Monetary Policy and the Credit Rationing Effects of Liquidity

Jonathan Swarbrick*

University of St Andrews

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Abstract

This paper studies monetary policy in a New Keynesian economy with frictional bank lending, rationalising evidence that lending conditions can remain tight despite liquidity injections. The model features a policy trade-off in which increases in banking sector liquidity can incentivise *more lending* by lowering the overnight rate and the marginal cost of funds, but can also incentivise *less lending* by compressing bank margins as interest rates approach the policy floor, worsening adverse selection and credit rationing. As a result, quantitative easing can exert a contractionary effect when the economy is away from the effective lower bound, with outcomes depending on borrower risk and the size of the programme. However, both channels raise inflation expectations, and so liquidity policies are always expansionary at the lower bound. Optimal policy features a deflation bias under credit rationing, while commitment to future accommodation eases current credit conditions and implies gradualism in quantitative tightening.

JEL: E5, E44, G21

Keywords: Monetary policy, quantitative easing, small business lending, credit rationing, bank liquidity

*University of St Andrews, Castlecliffe, The Scores, St Andrews, KY16 9AZ, UK. (jms48@st-andrews.ac.uk).

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1 Introduction

The monetary policy environment has evolved considerably since the global financial crisis (GFC). Many central banks expanded their menu of policies and now routinely use balance sheet policies to respond to crises. Yet the effects of these policies remain imperfectly understood, with implications both for the design of future interventions and for the unwinding of central bank balance sheets. While these policies have led to a dramatic rise in central bank balance sheets and banking sector liquidity, a central challenge has been ensuring that these expansions translate into improved lending conditions, particularly for small businesses. In practice, despite a sharp increase in banking sector liquidity after the GFC, euro area banks remained reluctant to lend to small businesses, even after the easing of the sovereign debt crisis. Figure 1 illustrates this disconnect, showing persistently sluggish loan growth (panel a) alongside a substantial and sustained rise in excess reserve balances (panel b).

This poses a puzzle as most macroeconomic theories of bank lending predict that liquidity expansions drive more lending and investment (see, e.g., Kashyap et al., 2002; Adrian and Shin, 2010; Carpenter et al., 2014), yet recent evidence suggests this is not always the case. For example, Diamond et al. (2024) find that bank reserve injections crowded out bank loans in the US, while Iyer et al. (2014) show that greater euro area liquidity from ECB balance sheet policies led to liquidity hoarding rather than increased small-business lending.[1]

In this paper, I rationalise this evidence using a New Keynesian macroeconomic model in which information frictions in small business lending interact with monetary policy implementation. The key mechanism is that policy-induced changes in bank profitability alter the opportunity cost of lending, generating endogenous credit rationing even in the presence of abundant liquidity.[2] The lending friction prevents banks from observing the risk characteristics of small firms in the economy, creating an adverse selection problem in credit markets. This generates a well-established effect, explored by Ikeda (2020) and Swarbrick (2023) in relation to the GFC, under

[1] Similarly, Acharya et al. (2019) find that the additional liquidity led to higher cash reserves and banks holding low-interest assets, but did not lead to more productive business lending. See also Gambacorta and Marques-Ibanez (2011), Joyce et al. (2011), Loutskina (2011), and Disyatat (2011). Mennuni (2025) develops a theory rationalising the buildup of excess bank liquidity.

[2] While some studies focus on bank conditions in affecting the bank-lending channel of monetary policy (see, e.g., Berger and Bouwman, 2009; Cornett et al., 2011; Martin et al., 2016), our focus follows evidence that adverse selection has been an important feature of European credit markets (see Albertazzi et al., 2021).

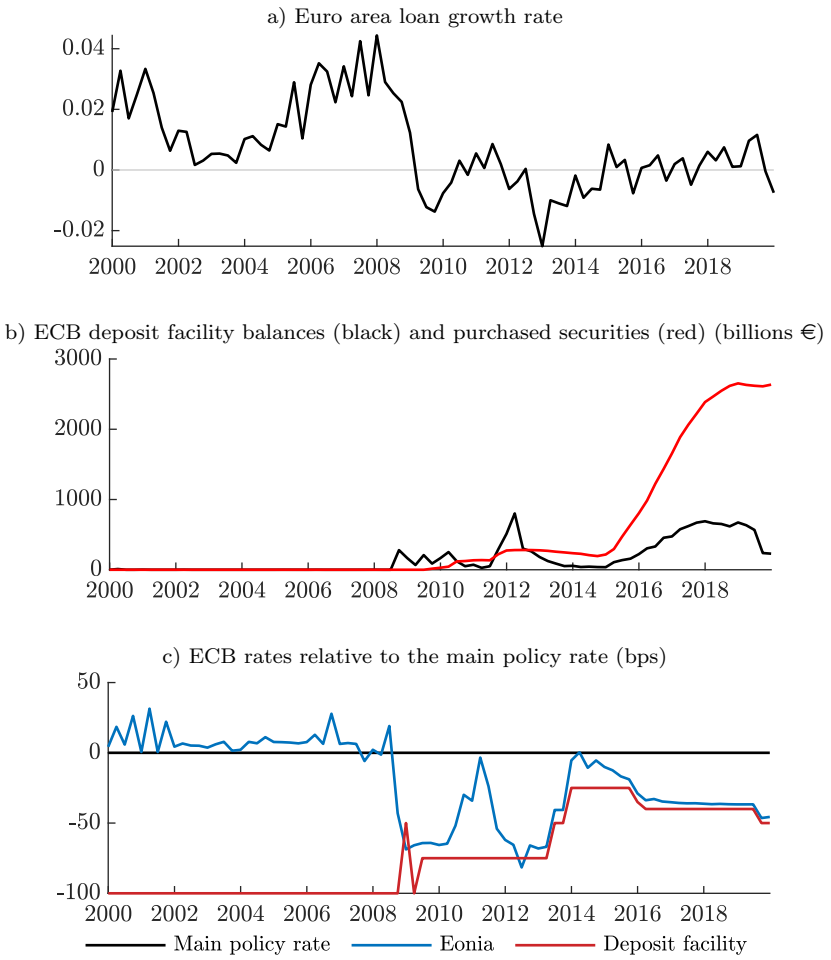


Figure 1: Source: ECB. Loans are those to euro area NFCs reported by MFIs. Purchases securities includes the stock of securities purchased as part of the various asset purchase programmes since 2009.

which banks ration credit and build up excess reserves when firm risk is high. Crucially, this adverse selection channel links monetary policy implementation to lending incentives, with the interest rate corridor providing a margin by which policy affects the opportunity cost of lending and therefore the incentive to ration credit to small businesses.

An important implication is that a narrowing of the interest corridor, such as that caused by the effective lower bound on nominal interest rates (ELB) in the euro area (Figure 1, panel c), can increase the incentive to ration small business loans, offsetting the expansionary effects of low rates and the unconventional policy measures. This creates a tension whereby when the bank funding rate falls, activity may be stimulated through the standard interest rate channel,

but lending incentives may simultaneously weaken as rates approach the floor, compressing net margins and increasing the relative attractiveness of holding reserves. This mechanism is supported by empirical evidence presented in Section 2, which shows that while central bank asset purchases and falling bank funding costs predict higher bank reserve balances and more lending in the euro area, a squeezing of the policy corridor is a significant and quantitatively important predictor of the opposite outcome.

A log-linear representation of a restricted version of the model takes the form of a 3+1 equation New Keynesian model, where the standard IS and Phillips curves are augmented by a credit wedge condition that captures endogenous lending distortions.[3] Because the credit wedge acts as a one-sided cost-push distortion, it generates a trade-off between inflation stabilisation and output stabilisation that is absent when lending is unconstrained. This trade-off has implications for optimal monetary policy design: under discretion, a policymaker facing credit rationing at the ELB optimally restrains liquidity provision to preserve bank lending margins, generating a deflation bias whose severity depends on the degree of adverse selection. Under commitment, forward guidance can ease credit conditions directly by raising expected lending profitability, and the obligation to deliver on past commitments provides a rationale for gradualism in the subsequent unwinding of balance sheet positions.

Much of the large literature examining the links between liquidity and macroeconomic outcomes focuses on how private provision of liquidity can tighten during crises, amplifying shocks and motivating a stabilizing role for liquidity easing policies (see, e.g. Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Holmström and Tirole, 1998; Adrian and Shin, 2010; Gertler and Karadi, 2011; Brunnermeier and Sannikov, 2014). Recent work has also emphasised the macroeconomic importance of adverse selection in external finance, which can distort the allocation of credit and investment (see e.g. Guo et al., 2024; Beyhaghi et al., 2026). Since this paper focuses on the rationing of credit to small businesses, the proposed model builds on Stiglitz and Weiss's (1981) work on adverse selection, which shows how credit markets can shut down due to information frictions.[4] The key contribution is to examine the role of liquidity and monetary policies

[3] This shares structural similarities with Sims et al. (2023), who derive a four-equation New Keynesian model in which QE and credit shocks enter both the IS and Phillips curves.

[4] This has been extended in many papers in the years since, including, for example Bester (1985), Mankiw (1986), Williamson (1986), De Meza and Webb (1987), Besanko and Thakor (1987), House (2006), Ikeda (2020).

when liquidity provision is abundant but lending remains constrained.

The approach of modelling information frictions faced by lenders relates to the literature studying the balance sheet channel of monetary policy (such as Bernanke et al., 1999; Christiano et al., 2005). Typically, models of financial frictions link monetary policy transmission to borrower balance sheets through a financial accelerator, whereby a monetary expansion raises asset prices and collateral values. In contrast, we abstract from the balance sheet channel. In our model, the effects of a monetary expansion are ambiguous as they depend more critically on how the expansion is implemented and how it affects banking sector liquidity, bank balance sheet composition, bank profitability, and the opportunity cost of lending. The key issue is how the shock or policy action interacts with banks' incentive to ration credit.

The theoretical model explicitly incorporates monetary policy implementation in the presence of information frictions. Banking sector liquidity is endogenous to macro-financial conditions, and banks may choose to hold excess reserves in equilibrium. Because of this, the interest rate paid on reserves affects bank profitability, liquidity demand, and ultimately lending behavior (see also Disyatat, 2011). While standard New Keynesian models abstract from implementation details and contain no endogenous excess reserves, this framework incorporates a corridor system explicitly and studies how its design affects equilibrium outcomes.

Several existing studies explicitly examine the role of the interest rate corridor (see, e.g., Bindseil, 2000; Whitesell, 2006). These typically focus on trade-offs between interest rate volatility, interbank market activity, and monetary transmission under frictions such as interbank market search costs (Bech and Monnet, 2016) and balance sheet risk (Whitesell, 2006; Bindseil and Jabłecki, 2011). In contrast, this paper shows how the corridor affects lending conditions through a profitability channel, even in the absence of interbank frictions. The underlying mechanism is that the floor rate competes with risky lending as an alternative use of liquidity, and this trade-off shapes banks' willingness to intermediate.

By highlighting the profitability channel, the analysis also relates to recent work on the contractionary effects of negative interest rates within corridor systems (e.g., Abadi et al., 2023; Eggertsson et al., 2023), where binding deposit rate floors compress margins and weaken mone-

tary policy transmission. In contrast, the channel here operates via adverse selection, providing a distinct profitability-based rationale for credit rationing as policy rates approach the floor.

This paper makes three main contributions to our understanding of monetary policy transmission in the presence of financial frictions. First, I provide novel empirical evidence on the relationship between the interest rate corridor and lending conditions in the euro area, documenting how corridor adjustments predict lending outcomes even controlling for standard policy measures. Second, I develop a theoretical framework that rationalizes why liquidity provision can fail to stimulate lending when information frictions are severe, highlighting that the design of monetary policy implementation itself becomes crucial for credit allocation. Third, I study optimal policy design, finding that credit rationing introduces a deflation bias as the policymaker seeks to preserve lending incentives, and that commitment to future accommodation calls for gradualism in quantitative tightening.

2 Motivating evidence from the euro area

In this section, we document a robust relationship between the spread to the floor and bank lending in the euro area, controlling for macroeconomic conditions, credit risk, and ECB asset purchases. These patterns motivate the theoretical mechanism developed in the subsequent section. Focusing on the euro area is insightful due to the important role of banks in providing credit to the real economy, with around double the share of business debt in the form of bank loans compared to the United States.[5]. Additionally, small and medium-sized enterprises account for around two thirds of private-sector employment in Europe compared to around one half in the United States (see Carbo-Valverde and Rodríguez-Fernández, 2016). These facts highlight the systemic importance of bank lending for credit conditions faced by small firms.

The euro area is also a useful laboratory because the ECB's operating framework has generated rich variation in the return on reserves. The ECB operates a corridor system and during periods of abundant liquidity, the overnight interbank market rate falls below the main policy rate, and the spread between the interbank rate and the policy floor becomes a key determinant of banks'

[5] 70% versus 35% (see Holm-Hadulla, Musso, Nicoletti and Tujula, 2022; Gambacorta, Yang and Tsatsaronis, 2014)

outside option. As a result, changes in the floor rate and the effective spread to the floor can affect lending incentives even when conventional policy rates move little (see Figure 1, panel c).[6]

2.1 Corridor Effects on Lending

To explore the relationship between the interest corridor and bank lending conditions, we estimate the following specification using monthly data between 2008–2019:

$$\ln X_t = \alpha_X + \beta_X s_t + \boldsymbol{\gamma}'_{\mathbf{X}} \cdot \mathbf{z}_t + u_{X,t} \tag{2.1}$$

for $X \in \{L, DF\}$, where L_t and DF_t are total euro area bank lending and excess reserves respectively, and s_t is either the corridor between the ECB's main refinancing rate and deposit facility rate, or the spread between the interbank rate and the deposit facility rate.[7] $\mathbf{z}$ is a vector of controls including GDP, inflation, the interbank rate (Eonia), credit risk, and ECB asset purchases.[8]

Table 1 reports the results. The central finding is that a 25 basis point wider ECB interest rate corridor predicts 2.2% higher lending and substantially lower excess reserve balances in the euro area.[9] Conditional on the interbank rate, a wider corridor predicts higher lending, pointing to the role of the interest on reserves in shaping banks' lending incentives. Conditional on the corridor, a 25 basis point higher interbank rate predicts 0.3% lower lending volumes and much lower reserve balances, consistent with tighter liquidity conditions driving higher interbank rates.

[6] The Federal Reserve did not operate an explicit corridor system, rather relied on open market operations to target the federal funds rate. Since 2008, the Fed began paying interest on excess reserves (IOER), moving towards a floor system. Other central banks including the Bank of England and Bank of Canada operated corridor systems and moved towards floor systems following the GFC, with corridor widths also subject to adjustment.

[7] Note that since October 2019, the ECB has targeted a new euro short-term rate, replacing Eonia as the interbank rate (see Nicoloso and Tsonchev, 2019, Box 1, p.23).

[8] L_t is the stock of loans by monetary and financial institutions (MFIs) to non-financial institutions (NFIs) in the euro area; and DF_t is the total use of the central bank deposit facility. Controls include total euro area log nominal GDP and CPI (HCIP) inflation, the Eonia rate (source ECB), Gilchrist and Mojon's (2018) measure of credit risk of euro area NFIs, and the stock of total asset purchase programmes (APPs), which includes all securities held by the eurosystem for monetary policy purposes.

[9] This is the case controlling for economic activity (growth or level), financial risk, the short-term interbank market interest rate (Eonia), inflation, and ECB asset purchases. The results are robust to dropping controls or additionally including the main policy interest rate and a time trend. Monthly data 2008M1–2019M12, where lending is measured as the loans vis-à-vis euro area NFCs reported by MFIs excl. ESCB, excess reserves is total balances at the ECB standing deposit facility, economic activity is log euro area GDP, and financial risk is proxied using Gilchrist and Mojon's (2018) spread between NFC rates and domestic sovereigns. See Appendix A for details.

	a) $s_t \equiv R_t^p - \underline{R}_t$			b) $s_t \equiv R_t - \underline{R}_t$		
	$\ln DF_t$	$\ln L_t$	LS_t	$\ln DF_t$	$\ln L_t$	LS_t
s_t	-0.481^{***}	0.0221^{***}	2.367	-0.931^{***}	0.0084^{**}	-3.769^{**}
	(0.134)	(0.002)	(1.758)	(0.144)	(0.0038)	(1.825)
R_t	-0.497^{***}	-0.0028^{***}	1.649^{***}	-0.294^{***}	-0.0041^{***}	2.654^{***}
	(0.025)	(0.0005)	(0.257)	(0.041)	(0.0011)	(0.506)
Obs	141	141	47	141	141	47
Adj. R^2	0.855	0.894	0.893	0.879	0.845	0.899

Notes: Standard errors in parentheses. Significance levels: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 1: Estimation results testing the impact of the ECB corridor width (Main Refinancing Rate R_t^p minus Deposit Facility Rate $\underline{R}_t$) and the spread to the floor (the interbank rate (Eonia) R_t minus Deposit Facility Rate), both measured per 25 bps, on bank excess reserves (ECB deposit facility use), bank lending to NFCs, and lending standards (net percentage of banks tightening or easing credit standards for SME loans). Excess reserves and lending regressions use monthly observations 2008M1–2019M12; lending standards regressions use quarterly observations over the same time period. Source: ECB. Excess reserves is use of the ECB Deposit Facility, lending is MFI loans to NFCs. Lending standards is from the ECB Bank Lending Survey.

Using quarterly bank-level balance sheet data over the same time period, I re-estimate specification A.1 with bank fixed effects.[10] This yields similar results: a 25 basis point wider corridor predicts 1.8% higher contemporaneous lending, and 5.5% lower cash and reserve balances.[11]

2.2 The Interbank Rate and the Policy Floor

The policy corridor is defined as the spread between the main policy rate and the interest on reserves. When there is surplus liquidity in the banking sector, the marginal bank funding cost, measured with the overnight interbank market rate, moves below the main policy rate. In this context, the key factor in determining lending conditions may be the spread between interbank rate and the floor rather than the official corridor.

Section (b) in Table 1 reports results using this alternative measure. The central results remain unchanged: A 25 basis point wider spread predicts 0.8% higher lending volumes.[12] Conditional

[10] Bank-level balance sheet data is from S&P Global Market Intelligence (SNL Financial Fundamentals). Further details and results are presented in Appendix A.

[11] Regression results reported in Table 2 in online Appendix A. The lower impact on reserves is due to a broader category which includes physical cash and required reserves. The relative increases in reserves are consequently much more muted.

[12] Using bank-level data, a 25 basis point wider spread predicts 1.0% higher lending volumes (Table 2 in online

on this spread, a higher interbank rate predicts lower lending volumes, consistent with tighter liquidity conditions.[13]

The lending response is also reflected in survey evidence on credit standards. In Table 1 panel (b), higher lending is accompanied by an easing of lending standards, which is consistent with a lending mechanism operating through screening and selection considerations. This motivates modelling information frictions as central drivers of bank lending conditions.

2.3 Threshold Effects and Competing Channels

These results reveal an important tension. When the interbank rate falls, perhaps due to quantitative easing increasing market liquidity, bank lending may rise as funding costs decline. However, if policy rates remain unchanged, the spread between the interbank rate and the policy floor contracts, which our results suggest reduces lending. The overall impact on lending is therefore ambiguous.

To investigate this further, we estimate how the effect of the interbank rate on lending varies depending on how close the interbank rate is to the floor:

$$\ln L_t = \alpha_L + \beta_{L,s<\mathcal{S}} i_t R_t + \beta_{L,s>\mathcal{S}} (1 - i_t) R_t + \boldsymbol{\gamma}'_{\mathbf{L}} \cdot \mathbf{z}_t + u_{L,t} \tag{2.2}$$

where $i = 1$ when the spread is below threshold $\mathcal{S}$, and zero otherwise.[14] The rationale is that when the spread is low and the economy operates as a floor system, variation in the spread becomes more important for lending incentives.

Figure 2 shows the results across different threshold values. When the spread between the interbank rate and the floor is small (blue points), versus when it is large (red points), the effects of the interbank rate on lending differ markedly. The key finding is striking: when the interbank rate is far above the floor, changes in this rate have no significant effect on lending. However, when the interbank rate is close to the floor (10-30 basis points above), a higher rate actually

Appendix A).

[13] There is a negative but insignificant relationship between the interbank rate (Eonia) and reserve balances using bank-level data. This reflects two offsetting drivers of reserve balances: changes in the deposit rate (which move the interbank rate in the same direction) and asset purchases (which increase reserves while lowering the interbank rate).

[14] The controls are as before, except with the inclusion of the main policy interest rate, R_t, and the floor, $\underline{R}_t$.

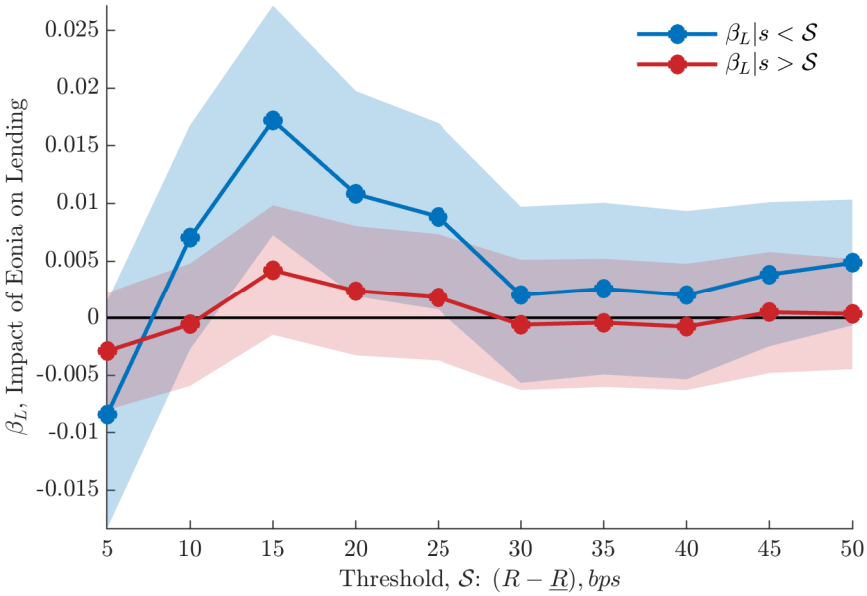


Figure 2: Estimated coefficients from specification (2.2) for a range of values of threshold, t. Shaded areas show the 95% confidence intervals.

predicts higher lending volumes—the opposite of what standard theory would predict.[15]

The empirical exercises reveal the following stylised facts. First, conditional on the market interest rate, a narrowing of either the policy corridor or the spread between the market rate and the floor predicts lower bank lending and higher bank reserve balances. Second, conditional on the corridor, a lower interbank market rate predicts higher lending volumes. Third, when the market rate is close to the floor, a fall in the market rate predicts lower lending, consistent with a dominant profitability outside-option channel. Fourth, changes in lending volumes are accompanied by systematic movements in lending standards, consistent with screening responding endogenously to banks' incentives.

These facts can be understood through the lens of the theoretical framework developed in the next section. With information frictions in small business lending, quantitative easing that drives a fall in the interbank rate generates two competing effects: the standard expansionary channel through lower bank funding costs, and a contractionary channel whereby reducing the spread

[15] An alternative explanation is the reversal interest rate, explained by the zero-lower bound on household deposit rates and the negative effects on bank profitability when the marginal funding rate goes negative (see Abadi et al., 2023).

to the floor disincentivises lending and can drive credit rationing. When the interbank rate is relatively close to the floor, this second channel becomes dominant.

3 A macroeconomic model of credit rationing

At the core of the model is a canonical New Keynesian (NK) closed economy model. The representative household chooses consumption, C_t, labour supply, H_t, and allocates savings across bank deposits, S_t, risk-free nominal bonds, B_t, and firm equity, E_t, to maximise expected lifetime utility $\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(C_{t+s}, H_{t+s})$. The consumption-savings decision yields the standard Euler equation,

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t, \tag{3.1}$$

where $\Lambda_{t,t+1} = \beta U'(C_{t+1})/U'(C_t)$ is the stochastic discount factor, R_t is the gross nominal return on deposits, and the return on bonds $R_t^B = R_t$ in equilibrium. The labour supply condition equates the real wage with the marginal rate of substitution: $W_t/P_t = -U'(H_t)/U'(C_t)$. Equity purchases are characterised by a zero-arbitrage condition outlined below.

A wholesale goods sector combines labour and purchases capital to produce a homogeneous intermediate good. This good is then sold to a retail sector under perfect competition. The retailers differentiate the intermediate good, selling their output under monopolistic competition subject to the Calvo (1983) price-setting friction.

Following Swarbrick (2023), the wholesale goods sector is characterised by information frictions in firm investment. This builds on two key assumptions. The first is that all investment projects require a single unit of productive capital for which firms need to raise a fixed quantity of external finance. This assumption ensures that firms rely on outside funding, but by treating all investment opportunities as a fixed size, we capture the inability of small firms being unable to diversify risk.

