Asymmetric Thin Markets

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Abstract

Anticipated supply and demand shocks —e.g., bond reopenings, passive investors' rebalancing— trigger V-shaped price paths characterized by time-series momentum and reversal. I explain these dynamics in a model where more risk-averse, smaller traders have endogenously lower price impact. As larger traders diversify slowly, smaller traders overshoot their inventory targets to clear the market during shocks, becoming the marginal pricers at short horizons. This generates a gradual risk-premium buildup and subsequent reversal, although smaller traders dynamically hedge against future misallocations. Altering the market structure can mitigate price distortions. A novel equilibrium representation characterizes the endogenous nondiversifiable risk driving these outcomes.

Keywords: Thin Markets, Asymmetric Price Impact, Anticipated Shocks.
JEL-Classification: D43, G12, G20, L12;

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1 Introduction

Changes in supply and demand are often pre-announced or easily predictable. For instance, European Treasuries pre-announce the schedule and magnitude of bond reopenings (Sigaux, 2023). Similarly, during QE episodes central banks pre-announce the timing and scale of asset purchases (D'Amico and King, 2013, An and Song, 2023). Passive investors have predictable rebalancing needs around index reconstitutions (e.g., Dick-Nielsen and Rossi, 2018) or futures contracts expiry dates (Bessembinder et al., 2016). Despite concerning the mere implementation of prior policy decisions or asset management mandates, and thus being largely uninformative, these events lead to V-shaped price paths: prices drift from the announcement until the shock realization, peak, and then gradually revert. These price dynamics are characterized by time-series momentum and reversal, generating price predictability around the events —a feature notoriously difficult to reconcile with rational behavior. These effects are widespread across asset classes and types of shocks and impose sizeable costs on issuers, central banks, and investors in index trackers (Lou et al., 2013, An and Song, 2023).

In this paper, I provide a market-structure-based explanation for these V-shaped price reactions. I analyze a market dominated by institutional traders accounting for their price impact and show that anticipated shocks lead to V-shaped price reactions, *provided* institutions differ in their risk bearing capacity. Obtaining this result requires extending the demand schedule competition literature to consider the interplay between market power, dynamic trading, and heterogeneity in risk aversion. Specifically, I characterize dynamic, nonstationary risk sharing in a complete-information setting among strategic traders submitting demand schedules and heterogeneous in risk aversion. New pricing and trading dynamics arise in this set up—even absent shocks. Shedding light on these dynamics requires departing from the standard representation of the equilibrium in terms of excess inventories. I derive a new representation to characterize endogenous nondiversifiable risk determined by trader-specific price impact in a dynamic setting.

Used as a laboratory to study anticipated shocks, the model qualitatively matches key features of empirical price reactions and institutional trading and delivers new testable predictions, tying trading strategies to price impact and the price path to the length of the announcement period. Further, the model suggests that altering the market structure can mitigate the V-shaped price reactions.

The model extends Rostek and Weretka (2015) by allowing for differences in risk aversion among n strategic traders, each endowed with exponential utility over final consumption with absolute risk aversion coefficient α^i, $i = 1, \ldots, n$. Trading occurs over multiple rounds

$t = 0, \ldots, T$ via complete information uniform-price auctions until the risky asset pays off. Owing to imperfect competition, each trader internalizes the impact of her demand on the equilibrium price. However, while all traders are nonnegligible, market power is asymmetric: more risk-tolerant traders will endogenously hold larger portfolios and have more price impact. I refer to these traders as large traders and to less risk-tolerant ones as small traders. Traders may represent institutions of varying sizes, such as small and large dealers, or distinct types of institutions (e.g., pension funds and insurers, dealers, etc.) coexisting in the market.[1]

In the baseline model, the supply of the risky asset is constant. I then introduce pre-announced supply and demand shocks. In line with the literature (Duffie (2010), Rostek and Weretka (2015)), I focus on nonstrategic shocks: blocks of shares sold/purchased by a price-taking issuer —a setting that can easily be tied to real-world issuance practices and passive rebalancings. As an extension, I consider strategic shocks, where traders anticipate endowment shocks that correlate with the asset payoff and strategically adjust their demand.

Absent shocks, the static model coincides with the centralized market equilibrium in Malamud and Rostek (2017). The dynamic model with homogeneous risk aversion ($\alpha^i = \alpha$) and nonstrategic shocks coincides with Rostek and Weretka (2015). However, the combination of dynamic trading and heterogeneity in risk aversion yields new insights, as dynamic trading alters the nature of nondiversifiable risk.

While nondiversifiable risk in competitive markets is given by the exogenous total quantity of risk (i.e., the supply of the risky asset, which equals aggregate inventories), it becomes an endogenous variable in thin markets. Indeed, as price impact impedes risk sharing, not all inventories are diversified in equilibrium and the total quantity of risk is typically not sufficient to characterize equilibrium risk sharing and price the risky asset. This point remained largely overlooked until Malamud and Rostek (2017)'s seminal paper, as prior literature focused on symmetric thin markets where strategic traders have identical risk aversion. In this special case, all traders shade their demand by the same magnitude, so that each trader diversifies an identical fraction of their initial inventories (see, e.g., Rostek and Weretka, 2015, Vayanos, 1999). As a result, the total volume diversified in equilibrium remains a frac-

[1]There is significant heterogeneity within and across institution type. Gabaix et al. (2003) report that assets under management in the mutual fund industry follow a power law. Similarly, the dealer industry is extremely concentrated, with 2% of broker-dealers controlling about 94% of total assets (Knyazeva and Bresler, 2025). Across institutions, portfolio size spans orders of magnitude. The largest sovereign and pension funds manage trillion-dollar portfolios (e.g., the Norwegian oil fund with 2tn, Calpers with about half a trillon), while the largest mutual funds have assets in the order of hundreds of billions of dollars (Cavani, Maggiori, and Schreger, 2025), and the largest dealers in the order of tens to hundreds of billions of dollars (Knyazeva and Bresler, 2025).

tion of aggregate inventories. Endogenous nondiversifiable risk thus remains proportional to aggregate inventories, and thus to exogenous nondiversifiable risk. Instead, when traders differ in risk aversion, each trader faces a different residual demand curve, and thus different risk sharing opportunities in equilibrium (Malamud and Rostek, 2017). Each trader diversifies a different fraction of her inventory, with the more risk averse diversifying larger fractions. Nondiversifiable risk is then endogenously determined by the joint distribution of initial inventories and risk aversion and captured by a portfolio in which traders' inventories are weighted by their price impact. Malamud and Rostek's insights, however, are limited to static asymmetric settings. In this paper, I present three novel results about dynamic asymmetric thin markets. First, absent shocks, I provide (i) a characterization of dynamic endogenous nondiversifiable risk and (ii) its implications for pricing and trading strategies. Then, extending (i) and (ii) to account for shocks, I derive V-shaped price paths and trading strategies around anticipated supply or demand shocks.

In my first main result, I show that – in contrast to the static case – the joint distribution of inventories and risk aversion in a given round is not sufficient to characterize endogenous nondiversifiable risk in the dynamic asymmetric setting. Instead, prices and trading strategies depend on the endogenous composition of the order flow from diversification trades at different horizons. Indeed, when trading is dynamic, traders differ not just in the fraction of the inventory they diversify but in the speed at which they trade, with smaller traders diversifying more aggressively in equilibrium, as they face endogenously lower price impact costs. The term structure of the order flow can then be summarized by what I call (the term structure of) diversified risk, $\mathbf{Q_t}^\top = \begin{pmatrix} Q_t^t & \dots & Q_t^T \end{pmatrix}$. Element $l \geq t$ of this vector gives the composition of the order flow at horizon l, with $Q_t^l \equiv \sum_j f_t^{jl} X_{t-1}^j$, where X_{t-1}^j is trader j's inventory before trading at t. Indeed, since trader j diversifies a fraction c_t^j of her inventory at round t, $f_t^{jl} \equiv (1-c_t^j)\dots(1-c_{l-1}^j)c_l^j$ is the fraction of her time-t inventory that she will diversify at a later time $l \geq t$.

Diversified risk is the (gross) risk transferred to the market, at different horizons. It measures endogenous nondiversifiable risk in asymmetric thin markets[2] and nests exogenous nondiversifiable risk (i.e., aggregate inventories) in special cases. Indeed, in competitive markets, all the risk is transferred in one round, so diversified risk coincides with aggregate inventories at the current round and is zero at longer horizons, precluding any term structure

[2]The terminology is not yet fixed in the literature for the determinants of risk premia in thin markets. I use diversified risk to emphasize that it's the inventories actually diversified that matter for pricing. In static contexts, Malamud and Rostek (2017) use aggregate risk instead of diversified risk, while Rostek and Yoon (2021) refer to it as systemic risk.

effects. Under symmetric market power, the term structure of the order flow is a redundant variable: since the total risk transferred at every round is a fraction of aggregate inventories, diversified risk at any horizon remains proportional to aggregate inventories.[3] The order flow composition remains constant over time, only determined by the initial distribution of inventories. Instead, as traders diversify at different speeds in asymmetric thin markets, the order flow composition changes over time.

The absence of term structure effects in symmetric thin markets implies that equilibrium trades can be represented as "excess inventories": traders diversify a fraction $c \in (0,1)$ of their inventory and replace it by the same fraction of the first best portfolio, effectively trading at rate c towards the first best. The same equilibrium representation can be used in static asymmetric settings, as trading occurs in a single round.[4] In dynamic, asymmetric thin markets, term structure effects require a new equilibrium representation: while a trader diversifies her inventory at rate $c_t^i \in (0,1)$, she gets exposure θ_t^{il} to diversified risk of horizon $l \geq t$:

$$x_t^i = \theta_{\mathbf{t}}^{\mathbf{i}} \mathbf{Q_t} - c_t^i X_{t-1}^i. \tag{1}$$

Correspondingly, pricing coefficients π_t^l measure the impact of round-l diversified risk on the risk premium:

$$D_t - p_t = \frac{\sigma^2}{q_t} \pi_{\mathbf{t}} \mathbf{Q_t}, \tag{2}$$

where D_t is the expected payoff, σ^2 is the per-period variance and q_t the aggregate risk bearing capacity adjusted for market power. I provide conditions for the existence of an equilibrium characterized by equations (1) and (2), which form the "diversified risk representation" of the equilibrium.

In my second main result, I derive the key implications of the term structure of order flow for trading and pricing. First, I show that the price is distorted relative to the first best, and that discounts/premia exhibit time-series momentum, especially at short horizon. Such dynamics are absent from symmetric thin markets, where traders – both net sellers and net buyers – uniformly shade their demand, so that prices coincide with the first best

[3] Formally, in the competitive case, the coefficients in the portfolio $\mathbf{Q_t}$ are $f_t^{it} = 1$ and $f_t^{il} = 0$, for $l > t$. In the symmetric market power case, $f_t^{il} = c(1-c)^{l-t}$, for $l \geq t$.

[4] Formally, if Q^π denotes the endogenous nondiversifiable risk portfolio, the equilibrium trade in the static case is $x_t^i = c^i \left(\frac{Q^\pi}{\alpha^i} - X_{t-1}^i \right)$.

(Rostek and Weretka, 2015). In asymmetric markets, small traders diversify more quickly in equilibrium, so they represent a large fraction of the order flow at short horizon. But since large traders absorb this flow slowly to control their price impact, smaller traders are the residual holders, and thus marginal pricers, of the asset at short horizons. This leads to a discount if small traders are over-exposed, and a premium otherwise. As time passes and some risk sharing occurs, this effect is progressively reduced, and the price reverts. Since the marginal benefit of risk sharing to small traders is stronger when the allocation is more inefficient, the reversal is stronger at short horizon.

The second key implication concerns trading strategies. All traders go long the current-round diversified risk (i.e., $\theta_t^{it} > 0$ in equation (1)), as this is how risk sharing takes place. However, smaller traders tend to short longer-horizon diversified risk (i.e., $\theta_t^{il} < 0$ for $l > t$), while larger traders go long. These strategies are determined by two opposite trading motives, as traders seek to hedge against limited diversification while exploiting price distortions. For each horizon, the coefficient θ_t^{il} can be decomposed into a hedging and a strategic components, with the former quantitatively dominating. Small traders' hedging component is negative, while their strategic component is positive for any horizon $l > t + 1$, and vice-versa for larger traders. Intuitively, as large traders have a more persistent order flow, smaller traders anticipate that they will have to absorb this flow far in the future, inducing them to hedge dynamically. Additionally, by distorting prices large traders' persistent order flow generates a buying opportunity to small traders at medium to long horizon. Traders with symmetric market power do not engage in such hedging, as there are no changes in the composition of order flow over time. Further, the price being equal to first best eliminates the strategic motive.

The momentum effect on the risk premium therefore results from complex dynamics. The hedging and strategic trading components induce traders to get different exposure to future-round diversified risk. These positions impact the price, because they change the composition of the order flow, and are also partly diversified away over time, which also affects the price, leading to new hedging and strategic trades, which further impact the price, etc.

Momentum and nonmonotonic trading strategies play a key role in generating V-shaped price patterns around anticipated shocks and delivering new empirical predictions. Generating this pattern from market structure effects is the third main result of the paper. To fix ideas, consider an anticipated (nonstrategic) supply shock (e.g., a bond reopening). The key effect that sets forces in motion is that due to market power asymmetry, smaller traders

must absorb an inefficient amount of new shares for the market to clear at the realization. Thus, the price overshoots the competitive level at realization. After the shock, the discount gradually reverts as the asset is progressively reallocated, based on the dynamics just described.

At the announcement, traders understand that the shock will modify the composition of future order flow. Smaller traders short ahead of the shock, anticipating being over-exposed at realization. The amount of hedging, however, is not sufficient to counterbalance the fact that risk will be inefficiently shared when the shock hits the market. This is because large traders (who need to take the other side) trade slowly and because the price overshoot also represents a buying opportunity for small traders. Thus, the more time passes, the more the upcoming misallocation gets reflected in the price, leading to a gradual increase in the risk premium until the shock realization. By contrast, in symmetric markets, traders have the same degree of market power, so each of them absorbs one n^{th} of the new supply at the realization, which is the efficient allocation. Thus, there is no need to hedge and there is no trading based on the shock before the realization. Since there are no order flow composition effects, the price does not exhibit momentum between the announcement and the realization.

The model yields new predictions about institutional trading around anticipated shocks as a function of price impact (Figure 1). Small traders (i.e. with lower price impact) first trade with the wind, then against the wind —absorbing a good deal of the new shares at realization— and with the wind again. Instead, large traders are contrarian throughout and their inventories undershoot their long-term level at realization. Medium-sized traders tend to first trade like small traders and then like large traders before the realization.

Small traders act as endogenous intermediaries in the model, stepping in at realization and reselling to larger traders afterwards. Their inventory dynamics mirror those of real-world dealers, who "make space on their balance sheet" ahead of shocks, provide immediacy at realization, thereby overshooting their target inventories, and mean-revert afterwards (Dick-Nielsen and Rossi, 2018, An and Song, 2023). While the pre-shock selling could be seen as front-running or anticipatory trading, the model suggests otherwise: small traders' hedging component leads them to sell and dominates the strategic component, which instead induces them to exploit the falling price by buying before the shock.[5]

Chemannur et al. (2009) find that some institutions short, while others long, and others

[5]The model also suggests a new take on intermediation. A common theme in the literature is that intermediation is constrained by regulations limiting risk bearing capacity (e.g., Duffie et al., (2023), Bao et al. (2018)). Here instead, it is the relatively limited risk bearing capacity of small traders that leads them to take the role of intermediaries: in my setting, limited risk bearing capacity is *inherent* to intermediation.

end up flat before shock realizations (SEOs). A good test of the theory would be to regress this pre-shock behaviour on trader price impact.

The model yields three more novel predictions. First, pre-announced shock have a smaller price impact at realization than unanticipated shocks, as the risk sharing occurring before the shock reduces misallocation when the shock takes place. By the same token, a longer announcement period is associated to smaller price effects at both announcement and realization. Second, numerical analysis suggests that the risk premium buildup between these two dates is always convex, a prediction that seems in line with visual evidence, but to the best of my knowledge has not been tested. Intuitively, the more imminent the realization of the shock, the more the price reflects that the small traders will be the marginal pricers at the realization. Third, a more uniform distribution of the risk bearing capacity among traders increases the price impact at announcement but reduces it at realization, with a less convex price path in between.

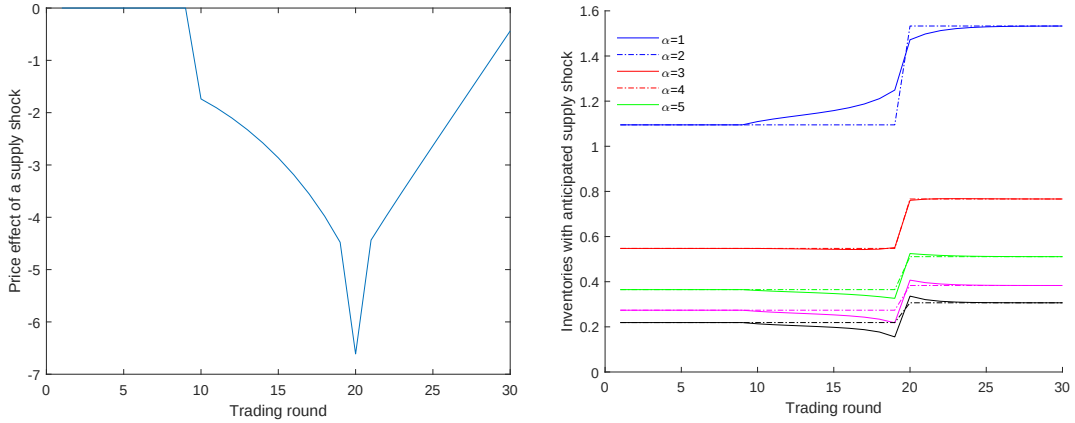
These effects rely solely on the market structure, absent other frictions such as private information or segmentation, suggesting that policies affecting the market structure may reduce the price impact of anticipated supply and demand shocks. For instance, regulators may use antitrust policies to limit the emergence of very large institutions concentrating most of the risk bearing capacity. This is at odds with many existing market structures. For instance, bond markets feature a dealership sector with decreasing risk bearing capacity (Duffie, 2025) along significantly larger players like insurers and pension funds. The model also points to a new trade-off between market-wide risk bearing capacity and its distribution across institutions: a more uniform distribution of risk bearing capacity may reduce V-shaped patterns even if it is associated with a reduction in aggregate risk bearing capacity.

Finally, in an extension, I consider anticipated endowment shocks to the traders. While these shocks may be more difficult to identify in the data, numerical analysis suggests that they trigger nonmonotonic price paths before shock realization, with different patterns depending on whether the larger or smaller traders are hit. For instance, when a shock adversely affects a small trader's demand, the price falls sharply at the announcement, briefly recovers before falling gradually again until realization, and reverting afterwards.

1.1 Related literature

The paper contributes to two strands of literature: (i) the literature on dynamic double auctions, and (ii) the literature on the price effects of anticipated shocks.

While the vast majority of the double auctions literature assumes homogeneous risk



(a) Price (b) Inventories

Figure 1: Effects of an Anticipated Supply Shocks on Prices and Inventories. The parameters are $T+1=30$, $t^a=10$, $t^r=20$, $n=5$, $\alpha^i=i$, $\Delta s_{t^r}=1$, $\sum_j X^j_{-1}=2.5$. Initial endowments are Pareto-optimal. On the right panel, dotted lines represent the competitive outcome.

aversion (symmetric market power), there are a few notable exceptions.[6] Among these, Sannikov and Skrzypacz (2016) and Rostek and Yoon (2021) analyze dynamic settings. Their focus, however, is on the interaction between asymmetric market power and private information, which introduces a complex dynamic inference problem. Instead, I retain a complete information setting, in which I isolate the effects of pre-announced supply and demand shocks, which neither paper considers.

Specifically, Sannikov and Skrzypacz (2016) analyze a conditional double auction in a stationary framework, in which traders can condition their bids on the order flow of others, while Rostek and Yoon (2021) retain the standard double auction and use a non-recursive approach to solve for a nonstationary equilibrium with private information. I similarly consider a standard double auction in a nonstationary framework, but assume complete information, so that the equilibrium remains solvable by standard recursive methods. As under homogeneous risk aversion (see Section 5 in Vayanos, 1999), the complete information equilibrium must be analyzed separately from the asymmetric information one: under complete information traders condition their demands on the inventories of others, leading to different equilibrium prices and allocations. To the best of my knowledge, this paper is thus the first to characterize dynamic risk sharing under complete information and heterogeneous risk aversion. Interestingly, while the standard "excess inventory" representation suffices under

[6]Papers featuring homogeneous risk aversion include, with different set ups, Vayanos (1999), Rostek and Weretka (2015), Kyle, Obizhaeva, and Wang (2017), Antill and Duffie (2020), Chen and Duffie (2021), Blonien (2024), Blonien and Ober (2025), etc. Malamud and Rostek (2017) and Gallien et al. (2023) allow for heterogeneous risk aversion in a static setting with possibly decentralized trading.

incomplete information (Rostek and Yoon, 2021), the complete information setting requires the new equilibrium representation derived in the paper.

The mechanism of this paper relies on the combination of market power and differences in risk aversion. Doing so, the paper complements and extends work, also in the complete information setting, by Rostek and Weretka (2015) and Pritsker (2009). In Rostek and Weretka's setting with symmetric market power, prices do not exhibit momentum and traders adjust their inventories once-and-for-all at the shock realization. In Pritsker (2009), asymmetric traders compete à la Cournot with a competitive fringe. In contrast to the Cournot setting, the equilibrium representation used here allows for (i) a tight characterization of trading as a function of the term structure of the order flow and (ii) a decomposition of trades into hedging and strategic motives. Both points permit a more tractable and intuitive interpretation of the complex strategies used by traders with different price impact.

Given that anticipated shocks are common knowledge, the only alternative explanation in the literature is based on search frictions or inattention (Duffie, 2010): inattentive investors rebalance only infrequently, leading to temporary asset misallocations and temporary increase and reversal in risk premia. Instead, in my framework, all investors are equally attentive, but trade at endogenously different speeds due to differences in risk aversion and imperfect competition. Further, my model predicts that trading strategies around anticipated shocks are determined by trader size or price impact, which are easier to observe and measure than investor attention.

1.2 Evidence about supply/demand shock effects

Before describing the model, I summarize the empirical evidence about the effects of supply and demand shocks on prices and inventories, highlighting three key empirical facts.

1.2.1 First fact: V-shaped Price Effects

Duffie (2010) collects evidence of V-shaped price effects around anticipated supply or demand shocks across different markets and asset classes. In fixed-income markets, these price effects are observed around pre-announced (i) Treasury bond issuances (Lou et al., 2013); (ii) sovereign bond reopenings (Sigaux, 2023); (iii) same-sector corporate bond issuances (Newman and Rierson, 2003); (iv) corporate bond index reconstitutions (Dick-Nielsen and Rossi, 2018)[7]; (v) central banks QE (An and Song, 2023).

[7]Some corporate bond indices rebalance their list only at the end of the month, so when a deletion follows from a downgrade that happened during the month, there is a clean separation between the announcement

Similarly, in equity markets, these price effects occur around (i) equity index reconstitutions (e.g., Greenwood and Sammon (2025) and references therein)[8]; (ii) mechanical sales or purchases by mutual funds experiencing large outflows or inflows (Lou, 2012) or a corporate event unrelated to asset fundamentals (trading scandal, merger, etc.)[9]; (iii) SEOs (Kulak, 2008). In derivatives markets, ETFs oil futures' roll generate the same type of V-shaped price pattern (Bessembinder et al., 2016).

1.2.2 Second and third facts: evidence about trading

While there is extensive evidence about V-shaped price effects of anticipated supply/demand shocks, evidence about institutionl trading before, at, and after the shock is more limited. Most papers focus on a given type of institutions and only report cross-sectional or time-series averages.

Dick-Nielsen and Rossi (2018) document *average* dealer inventories around corporate bond index deletions (see, e.g., Figure 4 in their paper). At the realization, dealers' inventories increase sharply, as they accommodate sales from index trackers seeking to limit their tracking errors. Inventories then typically revert in the weeks after the shock (but may sometimes remain above prior long-term levels). Prior to the shock, dealers tend to offload the bond from their inventories a week or two before the shock, and start increasing inventories in the days just before the shock[10]

An and Song (2023) also study dealers' inventories ahead of bond purchases by the Fed but distinguish between large and small dealers. They find that large dealers increase inventories ahead of the purchases more than small dealers, which allows them to sell more when the Fed buys.

Chemmanur et al. (2009) study trading by mutual funds and pension funds prior to issuances in SEOs. They report the cumulative order flow in the week before SEOs and not the time-series. They find that more than 83% of the SEO-institution pairs have net zero trading over that week, while about 10% are net buys and 6% net sales. This implies that even institutions selling before SEOs buy the new issue (percentages based on Table 4

and the realization of the shock (Dick-Nielsen and Rossi, 2018).

[8]While there are different rules across index providers, in many cases, the index inclusion or deletion is mechanical (e.g., Russell reconstitutions) and does not reveal fundamental information.

[9]Some of these events are surprise shocks rather than anticipated shocks.

[10]The increase in inventories starts earlier when the deletion is motivated by a downgrade than by the bond being close to maturity. In fact, Dick-Nielsen and Rossi (2018) find that on average dealers sell before the downgrades (shock announcement), which may be relatively easy to predict, and progressively buy until the exclusion (shock realization).

in Chemmanur et al. (2009)). Additionally, it is common for institutions to retrade their allocation post-SEOs.

We can group this evidence into two main facts.

Fact 2: Pre- and Post-shock Trading. Institutions do not delay their trades until shock realization. They trade in anticipation of the shocks and continue to adjust their portfolios afterwards.

Fact 3: Heterogeneity and Nonmonotonicity. Trading strategies around shocks show significant heterogeneity. While some institutions seem to employ simple, directional strategies, others – notably dealers– use more complex, nonmonotonic strategies.

2 Model

I begin with a formal description of the set up and then discuss the mapping of the model to actual shocks.

2.1 Set up

Time is discrete and goes from 0 to $T+1$. Trading takes place at $t = 0, \ldots, T$, and consumption at $T+1$.

The financial market consists of a risky asset and a risk-free asset. The risky asset trades at price p_t and pays off a normally distributed liquidating dividend D_{T+1} in the final round, with $D_{T+1} = D + \sum_{t=0}^{T+1} \varepsilon_t$, where $\varepsilon_t \sim N(0, \sigma^2)$. I denote $D_t \equiv \mathbb{E}_t(D_T) = D + \sum_{l=0}^{t} \varepsilon_l$ the conditional expected payoff of the risky asset at t. The risky asset is initially in positive net supply s. At time $t = 0, \ldots, T$, it incurs a supply shock $\Delta s_t \in \mathbb{R}$, so that the supply at time t is $s_t = s + \sum_{l=0}^{t} \Delta s_l$. The risk-free asset is in perfectly elastic supply and yields a return r_f normalised to 0.

There are $n \geq 3$ strategic traders, indexed by $i = 1, \ldots, n$, who maximise their expected utility of consumption, $u^i(C^i_{T+1}) = -\exp(-\alpha^i C^i_{T+1})$. Traders start with endowments X^i_{-1} and B^i_{-1} in the risky and risk-free assets, respectively. Without loss of generality, I set $B^i_{-1} = 0$. At each round $t = 1, \ldots, T+1$, traders receive endowment shocks $e^i_{t-1}\varepsilon_t$. These shocks correlate with the payoff of the risky asset and thus impact traders' willingness to hold it. For this reason, and for the sake of brevity, I refer to e^i_t as the demand shock.

