Beyond the Actual: The Macroeconomic Implications of the Perceived Taylor Rule

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Abstract

This paper establishes that the *perceived* monetary policy rule governs macroeconomic dynamics beyond the actual rule. I first identify a structural "covariance bias" causing downward bias in conventional interest rate forecast-based estimates of perceived rule and propose using interest rate nowcasts to recover unbiased perceptions. Through a Subjective Perception Markov-Switching New Keynesian framework, I show that equilibrium determinacy depends not on whether the central bank satisfies the Taylor principle, but on whether agents *believe* it does. Counterfactual analysis demonstrates that anchored perceptions were essential for macroeconomic stability during the Great Moderation and the post-COVID inflation surge.

Keywords: Perceived Monetary Policy Rule, Expectation Channel; Indeterminacy, Temporary Equilibrium, Perceived Taylor Principle

JEL Classification: E31, E52, E58, D84

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1 Introduction

Central to modern macroeconomic models is the role of agents' expectations, where perceptions about future policy could shape equilibrium dynamics. Standard Rational Expectations models typically assume agents perfectly observe the central bank's reaction function, imposing a highly restrictive coincidence between the actual and perceived policy rules. Consequently, the specific macroeconomic implications of a divergence between the private sector's *perception* of the monetary policy rule and the central bank's *actual* rule remain largely unexplored.

This paper establishes that the *perceived monetary policy rule* is what ultimately matters for equilibrium outcomes, effectively superseding the actual rule in governing macroeconomic dynamics. I make three key contributions: one methodological and two theoretical. First, I show that standard estimates of the perceived rule based on interest rate forecasts are structurally downward-biased relative to agents' actual perceptions due to a *perceived covariance* between the expected policy rule and macroeconomic aggregates. To address this issue, I propose an identification strategy using interest rate nowcasts. Second, I develop a *Subjective Perception Markov-Switching New Keynesian (SP-MSNK)* model in which the perceived Taylor rule differs from the actual rule implemented by the central bank. The model establishes that the perceived rule dominates the actual rule in shaping equilibrium outcomes. Specifically, the perceived rule operates primarily through private sector expectations (*Expectation Channel*) and proves more powerful than the contemporaneous interest rate through which the actual rule gets implemented (*Direct Policy Channel*). Third, the model demonstrates that equilibrium determinacy depends not on whether the central bank satisfies the Taylor principle, but on whether agents *believe* it does, which is referred to as *perceived Taylor principle*.

The identification of a perceived policy rule presents a non-trivial econometric challenge. When agents understand both the general equilibrium feedback of interest rates and the possibility of a shift in the monetary policy rule (a.k.a., a regime shift), their expectations of future interest rates incorporate a covariance term between the time-varying policy rule and the targeted variable (e.g., inflation). I show analytically and numerically that under this setup, regressing interest rate *forecasts* on inflation forecasts yields a downward-biased estimate of the perceived policy stance.[1] Specifically, if agents anticipate a hawkish regime shift that stabilizes inflation, the negative covariance between the policy parameter and expected inflation creates a downward bias in OLS estimates. As a resolution, I demonstrate

[1] Unlike the standard simultaneity bias driven by monetary policy shocks, which can be addressed using instrumental variables, the covariance bias identified here is structural and endogenous. Consequently, standard instruments satisfying the exclusion restriction are unavailable. See Section 3 for a detailed discussion.

that this bias vanishes in the limit of the nowcast, allowing the real-time perceived policy rule to be recovered using Time-Varying Parameter (TVP) estimates. I therefore apply TVP estimation to survey nowcast data to recover the unbiased, real-time evolution of the perceived reaction function.[2]

Empirically, the estimated perceived rule reveals significant time variation and distinct divergences from the actual rule implemented by the Fed, as estimated from Greenbook/Tealbook forecasts (Greenbook or GB onward). Notably, during the mid-1990s, a period characterized by the Federal Reserve's "preemptive tightening" and risk management under Chairman Greenspan (Greenspan, 2004; Bernanke, 2022), the actual rule departed from the conventional Taylor rule, in which interest rate decisions respond to the mean forecast of inflation and output. Specifically, the Fed tightened interest rates despite stable Greenbook inflation expectations. In contrast, the perceived monetary policy response to inflation expectation stayed intact, as professional forecasters continued to believe in a strong and stable inflation rule. To understand the implications of these divergences and the role of the perceived rule, I embed the estimated perceived rule into a Subjective Perception Markov-Switching New Keynesian model (SP-MSNK). In this environment, the agent forms expectation using their perceived law of motion based on a perceived monetary policy that may differ from the actual law of motion generated by the central bank's true policy rule.

This framework yields a sharp condition for equilibrium determinacy: in the constant parameter limit without regime-switching, the economy possesses a unique stable equilibrium if and only if the private sector *perceives* the Fed as satisfying the Taylor principle. Strikingly, an actual central bank may violate the Taylor principle (act passively) without inducing indeterminacy, provided the private sector *believes* the monetary authority is active. This result fundamentally reorients the conditions for sunspot equilibria from the *actual* conduct of policy to the *perception* regarding the policy regime.

Welfare analysis within this framework highlights critical policy implications regarding perception management. I find that a "Dovish" perception, or weaker response to inflation and output gap, generates strictly higher volatility in inflation and the output gap, regardless of the actual policy stance. However, the persistence of this perception turns out to be key. Even if the central bank is perceived as Dovish, welfare losses in terms of macroeconomic volatility are mitigated if agents view this regime as transitory. This suggests that the macroeconomic effect of monetary policy crucially depends on altering the perceived transition probabilities of the policy regime, thereby anchoring expectations even during periods of actual policy accommodation.

[2]The TVP estimation also incorporates stochastic volatility to avoid misattributing variation driven by volatility changes to movements in the time-varying parameters, as noted by Sims (2001).

Finally, counterfactual simulations quantify the historical importance of these perceptions. I show that the Great Moderation (1985–1999) was underpinned by a strongly anchored Hawkish perception; had the public perceived the Fed as Dovish, both the level and volatility of inflation would have remained elevated despite the Fed's actual anti-inflationary stance. Conversely, during the post-COVID inflation surge, the model suggests that inflation would have peaked around 12.5%, significantly higher than the actual realized path of inflation around 9%, had the public's perception unanchored toward a Dovish regime. These findings suggest that the efficacy of monetary policy is crucially dependent on the public's perceptions about the monetary policy rule.

Related Literature This paper contributes to multiple strands of literature. First, it contributes to the study of general equilibrium consequences of monetary policy perceptions. While Bauer, Pflueger, and Sunderam (2024) document the divergence between financial market perceptions and historical policy rules, their analysis is primarily restricted to asset pricing implications in a partial equilibrium setting. In contrast, this paper evaluates the general equilibrium consequences for inflation and output dynamics. Similarly, while Schorfheide (2005) studies the implications of private sector learning, the analysis is limited to learning about the inflation target. I propose a more flexible framework where agents hold beliefs about the full reaction function—including output gap and inertia. Hamilton, Pruitt, and Borger (2011) and Bocola et al. (2024) estimate perceptions about the monetary policy rule but analyze implications through a model in which actual and perceived rules coincide. This work departs from assuming such coincidence and proposes a framework where agents' perceived rule differs from the actual rule.

Second, the paper advances the econometric identification of perceived monetary policy rules, clarifying a distinct source of bias from that of actual rule. A vast literature estimates the central bank's actual reaction function using historical realizations or Greenbook forecasts (Clarida, Gali, and Gertler, 2000; Orphanides, 2001; Boivin, 2005; C.-J. Kim and Nelson, 2006; Carvalho, Nechio, and Tristao, 2021). A different strand utilizes survey or asset-price data to infer market-implied rules (Hamilton, Pruitt, and Borger, 2011; Bundick, 2015; Bauer, Pflueger, and Sunderam, 2024; Jia, Shen, and Zheng, 2023; Bocola et al., 2024). However, these latter studies typically rely on interest rate forecasts under the implicit assumption of a constant rule or abstract from general equilibrium feedback of monetary policy into macroeconmic variables. I demonstrate that in an environment where agents internalize possible time-variation of policy rule and its feedback effect into the macroeconomy, standard forecast-based regressions are subject to a structural covariance bias.[3] I provide a solution

[3]I use the term "structural" to distinguish this from standard econometric bias (e.g., omitted variable),

using nowcast data that properly identifies the real-time perceived reaction function.

Third, this work revisits the sources of equilibrium determinacy and the Great Moderation. A major view attributes the stabilization of the post-Volcker era to a shift in the *actual* conduct of monetary policy within a Full Information Rational Expectations (FIRE) framework (Lubik and Schorfheide, 2004; Coibion and Gorodnichenko, 2011; Bianchi, 2013).[4] I reframe this narrative by showing that equilibrium determinacy is governed by the *perceived* policy rule. Consequently, the Great Moderation is reinterpreted not merely as a consequence of changes in policy actions, but also as a result of *"Good Perceived Policy"*.

Fourth, the paper extends the literature on regime-switching DSGE models. Previous work has extensively analyzed the implications of recurring regime shifts under full information rational expectations (FIRE) (Davig and Leeper, 2007; Bianchi, 2013; Barthélemy and Marx, 2017; Bianchi, Lettau, and Ludvigson, 2022). I contribute to this literature by relaxing the FIRE assumption and its strict coincidence between the actual and perceived Taylor rules and isolating the distinct channel through which perceived rule propagates.

Finally, this paper connects to recent works on the post-pandemic monetary policy and central bank credibility. Nakamura, Riblier, and Steinsson (2025) propose a *"minimalist Taylor principle"* in which the central bank responds in a shock-contingent manner while retaining freedom to look through supply shocks under anchored long-run expectations. Bocola et al. (2025) show that in an imperfect information environment, low-credibility central banks must respond more aggressively to inflationary shocks to prevent de-anchoring, while high-credibility central banks enjoy a *"reputation dividend"* permitting more flexible responses. This paper offers a complementary insight through a different angle: When the public's beliefs about the monetary policy rule are firmly anchored, the central bank's scope for deviating from the Taylor principle is expanded, as determinacy depends on the *"perceived Taylor principle"*.

Outline The paper proceeds as follows. Section 2 establishes the theoretical motivation. Section 3 analytically identifies the covariance bias in standard estimations and validates the nowcast-based solution via simulation. Section 4 reports the empirical time-varying estimates of the perceived rule and documents its divergence from the actual policy rule. Section 5 develops the SP-MSNK model to analyze general equilibrium implications, focusing on determinacy, welfare, and historical counterfactuals. Section 6 concludes.

which can typically be addressed using instrumental variables. Here, the bias is intrinsic to the equilibrium expectation formation process. Consequently, no valid instruments exist that satisfy the exclusion restriction. See the discussion in Appendix B.3.

[4]There are other papers such as Sims and Zha (2006) that have pushed forward an alternative view of "Good Luck" in this period rather than "Good Policy".

2 Motivating Framework: Perceived vs. Actual Rule

This section establishes the theoretical motivation of why perceived monetary rule is important. It presents a simple framework to build intuition for the role of perceived policy rules, the equilibrium concept, and the macroeconomic implications that carry over to the full-fledged model in Section 5. To showcase the distinctive importance of the perceived monetary policy rule beyond the actual rule, consider a simple model where private agents hold beliefs about the reaction function that may differ from the rule actually implemented by the central bank.

The economy is governed by a Fisher equation and an exogenous real rate process r_t, which are structurally common to both the perceived and actual models:

$$i_t = E_t^P [\pi_{t+1}] + r_t \tag{1}$$

$$r_t = \rho_r r_{t-1} + v_t \tag{2}$$

where $E_t^P[\cdot]$ denotes the subjective expectation of the private sector implied by their perceived model. The distinction arises in the monetary policy rule. The private sector perceives the rule as:

$$i_t = \phi_p \pi_t + \varepsilon_{Mt} \tag{3}$$

whereas the Fed actually implements:

$$i_t = \phi_f \pi_t + \varepsilon_{Mt} \tag{4}$$

Perceived Law of Motion (PLM) The private sector agent is internally rational to her perceived model. Assuming $\phi_p > 1$, the Perceived Law of Motion (PLM) implied by equations (1)–(3) yields:

$$i_{t|t} = \frac{\phi_p}{\phi_p - \rho_r} r_t \tag{5}$$

$$\pi_{t|t} = \frac{1}{\phi_p - \rho_r} r_t - \frac{1}{\phi_p} \varepsilon_{Mt} \tag{6}$$

The subscript $_{t|t}$ denotes the nowcast implied by the agent's subjective model. Although the agent perfectly observes the realized exogenous states (r_t, ε_{Mt}), the misspecification of the policy rule leads to nowcasts of inflation and interest rate that may deviate from the actual realization. Furthermore, the private sector forms inflation expectations $E_t^P [\pi_{t+1}]$ based on

these perceived dynamics:

$$E_t^P[\pi_{t+1}] \equiv \pi_{t+1|t} = \frac{\rho_r}{\phi_p - \rho_r} r_t \tag{7}$$

This equation demonstrates that inflation expectations are inversely related to the perceived policy aggressiveness ϕ_p.

Actual Law of Motion (ALM) Given the subjective expectation (7), combining the actual structural equations (1), (2), and (4) yields the Actual Law of Motion (ALM):

$$i_t = \frac{\phi_p}{\phi_p - \rho_r} r_t \tag{8}$$

$$\pi_t = \frac{\phi_p}{\phi_f} \underbrace{\left[\frac{1}{\phi_p - \rho_r} r_t - \frac{1}{\phi_p} \varepsilon_{Mt} \right]}_{\text{FIRE}} \tag{9}$$

In this simple model, the actual interest rate dynamics (8) coincide with the perceived dynamics (5) because the subjective expectation in the Fisher equation is determined exogenously by the perceived model parameters.[5] However, equation (9) reveals that when $\phi_p \neq \phi_f$, equilibrium inflation dynamics diverge from the agent's perceived law of motion. The term underbraced as FIRE indicates the solution under Full Information Rational Expectations, obtained when the perception is consistent with the actual rule ($\phi_p = \phi_f$). Crucially, equation (9) shows that the ratio ϕ_p/ϕ_f acts as a scaling factor on this benchmark; specifically, if the actual rule falls short of the private sector's perceived rule ($\phi_p > \phi_f$), the impact of structural shocks on inflation is amplified relative to the full-information outcome.

Several insights from this model merit discussion. First, the equilibrium concept employed is within the confines of Temporary Equilibrium (TE, J. M. Grandmont, 1977; J.-M. Grandmont, 1987; Woodford, 2013) where the equilibrium is defined on the basis of the expectations that decision makers happen to entertain at each point in time. It shares resemblence with the other branches of TE, such as the restricted perception equilibrium (Sargent, 1991; Branch, 2006) and the internal rationality (Adam and Marcet, 2011), where agents optimize and form expectations subject to a PLM that differs from the implied ALM.[6] This

[5] If there were other endogenous components, such as in a standard IS curve, the interest rate nowcast would also differ from the realization.

[6] RPE typically emerges from under-parameterization, hidden variables, or linear approximations to nonlinear dynamics. The baseline model shares a resemblance: Agents correctly understand the economy's structure but misperceive the parameter values. The key difference is that RPE agents eventually learn and converge to an optimal model within the class of misspecified models, whereas we assume agents' perceptions remain fixed throughout.

concept is extended in Section 5 to a general equilibrium environment featuring a regime-switching in the monetary policy rule.

Second, equilibrium determinacy depends on the Taylor principle of the *perceived* rule ($\phi_p > 1$), rather than the actual rule (ϕ_f). Since the model relies on subjective expectations, a unique solution must exist for the subjective model. Thus, the relevant eigenvalues for the Blanchard and Kahn (1980) conditions are dictated by the PLM parameters. Consequently, a unique equilibrium exists even if the actual policy rule is passive ($\phi_f < 1$), provided the perceived Taylor principle is satisfied. In Section 5.2, I demonstrate that this result carries on even if the central bank operates a forward-looking Taylor rule with rational expectations.

Third, both ϕ_p and ϕ_f affect the transmission mechanism of structural shocks but through two distinct channels. The **Expectation Channel**, governed by the perceived parameter ϕ_p, determines the private sector's forward-looking inflation expectations ($E_t^P[\pi_{t+1}]$) and hence shape inflation dynamics. The **Direct Policy Channel**, governed by the actual parameter ϕ_f, affects inflation contemporaneously through the realized interest rate. As equation (9) illustrates, the interaction of these channels creates a structural wedge: if the private sector perceives the central bank to be more hawkish than it is in reality ($\phi_p > \phi_f$), the sensitivity of inflation to structural shocks is amplified relative to the FIRE benchmark. This underscores that the interaction between the contemporaneous policy stance and private sector perceptions about the rule jointly shape macroeconomic dynamics.

3 Identification of the Perceived Rule

This section investigates the econometric identification of the perceived monetary policy rule. Subsection 3.1 analytically demonstrates that standard forecast-based estimates are structurally biased in the presence of regime uncertainty, driven by a conditional covariance between policy parameters and inflation. Subsection 3.2 illustrates this mechanism through a Markov-Switching New Keynesian (MSNK) framework. Subsection 3.3 quantifies the magnitude of this bias through Monte Carlo simulations and Subsection 3.4 establishes that utilizing nowcasts is necessary to recover the true perceived reaction function.

3.1 Identification Challenge

For expositional clarity, the motivating framework in Section 2 assumed a constant monetary policy rule. However, a vast empirical literature documents substantial time variation in the monetary policy reaction function (C.-J. Kim and Nelson, 2006; Boivin, 2005; Coibion and Gorodnichenko, 2011), or discrete regime switching (Sims and Zha, 2006; Bianchi, 2013;

Bianchi, Lettau, and Ludvigson, 2022). Given this evidence, forward-looking economic agents may internalize this variation when forming expectations. Consequently, an identification strategy must account for an environment where private agents explicitly incorporate the probability of future policy shifts into their decision-making.

Specifically, this paper identifies the *perceived* policy rule in a setting where agents are aware of two fundamental features of the economy. First, agents understand the *general equilibrium* feedback from monetary policy actions to macroeconomic aggregates such as inflation and output. Second, agents recognize that the policy rule itself is subject to *change over time*.

Acknowledging this *joint awareness* is critical because existing literature implicitly imposes one while abstracting from the other to use interest rate *forecasts* for identification of the perceived rule. Some studies allow for perceived time variation in the policy rule but assume a partial equilibrium setting where interest rates do not affect macroeconomic outcomes (Bauer, Pflueger, and Sunderam, 2024).[7] Conversely, other studies embed monetary policy within a full general equilibrium framework but impose a constant policy rule for identification purposes (Hamilton, Pruitt, and Borger, 2011; Bocola et al., 2024).

This section relaxes both restrictions simultaneously. I show that when agents internalize *both* general equilibrium feedback and the possibility of regime changes, interest rate expectations ($h > 0$) embed a *perceived covariance* between future policy coefficients and macroeconomic variables. As a result, standard identification strategies relying on survey or asset-price-based interest rate *forecasts* are generally contaminated by this covariance term, yielding biased estimates. Furthermore, I establish that utilizing nowcasts ($h = 0$) enables consistent identification of the real-time perceived reaction function within a Time-Varying Parameter (TVP) framework, as the confounding covariance term vanishes in this limit.

To isolate the mechanism of this bias, I simplify the setup by assuming that the perceived and actual rules coincide. That is, the agent's expectation is fully rational ($E_t^P = \mathbb{E}_t$). This assumption is without loss of generality. The covariance bias is a general property of expectations in regime-switching environments, independent of any misalignment between perceived and actual policies.[8] Put differently, this section warns against the practice of

[7] Bauer, Pflueger, and Sunderam (2024) acknowledge this limitation and discuss the implications for econometric bias in their Appendix. However, the specific covariance bias analyzed here is not fully addressed in their framework.

[8] To be precise, the derivation requires only *internal rationality* (Adam and Marcet, 2011), not necessarily Full Information Rational Expectations (FIRE). Internal rationality implies that agents form expectations that are mathematically consistent with their own subjective model, which features interest rate feedback and the possibility of regime shifts. Whether these beliefs coincide with the objective data-generating process is immaterial for the derivation. The bias arises as long as agents internalize the joint distribution of policy parameters and equilibrium outcomes within their perceived model.

inferring perceived policy rules from interest rate *expectations*, whether or not those expectations are rational. The analysis of bias discussed in this section holds for the full-fledged model with misalinged perceived rule in Section 5.

Consider a simple contemporaneous Taylor rule with time-varying parameters:

$$i_t = \phi_{\pi,t}\pi_t + \varepsilon_{M,t} \tag{10}$$

The object of interest is the agent's perception of the policy coefficient $\phi_{\pi,t}$, defined at horizon h as $\mathbb{E}_t[\phi_{\pi,t+h}]$. At the nowcast horizon ($h = 0$), this expectation represents the operative parameter $\phi_{\pi,t|t}$ that governs the agent's current decisions. While I proceed under the simplifying assumption of full observability ($\phi_{\pi,t|t} = \phi_{\pi,t}$), the distinction between perfect and imperfect knowledge at $h = 0$ is immaterial for the derivation and identification. the covariance bias arises strictly from the internalization of future regime shifts ($h > 0$), whereas the nowcast simply identifies the rule currently in effect in the mind of the agent.

Assuming $\mathbb{E}_t[\varepsilon_{i,t+h}] = 0$, taking the conditional expectation of equation (10) at time t for horizon $t + h$ and decomposing the expectation of the product yields:

$$\mathbb{E}_t[i_{t+h}] = \mathbb{E}_t[\phi_{\pi,t+h}]\mathbb{E}_t[\pi_{t+h}] + \mathrm{Cov}_t\left(\phi_{\pi,t+h}, \pi_{t+h}\right). \tag{11}$$

Empirically, the perceived reaction function is typically estimated by projecting expected rates onto expected inflation using OLS or IV (Bauer, Pflueger, and Sunderam, 2024; Jia, Shen, and Zheng, 2023; Bocola et al., 2024). To further show the associated bias, it is useful to define the time-varying coefficient, $\beta_{t,h}$, that describes the relationship between expected rates and inflation:

$$\mathbb{E}_t[i_{t+h}] = \beta_{t,h}\mathbb{E}_t[\pi_{t+h}], \text{ where } \beta_{t,h} = \mathbb{E}_t[\phi_{\pi,t+h}] + \frac{\mathrm{Cov}_t\left(\phi_{\pi,t+h}, \pi_{t+h}\right)}{\mathbb{E}_t[\pi_{t+h}]} \tag{12}$$

Equation (12) demonstrates that $\beta_{t,h}$ provides a biased measure of the true perceived policy aggressiveness $\mathbb{E}_t[\phi_{\pi,t+h}]$. The wedge is driven by the conditional covariance between the policy parameter and inflation, scaled by the level of expected inflation. Thus, the direction of the bias depends on the sign of this covariance relative to expected inflation. For instance, if an agent with a positive inflation expectation anticipates that a shift to a hawkish regime will stabilize inflation (implying a negative covariance between $\phi_{\pi,t+h}$ and π_{t+h}), the simple ratio $\beta_{t,h}$ will understate the agent's true perception of the policy parameter at $h > 0$.

Under this setup, when the perceived rule is estimated using standard regression techniques, this perceived covariance translates into a specific bias in the OLS estimator. In the analysis that follows, the notation $x_{t+h|t} \equiv \mathbb{E}_t[x_{t+h}]$ is used interchangeably to denote the

conditional expectation.

Proposition 1. *Assume the data generating process for the forecasted interest rate is given by* $i_{t+h|t} = \beta_{t,h}\pi_{t+h|t}$. *If* $\mathbb{E}[\pi_{t+h|t}] = 0$, *the OLS estimator* β_{OLS} *decomposes as:*

$$\beta_{OLS,h} = \mathbb{E}[\beta_{t,h}] + \frac{\mathbb{E}\left[\left(\beta_{t,h} - \mathbb{E}\left[\beta_{t,h}\right]\right)\pi^2_{t+h|t}\right]}{\text{Var}\left(\pi_{t+h|t}\right)} \tag{13}$$

Proof. See Appendix A.

The first term in the decomposition represents the unconditional mean of the time-varying coefficient $\beta_{t,h}$. The second term reflects variance-adjusted weighting: because OLS seeks to minimize the sum of squared errors, the estimator effectively assigns greater weight to $\beta_{t,h}$ associated with periods of high inflation variance. Numerically, however, this variance-weighting effect proves to be negligible, and for intuition, suppose that $\beta_{OLS,h} \approx \mathbb{E}[\beta_{t,h}]$. Consequently, substituting the definition of $\beta_{t,h}$ from (12), the estimated coefficient approximates:

$$\beta_{OLS,h} \approx \mathbb{E}\left[\phi_{\pi,t+h}\right] + \mathbb{E}\left\{\frac{\text{Cov}_t\left[\phi_{\pi,t+h}, \pi_{t+h}\right]}{\mathbb{E}_t[\pi_{t+h}]}\right\} \tag{14}$$

This decomposition highlights again the structural drivers of the estimation bias: it is the perceived covariance between the policy parameter and inflation scaled by inflation expectation.

