A Dynamic Analysis of Parental Beliefs and Investments

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Abstract

I develop and estimate a dynamic model of parental investment under imperfect information about child cognitive skill. Parents choose investments based on subjective beliefs about child cognitive skill, which shape perceived returns; investments, in turn, affect skill accumulation and future beliefs. Parents update beliefs after observing noisy signals of child skill. I find that higher beliefs increase investment because skill and investment are complementary in production, implying higher perceived returns to investing in higher-skill children. Because investment also raises future beliefs, this creates a self-reinforcing dynamic in which early belief differences are amplified over time. I use the model to compare two policies for low socio-economic status (SES) families: an information intervention that removes initial belief bias (given that low SES under-estimate skills on average) and a one-time $3000 cash transfer in period 1. Both policies have modest effects on average investment and terminal skill, though the cash transfer produces larger gains. The information intervention nevertheless yields terminal-skill gains equivalent to a cash transfer of about $970 per child, implying that belief-based policies can complement, but are unlikely to replace, redistribution.

JEL codes: D83, I24, J13, J24, O15

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1 Introduction

High socio-economic status (SES) parents invest more in their children than low SES parents, and these investment differences contribute to disparities in children's human capital and later-life outcomes (Bolt, French, Maccuish, & O'Dea, 2024; Carneiro, Reis, & Toppeta, 2024; Caucutt, Lochner, & Park, 2017; Attanasio, Cattan, & Meghir, 2022; Becker & Tomes, 1979, 1986; Solon, 1999). A parent's investment decision depends not only on resources (Cunha, 2014; Caucutt et al., 2017), but also on what the parent believes about the child's skill, such as perceptions of academic performance (Dizon-Ross, 2019). Using U.S. longitudinal data, I document systematic SES differences in these beliefs: low SES parents under-estimate their child's skill on average, while high SES parents' beliefs are less biased on average. I also show that parental beliefs are strongly predictive of investments. These patterns suggest that informational frictions may be an important source of inequality in parental investments and, through them, inequality in skill formation.

This raises a policy question: should efforts to reduce skill gaps operate through household resources or through parental beliefs? Cash transfers are a standard tool for increasing parental investments by relaxing financial constraints (Del Boca, Flinn, & Wiswall, 2016). Information interventions, by contrast, may reduce belief distortions and thereby alter investment choices (Barrera-Osorio, Gonzalez, Lagos, and Deming (2020), Bergman (2021) and Dizon-Ross (2019)). Whether belief-based policies can substitute for, or complement, cash transfers is ultimately a quantitative question. The answer depends on the magnitude of belief distortions, the responsiveness of investments to beliefs, and the dynamic interaction between beliefs, investments, and skill accumulation.

To study this question, I develop and estimate a dynamic model of parental investment under imperfect information about child cognitive skill. In the model, there is a dynamic amplification mechanism: initial differences in beliefs affect investments which change future skill and future beliefs, leading small initial distortions to produce persistent differences in investment and skill accumulation. I estimate the model using panel data on parental beliefs, investments and children's skills. The estimates imply that cash transfers generate larger increases in children's terminal skills than belief-debiasing interventions. However, eliminating belief bias for low SES parents yields gains equivalent to a cash transfer of about $970 per child, implying that information frictions are quanti-

tatively meaningful and that belief-based policies can be a cost-effective complement to redistribution.

In the first part of the paper, using data from the National Longitudinal Survey of Youth, I present four empirical facts which motivate the research question and the model. Parental beliefs about the skill of the child are based on the mother's perceptions of the child's academic standing in class, the child's future prospects and the child's expected educational attainment. Investments are measured using time and goods inputs, while child skills are captured by achievement test scores. The four data patterns are as follows. Firstly, there are systematic differences in beliefs and investments by SES. High SES parents hold higher beliefs and invest more on average, even after accounting for child skill levels. At the same time, belief biases also vary by SES: between ages 9-14, low SES parents under-estimate their child's skill by roughly 0.19 to 0.29 years of education, whereas high SES parents' beliefs are, on average, closer to unbiased. These differences suggest that SES gaps in beliefs may help explain SES gaps in parental investments. Secondly, parental beliefs respond to changes in child skills: within-child changes in beliefs are positively correlated with within-child changes in child cognitive skills. Thirdly, parental investments are positively associated with parental beliefs, even after accounting for time-invariant characteristics of children. Fourthly, skill formation is dynamic: cognitive skills depend on earlier skills and investments. Together, these patterns point to a dynamic relationship between beliefs, investments, and skills, but they fall short of providing the longer term impact of shifting beliefs.

Building on these empirical patterns, I introduce a dynamic model of parental investment with imperfect information about child cognitive skill. Parents choose investments based on subjective beliefs because beliefs determine perceived returns. Investments then feed back into both skills and beliefs. First, higher investment raises expected next period skill, so parents enter the subsequent period with a higher prior belief. Second, investment increases realised skill, which improves the signals parents observe and thereby shifts updated (posterior) beliefs upward through the updating process. Together, these channels generate a dynamic interaction between beliefs, investments, and skill accumulation.

Having outlined the model's central mechanism, I now describe its structure in more detail. The model has three periods which span childhood to early adolescence. In each period, to maximise expected utility, parents allocate resources towards investment in

their child, consumption and saving. They make these choices subject to the income evolution process, belief evolution process and child skill accumulation process, all of which involve uncertainty. Child skill accumulates according to a translog production function, which allows the marginal productivity of investment to depend on the child's skill level. In periods when parents receive a noisy signal about child skills, beliefs are updated as follows: the updated (posterior) belief is a weighted average of the original (prior) belief and the signal. These dynamic processes depend on SES: high SES can differ from low SES in terms of income trajectory, productivity of investments in children and belief updating.

The model also incorporates unobserved preference heterogeneity: parents can be one of two discrete types, where the "high" type has a higher relative preference for child skill over consumption. The unobserved heterogeneity is included to account for unobserved differences across families and endogenous initial conditions. It is identified by variation in the investment choices that is unexplained by skills and resources, along with inter-temporal correlations in investment choices.

I estimate that higher parental beliefs increase parental investments, implying that beliefs and investments reinforce one another over time. This need not hold a priori: depending on preferences and the skill production technology, higher perceived skill could either raise or lower optimal investment. In this model with log preferences, a positive interaction between skills and investments in the production function implies higher perceived returns to investing in higher-skill children, so higher beliefs increase investment; a negative interaction implies the opposite. According to my estimates, this interaction term is positive, and higher beliefs therefore lead to higher investments.

I compare two policy levers targeted at low SES families: eliminating belief biases through information versus relaxing financial constraints through a cash transfer. I consider a one-time cash transfer of $3000 in period 1. This amount is approximately 10% of annual household income at the 25th percentile of the income distribution, and is comparable to amounts received through conditional cash transfer programmes in the United States. I contrast this transfer with an information intervention that eliminates low SES belief biases in period 1. This counterfactual should be interpreted as an upper bound, since in practice, providing information is unlikely to fully remove misperceptions. In the data, low SES parents systematically under-estimate their child's skill on average;

eliminating this bias therefore raises beliefs. Through the model's reinforcing dynamic between beliefs and investments, higher beliefs increase investments, which in turn raises future beliefs and future skills. This feedback enables the one-time belief shift to have effects beyond the initial period.

Overall, both policies have modest impacts on the average terminal skills of children, because neither policy induces large changes in investments. This reflects the estimated skill technology: with substantial persistence, sizeable increases in investment are needed to generate large skill gains. The cash transfer nonetheless produces larger improvements in skills than the information intervention, in part because average belief biases are not large, so debiasing translates into only limited investment responses. Even so, information provision may be highly cost-effective. The average terminal skill gain from the information intervention is comparable to that generated by a period 1 cash transfer of roughly \$970 per child. Thus, if information can be delivered at a per-child cost below this cash-equivalent amount, correcting beliefs is unlikely to substitute for redistribution but can be a cost-effective complement to cash transfers. This appears plausible in practice: Chikhale (2025) estimates that the intervention in List, Pernaudet, and Suskind (2021) costs approximately \$143–150 per participant.

This study joins the rising literature on parental subjective beliefs within Economics of Education. This literature highlights that parental beliefs on various dimensions, such as the return to investment (how parental investment translates into skills of children) and child skill level, may be inaccurate (Bergman, 2021; Dizon-Ross, 2019; Nicoletti, Sevilla, & Tonei, 2022), can be adjusted over time (Greaves, Hussain, Rabe, & Rasul, 2023; Nicoletti et al., 2022) and meaningfully affect investment decisions (Attanasio, Boneva, & Rauh, 2022; Cunha, Elo, & Culhane, 2022; Dizon-Ross, 2019; List et al., 2021). This work also documents systematic SES differences in beliefs (Attanasio & Kaufmann, 2009; Dizon-Ross, 2019; List et al., 2021). Several experimental studies (e.g. Barrera-Osorio et al. (2020), Dizon-Ross (2019)) establish that there are short-term effects of parental beliefs about child skill on investments and skills, but whether there are longer-term effects is unclear. I address this gap by developing and estimating a dynamic model in which beliefs and investments interact over time, and use it to quantify the longer-run effects of belief-based interventions and to benchmark them against cash transfers for reducing SES skill gaps.

Methodologically, this study builds on the learning and belief dynamics literature in Labour Economics (Zafar, 2011; Sanders, 2012; Stinebrickner & Stinebrickner, 2014; Kinsler & Pavan, 2016; Chikhale, 2025; Arcidiacono, Aucejo, Maurel, & Ransom, 2025), which provides frameworks for belief updating and for how subjective beliefs shape agents' choices. To the best of my knowledge, this study is the first to model the evolution of parental beliefs about child cognitive skill. A further distinction is that many learning models feature an unobserved state that is fixed over time (e.g. innate ability). Here, parents learn about an unobserved state (child skill) that evolves endogenously as a function of their investment choices.

Two other papers also estimate dynamic models of parental beliefs, investments and skills. However, their focus is different. Kinsler and Pavan (2016) concentrate on parental beliefs about the child's skill in the national population and show that these are distorted by academic performance of the local population (school/class of the child). Chikhale (2025) explores how parental beliefs about the return to investment contribute to inter-generational mobility. By contrast, I model parental beliefs about the child's cognitive skill level and use the estimated framework to compare belief-debiasing information interventions with cash transfers in reducing the SES skill gap.

The rest of this paper is organised as follows. Section 2 introduces the data. Section 3 presents patterns in the data which motivate the research question and the model. Section 4 introduces the dynamic model of parental investments with parental beliefs about child cognitive skill level. Section 5 describes the identification and estimation. Section 6 presents the model estimates and discusses model fit. Section 7 examines the impact of two different policies targeted at low SES families: an information intervention that eliminates belief biases and a cash transfer of $3000. Section 8 concludes the paper.

2 Data

Data sources are the National Longitudinal Survey of Youth 1979 (NLSY79) and the NLSY Child and Young Adult (CYA).

2.1 Parental Belief

Parental beliefs are constructed from the mother's responses to three survey questions. The first is about expected educational attainment: "How far do you think your child will go in school?". There are four categories: (1) Up to high school; (2) Some college or other training; (3) Graduate from college; and (4) More than college. The second is regarding the child's academic standing in class, with five categories: (1) Near the bottom of the class; (2) Below the middle; (3) In the middle; (4) Above the middle; or (5) One of the best students in class. This is a relative ranking rather than an absolute ranking, and there is evidence that relative rankings matter for child achievement (Kinsler & Pavan, 2021; Elsner, Isphording, & Zölitz, 2021). The third question is about future prospects, with the following responses: (1) Poor or fair; (2) Good; or (3) Excellent.

Overall, mothers have good perceptions of their children. More than half of children are expected to complete high school or beyond, more than half are ranked in the middle of the class or higher. In addition, more than half are perceived to have excellent future prospects.

Mothers change their perceptions over time. Among children with beliefs reported three times between ages 9 and 14, (1) around 57% experienced at least one change in expected educational attainment; (2) around 61% experienced at least one change in rating of academic standing and (3) around 45% experienced at least one change in rating of future prospects.

I assume that these three questions capture a latent belief about the difference between the child's cognitive skill level and the average cognitive skills of children of the same age[1]. This is extracted as a belief factor score predicted from a factor model estimated on the three measures. Using a factor model helps mitigate measurement error in any single item and aggregates the common signal across questions. The measure of expected educational attainment is used to link the factor over time, enabling the factor to be compared across ages of the child. Since latent factors have no natural scale (Anderson & Rubin, 1956), the belief factor is anchored to the years of education of the child at age 24 and above. Under this anchoring, a 1 unit increase in the latent factor corresponds to a 1 unit increase in the conditional expectation of the years of education

[1]This is an assumption. Beliefs elicited in absolute terms, such as expected educational attainment or future prospects, may nonetheless be formed in relative terms if parents view schooling and labor market outcomes as competitions for scarce slots (e.g., selective schools or jobs).

of the child at age 24 and above. Appendix B provides additional details on the factor model and anchoring.

To express beliefs in terms of the child's skill level at each age, I add the age-specific mean of log cognitive skill to the predicted belief factor score[2]. I refer to this sum as the anchored belief factor score. The age-specific mean log skill is the mean cognitive skill factor score (anchored) at the corresponding age. The construction of the cognitive skill factor score is described below.

In learning models, beliefs are naturally represented as subjective probability distributions over the unobserved state (e.g., Stinebrickner and Stinebrickner (2014); Arcidiacono et al. (2025)). In this framework, I interpret the anchored belief factor score as the mean of the parent's belief distribution.

2.2 Parental Investment

A latent factor of parental investment is constructed from components of the Home Observation Measurement of the Environment (HOME), which proxy time and goods investments. This method is based on Cunha, Heckman, and Schennach (2010). Some components include how often the mother reads to the child, how many books the child has and how often the child is brought to the museum. A complete list of the component measures is provided in Appendix A.3. The measure of the frequency of bringing the child to the museum is used to link the factor over time, so that it can be compared across ages. The investment factor does not have a natural location or scale (Anderson & Rubin, 1956). For the data patterns, I use the investment factor score standardised within the sample. In the model estimation, the original investment factor score is used. Appendix B provides additional details on the factor model for investment.

[2]Because the underlying belief measures are reported on bounded scales, some parents may already select the top category at young ages. In that case, belief factor scores may only remain the same or decrease as children grow older. In contrast, the skill factor score tends to rise as children grow because they achieve higher scores on achievement tests. The following narrative can rationalise why the belief factor score does not grow with age, while the skill factor score does. Parents perceive a child's skill as the sum of an (i) unobserved child-specific component that is constant over time and (ii) a common, age-specific skill component that is known to parents. Consequently, the belief factor score captures parents' assessment of the child-specific constant component and therefore need not trend upward with age. If this narrative holds, a parent's perceived level of skill at age t is the age-specific component plus the belief factor score. In implementation, I assume that the age-specific component is equal to the average log skill factor score at the corresponding age.

2.3 Child Cognitive Skill

Measurements of cognitive skill are raw scores on the Peabody Individual Achievement Tests (PIAT) in mathematics, reading recognition and reading comprehension, which were administered to children aged between 5 and 14. Higher raw scores indicate a higher skill level. Mothers typically do not observe PIAT scores directly; I assume instead that these test outcomes are correlated with the child's school grades[3].

I assume that the raw scores on the achievement tests are equal to the underlying log cognitive skill of the child plus measurement error, following Cunha et al. (2010). Using the achievement tests, I estimate a factor model and use it to predict cognitive skill factor scores for each child. The raw score on the mathematics achievement test is used to link the latent factor over time, allowing the latent skill to be compared across ages. Latent factors have no natural scale (Anderson & Rubin, 1956). Therefore, the cognitive skill factor is anchored to years of education of the child at age 24 and above. Under this anchoring, a 1 unit increase in the log cognitive skill corresponds to a 1 unit increase in the conditional expectation of the years of education of the child at age 24 and above. In this way, the skill factor has the same units as the belief factor. Appendix B provides additional details on the factor model and anchoring.

