Learning to Be Rich

How Expectations Amplify Wealth Inequality*

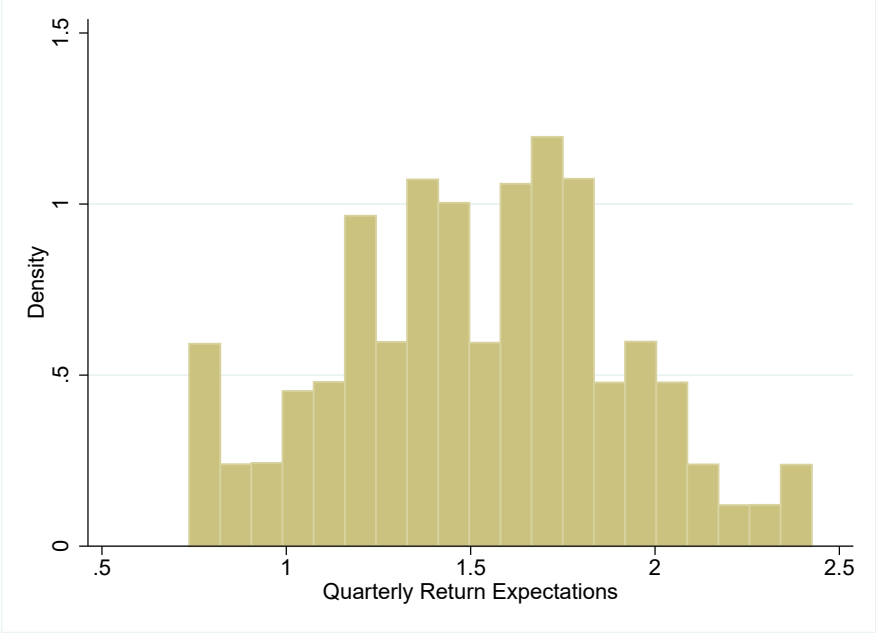
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*Only personal views.

Motivation

* Explaining the extreme wealth concentration at the top remains challenging.
* Recent literature: refocus from **labor** markets to **asset** markets.
    * Portfolio choice and Asset Pricing have come to the forefront.
* Typical assumption: **beliefs** behind portfolios and asset prices are the same for everyone ("Communism of beliefs").
* **Microdata**: measured **expectations** look more like a full **distribution**.



Cross-section of expected stock returns in the RAND survey.

Our questions

1. What are the main facts about the distribution of households expected stock returns?

2. How can subjective beliefs influence top concentration?

3. How can we model and solve HA models with HE? Are they even less tractable?

4. How much do HE influence Wealth Inequality?

5. Where do HE come from?

Our (tentative) answers

1. What are the key facts of the distribution of households expected stock returns?
    * Persistence disagreement (Giglio et al., 2021)
    * Strong positive correlation between expectations and wealth (New!)
2. How can subjective beliefs influence top concentration?
    * Accuracy maximizes expected wealth growth.
    * Over-optimism generates rare winners, increasing the Pareto tail.
3. How can we model and solve HA models with HE? Are they even less tractable?
    * No, they're simpler: prices become state variables.
    * Households directly forecast prices, not distributions.
4. How much do HE influence Wealth Inequality?
    * Increase the Top 1 by around +10pp (relative to homogeneous beliefs).
5. Where do HE come from?
    * Learning from one's own portfolio experience.

Contributions to the literature

1. Measured expectations:
   * Characterize the joint distribution of wealth and expectations (Greenwood and Shleifer, 2014; Coibion and Gorodnichenko, 2015; Kohlhas and Walther, 2021).
   * Model the dominance of individual fixed effects (Giglio et al., 2021).

2. Wealth inequality:
   * Microfoundation of heterogeneous returns and portfolios (Gabaix et al., 2016; Fagereng et al., 2020; Hubmer et al., 2021; Xavier, 2021; Fernández-Villaverde and Levintal, 2024).
   * New amplification mechanism based on expectations.

3. Solution method for HA models with aggregate risk: Internal Rationality & Deep Learning. Make the models easier to work with.

Theories of wealth inequality

1. Facts about the distribution of expectations

Measuring households' expectations

* Dataset: RAND - American Life Panel.
  * Representative US household survey.
  * 2008-2017, 61 Waves.
  * Unbalanced panel $\approx$ 3,000 participants
    * We keep households with at least 10 observations ($\approx 2,300$ households).
  * Rich sociodemographic information: age, gender, race, education, marital and employment status, income, wealth, and household composition.

* Only public panel data set that tracks wealth & stock return expectations.

* Questions yield a rough distribution of one year ahead (ex-dividend) returns for each agent:
  $P^i(R < 0.80)$, $P^i(0.80 < R < 1.00)$,$P^i(1.00 < R < 1.20)$, $P^i(R > 1.20)$.

Questions

RAND expectations show the familiar deviations from FIRE

1. Forecast errors predictability.
    * Extrapolation; over-reaction (consistent with e.g., Adam, Marcet & Beutel, 2017; Bordalo et al., 2020; Kohlhas and Walter, 2021).
        $\Rightarrow$ Many papers address this. More relevant for cyclical fluctuations.

2. **Persistent disagreement**.
    * Individual Fixed Effects explain most of the panel variation (Giglio et al., 2021).
        $\Rightarrow$ Fewer papers discuss this. We conjecture that this is more relevant for the (long-run) wealth distribution.

Regressions

Wealth shifts beliefs toward realistic returns

	$R < -20\%$	$-20\% < R < 0\%$	$0\% < R < 20\%$	$R > 20\%$
Wealth	0.001	-0.088^{***}	$\mathbf{0.103}^{***}$	0.014^{**}
Risky share	0.001	-0.007^{***}	$\mathbf{0.007}^{***}$	0.002^{***}
Controls	Home ownership, retirement acct., housing wealth, income, age			
Observations	12,025			
Mean returns	+14.88%	Min + 1.38%	Max +32.33%	

Note: coefficients are log-odds from fractional logits; bullets report odds changes.

* Wealth shifts beliefs toward the most realistic return range, 0–20%.
* Wealth reduces pessimism: richer households assign less probability to negative returns.
* Wealth also raises optimism: richer households place more probability on strongly positive returns.
* Risky-asset holdings show the same pattern; income and housing wealth point in the same direction, but weaker.