To explicitly characterize this covariance, I consider a general setup where the monetary policy rule follows a discrete regime-switching process ($\phi_{\pi,t} \equiv \phi_{\pi,s_t}$) with $s_t \in \{1, \ldots, S\}$. Inflation evolves according to a state-dependent linear system $\pi_t = \mathbf{C}_{\pi,s_t}\mathbf{X}_t$, with state variables following $\mathbf{X}_t = \mathbf{A}_{s_t}\mathbf{X}_{t-1} + \mathbf{B}_{s_t}\boldsymbol{\varepsilon}_t$. Restricting the analysis to a two-state Markov switching process for tractability yields the following proposition. Appendix B.2 shows that the bias discussed here also extends to endogenous regime switching, where transition probabilities depend on the inflation level.

Proposition 2. *Consider a two-state Markov switching process* $s_t \in \{1, 2\}$ *governing the policy parameter* ϕ_{π,s_t} *and inflation dynamics. The* h*-step ahead conditional covariance between the policy parameter and inflation, given the current state* $s_t = j$, *is:*

$$\text{Cov}\left(\phi_{\pi,s_{t+h}}, \pi_{t+h} \mid s_t = j\right) = P^h_{j1}\left(1 - P^h_{j1}\right)\left(\phi_{\pi,1} - \phi_{\pi,2}\right)\left(\pi^e_{1,t+h} - \pi^e_{2,t+h}\right) \tag{15}$$

where P^h_{jk} *is the* h*-step transition probability from state* j *to* k, *and* $\pi^e_{k,t+h} \equiv \mathbb{E}[\pi_{t+h} \mid s_t = j, s_{t+h} = k]$ *denotes the expected inflation conditional on realizing state* k *at time* $t + h$.

Proof. See Appendix A.

Proposition 2 yields three key insights regarding the sources of identification bias. First, under a constant policy rule ($\phi_{\pi,1} = \phi_{\pi,2}$), the covariance term vanishes, allowing interest rate forecasts to consistently identify the perceived reaction function. Second, even with regime-switching parameters ($\phi_{\pi,1} \neq \phi_{\pi,2}$), the bias disappears if agents perceive the future regime as deterministic (i.e., $P_{j1} \in \{0, 1\}$). For instance, if agents view the current regime as permanent ($P_{jj} = 1$), subsample regressions using forecasts of regime-specific subsamples can successfully recover the perceived reaction function. Third, the bias is eliminated in the absence of general equilibrium feedback. If the interest rate does not feed back into the inflation process, expected inflation becomes invariant to the monetary regime ($\pi^e_{1,t+h} = \pi^e_{2,t+h}$), thereby removing the covariance. In sum, the perceived covariance bias arises only when agents simultaneously internalize i) general equilibrium feedback, *and* ii) the possibility of future regime shifts.

3.2 Illustration of Bias: A Markov-Switching New Keynesian Model

To further characterize and quantitatively illustrate the bias identified in Subsection 3.1, I map the decomposition into a standard three-equation Markov-Switching New Keynesian (MSNK) model (Farmer, Waggoner, and Zha, 2011).[9] The linearized structural equations are given by:

$$x_t = \mathbb{E}_t [x_{t+1}] - \tau (i_t - \mathbb{E}_t [\pi_{t+1}]) + z_{Dt} \tag{16}$$

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa x_t + z_{St} \tag{17}$$

$$i_t = \phi_{\pi,s_t} \pi_t + z_{Mt} \tag{18}$$

where x_t denotes the output gap, π_t inflation, and i_t the nominal interest rate. The parameters β, κ, and τ represent the discount factor, the Phillips curve slope, and the intertemporal elasticity of substitution, respectively. The economy is driven by demand (z_{Dt}), supply (z_{St}), and monetary policy (z_{Mt}) shocks, which follow independent AR(1) processes.

The distinguishing feature of this framework is the regime-dependent policy coefficient ϕ_{π,s_t}, which evolves according to a discrete Markov chain. This structure allows for an explicit characterization of the conditional covariance between the future policy stance ($\mathbb{E}_t[\phi_{\pi,t+h}]$) and inflation ($\mathbb{E}_t[\pi_{t+h}]$).

Let $\mathbf{z}_t = [z_{Dt}, z_{St}, z_{Mt}]'$ be the vector of exogenous shocks with persistence diagonal matrix $\mathbf{R}$. The general result from Proposition 2 maps to the MSNK model as follows.

[9] This model is a nested special case of the full SP-MSNK model developed in Section 5.

Corollary 1. *In the MSNK model defined by equations (16)-(18), the conditional covariance given current state $s_t = j$ is:*

$$\text{Cov}\left(\phi_{\pi,s_{t+h}}, \pi_{t+h} \mid s_t = j\right) = P_{j1}^h \left(1 - P_{j1}^h\right) (\phi_{\pi,1} - \phi_{\pi,2}) (\mathbf{a}_1 - \mathbf{a}_2)^T \mathbf{R}^h \mathbf{z}_t \tag{19}$$

where $\mathbf{a}_k$ is the vector of inflation loadings on $\mathbf{z}_t$ conditional on the future regime being k. The term $(\mathbf{a}_1 - \mathbf{a}_2)^T \mathbf{R}^h \mathbf{z}_t$ represents the expected inflation differential between regimes, defined as $\mathbb{E}_t[\pi_{t+h} \mid s_{t+h} = 1] - \mathbb{E}_t[\pi_{t+h} \mid s_{t+h} = 2]$.

Proof. See Appendix A.

To establish the sign of the bias analytically, I simplify the transition probabilities to the limiting i.i.d. case. Furthermore, to clearly distinguish the bias arising from the covariance and to stay consistent with the full model in Section 5 where agents perceive the rule as purely systematic, I set the perceived monetary policy shock to zero ($z_{Mt} = 0$) for the following derivation and subsequent simulations. This will be discussed in more detail at the end of Section 3.

Corollary 2. *Assume the regime switching process is i.i.d. and let $\phi_{\pi,1} > \phi_{\pi,2}$ (Regime 1 is Hawkish). Under standard parameter restrictions ($\kappa, \tau > 0$) and the absence of monetary policy shocks ($z_{Mt} = 0$), the difference in inflation loadings for demand and supply shocks is strictly negative:*

$$a_{1,D} - a_{2,D} = \frac{-\kappa^2 \tau (\phi_{\pi,1} - \phi_{\pi,2})}{\vartheta_D} < 0 \tag{20}$$

$$a_{1,S} - a_{2,S} = \frac{-(1 - \rho_S) \kappa \tau (\phi_{\pi,1} - \phi_{\pi,2})}{\vartheta_S} < 0 \tag{21}$$

where ϑ_D and ϑ_S are positive scalars defined in Appendix A. Consequently, the conditional bias term is strictly negative:

$$\frac{\text{Cov}_t\left[\phi_{\pi,t+h}, \pi_{t+h}\right]}{\mathbb{E}_t[\pi_{t+h}]} < 0 \tag{22}$$

Proof. See Appendix A.

To build intuition for this result, consider the decomposition of the covariance term in the presence of supply shocks only:

$$\text{Cov}\left(\phi_{\pi,s_{t+h}}, \pi_{t+h} \mid s_t = j\right) = \underbrace{p(1-p)(\phi_{\pi,1} - \phi_{\pi,2})(a_{1S} - a_{2S}) \rho_S^h}_{(-)} z_{St} \tag{23}$$

Since $a_{1S} - a_{2S} < 0$, the underbraced coefficient is negative. Thus, the sign of the covariance

Table 1: Calibrated Parameters of MSNK model

Parameters	τ	κ	β	ϕ_π	ϕ_x	ρ	ρ_D	ρ_S	σ_D	σ_S
First regime	0.61	0.67	0.99	**2.19**	0.30	0.84	0.76	0.84	0.23	0.62
Second regime	0.61	0.67	0.99	**0.77**	0.17	0.60	0.74	0.84	0.23	0.62

Note: Parameter values are the original ones from Lubik and Schorfheide (2004) and Farmer, Waggoner, and Zha (2011).

is strictly opposite to the sign of the shock z_{St}.

The intuition for the downward bias follows directly. Consider a positive supply shock ($z_{St} > 0$), which generates positive expected inflation ($\mathbb{E}_t[\pi_{t+h}] > 0$). Agents understand that if the Hawkish regime ($s_{t+h} = 1$) realizes, the central bank will respond aggressively, resulting in lower inflation compared to the Dovish regime ($s_{t+h} = 2$). This implies a negative correlation between the realization of the policy parameter and the realization of inflation: states with higher ϕ_π yield lower π. Thus, the conditional covariance is negative, rendering the bias term negative. Conversely, a negative supply shock generates negative expected inflation but a positive covariance (as Hawkish policy mitigates the deflation), preserving the negative sign of the ratio. This structural covariance confirms that standard regression estimates will systematically understate the public's perceived responsiveness to inflation.

3.3 Numerical Illustration of Bias

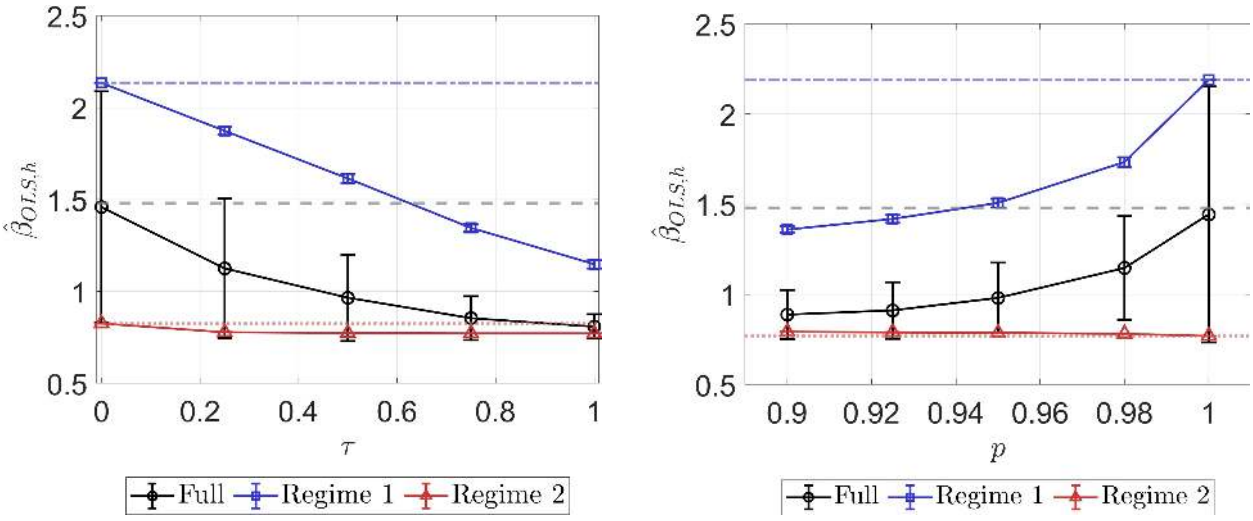
To quantify the theoretical bias derived above, I perform Monte Carlo simulations using the calibrated MSNK model. The structural parameters follow the calibration in Farmer, Waggoner, and Zha (2011) and Lubik and Schorfheide (2004), as summarized in Table 1. The transition probability matrix $\mathbf{P}_{\text{ms}}$ governing the regime switching process is:

$$\mathbf{P}_{\text{ms}} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix} \tag{24}$$

where p denotes the persistence of the current regime.

The model is solved and simulated using the method of Farmer, Waggoner, and Zha (2011). I then apply standard OLS estimation to the simulated h-period ahead forecasts. It is worth noting that instrumental variable (IV) methods do not resolve this bias as discussed in Appendix B.3.[10]

[10]This distinction is crucial: in conventional Taylor rule estimation, the error term is an exogenous monetary policy shock, which can be instrumented using lagged variables or other structural shocks. Here, the "error" is the structural covariance term derived in Proposition 1. Since this covariance is an endogenous function of the state variables, no valid instruments exist within the system to satisfy the exclusion



(a) Sensitivity to Interest Rate Feedback (τ)

(b) Sensitivity to Regime Persistence (p)

Figure 1: OLS Bias in the Perceived Taylor Rule Estimation from the MSNK Model (Univariate Rule)

Note: This figure reports the mean OLS coefficients and one standard deviation bands for equation (25) at $h = 2$. The left panel varies τ (fixing $p = 0.98$); the right panel varies p (fixing $\tau = 0.61$). The solid black line represents the OLS estimate on the full sample, while the grey dashed line represents the true unconditional expected rule coefficient. Blue and red lines represent OLS estimates on regime-specific subsamples.

Simple, Univariate Rule I begin with the simplest specification: a univariate Taylor rule without inertia or output gap response. The estimating equation is:

$$\mathbb{E}_t[i_{t+h}] = \beta_{\text{OLS},h}\mathbb{E}_t[\pi_{t+h}] + e_{t,h} \tag{25}$$

where $\beta_{\text{OLS},h}$ is the coefficient obtained by regressing h-period-ahead interest rate expectations on expected inflation. To conserve space, I report results for $h = 2$. These estimates are compared to the true conditional expectation of the policy coefficient, $\mathbb{E}_t[\phi_{\pi,s_{t+h}}]$.

Figure 1 summarizes the simulation results. The horizontal lines in both panels (a) and (b) establish the theoretical benchmarks: the top and bottom lines correspond to the regime-specific coefficients $\phi_{\pi,1}$ and $\phi_{\pi,2}$, while the middle line represents the unconditional ergodic mean $\mathbb{E}[\phi_{\pi,s_t}]$. Given the high persistence in the baseline calibration ($p \approx 1$), the regime-specific parameters closely approximate the true conditional expectations $\mathbb{E}_t[\phi_{\pi,s_{t+h}}]$. The solid black line plots the mean OLS coefficient estimated on the full sample, while the blue and red lines display the coefficients obtained from regime-specific subsample regressions.In other words, the blue (red) line is the OLS coefficients estimated on a sample of regime 1 (2) only.

restriction.

The left panel isolates the role of general equilibrium feedback (τ). Consistent with theoretical predictions, the OLS estimator is unbiased when monetary policy decouples from the real economy ($\tau = 0$), as the covariance term vanishes. However, as the strength of the feedback mechanism (τ) increases, the estimator deviates downward from the true parameter (and the conditional expectation, $\mathbb{E}_t[\phi_{\pi,s_{t+h}}]$), confirming that the bias is driven by the agent's awareness of the endogenous response of inflation to monetary policy.

The right panel illustrates the role of regime uncertainty (p). Under perfect persistence ($p = 1$), beliefs regarding the future regime become deterministic, thereby eliminating the covariance term and yielding unbiased estimates. However, as persistence declines, the downward bias intensifies.

Crucially, the results across both panels demonstrate that the regime-specific estimates (blue and red lines) remain biased even when the econometrician conditions on the true regime. This finding underscores that the bias is not an artifact of mechanically mixing data across regimes, nor is an actual regime switch a precondition for the bias. Instead, it arises because agents structurally internalize the regime uncertainty and the general equilibrium feedback of monetary policy, embedding a non-zero covariance between expected inflation and future policy parameters into their interest rate forecasts.

Multivariate, Inertial Rule This bias extends to the generalized inertial Taylor rule:

$$i_t = \Big(1 - \rho_{s_t}\Big)\Big(\phi_{\pi,s_t}\pi_t + \phi_{x,s_t}x_t\Big) + \rho_{s_t}i_{t-1}. \tag{26}$$

Replacing the simple rule (18) with the inertial rule, the corresponding estimation equation using simulated data is:

$$\mathbb{E}_t[i_{t+h}] = \beta_{\pi,h}\mathbb{E}_t[\pi_{t+h}] + \beta_{x,h}\mathbb{E}_t[x_{t+h}] + \beta_{i,h}\mathbb{E}_t[i_{t+h-1}] + e_{t,h} \tag{27}$$

Figure 2 displays the estimated coefficients across forecast horizons h as a function of the transition probability p. The green dashed line represents the benchmark of perfect persistence ($p = 1$). In the absence of regime uncertainty, the estimates are invariant to the horizon and uniformly recover the unconditional expectations (e.g., $\mathbb{E}[(1 - \rho_{s_t})\phi_{\pi,s_t}]$). Conversely, the introduction of regime uncertainty ($p < 1$) induces a significant downward bias in both the inflation and output gap coefficients. Notably, this bias generates a downward term structure in the estimated coefficients as the horizon h extends, qualitatively consistent with the empirical pattern from the actual survey data as documented in Appendix D.1.

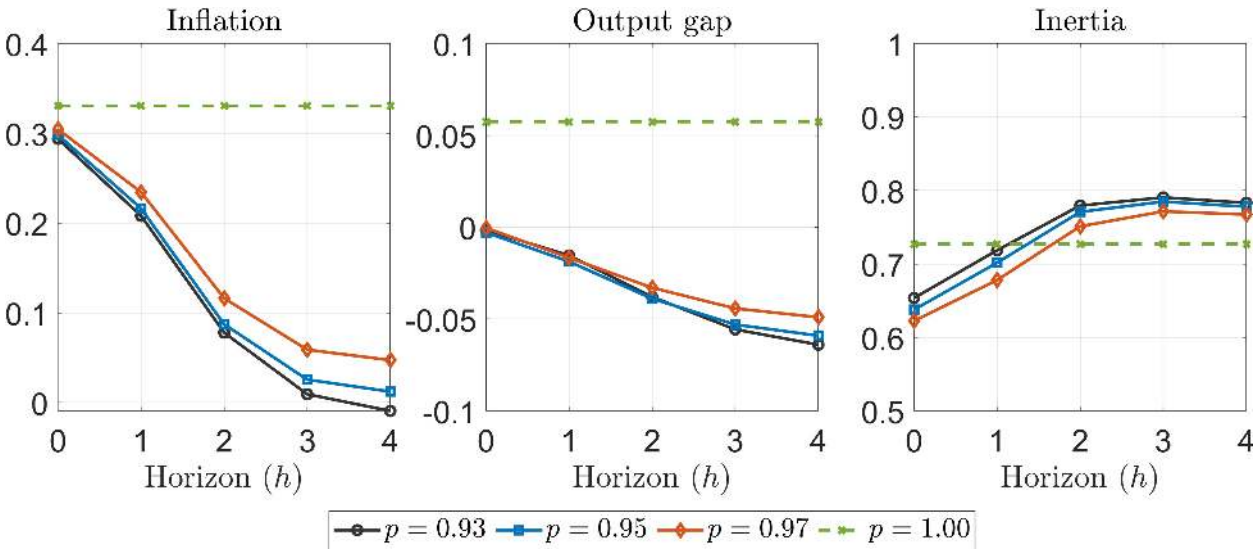


Figure 2: OLS Bias in the MSNK Model (Multivariate Case)

Note: This figure reports the mean OLS coefficients and one standard deviation bands for equation (27) at $h = 2$. The panels show the variation of the OLS coefficients for inflation (left), output gap (middle), and inertia (right) as a function of transition probability (p).

3.4 Recovering the Perceived Rule using Interest Rate Nowcasts

Finally, I demonstrate that applying TVP estimation to the nowcast ($h = 0$) enables the consistent recovery of the true perceived reaction function (e.g., ϕ_{π,s_t}). The estimation equation extends the constant-parameter specification (27) to allow for time-variation:

$$\mathbb{E}_t[i_{t+h}] = \beta_{\pi,t,h}\mathbb{E}_t[\pi_{t+h}] + \beta_{x,t,h}\mathbb{E}_t[x_{t+h}] + \beta_{i,t,h}\mathbb{E}_t[i_{t+h-1}] + e_{t,h}. \tag{28}$$

From these reduced-form time-varying coefficients, the implied long-run perceived responses are defined as:

$$\phi_{\pi,t,h} := \frac{\beta_{\pi,t,h}}{1-\beta_{i,t,h}}, \tag{29}$$

$$\phi_{x,t,h} := \frac{\beta_{x,t,h}}{1-\beta_{i,t,h}} \tag{30}$$

The estimation is performed on simulated data generated from a deterministic regime path: 100 periods of Regime 2 (Dovish) followed by 100 periods of Regime 1 (Hawkish).[11] The parameters are estimated using the method of Boivin (2005), following the convention in the time-varying Taylor rule literature (Coibion and Gorodnichenko, 2011).

[11] This regime frequency is consistent with the ergodic distribution of the Markov process $\mathbf{P}_{ms}$.

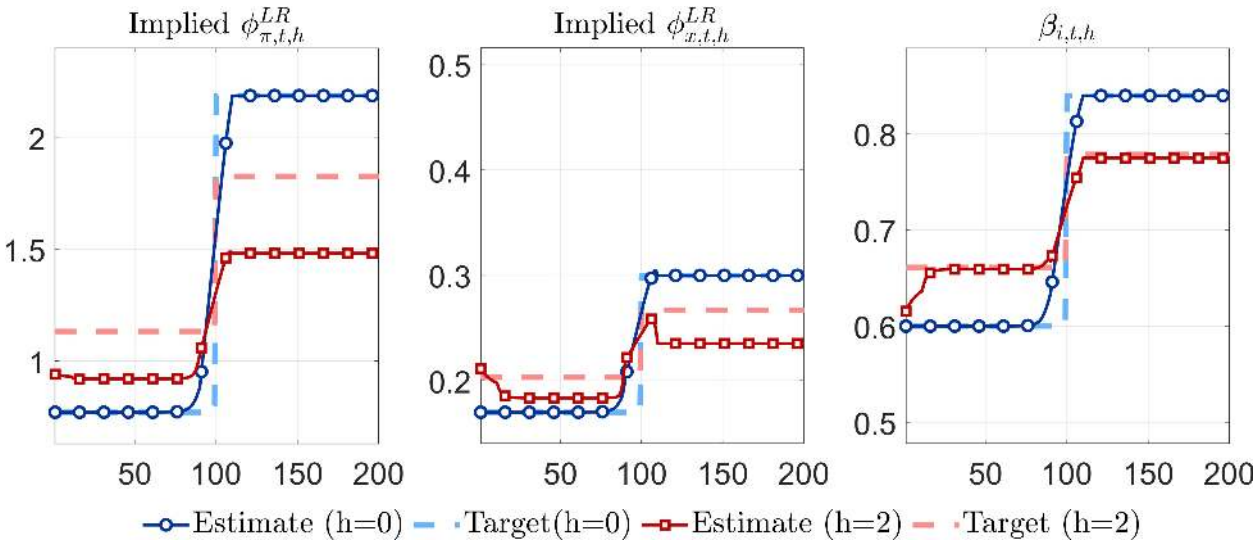


Figure 3: TVP Estimation with Regime Switching ($p = 0.85$)

Note: This figure compares the estimation results of equation (28) for $h = 0$ (blue) and $h = 2$ (red) under $p = 0.85$. The dashed lines indicate the target conditional expectations (e.g., $\mathbb{E}_t[\phi_{\pi,s_{t+h}}]$), generated based on the fixed regime path (100 periods of $s_t = 2$, followed by 100 periods of $s_t = 1$). The solid lines with markers indicate the estimated long-run coefficients.

Figure 3 presents the results for the baseline regime-switching economy ($p = 0.85$). The dashed lines represent the target conditional expectations (e.g., $\mathbb{E}_t[\phi_{\pi,s_{t+h}}]$), while the solid lines plot the estimated long-run coefficients. The forecast-based estimates ($h = 2$, red solid line) remain structurally biased relative to the true conditional expectation (red dashed line). In contrast, the nowcast-based estimates ($h = 0$, blue solid line) correctly track the true long-run parameters $\phi_{\pi,t}$. This result implies that while survey forecasts ($h > 0$) are contaminated by the perceived covariance bias, the survey nowcast ($h = 0$) provides a clean identification of the private sector's real-time perceived policy rule. In the Appendix, Figure B.1 demonstrates that the covariance bias vanishes under regime certainty, where interest rate forecasts successfully recover the perceived rule without covariance bias, analogous to the OLS case.

Several points merit discussion. First, although time-varying parameter (TVP) estimation applied to nowcasts corrects for this covariance bias, constant-parameter methods such as OLS or IV remain biased even when nowcasts are used. When the econometrician's sample spans multiple regimes, regressors become correlated with regime-specific policy rules. As a result, the OLS estimator places disproportionate weight on regimes associated with greater regressor variance and therefore fails to recover the unconditional mean of the perceived policy coefficients across regimes.

