiv \int_{\phi_{min}}^{\phi_{max}} \phi \; dG(\phi)$.

The first-order condition for optimal replacement is

$$f_t(1+\overline{\phi}_{\varepsilon,a,t}) = V_{\varepsilon,1,t} - V_{\varepsilon,a+1,t}, \tag{A34}$$

The age distribution evolves as follows:

$$m_{\varepsilon,a+1,t+1} = m_{\varepsilon,a,t}(1-\delta)\left(1-G(\overline{\phi}_{\varepsilon,a,t})\right) \tag{A35}$$

$$m_{\varepsilon,1,t+1} = \delta + (1-\delta)\sum_{a=1}^{\infty} m_{\varepsilon,a,t} G\left(\overline{\phi}_{\varepsilon,a,t}\right) \tag{A36}$$

The planner's shadow value of the distribution satisfies the following optimality condition:

$$\theta_{\varepsilon,a,t} = \beta\Bigg((1-\omega_{a,t+1}(q_{t+1-a}))z\varepsilon k_{a,t} + \delta\left(V_{\varepsilon,1,t+1} + \Delta^M_{t+1} - f_{t+1}(1+\Phi)\right) \tag{A37}$$

$$+(1-\delta)\Big(G(\overline{\phi}_{\varepsilon,a,t+1})\left(\theta_{\varepsilon,1,t+1} + \Delta^M_{t+1}\right) - f_{t+1}\left(G(\overline{\phi}_{\varepsilon,a,t+1}) + \Phi_{\varepsilon,a,t+1}\right)$$

$$+(1-G(\overline{\phi}_{\varepsilon,a,t+1}))\theta_{\varepsilon,a+1,t+1}\Big)\Bigg).$$

The first-order condition with respect to the replacement threshold is

$$f_t(1+\overline{\phi}_{\varepsilon,a,t}) = \theta_{\varepsilon,1,t} - \theta_{\varepsilon,a+1,t} + \Delta^M_t, \tag{A38}$$

The firm value function in the presence of subsidies is

$$V^S_{\varepsilon,a,t} = \beta\Bigg(z\varepsilon k_{a,t} + \delta\left(V_{\varepsilon,1,t+1} + S_{\varepsilon,a,t+1} - f_{t+1}(1+\Phi)\right) \tag{A39}$$
$$+(1-\delta)\max_{\overline{\phi}\in[\phi_{min},\phi_{max}]}\Bigg\{ - f_{t+1}\int_{\phi_{min}}^{\overline{\phi}}(1+\phi)dG(\phi)$$
$$+G(\overline{\phi})\left(V^S_{\varepsilon,1,t+1} + S_{\varepsilon,a,t+1}\right) + (1-G(\overline{\phi}))V^S_{\varepsilon,a+1,t+1}\Bigg\}\Bigg).$$

Hence, the first-order condition for optimal replacement in the presence of subsidies is

$$f_t(1+\overline{\phi}_{\varepsilon,a,t}) = V^S_{\varepsilon,1,t} - V^S_{\varepsilon,a+1,t} + S_{\varepsilon,a,t}. \tag{A40}$$

Plugging in the subsidy formulated in equation (A28) in the value function (A39), we get

$$V^S_{\varepsilon,a,t} = \beta\Big(z\varepsilon k_{a,t} - f_{t+1}\left(\delta\left(1+\Phi\right) + (1-\delta)\left(G\left(\overline{\phi}_{\varepsilon,a,t+1}\right) + \Phi_{\varepsilon,a,t+1}\right)\right) \tag{A41}$$
$$+\left(\delta + (1-\delta)G\left(\overline{\phi}_{\varepsilon,a,t+1}\right)\right)\left(\theta_{\varepsilon,1,t} - \theta_{\varepsilon,a+1,t+1} + \Delta^M_{t+1}\right) + V^S_{\varepsilon,a+1,t+1}\Big).$$

