Two Sides of the Same Coin: Expected Reaction Functions and Perceived Monetary Policy Shocks

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Abstract

To identify the causal effect of monetary policy, researchers often use either high-frequency or narrative monetary policy shock measures as instruments. By their nature, these instruments tend towards being either weak or endogenous, respectively. This paper proposes a third approach. First, we develop a framework that specifies the links between true and perceived FOMC reaction functions. Second, we use this framework to develop an econometric model to estimate the perceived FOMC reaction functions implied by monetary policy expectations. This allows us to extract both expected reaction function coefficient estimates and two perceived monetary policy expectations shock measures, which we show to be both relevant and exogenous instruments. Finally, using our two instruments, we demonstrate that the impulse response function estimates are consistent with New Keynesian theory.

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1 Introduction

Monetary policy shock measures are an indispensable tool used by macroeconometricians to identify the causal effect of monetary policy on the economy. The idea is to construct a relevant monetary policy shock measure to be used as an exogenous instrument for an endogenous measure of monetary policy. If successful, these instruments can facilitate valid estimation of impulse response functions that trace out the dynamic effects of monetary policy on macroeconomic aggregates.

The most popular approach to constructing such monetary policy shock measures is the high-frequency approach: to construct a monetary shock measure based on the changes in prices of futures contracts tied to the policy rate during short time windows that encompass monetary policy events, during which time the only (or main) plausible cause of those changes is monetary news. The approach was introduced by Kuttner (2001), but has been refined ever since (e.g. Gürkaynak, Sack, and Swanson (2005), Nakamura and Steinsson (2018)). Perhaps the current state-of-the-art measures were introduced by Miranda-Agrippino and Ricco (2021) and Bauer and Swanson (2023).

Another popular approach is the narrative approach, which began by using narrative evidence from FOMC meeting transcripts and the Fed's real-time Greenbook forecasts to construct a series of intended federal funds rate changes, then regressing these intentions on the Fed's forecasts to remove systematic policy responses (Romer and Romer (2004)). This approach has also been updated and refined ever since, with perhaps the current state-of-the-art measure introduced by Aruoba and Drechsel (2025).

By their nature, high-frequency shock measures will tend to be weak instruments because they capture only a small fraction of the updates to market participants' interest rate expectations (see Bianchi, Ludvigson, and Ma (2025) for related work). Conversely, narrative measures will tend to be endogenous because, although they capture a larger fraction of such changes, they may struggle to completely disentangle the endogenous and exogenous components. Further, to the extent that narrative measures focus on the federal funds rate, they may not be an ideal summary statistic for the stance of monetary policy, which is better captured by a term structure of interest rate expectations, not just one point.

So, this paper proposes a complementary approach (ideally we would utilize all three approaches) to estimating monetary policy shock measures that attempts to balance relevance and exogeneity. We develop a framework designed for the structural estimation of the perceived FOMC reaction functions implied by monetary policy expectations. The framework allows us to isolate separate information, macroeconomic response, forward guidance, smoothing/gradualism, and persistent judgment channels (where smoothing/gradualism and persistent judgment each contribute to monetary policy *inertia*). Private forecasts are likely to have been constructed using simple rules and heuristics (unlike the projections formed by central banks), which makes disentangling these channels more straightforward. We then extract the residuals from the expected reaction function estimates, and show that they have statistical properties consistent with valid econometric instruments.

This paper builds on Bauer, Pflueger, and Sunderam (2024), which also estimates the time-varying, perceived FOMC reaction functions implied by monetary policy expectations. By contrast, they do not extract monetary shocks. The authors argue that their error term "*contains forecaster j's expectation of the future monetary policy shock at t+h*," "*as well as possible measurement and specification errors affecting the funds rate forecasts*." This paper proposes strategies to manage these measurement and specification errors. First, our proposed central bank reaction function framework (§A) considers all forms of heterogeneity: the reaction function parameters vary by forecaster, by country, by forecast hori-

zon, and by time. As in Bauer, Pflueger, and Sunderam (2024), we allow for time-varying parameters using rolling windows. Then, we go further by estimating forecaster-expected, forward-looking reaction functions forecaster-by-forecaster and horizon-by-horizon. We also define gradualism/smoothing and central bank judgment channels, which allows us to both consider concepts such as forward guidance, and to account for the persistent nature of the residuals.

Based on this framework, we propose an econometric model (§2) that is designed to eliminate the many potential sources of estimation bias outlined in the reaction function framework: common latent factors, ensuing cross-sectional (forecaster) dependence, and failure to account for parameter heterogeneity. For instance, using Pesaran (2015) test statistics, we detect strong dependence between forecasters, precluding independent and identically distributed error terms. Failure to account for cross-sectional dependence causes the error term to be autocorrelated and the expected reaction function coefficient estimates to be biased. So, we utilize the Chudik and Pesaran (2015) dynamic panel estimator, which accounts for cross-sectional dependence. We parse the data by forecaster, country, time, and forecast horizon to account for multiples sources of documented forecast data heterogeneity (Patton and Timmermann (2010)). We argue our econometric approach allows us to treat the residuals as shocks to monetary policy expectations.

We apply this econometric model to the *Consensus Economics* U.S. forecast data to estimate forecaster-expected, forward-looking FOMC reaction functions and their implied monetary policy rate path shocks. Our method could be adapted to many other countries, not least because *Consensus Economics* provides standardized forecast data for over 100 countries.

We make a number of contributions to the empirical monetary economics literature. First, we provide estimates of the perceived FOMC reaction functions implied by monetary policy expectations purged of several sources of bias that had not yet been explored. Second, we provide estimates of shocks to monetary policy expectations at the three-month and twelve-month forecast horizons, which conceptually, are related to the target and path factors from Gürkaynak, Sack, and Swanson (2005). Crucially, unlike the high-frequency or narrative monetary shock measures available to date, at least, our shock measures are not weak instruments.

Third, we provide a framework that demonstrates that the reaction function estimates and the perceived policy rate path shocks are two sides of the same coin. Having both allows us to disentangle movements in interest rate forecasts into changes in: macroeconomic forecasts (inflation, growth, unemployment), expected responses to macroeconomic forecasts (coefficient estimates), expected smoothing, expected judgment (e.g. due to forward guidance), and finally, expected monetary policy rate path shocks.

Fourth, we argue that macroeconometricians should use Montiel Olea and Pflueger (2013) or Lewis and Mertens (2025) weak instruments tests when using monetary policy instruments (as Eberly, Stock, and Wright (2020) do). Doing so, we reject the hypothesis that our instruments are weak instruments at reasonable bias levels, while we cannot do the same for either high-frequency or narrative instruments (as alluded to in Nakamura and Steinsson (2018)). We also do not reject the hypothesis that our instruments are exogenous. Interestingly, while the residuals from the Bauer, Pflueger, and Sunderam (2024) regressions cannot be treated as shocks a priori, we find evidence that they are nonetheless a valid monetary policy instrument.

Finally, we use our monetary policy instruments to estimate impulse response functions based on the approach of Gertler and Karadi (2015). Our estimates are broadly consistent with New Keynesian theory,

as are those formed using the Aruoba and Drechsel (2025) shocks, and interestingly, as are those formed using Bauer, Pflueger, and Sunderam (2024) residuals.

The paper is structured as follows. Section (§) 2 presents the econometric model. §3 presents expected reaction function and perceived monetary policy shock estimates. §4 discusses instrument validity, and compares impulse response functions constructed using our instrument to those from the literature. §5 concludes. §A details the reaction function framework. We use the $\mathcal{A}$ symbol throughout to denote areas for which further details can be found in the online $\mathcal{A}$ppendix, which can be accessed by clicking the $\mathcal{A}$.