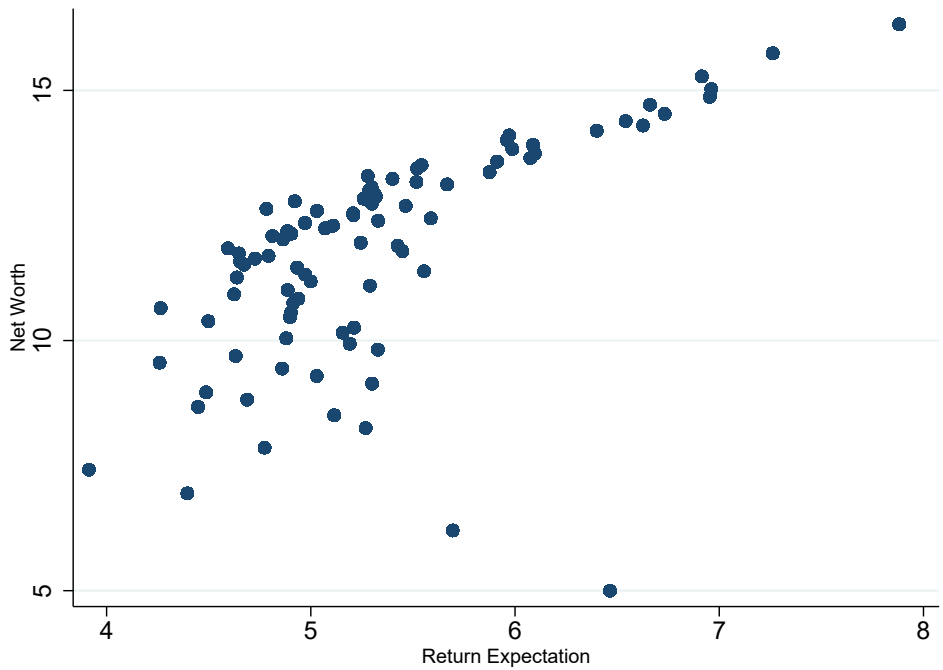
Logit Regression

An Integrated Dataset on Wealth & Expectations

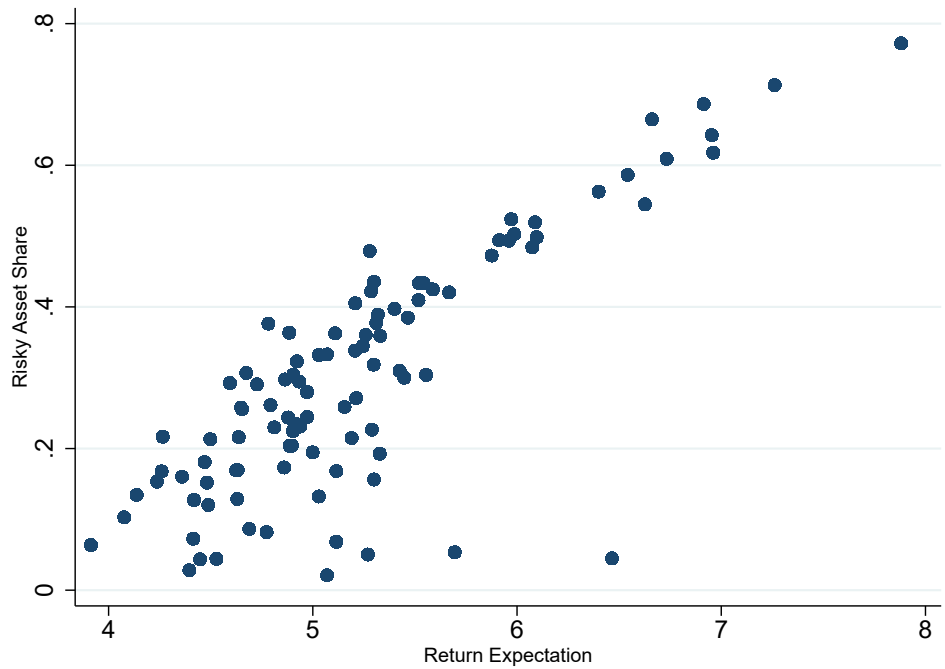
* Some shortcomings of RAND raw data
  * Coarse probability distribution $\neq$ expected returns.
  * Limited asset coverage, completely omitting directly-held financial assets.
* Convert raw data into expected returns (like Von Gaudecker and Wogrolly, 2022).
* Match RAND to the Survey of Consumer Finances (2016).
  * Use available links: Retirement Account, Equity share in the retirement account, Home value, Ownership states, Wage Income, Age to predict expected returns.
  * Key identifying assumption: households with identical observable characteristics form similar return expectations.

Details Back

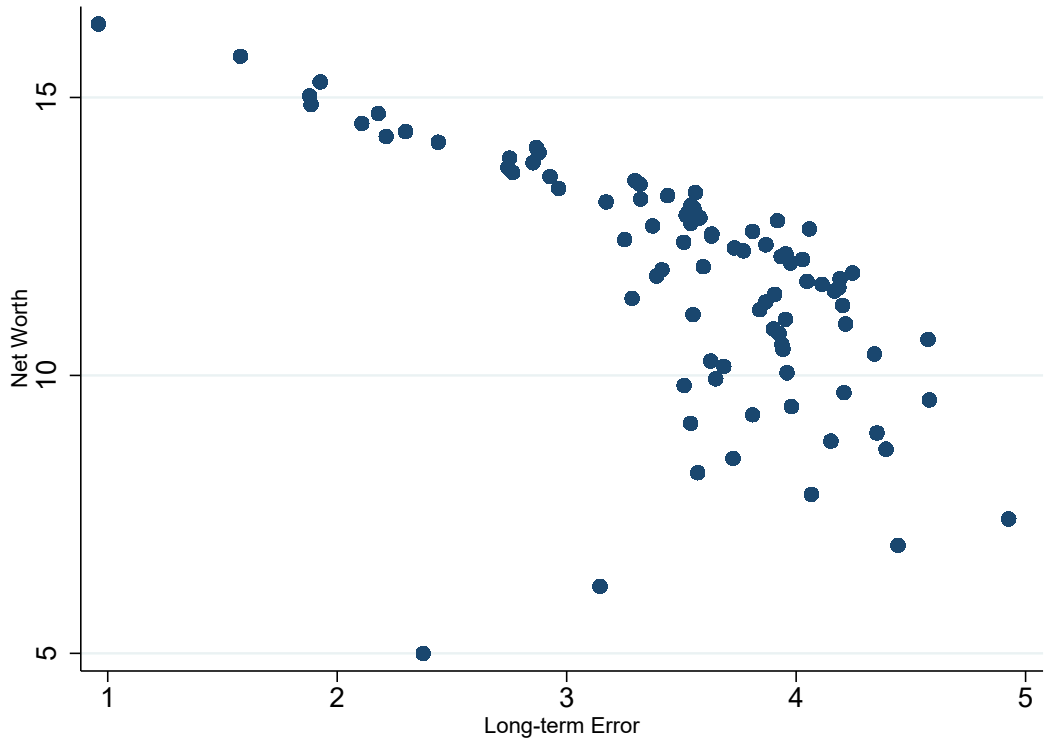
High wealth percentiles expect higher returns



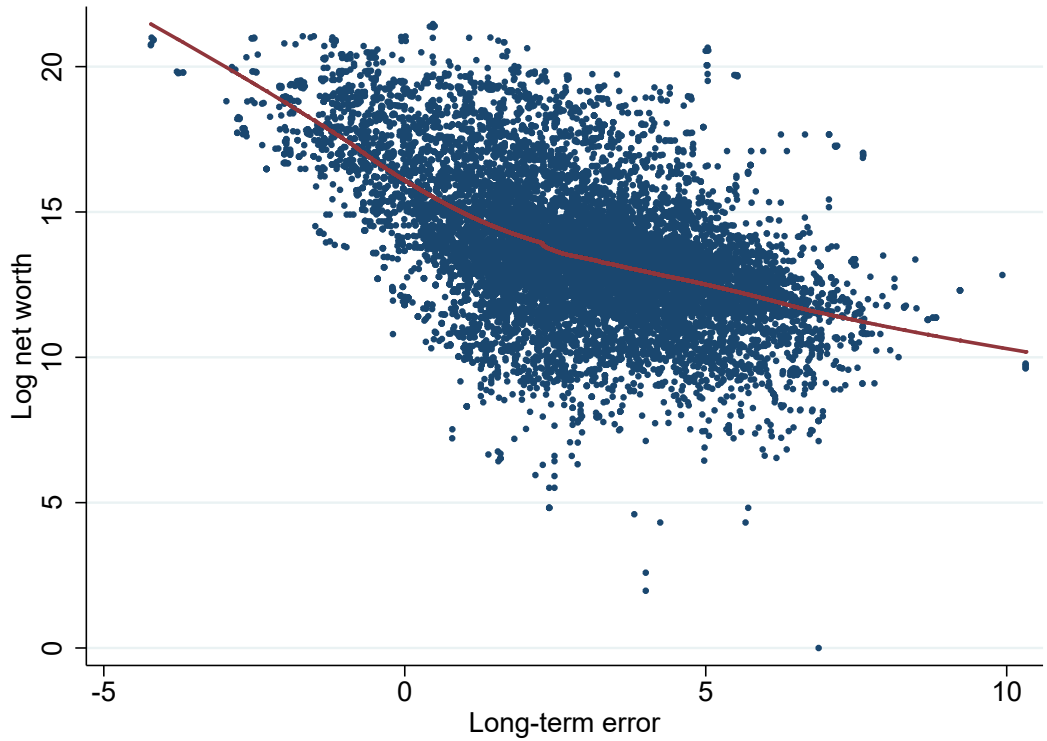
Higher expected return are associated with a higher risky share



High wealth percentiles are more accurate



The wealthiest households are over-optimistic



Summary of the empirics

1. Households persistently disagree about stock market returns.
2. Wealthier households are more accurate and invest more in risk.
3. The wealthiest are over-optimistic.