2.4 Family Income and Assets

Family income is the total net family income of the family. It sums the following components received by respondent and spouse or partner: wage income, farm/business income, military income, unemployment compensation, Aid to Families with Dependent Children (AFDC), food stamps, Supplemental Security Income (SSI)/welfare, child support, alimony, educational benefits and/or scholarships, fellowships and grants, veteran benefits and income from other sources. From survey year 2002, income from worker's compensation, disability and social security is also included. In every survey year, the top 2% of net family income values are top-coded.

Family assets are the total net wealth of the family. This is the sum of asset values minus debts. Assets include the market value of owned residential property, money assets like savings accounts, the market value of farm/business/other real estate and the total

[3]School grades are available in 1995-1996. In these years, national percentile ranks in the school achievement tests are correlated with the PIAT percentile ranks.

market value of all other assets worth more than $500. Debts include mortgages and back taxes owed on residential property, debts on farm/business/other property, debts on vehicles including automobiles and the total amount of other debts over $500. In each survey year, the top 2% of total net family wealth values are top-coded.

2.5 Definition of Socio-economic Status

High socio-economic status (SES) families are defined as those with family income above the sample median, where family income is measured as average household income when the child is aged 9-10. Families below the median are classified as low SES. The median threshold is computed using the children of females in the cross-section sample of the NLSY79.

2.6 Sample Summary Statistics

I focus on the 5,819 children of females in the NLSY79 cross-section sample. All biological children are included, therefore there can be siblings in the sample. Sample statistics of these children are provided in Table 1. There are fewer than 5,819 observations of certain variables because of missing values.

On average, mothers attained 13.42 years of education (note that 12 years corresponds to graduating from high school, provided no grades were repeated). The average percentile score of mothers on the Armed Forces Qualification Test (AFQT), a measure of cognitive skill, is 45.89. The mean age of the mother at birth is 26.79 years. Around 77% of the sample are Whites, 14% are Blacks and 9% are Hispanics. 52% of the children are male and the rest are female. At age 9-10, average family income is around $83,054.44 while average assets are around $171,402.31.

In this paper, several data patterns are presented to motivate the research question and the model. To maximise the sample, when producing each data pattern, child-year observations are included whenever the key variables of interest are available. Therefore, the sample of children used to produce each data pattern is a distinct subset of the 5,819 children presented in Table 1. Furthermore, although the model focuses on children aged between 9 and 14, data from age 5-8 will also be used to produce some data patterns, particularly when the estimator performs better with more time periods of data.

Table 1: Summary Statistics of Children of Females in Cross-Section Sample of NLSY79

	N	Mean	Median	Std. Dev.
Belief Factor Score Age 9-10	2,888	13.04	13.03	1.25
Skill Factor Score Age 9-10	4,116	13.07	13.14	0.80
Family Income Age 9-10	4,274	83,054.44	62,549.62	107,052.05
Asset Age 9-10	3,425	171,402.31	34,110.26	432,365.27
Mother's Years of Education	5,819	13.42	12.00	2.57
Mother's AFQT Percentile	5,547	45.89	44.06	28.87
Mother's Age at Birth	5,819	26.79	26.00	6.06
White	5,819	0.77	1.00	0.42
Black	5,819	0.14	0.00	0.35
Hispanic	5,819	0.09	0.00	0.28
Male	5,819	0.52	1.00	0.50

Notes: This table presents summary statistics of the 5,819 children born to females in the cross-section sample of the NLSY79. AFQT refers to Armed Forces Qualification Test and it is a measure of cognitive skill. Income and assets are adjusted to 2015 dollars using the Consumer Price Index series (CPI-U).

3 Data Patterns

This section presents several data patterns which motivate the research question and the model[4].

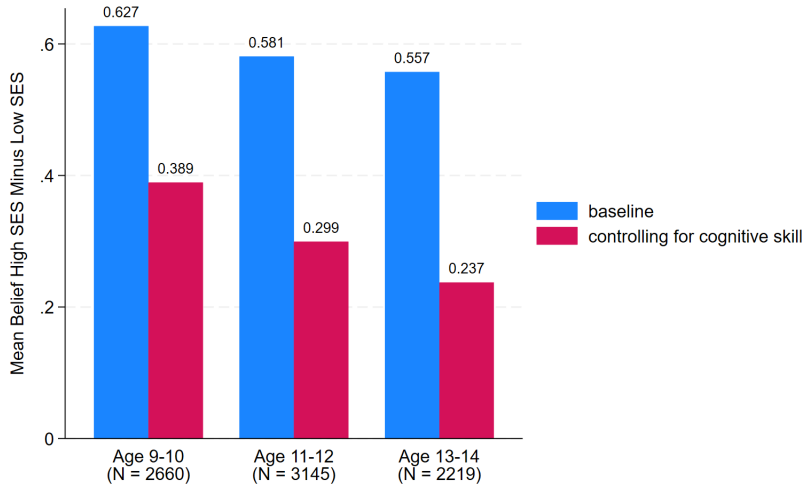
3.1 Systematic Differences in Beliefs and Investments by SES

Figure 1 presents the average difference in the belief factor score (anchored) between the high SES and the low SES parents at age 9-10, 11-12 and 13-14. At age 9-10, on average, high SES parents believe that their children will attain 0.627 years of education more than low SES parents. After accounting for skills, this figure is 0.389 years of education. The SES differences in average beliefs are statistically significant.

Parents may not know the skill level of their child: from Figure 2, the correlation

[4]Two of the belief measures are explicitly forward-looking. In addition to reflecting perceptions of current skill, they may also embed parents' plans for future investments or expectations about future shocks. Reassuringly, most descriptive patterns are similar when I instead use only the strictly contemporaneous measure: parents' rating of the child's current academic standing in class (see Appendix C). Moreover, the three belief measures are moderately correlated (polychoric correlations around 0.50-0.53), consistent with them capturing a common underlying component of perceived skill. In the structural model, I address the possibility that time-invariant unobservables jointly shape beliefs and choices such as investments (including systematic differences in expectations about the future) by allowing for unobserved preference heterogeneity. If such heterogeneity does not fully capture these confounding forces, the estimated impact of beliefs on investments should be interpreted as an upper bound.

Figure 1: Average Belief Factor Score (Anchored) by SES



Notes: This figure presents the mean belief factor score (anchored) for high SES parents and low SES parents at ages 9-10, 11-12 and 13-14. The unit of the belief factor score is years of education at age 24 and above. High socio-economic status (SES) parents refer to parents who have above median family income (defined based on average family income when the child is aged between 9 and 10). Otherwise, parents are low SES. At each age, all children with non-missing belief factor scores, cognitive skill factor scores and average family income between age 9-10 are included in the computation.

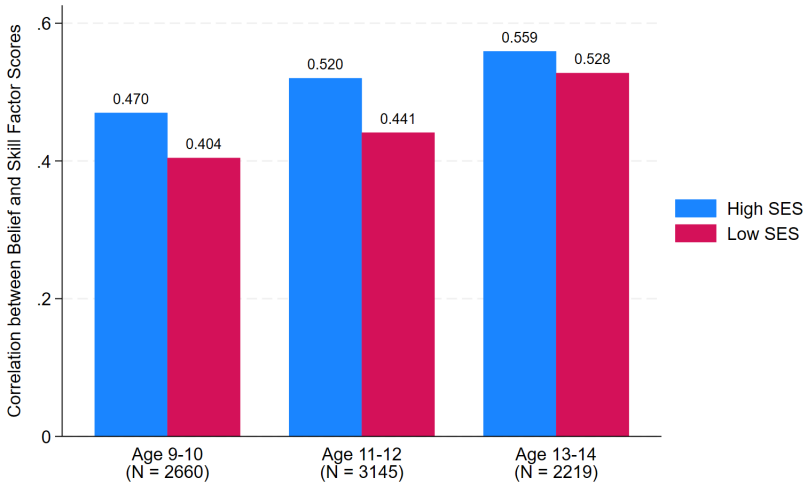
between the belief factor score and cognitive skill factor score is less than 1. Furthermore, the correlation is smaller for low SES parents, suggesting that on average, low SES parents hold more inaccurate beliefs. At ages 9-10 and 11-12, the SES differences in the correlations are statistically significant at the 5% level and 1% level respectively.

Figure 3 displays the average difference between the belief factor score and the skill factor score at ages 9-10, 11-12 and 13-14. On average, low SES parents under-estimate the skills of their child by around 0.19 to 0.26 years of education. In contrast, on average, high SES parents over-estimate the skills of their child by around 0.02 to 0.04 years of education at ages 9-10 and 11-12, while they under-estimate the skills of their child by 0.07 years of education at age 13-14[5]. These results are statistically significant, except for the over-estimate of the high SES at age 11-12. If beliefs positively influence investments, SES belief differences can contribute to SES skill disparities.

In addition, on average, high SES parents invest more in their children, even after controlling for differences in children's cognitive skills. Figure 4 displays the average difference in the investment factor score (standardised) between the high SES and the low SES parents. Both the unconditional difference (blue bars) and the difference conditional

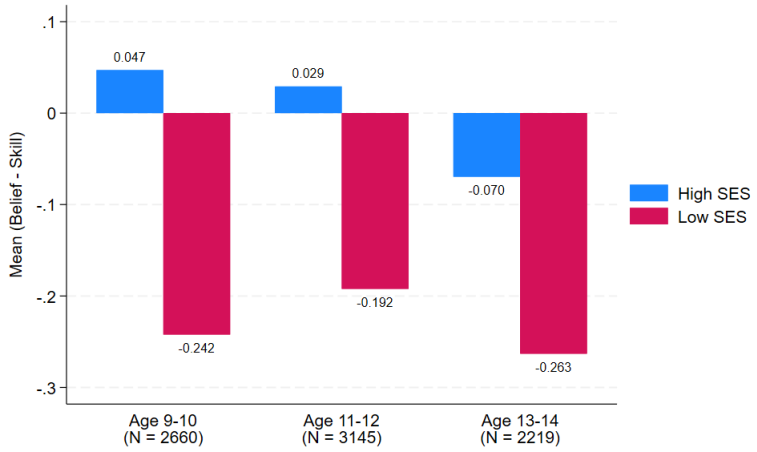
[5]The results on over-estimation and under-estimation rest on the assumptions made in the construction of the belief factor score and skill factor score.

Figure 2: Correlation between Belief Factor Score (Anchored) and Cognitive Skill Factor Score (Anchored) by SES



Notes: This figure presents the correlation between the belief factor score (anchored) and the cognitive skill factor score (anchored) at ages 9-10, 11-12 and 13-14. High socio-economic status (SES) parents refer to parents who have above median family income (defined based on average family income when the child is aged between 9 and 10). Otherwise, parents are low SES. At each age, all children with non-missing belief factor scores, cognitive skill factor scores and average family income between age 9-10 are included in the computation.

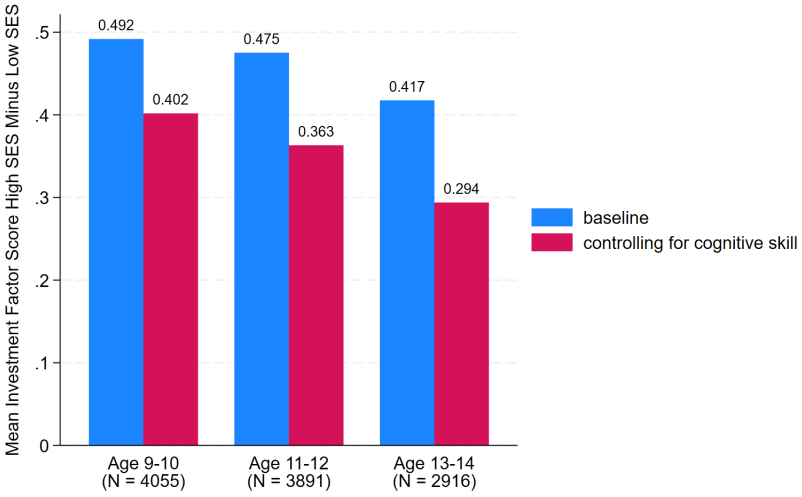
Figure 3: Average Difference between Parental Beliefs and Skills by SES



Notes: This figure presents the average difference between the belief factor score (anchored) and the skill factor score for high SES parents and low SES parents at ages 9-10, 11-12 and 13-14. The unit of the belief factor score is years of education at age 24 and above. High socio-economic status (SES) parents refer to parents who have above median family income (defined based on average family income when the child is aged between 9 and 10). Otherwise, parents are low SES. At each age, all children with non-missing belief factor scores, cognitive skill factor scores and average family income between age 9-10 are included in the computation.

on child skills (red bars) are positive, implying that high SES parents invest more in their children, even after accounting for child cognitive skills. On average, at age 9-10, investments of high SES parents are 0.492 standard deviations higher than low SES parents. The SES differences in mean investment factor scores are statistically significant.

Figure 4: Average Difference in Investment Factor Score (Standardised) Between High SES and Low SES



Notes: This figure presents the average difference in the investment factor score (standardised) between high SES and low SES parents at ages 9-10, 11-12 and 13-14. The blue bar presents the unconditional difference in investment factor scores, while the pink bar is the difference after controlling for the cognitive skill factor score. High socio-economic status (SES) parents refer to parents who have above median family income (defined based on average family income when the child is aged between 9 and 10). Otherwise, parents are low SES. At each age, all children with non-missing investment factor scores, cognitive skill factor scores and average family income between age 9-10 are included in the computation.

3.2 Parents Revise Beliefs When Cognitive Skills Change

This section estimates a belief updating model and shows that within-child changes in the belief factor score are correlated with within-child changes in the cognitive skill factor score.

Model: The belief updating model[6] (similar to Stinebrickner and Stinebrickner (2014)) is presented in Equation 1. I assume that the belief factor score of child j at period t (μ_{jt}) depends linearly on the belief factor score in period $t-1$ ($\mu_{j,t-1}$) and the cognitive skill factor score of the child at period t. X_{jt} represents a set of controls: dummies for region of residence (north, south, east, west), urban/rural residence, SMSA

[6]Though this may appear similar to Bayesian updating of the mean of a normal random variable, it is not exactly the same. The belief in period t is a different random variable from the belief in period $t-1$.

status (not in SMSA, SMSA but not in central city, SMSA in central city, SMSA in unknown central city), an indicator for whether mother of the child is employed, an indicator for whether the child's biological father lives with the child, the education of mother's spouse, mother's marital status, log family income and the child's age at time t. The child fixed effect δ_j accounts for time-invariant characteristics of the child which parents use to form beliefs.

$$\mu_{jt} = \alpha + \beta_1 \mu_{j,t-1} + \beta_2 cognitive_{jt} + \gamma X_{jt} + \delta_j + \epsilon_{jt} \quad (1)$$

I treat the cognitive skill factor score as a proxy for the information parents receive about their child's skill. Although parents do not observe the survey-administered test measures directly, these measures are likely correlated with observable indicators such as school grades and teacher feedback that parents do see.

Estimation: Given the short T panel, since Equation 1 contains both a lag dependent variable and a fixed effect, estimating this equation via fixed effects or first differences will yield inconsistent estimates of the coefficient on the lag dependent variable. To obtain consistent estimates, I employ the Blundell and Bond (1998) dynamic panel data estimator. This is a system generalised method of moments estimator which relies on moment conditions relating instruments to both the level equation (Equation 1) and the differenced equation (Equation 2).

$$\Delta\mu_{jt} = \beta_1 \Delta\mu_{j,t-1} + \beta_2 \Delta cognitive_{jt} + \gamma \Delta X_{jt} + \Delta\epsilon_{jt} \quad (2)$$

Δ is the first difference operator

Because the lag dependent variable is endogenous, only values of the dependent variable from time period $t-2$ and earlier are used as instruments in the differenced equation. Since there may be unobserved shocks which jointly affect both the child cognitive skills and the belief factor score, the cognitive factor score is also treated as a potentially endogenous variable. Consequently, like the lag dependent variable, only values of the cognitive skill factor score from time period $t-2$ and earlier are used as instruments in the differenced equation[7]. Since the model is over-identified, the validity of over-identifying

[7]Instruments used in the differenced equation: $\mu_{j,t-2}$, $\mu_{j,t-3}$, $\mu_{j,t-4}$; $cognitive_{j,t-2}$, $cognitive_{j,t-3}$,

restrictions can be assessed with the Sargan test.