Trading is organized as a (uniform-price) double auction. At every round, traders submit strictly downward-sloping demand schedules $x^i_t(p_t) : \mathbb{R} \to \mathbb{R}$. The equilibrium price $\tilde{p}_t$ is

the one that clears the market, i.e., such that the net aggregate demand equals the supply shock,

$$\sum_{j=1}^{n} x_t^j(\tilde{p}_t) = \Delta s_t. \tag{3}$$

Traders obtain equilibrium allocations (henceforth, equilibrium trades) $\tilde{x}_t^i = x_t^i(\tilde{p}_t)$, corresponding to their submitted schedule evaluated at the equilibrium price.[11]

In a given trading round, events occur in the following sequence. Trader i starts with an inventory X_{t-1}^i in the risky asset and a position B_{t-1}^i in the risk-free asset. Then (i) the dividend news ε_t is realized; (ii) traders receive endowment shocks $e_{t-1}^i \varepsilon_t$; (iii) they submit demand schedules; (iv) the auctioneer aggregates demand and determines the market-clearing price. After trading, positions in the risky and risk-free assets are

$$X_t^i = X_{t-1}^i + x_t^i \text{ and } B_t^i = B_{t-1}^i - x_t^i p_t + e_{t-1}^i \varepsilon_t, \tag{4}$$

where x_t^i is trader i's equilibrium trade in the risky asset.

Trader i thus chooses schedules $x_t^i(p_t)$, $t = 0, \ldots, T$ to maximize expected utility of consumption subject to other traders' schedules $x_t^j(p_t)$, $j \neq i$ and market clearing (3). The equilibrium is then defined as follows:

Definition 1 *Demand schedules* $\{x_t^i(p_t)\}_{i=1,\ldots,n}$, $t = 0, \ldots, T$, *constitute a Nash equilibrium if for each* i, $x_t^i(p_t)$ *solves*

$$\begin{aligned} \max_{x_t^i(p_t),\ t=0,\ldots,T} \quad & \mathbb{E}\left(-\exp(-\alpha^i C_{T+1}^i)\right) \\ s.t. \quad & x_t^i(p_t) + \sum_{j \neq i} x_t^j(p_t) = \Delta s_t, \end{aligned} \tag{5}$$

taking the other traders' strategies $\{x_t^j(p_t)\}_{j \neq i}$ *as given, where* $C_{T+1}^i = B_T^i + X_T^i D_{T+1}$ *and positions after trade evolve according to equation* (4).

I restrict attention to linear demand schedules:

$$x_t^i(p_t) = \beta_t^i(D_t - p_t) - c_t^i X_{t-1}^i - \sum_{j=1}^{n} d_t^{ij} X_{t-1}^j - \sum_{l=t}^{T} \delta_t^{il} \Delta s_l - \sum_{l=t}^{T} h_t^{il} e_l^i - \sum_{l=t}^{T} \sum_{j=1}^{n} g_t^{ijl} e_l^j \tag{6}$$

[11] Generally, to lighten notation, I don't use the tilde to denote the equilibrium price in the remainder of the paper, unless there is an ambiguity.

where the coefficients β_t^i, c_t^i, d_t^{ij}, δ_t^{il}, h_t^{il}, and g_t^{ijl} are to be determined in equilibrium.[12]

In equation (6), the slope of trader i's demand is given by β_t^i, while the intercept depends on the expected dividend, her inventory X_{t-1}^i at the beginning of round t, and on a combination of all traders' inventories and shocks.

The exact equilibrium dependence of the intercept on these variables varies with the market setting. In a static market, only the trader's own inventory and endowment shock will affect the intercept. In a dynamic market, strategic traders will additionally condition their demand on their future endowment shocks $\sum_{l=t}^T h_t^{il} e_l^i$, because of imperfect diversification. Further, whether they are strategic or not, in a dynamic market traders must also condition their demand on the distribution of inventories and upcoming shocks, represented by the portfolio $\sum_{j=1}^n d_t^{ij} X_{t-1}^j + \sum_{l=t}^T \delta_t^{il} \Delta s_l + \sum_{l=t}^T \sum_{j=1}^n g_t^{ijl} e_l^j$. This additional conditioning arises because the distribution of inventories and upcoming shocks serve as a sufficient statistics for future prices. While in a static setting, the future price is simply the liquidating dividend itself, in a dynamic market, these variables determine the future price path. The key challenge to solve the model in an intuitive way is to uncover the structure underlying the portfolio $\sum_{j=1}^n d_t^{ij} X_{t-1}^j + \sum_{l=t}^T \delta_t^{il} \Delta s_l + \sum_{l=t}^T \sum_{j=1}^n g_t^{ijl} e_l^j$.

To derive the equilibrium, instead of solving problem (5), I consider deviations Δx from the equilibrium trade, $\tilde{x}_t^i + \Delta x$, where $\tilde{x}_t^i$ is the equilibrium trade implied by schedules (6).[13] A deviating trader takes into account that her trades moves the equilibrium price from $\tilde{p}_t$ to $\tilde{p}_t + \lambda_t^i \Delta x$, where λ_t^i denotes trader i's price impact at t,

$$\lambda_t^i \equiv \frac{\partial p_t}{\partial x_t^i} = -\frac{1}{\sum_{j \neq i} \frac{\partial x_t^j}{\partial p_t}}. \tag{7}$$

I then derive conditions under which $\Delta x = 0$ is optimal for any trader. Since schedules (6) lead to this equilibrium trade and price, they are optimal.

As information is complete, however, all traders know with certainty the equilibrium price and trade and there exists an infinity of demand schedules passing through $(\tilde{x}_t^i, \tilde{p}_t)$. Absent uncertainty about the residual demand, traders are indifferent between all these schedules. I thus consider "trembles", as in Vayanos (1999), to select the equilibrium that is robust to small perturbations in the intercept of a trader's residual demand. Alternatively,

[12] To be precise, linear bids will be optimal for trader i given that other traders submit linear bids. A handful of papers in the literature allow for nonlinear schedules, including Klemperer and Meyer (1989), Du and Zhu (2017), and Glebkin, Malamud, and Teguia (2022).

[13] The expressions for the equilibrium risk premium and trade implied by bids (6) are given by equations (123) and (124) in the Appendix.

I could proceed as Rostek and Yoon (2021) and derive the equilibrium by (i) letting traders optimize taking their price impact λ_t^i as given, and (ii) imposing a consistency condition on price impact. This alternative method selects the same robust equilibrium. In the Online Appendix, I show the benefits of the "classic" approach to construct intuitively the diversified risk portfolios.

I call *asymmetric market power* the case with heterogeneity in risk aversion $\alpha^i \neq \alpha^j$ for at least one i. I refer to the homogeneous case $\alpha^i = \alpha$ for all i as *symmetric market power*.

I use bold font to denote vector and matrices. For a vector $\mathbf{y_t} = (y_t^i)_{i=1,\dots,n}$, I write $\bar{y}_t \equiv \sum_{i=1}^n y_t^i$. Section A in the Appendix collects all matrix notations.

2.2 Mapping the model to actual shocks

To capture predictable changes in supply or demand, I assume that current and future shocks are common knowledge, such that at time t all traders know the sequences $\{e_l^i\}_{i=1,\dots,n}$ and $\{\Delta s_l\}$ for $l \geq t$. In Section 4 on shock absorption, I consider a single shock announced at t^a and realized at $t^r \geq t^a$. I assume that the announcement occurs before the market opens for trading. When $t^a = t^r$, the shock is a surprise shock.[14]

2.2.1 Nonstrategic supply or demand shock

A shock Δs_{t^r} represents the net order flow from a nonstrategic trader in the secondary market at t^r. A positive shock corresponds to the sale of $\Delta s_{t^r} \geq 0$ new shares, while a negative shock $\Delta s_{t^r} < 0$ reflects a decrease in supply or an increase in demand such as a share buyback. This is the experiment considered in the literature (e.g., Duffie, 2010; Rostek and Weretka, 2015).

Nonstrategic sales/purchases of a block of shares on the secondary market can be tied to events such as (i) seasoned issuances (bond reopenings, SEOs),[15] (ii) QE programs by central banks, and (iii) bond or stock index reconstitutions and ETF futures rolls. Bond reopenings or issuances follow a regular and predictable pattern, where strategic considerations are

[14]The announcement refers to the announcement of the implementation (of Treasury issuances, central bank asset purchases, etc.), not of the policy (announcement of the budget deficit or of QE programs as a whole). In some cases, there may be some uncertainty about the size of the shock (e.g., passive investor rebalancing). It is straightforward to introduce uncertainty about the supply shock, and my key results would continue to hold. Two new effects would arise: before the shock, traders would face increased risk, which would simply strengthen the effects; after the shock, traders would need to further adjust their portfolios to account for the difference between the realization and the expectation of the shock.

[15]While SEOs may not be information-free, Chemmanur et al. (2009) argue that they convey less information than IPOs.

largely absent.[16] Similarly, during QE programs central banks maintain a market-neutral stance, with the stated objective to affect the composition of institutional portfolios rather than moving prices in a given direction.

Index reconstitutions trigger similarly uninformed and nonstrategic rebalancings by passive investors. For instance, when a bond is excluded from an index, ETFs and passive investors are likely to trade at the exclusion date to minimize tracking errors and thus to behave nonstrategically (Dick-Nielsen and Rossi, 2018). In the same vein, ETF futures' rolls trigger uninformed and easily predictable rebalancing by ETFs at expiry dates (see, e.g., Bessembinder et al., 2016).

2.2.2 Strategic demand adjustment

Strategic agents may also engage into predictable trades due to nonfundamental events such as large inflows or outflows, trading scandals, mergers, etc. For instance, following the leak of its positions in the CDS market during the "London Whale" episode, JPMorgan engaged in a series of unwinding trades. These demand shifts are captured via endowment shocks, which generate predictable demand adjustments by strategic traders. For instance, a permanent shift in trader i's demand is formalized by setting $e_l^i = e$, for any $l \geq t^r$, while setting $e_t^i \neq 0$ for a subset of trading rounds captures transitory demand shocks. I focus on nonstrategic shocks in the text and derive strategic demand adjustments as an extension in the Online Appendix (see also Section 4.4).

3 Dynamic risk sharing in asymmetric thin markets

In this section, I study the dynamic equilibrium with asymmetric market power, absent shocks ($\Delta s_l = 0$, $\forall l$, and $e_l^i = 0$, $\forall i, l$). I provide a new characterization of endogenous nondiversifiable risk in dynamic markets and delineate its properties relative to three benchmarks, each of which lacks one key component of the full model: (i) perfect competition (lacking market power), (ii) symmetric market power (lacking heterogeneity in risk aversion), and (iii) static asymmetric market power (lacking dynamics).

[16] "Regular and predictable issuance is a bedrock principle of Treasury debt management" (U.S. Department of the Treasury, 2023). European Treasuries typically uphold similar principles. Georgievska et al. (2025), among others, argue for a more strategic approach to reduce the cost of US debt.

3.1 Perfect competition and symmetric market power

In dynamic economies with perfect competition or symmetric market power, (exogenous) nondiversifiable risk determines asset prices and equilibrium trades. Exogenous nondiversifiable risk is given by the total supply s, which, by construction, equals aggregate inventories, $\sum_j X_{t-1}^j$. In a competitive economy, aggregate inventories are priced by the marginal utility of the representative agent. This agent has exponential utility with absolute risk aversion $\frac{1}{q^\star}$, where $q^\star \equiv \sum_j \frac{1}{\alpha^j}$ denotes the total risk bearing capacity of the market, defined as the sum of all traders' risk tolerances.

Throughout the paper, a * superscript denotes the competitive outcome. Whenever the context requires, I use $^{\mathsf{sym}}$ to distinguish symmetric from asymmetric market power. The symmetric market power coincides with the equilibrium in Rostek and Weretka (2015).

The equilibrium takes the following form, which I call the excess inventory representation:

Lemma 1 (Dynamic equilibrium in the benchmarks) *In both the competitive (with different risk aversion) and the symmetric market power benchmarks,*

1. *Exogenous nondiversifiable risk jointly determines the equilibrium price and traders' target portfolios $Q_t^{i\tau}$:*

$$D_t - p_t = \sigma_t^2 \frac{\sum_j X_{t-1}^j}{q^\star}, \tag{8}$$

$$Q_t^{i\tau} = \frac{\sum_{j=1}^n X_{t-1}^j}{\alpha^i q^\star}, \tag{9}$$

   where $\sigma_t^2 = \mathbb{V}_t(D_T) = \sigma^2(T-t+1)$ is the conditional variance at t.

2. *In both cases, trade can be expressed via the excess inventory representation:*

$$x_t^i = c\left(Q_t^{i\tau} - X_{t-1}^i\right), \tag{10}$$

   where the diversification speed c depends on the degree of competitiveness:

   - *Perfect competition: traders may differ in risk aversion (α^i, $i = 1, \ldots, n$); diversification is immediate ($c = c^\star = 1$) for all traders;*
   - *Symmetric market power: traders have identical risk aversion ($\alpha^i = \alpha$, $\forall i$); the diversification speed is limited due to price impact: $c = \frac{n-2}{n-1} \in (0,1)$.*

Competitive traders immediately diversify their inventories and replace them by their Pareto-optimal position $\frac{\sum_{j=1}^{n} X_{t-1}^{j}}{\alpha^i q^\star} = \frac{s}{\alpha^i q^\star}$, which consists of the nondiversifiable risk in proportion to their contribution to aggregate risk bearing capacity, $\frac{1}{\alpha^i q^\star}$. As such, competitive traders need only one trading round to exhaust all gains from trade and achieve perfect risk-sharing.

Traders with symmetric market power take into account their price impact. Thus, they slow down risk-sharing, and replace a fraction $c \in (0,1)$ of their inventory by the Pareto-optimal position at every round until T. Traders' inventories thus progressively tend towards the first-best.

As is well-known (Vayanos, 1999; Rostek and Weretka, 2015), although all traders understand that they have price impact and strategically reduce their demand, the price coincides with the competitive price under symmetric market power. Identical risk aversion implies that all traders face the same residual demand, so that all traders have the same price impact, and thus the same degree of market power. As a result, both net sellers (traders with an inventory larger than the equilibrium allocation) and net buyers (vice-versa) shade their supply/demand by the same magnitude, leaving the price unchanged relative to the first-best.[17] As we will see next, this logic breaks down when market power is asymmetric due to differences in risk aversion.

3.2 Static equilibrium

Suppose now that traders differ in risk aversion. The static equilibrium of the model coincides with the centralized market equilibrium in Malamud and Rostek (2017), who show that aggregate inventories are no longer sufficient to price asset due to differences in risk aversion among strategic traders.[18] For the sake of brevity, I relegate details about the existence and equilibrium value of the demand schedule coefficients to Proposition 4 in Appendix C.1 and focus here on prices and trades only.

Proposition 1 (Static equilibrium (Malamud and Rostek, 2017)) *Suppose that there is a single round of trading. Then a unique equilibrium exists in which the price and trades*

[17] Since $\alpha^i = \alpha$, the total risk bearing capacity is n/α and the equilibrium price under symmetric market power can be written as $D_t - p_t = \frac{\alpha}{n}\sigma_t^2 s$.

[18] Malamud and Rostek (2017) allow for privately observed inventories but the equilibrium is ex-post.

are given by

$$D_T - p_T = \frac{\sigma^2}{q_T} \sum_j c_T^j X_{T-1}^j, \tag{11}$$

$$x_T^i = -c_T^i X_{T-1}^i + \frac{\beta_T^i}{\bar{\beta}_T} \sum_j c_T^j X_{T-1}^j \cdot, \tag{12}$$

where $\bar{\beta}_T = \sum_j \beta_T^j$, $q_T = \sigma^2 \bar{\beta}_T$ *and* $c_T^i \in (0,1)$ *increases with* α^i.

As noted by Malamud and Rostek (2017), a key insight from the result is that the equilibrium price no longer depends on the total amount of risk, but on the joint distribution of risk aversion and inventories. Nondiversifiable risk becomes an endogenous outcome of the market structure. This is reflected in the price, which depends on the linear combination $\mathbf{c_T} \cdot \mathbf{X_{T-1}}$, where c_T^i is increasing in α^i. Intuitively, when traders differ in risk aversion, the larger traders shade their demand relative to the competitive case more than smaller ones. To clear the market, smaller traders must then absorb a larger fraction of the risk, leading to a larger impact of their inventory on the equilibrium risk premium.

3.3 Endogenous nondiversifiable risk in the dynamic model

Using $c_T^i = \alpha^i \sigma^2 \beta_T^i$ and defining $q_T \equiv \sigma^2 \bar{\beta}_T$, the equilibrium price and trade can be written using the same excess inventory representation as in the benchmarks:

$$x_T^i = c_T^i \left(Q_T^{i\tau} - X_{T-1}^i \right), \tag{13}$$

where $Q_T^{i\tau} \equiv \frac{\sum_j c_T^j X_{T-1}^j}{\alpha^i q_T}$and where, in analogy with $q^\star$, $q_T = \sum_j \frac{1}{\frac{\lambda_T^i}{\sigma^2} + \alpha^j}$ denotes the market power-adjusted total risk bearing capacity. Therefore, we might conjecture that the excess inventory representation may hold in the dynamic case, with $x_t^i = c_t^i \left(Q_t^{i\tau} - X_{t-1}^i \right)$ and $Q_t^{i\tau} = \frac{\sum_j c_t^j X_{t-1}^j}{\alpha^i q_t}$. This conjecture, however, does not hold.

To see why, note that when traders differ in risk aversion, what is priced in equilibrium is the risk actually diversified in the market, $\sum_j c_T^j X_{T-1}^j$ instead of aggregate inventories: the target portfolio Q_T^τ is simply a scaled version of this portfolio. With only one round, traders differ in the *fraction* of risk they transfer via the market. Where there are multiple rounds, asymmetric traders will differ in the *speed* at which they diversify. What is diversified in the *current* round thus no longer suffices to determine asset prices and target inventories.[19]

[19] See additional details and a more formal treatment in Section 2 of the Online Appendix.

In the next section, I guess and verify that the equilibrium risk premium and inventories in asymmetric thin markets can be expressed as a function of the *term structure* of diversified risk, as in equations (1) and (2), defined as follows:

Definition 2 (Term Structure of Diversified Risk) *Let* $\mathbf{Q_t^\top} = \left(Q_t^t, \ldots, Q_t^T\right)$ *denote the term structure of diversified risk, where the current-round and future-round diversified risk are defined as*

$$Q_t^t \equiv \sum_{j=1}^{n} c_t^j X_{t-1}^j, \tag{14}$$

$$Q_t^l \equiv \sum_{j=1}^{n} f_t^{jl} X_{t-1}^j, \quad \textit{for } l \geq t+1, \quad \textit{where } f_t^{jl} \equiv c_l^j \textstyle\prod_{s=t}^{l-1}(1-c_s^j). \tag{15}$$

Since at every round, traders diversify a fraction c_t^i of their endowment, f_t^{jl} is the fraction of the time-t inventory that will be diversified by trader j at a future time $l \geq t$. Taken together, the coefficients $(f_t^{jl})_j$ and initial inventories X_{t-1}^j determine the composition of the order flow generated by diversification trades at round $l \geq t$. Note that the current-round portfolio is a special case of the future-round portfolios for $l = t$, with the convention that $\prod_{s=t}^{t-1}(1-c_s^j) = 1$. For brevity, I sometimes refer to Q_t^t and Q_t^l $(l > t)$ as the static and dynamic portfolios. Further, instead of using the two expressions of exogenous and endogenous nondiversifiable risk, in the remainder I refer to the former simply as nondiversifiable risk and to the latter as diversified risk.

Using Definition 2, it is easy to verify that equation (2) holds in the static case. Let $\phi_T^i \equiv \frac{\beta_T^i}{\bar{\beta}_T}$ denote trader i's contribution to the market power-adjusted risk bearing capacity. Then,

$$D_T - p_T = \frac{\sigma^2}{q_T} Q_T^T \quad \text{and} \quad x_T^i = -c_T^i X_{T-1}^i + \phi_T^i Q_T^T,$$

which matches equation (2) with $\theta_T^{iT} = \phi_T^i$.

Further, the benchmark equilibria can also be represented in terms of diversified risk, facilitating a direct analytical comparison. Indeed, diversified risk nests aggregate inventories as a special case. In the competitive case, $c_t^{i\star} = 1$ at every round, so that $f_t^{jl\star} = 0$. Since all gains from trade occur in a single round, diversified risk in the current round corresponds to aggregate inventories and there is no diversified risk in future rounds, precluding term

structure effects:

$$\mathbf{Q_t^\star} = \left(\sum_j X_{t-1}^j, 0, \dots, 0\right). \tag{16}$$

In the symmetric case, all traders diversify at the same (and constant) speed $c_t^i = c$ (Lemma 1). Then $Q_t^l = f_t^l \sum_{j=1}^n X_{t-1}^j$, so that diversified risk remains proportional to aggregate inventories in every round,

$$\mathbf{Q_t^{sym}} = \left(f_t^t \sum_j X_{t-1}^j, \dots, f_t^T \sum_j X_{t-1}^j\right). \tag{17}$$

Diversified risk has a term structure in symmetric thin markets, but it is not relevant. What is diversified by trader i at round $l \geq t$ is a fraction f_t^l of her inventory. Similarly, what is diversified in aggregate at round l is a fraction f_t^l of aggregate inventories. Thus, the composition of the future order-flow is solely determined by the initial distribution of the inventories. For instance, in a static case, trader i represents $\frac{cX_{t-1}^i}{c\sum_j X_{t-1}^j} = \frac{X_{t-1}^i}{\sum_j X_{t-1}^j}$ of diversified risk. In a two-period case, trader i represents the same fraction of current- and future-round diversified risk: $\frac{cX_{t-1}^i}{c\sum_j X_{t-1}^j} = \frac{c(1-c)X_{t-1}^i}{c(1-c)\sum_j X_{t-1}^j}$.

By contrast, diversified risk matters in asymmetric thin markets, because different diversification speeds across traders modify the composition of the order-flow at different horizons. As we will see in Section 3.5, in the asymmetric equilibrium, if we take two traders i and k with different risk aversions, then $\frac{f_t^{il} X_{t-1}^i}{\sum_j f_t^{jl} X_{t-1}^j} \neq \frac{f_t^{kl} X_{t-1}^k}{\sum_j f_t^{jl} X_{t-1}^j}$.

3.4 Dynamic equilibrium

In the remainder of Section 3, I provide existence conditions for the equilibrium given by equations (1) and (2) and analyze the economic implications of dynamic trading under asymmetric price impact. The reader only interested in the implications can go directly to Section 3.5.

3.4.1 Dynamic trade-off

I begin by deriving the first-order condition. For trader i, the state of the economy at round t is summarized by two variables, her own inventory X_{t-1}^i, and diversified risk $\mathbf{Q_t}$. Using these state variables, I conjecture the following value function:

$$\Omega_t^i = v_t^i - \frac{\sigma^2}{2}\gamma_t^i (X_{t-1}^i)^2 - \alpha^i \sigma^2 X_{t-1}^i \mu_{\mathbf{t}}^{\mathbf{i}} \mathbf{Q_t} + \frac{\sigma^2}{2} \mathbf{Q_t}^\top \rho_{\mathbf{t}}^{\mathbf{i}} \mathbf{Q_t}, \tag{18}$$

where $v_t^i \equiv B_{t-1}^i + X_{t-1}^i D_t$ is the expected value of the risk-free and risky asset inventories, $\pi_{\mathbf{t}}$, $\theta_{\mathbf{t}}^{\mathbf{i}}$, $\mu_{\mathbf{t}}^{\mathbf{i}}$ are $1 \times (T-t+1)$ row-vectors, and $\rho_{\mathbf{t}}^{\mathbf{i}}$ a $(T-t+1) \times (T-t+1)$ matrix. Since the competitive and symmetric equilibria can be expressed in terms of diversified risk, the value function (18) nests the two benchmark cases.

Trader i solves the following problem at round t:

$$\Omega_t^i \equiv \max_{x_t^i(p_t)} \quad v_t^i + x_t^i(D_t - p_t) - \frac{\alpha^i \sigma^2}{2}(X_t^i)^2 + \mathbb{E}_t(\Omega_{t+1}^i)$$

In equilibrium, a trader must be indifferent between the benefit and the cost of acquiring a small extra amount of the risky asset:

$$\underbrace{D_t - p_t}_{\text{marginal benefit}} = \underbrace{\lambda_t^i x_t^i + \alpha^i \sigma^2 X_t^i - \frac{\partial \mathbb{E}_t(\Omega_{t+1}^i)}{\partial x_t^i}}_{\text{marginal cost}}.$$

The marginal benefit is the expected gain $D_t - p_t$ per unit. The marginal cost has three parts: (i) price impact costs λ_t^i, incurred on all the inframarginal units x_t^i traded in the current round; (ii) a marginal increase in risk $\alpha^i \sigma^2$, incurred on the post-trade position X_t^i; (iii) an intertemporal cost measured by the marginal impact on the expected value next period,

$$\frac{\partial \mathbb{E}_t(\Omega_{t+1}^i)}{\partial x_t^i} = -\alpha^i \sigma^2 \gamma_{t+1}^i X_t^i - \sigma^2 \left(\mu_{\mathbf{t+1}}^{\mathbf{i}} \mathbf{Q_{t+1}} + X_t^i \mu_{\mathbf{t+1}}^{\mathbf{i}} \frac{\partial \mathbf{Q_{t+1}}}{\partial x_t^i} \right) + \frac{\sigma^2}{2} \mathbf{Q_{t+1}^\top} (\rho_{\mathbf{t+1}}^{\mathbf{i}} + \rho_{\mathbf{t+1}}^{\mathbf{i}\top}) \frac{\partial \mathbf{Q_{t+1}}}{\partial x_t^i}.$$

A small change in the trade x_t^i impacts future utility via (i) a change in post-trade inventories X_t^i, holding $\mathbf{Q_{t+1}}$ constant and (ii) a change in $\mathbf{Q_{t+1}}$, holding X_t^i constant. Equations (1) and (2) show that a change in $\mathbf{Q_{t+1}}$ captures a change in future price and target inventory caused by a change in the distribution of inventories. Since the distribution is irrelevant in the competitive and symmetric cases, $\frac{\partial \mathbf{Q_{t+1}}}{\partial x_t^i} = 0$.

I then rewrite the first-order condition by grouping the terms that are proportional to X_t^i and to $\mathbf{Q_{t+1}}$ and using the definitions of the dynamic portfolios (Definition 2):

$$D_t - \sigma^2 \sum_{l=t+1}^{T} \varphi_{t+1}^{il} \sum_{j=1}^{n} f_{t+1}^{jl} X_t^j - p_t = (\lambda_t^i + \sigma^2 A_{t+1}^i) x_t^i + \sigma^2 A_{t+1}^i X_{t-1}^i, \tag{19}$$

where A_{t+1}^i and φ_{t+1}^{il} are defined as follows.

Definition 3

$$\textit{Let} \quad A^i_{t+1} \equiv \alpha^i + \alpha^i \gamma^i_{t+1} + \boldsymbol{\mu}^{\mathbf{i}}_{\mathbf{t+1}} \frac{\partial \mathbf{Q_{t+1}}}{\partial x^i_t}, \tag{20}$$

$$\textit{and} \quad \varphi^{il}_{t+1} \equiv \mu^{il}_{t+1} - r^{il}_{t+1}, \tag{21}$$

$$\textit{where} \quad \mathbf{r}^{\mathbf{i}}_{\mathbf{t+1}} \equiv \frac{1}{2}(\boldsymbol{\rho}^{\mathbf{i}}_{\mathbf{t+1}} + \boldsymbol{\rho}^{\mathbf{i}\top}_{\mathbf{t+1}}) \frac{\partial \mathbf{Q_{t+1}}}{\partial x^i_t}. \tag{22}$$

The coefficients A^i_{t+1} and φ^{il}_{t+1} denote, respectively, the effective risk aversion and the marginal expected cost to trader i of bearing an extra unit of diversified risk at horizon l. While A^i_{t+1} affects the slope of the demand, φ^{il}_{t+1} determines its intercept. To illustrate how asymmetric market power alters the dynamic trade-off, I first analyze equation (19) in the nested benchmark cases.