2 Econometric Model

2.1 Central Bank Reaction Function Framework

The foundation of the econometric model is a central bank reaction function framework, which is derived more formally in Appendix A, §A.1. Here, we present a less formal summary. First, we begin with a general specification of a *Federal Open Market Committee* (FOMC) reaction function, using this to develop our estimator (Eq. 44):

$$\underbrace{\tilde{r}_t}_{\substack{\text{Fed Funds}\\ \text{Rate}}} = \underbrace{\tilde{\rho}_t \tilde{r}_{t-\ell}}_{\substack{\text{Monetary policy}\\ \text{gradualism}}} + \underbrace{\tilde{\phi}_t^{r^*} r_t^*}_{\substack{\text{r-star}}} + \underbrace{\tilde{\phi}_t^{\pi^*} \pi_t^*}_{\substack{\text{Inflation}\\ \text{target}}} + \underbrace{\left(\tilde{\mathbb{E}}_{t^-}\left[\mathbf{x_{t+h_x}} - \mathbf{x^*_{t+h_x}} \;\middle|\; \tilde{\Omega}_{t^-}\right]\right)'}_{\substack{\text{Macroeconomic}\\ \text{projections}}} \underbrace{\tilde{\beta}_{\mathbf{t}}}_{\substack{\text{Macro}\\ \text{response}}} + \underbrace{\tilde{\varepsilon}_t}_{\substack{\text{Monetary policy}\\ \text{shock}}} \tag{1}$$

Notation. $\sim$ denotes objects attributable to The FOMC. t denotes the time period. ℓ denotes lag length. $*$ denotes long-run equilibrium variables. t^- is some time shortly before the monetary policy shock, $\tilde{\varepsilon}_t$, is realized at t. t^+ is some time shortly after the shock.

Gradualism. Following Clarida, Gali, and Gertler (2000), Woodford (2003), and particularly Coibion and Gorodnichenko (2012), we take the stance throughout this paper that our monetary policy rule should allow for gradualism (smoothing, partial adjustment) i.e. a lagged dependent variable $\tilde{\rho}_t \tilde{r}_{t-\ell}$, where we assume (but do not impose) that $|\tilde{\rho}_t| < 1$. As such, $\tilde{\phi}_t^{\pi^*}, \tilde{\phi}_t^{r^*}, \tilde{\beta}_{\mathbf{t}} \propto (1 - \tilde{\rho}_t)$, so estimates of $\tilde{\phi}_t^{\pi^*}$, $\tilde{\phi}_t^{r^*}$, and $\tilde{\beta}_{\mathbf{t}}$ are implicitly scaled by $(1 - \tilde{\rho}_t)$.

Variables. π_t^* denotes the inflation target. r_t^* denotes the natural rate of interest. $\mathbf{x_t}$ is a k x 1 vector of macroeconomic variables—the inflation rate, real GDP growth rate, and unemployment rate ($\mathcal{A}$). $\mathbf{x_t^*}$ is a matching vector that denotes the long-run equilibrium rates of inflation, real GDP growth, and the unemployment rate, respectively. h_x is a forecast horizon that determines how forward-looking the reaction function is.

Parameters. $\tilde{\beta}_{\mathbf{t}}$ is the FOMC's time-varying response to its macroeconomic projections. The usual assumption of constant parameters is too restrictive, so unlike in a standard specification, we remove this constraint, allowing for a more general reaction function. $\tilde{\varepsilon}_t$ is a traditional monetary policy shock, which is orthogonal to The FOMC's information set at t^-, $\tilde{\Omega}_{t^-}$, but is $\tilde{\Omega}_t$–measurable (at time t).[1]

Yet, at each FOMC meeting, members do not decide only on the Fed Funds Rate (FFR), but also on *a set*

[1] That is, $\tilde{\varepsilon}_t \perp\!\!\!\perp \tilde{\Omega}_{t^-}$ and $\tilde{\mathbb{E}}_{t^-}\left[\; \tilde{\varepsilon}_t \;\middle|\; \tilde{\Omega}_{t^-} \;\right] = 0$ (i.e. unpredictability of structural shocks).

of FFR projections—one for each projection horizon, h.[2] The FOMC updates its policy rate and policy rate projections in a recursive manner, which is detailed in Appendix A. Shifting Equation 1 forward h periods, and then taking the conditional expectation at time t:

$$\underbrace{\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h} \;\middle|\; \tilde{\Omega}_t\right]}_{\substack{\text{FOMC Fed Funds}\\ \text{Rate projection}}} = \tilde{\mathbb{E}}_t\left[\tilde{\rho}_{t+h}\tilde{r}_{t+h-\ell} + \tilde{\phi}^{r^*}_{t+h} r^*_{t+h} + \tilde{\phi}^{\pi^*}_{t+h}\pi^*_{t+h} + \left(\tilde{\mathbb{E}}_{t^-+h}\left[\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}} \;\middle|\; \tilde{\Omega}_{t^-+h}\right]\right)' \tilde{\beta}_{\mathbf{t+h}} \;\middle|\; \underbrace{\tilde{\Omega}_t}_{\substack{\text{Information}\\ \text{set (FOMC)}}}\right] \tag{2}$$

where $\tilde{\mathbb{E}}_t\left[\tilde{\varepsilon}_{t+h} \;\middle|\; \tilde{\Omega}_t\right] = 0$, and so drops out, and where $H = h_x + h$. Finally, observe that FOMC members are free to, and often do, add judgment to their FFR projections:

$$\underbrace{\tilde{\mathbb{E}}_{t^+}\left[\tilde{r}_{t+h}\right]}_{\substack{\text{FOMC Fed Funds}\\ \text{Rate projection}}} = \tilde{\mathbb{E}}_t\left[\tilde{\rho}_{t+h}\tilde{r}_{t+h-\ell} + \tilde{\phi}^{r^*}_{t+h} r^*_{t+h} + \tilde{\phi}^{\pi^*}_{t+h}\pi^*_{t+h} + \left(\tilde{\mathbb{E}}_{t^-+h}\left[\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right]\right)' \tilde{\beta}_{\mathbf{t+h}}\right] + \underbrace{\tilde{\lambda}^h_{t^+}}_{\substack{\text{FOMC}\\ \text{judgment}}} \tag{3}$$

where, henceforth, we suppress the conditioning sets for brevity only. $\tilde{\lambda}^h_{t^+}$ is known when the FFR projection is finalized (or, $\tilde{\lambda}^h_{t^+}$ is $\tilde{\Omega}_{t^+}$–measurable). This is important because we embed **a monetary policy expectations shock**, $\tilde{\varepsilon}^h_{t^+}$, in judgment ($\tilde{\varepsilon}^h_{t^+}$ is also $\tilde{\Omega}_{t^+}$–measurable). We postulate that judgment is persistent and takes the following form:

Assumption 1. ***FOMC-expected judgment***, *$\tilde{\lambda}^h_t$, evolves according to the following process:*

$$\tilde{\lambda}^h_t = \tilde{\theta}_t \tilde{\lambda}^h_{t-\ell} + \tilde{\varepsilon}^h_t \qquad \tilde{\varepsilon}^h_t \sim i.i.d.\left(0, \sigma^2_{\tilde{\varepsilon}_t}\right) \tag{4}$$

where $\tilde{\varepsilon}^h_t$ is a shock to expected judgment, and can equivalently be interpreted as a monetary policy expectations shock. It is assumed that $|\tilde{\theta}_t| < 1$, but this is not imposed. The AR(1) structure captures the idea that judgment that is persistent over time (due to, for example, ongoing forward guidance).[3] If new judgment enters through $\tilde{\varepsilon}^h_t$, $\tilde{\lambda}^h_{t-1}$ allows us to capture its persistent-yet-decaying nature:

$$\tilde{\lambda}^h_t = \tilde{\varepsilon}^h_t + \tilde{\theta}_t\tilde{\varepsilon}^h_{t-1} + \tilde{\theta}_t\tilde{\theta}_{t-1}\tilde{\varepsilon}^h_{t-2} + \tilde{\theta}_t\tilde{\theta}_{t-1}\tilde{\theta}_{t-2}\tilde{\varepsilon}^h_{t-3} + \cdots \tag{5}$$

What is FOMC-expected judgment? $\tilde{\lambda}^h_t$ represents other influences on the FFR projection not captured by the systematic component of the reaction function:

1. **Timing/rounding**. The FFR changes in (sometimes multiple) 25-basis-point increments, but a reaction function output will not neatly coincide with 25-basis point increments from zero. This can be conceptualised as timing/rounding choice. For example, if FOMC projections suggest that the target range for the FFR should be 3.375% to 3.625% (where there prevailing range is 3.5% to 3.75%), then the FOMC must decide whether to round up (hold the FFR now, and likely cut at the next meeting), or to round down (cut at the current meeting and likely hold at the next meeting). This shows up as positive or negative judgment, respectively, in the $\tilde{\lambda}^h_t$ term.

[2] González-Astudillo and Tanvir (2023) also use time-varying, forward-looking FOMC reaction functions.