We then add the environmental externality to both sides of the value function to write

$$V^S_{\varepsilon,a,t} - \beta\omega_{a,t+1}(q_{t+1-a})z\varepsilon k_{a,t} = \tag{A42}$$
$$\beta\left(z\varepsilon k_{a,t}(1-\omega_{a,t+1}(q_{t+1-a})) - f_{t+1}\left(\delta\left(1+\Phi\right) + (1-\delta)\left(G\left(\overline{\phi}_{\varepsilon,a,t+1}\right) + \Phi_{\varepsilon,a,t+1}\right)\right)\right.$$
$$\left.+\left(\delta + (1-\delta)G\left(\overline{\phi}_{\varepsilon,a,t+1}\right)\right)\left(\theta_{\varepsilon,1,t} - \theta_{\varepsilon,a+1,t+1} + \Delta^M_{t+1}\right) + V^S_{\varepsilon,a+1,t+1}\right).$$

Thus, equation (A29) holds and the optimal subsidy satisfies Proposition 1.

A.7 Adoption of New Technology

Constraints for the constrained-efficient allocation:

$$\begin{aligned} C_t &= \sum_{\varepsilon\in E}\sum_{a\in A}\Big(m^O_{\varepsilon,a,t}\Big(z\varepsilon k^O_{a,t} - (1-\rho)f^O_t\left(G\left(\overline{\phi}^O_{\varepsilon,a,t}\right)+\Phi^O_{\varepsilon,a,t}\right) \\ &\quad -\rho f^N_t\left(G\left(\overline{\phi}^{ON}_{\varepsilon,a,t}\right)+\Phi^{ON}_{\varepsilon,a,t}\right)\Big) \\ &\quad +m^N_{\varepsilon,a,t}\Big(z\varepsilon k^N_{a,t} - f^N_t\left(G\left(\overline{\phi}^N_{\varepsilon,a,t}\right)+\Phi^N_{\varepsilon,a,t}\right)\Big)\Big) \end{aligned} \tag{A43}$$

$$\Omega_t = \sum_{\varepsilon\in E}\sum_{a\in A}\left(\omega^O_{a,t}\left(q^O_{t-a}\right)m^O_{\varepsilon,a,t}z\varepsilon k^O_{a,t} + \omega^N_{a,t}\left(q^N_{t-a}\right)m^N_{\varepsilon,a,t}z\varepsilon k^N_{a,t}\right) \tag{A44}$$

$$q^O_t = H^O(q^O_{t-1},\psi^O_t) \tag{A45}$$

$$q^N_t = H^N(q^N_{t-1},\psi^N_t) \tag{A46}$$

$$f^O_t = F^O(q^O_{t-1},\psi^O_t) \tag{A47}$$

$$f^N_t = F^N(q^N_{t-1},\psi^N_t) \tag{A48}$$

$$\psi^O_t = \Psi^O\left(\sum_{\varepsilon\in E}\sum_{a\in A}m^O_{\varepsilon,a,t}(1-\rho)G\left(\overline{\phi}^O_{\varepsilon,a,t}\right)\right) \tag{A49}$$

$$\psi^N_t = \Psi^N\left(\sum_{\varepsilon\in E}\sum_{a\in A}m^O_{\varepsilon,a,t}\rho G\left(\overline{\phi}^{ON}_{\varepsilon,a,t}\right) + m^N_{\varepsilon,a,t}G\left(\overline{\phi}^N_{\varepsilon,a,t}\right)\right) \tag{A50}$$

Implementation:

$$S^{O}_{\varepsilon,a,t} = \theta^{O}_{\varepsilon,1,t} - V^{O,S}_{\varepsilon,1,t} - \left(\theta^{O}_{\varepsilon,a+1,t} - V^{O,S}_{\varepsilon,a+1,t}\right) + \Delta^{M,O}_{t} \tag{A51}$$

$$S^{ON}_{\varepsilon,a,t} = \theta^{N}_{\varepsilon,1,t} - V^{N,S}_{\varepsilon,1,t} - \left(\theta^{O}_{\varepsilon,a+1,t} - V^{O,S}_{\varepsilon,a+1,t}\right) + \Delta^{M,N}_{t} \tag{A52}$$