Estimates: Table 2 reports estimates from the belief-updating regression, focusing on the coefficients on lagged beliefs and contemporaneous cognitive skill. Lagged beliefs are a positive and statistically significant predictor of current beliefs: a one-unit increase in the lag belief factor score is associated with a 0.198 increase in the current belief factor score, consistent with persistence in parents' assessments. Current beliefs also track measured skill: a one-unit increase in the cognitive skill factor score is associated with a 0.871 increase in the belief factor score, indicating that parental beliefs respond to information about the child's cognitive performance. Finally, belief updating appears similar across SES groups. Estimating the regression separately by SES and conducting formal tests of coefficient equality, I do not reject equality of either the lag belief coefficient or the cognitive skill coefficient between high and low SES families. Appendix F reports the subgroup estimates and test results.

The validity of the over-identifying restrictions is rejected at the 10% level, suggesting that the model is misspecified and/or the instruments are invalid. In Appendix G, in an attempt to address the model misspecification, an additional lag of the belief factor score is also included as an independent variable. The belief factor score remains positively correlated with the lag belief factor score and the cognitive skill factor score. However, the validity of the over-identifying restrictions is still rejected, indicating that the expanded model may also be misspecified.

3.3 Investments are Correlated with Beliefs

This section shows that parental investments are correlated with parental beliefs, even after accounting for time-invariant characteristics of children and investment preferences of parents.

Empirical Strategy: To understand the relationship between beliefs and investments, the investment factor score I_{jt} of child j at time t is regressed on the belief factor score μ_{jt}. The model is presented in Equation 3. I include child fixed effects to account for child-specific time-invariant characteristics which jointly influence beliefs and investments. Furthermore, I control for the lag investment factor score to account for persistent investment preferences of parents. In addition, the model includes a set of time-varying

$cognitive_{j,t-4}$; ΔX_{jt}. Instruments used in the level equations are: $\Delta\mu_{j,t-1}$, $\Delta cognitive_{j,t-1}$.

Table 2: Dependent variable is Parent Belief Factor Score (Anchored)

	(1)
Lag Belief Factor Score	0.198***
	(0.030)
Cognitive Skill	0.871***
	(0.062)
Observations	5829
Number of Children	3334
Child FE	Yes
Cognitive Skill Potentially Endogenous	Yes
Sargan Test P-value	0.006

Notes: This table presents the estimated coefficients in a regression of the belief factor score (anchored) on the lag belief factor score (anchored) and the cognitive skill factor score. The construction of the belief factor score and the skill factor score is described in the data section. The model is estimated with the Blundell and Bond (1998) dynamic panel data estimator. The regression includes time-varying controls, as described in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between ages 5 and 14 are used.

controls denoted by X_{jt}: dummies for region of residence (north, south, east, west), urban/rural residence, SMSA status (not in SMSA, SMSA but not in central city, SMSA in central city, SMSA in unknown central city), an indicator for whether mother of the child is employed, an indicator for whether the child's biological father lives with the child, the education of mother's spouse, mother's marital status, log family income and the child's age at time t.

$$\underbrace{I_{jt}}_{\text{investment}} = \alpha + \beta \underbrace{\mu_{jt}}_{\text{belief}} + \gamma X_{jt} + \psi \underbrace{I_{j,t-1}}_{\text{lag investment}} + \underbrace{\delta_j}_{\text{child FE}} + \eta_{jt} \tag{3}$$

The investment equation is motivated by the literature on skill production functions (Cunha et al., 2010; Attanasio, Cattan, Fitzsimons, Meghir, & Rubio-Codina, 2020; Agostinelli & Wiswall, 2025), which highlights that the investment policy function depends on state variables such as the child skill level, family income and the skill level of parents. Since parents may not observe the skill of the child, the state variable is instead the parent's belief about the skill. In general, the investment policy function could be non-linear. For simplicity, it is approximated as a linear function of state variables.

Estimation: The equation contains both a lag dependent variable and a child fixed

effect. Given the short T panel, the coefficient on the lag dependent variable will be inconsistently estimated with fixed effects or first difference estimators. To obtain consistent estimates, I rely on the Blundell and Bond (1998) dynamic panel data estimator. This system generalised method of moments estimator relies on moment conditions relating instruments to both the level equation (Equation 3) and the differenced equation (Equation 4)[8]. As the model is over-identified, the validity of over-identifying restrictions can be assessed with the Sargan test.

$$\Delta \underbrace{I_{jt}}_{\text{investment}} = \beta \Delta \underbrace{\mu_{jt}}_{\text{belief}} + \gamma \Delta X_{jt} + \psi \Delta \underbrace{I_{j,t-1}}_{\text{lag investment}} + \Delta \eta_{jt} \tag{4}$$

Estimates: Table 3 presents estimates of the coefficient on the belief factor score (anchored) in a regression of the investment factor score (standardised) on the belief factor score (anchored). The estimate suggests that a 1 unit increase in the belief factor score (anchored) is associated with a 0.058 standard deviation increase in the investment factor score (standardised). This correlation suggests that there is an important relationship between these variables. It could be that parental beliefs influence investment decisions. Furthermore, the positive correlation suggests that on average, parents invest more when they hold higher beliefs, as in Dizon-Ross (2019). There is no evidence of SES differences in the relationship between beliefs and investments: when the model is estimated separately for high SES and low SES, the coefficient on the belief factor score is similar. Results are presented in Appendix H.

The validity of the over-identifying restrictions is rejected at the 5% level, indicating that the model is misspecified and/or the instruments are invalid. In an attempt to assess the sensitivity of the results to the model specification, alternative models with two or three lags of the investment factor score are estimated and presented in Appendix I. In these specifications, the investment factor score remains positively correlated with the belief factor score and I fail to reject the null hypothesis that the over-identifying restrictions are valid.

[8]The instruments used in the differenced equation are: $I_{j,t-2}$, $I_{j,t-3}$, $I_{j,t-4}$; $\Delta\mu_{jt}$; ΔX_{jt}.
The instruments used in the level equations are: $\Delta I_{j,t-1}$.

Table 3: Dependent variable is Investment Factor Score (Standardised)

	(1)
Belief Factor Score	0.058***
	(0.012)
Observations	9557
Number of Children	4136
Child FE	Yes
Lag Investment	Yes
Sargan Test P-value	0.019

Notes: This table reports coefficients on the belief factor score (anchored) in a regression of the investment factor score (standardised) on the belief factor score (anchored). The Blundell and Bond (1998) dynamic panel data estimator is used. The construction of the belief factor score and the investment factor score is provided in the data section. The regression includes time-varying controls which are explained in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between age 5 and 14 are used. This sample is larger than the one used to estimate the belief updating model, because children tend to have investments reported over a greater number of time periods than they do for beliefs.

3.4 Child Skills Evolve in Dynamic Process

This section points to dynamics in child skill formation: skills of children depend on earlier skills and investments.

Empirical Strategy: I estimate a reduced-form skill transition equation relating next-period skills to lagged skills and lagged parental investments. The model is presented in Equation 5. Mirroring a translog production function, the cognitive skill factor score of child i at time t, $cognitive_{it}$, is a function of the lag cognitive skill factor score $cognitive_{i,t-1}$, lag investment factor score $I_{i,t-1}$ and the interaction between the two. The interaction term allows the productivity of investment to depend on child skill level. To address endogeneity, in one specification, family fixed effects are included to account for time-invariant family characteristics.

$$cognitive_{jt} = A + \gamma_1 cognitive_{j,t-1} + \gamma_2 I_{j,t-1} + \gamma_3 cognitive_{j,t-1} \times I_{j,t-1} + \epsilon_{jt} \quad (5)$$

Estimates: Table 4 presents the estimates. Consistent with a dynamic accumulation process, lagged skills are strongly predictive of subsequent skills, and lagged investments predict higher subsequent skills. In the OLS specification, the lagged skill-investment

interaction is negative and statistically significant, but it becomes statistically indistinguishable from zero once family fixed effects are included. This pattern suggests that the negative interaction in OLS is driven largely by persistent between-family differences rather than by within-family dynamics in the skill production process. In particular, time-invariant family traits (such as parenting preferences, constraints, information, or stable aspects of the child's environment) may jointly influence both average investment levels and how parents adjust investments in response to the child's skill. An alternative, non-exclusive explanation is that the interaction is imprecisely estimated under fixed effects because identification relies only on within-family variation and is more sensitive to measurement error in skills and investments. This regression is not interpreted as identifying the structural production function: investments are endogenous and measured with error. However, it is a useful summary of the intertemporal association between parental inputs and child skill growth.

Table 4: Dependent variable is Cognitive Skill Factor Score

	(1)	(2)
Lag Cognitive Skill Factor Score	0.756***	0.573***
	(0.005)	(0.006)
Lag Investment	0.347***	0.166*
	(0.086)	(0.098)
Lag Cognitive Skill Factor Score x Lag Investment	-0.018***	-0.010
	(0.007)	(0.008)
Observations	10349	10349
Family FE	No	Yes
Number of Children	2034	2034

Notes: This table presents the estimated coefficients in a regression of the cognitive skill factor score on the lag investment factor score, lag cognitive skill factor score and the interaction between the lag investment factor score and the lag cognitive skill factor score. The construction of the skill factor score and the investment factor score is described in the data section. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). All available observations of the child between ages 9 and 14 are used.

3.5 Summary of Data Patterns and Link to Model

Four data patterns are presented. First, parental beliefs and investments differ systematically by socio-economic status (SES). On average, high SES parents hold higher beliefs

and invest more, even conditional on children's skill levels. Belief biases also vary by SES: low SES parents tend to underestimate their child's skill, whereas high SES parents' beliefs are, on average, less biased. The model is used to examine whether these SES differences in beliefs help account for SES differences in investments and skills. Second, there is suggestive evidence that parental beliefs respond to changes in child cognitive skill, motivating a belief-updating mechanism. Third, parental investments are positively associated with parental beliefs, even after controlling for time-invariant child characteristics, pointing to an important link between beliefs and investment choices. Fourth, skill formation is dynamic: current skills depend on prior skills and prior investments. The model incorporates these dynamics through the skill production process.

Taken alone, these descriptive patterns do not pin down how shifting beliefs at early ages propagates into subsequent beliefs, investments, and skill accumulation. Addressing that question requires a unified dynamic framework linking beliefs, investment choices, and skill production. The next section introduces such a framework.

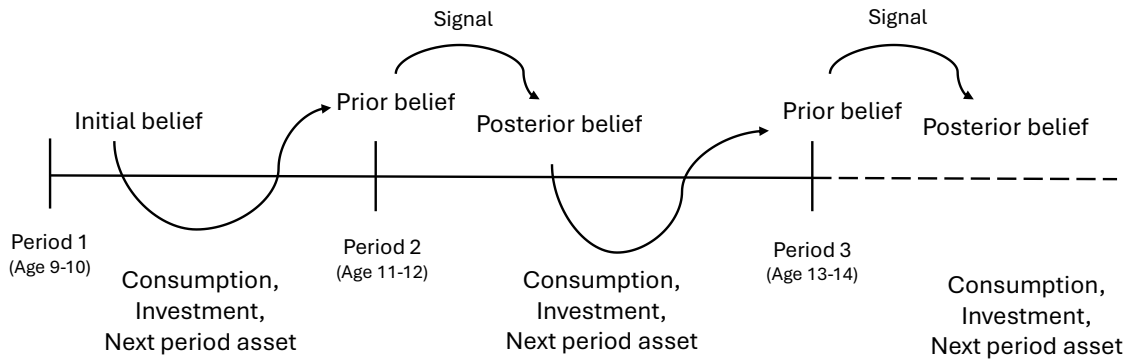
4 Model

4.1 Model Overview

This is a dynamic model of parental investments that incorporates parental beliefs about child cognitive skill level. Each parent has a single child, and the model spans three periods corresponding to ages 9-10, 11-12 and 13-14. Figure 5 summarises the sequence of events. In the first period, the parent enters with an initial belief about the child's cognitive skill and then makes choices for the period: consumption, investment and next period assets. In periods 2 and 3, parents enter with a prior belief about the cognitive skill of the child. After they receive a signal (noisy measure of the child's cognitive skill), they update their belief and obtain a posterior belief. Based on the posterior belief and the available resources, parents choose consumption, investment and next period assets. After they invest, they form an expectation of next period skill, which is their prior belief about skills in the next period.

Parents are forward-looking and are expected utility maximisers. Their utility depends on consumption and the perceived cognitive skill of their child (since skill is unobserved). Parents are motivated to invest, because investments produce skills, raising

Figure 5: Model Overview



their utility (Solon, 1999). Parents choose consumption, investments and next period assets to maximise their objective function, subject to the following processes: the child skill production function, the belief updating process, the resource constraint and the family income process. The central trade-off is between consumption and investment in child skill formation. Parents face uncertainty in the signal generation process, the skill production process and the income process.

The child skill production function, the belief updating process and the income process depend on socio-economic status (SES). This enables high and low SES families to differ in the productivity of investments, in the way they update beliefs, and in their expected future income trajectories.

The model incorporates unobserved heterogeneity in parental preferences. Parents are characterised by one of two discrete preference types, where the "high" type places a higher relative weight on child skill than on consumption. The probability of belonging to either type depends on the parent–child pair's initial conditions. Specifically, it depends on the child's initial skill level. Intuitively, parents with stronger preferences for child skill are likely to have invested more prior to the start of the model, leading to higher observed initial skills. Introducing unobserved preference heterogeneity serves two purposes. Firstly, it controls for unobservable differences across families. Secondly, it addresses the problem of endogenous initial conditions, as children's skills, parental beliefs, income, and assets at ages 9–10 are unlikely to be randomly assigned.

Next, I describe the utility function, the skill production function, beliefs and the

belief updating process, the resource constraint and the income process. Parent-child pairs are indexed by j and time t denotes the child's age. Socio-economic status is indexed by $s \in \{1, 2\}$, where $s = 1$ corresponds to low SES (up to median family income at age 9-10) and $s = 2$ corresponds to high SES (above median family income at age 9-10). Unobserved preference heterogeneity is captured by a discrete type $h \in \{1, 2\}$, where $h = 1$ denotes the "low" preference type and $h = 2$ the "high" preference type.

4.2 Utility

Parent utility depends on consumption c_{jt} and the perceived cognitive skill level of the child $\tilde{\theta}_{jt}$, since skill is unobserved. Parents are therefore motivated to invest because investments increase the perceived skill of the child, which raises utility. As in Del Boca, Flinn, and Wiswall (2014) and Del Boca, Flinn, Verriest, and Wiswall (2019), the skill level of the child is included in the flow utility.