[3] Campbell, Evans, Fisher, Justiniano, Calomiris, and Woodford (2012) capture a variant of this idea when they describe the central bank reaction function error as the sum of a contemporaneous policy rate shock and a series of past, predictable FG shocks (Eq.1, p.22). Miranda-Agrippino and Ricco (2021) also capture a similar idea (Eq.3, p.79): "*Also, the central bank can announce—or leak—at time $t-1$ a deviation from the monetary policy rule, $w_{t|t-1}$, that takes places at time t. This announced deviation—that can be thought of as due to either implicit or explicit forward guidance—creates a wedge between monetary policy rule residuals and monetary policy shocks*."

2. **Forward guidance** from the past and/or newly introduced.[4]

3. **Risk management** due to elevated projection uncertainty (e.g. anticipated insurance rate cuts due to asymmetric risks to the outlook).

4. **Model uncertainty** (e.g. about the monetary policy rule itself, or the r-star , etc.).

5. **Dissent** to make a point (to the market and/or to other FOMC participants).

6. **Persuasion** by judgment expressed in The Tealbook, staff presentations, or other FOMC participants during deliberations.

7. **An unanticipated event** not reflected in the macroeconomic projections or macro response ($\tilde{\beta}_{\mathbf{t}}$).

Given the sources of judgment, we hypothesize that the law of motion for judgment is a persistent process (Eq. 4). $\tilde{\lambda}^h_t$ is also intended to account for the role of persistent shocks in monetary policy rules discussed in Rudebusch (2002) and Coibion and Gorodnichenko (2012), and the serial correlation in shock measures discussed in Miranda-Agrippino and Ricco (2021) and Bauer and Swanson (2023).

2.2 The Expected Central Bank Reaction Function

Second, at all times t other than the four FOMC meetings per year at which projections are released, agents must rely on their *perception* of the FOMC's FFR projection, that is, how the projection may have changed in response to data and events. Because a FFR path projection is intrinsically forward-looking, it implies the existence of an *expected* reaction function. To derive this, we take expectations of Eq. 3 with respect to the information set of forecaster i at time t.

$$\ddot{\mathbb{E}}_{it^+}\left[\tilde{\mathbb{E}}_{t^+}\left[\tilde{r}_{t+h}\right]\right] = \ddot{\mathbb{E}}_{it^+}\left[\tilde{\mathbb{E}}_t\left[\tilde{\rho}_{t+h}\tilde{r}_{t+h-\ell} + \tilde{\phi}^{r^*}_{t+h}r^*_{t+h} + \tilde{\phi}^{\pi^*}_{t+h}\pi^*_{t+h} + \left(\tilde{\mathbb{E}}_{t^-+h}\left[\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right]\right)'\tilde{\beta}_{\mathbf{t+h}}\right] + \tilde{\lambda}^h_{t^+}\right] \quad (6)$$

where $\ddot{\mathbb{E}}_{it^+}\left[. \mid \ddot{\Omega}_{it^+}\right]$ is forecaster i's ($i = 1, ..., N$) expectation conditional on their information set, $\ddot{\Omega}_{it^+}$ at time t^+. We assume that both the FOMC and forecasters know their own expected coefficients ($\tilde{\Omega}_{t^+}$–measurable and $\ddot{\Omega}_{it^+}$–measurable, respectively—Assumption 10 in §A.2). This assumption implies that we can move those coefficients outside the the conditional expectations. Once outside, we denote those coefficients as, for example, ρ^h_{it}. The analogous definitions apply for all other coefficients in Eq. 8 below. Further, we denote:

$$\ddot{\mathbb{E}}_{it^+}\left[\tilde{\lambda}^h_{t^+}\right] := \lambda^h_{it^+} = \theta_{it^+}\lambda^h_{i,t^+-\ell} + \varepsilon^h_{it^+} \quad (7)$$

This is **forecaster-expected FOMC judgment**, which is the forecaster's expectation of FOMC-expected judgment (a more precise and technical definition can be found in §A.2, Equations 74 to 84). Finally, for notational simplicity, moving forward, we abstract from the notation for slight timing differences in expectation formation (t^-, t, t^+) without loss of generality. So, we can rewrite Eq. 6 as follows:[5]

$$\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h}\right]\right] = \rho^h_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-\ell}\right]\right] + \phi^{r^*}_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[r^*_{t+h}\right]\right] + \phi^{\pi^*}_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\pi^*_{t+h}\right]\right] + \ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)'\right]\right]\ddot{\beta}^{\mathbf{h}}_{\mathbf{it}} + \lambda^h_{it} \quad (8)$$

[4]The following forward guidance may contain this type of judgment "*In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.*" (source).

[5]In general, if $\mathcal{H} \subseteq \mathcal{G}$ (filtrations), and Z is integrable $\Rightarrow \mathbb{E}\left[\mathbb{E}\left[Z \mid \mathcal{G}\right] \mid \mathcal{H}\right] = \mathbb{E}\left[Z \mid \mathcal{H}\right]$, by the Law of Iterated Expectations. Since we can assume $\tilde{\Omega}_t \subseteq \tilde{\Omega}_{t^-+h}$ (filtrations) $\Rightarrow$ $\tilde{\mathbb{E}}_t\left[\tilde{\mathbb{E}}_{t^-+h}\left[Z\right]\right] = \tilde{\mathbb{E}}_t\left[Z\right]$. Then, if we are willing to assume that $\tilde{\beta}_{\mathbf{t+h}}$ is $\tilde{\Omega}_t$–measurable (deterministic conditional on $\tilde{\Omega}_t$, or, that The FOMC knows its own macro responsiveness), then it follows that $\ddot{\mathbb{E}}_{it^+}\left[\tilde{\mathbb{E}}_t\left[\left(\tilde{\mathbb{E}}_{t^-+h}\left[\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right]\right)'\tilde{\beta}_{t+h}\right]\right] = \ddot{\mathbb{E}}_{it^+}\left[\left(\tilde{\mathbb{E}}_t\left[\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right]\right)'\tilde{\beta}_{t+h}\right]$.

2.3 Mapping The Expected Reaction Function to the Data

Third, we take the forecaster-expected reaction function (Eq. 8), the parameters of which we seek to estimate, to the monthly (t) professional forecast panel data. So, henceforth, let t refer to monthly forecasting periods and set ℓ=1. Here, we use the *Consensus Economics* data, but the *Blue Chip Financial Forecasts* data have similar attributes, and are also suited to our framework. We begin by defining the relationship between the unobservable dependent variable from above, $\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h}\right]\right]$, and an observable-yet-noisy measure of the dependent variable, y^h_{it}. y^h_{it} is individual forecaster i's forecast of the risk-free, short-term interest rate (the three-month Treasury bill rate) at time t and forecast horizon h.

$$\underbrace{y^h_{it}}_{\substack{\text{Private interest}\\ \text{rate forecast}}} = \underbrace{\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h}\right]\right]}_{\substack{\text{Expected FOMC}\\ \text{FFR projection}}} + \underbrace{\ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right]}_{\substack{\text{Expected FOMC}\\ \text{FFR projection } \textit{error}}} + \underbrace{\ddot{\mathbb{E}}_{it}\left[\tau_{t+h}\right]}_{\substack{\text{Expected}\\ \text{spread}}} \tag{9}$$

$\ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right]$ is the expected FOMC FFR projection error. If $\ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right] \neq 0$, the forecaster thinks the FOMC is wrong about future policy rates as in Caballero and Simsek (2022). $\ddot{\mathbb{E}}_{it}\left[\tau_{t+h}\right]$ is the expected spread between the three-month Treasury bill rate and the expected FFR. It is an amalgam of the convenience yield, liquidity premium, and short-horizon term premium (although evidence suggests that this component is small at such short forecast horizons, e.g. Schmeling, Schrimpf, and Steffensen (2022)). Next, we substitute Eq. 8 into $\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h}\right]\right]$ in Eq. 9:

$$y^h_{it} = \underbrace{\rho^h_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] + \phi^{r^*}_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[r^*_{t+h}\right]\right] + \phi^{\pi^*}_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\pi^*_{t+h}\right]\right] + \ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)'\right]\right]\ddot{\boldsymbol{\beta}}^{\mathbf{h}}_{\mathbf{it}} + \lambda^h_{it}}_{\text{Expected FOMC FFR projection}} +$$
$$\ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right] + \ddot{\mathbb{E}}_{it}\left[\tau_{t+h}\right] \tag{10}$$

We now need to replace each expectation with *observable* forecast data to facilitate estimation. We can, similar to Eq. 9, define a relationship between the unobservable main independent variables in Eq. 10, $\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)'\right]\right]$, and an observable-yet-noisy measure of the main independent variables, $\mathbf{x^H_{it}}$. $\mathbf{x^H_{it}}$ is a vector of private forecaster i's macroeconomic forecasts (inflation, growth, unemployment rate) at time t and forecast horizon $H = h_x + h$.

$$\underbrace{\mathbf{x^H_{it}}}_{\substack{\text{Private macro}\\ \text{forecasts}}} := \underbrace{\ddot{\mathbb{E}}_{it}\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)\right]}_{\substack{\text{Private macro}\\ \text{forecasts}}} = \underbrace{\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)\right]\right]}_{\substack{\text{Expected FOMC}\\ \text{macro projections}}} + \underbrace{\ddot{\mathbb{E}}_{it}\left[\mathbf{e^x_{t+H}}\right]}_{\substack{\text{Expected FOMC}\\ \text{macro projection } \textit{errors}}} \tag{11}$$

However, $\ddot{\mathbb{E}}_{it}\left[\mathbf{e^x_{t+H}}\right]$ is also unobservable, so we must otherwise isolate $\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)\right]\right]$.