The second key assumption is that every period, each intermediate goods firm draws a project characterized by a production technology, productivity level and a risk profile. A proportion η of

firms, denoted *corporates*, have a perfectly observable project and so are suitable for raising funds via a bond market. The remaining $1-\eta$ firms have a privately observed project, a proportion λ of which have no idiosyncratic risk (*safe*), whereas the remaining $1-\lambda$ have a risky project that will only succeed with probability p_t (*risky*).

Because all loans are the same size, the interest rate is the only screening device. Under a decentralised bond market, safe firms would be rationed whenever the rate exceeds their expected return. The information asymmetry gives rise to a banking sector that can outperform bond markets by screening borrowers.[16] Despite not modelling differences in firm size, the fixed size of investment and information asymmetry captures salient features of small business: having private and being unable to diversify risk. For this reason, we refer to the non-corporates as small firms.

Before turning to the problem facing the borrowers and lenders in the economy, we first outline details about monetary policy. We will differ from the standard NK model by giving the central bank three instruments. The first is the standard tool: the main policy target rate, R_t^p, which the central bank charges for short-term liquidity operations. Second, the central bank can directly vary the amount of liquidity to influence the short-term market interest rate on household savings, R_t.[17] This can be targeted through private sector bond purchases, or direct liquidity provision to the bank sector.[18] The third instrument is the interest rates on standing deposit ($\underline{R}_t$) and lending ($\overline{R}_t$) facilities. Banks can hold excess reserve balances in the former and access additional liquidity as required via the latter.[19]

3.1 Banks

Bank assets consist of business loans and central bank reserves, while liabilities include central bank liquidity and household deposits. The composition of the liability side is addressed below. We begin by examining the asset side. To extend finance to firms, the banks can post the terms of

[16] Indeed, a major rationale for the existence of banks is to mitigate borrowers' financing frictions (see Flanagan, 2025).

[17] The first two instruments are implicitly the same, however to capture the policy tools used in practice, we model the implementation of the first via a Taylor rule, and the second to allow shifts of the market rate below the main policy rate towards the policy floor, $\underline{R}_t$.

[18] Either are equivalent as in equilibrium corporate bonds, household savings, and central bank loans to the banks will clear at the same interest rate.

[19] There is no regulatory reserve balance and so all bank reserve balances at the central bank are excess reserves.

a menu of loans that borrowers can choose on application.[20] Loans are single-period, with terms specifying both an interest rate and an approval probability. This flexibility allows banks to offer multiple loan options at different interest rates to separate borrowers. These options are designed as incentive-compatible, or self-selecting, contracts. For instance, a risky firm might accept a higher interest rate if it increases the probability of approval compared to a lower-interest loan.

Specifically, the lenders post contracts $c_t^i = \{\tau_t^i, x_t^i\}$ for $i \in \{s, r\}$, where τ_t^i is the nominal repayment rate, and x_t^i the financing, or approval probability. Letting R_t^i denote the gross nominal rate of return on capital for a type-i project and p_t the success probability of risky projects, lenders set contract terms subject to firms' individual rationality (IR) and incentive compatibility (IC) constraints. The relevant constraints can be written:

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] \tau_t^s \leq \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} R_{t+1}^s \right] \tag{3.2}$$

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} p_{t+1} \right] \tau_t^r \leq \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} p_{t+1} R_{t+1}^r \right] - \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} p_{t+1} \left(R_{t+1}^r - \tau_t^s \right) \right] \frac{x_t^s}{x_t^r}. \tag{3.3}$$

The first constraint ensures the participation of safe firms by promising weakly positive surplus. The second provides an incentive for the risky firm to reveal their type, even choosing a higher interest rate loan. This is achieved through a higher probability of loan approval. When the lending contracts separate borrowers, therefore, safe loans have low repayment rates but a lower approval rate, while risky loans have a higher repayment rate but higher approval rate.[21]

Free entry of banks implies a zero-arbitrage condition. Banks enter, access liquidity at rate R_t via household deposits or central bank loans, and post loan contracts. Matching to firms is proportional to bank balance sheet size.[22]

Denoting the volume of liquidity provision for bank j as S_t^j, the total volume of loans L_t^j, and the matched firms $f_t^j \equiv f_t S_t^j / \int S_t^j \mathrm{d}j$, the total *ex post* gross nominal rate of return to the

[20] Because firm-type is drawn every period, it is not possible that banks learn the firm type over time and so credit terms are not a function of a firm's history.

[21] The implications of these contracts are discussed further in the next section.

[22] Match uncertainty would generate a role for the interbank market and is explored separately.

representative bank is

$$\underline{R}_{t-1}\left(S_{t-1}^{j}-L_{t-1}^{j}\right)+\left[\lambda x_{t-1}^{s}\tau_{t-1}^{s}+(1-\lambda)\,x_{t-1}^{r}p_{t}\tau_{t-1}^{r}\right](1-\eta)\,f_{t-1}^{j}Q_{t-1}^{K}, \tag{3.4}$$

where $\underline{R}_{t-1}$ is the interest on reserves and $\left(S_{t-1}^{j}-L_{t-1}^{j}\right)$ the excess reserves of bank j.[23]

Each borrower will use the loan to purchase a single unit of capital at price Q_t^K. Therefore, total loans extended $L_t^j \equiv (1-\eta)\left(\lambda x_t^s + (1-\lambda)\,x_t^r\right) f_t^j Q_t^K$. Denoting the the loan supply-demand ratio $\phi_t^j \equiv \frac{S_t^j}{(1-\eta) f_t^j Q_t^K}$, the zero-profit condition will determine the equilibrium volume of banking sector liquidity:

$$1=\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\underline{R}_t+\left[\lambda x_t^s\left(\tau_t^s-\underline{R}_t\right)+(1-\lambda)\,x_t^r\left(p_{t+1}\tau_t^r-\underline{R}_t\right)\right]\frac{1}{\phi_t}\right)\right] \tag{3.5}$$

where we drop the bank index j since it follows that ϕ_t must be equal for all banks. Given that bank liabilities are risk-free deposits but assets are risky loans, it is possible for there to be *ex post* profits or losses in equilibrium. When there are profits, the household will receive a dividend, recapitalising the banks when there are losses.

Given their current level of liquidity, the banks choose the loan contract terms to maximise the expected next-period real surplus by solving:

$$\begin{aligned}
&\max_{c_t^s,c_t^r}\mathbb{E}_t\left[\Lambda_{t,t+1}\left\{\lambda x_t^s\left(\frac{\tau_t^s}{P_{t+1}}-\frac{\underline{R}_t}{P_{t+1}}\right)+(1-\lambda)\,x_t^r\left(p_{t+1}\frac{\tau_t^r}{P_{t+1}}-\frac{\underline{R}_t}{P_{t+1}}\right)\right\}\right]\\
&\text{s.t.}\quad 0\le x_t^s\le x_t^r\le 1\\
&\qquad \lambda x_t^s+(1-\lambda)\,x_t^r\le\phi_t
\end{aligned} \tag{3.6}$$

and subject to constraints (3.2) and (3.3). The $x_t^r \ge x_t^s$ also follows from the IC constraints and the inequality constraint (3.6) is a liquidity constraint. ϕ_t is determined by (3.5) and is less than one when loan demand exceeds available liquidity. This is not a hard constraint since banks can always access additional funds, but additional liquidity comes at the policy ceiling rate $\overline{R}_t$, making it unprofitable. This implies the possibility of effective liquidity constraints depending

[23] If negative, the bank must use the lending facility at rate $\overline{R}_t$. Without liquidity risk, this never occurs in equilibrium (see online appendix E.1).

on macro-financial conditions and the monetary policy stance.

When constraint (3.6) is slack, then there is excess liquidity in the banking sector, banks hold excess reserves which are held in the central bank deposit facility. The solution to the bank's problem gives

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(p_{t+1}R^r_{t+1}-\underline{R}_t\right)\right]=\varrho_t-\psi_t\frac{1}{1-\lambda}+\varphi^r_t\frac{1}{1-\lambda} \tag{3.7}$$

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\left(\lambda+(1-\lambda)\,p_{t+1}\right)R^s_{t+1}-\underline{R}_t\right)\right]=\varrho_t+\varphi^r_t-\varphi^s_t, \tag{3.8}$$

where ϱ_t is the Lagrange multiplier on the feasibility constraint, φ^s_t and φ^r_t those on x^s_t and $1-x^r_t$ respectively, and ψ_t is the Lagrange multiplier on $x^r_t-x^s_t$. These first-order conditions are also subject to complementary slackness conditions that include zero-lower bounds on the four Lagrange multipliers:[24]

$$\varphi^s_t,\varphi^r_t,\varrho_t,\psi_t\geq 0. \tag{3.9}$$

3.2 Intermediate goods firms

Intermediate goods firms draw their project type at the end of period t and seek external finance. If they draw an observable project, then they are characterised as a *corporate* and access funds in the bond market. Otherwise, as a *small firm*, they must apply for a bank loan. They may or may not be successful in securing funds; if firms are successful, they purchase the required unit of capital at nominal price Q^K_t, ready for production in the following period, otherwise, they must exit. New firms can enter but must pay fixed entry costs. Firm dynamics are illusrated in figure 3.

Of the funded risky projects, a proportion $1-p_{t+1}$ will fail in the next period before production begins, with all capital lost. Success probability, $p_{t+1}\in[0,1]$, is subject to aggregate risk. After production, the firm repays the loan, sells the depreciated capital back to the capital market at Q^K_{t+1}, and returns any surplus to the household.

[24]This is the nominal version of the conditions derived using the real business cycle framework in Swarbrick (2023).

We denote corporates, safe- and risky-project firms with superscript $i \in \{c, s, r\}$. A successfully funded project requires a single unit of capital that is converted into ω_t^i productive units where $\omega_t^r = 1/p_t > \omega_t^c = \omega_t^s = 1$.[25] The firm hires $h_t\left(\omega_t^i\right)$ units of labour and produces output using constant returns to scale technology,

$$y_t\left(\omega_t^i\right) = z_t\left[\omega_t^i\right]^{\alpha}\left[h_t\left(\omega_t^i\right)\right]^{1-\alpha}, \tag{3.10}$$

where aggregate total-factor productivity, z_t, follows a stationary stochastic process. Capital depreciates at rate δ, so the remaining capital after production will be $\omega_t^i(1-\delta)$. Denoting the final consumption good price P_t, the nominal market value of capital Q_t^K, and the wholesale good price P_t^W, we can write the real value of a successfully funded type-i firm as:

$$V_t^i = \max_{h_t\left(\omega_t^i\right)}\left\{\frac{P_t^W}{P_t}y_t\left(\omega_t^i\right) - \frac{W_t}{P_t}h_t\left(\omega_t^i\right) - \left(\tau_t^i\frac{Q_{t-1}^K}{P_t} - (1-\delta)\,\omega_t^i\frac{Q_t^K}{P_t}\right) + \overline{V_t}\right\}, \tag{3.11}$$

where τ_t^i is the aggregate-state indexed loan repayment rate and $\overline{V_t}$ represents the *ex-ante*, real continuation value of a firm, prior to drawing its type, given by:

$$\overline{V_t} \equiv \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\eta V_{t+1}^c + (1-\eta)\left(\lambda x_t^s V_{t+1}^s + (1-\lambda)\,x_t^r p_{t+1}^r V_{t+1}^r\right)\right)\right], \tag{3.12}$$

which is a probability-weighted average of future values conditional on the next period project draw. The solution to (3.11) yields a labour demand condition that equates the real wage with the marginal product of labour,

$$\frac{W_t}{P_t} = (1-\alpha)\frac{P_t^W}{P_t}\frac{y_t\left(\omega_t^i\right)}{h_t\left(\omega_t^i\right)}, \tag{3.13}$$

where it follows that output per worker y_t^i/h_t^i and the efficiency capital-labour ratio ω_t^i/h_t^i will be equal across all firms. We can then write the gross nominal return on capital used in the

[25] The assumption that $\omega_t^r = 1/p_t$ is not required but allows us to focus on the effects of risk, where a risk shock increases the default risk and the worsens the impact of the credit friction, but where there is no impact in the first-best economy.

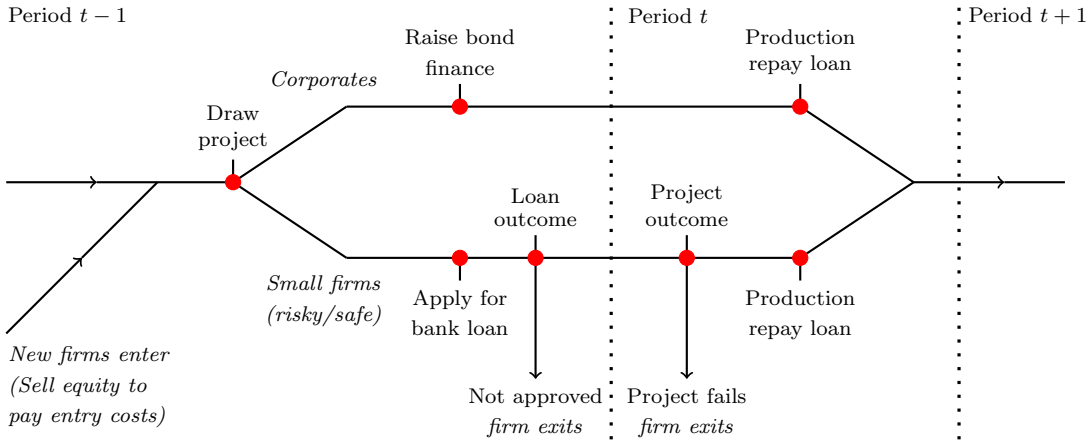


Figure 3: Firm dynamics.

previous section as

$$R_t^i \equiv \frac{\alpha \frac{P_t^W}{P_t} y_t^i + (1-\delta)\, q_t^K \omega_t^i}{q_{t-1}^K} \Pi_{t-1,t}, \tag{3.14}$$

where $q_t^K \equiv Q_t^K / P_t$ is the real price of capital. Noting that the gross return on efficiency units of capital, $\alpha \frac{y_t(\omega_t^i)}{\omega_t^i} + (1-\delta)$, is equal for all firms, it follows that $R_t^r = \omega_t^r R_t^c = \omega_t^r R_t^s$.

3.2.1 Firm entry

Firms can earn positive profits due to the information asymmetry and because each firm is a fixed size, the number of firms in the economy will matter for aggregate outcomes. To model firm dynamics, we assume firms pay a small fixed cost equal to F units of the final good to enter. The entry cost is a fee paid to households (a rebate), so it does not absorb real resources and enters household income as a lump-sum transfer.

To pay the entry costs, firms sell equity to households at nominal price Q_t^E. Total equity purchases, E_t, corresponds to the number of new firms invested in, where each share is a claim on the future profit streams of an entering firm.

$$E_t = \left(f_t - \left(\eta + (1-\eta)\left(\lambda x_{t-1}^s + (1-\lambda)\, x_{t-1}^r\right)\right) f_{t-1}\right).$$

Under this assumption, new firms will enter until the expected discounted profits $\overline{V}_t$, given by equation (3.12), equals the entry cost F.

3.3 Retailers and capital good producers

Firms in the retail sector differentiate a homogeneous intermediate good purchased at price P_t^W into final retail goods which are sold under monopolistic competition subject to a Calvo friction, whereby they face a fixed probability, ξ of being able to update each period. This leads to the usual conditions for the optimal price for price setters, P_t^*, and law of motion for inflation $\Pi_{t-1,t} \equiv P_t/P_{t-1}$, which are characterised by the usual NK Phillips curve.[26]

We also introduce capital producers that invest in new capital and buy and sell in a market for capital goods at the end of the period. Firms that secure funding seek to purchase enough capital to make up the required amount and exiting firms sell their remaining capital stock. Subject to the demand for capital and the market price, Q_t^K, capital producers purchase final retail goods, converting them into new capital goods to sell to firms under perfect competition. We assume that it is costly to adjust investment plans following Christiano et al. (2005). Because $\omega_t^r = 1/p_t$ and $\omega_t^c = \omega_t^s = 1$, in aggregate, the total units of capital sold is equal to the efficiency units in production, and therefore, the aggregate capital stock evolves according to: such that the aggregate capital stock evolves according to:

$$K_t = (1-\delta)K_{t-1} + I_t\left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]. \tag{3.15}$$

3.4 Market clearing and aggregation

Labour market clearing implies that total labour demanded by the three types of firm will equal the labour supplied by households, H_t. The perfect labour market implies that all firms will choose the same efficiency-capital-labour ratio and so, giving the capital markets clearing

[26] Since this part of the model is entirely standard, we leave details of the full derivation to the online appendix B

condition as

$$K_t \equiv \left[\eta + (1-\eta)\left(\lambda x_t^s + (1-\lambda)\, x_t^r\right)\right] f_t, \tag{3.16}$$

we can write the aggregate labour demand equation

$$\frac{W_t}{P_t} = (1-\alpha)\, MC_t z_t \left(\frac{K_{t-1}}{H_t}\right)^{\alpha}.$$

where $MC_t \equiv P_t^W / P_t$ is the real marginal cost in the retail sector. We can likewise give an aggregate intermediate good production function as

$$Y_t^W \equiv \int_i y_t^i \mathrm{d}i = z_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{3.17}$$

Finally, the goods market clears, implying the aggregate resource constraint

$$Y_t = C_t + I_t, \tag{3.18}$$

where $Y_t = Y_t^W / \Delta_t$, where Δ_t is a measure of price dispersion.

3.5 Monetary policy

A central bank can adjust the supply of liquidity in the economy in order to target the short-term rate, R_t. Denoting the total supply of liquid assets in the economy S_t, the central bank is able to engage in open market operations to perfectly target the market clearing interest rate, R_t.

Unlike the standard New Keynesian economy on which we build, the amount of liquidity in circulation affects the market interest rate through bank balance sheet and profitability constraints.[27] Suppose that the central bank increases the supply of liquidity through the purchase of corporate bonds. In the standard model, swapping one type of riskless asset (corporate bonds) for another (central bank reserves) would make no difference. However, here the central bank is replacing a higher interest earning bonds with lower interest rate reserves. In this case, the bond market

[27] A link between money volumes and interest rates can exist in the textbook case can be introduced via money in the utility. See, e.g., Galí (2015).

will only clear at a lower interest rate, shifting the market interest rate down. Any central bank profits are immediately paid out to households.[28]

To distinguish between the central bank interest rates, we assume that the main policy interest rate set to satisfy a Taylor-type rule:

$$\log\left(R_t^p\right) = \log\left(\frac{\Pi}{\beta}\right) + \gamma_\pi\left(\pi_{t-1,t} - \pi^*\right) + \gamma_y\left(y_t - y^*\right) \tag{3.19}$$

where $\pi_{t-1,t} \equiv \log\left(\Pi_{t-1,t}\right)$ and $y_t \equiv \log\left(Y_t\right)$, and the asterisks indicate a policy targets. In addition to the main policy interest rate, the central bank has two further tools. The first is the interest rate paid on excess reserves which is set equal to the main policy interest rate minus a desired corridor.

$$\underline{R}_t = R_t^p - c_t \tag{3.20}$$

The second additional policy are liquidity operations that separately move the market interest rate below the main policy interest rate. While this is essentially the same instrument as the main interest rate, we characterize the departure from the Taylor rule as a quantitative easing policy, whereby the market interest rate will move below the main policy rate towards the floor. This policy determines the spread between the main policy interest rate and the market interest rate:

$$R_t = R_t^p - q_t \tag{3.21}$$

Having outlined the model, we turn to discussing the model's key implications and some analytical results.

[28] Exactly equivalently is providing liquidity through central bank refinance operations, providing short term loans to the banking sector. In equilibrium, the interest on this liquidity will equal the rate paid on corporate bonds and household savings. Increasing liquidity provision will change the balance sheet position of banks, increasing the share in very low return reserves and so will only clear the money markets at a lower market interest rate.

4 Monetary Policy Transmission: Mechanisms and Quantitative Results

This section characterises how bank lending incentives depend jointly on macro-financial conditions and monetary policy. Theses incentives give rise to three equilibrium lending regimes, distinguished by whether banks finance all loan applications, face binding liquidity constraints, or actively ration credit.

A key implication is that liquidity policies operate through two competing channels: a standard expansionary demand channel and a contractionary credit-rationing channel that works through banks' lending incentives. Which channel dominates depends on risk and profitability conditions, and policy can shift the economy across regimes.

We first characterise equilibrium loan contracts and identify the three regimes. We then illustrate the steady-state thresholds using comparative statics, and finally quantify the competing channels using impulse responses simulations.

4.1 Loan contracts and equilibrium regimes

The bank first-order conditions (3.7) and (3.8) together with the (binding) IC and IR constraints, (3.3) and (3.2) imply a menu of loan contract specifying the loan repayment, τ_t^i, and loan approval probability, x_t^i.

Proposition 1 *If $x_t^s > 0$ ($\varphi_t^s = 0$) then $x_t^r = 1$ ($\varphi_t^r > 0$) in equilibrium.*[29]

Unless there is full rationing of safe firms, the approval rating on risky loans is 1 and all risky firms are funded. It follows that when banks wish to tighten lending standards due to changing macro-financial conditions, increased default risk for instance, they do so on price on risky loans and approval probability on safe loans.[30] This also implies that credit rationing will occur on safe loans (good types) rather than risky loans (bad types).

The bank first-order conditions, together with the firm and bank entry conditions, determine the

[29] All proofs in online Appendix E.

[30] This is consistent with evidence presented in Swarbrick (2023). Probit regressions using UK SME lending data reveal that low and average risk loans experienced lower approval rates during periods of credit tightening, whereas above average risk loans did not.

equilibrium level of bank sector liquidity and potential mismatch in loan supply and demand. Despite coming at a cost of lowering bank profitability, surplus liquidity can routinely arise in equilibrium and banks can hold excess reserves. This stems from a free entry condition in the banking sector. Providing liquidity has positive expected value, more will be demanded until the zero-profit condition holds. The opportunity cost of funds is equal to the interest on reserves $\underline{R}_t$.

While the banking sector can hold excess liquidity and provide enough loans to fulfill all firm financing needs, there are two other possible equilibrium outcomes. Either banks can become liquidity constrained when the marginal cost of funds exceeds the marginal return on lending, or banks can ration credit when the marginal information rent to risky firms exceeds the marginal revenue from safe loans.

Proposition 2 *There is no credit rationing in equilibrium if*

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \left(\lambda \left(R^s_{t+1} - \underline{R}_t \right) - (1-\lambda)(1-p_{t+1}) R^s_{t+1} \right) \right] > 0 \tag{4.1}$$

Using this threshold, we can identify three distinct regimes:

1. ***Abundant liquidity**: there is full approval under a pooling equilibrium ($\tau^s_t = \tau^r_t$ and $x^s_t = x^r_t = 1$) in which all firm financing needs are met if 4.1 is satisfied and:*

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \left[1 - (1-\lambda)(1-p_{t+1}) \right] R^s_{t+1} \right] \geq 1 \tag{4.2}$$

2. ***Scarce liquidity**: there is restricted lending due to binding liquidity constraints under a separating equilibrium ($\tau^s_t < \tau^r_t$ and $x^s_t < x^r_t = 1$) in which not all firms are financed if 4.1 is satisfied and:*

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \left[1 - (1-\lambda)(1-p_{t+1}) \right] R^s_{t+1} \right] < 1 \tag{4.3}$$

3. ***Credit rationing**: there is restricted lending due to credit rationing if*

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \left(\lambda \left(R^s_{t+1} - \underline{R}_t \right) - (1-\lambda)(1-p_{t+1}) R^s_{t+1} \right) \right] = 0 \tag{4.4}$$

Corollary 1 *The incentive to ration credit rises in: the share of risky firms,* $1-\lambda$*; the interest rate on reserves,* $\underline{R}_t$*; and the expected risky firm default rate,* $1-p_t$*. Given policy rates, the credit rationing incentive always falls in the return on capital,* R^s_{t+1}*, if* $\lambda > (1-\lambda)(1-p_t)$*.*[31]

Corollary 2 *Under non-rationing, the likelihood of liquidity constraints binding rises in the expected risky firm default rate,* $1-p_t$*, and the firm share of risky firms,* $1-\lambda$

In order to unpack proposition 2, first note from IC constraint (3.3), that the information rent earned by risky firms is equal to:

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(1-p_{t+1}\right)R^s_{t+1}\right]x^s_t, \tag{4.5}$$

which increases in the number of safe loans made. The credit rationing condition (4.4) is the difference between the marginal revenue from issuing a safe loan, $\lambda\left(R^s_{t+1}-\underline{R}_t\right)$, and the marginal cost from the resulting increase in the information rent. Once this cost outweighs the revenue, due to high default risk or high return on reserves, the banks will begin to ration credit until the marginal revenue and cost are equal.

The point at which rationing occurs depends both on how risky small business lending is but also on the interest on reserves since this determines the value of liquidity, or the opportunity cost of lending. As this rises, the incentive to ration credit increases and bank holdings of excess reserves rise.

Two features of this mechanism are worth emphasising. First, credit rationing emerges as a profit-maximising response, rather than as a consequence of insufficient funding. Banks are never mechanically constrained in their access to liquidity: they can always expand their balance sheets by holding additional reserves. Instead, lending is tightened because the opportunity cost of financing loans rises relative to holding reserves.