I assume that utility is additive and separable in terms of logs in both consumption and the perceived skill level of the child. κ_h is the relative value of child skill over consumption. Its value depends on the unobserved discrete type $h \in \{1, 2\}$. The "high" type ($h = 2$) has a higher relative value of child skill over consumption. The tilde over the expectations operator indicates that the expectations are computed using the parent's (subjective) belief about the skill level of their child, which will be explained in section 4.4.

$$u\left(\underbrace{c_{jt}}_{\text{consumption}}, \underbrace{\tilde{\theta}_{jt}}_{\text{perceived skill}}\right) = \ln c_{jt} + \underbrace{\kappa_h}_{\substack{\text{relative preference for} \\ \text{child skill over consumption}}} \times \underbrace{\tilde{E}_t \ln \theta_{jt}}_{\substack{\text{utility from} \\ \text{perceived skill}}} \tag{6}$$

4.3 Child Skill Production Technology

The child skill production function is translog. Next period log cognitive skills of child j, represented by $\ln \theta_{j,t+1}$, depend on current period log skills $\ln \theta_{jt}$ and current period log investments $\ln i_{jt}$ (Equation 7).

$$\ln \theta_{j,t+1} = A_t + \gamma_{s,1} \ln \theta_{jt} + \gamma_{s,2} \ln i_{jt} + \gamma_{s,3} \ln \theta_{jt} \times \ln i_{jt} + \eta_{j,t+1} \tag{7}$$

The age-specific total factor productivity A_t captures exogenous time-varying factors that influence child skills aside from parental investment (e.g. school education). I assume that total factor productivity is equal at age 11-12 and age 13-14. The production of skills is subject to an idiosyncratic shock η_{t+1}, which is normally distributed with mean 0 and variance $\sigma^2_{s,\eta}$.

The interaction between current log skill and log investment allows the marginal productivity of investment to vary with the child's skill level. When $\gamma_{s,3} > 0$, investments are more productive for higher-skill children; when $\gamma_{s,3} < 0$, the opposite holds. In this model with log utility and a translog production function, the sign of $\gamma_{s,3}$ therefore governs how beliefs translate into investment choices. If $\gamma_{s,3} > 0$, higher beliefs raise perceived marginal returns and lead parents to invest more; if $\gamma_{s,3} < 0$, higher beliefs reduce perceived marginal returns and lead parents to invest less[9].

Parameters of the skill production function depend on SES $s \in \{1, 2\}$, except for total factor productivity. This accommodates the possibility that high SES parents are more effective at translating a given level of investment into child skill gains.

Parents know the skill production technology (including the return to investment), but they do not observe their child's current skill, which enters the production function. As a result, when forecasting how today's investment will affect next period's log skill, they form expectations over $\ln \theta_{jt}$.

4.4 Belief about Child Cognitive Skill and Belief Updating

Parents do not observe the log cognitive skill of the child $\ln \theta_{jt}$. They enter each period with a subjective prior belief over the log skill. I summarise beliefs by their mean: because flow utility is linear in log skill (i.e. parents are risk neutral), investment decisions only depend on the expected log skill but not on higher moments such as the variance[10].

In each of the periods 2 and 3, parents receive a signal g_{jt}, a noisy measure of the log skill $\ln \theta_{jt}$. The idiosyncratic error term in the signal $\epsilon_{j,t+1}$ is normally distributed with

[9] This follows directly from the investment first-order condition in a simplified version of the model.
[10] The variance of the distribution matters if parents are risk averse or risk loving.

mean 0 and variance σ_ϵ^2 [11].

$$g_{j,t+1} = \ln \theta_{j,t+1} + \epsilon_{j,t+1}, \epsilon_{j,t+1} \sim \mathcal{N}(0,\, \sigma_\epsilon^2) \tag{8}$$

After observing the signal, parents update beliefs. The mean posterior (updated) belief μ_{jt}^{int} is a weighted average of the mean prior belief μ_{jt} and the signal g_{jt}[12], as in Equation 9. High SES parents may update beliefs differently from low SES parents[13]: the weights may differ by SES category s.

$$\mu_{jt}^{int} = \psi_s \mu_{jt} + (1 - \psi_s)\, g_{jt} \tag{9}$$

After choosing investment, parents form their prior belief for next period's cognitive skill. This prior equals their subjective expectation of next-period log skill conditional on the current posterior belief and the chosen investment level (Equation 10). The tilde on the expectation operator denotes that the expectation is taken with respect to the parent's posterior belief distribution over the child's current skill.

$$\begin{aligned} \mu_{j,t+1} &\equiv \tilde{E}\left[\ln \theta_{j,t+1} | \mu_{jt}^{int}, i_{jt}\right] \\ &= \tilde{E}\left[A_t + \gamma_{s,1} \ln \theta_{jt} + \gamma_{s,2} \ln i_{jt} + \gamma_{s,3} \ln \theta_{jt} \times \ln i_{jt} + \eta_{j,t+1} | \mu_{jt}^{int}, i_{jt}\right] \\ &= A_t + \gamma_{s,1} \mu_{jt}^{int} + \gamma_{s,2} \ln i_{jt} + \gamma_{s,3} \mu_{jt}^{int} \times \ln i_{jt} \end{aligned} \tag{10}$$

This structure delivers two channels through which investment raises beliefs. First, higher investment increases expected next period skill directly through the production technology, raising $\mu_{j,t+1}$. Second, by increasing realised skill, higher investment shifts the distribution of future signals upward, which feeds into posterior beliefs via the updating rule.

[11] I assume a common signal precision across parents, which may be restrictive. For instance, high SES parents may receive more precise signals if they spend more time with their children or if their children attend higher quality schools that provide more frequent and informative feedback on performance.

[12] These weights may be compatible with Bayesian updating of a normal random variable. Note, however, that the object being learned about changes over time: because log skill evolves, parents update beliefs about a new period-specific log skill each period rather than a fixed trait.

[13] Dizon-Ross (2019) provides evidence that belief updating depends on SES.

4.5 Resource Constraint and Income Process

There is a budget constraint in every period: consumption, investment and savings (in terms of assets in the next period) are equal to the available resources.

$$c_{jt} + i_{jt} + a_{j,t+1} = (1 + r)\, a_{jt} + y_{jt} \tag{11}$$

Log family income is a cubic function of the age of the child. The residuals of this equation follow an AR(1) process. These equations describe two-year income processes: each period is two years long in the model. The family income process depends on SES category $s \in \{1, 2\}$, because high SES families tend to earn higher income.

$$\ln(y_{j,t+1}) = \tau_{s,0} + \tau_{s,1} age_{j,t+1} + \tau_{s,2} age^2_{j,t+1} + \tau_{s,3} age^3_{j,t+1} + \nu_{j,t+1} \tag{12}$$

$$\nu_{j,t+1} = \rho_s \nu_{jt} + u_{y,jt}, u_{y,jt} \sim \mathcal{N}(0,\, \sigma_u^2) \tag{13}$$

4.6 Value Function

The state variables of the parent are the following: (1) mean posterior belief about the child cognitive skill μ_t^{int}, (2) income y_t, (3) assets a_t, (4) SES category $s \in \{1, 2\}$, where $s = 1$ denotes low SES (up to median family income at age 9-10) and $s = 2$ denotes high SES (above median family income at age 9-10) and (5) unobserved discrete type $h \in \{1, 2\}$. Type $h = 2$ is the "high" type parent, which holds a higher relative preference for child skill over consumption.

In the first and second period, the value function of the parent is shown in Equation 14. Parents choose a sequence of consumption, investment and next period assets to maximise the sum of their flow utility and the discounted expected value of the next period value function. The tilde on the expectation operator denotes that the expectation is taken with respect to the parent's posterior belief distribution over the child's current skill.

$$V_t\left(\mu_t^{int}, a_t, y_t, s, h\right) = \max_{c_t, i_t, a_{t+1}} \ln c_t + \kappa_h \tilde{E}_t \ln \theta_t + \beta \tilde{E}_t V_{t+1}\left(\mu_{t+1}^{int}, a_{t+1}, y_{t+1}, s, h\right) \quad (14)$$

Equation 15 defines the value function in the terminal period T. The investment choice in this period determines the child's terminal human capital θ_{T+1}. As in earlier periods, the tilde on the expectations operator indicates that expectations are taken with respect to the parent's subjective belief about the child's current skill. Because the horizon ends after period T, the parent consumes all remaining resources (with future utility discounted by β) and there is no subsequent income realisation or skill signal, so the continuation value does not involve uncertainty along these dimensions.

$$V_T\left(\mu_T^{int}, a_T, y_T, s, h\right) = \max_{c_T, i_T, a_{T+1}} \ln c_T + \kappa_h \tilde{E}_T \ln \theta_T + \beta \kappa_h \tilde{E}_T \ln \theta_{T+1} + \beta \ln\left((1+r)\, a_{T+1}\right) \quad (15)$$

4.7 Initial Conditions and Unobserved Heterogeneity in Preferences

To assess the role of parental beliefs, I compare the investments and skill trajectories across children whose parents hold higher versus lower beliefs. One concern is selection: parents with higher beliefs may differ systematically from those with lower beliefs in unobserved ways that also affect investment choices. For instance, parents with higher beliefs may place a greater weight on child skill relative to consumption, leading them to invest more regardless of their child's skill level. These latent differences could generate cross-family differences in both beliefs and investments. Moreover, because these unobserved traits influence decisions from early on, they also shape families' prior choices and thus the "initial conditions" of each parent-child pair. I address these concerns using the following strategies.

Firstly, following French and Jones (2011), I obtain the initial conditions of the parent-child pair (beliefs of parents, income of parents, assets of parents and skills of children) directly from the data. As a result, any empirical correlations among these baseline

variables (e.g. higher belief parents also have higher income or assets and children with higher initial skills) are preserved in the simulated sample used to estimate the model.

Secondly, I introduce unobserved preference heterogeneity in the style of Heckman and Singer (1984), Keane and Wolpin (1997) and Van der Klaauw and Wolpin (2008). Parents belong to one of two unobserved discrete types. The "high" type places a higher relative weight on child skill versus consumption. The types capture persistent, unobserved differences across families that jointly influence beliefs and choices. To allow this heterogeneity to be correlated with families' initial conditions, I model the probability of being the "high" type as a logistic function of the child's log skill in the first period, normalised by the mean of log skills[14] ($\overline{\ln \theta_{j1}}$). This specification reflects the idea that parents with stronger preferences for skill tend to have invested more prior to the first observation period, resulting in higher observed skills at baseline. The parameter governing the type probability, π_0, is estimated jointly with the other model parameters in the method of simulated moments procedure described in the next section.

$$Pr\left(\text{Unobserved Type } h = 2|\overline{\ln \theta_{j1}}\right) = \frac{exp\left(\pi_0 \overline{\ln \theta_{j1}}\right)}{1 + exp\left(\pi_0 \overline{\ln \theta_{j1}}\right)} \tag{16}$$

5 Identification and Estimation

This section discusses identification of the model, data preparation followed by the estimation of the model.

5.1 Discussion of Identification

The parameters in the family income process are identified directly from moments in the data. Next, I discuss the identification of the remaining components, beginning with the distribution of the latent factors (beliefs, log skills and investment) and the skill production function parameters.

At each time period t, there are $L_t^I \geq 3$ noisy measurements of investments for child j. I assume that each measurement b_{jkt} for child j at time period t is related to the latent investment Φ_{jt} according to Equation 17. I also assume that errors v_{jkt} are independent

[14] In an earlier version of the model, probability depended on the full set of initial conditions (first-period beliefs, parental income, parental assets and child skill). The coefficients on the first period beliefs, income and assets were very small, and were not well identified.

across individuals and measures.

$$b_{jkt} = \alpha_{kt} + \lambda_{kt}\Phi_{jt} + v_{jkt}, k \in \left\{1, 2, ...L_t^I\right\} \tag{17}$$

In a similar way, there are $L_t^\theta \geq 3$ measurements of the latent log cognitive skills $\ln \theta_{jt}$ and $L_t^\mu \geq 3$ measurements of the latent parental belief μ_{jt}^{int} for $t = 1, 2, 3$. Given this, the distribution of $\left(\{\Phi_{jt}\}_{t=1}^3, \{\ln \theta_{jt}\}_{t=1}^3, \left\{\mu_{jt}^{int}\right\}_{t=1}^3\right)$ can be identified according to arguments in Cunha et al. (2010).

The belief factors and log skill factors do not have natural location and scale. Following Cunha et al. (2010), I anchor these factors to the years of education of the child at age 24 and above. In addition, the investment factor score Φ_{jt} is not expressed in monetary units, while the model's investment choice is monetary. To make both measures comparable, following Caucutt and Lochner (2020), I assume a parametric mapping function $\phi(.)$ between the investment factor score Φ_{jt} and monetary investment i_{jt}. That is, $\Phi_{jt} = \phi(i_{jt})$, where $\phi(.)$ is monotone, so that higher factor scores correspond to higher monetary investments. Under this parametric assumption, the mapping can be inverted to recover investment in monetary units i_t, and hence the distribution of $\ln i_t$.

Because $\phi(.)$ is a known function, $i_{jt} = \phi^{-1}(\Phi_{jt})$ can be substituted directly into the skill production function. Given the joint distribution of the latent factors and the parametric form of the production technology, the parameters governing the skill production function are identified.

To see this, note:

$$\begin{aligned} &E\left[\ln \theta_{j,t+1} | \ln \theta_{jt}, \Phi_{jt}\right] \\ &\quad = E\left[A_t + \gamma_{s,1} \ln \theta_{jt} + \gamma_{s,2} \ln \phi^{-1}(\Phi_{jt}) + \gamma_{s,3} \ln \theta_{jt} \times \ln \phi^{-1}(\Phi_{jt}) + \eta_{jt} | \ln \theta_{jt}, \Phi_{jt}\right] \end{aligned}$$

The variance of the production shock can be recovered from the residual variation in next period skills after accounting for the predictable component implied by current skills and investments: $\sigma_{s,\eta}^2 = Var(\ln \theta_{j,t+1}) - Var(\gamma_{s,1} \ln \theta_{jt} + \gamma_{s,2} \ln \phi^{-1}(\Phi_{jt}) + \gamma_{s,3} \ln \theta_{jt} \times \ln \phi^{-1}(\Phi_{jt}))$. Both terms on the right hand side are pinned down by the estimated joint distribution of the latent log skill and investment factors.

The identification argument relies on the assumption that the innovation in the skill technology is orthogonal to the inputs. Identification may fail if the production shock is

correlated with current skill or investment, for example, if parents adjust investments in response to unobserved shocks to the child's skill.

I now discuss identification of the remaining parameters: belief updating parameters and the type probability parameter. While these parameters are jointly identified by the full set of moments used in estimation, some moments may be particularly informative.

The preference parameters governing the relative weight on child skill versus consumption, $\{\kappa_1, \kappa_2\}$, are primarily disciplined by moments that link current investment to subsequent saving: moments involving the ratio of next period assets to the current period investment factor score. Intuitively, parents who place greater value on child skill allocate more resources toward investments and less to savings, implying higher investment today and low assets tomorrow. This implication follows from the first-order conditions of a simplified static version of the model. This relies on my assumption that the price of monetary investment is equal to the price of consumption (both normalised to 1), so relative prices do not enter the optimality condition. Moments regarding the mean and variance of investment factor scores further inform the preference parameters, because stronger preferences translate into higher investment.

The belief updating parameters, the weight on the prior belief $\{\psi_{s=1}, \psi_{s=2}\}$ and the variance of noise in the signal σ_ϵ^2 are particularly informed by auxiliary moments that mirror the model's updating rule. Specifically, I use the coefficients and residuals of a regression of the belief factor score on the lag skill factor score, lag investment factor score and the skill factor score. This regression is derived from an approximation of the model's belief updating rule. Details are provided in Appendix E.3.

Finally, the type probability parameter π_0 is particularly informed by two sets of moments that isolate persistent unobserved heterogeneity in investment choices. First, I use the standard deviation of residuals from a regression of the investment factor score on observed variables (the skill factor score, log income and log assets). Conditional on these observables, residual variation in investment is attributed to differences in unobserved type. Second, I use the conditional autocorrelation of the investment factor score within quartiles of child log skill, which captures the persistence generated by type-specific preferences and therefore provides direct information on the strength of the "fixed effect" component of unobserved preferences.

Table 5 summarises the data features which are most informative for the estimation

of each of the parameters. In the structural estimation, I treat the belief, skill and investment factor scores as observed inputs, even though they are themselves generated regressors obtained from factor models[15].