Second, the derivations and simulations in this section have abstracted from the monetary policy shocks. The underlying assumption is that the perceived monetary policy shocks do not substantially confound the estimation of the perceived Taylor rule when nowcasts are used. This assumption is based on the consensus in the literature that monetary policy shocks introduce only negligible bias in Taylor-rule estimation (Coibion and Gorodnichenko, 2011; Carvalho, Nechio, and Tristao, 2021; Bauer, Pflueger, and Sunderam, 2024). In Appendix D.6, I show that this consensus also extends to the perceived Taylor rule by following the approach of Carvalho, Nechio, and Tristao (2021).

4 Empirical Results

This section presents the empirical estimation of the perceived monetary policy rule. Subsection 4.1 details the real-time survey data employed in the analysis. Subsection 4.2 implements the aforementioned identification strategy, utilizing a Time-Varying Parameter model with Stochastic Volatility (TVP-SV) on nowcast data to recover the unbiased evolution of the perceived reaction function. Subsection 4.3 contrasts these perceptions with the actual policy rule estimated from Greenbook forecasts to highlight historical divergences. Several robustness checks related to empirical results are documented in Appendix D.

4.1 Data

To identify the perceived monetary policy rule, I utilize survey-based expectations data from the Blue Chip Economic Indicators (BCEI) and the Survey of Professional Forecasters (SPF). These datasets enable the construction of a real-time measure of private sector beliefs regarding future economic conditions and policy actions. Specifically, I employ the consensus forecasts for the short-term interest rate (Three-Month Treasury Bill), inflation (GDP deflator), and the unemployment rate.[12]

To estimate the actual monetary policy reaction function, I rely on the Greenbook/Tealbook forecasts prepared by Federal Reserve staff prior to each FOMC meeting. This choice is motivated by the substantial literature arguing that the central bank sets policy based on its own internal forecasts available in real-time (Orphanides, 2004; Boivin, 2005; Coibion and Gorodnichenko, 2011). To ensure comparability with the perceived rule estimation, I utilize the Greenbook forecasts for the GDP deflator and the unemployment rate, aligning

[12]Quarterly inflation forecasts are transformed into year-over-year rates to align with the actual practice of the monetary policy. This transformation utilizes the Real-Time Data Set for Macroeconomists (RTDSM) maintained by the Federal Reserve Bank of Philadelphia, following Bauer, Pflueger, and Sunderam (2024). Robustness checks confirm that the estimation results are similar if quarterly inflation series are used directly.

the forecast horizons with those in the private sector surveys. The policy instrument is defined as the target federal funds rate established at each corresponding FOMC meeting.

4.2 TVP Estimation of the Perceived Rule

The takeaway from Section 3 was to use interest rate nowcasts to infer the real-time perceived Taylor rule parameters used by the agent. To precisely identify the time-varying perceived reaction function at a quarterly frequency, I pool information from multiple surveys available within each quarter. Specifically, I stack the monthly Blue Chip Economic Indicators (BCEI) nowcasts and forecasts alongside the quarterly Survey of Professional Forecasters (SPF) nowcasts and forecasts. This approach increases the number of observations per period (quarter), improving the precision of the quarterly parameter estimates while maintaining the real-time nature of the identification.[13]

Let t index the survey period and h denote the forecast horizon, both measured in quarters. I formulate the model as a system where the observables are vectors containing all available survey observations within that quarter for a specific horizon h. The measurement equation is given by:

$$\begin{aligned}\mathbf{i}^{BC}_{t+h|t} &= \beta_{\pi,t,h}\boldsymbol{\pi}^{BC}_{t+h+j|t} + \beta_{x,t,h}\left(-2\cdot\mathbf{u}^{BC}_{t+h+j|t}\right) + \beta_{i1,t,h}\mathbf{i}^{BC}_{t+h-1|t} + \beta_{i2,t,h}\mathbf{i}^{BC}_{t+h-2|t} + c_{t,h}\mathbf{1} + \boldsymbol{\varepsilon}_{t,h} \\ &= \mathbf{X}_{t,h,j}\boldsymbol{\Theta}_{t,h} + \boldsymbol{\varepsilon}_{t,h} \end{aligned} \tag{31}$$

where bold variables $(\mathbf{i}, \boldsymbol{\pi}, \mathbf{u})$ denote vectors of stacked survey expectations (BCEI and SPF) for quarter t, and $\boldsymbol{\Theta}_{t,h} = [\beta_{\pi,t,h}, \beta_{x,t,h}, \beta_{i1,t,h}, \beta_{i2,t,h}, c_{t,h}]'$ represents the vector of time-varying coefficients. The case $h = 0$ corresponds to the nowcast specification, while $h > 0$ corresponds to forecasts. The term $c_{t,h}$ denotes the time-varying constant, which controls for time variation in long-run target values as in Coibion and Gorodnichenko (2011). To conserve space, I report the result based on a forward-looking horizon of $j = 1$. In Appendix D, I assess the sensitivity of the empirical results to alternative horizons j.

The time-varying parameters are assumed to be common across the stacked observations and evolve according to a random walk:

$$\boldsymbol{\Theta}_{t,h} = \boldsymbol{\Theta}_{t-1,h} + \boldsymbol{\nu}_{t,h}, \quad \boldsymbol{\nu}_{t,h} \sim \mathcal{N}(0, \boldsymbol{\Sigma}_{\nu,h}) \tag{32}$$

I account for heteroskedasticity by allowing the variance of the observation vector to follow

[13] In Appendix D, the model is re-estimated at a monthly frequency using only the BCEI monthly consensus forecasts. The resulting monthly time series closely tracks the quarterly baseline,

a stochastic volatility process:

$$\boldsymbol{\varepsilon}_{t,h} \sim \mathcal{N}(\mathbf{0}, \sigma_{t,h}\mathbf{I}), \quad \sigma_{t,h} \equiv e^{\eta_{t,h}}, \quad \eta_{t,h} = \eta_{t-1,h} + \xi_{t,h}, \quad \xi_{t,h} \sim \mathcal{N}(0, \sigma^2_{\xi,h}) \tag{33}$$

For estimation, I follow the established method in time-varying Taylor rule literature of Boivin (2005) and Coibion and Gorodnichenko (2011). The method allows for structural breaks in the variance-covariance matrix of innovations to the time-varying parameters. One innovation in this specification is the inclusion of stochastic volatility in equation (33), which follows the methodology by S. Kim, Shephard, and Chib (1998). As argued by Sims (2001), omitting stochastic volatility can lead to spurious time variation in coefficients, as the model may misattribute changes in shock volatility to changes in the time-varying parameters. A detailed description of the estimation procedure is provided in Appendix C.

4.2.1 Comparison of Horizons

A key prediction of the constant-parameter benchmark is that if agents operated under a constant rule or believed the current regime to be permanent, the estimated time-varying coefficients should be invariant across interest rate forecast horizons.

Figure 4 displays the estimated coefficients from equation (31) and compares the nowcast specification ($h = 0$) with the 2-quarter ahead forecast specification ($h = 2$).[14] The coefficients are presented with 16-84th percentile bands. Due to Zero Lower Bound (ZLB) period where total inertia exceeds unity in $h = 2$ results, I compare the estimated short-run coefficients directly rather than the implied long-run coefficients.[15] The results reveal that the forecast-based estimates are systematically and statistically different from the nowcast-based estimates. This finding rejects the null hypothesis of a constant rule or a permanent belief in the current regime in favor of a time-varying rule. Furthermore, the forecast-based estimates for inflation and the output gap rules are attenuated compared to the nowcast-based estimates, even exhibiting spurious negative responses in certain periods. Such an attenuated pattern in perceived responses, especially for inflation, has similarly been observed in Bauer, Pflueger, and Sunderam (2024), which use microdata on interest rate forecasts. This confirms that relying on interest rate forecasts can yield misleading conclusions regarding the perceptions about the monetary rule, contrasting sharply with the results obtained from interest rate nowcasts.

One potential concern is that this attenuation at longer h is mechanical, driven by the smaller variation in longer-horizon expectations of inflation and output gap (unemployment

[14] The results are qualitatively similar across different forecast horizons (h).

[15] The implied long-run coefficients for forecast-based estimates are still attenuated compared to the nowcast-based estimates even if we focus on the pre-ZLB period.

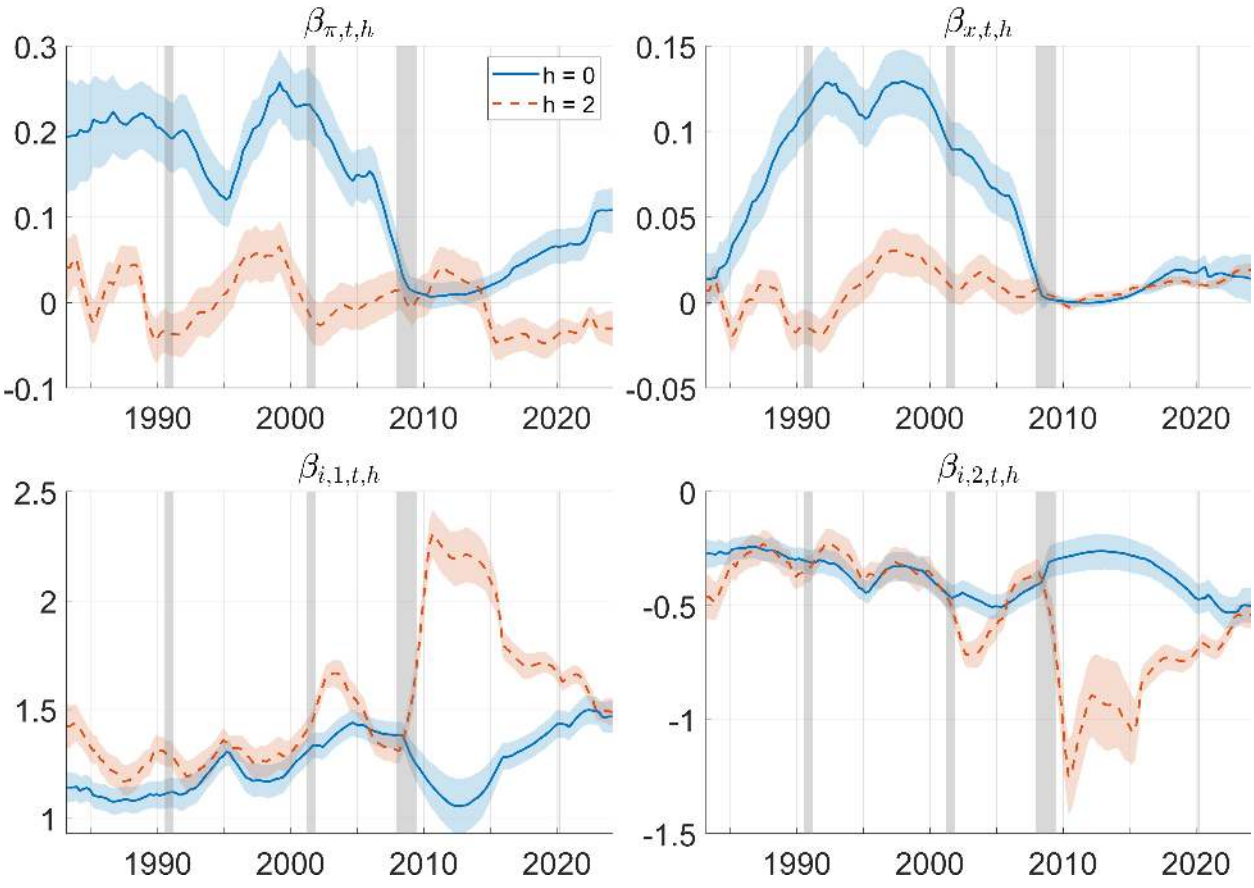


Figure 4: Perceived Taylor rule estimation for different interest rate horizon h

Note: The figure compares the short-run inflation, output gap, and inertia coefficients estimated using nowcasts ($h = 0$, blue line) against the coefficients estimated using 2-quarter ahead forecasts ($h = 2$, red line) from the estimating equation (31). Shaded areas are 16-84th percentile bands.

rate) as they converge to long-run targets. However, this statistical interpretation is unlikely. In Appendix D.2, interest rate *nowcasts* ($h = 0$) are regressed on longer-horizon inflation and unemployment forecasts but yields no attenuation, implying that the decline is driven by the properties of the dependent variable (the interest rate forecast), not the reduced variation of the regressors.

4.2.2 The Real-Time Perceived Rule using Interest Rate Nowcasts

Building on the identification argument established in Section 3.4, the nowcast specification ($h = 0$) provides the consistent measure of the private sector's real-time perception of the monetary policy rule. Using the estimated coefficients from equation (31) at $h = 0$, I define

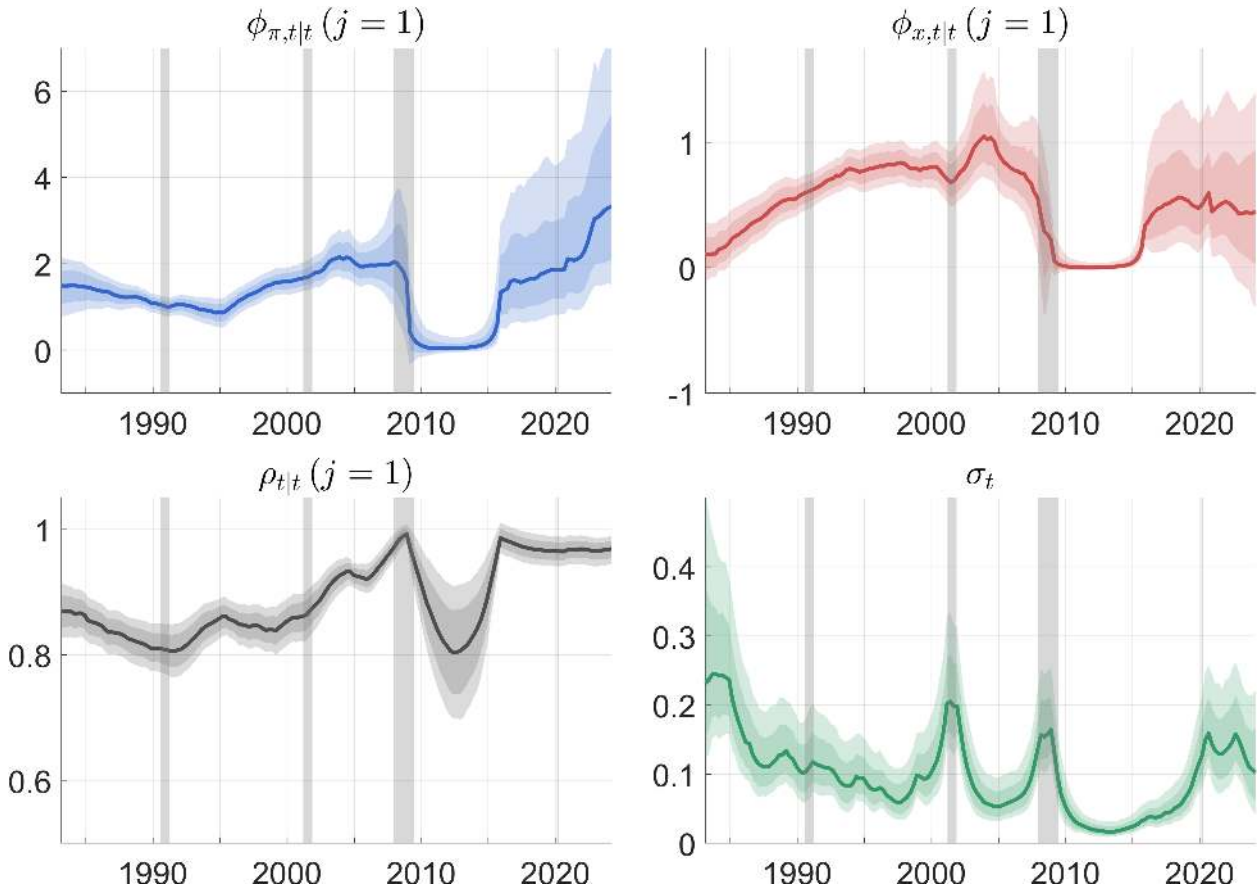


Figure 5: Time-Varying Estimates of the Perceived Policy Rule (Nowcast Specification)

Note: The figure plots the posterior median (solid line) along with the 68% and 90% confidence intervals (shaded regions) for the long-run response coefficients to inflation and the output gap, the total inertia parameter, and the stochastic volatility of the policy shock.

the total inertia and the implied long-run *perceived* responses as:

$$\rho_{t|t} := \beta_{i1,t,0} + \beta_{i2,t,0} \tag{34}$$

$$\phi_{\pi,t|t} := \frac{\beta_{\pi,t,0}}{1-\rho_{t|t}} \tag{35}$$

$$\phi_{x,t|t} := \frac{\beta_{x,t,0}}{1-\rho_{t|t}} \tag{36}$$

Figure 5 presents the filtered evolution of these perceived policy coefficients and the stochastic volatility. The plots display posterior medians alongside 68% and 90% credible intervals. The estimates exhibit substantial time variation, underscoring the inadequacy of constant-parameter approximations for capturing private sector beliefs.

First, the pre-ZLB era provides compelling support for the "Good Perceived Policy" hypothesis regarding the Great Moderation. Crucially, even after controlling for stochastic volatility, the perceived inflation coefficient $\phi_{\pi,t|t}$ consistently satisfies the Taylor principle throughout the period. This robustness confirms that the private sector's anchored belief in the central bank's active stabilization stance was a structural feature of the regime rather than a statistical artifact driven by periods of varying shock volatility.

Second, the steady rise in the output gap coefficient $\phi_{x,t|t}$ throughout the pre-crisis period suggests a structural shift in private beliefs toward a dual mandate framework.[16] Third, the dynamics of the inertia parameter $\rho_{t|t}$ during the ZLB episode reflect evolving expectations regarding lift-off. The initial sharp decline suggests forecasters viewed the effective lower bound as a temporary constraint at the onset, whereas the subsequent rebound around 2011 is consistent with agents learning that the constraint would be long-lasting. Finally, in the post-ZLB period, perceived inertia remains entrenched at a level structurally higher than its pre-crisis average, likely reflecting the private sector's internalization of the central bank's active reliance on forward guidance (Bauer, Pflueger, and Sunderam, 2024).

4.3 Actual vs. Perceived Rule

To compare the private sector's perception against the central bank's actual conduct, I estimate the actual monetary policy rule using Greenbook forecasts, following the established literature (Orphanides, 2001; Boivin, 2005; Coibion and Gorodnichenko, 2011; Coibion, 2012). The specification mirrors the quarterly stacked model used for the perceived rule but utilizes Greenbook data at the FOMC meeting frequency (denoted by superscript GB):

$$i_t = (1 - \rho_{1,t} - \rho_{2,t}) \left[\phi_{\pi,t} \pi^{GB}_{t+j|t} + \phi_{x,t} \left(-2 \cdot u^{GB}_{t+j|t} \right) \right] + \rho_{1,t} i_{t-1} + \rho_{2,t} i_{t-2} + c_t + \varepsilon_t \tag{37}$$

Here, i_t corresponds to the target federal funds rate, as opposed to the interest rate nowcast $i_{t|t}$ used for perceived rule estimation. This estimation yields the time path of the actual policy coefficients $\phi_{\pi,t}$ and $\phi_{x,t}$, which I then contrast with the perceived coefficients $\phi_{\pi,t|t}$ and $\phi_{x,t|t}$ recovered from the survey data. I maintain $j = 1$ as the baseline horizon to ensure strict comparability with the perceived rule estimates. The sample period runs from 1969 to 2019 due to publication lag of Greenbook data. In Appendix D.5, I also re-estimate the actual policy rule with the shadow rate of Wu and Xia (2016) for possible concern related to ZLB, but the resulting time series of coefficients are very similar to this baseline.

[16] Menon (2025) uses a structural model where the private sector learns about the central bank's objective. The paper documents that the perception of the Fed's weight on unemployment-gap stabilization has been increasing over time.

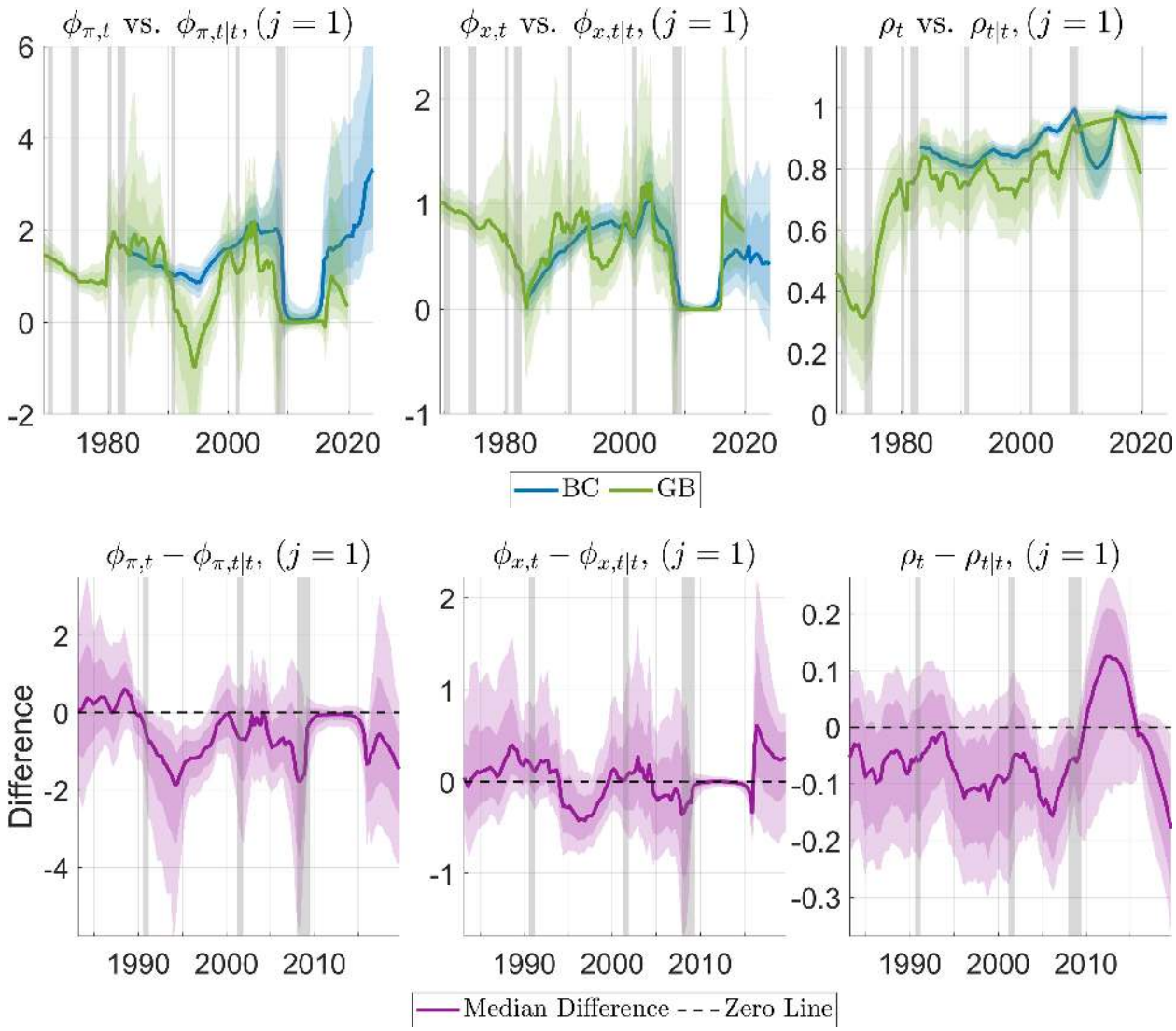


Figure 6: Actual vs. Perceived Monetary Policy Rules

Note: **Top panel:** The figure superimposes the estimated long-run implied long-run inertia coefficients from the actual rule (green) from the Greenbook forecasts and the perceived rule (blue) from Blue Chip and SPF survey data. The confidence bands are 68% and 90% respectively. **Bottom panel:** The figure plots the difference $\phi_{\pi,t} - \phi_{\pi,t|t}$ with 68% and 90% confidence bands. Positive values indicate the Fed was more aggressive than perceived. The differences are calculated for estimates observed at March, June, September, and December. The inertia coefficient estimated from Greenbook data has been adjusted from the FOMC to the quarterly frequency following Coibion and Gorodnichenko (2011).

Figure 6 superimposes the estimated time-varying coefficients for the actual and perceived rules. While the two series generally comove closely, some periods of divergence are evident. To rigorously assess these discrepancies, the bottom panel of the figure plots the difference in these coefficients along with the 68% and 90% credible intervals.