Second, this distinction clarifies the interpretation of "scarce liquidity" in the model. Scarcity arises for the real economy, not for banks. When lending margins are compressed, either by elevated borrower risk or a high policy floor, banks optimally reallocate their balance sheets

[31] This holds if the number of safe firms in the economy outnumber the number defaulting risky firms. This condition will be satisfied under any reasonable calibration.

away from loans and towards reserves. Aggregate bank liquidity may increase, yet the volume of credit extended to firms decline.[32]

Finally, Proposition 2 highlights a channel through which excess reserves can arise even in low-risk environments. When firm entry costs are high and default risk is low, information rents are small and a pooling equilibrium emerges. However, low firm profitability discourages entry, reducing aggregate loan demand. In this case, bank liquidity exceeds profitable lending opportunities, and banks optimally hold excess reserves despite the absence of severe informational frictions. Excess liquidity therefore reflects weak loan demand rather than impaired loan supply, and is associated with a larger credit wedge driven by subdued firm entry rather than by credit rationing.

These results establish that both liquidity constraints and credit rationing are equilibrium outcomes shaped jointly by macro-financial risk and monetary policy. The policy floor plays a central role by governing the opportunity cost of lending, but the allocation of liquidity between loans and reserves is ultimately determined by bank profitability and lending incentives, rather than the availability of funds.

4.2 Comparative statics

Figure 4 provides a steady-state illustration of the lending regimes characterised in Proposition 2. It shows how equilibrium outcomes vary with borrower risk and the width of the policy corridor, defined as the spread between the main policy rate and the interest on reserves. Movements along either dimension shift banks' lending incentives by altering the profitability of extending loans relative to holding reserves, generating transitions between abundant liquidity, scarce liquidity, and credit rationing.

In addition to borrower risk and the policy corridor, the three regions depend only on a small set of structural parameters. The overall share of small firms in the economy, $1 - \eta$, plays a minor role, whereas the composition of small firms between safe and risky types, $1 - \lambda$, is central. A higher share of risky firms worsens adverse selection, raising expected information

[32]This departs from the standard banks-as-intermediaries view. A reduction in lending therefore does not translate one-for-one into an increase in reserves: because reserves yield a lower return than loans, substituting away from lending reduces bank profitability and limits balance-sheet expansion. A $1 fall in lending will therefore typically lead to an increase in reserves of less than $1.

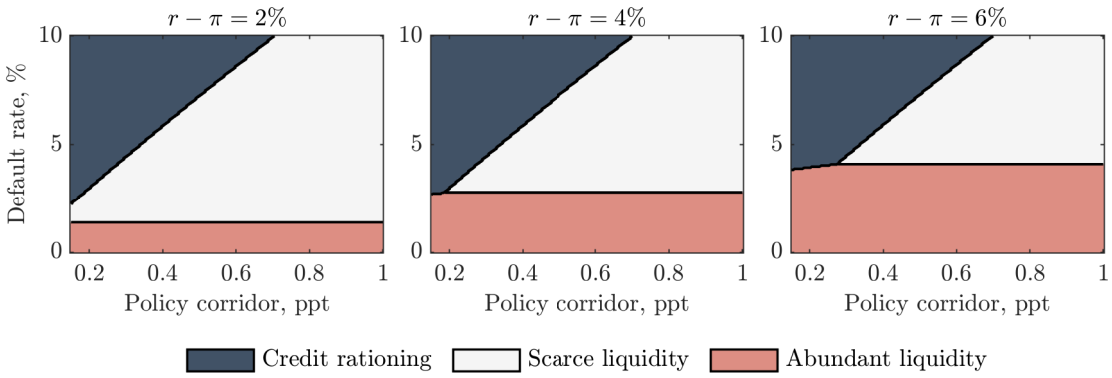


Figure 4: Comparative statics showing the role of risk (risky firm default rate), the policy corridor and the steady-state real interest rate. Other parameter values: $\eta = 0.5$, $\lambda = 0.775$, $F = 0.15$, $\beta = 0.9951, 0.9902, 0.9655$.

rents and increasing the likelihood of credit rationing. The remaining parameters determining the steady-state regime are the discount factor, β, and firm entry costs, F, both of which govern the profitability of firm entry and therefore the demand for credit.

The effect of varying β is illustrated across the panels of figure 4. A higher discount factor raises the present value of future firm profits and encourages entry for a given entry cost, increasing the demand for external finance. As a result, higher β (or lower F) makes binding liquidity constraints and credit rationing more likely, shrinking the region in which a pooling equilibrium with abundant liquidity emerges. Secular declines in real interest rates therefore influence steady-state investment and output not only through standard intertemporal substitution, but also by affecting firm entry and lending conditions. When firm entry is low and liquidity abundant, lower real rates stimulate entry and investment. When risk is high and lending conditions are already tight, however, lower real rates can tighten effective liquidity constraints. Long-run growth effects therefore depend on prevailing macro-financial conditions.

Figure 5 illustrates how the regimes map into macro-financial outcomes. Although average loan approval rates are lower when liquidity is scarce (the white region in Figure 4) than when liquidity is abundant (the red region), macroeconomic performance need not be worse. In the abundant liquidity region, low default risk and low information rents imply weak firm profitability and subdued entry. This reduces aggregate investment and capital accumulation, generating a lower

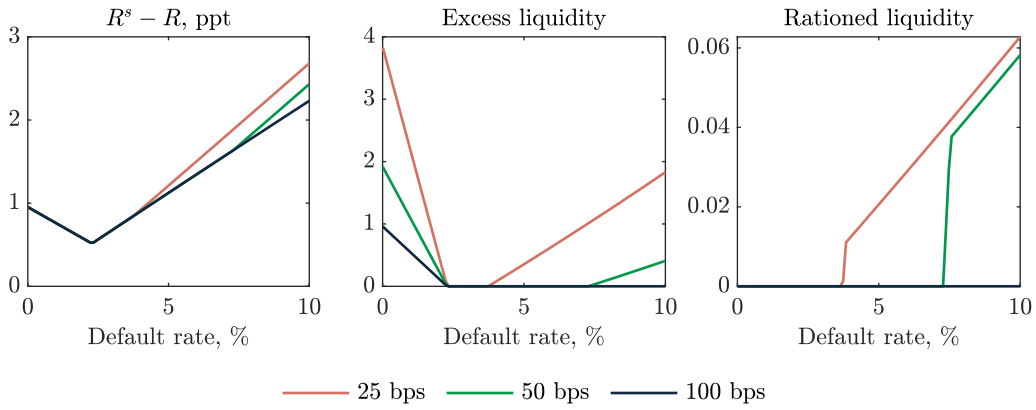


Figure 5: Comparative statics showing the role of risk (risky firm default rate) and the policy corridor on key macro-financial outcomes. Parameter values: $\eta = 0.5$, $\lambda = 0.775$, $F = 0.15$, $\beta = 0.992$.

steady-state output and a positive credit wedge, defined here as the spread between the risky return on capital and the risk-free rate. In contrast, when liquidity constraints bind, higher firm entry raises investment and output potential, even though not all firms receive funding.

More generally, inefficiencies can arise on either side of the regime boundaries. When liquidity constraints bind, productive firms remain unfunded because the marginal cost of funds exceeds the marginal return on lending. When liquidity is abundant, banks optimally hold excess reserves because profitable lending opportunities are scarce, reflecting weak loan demand rather than impaired loan supply. The credit wedge is minimised at the boundary where condition (4.1) is satisfied and the pooling condition (4.2) binds. Monetary policy can directly influence the former through the policy floor, but cannot affect the latter in steady state, as it is determined by secular features such as firm entry costs and the long-run real interest rate.

Monetary policy plays an asymmetric role across these margins. By lowering the policy floor or widening the corridor, policymakers can directly affect the credit rationing threshold by reducing the opportunity cost of lending. However, steady-state liquidity constraints are determined by secular factors such as the natural rate of interest and firm entry costs, and cannot be eliminated through interest rate policy alone. Figure 5 highlights this distinction by tracing the risk threshold implied by (4.1): a lower interest on reserves shifts this threshold outward, allowing the economy to sustain higher borrower risk without triggering credit rationing.

4.3 Liquidity policies: two channels

We now study the transmission of liquidity policies in the presence of the three lending regimes. We model a quantitative easing (QE) programme as an open-market operation that increases reserve balances and pushes the overnight market rate R_t below the main policy rate R_t^p, towards the policy floor $\underline{R}_t$.[33] In the baseline simulations, the policy is implemented as an unexpected, temporary reduction in R_t of 25 basis points (annualised), corresponding to a move halfway from the policy rate to the floor, and following an AR(1) process with persistence of 0.75. Although this resembles a conventional rate cut from the perspective of households, because it lowers the market interest rate, it also alters banks' incentives by compressing the spread between the return on lending and the interest on reserves.

In the model, QE therefore operates through two competing channels. The first is the standard *expansionary* demand channel: a fall in R_t raises consumption and increases the incentive to invest. The second is a *contractionary* credit-rationing channel: by moving R_t closer to $\underline{R}_t$, the policy increases the opportunity cost of funding loans (relative to holding reserves), lowering banks' lending incentives and worsening the adverse-selection friction. When this channel dominates, banks ration credit, optimally reducing approval rates for safe loans. This causes credit spreads to rise, and investment falls even though household demand strengthens.

To evaluate the dynamic implication of the policy, we simulate the model using the King, Plosser and Rebelo (1988, KPR) class of utility with external habits in consumption and a standard New Keynesian calibration.[34] For the new parameters, we set $\eta = 0.5$ so half of firms are corporates, set $\lambda = 0.775$ and $F = 0.15$, and unless otherwise stated, set the steady-state annual risky firm default rate to 6% ($\bar{p} = 0.985$). We focus on the role of borrower risk and the size of the liquidity programme, since these are the key determinants of whether the economy remains in the non-rationing region or crosses into the rationing regime.

Figure 6 compares impulse responses to the same QE shock in two economies that differ only in

[33] This pattern is consistent with the empirical response of short-term market rates to asset purchase programmes; see Figure 1.

[34] Full details of the calibration is shown in table 3 in online appendix B.5. Habits in consumption mainly affect the persistence of aggregate demand and have little impact on the transmission of liquidity policies away from the effective lower bound. When nominal rates are constrained, however, they become quantitatively relevant by generating more persistent declines in demand following adverse shocks.

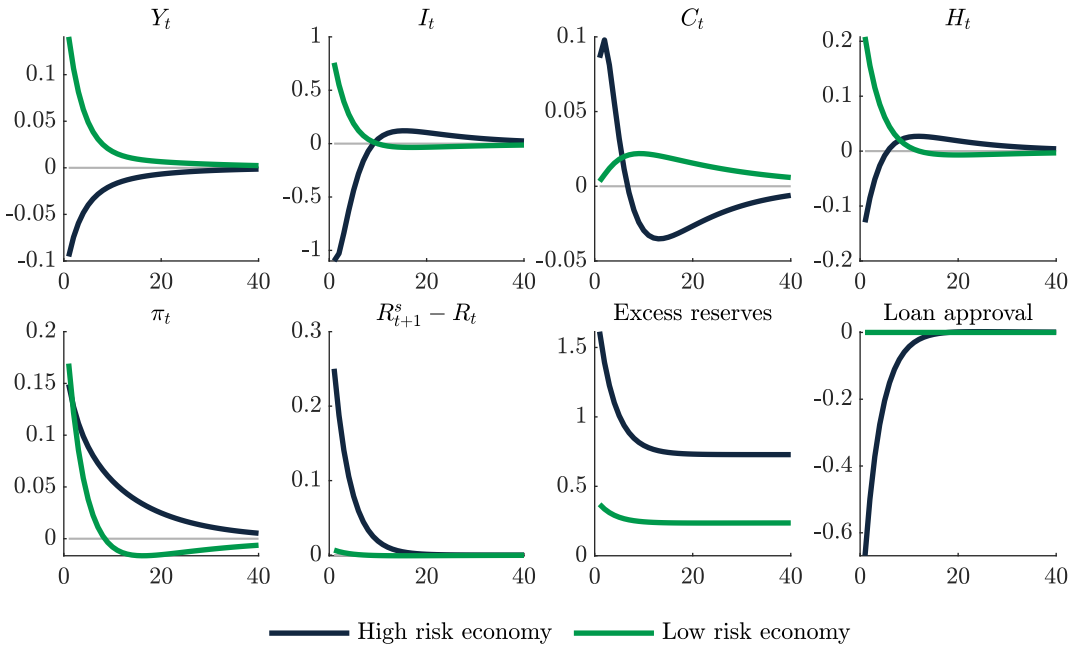


Figure 6: Impulse response functions to an unexpected temporary 'QE' programme. Plots in the top row show % deviation from steady state. The bottom row shows ppt deviation except for excess reserves with plots the level response. Excess reserves is given as the ratio of reserves to stock of loans. Per annum risky firm default rate is 12% (high risk) and 2% (low risk).

steady-state borrower risk. In the low-risk economy, the programme behaves like a conventional expansionary monetary policy shock. A lower R_t raises aggregate demand through the usual intertemporal-substitution channel, increasing consumption and investment. Lending standards remain unchanged because the economy stays away from the rationing threshold, so the demand channel dominates.[35]

In the high-risk economy, the same policy produces the opposite short-run output response. Because banks are already within the rationing region, compressing the spread between R_t and $\underline{R}_t$ strengthens the incentive to hold reserves rather than expand lending. Banks tighten lending standards by reducing safe-loan approval, credit spreads rise, and investment contracts. Reserves increase sharply at the same time, reflecting the endogenous reallocation of bank balance sheets towards low-return liquid assets. The economy therefore displays strongly state-dependent trans-

[35] A subtle point is that while the compressed spread moves the economy closer to the credit-rationing threshold, it simultaneously moves it away from the scarce-liquidity region. Reserves therefore rise endogenously as lending expands, reflecting higher bank liquidity rather than tighter lending incentives. This distinction underscores that liquidity scarcity in the model is driven by lending profitability rather than by funding availability.

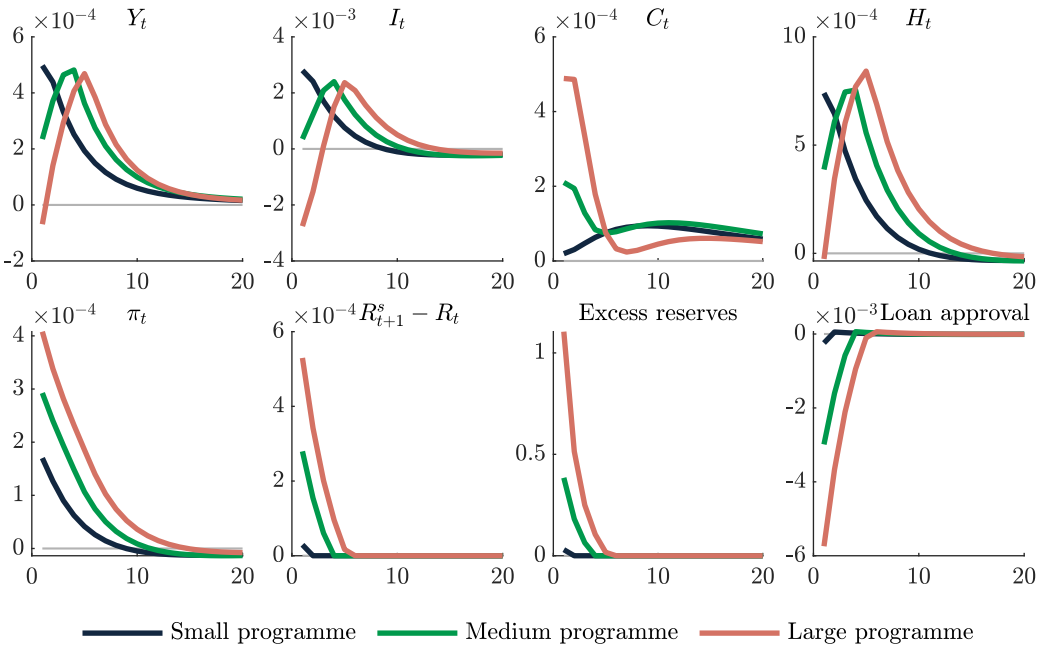


Figure 7: Impulse response functions to a temporary 'QE' programme. Plots in the top row show % deviation from steady state. The bottom row shows ppt deviation except for excess reserves which plots the ratio of reserves to stock of loans.

mission: the same liquidity operation is expansionary when risk is low, but can be contractionary when risk is high because it pushes lending incentives into the rationing regime. The threshold separating these outcomes is given by condition (4.1). Providing this remains slack, the policy works through the demand channel. However, when the constraint binds, banks will choose to tighten lending standards, and the negative investment effects offset (and can dominate) the expansionary effect from the intertemporal-substitution channel.

Figure 7 shows the same mechanism by varying the size of the QE programme: one lowering R_t by 10 bps relative to R_t^p, one by 20 bps, and one by 30 bps. A small programme leaves the economy in the non-rationing region, so responses scale approximately proportionally with shock size. Larger programmes, however, move R_t sufficiently close to $\underline{R}_t$ that condition (4.1) binds and banks begin to ration credit. This raises the opportunity cost of lending, squeezing the marginal value of credit to safe firms. Once this regime shift occurs, loan approval falls and spreads rise endogenously; investment declines sharply and output can contract on impact, despite higher consumption demand. The programme size therefore matters nonlinearly: beyond

threshold (4.1), additional easing disproportionately strengthens the credit-rationing channel.

4.4 Robustness: investment adjustment costs and preferences

The strength of the two competing transmission channels depends on how aggregate demand and investment respond to interest-rate movements. We therefore assess robustness along two dimensions: the presence of investment adjustment costs and alternative preference specifications that alter labour supply and consumption responses.

Liquidity policies affect the real economy through an expansionary demand channel and a contractionary credit-rationing channel that operates through investment. Introducing investment adjustment costs dampens the responsiveness of investment to changes in lending conditions. As a result, the contractionary credit-rationing channel is weakened, allowing the expansionary demand channel to dominate even in high-risk economies.[36]

Another margin is the strength of the wealth effect on labour supply. Adopting preferences of the type proposed by Jaimovich and Rebelo (2009, JR), which allow for a weak short-run wealth effect on labour supply, can substantially dampen the consumption response to lower interest rates. As a result, the expansionary demand channel is weakened. In this case, liquidity programmes can generate much larger and more persistent contractions in output, as the decline in investment driven by credit rationing is no longer offset by higher consumption.[37]

Finally, combining JR preferences with investment adjustment costs reverses this result. Adjustment costs limit the contractionary investment response induced by tighter lending standards, while the weak wealth effect prevents excessive labour supply responses. Together, these forces restore the dominance of the demand channel, so that even large liquidity programmes generate positive output responses despite increased credit rationing.[38]

[36] This mechanism is illustrated in figure 9 in online appendix D, which shows that with adjustment costs, the same QE programme generates a positive output response despite tighter lending standards.

[37] See figure 10 in online appendix D. Working in the same direction, increasing the degree of habit formation with the original KPR preferences will also dampen the offsetting effects of consumption demand. However, quantitatively, the effect is insufficiently strong to alter the key results.

[38] See figure 11 in online appendix D.

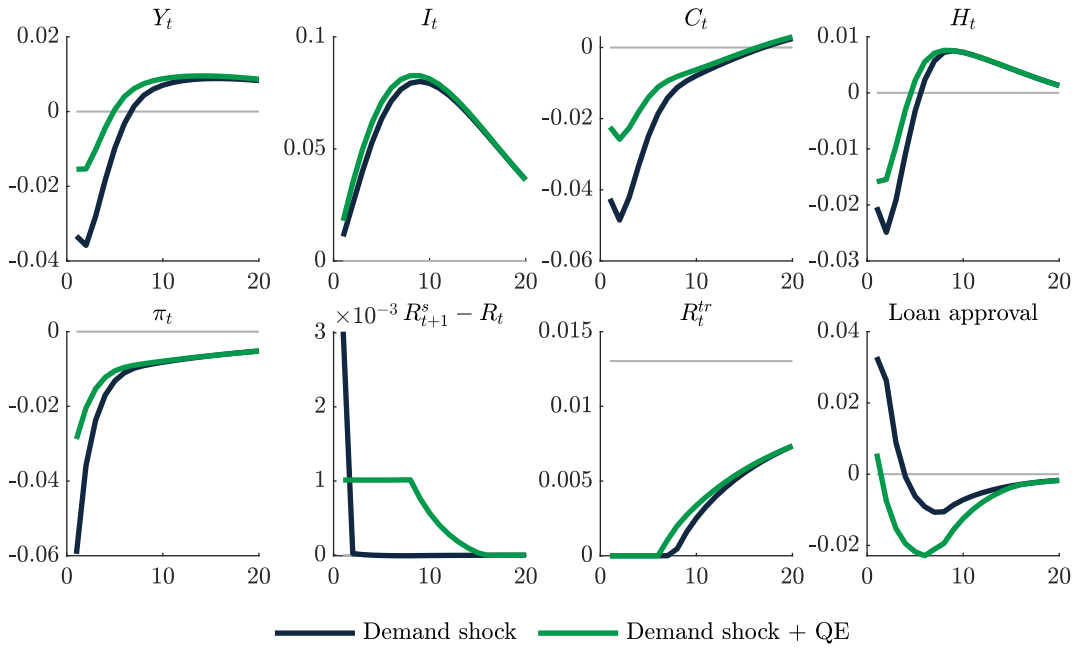


Figure 8: Impulse response functions to a demand shock with and without a 'QE' response and an economy with investment adjustment costs ($\phi_K = 4$). Plots in the top row show % deviation from steady state. The bottom row shows ppt deviation except for excess reserves which plots the ratio of reserves to stock of loans.

4.5 Liquidity policies at the effective lower bound

In practice, liquidity policies are most often used when the main policy interest rate, R^p_t, is constrained by the effective lower bound (ELB). This raises the question of how the ELB alters the balance between the expansionary demand channel and the contractionary credit-rationing channel. The key mechanism in this environment is the response of inflation expectations.

Figure 8 plots impulse response functions to a negative demand shock, with and without a QE response, in an economy with investment adjustment costs.[39] The demand shock is modelled as an unexpected increase in the household discount factor, followed by its gradual return to steady state.[40]

The shock pushes the policy rate to the ELB, where it remains for several periods. In the absence of QE, the inability of monetary policy to offset the decline in demand leads to a sharp fall in

[39]Investment adjustment costs are included to ensure that the shock generates a contraction in aggregate demand. Without adjustment costs, households would substitute away from consumption towards investment and labour, partially offsetting the decline in demand.

[40]The shock increases β by 0.01, and follows an AR(1) process with persistence 0.9.

inflation. Higher household saving increases bank liquidity and results in a temporary easing of lending standards, but this effect is insufficient to stabilise aggregate activity.

When QE is implemented alongside the demand shock, the central bank expands reserve balances, lowering the overnight market rate below the policy rate and all the way to the policy floor.[41] As in the previous section, this activates two competing channels: an expansionary demand channel and a contractionary credit-rationing channel. Crucially, however, both channels are inflationary.

As shown analytically in the next section, tighter lending conditions raise firms' marginal costs, introducing a cost-push effect. Even when credit standards tighten and investment is partially restrained, inflation expectations rise. At the ELB, this increase in expected inflation reduces real interest rates and brings forward policy lift-off, providing additional stimulus over the horizon of the shock. As a result, QE remains expansionary on net, despite a tightening in lending standards.

5 Optimal monetary policy design

This section studies the implications of credit frictions for welfare and optimal monetary policy in a simplified environment. We derive the log-linear New Keynesian representation in which endogenous lending distortions enter as a credit wedge, and obtain the associated quadratic welfare loss function. Building on this structure, we study optimal monetary policy through targeting rules under discretion and commitment, and examine the role of liquidity policy when nominal interest rates are constrained by the effective lower bound. The analysis highlights how credit frictions act as endogenous cost-push shocks and alter the inflation–output trade-off faced by the central bank.

5.1 Restricted environment and the efficient allocation

Assumption 1 (Restricted version) *We consider a restricted version without physical capital and with a fixed number of firms, f. Firms require external finance to operate, but we abstract from entry dynamics in order to focus on the allocation and policy implications of credit*

[41] As before, the policy is implemented as a direct reduction in the market interest rate, here falling by 50 basis points relative to the policy rate, with persistence 0.8.

distortions. Preferences are additive-separable in consumption and labour:

$$U\left(C_t, H_t\right) = \ln\left(C_t\right) - \varphi_H \frac{H_t^{1+\varphi}}{1+\varphi} \tag{5.1}$$

Additionally, the following parameter restrictions are imposed: $\eta = \pi^* = 0$.