Consider first a competitive economy and recall that $D_t = \mathbb{E}_t(D_{t+1})$. The left-hand side of equation (19) boils down to:[20]

$$D_t - \sigma^2 \sum_{l=t+1}^{T} \varphi^{il\star}_{t+1} \sum_{j=1}^{n} f^{jl\star}_{t+1} X^j_t = D_t - \frac{\sigma^2}{q^\star}(T-t) \sum_j X^j_t = \mathbb{E}_t(p^\star_{t+1}).$$

Since the competitive market is perfectly liquid, the marginal expected value of the asset for any trader is just the expected price next period, $\mathbb{E}_t(p^\star_{t+1})$. On the right-hand side of equation (19), A^i_{t+1} boils down to α^i. This is because $\alpha^i \gamma^i_{t+1}$ is the marginal cost of under-diversification. In the first-best, traders reach Pareto-optimal holdings after one trade, so they are optimally diversified and $\gamma^{i\star}_{t+1} = 0$. In the competitive case, equation (19) thus reduces to the familiar first-order condition in a CARA-Gaussian setting:

$$\mathbb{E}_t(p^\star_{t+1}) - p_t = \alpha^i \sigma^2 (x^i_t + X^i_{t-1}), \tag{23}$$

i.e., to hold a larger inventory, the trader requires a larger expected capital gain.

Consider next symmetric market power. The left-hand side of equation (19) is given by (see Appendix B.2.3):

$$D_t - \sigma^2 \sum_{l=t+1}^{T} \varphi^{il}_{t+1} \sum_{j=1}^{n} f^{jl}_{t+1} X^j_t = \mathbb{E}_t(p^\star_{t+1}) - \mathcal{D}^{\mathsf{sym}}_t, \tag{24}$$

[20] This follows from (i) derivations in Appendix B showing that $\varphi^{il\star}_{t+1} = \mu^{il\star}_{t+1} = \frac{T-l+1}{q^\star}$ and (ii) using $Q^{l\star}_{t+1} = 0$ for $l \geq t+2$.

where $\mathcal{D}_t^{\mathsf{sym}}$, defined by equation (107), is a discount for limited diversification. For a non-competitive trader, the marginal expected value of the asset is the competitive price adjusted for the cost of limited diversification. On the right-hand side, $A_{t+1}^i = \alpha(1+\gamma_{t+1})$. With market power, diversification speed is always strictly less than 1, leading to under-diversification relative to the first-best, so $\gamma_{t+1}^i > 0$. Equation (19) then gives:

$$\mathbb{E}_t(p_{t+1}^\star) - p_t - \mathcal{D}_t^{\mathsf{sym}} = \lambda_t x_t^i + \alpha(1+\gamma_{t+1})\sigma^2 X_t^i \tag{25}$$

The left-hand side has the same structure as in the competitive case, except for the limited diversification discount. The right-hand side shows that the trader must now be compensated not only for holding an inventory X_t^i, with effective risk $\alpha\sigma^2(1+\gamma_{t+1})$, but also for price impact on the units traded at t.

In the asymmetric case, traders additionally account for their impact on the distribution of inventories. Thus, A_{t+1}^i and φ_{t+1}^{il} now include new terms in $\frac{\partial \mathbf{Q_{t+1}}}{\partial x_t^i}$ (see Definition 3) and cannot be written in closed-form. But the logic remains the same. The marginal expected value of the asset includes a discount for underdiversification as in the symmetric case, and an extra discount or premium for distributional effects. Indeed, the term φ_{t+1}^{il} has two components, so that

$$\begin{aligned} D_t - \sigma^2 \sum_{l=t+1}^{T} \varphi_{t+1}^{il} \sum_{j=1}^{n} f_{t+1}^{jl} X_t^j = & \underbrace{D_t - \sigma^2 \sum_{l=t+1}^{T} \mu_{t+1}^{il} \sum_{j=1}^{n} f_{t+1}^{jl} X_t^j}_{\text{expected price - discount for limited diversification}} \\ & + \underbrace{\sigma^2 \sum_{l=t+1}^{T} r_{t+1}^{il} \sum_{j=1}^{n} f_{t+1}^{jl} X_t^j}_{\text{discount or premium for change in inventory distribution}} . \end{aligned} \tag{26}$$

Equation (19) then gives, for every trader i:

$$D_t - \sigma^2 \sum_{l=t+1}^{T} \mu_{t+1}^{il} \sum_{j=1}^{n} f_{t+1}^{jl} X_t^j + \sigma^2 \sum_{l=t+1}^{T} r_{t+1}^{il} \sum_{j=1}^{n} f_{t+1}^{jl} X_t^j - p_t = \lambda_t^i x_t^i + \sigma^2 A_{t+1}^i X_t^i \tag{28}$$

A key departure from the benchmark cases is that both the diversification and the distribution discounts are now trader-specific. In the competitive case, despite heterogeneous risk aversion, all traders assign the same expected value to the asset and the left-hand side of the indifference condition is independent of i. Under symmetric market power, while the marginal cost (RHS of equation (25)) is trader-specific because of differences in inventories,

the marginal benefit remains the same across traders as it only depends on aggregate inventories. Under asymmetric market power, two traders, i and j, do not have the same effect on the distribution of inventories when they vary their trading intensity x_t^i, leading to different expected marginal values, and thus different marginal benefits.[21]

This fact complicates the derivation of equilibrium demand schedules. In the competitive and symmetric cases, we can directly read the equilibrium schedules from equations (23) and (25), because total inventories do not change over time: $\sum_j X_t^j = \sum_j X_{t-1}^j$. Instead, in the asymmetric case, the marginal benefit of an extra unit of the asset to trader i depends on how other traders trade. Formally, trader i's equilibrium discounts depend on linear combinations $\sum_j (\boldsymbol{\varphi}^{\mathbf{i}}_{\mathbf{t+1}} \cdot \mathbf{f}^{\mathbf{j}}_{\mathbf{t+1}}) X_t^j$ and one must first invert a system of equations to obtain her demand schedule. Indeed, equation (19) can be written as follows:

$$\mathcal{M}_{\mathbf{t}} \mathbf{x_t} = (D_t - p_t)\mathbf{1}_n - \sigma^2 \mathcal{A}^{\mathbf{X}}_{\mathbf{t}} \mathbf{X_{t-1}},$$

where $\mathbf{x_t}^\top = \begin{pmatrix} x_t^1 & \dots x_t^n \end{pmatrix}$ is the vector of trades, $\mathbf{1}_n$ is a column-vector of ones of isze n, $\mathbf{X_{t-1}}$ the vector of initial inventories and $\mathcal{M}_{\mathbf{t}}$ is defined by equation (130).

3.4.2 Existence conditions

To form an equilibrium, equation (19) must be paired with the second-order condition $\lambda_t^i + A_{t+1}^i + \boldsymbol{\varphi}^{\mathbf{i}}_{\mathbf{t+1}} \cdot \mathbf{f}^{\mathbf{i}}_{\mathbf{t+1}} \geq 0$. In the remainder, I will assume that the following conditions are satisfied for every agent:

Condition 1

$$A_{t+1}^i \geq 0, \quad A_{t+1}^i + \boldsymbol{\varphi}^{\mathbf{i}}_{\mathbf{t+1}} \cdot \mathbf{f}^{\mathbf{i}}_{\mathbf{t+1}} \geq 0.$$

These conditions were satisfied in all numerical examples I considered. Under these conditions, the next result shows whether and when a unique equilibrium exists. To simplify expressions, I use the following notation:

$$F_{t+1}^{ik} = \boldsymbol{\varphi}^{\mathbf{i}}_{\mathbf{t+1}} \cdot \mathbf{f}^{\mathbf{k}}_{\mathbf{t+1}}. \tag{29}$$

[21] An analogy with classical asset pricing in complete/incomplete markets may also be useful. In complete markets, state prices are unique, and assets can be valued unambiguously. In incomplete markets, several state prices can satisfy no-arbitrage conditions and different investors may assign different values to the same cash flow. As market power induces a form of endogenous incompleteness, it is not surprising that the expected marginal value of the asset becomes trader-specific.

Proposition 2 (Dynamic equilibrium) *Suppose that Condition 1 holds.*

1. *A dynamic equilibrium in which equation (2) holds, exists and is unique if and only if there exists a unique vector* $\left(L_t, \mathcal{W}_t^{1L}, \ldots, \mathcal{W}_t^{nL}\right)$*, with* $L_t > 0$ *and* $\mathcal{W}_t^{iL} \in \mathbb{R}$*, solving*

$$L_t = \sum_j l_t^j, \tag{30}$$

$$\mathcal{W}_t^{iL} = \sum_j F_{t+1}^{ik} l_t^k, \tag{31}$$

$$\text{where } l_t^i = \frac{(n-1)(2+\mathcal{W}_t^{iL}) - (A_{t+1}^i + \sum_k F_{t+1}^{ik})L_t + \sqrt{D_t^i(L_t, \mathcal{W}_t^{iL})}}{2(n-1)A_{t+1}^i} > 0, \tag{32}$$

   and $\mathcal{D}_t^i = \left[(A_{t+1}^i + \sum_k F_{t+1}^{ik})L_t - (n-1)(2+\mathcal{W}_t^{iL})\right]^2 + 4(n-1)A_{t+1}^i L_t$*.*

2. *If such a solution exists, then price impact is proportional to* σ^2*, with*

$$\lambda_t^i = \frac{\sigma^2}{l_t^i}, \tag{33}$$

   and the other demand schedule coefficients are given by

$$c_t^i = \frac{A_{t+1}^i}{(l_t^i)^{-1} + A_{t+1}^i} \in (0,1), \tag{34}$$

$$d_t^{ij} = (1 - c_t^j) \sum_{l=t+1}^{T} f_{t+1}^{jl} \delta_t^{il}, \tag{35}$$

$$\delta_t^{il} = \sum_{k=1}^{n} \omega_t^{ik} \varphi_{t+1}^{kl}, \tag{36}$$

   where ω_t^{ik} *are the entries of* ${\mathbf{M_t}}^{-1}$*, with* $\mathbf{M_t}$ *defined by* $\sigma^2 \mathbf{M_t} = \mathcal{M}_\mathbf{t}$*.*

3. *Value function coefficients are defined recursively by equations (149)-(151) in the Appendix.*

If one of the conditions in Condition 1 does not hold, either the equilibrium does not exist or it is not unique.

Equation (35) is key for the representation of the equilibrium in terms of diversified risk. The reason is that by Definition 2, $(1 - c_t^j) f_{t+1}^{jl} = f_t^{jl}$. Thus, if the coefficient d_t^{ij} in the equilibrium schedules (6) is a linear combination of f_{t+1}^{jl} at $t+1$, then it is so at t as well. As

I emphasize in the proof, scaling price impact by the variance is natural but not necessary for the result. [22]

3.5 Implications

I now turn to the main economic implications for pricing and trading strategies in asymmetric thin markets.

3.5.1 Pricing

I begin by collecting the observations made so far about pricing in thin vs competitive markets. First, recall that when the market is competitive, pricing is determined in a CAPM fashion by (exogenous) nondiversifiable risk. Nondiversifiable risk corresponds to the supply s of the risky asset: traders must eventually hold the risk and require compensation for it. For accounting reasons, the supply s coincides with the aggregate inventories in the market, $\sum_j X^j_{t-1} = s$. When traders are competitive, they diversify their entire inventories, so diversified risk corresponds to aggregate inventories. Thus, in competitive markets, the following equality holds: priced risk = nondiversifiable risk = aggregate inventories = diversified risk, and the notion of diversified risk is redundant. In thin markets, this equality breaks down: pricing is solely determined by diversified risk. Nondiversifiable risk still corresponds to aggregate inventories, but these are no longer sufficient to determine pricing, except when market power is symmetric. In that special case, diversified risk is proportional to aggregate risk. To sum up:

Corollary 1 (Priced risk in competitive vs thin markets)

1. *In competitive markets,*

$$\textit{priced risk} = \textit{nondiversifiable risk} = \textit{aggregate inventories} = \textit{diversified risk.}$$

2. *In symmetric thin markets,*

$$\begin{aligned} \textit{priced risk} &= \textit{diversified risk} \\ \textit{diversified risk} &\propto \textit{aggregate inventories} \\ \textit{aggregate inventories} &= \textit{nondiversifiable risk} \end{aligned}$$

[22] In Proposition 2, the equilibrium risk premium and trade are characterized by backward induction. Corollary 4 in Appendix C.3 gives the equilibrium as a function of primitives.

3. In asymmetric thin markets,

$$\begin{aligned} \textit{priced risk} &= \textit{diversified risk} \\ \textit{nondiversifiable risk} &= \textit{aggregate inventories.} \end{aligned}$$

These results generalize Malamud and Rostek (2017)'s insights obtained in a static setting to a dynamic, nonstationary framework. With a single trading round, diversified risk boils down to the static portfolio. With multiple rounds, order flow composition changes over time, so diversified risk has a term structure, as it is necessary to account for the trajectory of diversification trades of each trader.

In equilibrium, large traders diversify at lower speed, so their order flow is more persistent: while small traders "front-load" by diversifying aggressively early on, large traders "back load". This implies that for given initial inventories, large traders order flow is likely to represent a larger fraction of diversified risk at later rounds than early ones. Formally, $\frac{f_t^{il}}{f_t^l}$ is likely to be small for a large trader when l is close to t and larger as $l-t$ goes up. This can already be seen in a simple two-period case.

Consider a market with five strategic traders, with $\alpha^i = i$, $i = 1, \ldots, 5$ and identical initial endowments. If there is a single trading round, the diversification speeds are given by:

	Trader 1	Trader 2	Trader 3	Trader 4	Trader 5
c_t^i	0.498	0.696	0.787	0.837	0.868
$\frac{c_t^i}{\bar{c}_t}$	14%	19%	21%	23%	24%

Thus, in a single round, traders with higher risk aversion represent a larger fraction of the order flow. With two rounds, the same holds in the first but not in the second round, where larger traders account for a larger share of the order flow.

	Trader 1	Trader 2	Trader 3	Trader 4	Trader 5
$f_t^{it} = c_t^i$	0.440	0.606	0.695	0.752	0.791
$\frac{f_t^{it}}{f_t^t}$	13%	18%	22%	23%	24%
$f_t^{it+1} = c_{t+1}^i(1-c_t^i)$	0.280	0.274	0.240	0208	0.182
$\frac{f_t^{it+1}}{f_t^{t+1}}$	24%	23%	20%	18%	15%

Of course, the fact would be compounded if instead of equal initial inventories, large traders started with larger ones.[23]

[23]This mechanism does not depend on the particular distribution of risk aversion in the example. The

This term structure of diversified risk has important consequences for pricing. Smaller traders' diversification trades represent a larger fraction of the order flow in the current round and at short horizon. However, large traders absorb this flow only slowly as they trade less aggressively. This means that in equilibrium, in the current round and at short horizon, smaller traders are likely to remain the marginal holders, and thus the marginal pricers, of the asset. Thus, when the inventory distribution is initially geared towards smaller traders, the price exhibits a discount. If instead small traders are initially under-exposed to the risky asset, it trades at a premium. These price distortions progressively revert as the asset is reallocated towards larger traders. The reversal is stronger at the beginning because the marginal benefit of the reallocation to small traders is the largest then. Asymmetric market power thus leads to time-series momentum, especially at short horizon (see Figure 2).

In equation (2) for the risk premium, the impact of diversified risk on the price depends on the market power-adjusted risk bearing capacity q_t and on the coefficients $\pi_{\mathbf{t}}$. For current-period diversified risk, $\pi_t^t = 1$. For future-period diversified risk, $\pi_t^l = \bar{\delta}_t^l = \sum_i \delta_t^{il}$, where $\delta_t^{il} = \sum_k \omega_t^{ik} \varphi_{t+1}^{kl}$ measures the marginal cost, in expected utility terms, to trader i of being exposed to an extra unit of diversified risk at round l. In equilibrium δ_t^{il} depends – in a different way for each trader – on a combination of all traders' diversification and inventory discounts. Since $\pi_t^l = \bar{\delta}_t^l$, the price simply aggregates all these trader-specific marginal costs.

While these coefficients cannot be directly expressed as a function of primitives, extensive numerical analysis shows that for $l > t$, $\pi_t^l \in (0, 1)$ and provides two key comparative statics: (i) holding t constant, $\pi_t^l > pi_t^{l'}$ for $l' > l$, so that shorter-horizon order flow has a larger price impact ; (ii) holding $l \geq t$ constant, $\frac{\pi_{t+1}^l}{q_{t+1}} > \frac{\pi_t^l}{q_t}$, so that the price impact of a marginal increase in diversified risk at a given round $l > t$ increases over time. Intuitively, as round l approaches, the price increasingly reflects the fact that small traders will be the marginal pricers. Essentially, if we could price the asset by buidling a representative agent, her social welfare function would initially give more weight to smaller traders. These effects are absent in competitive and symmetric thin markets:

Lemma 2 *In both competitive and symmetric thin markets, the price impact of a marginal increase in diversified risk at round $l > t$ is constant over time and depends on the aggregate marginal utility at l.*

In competitive markets, the price impact an increase in diversified risk at $l > t$ is given by $-\frac{(T-l+1)\sigma^2}{q^\star}$ (see equation (51) in Appendix B). Since there are no term structure effects and risk is always perfectly diversified, a shock to diversified risk at l is priced at the aggregate

same effect arises when a single trader differs in risk aversion from the others.

marginal utility *in that round* and is thus independent both of t and of the distance to the shock $l-t$.[24] A similar effect occurs in symmetric thin markets, because the composition of the order flow is constant over time (see equations (69) and (77) in Appendix B). The marginal impact is given by $\frac{\partial p_t}{\partial Q_t^l} = -\sigma^2 \mu_l^l = -\sigma^2 \frac{1}{q^\star}\left(T-l+1+(1-c)(1+\gamma_{l+1})\right)$, and is thus also independent of t and of $l-t$.[25]

In symmetric thin markets, the marginal impact of an increase in diversified risk is given by the sum of the competitive risk premium at round l, $-\sigma^2 \frac{T-l+1}{q^\star}$ and of a hedging component caused by limited diversification, $-\sigma^2 \frac{1}{q^\star}(1-c)(1+\gamma_{l+1})$. In asymmetric thin markets the price effect of an increase in future-round diversified includes an extra strategic component linked to the inventory distribution, as discussed at the individual trader level in Section 3.4: Definition 3 and equation (21) imply that, for $l \geq t+1$, $\pi_t^l = \sum_k \bar{\omega}_t^k \mu_{t+1}^{kl} - \sum_k \bar{\omega}_t^k r_{t+1}^{kl} \equiv \pi_t^{l,div} + \pi_t^{l,inv}$. The risk premium can then be decomposed into three terms:

$$D_t - p_t = \frac{\sigma^2}{q_t}\left(\underbrace{\pi_t^t Q_t^t}_{static} + \underbrace{\sum_{l=t+1}^{T} \pi_t^{l,div} Q_t^l}_{diversification} + \underbrace{\sum_{l=t+1} \pi_t^{l,inv} Q_t^l}_{inventory}\right). \tag{37}$$

Numerical analysis shows that the strategic component pushes the price up: traders benefit from being able to influence the risk premium and target portfolio by affecting the distribution of inventories. Instead, the hedging component lowers the price, and so does the static component (Figure 2).

3.5.2 Trading strategies

The change in the composition of the order flow at different horizons also has important consequences for the dynamic trading strategies in asymmetric thin markets.

Corollary 2 (Optimally myopic trading vs dynamic strategies)

1. *Under the conditions of Propositon 2, when market power is asymmetric,*

   (a) Traders always go long current-round diversified risk and may go long or short

[24] The aggregate marginal utility depends on aggregate risk aversion $1/q^\star$ and the distance to maturity, because it determines the risk at t. This is because fundamenal news is realized at every round, so the conditional variance of the asset payoff decreases as time passes.

[25] This follows from equation (102) and $q^\star = \frac{n}{\alpha}$ when $\alpha^i = \alpha$ for all traders.

future-round diversified risk:

$$\theta_t^{it} = \phi_t^i \equiv \frac{\beta_t^i}{\bar{\beta}_t} \in (0,1), \quad \theta_t^{il} \in \mathbb{R}, \quad \forall i, \ \forall l \geq t+1. \tag{38}$$

(b) The positions in future-round diversified risk are "speculative", in the sense that they sum up to zero, while positions in the static portfolios sum up to one:

$$\sum_i \theta_t^{it} = 1, \qquad \sum_i \theta_t^{il} = 0, \quad \forall l \geq t+1. \tag{39}$$

2. *In the competitive and symmetric cases, trading is only (but optimally) myopic:* $\theta_t^{il} = 0$, $\forall l \geq t+1$.

Results 1a and 1b provide essential insights into the trading strategies of traders with asymmetric market power. All traders get positive exposure to current-round diversified risk and traders representing a larger fraction of the market power-adjusted risk bearing capacity (i.e., more risk tolerant ones, with higher ϕ_t^i) buy a larger fraction of this portfolio. Exposure to future-round diversified risk is given by

$$\theta_t^{il} = \phi_t^i \bar{\delta}_t^l - \delta_t^{il}, \quad \forall l \geq t+1. \tag{40}$$

This expression shows that a trader with a marginal valuation close to the "average" will have a small θ_t^{il} in absolute value, while traders with either a higher or lower than average risk tolerance / marginal valuation will take larger long or short positions.

As for π_t^l, the θ_t^{il}'s can be decomposed into a hedging and a strategic components capturing limited diversification and inventory distribution effects: $\theta_t^{il} = \theta_t^{il,div} + \theta_t^{il,inv}$, for any $l \geq t+1$. Numerical analysis shows that the hedging component quantitatively dominates and that exposure to round-l diversified risk results from two conflicting effects. For instance, small traders' hedging component is negative, while their strategic component is positive (but smaller) at horizon $l > t+1$. It is the opposite for larger traders who take the other side (Figure 8). Intuitively, larger traders diversify more slowly than smaller ones, so their diversification trades will account for a larger part of future diversified risk. Thus, small traders will continue to absorb the order flow from today's inventories of large traders at later rounds, and seek to hedge against this, leading to a negative $\theta_t^{il,div}$, for $l > t$. However, large traders' persistent order flow also generates trading opportunities as it continues to distort the price at mid-to-long horizons. Small traders exploit these distortions, leading to

a postive $\theta_t^{il,div}$ for $l > t+1$.

To sum up, traders trade future-round diversified risk both for strategic reasons because the price is not first-best and to hedge their future over- or under-exposure to the asset caused by the change in the composition of order-flow over time.

As point 2 in the Corollary shows, both market power and differences in risk aversion are necessary for traders to engage in this dynamic trading. There are neither hedging nor strategic components in competitive markets as prices and risk sharing are first best. When traders have symmetric market power, the composition of the order-flow is constant over time, and this eliminates the hedging component. Further, the price depends on aggregate inventories, i.e., the total supply, so traders do not take into account their impact on the distribution of inventories, eliminating the strategic component.

4 Shock absorption in asymmetric thin markets

In this section, I introduce anticipated supply and demand shocks. I first derive the predictions of the competitive and symmetric market power benchmarks and contrast them to the facts collected in Section 1.2. Then I study price reaction to the shocks and trading strategies in asymmetric thin markets.

4.1 Extending the definition of diversified risk

Since future shifts in supply or demand affect the future order flow, one can easily extend the definition of diversified risk to include nonstrategic and strategic supply and demand shocks. For the sake of brevity, I focus on nonstrategic shocks in the text and provide a formal treatment of strategic demand adjustments, which require somewhat heavier notations, in the Online Appendix.

Lemma 3 (Diversified Risk with Nonstrategic Shocks) *In the presence of nonstrategic supply and demand shocks, define diversified risk as*

$$\begin{aligned} Q_t^t &= \sum_{j=1}^{n} c_t^j X_{t-1}^j + c_t^{s_t} \Delta s_t \\ Q_t^l &= \sum_{j=1}^{n} f_t^{jl} X_{t-1}^j + c_l^{s_l} \Delta s_l \quad \textit{for } l \geq t+1, \end{aligned} \tag{41}$$

where $f_t^{jl} = c_l^j \prod_{s=t}^{l-1}(1-c_s^j)$, $(l \geq t)$ *and* $c_l^{s_l} = 1$. *Then, all the results derived in Section 3.4, including equations (1) and (2), the existence conditions, and Corollary 2 hold under this extended definition.*

The extension of the definition of diversified risk is intuitive. The coefficient of the shock is $c_l^{s_l} = 1$, $l \geq t$. This is because, in line with the literature (Duffie, 2010, Rostek and Weretka, 2015), I model a nonstrategic agent selling (or purchasing) a block Δs_l on the market at time l. This agent is thus equivalent to a competitive trader entering the market at time l to diversify Δs_l.

4.2 Benchmark effects

I now review the effects of anticipated non strategic supply/demand shocks in the benchmark models, assuming that a single shock, announced at t^a, hits the market at t^r. To fix ideas, I assume that $\Delta s_{t^r} > 0$.

4.2.1 Competitive model

The impact of the shock on prices and trading strategies can be directly understood from Lemma 2 and Corollary 2 (see also Lemma 4, equations (52)-(48), and Figure 3). At the announcement, the price adjusts to the level implied by the marginal value of the asset at t^r. There is no further price effect. Traders are optimally "myopic" and do not trade based on the shock until the realization. At realization, they immediately adjust their portfolios and reach the new Pareto-optimal allocation in one trade.

4.2.2 Symmetric thin markets

Under symmetric market power, the price effects of the anticipated shock differ somewhat from the competitive benchmark, although prices coincide in the absence of shocks. At the announcement, the price overshoots the competitive price. There is no further effect until the realization, at which point the price drops further. One round later, the price rebounds to the competitive level (Figure 3, and Lemmata 5 and 3).

The price overshoot at realization results from the introduction of a competitive issuer, which eliminates the symmetry in market power in that round. Indeed, as mentioned in Lemma 3, $c_l^{s_l} = 1 = c_l^\star$, while for strategic traders, $c^i = c \in (0,1)$, reflecting the fact that strategic traders, who are net buyers, have more market power than the issuer, who is a net

seller. As in the asymmetric setting, the price must therefore drop for the market to clear.[26] This asymmetry disappears one round later, as only strategic traders remain in the market, explaining why the price immediately returns to the competitive level.

The absence of price drift between the announcement and the realization follows from Lemma 2. Now consider the time between the announcement and the realization. The price does not fully drop at the announcement, because the market is now imperfectly liquid: the difference between the price at t^a and the price at t^r reflects the cost of a round-trip trade induced by price impact.

There is no trade in anticipation of the shock between the announcement and the realization. This is a direct consequence of Point 2 in Corollary 2. Intuitively, since all traders have the same degree of market power, the shock is efficiently allocated at realization, eliminating the need to hedge ex-ante.

Clearly, both benchmark models are unable to account, even qualitatively, for the observed price effects (Fact 1 in Section 1.2). The competitive model can only explain the price reaction at announcement. The symmetric market power model can explain the reaction at announcement and at realization, but fails to explain the drift between the announcement and the realization and the slow subsequent reversal. Both benchmarks predict that investors trade only at the realization of the shock, at odds with Facts 2 and 3 in Section 1.2.

4.3 Anticipated shocks in asymmetric thin markets

When market power is asymmetric, the model can account for the three key facts and delivers new, specific predictions, both about the price path and trading dynamics. To highlight the differences with the symmetric market power benchmark, I consider a minimal departure from the symmetric case, using a binary structure with two groups of traders differing in risk aversion ($n-m$ $a$verse with risk aversion a and $m \geq 1$ $b$older traders with risk aversion $b < a$, so that a-traders are smaller institutions and b-traders larger ones). I use a mix of analytical and numerical results to compare the effects to the symmetric case. The impact of the anticipated shock on the risk premium, inventories, and diversified risk is denoted $\Delta(D_t - p_t)$, ΔX_t, and ΔQ^l_t, respectively.

A key difference between the models concerns their ability to generate momentum before the shock (i.e., a drift between the announcement and the realization) and reversal afterwards (i.e., a progressive –as opposed to immediate– rebound in the price). To generate such effects,

[26] Formally, bid-shading leads to a decrease in risk bearing capacity relative to the competitive one: using equation (90), $q^{\mathsf{sym}}_{t^r} = \frac{nc}{\alpha\sigma^2(1+\gamma_{t+1})} < \frac{n}{\alpha} = q^\star$, so $\Delta(D_{t^r} - p_{t^r})^{\mathsf{sym}} = \frac{1+\gamma_{t+1}}{c}\Delta(D_{t^r} - p^\star_{t^r}) > \Delta(D_{t^r} - p^\star_{t^r})$.

it is necessary to have at least two trading rounds before the realization, and two trading rounds after the realization, thus I assume that $t^r - t^a \geq 2$ and $T - t^r \geq 2$.