Table 5: Summary of Identification of Estimated Parameters

Parameter	Identifying Data Features
$\kappa_{h=1}, \kappa_{h=2}$	Ratio of next period assets to investment factor score, moments of investment factor score
$A_1, A_2, \gamma_{s=1,1}, \gamma_{s=1,2}, \gamma_{s=1,3}, \gamma_{s=2,1}, \gamma_{s=2,2}, \gamma_{s=2,3}, \sigma^2_{s=1,\eta}, \sigma^2_{s=2,\eta}$	Auxiliary model: Regression of log skill factor score on lag log skill factor score, lag investment factor score, an interaction between lag log skill factor score and lag investment factor score, along with current log skill factor score
$\psi_{s=1}, \psi_{s=2}, \sigma^2_\epsilon$	Auxiliary model: Regression of the belief factor score on lag log skill factor score, lag investment factor score and the current log skill factor score (this specification is motivated by an approximation to the belief updating rule)
π_0	(1) Standard deviation of residuals from a regression of investment factor score on log skill factor score, log income and log assets and (2) the autocorrelation of the investment factor score conditional on the child's log skill quartile

Notes: This table summarises the moments that are particularly informative for identifying the parameters estimated in the method of simulated moments step. In the method of simulated moments procedure, the belief, log skill and investment factor scores are treated as observed inputs, although they are predicted factor scores obtained from first-stage factor models.

5.2 Data Preparation

Sample preparation for model estimation and calculation of data moments: I draw initial conditions from the sub-sample for which I observe all initial conditions: the belief factor score, income, assets and child skills. Because the model rules out borrowing

[15]Treating predicted factor scores as data affects the reported standard errors. A natural extension is to account for this first-stage uncertainty via a bootstrap that resamples the data and re-estimates the factor models, or by drawing each individual's factor score from its estimated posterior distribution in each bootstrap replication.

(assets must be weakly positive), I exclude observations with negative assets at age 9-10, 11-12 or 13-14. I also drop children from families reporting zero income at ages 9-10. Finally, I restrict the sample to children whose mothers were married throughout ages 5-14 to focus on intact families and to avoid additional volatility in the income process induced by marital transitions. Assuming that missingness is at random, I compute each data moment using all available (non-missing) observations for that variable and age. For example, the average belief factor score at age 11-12 is calculated over all non-missing belief observations at ages 11-12. 655 parent-child pairs are used in the estimation.

Factor models and anchoring: I use factor models to recover the latent parental beliefs, investments and child's log cognitive skill from multiple noisy measurements of each construct. The estimated factor models yield predicted factor scores for beliefs, investments, and log skill, which I then treat as data in the two-step estimation procedure described in Section 5.3. Because both the belief and skill factors are anchored to the child's completed years of education at age 24 and above, they are comparable in levels.

Beliefs in the data and the model: I interpret the belief factor score as the mean of parents' posterior (updated) belief distribution rather than their prior. The belief measures are elicited at a point when parents have likely already observed information about the child's skill (through grades, teacher feedback, or day-to-day interactions) so the reported beliefs are best viewed as beliefs after incorporating such signals.

Converting investment factor scores to investment in monetary units: To map the model's monetary investment choice to the empirical investment factor score, I introduce a parametric link between the two. Following Caucutt and Lochner (2020), I assume a quadratic mapping from monetary investment i_{jt} to the observed investment factor score Φ_{jt} for child j in period t: $\Phi_{jt} = \phi(i_{jt}) = \phi_0 + \phi_1 i_{jt} + \phi_2 i_{jt}^2$. I impose monotonicity, $\phi'(.) > 0$, such that higher factor scores correspond to higher monetary investment. In this analysis, the parameters of this function are set at the following values: $\phi_0 = -18.23 \times 10^{-2}$, $\phi_1 = 4.46 \times 10^{-6}$ and $\phi_2 = 5.62 \times 10^{-14}$.

5.3 Estimation

Estimation proceeds in two steps. First, a subset of parameters is calibrated or estimated directly from the data. These values are then taken as given in the second step, where the remaining parameters are identified and estimated using the method of simulated

moments.

5.3.1 Step One: Calibration and External Estimation

Following Del Boca et al. (2014), I set the discount factor for households to $\beta = 0.95$, which implies an interest rate of $r = \frac{1}{\beta} - 1$. The parameters governing the family income process are estimated directly from the data in the first step and then treated as inputs in the second step of the estimation procedure.

$$\left\{\tau_{s=1,0}, \tau_{s=1,1}, \tau_{s=1,2}, \tau_{s=1,3}, \rho_{s=1}, \sigma^2_{s=1,u}, \tau_{s=2,0}, \tau_{s=2,1}, \tau_{s=2,2}, \tau_{s=2,3}, \rho_{s=2}, \sigma^2_{s=2,u}\right\}$$

5.3.2 Step Two: Method of Simulated Moments

In step two, I estimate the remaining parameters by the method of simulated moments. These include (i) the preference parameters governing the relative weight on child skill versus consumption, (ii) the skill production function parameters, (iii) the parameters governing beliefs and belief updating, and (iv) the parameter governing the type probability function. Let Γ denote the vector of parameters to be estimated in the second step (defined below). Socio-economic status (SES) is indexed by s, with $s = 1$ for low SES and $s = 2$ for high SES. The unobserved discrete type is denoted by h, where $h = 2$ denotes the "high" type parent who places a higher relative weight on child skill than consumption.

$$\begin{aligned}\Gamma \equiv \Big\{&\kappa_{h=1}, \kappa_{h=2}, A_1, A_2, \gamma_{s=1,1}, \gamma_{s=1,2}, \gamma_{s=1,3}, \sigma^2_{s=1,\eta}, \dots \\ &\gamma_{s=2,1}, \gamma_{s=2,2}, \gamma_{s=2,3}, \sigma^2_{s=2,\eta}, \sigma^2_{\epsilon}, \psi_{s=1}, \psi_{s=2}, \pi_0\Big\}\end{aligned}$$

For a candidate parameter vector Ω, I solve the parent's dynamic optimisation problem numerically by backward induction, starting from the terminal period T. This yields policy functions for consumption, investment and next period assets (savings). I then forward simulate $5 \times$ N parent–child trajectories. At the start of each simulation, I draw an unobserved type for the parent. In each period, I draw the relevant shocks and use the policy functions to generate choices and update the state variables for the next period.

I compute the resulting simulated moments $M(\Omega)$ and choose $\hat{\Omega}$ to minimise the GMM distance between simulated and empirical moments, iterating on Ω until convergence.

$$\hat{\Omega} = \arg\min_{\Omega} \left(M_D - M(\Omega)\right)' W \left(M_D - M(\Omega)\right) \tag{18}$$

To construct the weighting matrix, I estimate the variance-covariance matrix of the data moments, S, by bootstrap with $K = 200$ replications. Let M_k denote the moment vector computed from bootstrap replication k. Then

$$S = \frac{1}{K} \sum_{k=1}^{K} \left(M_k - M_D\right) \left(M_k - M_D\right)' \tag{19}$$

Although the asymptotically optimal weighting matrix is S^{-1}, it may not be ideal in small samples (Altonji & Segal, 1996). Therefore, I use a diagonal weighting matrix constructed from S^{-1}: the weighting matrix W retains only the diagonal elements of S^{-1} and sets the off-diagonal entries to zero. Standard errors for the parameter estimates are computed using the formula derived by Gourieroux, Monfort, and Renault (1993), which accounts for simulation noise.

Estimation targets a set of moments and auxiliary model statistics that summarise the joint dynamics of beliefs, skills, investments and resources. These include means and standard deviations of the skill factor score, investment factor score, and assets. I also target contemporaneous covariances or correlations among skills, investments, and beliefs, along with autocorrelations. In addition, I match the coefficients from auxiliary regressions that approximate the key relationships implied by the model. For example, a regression of the skill factor score on lagged skill, lagged investment, and their interaction. Appendix J provides the complete list of targeted moments and auxiliary models.

6 Model Estimates

This section presents the parameter estimates and discusses the model fit.

6.1 Parameter Estimates

The parameters in the income process are estimated directly from the data (step one). The remaining parameters are estimated via method of simulated moments (step two).

Preferences and Type Probabilities: Table 6 reports estimates of the preference parameters and the parameter governing unobserved type probability. In the model, parents belong to one of two unobserved discrete types. The "high" type ($h = 2$) places a greater relative weight on child skill over consumption. According to model estimates, more parents are the "high" type: the implied share of the "high" type in the simulated population is around 65.8%. Consistent with meaningful preference heterogeneity, $\kappa_{h=1}$ differs from $\kappa_{h=2}$ and the difference is statistically significant at the 5% level. Appendix D reports the corresponding test statistic and p-value.

The estimated parameter governing type probability also implies that parents of children with higher baseline (period 1) log skills are more likely to be the "high" type: the coefficient π_0 on initial log skill is positive. This pattern is consistent with the model's interpretation of types as persistent preference differences: parents who place a higher weight on child skill would have invested more prior to the first observed period, leading their children to enter the sample with higher skills.

Table 6: Preference Parameters and Type Probability Parameter

Parameter	Description	Estimate	S.E.
$\kappa_{h=1}$	Relative preference for skill over consumption: Type 1	2.609	0.00689
$\kappa_{h=2}$	Relative preference for skill over consumption: Type 2	3.253	0.27638
π_0	Type probability logistic: coefficient on log skill	0.650	0.31521

Skill Production Function: Table 7 presents the estimated parameters of the skill production function. Cognitive skills are highly persistent: the weight on the current log skill in the production function is 0.77 or higher. This degree of persistence is in line with estimates in the child human capital literature (e.g. Cunha et al. (2010) and Attanasio, Meghir, and Nix (2020)). Given this persistence, sizeable increases in investment are required to generate large gains in skills, a feature that will be important for interpreting the policy counterfactuals.

The estimates imply that skills and investments are complements in the production function, so the marginal productivity of investment is higher for children with higher skills. The coefficient on the interaction term between log skill and log investment ($\gamma_{s,3}$) is positive for both low SES ($s = 1$) and high SES ($s = 2$) families. However, complementarity between skills and investments in the production function is not high, like other stud-

ies (see Cunha et al. (2010), Carneiro, Cruz-Aguayo, Pachon, and Schady (2022), Goff, Malamud, Pop-Eleches, and Urquiola (2023), Agostinelli and Wiswall (2025)). The coefficient on the interaction term is a key parameter that determines the direction of the relationship between beliefs and investments.

In general, depending on preferences and skill production technology, higher beliefs could either raise or lower optimal investment. In the present model, with log utility and a translog production function, the direction is determined by the interaction between skills and investments in the production function. A positive interaction implies higher perceived returns to investing in higher-skill children, so higher beliefs increase investment; a negative interaction implies the opposite. According to the estimates, the interaction term is positive, and higher beliefs therefore lead to higher investments. Since investments in turn raise subsequent beliefs, the resulting feedback creates a self-reinforcing dynamic that amplifies initial changes in beliefs.

Finally, the production technology differs by SES. The interaction between skills and investments is stronger for low SES families (a larger coefficient on interaction term). There is no significant difference in the weight on log skill and weight on log investment between the high SES and the low SES. Appendix D reports tests of equality of the production function parameters across SES groups, along with the associated p-values.

Table 7: Child Skill Production Function Parameters

Parameter	Description	Estimate	S.E.
A_1	Age 9-10 total factor productivity	0.541	0.05814
A_2	Age 11-12, 13-14 total factor productivity	0.404	0.06940
Low SES ($s = 1$)			
$\gamma_{s=1,1}$	Weight on log skill	0.783	0.01307
$\gamma_{s=1,2}$	Weight on log investment	0.030	0.02059
$\gamma_{s=1,3}$	Weight on interaction between log skill and log investment	0.017	0.00141
$\sigma^2_{s=1,\eta}$	Variance of idiosyncratic shock	0.198	0.02395
High SES ($s = 2$)			
$\gamma_{s=2,1}$	Weight on log skill	0.778	0.00614
$\gamma_{s=2,2}$	Weight on log investment	0.053	0.00366
$\gamma_{s=2,3}$	Weight on interaction between log skill and log investment	0.014	0.00018
$\sigma^2_{s=2,\eta}$	Variance of idiosyncratic shock	0.129	0.01721

Belief Updating: Table 8 presents the estimates of the parameters pertaining to belief updating. The weight on the prior belief is around 0.55-0.58, implying that parents place roughly equal weight on their prior beliefs and on the noisy signal about the child's skill.

The updating rule is similar across socio-economic groups: I find no evidence that the weight on the prior differs between low and high SES parents. Appendix D reports a test of equality of the belief updating weight across SES groups, along with the corresponding p-values.

Table 8: Belief Parameters

Parameter	Description	Estimate	S.E.
σ^2_ϵ	Variance of error in signal	3.884	0.61686
$\psi_{s=1}$	Weight on prior (Low SES)	0.552	0.03187
$\psi_{s=2}$	Weight on prior (High SES)	0.583	0.02574

Income Process: Table 9 presents the estimates of the family income process, which depends on SES.

Table 9: Family Income Process Parameters

Parameter	Description	Estimate	S.E.
Low SES ($s = 1$)			
$\tau_{s=1,0}$	Intercept term	10.747	0.44918
$\tau_{s=1,1}$	Coefficient on age	0.088	0.15816
$\tau_{s=1,2}$	Coefficient on age squared	-0.009	0.01749
$\tau_{s=1,3}$	Coefficient on age cubed	3.605×10^{-4}	0.00061
$\rho_{s=1}$	AR(1) residual persistence parameter	0.633	0.03862
$\sigma^2_{s=1,u}$	AR(1) residual variance	0.230	0.03265
High SES ($s = 2$)			
$\tau_{s=2,0}$	Intercept term	11.725	0.44912
$\tau_{s=2,1}$	Coefficient on age	-0.129	0.15652
$\tau_{s=2,2}$	Coefficient on age squared	0.018	0.01719
$\tau_{s=2,3}$	Coefficient on age cubed	-6.781×10^{-4}	0.00060
$\rho_{s=2}$	AR(1) residual persistence parameter	0.525	0.01326
$\sigma^2_{s=2,u}$	AR(1) residual variance	0.372	0.03942

Notes: Robust standard errors reported for the log family income regression. Standard errors for the AR(1) process are computed via bootstrap with 100 replications.

6.2 Model Fit

To illustrate model fit, Table 10 reports a selected set of targeted moments: mean beliefs and mean log skills by family income quartile at ages 11–12 and 13–14. In both age ranges, the model modestly underpredicts average beliefs and skills in the top income quartile. At ages 11–12, it slightly overpredicts beliefs and skills in the bottom income quartile. Appendix J reports the full set of targeted moments for a comprehensive assessment of fit. The model fits the following moments less well: final period assets, and moments related

to the intertemporal correlation of investment factor scores and the standard deviation of the residual from the regression of the investment factor score on observables.

Table 10: Selected Moments

Moment	Data	Model
Mean skill age 11-12: Family income Q1	13.474	13.671
Mean skill age 11-12: Family income Q2	13.685	13.707
Mean skill age 11-12: Family income Q3	13.763	13.736
Mean skill age 11-12: Family income Q4	14.040	13.867
Mean skill age 13-14: Family income Q1	13.816	13.880
Mean skill age 13-14: Family income Q2	13.944	13.932
Mean skill age 13-14: Family income Q3	14.104	14.083
Mean skill age 13-14: Family income Q4	14.318	14.186
Mean belief age 11-12: Family income Q1	13.320	13.640
Mean belief age 11-12: Family income Q2	13.633	13.676
Mean belief age 11-12: Family income Q3	13.874	13.708
Mean belief age 11-12: Family income Q4	14.226	13.969
Mean belief age 13-14: Family income Q1	13.775	13.813
Mean belief age 13-14: Family income Q2	13.761	13.875
Mean belief age 13-14: Family income Q3	14.187	14.128
Mean belief age 13-14: Family income Q4	14.454	14.165

7 Policy Counterfactuals: Information Intervention and Cash Transfer

I compare a belief-based information intervention to a cash transfer targeted to low socio-economic status (SES) families in period 1. The information intervention is modelled as fully eliminating the average belief bias among low SES parents. Because real-world information provision is unlikely to remove distortions completely, this counterfactual should be interpreted as an upper bound. I benchmark the transfer at $3000—about 10% of household income at the 25th percentile—to reflect a policy-relevant magnitude that is broadly comparable to the scale of large U.S. transfer programmes and conditional cash transfer style payments.