The social planner's problem is to choose allocations C_t, H_t and A_t to maximise household utility $\sum_t \beta^t U\left(C_t, H_t\right)$ subject to the resource constraint $C_t = Y_t$ and technology $Y_t = A_t H_t^{1-\alpha}$, where $A_t = \exp\left(z_t\right)\left[f x_{t-1}\right]^{\alpha}$, and where $x_t \equiv \lambda x_t^s + (1-\lambda)\, x_t^r$ is the total share of projects that are funded.[42]

The solution to the social planner problem yields $x_t = 1$ and $U_{H,t}/U_{C,t} = -(1-\alpha)A_t H_t^{-\alpha}$. This is equal to a full-lending (first-best), flexible-price economy where labour subsidies are imposed to remove the steady-state distortion stemming from monopolistic competition. In the optimal policy exercises that follow, we include constant labour subsidies and lending subsidies to impose the efficient steady-state allocation.[43]

5.2 The New Keynesian model with a credit wedge

Taking a first-order approximation around the efficient allocation yields the New Keynesian IS relationship and Phillips curve:

$$\pi_t = \beta \mathbb{E}_t\left[\pi_{t+1}\right] + \kappa \tilde{y}_t - \kappa\alpha \ln(x_{t-1}) \tag{5.2}$$

$$\tilde{y}_t = \mathbb{E}_t\left[\tilde{y}_{t+1}\right] - \left(r_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^n\right) \tag{5.3}$$

where

$$r_t^n = \rho + \mathbb{E}_t\left[\Delta z_{t+1}\right], \qquad \kappa = \frac{(1-\xi)\,(1-\xi\beta)}{\xi}\frac{1+\varphi}{1-\alpha} \tag{5.4}$$

[42] Since there are many projects and each has the same expected return, a pool of risky projects has the same net present value as a pool of safe projects. Therefore, we do not discriminate and can focus only on x_t.

[43] Model derivations and details of the steady-state lending subsidy are provided in online appendix C.

and $\tilde{y}_t$ is the efficient output gap, π_t the log of inflation, and r_t the log nominal market interest rate.[44]

The key innovation relative to the benchmark New Keynesian model (e.g. Clarida et al., 1999) is that the marginal cost depends endogenously on credit allocation. When lending is constrained, effective productivity falls, generating a cost-push term in the Phillips curve. This mechanism mirrors the regime structure analysed earlier. Liquidity constraints and credit rationing reduce the funded share of projects, generating a supply-side distortion that feeds directly into inflation dynamics.[45]

5.2.1 Two policy instruments and two channels

In the full model, the policymaker has access to both the market interest rate r_t and the interest on reserves $\underline{r}_t$. We define the spread to the floor as $\nu_t \equiv r_t - \underline{r}_t$. These two instruments affect lending through distinct channels. The log-linearised bank first-order condition under rationing yields:[46]

$$\ln(x_t) = \Lambda_0 - \Lambda^D (r_t - \rho) + \Lambda^\nu \hat{\nu}_t + \Lambda_y \mathbb{E}_t [\tilde{y}_{t+1}] + \Lambda_\pi \mathbb{E}_t [\pi_{t+1}] + u_{r,t} \tag{5.5}$$

where $\hat{\nu}_t \equiv \nu_t - \nu$ denotes the deviation of the spread from steady state, $u_{r,t}$ collects exogenous shocks, and:

$$\Lambda^D \equiv \frac{1-\alpha}{1+\alpha\varphi} \frac{R^s}{R^s - 1}, \qquad \Lambda^\nu \equiv \frac{1-\alpha}{1+\alpha\varphi} \frac{\lambda \underline{R}}{(R^s - 1)[\lambda - (1-p^r)(1-\lambda)]}. \tag{5.6}$$

The two coefficients capture the two competing channels of monetary policy on lending identified in Section 4.3. The demand channel, governed by $\Lambda^D > 0$, captures how a lower interest rate raises aggregate demand and the expected return on lending, increasing banks' willingness to lend. The credit-rationing channel, governed by $\Lambda^\nu > 0$, captures how a narrower spread to the floor increases the opportunity cost of lending, reducing banks' lending incentive.

[44] The derivation of the log-linear approximation are given in online Appendix C.

[45] Since $\ln(x_{t-1}) \leq 0$, the credit wedge acts as a one-sided cost-push shock: it raises inflation when lending is constrained but never lowers it. In the stochastic equilibrium, this implies a positive unconditional mean of inflation relative to the standard New Keynesian model, even under optimal policy.

[46] In general, the condition for Λ^ν should also include the lending subsidy, however, in all reasonable parameterisations, a subsidy is not required for $x = 1$ and so we drop this here for brevity.

Lending is determined by three inequality constraints corresponding to the regimes identified in Proposition 2:[47]

$$\ln(x_t) \leq \Lambda_0 - \Lambda^D(r_t - \rho) + \Lambda^\nu \hat{\nu}_t + \Lambda_y \mathbb{E}_t[\tilde{y}_{t+1}] + \Lambda_\pi \mathbb{E}_t[\pi_{t+1}] + u_{r,t} \qquad \text{(Rationing)} \qquad (5.7)$$

$$\ln(x_t) \leq \Upsilon_0 - \Upsilon_r(r_t - \rho) + \Upsilon_y \mathbb{E}_t[\tilde{y}_{t+1}] + \Upsilon_\pi \mathbb{E}_t[\pi_{t+1}] + u_{l,t} \qquad \text{(Liquidity)} \qquad (5.8)$$

$$\ln(x_t) \leq 0 \qquad \text{(Feasibility)} \qquad (5.9)$$

Banks ration credit when (5.7) binds, lending is constrained by scarce liquidity when (5.8) binds, and there is full lending under abundant liquidity when only the feasibility constraint (5.9) binds.[48]

Proposition 3 (No-ELB benchmark) *Away from the effective lower bound on the floor, the policymaker can always set ν_t sufficiently large to ensure that credit rationing does not occur. The corridor instrument ν_t neutralises the credit-rationing channel, though lending may still be constrained by low bank profitability through the liquidity channel.*[49]

Proposition 3 establishes that the corridor is sufficient to address credit rationing but not constrained lending more broadly. When $\underline{r}_t \geq \underline{r}^{ELB}$ binds, $\nu_t = r_t - \underline{r}^{ELB}$ and the spread is no longer a free instrument. The two channels become yoked: lowering r_t both stimulates demand and compresses ν_t, which can worsen banks' lending incentive. The credit wedge then depends on r_t through the net coefficient $\Lambda_r \equiv \Lambda^D - \Lambda^\nu$, which is positive at the efficient steady state, reflecting the local dominance of the demand channel. However, $\Lambda_r = 0$ at the boundary of the rationing regime (see Proposition 2) and is negative under credit rationing, where the credit-rationing channel dominates.[50]

[47] See Appendix C.3 for derivations.

[48] The absence of $\hat{\nu}_t$ in (5.8) reflects the fact that when bank liquidity is scarce, there are no excess reserves and the interest on reserves is irrelevant to bank profitability. The corridor instrument ν_t therefore affects only the rationing margin, consistent with Corollary 1.

[49] All proofs in Appendix E.

[50] The linear approximation does not capture this reversal, which would require a global solution method. The qualitative insight that the ELB on the floor creates a policy-relevant interaction between the two channels is nonetheless robust.

5.3 Welfare loss function and optimal targeting rules

Taking a second-order approximation to household utility around the efficient undistorted steady state yields the loss function:[51]

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(-\lambda_x \frac{1-\alpha}{1+\varphi} \alpha \ln(x_{t-1}) + \frac{1}{2} \left[\pi_t^2 + \lambda_x \left(\tilde{y}_t - \alpha \ln(x_{t-1}) \right)^2 \right] \right) \tag{5.10}$$

where

$$\lambda_x = \frac{1+\varphi}{1-\alpha} \frac{(1-\beta\xi)(1-\xi)}{\varepsilon\xi} = \frac{\kappa}{\varepsilon} \tag{5.11}$$

Without the credit friction, $\ln(x_{t-1}) = 0$ and the loss function collapses to the standard case, which is linear-quadratic in inflation and the output gap. The possibility of credit rationing introduces additional sources of welfare loss.

First, reduced lending lowers effective productivity and output. This appears as a linear welfare loss through the term in $\ln(x_{t-1})$, capturing the direct cost of financing distortions. Second, there is a quadratic term in $\tilde{y}_t - \alpha \ln(x_{t-1})$, which is the gap between output and its natural (flex-price) level taking the credit allocation as given. To see this, note that real marginal cost can be written as

$$\widehat{mc}_t = \frac{1+\varphi}{1-\alpha} \left(\tilde{y}_t - \alpha \ln(x_{t-1}) \right) \tag{5.12}$$

so that $\tilde{y}_t - \alpha \ln(x_{t-1}) = 0$ when prices are flexible. The welfare-relevant output gap therefore absorbs the credit wedge into the definition of potential output. Stabilising the marginal cost stabilises inflation, but under credit rationing this requires the policymaker to engineer an off-setting output gap since closing the gap to efficient output ($\tilde{y}_t = 0$) would leave marginal costs elevated, generating inflation. The loss function thus captures a trade-off between stabilising inflation and closing the efficient output gap that is absent under full lending.

[51] Derivations in Appendix C.5.

5.4 Optimal policy under discretion

To evaluate optimal policy, we minimise period welfare loss subject to the Phillips curve (5.2), the credit wedge conditions (5.7)–(5.9), and the effective lower bound on the floor, $\underline{r}_t \geq \underline{r}^{ELB}$. When the floor is unconstrained, Proposition 3 establishes that the policymaker can independently prevent credit rationing by adjusting the spread ν_t. When the floor is constrained at the ELB, however, the policymaker must rely on liquidity operations that lower the market rate r_t below the main policy rate towards the floor.[52] In either case, lending may also be constrained by low bank profitability through the liquidity channel, which depends on lending returns rather than the corridor and must be addressed through the market rate r_t directly.

Proposition 4 (No current rationing) *If credit is not rationed in period t ($\ln(x_t) = 0$), the discretionary targeting rule is:*

$$\pi_t = -\frac{\lambda_x}{\kappa}\left(\tilde{y}_t - \alpha \ln\left(x_{t-1}\right)\right) \tag{5.13}$$

which reduces to the standard rule $\pi_t = -(\lambda_x/\kappa)\tilde{y}_t$ when $\ln(x_{t-1}) = 0$.

When past lending was constrained ($\ln(x_{t-1}) < 0$), the credit wedge enters the targeting rule as an endogenous cost-push shock through the marginal cost channel (5.12). The central bank must tolerate a larger negative output gap for a given inflation rate. Importantly, the targeting rule itself is unaffected by whether the ELB currently binds: the policymaker uses conventional policy away from the ELB, or liquidity policies at the ELB, to satisfy (5.13).

Proposition 3 establishes that credit rationing will not occur when the floor is unconstrained. When the ELB is binding, however, the policymaker may be unable to prevent credit rationing. In this case, the policymaker faces a choice about the scale of liquidity provision, knowing that any reduction in r_t operates through both the expansionary demand channel and the contractionary credit-rationing channel identified in Section 4.3.

Proposition 5 (Credit rationing at the ELB) *When the ELB on the floor binds and credit*

[52] In the model, this is equivalent to choosing r_t directly, but these operations simultaneously compress ν_t, activating the credit-rationing channel. The optimal policy problem therefore requires balancing the demand benefits of liquidity provision against its effects on bank lending incentives.

is rationed ($\ln(x_t) < 0$*), the discretionary targeting rule is:*

$$\pi_t = -\frac{\lambda_x}{\kappa}\left(\tilde{y}_t - \alpha \ln\left(x_{t-1}\right)\right) + \frac{\Lambda_r}{\kappa}\xi_{r,t} \tag{5.14}$$

where $\xi_{r,t} > 0$ *is the shadow value of relaxing the credit-rationing constraint, satisfying:*[53]

$$\xi_{r,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t\left[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}\right] \tag{5.15}$$

The final term in (5.14) is an inflation bias whose sign is governed by Λ_r. Since Proposition 5 is conditioned on active credit rationing, and $\Lambda_r \leq 0$ in this region, the bias is negative: the policymaker restrains liquidity provision, accepting lower inflation and a wider output gap to limit the degree of credit rationing.

The multiplier $\xi_{r,t}$ captures the (discounted) marginal welfare gain from easing the credit constraint. The first term in (5.15) is the direct productivity benefit of additional lending. The remaining terms reflect how the expected future lending regime affects current policy through the expectations channel. The mechanism operates through the targeting rule (5.14): binding constraints in $t+1$ create inflation or deflation biases that feed back into current inflation expectations and the current output gap through the Phillips and IS curves.

Expected future rationing ($\xi_{r,t+1} > 0$) creates a future deflation bias (since $\Lambda_r \leq 0$), lowering $\mathbb{E}_t[\pi_{t+1}]$. Through the IS curve, lower expected inflation raises the real interest rate and depresses the current output gap; through the Phillips curve, it directly reduces current inflation. Both effects worsen the current deflationary environment, leading the policymaker to ease more aggressively. This appears as an attenuated deflation bias (lower $\xi_{r,t}$). The deflation bias is therefore weakest when rationing is expected to persist.

Expected future constrained lending ($\xi_{l,t+1} > 0$) has the opposite effect. This regime occurs when bank profitability is low, causing reduced liquidity for lending and a negative expected $t+1$ output gap. In this case, the main policy interest rate, r_t, is the opportunity cost of

[53] Equation (5.15) holds under active rationing. The general condition, which holds in all regimes, is $\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}]$, where $\xi_{l,t} \geq 0$ and $\xi_{x,t} \geq 0$ are the multipliers on the liquidity and feasibility constraints respectively. See Appendix C.6.

funds rather than the interest on reserves. Since there is no credit rationing channel present, the policymaker can improve lending conditions through monetary accommodation, generating expected inflation to satisfy the targeting rule in Proposition 4. Expected future constrained lending ($\xi_{l,t+1} > 0$), therefore, generates an inflation bias (since $\Upsilon_r > 0$), raising $\mathbb{E}_t[\pi_{t+1}]$. Higher expected inflation lowers the real rate and stimulates the current output gap. Since expectations are already providing stimulus, the policymaker needs to ease less, strengthening the deflation bias (higher $\xi_{r,t}$). The deflation bias is therefore strongest when the policymaker expects the economy to transition from rationing to constrained lending.

From the decomposition in (5.6), $|\Lambda_r|$ increases with the severity of adverse selection: higher borrower risk (lower p^r) or a larger share of risky firms (lower λ) strengthens the credit-rationing channel relative to the demand channel, requiring a larger deviation from the standard targeting rule. The bias vanishes at the boundary of the rationing regime where $\Lambda_r = 0$, and increases with α, the importance of credit for productivity. In economies with more severe information frictions, the policymaker therefore faces a sharper trade-off between stimulating demand through liquidity provision and preserving bank lending incentives — consistent with the quantitative results in Figure 6, where liquidity policies become contractionary under high borrower risk.

5.5 Optimal policy under commitment

Under commitment from a timeless perspective, the policymaker internalises how current credit conditions affect future welfare through $\ln(x_{t-1})$ in the Phillips curve and loss function, and how promised future policy affects current lending incentives through $\mathbb{E}_t[\tilde{y}_{t+1}]$ and $\mathbb{E}_t[\pi_{t+1}]$ in the rationing condition (5.5).

Proposition 6 (Commitment targeting rule) *The commitment targeting rule is:*

$$\pi_t + \frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha \ln(x_{t-1})) = \mu_{\pi,t-1} + \frac{\Lambda_r}{\kappa}\xi_{r,t} + \frac{\Upsilon_r}{\kappa}\xi_{l,t} - \Omega_r \xi_{r,t-1} - \Omega_l \xi_{l,t-1} \tag{5.16}$$

where Ω_r, Ω_l are composite coefficients defined in Appendix C.7, and $\xi_{r,t}$, $\xi_{l,t}$ are the shadow values of the rationing and liquidity constraints. Under credit rationing ($\xi_{r,t} > 0$, $\xi_{l,t} = 0$), the

shadow value satisfies:[54]

$$\xi_{r,t} = \frac{1}{\Gamma_r}\left[\beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}]\right] \tag{5.17}$$

When $\ln(x_t) = 0$ *and* $\ln(x_{t-1}) = \xi_{r,t-1} = \xi_{l,t-1} = 0$*, the rule reduces to the standard price-level targeting rule* $\pi_t = -(\lambda_x/\kappa)(\tilde{y}_t - \tilde{y}_{t-1})$.

The commitment targeting rule (5.16) differs from the discretionary rule (5.14) in two respects. First, the lagged multipliers $\mu_{\pi,t-1}$, $\xi_{r,t-1}$, and $\xi_{l,t-1}$ introduce history dependence. The lagged NKPC multiplier $\mu_{\pi,t-1}$ generates the standard price-level targeting property. The terms in $\xi_{r,t-1}$ and $\xi_{l,t-1}$ propagate past lending constraints into current policy. Since $\Omega_r < 0$ under rationing,[55] past rationing ($\xi_{r,t-1} > 0$) raises inflation through $-\Omega_r\xi_{r,t-1} > 0$: the policymaker delivers accommodation that was committed to during the rationing episode in order to raise inflation expectations and ease the credit constraint at that time. It is the anticipation of this accommodation that reduces the severity of rationing under commitment relative to discretion.

Second, the shadow value $\xi_{r,t}$, which governs the deflation bias $(\Lambda_r/\kappa)\xi_{r,t}$, is amplified by $1/\Gamma_r > 1$ relative to discretion. The scaling arises because the commitment policymaker recognises that current rationing obliges future accommodation, making each unit of rationing more costly than under discretion. The deflation bias is therefore larger under commitment. More broadly, $1/\Gamma_r$ reflects a positive feedback loop: since $\Lambda_\pi, \Lambda_y > 0$, the promised future accommodation credibly raises $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[\tilde{y}_{t+1}]$, which raises the profitability of lending and relaxes the credit constraint for any given r_t. The amplification factor scales the entire recursion, including $\beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1}]$: when future rationing is expected, the future deflation bias lowers expected inflation, reducing the effectiveness of this forward-guidance channel. The benefit of commitment is therefore largest when rationing is expected to be temporary and smallest when it is expected to persist.

Corollary 3 *The optimal degree of credit rationing under commitment is less severe than under discretion:* $x_t^{\text{commit}} > x_t^{\text{discr}}$.

[54] Under discretion, $\Gamma_r = 1$. Under commitment, $\Gamma_r = 1 - \alpha(\Lambda_y - \Lambda_r) < 1$ since $\Lambda_y > \Lambda_r$. See Appendix C.7.

[55] From the definition $\Omega_r \equiv [(1+\kappa)\Lambda_r - \Lambda_y - \kappa\Lambda_\pi]/(\beta\kappa)$. Since $\Lambda_r \leq 0$, $\Lambda_y > 0$, and $\Lambda_\pi > 0$, $\Omega_r < 0$. See Appendix C.7.

This provides a new channel through which commitment matters at the ELB, beyond the standard real-rate channel. Forward guidance not only reduces real interest rates; it also directly eases credit conditions by raising the expected return on lending relative to the floor, weakening the credit-rationing friction.[56]

The history dependence of the commitment solution also implies gradualism in the withdrawal of accommodation. The lagged multiplier $\xi_{r,t-1}$ propagates past rationing episodes into the targeting rule, generating above-normal accommodation even after full lending is restored. Through the price-level targeting property of commitment, this accommodation is persistent: the rationing episode permanently shifts the implicit price-level target upward, generating a protracted period of above-normal inflation during the recovery. In the context of unconventional policy, this provides a rationale for gradual quantitative tightening: the promised future accommodation that made aggressive liquidity provision optimal during the crisis must be delivered during the recovery to maintain credibility and support the transition back to full lending.

6 Concluding remarks

This paper makes three broad contributions. First, it provides empirical evidence consistent with a link between interest-rate compression and bank lending incentives. Using euro area bank-level and aggregate data, the analysis documents that narrower spreads between market rates and the policy floor are associated with tighter lending standards and reduced credit to small firms, consistent with the credit-rationing channel developed in the theoretical model.

Second, the paper develops a structural model that rationalises these findings. By embedding adverse selection into a New Keynesian framework with bank balance sheets and a policy corridor, the model delivers endogenous lending regimes and state-dependent transmission of liquidity policies. Liquidity injections operate through two opposing channels: a conventional demand channel and a credit-rationing channel that works through banks' lending incentives. Which channel dominates depends on risk, entry conditions, and the opportunity cost of lending, rather

[56] This mirrors the analytical result in Section 4.5: both the demand channel and the credit-rationing channel raise inflation expectations, so QE remains expansionary at the ELB. Under commitment, the policymaker exploits this mechanism optimally.

than on banks' access to funding. As a result, credit scarcity can arise for the real economy even as bank liquidity increases.

Third, the paper draws implications for monetary policy design. Endogenous lending distortions enter the welfare-relevant loss function as a credit wedge, generating a trade-off between stabilising inflation and closing the efficient output gap that is absent under full lending. Under discretion, optimal policy at the effective lower bound features a deflation bias: the policymaker restrains liquidity provision to preserve bank lending margins, with the bias increasing in the severity of adverse selection. Expected future lending regimes feed back into current policy through the expectations channel, with the risk of future rationing moderating the bias. Under commitment, the policymaker can ease credit conditions through forward guidance, by credibly promising future accommodation that raises lending profitability. This commitment generates history dependence in optimal policy: past rationing episodes oblige future accommodation, providing a rationale for gradualism in quantitative tightening. The promised accommodation that made aggressive liquidity provision optimal during the crisis must be delivered during the recovery, implying a protracted period of above-normal inflation to support the transition back to full lending.

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Online Appendices for "*Monetary Policy and the Credit Rationing Effects of Liquidity*"

Jonathan Swarbrick

University of St Andrews

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Appendix A Empirical Results

The following specification is estimated using monthly data between 2008–2019:

$$\ln X_t = \alpha_X + \beta_X s_t + \boldsymbol{\gamma}'_{\mathbf{X}} \cdot \mathbf{z}_t + u_{X,t} \tag{A.1}$$

for $X \in \{L, DF\}$, where L_t is the stock of loans by monetary and financial institutions (MFIs) to non-financial institutions (NFIs) in the euro area; and DF_t is the total use of the central bank deposit facility (both source ECB). s_t is either the corridor between the ECB's main refinancing rate and deposit facility rate, or the spread between the interbank rate (Eonia) and the deposit facility rate (both source ECB).[1] $\mathbf{z}$ is a vector of controls including GDP, inflation, the interbank rate (Eonia), credit risk, and ECB asset purchases. Controls include total euro area log nominal GDP and CPI (HCIP) inflation, the Eonia rate (source ECB), Gilchrist and Mojon's (2018) measure of credit risk of euro area NFIs, and the stock of total asset purchase programmes (APPs), which includes all securities held by the eurosystem for monetary policy purposes (source ECB). Table 1 reports the results.

The following specification is estimated using quarterly bank-level balance sheet data over 2008Q1–

[1] Note that since October 2019, the ECB has targeted a new euro short-term rate, replacing Eonia as the interbank rate (see Nicoloso and Tsonchev, 2019, Box 1, p.23).

2019Q4:

$$\ln X_{i,t+h} = \alpha_{X,h} + \gamma_{X,i,h} + \beta_{X,h} s_t + \boldsymbol{\gamma}'_{\mathbf{X},\mathbf{h}} \cdot \mathbf{z}_t + u_{X,i,t} \tag{A.2}$$

Bank-level balance sheet data is from S&P Global Market Intelligence (SNL Financial Fundamentals). We use all available data for banks operating in Austria, Belgium, France, Germany, Ireland, Italy, Netherlands, Portugal, Slovenia and Spain. Bank-level lending is first normalized by reporting as a ratio with the bank's average lending volume over 2008Q1–2019Q4. In order to ensure larger banks play a more important role, we use the size of the bank's balance sheet as a sampling weight in the estimation. We include bank fixed effects and cluster standard errors at the bank level.

Table 2 reports the results using this specification. Different from the aggregate data, the cash and reserves includes all cash, central bank deposits and central bank reserves, including both required and excess reserves. Therefore, because it also includes required reserves, the relative changes are much smaller.

	a) $s_t \equiv R_t - \underline{R}_t$		b) $s_t \equiv R^*_t - \underline{R}_t$	
	$\ln Reserves_t$	$\ln L_t$	$\ln Reserves_t$	$\ln L_t$
s_t	-0.055^{***}	0.0179^{***}	-0.042^{**}	0.0099^{**}
	(0.011)	(0.005)	(0.019)	(0.0039)
R^*_t	-0.0732^{***}	-0.0163^{**}	-0.0347	-0.0251^{***}
	(0.020)	(0.007)	(0.025)	(0.0073)
Obs	5804	4206	5804	4206
Adj. R^2	0.385	0.053	0.381	0.042

Notes: Standard errors in parentheses. Significance levels: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 2: Estimation results testing the impact of the ECB corridor width (Main Refinancing Rate R^p_t minus Deposit Facility Rate $\underline{R}_t$) and the spread to the floor (the interbank rate (Eonia) R_t minus Deposit Facility Rate), both measured per 25 bps, on bank balance sheet data: 'Cash and balances with Central Banks' and 'Total Loans'. Source: Capital IQ: SNL Financial Fundementals.