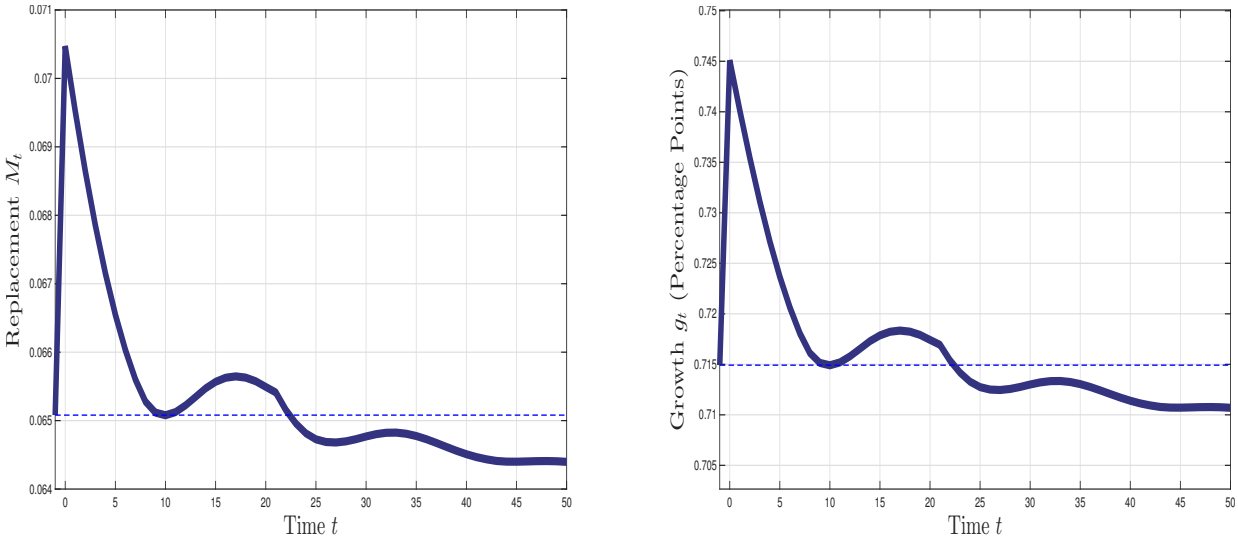
$$S^{N}_{\varepsilon,a,t} = \theta^{N}_{\varepsilon,1,t} - V^{N,S}_{\varepsilon,1,t} - \left(\theta^{N}_{\varepsilon,a+1,t} - V^{N,S}_{\varepsilon,a+1,t}\right) + \Delta^{M,N}_{t} \tag{A53}$$

$$\theta^{O}_{\varepsilon,a,t} - V^{O,S}_{\varepsilon,a,t} = -\beta\omega^{O}_{a,t+1}(q^{O}_{t+1-a})z\varepsilon k^{O}_{a,t} + \beta\left(\theta^{O}_{\varepsilon,a+1,t+1} - V^{O,S}_{\varepsilon,a+1,t+1}\right) \tag{A54}$$

$$\theta^{N}_{\varepsilon,a,t} - V^{N,S}_{\varepsilon,a,t} = -\beta\omega^{N}_{a,t+1}(q^{N}_{t+1-a})z\varepsilon k^{N}_{a,t} + \beta\left(\theta^{N}_{\varepsilon,a+1,t+1} - V^{N,S}_{\varepsilon,a+1,t+1}\right) \tag{A55}$$