We want to reproduce these patterns in a model,
and understand how much they contribute to overall inequality.

2. A Heterogeneous Agent model with Heterogeneous Expectations

Structure

* Idiosyncratic wage risk and incomplete markets (à la Aiyagari-Bewley-Huggett).
* Aggregate risk (à la Krusell-Smith).
* Endowment economy (no production).
* Portfolio choice between safe and risky assets.
* Internal Rationality (à la Adam-Marcet) and imperfect information.

Households problem

* Savings-consumption + portfolio choice problem:

$$\max_{\{C_t^i, A_t^i, v_t^i\}_{t=0}^{\infty}} \mathbb{E}_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \beta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \tag{1}$$

* Subject to the budget constraint

$$C_t^i + A_t^i = W_t^i + R_t^{A,i}(v_{t-1}^i) A_{t-1}^i \tag{2}$$

where assets are stocks and bonds

$$A_t^i = P_t S_t^i + B_t^i \tag{3}$$

and the return to wealth is

$$R_t^{A,i}(v_{t-1}^i) = v_{t-1}^i R_t^S + (1 - v_{t-1}^i) R_t^B; \quad v_{t-1}^i = \frac{P_{t-1} S_{t-1}^i}{A_{t-1}^i} \tag{4}$$

* Subject to asset holding limits

$$\begin{aligned} A_t^i &\geq 0 \\ 0 \leq v_t^i &\leq \bar{v} \end{aligned} \tag{5}$$

* Given initial endowments $S_0^i = 1$ and $B_0^i = 0$.

Subjective probability measure $\mathcal{P}^i$

* Internal Rationality: agents maximize expected utility given a well-specified subjective probability measure $\mathcal{P}^i$.

* $\mathcal{P}^i$ is a primitive of the model (like e.g., $U(C_t^i)$).

* $\mathcal{P}^i$ is defined over payoff-relevant variables that are external to the household, including market outcomes such as prices.
    * Underlying probability space $(\mathbf{\Omega}, \mathcal{B}, \mathcal{P}^i)$ with a typical element $\omega \in \mathbf{\Omega}$ with $\omega = \{D_t, W_t^i, P_t, R_t^B\}_{t=0}^{\infty}$.

* Rational Expectations is a particular case that emerges when agents are endowed with Full Information.

* We deviate from FIRE to model heterogeneous beliefs:
    * Full Information about income processes.
    * Imperfect market knowledge prevents agents from deducing the equilibrium stochastic process for asset prices.

Subjective model for risky prices

* Remember a key empirical pattern: persistent disagreement.
* We start with "brute force".
* Subjective model of capital gains:

$$\frac{P_t}{P_{t-1}} = \zeta^i \varepsilon_t^{P,i}, \quad \ln \varepsilon_t^{P,i} \sim i.i.\mathcal{N}(-\frac{1}{2}\sigma_P^2, \sigma_P^2) \tag{6}$$

* ζ^i is a form of ex-ante heterogeneity. Calibrate it to match survey's individual fixed effects.
* It's easy to make ζ^i time-varying to match time series deviation from FIRE.
* We endogenize ζ^i later!
* The subjective one-period bond return is i.i.d. and homogeneous.

3. Analytical results

A stripped-down version

* Extra assumptions:
  * Wages are constant, positive, but small.
  * No limits on asset holdings.
  * Non-pledgeable human wealth.
  * The actual laws of motion for stock and bond returns are exogenous (no equilibrium).
* Approximate Merton-Samuelson solutions for the savings rate

$$\kappa_i = \left[\delta \mathbb{E}^{\mathcal{P}^i}\left[\left(R^i_{p,t+1}\right)^{1-\gamma_i}\right]\right]^{1/\gamma_i}$$

and the risky share

$$\nu_i = \frac{\mathbb{E}^{\mathcal{P}^i}[R^S_{t+1}] - R^B_t}{\gamma_i \sigma^2_S}$$

* Individual wealth evolves as a Kesten process:

$$A^i_t = \kappa^i R^{A,i}_t A^i_{t-1} + \kappa^i \bar{W}^i$$

Accuracy maximizes expected wealth growth

* Subjective expected excess return: $\Delta^i \equiv \mathbb{E}_t^{\mathcal{P}^i}[R_{t+1} - R_t^B]$. Objective one is Δ^0.
* Expected log growth of wealth:

$$\lambda^i \equiv \mathbb{E}\left[\log\left(\kappa_i R_t^{A,i}\right)\right] = \frac{1}{\gamma}\left[\log\delta + r_B + \frac{(\Delta^0)^2}{2\sigma^2} - \frac{(\Delta^i - \Delta^0)^2}{2\sigma^2}\right]$$

* Three immediate results:
  1. Accurate beliefs maximize objective expected log wealth growth (as Martin and Papadimitriou, 2022, and RAND-SCF data).
  2. Wealth growth is hump-shaped in optimism.
  3. Beliefs errors make the Kesten stationarity condition (i.e., $\lambda^i < 0$) looser.

Optimism increase the Pareto tail

* Under the usual Kesten–Goldie conditions, the stationary right tail is Pareto with exponent $\xi_i > 0$ satisfying (normalized by growth):

$$\mathbb{E}\Big[\exp\Big\{\xi_i(r_t^A - g_t - c^i)\Big\}\Big] = 1, \quad c^i \equiv -\log \kappa^i$$

* Approximate closed-form solution:

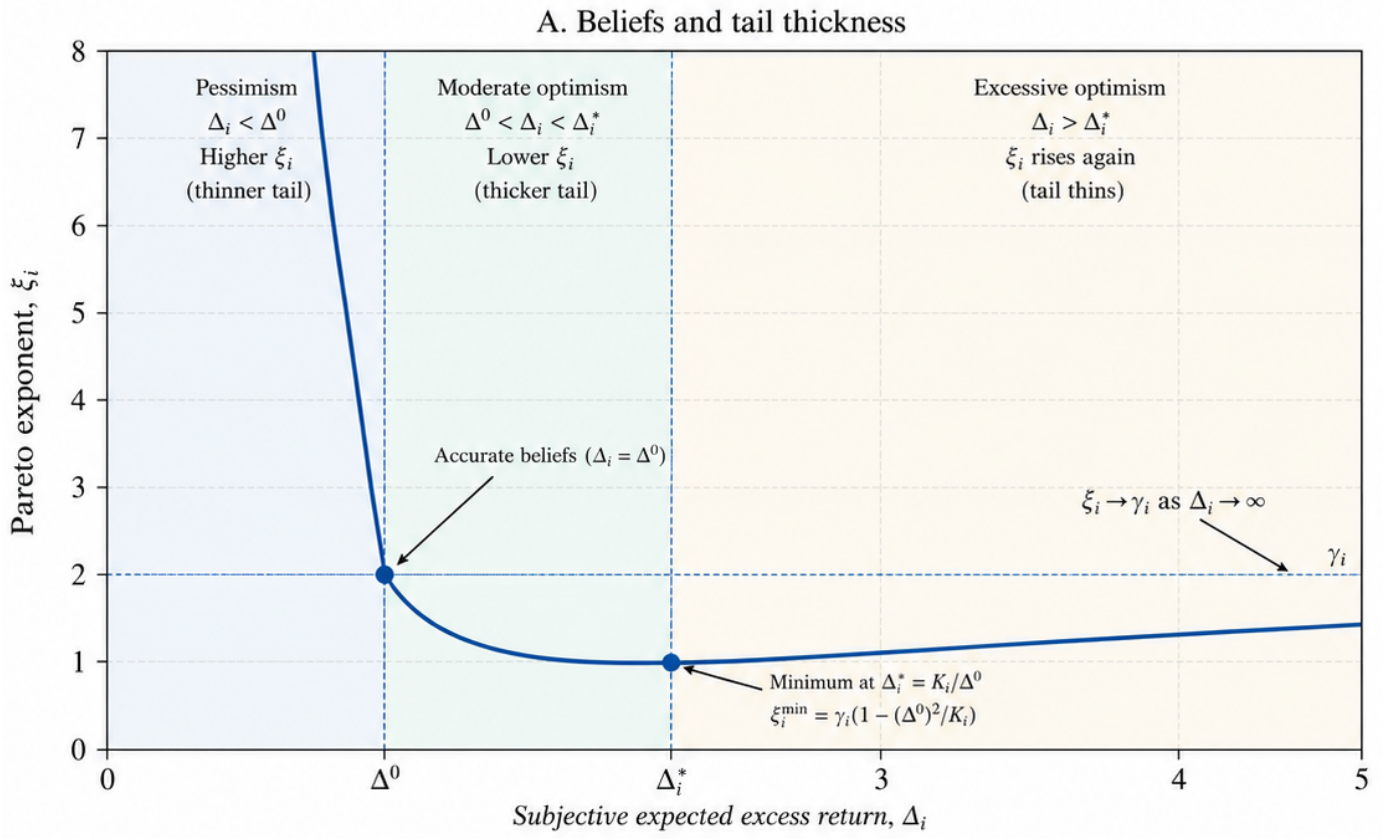
$$\xi_i = \gamma_i \frac{(\Delta^i)^2 - 2\Delta^0\Delta^i + 2\sigma^2\big(\gamma g - \log\delta - r_B\big)}{(\Delta^i)^2}$$

* The Pareto exponent is minimized at

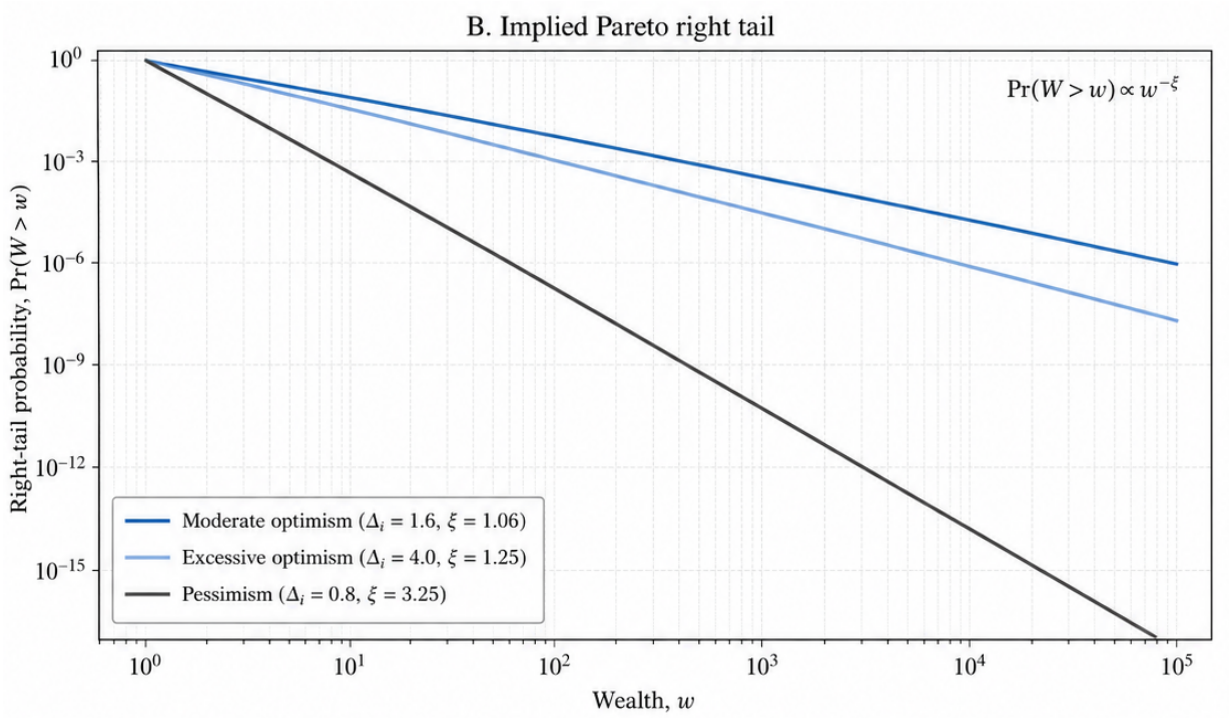
$$(\Delta^i)^\star = \frac{2\sigma^2\big(\gamma_i g - \log\delta - r_B\big)}{\Delta^0} > \Delta^0$$

(the inequality follows from assuming the accurate household is stationary).

Moderate optimism lowers the Pareto exponent



Moderate optimism increase the Pareto tail



* The tail depends on two forces:

$$\underbrace{\text{lower mean growth}}_{\text{thins the tail}} \quad \textit{vs.} \quad \underbrace{\text{higher return variance}}_{\text{thickens the tail}}$$

* Accurate households perform best along the typical long-run history.
* Optimistic households overinvest in risk, lowering their typical growth but creating more extreme winners.
* The Pareto tail is determined by rare sequences of exceptionally high multiplicative returns rather than by average performance.

General Equilibrium brings dampening forces

* Before: beliefs move portfolios, not prices. Now, beliefs also move prices.
* If optimistic households demand more equity, market clearing requires:

$$P_t \uparrow \quad \Rightarrow \quad \mathbb{E}_t R_{S,t+1} \downarrow$$

  and, with bonds in zero net supply, $R_{B,t} \uparrow$
* Hence, the objective excess return compresses: $\Delta_t^0 \equiv \mathbb{E}_t^0 r_{S,t+1} - r_{B,t} \downarrow$
* The non-equilibrium force remains, but is attenuated:

$$\text{optimism} \Rightarrow v_i \uparrow \Rightarrow P_t \uparrow,\ r_{B,t} \uparrow,\ \Delta_t^0 \downarrow \Rightarrow \text{weaker tail amplification.}$$

* GE introduces a self-correcting price channel, but does not imply a general ordering $\xi^{i,GE} > \xi^i$.
  * GE may lower the mean return but also change risk exposure and savings rate.
  * The net effect on ξ_i is therefore quantitative.

4. Solving the Quantitative Model

Recursive Formulation

* Specify the aggregate and individual state variables.
* Recall the budget constraint

$$C_t^i + A_t^i = W_t^i + \left(v_{t-1}^i R_t^S + (1 - v_{t-1}^i) R_t^B\right) A_{t-1}^i \tag{7}$$

* Household i position at the beginning of each period

$$\boldsymbol{x}_t^i = (W_t^i, A_{t-1}^i, v_{t-1}^i, D_t, P_t, Q_t) \tag{8}$$

with $R_{t+1}^B = 1/Q_t$.