Appendix B Model derivation and equilibrium conditions

This appendix outlines sections of the model which are not fully detailed in the main text. The full set of equilibrium conditions is also outlined

B.1 Retailers

Firms in the retail sector differentiate a homogeneous intermediate good purchased at price P_t^W into final retail goods which are sold under monopolistic competition. Following the standard approach, preferences for retail goods is characterized by constant elasticity of substitution between the differentiated goods indexed $j \in (0, 1)$, for example for consumption goods:

$$C_t = \left(\int_0^1 \left[C_t(j) \right]^{\frac{\varepsilon - 1}{\varepsilon}} \mathrm{d}j \right)^{\frac{\varepsilon}{\varepsilon - 1}}. \tag{B.1}$$

For a given amount of total consumption, households choose the amount of each variety in order to minimize total expenditure, $P_t C_t = \int_0^1 P_t(j) C_t(j) \mathrm{d}j$. This leads to demand schedules $C_t(j) = \left(P_t(j)/P_t \right)^{-\varepsilon} C_t$ of which there are equivalent conditions for investment demand.

The retailers set their price to maximize the present value of future profits but do so subject to Calvo friction, whereby they face a fixed probability, ξ of being able to update each period. This leads to the usual conditions for the optimal price, and the law of motion for inflation $\Pi_{t-1,t} \equiv P_t / P_{t-1}$ and price dispersion Δ_t:

$$1 = \xi \Pi_{t-1,t}^{\zeta - 1} + (1 - \xi) \left(\frac{\Omega_{1,t}}{\Omega_{2,t}} \right)^{1 - \zeta} \tag{B.2}$$

$$\Delta_t = \xi \Pi_{t-1,t}^{\zeta} \Delta_{t-1} + (1 - \xi) \left(\frac{\Omega_{1,t}}{\Omega_{2,t}} \right)^{-\zeta} \tag{B.3}$$

where

$$\Omega_{1,t} = Y_t \frac{1}{1 - \frac{1}{\zeta}} MC_t + \xi \beta \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} \Omega_{1,t+1} \right] \tag{B.4}$$

$$\Omega_{2,t} = Y_t + \xi \beta \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta - 1} \Omega_{2,t+1} \right] \tag{B.5}$$

Price dispersion generates an output loss in aggregate, so that the total intermediate output is

related to total retail output according to:

$$Y_t = \frac{1}{\Delta_t} \int_i y_t^i \mathrm{d}i \tag{B.6}$$

over intermediate goods firms indexed i.

B.2 Capital producers

Capital producers purchase final retail goods, converting them into new capital goods to sell to firms under perfect competition at real price q_t^k. We assume that it is costly to adjust investment plans following Christiano et al. (2005), such that the representative capital producer seeks to maximise the stream of real profits:

$$\max_{I_t} \sum_{s=0}^{\infty} \mathbb{E}_t \left[\Lambda_{t,t+s} \left(q_{t+s}^k I_{t+s} \left[1 - s\left(\frac{I_{t+s}}{I_{t-1+s}} \right) \right] - I_{t+s} \right) \right] \tag{B.7}$$

We assume a quadratic form of the cost function, so

$$s\left(\frac{I_t}{I_{t-1}} \right) = \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \tag{B.8}$$

from which we arrive at a first-order condition that determines investment in period t

$$1 = q_t^K \left(\left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] - \phi \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + \mathbb{E}_t \left[\Lambda_{t,t+1} q_{t+1}^k \phi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right].$$

Because $\omega_t^r = 1/p_t$ and $\omega_t^c = \omega_t^s = 1$, in aggregate, the total units of capital sold is equal to the efficiency units in production, and therefore, the aggregate capital stock evolves according to:

$$K_t = (1-\delta) K_{t-1} + I_t \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right]. \tag{B.9}$$

B.3 Equilibrium conditions

The competitive general equilibrium is determined by the following conditions:

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t \tag{B.10}$$

$$\frac{W_t}{P_t} = -\frac{U'(H_t)}{U'_t(C_t)} \tag{B.11}$$

$$\frac{W_t}{P_t} = (1-\alpha) MC_t z_t \left(\frac{K_{t-1}}{H_t} \right)^{\alpha} \tag{B.12}$$

$$R_t^s = \frac{\alpha MC_t z_t \left(\frac{K_{t-1}}{H_t} \right)^{\alpha-1} + (1-\delta) q_t^K}{q_{t-1}^K} \Pi_{t-1,t}, \tag{B.13}$$

$$R_t^r = \omega_t^r R_t^s \tag{B.14}$$

$$K_t = (1-\delta) K_{t-1} + I_t \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] \tag{B.15}$$

$$1 = q_t^K \left(\left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] - \phi \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + \mathbb{E}_t \left[\Lambda_{t,t+1} q_{t+1}^k \phi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]. \tag{B.16}$$

$$Y_t^W = z_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{B.17}$$

$$Y_t = \frac{Y_t^W}{\Delta_t} \tag{B.18}$$

$$Y_t = C_t + I_t \tag{B.19}$$

$$1 = \xi \Pi_{t-1,t}^{\zeta-1} + (1-\xi) \left(\frac{\Omega_{1,t}}{\Omega_{2,t}} \right)^{1-\zeta} \tag{B.20}$$

$$\Delta_t = \xi \Pi_{t-1,t}^{\zeta} \Delta_{t-1} + (1-\xi) \left(\frac{\Omega_{1,t}}{\Omega_{2,t}} \right)^{-\zeta} \tag{B.21}$$

$$\Omega_{1,t} = Y_t \frac{1}{1-\frac{1}{\zeta}} MC_t + \xi \beta \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} \Omega_{1,t+1} \right] \tag{B.22}$$

$$\Omega_{2,t} = Y_t + \xi \beta \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta-1} \Omega_{2,t+1} \right] \tag{B.23}$$

$$R_t = \frac{\Pi^*}{\beta} \left(\frac{Y_t}{\overline{Y}} \right)^{\gamma_y} \left(\frac{\Pi_{t-1,t}}{\Pi^*} \right)^{\gamma_\pi} \tag{B.24}$$

In the model without credit frictions or firm-entry costs, the model is closed with

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} R_{t+1}^s \right]. \tag{B.25}$$

Otherwise, we have the firm entry condition, and the capital market clearing linking the number of firms and capital:

$$F = \frac{\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\eta + (1-\eta)(1-\lambda)(1-p_{t+1})\,x_t^s\right)R_{t+1}^s\right] - \eta}{1 - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\eta + (1-\eta)\left(\lambda x_t^s + (1-\lambda)\,x_t^r p_{t+1}^r\right)\right)\right]} q_t^K \tag{B.26}$$

$$K_t = \left(\eta + (1-\eta)\left(\lambda x_t^s + (1-\lambda)\,x_t^r\right)\right) f_t \tag{B.27}$$

The bank entry condition for liquidity ϕ:

$$1 = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\underline{R}_t\left(1 - \left[\lambda x_t^s + (1-\lambda)\,x_t^r\right]\frac{1}{\phi_t}\right) + \left(\lambda x_t^s + (1-\lambda)\left(x_t^r - (1-p_{t+1})\,x_t^s\right)\right)\frac{1}{\phi_t}R_{t+1}^s\right)\right] \tag{B.28}$$

and conditions for lending (x_t^r and x_t^s):

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(R_{t+1}^s - \underline{R}_t\right)\right] = \varrho_t - \psi_t\frac{1}{1-\lambda} + \varphi_t^r\frac{1}{1-\lambda} \tag{B.29}$$

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\left(\lambda + (1-\lambda)\,p_{t+1}\right)R_{t+1}^s - \underline{R}_t\right)\right] = \varrho_t + \varphi_t^r - \varphi_t^s \tag{B.30}$$

$$\varphi_t^s, \varphi_t^r, \varrho_t, \psi_t \geq 0 \tag{B.31}$$

$$\varphi_t^s x_t^s = 0 \tag{B.32}$$

$$\varphi_t^r\left(1 - x_t^r\right) = 0 \tag{B.33}$$

$$\varrho_t\left(\bar{x}_t - \lambda x_t^s - (1-\lambda)\,x_t^r\right) = 0 \tag{B.34}$$

$$\psi_t\left(x_t^r - x_t^s\right) \tag{B.35}$$

$$\bar{x}_t = \min\left\{1, \phi_t\right\} \tag{B.36}$$

B.4 Utility function

For the baseline numerical simulations, we use the King, Plosser and Rebelo (1988, KPR) class of utility with external habits in consumption:

$$U\left(C_t, H_t\right) = \frac{\left(\left[C_t - h\bar{C}_{t-1}\right]^{1-\chi}\left(1-H_t\right)^{\chi}\right)^{1-\sigma}}{1-\sigma} \tag{B.37}$$

For robustness exercises, we also consider preferences of the type proposed by Jaimovich and Rebelo (2009, JR), which allow for a weak short-run wealth effect on labour supply. These take the form:

$$U(C_t, H_t) = \frac{1}{1-\sigma}\left(C_t - \chi H_t^{\theta} X_t\right)^{1-\sigma} \tag{B.38}$$

where $X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}$. In the numerical simulations these are calibrated with $\gamma = 0.2$ and an inverse Frisch elasticity $\theta = 2.5$.

B.5 Baseline parameterisation for numerical simulations

For numerical simulations, we use a standard New Keynesian calibration shown in Table 3. Parameters for the novel part of the model (share of corporates, share of risky smalls, risky firm default) come from the calibration in Swarbrick (2023).

Appendix C Simplified version and Linear-Quadratic Welfare

The simplified version abstracts from capital accumulation and firm entry. The number of firms is fixed at f. Firms still, by assumption, require a single unit of outside finance. Firm i still draws a type, then seeks credit, converting it into an idea (or productivity) ω as before. The only difference is that there is no real cost, they do not need to purchase capital. To keep things as close to the large model as possible, we treat liquidity directly as 'productive capacity'. If firm i receives credit (capacity) and is successful in their project, the credit converts into ω_t^i units of

Parameter	Description	Value
β	Household discount factor	0.992
σ	Coefficient of relative risk aversion	1
χ	Consumption–leisure weight	$\bar{H} = 1/3$
α	Capital share in production	0.30
δ	Capital depreciation rate (quarterly)	0.023
ξ	Calvo price stickiness	0.75
ζ	Elasticity of substitution across goods	7
γ_π	Taylor rule inflation coefficient	1.5
γ_y	Taylor rule output gap coefficient	0.25
π^*	Inflation target (quarterly)	0.005
$\underline{R}$	Policy floor relative to policy rate (annualised)	-50 bps
η	Share of corporate firms	0.50
λ	Share of safe small firms	0.775
F	Firm entry cost	0.15
$\bar{p}$	Steady-state risky firm default rate (annual)	6%
ϕ_K	Investment adjustment cost parameter	0
h	Habits in consumption	0.5

Table 3: Baseline parameter values

productive capacity which can be returned to the lender for future use.

We further assume $\pi^* = \eta = 0$. Preferences take the form:

$$U\left(C_t, H_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + \varphi_H \frac{H_t^{1+\varphi}}{1+\varphi} \tag{C.1}$$

The production function and the labour demand equation are the same except we allow for the inclusion of a steady-state labour subsidy, s_W. The firm requires one real unit of external finance in $t-1$ for production at t. The firms to transform this liquidity into into ω_t^i units of productive capacity, producing output using :

$$y_t^i = Z_t \left[\omega_t^i\right]^\alpha \left[h_t^i\right]^{1-\alpha} \tag{C.2}$$

$$(1-s_W)\frac{W_t}{P_t} = (1-\alpha)\frac{P_t^W}{P_t}\frac{y_t\left(\omega_t^i\right)}{h_t\left(\omega_t^i\right)}, \tag{C.3}$$

$$\Rightarrow (1-s_W)\frac{W_t}{P_t} = (1-\alpha)\, MC_t \frac{z_t \left[\omega_t^i\right]^\alpha \left[h_t^i\right]^{1-\alpha}}{h_t^i} \tag{C.4}$$

where $\frac{y_t^i}{\omega_t^i}$ is the same for all firms due to flexible labour markets. This implies:

$$\frac{[\omega_t^s]^\alpha [h_t^s]^{1-\alpha}}{h_t^s} = \frac{[\omega_t^r]^\alpha [h_t^r]^{1-\alpha}}{h_t^r} \tag{C.5}$$

$$\Rightarrow \omega_t^r h_t^s = h_t^r \tag{C.6}$$

Therefore, we define gross nominal return on liquidity as:

$$R_t^i P_{t-1} = \omega_t^i P_t + P_t^W y_t\left(\omega_t^i\right) - (1-s_W) W_t h_t\left(\omega_t^i\right) \tag{C.7}$$

$$= \omega_t^i P_t + P_t^W Z_t \left[\omega_t^i\right]^\alpha \left[h_t^i\right]^{1-\alpha} - (1-\alpha) P_t^W Z_t \left[\omega_t^i\right]^\alpha \left[h_t^i\right]^{1-\alpha} \tag{C.8}$$

$$= \omega_t^i P_t + \alpha(1+s_W) P_t^W Z_t \left[\omega_t^i\right]^\alpha \left[h_t^i\right]^{1-\alpha} \tag{C.9}$$

where we have used the labour demand equation. Then using $\omega_t^s = 1$ and $\omega_t^r h_t^s = h_t^r \Rightarrow h_t^i = \omega_t^i h_t^s$:

$$R_t^i = \omega_t^i \left[1 + \alpha(1+s_W) MC_t Z_t \left[h_t^s\right]^{1-\alpha}\right] \frac{P_t}{P_{t-1}} \tag{C.10}$$

from which is follows: $R_t^r = \omega_t^r R_t^s$.

The labour market clearing implies

$$H_t = \eta f h_t^s + (1-\eta)\lambda x_{t-1}^s f h_t^s + (1-\eta)(1-\lambda) p_t^r f h_t^r \tag{C.11}$$

Noting that $[h_t^s]^{1-\alpha} = \frac{[\omega_t^r]^\alpha [h_t^r]^{1-\alpha}}{\omega_t^r} \Rightarrow h_t^s = h_t^r/\omega_t^r = p_t^r h_t^r$, this is:

$$H_t = \left[\eta + (1-\eta)\lambda x_{t-1}^s + (1-\eta)(1-\lambda)\right] f h_t^s \tag{C.12}$$

then:

$$R_t^s = \left[1 + \alpha(1+s_W) MC_t z_t \left[\frac{H_t}{f\left[\lambda x_{t-1}^s + 1 - \lambda\right]}\right]^{1-\alpha}\right] \Pi_{t-1,t} \tag{C.13}$$

Firm output is:

$$y_t^i = Z_t \left[\omega_t^i\right]^{\alpha} \left[h_t^i\right]^{1-\alpha}$$
$$\Rightarrow\ y_t^s = Z_t \left[h_t^s\right]^{1-\alpha}$$
$$y_t^r = Z_t \left[\omega_t^r\right]^{\alpha} \left[\omega_t^r h_t^s\right]^{1-\alpha} = Z_t \left[\omega_t^r\right] \left[h_t^s\right]^{1-\alpha}$$

Total output is then:

$$Y_t = Z_t f \left(\lambda x_{t-1}^s + 1 - \lambda\right) \left[\frac{H_t}{\left(\lambda x_{t-1}^s + 1 - \lambda\right) f}\right]^{1-\alpha} \tag{C.14}$$

then by denoting aggregate productivity:

$$A_t \equiv Z_t \left[f \left(\lambda x_{t-1}^s + 1 - \lambda\right)\right]^{\alpha}, \tag{C.15}$$

we give aggregate output, labour demand and gross return on bank funding:

$$Y_t = A_t H_t^{1-\alpha} \tag{C.16}$$
$$\frac{W_t}{P_t} = (1 + s_W)(1 - \alpha) MC_t \frac{Y_t}{H_t} \tag{C.17}$$
$$R_t^s = \left[1 + \alpha(1 + s_W) MC_t \frac{Y_t}{f \left(\lambda x_{t-1}^s + 1 - \lambda\right)}\right] \Pi_{t-1,t} \tag{C.18}$$

C.1 Lending subsidy

As well as production subsidies which remove the steady-state distortion stemming from monopolistic competition in goods markets, we introduce a lending subsidy, so for every dollar returned on loans, the bank receives $1 + s_L$ dollars. In doing so, the government can ensure that the pooling condition ($x = 1$) is satisfied in steady-state, namely:

$$\beta \left[1 - (1 - \lambda)(1 + p)\right] (1 + s_L) R^s \geq 1 \tag{C.19}$$
$$\beta \left(\lambda \left((1 + s_L) R^s - \underline{R}\right) - (1 - \lambda)(1 - p)(1 + s_L) R^s\right) > 0 \tag{C.20}$$

The first ensures there is sufficient liquidity in the banking sector, and the second that banks do not rationing credit. The first is not sufficient as this can be satisfied and yet banks still ration credit. We can see this by rearranging the second and taking the first to just bind, writing:

$$\frac{R}{\underline{R}} > \lambda \frac{[1-(1-\lambda)(1+p)]}{[1-(1-\lambda)(1+p)]\,(1+s_L)-(1-\lambda)} \tag{C.21}$$

Noting we have used $R = 1/\beta$. The subsidy therefore must be set sufficiently high that both conditions are satisfied. Setting $\underline{R}$ at an effective lower bound, taken to be zero for convenience, so $\underline{R} = 1$, and setting the subsidy so the constraint just binds if required and zero otherwise implies:

$$s_L = \max\left\{0 \quad , \quad \frac{\beta\lambda[1-(1-\lambda)(1-p)]+1-\lambda}{1-(1-\lambda)(1-p)} - 1\right\} \tag{C.22}$$

C.2 Full non-linear equilibrium conditions

$$Y_t = \frac{1}{\Delta_t} A_t H_t^{1-\alpha} \tag{C.23}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t,t+1}} \right] \tag{C.24}$$

$$\frac{W_t}{P_t} = \varphi_H C_t^{\sigma} H_t^{\varphi} \tag{C.25}$$

$$C_t = Y_t \tag{C.26}$$

$$\frac{W_t}{P_t} = (1+s_W)(1-\alpha) MC_t \frac{\Delta_t Y_t}{H_t} \tag{C.27}$$

$$A_t = \exp(z_t) \left[f x_{t-1} \right]^{\alpha} \tag{C.28}$$

$$R_t^s = \left[1 + \alpha(1+s_W) MC_t \frac{\Delta_t Y_t}{f x_{t-1}} \right] \Pi_{t-1,t} \tag{C.29}$$

$$\tag{C.30}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\underline{R}_t + \mathbb{E}_t \left[(\lambda x_t^s (1+s_L) \left(R_{t+1}^s - \underline{R}_t \right) \right. \right. \right.$$

$$+(1-\lambda)\left((1+s_L)R^s_{t+1}-\left(1-p^r_{t+1}\right)(1+s_L)R^s_{t+1}x^s_t-\underline{R}_t\right)\Big)\Big]\frac{1}{\phi_t}\Big)\frac{1}{\Pi_{t,t+1}}\Bigg]$$

(Bank free-entry condition, $\hat{\phi}_t$) (C.31)

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left[\left(\lambda-(1-\lambda)\left(1-p^r_{t+1}\right)\right)(1+s_L)R^s_{t+1}-\lambda\underline{R}_t\right]\right]-\varrho_t\lambda-\psi_t=0 \tag{C.32}$$

$$\min\left\{\varrho_t,(\phi_t-\lambda x^s_t-(1-\lambda))\right\}=0 \tag{C.33}$$

$$\min\left\{\psi_t,(1-x^s_t)\right\}=0 \tag{C.34}$$

(Bank FOC, $\hat{x}^s_t$)

We can define:

$$\nu_t\equiv\ln\left(R_t/\underline{R}_t\right) \tag{C.35}$$

$$\varsigma_t\equiv\ln\left(R^s_t/R_{t-1}\right) \tag{C.36}$$

$$x_t\equiv\lambda x^s_t+1-\lambda \tag{C.37}$$

The first is the spread between the risk-free market interest rate and the floor, the second is the spread between the risky return on capital and the risk-free rate, and the last is the overall lending margin. The last allows us to write the constraints:

$$\phi_t\geq\lambda x^s_t-(1-\lambda)\;\Rightarrow\;\phi_t\geq x_t \tag{C.38}$$

$$1\geq x^s_t\;\Rightarrow\;1\geq x_t \tag{C.39}$$

To characterize the inequality constraints, we can rearrange the first-order condition to:

$$x_t=(1+s_L)\frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left[\left(\lambda-(1-\lambda)\left(1-p^r_{t+1}\right)\right)\alpha(1+s_W)MC_{t+1}\frac{\Delta_{t+1}Y_{t+1}}{f}\right]\right]}{\varrho_t\lambda+\psi_t+\mathbb{E}_t\left[\Lambda_{t,t+1}\left[\lambda\frac{R_t}{\Pi_{t,t+1}}-(1+s_L)\left(\lambda-(1-\lambda)\left(1-p^r_{t+1}\right)\right)\right]\right]} \tag{C.40}$$

When the inequality constraints are both slack, there is surplus liquidity and credit rationing.

The multipliers $\varrho_t = \psi_t = 0$. We can give x_t with:

$$x_t = \min\left\{\frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left[\left(\lambda-(1-\lambda)\left(1-p^r_{t+1}\right)\right)\alpha(1+s_W)MC_{t+1}\frac{\Delta_{t+1}Y_{t+1}}{f}\right]\right]}{\mathbb{E}_t\left[\Lambda_{t,t+1}\left[\lambda\frac{\underline{R}_t}{\Pi_{t,t+1}}\frac{1}{1+s_L}-\left(\lambda-(1-\lambda)\left(1-p^r_{t+1}\right)\right)\right]\right]},\phi_t,1\right\} \tag{C.41}$$

C.3 Log-linear approximation

We derive a log-linear approximation around a zero-inflation steady state. The policymaker has two instruments: the market interest rate r_t and the spread to the floor $\nu_t \equiv r_t - \underline{r}_t$, where $\underline{r}_t$ is the interest on reserves. In steady state $x = 1$:[2]

$$R = 1/\beta, \qquad \underline{R} = R\exp(-\nu), \qquad R^s = R\exp(\varsigma) \tag{C.42}$$

Denoting log deviations with a hat ($\hat{x}_t \equiv \ln x_t - \ln x$) and level deviations with a tilde ($\tilde{x}_t \equiv x_t - x$), the average funding share satisfies:

$$x_t = \lambda x^s_t + 1 - \lambda, \qquad \hat{x}_t = \frac{\lambda x^s}{x}\hat{x}^s_t \tag{C.43}$$

C.3.1 Return on lending

The return on safe-firm projects satisfies $R^s_t = [1 + \alpha(1 + s_W)MC_tY_t/(fx_{t-1})]\Pi_{t-1,t}$. Taking logs and using $\varsigma_t \equiv r^s_t - r_{t-1}$, a first-order Taylor approximation gives:

$$\hat{r}^s_t - \pi_t = \frac{R^s - 1}{R^s}\left[\widehat{mc}_t + \hat{y}_t - \hat{x}_{t-1}\right] \tag{C.44}$$

The expected spread is therefore:

$$\mathbb{E}_t[\hat{\varsigma}_{t+1}] = \frac{R^s - 1}{R^s}\left(\mathbb{E}_t[\widehat{mc}_{t+1} + \hat{y}_{t+1}] - \hat{x}_t\right) - \left(r_t - \mathbb{E}_t[\pi_{t+1}] - \rho\right) \tag{C.45}$$

[2]This is the case under most reasonable parameterisations. We show above how a constant lending subsidy can impose this were there constrained lending in steady state.

C.3.2 NK core

Standard conditions with $\hat{c}_t = \hat{y}_t$ (goods market clearing) and $\hat{a}_t = z_t + \alpha \hat{x}_{t-1}$:

$$\hat{y}_t = \hat{a}_t + (1-\alpha)\hat{h}_t \tag{C.46}$$

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(r_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \tag{C.47}$$

$$\hat{w}_t - \hat{p}_t = \sigma \hat{c}_t + \varphi \hat{h}_t \tag{C.48}$$

$$\hat{w}_t - \hat{p}_t = \widehat{mc}_t + \hat{y}_t - \hat{h}_t \tag{C.49}$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \frac{(1-\xi)(1-\xi\beta)}{\xi}\widehat{mc}_t \tag{C.50}$$

C.3.3 Bank free-entry condition

Starting from the non-linear bank free-entry condition and using $\underline{R}_t = R_t \exp(-\nu_t)$, $x_t^s = (x_t - 1 + \lambda)/\lambda$, and $R_t^s = R_{t-1}\exp(\varsigma_t)$:

$$\phi_t = [\phi_t - x_t]\exp(-\nu_t) + \left[x_t - \tfrac{1-\lambda}{\lambda}(1-p^r_{t+1})(x_t - 1 + \lambda)\right](1+s_L)\exp(\varsigma_{t+1}) \tag{C.51}$$

With $\eta = 0$ imposed, taking a log-linear approximation:

$$\begin{aligned}\phi[1-\exp(-\nu)]\hat{\phi}_t = {} & \left[(1+s_L)\exp(\varsigma)\left(1 - \tfrac{1-\lambda}{\lambda}(1-p^r)\right) - \exp(-\nu)\right] x\hat{x}_t \\ & - [\phi - x]\exp(-\nu)\,\hat{\nu}_t + \tfrac{1-\lambda}{\lambda}p^r(x-1+\lambda)(1+s_L)\exp(\varsigma)\,\mathbb{E}_t[\hat{p}^r_{t+1}] \\ & + \left[x - \tfrac{1-\lambda}{\lambda}(1-p^r)(x-1+\lambda)\right](1+s_L)\exp(\varsigma)\,\mathbb{E}_t[\hat{\varsigma}_{t+1}]\end{aligned} \tag{C.52}$$

Using $R\exp(\varsigma) = R^s$, $\exp(\varsigma) = R^s/R = \beta R^s$, and $\exp(-\nu) = \underline{R}/R = \beta\underline{R}$, this can be written with composite coefficients, simplified further under the efficient steady state ($x = 1$, with lending subsidies).