B Additional Quantitative Results

B.1 Vehicles

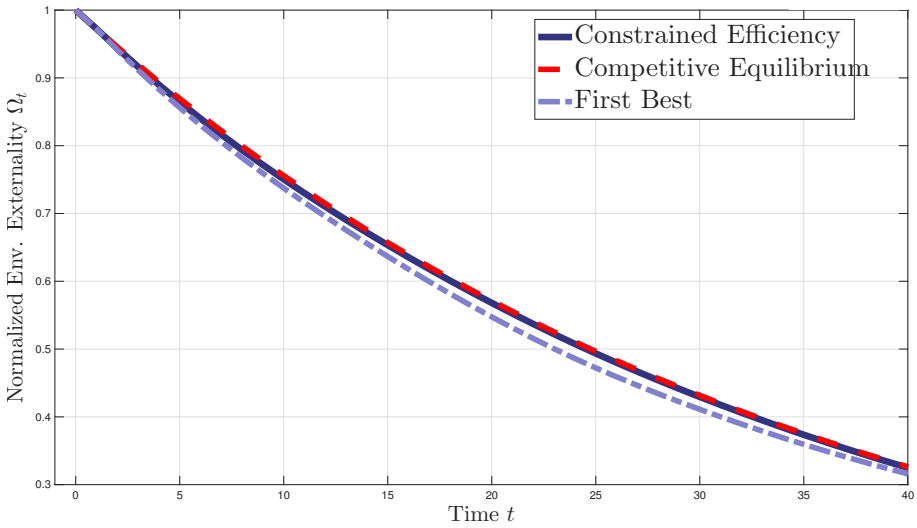


(a) Replacement (b) Quality Growth

Notes: The figure displays the fraction of firms replacing their capital M_t (left panel) and the growth rate of new capital quality g_t (right panel) after implementing the constrained-efficient optimal policy in our calibration of the model to vehicles (single technology). The economy is assumed to be on the decentralized-equilibrium BGP at $t = -1$ (represented by the horizontal dashed blue line) and the optimal policy is implemented at $t = 0$.

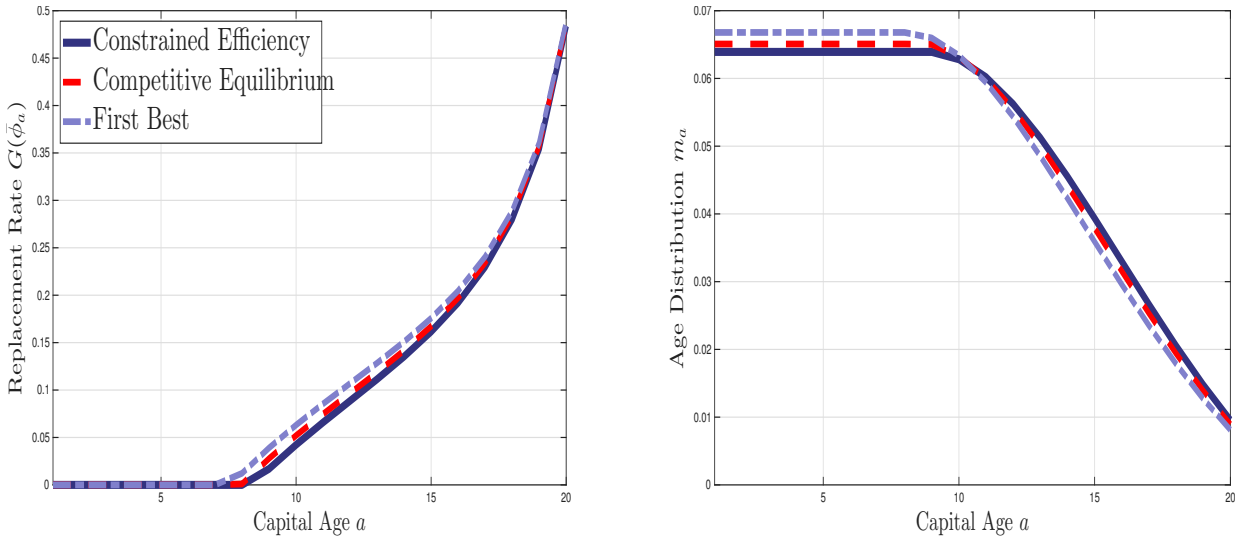
B.2 Aggregate Capital

Figure B2: Environmental Externality (Vehicles)



Notes: The figure displays the dynamics of the aggregate environmental externality Ω_t in the constrained-efficient equilibrium (solid blue line), in the competitive equilibrium (red dashed line), and in the first best (dash-dotted light-blue line) for the vehicles calibration of the single-technology model. For all the allocations, we normalize the aggregate externality Ω_t to 1 in the initial period $t = -1$, when we assume that the economy is in the competitive-equilibrium BGP. For the constrained-efficient and first-best allocations, we assume that the optimal policies are implemented at $t = 0$.