A key driver of the V-shaped pattern is what happens at the realization of the shock: unlike the symmetric market power case, the shock is not optimally shared among traders, leading to both pre- and post-shock trading and the V-shaped price reaction. To illustrate this shock misallocation, I begin with a surprise shock, i.e., $t^a = t^r$.[27]

4.3.1 Surprise shock: shock misallocation, retrading, and reversal

When a shock hits the market by surprise, initially the order flow composition is modified only in the current round, as an extra trader (the issuer) liquidates a block Δs_{t^r}. Since the issuer sells in one go and traders have not yet adjusted their inventories, future order flow composition remains unchanged at t^r. Hence, the change in diversified risk is

$$\Delta Q^{t^r}_{t^r} = \Delta s_{t^r} \quad \text{and} \quad \Delta Q^{l}_{t^r} = 0,\ l \geq t^r + 1.$$

The only effect on inventories at the realization is thus related to how traders absorb the shock, with the constraint that the market must clear. The change in inventories at t^r is given by

$$\Delta X^a_{t^r} = (1 - c^a_{t^r})\Delta X^a_{t^r-1} + \sum_{l=t^r}^{T} \theta^{al}_{t^r} \Delta Q^l_{t^r} = \phi^a_{t^r} \Delta s_{t^r}, \tag{42}$$

becaues $\theta^{at^r}_{t^r} = \phi^a_{t^r}$ in $(0, 1)$. Similarly, for larger traders, $\Delta X^b_{t^r} = \phi^b_{t^r} \Delta s_{t^r}$, where $(n-m)\phi^a_{t^r} + m\phi^b_{t^r} = 1$. The ϕ^i_t coefficients determine how the additional risk is shared at realization, as $\phi^i_t \equiv \frac{\beta^i_t}{\bar{\beta}_t}$ is trader i's risk bearing capacity adjusted for market power, with $\phi^i_t \neq \phi^{i\star}_t \equiv \frac{1}{\alpha^i q^\star}$. Here, $\phi^a_{t^r} < \phi^b_{t^r}$, but $\phi^a_{t^r} > \phi^{a\star}_{t^r}$. Instead, in symmetric markets, at any round $\phi^i_t = \frac{1}{n} \equiv \phi^\star_t$.

How strategic traders share risk at realization determines the post-realization price path. In symmetric markets, the price overshoots the competitive price at realization because buyers (strategic traders) have more market power than the seller (the non strategic issuer) in that round. In asymmetric thin markets, there is asymmetry in market power between the seller and the buyers at realization, and also between the buyers themselves, both at realization and afterwards. Thus, the shock is initially misallocated, which leads to re-trading, both for diversification purposes, and for strategic purposes, as the price is distorted.

[27] Lemma 12 in the Appendix provides results for both anticipated and surprise shocks.

At round t^r+1, the impact on round-l diversified risk is given by

$$\Delta Q^l_{t^r+1} = \sum_j f^{jl}_{t^r+1} X^j_{t^r} = \left[(n-m) f^{al}_{t^r} \phi^a_{t^r} + m f^{bl}_{t^r} \phi^b_{t^r}\right] \Delta s_{t^r} \equiv PSM^l_{t^r} \Delta s_{t^r},$$

where PSM stands for projected shock misallocation. For l close to t^r, the order-flow is tilted towards an inefficient allocation since $f^{at}_t > f^{bt}_t$ and $\phi^a_t < \phi^{a\star}_t$. For l closer to T, $f^{al}_t < f^{bl}_t$, so the order flow is tilted towards more efficient allocations. The intuition is the same as in Section 3.5.1: small traders will quickly resell their extra inventory, but this order flow is only slowly absorbed by large traders, perpetuating the misallocation at short horizon. This leads to a gradual rebound, as small traders remain the marginal pricers at short horizon.

Meanwhile, the simplicity of the single shock setting reveals complicated 2^{nd}, 3^{rd}, ..., n^{th} round effects governing the price dynamics that were not transparent in the recursive characterization of Section 3.5.1: as the initial misallocation is not immediately eliminated, the price distortion persists, which leads to additional hedging and strategic trading motives, which in turn move the price, leading to further hedging and strategic trading, and so on. These further-round effects are captured by the second term in the bracket in the following expression for diversified risk at round $t > t^r + 1$:

$$\Delta Q^l_t = \left[PSM^l_{t^r} + m(n-m) \sum_{k=t^r+1}^{t} \sum_{l'=k}^{T} \hat{\theta}^{l'}_k PSM^{l'}_{t^r} (f^{al}_t z^a_{t-1k} - f^{bl}_t z^b_{t-1k}) \right] \Delta s_{t^r},$$

Yet, as time passes, just like the initial misallocation gets progressively corrected, these further round effects die out and the price gradually rebounds. Corollary 6 in the Appendix gives a formal condition.

4.3.2 Anticipated shock

I now assume that the shock is pre-announced at $t^a < t^r$, with $t^r - t^a \geq 2$ and show that despite pre-shock trading, the post-shock dynamics will remain qualitatively unchanged.

As the shock is announced, traders understand that the composition of the future order flow will change at round $l = t^r$, when the shock hits. The composition of the order flow at other rounds is unchanged, because traders have not yet adjusted their inventories, so $\Delta Q^l_{t^a} = \Delta s_{t^r} \mathbb{1}_{l=t^r}$, where $\mathbb{1}_{l=t^r}$ is a dummy variable that takes value 1 when $l = t^r$ and 0 otherwise. The change in their inventories at t^a is thus driven only by change in diversified

risk:

$$\Delta X^a_{t^a} = (1 - c_{t^a})\Delta X^a_{t^a-1} + \sum_{l=t^a}^{T} \theta^{al}_{t^a} \Delta Q^l_{t^a} = \theta^{at^r}_{t^a} \Delta s^r_t.$$

With two levels of risk aversion, traders' dynamic trading/hedging is driven by a common factor $\hat{\theta}^l_t$, for any $l \geq t+1$: $\theta^{al}_t = m\hat{\theta}^l_t$ and $\theta^{bl}_t = -(n-m)\hat{\theta}^l_t$ (see Lemma 5 in Appendix C.4). As more risk averse traders short future-round diversified risk, $\theta^{al}_t < 0$ and $\hat{\theta}^l_t < 0$. At the shock announcement, the change in inventory is thus given by

$$\Delta X^a_{t^a} = m\hat{\theta}^{t^r}_{t^a} \Delta s_{t^r} < 0 \quad \text{and} \quad \Delta X^b_{t^a} = -(n-m)\hat{\theta}^{t^r}_{t^a} \Delta s_{t^r} > 0,$$

i.e., small traders short as they expect being over-exposed to the risky asset due to the asymmetry in market power at t^r, and large traders take the other side. This leads to an increase in the risk premium,

$$\Delta(D_{t^a} - p_{t^a}) = \sigma^2 \frac{\pi^{t^r}_{t^a}}{q_{t^a}} \Delta s_{t^r}.$$

which measures the announcement effect. Since traders change their inventories at the announcement and will later diversify these new positions, the composition of future order flow is modified in the rounds that follows, which then modifies the target inventories themselves. Thus, as with a surprise shock, trading begets trading.

Formally, at round $t > t^a$, inventories and the risk premium are given by:

$$\Delta X^a_t = m \left(\sum_{k=t^a}^{t} \hat{\theta}^{t^r}_k z^a_{tk} \right) \Delta s_{t^r} \tag{43}$$

$$\Delta X^b_t = -(n-m) \left(\sum_{k=t^a}^{t} \hat{\theta}^{t^r}_k z^b_{tk} \right) \Delta s_{t^r} \tag{44}$$

$$\Delta(D_t - p_t) = \frac{\sigma^2}{q_t} \pi^{t^r}_t \Delta s_{t^r} + \frac{\sigma^2}{q_t} m(n-m) \Delta s_{t^r} \left(\sum_{k=t^a}^{t} \hat{\theta}^{t^r}_k \sum_{l=t}^{T} \pi^l_t (f^{al}_t z^a_{t-1k} - f^{bl}_t z^b_{t-1k}) \right) \tag{45}$$

Equations (43) and (44) show that inventories change due to a series of dynamic hedging and strategic trades that are partly diversified away and strategically exploited as time passes (this is the meaning of the dampening coefficients $z \geq 0$, defined recursively by equation (174), with $z^a_{tt} = z^b_{tt} = 1$). Since the common factor $\hat{\theta}^{t^r}_t$ does not change sign over time,

smaller traders keep shorting and larger ones buying before the shock.

Equation (45) helps understanding the dynamics of the risk premium between the announcement and the realization. There are two conflicting effects, captured by the two terms in the equation. One the one hand, as time passes, the price reflects more and more that small traders will be the marginal holders and pricers at the realization (first term). This effect leads to a gradual buildup in the risk premium. On the other hand, as time passes, traders engage in a series of hedging and strategic trades, which contribute to improving risk sharing (second term). This effect tends to reduce the risk premium as time passes and trade takes place.

To see that the second term is negative, recall that $\hat{\theta}_k^{t^r} < 0$ and that $\pi_t^l > \pi_t^{l'}$ for a given t and $l' > l$. Further, $f_t^{al} z_{t-1k}^a - f_t^{bl} z_{t-1k}^b$ is positive for l close to t, and negative for $l >> t$. However, quantitatitvely the terms close to t dominate. Thus, $\sum_{l=t}^{T} \pi_t^l (f_t^{al} z_{t-1k}^a - f_t^{bl} z_{t-1k}^b)$ is positive, and $\sum_{k=t^a}^{t} \hat{\theta}_k^{t^r} \sum_{l=t}^{T} \pi_t^l (f_t^{al} z_{t-1k}^a - f_t^{bl} z_{t-1k}^b)$ is negative, showing that dynamic trading reduces the risk premium.

Extensive numerical analysis suggests that the first term always dominates.[28] First, as large traders prefer to trade slowly, the amount of risk-sharing before the shock is never enough to reduce the risk premium before the shock. Second, the anticipated shock represents a buying opportunity for smaller traders (strategic motive), which fruther limits pre-shock risk-sharing. Consequently, the risk premium gradually increases before the realization and gradually decreases afterwards, an (inverted) V-shaped pattern.[29]

We can study how the V shape is deformed as heterogeneity in risk aversion decreases. An important caveat is that the symmetric model is not the limit of the asymmetric model for $\alpha^i \to \alpha$. This is because making all risk aversions identical *after* solving the fixed point does not yield the same result as assuming that all risk aversions are identical and then solving the simpler fixed point. Nevertheless, it is interesting to study numerically the effects of bringing risk aversion a and b closer and closer to each other, while maintaining $a \neq b$: the price reaction in the asymmetric market becomes more comparable to the symmetric case, with a stronger reaction at the announcement, a more muted drift between the announcement and the realization, and a lower price distortion at the realization. Intuitively, as the asymmetries in market power become smaller, the shock is more optimally shared at realization, reducing pre-shock trading and the drift (see Figure 7).

[28] Corollary 6 in the Appendix gives a formal condition for this to be the case.

[29] The pre-shock trading leads to a lower price reaction at the realization than when the shock is not anticipated, and equations derived in the previous section with surprise shocks must be adjusted to include a pre-shock trading term – see Lemma 12. Qualititatively, these new terms do not change the dynamics, and the risk premium reverts after the realization of the shock.

4.3.3 Implications for design

Given the cost of trading mispriced securities for, e.g., taxpayers (as Treasuries issue or reissue underpriced bonds) or investors in index trackers (who rebalance at distorted prices when indices are recomposed), it is important to understand how to mitigate the V-shaped price effects. The analysis yields at least two interesting implications about the design of nonstrategic shocks.

First, pre-announcing the shock leads to a lower impact at realization, and thus more revenue for the seller of the block Δs_{t^r}. Note that the mechanism is different from the sunshine trading literature, where pre-announcing improves liquidity at realization by attracting liquidity suppliers on that date. Here the improvement comes from additional risk-sharing before the realization.

Second, reducing market power asymmetry also leads to higher revenues for the seller. However, existing market structures are often very concentrated and institution size within a given category of funds follows a power law distribution (Gabaix et al., 2003).

Preventing the concentration of risk bearing capacity into very large institutions through consolidation, for instance via antitrust policies, might become a new regulatory objective, beyond simple measures of the degree of competition. If risk aversion proxies for balance sheet costs, then regulations affecting these costs (SLR, capital requirements, etc.) could become size or price impact-dependent, beyond the current practice of one-size-fits-all. The model also suggests a new potential trade off between total risk bearing capacity and heterogeneity in risk bearing capacity. For instance, lower risk bearing capacity, which potentially increases the price overshoot, could be compensated by reducing asymmetry among traders. Perhaps policies strengthening regulatory requirements on the largest, systemically important institutions can be evaluated through this lens.

Such changes are not neutral, both for smaller and larger traders. On the one hand, larger traders, not surprisingly, are better off at realization when shocks are not pre-announced, as they reap larger benefits from their stronger market power (Figure 6). On the other hand, smaller traders would be worse-off in a more uniform market structure (Figure 5). Indeed, they would have higher price impact, impeding their risk-sharing opportunities. It is thus unrealistic to expect concentrated market structures to evolve towards situations that increase the seller's revenues without regulatory involvement.

4.3.4 New empirical implications

The asymmetric model yields the correct price pattern, pre- and post- shock trading, and trading heterogeneity, including nonmontonic strategies, matching three key observations. However, the anlysis provides at least four additional implications.

First, the model predicts very specific inventory patterns as a function of trader size/price impact.[30] Smaller traders trade first with the wind, then against the wind, then with the wind again. Their inventories overshoot the competitive level at realization and revert afterwards. Larger traders are contrarians throughout and undershoot their competitive inventory level at realization. Before the shock, medium-sized traders first trade like larger traders and then like smaller traders, ending with an almost flat cumulative inventory change at realization (Figure 1).

The predictions about small traders' strategies are remarkably consistent with empirical evidence about dealers' behaviour around shocks: they "make space on their balance sheet" ahead of supply shocks (and instead build up inventories ahead of demand shocks) and provide immediacy when shocks hit, overshooting their long-term inventories (Dick-Nielsen and Rossi, 2018, An and Song, 2023). Dealers' pre-shock trading could be interpreted as a form of front-running. Seen through the lens of the model, it relates to sound risk management. It is small traders' hedging component that dictates selling ahead of the supply shock, while their strategic component dictates buying to exploit the decreasing price. As small traders are more risk averse, hedging dominates. While Duffie (2012) argues that proprietary trading and risk management are indistinguishable in market making, the paper offers a decomposition between these two dimensions that could appeal to structural economists.

More generally, these predictions may guide empiricists studying the cross-section of institutional trading around anticipated shocks, beyond the current focus on cross-sectional or time-series *average* effects for one type of institution at a time.[31] The analysis also suggests a follow-up test on the evidence in Chemmanur et al. (2009), who study pre-SEO trading by insurers and mutual funds and show that the average cumulative order flow is positive for some institutions, negative for some, and about flat for others. Testing whether the sign of the cumulative order flow depends on institutions price impact would provide a test of the theory.

[30] As size can be influenced by misallocated inventories, price impact might a more reliable empirical proxy.

[31] An exception is Dick-Nielsen and Rossi who study dealer behaviour around bond index exclusions and, in addition, provide some evidence about mutual funds and insurers, but only at the shock realization.

Second, all the numerical examples of the model generate a convex increase in the risk premium before the shock (or, equivalently concave decreasing price). As time passes, the marginal cost $\pi_t^{t^r}$ of an extra unit of diversified risk at t^r increases as the time remaining until realization decreases, and increases even more if the remaining time is shorter. This is intuitive. If there are many trading opportunities until the shock, removing one trading opportunity should have little effect. However, the effect should be more pronounced as one approaches the shock. While this prediction seems in line with visual evidence reported in the literature, I am not aware of any formal test.

Third, owing to a reduction in pre-shock trading, a shorter pre-announcement period is associated with a larger price effect at announcement and at realization (Figure 4).

Fourth, in a more uniform structure, anticipated shocks have a larger price effect at the announcement and smaller at realization.

4.4 Strategic demand adjustment in asymmetric thin markets

Introducing strategic shocks does not change the fundamental result that V shapes occur only when traders differ in risk aversion. However, new dynamics arise due to strategic traders' optimal response to shocks. In my main experiment, an endowment shock with constant exposure $e^i > 0$ (implying a drop in demand) hits trader i from t^r to T, with an announcement at $t^a < t^r$.

4.4.1 Benchmarks

In competitive markets, a constant endowment shock has the same effect as a nonstrategic supply shock, holding shock size constant. In settings with symmetric market power, a constant endowment shock yields the same price effect as in competitive markets: a once-and-for-all adjustment at the announcement (Figure 1 in the Online Appendix). Thus, the dynamics are less interesting than with a nonstrategic supply shock, where the price reacts both at announcement and at realization. The reason for this difference is that there is no market power asymmetry when traders receive endowment shocks, while this asymmetry plays a key role when a competitive agent sells a block of shares to strategic traders. Further, in these benchmarks, it does not matter whether all the traders or a subset of them receive shocks, only the aggregate shock matters.

4.4.2 Trader-specific, constant shocks in asymmetric markets

In asymmetric thin markets, the price effects are gradual, as with nonstrategic shocks. However, in contrast to the nonstrategic shocks, where the price-taking agent sells a block in a single round, traders receiving endowment shocks trade optimally over time. Further, traders with different risk aversion naturally follow different strategies. This mechanism generates two novel results: (i) the model predicts quite different price and inventory paths depending on *which* trader or subset of traders receives a shock. (ii) Numerical analysis suggests that, while nonstrategic shocks are associated with two distinct phases (a gradual increase in risk premium followed by a gradual decrease), strategic shocks lead to three distinct phases for the risk premium: two phases before realization —implying a nonmonotonic price path— and a third, largely unchanged, reversal phase after realization.

Consider first the case where the smallest trader receives an endowment shock. Figure 9 shows that after a large price drop at announcement, the price briefly and slightly recovers and then monotonically decreases until realization. After realization, the price simply rebounds. The smallest trader sells in three distinct phases until the shock: aggressively for a few rounds after the announcement, then more slowly, and then aggressively again as the realization nears. Other small traders tend to short ahead of realization, in anticipation of an inventory overshoot at realization, when they absorb the smallest trader's sales. Instead, larger traders buy throughout. Medium-sized traders first buy and then sell until realization. Analysis of inventory dynamics reveals the critical role played by these traders (e.g., trader 3, see Figures 2 to 4 in the Online Appendix). The first phase in the risk premium corresponds to the time during which medum-sized traders emulate the largest trader by buying the asset. When they switch side and start selling, the risk premium builds up until the realization. Medium-sized traders' buying during this first phase is dictated by the strategic motive: they exploit the large price drop at announcement. The smallest trader takes advantage of the initial buying by medium-sized traders; this is what drives her aggressive selling in the first phase. The smallest trader then strategically reduces her selling rate as medium-sized traders switch to selling, as their hedging motive becomes dominant. The second phase of aggressive selling occurs at realization when the small and medium-sized traders provide immediacy along with the largest one, who is buying throughout.[32]

When a large trader is shocked, the pre-shock dynamics are reversed: most of the price reaction occurs at the announcement, followed by a short further price fall and a rebound.

[32]The different phases in the trading strategies bear some resemblance with the "rat race" and "waiting phase" in Foster and Viswanathan (1996), but are solely driven by differences in risk aversion, not in information.

The rebound starts before the realization and accelerates afterwards (Figure 10). Thus, in contrast to nonstrategic shocks, or to strategic shocks to the smallest trader, the risk premium pre-shock trajectory is concave, not convex. Figures 2 to 4 in the Online Appendix show that the largest trader also sells in three distinct phases, but these differences are less pronounced. Intuitively, due to higher price impact, the largest trader tends to spread her liquidation more equally over time, rather than concentrating it in specific periods. The largest trader sells at a somewhat higher rate after the announcement and close to realization. Doing so, she attempts to exploit the periods when the smaller traders (traders 2 to 5) are buying more aggresively. After the announcement, this buying pressure is not enough to offset the largest trader's footprint: as a result, the price drifts downwards for a few rounds. As the largest trader's selling abates, the price starts recovering. As realization nears, smaller traders are buying more aggressively than after the announcement and the rebound accelerates.

4.4.3 Market-wide, constant shocks

Holding total shock size constant, market-wide shocks do not have the same effect as trader-specific shocks. If all traders receive the same anticipated constant shock, then smaller traders hedge by reducing inventories, while larger ones increase their inventories to clear the market. The price reaction is similar to the case where the smallest trader receives a shock. Intuitively, since all traders receive the same shock, it is disproportionaly larger for smaller ones (Figures 5 and 6, Online Appdendix).

4.4.4 Transitory shocks

While a shock of constant intensity permanently reduces a trader's willingness to hold risk, transitory shocks capture a temporary decrease in appetite for the risky asset, e.g., following a liquidity shock such as a loss in a correlated portfolio. The effects of transitory shocks similarly depend on whom is affected. Transitory socks to small traders trigger a gradual increase in the risk premium until realization, followed by a sharp reversal which overshoots when the endowment shock ceases. The intuition for the overshoot on way back is the same as on the way down: when the shock hits, small traders are over-exposed to the asset due to limited pre-shock risk sharing. When the shock stops, small traders need to buy back. As large traders trade slowly, small traders are restrained and buy less in equilibrium than they wish, leading to the overshoot. Transitory shocks to large traders, instead, do not generate an overshoot when the price rebounds (Figures 8 and 9, Online Appdendix).

5 Conclusion

Empirically, shifts in supply or demand lead to V-shaped price reactions, even when their magnitude and timing are well-anticipated. These effects are widespread across asset classes and types of shocks (security issuances, changes in index-trackers' demand, pre-announced trades by central banks or other institutions, etc.), and are at odds with an efficient market response.

In this paper, I show that in the presence of strategic institutions, anticipated supply and demand shocks lead to V-shaped price reactions, provided institutions differ in their risk bearing capacity. To this end, I derive the nonstationary equilibrium of a model where traders with heterogeneous risk aversion compete in demand schedules and characterize dynamic risk sharing via a new equilibrium representation.

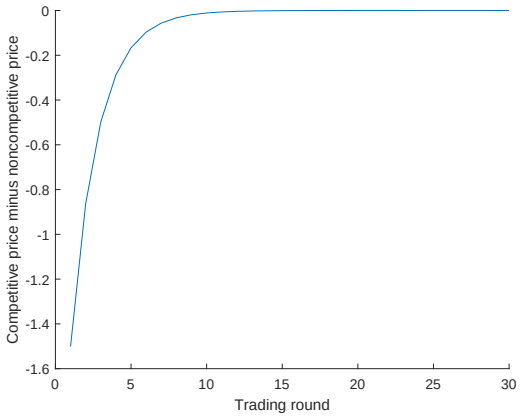
The key mechanism of the model is that traders diversify their inventories at different speeds in equilibrium, with more risk averse traders (i.e., smaller traders) diversifying more aggressively, as they face endogenously smaller price impact costs. Differences in diversification speeds imply that prices and trading strategies cannot be summarized simply by the total quantity of the risky asset, nor even by the joint distribution of inventories and risk aversion in a given round. Instead, the new "diversified risk" representation reveals that prices and trading strategies depend on the endogenous composition of the order flow from diversification trades at different horizons.

This mechanism leads to new pricing and trading dynamics, including short-run momentum in the risk premium and non-monotonic trading strategies for smaller traders, who in the presence of shocks endogenously arise as intermediaries between the agent selling –or buying back– new shares and larger traders. Anticipated supply or demand shocks generate time-series momentum and reversal marked by a gradual increase in the risk premium until the shock realization and a gradual reversal afterwards. The model qualtitatively matches key features of the data and predicts institutional trading as a function of price impact. The analysis also shows that the V-shaped price patterns, which are costly for issuers and passive investors, can be mitigated by making the distribution of the risk bearing capacity more uniform across institutions.

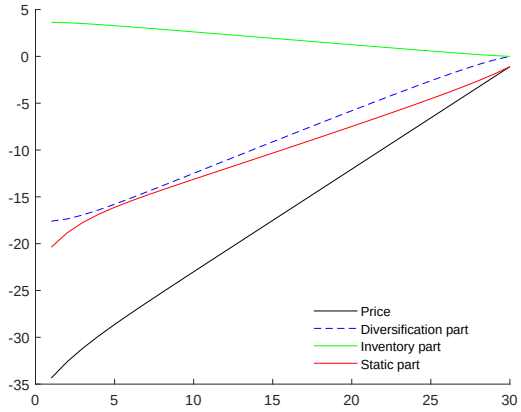
While markets differ along many dimensions, the dominance of large institutions is a common feature of many asset classes. In my set up, imperfect competition among institutions differing in risk aversion is the sole departure from frictionless markets. Accordingly, my analysis stresses that the market structure *per se* generates V-shaped price patterns around anticipated shocks. This matters both conceptually and from a regulatory perspec-

tive, as it may be easier for regulators to influence the market structure than, say, the flow of private information. In practice, however, markets are neither fully transparent, nor fully integrated. Developing a dynamic framework jointly allowing for market structure effects, information asymmetries, and segmentation is an important avenue for future research.

Appendix

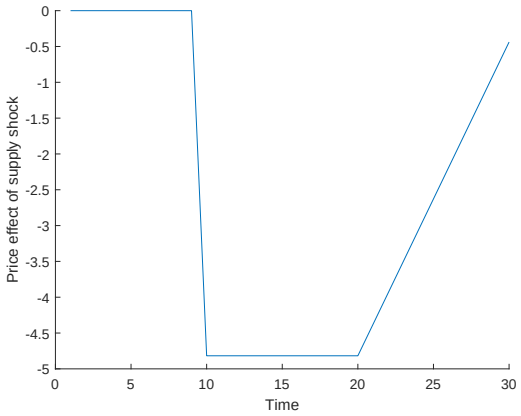


(a) Short run time-series momentum

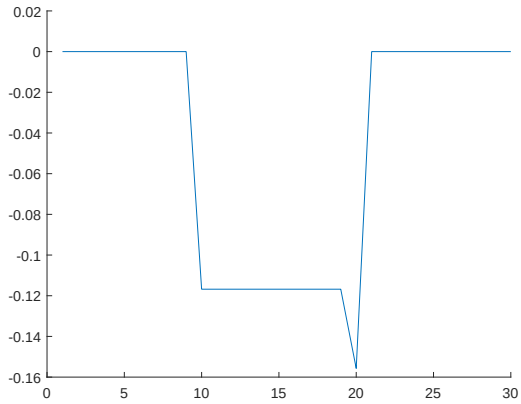


(b) Decomposition of the risk premium

Figure 2: Time-series momentum at short horizon and decomposition of the risk premium into three components. The parameters are the same as in Figure 1, except initial enowments, which are given by $\left(2.5, 1.5, 0, -0.5, -1\right)$. The left panel is the difference between the competitive and the noncompetitive price when small traders are initially over-exposed to the risky asset. The distribution of initial inventories is inverted relative to Pareto optimal endowments. The right panel illustrates equation (37).



(a) Perfect competition



(b) Symmetric market power

Figure 3: Average price reaction to an anticipated supply shock under perfect competition and symmetric market power. The parameters are the same as in Figure 1. Under perfect competition, the price has a drift after the shock due nonstationarity of the model. For symmetric market power risk aversion is set so as to keep the same total risk bearing capacity as in the competitive case. The right panel illustrates Rostek and Weretka (2015), showing only the liquidity effect by substracting the competitive price reaction.