Assumption 2. *Given that both the FOMC and our sample group are sophisticated forecasters forecasting the same common macro aggregates, and that expected FOMC projections are anchored by the publicly-released Summary of Economic Projections every three months (while private forecasts are comparatively unanchored), we can infer that:*

$$span\left\{\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)\right]\right]\right\} \subseteq span\left\{\ddot{\mathbb{E}}_{it}\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)\right]\right\} \tag{12}$$

That is, the private-sector macroeconomic forecasts span those same forecasters' expected FOMC macroeconomic projections (that the space generated by private forecasts contains the space generated by the

perceived FOMC projections). This implies that (where $\mathbf{A^H_{it}}$ is a k x k nonsingular matrix):

$$\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_{t+H}}\right)\right]\right] = \mathbf{A^H_{it}}\mathbf{x^H_{it}} + \upsilon^{\mathbf{H}}_{\mathbf{it}} \quad s.t. \quad \ddot{\mathbb{E}}_{it}\left[\ \upsilon^{\mathbf{H}}_{\mathbf{it}} \mid \mathbf{x^H_{it}}\ \right] = \mathbf{0} \tag{13}$$

Assumption 3. *Third, we treat the inflation target, neutral rate, and T-bill spread in Eq. 10 as common-knowledge Martingales:*

$$\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\pi^*_{t+h}\right]\right] = \ddot{\mathbb{E}}_{it}\left[\pi^*_{t+h}\right] = \tilde{\mathbb{E}}_t\left[\pi^*_{t+h}\right] := \pi^*_t \tag{14}$$

which is the prevailing inflation target, which is observable, and can be substituted directly into Eq. 10. Similarly:

$$\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[r^*_{t+h}\right]\right] = \ddot{\mathbb{E}}_{it}\left[r^*_{t+h}\right] = \tilde{\mathbb{E}}_t\left[r^*_{t+h}\right] := r^*_t \tag{15}$$

where r^*_t could, in principle, be the last-published estimates from The FOMC Summary of Economic Projections (SEP). Yet, our data frequency is monthly, and the SEP data are only available from 2012. So, instead, we use the widely-used Laubach and Williams (2003) r-star estimates (updated versions of which are provided Holston, Laubach, and Williams (2023) and available on the FRBNY website). These are observable, and so can be substituted directly into Eq. 10. Consistent with the literature on monetary policy rules, the following is implied in Eq. 10, $\phi^{\pi^*}_{it} \propto \left(1 - \rho^h_{it}\right)$, $\phi^{r^*}_{it} \propto \left(1 - \rho^h_{it}\right)$. Finally,

$$\ddot{\mathbb{E}}_{it}\left[\tau_{t+h}\right] := \tau_t \tag{16}$$

Our observable measure of τ_t is the spread between the three-month Treasury bill rate and the three-month Overnight Indexed Swap (OIS) rate.

We can now substitute Eq. 13, 14, 15, 16 into Eq. 10, which moves us towards an equation we can estimate with observable data:[6]

$$y^h_{it} = \rho^h_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] + \phi^{r^*}_{it} r^*_t + \phi^{\pi^*}_{it}\pi^*_t + \phi^{\tau}_{it}\tau_t + \mathbf{x^{H\prime}_{it}}\beta^{\mathbf{h}}_{\mathbf{it}} + \ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right] + \upsilon^{\mathbf{H\prime}}_{\mathbf{it}}\ddot{\beta}^{\mathbf{h}}_{\mathbf{it}} + \lambda^h_{it} \tag{17}$$

$$s.t. \qquad \beta^{\mathbf{h}}_{\mathbf{it}} := \mathbf{A^{H\prime}_{it}}\ddot{\beta}^{\mathbf{h}}_{\mathbf{it}} \tag{18}$$

Assumption 4. *Drawing on Chudik and Pesaran (2015), Stock and Watson (2016), and Giacomini, Lu, and Smetanina (2024), the vector of private macroeconomic forecasts, $\mathbf{x^H_{it}}$, can be described by the following linear covariance stationary dynamic heterogeneous panel data model:*

$$\mathbf{x^H_{it}} := \ddot{\mathbb{E}}_{it}\left[\left(\mathbf{x_{t+H}} - \mathbf{x^*_t}\right)\right] = \mathbf{c}_{xi} + \mathbf{\Gamma^H_{it}}\mathbf{f_t} + \mathbf{v^H_{it}} \qquad s.t. \tag{19}$$

$$\mathbf{\Gamma^H_{it}} = \bar{\mathbf{\Gamma}}^{\mathbf{H}}_{\mathbf{t}} + \ddot{\mathbf{\Gamma}}^{\mathbf{H}}_{\mathbf{it}}, \qquad \frac{1}{N}\sum_i \ddot{\mathbf{\Gamma}}^{\mathbf{H}}_{\mathbf{it}} = \mathbf{0}, \qquad rank\left(\frac{1}{N}\sum_i \ddot{\mathbf{\Gamma}}^{\mathbf{H}}_{\mathbf{it}}\ddot{\mathbf{\Gamma}}^{\mathbf{H}\prime}_{\mathbf{it}}\right) = m \tag{20}$$

$\mathbf{c}_{xi}$ is an individual fixed effect. $\mathbf{f_t}$ is an m x 1 vector of latent factors. $\mathbf{\Gamma^H_{it}}$ is an k x m matrix of factor loadings. The factor structure reflects the effect of contemporaneous structural shocks on expectations of future macroeconomic gaps. $\mathbf{v^H_{it}}$ follows a general linear covariance stationary process distributed independently of the idiosyncratic errors in Eq. 7, ε^h_{it}.

Assumption 5. *The vector $\mathbf{f_t}$ is composed of two structural shocks: $\mathbf{f_t}' = \left[f^s_t, f^d_t\right]$ where f^s_t is a supply factor, and f^d_t is a demand factor (where a monetary policy expectations shock, ε^h_{it}, was introduced in Eq. 7 earlier).*

[6] Where we have applied the notation from Eq. 11, $\mathbf{x^H_{it}}$, to Eq. 13.

Giacomini, Lu, and Smetanina (2024), among others, explain how we can represent macroeconomic forecast survey data using time-varying (dynamic) factor models. Stock and Watson (2016) provide strong empirical support that such factor models capture macroeconomic time series well—that is, an $m \times 1$ vector of latent common factors, which are contemporaneously orthogonal and possibly serially correlated. $\mathbf{f_t}$ can be interpreted as perceived structural shocks—the interpretation of which requires additional assumptions, such as those provided in our reaction function framework here, or those based on impulse response function interpretation.[7] Accordingly, we postulate that we can approximate a number of the elements of the expected reaction functions using time-varying dynamic factor models.