* In equilibrium, stock demand functions $h^s(\boldsymbol{x}_t^i)$.
* Asset markets equilibrium:

$$\begin{aligned} \sum_i \mu^i h^s(W_t^i, A_{t-1}^i, v_{t-1}^i, D_t, P_t, Q_t) &= 1 \\ \sum_i \mu^i h^b(W_t^i, A_{t-1}^i, v_{t-1}^i, D_t, P_t, Q_t) &= 0 \end{aligned} \tag{9}$$

* Solving that equation for prices

$$(P_t, Q_t) = p(\bar{\Gamma}_t, D_t) \tag{10}$$

* Conventionally, $\bar{\Gamma}$ is the cross-sectional distribution over stocks, bonds and wages:

$$\bar{\Gamma}_t \equiv \bar{\Gamma}(s, b, w) = \sum_{i=1}^{I} \mu^i \mathbf{1}\left\{A_{t-1}^i \leq a, v_{t-1}^i \leq v, W_t^i \leq w\right\} \tag{11}$$

Recursive Formulation: RE vs. IR

* RE: agents know $(P_t, Q_t) = p(\bar{\Gamma}_t, D_t)$.
* Vector of states becomes

$$(\boldsymbol{x}_t^i)^{RE} = (W_t^i, A_{t-1}^i, v_{t-1}^i, D_t, p(\bar{\Gamma}_t, D_t)) \tag{12}$$

* Problematic: deal with a highly-dimensional object.
* Under Internal Rationality (with imperfect info):

$$(\boldsymbol{x}_t^i)^{IR} = (W_t^i, A_{t-1}^i, v_{t-1}^i, D_t, P_t, Q_t, \zeta_t^i) \tag{13}$$

* Don't need to keep track of the distribution to solve households' problems!
* Prices still depend on the cross-sectional distribution

$$(P_t, Q_t) = p(\Gamma_t, D_t) \tag{14}$$

$$\Gamma_t \equiv \Gamma(s, b, w, \zeta) = \sum_{i=1}^{I} \mu^i \mathbf{1}\left\{A_{t-1}^i \leq a, v_{t-1}^i \leq v, W_t^i \leq w, \zeta_t^i \leq \zeta\right\} \tag{15}$$

Recursive Competitive Equilibrium under IR

Given $\{\mathcal{P}^i, S^i_{-1}, B^i_{-1}\}_i$, a RCE is given by

* The policy functions $\boldsymbol{h}_i$ for all i:

$$\begin{pmatrix} C^i_t \\ v^i_t \end{pmatrix} = h_i((\boldsymbol{x}^i_t)^{IR}) \tag{16}$$

* The pricing function p:

$$(P_t, Q_t) = p(\Gamma_t, D_t) \tag{17}$$

Such that

1. The first-order conditions, budget constraints and asset holdings limits of each household are satisfied.
2. Markets clear.

Recursive Competitive Equilibrium under IR: 2 stage approach

Under Internal Rationality, computing the equilibrium can be separated into two steps

1. Solve the individual problems conditional on simulated price paths drawn from $\mathcal{P}^i$.

2. Use optimal demands to compute the market-clearing prices.
    * Step 2 recovers the implied equilibrium pricing function $p(\Gamma, \cdot)$.

New solution algorithm: 2 Stage Deep Learning (2SDL). Details

5. Quantitative Analysis

Calibration

* Calibration:
  * Dividend growth matches US stock-market data. Dividends
  * Labor income follows the Fernández-Villaverde–Levintal two-state wage process. Wages
  * Ex-ante types jointly determine beliefs, permanent wages, and risk aversion, matching RAND–SCF rank correlations and wealth-group belief gradients. Types
* SMM for common wage-dividend scale. Targets
* All portfolio choices, asset prices and income and wealth distributions are endogenous.
* We assess the model's stationary wealth distribution against RAND–SCF 2016.
  * Focus on financial wealth (excluding housing and other nonfinancial assets).
* Results are preliminary.

Distributions

	Top 1%		Top 10%		Bottom 90%	
	Model	**Data**	**Model**	**Data**	**Model**	**Data**
Wealth share	41.79	37.60	59.87	75.70	40.13	24.20
Income share	14.40	18.40	37.07	43.10	62.93	56.80
Wage share	12.36	12.30	35.12	33.90	64.88	66.00
Risky asset share	86.42	77.30	85.49	69.75	33.20	38.36
Expected stock return	7.78	7.88	5.65	6.89	4.69	5.05
Portfolio return	8.65	8.30	8.59	7.20	5.13	3.62

Heterogeneous beliefs increase concentration at the top

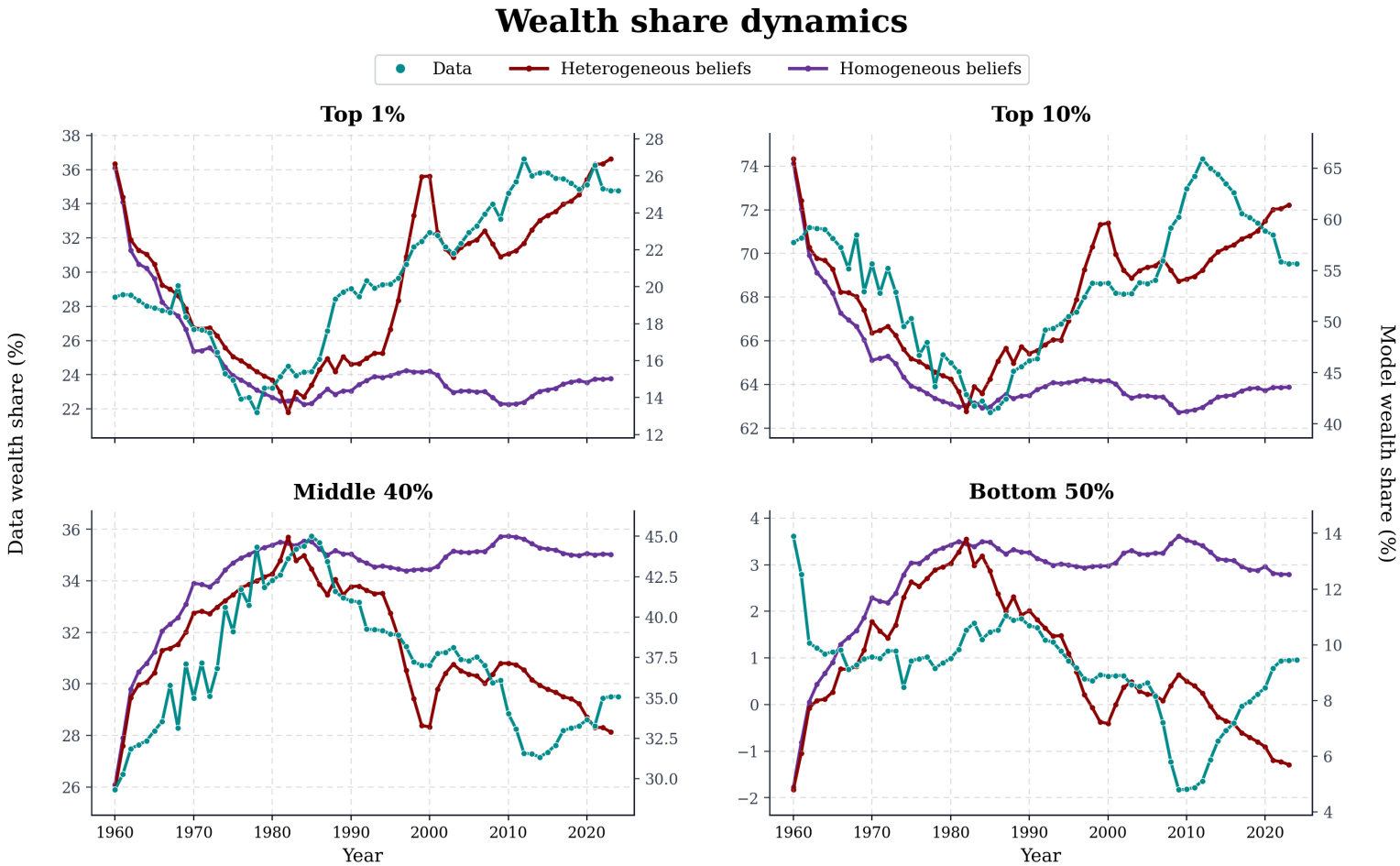
	Top 1%		Top 10%		Bottom 90%	
	Hetero	**Homo**	**Hetero**	**Homo**	**Hetero**	**Homo**
Wealth share	41.79	31.36	59.87	61.47	40.13	38.53
Income share	14.40	12.69	37.07	36.88	62.93	63.12
Wage share	12.36	12.53	35.12	35.42	64.88	64.58
Risky asset share	86.42	63.14	85.49	72.80	33.20	41.48
Expected stock return	7.78	5.68	5.65	5.68	4.69	5.68
Portfolio return	8.65	1.08	8.59	3.47	5.13	−3.83

Historical Transitional Dynamics

Exercise: feed the model the realized U.S. asset-price history and ask how heterogeneous beliefs reshape the evolution of portfolios and wealth inequality.