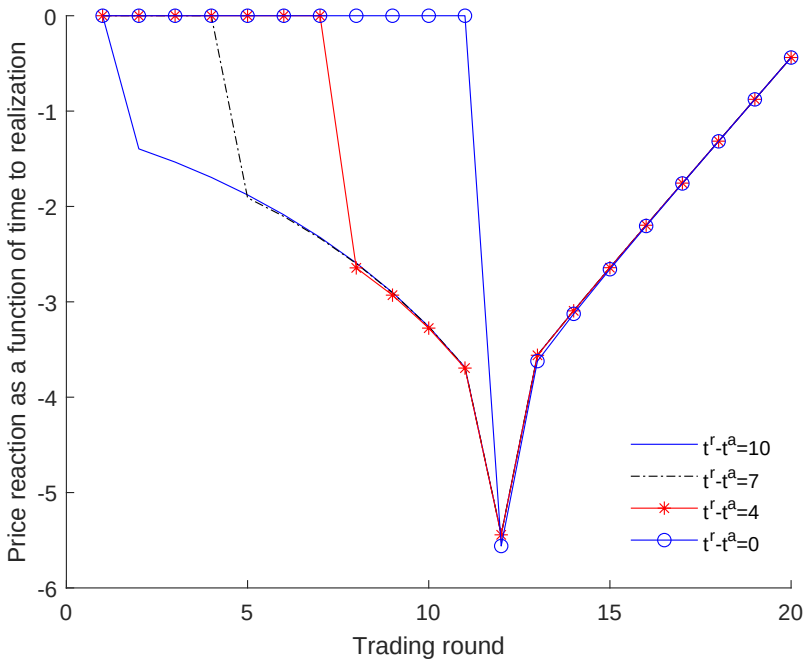


Figure 4: Average price effect of an anticipated supply shock as a function of the time to realization. The parameters for risk aversion and number of trades are the same as in Figure 1, but $T = 20$. The realization date $t^r = 11$ is fixed. When $t^a = t^r$, the shock is a surprise shock.

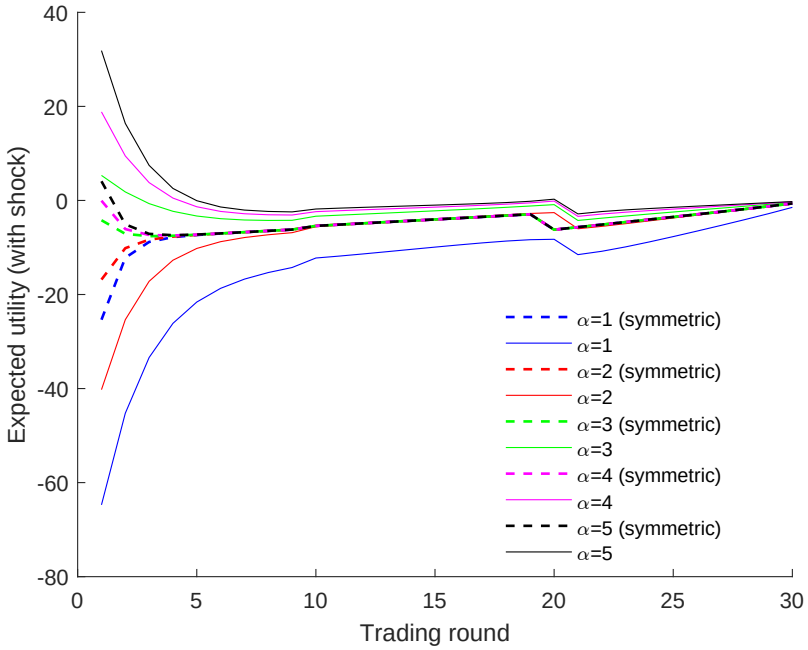
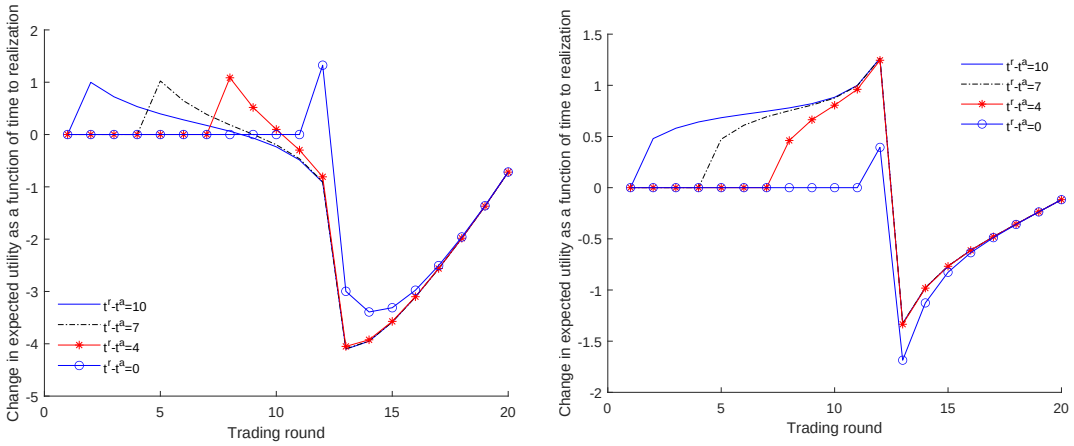


Figure 5: Expected utility in the symmetric vs asymmetric case. The parameters for risk aversion and number of trades are the same as in Figure 4, except that $T = 30$.



(a) Largest trader

(b) Smallest trader

Figure 6: Change in expected utility due to an anticipated supply shock as a function of risk aversion. The parameters are the same as in Figure 4. The left panel shows the largest trader ($\alpha = 1$), the right panel the smallest trader ($\alpha = 5$).

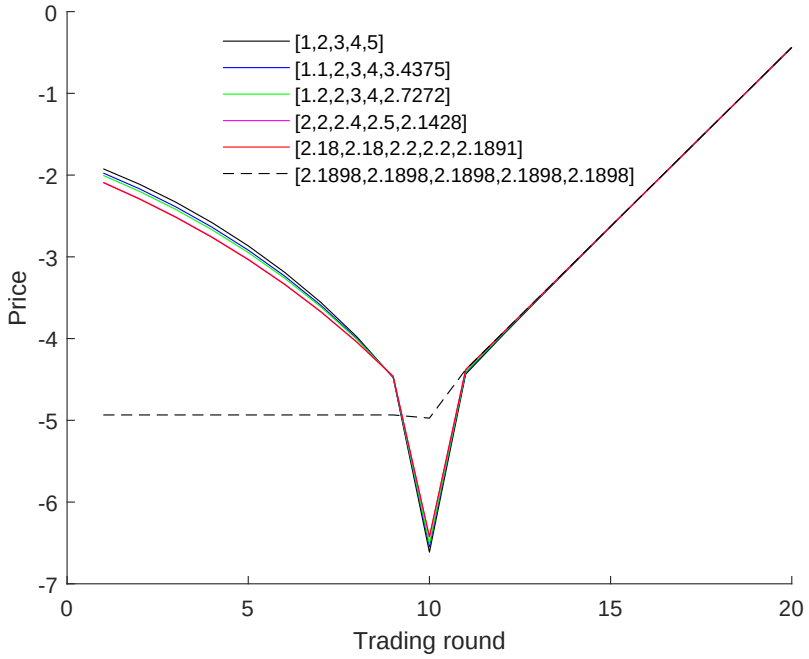
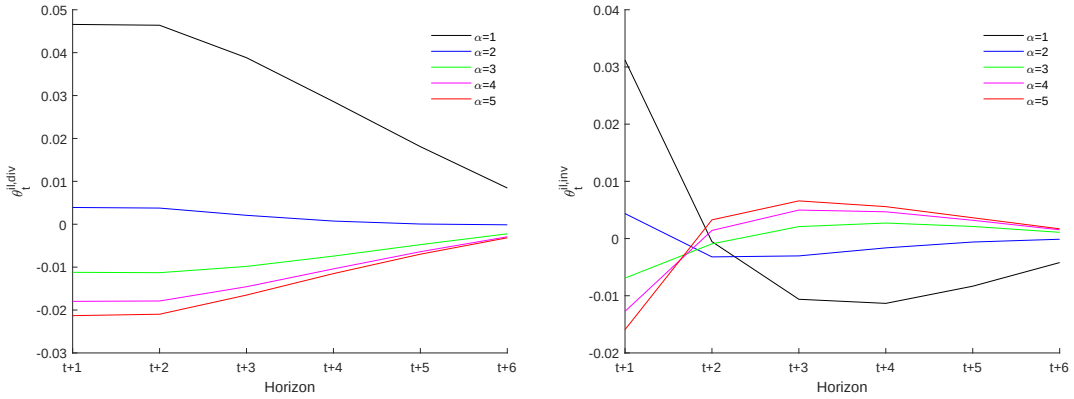


Figure 7: Average price effect of an anticipated supply shock as a function of heterogeneity in risk aversion. The numbers in brackets give the risk aversion. The shock is announced at $t^a = 0$. The total risk bearing capacity is kept constant, equal to $\frac{n}{\alpha}$, where $n = 5$ and $\alpha = 2.1898$.

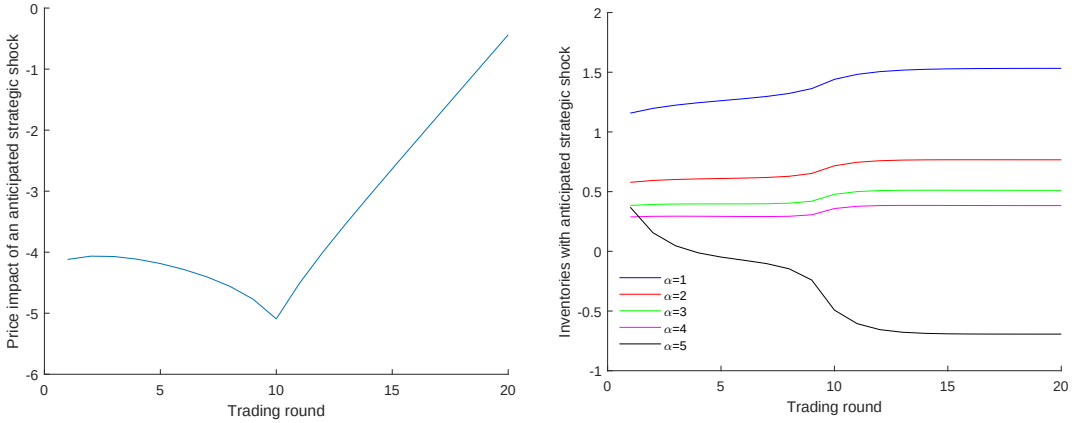
A Matrix notations

- $\mathbf{A}^\top$ denotes the transpose of $\mathbf{A}$.



(a) Hedging component (b) Strategic component

Figure 8: Decompostion of θ_t^{il} into a hedging and a strategic components. The risk aversion parameters are the same as in Figure 1.



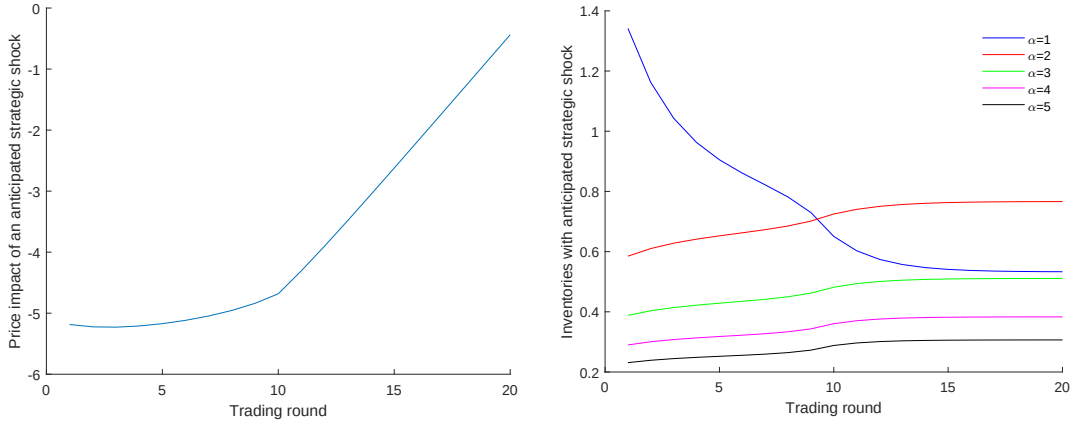
(a) Price (b) Inventories

Figure 9: Average price and inventory effects of an anticipated endowment shock to the smallest trader ($\alpha^5 = 5$). Parameters are the same as in Figure 1, except that $T + 1 = 20$, $t^a = 0$, $t^r = 10$. The shock is of the same magnitude ($e^5 = 1$) as in Figure 1. Initial inventories are Pareto-optimal.

- $\mathbf{I}_n$ denotes the identity matrix of size n.
- For a vector $\mathbf{y_t} = (y_t^i)_i$, I write $\bar{y}_t = \sum_i y_t^i$. Similarly, for a matrix $\mathbf{a_t} = (a_t^{ij})_{i,j}$, I write $(\bar{a}_t^j)_j = \left(\sum_i a_t^{ij}\right)_j$.

B Benchmarks

In this section, I derive the competitive and symmetric equilibria. While this material is standard, deriving it using the diversified risk representation facilitates comparison with the



(a) Price (b) Inventories

Figure 10: Average price and inventory effects of an anticipated endowment shock to the largest trader ($\alpha^1 = 1$). Parameters are in Figure 9. The shock is of the same magnitude ($e^1 = 1$). Initial inventories are Pareto-optimal.

asymmetric market power setting.

B.1 Perfect competition

As shown in the text, the static and dynamic portfolios boil down to

$$Q_t^{t\star} = \sum_{j=1}^{n} c_t^{j\star} X_{t-1}^j + \Delta s_t, \quad Q_t^{l\star} = \Delta s_l, \quad l \geq t+1. \tag{46}$$

Although the diversification speed $c_t^{i\star}$ will always be equal to 1, I often keep this notation to emphasize the parallel with the noncompetitive case. The next result expresses the competitive equilibrium in terms of diversfied risk.

Lemma 4 (Competitive equilibrium) *In the competitive equilibrium:*

1. *The trade and risk premium are:*

$$x_t^{i\star} = -c_t^{i\star} X_{t-1}^i + \phi_t^{i\star} Q_t^{t\star}, \tag{47}$$

$$D_t - p_t^\star = \frac{\sigma^2}{q^\star} \sum_{l=t}^{T} (T - l + 1) Q_t^{l\star}, \quad \forall t \geq 0, \tag{48}$$

2. *Traders diversfiy their inventories immediately ($c_t^{i\star} = 1, \forall i$) and $\phi_t^{i\star}$ denotes the trader's contribution to the total risk bearing capacity, $\phi_t^{i\star} = \frac{1}{\alpha^i q^\star}$.*

3. The equilibrium demand schedule coefficients are:

$$\beta_t^{i\star} = \frac{1}{\alpha^i \sigma^2} \ (t \leq T), \quad d_t^{ij\star} = \frac{(T-t)c_t^{j\star}}{\alpha^i q^\star} = \frac{T-t}{\alpha^i q^\star} = d_t^{i\star} \ (t < T), \tag{49}$$

$$\delta_t^{il\star} = \frac{T-l+1}{\alpha^i q^\star}, \ l \leq t < T, \quad \delta_T^{iT\star} = 0. \tag{50}$$

4. Value function coefficients are:

$$\gamma_t^{i\star} = 0, \quad \mu_t^{il\star} = \frac{T-l+1}{q^\star}. \tag{51}$$

The demand and value function coefficients are easy to interpret. Competitive traders immediately trade towards Pareto-optimal positions, implying that there is not under-diversification ($\gamma_t^i = 0$) for all traders and trading rounds. Because risk is always fully diversified, the slope of the demand schedule is equal to the trader's risk bearing capacity times the amount of per-period risk in all trading rounds. Since traders are price-takers, the marginal effect of a change in position on their equilibrium expected utility is simply the expected risk premium, $\frac{\partial \mathbb{E}_t(\Omega_{t+1}^i)}{\partial X_t^i} = \sigma^2 \mu_{\mathbf{t+1}}^{\mathbf{i}} \mathbf{Q_{t+1}} = \mathbb{E}_t(D_{t+1} - p_{t+1})$. This implies that the demand is given by $x_t^i = \frac{\mathbb{E}_t(p_{t+1}-p_t)}{\alpha^i \sigma^2} - X_{t-1}^i$. Hence, to hold an extra unit of the risky asset, a trader must be compensated by an expected capital gain $\mathbb{E}_t(p_{t+1} - p_t)$.

Since diversified risk boils down to aggregate inventories, which are equal to the supply, the trade and risk premium can be rewritten as:

$$x_t^{i\star} = \frac{s_t}{\alpha^i q^\star} - X_{t-1}^i, \tag{52}$$

$$D_t - p_t^\star = \frac{\sigma^2}{q^\star} \sum_{l=t}^{T} s_l, \quad \forall t \geq 0. \tag{53}$$

Thus, in the competitive case, traders fully diversify their endowments at time 0, $x_0^{i\star} = \frac{s_0}{\alpha^i q^\star} - X_{-1}^i$ and replace them with the Pareto-optimal position. In the subsequent rounds, they immediately adjust their position towards the new Pareto optimum when a supply shock hits the market: $x_t^{i\star} = \frac{\Delta s_t}{\alpha^i q^\star}$. The price reflects that from t onwards traders will have to hold aggregate risk in quantity s_l at each round l, for every $l \geq t$, hence the cumulative effect.

Proof. The proof proceeds recursively. At the final round ($t = T$), traders solve:

$$\max_{x_T^i} \; B_{T-1}^i + X_{T-1}^i D_T + x_T^i (D_T - p_T) - \frac{\alpha^i \sigma^2}{2} (X_T^i)^2. \tag{54}$$

The problem is concave, so that the first-order condition is necessary and sufficient. It leads to

$$x_T^i = \frac{D_T - p_T}{\alpha^i \sigma^2} - X_{T-1}^i. \tag{55}$$

Thus, the demand schedule coefficients are $\beta_T^{i\star} = \frac{1}{\alpha^i \sigma^2}$, $c_T^{i\star} = 1$, and $d_T^{ij\star} = 0$, which match equations (49)-(50) for $t = T$. The term in Δs_T is equal to 0.

Aggregate schedules (55) over i and use market clearing at T (equation (3)) to get the equilibrium risk premium at T:

$$D_T - p_T^\star = \frac{\sigma^2}{q^\star} \left(\sum_{j=1}^n c_T^{j\star} X_{T-1}^j + \Delta s_T \right) = \frac{\sigma^2}{q^\star} Q_T^{T\star}. \tag{56}$$

Subsitute back the risk premium into the schedule (55) to obtain the equilibrium trade:

$$x_T^{i\star} = \frac{1}{\alpha^i q^\star} Q_T^{T\star} - X_{T-1}^i. \tag{57}$$

Then, substitute the equilibrium trade (57) and risk premium (56) into the objective (54) to get the value function:

$$\Omega_T^{i\star} = -\sigma^2 X_{T-1}^i \frac{Q_T^{T\star}}{q^\star} + \frac{\sigma^2}{2\alpha^i (q^\star)^2} (Q_T^{T\star})^2. \tag{58}$$

This gives: $\hat{\mu}_T^{iT\star} = \frac{1}{q^\star}$ and $\hat{\rho}_T^{iT\star} = \frac{1}{\alpha^i (q^\star)^2}$, which matches equations (51) for $t = T$.

Assume now that equations (18) and (47)-(48) hold from $t+1$ on. Then at time t, traders solve:

$$\max_{x_t^i} \; v_t^i + x_t^i (D_t - p_t) - \frac{\alpha^i \sigma^2}{2} (X_t^i)^2 - \frac{\alpha^i \sigma^2}{2} \gamma_t^i (X_t^i)^2 - \sigma^2 X_t^i \boldsymbol{\mu_{t+1}^i Q_{t+1}} + \frac{\sigma^2}{2} \mathbf{Q_{t+1}^\top} \boldsymbol{\rho_{t+1}^i Q_{t+1}}. \tag{59}$$

The first-order condition gives:

$$D_t - p_t - \alpha^i \sigma^2 X_t^i - \frac{\sigma^2}{q^\star} \sum_{l=t+1}^{T} (T-l+1) Q_{t+1}^{l\star} = 0, \quad \text{i.e.,}$$

$$x_t^i = -X_{t-1}^i + \frac{D_t - p_t}{\alpha^i \sigma^2} - \frac{1}{\alpha^i q^\star} \sum_{l=t+1}^{T} (T-l+1) Q_{t+1}^{l\star}. \tag{60}$$

Since $Q_{t+1}^{t+1\star} = \sum_{j=1}^n c_{t+1}^{j\star} X_t^j + \Delta s_{t+1} = \sum_{j=1}^n X_{t-1}^j + \Delta s_t + \Delta s_{t+1}$, as $c_{t+1}^{j\star} = 1$, and given that $Q_{t+1}^{l\star} = \Delta s_l$ for $l \geq t+2$, schedules (60) can be rewritten as

$$x_t^i = -X_{t-1}^i + \frac{D_t - p_t}{\alpha^i \sigma^2} - \frac{1}{\alpha^i q^\star} \left((T-t) \sum_{j=1}^{n} X_{t-1}^j + (T-t) \Delta s_t + \sum_{l=t+1}^{T} (T-l+1) \Delta s_l \right). \tag{61}$$

Demand schedule coefficients (49)-(50) given in the result can be read directly from this equation. Next, aggregating equation (60) and using market clearing (3) at t gives:

$$\begin{aligned} D_t - p_t &= \frac{\sigma^2}{q^\star} \left(\sum_{j=1}^{n} c_t^{j\star} X_{t-1}^j + \Delta s_t + \sum_{l=t+1}^{T} (T-l+1) Q_{t+1}^{l\star} \right) \\ &= \frac{\sigma^2}{q^\star} \left(Q_t^{t\star} + \sum_{l=t+1}^{T} (T-l+1) Q_{t+1}^{l\star} \right). \end{aligned} \tag{62}$$

Substitute this expression into the demand schedule (60) to get the equilibrium trade (47) given in the result. Using (47), it is possible to express the dynamic portfolios as a function of time-$t-1$ positions:

$$Q_{t+1}^{t+1\star} = \sum_{j=1}^{n} c_{t+1}^{j\star} (X_{t-1}^j + \Delta s_t) + \Delta s_{t+1} = Q_t^{t\star} + \Delta s_{t+1} \tag{63}$$

$$Q_{t+1}^{l\star} = \Delta s_l = Q_t^{l\star}, \quad l \geq t+2 \tag{64}$$

Substituting equations (63)-(64) into equation (62) then gives the equilibrium risk premium (48). Then, plug equations (63), (64), (47), and (48) into the objective (59) to write the value function in the form (18) and obtain coefficients (51) given in the result. ■

B.2 Symmetric market power

I now derive the equilibrium in Rostek and Weretka (2015)'s symmetric market power setting where all traders have the same risk aversion ($\alpha^i = \alpha$, $\forall i$). I express their findings using the term structure of diversified risk.

Since all traders are identical, the value function coefficients are now independent of i and the candidate equilibrium demand schedule boil down to

$$x_t^i(p_t) = \beta_t(D_t - p_t) - c_t X_{t-1}^i - d_t \sum_{j=1}^{n} X_{t-1}^j - \sum_{l=t}^{T} \delta_t^l \Delta s_l. \tag{65}$$

If all traders submit schedules of this form in equilibrium, the following price and trade arise:

$$D_t - p_t = \frac{\sum_{j=1}^{n}(c_t + nd_t)X_{t-1}^j + (1 + n\delta_t^t)\Delta s_t + \sum_{l=t+1}^{T} n\delta_t^l \Delta s_l}{n\beta_t} \tag{66}$$

$$\begin{aligned} x_t^i &= -c_t X_{t-1}^i + \phi_t \left(\sum_{j=1}^{n} c_t X_{t-1}^j + \Delta s_t \right) \\ &= -c_t X_{t-1}^i + \phi_t Q_t^t, \end{aligned} \tag{67}$$

$$\text{where} \quad \phi_t = \frac{1}{n}. \tag{68}$$

There are two key results in this special case: first, traders do not trade the dynamic portfolios; second, the dynamic portfolios have a constant impact on the price up to one round before the shock takes place (corresponding to Lemma 2 in the main text).

Equation (67) shows that the first result can be derived simply from looking at the trade implied by the candidate equilibriums schedules. If traders have symmetric market power (and provided they submit schedules of type (65) in equilibrium), they only trade the current-round diversified risk, and not the dynamic ones. Each of them buys $1/n^{th}$ of this portfolio.

To see why dynamic portolios are not traded, suppose that the asset is in zero net supply that all traders have zero initial inventories, and consider a change in the aggregate inventories from 0 to something strictly positive. Then the net effect on the equilibrium trade depends on two forces. First, there is a direct effect, whereby traders' demand adjusts by $-d_t$. Positive aggregate inventories implyy a lower prices tomorrow, so today's demand decreases. However, there is a second, positive, and indirect effect on demand through price changes. As aggregate inventories increase, today's price is reduced by $(c_t + nd_t)/(n\beta_t)$.

Thus, traders increase their demand by $\beta_t(c_t + nd_t)/(n\beta_t) = \frac{c_t}{n} + d_t$. The net effect is thus $-d_t + \frac{c_t}{n} + d_t = \phi_t c_t$. The only remaining term, in c_t, relates to the static portfolio (current-round diversified risk). Dynamic portfolios relate to the term in d_t, which is zero. In this special case, the indirect effect on demand of a price change exactly offsets the effect on future marginal utility.

The second main result about the constant price effect of dynamic portfolios depends on value function coefficients. It is already apparent in point 3 of Proposition 3 below.

The special case of identical risk aversion is very tractable, as the fixed point in price impact is simple. As a result, all demand schedule parameters can be directly expressed as a function of the diversification speed c. I first derive the system characterizing value function coefficients, and then solve it in closed form.

B.2.1 Deriving the system for demand and value function coefficients

The next result provides the first step in characterizing the equilibrium.

Proposition 3 (Symmetric market power) *When traders have identical risk aversion, the following holds:*

1. *There exists a unique dynamic equilibrium in which traders diversify at constant speed $c = \frac{n-2}{n-1}$ and the price and equilibrium trade are*

$$D_t - p_t = \sigma^2 \left[\left\{ \frac{1}{q_t} + \sum_{l=t+1}^{T} f_{t+1}^l \mu_{t+1}^l \right\} Q_t^t + \sum_{l=t+1}^{T} \mu_{t+1}^l Q_t^l \right], \ t < T \tag{69}$$

$$x_t^i = -cX_{t-1}^i + \phi_t Q_t^t, \quad \textit{where} \quad \phi_t = \frac{1}{n}, \quad q_t = \frac{nc}{\alpha(1+\gamma_{t+1})}, \ t \leq T. \tag{70}$$

2. *Other demand schedule coefficients are:*

$$\beta_t = \frac{c}{\alpha\sigma^2(1+\gamma_{t+1})}, \quad d_t = \delta_t^t = \sum_{l=t+1}^{T} f_{t+1}^l \delta_t^l \tag{71}$$

$$\delta_t^l = \hat{\mu}_{t+1}^l \sigma^2 \beta_t. \tag{72}$$

3. *The law of motion for the state variables is*

$$X_t^i = (1-c)X_{t-1}^i + \phi_t Q_t^t \tag{73}$$

$$\mathbf{Q_{t+1}} = \mathbf{\Theta_t^Q Q_t}. \tag{74}$$

4. Value function coefficients are defined recursively by the following system:

$$\gamma_t = (1-c)^2(1+\gamma_{t+1}), \tag{75}$$

$$\mu_t^t = \frac{c}{q_t} + \frac{\alpha}{n}(1-c)(1+\gamma_{t+1}) + \sum_{l=t+1}^{T} c(1-c)^{l-t-1}\mu_{t+1}^l, \tag{76}$$

$$\mu_t^l = \mu_{t+1}^l, \quad l \geq t+1. \tag{77}$$

with initial conditions $\mu_T^T = \frac{\alpha}{n}(2-c)$ *and* $\gamma_T = (1-c)^2$.

Proof. The proof proceeds recursively.

Step 1: Time T.