Next, we address a remaining unobservable term in Eq. 17, the expected (forward-looking) monetary policy gradualism term, $\rho^h_{it}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right]$. Observe that we can shift Eq. 9 back one period:

$$y^h_{i,t-1} = \ddot{\mathbb{E}}_{i,t-1}\left[\tilde{\mathbb{E}}_{t-1}\left[\tilde{r}_{t+h-1}\right]\right] + \ddot{\mathbb{E}}_{i,t-1}\left[e^y_{t+h-1}\right] + \ddot{\mathbb{E}}_{i,t-1}\left[\tau_{t+h-1}\right] \tag{21}$$

which corresponds to the h-period-ahead forecast formed at time $t-1$. Decomposing the first term on the right-hand-side:

$$y^h_{i,t-1} = \underbrace{\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] - \Delta\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right]}_{=\ \ddot{\mathbb{E}}_{i,t-1}\left[\tilde{\mathbb{E}}_{t-1}\left[\tilde{r}_{t+h-1}\right]\right]} + \ddot{\mathbb{E}}_{i,t-1}\left[e^y_{t+h-1}\right] + \ddot{\mathbb{E}}_{i,t-1}\left[\tau_{t+h-1}\right] \tag{22}$$

Rearranging to isolate $\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right]$:

$$\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] = y^h_{i,t-1} + \Delta\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] - \ddot{\mathbb{E}}_{i,t-1}\left[e^y_{t+h-1}\right] - \ddot{\mathbb{E}}_{i,t-1}\left[\tau_{t+h-1}\right] \tag{23}$$

Using Lemma A.1:

$$\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] = y^h_{i,t-1} + \underbrace{\left(\varphi^{r^*}_{it} r^*_t + \varphi^{\pi^*}_{it}\pi^*_t + \mathbf{\Psi^h_{it}}\mathbf{f_t} + \mathbf{\check{\Psi}^h_{it}}\mathbf{\Delta f_t} + \Xi^h_{it}\right)}_{=\ \Delta\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] \text{ by Lemma A.1}} - \ddot{\mathbb{E}}_{i,t-1}\left[e^y_{t+h-1}\right] - \ddot{\mathbb{E}}_{i,t-1}\left[\tau_{t+h-1}\right] \tag{24}$$

We then substitute this into Eq. 17 as desired:

$$\begin{aligned} y^h_{it} = \rho^h_{it}\Big(y^h_{i,t-1} + \left(\varphi^{r^*}_{it} r^*_t + \varphi^{\pi^*}_{it}\pi^*_t + \mathbf{\Psi^h_{it}}\mathbf{f_t} + \mathbf{\check{\Psi}^h_{it}}\mathbf{\Delta f_t} + \Xi^h_{it}\right) - \ddot{\mathbb{E}}_{i,t-1}\left[e^y_{t+h-1}\right] - \ddot{\mathbb{E}}_{i,t-1}\left[\tau_{t+h-1}\right]\Big) + \\ \phi^{r^*}_{it} r^*_t + \phi^{\pi^*}_{it}\pi^*_t + \phi^{\tau}_{it}\tau_t + \mathbf{x^{H'}_{it}}\beta^{\mathbf{h}}_{\mathbf{it}} + \ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right] + \upsilon^{\mathbf{H'}}_{\mathbf{it}}\ddot{\beta}^{\mathbf{h}}_{\mathbf{it}} + \lambda^h_{it} \end{aligned} \tag{25}$$

We collect like variables and parameters in vectors to compress the notation, and apply Eq. 16 again:

$$\begin{aligned} y^h_{it} = \rho^h_{it} y^h_{i,t-1} + \underbrace{\rho^h_{it}\varphi^{r^*}_{it} r^*_t + \rho^h_{it}\varphi^{\pi^*}_{it}\pi^*_t + \phi^{r^*}_{it} r^*_t + \phi^{\pi^*}_{it}\pi^*_t + \phi^{\tau}_{it}\tau_t - \rho^h_{it}\phi^{\tau}_{i,t-1}\tau_{t-1}}_{:=\ \mathbf{\Phi_{it}}\mathbf{z_t}} \quad + \quad \mathbf{x^{H'}_{it}}\beta^{\mathbf{h}}_{\mathbf{it}} \quad + \\ \underbrace{\rho^h_{it}\mathbf{\Psi^h_{it}}\mathbf{f_t} + \rho^h_{it}\mathbf{\check{\Psi}^h_{it}}\mathbf{\Delta f_t} + \ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right] - \rho^h_{it}\ddot{\mathbb{E}}_{i,t-1}\left[e^y_{t+h-1}\right]}_{:=\ \sum_{\ell=0}^{L}\left(\gamma^{\ell}_{\mathbf{it}}\right)'\mathbf{f_{t-\ell}}} + \underbrace{\upsilon^{\mathbf{H'}}_{\mathbf{it}}\ddot{\beta}^{\mathbf{h}}_{\mathbf{it}} + \rho^h_{it}\Xi^h_{it}}_{:=\ \zeta_{it}} + \lambda^h_{it} \end{aligned} \tag{26}$$

where $\mathbf{\Phi_{it}} = \left[\rho^h_{it}\varphi^{r^*}_{it} + \phi^{r^*}_{it}, \rho^h_{it}\varphi^{\pi^*}_{it} + \phi^{\pi^*}_{it}, \phi^{\tau}_{it}, -\rho^h_{it}\phi^{\tau}_{i,t-1}\right]$, $\mathbf{z_t}' = \left[r^*_t, \pi^*_t, \tau_t, \tau_{t-1}\right]$. Using the identity $\mathbf{\Delta f_t = f_t - f_{t-1}}$, the factor innovations can be expressed as a distributed lag in $\mathbf{f_t}$. $\ddot{\mathbb{E}}_{it}\left[e^y_{t+h}\right]$ is assumed to be spanned by the factor space. ζ_{it} is expected to be small under the conditions of Lemma A.1.

[7] See Giacomini, Lu, and Smetanina (2024), p.10. The elements of $\mathbf{f_t}$ *can* be serially correlated (see p.15).

2.4 Identification Strategy

In this section, we develop our estimator for Eq. 26, which, based on the derivations flowing through §2.1, §2.2, and §2.3, begins as follows:

$$y_{it} = \rho_{it} y_{i,t-1} + \mathbf{\Phi_{it}} \mathbf{z_t} + \mathbf{x_{it}^{H'}} \beta_{\mathbf{it}} + \sum_{\ell=0}^{L} (\gamma_{\mathbf{it}\ell})' \mathbf{f_{t-\ell}} + u_{it} \tag{27}$$

$$u_{it} = \zeta_{it} + \lambda_{it} \tag{28}$$

$$\lambda_{it} = \theta_{it} \lambda_{i,t-1} + \varepsilon_{it} \tag{29}$$

Assumption 6. *The parameters of interest follow a random coefficient model:*

$$\rho_{\mathbf{it}} = \rho + \eta_{\rho\mathbf{i}} + \nu_{\rho\mathbf{t}}, \quad \eta_{\rho\mathbf{i}} \sim \mathbf{iid}\left(\mathbf{0}, \sigma_{\eta_\rho}\right), \quad \nu_{\rho\mathbf{t}} \sim \mathbf{iid}\left(\mathbf{0}, \sigma_{\nu_\rho}\right) \tag{30}$$

$$\beta_{\mathbf{it}} = \beta + \eta_{\beta\mathbf{i}} + \nu_{\beta\mathbf{t}}, \quad \eta_{\beta\mathbf{i}} \sim \mathbf{iid}\left(\mathbf{0}, \mathbf{\Sigma}_{\eta_\beta}\right), \quad \nu_{\beta\mathbf{t}} \sim \mathbf{iid}\left(\mathbf{0}, \mathbf{\Sigma}_{\nu_\beta}\right) \tag{31}$$

$$\gamma_{\mathbf{it}\ell} = \gamma_\ell + \eta_{\gamma\mathbf{i}\ell} + \nu_{\gamma\mathbf{t}\ell}, \quad \eta_{\gamma\mathbf{i}\ell} \sim \mathbf{iid}\left(\mathbf{0}, \mathbf{\Sigma}_{\eta_\gamma}\right), \quad \nu_{\gamma\mathbf{t}\ell} \sim \mathbf{iid}\left(\mathbf{0}, \mathbf{\Sigma}_{\nu_\gamma}\right) \tag{32}$$

$$\theta_{\mathbf{it}} = \theta + \eta_{\theta\mathbf{i}} + \nu_{\theta\mathbf{t}}, \quad \eta_{\theta\mathbf{i}} \sim \mathbf{iid}\left(\mathbf{0}, \sigma_{\eta_\theta}\right), \quad \nu_{\theta\mathbf{t}} \sim \mathbf{iid}\left(\mathbf{0}, \sigma_{\nu_\theta}\right) \tag{33}$$

where each parameter fluctuates around a global mean. This helps clarify that, in the model, there are **two main sources of perceived monetary policy shocks**: shocks to the perceived reaction function **parameters**—the $\eta_{\mathbf{i}}$ and $\nu_{\mathbf{t}}$ terms, and shocks to perceived FOMC **judgment** (λ_{it})—the ε_{it} terms.