A prominent episode of divergence emerges during the "preemptive tightening" of the

mid-1990s. Under Chairman Greenspan, the actual response to the mean inflation forecast was significantly less aggressive than the private sector perceived. This discrepancy arose from the Federal Reserve's adoption of a "risk management" approach to monetary policy, in which the target rate was adjusted to mitigate potential tail risks despite stable baseline macroeconomic expectations (Greenspan, 2004; Bernanke, 2022). This has led to a weaker statistical relationship between the target rate and the Greenbook forecasts. However, while this discretionary strategy manifested as a systematic deviation in the actual reaction function, the private sector's perceived rule remained structurally intact.

5 Theory of Perceived vs. Actual Rule

This section develops the general equilibrium framework used to analyze the macroeconomic consequences of misperception. Subsection 5.1 establishes the general economic environment. Subsection 5.3 specifies the Subjective Perception Markov-Switching New Keynesian (SP-MSNK) model, incorporating second-order interest rate inertia to match the empirical evidence. Subsection 5.4 conducts a welfare analysis to disentangle the distinct effects of the policy rule's coefficients and its perceived persistence. Finally, Subsection 5.5 presents counterfactual simulations to quantify the historical impact of private sector perceptions during the Great Moderation and the post-COVID inflation surge.

5.1 General Framework

5.1.1 Setup

Consider an economy described by a vector of endogenous variables $\mathbf{y}_t \in \mathbb{R}^n$ and a vector of actual structural shocks $\boldsymbol{\varepsilon}_t \in \mathbb{R}^m$. The economy is subject to two distinct regime-switching processes. First, the *perceived regime*, denoted by $s_t^p \in \mathcal{S}^p = \{1, 2, \ldots, S_p\}$, represents the state of the economy perceived by the private sector. Agents believe s_t^p evolves according to a subjective transition probability matrix $\widehat{\mathbf{P}}$ with elements $\widehat{P}_{jk} = \Pr\left(s_{t+1}^p = k \mid s_t^p = j\right)$. Second, the *central bank's actual regime*, denoted by $s_t^f \in \mathcal{S}^f = \{1, 2, \ldots, S_f\}$, characterizes the true policy stance implemented by the monetary authority. Consequently, the *actual* state of the economy is the tuple $s_t = \left(s_t^p, s_t^f\right)$ within the joint state space $\mathcal{S} = \mathcal{S}^p \times \mathcal{S}^f$, where the total number of joint states is $S = S_p \times S_f$. For notational convenience, I index the joint state by a single integer $\ell \in \{1, 2, \ldots, S\}$. The economy evolves according to the actual transition probability matrix $\mathbf{P}$ with elements $P_{\ell\ell'} = \Pr\left(s_{t+1} = \ell' \mid s_t = \ell\right)$.[17]

[17] The dimension of $\mathbf{P}$ is $S \times S$, whereas the dimension of $\widehat{\mathbf{P}}$ is $S_p \times S_p$.

I distinguish between the full vector of structural shocks hitting the economy, $\boldsymbol{\varepsilon}_t$, and the subset of shocks observed by the private sector, denoted by $\boldsymbol{\varepsilon}_t^p$. Let $\mathbf{Q}$ be a selection matrix such that:

$$\boldsymbol{\varepsilon}_t^p = \mathbf{Q}\boldsymbol{\varepsilon}_t \tag{38}$$

For instance, in the specific model developed later, I assume agents perceive the Taylor rule as purely systematic and cannot observe the true monetary policy shock. This restriction reflects the information constraint that agents cannot empirically distinguish whether the discrepancy between their interest rate nowcast and the realized rate arises from a systematic deviation in the reaction function or a temporary monetary policy shock. Given this signal extraction problem, the agent cannot tell the source of their forecast error, having no incentive to detect and change their misspecified model. In this case, the perceived shock vector $\boldsymbol{\varepsilon}_t^p$ contains only non-policy shocks (e.g., demand and supply), whereas the actual shock vector $\boldsymbol{\varepsilon}_t$ includes the monetary policy shock.

5.1.2 Perceived Law of Motion

Conditional on the perceived state $s_t^p = j$, the private sector assumes the economy evolves according to the linear structural system:

$$\mathbf{\Gamma}_{0,j}^{\mathrm{P}} E_t^{\mathrm{P}}\left[\mathbf{y}_{t+1}\right] + \mathbf{\Gamma}_{1,j}^{\mathrm{P}}\mathbf{y}_t + \mathbf{\Gamma}_{2,j}^{\mathrm{P}}\mathbf{y}_{t-1} + \mathbf{\Gamma}_{3,j}^{\mathrm{P}}\boldsymbol{\varepsilon}_t^p = \mathbf{0} \tag{39}$$

where $E_t^{\mathrm{P}}[\cdot]$ denotes the expectation operator conditional on the agents' information set and the subjective transition probabilities $\widehat{\mathbf{P}}$.

Assumption 1 (MSV Solution). I assume that for the system defined by $\left\{\mathbf{\Gamma}_{i,j}^{\mathrm{P}}\right\}$ and $\widehat{\mathbf{P}}$, there exists a mean-square stable Minimum State Variable (MSV) solution. This implies the Perceived Law of Motion (PLM) takes the form of a Markov-Switching VAR driven by the perceived shocks:

$$\mathbf{y}_{t|t} = \widehat{\mathbf{A}}_{s_t^p}\mathbf{y}_{t-1} + \widehat{\mathbf{B}}_{s_t^p}\boldsymbol{\varepsilon}_t^p \tag{40}$$

The matrices $\widehat{\mathbf{A}}_j$ and $\widehat{\mathbf{B}}_j$ are structurally fundamental to the agent's belief system and are obtained via standard MS-DSGE solution algorithms (e.g., Maih, 2015). Note that $\mathbf{y}_{t|t}$ denotes the private sector's *nowcast* of the endogenous variables given the state variables $\mathbf{y}_{t-1}$ and the perceived shocks $\boldsymbol{\varepsilon}_t^p$. This nowcast is consistent with the perceived law of motion but may differ from the actual realization $\mathbf{y}_t$ observed in the economy, which is generated by actual model of the economy and full vector of actual shocks $\boldsymbol{\varepsilon}_t$.

Agents form expectations using their subjective model. The one-period ahead forecast given the current perceived state j is:

$$E_t^{\mathrm{P}}\left[\mathbf{y}_{t+1}\right] = \sum_{k=1}^{S_p} \widehat{P}_{jk} E_t^{\mathrm{P}}\left[\mathbf{y}_{t+1} \mid s_{t+1}^p = k\right] \tag{41}$$

$$= \sum_{k=1}^{S_p} \widehat{P}_{jk} \widehat{\mathbf{A}}_k \mathbf{y}_{t|t} \tag{42}$$

$$= \tilde{\mathbf{A}}_j \left(\widehat{\mathbf{A}}_j \mathbf{y}_{t-1} + \widehat{\mathbf{B}}_j \boldsymbol{\varepsilon}_t^p\right) \tag{43}$$

where $\tilde{\mathbf{A}}_j \equiv \sum_{k=1}^{S_p} \widehat{P}_{jk} \widehat{\mathbf{A}}_k$ is the expected transition matrix. I define the sensitivity of expectations to history and perceived shocks as $\boldsymbol{\Gamma}_{y,j} \equiv \tilde{\mathbf{A}}_j \widehat{\mathbf{A}}_j$ and $\boldsymbol{\Gamma}_{\varepsilon,j} \equiv \tilde{\mathbf{A}}_j \widehat{\mathbf{B}}_j$, respectively. Thus, the behavioral equation for the agent's forecast is given by $E_t^{\mathrm{P}}\left[\mathbf{y}_{t+1}\right] = \boldsymbol{\Gamma}_{y,j} \mathbf{y}_{t-1} + \boldsymbol{\Gamma}_{\varepsilon,j} \boldsymbol{\varepsilon}_t^p$.

5.1.3 Actual Law of Motion

The equilibrium is characterized as a Temporary Equilibrium (J. M. Grandmont, 1977; J.-M. Grandmont, 1987; Woodford, 2013) extended to a Markov-switching environment. At each point in time, equilibrium is defined by the expectations agents form based on their perceived law of motion (PLM), which in turn determines the actual law of motion (ALM) for aggregate variables.

The actual evolution of the economy is governed by the joint state s_t, indexed by ℓ, and the actual shocks $\boldsymbol{\varepsilon}_t$. The actual model of the economy consists of agents with heterogeneous expectations operators. For example, private sector agents may possess subjective expectations based on their misspecified model, whereas the central bank may hold rational expectations, essentially having superior information than the private sector:

$$\boldsymbol{\Gamma}_{0,\ell}^{\mathrm{A}} \mathbb{E}_t\left[\mathbf{y}_{t+1}\right] + \boldsymbol{\Gamma}_{1,\ell}^{\mathrm{A}} E_t^{\mathrm{P}}\left[\mathbf{y}_{t+1}\right] + \boldsymbol{\Gamma}_{2,\ell}^{\mathrm{A}} \mathbf{y}_t + \boldsymbol{\Gamma}_{3,\ell}^{\mathrm{A}} \mathbf{y}_{t-1} + \boldsymbol{\Gamma}_{4,\ell}^{\mathrm{A}} \boldsymbol{\varepsilon}_t = \mathbf{0} \tag{44}$$

Substituting the expectation formation mechanism derived in (43) into the actual structural system and utilizing the mapping $\boldsymbol{\varepsilon}_t^p = \mathbf{Q}\boldsymbol{\varepsilon}_t$ yields:

$$\boldsymbol{\Gamma}_{0,\ell}^{\mathrm{A}} \mathbb{E}_t\left[\mathbf{y}_{t+1}\right] + \boldsymbol{\Gamma}_{1,\ell}^{\mathrm{A}} \left(\boldsymbol{\Gamma}_{y,j} \mathbf{y}_{t-1} + \boldsymbol{\Gamma}_{\varepsilon,j} \mathbf{Q} \boldsymbol{\varepsilon}_t\right) + \boldsymbol{\Gamma}_{2,\ell}^{\mathrm{A}} \mathbf{y}_t + \boldsymbol{\Gamma}_{3,\ell}^{\mathrm{A}} \mathbf{y}_{t-1} + \boldsymbol{\Gamma}_{4,\ell}^{\mathrm{A}} \boldsymbol{\varepsilon}_t = \mathbf{0} \tag{45}$$

Rearranging this system yields the canonical state-space form:

$$\boldsymbol{\Gamma}_{0,\ell}^{\mathrm{A}} \mathbb{E}_t\left[\mathbf{y}_{t+1}\right] + \mathbf{H}_\ell \mathbf{y}_t + \mathbf{F}_\ell \mathbf{y}_{t-1} + \mathbf{C}_\ell \boldsymbol{\varepsilon}_t = \mathbf{0} \tag{46}$$

where $\mathbf{H}_\ell = \boldsymbol{\Gamma}_{2,\ell}^{\mathrm{A}}$, $\mathbf{F}_\ell = \boldsymbol{\Gamma}_{1,\ell}^{\mathrm{A}} \boldsymbol{\Gamma}_{y,j} + \boldsymbol{\Gamma}_{3,\ell}^{\mathrm{A}}$, and $\mathbf{C}_\ell = \boldsymbol{\Gamma}_{1,\ell}^{\mathrm{A}} \boldsymbol{\Gamma}_{\varepsilon,j} \mathbf{Q} + \boldsymbol{\Gamma}_{4,\ell}^{\mathrm{A}}$ aggregates terms from

both the expectation channel and the direct structural impact. Then, applying the standard MS-DSGE solution algorithm yields the Actual Law of Motion (ALM):

$$\mathbf{y}_t = \mathbf{A}_\ell^{\text{ALM}} \mathbf{y}_{t-1} + \mathbf{B}_\ell^{\text{ALM}} \boldsymbol{\varepsilon}_t \tag{47}$$

This system characterizes the dynamics of an economy where agents possess a persistent misperception of the true model of the economy.

5.2 Perceived Taylor Principle and Determinacy

A central question in monetary economics concerns how central banks should conduct policy to prevent self-fulfilling equilibria. In standard Full Information Rational Expectations (FIRE) models, determinacy requires the central bank to satisfy the Taylor principle: the policy rate must respond more than one-for-one to inflation. This principle was essential to the transition from the Great Inflation of the 1970s–mid-1980s to the Great Moderation of the 1990s. However, recent developments, characterized by the Fed's delayed response to the post-pandemic inflation surge and the subsequent soft landing, have reignited this debate. For instance, Nakamura, Riblier, and Steinsson (2025) propose a *"Minimal Taylor Principle"* in which central banks can rely on established credibility and respond in a shock-contingent manner without strictly adhering to the conventional Taylor principle.

In this subsection, I demonstrate that relaxing the FIRE assumption to allow for a wedge between agents' perceptions and the actual policy rule fundamentally alters determinacy conditions. Specifically, once agents *believe* the Taylor principle holds (*"perceived Taylor principle"*), the central bank gains significant flexibility in its actual rule implementation without inducing indeterminacy.

To establish this result analytically, I consider a constant-parameter limit case ($S_p = S_f = 1$) nested within the full SP-MSNK model developed in Section 5.3, where determinacy conditions are well established. The private sector perceives the economy to evolve according to:

$$x_t = E_t^{\text{P}}\left[x_{t+1}\right] - \tau\left(i_t - E_t^{\text{P}}\left[\pi_{t+1}\right]\right) + z_{Dt} \tag{48}$$

$$\pi_t = \beta E_t^{\text{P}}\left[\pi_{t+1}\right] + \kappa x_t + z_{St} \tag{49}$$

$$i_t = \phi_\pi^{\text{P}} E_t^{\text{P}}\left[\pi_{t+1}\right] + \phi_x^{\text{P}} E_t^{\text{P}}\left[x_{t+1}\right] \tag{50}$$

For tractability, consider a simple rule without inertia. The actual rule implemented by the

central bank is:

$$i_t = \phi_\pi^{\mathrm{F}} \mathbb{E}_t \left[\pi_{t+1}\right] + \phi_x^{\mathrm{F}} \mathbb{E}_t \left[x_{t+1}\right] + \varepsilon_{Mt} \tag{51}$$

where $\mathbb{E}_t[\cdot]$ denotes the mathematical rational expectation based on the actual law of motion. In other words, the central bank has access to all the information in the economy, including the mispecification of the private sector. The following proposition characterizes the conditions under which this economy admits a unique bounded equilibrium:

Proposition 3 (Determinacy under Subjective Perception)**.** *Consider the constant-parameter SP-NK model defined by equations* (48)—(51)*. There is a unique bounded equilibrium if and only if the following two conditions are jointly satisfied:*

1. ***Perceived Taylor Principle:***

$$\phi_\pi^P + \phi_x^P \frac{1-\beta}{\kappa} > 1 \tag{52}$$

$$\phi_\pi^P + \phi_x^P \frac{1+\beta}{\kappa} < 1 + \frac{2(1+\beta)}{\tau\kappa} \tag{53}$$

2. ***Actual Policy Restriction:***

$$\tau \left(\phi_x^F + \kappa \phi_\pi^F\right) < 1 \tag{54}$$

Proof. See Appendix A.4.

Proposition 3 reorients the determinacy conditions relative to standard FIRE models. The first condition that consists of (52) and (53) corresponds to the Taylor principle under the forward-looking rule (Galí, 2015). It requires that the private sector *perceives* the central bank to satisfy the Taylor principle, thereby anchoring subjective expectations $E_t^{\mathrm{P}}[\cdot]$ and eliminating the sunspot equilibria in the subjective model. Crucially, this condition depends entirely on beliefs rather than actual conduct. The second condition (54) imposes an upper bound on the actual policy response, but this restriction is very loose considering the conventional calibrations (e.g., $\kappa \approx 0, \phi_x \in (0, 0.5)$). Moreover, this actual policy restriction will not be required if the central bank uses a contemporaneous rule or responds to the subjective expectations in a forward-looking rule.

The policy implications are striking. Determinacy hinges on the *perceived Taylor principle* rather than the actual Taylor principle, implying that a central bank may violate the Taylor principle in practice ($\phi_\pi^{\mathrm{F}} < 1$) without inducing self-fulfilling fluctuations if the private sector

continues to *believe* in an active regime. This potentially explains why and how the Fed achieved the soft landing during the period of post-pandemic inflation surge, despite a more measured interest rate responses. This underscores the critical importance of managing the private sector perceptions about the monetary policy rule.

5.3 Subjective Perception Markov Switching New Keynesian Model

To formally analyze the macroeconomic effects of the discrepancy between the perceived and actual monetary policy rules, I develop a concrete Subjective Perception Markov-Switching New Keynesian (SP-MSNK) model. This specification fits the general framework and adheres to a standard three-equation New Keynesian structure extended to include behavioral insight that mitigates the forward-guidance puzzle.

5.3.1 Setup

Assume the economy is characterized by S_p perceived states of monetary policy, $s_t^p \in \mathcal{S}^p$. The representative agent's perceived law of motion (PLM) is given by the following system:

$$x_t = \theta E_t^{\mathrm{P}}\left[x_{t+1}\right] - \tau\left(i_t - E_t^{\mathrm{P}}\left[\pi_{t+1}\right]\right) + z_{Dt} \tag{55}$$

$$\pi_t = \beta E_t^{\mathrm{P}}\left[\pi_{t+1}\right] + \kappa x_t + z_{St} \tag{56}$$

$$i_t = \left(1-\rho_{s_t^p}^{\mathrm{P}}\right)\left[\phi_{\pi,s_t^p}^{\mathrm{P}} E_t^{\mathrm{P}}\left[\pi_{t+1}\right] + \phi_{x,s_t^p}^{\mathrm{P}} E_t^{\mathrm{P}}\left[x_{t+1}\right]\right] + \rho_{1,s_t^p}^{\mathrm{P}} i_{t-1} + \rho_{2,s_t^p}^{\mathrm{P}} i_{t-2} \tag{57}$$

$$z_{Dt} = \rho_D z_{Dt-1} + \varepsilon_{D,t} \tag{58}$$

$$z_{St} = \rho_S z_{St-1} + \varepsilon_{S,t} \tag{59}$$

The perceived state evolves according to the transition probability matrix $\widehat{\mathbf{P}}$, where $\widehat{P}_{jk} = \Pr(s_{t+1}^p = k \mid s_t^p = j)$. The total perceived inertia is defined as $\rho_{s_t^p}^{\mathrm{P}} \equiv \rho_{1,s_t^p}^{\mathrm{P}} + \rho_{2,s_t^p}^{\mathrm{P}}$.

Defining the vector of state variables as $\mathbf{y}_t = [x_t, \pi_t, i_t, i_{t-1}, z_{Dt}, z_{St}]'$, the system yields a state-space representation $\mathbf{y}_{t|t} = \widehat{\mathbf{A}}_{s_t^p}\mathbf{y}_{t-1} + \widehat{\mathbf{B}}_{s_t^p}\boldsymbol{\varepsilon}_t^p$. Notably, the perceived model does not contain a structural monetary policy shock. From the agent's perspective, deviations of the realized interest rate i_t from the rule-implied rate could be due to systematic rule or monetary policy shocks. The AR(2) smoothing structure is included to maintain consistency with the empirical specification presented in Section 4.

The Federal Reserve conducts monetary policy according to an *actual* Taylor rule. This rule is governed by the state $s_t^f \in \mathcal{S}^f$ and may differ from the perceived rule:

$$i_t = \left(1-\rho_{s_t^f}^{\mathrm{F}}\right)\left[\phi_{\pi,s_t^f}^{\mathrm{F}} E_t^{\mathrm{P}}\left[\pi_{t+1}\right] + \phi_{x,s_t^f}^{\mathrm{F}} E_t^{\mathrm{P}}\left[x_{t+1}\right]\right] + \rho_{1,s_t^f}^{\mathrm{F}} i_{t-1} + \rho_{2,s_t^f}^{\mathrm{F}} i_{t-2} + \varepsilon_{Mt} \tag{60}$$

where $\rho^{\mathrm{F}}_{s^f_t} \equiv \rho^{\mathrm{F}}_{1,s^f_t} + \rho^{\mathrm{F}}_{2,s^f_t}$. The actual state of the economy is the tuple $s_t = \left(s^p_t, s^f_t\right)$ indexed by ℓ.[18] I assume that the Fed observes and responds to private sector expectations. This assumption is introduced to mainly isolate the effect of disagreement about the policy rule without confounding the expectational disagreement. In Appendix B.4, I relax this assumption by allowing the Fed to use a forward-looking Taylor rule with rational expectations, which yields quantitatively similar results.

Solving for the expectation coefficients $\tilde{\mathbf{A}}_j$ (where j corresponds to the perceived component of ℓ) and substituting them into the structural equations yields the Actual Law of Motion (ALM):

$$\mathbf{y}_t = \mathbf{A}^{\mathrm{ALM}}_{\ell}\mathbf{y}_{t-1} + \mathbf{B}^{\mathrm{ALM}}_{\ell}\boldsymbol{\varepsilon}_t \tag{61}$$

where the matrices $\mathbf{A}^{\mathrm{ALM}}_{\ell}$ and $\mathbf{B}^{\mathrm{ALM}}_{\ell}$ are defined and derived explicitly in Appendix A.5. This system serves as the laboratory for analyzing the implications of the perceived rule and empirical findings in Section 4.

Calibration The model parametrization adheres to the standard New Keynesian literature. The cognitive discounting parameter θ is used to attenuate the forward guidance puzzle (Gabaix, 2020; Pfäuti and Seyrich, 2022) and to avoid overstating the general equilibrium impact of the perceived monetary policy rule. I set $\theta = 0.85$ and the Phillips curve slope $\kappa = 0.057$. These values are consistent with the estimates in Pfäuti (2023). The remaining structural parameters follow standard benchmarks. The discount factor is $\beta = 0.99$, and the intertemporal elasticity of substitution is $\tau = 1.00$. The shock processes assume persistence $\rho_D = \rho_S = 0.85$.

The transition probabilities are calibrated by maximum likelihood based on the historical frequency of regimes of actual rule in the 1983–2019 sample. This yields a persistence probability for the Hawkish regime of $\hat{p}_{11} = 0.9780$ and for the Dovish regime of $\hat{p}_{22} = 0.9643$. Given the transition probabilities, the regime-dependent policy parameters are calibrated to match the first and second moments of the time-varying perceived policy rule estimates in Figure 6.[19] The Hawkish regime ($s = 1$) is characterized by strong inflation stabilization ($\phi_{\pi,1} = 1.7226$), a responsive output gap coefficient ($\phi_{x,1} = 0.6991/4$), and inertia coefficients of $\rho_{1,1} = 1.2872$ and $\rho_{2,1} = -0.3912$. In contrast, the Dovish regime ($s = 2$) exhibits a passive inflation response ($\phi_{\pi,2} = 0.4512$), a muted output response ($\phi_{x,2} = 0.2033/4$), and altered inertia parameters ($\rho_{1,2} = 1.1299, \rho_{2,2} = -0.2824$).

[18] For example, in a two-regime case, $\ell = 1, 2, 3, 4$ indicates $\{(1,1), (2,2), (1,2), (2,1)\}$.

[19] The time series of four perceived rule parameters were approximated jointly with a two-state Markov chain by matching the unconditional mean and the first principal component of the covariance matrix.

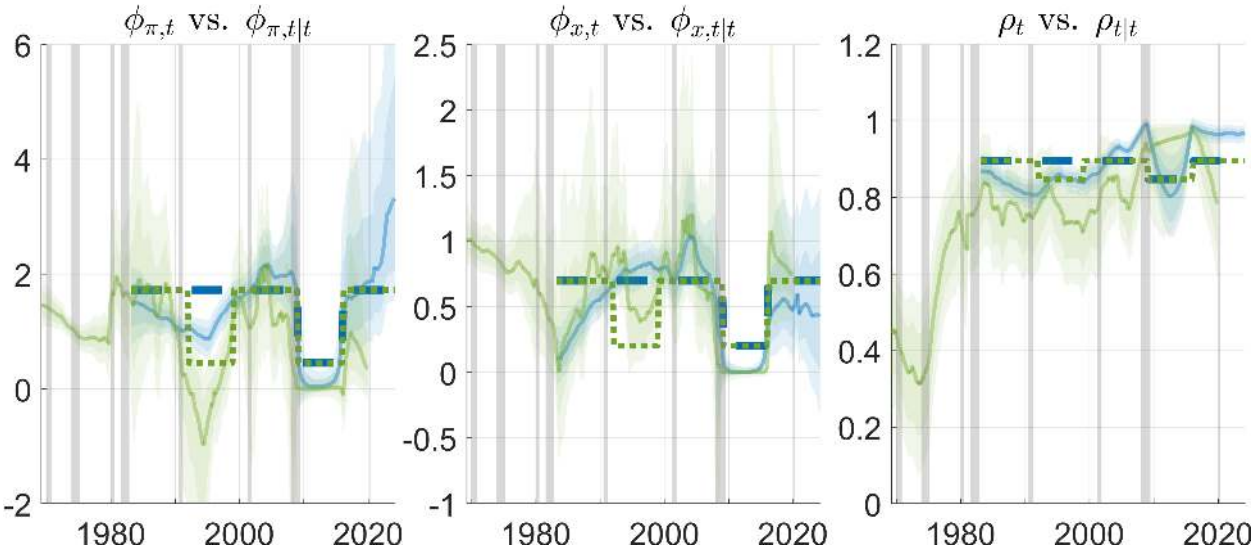


Figure 7: Regime Approximation of Perceived and Actual Rules

Note: The figure displays the approximated regime sequence alongside the estimated time series of perceived and actual Taylor rules with AR(2) inertia. The blue and green dashed lines indicate the approximated regime sequence for the perceived and actual rules.