1. Historical inputs, 1960–2024: observed stock prices, dividends, and calibrated price moments for the period.
2. Start from the observed initial wealth distribution.
3. Subjective expected capital gains match the RAND gradient.
4. Model output: endogenous portfolio shares, wealth shares, capital-income shares, and inequality statistics over 1960–2024.

Historical Transitional Dynamics



Table

5. Where do Heterogeneous Expectations come from?

Beyond ex-ante heterogeneity

* So far, a reduced-form approach:
  * Impose persistent fixed effects in households' subjective models of prices.

* This matches the observed heterogeneity, but leaves its origin unexplained.

* Taken at face value, some households are simply endowed with optimism and become rich partly because of it.
  * A similar issue arises when persistent heterogeneity in discount factors or risk aversion is treated as primitive.

* Can we explain the origins of persistent belief differences?

Learning from own portfolio experience

* Household i perceives an unobserved expected return state:

$$\underbrace{\log R^S_{t+1}}_{\text{stock return}} = \underbrace{M^i_t}_{\text{latent expected log return}} + \varepsilon^{S,i}_{t+1}$$

* Portfolio performance is interpreted as a noisy signal $\mathcal{S}_t$ about this state:

$$\mathcal{S}^i_t = M^i_t + \nu^i_t; \quad M^i_t = M^i_{t-1} + \varepsilon^i_t$$

where the experience signal is given by

$$\mathcal{S}^i_t = \kappa_L + \mu_L \log R^{A,i}_t + (1 - \mu_L) \log R^S_t$$

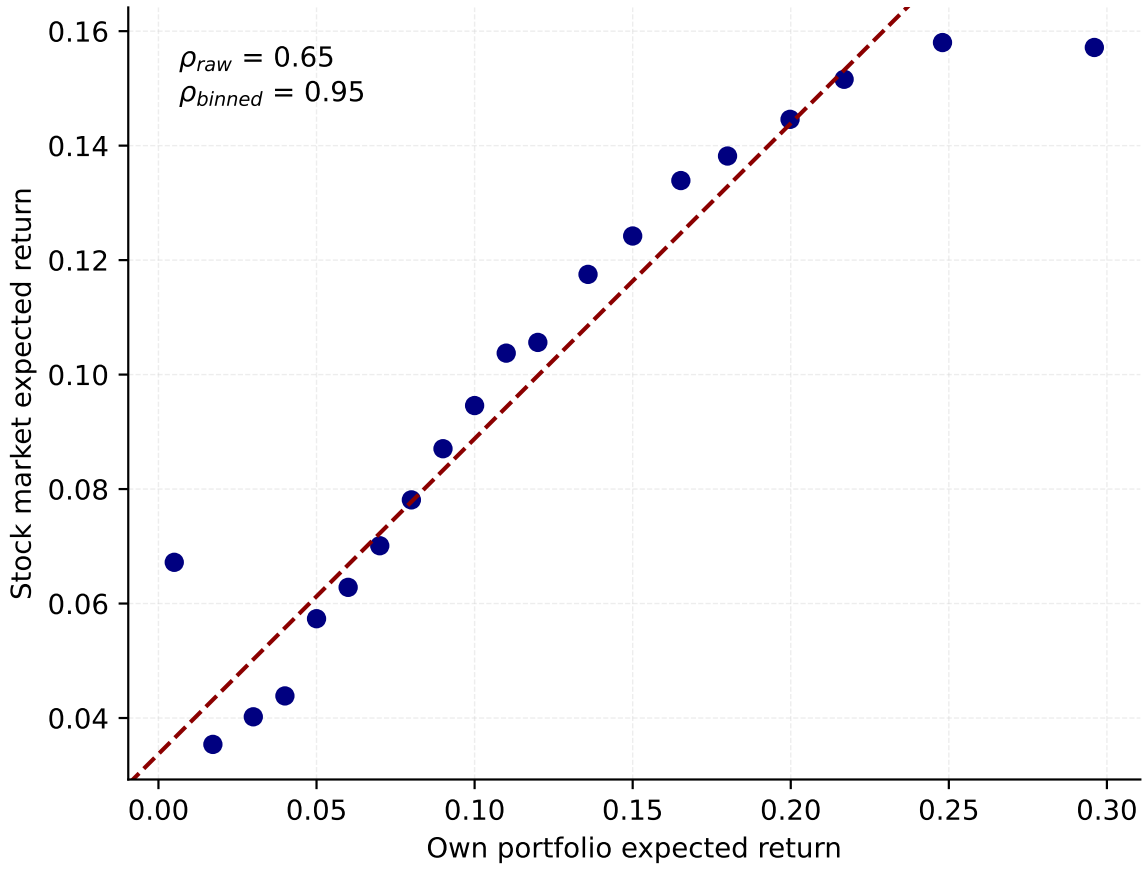
* Let $m^i_t \equiv \mathbb{E}^i_{t-1}[M^i_t]$ denote household i's estimate of the latent state.
* A steady-state Kalman filter gives the constant-gain updating rule

$$m^i_{t+1} = (1 - g_L) m^i_t + g_L \left[\kappa_L + \mu_L \log R^{A,i}_t + (1 - \mu_L) \log R^S_t \right]$$

High portfolio return $\Longrightarrow$ optimism $\Longrightarrow$ higher risky share $\Longrightarrow$ higher future portfolio returns

Learning from own portfolio returns

* UBS Gallup survey contain own portfolio and market expected returns.
* Past own portfolio returns predicts expected market returns.



Learning from own portfolio return generates the distribution of beliefs across wealth groups

	Top 1%		Next 9%		Middle 40%		Bottom 50%	
	Model	**Data**	**Model**	**Data**	**Model**	**Data**	**Model**	**Data**
Wealth share	40.22	37.60	33.94	38.10	17.18	22.80	8.65	1.40
Income share	18.35	18.40	25.86	24.70	33.57	35.20	22.22	21.60
Wage share	9.58	12.30	23.80	21.60	39.55	38.50	27.07	27.50
Risky asset share	89.32	77.30	70.11	62.30	33.53	39.60	13.97	18.10
Expected stock return	8.60	7.88	8.14	6.78	6.72	5.39	5.09	4.77
Portfolio return	12.09	8.30	8.97	6.12	4.72	3.63	2.53	3.40

Aggregates

6. Conclusions

Main results

1. Key facts from microdata:
   * Persistent disagreement.
   * Strong association (expected returns, wealth).

2. Accuracy maximizes expected wealth growth, but over-optimism maximizes the Pareto tail.

3. Internal Rationality makes HA models with aggregate risk
   * Consistent with the microdata.
   * Easier to solve.

4. Heterogeneous Expectations = amplifiers of ex-ante inequality, helping to explain the extreme concentration of wealth at the top.

Appendix

What explains Wealth Inequality?