Table 11 reports percentage changes from the baseline in posterior beliefs, investments, skills, consumption, and assets. Column (1) shows the information intervention. Since low SES parents under-estimate child skill on average, eliminating belief bias raises period 1 beliefs. Through the model's belief–investment feedback, this initial shift propagates forward: average posterior beliefs increase by 0.862% in period 2 and 0.459% in

period 3. Investments rise modestly: they rise by 0.469% in period 1, 0.109% in period 2 and by 0.048% in period 3. Overall, there are small but positive gains in skills: average log skills increase by 0.016% at ages 11–12, by 0.021% at ages 13–14, and terminal log skills (skills after parent makes investment decision in the third and final period) by 0.021%. These gains are accompanied by slightly lower consumption and assets relative to baseline, reflecting reallocation toward investment.

Table 11: Percentage Change from Baseline

	Information	Cash Transfer
Mean log skill age 11-12	0.016	0.066
Mean log skill age 13-14	0.021	0.066
Mean log skill terminal	0.021	0.065
Mean posterior belief age 11-12	0.862	0.065
Mean posterior belief age 13-14	0.459	0.066
Mean investment age 9-10	0.469	2.692
Mean investment age 11-12	0.109	0.157
Mean investment age 13-14	0.048	0.125
Mean consumption age 9-10	-0.265	1.779
Mean consumption age 11-12	-0.367	0.131
Mean consumption age 13-14	-0.213	0.091
Mean next period assets choice age 9-10	-0.339	0.397
Mean next period assets choice age 11-12	-0.347	0.292
Mean next period assets choice age 13-14	-0.169	0.104

Notes: This table presents the percentage change from the baseline in the mean log skills, mean posterior beliefs, mean investment, mean consumption and mean next period assets choice.

Column (2) reports the $3000 transfer. The transfer raises beliefs, investments, and skills in the same direction as the information intervention, but it also increases consumption and assets because households do not allocate the full transfer to child investments.

Two broad lessons emerge. First, both policies generate modest average gains in terminal skills because neither induces large changes in investments. This reflects the estimated production technology: with highly persistent skills and modest complementarity, substantial increases in investment are required to generate large improvements in skill. Second, the transfer produces larger gains than the information intervention, in part because average belief distortions are not large enough that debiasing generates sizeable investment responses.

Although the information intervention generates smaller average skill gains than a $3000 transfer, it may still be attractive on cost-effectiveness grounds. The average increase in terminal skills it produces is comparable to that generated by a period 1 cash transfer of about $970 per child, yet the marginal cost of delivering information can plausibly be well below this cash-equivalent amount. For example, Chikhale (2025) places the per-person cost of the intervention in List et al. (2021) at roughly $143–150. In a different setting, Bergman (2021) shows that providing parents with information about their child's assignment completion improves academic performance and estimates that, with automation, the cost can be less than $10 per student for a 0.10 standard deviation increase in grade point average. Taken together, these figures suggest that belief-based policies are unlikely to substitute for redistribution, but can be a cost-effective complement to cash transfers. These estimates should be interpreted with care because some belief measures, such as forward-looking expectations about educational attainment or future prospects, may partly capture plans rather than beliefs about current skill, potentially making the estimated effect an upper bound.

8 Conclusion

I develop and estimate a dynamic model of parental investments that incorporates parental beliefs about child cognitive skill level. Model estimates reveal a reinforcing dynamic between beliefs and investments: parents with higher beliefs invest more, because they perceive higher returns to investment, since investments are more productive in children with higher skills. Investing more leads parents to hold higher beliefs in the following period, creating a reinforcing cycle.

I use the model to evaluate a policy-relevant trade-off: should policymakers rely on belief-based interventions or cash transfers to reduce socio-economic status (SES) skill gaps? In the counterfactuals, an information intervention that eliminates belief distortions among low SES parents delivers modest improvements in child skills relative to a $3000 cash transfer in period 1. Even so, information may be highly cost-effective: its estimated average gain in terminal skill is comparable to that produced by a period 1 transfer of roughly $970 per child, while the per-child cost of providing information can plausibly be much lower. Thus, belief-based policies are unlikely to replace redistribution,

but they can be a cost-effective complement to cash transfers.

Future research could extend the model to include multiple skill dimensions, additional dimensions of parental beliefs (e.g. return to investment), and contextual factors such as neighbourhood quality and school inputs. It would also be valuable to examine whether similar patterns hold in different settings, particularly in developing countries or among younger age cohorts. Given that some skills are more malleable at younger ages (Kautz, Heckman, Diris, ter Weel, & Borghans, 2014), adjusting beliefs at earlier ages may yield larger impacts on skills and the SES skill gap.

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A Data Appendix

A.1 Further Information on the Data and Representativeness

The analysis uses the sample of children born to women in the cross-section sample of the NLSY79, comprising 5,819 children. Because the NLSY79 records all biological children of participating women, siblings appear in the data whenever a mother has multiple children.

The CYA provides a broadly representative sample of children born to women from the 1957–1964 U.S. birth cohorts. However, the sample of children of NLSY79 women need not be representative of children in the same birth cohorts. This reflects selection into fertility and into the timing of births. In particular, children in earlier birth cohorts were born to younger mothers and tend to be negatively selected on maternal characteristics. For example, their mothers have, on average, lower schooling and cognitive skills and they are more likely to have grown up in single-parent households.

A.2 Additional Information on Measures of Parental Belief

For the question "How far do you think your child will go in school?", the survey offers six response categories: (1) leave high school before graduation; (2) graduate from high school; (3) get some college or other training; (4) graduate from college; (5) get more than 4 years of college/further training after college; and (6) something else. In the data, very few mothers select option (1), so I combine options (1) and (2) into a single category, "Up to High School". Because option (6) is vague and rarely chosen, I treat it as a missing response.

For the mother's assessment of the child's future prospects, the response categories are: (1) poor; (2) fair; (3) good; and (4) excellent. Since "poor" is infrequently selected, I pool "poor" and "fair" into a single category.

A.3 Measures of Parental Investments

Components of the Home Observation Measurement of the Environment (HOME) used to construct the parental investment factor are as follows:

- How many magazines the family gets regularly

- Whether the child has the use of a record player, tape deck or CD player at home and at least 5 children's records or tapes
- The number of books the child has
- How often the mother reads to child
- Whether the family has a musical instrument which the child can use at home
- Whether the family receives a daily newspaper
- Whether the family encourages the child to pursue hobbies
- Whether the child attends special lessons
- How often a family member gets a chance to take the child on any kind of outing
- How often the child is brought to the museum
- How often the child is brought to music/theatre performance
- How often the family gets together
- How often the child eats with mom and dad

B Factor Models

B.1 Factor Model for Parental Investment

The measures used to construct the investment latent factor are either binary indicators or ordered categorical variables.

For the binary indicator m_{ijt}, I assume it is generated by an underlying continuous latent propensity m^*_{ijt} according to a threshold rule:

$$m_{ijt} = \begin{cases} 0 \text{ if } m^*_{ijt} \leq 0 \\ 1 \text{ if } m^*_{ijt} > 0 \end{cases}$$

For ordered discrete variables, it is assumed that each of these categorical measurements m_{ijt} depend on an unobserved continuous latent variable m^*_{ijt} according to a threshold model. As an example, suppose measurement m_{ijt} takes 4 values. I model it as a

discretised version of an underlying latent continuous variable m^*_{ijt}, linked through a standard threshold (ordered-response) model:

$$m_{ijt} = \begin{cases} 0 \text{ if } m^*_{ijt} \leq \tau_{1,jt} \\ 1 \text{ if } m^*_{ijt} \in (\tau_{1,jt}, \tau_{2,jt}] \\ 2 \text{ if } m^*_{ijt} \in (\tau_{2,jt}, \tau_{3,jt}] \\ 3 \text{ if } m^*_{ijt} > \tau_{3,jt} \end{cases}$$

Let I_t denote the latent investment factor in period t. For expositional simplicity, I write the measurement system with three measurements, although in the empirical implementation the factor is estimated using more than three measurements in each period.

Let m^*_1, m^*_2 and m^*_3 denote latent continuous "propensity" variables underlying the observed (binary or ordered) investment measures. Each latent propensity is assumed to depend linearly on I_t. For example, measurement m^*_{1t} loads on I_t through an intercept $\alpha_{m1,t}$ and a factor loading $\lambda_{m1,t}$.

Equations for the latent factor model of investment for $t = 1, 2, 3, 4, 5$

$$m^*_{1t} = \alpha_{m1,t} + \lambda_{m1,t} I_t + \epsilon_{m1,t} \tag{20}$$

$$m^*_{2t} = \alpha_{m2,t} + \lambda_{m2,t} I_t + \epsilon_{m2,t} \tag{21}$$

$$m^*_{3t} = \alpha_{m3,t} + \lambda_{m3,t} I_t + \epsilon_{m3,t} \tag{22}$$

The measurement errors $\epsilon_{ml,t}$ are assumed to be normally distributed, uncorrelated across measurements and uncorrelated across time. One of the measurements—in this case, the frequency of bringing the child to the museum—is used to link the factor over time. This is achieved by restricting the value of the first threshold and the factor loading of this measurement to be the same in all time periods. The intercepts and the factor loadings of other measurements are unrestricted.

B.2 Factor Model for Cognitive Skills

$\ln \theta_t$ denotes the child's latent log cognitive skill in period t. Let Q denote an adult outcome used for anchoring. Here, it is the completed years of schooling measured at age 24 and above. The observed measures m_1, m_2 and m_3 are noisy indicators of $\ln \theta_t$:

the raw scores on achievement tests in mathematics, reading recognition and reading comprehension (continuous variables). I assume that each test score depends linearly on the latent skill through an intercept and a factor loading. For example, measurement m_{1t} loads on $\ln \theta_t$ with intercept $\alpha_{m1,t}$ and loading $\lambda_{m1,t}$.

Equations for the latent factor model of skills for $t = 1, 2, 3, 4, 5$

$$
\begin{aligned}
Q &= \alpha + \ln \theta_T + \eta_Q && (23)\\
m_{1t} &= \alpha_{m1,t} + \lambda_{m1,t} \ln \theta_t + \epsilon_{m1,t} && (24)\\
m_{2t} &= \alpha_{m2,t} + \lambda_{m2,t} \ln \theta_t + \epsilon_{m2,t} && (25)\\
m_{3t} &= \alpha_{m3,t} + \lambda_{m3,t} \ln \theta_t + \epsilon_{m3,t} && (26)
\end{aligned}
$$

The measurement errors $\epsilon_{ml,t}$ are assumed to be normally distributed, uncorrelated across measurements and uncorrelated over time.

The first equation anchors the scale of the latent factor by linking terminal period skill to the adult outcome. I normalise the loading on $\ln \theta_T$ to one, so that a one-unit increase in $\ln \theta_T$ corresponds to a one-unit increase in the conditional expectation of Q.

To link $\ln \theta_t$ across periods, I use one measurement (achievement test in mathematics) as the time anchor by restricting its intercept and factor loading to be constant over t. Under this restriction, changes in the math raw score over age are informative about changes in the level of latent skill. Intercepts and factor loadings for the remaining test measures are allowed to vary flexibly across periods.

B.3 Factor Model for Beliefs

The measures used to construct the belief latent factor are ordered categorical variables. I model each observed response as a discretised version of an underlying continuous latent "propensity" variable via a standard threshold (ordered-response) model. The three belief measures are: (i) the mother's rating of the child's future prospects (3 categories), (ii) expected educational attainment (4 categories), and (iii) the mother's rating of the child's academic standing in class (5 categories).

Let $m_{ijt} \in \{0, 1, ..., K-1\}$ denote an observed categorical response for item i for child j at time t and let m^*_{ijt} denote the corresponding latent continuous propensity.

Suppose measurement m_{ijt} takes 3 values. We assume that it is mapped to unobserved

latent continuous variable m^*_{ijt} according to a threshold model in this way:

$$m_{ijt} = \begin{cases} 0 \text{ if } m^*_{ijt} \leq \tau_{1,jt} \\ 1 \text{ if } m^*_{ijt} \in (\tau_{1,jt}, \tau_{2,jt}] \\ 2 \text{ if } m^*_{ijt} > \tau_{2,jt} \end{cases}$$

Suppose measurement m_{ijt} takes 4 values. We assume that it is mapped to unobserved latent continuous variable m^*_{ijt} according to a threshold model in this way:

$$m_{ijt} = \begin{cases} 0 \text{ if } m^*_{ijt} \leq \tau_{1,jt} \\ 1 \text{ if } m^*_{ijt} \in (\tau_{1,jt}, \tau_{2,jt}] \\ 2 \text{ if } m^*_{ijt} \in (\tau_{2,jt}, \tau_{3,jt}] \\ 3 \text{ if } m^*_{ijt} > \tau_{3,jt} \end{cases}$$

Suppose measurement m_{ijt} takes 5 values. We assume that it is mapped to unobserved latent continuous variable m^*_{ijt} according to a threshold model in this way:

$$m_{ijt} = \begin{cases} 0 \text{ if } m^*_{ijt} \leq \tau_{1,jt} \\ 1 \text{ if } m^*_{ijt} \in (\tau_{1,jt}, \tau_{2,jt}] \\ 2 \text{ if } m^*_{ijt} \in (\tau_{2,jt}, \tau_{3,jt}] \\ 3 \text{ if } m^*_{ijt} \in (\tau_{3,jt}, \tau_{4,jt}] \\ 4 \text{ if } m^*_{ijt} > \tau_{4,jt} \end{cases}$$

Let μ_t denote the latent belief factor in period t. Let Q denote an adult outcome used for anchoring. Here, it is the completed years of schooling measured at age 24 and above. The latent propensities m^*_1, m^*_2 and m^*_3 correspond to the three belief items: future prospects, expected educational attainment, and academic standing. Each of these load linearly on μ_t.

Equations for the latent factor model of belief for $t = 1, 2, 3, 4, 5$

$$Q = \alpha + \mu_T + \nu_Q \tag{27}$$

$$m^*_{1t} = \alpha_{m1,t} + \lambda_{m1,t}\mu_t + \epsilon_{m1,t} \tag{28}$$

$$m^*_{2t} = \alpha_{m2,t} + \lambda_{m2,t}\mu_t + \epsilon_{m2,t} \tag{29}$$

$$m^*_{3t} = \alpha_{m3,t} + \lambda_{m3,t}\mu_t + \epsilon_{m3,t} \tag{30}$$

The measurement errors $\epsilon_{ml,t}$ are assumed to be normally distributed, uncorrelated across measurements and uncorrelated across time.

The first equation anchors the scale of beliefs by linking terminal period beliefs μ_T to the adult outcome Q. I normalise the loading on μ_T to one, so that a one unit increase in μ_T corresponds to a one unit increase in the conditional expectation of completed schooling Q.

To link μ_t across periods, I use the expected educational attainment item as the time anchor by restricting its intercept and factor loading to be constant across t. Intercepts and factor loadings for the remaining belief items are allowed to vary freely over time.