Figure 7 displays the resulting regime sequence under the baseline calibration. The perceived regime is classified as Hawkish throughout the sample with the exception of the ZLB episode (2009–2015). The Fed's actual policy regime follows the Hawkish path but deviates to the Dovish regime (in terms of response to mean inflation expectation) during the 1992–1998 period of risk-management policy period and the ZLB episode around the Global Financial Crisis. Although Greenbook data availability limits estimation of the actual rule beyond 2019, I assume for the counterfactual exercises that the perceived and actual regimes coincide at the hawkish regime from 2020 onward.

5.4 Welfare Analysis and Policy Implications

This section evaluates the model's implications for monetary policy by simulating the model under four distinct fixed-regime scenarios. These scenarios correspond to the four possible realizations of the joint state tuple $s_t = (s_t^p, s_t^f)$, where the perceived and actual regimes can independently be Hawkish (1) or Dovish (2). Specifically, I consider:

1. **HH (Regime 1)**: Agents perceive a Hawkish regime, and the Fed implements a Hawkish rule. This corresponds to the standard rational expectation equilibrium under an active policy.

2. **DD (Regime 2)**: Agents perceive a Dovish regime, and the Fed implements a Dovish

rule. This represents the standard rational expectation equilibrium under a passive policy.

3. **HD (Regime 3)**: Agents perceive a Hawkish regime, but the Fed actually implements a Dovish rule. This captures the case where the central bank's commitment to stability is overestimated by the public.

4. **DH (Regime 4)**: Agents perceive a Dovish regime, but the Fed actually implements a Hawkish rule. This represents a scenario where the central bank attempts to stabilize the economy against anchored beliefs of policy passivity.

It is important to clarify the distinction between the *realized* regime path and the *perceived* regime dynamics in this exercise. While the actual law of motion (ALM) is simulated conditional on the joint state s_t, the private sector forms expectations based on the belief that the perceived state is s_t^p, evolving stochastically according to the transition matrix $\widehat{\mathbf{P}}$. In other words, this is not a constant-parameter model from the agent's perspective. Subjective expectations $E_t^{\mathrm{P}}[\cdot]$ continue to incorporate the probability of future regime shifts. This setup isolates the welfare effects of the current regime realization while preserving the agents' awareness of possible regime shifts in the future. For this exercise, I assume that the transition probability matrix $\widehat{\mathbf{P}}$ is symmetric, with the diagonal elements denoted as $\widehat{p}$, and keep all other parameters at their calibrated values. Later, I explicitly analyze how changes in the perceived persistence affect welfare losses within these fixed regimes.

Methodology Conditional on the structural parameters and the specific policy regime ℓ, I construct the state-space matrices $\mathbf{A}_\ell^{\mathrm{ALM}}$ and $\mathbf{B}_\ell^{\mathrm{ALM}}$ governing the Actual Law of Motion (ALM), as derived in Equation (47). Then, I simulate the economy for each $\ell \in \{1, 2, 3, 4\}$, and assess the welfare loss using a standard quadratic loss function defined as the expected discounted sum of squared deviations in inflation and the output gap:

$$\mathcal{L} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_x x_t^2 \right] \tag{62}$$

where the relative weight on output gap stabilization is set to $\lambda_x = 0.1$.

The Role of the Perceived Rule and Regime Persistence Figure 8 illustrates the unconditional variance trade-offs between inflation and the output gap across the four policy regimes. The regimes are denoted by a two-letter code where the first letter represents the perceived regime and the second represents the actual policy rule; circle markers identify

the Full Information Rational Expectations (FIRE) benchmarks (HH and DD), while cross markers denote the misperception regimes (HD and DH).

The left panel, calibrated to a high perceived persistence ($\widehat{p} = 0.98$), establishes the baseline stability properties of misperception. Regimes characterized by a Dovish perception (DD, DH) generate strictly higher volatility in both inflation and the output gap compared to their Hawkish perception counterparts (HD, HH). Consequently, The Dovish-perception equilibria cluster to the north-east of the Hawkish-perception equilibria regardless of the actual policy rule in place. This separation underscores the primacy of the Expectation Channel. Since private agents form expectations ($E_t^P[\cdot]$) based on the perceived Taylor rule, a belief in passive policy unanchors expectations in the IS and Phillips curves. Consequently, even if the Federal Reserve actually adheres to the Taylor principle (Regime DH), it cannot fully offset the destabilizing feedback loop generated by the private sector's belief in accommodation. This shows that the *perceived* reaction function are the primary determinant of equilibrium dynamics beyond the actual policy rate.

The middle ($\widehat{p} = 0.50$) and right ($\widehat{p} = 0.01$) panels elucidate the role of belief about regime persistence by illustrating how these trade-offs evolve when the perceived regime is viewed as transitory. Comparing the trajectory across the three panels reveals a convergence property. As perceived persistence declines, the welfare outcomes in the misperception regimes (HD and DH) and their respective FIRE benchmarks (HH and DD) converge toward each other. The mechanism driving this result is also the role of expectations. When agents view the current regime as transitory (low $\widehat{p}$), their forward-looking expectations are weighted toward the alternative regime rather than the current regime they perceive to be located in. Consequently, the destabilizing (stabilizing) effects of a perceived Dovish (Hawkish) regime are significantly attenuated (strengthened) if the private sector anticipates a rapid reversion to the Hawkish (Dovish) regime. This finding highlights an important dimension of monetary communication: effective stabilization requires managing not only the perceived *stance* of the current rule (e.g., ϕ_{π,s_t}) but also its perceived *duration* (p), ensuring that necessary accommodative deviations are viewed as strictly transitory.

5.5 Counterfactual Analysis

This section quantifies the historical implications of misperceived monetary policy rules by simulating counterfactual scenarios for two key periods: the Great Moderation (1985–1999) and the post-ZLB period (2016–2023). The objective is to disentangle the macroeconomic consequences of the private sector's perception of that rule from those of the central bank's actual policy rule.

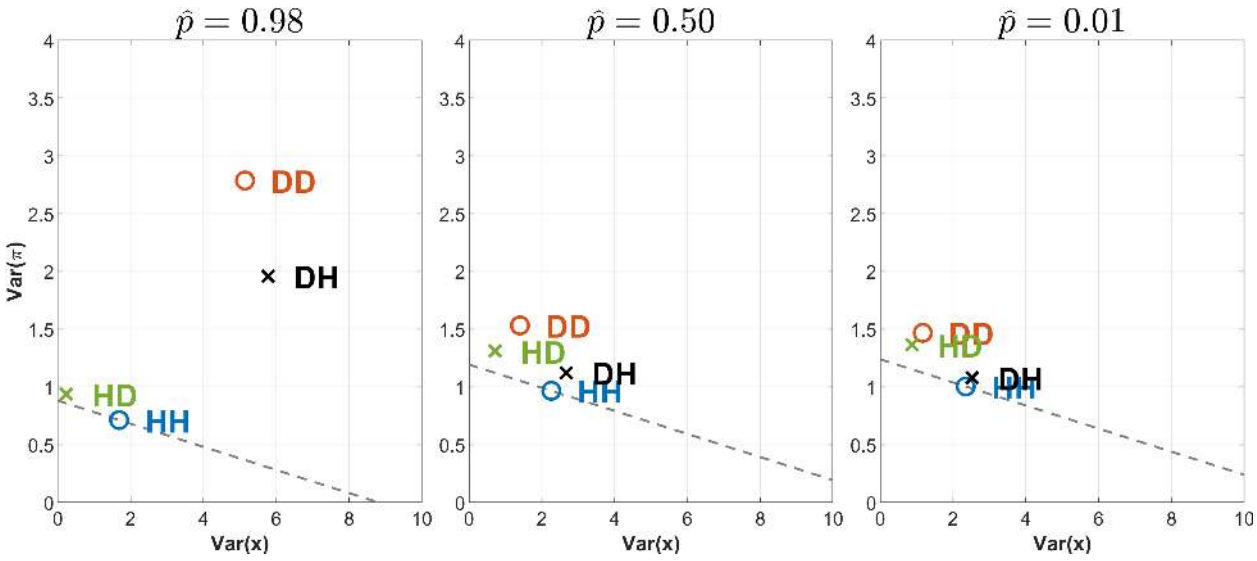


Figure 8: Policy Trade-offs and Perceived Regime Persistence (p)

Note: The figure plots welfare loss, defined as the weighted sum of the unconditional variance of inflation and the output gap, under varying perceived transition probabilities $\widehat{p}_{11} = \widehat{p}_{22} = \widehat{p}$. The panels assume high persistence ($\widehat{p} = 0.98$), moderate persistence ($\widehat{p} = 0.50$), and low persistence ($\widehat{p} = 0.01$), respectively. Blue circles denote the Hawk-Hawk (HH) regime, red circles denote Dove-Dove (DD), green crosses denote Hawk-Dove (HD), and black crosses denote Dove-Hawk (DH). The dashed line indicates the iso-loss curve for the minimum loss outcome.

Methodology The counterfactual simulation proceeds in three steps. First, I recover the sequence of structural shocks $\boldsymbol{\varepsilon}_t = [\varepsilon_{D,t}, \varepsilon_{S,t}, \varepsilon_{M,t}]'$ from the observed data. I treat the realized output gap, inflation, and policy rate as observables[20] and apply the Kalman filter and smoother to the state-space representation of the model. Specifically, the underlying model for this identification is the actual law of motion (61) where the sequence of regime-specific state-space matrices $\mathbf{A}_\ell^{\text{ALM}}$ and $\mathbf{B}_\ell^{\text{ALM}}$ follow the ones in Figure 7. Second, I construct counterfactual sequence of regimes by selectively altering the ALM matrices for periods of interest (1985-1998, 2016-2023) while holding the other parts of the sequence fixed. Third, I feed the identified structural shocks into this counterfactual sequence to generate hypothetical trajectories for inflation, output gap, and interest rates. The resulting differences are attributed to the difference between the original ALM matrices and the replacing counterfactual ALM matrices defined under alternative perceived or actual policy rules.

Counterfactual Scenarios Table 2 details the specific regime assumptions for the historical baseline and the two counterfactual exercises. The **Actual History** baseline reflects the

[20] I employ the shadow rates of Wu and Xia (2016) during the ZLB periods (2009Q1–2015Q4, 2020Q2–2022Q1) to avoid censoring issues.

Table 2: Counterfactual Exercise Design: Regime Assumptions

	Actual History (Baseline)		**Counterfactual 1** (Actual Dovish)		**Counterfactual 2** (Perceived Dovish)	
Period	Perc.	Fed	Perc.	Fed	Perc.	Fed
A. Great Moderation						
1985–1991	H	H	H	**D**	**D**	H
1992–1998	H	D	H	**D**	**D**	D
1999	H	H	H	**D**	**D**	H
B. Post-ZLB						
2016–2023	H	H	H	**D**	**D**	H

Note: **H** denotes a Hawkish regime and **D** denotes a Dovish regime. Bold entries indicate the regime parameter that is counterfactually altered from the historical baseline. **Actual History** follows the estimated structural break where the Fed temporarily deviated between 1992 and 1998. **Counterfactual 1** assumes the Fed adopts a consistent Dovish rule ($Fed = D$), but the private sector maintains its historical Hawkish perception. **Counterfactual 2** assumes the private sector consistently misperceives the policy as Dovish ($Perc. = D$), while the Fed follows its historical path.

estimated regime chronology in Figure 7: the private sector maintains a Hawkish perception throughout both sample periods, while the Federal Reserve temporarily deviates to a Dovish rule during the mid-1990s (1992–1998), consistent with the risk-management narratives of that era. This history constitutes the baseline sequence of ALM matrices $\mathbf{A}_\ell^{\text{ALM}}$ and $\mathbf{B}_\ell^{\text{ALM}}$ that identify the shocks.

Against this baseline, I evaluate two alternative scenarios:

1. **Counterfactual 1 (Actual Dovish):** I isolate the impact of the actual policy rule by assuming the Federal Reserve adopts a consistent Dovish stance ($s^f = 2$) (toward mean inflation expectation) throughout the entire period of interest, while private sector perceptions remain historically anchored ($s^p = 1$). This scenario tests whether a passive central bank can maintain stability if the public continues to believe in an active regime.

2. **Counterfactual 2 (Perceived Dovish):** I isolate the impact of expectations by assuming the private sector consistently misperceives the policy as Dovish ($s^p = 2$), while keeping the actual path of the Federal Reserve stance as Hawkish. This scenario quantifies the destabilizing cost of unanchored expectations, even when the central bank is actually fighting inflation.

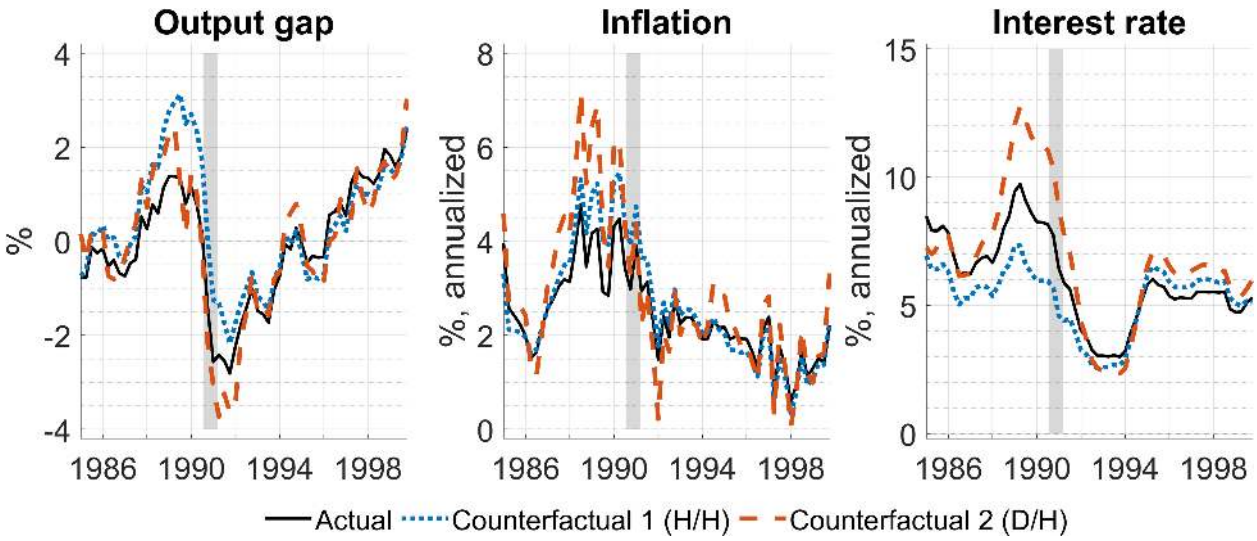


Figure 9: Counterfactual Dynamics 1 (Great Moderation, 1985-1999)

Note: The black solid line represents the actual data that is assumed to have been generated under the baseline Actual History in Table 2. The blue dotted line shows the counterfactual path under Actual Dovish (Counterfactual 1), and the red dashed line shows the path under Perceived Dovish (Counterfactual 2).

Results Figures 9 and 10 present the counterfactual trajectories for the Great Moderation (1985–1999) and the post-ZLB period, respectively. In each panel, the solid black line depicts the actual historical data. The dotted blue line plots **Counterfactual 1** (Actual Dovish), where the central bank implements a passive rule while perceptions remain historically anchored. The dashed red line plots **Counterfactual 2** (Perceived Dovish), where the private sector misperceives the policy as passive while the central bank follows its actual historical conduct.

For the Great Moderation (1985–1999), the historical baseline (solid black) is characterized by anchored Hawkish perceptions ($s^p = 1$), with the actual policy rule remaining Hawkish except for the 1992–1998 deviation. The dashed red line reveals that had the public perceived the Fed as Dovish ($s^p = 2$)—particularly during the late 1980s—inflation would have been significantly more volatile, despite the Fed's actual Hawkish behavior and the consequently tighter path of realized interest rates. While the actual level of inflation hovered around 4%, the counterfactual path is elevated around 7%. Similarly, the dotted blue line indicates that had the Fed adopted a consistent Dovish stance throughout the entire period (abandoning the Hawkish rule pre-1992), inflation volatility would also have increased. However, the significantly larger magnitude of the destabilization in the Perceived Dovish scenario highlights that "Good Perceived Policy" was a necessary condition for the onset of the Great Moderation.

While both counterfactuals generate excess volatility, the transmission mechanisms differ

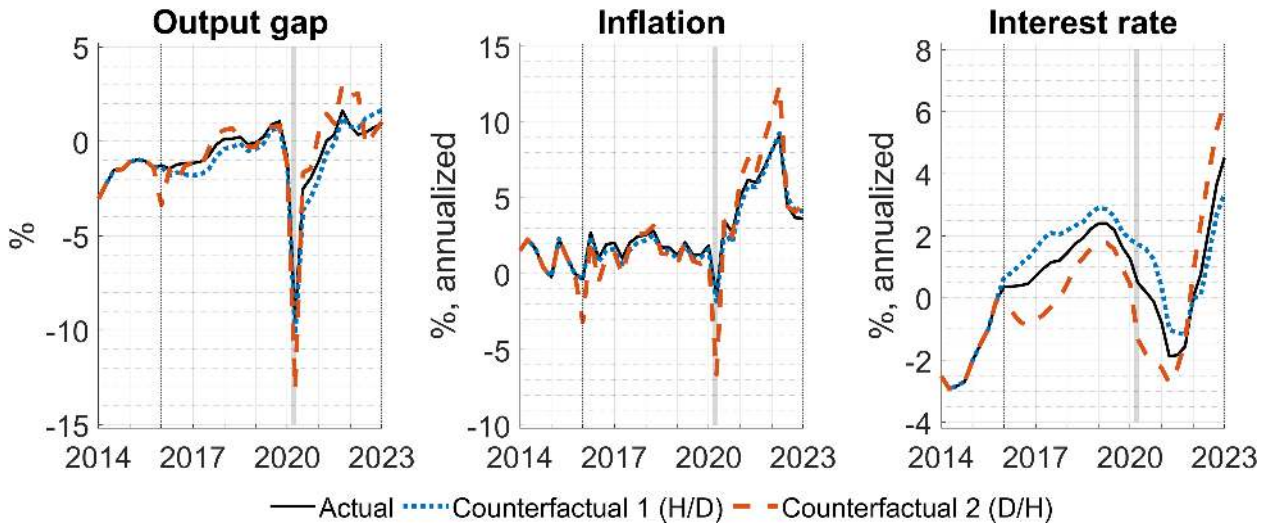


Figure 10: Counterfactual Dynamics 2 (Post-ZLB, 2016-2022)

Note: The black solid line represents the actual data that is assumed to have been generated under the baseline Actual History in Table 2. The blue dotted line shows the counterfactual path under Actual Dovish (Counterfactual 1), and the red dashed line shows the path under Perceived Dovish (Counterfactual 2).

fundamentally. In **Counterfactual 2** (Perceived Dovish), the instability is driven by the *expectation channel*. The belief in a passive policy regime unanchors private sector expectations $E_t^P[\pi_{t+1}]$ and $E_t^P[x_{t+1}]$, which feed directly into realized outcomes via the forward-looking terms in the IS and Phillips curves. In contrast, the volatility in **Counterfactual 1** (Actual Dovish) operates through the *direct policy channel*. The implementation of a passive rule results in a lower realized nominal interest rate i_t, which mechanically stimulates aggregate demand through the contemporaneous intertemporal substitution term $(-\tau i_t)$ and subsequently drives inflation through the Phillips curve (κx_t). The quantitative dominance of the inflation surge in Counterfactual 2 underscores the critical importance of the perceived monetary policy rule, establishing the expectation channel as the primary driver of equilibrium dynamics. Crucially, this suggests that the stability of the Great Moderation relied less on the implementation of the Hawkish actual rule itself, and more on the successful anchoring of beliefs through a Hawkish perception.

For the post-ZLB period (2016–2023), the analysis reinforces the dominance of the expectation channel, albeit through distinct transmission dynamics. **Counterfactual 2** (Perceived Dovish, red dashed line) illustrates the severe cost of unanchored expectations: had the public perceived the policy regime as Dovish during the inflationary shocks of 2021, the expectation channel would have amplified the transmission of shocks to inflation, shifting the peak of inflation from the realized 9% to approximately 12.5%. The underlying mechanism mirrors the Great Moderation scenario: the belief in passivity unanchors inflation expecta-

tions, effectively shifting the Phillips curve upward. Furthermore, the initial recessionary Covid shock in 2020 would have resulted in a temporary but deep deflation.

In contrast, **Counterfactual 1** (Actual Dovish, blue dotted line) reveals a striking insensitivity of inflation to the central bank's actual instrument setting. Counterintuitively, the counterfactual interest rate path in this scenario is *higher* than the historical baseline for the majority of the period. This reversal arises because the post-ZLB recovery was defined by persistent economic slack (a negative output gap). In such an environment, a Passive (Dovish) rule prescribes *less* aggressive accommodation (i.e., higher rates) than the Active (Hawkish) rule actually pursued by the Fed to close the gap. Yet, despite this structurally tighter counterfactual rate path operating through the direct policy channel, the inflation trajectory remains virtually indistinguishable from the actual history. This disconnect confirms that as long as perceptions remain anchored, shifts in the actual policy rule, even those that mechanically tighten the interest rate path, may have a limited impact on price stability. Consequently, the analysis suggests that the post-COVID inflation surge would have been exacerbated had the anchored Hawkish perception weakened.

6 Conclusion

This paper establishes that the private sector's perception of the monetary policy rule is the main driver of macroeconomic dynamics beyond the influence of the actual policy rule alone. Empirically, a central contribution is the identification of a structural covariance bias in standard forecast-based estimates. By considering a general environment where agents internalize both general equilibrium feedback and regime uncertainty, I show that traditional forecast-based regressions systematically underestimate the public's belief in policy responsiveness. Using a robust nowcast-based identification strategy, I recover the unbiased evolution of the perceived rule.

Theoretically, analysis through the lens of a Subjective Perception Markov-Switching New Keynesian framework reorients the conditions for stability. Equilibrium determinacy hinges on the *perceived Taylor principle* rather than the actual Taylor principle. Furthermore, the perceived rule affects macroeconomic dynamics in a manner distinct from the actual rule. To explain this result, I decompose policy transmission into two mechanisms. The first is the *Expectation Channel* governed by perceived rule. The second is the *Direct Policy Channel* governed by the actual rule. The analysis identifies the Expectation Channel as the decisive driver of equilibrium outcomes. Consequently, a passive actual rule can coexist with a determinate equilibrium if the private sector believes policy is active. Welfare analysis sheds further light on this by showing that the cost of misperception is endogenous to perceived

persistence. Even a perceived Dovish stance generates limited welfare losses if agents view the regime as strictly transitory.

Quantitatively, counterfactual simulations validate the importance of the Expectation Channel. I find that the Great Moderation was underpinned by strongly anchored Hawkish perceptions rather than the actual path and stance of policy rates alone. Conversely, a shift to a Dovish perception during 2021-2023 would have amplified the inflationary shocks via the Expectation Channel. These findings suggest that the success of modern monetary policy relies not merely on the mechanical adjustment of rates but on the successful management of private sector perceptions of the monetary policy rule.