I first show that the result holds in the final period. At time T, traders solve:

$$\max_{x_T^i} \; B_{T-1}^i + X_{T-1}^i D_T + x_T^i(D_T - p_T) - \frac{\alpha^i\sigma^2}{2}(X_T^i)^2. \tag{78}$$

The problem is concave, so that the first-order condition is necessary and sufficient.

$$D_T - p_T - \lambda_T x_T^i - \alpha\sigma^2 X_T^i = 0. \tag{79}$$

It leads to

$$x_T^i = \frac{D_T - p_T}{\lambda_T + \alpha\sigma^2} - \frac{\alpha\sigma^2}{\lambda_T + \alpha\sigma^2} X_{T-1}^i. \tag{80}$$

Thus, the demand schedule coefficients are $\beta_T = \frac{1}{\lambda_T + \alpha\sigma^2}$, $c_T = \frac{\alpha\sigma^2}{\lambda_T + \alpha\sigma^2}$, and $d_T = 0$, which match equations (49)-(50) for $t = T$. The term in Δs_T is equal to 0.

To prove existence, solve the system given by $\beta_T = \frac{1}{\lambda_T + \alpha\sigma^2}$ and $\lambda_T = \frac{1}{(n-1)\beta_T}$, which leads to

$$\lambda_T = \frac{\alpha\sigma^2}{n-2}, \quad \beta_T = \frac{c}{\alpha\sigma^2}, \quad c_T = c = \frac{n-2}{n-1} \tag{81}$$

Then substitute the value of the coefficients in the risk premium, leading to

$$D_T - p_T = \frac{\sigma^2}{q_T}\left(\sum_{j=1}^{n} c X_{T-1}^j + \Delta s_T\right) = \frac{\sigma^2}{q_T} Q_T^T. \tag{82}$$

Subsitute back the risk premium into the schedule (80) to obtain the equilibrium trade:

$$x_T^i = \frac{1}{n} Q_T^T - c X_{T-1}^i. \tag{83}$$

Then, substitute the equilibrium trade (83) and risk premium (82) into the objective (78) to get the value function:

$$\Omega_T^i = v_T^i - \frac{\alpha\sigma^2}{2}\gamma_T (X_{T-1}^i)^2 + \sigma^2(1-\gamma_T)\left(\frac{1}{2\alpha(q_T)^2}(Q_T^T)^2 - X_{T-1}^i \frac{Q_T^T}{q_T}\right). \tag{84}$$

This gives: $\gamma_T = (1-c)^2$, $\mu_T^T = \frac{1-\gamma_T}{q_T} = \frac{\alpha}{n}(2-c)$ and $\rho_T^T = \frac{1-\gamma_T}{\alpha(q_T)^2}$, which matches equations (75)-(96) for $t = T$, since $\gamma_{T+1} = 0$.

Step 2: from $t+1$ to t.

Assume now that equations (18) and (70)-(69) hold from $t+1$ on.

a. Optimization given price impact and equilibrium price. At time t, traders solve:

$$\max_{x_t^i} \; v_t^i + x_t^i(D_t - p_t) - \frac{\alpha^i\sigma^2}{2}(X_t^i)^2 - \frac{\alpha^i\sigma^2}{2}\gamma_{t+1}(X_t^i)^2 - \sigma^2 X_t^i \boldsymbol{\mu_{t+1}} \mathbf{Q_{t+1}} + \frac{\sigma^2}{2}\mathbf{Q_{t+1}^\top}\boldsymbol{\rho_{t+1}}\mathbf{Q_{t+1}}, \tag{85}$$

taking their price impact λ_t as given. The first-order condition gives:

$$\begin{aligned} &D_t - p_t - \lambda_t x_t^i - \alpha\sigma^2(1+\gamma_{t+1})X_t^i - \sigma^2 \sum_{l=t+1}^{T} \mu_{t+1}^l Q_{t+1}^l = 0, \quad \text{i.e.,} \\ &x_t^i = -\frac{\alpha\sigma^2(1+\gamma_{t+1})}{\lambda_t + \alpha\sigma^2(1+\gamma_{t+1})} X_{t-1}^i + \frac{D_t - p_t}{\lambda_t + \alpha\sigma^2(1+\gamma_{t+1})} - \frac{\sigma^2}{\lambda_t + \alpha\sigma^2(1+\gamma_{t+1})} \sum_{l=t+1}^{T} \mu_{t+1}^l Q_{t+1}^l. \end{aligned} \tag{86}$$

This expression holds for any price p_t. Use the definition of Q_{t+1}^l and the identity $\sum_j X_t^j = \sum_j X_{t-1}^j + \Delta s_t$ to identify schedule coefficients:

$$\beta_t = \frac{1}{\lambda_t + \alpha\sigma^2(1+\gamma_{t+1})}, \quad c_t = \frac{\alpha\sigma^2(1+\gamma_{t+1})}{\lambda_t + \alpha\sigma^2(1+\gamma_{t+1})}, \tag{87}$$

$$d_t = \delta_t^t = \sigma^2 \sum_{l=t+1}^{T} c(1-c)^{l-t-1}\mu_{t+1}^l \beta_t, \tag{88}$$

$$\delta_t^l = \sigma^2 \beta_t \mu_{t+1}^l, \quad l \geq t+1. \tag{89}$$

An equilibrium exists iff there exists a solution to the system given by $\beta_t = \frac{1}{\lambda_t + \alpha\sigma^2(1+\gamma_{t+1})}$ and $\lambda_t = \frac{1}{(n-1)\beta_t}$. There is a unique solution

$$\beta_t = \frac{c}{\alpha\sigma^2(1+\gamma_{t+1})}, \quad c_t = c = \frac{n-2}{n-1}, \tag{90}$$

$$\lambda_t = \frac{\alpha\sigma^2(1+\gamma_{t+1})}{n-2}. \tag{91}$$

To derive the equilibrium price, first note that $d_t = \delta_t^t = \sum_{l=t+1}^{T} c(1-c)^{l-t-1}\delta_t^l$. This implies that:

$$\sum_j d_t X_{t-1}^j + \sum_{l=t}^{T} \delta_t^l \Delta s_l = \sum_{l=t+1}^{T} \delta_t^l \left(c(1-c)^{l-t-1} \left(\sum_j X_{t-1}^j + \Delta s_t \right) + \Delta s_l \right) \tag{92}$$

Then, using $c(1-c)^{l-t-1} = c(1-c)^{l-t} + c^2(1-c)^{l-t-1}$ leads to

$$\begin{aligned} c(1-c)^{l-t-1} &\left(\sum_j X_{t-1}^j + \Delta s_t \right) + \Delta s_l \\ &= c(1-c)^{l-t} \sum_j X_{t-1}^j + \Delta s_l + c^2(1-c)^{l-t-1} \sum_j X_{t-1}^j + c(1-c)^{l-t-1}\Delta s_t \\ &= Q_t^l + c(1-c)^{l-t-1} Q_t^t \end{aligned}$$

Substituting back into equation (92):

$$\sum_j d_t X_{t-1}^j + \sum_{l=t}^{T} \delta_t^l \Delta s_l = \sum_{l=t+1}^{T} \delta_t^l Q_t^l + \left(\sum_{l=t+1}^{T} c(1-c)^{l-t-1} \delta_t^l \right) Q_t^t$$

Then given the implied equilibrium price (66), the previous expression must be multiplied by $1/(n\beta_t) = 1/\bar{\beta}_t$. Using equation (89) and $q_t \equiv \frac{\bar{\beta}_t}{\sigma^2}$ leads to $\bar{\delta}_t^l = \mu_{t+1}^l q_t$. Thus, the risk premium boils down to

$$D_t - p_t = \frac{\sigma^2}{q_t} Q_t^t + \sigma^2 \left(\sum_{l=t+1}^{T} \mu_{t+1}^l Q_t^l + \left(\sum_{l=t+1}^{T} c(1-c)^{l-t-1} \mu_{t+1}^l \right) Q_t^t \right),$$

which gives equation (69) in the result.

b. Transition matrix and value function. To compute the value function, it is first necessary to express Q_{t+1}^l as a function of time-$t-1$ variables. Given the equilibrium trade (70), for

any $l \geq t+1$,

$$\begin{aligned} Q_{t+1}^l &= \sum_j c(1-c)^{l-t-1} X_t^j + \Delta s_l \\ &= \sum_j c(1-c)^{l-t-1} \left((1-c) X_{t-1}^j + \frac{1}{n} Q_t^t \right) + \Delta s_l \\ &= Q_t^l + c(1-c)^{l-t-1} Q_t^t. \end{aligned}$$

Therefore, time-$t+1$ portfolios can be expressed as

$$\mathbf{Q_{t+1}} = \begin{pmatrix} Q_{t+1}^{t+1} \\ \vdots \\ Q_{t+1}^T \end{pmatrix} = \mathbf{\Theta_t^Q Q_t}, \tag{93}$$

where the transition matrix is given by:

$$\mathbf{\Theta_t^Q} = \begin{pmatrix} c & 1 & 0 & \dots & 0 \\ c(1-c) & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c(1-c)^{T-t-1} & 0 & 0 & \dots & 1 \end{pmatrix}, \tag{94}$$

Then the value function can be computed and value function coefficients derived by using the equlibrium trade, risk premium, and the law of motion for $\mathbf{Q_t}$ (93). ■

B.2.2 Solving the system

The system defined by equations (75)-(77) can be solved in closed form. To do so, it is useful to introduce the following auxiliary variable:

$$\mu_t^X = c\mu_t^t + \cdots + c(1-c)^{T-t} \mu_t^T = \sum_{l=t}^T c(1-c)^{l-t} \mu_t^l \tag{95}$$

Then the system (75)-(77) can be rewritten as

$$\gamma_t = (1-c)^2(1+\gamma_{t+1}), \tag{96}$$
$$\mu_t^X = \frac{\alpha}{n}c(2-c)(1+\gamma_{t+1}) + \mu_{t+1}^X, \tag{97}$$
$$\mu_t^t = \frac{\alpha}{n}(2-c)(1+\gamma_{t+1}) + \mu_{t+1}^X, \tag{98}$$
$$\mu_t^l = \mu_l^l, \quad l \geq t+1, \tag{99}$$

with initial conditions $\mu_T^X = c\mu_T^T = \frac{\alpha}{n}c(2-c)$ and $\gamma_T = (1-c)^2$. The next result provides the solution to the system and shows that the effect of dynamic portfolios on the equilibrium price is constant before the realization.

Lemma 5 *Value function coefficients are functions of the trading speed c. The coefficient γ_t aggregates all the under-diversification expected utility costs from t to T. The other coefficients combine this cost with a linear factor $T-t+1$ that decreases as the dividend payment approaches:*

$$\gamma_t = \sum_{k=1}^{T-t+1} (1-c)^{2k}, \tag{100}$$
$$\mu_t^X = \frac{\alpha}{n}(T-t+1-\gamma_t), \tag{101}$$
$$\mu_t^t = \frac{\alpha}{n}\left(T-t+1+(1-c)(1+\gamma_{t+1})\right). \tag{102}$$

A change in round-l diversified risk has a constant effect on the price:

$$D_t - p_t = \sigma^2\left[\left\{\frac{\alpha}{n}\frac{1+\gamma_{t+1}}{c} + \mu_{t+1}^X\right\}Q_t^t + \sum_{l=t+1}^{T}\mu_l^l Q_t^l\right] \tag{103}$$

Equation (103) expresses the price in terms of diversified risk. Using Definition 2, the price can be rewritten as in Rostek and Weretka's (2015) paper:

Corollary 3 (Rostek and Weretka (2015)) *In the symmetric model:*

1. *The price effect of a sequence of anticipated shocks $\{\Delta s_l\}_{l\geq t}$ can be decomposed into a fundamental effect (corresponding to the competitive price adjusted for the shock) and*

a liquidity effect:

$$D_t - p_t = D_t - p_t^\star + \Delta^{L_t} + c \sum_{k=t+1}^{T} \Delta^{L_k} \tag{104}$$

where $\Delta^{L_k} = (1-c)(1+\gamma_{k+1})\Delta s_k$.

2. *The liquidity effect is constant from the announcement to the realization of the shock, but muted by a factor c. It reaches its full effect at the realization and disappears after one trading round.*

Proof. I first derive the system (96)-(99) Using the definition (95), the expression for q_t, the difference equation (77) for μ_t^t can rewritten as

$$\mu_t^t = \frac{\alpha}{n}(2-c)(1+\gamma_{t+1}) + \mu_{t+1}^X \tag{105}$$

Equation (95) also implies that $\mu_t^X = c\mu_t^t + (1-c)\mu_{t+1}^X$. Substitute equation (105) into this expression to get (77). The other two equations are as before. The inital value for μ_t^X is $\mu_T^X = c\mu_T^T$. From the system, we get immediately the value of γ_t as a function of c. For μ_t^X and μ_t^t, it is easy to verify that the general expressions (101) and (102) hold at T; then, assuming that they hold at $t+1$, it is straightforward to show that they also hold at t. Equation (104) follows from substituting the expression for γ_t, μ_t^t, and μ_t^X into equation (103) and rearranging the terms. ■

B.2.3 Deriving equation (24)

Under symmetric market power, $\varphi_{t+1}^{il} = \mu_{t+1}^{il} = \mu_l^l = \frac{\alpha}{n}(T-l+1) + \frac{\alpha}{n}(1-c)(1+\gamma_{l+1})$ (equation (102)), so that

$$\begin{aligned}
\sum_{l=t+1}^{T} \varphi_{t+1}^{il} Q_{t+1}^l &= \sum_{l=t+1}^{T} \mu_{t+1}^l \sum_j f_{t+1}^{jl} X_t^j \\
&= \frac{\alpha}{n} \sum_{l=t+1}^{T} (T-l+1) c (1-c)^{l-(t+1)} \sum_j X_t^j \\
&\quad + \frac{\alpha}{n}(1-c) \sum_{l=t+1}^{T} (1+\gamma_{l+1}) c (1-c)^{l-(t+1)} \sum_j X_t^j,
\end{aligned}$$

where I used the definition of f^{jl}_{t+1}. Using a change of variables and breaking up terms leads to

$$\sum_{l=t+1}^{T} \varphi^{il}_{t+1} Q^{l}_{t+1} = \frac{\alpha}{n} c \sum_j X^j_t \left[(T-t) \sum_{l=0}^{T-t-1} (1-c)^l - \sum_{l=0}^{T-t-1} l(1-c)^l + \sum_{l=0}^{T-t-1} (1-c)^{l+1} + \sum_{l=t+1}^{T} (1-c)^{l-t} \gamma_{l+1} \right],$$

where $\gamma_{T+1} \equiv 0$. The second term in the bracket can be calculated by noting that

$$\begin{aligned}
\sum_{l=0}^{T-t-1} l(1-c)^l &= -(1-c) \sum_{l=0}^{T-t-1} \frac{\partial((1-c)^l)}{\partial c} = -(1-c) \frac{\partial}{\partial c} \left(\sum_{l=0}^{T-t-1} (1-c)^l \right) \\
&= -(1-c) \frac{\partial}{\partial c} \left(\frac{1-(1-c)^{T-t}}{c} \right) \\
&= -(T-t) \frac{(1-c)^{T-t}}{c} + \frac{1-c}{c} \frac{1-(1-c)^{T-t}}{c}
\end{aligned}$$

Then, putting the different expressions together, using $\gamma_T \equiv (1-c)^2$, and simplifying gives:

$$\sum_{l=t+1}^{T} \varphi^{il}_{t+1} Q^{l}_{t+1} = \frac{\alpha}{n} \sum_j X^j_t \left(T - t + c \sum_{l=t+1}^{T} (1-c)^{l-t} \gamma_{l+1} - \gamma_T \sum_{l=t+1}^{T} (1-c)^{l-t-1} \right). \tag{106}$$

Equation (24) follows after defining

$$\mathcal{D}^{\mathsf{sym}}_t = \frac{\alpha \sigma^2}{n} \left[\sum_{l \geq t+1} \gamma_{l+1} c (1-c)^{l-t} \sum_j X^j_t - \gamma_T \sum_{l \geq t+1} (1-c)^{l-t-1} \sum_j X^j_t \right]. \tag{107}$$

C Asymmetric market power

In this section, I derive the equilibrium and its properties when traders differ in risk aversion.

C.1 Static model (Proposition 1)

I begin with the static equilibrium. The next result encompasses Proposition 1 given in the text.

Proposition 4 (Static equilibrium) *Suppose that there is a single round of trading.*

1. *There exists a unique equilibrium in linear demand schedules in which the equilibrium price impact is* $\lambda^i_T = \alpha^i \sigma^2 k^i_T$, *with* $k^i_T = \frac{2 - \alpha^i B + \sqrt{\Delta^i_T}}{2 \alpha^i B}$, $\Delta^i_T = 4 + (\alpha^i B)^2$, *and where* B

is the unique positive solution to:

$$\sum_i \frac{1}{2+\alpha^i B+\sqrt{\Delta_T^i}}=\frac{1}{2}. \tag{108}$$

2. The coefficients of the demand schedules are:

$$\beta_T^i=\frac{1}{\lambda_T^i+\alpha^i\sigma^2},\quad c_T^i=\alpha^i\sigma^2\beta_T^i,\quad d_T^{ij}=\delta_T^{iT}=0.$$

3. The equilibrium price and trades are determined by the static portfolio:

$$D_T-p_T=\frac{\sigma^2}{q_T}\sum_j c_T^j X_{T-1}^j, \tag{109}$$

$$x_T^i=-c_T^i X_{T-1}^i+\frac{\beta_T^i}{\bar{\beta}_T}\sum_j c_T^j X_{T-1}^j., \tag{110}$$

where $\bar{\beta}_T=\frac{q_T}{\sigma^2}$.

Proof. The proof has three steps: first, I solve the optimization problem of a trader; second, I study existence; third, I obtain the equilibrium price, trade, and value function.

A. Optimization.

Trader i's problem is

$$\max_{x_T^i}\quad B_{T-1}^i+X_{T-1}^i D_T+x_T^i(D_T-p_T)-\frac{\alpha^i\sigma^2}{2}(X_T^i+e_T^i)^2 \tag{111}$$

The necessary and sufficient FOC gives:

$$D_T-p_T-\lambda_T^i x_T^i-\alpha^i\sigma^2(X_T^i+e_T^i)=0,$$

which in turn leads to

$$x_T^i=\frac{1}{\lambda_T^i+\alpha^i\sigma^2}(D_T-p_T-\alpha^i\sigma^2(X_T^i+e_T^i)),\quad \forall i. \tag{112}$$

This equation holds for every p_T, thus identifying pointwise gives the coefficients of the

postulated demand schedule:

$$\beta_T^i = \frac{1}{\lambda_T^i + \alpha^i \sigma^2}, \tag{113}$$

$$c_T^i = \frac{\alpha^i \sigma^2}{\lambda_T^i + \alpha^i \sigma^2}, \quad d_T^{i,j} = 0, \quad \delta_T^{i,T} = 0 \tag{114}$$

These conditions are necessary and sufficient.

B. Existence. Existence of an equilibrium depends on existence of a solution to fixed-point in price impact, given by equation (113). Indeed, from the market clearing equation $\sum_i x_T^i(p_T) = \Delta s_T$,

$$\lambda_T^i = -\frac{1}{\sum_{j\neq i} \frac{\partial x_T^j}{\partial p_T}} = \frac{1}{\sum_{j\neq i} \beta_T^j} = \frac{1}{\sum_{j\neq i} \frac{1}{\alpha^j\sigma^2 + \lambda_T^j}} = \frac{1}{\sum_j \frac{1}{\alpha^j\sigma^2+\lambda_T^j} - \frac{1}{\alpha^i\sigma^2+\lambda_T^i}}. \tag{115}$$

Let $\Lambda_T = \sum_j \frac{1}{\alpha^j\sigma^2+\lambda_T^j}$. The equilibrium condition becomes:

$$\lambda_T^i \left(\Lambda_T - \frac{1}{\alpha^i\sigma^2 + \lambda_T^i} \right) = 1. \tag{116}$$

Malamud and Rostek (2017) show that any solution to the fixed point problem takes the form $\lambda_T^i = \alpha^i k_T^i \sigma^2$. Then, $\Lambda_T = \frac{1}{\sigma^2} \sum_j \frac{1}{\alpha^j(1+k_T^j)} \equiv \frac{B}{\sigma^2}$, and equation (116) becomes:

$$\alpha^i B (k^i)^2 + k^i(\alpha^i B - 2) - 1 = 0. \tag{117}$$

The discriminant Δ_T^i, given in the proposition, is always strictly positive. There is a single positive root given by $k^i = \frac{2-\alpha^i B + \sqrt{\Delta_T^i}}{2\alpha^i B}$. Substituting this equation in the definition of Λ_t gives equation (108).

C. Equilibrium trade, price, and value function.
Using market clearing, $\sum_i x_T^i(p_T) = \Delta s_T$ and the candidate equilibrium schedule with $d_T^{i,j} = 0$ and $\delta_T^{i,T} = 0$ leads to

$$D_T - p_T = \sum_j \frac{c_T^j}{\bar{\beta}_T}(X_{T-1}^j + e_T^j) + \frac{1}{\beta_T}\Delta s_T \tag{118}$$

$$x_T^i = \frac{\beta_T^i}{\bar{\beta}_T} \left(\Delta s_T + \sum_j c_T^j (X_{T-1}^j + e_T^j) \right) - c_T^i (X_{T-1}^i + e_T^i) \tag{119}$$

Substitute these expressions into the objective function (111) to obtain the equilibrium value function. The coefficients are defined by

$$\gamma_T^i = (1 - c_T^i)^2, \tag{120}$$

$$\mu_T^i = \frac{1 - \gamma_T^i}{q_T}, \tag{121}$$

$$\frac{1}{2}\rho_T^i = \frac{1}{2}\frac{1 - \gamma_T^i}{\alpha^i (q_T)^2}. \tag{122}$$

■

C.2 Dynamic model (Proposition 2)

In this section, I show under which conditions the value function (18) and demand schedules (6) arise in equilibrium with multiple trading rounds. The proof proceeds recursively. Proposition 4 shows that the result holds at $t = T$. I now show that if equations (1), (2), and (18) hold at $t + 1$, then it also holds at t. I break the argument in three lemmata. Proposition 2 in the text follows.

Before giving the results, I derive the risk premium and trade implied by the candidate schedules (6) (without endowment shocks, treated in the Online Appendix). If all traders submit schedules of the form (6), then the risk premium and equilibrium trade will be equal to:

$$D_t - p_t = \frac{\sum_{j=1}^n (c_t^j + \bar{d}_t^j) X_{t-1}^j + \Delta s_t + \sum_{l=t+1}^T \bar{\delta}_t^l \Delta s_l}{\bar{\beta}_t} \tag{123}$$

$$x_t^i = -c_t^i X_{t-1}^i + \sum_{j=1}^n \left(\frac{\beta_t^i}{\bar{\beta}_t}(c_t^j + \bar{d}_t^j) - d_t^{ij} \right) X_{t-1}^j + \frac{\beta_t^i}{\bar{\beta}_t} \Delta s_t + \sum_{l=t+1}^T \left(\frac{\beta_t^i}{\bar{\beta}_t} \bar{\delta}_t^l - \delta_t^{il} \right) \Delta s_l. \tag{124}$$

The first result characterizes the system defining equilibrium demand schedule coefficients and the second-order condition from traders' optimization.

Lemma 6 (System for demand schedule coefficients) *Suppose that at $t+1$, the value*

function is given by equation (18)*. Then traders submit demand schdules* (6) *if and only if*

$$(\mathcal{S}^D): \quad \begin{cases} (\lambda_t^i + \sigma^2 A_{t+1}^i)\beta_t^i + \sigma^2 \sum_{k=1}^n (\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{i}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{k}})\beta_t^k = 1, \quad \forall i \\ \lambda_t^i = \frac{1}{\sum_{j\neq i}\beta_t^j}, \quad \forall i \\ (\lambda_t^i + \sigma^2 A_{t+1}^i)d_t^{ij} + \sigma^2 \sum_{k=1}^n (\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{i}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{k}})d_t^{kj} = (1-c_t^j)\sigma^2 \sum_{l=t+1}^T \varphi_{t+1}^{il} f_{t+1}^{jl}, \quad \forall i,j \\ (\lambda_t^i + \sigma^2 A_{t+1}^i)\delta_t^{il} + \sigma^2 \sum_{k=1}^n (\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{i}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{k}})\delta_t^{kl} = \sigma^2 \varphi_{t+1}^{il}, \quad \forall i, l \geq t+1 \\ c_t^i = \frac{\sigma^2 A_{t+1}^i}{\lambda_t^i + \sigma^2 A_{t+1}^i}, \quad \forall i. \end{cases}$$

An equilibrium in demand schedule exists at t *if and only if the previous system has a solution and the second-order condition holds:*

$$\lambda_t^i + \sigma^2 A_{t+1}^i + \sigma^2 \boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{i}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{i}} \geq \mathbf{0} \tag{125}$$

Proof. The proof has two steps.

Step 1: Traders' optimization.

At time t, traders solve:

$$\max_{x_t^i} \; v_t^i + x_t^i(D_t - p_t) - \frac{\alpha^i \sigma^2}{2}(1+\gamma_{t+1}^i)(X_t^i)^2 - \sigma^2 X_t^i \boldsymbol{\mu}_{\mathbf{t+1}}^{\mathbf{i}} \mathbf{Q}_{\mathbf{t+1}} + \frac{\sigma^2}{2} \mathbf{Q}_{\mathbf{t+1}}^\top \boldsymbol{\rho}_{\mathbf{t+1}}^{\mathbf{i}} \mathbf{Q}_{\mathbf{t+1}} \tag{126}$$

The first-order condition leads to

$$D_t - p_t - \lambda_t^i x_t^i - \alpha^i \sigma^2 (1+\gamma_{t+1}^i) X_t^i - \sigma^2 \left(\boldsymbol{\mu}_{\mathbf{t+1}}^{\mathbf{i}} \mathbf{Q}_{\mathbf{t+1}} + X_t^i \boldsymbol{\mu}_{\mathbf{t+1}}^{\mathbf{i}} \frac{\partial \mathbf{Q}_{\mathbf{t+1}}}{\partial x_t^i} \right) + \frac{\sigma^2}{2} \mathbf{Q}_{\mathbf{t+1}}^\top (\boldsymbol{\rho}_{\mathbf{t+1}}^{\mathbf{i}} + \boldsymbol{\rho}_{\mathbf{t+1}}^{\mathbf{i}\top}) \frac{\partial \mathbf{Q}_{\mathbf{t+1}}}{\partial x_t^i} = 0$$

Note that

$$\frac{\partial \mathbf{Q}_{\mathbf{t+1}}}{\partial x_t^i} = \begin{pmatrix} \frac{\partial Q_{t+1}^{t+1}}{\partial x_t^i} \\ \vdots \\ \frac{\partial Q_{t+1}^{T}}{\partial x_t^i} \end{pmatrix} = \begin{pmatrix} c_{t+1}^i \\ \vdots \\ c_T^i (1-c_{T-1}^i) \dots (1-c_{t+1}^i) \end{pmatrix} = \begin{pmatrix} f_{t+1}^{it+1} \\ \vdots \\ f_{t+1}^{iT} \end{pmatrix}.$$

Therefore,

$$\mu^{\mathbf{i}}_{\mathbf{t+1}}\frac{\partial \mathbf{Q_{t+1}}}{\partial x^i_t} = \sum_{l=t+1}^{T}\mu^{il}_{t+1}f^{il}_{t+1}$$

$$\frac{1}{2}(\rho^{\mathbf{i}}_{\mathbf{t+1}}+\rho^{\mathbf{i}\top}_{\mathbf{t+1}})\frac{\partial \mathbf{Q_{t+1}}}{\partial x^i_t} = \frac{1}{2}\begin{pmatrix}\sum_{l=t+1}^{T}(\rho^{it+1l}_{t+1}+\rho^{ilt+1}_{t+1})f^{il}_{t+1}\\ \vdots \\ \sum_{l=t+1}^{T}(\rho^{iTl}_{t+1}+\rho^{ilT}_{t+1})f^{il}_{t+1}\end{pmatrix} = \begin{pmatrix}r^{it+1}_{t+1}\\ \vdots \\ r^{iT}_{t+1}\end{pmatrix},$$

where I used Notation 3. Substituting these expressions into the first-order conditions and grouping terms leads to:

$$D_t - p_t - \lambda^i_t x^i_t - \left[\alpha^i\sigma^2(1+\gamma^i_{t+1}) + \sigma^2\mu^{\mathbf{i}}_{\mathbf{t+1}}\frac{\partial \mathbf{Q_{t+1}}}{\partial x^i_t}\right]X^i_t - \sigma^2\mathbf{Q}^{\top}_{\mathbf{t+1}}\left(\mu^{\mathbf{i}}_{\mathbf{t+1}} - \mathbf{r^i_{t+1}}\right) = 0$$

$$D_t - p_t - (\lambda^i_t + \sigma^2 A^i_{t+1})x^i_t - \sigma^2 A^i_{t+1}X^i_{t-1} - \sigma^2\varphi^{\mathbf{i}}_{\mathbf{t+1}}\mathbf{Q_{t+1}} = 0,$$

where the second equation makkes use of Notation 3 again. Then, using the definition of $\mathbf{Q_{t+1}}$, the first-order condition can be expressed as a function of holdings and supply shocks:

$$D_t - p_t - (\lambda^i_t + \sigma^2 A^i_{t+1})x^i_t - \sigma^2 A^i_{t+1}X^i_{t-1} - \sigma^2\sum_{l=t+1}^{T}\varphi^{il}_{t+1}\sum_{j=1}^{n}f^{jl}_{t+1}X^j_t - \sigma^2\sum_{l=t+1}^{T}\varphi^{il}_{t+1}\Delta s_l = 0, \quad (127)$$

The second-order condition (125) follows directly from equation (127).