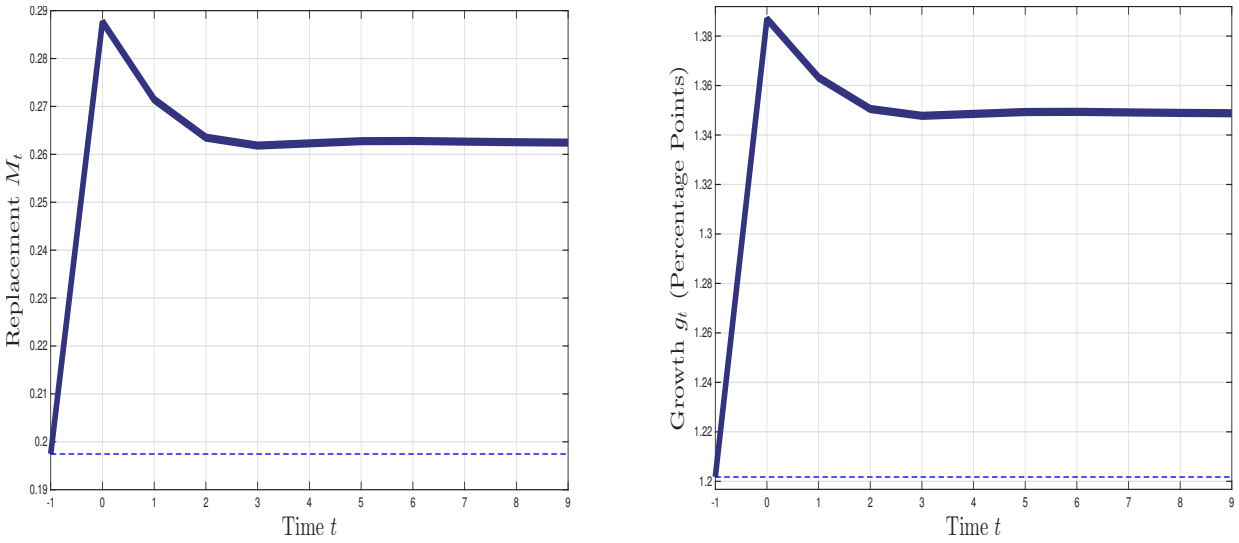
Figure B3: BGP Distribution (Vehicles)



(a) Replacement Rate (b) Quality Growth

Notes: The figure represents the age-profile of the replacement rate $G(\overline{\phi}_a)$ (panel a) and of the firm distribution m_a (panel b) in the BGP of the constrained-efficient optimal allocation (solid blue line), of the competitive equilibrium (red dashed line), and of the first-best allocation (light blue dashed-dotted line), for the single-technology, vehicles calibration of the model.

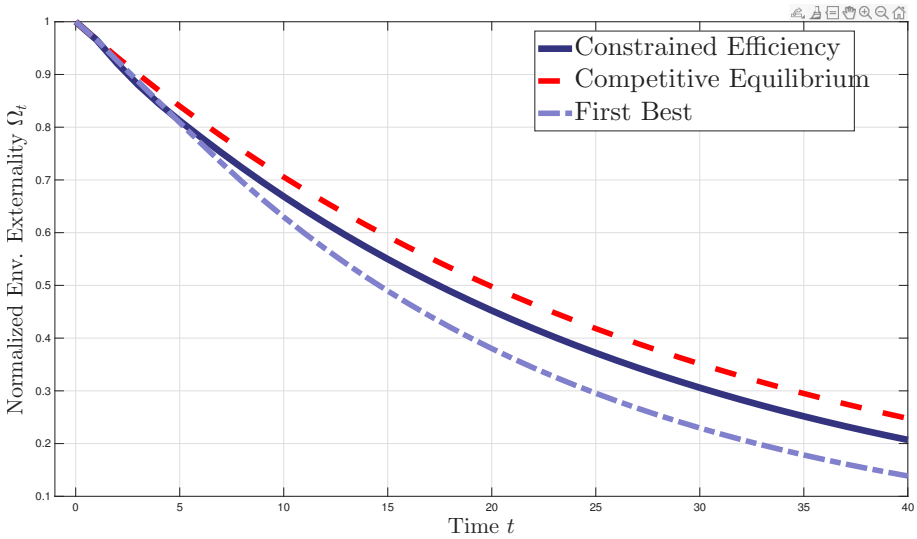
Figure B4: Replacement and Innovation (Aggregate Capital)