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A Proofs and Derivations

A.1 Proof of Proposition 1

Let the data generating process be $i_{t+h|t} = \beta_{t,h}\pi_{t+h|t}$, where $\pi_{t+h|t} \equiv \mathbb{E}_t[\pi_{t+h}]$. The OLS estimator is defined as:

$$\beta_{OLS} = \frac{\text{Cov}\left(\beta_{t,h}\pi_{t+h|t}, \pi_{t+h|t}\right)}{\text{Var}\left(\pi_{t+h|t}\right)} \tag{63}$$

Expanding the covariance term in the numerator:

$$\begin{aligned}\text{Cov}\left(\beta_{t,h}\pi_{t+h|t}, \pi_{t+h|t}\right) &= \mathbb{E}\left[\beta_{t,h}\right]\text{Cov}\left(\pi_{t+h|t}, \pi_{t+h|t}\right) \\ &+ \mathbb{E}\left[\pi_{t+h|t}\right]\text{Cov}\left(\beta_{t,h}, \pi_{t+h|t}\right) + \mathbb{E}\left[\left(\beta_{t,h} - \mathbb{E}\left[\beta_{t,h}\right]\right)\left(\pi_{t+h|t} - \mathbb{E}\left[\pi_{t+h|t}\right]\right)^2\right]\end{aligned} \tag{64}$$

Assuming $\mathbb{E}\left[\pi_{t+h|t}\right] = 0$, the middle term vanishes, and the expression simplifies to:

$$\beta_{OLS} = \frac{\mathbb{E}\left[\beta_{t,h}\right]\text{Var}(\pi_{t+h|t}) + \mathbb{E}\left[\left(\beta_{t,h} - \mathbb{E}\left[\beta_{t,h}\right]\right)\pi_{t+h|t}^2\right]}{\text{Var}\left(\pi_{t+h|t}\right)} \tag{65}$$

$$= \mathbb{E}\left[\beta_{t,h}\right] + \frac{\mathbb{E}\left[\left(\beta_{t,h} - \mathbb{E}\left[\beta_{t,h}\right]\right)\pi_{t+h|t}^2\right]}{\text{Var}\left(\pi_{t+h|t}\right)} \tag{66}$$

This concludes the proof. ■

A.2 Proof of Proposition 2

Let the state space system be defined by $\pi_t = \mathbf{C}_{\pi,s_t}\mathbf{X}_t$ and $\mathbf{X}_t = \mathbf{A}_{s_t}\mathbf{X}_{t-1} + \mathbf{B}_{s_t}\boldsymbol{\varepsilon}_t$. The objective is to compute the conditional covariance $\text{Cov}\left[\phi_{\pi,s_{t+h}}, \pi_{t+h} \mid s_t = j\right]$. First, the conditional expectation of inflation given the future state k is defined as follows:

$$\pi_{k,t+h}^e \equiv \mathbb{E}\left[\pi_{t+h} \mid s_t = j, s_{t+h} = k\right] \tag{67}$$

$$= \mathbf{C}_{\pi,k}\mathbb{E}\left[\mathbf{X}_{t+h} \mid s_t = j, s_{t+h} = k\right] \tag{68}$$

$$= \mathbf{C}_{\pi,k}\mathbf{M}_h^{(j\to k)}\mathbf{X}_t \tag{69}$$

where $\mathbf{M}_h^{(j\to k)}$ is the expected propagation matrix conditional on starting in state j and ending in state k:

$$\mathbf{M}_h^{(j\to k)} = \frac{1}{P_{jk}^h} \sum_{m_1,\cdots,m_{h-1}} \left(\prod_{\ell=0}^{h-1} P_{m_\ell, m_{\ell+1}} \right) \mathbf{A}_{m_h} \mathbf{A}_{m_{h-1}} \cdots \mathbf{A}_{m_1} \tag{70}$$

with $m_0 = j$ and $m_h = k$. Using the definition of covariance:

$$\text{Cov}\left[\phi_{\pi,s_{t+h}}, \pi_{t+h} \mid s_t = j\right] = \mathbb{E}\left[\phi_{\pi,s_{t+h}} \pi_{t+h} \mid s_t = j\right] - \mathbb{E}\left[\phi_{\pi,s_{t+h}} \mid s_t = j\right] \mathbb{E}\left[\pi_{t+h} \mid s_t = j\right] \tag{71}$$

Expanding the terms using the law of iterated expectations over the future state $k \in \{1, 2\}$:

$$\mathbb{E}\left[\phi_{\pi,s_{t+h}} \pi_{t+h} \mid s_t = j\right] = \sum_k P_{jk}^h \phi_{\pi,k} \pi_{k,t+h}^e \tag{72}$$

$$\mathbb{E}\left[\pi_{t+h} \mid s_t = j\right] = \sum_k P_{jk}^h \pi_{k,t+h}^e \tag{73}$$

$$\mathbb{E}\left[\phi_{\pi,s_{t+h}} \mid s_t = j\right] = \sum_k P_{jk}^h \phi_{\pi,k} \tag{74}$$

Substituting these back into the covariance expression:

$$\text{Cov}_t = \sum_k P_{jk}^h \phi_{\pi,k} \pi_{k,t+h}^e - \left(\sum_k P_{jk}^h \pi_{k,t+h}^e \right) \left(\sum_k P_{jk}^h \phi_{\pi,k} \right) \tag{75}$$

$$\begin{aligned} &= P_{j1}^h \phi_{\pi,1} \pi_{1,t+h}^e + P_{j2}^h \phi_{\pi,2} \pi_{2,t+h}^e \\ &\quad - \left(P_{j1}^h \pi_{1,t+h}^e + P_{j2}^h \pi_{2,t+h}^e \right) \left(P_{j1}^h \phi_{\pi,1} + P_{j2}^h \phi_{\pi,2} \right) \end{aligned} \tag{76}$$

Expanding and collecting terms, noting that $P_{j2}^h = 1 - P_{j1}^h$:

$$\begin{aligned} \text{Cov}_t &= P_{j1}^h \pi_{1,t+h}^e \left[\phi_{\pi,1} - \left(P_{j1}^h \phi_{\pi,1} + P_{j2}^h \phi_{\pi,2} \right) \right] \\ &\quad + P_{j2}^h \pi_{2,t+h}^e \left[\phi_{\pi,2} - \left(P_{j1}^h \phi_{\pi,1} + P_{j2}^h \phi_{\pi,2} \right) \right] \end{aligned} \tag{77}$$

$$= P_{j1}^h \pi_{1,t+h}^e \left[P_{j2}^h (\phi_{\pi,1} - \phi_{\pi,2}) \right] + P_{j2}^h \pi_{2,t+h}^e \left[P_{j1}^h (\phi_{\pi,2} - \phi_{\pi,1}) \right] \tag{78}$$

$$= P_{j1}^h P_{j2}^h (\phi_{\pi,1} - \phi_{\pi,2}) \pi_{1,t+h}^e - P_{j1}^h P_{j2}^h (\phi_{\pi,1} - \phi_{\pi,2}) \pi_{2,t+h}^e \tag{79}$$

$$= P_{j1}^h (1 - P_{j1}^h)(\phi_{\pi,1} - \phi_{\pi,2})(\pi_{1,t+h}^e - \pi_{2,t+h}^e) \tag{80}$$

This concludes the proof. ■

A.3 Derivation of Corollary 2 (i.i.d. Case)

Using the method of undetermined coefficients, the inflation and output gap loadings on the exogenous shocks are solved under the assumption of i.i.d. regime switching. It is conjectured that the equilibrium laws of motion for inflation and the output gap are linear in the exogenous state vector $\mathbf{z}_t = [z_{Dt}, z_{St}, z_{Mt}]'$ with regime-dependent coefficients:

$$\pi_t = \mathbf{a}_{s_t}\mathbf{z}_t \tag{81}$$

$$x_t = \mathbf{b}_{s_t}\mathbf{z}_t \tag{82}$$

where $\mathbf{a}_{s_t}$ and $\mathbf{b}_{s_t}$ are 1×2 vectors corresponding to the regime $s_t \in \{1, 2\}$. Specifically, for the demand and supply components, the scalars $a_{s_t,k}$ and $b_{s_t,k}$ for $k \in \{D, S\}$ are solved. The resulting coefficients for the demand shock (z_D) are:

$$\begin{bmatrix} a_{1,D} \\ a_{2,D} \\ b_{1,D} \\ b_{2,D} \end{bmatrix} = \kappa\vartheta_D^{-1} \begin{bmatrix} 1 + \kappa\tau\phi_{\pi,2} \\ 1 + \kappa\tau\phi_{\pi,1} \\ (1 + \kappa\tau\phi_{\pi,2}) - \beta\rho_D[p(1 + \kappa\tau\phi_{\pi,2}) + (1-p)(1 + \kappa\tau\phi_{\pi,1})] \\ (1 + \kappa\tau\phi_{\pi,1}) - \beta\rho_D[p(1 + \kappa\tau\phi_{\pi,2}) + (1-p)(1 + \kappa\tau\phi_{\pi,1})] \end{bmatrix} \tag{83}$$

where the scalar denominator ϑ_D is defined as:

$$\vartheta_D = (1 + \kappa\tau\phi_{\pi,1})(1 + \kappa\tau\phi_{\pi,2}) + \rho_D(-\kappa\tau - \zeta)[(1-p)(1 + \kappa\tau\phi_{\pi,1}) + p(1 + \kappa\tau\phi_{\pi,2})] \tag{84}$$

Similarly, for the supply shock (z_S), the coefficients are given by:

$$\begin{bmatrix} a_{1,S} \\ a_{2,S} \\ b_{1,S} \\ b_{2,S} \end{bmatrix} = \begin{bmatrix} \frac{(1-\rho_S)(1+\kappa\tau\phi_{\pi,2})}{\vartheta_S} \\ \frac{(1-\rho_S)(1+\kappa\tau\phi_{\pi,1})}{\vartheta_S} \\ \frac{(1-\rho_S)[(1-\beta p\rho_S)(1+\kappa\tau\phi_{\pi,2})-\beta(1-p)\rho_S(1+\kappa\tau\phi_{\pi,1})]-\vartheta_S}{\kappa\vartheta_S} \\ \frac{(1-\rho_S)[-\beta p\rho_S(1+\kappa\tau\phi_{\pi,2})+(1-\beta(1-p)\rho_S)(1+\kappa\tau\phi_{\pi,1})]-\vartheta_S}{\kappa\vartheta_S} \end{bmatrix} \tag{85}$$

where the denominator ϑ_S is defined as:

$$\vartheta_S = (1 + \kappa\tau\phi_{\pi,1})(1 + \kappa\tau\phi_{\pi,2}) + \rho_S\zeta_S[p(1 + \kappa\tau\phi_{\pi,2}) + (1-p)(1 + \kappa\tau\phi_{\pi,1})] \tag{86}$$

Substituting these coefficients into the expressions for $a_{1,D} - a_{2,D}$ and $a_{1,S} - a_{2,S}$ yields the results presented in Corollary 2. ■

A.4 Proof of Proposition 3

The private sector's perceived dynamics are governed by equations (48)—(50). The subjective model is determinate under the perceived Taylor principle (52) and (53). The proofs are straightforward as noted in Galí (2015). WLOG suppose there is only demand shock $x_{t|t} = a_1 z_{Dt}$ and $\pi_{t|t} = b_1 z_{Dt}$, where a_1 and b_1 are coefficients determined by the perceived parameters. Given the AR(1) structure of the demand shock, the subjective expectations are:

$$E_t^{\mathrm{P}}\left[x_{t+1}\right] = a_1 \rho_D z_{Dt}, \quad E_t^{\mathrm{P}}\left[\pi_{t+1}\right] = b_1 \rho_D z_{Dt} \tag{87}$$

The actual economy features the same IS and Phillips curves but with the Fed implementing the actual Taylor rule (51). Substituting the subjective expectations into the IS curve:

$$x_t = a_1 \rho_D z_{Dt} - \tau\left(\phi_\pi^{\mathrm{F}} \mathbb{E}_t\left[\pi_{t+1}\right] + \phi_x^{\mathrm{F}} \mathbb{E}_t\left[x_{t+1}\right] - b_1 \rho_D z_{Dt}\right) + z_{Dt} \tag{88}$$

Rearranging:

$$\tau\left(\phi_\pi^{\mathrm{F}} \mathbb{E}_t\left[\pi_{t+1}\right] + \phi_x^{\mathrm{F}} \mathbb{E}_t\left[x_{t+1}\right]\right) = -x_t + \left(a_1 \rho_D + \tau b_1 \rho_D + 1\right) z_{Dt} \tag{89}$$

From the Phillips curve, $\pi_t = \beta b_1 \rho_D z_{Dt} + \kappa x_t$, which implies:

$$x_t = \frac{1}{\kappa}\pi_t - \frac{\beta b_1 \rho_D}{\kappa} z_{Dt} \tag{90}$$

Combining these equations with the law of motion for the demand shock yields the state-space system:

$$\begin{bmatrix} \tau\phi_x^{\mathrm{F}} & \tau\phi_\pi^{\mathrm{F}} & -\left(a_1\rho_D + \tau b_1 \rho_D + 1\right) \\ 0 & 0 & -\frac{\beta b_1 \rho_D}{\kappa} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t x_{t+1} \\ \mathbb{E}_t \pi_{t+1} \\ z_{Dt} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -\frac{1}{\kappa} & 0 \\ 0 & 0 & \rho_D \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ z_{Dt-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \varepsilon_{Dt} \tag{91}$$

To compute the eigenvalues, invert the right-hand side matrix and multiply through:

$$\begin{bmatrix} -1 & 0 & 0 \\ 1 & -\frac{1}{\kappa} & 0 \\ 0 & 0 & \rho_D \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ -\kappa & -\kappa & 0 \\ 0 & 0 & \frac{1}{\rho_D} \end{bmatrix} \tag{92}$$

This gives:

$$\begin{bmatrix} -\tau\phi_x^{\mathrm{F}} & -\tau\phi_\pi^{\mathrm{F}} & (a_1\rho_D + \tau b_1\rho_D + 1) \\ -\kappa\tau\phi_x^{\mathrm{F}} & -\kappa\tau\phi_\pi^{\mathrm{F}} & \kappa\left(a_1\rho_D + \tau b_1\rho_D + 1\right) + \beta b_1\rho_D \\ 0 & 0 & \frac{1}{\rho_D} \end{bmatrix} \begin{bmatrix} \mathbb{E}_t x_{t+1} \\ \mathbb{E}_t \pi_{t+1} \\ z_{Dt} \end{bmatrix} = \begin{bmatrix} x_t \\ \pi_t \\ z_{Dt-1} \end{bmatrix} + \cdots \tag{93}$$

The eigenvalues of this system are:

$$\lambda_1 = \frac{1}{\rho_D} > 1, \quad \lambda_2 = 0 < 1, \quad \lambda_3 = -\tau\left(\phi_x^{\mathrm{F}} + \kappa\phi_\pi^{\mathrm{F}}\right) \tag{94}$$

Given two jump variables, we need two eigenvalues inside the unit circle. The determinacy condition reduces to:

$$\left|\tau\left(\phi_x^{\mathrm{F}} + \kappa\phi_\pi^{\mathrm{F}}\right)\right| < 1 \tag{95}$$

Under standard parameter restrictions ($\tau, \kappa, \phi_x^{\mathrm{F}}, \phi_\pi^{\mathrm{F}} > 0$), this simplifies to $0 < \tau\left(\phi_x^{\mathrm{F}} + \kappa\phi_\pi^{\mathrm{F}}\right) < 1$. Combined with the perceived Taylor principle (52), this establishes the determinacy conditions in Proposition 3. ■

A.5 Derivation of the Actual Law of Motion for the MSNK Model

In this appendix, the structural matrices for the actual law of motion described in Section 5.2 are derived. Since the actual Taylor rule is also based on the private sector's expectation, there is no rational expectation component in the state-space. This means that after substituting the subjective expectations, we are left with

$$\mathbf{H}_\ell \mathbf{y}_t + \mathbf{F}_\ell \mathbf{y}_{t-1} + \mathbf{C}_\ell \boldsymbol{\varepsilon}_t = \mathbf{0} \tag{96}$$

Provided that $\mathbf{H}_\ell$ is non-singular, the Actual Law of Motion (ALM) is simply given by:

$$\mathbf{y}_t = \underbrace{-\mathbf{H}_\ell^{-1}\mathbf{F}_\ell}_{\mathbf{A}_\ell^{\mathrm{ALM}}} \mathbf{y}_{t-1} + \underbrace{-\mathbf{H}_\ell^{-1}\mathbf{C}_\ell}_{\mathbf{B}_\ell^{\mathrm{ALM}}} \boldsymbol{\varepsilon}_t \tag{97}$$

Hence, it boils down to specifying the three loading matrices.

Let the actual joint state be $\ell \in \mathcal{S}$, corresponding to the pair $(s_t^p = j, s_t^f = k)$. The private sector forms expectations based on the perceived state j. Using the derived behavioral equation $E_t^{\mathrm{P}}[\mathbf{y}_{t+1}] = \boldsymbol{\Gamma}_{y,j}\mathbf{y}_{t-1} + \boldsymbol{\Gamma}_{\varepsilon,j}\boldsymbol{\varepsilon}_t$, I decompose the inflation and output gap expectations.

Let e_π and e_x be selection vectors such that $\pi = e_\pi \mathbf{y}$ and $x = e_x \mathbf{y}$. The expectations are:

$$E_t^{\mathrm{P}}\left[\pi_{t+1}\right] = e_\pi\left(\mathbf{\Gamma}_{y,j}\mathbf{y}_{t-1} + \mathbf{\Gamma}_{\varepsilon,j}\boldsymbol{\varepsilon}_t\right) \tag{98}$$

$$E_t^{\mathrm{P}}\left[x_{t+1}\right] = e_x\left(\mathbf{\Gamma}_{y,j}\mathbf{y}_{t-1} + \mathbf{\Gamma}_{\varepsilon,j}\boldsymbol{\varepsilon}_t\right) \tag{99}$$

I define the aggregate expectation coefficients for the structural equations as:

$$\tilde{\mathbf{\Gamma}}_A = \theta e_x \mathbf{\Gamma}_{y,j} + \tau e_\pi \mathbf{\Gamma}_{y,j} \qquad \tilde{\mathbf{\Gamma}}_B = \theta e_x \mathbf{\Gamma}_{\varepsilon,j} + \tau e_\pi \mathbf{\Gamma}_{\varepsilon,j} \tag{100}$$

$$\tilde{\mathbf{\Gamma}}_C = \beta e_\pi \mathbf{\Gamma}_{y,j} \qquad \tilde{\mathbf{\Gamma}}_D = \beta e_\pi \mathbf{\Gamma}_{\varepsilon,j} \tag{101}$$

and for the Taylor Rule (with smoothing parameter $\Lambda_\ell = 1 - \rho_{1,\ell}^{\mathrm{F}} - \rho_{2,\ell}^{\mathrm{F}}$):

$$\tilde{\mathbf{\Gamma}}_E = \Lambda_\ell\left(\phi_{\pi,\ell}^{\mathrm{F}} e_\pi \mathbf{\Gamma}_{y,j} + \phi_{x,\ell}^{\mathrm{F}} e_x \mathbf{\Gamma}_{y,j}\right) \qquad \tilde{\mathbf{\Gamma}}_F = \Lambda_\ell\left(\phi_{\pi,\ell}^{\mathrm{F}} e_\pi \mathbf{\Gamma}_{\varepsilon,j} + \phi_{x,\ell}^{\mathrm{F}} e_x \mathbf{\Gamma}_{\varepsilon,j}\right) \tag{102}$$

These expressions are substituted into the actual structural equations.

1. IS Curve:

$$\begin{aligned} x_t &= \theta E_t^{\mathrm{P}}\left[x_{t+1}\right] - \tau\left(i_t - E_t^{\mathrm{P}}\left[\pi_{t+1}\right]\right) + z_{Dt} \\ 0 &= -x_t - \tau i_t + z_{Dt} + \tilde{\mathbf{\Gamma}}_A \mathbf{y}_{t-1} + \tilde{\mathbf{\Gamma}}_B \boldsymbol{\varepsilon}_t \end{aligned} \tag{103}$$

2. Phillips Curve:

$$\begin{aligned} \pi_t &= \beta E_t^{\mathrm{P}}\left[\pi_{t+1}\right] + \kappa x_t + z_{St} \\ 0 &= \kappa x_t - \pi_t + z_{St} + \tilde{\mathbf{\Gamma}}_C \mathbf{y}_{t-1} + \tilde{\mathbf{\Gamma}}_D \boldsymbol{\varepsilon}_t \end{aligned} \tag{104}$$

3. Actual Taylor Rule: The actual rule includes the explicit monetary policy shock ε_{Mt} which is absent from $\boldsymbol{\varepsilon}_t$.

$$\begin{aligned} i_t &= \Lambda_\ell\left[\phi_{\pi,\ell}^{\mathrm{F}} E_t^{\mathrm{P}}\left[\pi_{t+1}\right] + \phi_{x,\ell}^{\mathrm{F}} E_t^{\mathrm{P}}\left[x_{t+1}\right]\right] + \rho_{1,\ell}^{\mathrm{F}} i_{t-1} + \rho_{2,\ell}^{\mathrm{F}} i_{t-2} + \varepsilon_{Mt} \\ 0 &= -i_t + \tilde{\mathbf{\Gamma}}_E \mathbf{y}_{t-1} + \tilde{\mathbf{\Gamma}}_F \boldsymbol{\varepsilon}_t + \rho_{1,\ell}^{\mathrm{F}} i_{t-1} + \rho_{2,\ell}^{\mathrm{F}} i_{t-2} + \varepsilon_{Mt} \end{aligned} \tag{105}$$

This system is written in the matrix form $\mathbf{0} = \mathbf{H}_\ell \mathbf{y}_t + \mathbf{F}_\ell \mathbf{y}_{t-1} + \mathbf{C}_\ell \boldsymbol{\varepsilon}_t$. The matrix $\mathbf{H}_\ell$ is:

$$\mathbf{H}_\ell = \begin{bmatrix} -1 & 0 & -\tau & 0 & 1 & 0 \\ \kappa & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{106}$$

The matrix $\mathbf{F}_\ell$ incorporates the expectation effects plus the structural persistence:

$$\mathbf{F}_\ell = \begin{bmatrix} \tilde{\mathbf{\Gamma}}_A \\ \tilde{\mathbf{\Gamma}}_C \\ \tilde{\mathbf{\Gamma}}_E + \mathbf{R}_{\text{lag}} \\ \mathbf{I}_{i_{t-1}} \\ \mathbf{I}_D \\ \mathbf{I}_S \end{bmatrix} \tag{107}$$

where $\mathbf{R}_{\text{lag}}$ adds $\rho^{\text{F}}_{1,\ell}$ to the i_{t-1} column and $\rho^{\text{F}}_{2,\ell}$ to the i_{t-2} column. The lower blocks $\mathbf{I}_{(\cdot)}$ represent the AR processes for i_{t-1}, z_D, and z_S (e.g., $i_{t-1} = i_{t-1}$, $z_{Dt} = \rho_D z_{Dt-1} + \varepsilon_{Dt}$).

Finally, the shock matrix $\mathbf{C}_\ell$ is:

$$\mathbf{C}_\ell = \begin{bmatrix} \tilde{\mathbf{\Gamma}}_{B,D} & \tilde{\mathbf{\Gamma}}_{B,S} & 0 \\ \tilde{\mathbf{\Gamma}}_{D,D} & \tilde{\mathbf{\Gamma}}_{D,S} & 0 \\ \tilde{\mathbf{\Gamma}}_{F,D} & \tilde{\mathbf{\Gamma}}_{F,S} & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{108}$$

where $\tilde{\mathbf{\Gamma}}_{B,D}$ denotes the column of $\tilde{\mathbf{\Gamma}}_B$ corresponding to the demand shock. Note that the $(3,3)$ element is exactly 1, representing the actual monetary policy shock, while the perceived impact of this shock via $\tilde{\mathbf{\Gamma}}_F$ is zero. ■

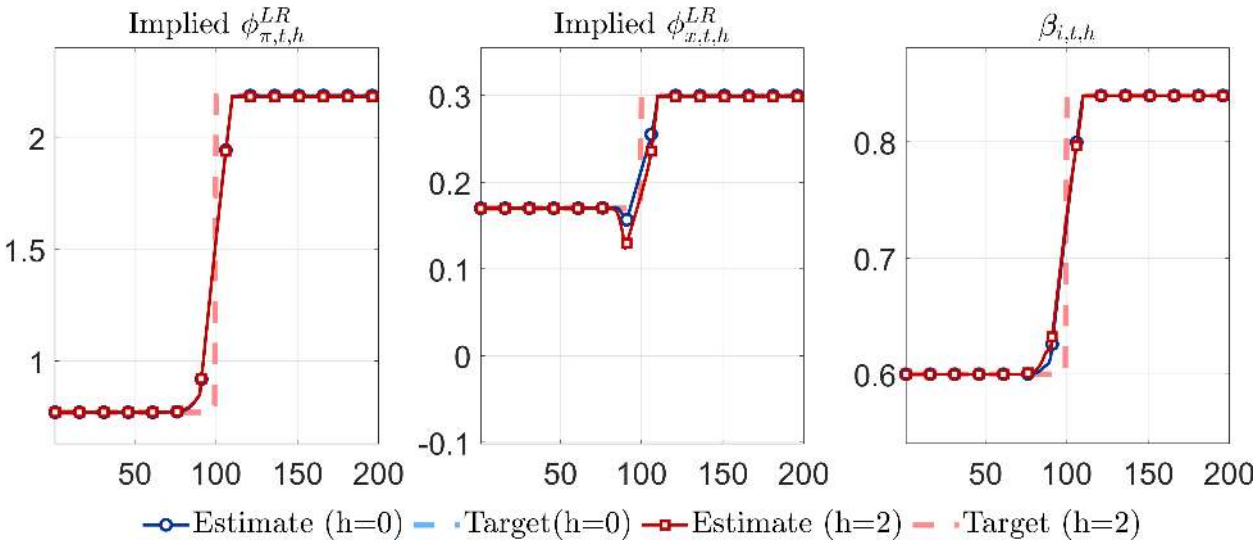


Figure 11: TVP Estimation with No Switching ($p = 1.00$)

Note: This figure compares the estimation results of equation (28) for $h = 0$ (blue) and $h = 2$ (red) under $p = 1.00$. The dashed lines indicate the target conditional expectations $\mathbb{E}_t[\phi_{\pi,s_{t+h}}]$. The solid lines with markers indicate the estimated long-run coefficients.