C Data Patterns with Academic Standing of Child in Class

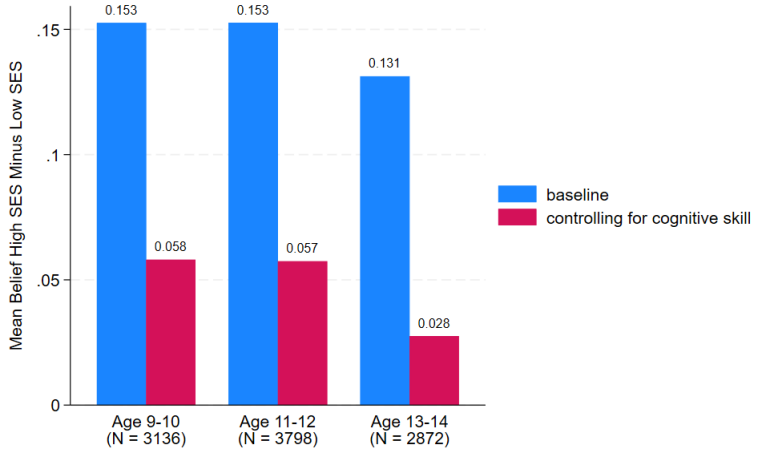
This section replicates the main descriptive patterns using a simpler, strictly contemporaneous measurement of beliefs. Specifically, I replace the belief factor score with a binary indicator, "above the middle in class", which equals one if the child is rated as above the middle or one of the best students in the class, and zero otherwise.

Figure C.1 presents the average gap between the high SES and the low SES in this indicator at age 9-10, 11-12 and 13-14. The SES differences are statistically significant.

Figure C.2 presents the correlation between the "above the middle in class" indicator and the cognitive skill factor score. The correlation is weaker among low SES families than among high SES families, with statistically significant differences at ages 9-10 and 13-14.

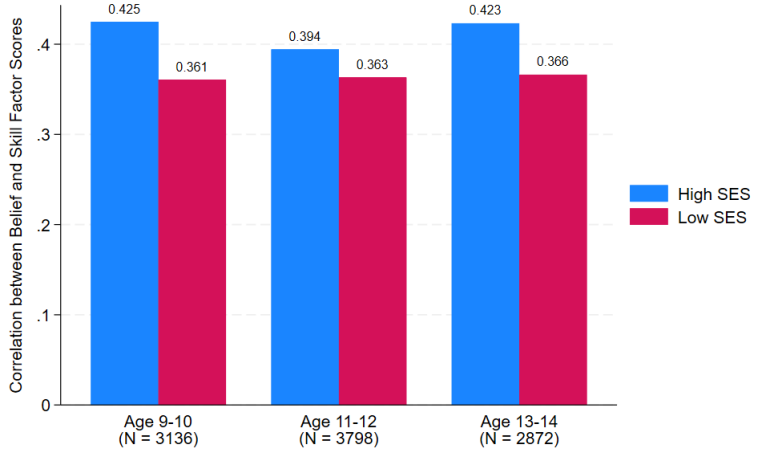
Table C.1 reports estimates from an updating regression analogous to that in Section 3.2, but with the belief factor score replaced by the binary "above the middle in class" indicator; all other variables are defined as before (Equation 31). The lagged indicator is a positive and statistically significant predictor of the current indicator, indicating persistence in parents' assessments. Contemporaneous cognitive skill is also positive and significant, consistent with parents' ratings responding to information about the child's performance.

Figure C.1: Average Belief Factor Score (Anchored) by SES



Notes: This figure presents the mean value of "above the middle in class" indicator for high SES parents and low SES parents at ages 9-10, 11-12 and 13-14. High socio-economic status (SES) parents refer to parents who have above median family income (defined based on average family income when the child is aged between 9 and 10). Otherwise, parents are low SES. At each age, all children with non-missing academic standing ratings, cognitive skill factor scores and average family income between age 9-10 are included in the computation.

Figure C.2: Correlation between Above Middle in Class and Cognitive Skill Factor Score (Anchored) by SES



Notes: This figure presents the correlation between the "above the middle in class" indicator and the cognitive skill factor score (anchored) at ages 9-10, 11-12 and 13-14. High socio-economic status (SES) parents refer to parents who have above median family income (defined based on average family income when the child is aged between 9 and 10). Otherwise, parents are low SES. At each age, all children with non-missing academic standing ratings, cognitive skill factor scores and average family income between age 9-10 are included in the computation.

$$abovemiddle_{jt} = \alpha + \beta_1 abovemiddle_{j,t-1} + \beta_2 cognitive_{jt} + \gamma X_{jt} + \delta_j + \epsilon_{jt} \quad (31)$$

Table C.1: Dependent variable is Above Middle in Class Indicator

	(1)
Lag Above Middle in Class	0.195***
	(0.021)
Cognitive Skill	0.190***
	(0.023)
Observations	7100
Number of Children	3691
Child FE	Yes
Cognitive Skill Potentially Endogenous	Yes
Sargan Test P-value	0.369

Notes: This table presents the estimated coefficients in a regression of the "above middle in class" indicator on the lag "above middle in class" indicator and the cognitive skill factor score. The construction of the belief factor score and the skill factor score is described in the data section. The model is estimated with the Blundell and Bond (1998) dynamic panel data estimator. The regression includes time-varying controls, as described in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between ages 5 and 14 are used.

Table C.2 reports estimates from a regression analogous to that in Section 3.3, replacing the belief factor score with the binary "above the middle in class" indicator; all other variables are defined as before (Equation 32). The estimated coefficient is positive, indicating that children rated above the middle in class are associated with higher investment levels.

$$\underbrace{I_{jt}}_{\text{investment}} = \alpha + \beta abovemiddle_{jt} + \gamma X_{jt} + \psi \underbrace{I_{j,t-1}}_{\text{lag investment}} + \underbrace{\delta_j}_{\text{child FE}} + \eta_{jt} \quad (32)$$

Table C.2: Dependent variable is Investment Factor Score (Standardised)

	(1)
Above Middle in Class	0.113***
	(0.025)
Observations	11134
Number of Children	4400
Child FE	Yes
Lag Investment	Yes
Sargan Test P-value	0.011

Notes: This table reports coefficients on the "above middle in class" indicator variable in a regression of the investment factor score (standardised) on the "above middle in class" indicator variable. The Blundell and Bond (1998) dynamic panel data estimator is used. The construction of the investment factor score is provided in the data section. The regression includes time-varying controls which are explained in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between age 5 and 14 are used. This sample is larger than the one used to estimate the belief updating model, because children tend to have investments reported over a greater number of time periods than they do for beliefs.

D Parameter Equality Test

Table D.1: Test of Equality of Relative Preference for Child Skill Over Consumption Between Unobserved Discrete Types

Test	Description	Test Statistic	p-value
$\kappa_{h=1} = \kappa_{h=2}$	Relative preference for skill over consumption	2.332	0.01971

Table D.2: Test of Equality of Skill Production Function Parameters Between High SES and Low SES

Test	Description	Test Statistic	p-value
$\gamma_{s=1,1} = \gamma_{s=2,1}$	Weight on log skill	0.333	0.73901
$\gamma_{s=1,2} = \gamma_{s=2,2}$	Weight on log investment	1.071	0.28400
$\gamma_{s=1,3} = \gamma_{s=2,3}$	Weight on interaction between log skill and log investment	2.110	0.03490
$\sigma^2_{s=1,\eta} = \sigma^2_{s=2,\eta}$	Variance of idiosyncratic shock	2.367	0.01794

Table D.3: Test of Equality of Weight on Prior Belief Between High SES and Low SES

Test	Description	Test Statistic	p-value
$\psi_{s=1} = \psi_{s=2}$	Weight on Prior	0.763	0.44559

E Further Details of Estimation

E.1 Sample Selection

This analysis uses the sample of children born to women in the cross-section sample of the NLSY79. To enter the estimation, a child must have non-missing baseline (ages 9-10) values for all model initial conditions: the belief factor score, family income, parental assets and the child's skill factor score. Because the model rules out borrowing (assets must be weakly positive), I exclude observations with negative assets in any of the three periods (ages 9-10, 11-12, or 13-14). I also drop children from families reporting zero income at ages 9-10. Finally, I restrict attention to children whose mothers were married throughout ages 5-14 to focus on intact families.

Table E.1 summarises sample selection. The starting sample comprises all children of women in the NLSY79 cross-section. High SES families are defined as those with above median family income, where income is measured as average household income at ages 9-10; families below the median are classified as low SES. "Married age 5-14" indicates that the mother is observed married throughout ages 5-14. "Initial conditions available" requires non-missing beliefs, income, assets, and skills at ages 9-10. "Assets non-negative" imposes non-negative assets at ages 9-10, 11-12, and 13-14. "Family income positive age 9-10" requires strictly positive income at ages 9-10.

As Table E.1 shows, these restrictions reduce the sample, with larger attrition among low SES families than high SES families.

E.2 Method of Simulated Moments

This section provides additional details of the method of simulated moments estimation step.

Discretisation: The dynamic program has three continuous state variables: the mean posterior belief μ^{int}, assets a, and family income y. I discretise μ^{int} using a 4-point grid and discretise a and y using 7-point grids. The asset grid is denser at low asset levels, where policy functions are more likely to be nonlinear. Choice variables are discretised on a 100-point consumption grid and a 10-point investment grid.

Interpolation/Extrapolation: I use linear interpolation to evaluate the value function and policy functions at off-grid points and linear extrapolation when simulated states

Table E.1: Sample Selection for Model Estimation

	All	High SES	Low SES
Original Sample	5,819	3,682	2,137
Married Age 5-14	1,988	1,552	436
Initial Conditions Available	731	510	221
Assets Non-Negative	657	485	172
Family Income Positive Age 9-10	655	485	170

Notes: This table presents the observations remaining after each sample selection criterion is applied. Married age 5-14 means that the child's mother was married when the child was age 5-14. Initial conditions refer to the belief factor score at age 9-10, family income at age 9-10, assets at age 9-10 and skill factor score at age 9-10. High socio-economic status (SES) parents refer to parents who have above median family income (defined based on average family income when the child is aged between 9 and 10). Otherwise, parents are low SES.

fall outside the grid bounds. This is required both to compute expected continuation values during solution and to obtain optimal choices during simulation, since the simulated state variables generally do not lie exactly on grid points.

Integration: Expectations over stochastic shocks (the signal noise and the production shock) are computed using 5-node Gauss-Hermite quadrature.

Solving for Value Functions and Policy Functions by Backward Induction: I solve the model by backward induction. For each period t and each grid point (μ^{int}, a, y), I evaluate the objective function over feasible consumption and investment choices on the discretised choice grids. The feasible sets are endogenous: the upper bound on consumption is total resources, and the upper bound on investment is residual resources after the chosen consumption level. For each candidate (c, i), next period assets are pinned down by the budget constraint, and I compute the current payoff plus the discounted expected continuation value. The maximiser (c^*, i^*) yields the value function and the associated policy rules at that state. I start from the terminal period T and iterate backward to period 1.

Optimisation Algorithm: The minimisation algorithm used in the optimisation is Bound Optimisation by Quadratic Approximation (BOBYQA) by Powell (2009), implemented via NLOpt in Julia.

Programming Language: The program is written and implemented in Julia.

E.3 Auxiliary Model for Belief Parameters

To identify the belief updating weight ψ and the variance of the error term in the signal σ_ϵ^2, I use an auxiliary approximation to the updating rule in Equation 9. The key difficulty is that the prior mean μ_t is not observed. I therefore proxy μ_t with the predictable component of skill implied by the production function. For ease of exposition, I drop the child index j.

$$\begin{aligned}
\mu_t^{int} &= \psi\mu_t + (1-\psi)g_t \\
&\approx \psi\left[A_{t-1} + \gamma_{s,1}\ln\theta_{t-1} + \gamma_{s,2}\ln i_{t-1} + \gamma_{s,3}\ln\theta_{t-1}\times\ln i_{t-1}\right] + (1-\psi)\left(\ln\theta_t + \epsilon_t\right) \\
&= \psi A_{t-1} + \psi\gamma_{s,1}\ln\theta_{t-1} + \psi\gamma_{s,2}\Phi_{t-1} + \psi\gamma_{s,3}\Phi_{t-1}\ln\theta_{t-1} + (1-\psi)\ln\theta_t \\
&\quad + \psi\gamma_{s,2}\left(\Phi_{t-1} - \ln i_{t-1}\right) - \psi\gamma_{s,3}\left(\Phi_{t-1}\ln\theta_t - \ln\theta_t\times\ln i_{t-1}\right) + (1-\psi)\,\epsilon_t \quad (33)
\end{aligned}$$

Equation 33 motivates an auxiliary regression of the belief factor score μ_t^{int} on a constant, lagged skill, lagged investment, the lagged skill-investment interaction, and current skill. The coefficients in this regression are informative about ψ. In addition, the residual variance of this auxiliary regression is informative about σ_ϵ^2. Finally, the regression residual may exhibit serial correlation, but this does not pose a problem for identification in the method of simulated moments framework: the same persistence will be reflected in the corresponding simulated auxiliary moments when the model is correctly specified.

F Belief Updating by Socio-economic Status

The main analysis pools all children. To examine whether belief updating differs by SES, I re-estimate the updating regression separately for low SES and high SES families. Table F.1 reports the coefficients on the lagged beliefs and cognitive skill. The estimates are similar across the two subsamples: I conduct formal tests of coefficient equality across SES groups and do not reject equality for either the coefficient on the lagged belief factor score or the coefficient on the cognitive skill.

Table F.1: Dependent variable is Parent Belief Factor Score (Anchored)

	(1) High SES	(2) Low SES
Lag Belief Factor Score	0.184***	0.278***
	(0.038)	(0.053)
Cognitive Skill	0.925***	0.724***
	(0.081)	(0.101)
Observations	3477	2352
Number of Children	1941	1393
Child FE	Yes	Yes
Cognitive Skill Potentially Endogenous	Yes	Yes
Sargan Test P-value	0.057	0.003

Notes: This table presents the estimated coefficients in a regression of the belief factor score (anchored) on the lag belief factor score (anchored) and the cognitive skill factor score. The construction of the belief factor score and the skill factor score is described in the data section. The model is estimated with the Blundell and Bond (1998) dynamic panel data estimator. The regression includes time-varying controls, as described in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between ages 5 and 14 are used.

G Belief Updating: Extended Model

The validity of the Blundell and Bond (1998) estimator hinges on several conditions: (i) the error term is not serially correlated and (ii) the excluded instruments are exogenous. In the main specification, I reject the null hypothesis that the over-identifying restrictions are valid (Sargan test). This indicates that the model is misspecified and/or the instruments are invalid. One plausible source of failure is persistent unobserved shocks (e.g., to latent child skill), which would generate serial correlation in the structural error term and undermine the moment conditions.

$$\mu_{jt} = \alpha + \tilde{\beta}_1 \mu_{j,t-1} + \tilde{\beta}_2 \mu_{j,t-2} + \tilde{\beta}_3 cognitive_{jt} + \tilde{\gamma} X_{jt} + \delta_j + \epsilon_{jt} \quad (34)$$

In an attempt to address the misspecification, I estimate an augmented belief-updating specification that includes two lags of beliefs. Table G.1 reports the coefficients on the first lag of the belief factor score, the second lag, and the cognitive skill factor score. The sample size is smaller than in the baseline regression because the two-lag specification

requires beliefs to be observed in three consecutive periods.

The qualitative results are unchanged: both lagged beliefs and cognitive skill remain positive and statistically significant predictors of current beliefs, consistent with persistence and responsiveness to skill signals. However, the null of valid over-identifying restrictions is also rejected in this expanded specification, indicating that the additional lag does not resolve the underlying issue.

Table G.1: Dependent variable is Parent Belief Factor Score (Anchored)

	(1)
Lag Belief Factor Score	0.227***
	(0.057)
Second Lag Belief Factor Score	0.029
	(0.043)
Cognitive Skill	0.852***
	(0.164)
Observations	2833
Number of Children	1938
Child FE	Yes
Skills Potentially Endogenous	Yes
Sargan Test P-value	0.032

Notes: This table presents the estimated coefficients in a regression of the belief factor score (anchored) on the lag belief factor score (anchored), the second lag belief factor score (anchored) and the cognitive skill factor score. The construction of the belief factor score and the skill factor score is described in the data section. The model is estimated with the Blundell and Bond (1998) dynamic panel data estimator. The regression includes time-varying controls, as described in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between ages 5 and 14 are used.