* **Labour income inequality:** insufficient to explain wealth inequality at the top De Nardi and Fella (2017)
* **Bequests:** De Nardi (2004)
* **Preference heterogeneity**: discount rate (Krusell and Smith (1998)), risk-aversion (Fernandez-Villaverde and Levintal (2024))
* **Earning risk:** helps in matching bottom 60% De Nardi et al. (2016)
* **Heterogeneous capital/wealth returns:** Gabaix et al. (2016), Hubmer et al. (2020) , Xavier (2021)
* **Entrepreneurship:** Quadrini (1999, 2000), Cagetti and De Nardi (2006)
* **Decline in tax progressivity** Hubmer et al. (2020)

Back

RAND Expectations Questions

1. *By next year at this time, what are the chances that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be* **worth more** *than they are today?*

2. *By next year at this time, what are the chances that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have* **increased** *in value by* **more than 20 percent** *compared to what they are worth today?*

3. *By next year at this time, what are the chances that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have* **fallen** *in value by* **more than 20 percent** *compared to what they are worth today?*

A typical answer would be 1. 65%, 2. 15%, 10%.
From it, we can back out:
$P(1 < R < 1.2) = 65\% - 15\% = 50\%$;
$P(0.8 < R < 1) = (100\% - 65\%) - 10\% = 25\%$.
Internally inconsistent (negative probabilities) observations are dropped.

Back

Features of measured expectations

1. Extrapolation: $\underbrace{R_{t+k} - \mathbb{E}_t^{\mathcal{S}^i}(R_{t+k})}_{\text{Individual forecast error}} = \alpha^i + b \underbrace{R_t}_{\text{Current market return}} + \varepsilon_t^i$
2. Reaction to news: $R_{t+k} - \mathbb{E}_t^{\mathcal{S}^i}(R_{t+k}) = \alpha^i + d \underbrace{(\bar{\mathbb{E}}_t^{\mathcal{S}}(R_{t+k}) - \bar{\mathbb{E}}_{t-1}^{\mathcal{S}}(R_{t+k}))}_{\text{Average Forecast revision}} + \varepsilon_t^i$

   FIRE null hypothesis: $b = d = 0$.
3. Persistent disagreement: $\mathbb{E}_t^{\mathcal{S}^i}(R_{t+k}) = \alpha^i + \mu_t + v_t^i$

	1. Forecast Error Predictability		
Individual Forecast Error	(1)	(2)	(3)
R_t	-0.13***	-	-0.14***
$\bar{\mathbb{E}}_t^{\mathcal{S}}(R_{t+k}) - \bar{\mathbb{E}}_{t-1}^{\mathcal{S}}(R_{t+k})$	-	-0.50***	-0.51***
Obs.	64749	64749	64749
	2. Persistent Disagreement		
Fixed Effects	μ_t	α^i	μ_t, α^i
R^2	0.01	0.50	0.52

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Regression specification

For each return bin j, estimate a separate fractional logit:

$$p_{ijt} = \Lambda_j\left(\alpha_j + X_{it}'\beta_j + \delta_t\right)$$

* Dependent variables: subjective probabilities assigned to four bins:

$$R < -20\%, \quad -20\% < R < 0\%, \quad 0\% < R < 20\%, \quad R > 20\%.$$

* Estimation: GLM, binomial family, logit link.
* Sample: 12,025 household-wave observations.
* Interpretation: coefficients are not percentage-point effects; they measure changes in log-odds.

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Matching Expectations and Portfolios

$$\mathbb{E}_t^{\mathcal{S}^i}[R_{t+1}] = \alpha + (\boldsymbol{Z}_t^i)'\boldsymbol{\beta} + \epsilon_t^i$$

Predictor	Expected Returns	Forecast Error
Constant	-8.17	6.3^{***}
Owns home?	-2.1	0.7
Has Retirement Account?	1.66	-0.27
Retirement Account (value)	0.4^{***}	-0.1^{***}
Stock Share in RA	0.04^{***}	-0.01^{***}
Home Value	0.07	-0.02
Wage Income	0.5	-0.21
Age	0.002	-0.01

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2 Stage Deep Learning algorithm

1. **Stage 1: Solve household policies under subjective beliefs** Graph
    * Train a policy network for the saving rate and risky portfolio share:

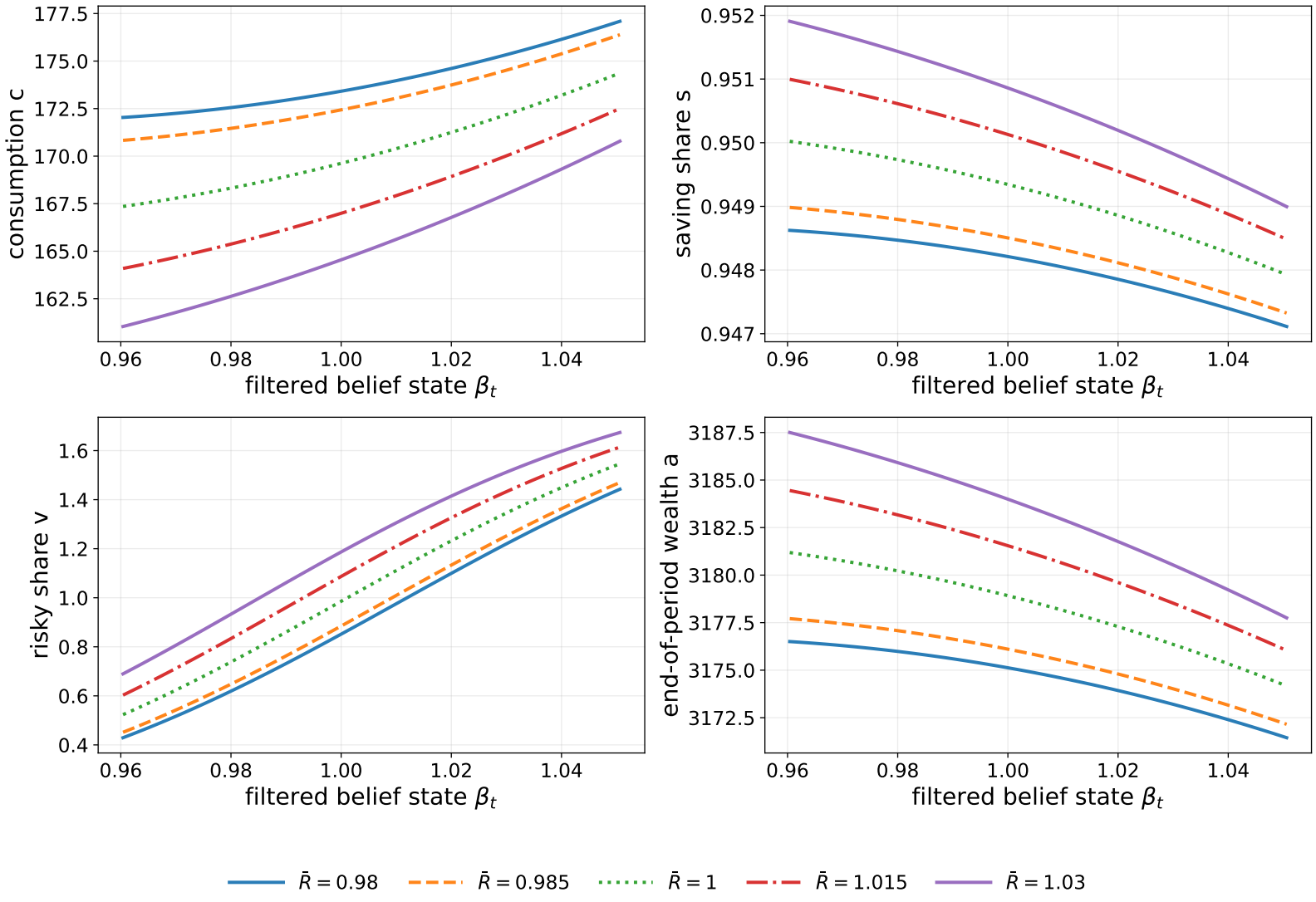
$$(\kappa_t^i, v_t^i) = \mathcal{N}_\theta\left(\boldsymbol{x}_t^i\right)$$

    * Initialize the network at the analytical Merton–Samuelson policy; the network learns a nonlinear correction around it.
    * Generate training states by simulating histories under households' subjective measure, without imposing market clearing.
    * Enrich the simulated histories with off-path collocation states, placing additional mass near the saving constraint.
    * Minimize the Euler–KKT loss: Fischer–Burmeister complementarity residuals for $a_t^i \geq 0$ and $0 \leq v_t^i \leq \bar{v}$, with a penalty for infeasible current or future resources.