C.3.4 Bank FOC (credit wedge)

The linearised bank first-order condition, with $\eta = 0$ and using $\underline{R}_t/R_t = \exp(-\nu_t)$:

$$(1+s_L)\exp(\varsigma)[\lambda-(1-p^r)(1-\lambda)]\,\mathbb{E}_t[\hat{\varsigma}_{t+1}]+(1+s_L)\exp(\varsigma)p^r(1-\lambda)\,\mathbb{E}_t[\hat{p}^r_{t+1}]$$
$$+\lambda\exp(-\nu)\,\hat{\nu}_t=\lambda\tilde{\varrho}_t+\tilde{\psi}_t \quad \text{(C.53)}$$

where $\tilde{\varrho}_t \equiv \varrho_t - \varrho$ and $\tilde{\psi}_t \equiv \psi_t - \psi$.

Substituting in the expected spread (C.45):

$$(1+s_L)\exp(\varsigma)[\lambda-(1-p^r)(1-\lambda)]\frac{R^s-1}{R^s}\left(\mathbb{E}_t[\widehat{mc}_{t+1}+\hat{y}_{t+1}]-\hat{x}_t\right)$$
$$-(1+s_L)\exp(\varsigma)[\lambda-(1-p^r)(1-\lambda)](r_t-\mathbb{E}_t[\pi_{t+1}]-\rho)$$
$$+(1+s_L)\exp(\varsigma)p^r(1-\lambda)\,\mathbb{E}_t[\hat{p}^r_{t+1}]+\lambda\exp(-\nu)\,\hat{\nu}_t=\lambda\tilde{\varrho}_t+\tilde{\psi}_t \quad \text{(C.54)}$$

Using $\exp(\varsigma)/R^s = 1/R$ and solving for $\hat{x}_t$:

$$\hat{x}_t=\mathbb{E}_t[\widehat{mc}_{t+1}+\hat{y}_{t+1}]-\frac{R^s}{R^s-1}(r_t-\mathbb{E}_t[\pi_{t+1}]-\rho)$$
$$+\frac{R^s}{R^s-1}\frac{p^r(1-\lambda)}{[\lambda-(1-p^r)(1-\lambda)]}\,\mathbb{E}_t[\hat{p}^r_{t+1}]+\frac{1}{1+s_L}\frac{1}{R^s-1}\frac{\lambda}{[\lambda-(1-p^r)(1-\lambda)]}\,\hat{\nu}_t$$
$$-\frac{R}{(1+s_L)(R^s-1)}\frac{\lambda\tilde{\varrho}_t+\tilde{\psi}_t}{[\lambda-(1-p^r)(1-\lambda)]} \quad \text{(C.55)}$$

Note the two channels of the interest rate on lending. The first term in r_t (via the real rate, with coefficient $-R^s/(R^s-1)$) is the demand channel: lower r_t raises demand and the expected return on lending. The second term in $\hat{\nu}_t$ is the credit-rationing channel: a wider spread to the floor raises lending incentives by increasing the gap between the return on lending and the return on reserves.

Under credit rationing, the mulitpliers are zero, so $\tilde{\varrho}_t = -\varrho$, $\tilde{\psi}_t = -\psi$. Using the steady-state bank FOC $\lambda\varrho+\psi=\beta[\lambda((1+s_L)R^s-1)-(1-\lambda)(1-p^r)(1+s_L)R^s]$ to substitute out the steady-state multipliers gives:

$$\hat{x}_t=\mathbb{E}_t[\widehat{mc}_{t+1}+\hat{y}_{t+1}]-\frac{R^s}{R^s-1}(r_t-\mathbb{E}_t[\pi_{t+1}]-\rho)$$
$$+\frac{R^s}{R^s-1}\frac{p^r(1-\lambda)}{[\lambda-(1-p^r)(1-\lambda)]}\,\mathbb{E}_t[\hat{p}^r_{t+1}]+\frac{\lambda\underline{R}}{(1+s_L)(R^s-1)[\lambda-(1-p^r)(1-\lambda)]}\,\hat{\nu}_t$$

$$+\left(\frac{R^s}{R^s-1}-\frac{1}{1+s_L}\frac{1}{R^s-1}\frac{\lambda}{[\lambda-(1-p^r)(1-\lambda)]}\right) \quad \text{(C.56)}$$

where the last term is the constant capturing the distance from the rationing threshold (positive when the steady state is non-rationing).

C.3.5 Piecewise credit wedge

The liquidity constraint $x_t \leq \phi_t$ and feasibility $x_t \leq 1$ in log-deviation form:

$$\hat{x}_t \leq \frac{\phi}{x}\hat{\phi}_t + \frac{\phi}{x} - 1, \qquad \hat{x}_t \leq -\ln(x) \quad \text{(C.57)}$$

The piecewise condition is:

$$\hat{x}_t = \min\left\{\begin{array}{c} \mathbb{E}_t[\widehat{mc}_{t+1}+\hat{y}_{t+1}] - \frac{R^s}{R^s-1}(r_t - \mathbb{E}_t[\pi_{t+1}] - \rho) + \frac{R^s}{R^s-1}\frac{p^r(1-\lambda)}{[\lambda-(1-p^r)(1-\lambda)]}\mathbb{E}_t[\hat{p}^r_{t+1}] \\ +\frac{\lambda\underline{R}}{(1+s_L)(R^s-1)[\lambda-(1-p^r)(1-\lambda)]}\hat{\nu}_t + \left(\frac{R^s}{R^s-1}-\frac{1}{1+s_L}\frac{1}{R^s-1}\frac{\lambda}{[\lambda-(1-p^r)(1-\lambda)]}\right) \quad , \\ \frac{\phi}{x}\hat{\phi}_t + \frac{\phi}{x} - 1 \quad , \quad -\ln(x) \end{array}\right\} \quad \text{(C.58)}$$

C.4 Minimum state space representation

From the labour supply (C.48), labour demand (C.49), goods clearing, and the production function (C.46):

$$\widehat{mc}_t = \frac{(\sigma-1)(1-\alpha)+1+\varphi}{1-\alpha}\hat{y}_t - \frac{1+\varphi}{1-\alpha}\hat{a}_t \quad \text{(C.59)}$$

The efficient output level (flex-price, full lending) is:

$$\hat{y}^e_t = \frac{1+\varphi}{(\sigma-1)(1-\alpha)+1+\varphi}z_t \quad \text{(C.60)}$$

The efficient output gap:

$$\tilde{y}^e_t \equiv \hat{y}_t - \hat{y}^e_t + y - y^e, \qquad \tilde{y}^e = \frac{1+\varphi}{(\sigma-1)(1-\alpha)+1+\varphi}\alpha\ln x \quad \text{(C.61)}$$

With the efficient steady state ($x = 1$): $\tilde{y}^e = 0$ and $\tilde{y}^e_t = \hat{y}_t - \hat{y}^e_t$.

The natural (flex-price, taking credit as given) output level:

$$\hat{y}_t^n = \frac{1+\varphi}{(\sigma-1)(1-\alpha)+1+\varphi}\hat{a}_t = \hat{y}_t^e + \frac{1+\varphi}{(\sigma-1)(1-\alpha)+1+\varphi}\alpha\hat{x}_{t-1} \tag{C.62}$$

The natural output gap: $\tilde{y}_t^n = \hat{y}_t - \hat{y}_t^n = \tilde{y}_t^e - \alpha\Theta\hat{x}_{t-1}$ where $\Theta \equiv \frac{1+\varphi}{(\sigma-1)(1-\alpha)+1+\varphi}$.

With $\sigma = 1$: $\Theta = 1$ and $\tilde{y}_t^n = \tilde{y}_t^e - \alpha\hat{x}_{t-1}$. Furthermore:

$$\hat{y}_t^e = z_t, \qquad \tilde{y}_t^e = \hat{y}_t - z_t, \qquad \hat{y}_t^n = z_t + \alpha\hat{x}_{t-1} = \hat{a}_t \tag{C.63}$$

Marginal cost simplifies to:

$$\widehat{mc}_t = \frac{1+\varphi}{1-\alpha}(\hat{y}_t - \hat{a}_t) = \frac{1+\varphi}{1-\alpha}\tilde{y}_t^n = \frac{1+\varphi}{1-\alpha}(\tilde{y}_t^e - \alpha\hat{x}_{t-1}) \tag{C.64}$$

C.4.1 IS curve

Substituting the output gap definitions into the Euler equation (C.47):

$$\tilde{y}_t^e = \mathbb{E}_t[\tilde{y}_{t+1}^e] - \frac{1}{\sigma}(r_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) \tag{C.65}$$

where

$$r_t^n \equiv \rho + \frac{\sigma(1+\varphi)}{(\sigma-1)(1-\alpha)+1+\varphi}\mathbb{E}_t[\Delta z_{t+1}] \tag{C.66}$$

With $\sigma = 1$: $r_t^n = \rho + \mathbb{E}_t[\Delta z_{t+1}]$.

C.4.2 Phillips curve

Substituting (C.59) into (C.50):

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa\tilde{y}_t^e - \kappa\alpha\hat{x}_{t-1} \tag{C.67}$$

where $\kappa \equiv \frac{(1-\xi\beta)(1-\xi)}{\xi}\frac{1+\varphi}{1-\alpha}$ (using $\sigma = 1$).

Equivalently, in terms of the natural output gap:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \tilde{y}_t^n \tag{C.68}$$

C.4.3 Credit wedge under rationing

To express the rationing condition (C.56) in terms of $(\tilde{y}_t^e, \pi_t, \hat{x}_t)$, we substitute out $\mathbb{E}_t[\widehat{mc}_{t+1} + \hat{y}_{t+1}]$.

With $\sigma = 1$:

$$\widehat{mc}_{t+1} + \hat{y}_{t+1} = \frac{1+\varphi+1-\alpha}{1-\alpha}\hat{y}_{t+1} - \frac{1+\varphi}{1-\alpha}\hat{a}_{t+1} = \left(1 + \frac{1+\varphi}{1-\alpha}\right)\hat{y}_{t+1} - \frac{1+\varphi}{1-\alpha}(z_{t+1} + \alpha \hat{x}_t) \tag{C.69}$$

Using $\hat{y}_{t+1} = \tilde{y}_{t+1}^e + z_{t+1}$:

$$\mathbb{E}_t[\widehat{mc}_{t+1} + \hat{y}_{t+1}] = \left(1 + \frac{1+\varphi}{1-\alpha}\right)\mathbb{E}_t[\tilde{y}_{t+1}^e] + \frac{1}{1-\alpha}\mathbb{E}_t[z_{t+1}] - \frac{1+\varphi}{1-\alpha}\alpha \hat{x}_t \tag{C.70}$$

And the real rate gap from the IS:

$$r_t - \mathbb{E}_t[\pi_{t+1}] - \rho = \mathbb{E}_t[\tilde{y}_{t+1}^e] - \tilde{y}_t^e + \mathbb{E}_t[\Delta z_{t+1}] \tag{C.71}$$

Substituting both into (C.56) and rearranging leads to:

$$\hat{x}_t = \Lambda_0 - \Lambda^D(r_t - \rho) + \Lambda^\nu \hat{\nu}_t + \Lambda_y \mathbb{E}_t[\tilde{y}_{t+1}^e] + \Lambda_\pi \mathbb{E}_t[\pi_{t+1}] + u_{r,t} \tag{C.72}$$

where

$$\Lambda^D \equiv \frac{1-\alpha}{1+\alpha\varphi}\frac{R^s}{R^s - 1} \tag{C.73}$$

$$\Lambda^\nu \equiv \frac{1-\alpha}{1+\alpha\varphi}\frac{\lambda \underline{R}}{(1+s_L)(R^s-1)[\lambda - (1-p^r)(1-\lambda)]} \tag{C.74}$$

$$\Lambda_y \equiv \frac{(1-\alpha)+(1+\varphi)}{1+\alpha\varphi} \tag{C.75}$$

$$\Lambda_\pi \equiv \frac{1-\alpha}{1+\alpha\varphi}\frac{R^s}{R^s-1} = \Lambda^D \tag{C.76}$$

and

$$u_{r,t} \equiv \frac{1-\alpha}{1+\alpha\varphi}\mathbb{E}_t[z_{t+1}] + \frac{R^s}{R^s-1}\frac{1-\alpha}{1+\alpha\varphi}\frac{p^r(1-\lambda)}{[\lambda-(1-p^r)(1-\lambda)]}\mathbb{E}_t[\hat{p}^r_{t+1}] \tag{C.77}$$

and Λ_0 is the constant capturing the steady-state distance from the rationing threshold.

When the floor is fixed ($\hat{\nu}_t = r_t - \rho$ since $\nu = \rho$ with $\underline{R} = 1$), the credit wedge depends on r_t through the net coefficient $\Lambda_r \equiv \Lambda^D - \Lambda^\nu$, confirming the decomposition into demand and credit-rationing channels.

C.4.4 Liquidity constraint

Starting from the linearised bank free-entry condition (C.52), substituting in the expected spread (C.45), then the marginal cost expression, solving for $\hat{\phi}_t$, then substituting in the binding liquidity constraint $\hat{x}_t = \frac{\phi}{x}\hat{\phi}_t + \frac{\phi}{x} - 1$ yields:

$$\hat{x}_t = \Upsilon_0 - \Upsilon_r(r_t - \rho) + \Upsilon_\nu\hat{\nu}_t + \Upsilon_y\mathbb{E}_t[\tilde{y}^e_{t+1}] + \Upsilon_\pi\mathbb{E}_t[\pi_{t+1}] + u_{l,t} \tag{C.78}$$

with composite Υ-coefficients.

C.4.5 Summary: the 3+1 system

Focusing on the efficient steady-state case ($x = 1$), for brevity and clarity, we can denote the efficient output gap $\tilde{y}_t \equiv \tilde{y}^e_t$, and use $\ln(x_t) \equiv \hat{x}_t$ The model is characterised by three standard equations plus the piecewise credit wedge:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - (r_t - \mathbb{E}_t[\pi_{t+1}] - r^n_t) \tag{C.79}$$

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa\tilde{y}_t - \kappa\alpha\ln(x_{t-1}) \tag{C.80}$$

$$\ln(x_t) = \min\left\{\begin{array}{c} \Lambda_0 - \Lambda^D(r_t-\rho) + \Lambda^\nu\hat{\nu}_t + \Lambda_y\mathbb{E}_t[\tilde{y}_{t+1}] + \Lambda_\pi\mathbb{E}_t[\pi_{t+1}] + u_{r,t} \quad , \\ \Upsilon_0 - \Upsilon_r(r_t-\rho) + \Upsilon_\nu\hat{\nu}_t + \Upsilon_y\mathbb{E}_t[\tilde{y}_{t+1}] + \Upsilon_\pi\mathbb{E}_t[\pi_{t+1}] + u_{l,t} \quad , \quad 0 \end{array}\right\} \tag{C.81}$$

r_t and ν_t are policy instruments, subject to $\underline{r}_t = r_t - \nu_t \geq \underline{r}^{ELB}$. r_t^n, $u_{r,t}$ and $u_{r,t}$ are (linear combinations) of exogenous shocks.

C.5 Linear-quadratic welfare approximation

We derive a second-order approximation to household utility around the efficient undistorted steady state, where labour subsidies remove the monopolistic competition distortion and lending subsidies ensure $x = 1$.

C.5.1 Second-order expansion of utility

With separable preferences $U(C_t, H_t) = C_t^{1-\sigma}/(1-\sigma) + \varphi_H H_t^{1+\varphi}/(1+\varphi)$ and $U_{CH} = 0$:

$$U(C_t, H_t) = U(C, H) + U_C C\left(\hat{c}_t + \tfrac{1}{2}\hat{c}_t^2\right) + U_H H\left(\hat{h}_t + \tfrac{1}{2}\hat{h}_t^2\right) + \tfrac{1}{2}\left[U_{CC}C^2\hat{c}_t^2 + U_{HH}H^2\hat{h}_t^2\right] \tag{C.82}$$

using $\frac{C_t - C}{C} = \hat{c}_t + \frac{1}{2}\hat{c}_t^2$ to second order. With $\hat{c}_t = \hat{y}_t$ and $\hat{h}_t = \frac{1}{1-\alpha}(\hat{y}_t - \hat{a}_t + \hat{\delta}_t)$:

$$\begin{aligned}\frac{U(C_t, H_t) - U(C, H)}{U_C Y} = &\left(1 + \frac{U_H}{U_C}\frac{H}{Y}\frac{1}{1-\alpha}\right)\hat{y}_t + \frac{U_H}{U_C}\frac{H}{Y}\frac{1}{1-\alpha}(\hat{\delta}_t - \hat{a}_t) \\ &+ \frac{1}{2}\left[\left(\frac{U_{CC}}{U_C}Y + 1\right)\hat{y}_t^2 + \left(\frac{U_{HH}}{U_C}\frac{H^2}{Y} + \frac{U_H}{U_C}\frac{H}{Y}\right)\frac{1}{(1-\alpha)^2}(\hat{y}_t - \hat{a}_t + \hat{\delta}_t)^2\right]\end{aligned} \tag{C.83}$$

In the efficient steady state: $-U_H/U_C = (1-\alpha)Y/H$, so $\frac{1}{1-\alpha}\frac{H}{Y}\frac{U_H}{U_C} = -1$, and the linear term in $\hat{y}_t$ vanishes. Using the specific functional forms ($U_{CC}/U_C = -\sigma/Y$, $U_{HH}/U_H = \varphi/H$):

$$\widehat{U}_t \equiv \frac{U(C_t, H_t) - U(C, H)}{U_C Y} = -(\hat{\delta}_t - \hat{a}_t) + \frac{1}{2}\left[(1-\sigma)\hat{y}_t^2 - \frac{1+\varphi}{1-\alpha}(\hat{y}_t - \hat{a}_t + \hat{\delta}_t)^2\right] \tag{C.84}$$

Noting $\hat{\delta}_t = \mathcal{O}(\pi_t^2)$, so $\hat{\delta}_t^2$ and cross-terms with $\hat{\delta}_t$ are third/fourth order:

$$\widehat{U}_t = -(\hat{\delta}_t - \hat{a}_t) + \frac{1}{2}\left[(1-\sigma)\hat{y}_t^2 - \frac{1+\varphi}{1-\alpha}(\hat{y}_t - \hat{a}_t)^2\right] + \text{t.i.p.} \tag{C.85}$$

where t.i.p. denotes terms independent of policy.

C.5.2 Imposing $\sigma = 1$

With $\sigma = 1$, the $\hat{y}_t^2$ term vanishes and:

$$\widehat{U}_t = -(\hat{\delta}_t - \hat{a}_t) - \frac{1}{2}\frac{1+\varphi}{1-\alpha}(\hat{y}_t - \hat{a}_t)^2 + \text{t.i.p.} \tag{C.86}$$

Since $\hat{y}_t - \hat{a}_t = \tilde{y}_t^n = \tilde{y}_t - \alpha \ln(x_{t-1})$ (the natural output gap), this is:

$$\widehat{U}_t = -(\hat{\delta}_t - \hat{a}_t) - \frac{1}{2}\frac{1+\varphi}{1-\alpha}(\tilde{y}_t - \alpha\hat{x}_{t-1})^2 + \text{t.i.p.} \tag{C.87}$$

The quadratic part collapses to the natural (flex-price) output gap $\tilde{y}_t - \alpha \ln(x_{t-1})$.

C.5.3 Price dispersion and the linear term

Using the standard Calvo result:

$$\sum_{t=0}^{\infty} \beta^t \hat{\delta}_t = \frac{\varepsilon\xi}{2(1-\beta\xi)(1-\xi)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \tag{C.88}$$

and $\hat{a}_t = z_t + \alpha \ln(x_{t-1})$, so $-\hat{a}_t = -z_t - \alpha \ln(x_{t-1})$:

$$-(\hat{\delta}_t - \hat{a}_t) = -\hat{\delta}_t + z_t + \alpha \ln(x_{t-1}) \tag{C.89}$$

Defining the welfare loss $\mathcal{L} \equiv -\mathbb{E}_0 \sum \beta^t \widehat{U}_t$ (minus t.i.p.) and normalising by $\frac{\varepsilon\xi}{(1-\beta\xi)(1-\xi)} = \frac{\kappa}{\lambda_x}^{-1}$:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[-\lambda_x \frac{1-\alpha}{1+\varphi} \alpha \ln(x_{t-1}) + \frac{1}{2}\left(\pi_t^2 + \lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1}))^2\right) \right] \tag{C.90}$$

where

$$\lambda_x \equiv \frac{1+\varphi}{1-\alpha}\frac{(1-\beta\xi)(1-\xi)}{\varepsilon\xi} = \frac{\kappa}{\varepsilon} \tag{C.91}$$

Without the credit friction ($\ln(x_{t-1}) = 0$), the loss function collapses to the standard $\frac{1}{2}[\pi_t^2 + \lambda_x(\tilde{y}_t)^2]$.

With credit frictions, the linear term $-\lambda_x \frac{1-\alpha}{1+\varphi}\alpha \ln(x_{t-1})$ captures the direct productivity cost of reduced lending. Since $\ln(x_{t-1}) \leq 0$ under rationing, this is a welfare loss.

The quadratic term takes the form of $\tilde{y}_t - \alpha \ln(x_{t-1})$, which is the natural (flex-price) output gap taking the credit allocation as given. This follows from the marginal cost expression (C.64): $\widehat{mc}_t = \frac{1+\varphi}{1-\alpha}(\tilde{y}_t - \alpha \ln(x_{t-1}))$, so $\tilde{y}_t - \alpha \ln(x_{t-1}) = 0$ when prices are flexible. The welfare-relevant output gap absorbs the credit wedge into the definition of potential output.

C.6 Optimal policy under discretion

The policymaker minimises period welfare loss taking expectations as given (Markov-perfect equilibrium), but internalising the effect of current choices on future welfare through the state variable $\ln(x_t)$.

The per-period welfare loss is:

$$\ell_t = -\lambda_x \frac{1-\alpha}{1+\varphi}\alpha \ln(x_{t-1}) + \frac{1}{2}\left[\pi_t^2 + \lambda_x \left(\tilde{y}_t - \alpha \ln(x_{t-1})\right)^2\right] \tag{C.92}$$

The Bellman equation is:

$$V\left(\ln(x_{t-1})\right) = \min_{\pi_t, \tilde{y}_t, \ln(x_t), r_t} \left\{\ell_t + \beta \mathbb{E}_t\left[V\left(\ln(x_t)\right)\right]\right\} \tag{C.93}$$

Optimisation is subject to the Phillips curve (multiplier $\mu_{\pi,t}$):

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \tilde{y}_t - \kappa\alpha \ln(x_{t-1}); \tag{NKPC}$$

the IS curve (multiplier $\mu_{y,t}$):

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - (r_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n); \tag{IS}$$

the rationing constraint (multiplier $\xi_{r,t} \geq 0$). At the ELB, $\underline{r}_t = \underline{r}^{ELB}$, so $\hat{\nu}_t = (r_t - \underline{r}^{ELB}) - \nu$:

$$\ln(x_t) \leq \Lambda_0' - \Lambda_r(r_t - \rho) + \Lambda_y \mathbb{E}_t[\tilde{y}_{t+1}] + \Lambda_\pi \mathbb{E}_t[\pi_{t+1}] + u_{r,t} \tag{Rationing}$$

where $\Lambda_0' \equiv \Lambda_0 + \Lambda^\nu(\rho - \underline{r}^{ELB} - \nu)$ and $\Lambda_r \equiv \Lambda^D - \Lambda^\nu$; the liquidity constraint (multiplier $\xi_{l,t} \geq 0$):

$$\ln(x_t) \leq \Upsilon_0' - \Upsilon_r(r_t - \rho) + \Upsilon_y \mathbb{E}_t[\tilde{y}_{t+1}] + \Upsilon_\pi \mathbb{E}_t[\pi_{t+1}] + u_{l,t}; \qquad \text{(Liquidity)}$$

and the feasibility constraint (multiplier $\xi_{x,t} \geq 0$):

$$\ln(x_t) \leq 0; \qquad \text{(Feasibility)}$$

With complementary slackness: $\xi_{r,t} \geq 0$, $\xi_{l,t} \geq 0$, $\xi_{x,t} \geq 0$.