H Correlation of Investments and Beliefs by Socioeconomic Status

In this section, I estimate the relationship between the investment factor score and the belief factor score separately for low SES and high SES families. Table H.1 reports the estimated coefficient on beliefs for each group. In both subsamples, the investment factor score is positively correlated with the belief factor score, indicating that higher parental beliefs are associated with higher investments for both low and high SES families. A

formal test fails to reject equality of the belief coefficients between low and high SES families.

Table H.1: Dependent variable is Investment Factor Score (Standardised)

	(1) High SES	(2) Low SES
Belief Factor Score	0.068***	0.051**
	(0.015)	(0.020)
Observations	5567	3990
Number of Children	2407	1729
Lag Investment	Yes	Yes
Sargan Test P-value	0.003	0.561

Notes: This table reports coefficients on the belief factor score (anchored) in a regression of the investment factor score (standardised) on the belief factor score (anchored). The Blundell and Bond (1998) dynamic panel data estimator is used. The construction of the belief factor score and the investment factor score is provided in the data section. The regression includes time-varying controls which are explained in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between age 5 and 14 are used. This sample is larger than the one used to estimate the belief updating model, because children tend to have investments reported over a greater number of time periods than they do for beliefs.

I Correlation of Investments and Beliefs: Extended Model

In the baseline specification, I reject the null hypothesis that the over-identifying restrictions are valid, suggesting that the model is misspecified and/or the instruments are invalid. To assess robustness, I consider extended specifications that add additional lags of the investment factor score as regressors, potentially absorbing persistence that could otherwise appear as serial correlation in the error.

Specifically, I estimate versions with two and three lags of the investment factor score. These specifications use smaller samples than the baseline because they require children to have investment measures observed in three (or more) consecutive periods. Table I.1 reports the estimated coefficients on the belief factor score. Across specifications, the investment factor score remains positively associated with the belief factor score. Moreover, in the models that include two or three investment lags, I do not reject the

null hypothesis that the over-identifying restrictions are valid.

$$\underbrace{I_{jt}}_{\text{investment}} = \alpha + \beta \underbrace{\mu_{jt}}_{\text{belief}} + \gamma X_{jt} + \psi_1 \underbrace{I_{j,t-1}}_{\text{lag investment}} + \psi_2 \underbrace{I_{j,t-2}}_{\text{second lag of investment}} + \psi_3 \underbrace{I_{j,t-3}}_{\text{third lag of investment}} + \underbrace{\delta_j}_{\text{child FE}} + \eta_{jt} \quad (35)$$

Table I.1: Dependent variable is Investment Factor Score (Standardised)

	(1)	(2)
Belief Factor Score	0.055***	0.053***
	(0.015)	(0.018)
Observations	7836	5828
Number of Children	3649	3110
2 Lags of Investment	Yes	No
3 Lags of Investment	No	Yes
Sargan Test P-value	0.202	0.456

Notes: This table reports coefficients on the belief factor score (anchored) in a regression of the investment factor score (standardised) on the belief factor score (anchored). Either two or three lags of the investment factor score are included as independent variables. The Blundell and Bond (1998) dynamic panel data estimator is used. The construction of the belief factor score and the investment factor score is provided in the data section. The regression includes time-varying controls which are explained in the empirical strategy. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). One-step GMM standard errors are used. All available observations of the child between age 5 and 14 are used. This sample is larger than the one used to estimate the belief updating model, because children tend to have investments reported over a greater number of time periods than they do for beliefs.

J Model Fit

Table J.1-J.9 report the moments targeted in the estimation. For each moment, the tables list (i) the empirical moment, (ii) the corresponding model-implied (simulated) moment, (iii) the standard error of the empirical moment, and (iv) the standardised difference between data and model (the moment discrepancy divided by its standard error). In total, the estimator matches 114 moments, spanning beliefs (Table J.1 and Table J.2), investment factor scores (Table J.3 and Table J.4), child skills (Table J.5 and Table J.6), family assets (Table J.7), correlations and covariances (Table J.8), and the ratio of next period assets to the investment factor score (Table J.9).

Table J.1: Belief Moments Part 1

Moment	Data	Model	SE Data	SE Diff
Mean belief age 11-12: Family income Q1	13.320	13.640	0.102	3.144
Mean belief age 11-12: Family income Q2	13.633	13.676	0.105	0.406
Mean belief age 11-12: Family income Q3	13.874	13.708	0.099	1.683
Mean belief age 11-12: Family income Q4	14.226	13.969	0.111	2.310
Mean belief age 13-14: Family income Q1	13.775	13.813	0.124	0.301
Mean belief age 13-14: Family income Q2	13.761	13.875	0.163	0.701
Mean belief age 13-14: Family income Q3	14.187	14.128	0.150	0.392
Mean belief age 13-14: Family income Q4	14.454	14.165	0.156	1.860
Correlation (low SES): Belief age 9-10, Belief age 11-12	0.695	0.656	0.053	0.731
Correlation (high SES): Belief age 9-10, Belief age 11-12	0.656	0.655	0.033	0.019
Correlation (low SES): Belief age 11-12, Belief age 13-14	0.693	0.674	0.058	0.319
Correlation (high SES): Belief age 11-12, Belief age 13-14	0.735	0.689	0.029	1.586

Notes: Belief refers to belief factor score. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.2: Belief Moments Part 2

Moment	Data	Model	SE Data	SE Diff
Regression of belief on lag investment, lag skill and current skill (low SES): Constant	2.054	2.094	1.722	0.023
Regression of belief on lag investment, lag skill and current skill (low SES): Coefficient on lag skill	0.136	0.294	0.156	1.018
Regression of belief on lag investment, lag skill and current skill (low SES): Coefficient on lag investment	-1.062	6.916	2.257	3.535
Regression of belief on lag investment, lag skill and current skill (low SES): Coefficient on lag investment x lag skill	0.111	-0.455	0.170	3.327
Regression of belief on lag investment, lag skill and current skill (low SES): Coefficient on current skill	0.695	0.548	0.170	0.866
Std residual of regression of belief on lag investment, lag skill and current skill (low SES)	1.018	1.041	0.054	0.426
Regression of belief on lag investment, lag skill and current skill (high SES): Constant	1.773	2.658	1.517	0.583
Regression of belief on lag investment, lag skill and current skill (high SES): Coefficient on lag skill	0.228	0.254	0.141	0.184
Regression of belief on lag investment, lag skill and current skill (high SES): Coefficient on lag investment	0.192	0.725	1.599	0.333
Regression of belief on lag investment, lag skill and current skill (high SES): Coefficient on lag investment x lag skill	0.025	-0.038	0.118	0.537
Regression of belief on lag investment, lag skill and current skill (high SES): Coefficient on current skill	0.638	0.556	0.110	0.744
Std residual of regression of belief on lag investment, lag skill and current skill (high SES)	1.014	0.986	0.037	0.756

Notes: Belief, skill and investment refer to belief factor score, skill factor score and investment factor score, respectively. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.3: Investment Factor Score Moments Part 1

Moment	Data	Model	SE Data	SE Diff
Std investment age 9-10	0.590	0.564	0.018	1.442
Std investment age 11-12	0.568	0.563	0.017	0.288
Std investment factor Age 13-14	0.635	0.417	0.021	10.565
Mean investment age 9-10: Family income Q1	0.250	0.137	0.052	2.191
Mean investment age 9-10: Family income Q2	0.464	0.231	0.048	4.869
Mean investment age 9-10: Family income Q3	0.543	0.404	0.043	3.265
Mean investment age 9-10: Family income Q4	0.666	0.915	0.047	5.314
Mean investment age 11-12: Family income Q1	0.272	0.063	0.050	4.205
Mean investment age 11-12: Family income Q2	0.480	0.222	0.047	5.470
Mean investment age 11-12: Family income Q3	0.486	0.405	0.048	1.719
Mean investment age 11-12: Family income Q4	0.686	0.759	0.044	1.637
Mean investment age 13-14: Family income Q1	0.230	-0.027	0.067	3.854
Mean investment age 13-14: Family income Q2	0.306	0.069	0.068	3.475
Mean investment age 13-14: Family income Q3	0.420	0.170	0.058	4.321
Mean investment age 13-14: Family income Q4	0.625	0.432	0.053	3.634

Notes: Investment refers to investment factor score. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.4: Investment Factor Score Moments Part 2

Moment	Data	Model	SE Data	SE Diff
Correlation (Skill Q1): investment age 9-10, investment age 11-12	0.593	0.903	0.057	5.404
Correlation (Skill Q2): investment age 9-10, investment age 11-12	0.560	0.928	0.062	5.956
Correlation (Skill Q3): investment age 9-10, investment age 11-12	0.620	0.915	0.060	4.879
Correlation (Skill Q4): investment age 9-10, investment age 11-12	0.593	0.927	0.071	4.693
Correlation (Skill Q1): investment age 11-12, investment age 13-14	0.644	0.836	0.051	3.747
Correlation (Skill Q2): investment age 11-12, investment age 13-14	0.612	0.908	0.055	5.403
Correlation (Skill Q3): investment age 11-12, investment age 13-14	0.664	0.841	0.060	2.941
Correlation (Skill Q4): investment age 11-12, investment age 13-14	0.609	0.926	0.059	5.331
Std residual of regression of investment on skill, income and asset age 9-10	0.547	0.362	0.016	11.837
Std residual of regression of investment on skill, income and asset age 11-12	0.514	0.413	0.025	4.137
Std residual of regression of investment on skill, income and asset age 13-14	0.593	0.317	0.027	10.151

Notes: Investment and skill refer to investment factor score and skill factor score respectively. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.5: Skill Moments Part 1

Moment	Data	Model	SE Data	SE Diff
Std skill age 11-12	0.736	0.829	0.025	3.697
Std skill age 13-14	0.789	0.946	0.031	4.997
Mean skill age 11-12: Family income Q1	13.474	13.671	0.070	2.827
Mean skill age 11-12: Family income Q2	13.685	13.707	0.066	0.343
Mean skill age 11-12: Family income Q3	13.763	13.736	0.054	0.501
Mean skill age 11-12: Family income Q4	14.040	13.867	0.062	2.768
Mean skill age 13-14: Family income Q1	13.816	13.880	0.073	0.884
Mean skill age 13-14: Family income Q2	13.944	13.932	0.081	0.146
Mean skill age 13-14: Family income Q3	14.104	14.083	0.073	0.286
Mean skill age 13-14: Family income Q4	14.318	14.186	0.066	2.021
Correlation (low SES): Skill age 9-10, Skill age 11-12	0.873	0.843	0.024	1.275
Correlation (high SES): Skill age 9-10, Skill age 11-12	0.799	0.882	0.031	2.633
Correlation (low SES): Skill age 11-12, Skill age 13-14	0.865	0.873	0.027	0.283
Correlation (high SES): Skill age 11-12, Skill age 13-14	0.823	0.899	0.029	2.604
Correlation (low SES): Skill age 9-10, Skill age 13-14	0.851	0.724	0.021	5.960
Correlation (high SES): Skill age 9-10, Skill age 13-14	0.754	0.772	0.030	0.629

Notes: Skill refers to skill factor score. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.6: Skill Moments Part 2

Moment	Data	Model	SE Data	SE Diff
Regression skill production function (low SES): Constant 1	2.033	0.885	0.530	2.165
Regression skill production function (low SES): Constant 2	1.870	0.733	0.545	2.084
Regression skill production function (low SES): Coefficient on skill	0.877	0.967	0.041	2.203
Regression skill production function (low SES): Coefficient on investment	-0.698	2.268	0.534	5.557
Regression skill production function (low SES): Coefficient on skill x investment	0.061	-0.120	0.041	4.432
Std residual skill production function (low SES)	0.365	0.434	0.029	2.390
Regression skill production function (high SES): Constant 1	2.621	0.744	0.755	2.485
Regression skill production function (high SES): Constant 2	2.554	0.579	0.771	2.562
Regression skill production function (high SES): Coefficient on skill	0.836	0.968	0.057	2.320
Regression skill production function (high SES): Coefficient on investment	0.923	0.506	0.608	0.685
Regression skill production function (high SES): Coefficient on skill x investment	-0.062	-0.018	0.046	0.973
Std residual skill production function (high SES)	0.428	0.363	0.029	2.240

Notes: Investment and skill refer to investment factor score and skill factor score respectively. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.7: Asset Moments

Moment	Data	Model	SE Data	SE Diff
Mean asset age 11-12	0.48	0.44	0.04	0.942
Std asset age 11-12	0.84	0.84	0.11	0.007
Mean asset age 13-14	0.78	0.29	0.06	8.266
Std asset age 13-14	1.22	0.55	0.12	5.355
Interquartile range of assets age 11-12	0.20	0.18	0.03	0.791
Interquartile range of assets age 13-14	0.39	0.11	0.02	13.621

Notes: Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.8: Correlations and Covariances

Moment	Data	Model	SE Data	SE Diff
Correlation (low SES): Belief age 11-12, Skill age 11-12	0.523	0.521	0.059	0.031
Correlation (high SES): Belief age 11-12, Skill age 11-12	0.519	0.524	0.041	0.118
Correlation (low SES): Belief age 13-14, Skill age 13-14	0.643	0.607	0.046	0.769
Correlation (high SES): Belief age 13-14, Skill age 13-14	0.543	0.614	0.049	1.433
Correlation (low SES): Belief age 9-10, Skill age 11-12	0.510	0.420	0.061	1.488
Correlation (high SES): Belief age 9-10, Skill age 11-12	0.500	0.420	0.039	2.052
Correlation (low SES): Belief age 11-12, Skill age 13-14	0.564	0.509	0.072	0.761
Correlation (high SES): Belief age 11-12, Skill age 13-14	0.488	0.509	0.053	0.399
Covariance: Investment age 9-10, Belief age 9-10	0.256	0.116	0.032	4.337
Covariance: Investment age 11-12, Belief age 11-12	0.231	0.161	0.033	2.113
Covariance: Investment age 13-14, Belief age 13-14	0.336	0.149	0.047	3.976
Covariance: Investment age 9-10, Skill age 9-10	0.110	0.079	0.020	1.599
Covariance: Investment age 11-12, Skill age 11-12	0.106	0.130	0.018	1.341
Covariance: Investment age 13-14, Skill age 13-14	0.126	0.134	0.026	0.303
Covariance: Investment age 9-10, Belief age 11-12	0.277	0.156	0.037	3.307
Covariance: Investment age 11-12, Belief age 13-14	0.293	0.235	0.046	1.272

Notes: Belief, skill and investment refer to belief factor score, skill factor score and investment factor score, respectively. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.

Table J.9: Ratio of Next Period Assets to Investment Factor Score

Moment	Data	Model	SE Data	SE Diff
Ratio of Assets Age 11-12 to Investment age 9-10: Skill Q1	-1.06	0.24	1.06	1.235
Ratio of Assets Age 11-12 to Investment age 9-10: Skill Q2	-1.92	0.24	2.02	1.067
Ratio of Assets Age 11-12 to Investment age 9-10: Skill Q3	-40.49	0.34	42.42	0.963
Ratio of Assets Age 11-12 to Investment age 9-10: Skill Q4	0.20	0.35	0.11	1.379
Ratio of Assets Age 13-14 to Investment age 11-12: Skill Q1	-0.28	-0.11	0.51	0.342
Ratio of Assets Age 13-14 to Investment age 11-12: Skill Q2	1.12	-0.08	1.24	0.972
Ratio of Assets Age 13-14 to Investment age 11-12: Skill Q3	-0.17	-0.08	0.45	0.214
Ratio of Assets Age 13-14 to Investment age 11-12: Skill Q4	0.47	0.47	0.15	0.003

Notes: Investment refers to investment factor score. Data refers to the data moment, Model refers to the moment from the simulated data, SE Data refers to the standard error of the data moment and SE Diff is the normalised (by standard error) difference between the data moment and the model moment.