B Additional Simulation Results

B.1 TVP estimation with no regime uncertainty

Figure 11 serves as a consistency check for the case of permanent regimes ($p = 1.00$). In the absence of regime uncertainty, both the nowcast and forecast specifications recover the true parameters. However, as documented in Section 4, the empirical data strongly reject the null hypothesis of constant parameters.

B.2 Endogenous Regime Switching

This section explores the robustness of the identification strategy to endogenous regime switching, where the probability of a policy regime shift depends on the state of the economy. Specifically, I consider a specification where the transition probabilities are functions of the current inflation level.

Simulation Setup The simulation follows the baseline Subjective Perception MSNK model with a state-dependent transition matrix $\mathbf{P}(\pi_t)$. The diagonal elements $p_{jj}(\pi_t)$, representing regime persistence, are modeled using a logistic function centered around the inflation target. Let $L(\pi_t)$ be the logistic component:

$$L(\pi_t) = \frac{1}{1 + \exp\left(-\frac{g}{\sigma}(\pi_t - c_\pi)\right)} \tag{109}$$

where g governs the steepness and σ is a scaling parameter. The regime-specific persistence probabilities are defined as:

$$p_{11}(\pi_t) = \text{clip}\left(\bar{p} + \delta_p \left(L(\pi_t) - 0.5\right)\right) \tag{110}$$

$$p_{22}(\pi_t) = \text{clip}\left(\bar{p} - \delta_p \left(L(\pi_t) - 0.5\right)\right) \tag{111}$$

where the function $\text{clip}(x) = \min(\max(x, 0.001), 0.999)$ ensures probabilities remain within valid bounds. $\bar{p}$ is the steady-state persistence (0.95), and δ_p controls the maximum amplitude of the probability deviation. Under this specification, higher inflation ($\pi_t > c_\pi$) increases the persistence of the Hawkish regime (p_{11}) and decreases the persistence of the Dovish regime (p_{22}), making a switch to (or retention of) active policy more likely.

Results The TVP estimation results on the simulated data confirm the findings from the exogenous switching case:

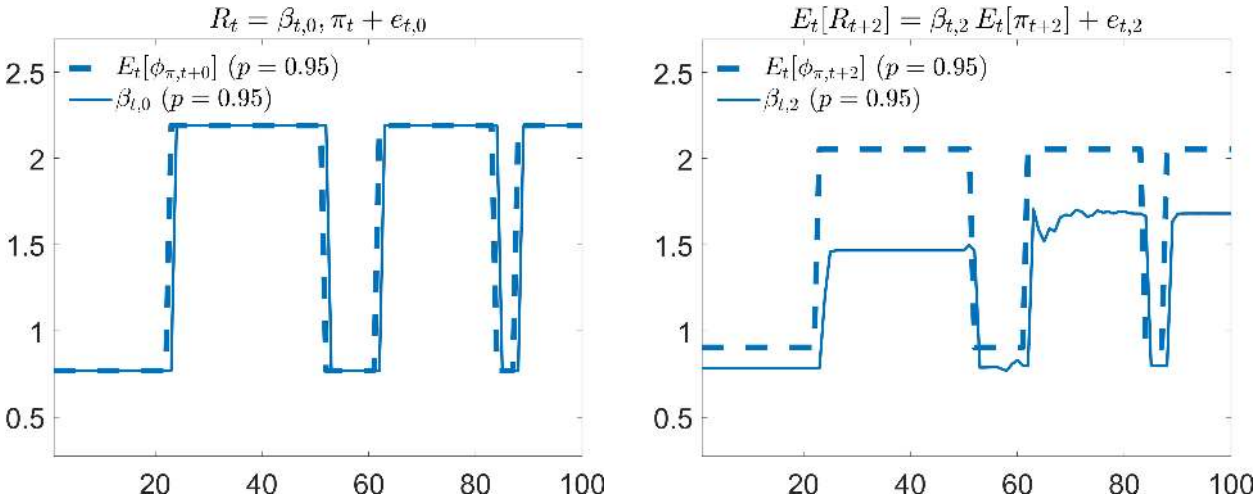
1. **Nowcast ($h = 0$):** The nowcast-based estimator successfully recovers the true time-varying perceived policy coefficient $\phi_{\pi,t}$ without systematic bias.

2. **Forecast ($h = 2$):** The forecast-based estimates exhibit a significant bias, confirming that the "perceived covariance" bias persists when agents internalize endogenous regime dynamics.

B.3 Instrumental Variable Estimation

A potential concern with the OLS results presented in Section 3 is that the bias might stem from standard simultaneity issues—specifically, the correlation between inflation and monetary policy shocks—rather than the structural "perceived covariance" channel. To address this, I investigate whether standard Instrumental Variable (IV) strategies can recover the true policy parameter in the presence of regime switching.

Methodology I estimate the policy rule on data simulated from the MSNK model using the Generalized Method of Moments (GMM), following the standard approach in the literature (Clarida, Gali, and Gertler, 2000). The estimating equation is specified in terms of



(a) Nowcast ($h = 0$) (b) Forecast ($h = 2$)

Figure 12: TVP Estimation under Endogenous Regime Switching

Note: This figure displays the TVP estimation results on data simulated from the endogenous regime switching model described in Appendix E. The left panel shows the estimates using nowcasts ($h = 0$), and the right panel shows the estimates using 2-period ahead forecasts ($h = 2$). The solid colored lines represent the posterior median of the estimated time-varying coefficient $\beta_{t,h}$, while the dashed lines represent the true model-implied conditional expectation $E_t[\phi_{\pi,t+h}]$. Shaded regions indicate 68% confidence intervals.

h-period ahead expectations, consistent with the identification of the perceived rule:

$$\mathbb{E}_t[i_{t+h}] = \beta_{IV}\mathbb{E}_t[\pi_{t+h}] + u_{t,h} \tag{112}$$

I consider two distinct sets of instruments $\mathbf{Z}_t \in \mathcal{I}_t$:

1. **Lagged Variables:** $\mathbf{Z}_t = [\pi_{t-1}, \pi_{t-2}]'$. This follows the standard New Keynesian identification strategy assuming that lagged variables are orthogonal to the current monetary policy shock (Clarida, Gali, and Gertler, 2000).

2. **Structural Shocks:** $\mathbf{Z}_t = [z_{D,t}, z_{S,t}]'$. This set uses the realized non-monetary structural shocks (demand and supply) as instruments. This approach mirrors the identification logic of Carvalho, Nechio, and Tristao (2021), who use identified exogenous shocks to instrument for inflation.

Results To conserve space, result for $h = 2$ is reported. Figure 13 compares the OLS estimates with the two GMM-IV estimators across varying degrees of regime persistence p. The results demonstrate that the IV estimates (dashed lines) are virtually indistinguishable from the biased OLS estimates (solid blue line). As regime uncertainty increases (p decreases), all estimators systematically diverge from the true unconditional mean (black dashed line).

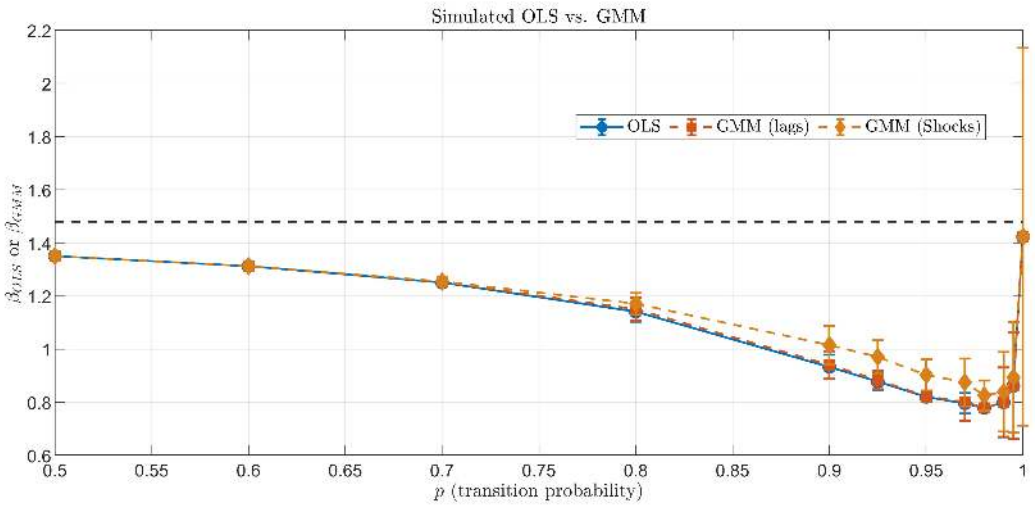


Figure 13: Comparison of OLS and IV Estimates under Regime Uncertainty

Note: The figure displays Monte Carlo estimates of the policy coefficient β across varying degrees of regime persistence p for the forecast horizon $h = 2$. The blue line represents OLS estimates. The red and yellow dashed lines represent GMM estimates using lagged variables and structural shocks as instruments, respectively. The black dashed line indicates the true unconditional mean of the policy parameter.

This failure of IV estimation confirms that the bias is not driven by standard simultaneity between $\mathbb{E}_t[\pi_{t+h}]$ and an exogenous policy shock. Instead, as derived in Proposition 1, the effective "error term" in this regression contains the conditional covariance between the policy parameter and inflation, $\text{Cov}_t(\phi_{\pi,t+h}, \pi_{t+h})$. Because this covariance is an endogenous object that depends on the state of the economy, it is correlated with both the regressors (expected inflation) *and* the valid instruments (lags or supply shocks). Consequently, the exclusion restriction is structurally violated, rendering standard IV methods ineffective for correcting the perceived covariance bias.

B.4 SP-MSNK Model with Rational Expectation of the Fed

In the baseline model presented in Section 5, the Fed is assumed to respond to the private sector's expectations ($E_t^{\text{P}}[\cdot]$). This assumption simplifies the solution of the actual law of motion to a recursive, backward-looking system. In this section, I relax this assumption and consider a central bank that operates under Full Information Rational Expectations (FIRE). Specifically, the Fed knows the true structure of the economy, including the agents' misperceived law of motion, and responds to the mathematical rational expectation of future variables ($\mathbb{E}_t[\cdot]$). This case is meaningful as the only departure from the standard FIRE 3-eq NK model would be the misspecified monetary policy rule. The change in determinacy codntions will be solely attributed to the separaton of perceived and actual rules.

Model Setup The private sector's behavior remains identical to the baseline. Agents form expectations using the Perceived Law of Motion (PLM) defined by the matrices $\widehat{\mathbf{A}}_{s_t^p}$ and $\widehat{\mathbf{B}}_{s_t^p}$, and the subjective transition matrix $\widehat{\mathbf{P}}$. Consequently, the behavioral equations for private expectations are predetermined linear functions of the state vector:

$$E_t^{\mathrm{P}}\left[\mathbf{y}_{t+1}\right] = \mathbf{\Gamma}_{y,s_t^p}\mathbf{y}_{t-1} + \mathbf{\Gamma}_{\varepsilon,s_t^p}\boldsymbol{\varepsilon}_t \tag{113}$$

where $\boldsymbol{\varepsilon}_t$ is the vector of actual structural shocks.

The actual economy is governed by the joint state $s_t = (s_t^p, s_t^f)$. The IS and Phillips curves are structurally identical to the baseline, as they describe private sector decision-making rules:

$$x_t = \theta E_t^{\mathrm{P}}\left[x_{t+1}\right] - \tau\left(i_t - E_t^{\mathrm{P}}\left[\pi_{t+1}\right]\right) + z_{Dt} \tag{114}$$

$$\pi_t = \beta E_t^{\mathrm{P}}\left[\pi_{t+1}\right] + \kappa x_t + z_{St} \tag{115}$$

However, the Federal Reserve's reaction function is now specified as:

$$i_t = \left(1 - \rho_{s_t^f}^{\mathrm{F}}\right)\left[\phi_{\pi,s_t^f}^{\mathrm{F}}\mathbb{E}_t\left[\pi_{t+1}\right] + \phi_{x,s_t^f}^{\mathrm{F}}\mathbb{E}_t\left[x_{t+1}\right]\right] + \rho_{1,s_t^f}^{\mathrm{F}} i_{t-1} + \rho_{2,s_t^f}^{\mathrm{F}} i_{t-2} + \varepsilon_{Mt} \tag{116}$$

where $\mathbb{E}_t[\cdot]$ denotes the mathematical expectation operator conditional on the actual joint regime s_t and the actual transition matrix $\mathbf{P}$. I assume that the Fed's state s_t^f transitions with the same probability as $\widehat{\mathbf{P}}$, which gives an implied actual transition probability matrix $\mathbf{P}$. However, the results are robust to alternative characterization of the actual transition probability matrix $\mathbf{P}$ since the perceived transition probability is what eventually matters for expectation.

Derivation and Solution Substituting the private sector's predetermined expectations into the IS and Phillips curves allow us to express equations (114) and (115) as:

$$0 = -x_t - \tau i_t + z_{Dt} + \tilde{\mathbf{\Gamma}}_{A,\ell}\mathbf{y}_{t-1} + \tilde{\mathbf{\Gamma}}_{B,\ell}\boldsymbol{\varepsilon}_t \tag{117}$$

$$0 = \kappa x_t - \pi_t + z_{St} + \tilde{\mathbf{\Gamma}}_{C,\ell}\mathbf{y}_{t-1} + \tilde{\mathbf{\Gamma}}_{D,\ell}\boldsymbol{\varepsilon}_t \tag{118}$$

where ℓ indexes the joint state s_t. The aggregate coefficient matrices $\tilde{\mathbf{\Gamma}}$ are defined analogously to Appendix A.4 (e.g., $\tilde{\mathbf{\Gamma}}_{A,\ell} = \theta e_x \mathbf{\Gamma}_{y,s_t^p} + \tau e_\pi \mathbf{\Gamma}_{y,s_t^p}$).

Combining these with the RE Taylor rule (116), the full system can be written in a

standard Markov-Switching Rational Expectations form:

$$\mathbf{A}_{\ell,0}\mathbf{y}_t = \mathbf{A}_{\ell,1}\mathbf{y}_{t-1} + \mathbf{A}_{\ell,2}\mathbb{E}_t[\mathbf{y}_{t+1}] + \mathbf{A}_{\ell,3}\boldsymbol{\varepsilon}_t \tag{119}$$

Unlike the baseline model, where the actual law of motion reduces to a purely backward-looking system, this system retains forward-looking elements due to the Fed's reaction function. I solve this system using the algorithm of Maih (2015).

Figure 14 and 15 compare the counterfactual simulation results of the baseline model (Fed responds to E^P) against the RE-Fed model (Fed responds to $\mathbb{E}$). PE Fed refers to the Fed responding to the private sector's subjective expectation (dashed blue or red). The dashed green line represents the result under the rational expectation of the actual rule. The dynamics are virtually indistinguishable. The marginal impact of the Fed adopting Rational Expectations and, hence, the disagreement in expectation is negligible compared to the impact of private sector misperceptions about the rule.

B.5 Robustness to Calibration

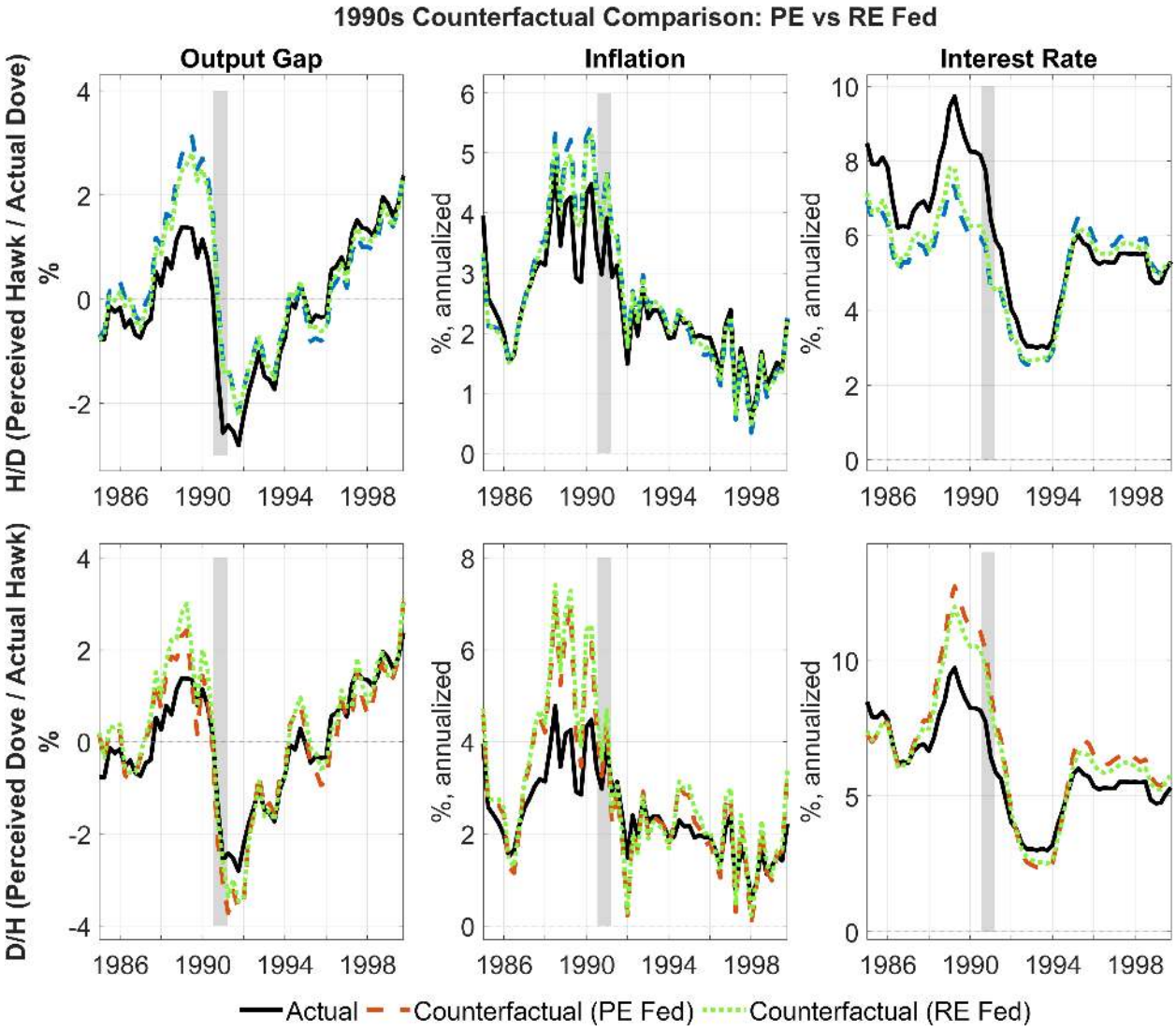


Figure 14: Counterfactual simulations: Fed Responding to E^P vs. $\mathbb{E}$

Note: The figure compares impulse responses to a demand shock under the baseline specification (blue solid line), where the Fed responds to private sector expectations E_t^P, and the alternative specification (red dashed line), where the Fed responds to rational expectations $\mathbb{E}_t$.

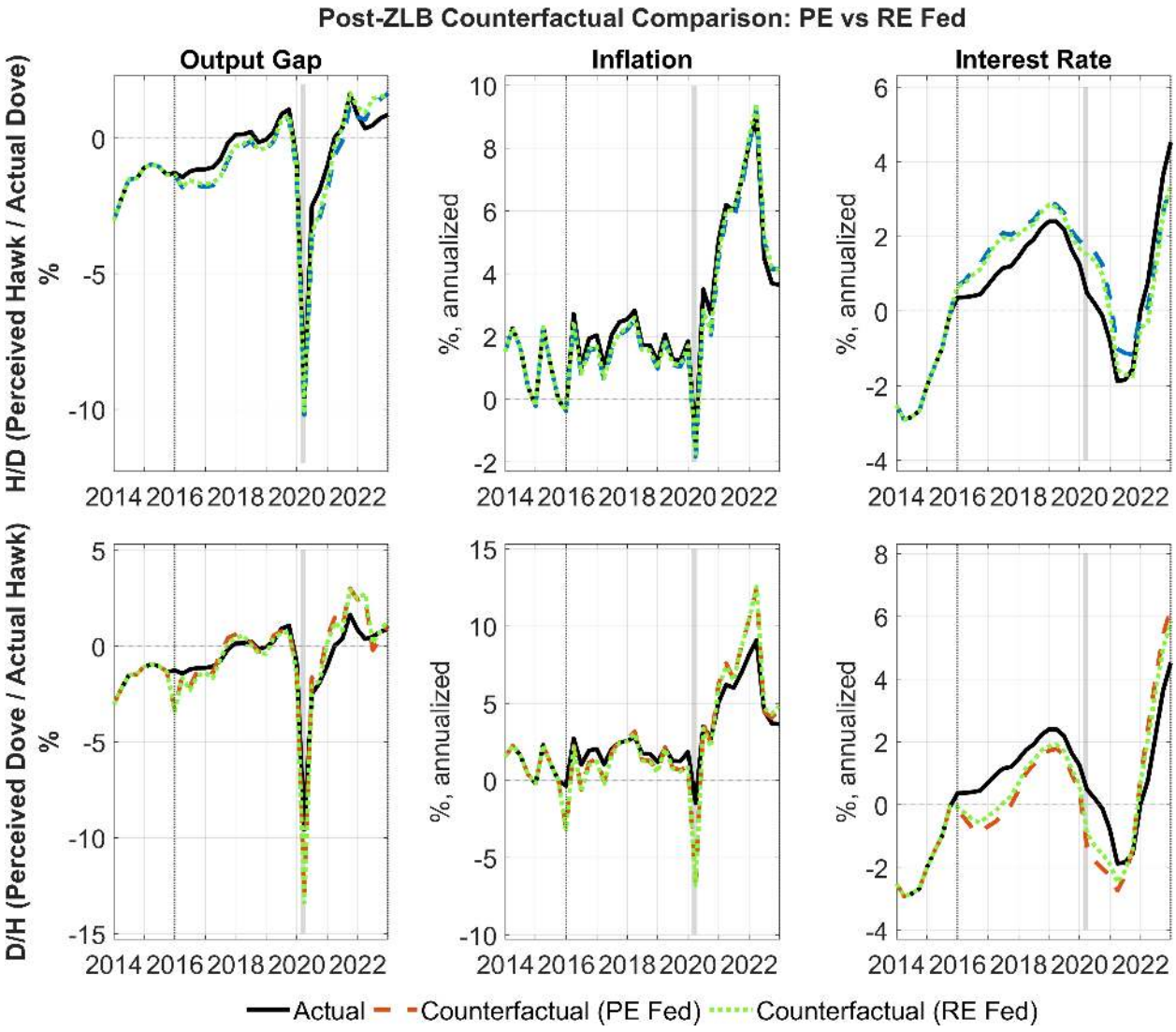


Figure 15: Counterfactual simulations: Fed Responding to E^P vs. $\mathbb{E}$

Note: The figure replicates the Perceived Dovish counterfactual (Counterfactual 2) from Section 5.5. The red dashed line represents the baseline counterfactual (Fed responds to E^P), and the green dotted line represents the case where the Fed responds to $\mathbb{E}_t$.