Identification challenge 1. Overparameterization. This generalized forecaster-expected reaction function is plausibly an accurate representation of the true expected reaction function, but as such, it is overparamaterized because we know that parameters vary by forecaster, i (Altavilla and Giannone (2017)), by country, c (Sutherland (2023)), time, t (Bauer, Pflueger, and Sunderam (2024)), and forecast horizon, h (Hamilton, Pruitt, and Borger (2011)). So, it cannot be estimated without further assumptions. Specifically, we will need to pool data across one of the three main dimensions of the data: i, t, or h.

Identification challenge 2. Forecaster (i) heterogeneity. We cannot pool data across i. If the assumption of parameter homogeneity across forecasters does not hold in a *dynamic* model such as ours, then Pesaran and Smith (1995) shows that a pooled OLS estimator leads to inconsistent and potentially highly misleading estimates of the coefficients. To account for forecaster parameter heterogeneity, we estimate the model forecaster by forecaster and compute the cross-sectional average of the estimated coefficients, which is a novel approach. This is the mean-group estimator, which Pesaran and Smith (1995) show is a consistent estimator of the mean coefficient. By contrast, the pooled estimator imposes parameter homogeneity and instead converges to a variance-weighted average of the individual coefficients, where the weights depend on the regressor variation for each forecaster (and, so, implicitly, the sample size of each forecaster in each window). And, as this paper shows, the two approaches can yield very different estimates, especially with respect to monetary policy inertia (see Figures 2 and 3).

Identification challenge 3. Forecast horizon (h) heterogeneity. We opt not to pool data across h. As such, we have abstracted from the h superscript in Eq. 27 because we will estimate the model separately for each h. Arguably, imposing parameter homogeneity across forecast horizons is a fairly innocuous assumption (as used in Bauer, Pflueger, and Sunderam (2024)), yet in our structural setting, it would also be a missed opportunity: we can extract two separate monetary policy rate path shocks at two forecast horizons (3-months and 12-months) that gives us two separate econometric instruments—each with a

slightly different interpretation (§3.5), akin to the target and path factors from Gürkaynak, Sack, and Swanson (2005). In turn, when estimating impulse response functions, having two instruments for a single endogenous regressor yields an overidentified model (see Eq. 52), which allows us to test the exogeneity of the instruments, and which leads to new insights (see §4.1 and §4.2).

Identification challenge 4. Time-varying (t) parameters. We opt to pool data across t, and use seven-year rolling windows. This forces us to average away some of the time variation, yet still allows for a high degree of parameter variation. Bauer, Pflueger, and Sunderam (2024) also use seven-year rolling windows (T=84) following the same reasoning.[8] Hence, we can further simplify Eq. 27 by focusing on the equation for each rolling window. So, coefficients no longer vary by *t within* a given window, but are time-varying because they can vary *by* rolling window:

$$y_{it} = \rho_i y_{i,t-1} + \mathbf{\Phi_i z_t} + \mathbf{x_{it}^{H}}^{'} \beta_{\mathbf{i}} + \sum_{\ell=0}^{L} (\gamma_{\mathbf{i}\ell})^{'} \mathbf{f_{t-\ell}} + u_{it} \tag{34}$$

Accordingly, the heterogeneous coefficients now follow a simpler random coefficients model within each window:[9]

$$\rho_{\mathbf{i}} = \rho + \eta_{\rho\mathbf{i}}, \quad \eta_{\rho\mathbf{i}} \sim \mathbf{iid}\left(\mathbf{0}, \sigma_\rho\right) \tag{35}$$

$$\beta_{\mathbf{i}} = \beta + \eta_{\beta\mathbf{i}}, \quad \eta_{\beta\mathbf{i}} \sim \mathbf{iid}\left(\mathbf{0}, \mathbf{\Sigma}_\beta\right) \tag{36}$$

$$\gamma_{\mathbf{i}\ell} = \gamma_\ell + \eta_{\gamma\mathbf{i}\ell}, \quad \eta_{\gamma\mathbf{i}\ell} \sim \mathbf{iid}\left(\mathbf{0}, \mathbf{\Sigma}_\gamma\right) \tag{37}$$

$$\theta_{\mathbf{i}} = \theta + \eta_{\theta\mathbf{i}}, \quad \eta_{\theta\mathbf{i}} \sim \mathbf{iid}\left(\mathbf{0}, \sigma_\theta\right) \tag{38}$$

Identification challenge 5. Recovering the structural beta parameters. Comparison of Eq. 17 and Eq. 27 illustrates that the $\beta_{\mathbf{i}}$ must correspond to the following:[10]

$$\beta_{\mathbf{i}} := (\mathbf{A_i})^{'} \ddot{\beta}_{\mathbf{i}}^{\mathbf{h}} \tag{39}$$

As such, to recover the structural parameters of interest, $\ddot{\beta}_{\mathbf{i}}^{\mathbf{h}}$, we also need to identify $\mathbf{A_i}$.[11]

Identification challenge 6. Cross-sectional dependence. A priori, the presence of $\mathbf{f_t}$ in Eq. 34 will tend to introduce cross-sectional dependence between forecasts; this leads to substantial bias and size distortions if ignored (Pesaran (2006), p.970). Unsurprisingly, in rolling window regressions, we invariably detect cross-sectional dependence across forecasts using the test from Pesaran (2015).[12]

To account for cross-sectional dependence across forecasters, that is, exposure to common shocks ($\mathbf{f_t}$), we employ the Common Correlated Effects (CCE) estimator developed by Pesaran (2006). This approach controls for unobserved common factors by augmenting each forecaster's regression with cross-sectional averages (across all forecasters that period) of the dependent and independent variables.

[8] Bauer, Pflueger, and Sunderam (2024) also estimate a series of cross-sectional regressions with $N \approx 100$.

[9] Where each $\mathbf{\Sigma}$ is a symmetric nonnegative definite matrix, and where each of the $\eta_{\mathbf{i}}$ are independently distributed of $\gamma_{\mathbf{j}}, \mathbf{\Gamma_j}, \mathbf{v_{jt}}, \mathbf{f_t}, \varepsilon_{jt} \;\; \forall i, j, t$. Further, ρ_i lies strictly inside the unit circle.

[10] $\beta_{\mathbf{it}}$ is identified by the variation in $\mathbf{v_{it}^{h}}$, which is the idiosyncratic movement in $\mathbf{x_{it}^{H}}$. See below for the rationale.

[11] This has not yet been done in this draft, but will be straightforward to address in the next draft of this paper.

[12] That is, we invariably reject weak cross-sectional dependence during estimation.

The *Dynamic* Common Correlated Effects (Chudik and Pesaran (2015)) estimator extends the CCE framework to allow for lagged dependent variables (in addition to weakly exogenous regressors), which is crucial here. The DCCE estimator has excellent small-sample properties given even substantial heterogeneity, dynamics, and relatively small values of N and T (e.g. $N = 20$, $T = 20$). DCCE allows one to account for both common effects and parameter heterogeneity.

In the DCCE framework, the $\mathbf{f_t}$ can exhibit an arbitrary degree of correlation among themselves and with the individual-specific regressors, $\mathbf{x_{it}}$. The factors (or cross-sectional averages) are assumed to span the common component, although later, we allow for additional factors ($\mathbf{g_{it}}$) to be certain. The ε_{it} in λ_{it} can be spatially correlated. So, the $\mathbf{x}_{it}$, which are weakly exogenous regressors within the broader linear covariance stationary dynamic heterogeneous panel data model, are assumed to be generated by the following:

$$\omega_{\mathbf{it}} = \begin{bmatrix} \mathbf{x_{it}} \\ \mathbf{g_{it}} \end{bmatrix} = \mathbf{c}_{\omega i} + \mathbf{\Gamma_i'} \mathbf{f_t} + \mathbf{v_{it}} \tag{40}$$

The $mx1$ vector of unobserved common factors, $\mathbf{f_t}$, is assumed to be covariance stationary, and distributed independently of ε_{it} and $\mathbf{v_{it}}$ $\forall i, t, t'$. $\mathbf{\Gamma_i}$ is an m x k matrix of factor loadings. $\gamma_\mathbf{i}$ and $\mathbf{\Gamma_i}$ are independently and identically distributed across i, and of $\mathbf{f_t}$, for all i and t, with mean γ and $\mathbf{\Gamma}$, respectively.[13] $\mathbf{g_{it}}$ is a vector of variables, such as consumption expenditure and business investment forecasts, that do not enter Eq. 34 directly, but that can be used to proxy for additional factors if necessary. $\mathbf{v_{it}}$ follows general linear covariance stationary process distributed independently of the i.i.d. errors, ε_{it}.