Step 2: Deriving system $\mathcal{S}^D$.

When traders submit schedules (6), the implied equilibrium price and trade are given by equations (123)-(124). Further,

$$\sum_{j=1}^{n}f^{jl}_{t+1}X^j_t = \sum_{j=1}^{n}\left\{(1-c^j_t)f^{jl}_{t+1} + \left(\sum_k f^{kl}_{t+1}\beta^k_t\right)\frac{c^j_t+\bar{d}^j_t}{\bar{\beta}_t} - \sum_k f^{kl}_{t+1}d^{kj}_t\right\}X^j_{t-1}$$
$$+\left(\frac{\sum_k f^{kl}_{t+1}\beta^k_t}{\bar{\beta}_t}\right)\Delta s_t + \sum_{u=t+1}^{T}\left(\sum_k f^{kl}_{t+1}\left(\frac{\beta^k_t}{\bar{\beta}_t}\bar{\delta}^u_t - \delta^{ku}_t\right)\right)\Delta s_u \quad (128)$$

The first-order condition is necessary. For equations (123), (124) and (128) to hold, it suffices

that coefficients satisfy sytem $\mathcal{S}^D$, where I used the fact that

$$\sum_{l=t+1}^{T}\varphi_{t+1}^{il}\sum_{k=1}^{n}f_{t+1}^{kl}\beta_t^k=\sum_{k=1}^{n}\sum_{l=t+1}^{T}\varphi_{t+1}^{il}f_{t+1}^{kl}\beta_t^k=\sum_{k}\left(\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{i}}\cdot\mathbf{f}_{\mathbf{t+1}}^{\mathbf{k}}\right)\beta_t^k \tag{129}$$

■

The system $\mathcal{S}^D$ given in Lemma 6 can be written in matrix form as follows:

$$(\mathcal{S}^D):\quad\begin{cases}\mathcal{M}_{\mathbf{t}}(\lambda_{\mathbf{t}})\beta_{\mathbf{t}}=1\\ \mathcal{M}_{\mathbf{t}}(\lambda_{\mathbf{t}})\mathbf{d}_{\mathbf{t}}^{\mathbf{j}}=\mathcal{A}_{\mathbf{d}}^{\mathbf{j}},\quad\forall j\\ \mathcal{M}_{\mathbf{t}}(\lambda_{\mathbf{t}})\delta_{\mathbf{t}}^{\mathbf{l}}=\mathcal{A}_{\delta}^{\mathbf{l}},\quad l\geq t+1\\ \lambda_t^i=\frac{1}{\sum_{j\neq i}\beta_t^j},\quad\forall i\\ c_t^i=\frac{\sigma^2A_{t+1}^i}{\lambda_t^i+\sigma^2A_{t+1}^i},\quad\forall i,\end{cases}$$

where $\mathcal{M}_{\mathbf{t}}(\lambda_{\mathbf{t}})$ is given by

$$\mathcal{M}_{\mathbf{t}}=\begin{pmatrix}\lambda_t^1+\sigma^2A_{t+1}^1+&\dots&0\\ \vdots&\ddots&\vdots\\ 0&\dots&\lambda_t^n+\sigma^2A_{t+1}^n\end{pmatrix}+\begin{pmatrix}\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}}\cdot\mathbf{f}_{\mathbf{t+1}}^{\mathbf{k}}&\dots&\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}}\cdot\mathbf{f}_{\mathbf{t+1}}^{\mathbf{n}}\\ \vdots&\ddots&\vdots\\ \boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{n}}\cdot\mathbf{f}_{\mathbf{t+1}}^{\mathbf{1}}&\dots&\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{n}}\cdot\mathbf{f}_{\mathbf{t+1}}^{\mathbf{n}}\end{pmatrix}. \tag{130}$$

In the next result, I show that under certain conditions the price impact is proportional to σ^2. To establish these conditions, it is convenient to work with the inverse of price impact. To this end, I introduce the variable l_t^i defined by

$$l_t^i=\frac{1}{\frac{\lambda_t^i}{\sigma^2}}. \tag{131}$$

When normalizing the variables by σ^2, system $\mathcal{S}^D$ can be rewritten using matrix $\mathbf{M}_{\mathbf{t}}(\mathbf{l}_{\mathbf{t}})$.

Lemma 7 (Existence) *Suppose that the second-order condition (125) holds. Then,*

1. *If $A_{t+1}^i\geq 0$, the equilibrium at t exists and is unique if and only if there exists a unique*

vector $\left(L_t, \mathcal{W}_t^{1L}, \ldots, \mathcal{W}_t^{nL}\right)$*, with* $L_t > 0$ *and* $\mathcal{W}_t^{iL} \in \mathbb{R}$*, solving*

$$L_t = \sum_j l_t^j, \tag{132}$$

$$\mathcal{W}_t^{iL} = \sum_j F_{t+1}^{ik} l_t^k, \tag{133}$$

$$\text{where } l_t^i = \frac{(n-1)(2+\mathcal{W}_t^{iL}) - (A_{t+1}^i + \sum_k F_{t+1}^{ik})L_t + \sqrt{D_t^i(L_t, \mathcal{W}_t^{iL})}}{2(n-1)A_{t+1}^i}, \tag{134}$$

and $\mathcal{D}_t^i = \left[(A_{t+1}^i + \sum_k F_{t+1}^{ik})L_t - (n-1)(2+\mathcal{W}_t^{iL})\right]^2 + 4(n-1)A_{t+1}^i L_t$*. If such a solution exists, then price impact is proportional to* σ^2*, with*

$$\lambda_t^i = \frac{\sigma^2}{l_t^i}, \tag{135}$$

and other coefficients are given by

$$c_t^i = \frac{A_{t+1}^i}{(l_t^i)^{-1} + A_{t+1}^i}, \tag{136}$$

$$d_t^{ij} = (1 - c_t^j) \sum_{l=t+1}^{T} f_{t+1}^{jl} \delta_t^{il}, \tag{137}$$

$$\delta_t^{il} = \sum_{k=1}^{n} \omega_t^{ik} \varphi_{t+1}^{kl}, \quad l \geq t+1, \tag{138}$$

where ω_t^{ik} *are the elements of* $\mathbf{M_t}^{-1}$*.*

2. *If* $A_{t+1}^i < 0$*, the equilibrium is not unique or may fail to exist.*

Proof. Suppose that $A_{t+1}^i \geq 0$, $\forall i$. The result is based on the following four points.

1. Existence of a fixed point in price impact is equivalent to $\mathcal{M}_\mathbf{t}(\lambda_\mathbf{t})$ *being of full rank.* Proceed by contradiction: suppose that there exists a fixed point λ_t^* but $\mathcal{M}_\mathbf{t}(\lambda_\mathbf{t}^*)$ is not of fulll rank, i.e., at least two rows in the matrix are proportional to each other. Suppose without loss of generality that the second row is z times the first one. Then the first two rows of $\mathcal{M}_\mathbf{t}(\lambda_\mathbf{t}^*)\beta_\mathbf{t}^*$ are

$$(\lambda_t^{1*} + \sigma^2 A_{t+1}^1 + \sigma^2 \boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{1}})\beta_t^{1*} + \sigma^2(\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{2}})\beta_t^{2*} + \cdots + \sigma^2(\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{n}})\beta_t^{n*} = 1$$

$$z(\lambda_t^{1*} + \sigma^2 A_{t+1}^1 + \sigma^2 \boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{1}})\beta_t^{1*} + z\sigma^2(\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{2}})\beta_t^{2*} + \cdots + z\sigma^2(\boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{n}})\beta_t^{n*} = 1$$

This implies that $z = 1$. But then it must be that

$$\begin{aligned}\lambda_t^{1*} + \sigma^2 A_{t+1}^1 + \sigma^2(\varphi_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{1}}) &= \sigma^2(\varphi_{\mathbf{t+1}}^{\mathbf{2}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{1}}) \\ \sigma^2(\varphi_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{2}}) &= \lambda_t^{2*} + \sigma^2 A_{t+1}^2 + \sigma^2(\varphi_{\mathbf{t+1}}^{\mathbf{2}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{2}}) \\ &\vdots \\ \sigma^2(\varphi_{\mathbf{t+1}}^{\mathbf{1}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{n}}) &= \sigma^2(\varphi_{\mathbf{t+1}}^{\mathbf{2}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{n}}) \end{aligned} \tag{139}$$

The last $n-2$ rows imply that $\varphi_{\mathbf{t+1}}^{\mathbf{1}} = \varphi_{\mathbf{t+1}}^{\mathbf{2}}$. Then the first equation becomes $\lambda_t^{1*} + \sigma^2 A_{t+1}^1 = 0$, a contradiction, since $\lambda_t^{1*} > 0$ in equilibrium and $A_{t+1}^1 \geq 0$ by assumption.

2. Price impact is proportional to σ^2. Suppose that equilibrium price impact is of the form $\lambda_t^{i*} = k_t^i \sigma^2$ and let β_t^{i*} be the associated demand slopes. Then this ansatz is a fixed point if it satisfies

$$\begin{aligned}(k_t^i + A_{t+1}^i)\beta_t^{i*} + \sum_{k=1}^{n}(\varphi_{\mathbf{t+1}}^{\mathbf{i}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{k}})\beta_t^{k*} &= \frac{1}{\sigma^2}, \quad \forall i, \\ \text{i.e.} \quad \mathbf{M_t}(\mathbf{k_t})\beta_{\mathbf{t}}^* &= \frac{\mathbf{1}_n}{\sigma^2}, \end{aligned} \tag{140}$$

where $\mathbf{1}_n$ is a column vector of ones of size n. If λ_t^* is a solution, $\mathcal{M}_{\mathbf{t}}(\lambda_{\mathbf{t}}^*)$ is invertible (previous point), and by the same token $\mathbf{M_t}(\mathbf{k_t})$ is invertible. Let $(\omega_t^{ij})_{i,j}$ denote the elements of $\mathbf{M_t^{-1}}$. By construction, these elements do not depend on σ^2. The demand slopes are then given by

$$\begin{aligned}\beta_{\mathbf{t}}^* &= \mathbf{M_t^{-1}}\frac{\mathbf{1}_n}{\sigma^2}, \\ \text{i.e., } \beta_t^{i*} &= \frac{\sum_k \omega_t^{ik}}{\sigma^2} \equiv \frac{q_t^i}{\sigma^2}, \end{aligned} \tag{141}$$

This implies that $\bar{\beta}_t^* = \sum_i \beta_t^{i*} = \frac{\sum_i q_t^i}{\sigma^2} \equiv \frac{q_t}{\sigma^2}$. But price impact is defined as $\lambda_t^i = (\bar{\beta}_t - \beta_t^i)^{-1}$. Hence, $\lambda_t^{i*} = \sigma^2(q_t - q_t^i)^{-1}$. This confirms that λ_t^{i*} is indeed of the form $\sigma^2 k_t^i$, as guessed.

Conversely, if there is a solution, it must be proportional to σ^2. Suppose for instance that price impact is of the form $\lambda_t^{i*} = k_t^i g(\sigma^2)$, where g is a non-linear function. Then proceeding as before,

$$\begin{aligned}(k_t^i g(\sigma^2) + \sigma^2 A_{t+1}^i)\beta_t^{i*} + \sigma^2 \sum_{k=1}^{n}(\varphi_{\mathbf{t+1}}^{\mathbf{i}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{k}})\beta_t^{k*} &= 1, \quad \forall i, \\ \text{i.e.} \quad \mathbf{M_t^g}\beta_{\mathbf{t}}^* &= \mathbf{1}_n. \end{aligned} \tag{142}$$

$\mathbf{M_t^g}$ is invertible, but the elements of its inverse are now a combination of σ^2 and $g(\sigma^2)$ (this can be seen already in the case with two levels of risk aversion where it is easy to compute the inverse). Therefore, in general, the resulting slopes β_t^{i*} are not of the form $\frac{b_t^i}{g(\sigma^2)}$. But the assumed solution $\lambda_t^{i*} = k_t^i g(\sigma^2)$ implies that slope should take this form. Indeed, since $\bar{\beta}_t - \beta_t^i = \frac{1}{\lambda_t^i}$, the assumed solution leads to $\bar{\beta}_t - \beta_t^i = \frac{1}{k_t^i g(\sigma^2)}$. Summing over i and rearranging the terms, $\bar{\beta}_t = \frac{1}{(n-1)g(\sigma^2)} \sum_i \frac{1}{k_t^i} \equiv \frac{B_t}{g(\sigma^2)}$. Then, $\beta_t^i = \frac{B_t - \frac{1}{k_t^i}}{g(\sigma^2)}$, which is of the form $\frac{b_t^i}{g(\sigma^2)}$. Thus, assuming that price impact is of the form $\lambda_t^{i*} = k_t^i g(\sigma^2)$ leads to a contradiction.

3. Existence conditions as a function of the sign of A_{t+1}^i. Suppose that price impact is proportional to σ^2 and is given by equation (135) (working with λ^{-1} is analytically more convenient). Then, denoting $L_t = \sum_i l_t^i$ and using the same arguments as in point 2, it is easy to derive the following relationship between the variables:

$$\bar{\beta}_t = \frac{L_t}{(n-1)\sigma^2}, \quad \beta_t^i = \frac{L_t}{(n-1)\sigma^2} - \frac{l_t^i}{\sigma^2}$$

After substituting these expressions in the first equation of $\mathcal{S}^D$, the terms in σ^2 cancel out, leading to:

$$\left(\frac{1}{l_t^i} + A_{t+1}^i\right)\left(L_t - (n-1)l_t^i\right) + \sum_k F_{t+1}^{ik}\left(\frac{L_t}{n-1} - l_t^k\right) = 1,$$

where I used notation (29). Multiplying both sides by $(n-1)l_t^i$ and grouping terms leads to a second-order equation in l_t^i:

$$-(n-1)A_{t+1}^i (l_t^i)^2 + l_t^i\left(L_t\left(A_{t+1}^i + \sum_k F_{t+1}^{ik}\right) - (n-1)\left(2 + \mathcal{W}_t^{iL}\right)\right) + L_t = 0 \quad (143)$$

In equilibrium, L_t must be strictly positive. Thus, it suffices that $A_{t+1}^i \geq 0$ for a unique positive solution to equation (143) to exist. This solution is given by equation (134). Thus, proving existence and uniqueness is equivalent to showing that there exists a unique vector $\left(L_t, \mathcal{W}_t^{1L}, \ldots, \mathcal{W}_t^{nL}\right)$, with $L_t > 0$ and $\mathcal{W}_t^{iL} \in \mathbb{R}$, solving

$$L_t = \sum_j l_t^j,$$
$$\mathcal{W}_t^{iL} = \sum_j F_{t+1}^{ik} l_t^k,$$

where l_t^i is given by equation (134).

If $A_{t+1}^i < 0$, then

1. If $\mathcal{D}_t^i \geq 0$, where $\mathcal{D}_t^i$ is the discriminant (given in the result), there are either two positive roots if $L_t\left(A_{t+1}^i + \sum_k F_{t+1}^{ik}\right) < (n-1)\left(2 + \mathcal{W}_t^{iL}\right)$, and thus two potential equilibria, or two negative roots, and thus no equilibria, otherwise.

2. If the discriminant $\mathcal{D}_t^i < 0$, an equilibrium does not exist.

4. Other coefficients when the fixed point is unique. Once the fixed point problem is solved, all the other coefficients can be expressed by solving a linear system. Let $\mathbf{M}_t^*$ denote the matrix associated with the fixed point, then:

$$\begin{aligned}
\mathbf{M}_t^* \boldsymbol{\delta}_{\mathbf{t}}^{\mathbf{l}} &= \boldsymbol{\varphi}_{\mathbf{t+1}}^{\mathbf{l}} \\
\mathbf{M}_t^* \mathbf{d}_{\mathbf{t}}^{\mathbf{j}} &= (1 - c_t^j) \begin{pmatrix} \varphi_{\mathbf{t+1}}^{\mathbf{n}} \mathbf{f}_{\mathbf{t+1}}^{\mathbf{j}} \\ \vdots \\ \varphi_{\mathbf{t+1}}^{\mathbf{1}} \mathbf{f}_{\mathbf{t+1}}^{\mathbf{j}} \end{pmatrix}.
\end{aligned}$$

So inverting the system gives $\delta_t^{il} = \sum_{k=1}^n \omega_t^{ik} \varphi_{t+1}^{kl}$ and

$$\begin{aligned}
d_t^{ij} &= (1 - c_t^j) \sum_{k=1}^n \omega_t^{ik} \sum_{l=t+1}^T \varphi_{t+1}^{kl} f_{t+1}^{jl} \\
&= (1 - c_t^j) \sum_{l=t+1}^T \left(\sum_{k=1}^n \omega_t^{ik} \varphi_{t+1}^{kl} \right) f_{t+1}^{jl} \\
&= (1 - c_t^j) \sum_{l=t+1}^T f_{t+1}^{jl} \delta_t^{il}. \qquad (144)
\end{aligned}$$

Recall that by definition, $f_t^{jl} = (1 - c_t^j) f_{t+1}^{jl}$. It is key for the decomposition in static and dynamic portfolios that d_t^{ij} remains a linear combination of f_t^{jl}. This point is implied by the first-order condition of the problem and the fact that this linear combination occurs at the final period. Then the linear combination at $t+1$ implies the same structure at t. The result does not depend on price impact, A^i and φ^i being proportional to σ^2 or any other kind of scaling. Indeed, denote $l_t^{i\sigma} = \frac{l_t^i}{\sigma^2}$, $A_{t+1}^{i\sigma} = \sigma^2 A_{t+1}^i$ and $\varphi_{t+1}^{il\sigma} = \sigma^2 \varphi_{t+1}^{il}$. Then the first-order

condition (127) remains the same, and using these notations, it is given by:

$$D_t - p_t - (\lambda_t^i + A_{t+1}^{i\sigma})x_t^i - A_{t+1}^{i\sigma}X_{t-1}^i - \sum_{l=t+1}^{T}\varphi_{t+1}^{il\sigma}\sum_{j=1}^{n}f_{t+1}^{jl}X_t^j - \sum_{l=t+1}^{T}\varphi_{t+1}^{il\sigma}\Delta s_l = 0.$$

All the arguments in point 3 then go through with these variables.

■

When a unique equilibrium exists at time t (under the conditions given in Lemma 7), the risk premium and trade take a simple expression. We can then derive the system for value function coefficients ensuring that the guess for the value function is verified at t. Point 1 in Corollary 2 followss from the next result and derivations in the proof.

Lemma 8 *Under the conditions of point 1 in Lemma 7:*

1. *The equilibrium risk premium and trade can be written as*

$$D_t - p_t = \frac{\sigma^2}{q_t}\left(Q_t^t + \sum_{l=t+1}^{T}\pi_t^l Q_t^l\right) = \frac{\sigma^2}{q_t}\pi_{\mathbf{t}}\mathbf{Q_t}, \tag{145}$$

$$x_t^i = -c_t^i X_{t-1}^i + \phi_t^i Q_t^t + \sum_{l=t+1}^{T}\theta_t^{il}Q_t^l = -c_t^i X_{t-1}^i + \boldsymbol{\theta}_{\mathbf{t}}^{\mathbf{i}}\mathbf{Q_t}, \tag{146}$$

   where $\theta^{it} = \phi_t^i > 0$ *and* $\sum_j \theta_t^{jl} = 0$, *for any* $l \geq t+1$.

2. *The law of motion for the state variables is*

$$X_t^i = (1 - c_t^i)X_{t-1}^i + \boldsymbol{\theta}_{\mathbf{t}}^{\mathbf{i}}\mathbf{Q_t} \tag{147}$$

$$\mathbf{Q_{t+1}} = \boldsymbol{\Theta}_{\mathbf{t}}^{\mathbf{Q}}\mathbf{Q_t}. \tag{148}$$

3. *The value function is given by equation (18) at t if and only if its coefficients solve:*

$$\gamma_t^i = (1 - c_t^i)^2(1 + \gamma_{t+1}^i), \tag{149}$$

$$\boldsymbol{\mu}_{\mathbf{t}}^{\mathbf{i}} = \frac{c_t^i}{q_t}\pi_{\mathbf{t}} + (1 - c_t^i)\alpha^i(1 + \gamma_{t+1}^i)\boldsymbol{\theta}_{\mathbf{t}}^{\mathbf{i}} + (1 - c_t^i)\boldsymbol{\mu}_{\mathbf{t+1}}^{\mathbf{i}}\boldsymbol{\Theta}_{\mathbf{t}}^{\mathbf{Q}}, \tag{150}$$

$$\frac{1}{2}\boldsymbol{\rho}_{\mathbf{t}}^{\mathbf{i}} = \frac{1}{q_t}\boldsymbol{\theta}_{\mathbf{t}}^{\mathbf{i}\top}\pi_{\mathbf{t}} - \frac{\alpha^i}{2}(1 + \gamma_{t+1}^i)\boldsymbol{\theta}_{\mathbf{t}}^{\mathbf{i}\top}\boldsymbol{\theta}_{\mathbf{t}}^{\mathbf{i}} - \boldsymbol{\theta}_{\mathbf{t}}^{\mathbf{i}\top}\boldsymbol{\mu}_{\mathbf{t+1}}^{\mathbf{i}}\boldsymbol{\Theta}_{\mathbf{t}}^{\mathbf{Q}} + \frac{1}{2}\boldsymbol{\Theta}_{\mathbf{t}}^{\mathbf{Q}\top}\boldsymbol{\rho}_{\mathbf{t+1}}^{\mathbf{i}}\boldsymbol{\Theta}_{\mathbf{t}}^{\mathbf{Q}}. \tag{151}$$

Proof. *1. Equilibrium risk premium and trade.* In equation (123), it is easy to recognize Q_t^t

in the numerator by grouping the terms in c_t^j and Δs_t. The other term in the numerator is $\sum_j \bar{d}_t^j X_{t-1}^j + \sum_{l=t+1}^T \bar{\delta}_t^l \Delta s_l$. Starting from equation (137) and summing over i gives:

$$\bar{d}_t^j = \sum_i d_t^{ij} = (1 - c_t^j) \sum_{l=t+1}^T f_{t+1}^{jl} \bar{\delta}_t^l. \tag{152}$$

Therefore, using this expression and equation (138) leads to

$$\begin{aligned}
\sum_j \bar{d}_t^j X_{t-1}^j + \sum_{l=t+1}^T \bar{\delta}_t^l \Delta s_l &= \sum_j \left\{ (1 - c_t^j) \sum_{l=t+1}^T f_{t+1}^{jl} \bar{\delta}_t^l \right\} X_{t-1}^j + \sum_{l=t+1}^T \bar{\delta}_t^l \Delta s_l \\
&= \sum_{l=t+1}^T \bar{\delta}_t^l \left\{ \sum_j (1 - c_t^j) f_{t+1}^{jl} X_{t-1}^j + \Delta s_l \right\} \\
&= \sum_{l=t+1}^T \bar{\delta}_t^l Q_t^l,
\end{aligned}$$

since $(1 - c_t^j) f_{t+1}^{jl} = f_t^{jl}$. Then, equation (145) follows by defining

$$\pi_t^t = 1, \quad \pi_t^l = \bar{\delta}_t^l, \quad l \geq t + 1. \tag{153}$$

Next, turn to the equilibrium trade. Note that the term based on future shocks in equation (124) is given by $\phi_t^i \bar{\delta}_t^l - \delta_t^{il}$. Summing equation (138) over i leads to $\bar{\delta}_t^l = \sum_k \bar{\omega}_t^k \varphi_{t+1}^{kl}$. Thus,

$$\begin{aligned}
\phi_t^i \bar{\delta}_t^l - \delta_t^{il} &= \phi_t^i \sum_k \bar{\omega}_t^k \varphi_{t+1}^{kl} - \sum_k \omega_t^{ik} \varphi_{t+1}^{kl} \\
&= \sum_k (\phi_t^i \bar{\omega}_t^k - \omega_t^{ik}) \varphi_{t+1}^{kl} \\
&\equiv \theta_t^{il}, \quad l \geq t + 1.
\end{aligned} \tag{154}$$

Similarly, in equation (124), the term based on X_{t-1}^j is $\phi_t^i \bar{d}_t^j - d_t^{ij}$. Summing equation (137) over i gives

$$\bar{d}_t^j = (1 - c_t^j) \sum_{l=t+1}^T f_{t+1}^{jl} \sum_i \delta_t^{il} = (1 - c_t^j) \sum_{l=t+1}^T f_{t+1}^{jl} \bar{\delta}_t^l.$$

This leads to

$$\phi_t^i \bar{d}_t^j - d_t^{ij} = \sum_{l=t+1}^{T} f_t^{jl}(\phi_t^i \bar{\delta}_t^l - \delta_t^{il}).$$

Therefore,

$$\begin{aligned}\sum_j \left(\phi_t^i \bar{d}_t^j - d_t^{ij}\right) X_{t-1}^j + \sum_{l=t+1}^{T} (\phi_t^i \bar{\delta}_t^l - \delta_t^{il}) \Delta s_l &= \sum_{l=t+1}^{T} (\phi_t^i \bar{\delta}_t^l - \delta_t^{il}) \left(\sum_j f_t^{jl} X_{t-1}^j + \Delta s_l\right) \\ &= \sum_{l=t+1}^{T} \theta_t^{il} Q_t^l,\end{aligned}$$

where the last step follows from the definition of dynamic portfolios and equation (154). This leads to equation (146) with the definition $\theta_t^{it} = \phi_t^i$.

Consider the sign of the elements of $\theta_{\mathbf{t}}^{\mathbf{i}}$. The first element $\theta_t^{it} = \phi_t^i \equiv \frac{\beta_t^i}{\bar{\beta}_t} \in (0,1)$ is stricly positive. The coefficients θ_t^{il}, defined by equation (154), can be positive or negative for $l \geq t+1$, because $\phi_t^i \in (0,1)$ and $\sum_i \omega_t^{ik} = \bar{\omega}_t^k$. Further,

$$\begin{aligned}\sum_i \theta_t^{it} &= \sum_i \phi_t^i = \sum_i \frac{\beta_t^i}{\bar{\beta}_t} = 1 \\ \sum_i \theta_t^{il} &= \sum_k \left(\sum_i \phi_t^i \bar{\omega}_t^k - \sum_i \omega_t^{ik}\right) \varphi_{t+1}^{kl} = 0,\end{aligned}$$

since $\sum_i \phi_t^i = 1$ and $\sum_i \omega_t^{ik} = \bar{\omega}_t^k$.