2. **Stage 2: General Equilibrium**
    * Simulate thousands of households with agent-specific exogenous realizations (wages, belief anchors).
    * Use the trained policy network inside the full GE simulation.
    * At each date, solve equilibrium prices from market clearing.
    * Update household states and repeat along the simulated GE path.

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A look into the policy functions



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Aggregate Processes and Asset Returns

* The risky Lucas-tree dividend is the numeraire. Dividend growth is lognormal:

$$g_{t+1} \equiv \frac{D_{t+1}}{D_t} = \bar{g}_D \exp\left(-\frac{1}{2}\sigma_D^2 + \sigma_D \varepsilon_{t+1}^D\right), \qquad \varepsilon_{t+1}^D \sim \mathcal{N}(0,1).$$

* Calibration targets: annual mean dividend growth 2.84% and annual volatility 6.4%.
* The risky return and safe return realized in the GE simulation are

$$R_{t+1}^S = g_{t+1}\frac{pd_{t+1}+1}{pd_t}, \qquad R_{t+1}^B = \frac{1}{Q_t}.$$

* Aggregate return anchors: mean real stock return 9.29% and mean real safe rate 0.75% per year.

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Idiosyncratic Wages: Two-State Process

* Wage persistence follows Fernández-Villaverde and Levintal, 2024; the two wage levels are disciplined by 2016 SCF wage-income shares.
* Every household faces the same transitory two-state wage process:

$$w_t^i = \bar{w}\,\varepsilon(z_t^i), \qquad z_t^i \in \{L, H\}.$$

  There is no permanent household-specific wage type.
* Annual wage-state transition matrix:

$$P^A = \begin{pmatrix} 0.9972 & 0.0028 \\ 0.0252 & 0.9748 \end{pmatrix}, \qquad \pi = (0.9, 0.1).$$

* The quarterly model uses the exact stochastic fourth root:

$$P^Q \simeq \begin{pmatrix} 0.999293 & 0.000707 \\ 0.006367 & 0.993633 \end{pmatrix}.$$

* Mean-one wage levels implied by the SCF shares are

$$\varepsilon_L = \frac{0.661}{0.9} = 0.7344, \qquad \varepsilon_H = \frac{0.339}{0.1} = 3.39.$$

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Six Permanent Belief–Preference Types

* Each household draws a permanent ex-ante type $\tau_i \in \{1, \ldots, 6\}$, which determines

$$\tau_i \longmapsto \left(\mathbb{E}^i[R^S],\, \delta_i,\, \gamma_i \right).$$

* The wage state is a separate transitory state:

$$z_t^i \in \{L, H\}, \qquad z_{t+1}^i \sim P^Q(z_t^i, \cdot),$$

  and is independent of the permanent belief–preference type.

* Hence the full idiosyncratic state is (τ_i, z_t^i): six permanent types crossed with two wage states. Only z_t^i changes over time.

* The six-type support is selected to reproduce the empirical distribution of beliefs and its association with risk tolerance. The wage distribution is calibrated separately through the two-state process.

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Six-Type Support Used in the Baseline

Type	Mass	$\mathbb{E}^i[R^S] - 1$	δ_i	γ_i
1	39.1%	4.77%	0.952	13.00
2	12.2%	5.39%	0.973	1.75
3	22.0%	5.80%	0.956	4.25
4	16.7%	6.78%	0.951	9.50
5	9.0%	7.50%	0.954	6.25
6	1.0%	7.88%	0.975	2.10

* Expected returns are annualized subjective stock-return means.
* The permanent type assigns beliefs, patience, and risk aversion—not wages.
* Every type faces the same low and high wage states and the same transition matrix.

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Stage-2 Targets and Diagnostics

* Aggregate targets:

Capital-income share	27.0%	Wealth-income ratio	491%
Mean risky share	30.5%	Wealth-weighted risky share	70.8%
Saving rate	6.7%	Mean stock return	9.29%
Mean safe rate	0.75%	Wage-income share	diagnostic

* Distributional targets by wealth group: wealth, income, wages, capital-income shares, risky shares, expected returns, and portfolio returns.
* Forecast errors and the alternative risky-share definition based on net financial wealth are left as diagnostics.
* Final refinement holds $\bar{w}$ fixed and adjusts only B in a longer simulation to hit the annual safe-rate target $0.75\% \pm 0.20$ pp.

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Historical transitional dynamics: Distributions

	Model: Heterogeneous Beliefs				Model: Homogeneous Beliefs			
By wealth percentile	**Top 1%**	**Next 9%**	**Mid 40%**	**Bot 50%**	**Top 1%**	**Next 9%**	**Mid 40%**	**Bot 50%**
Wealth	22.18	32.88	36.68	8.25	14.74	28.51	43.76	13.00
Wages	7.08	18.35	37.03	37.55	6.63	18.35	37.14	37.88
Risky asset share	86.79	74.68	60.52	41.94	65.48	60.56	55.08	50.84
Expected return	8.35	6.63	5.42	4.40	5.04	5.04	5.04	5.04
Forecast error	-1.07	0.65	1.86	2.88	2.25	2.25	2.25	2.25
Portfolio return	7.46	6.43	5.14	2.16	5.39	5.05	4.60	2.26

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Evidence consistent with learning from own returns

$$\ln \mathbb{E}_t^i(R_{t+1}^m) = \alpha_m + \gamma_m \ln R_{t-1}^i + \delta_m \ln \mathbb{E}_t^i(R_{t+1}^i) + X_t^{i\prime} \lambda_m + u_{m,t}^i$$

$$H_0 : \gamma_m = 0 \qquad \text{vs.} \qquad H_1 : \gamma_m > 0$$

	(1)	(2)	(3)	(4)
	$\ln \mathbb{E}_t^i(R_{t+1}^m)$			
$\ln(R_{t-1}^i)$	0.1304***	0.1206***	0.3834***	0.3852***
	(0.025)	(0.025)	(0.007)	(0.008)
$\ln \mathbb{E}_t^i(R_{t+1}^i)$	0.6183***	0.6152***	0.6166***	0.6148***
	(0.007)	(0.007)	(0.007)	(0.008)
Controls	–	✓	–	✓
Restriction $\delta_m = 1 - \gamma_m$	–	–	✓	✓
R^2	0.438	0.441	0.201	0.203
Obs.	26,607	26,607	26,607	26,607

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Aggregate variables in the model with learning from own portfolio returns

Moment	Model	Data
Household aggregates		
Capital-income share (%)	19.78	27.00
Wage-income share (%)	80.22	73.00
Wealth-to-income ratio (%)	236.42	491.00
Average risky asset share (%)	25.35	30.50
Risky assets / aggregate wealth (%)	69.40	70.80
Saving rate (%)	5.53	6.70
Aggregate returns (annual, %)		
Mean stock return	12.77	9.29
Mean safe rate	1.22	0.75
Mean dividend growth	2.77	2.84
SD dividend growth	6.32	6.40

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