If the errors, u_{it}, are cross-sectionally weakly dependent, and $\mathbb{E}[\gamma_\mathbf{i}] = \mathbb{E}[\gamma]$, the lagged dependent variable is no longer strictly exogenous. Yet, Chudik and Pesaran (2015) show that the estimator can become consistent if it is augmented with cross-sectional (weighted) averages of the dependent variable and the individual-specific regressors. Specifically, the floor of $\sqrt[3]{T}$ lags of the cross-section averages of the regressors are added to Eq. 34.[14] We denote the number of lags needed by $p_T = \sqrt[3]{T}$.[15] We also need at least as many (independent) cross-sectional averages as we have latent factors. That condition is satisfied here because we use the cross-sectional averages of the very variables from which the factors originate as well as two others.

$$y_{it} = \rho_i y_{i,t-1} + \mathbf{\Phi_i z_t} + \mathbf{x_{it}^{H'}} \beta_\mathbf{i} + \sum_{\ell=0}^{p_T} \bar{\mathbf{x}}_{t-\ell}' \delta_{\mathbf{i}\ell} + \sum_{\ell=0}^{p_T} \bar{\mathbf{g}}_{t-\ell}' \kappa_{\mathbf{i}\ell} + e_{it} \tag{41}$$

$$e_{it} = u_{it} + \sum_{\ell=p_T+1}^{\infty} \bar{\mathbf{x}}_{t-\ell}' \delta_{\mathbf{i}\ell} + \sum_{\ell=p_T+1}^{\infty} \bar{\mathbf{g}}_{t-\ell}' \kappa_{\mathbf{i}\ell} + O_p\left(N^{-1/2}\right) \tag{42}$$

In doing so, we filter the individual-specific regressors by using cross-section averages (denoted with an overbar) such that, asymptotically, the differential effects of unobserved common factors are eliminated.[16] That is, the cross-sectional averages and their lags are used as proxies for the unobserved

[13] Where $vec(\mathbf{\Gamma_i}) = vec(\mathbf{\Gamma}) + \eta_{\mathbf{\Gamma i}}, \quad \eta_{\mathbf{\Gamma i}} \sim iid\,(\mathbf{0}, \mathbf{\Omega_\Gamma})$, for $i = 1, 2, \ldots, N$, and where $\mathbf{0}$ is km x 1.

[14] See Ditzen (2018), p. 587 for more details.

[15] In practice, Chudik and Pesaran (2015) recommend restricting the number of cross-sectional lags to maintain a sufficient number of degrees of freedom, which we do.

[16] We omit cross-sectional averages of lagged dependent variables. The dependent variable is an expectation, not a realized macro outcome. Any common persistent component in expectations operates through observable macroeconomic fundamentals, which are controlled for using cross-sectional averages of regressors and, therefore, factor structures. Including lagged cross-sectional averages of expectations would over-partial common belief dynamics and obscure the structural interpretation of the lagged dependent variable coefficients. We also could not include $\bar{y}_{it}$ because we would purge the very shock measure we seek to extract.

common factors. The estimator relies on $(N,T)\to\infty$ jointly for consistency, with the number of lags of cross-sectional averages growing slowly with the time dimension.[17] Crucially, Chudik and Pesaran (2015) showed that this CCE estimator has good small-sample properties despite substantial heterogeneity and dynamics, and for relatively small values of N and T.

We also allow for the possibility that there may be other, outside latent factors present in the data, which could be proxied by other forecasts made by forecaster i at time t, such as consumption expenditure and business investment forecasts.

Thus far, the key difference from Chudik and Pesaran (2015) has been that our error term, λ_{it}^{h}, is serially correlated by definition (Eq. 5). Yet, if we reparameterize Eq. 41 as follows:[18]

$$y_{it}-\theta_i y_{i,t-1}=\left(\rho_i y_{i,t-1}+\mathbf{\Phi_i z_t}+\mathbf{x_{it}^{H}}'\beta_{\mathbf{i}}+\sum_{\ell=0}^{p_T}\overline{\mathbf{x}}_{t-\ell}'\delta_{\mathbf{i}\ell}+\sum_{\ell=0}^{p_T}\overline{\mathbf{g}}_{t-\ell}'\kappa_{\mathbf{i}\ell}+e_{it}\right)- \\ \theta_i\left(\rho_i y_{i,t-2}+\mathbf{\Phi_i z_{t-1}}+\mathbf{x_{i,t-1}^{H}}'\beta_{\mathbf{i}}+\sum_{\ell=0}^{p_T}\overline{\mathbf{x}}_{t-1-\ell}'\delta_{\mathbf{i}\ell}+\sum_{\ell=0}^{p_T}\overline{\mathbf{g}}_{t-1-\ell}'\kappa_{\mathbf{i}\ell}+e_{i,t-1}\right) \quad (43)$$

Rearranging gives the final form of our estimator:

$$y_{it}=(\theta_i+\rho_i)y_{i,t-1}-\theta_i\rho_i y_{i,t-2}+\mathbf{\Phi_i z_t}-\theta_i\mathbf{\Phi_i z_{t-1}}+\mathbf{x_{it}^{H}}'\beta_{\mathbf{i}}-\theta_i\mathbf{x_{i,t-1}^{H}}'\beta_{\mathbf{i}} \\ +\left(\sum_{\ell=0}^{p_T}\overline{\mathbf{x}}_{t-\ell}'\delta_{\mathbf{i}\ell}-\theta_i\sum_{\ell=0}^{p_T}\overline{\mathbf{x}}_{t-1-\ell}'\delta_{\mathbf{i}\ell}\right)+\left(\sum_{\ell=0}^{p_T}\overline{\mathbf{g}}_{t-\ell}'\kappa_{\mathbf{i}\ell}-\theta_i\sum_{\ell=0}^{p_T}\overline{\mathbf{g}}_{t-1-\ell}'\kappa_{\mathbf{i}\ell}\right)+(e_{it}-\theta_i e_{i,t-1}) \quad (44)$$

Expanding the error term gives:

$$(e_{it}-\theta_i e_{i,t-1})=(u_{it}-\theta_i u_{i,t-1}) \quad + \\ \left(\sum_{\ell=p_T+1}^{\infty}\overline{\mathbf{x}}_{t-\ell}'\delta_{\mathbf{i}\ell}+\sum_{\ell=p_T+1}^{\infty}\overline{\mathbf{g}}_{t-\ell}'\kappa_{\mathbf{i}\ell}+O_p\left(N^{-1/2}\right)\right)-\theta_i\left(\sum_{\ell=p_T+1}^{\infty}\overline{\mathbf{x}}_{t-1-\ell}'\delta_{\mathbf{i}\ell}+\sum_{\ell=p_T+1}^{\infty}\overline{\mathbf{g}}_{t-1-\ell}'\kappa_{\mathbf{i}\ell}+O_p\left(N^{-1/2}\right)\right) \quad (45)$$

where we can assume that the terms in the second line (essentially, the differenced effect of distant lagged regressors) above are vanishingly small.[19] So, then:

$$e_{it}-\theta_i e_{i,t-1}\approx u_{it}-\theta_i u_{i,t-1}\Rightarrow \quad (46)$$

And, by Eq. 28:

$$u_{it}-\theta_i u_{i,t-1}=(\zeta_{it}+\lambda_{it})-\theta_i(\zeta_{i,t-1}+\lambda_{i,t-1})\Rightarrow \quad (47)$$

And, similarly, because $\zeta_{it}-\theta_i\zeta_{i,t-1}$ should also be vanishingly small (by Lemma A.1), then (rearranging):

$$\lambda_{it}-\theta_i\lambda_{i,t-1}+(\zeta_{it}-\theta\zeta_{i,t-1})\approx\lambda_{it}-\theta_i\lambda_{i,t-1}\Rightarrow \quad (48)$$

[17] This requires the matrix $\mathbf{C}=(\gamma,\mathbf{\Gamma})'$ to have full column rank (Chudik and Pesaran (2015), p. 397).

[18] This reparameterization is reminiscent of quasi-differencing, which is often used as a statistical device to eliminate serial correlation in the error process by estimating and filtering out an autoregressive component (Prais and Winsten (1954)). Here the use of quasi-differencing is theory-driven: we impose a structural persistence parameter implied by the model rather than estimating it from the data. This transformation should therefore be understood not as a statistical correction but rather as part of the identifying structure of the model, rendering the residuals interpretable as structural shocks.