2. Transition matrix. Using the equilibrium trade just derived, and the definition of static and dynamic portfolios:

$$Q_{t+1}^l = \sum_j f_{t+1}^{jl} X_t^j + \Delta s_l = Q_t^l + \left(\sum_j f_{t+1}^{jl} \phi_t^j\right) Q_t^t + \sum_{u=t+1}^{T} \left(\sum_j f_{t+1}^{jl} \theta_t^{ju}\right) Q_t^u, \quad l \geq t+1.$$

To write this in matrix form, as in the result, $\mathbf{Q_{t+1}} = \mathbf{\Theta_t^Q Q_t}$, define

$$\mathbf{\Theta_t^Q} = \begin{pmatrix}\mathbf{0}_{(T-t)\times 1} & \mathbf{I}_{(T-t)\times(T-t)}\end{pmatrix} + \begin{pmatrix}\sum_j f_{t+1}^{jt+1}\phi_t^j & \sum_j f_{t+1}^{jt+1}\theta_t^{jt+1} & \cdots & \sum_j f_{t+1}^{jt+1}\theta_t^{jT} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_j f_{t+1}^{jT}\phi_t^j & \sum_j f_{t+1}^{jT}\theta_t^{jt+1} & \cdots & \sum_j f_{t+1}^{jT}\theta_t^{jT}\end{pmatrix}. \quad (155)$$

3. Value function. This part is simply algebra. To obtain equations (149)-(151) for the value function coefficients, plug the risk premium (145), equilibrium trade (146), and the law of motion equation (148) into the objective (126) and regroup the terms. ■

C.3 Equilibrium as a function of primitives

In Proposition 2, the equilibrium risk premium and trade are characterized by backward induction. Moving forward from time 0 to time T, we can then express equilibrium prices and quantities in terms of primitives:

Corollary 4 (Equilibrium as a function of primitives) *Under the conditions of Propositon 2, the risk premium and inventories are given by:*

$$D_t - p_t = \frac{\sigma^2}{q_t}\pi_t \mathbf{\Pi^Q_{0,t-1}} \mathbf{Q_0} \tag{156}$$

$$X^i_t = \Pi^{ic}_{0,t} X^i_{-1} + \mathbf{\Xi^i_{0,t-1}} \mathbf{Q_0} + \theta^i_t \mathbf{\Pi^Q_{0,t-1}} \mathbf{Q_0}, \tag{157}$$

where $\Pi^{ic}_{0,t} = \prod_{u=0}^{t}(1-c^i_u)$, $\mathbf{\Pi^Q_{0,t-1}} = \prod_{u=0}^{t-1} \mathbf{\Theta^Q_u}$, *where* $\mathbf{\Theta^Q_u}$ *is given by equation* (170), *and*

$$\mathbf{\Xi^i_{0,t-1}} = \sum_{l=0}^{t-1} \Pi^{ic}_{l+1,t} \theta^i_l \mathbf{\Pi^Q_{0,l-1}}. \tag{158}$$

Equations (156) and (157) have simple interpretations. The projection $\mathbf{\Pi^Q_{0,t-1}}$ helps "resize" time-t portfolios in terms of time-0 variables. This allows to express the risk premium (156) as a function of initial portfolios $\mathbf{Q_0}$. Inventories are made of three terms. The first term, $\Pi^{ic}_{0,t} X^i_{-1}$, is what remains of the initial inventory after $t+1$ rounds of trade. The second term, $\mathbf{\Xi^i_{0,t-1}} \mathbf{Q_0}$, is what remains of the target portfolios acquired between 0 and $t-1$ and later partially diversified (term $\Pi^{ic}_{l+1,t}\theta^i_l$ in equation (158)), measured in terms of time-0 variables (term $\mathbf{\Pi^Q_{0,l-1}}$). The third term, $\theta^i_t \mathbf{\Pi^Q_{0,t-1}} \mathbf{Q_0}$, expresses the target trade of time-t as a function of time-0 variables. Thus, after $t+1$ rounds of trade, a trader holds a combination of her initial inventory, her past (under-diversified) target portfolios, and her current target portfolio.

C.4 Special case: two levels of risk aversion

In this section, I assume that α^i can take only two values: $\alpha^i = a$ for $i = 1, \ldots, n-m$ and $\alpha^i = b$ for $i = n-m+1, \ldots, n$, i.e., there are $n-m$ traders with risk aversion a and $m \geq 1$

traders with risk aversion $b \neq a$. I denote $\mathcal{A}$ and $\mathcal{B}$ the set of traders of each type. The aggregate inventory at the beginning of round t can thus be written as:

$$\sum_{j=1}^{n} X_{t-1}^{j} = \sum_{j \in \mathcal{A}} X_{t-1}^{j} + \sum_{j \in \mathcal{B}} X_{t-1}^{j}.$$

The system $(\mathcal{S}^D)$ can then be simplified; in particular, the fixed point for β boils down to two equations. All the value function and demand schedule coefficients also take only two values, e.g., $c_t^i = c_t^a$ for $i \in \mathcal{A}$, and $c_t^i = c_t^b$ if $i \in \mathcal{B}$. As a result, diversified risk is driven by the aggregate inventories of both groups of traders, for any $t \leq T$:

$$Q_t^t = c_t^a \sum_{j \in \mathcal{A}} X_{t-1}^{j} + c_t^b \sum_{j \in \mathcal{B}} X_{t-1}^{j} + \Delta s_t, \tag{159}$$

$$Q_t^l = f_t^{al} \sum_{j \in \mathcal{A}} X_{t-1}^{j} + f_t^{bl} \sum_{j \in \mathcal{B}} X_{t-1}^{j} + \Delta s_l, \quad l \geq t+1. \tag{160}$$

It is useful to adjust the notations: for $\alpha \in \{a, b\}$, $k_t^\alpha = \frac{\lambda_t^\alpha}{\sigma^2}$, $B_t^\alpha = \beta_t^\alpha \sigma^2$, $F_{t+1}^{aa} = \varphi_{\mathbf{t+1}}^{\mathbf{a}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{a}}$, $F_{t+1}^{ab} = \varphi_{\mathbf{t+1}}^{\mathbf{a}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{b}}$, etc.

C.4.1 Equilibrium at T

In the general case, I write the fixed point in terms of $l_t^i = \sigma^2(\lambda_t^i)^{-1}$. With two levels of risk aversion, I write the fixed point directly in terms of the demand schedule slopes $\beta_t^i = \frac{B_t^i}{\sigma^2}$. The next result determines the slopes of the groups of traders in the final round.

Lemma 9 (Equilibrium at T with two levels of risk aversion) *In the final round, there exists a unique equilibrium, as follows:*

- *If $m = 1$,*

$$B_T^a = \frac{(n-1)(n-3)b - (2n-3)a + \sqrt{\Delta_a}}{2(n-1)(n-2)ab}, \tag{161}$$

$$\text{where} \quad \Delta_a = (2n-3)^2 a^2 + (n-1)^2(n-3)^2 b^2 + 2(n-1)(n^2 - 7(n-1))ab \tag{162}$$

$$B_T^b = \frac{(n-1)B_T^a}{1 + b(n-1)B_T^a} \tag{163}$$

- *If* $m > 1$, (B_T^a, B_T^b) *solves*

$$\begin{cases} (n-m-1)a(B_T^a)^2 + B_T^a\left(maB_T^b + m + 2 - n\right) - mB_T^b = 0 \\ (m-1)b(B_T^b)^2 + B_T^b\left((n-m)bB_T^a + 2 - m\right) - (n-m)B_T^a = 0 \end{cases}$$

Further, $q_T = (n-m)B_t^a + mB_t^b$ *and* $c_T^\alpha = \frac{\alpha}{k_T^\alpha + \alpha}, \quad \alpha \in \{a, b\}$.

Proof. This follows from the system given by equations (113)-(115), using the scaled variables k_T and B_T and rearranging. ■

C.4.2 Equilibrium at $t < T$

With multiple rounds, the fixed point for traders' slopes leads to the following conditions:

Lemma 10 (Existence at t with two levels of risk aversion) *For $m \geq 1$, existence of an equilibrium at t is equivalent to the existence of $(B_t^a, B_t^b) \in \mathbb{R}_{++}^2$ such that*

$$\begin{aligned} &(n-m-1)\left(A_{t+1}^a + (n-m)F_{t+1}^{aa}\right)(B_t^a)^2 \\ &+\left[m\{A_{t+1}^a + (n-m-1)F_{t+1}^{ab} + (n-m)F_{t+1}^{aa}\}B_t^b - (n-m-2)\right]B_t^a + mB_t^b(mF_{t+1}^{ab}B_t^b - 1) = 0, \\ &(m-1)\left(A_{t+1}^b + mF_{t+1}^{bb}\right)(B_t^b)^2 + \left[(n-m)\{A_{t+1}^b + (m-1)F_{t+1}^{ba} + mF_{t+1}^{bb}\}B_t^a - (m-2)\right]B_t^b \\ &\qquad + (n-m)B_t^a((n-m)F_{t+1}^{ba}B_t^a - 1) = 0. \end{aligned}$$

Proof. Start from the system given by first two lines in system $\mathcal{S}^D$ given in Lemma 6 and replace λ_t^α by $\sigma^2 k_t^\alpha$ and $\beta^\alpha = \frac{B_t^\alpha}{\sigma^2}$, for $\alpha \in \{a, b\}$. Then use the conditions $B_t^a = (k_t^a + A_{t+1}^a)^{-1}$ and $B_t^b = (k_t^b + A_{t+1}^b)^{-1}$. Simple algebra then yields the two conditions in the result. ■

These conditions nest those of the static model given in Lemma 9, since with a single round, the variables simplify as follows: $A_{t+1}^\alpha = \alpha$, for $\alpha \in \{a, b\}$, and $F_{t+1}^{ij} = 0$. The next result describes pricing and trading with two levels of risk aversion.

Corollary 5 *With two levels of risk aversion:*

1. *The pricing vector* $\pi_\mathbf{t}$ *is is given by*

$$\begin{aligned} \pi_t^t &= 1 \\ \pi_t^l &\equiv \bar{\delta}_t^l, \quad l \geq t+1 \\ &= \frac{1}{\det M_t}\left[(n-m)K_t^b\varphi_{t+1}^{al} + mK_t^a\varphi_{t+1}^{bl} + m(n-m)\sum_{k=t+1}^{T}(f_{t+1}^{ak} - f_{t+1}^{bk})\begin{vmatrix}\varphi_{t+1}^{ak} & \varphi_{t+1}^{bk} \\ \varphi_{t+1}^{al} & \varphi_{t+1}^{bl}\end{vmatrix}\right] \end{aligned}$$

where $K_t^\alpha = k_t^\alpha + A_{t+1}^\alpha$, *for* $\alpha \in \{a, b\}$.

2. *The exposure to future-round diversified risk of the two groups of traders is driven by a common component* $\hat{\theta}_t^l$,

$$\theta_t^{al} = m\hat{\theta}_t^l, \quad \theta_t^{bl} = -(n-m)\hat{\theta}_t^l, \tag{164}$$

where $\hat{\theta}_t^l$ *is given by equation* (169).

Proof. As a preliminary step, I show that with two levels of risk aversion the coefficients $\delta_{\mathbf{t}}$ and $\mathbf{d_t}$ are given by:

$$\begin{pmatrix} \delta_t^{al} \\ \delta_t^{bl} \end{pmatrix} = \frac{1}{\det(M_t)} \begin{pmatrix} K_t^b \varphi_{t+1}^{al} - m \sum_{k=t+1}^T f_{t+1}^{bk} \begin{vmatrix} \varphi_{t+1}^{ak} & \varphi_{t+1}^{bk} \\ \varphi_{t+1}^{al} & \varphi_{t+1}^{bl} \end{vmatrix} \\ K_t^a \varphi_{t+1}^{bl} + (n-m) \sum_{k=t+1}^T f_{t+1}^{ak} \begin{vmatrix} \varphi_{t+1}^{ak} & \varphi_{t+1}^{bk} \\ \varphi_{t+1}^{al} & \varphi_{t+1}^{bl} \end{vmatrix} \end{pmatrix} \tag{165}$$

$$\begin{pmatrix} d_t^{aa} \\ d_t^{ab} \end{pmatrix} = (1-c_t^a) \begin{pmatrix} \sum_{l=t+1}^T f_{t+1}^{al} \delta_t^{al} \\ \sum_{l=t+1}^T f_{t+1}^{al} \delta_t^{bl} \end{pmatrix}, \quad \begin{pmatrix} d_t^{ba} \\ d_t^{bb} \end{pmatrix} = (1-c_t^b) \begin{pmatrix} \sum_{l=t+1}^T f_{t+1}^{bl} \delta_t^{al} \\ \sum_{l=t+1}^T f_{t+1}^{bl} \delta_t^{bl} \end{pmatrix} \tag{166}$$

This is because

$$M_t = \begin{pmatrix} K_t^a + (n-m)F_{t+1}^{aa} & mF_{t+1}^{ab} \\ (n-m)F_{t+1}^{ba} & K_t^b + mF_{t+1}^{bb} \end{pmatrix}, \tag{167}$$

leading to

$$M_t^{-1} = \frac{1}{\det M_t} \begin{pmatrix} K_t^b + mF_{t+1}^{bb} & -mF_{t+1}^{ab} \\ -(n-m)F_{t+1}^{ba} & K_t^a + (n-m)F_{t+1}^{aa} \end{pmatrix} \equiv \begin{pmatrix} \omega_t^{aa} & \omega_t^{ab} \\ \omega_t^{ba} & \omega_t^{bb} \end{pmatrix}, \tag{168}$$

Then $\begin{pmatrix} \delta_t^{al} \\ \delta_t^{bl} \end{pmatrix} = M_t^{-1} \begin{pmatrix} \varphi_{t+1}^{al} \\ \varphi_{t+1}^{bl} \end{pmatrix}$, which gives equation (165). Then it is simple to calculate $\bar{\delta}_t^l = (n-m)\delta_t^{al} + m\delta_t^{bl}$ and obtain π_t^l (point 1 in the result).

Similarly, $\begin{pmatrix} d_t^{aa} \\ d_t^{ba} \end{pmatrix} = (1-c_t^a) M_t^{-1} \begin{pmatrix} \varphi_{\mathbf{t+1}}^{\mathbf{a}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{a}} \\ \varphi_{\mathbf{t+1}}^{\mathbf{b}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{a}} \end{pmatrix}$ and $\begin{pmatrix} d_t^{ba} \\ d_t^{bb} \end{pmatrix} = (1-c_t^b) M_t^{-1} \begin{pmatrix} \varphi_{\mathbf{t+1}}^{\mathbf{a}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{b}} \\ \varphi_{\mathbf{t+1}}^{\mathbf{b}} \cdot \mathbf{f}_{\mathbf{t+1}}^{\mathbf{b}} \end{pmatrix}$.

So, usnig equation (168),

$$\begin{aligned} d_t^{bb} &= (1-c_t^b)\left(\omega_t^{ba}\varphi_{\mathbf{t+1}}^{\mathbf{a}}\cdot\mathbf{f}_{\mathbf{t+1}}^{\mathbf{b}}+\omega_t^{bb}\varphi_{\mathbf{t+1}}^{\mathbf{b}}\cdot\mathbf{f}_{\mathbf{t+1}}^{\mathbf{b}}\right) \\ &= (1-c_t^b)\sum_{l=t+1}^{T} f_{t+1}^{bl}(\omega_t^{ba}\varphi_{t+1}^{al}+\omega_t^{bb}\varphi_{t+1}^{bl}) \\ &= (1-c_t^b)\sum_{l=t+1}^{T} f_{t+1}^{bl}\delta_t^{bl}. \end{aligned}$$

The other coefficients of equation (166) follow by the same calculation.

Next, I turn to equilibrium trades and the coefficients $\theta_t^{\alpha l}$, $\alpha\in\{a,b\}$:

$$\theta_t^{al} = \frac{\beta_t^a}{\bar{\beta}_t}\bar{\delta}_t^l - \delta_t^{al} = m(\phi_t^a\delta_t^{bl}-\phi_t^b\delta_t^{al}) \equiv m\hat{\theta}_t^l.$$

Similarly, for agent b, we get $\theta_t^{bl} = -(n-m)\hat{\theta}_t^l$. The common component $\hat{\theta}_t^l$ is given by:

$$\begin{aligned} \hat{\theta}_t^l &\equiv \phi_t^a\delta_t^{bl}-\phi_t^b\delta_t^{al} \\ &= \frac{1}{\det M_t}\left(\phi_t^a K_t^a\varphi_{t+1}^{bl}-\phi_t^b K_t^b\varphi_{t+1}^{al}+\sum_{k=t+1}^{T}\{(n-m)\phi_t^a f_{t+1}^{ak}+m\phi_t^b f_{t+1}^{bk}\}\begin{vmatrix}\varphi_{t+1}^{ak} & \varphi_{t+1}^{bk}\\ \varphi_{t+1}^{al} & \varphi_{t+1}^{bl}\end{vmatrix}\right), \end{aligned} \tag{169}$$

where I used equation (165). Finally, I calculate the coefficients of $\sum_{j\in\mathcal{A}}X_{t-1}^j$ and $\sum_{j\in\mathcal{B}}X_{t-1}^j$ in the equilibrium trade:

$$\begin{aligned} \frac{\beta_t^a}{\bar{\beta}_t}\bar{d}_t^a - d_t^{aa} &= m(\phi_t^a d_t^{ba}-\phi_t^b d_t^{aa}) \\ &= m\sum_{l=t+1}^{T} f_t^{al}(\phi_t^a\delta_t^{bl}-\phi_t^b\delta_t^{al}) \\ &= m\sum_{l=t+1}^{T} f_t^{al}\hat{\theta}_t^l \end{aligned}$$

The other coefficients can be calculated in the same way.

Last, the transition matrix (170) can be simplified into:

$$\mathbf{\Theta_t^Q} = \begin{pmatrix} \mathbf{0}_{(T-t)\times 1} & \mathbf{I}_{(T-t)\times(T-t)} \end{pmatrix} + \begin{pmatrix} f_{t+1}^{bt+1} + (n-m)\phi_t^a(f_{t+1}^{at+1} - f_{t+1}^{bt+1}) & (n-m)\theta_t^{at+1}(f_{t+1}^{at+1} - f_{t+1}^{bt+1}) & \dots & (n-m)\theta_t^{aT}(f_{t+1}^{at+1} - f_{t+1}^{bt+1}) \\ \vdots & \vdots & \ddots & \vdots \\ f_{t+1}^{bT} + (n-m)\phi_t^a(f_{t+1}^{aT} - f_{t+1}^{bT}) & (n-m)\theta_t^{at+1}(f_{t+1}^{aT} - f_{t+1}^{bT}) & \dots & (n-m)\theta_t^{aT}(f_{t+1}^{aT} - f_{t+1}^{bT}) \end{pmatrix}.$$

■

C.4.3 Effects of an anticipated shock

Here I derive the effect of a single anticipated shock, realized at $t^r \leq T-2$ and announced at $t^a \leq t^r - 2$. I begin with the effects before the realization.

Lemma 11 (Pre-shock trading with two levels of risk aversion) *For $t \in \{t^a, \dots, t^r - 1\}$, prices and inventories are affected as follows by an anticipated shock:*

$$\Delta X_t^a = m \sum_{k=t^a}^{t} \hat{\theta}_k^{t^r} z_{tk}^a \Delta s_{t^r} \tag{170}$$

$$\Delta X_t^b = -(n-m) \sum_{k=t^a}^{t} \hat{\theta}_k^{t^r} z_{tk}^b \Delta s_{t^r} \tag{171}$$

$$\Delta Q_t^l = \mathbb{1}_{l=t^r} \Delta s_{t^r} + m(n-m) \sum_{k=t^a}^{t} \hat{\theta}_k^{t^r} (f_t^{al} z_{t-1k}^a - f_t^{bl} z_{t-1k}^b) \Delta s_{t^r} \tag{172}$$

$$\Delta(D_t - p_t) = \frac{\sigma^2 \Delta s_{t^r}}{q_t} \left[\pi_t^{t^r} + m(n-m) \sum_{k=t^a}^{t} \hat{\theta}_k^{t^r} \sum_{l=t}^{t^r} \pi_t^l (f_t^{al} z_{t-1k}^a - f_t^{bl} z_{t-1k}^b) \right], \tag{173}$$

where the z coefficients are defined recursively as follows, for $\alpha \in \{a, b\}$:

$$z_{tt}^\alpha = 1$$

$$z_{tk}^\alpha = (1 - c_t^\alpha) z_{t-1k}^\alpha + m(n-m) \sum_{l=t}^{T} \hat{\theta}_t^l (f_t^{al} z_{t-1k}^a - f_t^{bl} z_{t-1k}^b). \tag{174}$$

Proof. The expressions for the change in diversified risk and inventories are obtained simply by starting from the changes at t^a (given in the text) and iterating forward, using each time the fact that $\theta_t^{al} = m\hat{\theta}_t^l$ and $\theta_t^{bl} = -(n-m)\hat{\theta}_t^l$ and the definitions of the z coefficients. The price follows from applying the definition in equation (2). ■

I now turn to the expressions for the price and inventories after the realization. The result nests the case with a surprise shock by setting $t^a = t^r$.

Lemma 12 (Post-shock trading with two levels of risk aversion) *At realization, the change in diversified risk and the risk premium are given by equations* (172) *and*(173) *at* $t = t^r$. *The change in inventories is given by:*

$$\Delta X^a_{t^r} = \phi^a_{t^r} \Delta s_{t^r} + m \sum_{k=t^a}^{t^r-1} \hat{\theta}^{t^r}_k z^a_{t^r k} \Delta s_{t^r} \tag{175}$$

$$\Delta X^b_{t^r} = \phi^b_{t^r} \Delta s_{t^r} - (n-m) \sum_{k=t^a}^{t^r-1} \hat{\theta}^{t^r}_k z^b_{t^r k} \Delta s_{t^r} \tag{176}$$

After the realization, $t > t^r$, *the corresponding changes are:*

$$\Delta X^a_{t^r} = \prod_{k=t^r+1}^{t} (1-c^a_k)\phi^a_{t^r} \Delta s_{t^r} + m \sum_{k=t^a}^{t^r-1} \hat{\theta}^{t^r}_k z^a_{tk} \Delta s_{t^r} + m \sum_{k=t^r+1}^{t} \sum_{l=k} \hat{\theta}^l_k z^a_{tk} PSM^l_{t^r} \Delta s_{t^r} \tag{177}$$

$$\Delta X^b_{t^r} = \prod_{k=t^r+1}^{t} (1-c^b_k)\phi^b_{t^r} \Delta s_{t^r} - (n-m) \sum_{k=t^a}^{t^r-1} \hat{\theta}^{t^r}_k z^b_{tk} \Delta s_{t^r} - (n-m) \sum_{k=t^r+1}^{t} \sum_{l=k} \hat{\theta}^l_k z^b_{tk} PSM^l_{t^r} \Delta s_{t^r} \tag{178}$$

$$\begin{aligned} \Delta Q^l_t = {} & PSM^l_{t^r} \Delta s_{t^r} + m(n-m) \sum_{k=t^a}^{t^r-1} \hat{\theta}^{t^r}_k (f^{al}_t z^a_{t-1k} - f^{bl}_t z^b_{t-1k}) \Delta s_{t^r} \\ & + m(n-m) \sum_{k=t^r+1}^{t} \sum_{l'=k}^{T} \hat{\theta}^{l'}_k PSM^{l'}_{t^r} (f^{al}_t z^a_{t-1k} - f^{bl}_t z^b_{t-1k}) \Delta s_{t^r} \end{aligned} \tag{179}$$

$$\begin{aligned} \Delta(D_t - p_t) = {} & \frac{\sigma^2 \Delta s_{t^r}}{q_t} \left[\sum_{l=t}^{T} \pi^l_t PSM^l_{t^r} + m(n-m) \sum_{k=t^a}^{t^r-1} \hat{\theta}^{t^r}_k \sum_{l=t}^{T} \pi^l_t \mathcal{R}^l_{t,k} \right] \\ & + \frac{\sigma^2 \Delta s_{t^r}}{q_t} m(n-m) \sum_{l=t}^{T} \pi^l_t \sum_{k=t^r+1}^{t} \sum_{l'=k}^{T} \hat{\theta}^{l'}_k PSM^{l'}_{t^r} \mathcal{R}^l_{t,k}, \end{aligned} \tag{180}$$

where the following notation is used:

$$PSM^l_{t^r} = (n-m)\phi^a_{t^r} f^{al}_{t^r+1} + m\phi^b_{t^r} f^{bl}_{t^r+1} \tag{181}$$

$$\mathcal{R}^l_{t,k} = f^{al}_t z^a_{t-1k} - f^{bl}_t z^b_{t-1k}. \tag{182}$$

Proof. Inventory changes at t^r are derived in the text. From there on, it is simple to iterate

forward as in the pre-trading case. In the expression for the change in diversified risk, the term in $PSM^l_{t^r}$ arises at every round because of the recursive definition of the f^{il}_t coefficients. For instance, at t^r+2, the following term shows up:

$$(n-m)\phi^a_{t^r}(1-c^a_{t^r+1})f^{al}_{t^r+2}+m\phi^b_{t^r}(1-c^b_{t^r+1})f^{bl}_{t^r+2}=(n-m)\phi^a_{t^r}f^{al}_{t^r+1}+m\phi^b_{t^r}f^{bl}_{t^r+1}\equiv PSM^l_{t^r}.$$

The first equality follows from the fact that by Definition 2, $(1-c^\alpha_{t^r+1})f^{\alpha l}_{t^r+2}=f^{\alpha l}_{t^r+1}$, for $\alpha\in\{a,b\}$. ■

From the previous results, it is immediate to derive conditions for time-series momentum and reversal.

Corollary 6 (Conditions for time-series momentum and reversal) *When a shock is announced:*

1. *The risk premium increases monotonically between the announcement and the realization if for all $t\in(t^a,t^r]$, the following condition holds:*

$$\Delta\left(\frac{\pi^{t^r}_{t+1}}{q_{t+1}}\right)>m(n-m)\left(\sum_{k=t^a}^{t+1}\frac{\hat\theta^{t^r}_k}{q_{t+1}}\sum_{l=t+1}^{T}\pi^l_{t+1}\mathcal{R}^l_{t+1,k}-\sum_{k=t^a}^{t}\frac{\hat\theta^{t^r}_k}{q_t}\sum_{l=t}^{T}\pi^l_t\mathcal{R}^l_{t,k}\right), \quad (183)$$

   where $\Delta\left(\frac{\pi^{t^r}_{t+1}}{q_{t+1}}\right)=\frac{\pi^{t^r}_{t+1}}{q_{t+1}}-\frac{\pi^{t^r}_t}{q_t}$.

2. *The risk premium monotonically decreases after the shock if for all $t>t^r$, the following holds:*

$$\Delta\mathcal{C}_{t+1}+\Delta\mathcal{H}_{t+1}<0, \quad (184)$$

   where

$$\Delta\mathcal{H}_{t+1}=m(n-m)\sum_{k=t^a}^{t^r-1}\hat\theta^{t^r}_k\left(\sum_{l=t+1}^{T}\frac{\pi^l_{t+1}}{q_{t+1}}\mathcal{R}^l_{t+1k}-\sum_{l=t}^{T}\frac{\pi^l_t}{q_t}\mathcal{R}^l_{tk}\right), \quad (185)$$

$$\begin{aligned}\Delta\mathcal{C}_{t+1}=&\left(\sum_{l=t+1}^{T}\frac{\pi^l_{t+1}}{q_{t+1}}PSM^l_{t^r}-\sum_{l=t}^{T}\frac{\pi^l_t}{q_t}PSM^l_{t^r}\right)\\&+m(n-m)\left(\sum_{l=t+1}^{T}\frac{\pi^l_{t+1}}{q_{t+1}}\sum_{k=t^r+1}^{t+1}\mathcal{R}^l_{t+1,k}\sum_{l'=k}^{T}\hat\theta^{l'}_kPSM^{l'}_{t^r}-\sum_{l=t}^{T}\frac{\pi^l_t}{q_t}\sum_{k=t^r+1}^{t}\mathcal{R}^l_{t,k}\sum_{l'=k}^{T}\hat\theta^{l'}_kPSM^{l'}_{t^r}\right)\end{aligned} \quad (186)$$

The condition for momentum is simply that the anticipatory component in the price (LHS of equation (183)) increases faster than the amount of pre-shock risk sharing. In condition (184) for reversal the first term on the LHS captures the change in the price impact of pre-shock trading (which continues to be diversified), while the second term captures the change in the price impact of current trading on the misallocation at realization. The sum of these effects need to be negative for the risk premium to abate over time.

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