The Lagrangian is:

$$\begin{aligned}
\mathcal{L}_t = &-\lambda_x \frac{1-\alpha}{1+\varphi}\alpha \ln(x_{t-1}) + \frac{1}{2}\left[\pi_t^2 + \lambda_x\left(\tilde{y}_t - \alpha\ln(x_{t-1})\right)^2\right] + \beta\mathbb{E}_t[V(\ln(x_t))] \\
&+ \mu_{\pi,t}\left[\pi_t - \beta\mathbb{E}_t[\pi_{t+1}] - \kappa\tilde{y}_t + \kappa\alpha\ln(x_{t-1})\right] \\
&+ \mu_{y,t}\left[\tilde{y}_t - \mathbb{E}_t[\tilde{y}_{t+1}] + r_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n\right] \\
&+ \xi_{r,t}\left[\ln(x_t) - \text{RHS(Rationing)}\right] \\
&+ \xi_{l,t}\left[\ln(x_t) - \text{RHS(Liquidity)}\right] \\
&+ \xi_{x,t}\ln(x_t)
\end{aligned} \qquad \text{(C.94)}$$

FOC w.r.t. π_t:

$$\pi_t + \mu_{\pi,t} = 0 \quad \Rightarrow \quad \mu_{\pi,t} = -\pi_t \qquad \text{(C.95)}$$

FOC w.r.t. $\tilde{y}_t$:

$$\lambda_x\left(\tilde{y}_t - \alpha\ln(x_{t-1})\right) - \kappa\mu_{\pi,t} + \mu_{y,t} = 0 \qquad \text{(C.96)}$$

Substituting (C.95):

$$\lambda_x\left(\tilde{y}_t - \alpha\ln(x_{t-1})\right) + \kappa\pi_t + \mu_{y,t} = 0 \qquad \text{(C.97)}$$

FOC w.r.t. r_t:

$$\mu_{y,t} + \Lambda_r \xi_{r,t} + \Upsilon_r \xi_{l,t} = 0 \quad \Rightarrow \quad \mu_{y,t} = -\Lambda_r \xi_{r,t} - \Upsilon_r \xi_{l,t} \tag{C.98}$$

FOC w.r.t. $\ln(x_t)$:

$$\beta \mathbb{E}_t \left[V' \left(\ln(x_t) \right) \right] + \xi_{r,t} + \xi_{l,t} + \xi_{x,t} = 0 \tag{C.99}$$

We have:

$$V' \left(\ln(x_{t-1}) \right) = -\lambda_x \frac{1-\alpha}{1+\varphi} \alpha - \lambda_x \alpha \left(\tilde{y}_t - \alpha \ln(x_{t-1}) \right) + \kappa \alpha \mu_{\pi,t} \tag{C.100}$$

so:

$$\beta \mathbb{E}_t \left[V' \left(\ln(x_t) \right) \right] = -\beta \lambda_x \frac{1-\alpha}{1+\varphi} \alpha - \beta \lambda_x \alpha \mathbb{E}_t \left[\tilde{y}_{t+1} - \alpha \ln(x_t) \right] - \beta \kappa \alpha \mathbb{E}_t [\pi_{t+1}] \tag{C.101}$$

Substituting (C.101) into the FOC (C.99):

$$\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta \lambda_x \frac{1-\alpha}{1+\varphi} \alpha + \beta \lambda_x \alpha \mathbb{E}_t \left[\tilde{y}_{t+1} - \alpha \ln(x_t) \right] + \beta \kappa \alpha \mathbb{E}_t [\pi_{t+1}] \tag{C.102}$$

This holds in all regimes. The RHS is the (discounted) total shadow value of an additional unit of lending; the LHS shows how it is allocated across the three constraint multipliers.

C.6.1 General targeting rule

From (C.97) and (C.98):

$$\pi_t = -\frac{\lambda_x}{\kappa} \left(\tilde{y}_t - \alpha \ln(x_{t-1}) \right) + \frac{\Lambda_r}{\kappa} \xi_{r,t} + \frac{\Upsilon_r}{\kappa} \xi_{l,t} \tag{C.103}$$

This holds whenever r_t is unconstrained.

C.6.2 Lending multipliers

The targeting rule (C.103) taken forward one period:

$$\lambda_x \left(\tilde{y}_{t+1} - \alpha \ln(x_t)\right) + \kappa \pi_{t+1} = \Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1} \tag{C.104}$$

Substituting this nto (C.102) yields:

$$\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta \lambda_x \frac{1-\alpha}{1+\varphi} \alpha + \beta \alpha \mathbb{E}_t \left[\Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1}\right] \tag{C.105}$$

C.6.3 Case analysis

At most one inequality constraint binds at a time.

Case 1: Full lending ($\ln(x_t) = 0$). Feasibility binds: $\xi_{x,t} \geq 0$. Rationing and liquidity slack: $\xi_{r,t} = \xi_{l,t} = 0$.

From (C.105):

$$\xi_{x,t} = \beta \lambda_x \frac{1-\alpha}{1+\varphi} \alpha + \beta \alpha \mathbb{E}_t \left[\Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1}\right] \tag{C.106}$$

From (C.103):

$$\pi_t = -\frac{\lambda_x}{\kappa} \left(\tilde{y}_t - \alpha \ln(x_{t-1})\right) \tag{C.107}$$

Case 2: Credit rationing ($\ln(x_t) < 0$, rationing constraint binds). Rationing binds: $\xi_{r,t} > 0$. Liquidity and feasibility slack: $\xi_{l,t} = \xi_{x,t} = 0$.

From (C.105):

$$\xi_{r,t} = \beta \lambda_x \frac{1-\alpha}{1+\varphi} \alpha + \beta \alpha \mathbb{E}_t \left[\Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1}\right] \tag{C.108}$$

From (C.103):

$$\pi_t = -\frac{\lambda_x}{\kappa}\left(\tilde{y}_t - \alpha \ln(x_{t-1})\right) + \frac{\Lambda_r}{\kappa}\xi_{r,t} \tag{C.109}$$

Case 3: Liquidity constraint ($\ln(x_t) < 0$, liquidity constraint binds). Liquidity binds: $\xi_{l,t} > 0$. Rationing and feasibility slack: $\xi_{r,t} = \xi_{x,t} = 0$.

From (C.105):

$$\xi_{l,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t\left[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}\right] \tag{C.110}$$

From (C.103):

$$\pi_t = -\frac{\lambda_x}{\kappa}\left(\tilde{y}_t - \alpha \ln(x_{t-1})\right) + \frac{\Upsilon_r}{\kappa}\xi_{l,t} \tag{C.111}$$

C.7 Optimal policy under commitment

The policymaker commits at $t = 0$ to a state-contingent plan $\{\pi_t, \tilde{y}_t, \ln(x_t), r_t\}_{t=0}^{\infty}$ to minimise the expected discounted welfare loss:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[-\lambda_x \frac{1-\alpha}{1+\varphi}\alpha \ln(x_{t-1}) + \frac{1}{2}\left(\pi_t^2 + \lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1}))^2\right)\right] \tag{C.112}$$

subject to, for all $t \geq 0$:

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa\tilde{y}_t - \kappa\alpha \ln(x_{t-1}) \qquad [\mu_{\pi,t}] \qquad \text{(NKPC)}$$

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - (r_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) \qquad [\mu_{y,t}] \qquad \text{(IS)}$$

$$\ln(x_t) \leq \Lambda_0' - \Lambda_r(r_t - \rho) + \Lambda_y\mathbb{E}_t[\tilde{y}_{t+1}] + \Lambda_\pi\mathbb{E}_t[\pi_{t+1}] + u_{r,t} \qquad [\xi_{r,t} \geq 0] \qquad \text{(Rat)}$$

$$\ln(x_t) \leq \Upsilon_0' - \Upsilon_r(r_t - \rho) + \Upsilon_y\mathbb{E}_t[\tilde{y}_{t+1}] + \Upsilon_\pi\mathbb{E}_t[\pi_{t+1}] + u_{l,t} \qquad [\xi_{l,t} \geq 0] \qquad \text{(Liq)}$$

$$\ln(x_t) \leq 0 \qquad [\xi_{x,t} \geq 0] \qquad \text{(Feas)}$$

with complementary slackness on the inequality constraints.

C.7.1 Lagrangian

Under commitment, the policymaker internalises how current choices affect past constraints through expectations. By the law of iterated expectations, $\mathbb{E}_0[\beta^t \mu_{\pi,t} \beta \mathbb{E}_t[\pi_{t+1}]] = \mathbb{E}_0[\beta^{t+1} \mu_{\pi,t} \pi_{t+1}]$, so we can replace conditional expectations with realisations inside the outer $\mathbb{E}_0$. The Lagrangian is therefore:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[& \underbrace{-\lambda_x \frac{1-\alpha}{1+\varphi} \alpha \ln(x_{t-1}) + \frac{1}{2}\left[\pi_t^2 + \lambda_x (\tilde{y}_t - \alpha \ln(x_{t-1}))^2\right]}_{\ell_t} \\ & + \mu_{\pi,t}\left[\pi_t - \beta \pi_{t+1} - \kappa \tilde{y}_t + \kappa \alpha \ln(x_{t-1})\right] \\ & + \mu_{y,t}\left[\tilde{y}_t - \tilde{y}_{t+1} + r_t - \pi_{t+1} - r_t^n\right] \\ & + \xi_{r,t}\left[\ln(x_t) - \Lambda_0' + \Lambda_r (r_t - \rho) - \Lambda_y \tilde{y}_{t+1} - \Lambda_\pi \pi_{t+1} - u_{r,t}\right] \\ & + \xi_{l,t}\left[\ln(x_t) - \Upsilon_0' + \Upsilon_r (r_t - \rho) - \Upsilon_y \tilde{y}_{t+1} - \Upsilon_\pi \pi_{t+1} - u_{l,t}\right] \\ & + \xi_{x,t} \ln(x_t) \Bigg] \end{aligned} \tag{C.113}$$

C.7.2 First-order conditions

FOC w.r.t. π_t:

$$\pi_t + \mu_{\pi,t} - \mu_{\pi,t-1} - \frac{1}{\beta}\mu_{y,t-1} - \frac{\Lambda_\pi}{\beta}\xi_{r,t-1} - \frac{\Upsilon_\pi}{\beta}\xi_{l,t-1} = 0 \tag{C.114}$$

FOC w.r.t. $\tilde{y}_t$:

$$\lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1})) - \kappa \mu_{\pi,t} + \mu_{y,t} - \frac{1}{\beta}\mu_{y,t-1} - \frac{\Lambda_y}{\beta}\xi_{r,t-1} - \frac{\Upsilon_y}{\beta}\xi_{l,t-1} = 0 \tag{C.115}$$

FOC w.r.t. r_t:

$$\mu_{y,t} + \Lambda_r \xi_{r,t} + \Upsilon_r \xi_{l,t} = 0 \tag{C.116}$$

This is identical to the discretion case. The IS multiplier is:

$$\mu_{y,t} = -\Lambda_r \xi_{r,t} - \Upsilon_r \xi_{l,t} \tag{C.117}$$

FOC w.r.t. $\ln(x_t)$**:**

$$\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\lambda_x \alpha \mathbb{E}_t[\tilde{y}_{t+1} - \alpha \ln(x_t)] - \beta\kappa\alpha\mathbb{E}_t[\mu_{\pi,t+1}] \tag{C.118}$$

Complementary slackness: $\xi_{r,t}, \xi_{l,t}, \xi_{x,t} \geq 0$

Comparison with discretion

Under discretion, $\mu_{\pi,t+1} = -\pi_{t+1}$ from the period-$(t{+}1)$ FOC. Under commitment, $\mu_{\pi,t+1}$ is *not* equal to $-\pi_{t+1}$; it carries the full history of past commitments through (C.114). This is the key source of the difference between commitment and discretion in the $\ln(x_t)$ FOC.

C.7.3 General targeting rule

We eliminate $\mu_{\pi,t}$ and $\mu_{y,t}$ from (C.114) and (C.115) to obtain a single condition in observables and credit multipliers.

From (C.115):

$$\kappa\mu_{\pi,t} = \lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1})) + \mu_{y,t} - \frac{1}{\beta}\mu_{y,t-1} - \frac{\Lambda_y}{\beta}\xi_{r,t-1} - \frac{\Upsilon_y}{\beta}\xi_{l,t-1} \tag{C.119}$$

From (C.116): $\mu_{y,t} = -\Lambda_r \xi_{r,t} - \Upsilon_r \xi_{l,t}$ and $\mu_{y,t-1} = -\Lambda_r \xi_{r,t-1} - \Upsilon_r \xi_{l,t-1}$. Substituting into (C.119):

$$\begin{aligned}\kappa\mu_{\pi,t} = \lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1})) - \Lambda_r \xi_{r,t} - \Upsilon_r \xi_{l,t} \\ + \frac{\Lambda_r}{\beta}\xi_{r,t-1} + \frac{\Upsilon_r}{\beta}\xi_{l,t-1} - \frac{\Lambda_y}{\beta}\xi_{r,t-1} - \frac{\Upsilon_y}{\beta}\xi_{l,t-1}\end{aligned} \tag{C.120}$$

Collecting the coefficients on $\xi_{r,t-1}$ and $\xi_{l,t-1}$:

$$\kappa\mu_{\pi,t} = \lambda_x(\tilde{y}_t - \alpha\ln(x_{t-1})) - \Lambda_r\xi_{r,t} - \Upsilon_r\xi_{l,t} + \frac{\Lambda_r - \Lambda_y}{\beta}\xi_{r,t-1} + \frac{\Upsilon_r - \Upsilon_y}{\beta}\xi_{l,t-1} \tag{C.121}$$

From (C.114): $\mu_{\pi,t} = -\pi_t + \mu_{\pi,t-1} + \frac{1}{\beta}\mu_{y,t-1} + \frac{\Lambda_\pi}{\beta}\xi_{r,t-1} + \frac{\Upsilon_\pi}{\beta}\xi_{l,t-1}$. Multiplying by κ:

$$\kappa\mu_{\pi,t} = -\kappa\pi_t + \kappa\mu_{\pi,t-1} + \frac{\kappa}{\beta}\mu_{y,t-1} + \frac{\kappa\Lambda_\pi}{\beta}\xi_{r,t-1} + \frac{\kappa\Upsilon_\pi}{\beta}\xi_{l,t-1} \tag{C.122}$$

Substituting $\mu_{y,t-1} = -\Lambda_r\xi_{r,t-1} - \Upsilon_r\xi_{l,t-1}$:

$$\kappa\mu_{\pi,t} = -\kappa\pi_t + \kappa\mu_{\pi,t-1} + \frac{\kappa(\Lambda_\pi - \Lambda_r)}{\beta}\xi_{r,t-1} + \frac{\kappa(\Upsilon_\pi - \Upsilon_r)}{\beta}\xi_{l,t-1} \tag{C.123}$$

Equate (C.121) and (C.123):

$$\begin{aligned}\lambda_x(\tilde{y}_t - \alpha\ln(x_{t-1})) - \Lambda_r\xi_{r,t} - \Upsilon_r\xi_{l,t} + \frac{\Lambda_r - \Lambda_y}{\beta}\xi_{r,t-1} + \frac{\Upsilon_r - \Upsilon_y}{\beta}\xi_{l,t-1} \\ = -\kappa\pi_t + \kappa\mu_{\pi,t-1} + \frac{\kappa(\Lambda_\pi - \Lambda_r)}{\beta}\xi_{r,t-1} + \frac{\kappa(\Upsilon_\pi - \Upsilon_r)}{\beta}\xi_{l,t-1}\end{aligned} \tag{C.124}$$

Rearranging:

$$\begin{aligned}\kappa\pi_t + \lambda_x(\tilde{y}_t - \alpha\ln(x_{t-1})) = \kappa\mu_{\pi,t-1} + \Lambda_r\xi_{r,t} + \Upsilon_r\xi_{l,t} \\ + \frac{1}{\beta}\left[\kappa(\Lambda_\pi - \Lambda_r) - (\Lambda_r - \Lambda_y)\right]\xi_{r,t-1} + \frac{1}{\beta}\left[\kappa(\Upsilon_\pi - \Upsilon_r) - (\Upsilon_r - \Upsilon_y)\right]\xi_{l,t-1}\end{aligned} \tag{C.125}$$

The coefficient on $\xi_{r,t-1}$:

$$\begin{aligned}\frac{1}{\beta}\left[\kappa\Lambda_\pi - \kappa\Lambda_r - \Lambda_r + \Lambda_y\right] &= \frac{1}{\beta}\left[\kappa\Lambda_\pi + \Lambda_y - (1+\kappa)\Lambda_r\right] \\ &= -\frac{1}{\beta}\left[(1+\kappa)\Lambda_r - \Lambda_y - \kappa\Lambda_\pi\right]\end{aligned} \tag{C.126}$$

Define:

$$\Omega_r \equiv \frac{(1+\kappa)\Lambda_r - \Lambda_y - \kappa\Lambda_\pi}{\beta\kappa} \tag{C.127}$$

so the coefficient on $\xi_{r,t-1}$ is $-\kappa\Omega_r$. Similarly, for $\xi_{l,t-1}$:

$$\Omega_l \equiv \frac{(1+\kappa)\Upsilon_r - \Upsilon_y - \kappa\Upsilon_\pi}{\beta\kappa} \tag{C.128}$$

Dividing (C.125) by κ and using the Ω definitions, we arrive at our targeting rule:

$$\pi_t + \frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha\ln(x_{t-1})) = \mu_{\pi,t-1} + \frac{\Lambda_r}{\kappa}\xi_{r,t} + \frac{\Upsilon_r}{\kappa}\xi_{l,t} - \Omega_r\xi_{r,t-1} - \Omega_l\xi_{l,t-1} \tag{C.129}$$

Comparison with discretion

The discretionary targeting rule is $\pi_t = -\frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha\ln(x_{t-1})) + \frac{\Lambda_r}{\kappa}\xi_{r,t} + \frac{\Upsilon_r}{\kappa}\xi_{l,t}$. Commitment adds three sources of history dependence:

1. $\mu_{\pi,t-1}$: the standard price-level targeting term, capturing past inflation commitments.
2. $-\Omega_r\xi_{r,t-1}$: the lagged rationing multiplier, capturing past credit-rationing commitments.
3. $-\Omega_l\xi_{l,t-1}$: the lagged liquidity multiplier, capturing past constrained-lending commitments.

C.7.4 Law of motion for $\mu_{\pi,t}$

From (C.121), the NKPC multiplier evolves as:

$$\mu_{\pi,t} = \frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha\ln(x_{t-1})) - \frac{\Lambda_r}{\kappa}\xi_{r,t} - \frac{\Upsilon_r}{\kappa}\xi_{l,t} + \frac{\Lambda_r - \Lambda_y}{\beta\kappa}\xi_{r,t-1} + \frac{\Upsilon_r - \Upsilon_y}{\beta\kappa}\xi_{l,t-1} \tag{C.130}$$

Under discretion, $\mu_{\pi,t} = -\pi_t$. Using the discretionary targeting rule, this equals $\frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha\ln(x_{t-1})) - \frac{\Lambda_r}{\kappa}\xi_{r,t} - \frac{\Upsilon_r}{\kappa}\xi_{l,t}$, which matches (C.130) when the lagged terms vanish.

C.7.5 Recursion for the credit multipliers

We now derive the recursion by substituting the forward targeting rule into (C.118).

Taking (C.129) forward one period:

$$\lambda_x(\tilde{y}_{t+1} - \alpha\ln(x_t)) + \kappa\pi_{t+1} = \kappa\mu_{\pi,t} + \Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1} - \kappa\Omega_r\xi_{r,t} - \kappa\Omega_l\xi_{l,t} \tag{C.131}$$

From (C.114) shifted forward one period:

$$\mu_{\pi,t+1} = -\pi_{t+1} + \mu_{\pi,t} + \frac{1}{\beta}\mu_{y,t} + \frac{\Lambda_\pi}{\beta}\xi_{r,t} + \frac{\Upsilon_\pi}{\beta}\xi_{l,t} \tag{C.132}$$

Therefore:

$$-\kappa\mu_{\pi,t+1} = \kappa\pi_{t+1} - \kappa\mu_{\pi,t} - \frac{\kappa}{\beta}\mu_{y,t} - \frac{\kappa\Lambda_\pi}{\beta}\xi_{r,t} - \frac{\kappa\Upsilon_\pi}{\beta}\xi_{l,t} \tag{C.133}$$

(C.118) requires $\beta\lambda_x\alpha\mathbb{E}_t[\tilde{y}_{t+1} - \alpha\ln(x_t)] - \beta\kappa\alpha\mathbb{E}_t[\mu_{\pi,t+1}]$. To get this, we add (C.131) and (C.133):

$$\begin{aligned}\lambda_x(\tilde{y}_{t+1} - \alpha\ln(x_t)) - \kappa\mu_{\pi,t+1} = \Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1} \\ - \kappa\Omega_r\xi_{r,t} - \kappa\Omega_l\xi_{l,t} - \frac{\kappa}{\beta}\mu_{y,t} - \frac{\kappa\Lambda_\pi}{\beta}\xi_{r,t} - \frac{\kappa\Upsilon_\pi}{\beta}\xi_{l,t}\end{aligned} \tag{C.134}$$

Substituting $\mu_{y,t} = -\Lambda_r\xi_{r,t} - \Upsilon_r\xi_{l,t}$:

$$\begin{aligned}\lambda_x(\tilde{y}_{t+1} - \alpha\ln(x_t)) - \kappa\mu_{\pi,t+1} = \Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1} \\ + \left[\frac{\kappa\Lambda_r}{\beta} - \kappa\Omega_r - \frac{\kappa\Lambda_\pi}{\beta}\right]\xi_{r,t} + \left[\frac{\kappa\Upsilon_r}{\beta} - \kappa\Omega_l - \frac{\kappa\Upsilon_\pi}{\beta}\right]\xi_{l,t}\end{aligned} \tag{C.135}$$

For $\xi_{r,t}$, substituting $\Omega_r = \frac{(1+\kappa)\Lambda_r - \Lambda_y - \kappa\Lambda_\pi}{\beta\kappa}$:

$$\begin{aligned}\frac{\kappa\Lambda_r}{\beta} - \kappa\Omega_r - \frac{\kappa\Lambda_\pi}{\beta} &= \frac{\kappa\Lambda_r}{\beta} - \frac{(1+\kappa)\Lambda_r - \Lambda_y - \kappa\Lambda_\pi}{\beta} - \frac{\kappa\Lambda_\pi}{\beta} \\ &= \frac{\kappa\Lambda_r - (1+\kappa)\Lambda_r + \Lambda_y + \kappa\Lambda_\pi - \kappa\Lambda_\pi}{\beta} \\ &= \frac{-\Lambda_r + \Lambda_y}{\beta}\end{aligned} \tag{C.136}$$

Similarly, for $\xi_{l,t}$:

$$\frac{\kappa\Upsilon_r}{\beta} - \kappa\Omega_l - \frac{\kappa\Upsilon_\pi}{\beta} = \frac{-\Upsilon_r + \Upsilon_y}{\beta} \tag{C.137}$$

Therefore:

$$\lambda_x(\tilde{y}_{t+1} - \alpha \ln(x_t)) - \kappa\mu_{\pi,t+1} = \Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1} + \frac{\Lambda_y - \Lambda_r}{\beta}\xi_{r,t} + \frac{\Upsilon_y - \Upsilon_r}{\beta}\xi_{l,t} \quad \text{(C.138)}$$

(C.118) is:

$$\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t\left[\lambda_x(\tilde{y}_{t+1} - \alpha\ln(x_t)) - \kappa\mu_{\pi,t+1}\right]$$

Substituting in (C.138):

$$\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}] + \alpha(\Lambda_y - \Lambda_r)\xi_{r,t} + \alpha(\Upsilon_y - \Upsilon_r)\xi_{l,t} \quad \text{(C.139)}$$

Moving the $\xi_{r,t}$ and $\xi_{l,t}$ terms to the LHS:

$$\left[1 - \alpha(\Lambda_y - \Lambda_r)\right]\xi_{r,t} + \left[1 - \alpha(\Upsilon_y - \Upsilon_r)\right]\xi_{l,t} + \xi_{x,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}] \quad \text{(C.140)}$$

Define:

$$\Gamma_r \equiv 1 - \alpha(\Lambda_y - \Lambda_r), \qquad \Gamma_l \equiv 1 - \alpha(\Upsilon_y - \Upsilon_r) \quad \text{(C.141)}$$

so or final multiplier recursion is:

$$\Gamma_r\xi_{r,t} + \Gamma_l\xi_{l,t} + \xi_{x,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}] \quad \text{(C.142)}$$

Comparison with discretion

Under discretion, the corresponding condition is:

$$\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1}]$$

i.e. $\Gamma_r = \Gamma_l = 1$. The commitment recursion differs through $\Gamma_r \neq 1$ and $\Gamma_l \neq 1$, reflecting the commitment policymaker's internalisation of how current credit conditions feed back into the future targeting rule through the lagged multipliers.