C Estimation Procedure

The perceived monetary policy rule is estimated using a Time-Varying Parameter (TVP) model with Stochastic Volatility (SV). I employ a "stacked" measurement equation pooling monthly and quarterly survey data within each period to improve identification. The estimation proceeds in three steps:

1. **State-Space Representation:** Let $t = 1, \ldots, T$ index quarters and N be the number of survey observations in quarter t. The measurement equation stacks these observations:

$$\mathbf{i}_{t+h|t} = \mathbf{X}_{t,h}\boldsymbol{\Theta}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}\left(\mathbf{0}, e^{h_t}\mathbf{I}_N\right) \tag{120}$$

   where $\boldsymbol{\Theta}_t$ follows a random walk with regime-dependent covariance $\boldsymbol{\Sigma}_{\nu,h,s}$, and the log-volatility h_t follows a geometric random walk with variance σ_h^2.

2. **Prior Calibration:** Priors are calibrated using a training sample of the first 48 observations, following Boivin (2005):

   - **Initial States $(\boldsymbol{\Theta}_0, h_0)$:** Set using OLS point estimates and residual variances from the training sample (Primiceri, 2005).
   - **Coefficient Covariance $(\boldsymbol{\Sigma}_{\nu,h,s})$:** The scale is calibrated via the Stock and Watson (1998) median-unbiased estimator based on the Quandt Likelihood Ratio (QLR) statistic following Boivin (2005). Subscript s stands for possible breaks in the variance of innovation to the TVPs. I give structural break points around ZLB. For the actual rule using Greenbook. I give additional break around the inauguration of Paul Volcker similar to Coibion and Gorodnichenko (2011).

3. **Gibbs Sampling Algorithm:** The posterior is approximated using a Gibbs sampler with $M = 10,000$ iterations (5,000 burn-in)[21]:

   (a) **Coefficients $(\boldsymbol{\beta}_{1:T})$:** Drawn using the multimove simulation smoother of Carter and Kohn (1994). The stacked observations are handled by setting the measurement error covariance to $R_t = e^{h_t}\mathbf{I}_N$.

   (b) **Stochastic Volatility $(h_{1:T})$:** Drawn using the auxiliary mixture sampler of S. Kim, Shephard, and Chib (1998). I construct a quarterly volatility proxy from the stacked residuals to isolate the aggregate signal.

[21] The results are robust to a higher number of simulations. Increasing the number of simulations only adds to the computation time.

D Robustness and Additional Empirical Results

This appendix provides a set of robustness checks and additional empirical results that supplement the main empirical findings.

D.1 OLS Estimation of perceived rule

In this subsection, I present results obtained via constant-parameter Ordinary Least Squares (OLS). This exercise mirrors the simulation analysis in Section 3, establishing a baseline to assess the empirical consequences of neglecting time-variation. As demonstrated below, the resulting estimates exhibit a downward-sloping term structure for the inflation and output gap coefficients that qualitatively replicates the simulation findings, thereby providing empirical evidence of the covariance bias.

Using the Blue Chip consensus data, I estimate the following constant-parameter reaction function for forecast horizons $h \geq 0$:

$$i^{BC}_{t+h|t} = \beta_{\pi,h}\pi^{BC}_{t+h+j|t} + \beta_{x,h}\left(-2 \cdot u^{BC}_{t+h+j|t}\right) + \beta_{i1,h}i^{BC}_{t+h-1|t} + \beta_{i2,h}i^{BC}_{t+h-2|t} + c_h + \varepsilon_{t,h} \tag{121}$$

where $i^{BC}_{t+h|t}$, $\pi^{BC}_{t+h+j|t}$, and $u^{BC}_{t+h+j|t}$ denote the Blue Chip consensus forecasts for the interest rate, inflation, and unemployment rate at horizon h, respectively. The subscript j denotes the policy targeting horizon (e.g., $j \geq 1$ for forward-looking rules).[22] The case $h = 0$ corresponds to the nowcast specification. The unemployment rate is scaled by -2 to interpret the coefficient $\beta_{x,h}$ as the response to the output gap, consistent with a standard Okun's law relationship. Comparing estimates across horizons h allows for an empirical evaluation of the theoretical predictions regarding the covariance bias discussed in Section 3. In the results below, I report estimates for the forward-looking horizon $j = 1$.[23] Results for other horizons are qualitatively similar and are reported in the Appendix.

Figure 16 presents the estimation results. A distinct term structure is evident in the OLS coefficients across forecast horizons h. Under the null hypothesis of a constant policy rule, these coefficients should be invariant to h. The observed variation thus leads to a rejection of the constant parameter hypothesis. Moreover, the qualitative pattern of this term structure aligns with the simulation results in Section 3. This suggests that using interest rate *forecasts* to infer the perceptions about the policy rule is not an innocuous exercise.

[22] Although the simulation analysis in Section 3 utilized a contemporaneous rule, robustness checks confirm that a similar downward-sloping term structure emerges in alternative empirical specifications with varying targeting horizons j.

[23] Boivin (2005) and Coibion and Gorodnichenko (2011) document that actual monetary policy rules are described better by a forward-looking rule rather than a contemporaneous rule based on statistical tests. I also find similar evidence for the perceived rule as well.

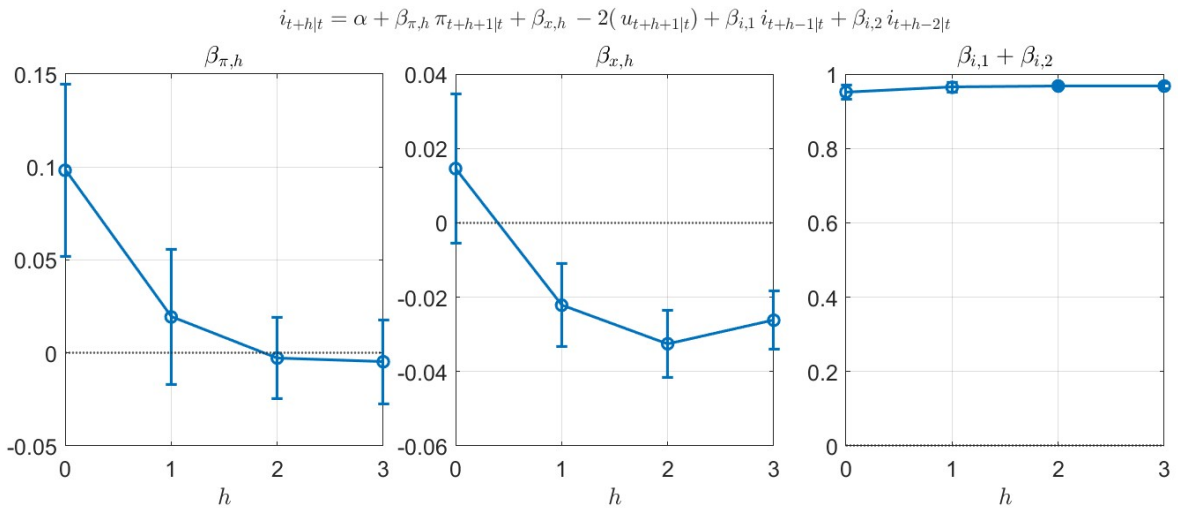


Figure 16: OLS Estimates of Policy Coefficients Across Forecast Horizons (h)

Note: The figure displays the estimated OLS coefficients of equation (121) using Blue Chip consensus forecasts at various horizons h.

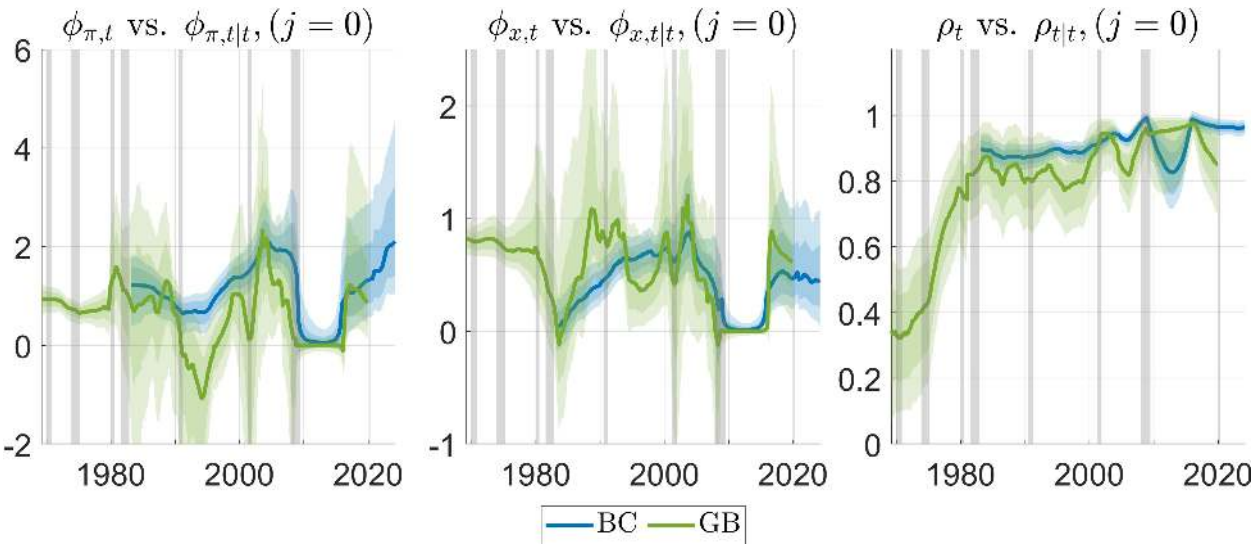
D.2 Forecasting Horizon on the RHS

The baseline estimation assumes a fixed targeting horizon of $j = 1$ (i.e., the central bank responds to 1-quarter ahead expectations). However, the specific horizon at which market participants believe the central bank targets inflation is an empirical question. If the assumed horizon j is misspecified, the resulting estimates of the perceived policy rule could be biased. To assess this, I re-estimate the perceived rule allowing for alternative targeting horizons. To conserve space, I report the results for $j = 0$ (contemporaneous rule) and $j = 2$ (medium-horizon rule).

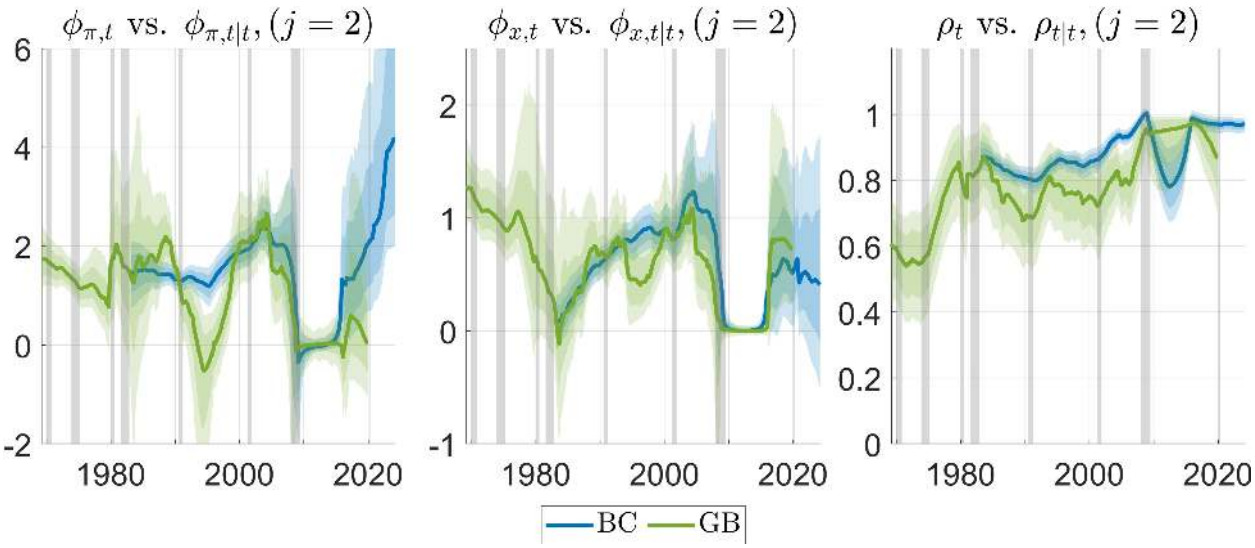
Figure 17 compares these alternative specifications. The estimated patter of the actual and perceived coefficients remains qualitatively similar across different values of j. Notably, the divergence during the mid-1990s and the stability of the *perceived* Taylor principle during the Great Moderation are present in all specifications.

D.3 Monthly Frequency of Estimation

The baseline perceived rules were estimated at a quarterly frequency by stacking monthly Blue Chip surveys with quarterly SPF data. A potential concern is that this temporal aggregation might mask high-frequency updates in perceptions or artificially smooth transitions within a quarter. To ensure that the quarterly aggregation does not distort the timing or magnitude of the perceived regime shifts, I re-estimate the model at a monthly frequency



(a) Estimation with horizon $j = 0$



(b) Estimation with horizon $j = 2$

Figure 17: Robustness to Targeting Horizon (j)

Note: The figure compares the time-varying coefficients of the Perceived Rule (Blue Chip/SPF, blue line) and the Actual Rule (Greenbook, green line) under alternative assumptions for the targeting horizon on the right-hand side (RHS) of the reaction function. Panel (a) displays estimates assuming contemporaneous targeting ($j = 0$), while Panel (b) displays estimates assuming forward-looking targeting ($j = 2$). The solid lines represent the posterior medians, and the shaded regions indicate 68% credible intervals.

using only the disjoint BCEI monthly consensus forecasts.

Figure 18 plots the resulting monthly time series of the perceived inflation coefficient against the quarterly baseline. The monthly estimates closely track the quarterly results, capturing the same turning points and structural shifts. This congruence confirms that the quarterly stacked approach efficiently utilizes all available information without inducing

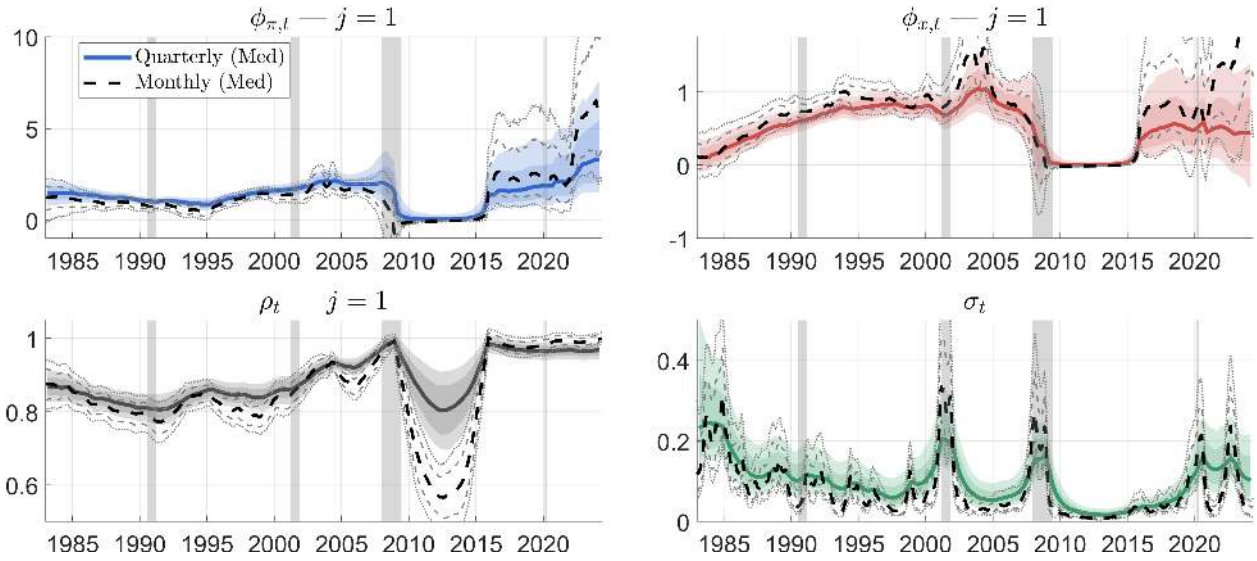


Figure 18: Robustness to Estimation Frequency: Quarterly vs. Monthly

Note: The figure compares the estimated long-run inflation coefficient of the perceived rule obtained from the baseline quarterly stacked model (blue) against the estimates obtained from a purely monthly model using Blue Chip data (red).

much aggregation bias.

D.4 Variation at Longer Horizons

In Section 4, I documented a significant attenuation in the estimated policy coefficients when using interest rate forecasts (LHS horizon $h > 0$) compared to nowcasts ($h = 0$). A potential econometric concern is that this attenuation might be mechanical rather than structural. Specifically, as the forecast horizon extends, the variance of the predictors (inflation and output gap forecasts) tends to decline as they revert to their long-run means. If the dependent variable (interest rate forecast) variance does not decline at the same rate, or if measurement error becomes relatively larger, this could mechanically drive coefficients toward zero.

However, further analysis suggests this mechanical explanation is insufficient. The critical distinction lies in whether the horizon variation applies to the dependent variable (LHS) or the independent variables (RHS). As shown in Appendix D.2, varying the targeting horizon j on the RHS (regressing interest rate nowcasts on longer-horizon inflation forecasts) does not lead to systematic attenuation. The estimates remain robust whether agents project rates based on current or future inflation.

In contrast, the attenuation is specific to increasing the horizon of the LHS variable (the interest rate expectation itself). This pattern is consistent with the structural covariance bias identified in Section 3: when agents forecast the interest rate itself ($h > 0$), they internalize

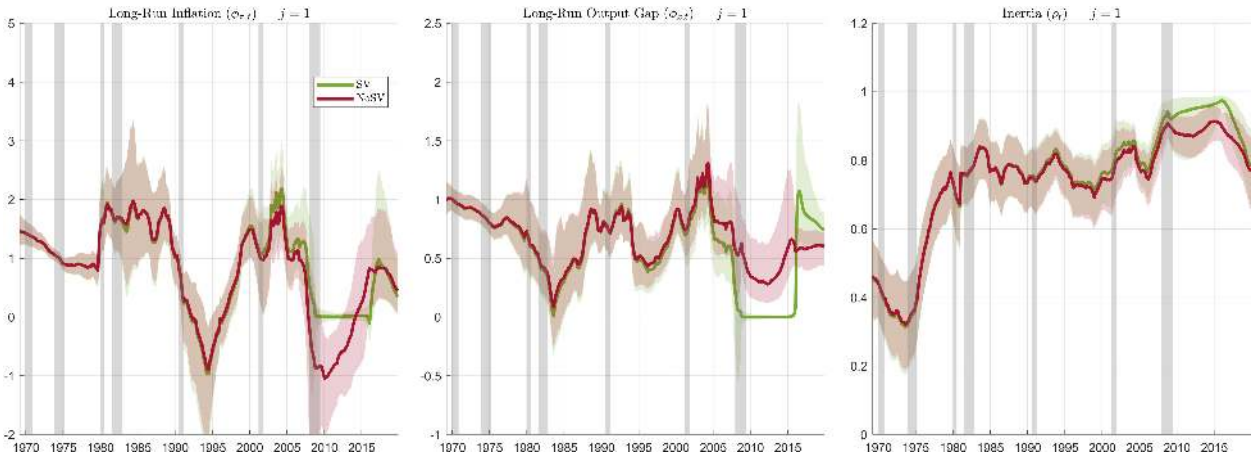


Figure 19: Robustness to ZLB Censoring: Effective Rate vs. Shadow Rate

Note: The figure compares the time-varying estimates of the actual policy rule using the effective federal funds rate (green) versus the shadow rate of Wu and Xia (2016) (red).

the conditional covariance between the future policy parameter and future inflation. This covariance term is absent in the nowcast ($h = 0$) but present in the forecast, regardless of the variance properties of the RHS variables. Thus, the attenuation is a result of the perceived covariance bias, not the statistical properties of long-horizon forecasts.

D.5 Zero Lower Bound Censoring and Shadow Rate

The estimation of the central bank's *actual* reaction function relies on the federal funds target rate as the dependent variable. During the Zero Lower Bound (ZLB) episodes (2009–2015 and 2020–2022), the inability of the federal funds rate to drop below zero creates a censoring problem. Standard estimation might mechanically bias the reaction coefficients ($\phi_{\pi,t}$) downward, as the policy rate appears unresponsive to negative output gaps or below-target inflation.

To address this, I re-estimate the actual policy rule replacing the effective federal funds rate with the shadow rate constructed by Wu and Xia (2016). The shadow rate is unconstrained and can take negative values, providing a better measure of the intended policy stance during unconventional monetary policy periods.

Figure 19 compares the baseline actual rule estimates with those obtained using the shadow rate. While the shadow rate implies a slightly more active policy during the depths of the ZLB, the qualitative trajectory of the actual rule remains similar. Crucially, the major divergence identified in the mid-1990s (pre-ZLB) is unaffected, and the recent post-ZLB dynamics are qualitatively robust. This confirms that the key findings regarding the gap between actual and perceived rules are not driven by censoring effects at the lower bound.

D.6 OLS vs. GMM Estimation of Perceived Taylor Rule

One assumption in estimating the perceived Taylor rule is that the perceived monetary policy shock does not give rise to a substantial endogeneity bias, as is the case in the conventional Taylor rule estimation. To test the validity of this assumption, I compare OLS estimates with Generalized Method of Moments (GMM) estimates, following the approach of Carvalho, Nechio, and Tristao (2021). If the endogeneity bias is severe, we should observe a statistically meaningful difference between OLS and GMM estimates.

The estimating equation is:

$$i^{BC}_{t|t} = \beta_\pi \pi^{BC}_{t+j|t} + \beta_x \left(-2 \cdot u^{BC}_{t+j|t}\right) + \rho_1 i^{BC}_{t-1|t} + \rho_2 i^{BC}_{t-2|t} + \alpha + e_t \tag{122}$$

where $i^{BC}_{t|t}$ denotes the Blue Chip consensus interest rate nowcast, $\pi^{BC}_{t+j|t}$ and $u^{BC}_{t+j|t}$ are the consensus inflation and unemployment forecasts at horizon j, and α is a constant term. The unemployment rate is scaled by -2 to interpret β_x as the response to the output gap under Okun's law. I report long-run coefficients derived as $\phi_\pi = \beta_\pi/(1-\rho_1-\rho_2)$ and $\phi_x = \beta_x/(1-\rho_1-\rho_2)$. Standard errors for the short-run coefficients are obtained by the Newey-West standard errors, and standard errors for the long-run coefficients are obtained via the delta method.

For GMM estimation, I follow Clarida, Gali, and Gertler (2000) and Carvalho, Nechio, and Tristao (2021) and use a k-step estimator. The instruments are lagged forecasts in terms of survey months. To avoid overlapping horizons, I use lags with at least three months. In each specification, the number of lags and hence instruments is chosen such that they have sufficiently high first-stage F-statistics ($F > 10$). The Hansen J-test evaluates the validity of the overidentifying restrictions.

Table 3 reports the estimation results across four alternative specifications depending on the targeting horizon j. The OLS and GMM estimates are similar and statistically indistinguishable within a standard error band. The long-run inflation coefficient ϕ_π ranges from 1.83 to 2.22 under GMM, closely tracking the OLS estimates. The output gap coefficient ϕ_x exhibits comparable magnitudes as well. The Hansen J-test p-values range from 0.280 to 0.599, failing to reject the validity of the overidentifying restrictions at conventional significance levels. This indicates that the instruments are valid and that endogeneity due to perceived monetary policy shocks is not a major concern.

Table 3: Perceived Taylor Rule Estimation: OLS vs. GMM

	Contemporaneous $(\pi_{t\|t},\ u_{t\|t})$		Forward-looking 1 $(\pi_{t+1\|t},\ u_{t\|t})$		Forward-looking 2 $(\pi_{t+1\|t},\ u_{t+1\|t})$		Forward-looking 3 $(\pi_{t+2\|t},\ u_{t+1\|t})$	
	OLS	GMM	OLS	GMM	OLS	GMM	OLS	GMM
ϕ_π	1.95	1.83	2.21	2.02	2.22	2.01	2.18	2.19
	(0.54)	(0.41)	(0.48)	(0.40)	(0.48)	(0.40)	(0.46)	(0.37)
ϕ_x	0.28	0.20	0.26	0.19	0.27	0.17	0.30	0.22
	(0.17)	(0.11)	(0.14)	(0.11)	(0.15)	(0.11)	(0.13)	(0.10)
$\rho = \rho_1 + \rho_2$	0.96	0.97	0.96	0.96	0.96	0.96	0.96	0.96
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
N	497	497	497	497	497	497	497	497
J-test p-val		0.599		0.467		0.422		0.280