(a) Replacement (b) Quality Growth

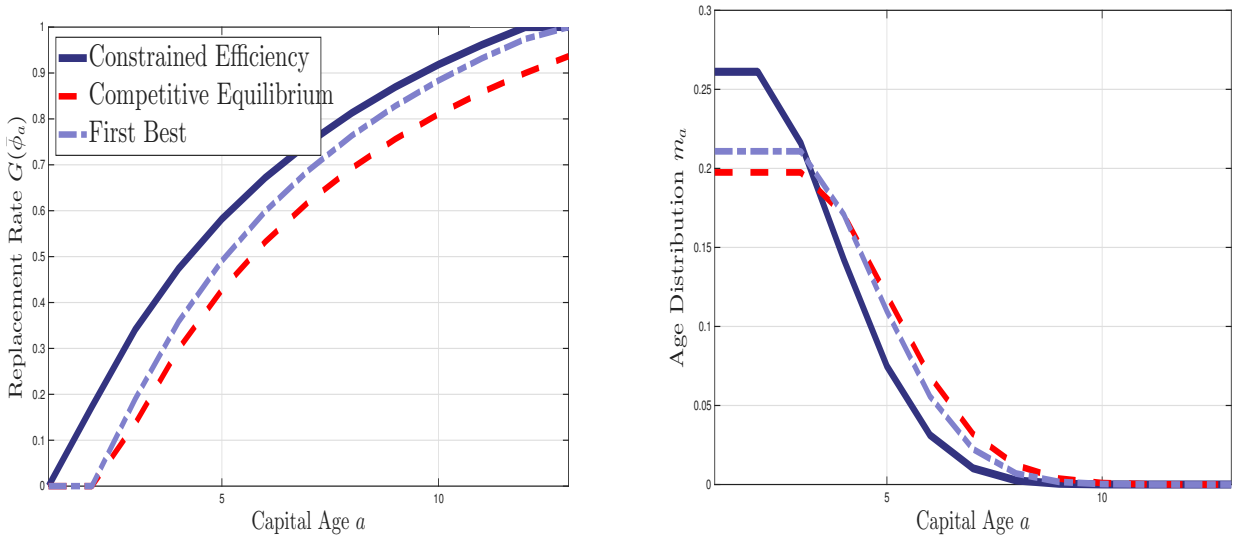
Notes: The figure displays the fraction of firms replacing their capital M_t (left panel) and the growth rate of new capital quality g_t (right panel) after implementing the constrained-efficient optimal policy in our calibration of the model to aggregate capital. The economy is assumed to be on the decentralized-equilibrium BGP at $t = -1$ (represented by the horizontal dashed blue line) and the optimal policy is implemented at $t = 0$.

Figure B5: Environmental Externality (Aggregate Capital)



Notes: The figure displays the dynamics of the aggregate environmental externality Ω_t in the constrained-efficient equilibrium (solid blue line), in the competitive equilibrium (red dashed line), and in the first best (dash-dotted light-blue line) for the calibration of the model to aggregate capital. For all the allocations, we normalize the aggregate externality Ω_t to 1 in the initial period $t = -1$, when we assume that the economy is in the competitive-equilibrium BGP. For the constrained-efficient and first-best allocations, we assume that the optimal policies are implemented at $t = 0$.

Figure B6: BGP Distribution (Aggregate Capital)



(a) Replacement Rate (b) Quality Growth

Notes: The figure represents the age-profile of the replacement rate $G(\overline{\phi}_a)$ (panel a) and of the firm distribution m_a (panel b) in the BGP of the constrained-efficient optimal allocation (solid blue line), of the competitive equilibrium (red dashed line), and of the first-best allocation (light blue dashed-dotted line), for the calibration of the model to aggregate capital.