C.7.6 Case analysis

Case 1: Full lending ($\ln(x_t) = 0$). Feasibility binds: $\xi_{x,t} \geq 0$, $\xi_{r,t} = \xi_{l,t} = 0$.

Targeting rule. From (C.129):

$$\pi_t + \frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha \ln(x_{t-1})) = \mu_{\pi,t-1} - \Omega_r \xi_{r,t-1} - \Omega_l \xi_{l,t-1} \tag{C.143}$$

Special case: no past constrained lending ($\ln(x_{t-1}) = 0$, $\xi_{r,t-1} = \xi_{l,t-1} = 0$). Then (C.143) gives $\pi_t + \frac{\lambda_x}{\kappa}\tilde{y}_t = \mu_{\pi,t-1}$. From (C.130) with zero credit multipliers: $\mu_{\pi,t} = \frac{\lambda_x}{\kappa}\tilde{y}_t$. Therefore $\mu_{\pi,t-1} = \frac{\lambda_x}{\kappa}\tilde{y}_{t-1}$, yielding the standard price-level targeting rule:

$$\pi_t = -\frac{\lambda_x}{\kappa}(\tilde{y}_t - \tilde{y}_{t-1}) \tag{C.144}$$

Transition from constrained lending. If $\xi_{r,t-1} > 0$ or $\xi_{l,t-1} > 0$ (past credit constraints were active), the lagged multiplier terms propagate past credit conditions into the current targeting rule. This forward-guidance channel is absent under discretion.

Case 2: Credit rationing ($\ln(x_t) < 0$, rationing binds). $\xi_{r,t} > 0$, $\xi_{l,t} = \xi_{x,t} = 0$.

Targeting rule. From (C.129):

$$\pi_t + \frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha \ln(x_{t-1})) = \mu_{\pi,t-1} + \frac{\Lambda_r}{\kappa}\xi_{r,t} - \Omega_r \xi_{r,t-1} - \Omega_l \xi_{l,t-1} \tag{C.145}$$

Multiplier. From (C.142) with $\xi_{l,t} = \xi_{x,t} = 0$:

$$\xi_{r,t} = \frac{1}{\Gamma_r}\left[\beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}]\right] \tag{C.146}$$

This has the same RHS as the discretionary recursion, but scaled by $1/\Gamma_r$.

Case 3: Constrained lending ($\ln(x_t) < 0$, liquidity binds). $\xi_{l,t} > 0$, $\xi_{r,t} = \xi_{x,t} = 0$.

Targeting rule. From (C.129):

$$\pi_t + \frac{\lambda_x}{\kappa}(\tilde{y}_t - \alpha\ln(x_{t-1})) = \mu_{\pi,t-1} + \frac{\Upsilon_r}{\kappa}\xi_{l,t} - \Omega_r\xi_{r,t-1} - \Omega_l\xi_{l,t-1} \tag{C.147}$$

Multiplier. From (C.142) with $\xi_{r,t} = \xi_{x,t} = 0$:

$$\xi_{l,t} = \frac{1}{\Gamma_l}\left[\beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r\xi_{r,t+1} + \Upsilon_r\xi_{l,t+1}]\right] \tag{C.148}$$

C.7.7 Properties of the commitment solution

History dependence. The commitment targeting rule (C.129) contains five RHS terms absent under discretion: $\mu_{\pi,t-1}$, which captures past inflation commitments (standard price-level targeting); $\xi_{r,t-1}$ and $\xi_{l,t-1}$, which capture past credit-condition commitments; and the modification of $\xi_{r,t}$ and $\xi_{l,t}$ through Γ_r and Γ_l.

Forward guidance eases credit conditions. Since $\Lambda_\pi, \Lambda_y > 0$, credible promises to raise future inflation and output directly relax the rationing constraint $\ln(x_t) = \Lambda_0' - \Lambda_r(r_t - \rho) + \Lambda_y\mathbb{E}_t[\tilde{y}_{t+1}] + \Lambda_\pi\mathbb{E}_t[\pi_{t+1}] + u_{r,t}$. Under commitment, the lagged multipliers provide a mechanism for generating these higher expectations, yielding less severe rationing: $|\ln(x_t)^{\text{commit}}| < |\ln(x_t)^{\text{discr}}|$.

The Γ coefficients. Since $\Lambda_y - \Lambda_r = (\Lambda_y - \Lambda^D) + \Lambda^\nu$ and $\Lambda^\nu > 0$, we have $\Gamma_r < 1$ for all parameter values. This reflects a positive feedback loop present only under commitment: current rationing generates a commitment to future accommodation (through the lagged multiplier $\xi_{r,t}$

in the targeting rule), which raises expected inflation and output, which in turn partially relaxes the current rationing constraint (since $\Lambda_\pi, \Lambda_y > 0$). The factor $1/\Gamma_r$ captures the strength of this feedback.

The sign of Ω_r is unambiguous under rationing: since $\Lambda_r \leq 0$, $\Lambda_y > 0$, and $\Lambda_\pi > 0$, we have $\Omega_r < 0$, ensuring that past rationing ($\xi_{r,t-1} > 0$) raises current inflation through $-\Omega_r \xi_{r,t-1} > 0$.

Appendix D Additional simulations and figures

This appendix contains the results from further simulations. Figure 9 repeats the high vs low risk economy simulations to a QE programme with investment adjustment costs.

We next consider preferences of the type proposed by Jaimovich and Rebelo (2009, JR), which allow for a weak short-run wealth effect on labour supply. These take the form:

$$U(C_t, H_t) = \frac{1}{1-\sigma}\left(C_t - \chi H_t^{\theta} X_t\right)^{1-\sigma} \tag{D.1}$$

where $X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}$. When calibrated with ($\gamma = 0.2$) and an inverse Frisch elasticity $\theta = 2.5$, these preferences substantially dampen the consumption response to lower interest rates. Figures 10 and 11 shows the small/medium/large simulations with JR preferences without and with investment adjustment costs, respectively.

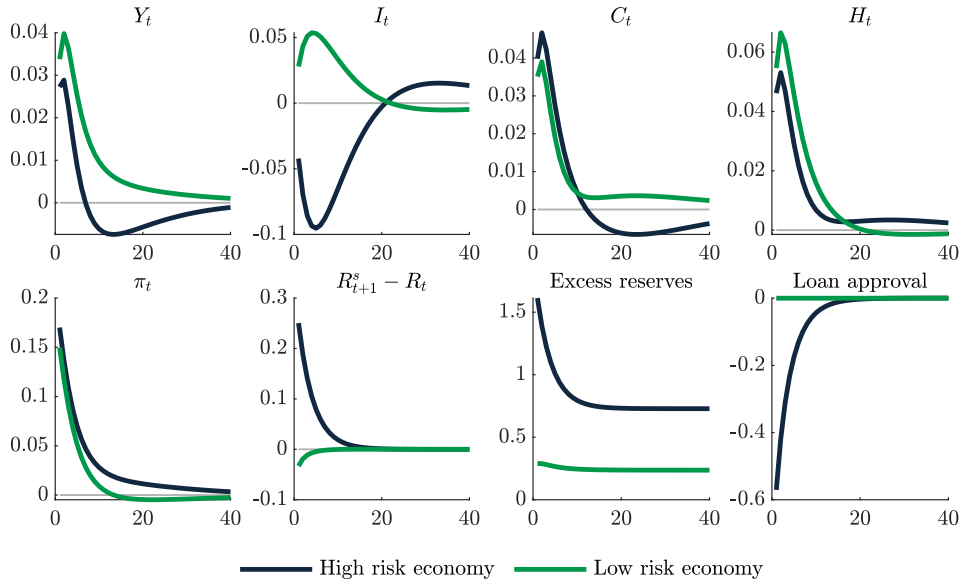


Figure 9: Impulse response functions to an unexpected temporary 'QE' programme in model with investment adjustment costs ($\phi_K = 4$). Plots in the top row show % deviation from steady state. The bottom row shows ppt deviation except for excess reserves which plots the ratio of reserves to stock of loans. Per annum risky firm default rate is 12% (high risk) and 2% (low risk).

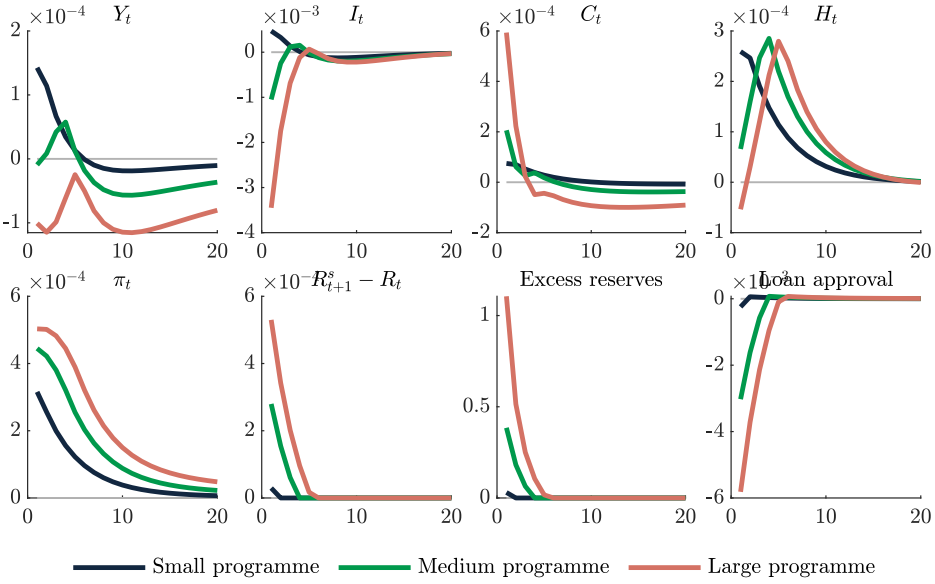


Figure 10: Impulse response functions to a temporary 'QE' programme with Jaimovich-Rebelo preferences, calibrated with a weak wealth effect on labour. Plots in the top row show % deviation from steady state. The bottom row shows ppt deviation except for excess reserves which plots the ratio of reserves to stock of loans.

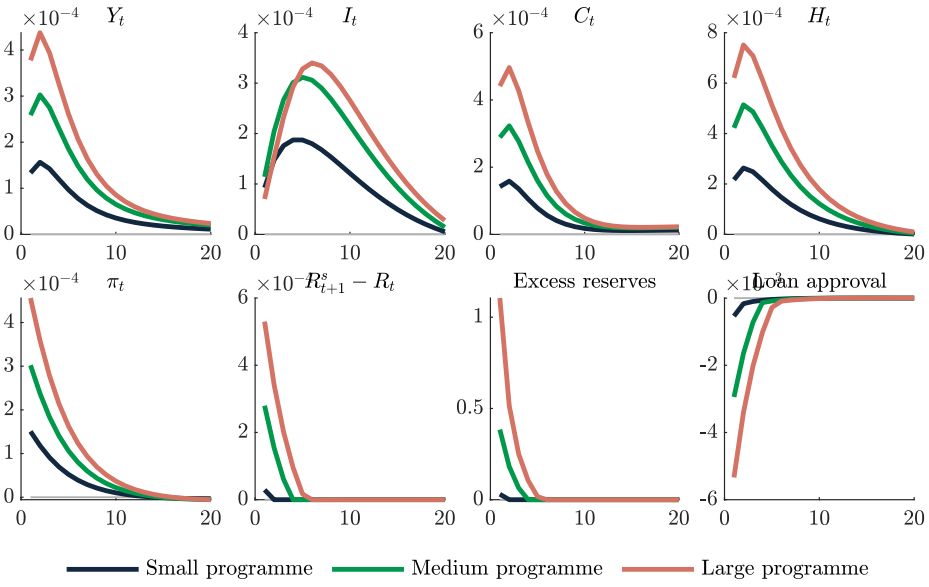


Figure 11: Impulse response functions to a temporary 'QE' programme with Jaimovich-Rebelo preferences, calibrated with a weak wealth effect on labour, and investment adjustment costs ($\phi_K = 4$). Plots in the top row show % deviation from steady state. The bottom row shows ppt deviation except for excess reserves which plots the ratio of reserves to stock of loans.

Appendix E Derivations and proofs

E.1 Lending facility use

Proposition 7 *Banks never use the central bank standing lending facility if* $\overline{R}_t > R_t^p$*. However, banks will use the deposit (reserve) facility in equilibrium and so the opportunity cost of lending is* $\underline{R}_t \forall t$*.*

Proposition 7 establishes that without a source of liquidity risk, the banks will never resort to using the central bank lending facility if this charges a higher interest rate than the main liquidity operation. So it truly is a last resort facility.

Proof: Without a source of liquidity risk, banks know their liquidity requirements with certainty at the time of portfolio allocation. Since funds are available through the main refinancing operation at rate $R_t^p < \overline{R}_t$, a bank facing a liquidity shortfall can always obtain funds at R_t^p rather than borrowing at the marginal lending facility rate $\overline{R}_t$. The lending facility is therefore strictly dominated and is never used in equilibrium. □

E.2 Proof of proposition 1

The bank's first-order conditions for x_t^r and x_t^s are:

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(R_{t+1}^s - \underline{R}_t\right)\right] = \varrho_t - \frac{\psi_t}{1-\lambda} + \frac{\varphi_t^r}{1-\lambda} \tag{E.1}$$

$$\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left((\lambda + (1-\lambda)p_{t+1})R_{t+1}^s - \underline{R}_t\right)\right] = \varrho_t + \varphi_t^r - \varphi_t^s \tag{E.2}$$

Subtracting (E.1) from (E.2):

$$-(1-\lambda)\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}(1-p_{t+1})R_{t+1}^s\right] = -\varphi_t^s + \frac{\psi_t}{1-\lambda} - \frac{\lambda}{1-\lambda}\varphi_t^r \tag{E.3}$$

The left-hand side is strictly negative since $(1-p_{t+1}) > 0$, $R_{t+1}^s > 0$, and $\Lambda_{t,t+1}/\Pi_{t,t+1} > 0$. If $x_t^s > 0$ then $\varphi_t^s = 0$ by complementary slackness, giving:

$$\frac{\psi_t}{1-\lambda} - \frac{\lambda}{1-\lambda}\varphi_t^r < 0 \quad \Rightarrow \quad \psi_t < \lambda\varphi_t^r \tag{E.4}$$

Since $\psi_t \geq 0$, this requires $\varphi_t^r > 0$, which by complementary slackness implies $x_t^r = 1$.

E.3 Proof of proposition 2

By Proposition 1, $x_t^r = 1$ whenever $x_t^s > 0$, so $\varphi_t^r > 0$ in all three regimes. Under the maintained assumption $\varphi_t^s = 0$, combining the bank's first-order conditions for x_t^r and x_t^s yields:

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \left(\lambda(R_{t+1}^s - \underline{R}_t) - (1-\lambda)(1-p_{t+1})R_{t+1}^s \right) \right] = \lambda \varrho_t + \psi_t \tag{E.5}$$

Since $\varrho_t, \psi_t \geq 0$, the LHS is non-negative.

No credit rationing. When (4.1) holds strictly, the LHS of (E.5) is positive, requiring $\lambda\varrho_t + \psi_t > 0$: either the liquidity constraint binds ($\varrho_t > 0$) or there is pooling ($\psi_t > 0$). In either case, rationing does not occur.

Regime 3 (Credit rationing). When (4.1) holds with equality, $\lambda\varrho_t + \psi_t = 0$, so $\varrho_t = \psi_t = 0$: there is no pooling and the liquidity constraint is slack (banks hold excess reserves). Banks optimally set $x_t^s < 1$ because the marginal revenue from a safe loan, $\lambda(R_{t+1}^s - \underline{R}_t)$, exactly equals the marginal information rent cost, $(1-\lambda)(1-p_{t+1})R_{t+1}^s$.

Regimes 1 and 2. When no rationing occurs, the regime depends on whether equilibrium bank liquidity is sufficient to fund all firms. The bank free-entry condition determines ϕ_t. When $x_t^s = 1$ and $\phi_t \geq 1$, evaluating the free-entry condition yields condition (4.2).

Regime 1 (Abundant liquidity). When both (4.1) and (4.2) hold, $\phi_t \geq x_t = 1$ and $\psi_t > 0$ enforces pooling. All firms are funded with excess reserves ($\varrho_t = 0$).

Regime 2 (Scarce liquidity). When (4.1) holds but (4.2) fails, the expected return on the bank's portfolio is insufficient to attract enough entry for $\phi_t \geq 1$. The liquidity constraint binds ($\varrho_t > 0$, $\phi_t = x_t < 1$), restricting $x_t^s < 1$ despite positive lending incentives ($\psi_t = 0$). □

E.4 Proof of proposition 3

From Proposition 2, credit rationing occurs when:

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \left(\lambda(R^s_{t+1} - \underline{R}_t) - (1-\lambda)(1-p_{t+1})R^s_{t+1} \right) \right] = 0 \tag{E.6}$$

The interest on reserves $\underline{R}_t$ enters this condition with coefficient $-\lambda < 0$. For any given R^s_{t+1} and p_{t+1}, reducing $\underline{R}_t$ (equivalently, raising $\nu_t = r_t - \underline{r}_t$) strictly increases the LHS. Away from the effective lower bound, the policymaker can always lower $\underline{R}_t$ sufficiently to make the LHS strictly positive, ensuring condition (4.1) holds and credit rationing does not occur.

The liquidity constraint threshold (4.2):

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} [1 - (1-\lambda)(1-p_{t+1})] R^s_{t+1} \right] \geq 1 \tag{E.7}$$

does not depend on $\underline{R}_t$. The corridor instrument therefore cannot prevent this constraint from binding.

The linearised model confirms this structure: the rationing condition (5.7) contains $\Lambda^\nu \hat{\nu}_t$ with $\Lambda^\nu > 0$, so widening the corridor relaxes the constraint, while the liquidity condition (5.8) is independent of $\hat{\nu}_t$. □

E.5 Proof of proposition 4

When $\ln(x_t) = 0$, the feasibility constraint binds ($\xi_{x,t} \geq 0$) while the rationing and liquidity constraints are slack ($\xi_{r,t} = \xi_{l,t} = 0$). From the FOC for r_t (Appendix C.6): $\mu_{y,t} = -\Lambda_r \xi_{r,t} - \Upsilon_r \xi_{l,t} = 0$. From the FOC for π_t: $\mu_{\pi,t} = -\pi_t$. Substituting both into the FOC for $\tilde{y}_t$:

$$\lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1})) + \kappa \pi_t = 0 \tag{E.8}$$

which gives (5.13). When $\ln(x_{t-1}) = 0$, this reduces to $\pi_t = -(\lambda_x/\kappa)\tilde{y}_t$. □

E.6 Proof of proposition 5

When the ELB binds, $\nu_t = r_t - \underline{r}^{ELB}$ and the rationing constraint depends on r_t through $-\Lambda_r(r_t - \rho)$. Under active rationing ($\xi_{r,t} > 0$) with the liquidity constraint slack ($\xi_{l,t} = 0$) and $\ln(x_t) < 0$ ($\xi_{x,t} = 0$), the discretionary FOCs (Appendix C.6) give: $\mu_{\pi,t} = -\pi_t$; $\lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1})) + \kappa\pi_t + \mu_{y,t} = 0$; and $\mu_{y,t} = -\Lambda_r \xi_{r,t}$. Combining yields (5.14).

For the recursion, since $\ln(x_t)$ affects welfare only from $t+1$ onwards, the FOC for $\ln(x_t)$ is:

$$\xi_{r,t} = \beta\lambda_x \frac{1-\alpha}{1+\varphi}\alpha + \beta\lambda_x \alpha \mathbb{E}_t[\tilde{y}_{t+1} - \alpha \ln(x_t)] + \beta\kappa\alpha\mathbb{E}_t[\pi_{t+1}]$$

The targeting rule holds in all future states, giving $\lambda_x(\tilde{y}_{t+1} - \alpha \ln(x_t)) + \kappa\pi_{t+1} = \Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1}$. Substituting yields (5.15). □

E.7 Proof of proposition 6

Under commitment, π_t and $\tilde{y}_t$ appear in period-$(t-1)$ constraints through expectations, generating lagged multiplier terms absent under discretion. The FOCs are (Appendix C.7):

$$\pi_t + \mu_{\pi,t} - \mu_{\pi,t-1} - \tfrac{1}{\beta}\mu_{y,t-1} - \tfrac{\Lambda_\pi}{\beta}\xi_{r,t-1} - \tfrac{\Upsilon_\pi}{\beta}\xi_{l,t-1} = 0 \tag{E.9}$$

$$\lambda_x(\tilde{y}_t - \alpha \ln(x_{t-1})) - \kappa\mu_{\pi,t} + \mu_{y,t} - \tfrac{1}{\beta}\mu_{y,t-1} - \tfrac{\Lambda_y}{\beta}\xi_{r,t-1} - \tfrac{\Upsilon_y}{\beta}\xi_{l,t-1} = 0 \tag{E.10}$$

$$\mu_{y,t} + \Lambda_r \xi_{r,t} + \Upsilon_r \xi_{l,t} = 0 \tag{E.11}$$

Multiplying (E.9) by κ, adding (E.10), and using (E.11) to eliminate $\mu_{y,t}$ and $\mu_{y,t-1}$ yields (5.16), where $\Omega_r \equiv [(1+\kappa)\Lambda_r - \Lambda_y - \kappa\Lambda_\pi]/(\beta\kappa)$ and Ω_l is defined analogously.

For the recursion, the FOC for $\ln(x_t)$ is:

$$\xi_{r,t} + \xi_{l,t} + \xi_{x,t} = \beta\lambda_x \tfrac{1-\alpha}{1+\varphi}\alpha + \beta\lambda_x\alpha\mathbb{E}_t[\tilde{y}_{t+1} - \alpha\ln(x_t)] - \beta\kappa\alpha\mathbb{E}_t[\mu_{\pi,t+1}] \tag{E.12}$$

Using the forward targeting rule and the forward (E.9) to express $\lambda_x(\tilde{y}_{t+1} - \alpha \ln(x_t)) - \kappa\mu_{\pi,t+1}$

in terms of multipliers, the $\mu_{\pi,t}$ and π_{t+1} terms cancel, leaving (under rationing, $\xi_{l,t} = \xi_{x,t} = 0$):

$$\Gamma_r \xi_{r,t} = \beta\lambda_x \tfrac{1-\alpha}{1+\varphi}\alpha + \beta\alpha\mathbb{E}_t[\Lambda_r \xi_{r,t+1} + \Upsilon_r \xi_{l,t+1}]$$

where $\Gamma_r = 1 - \alpha(\Lambda_y - \Lambda_r)$, giving (5.17). For the special case, setting $\xi_{r,t} = \xi_{l,t} = \xi_{r,t-1} = \xi_{l,t-1} = 0$ and $\ln(x_{t-1}) = 0$ in (5.16) gives $\pi_t + (\lambda_x/\kappa)\tilde{y}_t = \mu_{\pi,t-1}$. From (E.10) with zero credit multipliers: $\mu_{\pi,t} = (\lambda_x/\kappa)\tilde{y}_t$, so $\mu_{\pi,t-1} = (\lambda_x/\kappa)\tilde{y}_{t-1}$, yielding $\pi_t = -(\lambda_x/\kappa)(\tilde{y}_t - \tilde{y}_{t-1})$. □

E.8 Condition for excess liquidity under pooling

Substituting the (binding) constraints (3.3) and (3.2) into (3.4) then (3.5) yields:

$$1 = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\underline{R}_t\left(1 - \left[\lambda x_t^s + (1-\lambda)\, x_t^r\right]\frac{1}{\phi_t}\right)\right.\right.$$
$$\left.\left. + \left(\lambda x_t^s + (1-\lambda)\left(x_t^r - (1-p_{t+1})\, x_t^s\right)\right)\frac{1}{\phi_t}R_{t+1}^s\right)\right] \quad \text{(E.13)}$$

Under pooling, with $\bar{x}_t = 1$, $x_t^r = x_t^s = 1$, at the point at which $\phi_t = 1$, this becomes:

$$1 = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\left(\lambda + (1-\lambda)\left(1 - (1-p_{t+1})\right)\right)\frac{1}{\phi_t}R_{t+1}^s\right)\right] \quad \text{(E.14)}$$

or

$$\phi_t = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\lambda + (1-\lambda)\, p_{t+1}\right)R_{t+1}^s\right] \quad \text{(E.15)}$$

Likewise, substituting the zero-arbitrage condition $V_t = q_t^K F$ into (3.12) with $\phi_t = 1$, $x_t^r = x_t^s = 1$ yields:

$$F = \frac{\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\left(\eta + (1-\eta)\left(1-\lambda\right)\left(1-p_{t+1}\right)\right)R_{t+1}^s\right] - \eta}{1 - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\eta + (1-\eta)\left(\lambda + (1-\lambda)\, p_{t+1}^r\right)\right)\right]}q_t^K \quad \text{(E.16)}$$