[19] See Chudik and Pesaran (2015) p.397.

Now, substituting in Eq. 29 (while suppressing the t parameter subscripts) into the first term only gives:

$$\lambda_{it} - \theta_i \lambda_{i,t-1} = \left(\theta_i \lambda_{i,t-1} + \varepsilon_{it}\right) - \theta_i \lambda_{i,t-1} = \varepsilon_{it} \Rightarrow \tag{49}$$

$$e_{it} - \theta_i e_{i,t-1} \approx u_{it} - \theta_i u_{i,t-1} \approx \lambda_{it} - \theta_i \lambda_{i,t-1} = \varepsilon_{it} \tag{50}$$

and so, as a close approximation, ε_{it} becomes the new error term, which is not serially correlated. As such, we can estimate Eq. 44 with the adapted DCCE estimator. Then, the forecast horizon-specific (h) mean perceived shock estimates are:

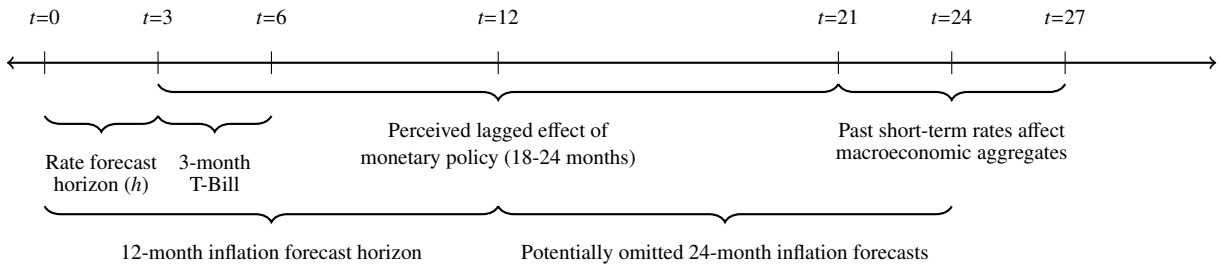
$$\widehat{\varepsilon_t^h} = \frac{1}{N} \sum_i^N \widehat{\varepsilon_{it}^h} \tag{51}$$

where the forecaster-specific parameter and ε_{it}^h estimates come from the rolling-window estimate whose last observation is t (and first observation is $t-W+1$), where W is window length in months. The mean-group coefficients and ε_{it}^h estimates are averaged in the same manner.

Identification challenge 7. Macro forecast horizons: simultaneity vs. omitted variable bias.

Monetary policy is forward-looking (as reflected in Eq. 1 onwards). This presents an identification challenge for our estimator (Eq. 44). The dependent variable is a forecast of an interest rate either three or twelve months from now ($h \in \{3, 12\}$). The key independent variable is a forecast of a macroeconomic variable, such as inflation, 12 months from now (H=12). Depending on how the forecast horizons are calibrated, endogeneity may be introduced to the estimator in the form of either simultaneity bias or omitted variable bias.

Figure 1: **Forecast Horizons (t months) with h=3, H=12.**



Aruoba and Drechsel (2024) find that, after an exogenous monetary policy contraction, the headline *Personal Consumption Expenditures Price Index* displays a gradual decline, with a significantly negative response at conventional levels only after 43 months and the peak reduction occurs after 54 months (consistent with Christiano, Eichenbaum, and Evans (1999), and tangentially, Nakamura and Steinsson (2018)). However, because we are using expectations data, what matters is not the true long and variable lags of monetary policy but the *perception* thereof. A more conventional period would be 18 to 24 months.

As long as the *12-month inflation forecast horizon* window in Figure 1 does not overlap the *Past short-term rates affect macroeconomic aggregates* window in Figure 1, then we avoid endogeneity due to simultaneity. In Figure 1 with h=3, there is a buffer of nine months; with h=12, there is a slight overlap of three months, but only due to our conservative assumption about perceived monetary policy lags.

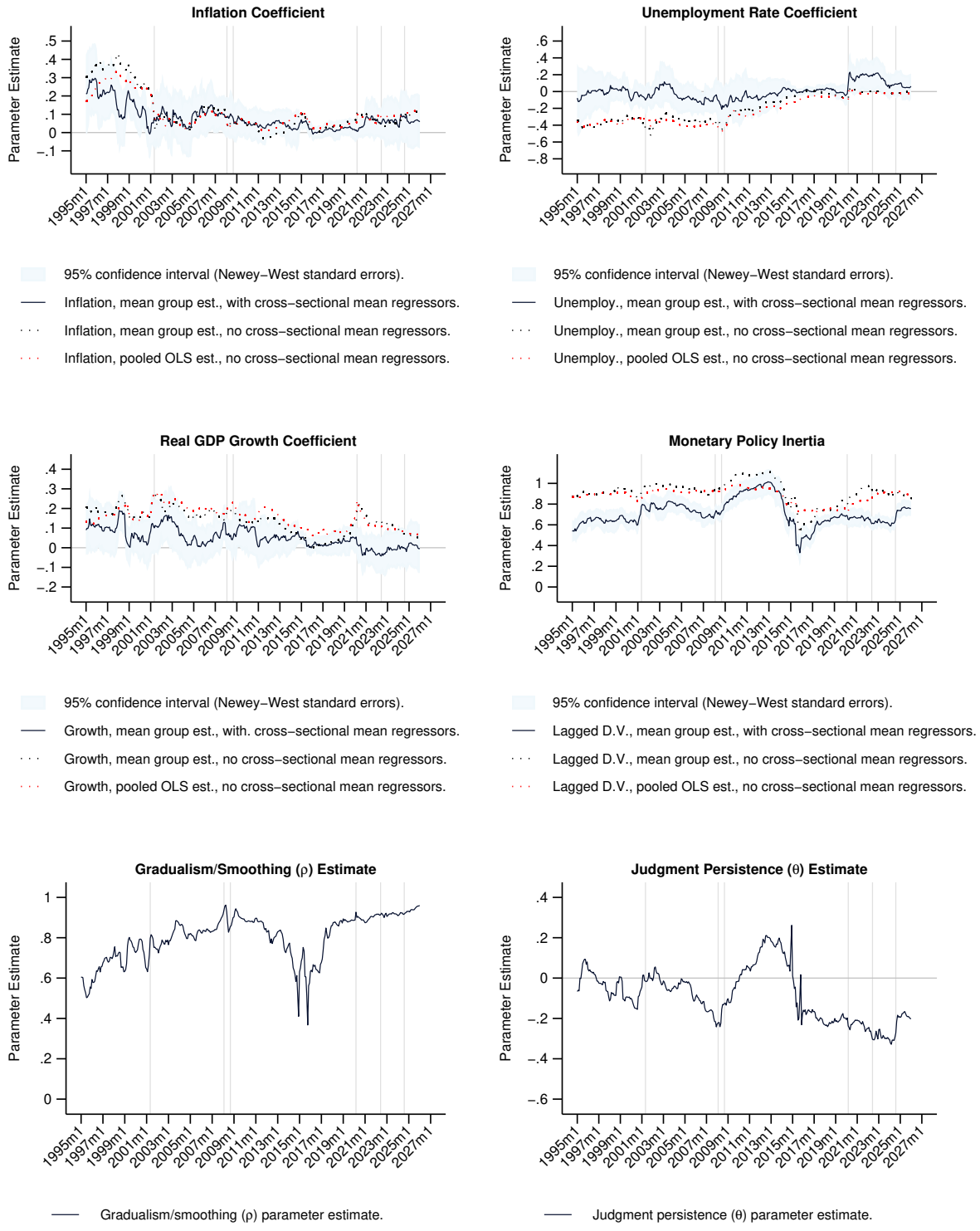
Conversely, if the inflation forecasts are not forward-looking enough, then we may have some omitted variable bias. This is captured in Figure 1 by considering the possibility that forecasters use inflation forecasts as far as 24 months ahead to form their policy rate forecasts. We can check for this by including both current-year and next-year inflation forecasts, which at their peak in January, look forward 24 months. When we do so, our results are not materially different.

3 Estimation

In this section, we present the forecast data, the forecaster-expected reaction function parameter estimates, and the monetary shock estimates. We then consider three case studies to help interpret the results. Estimation is performed with the support of the xtdcce2 package in Stata, which should facilitate straightforward replication and adaptation of our empirical approach and code to one of the over 100 countries for which data are available from *Consensus Economics*.

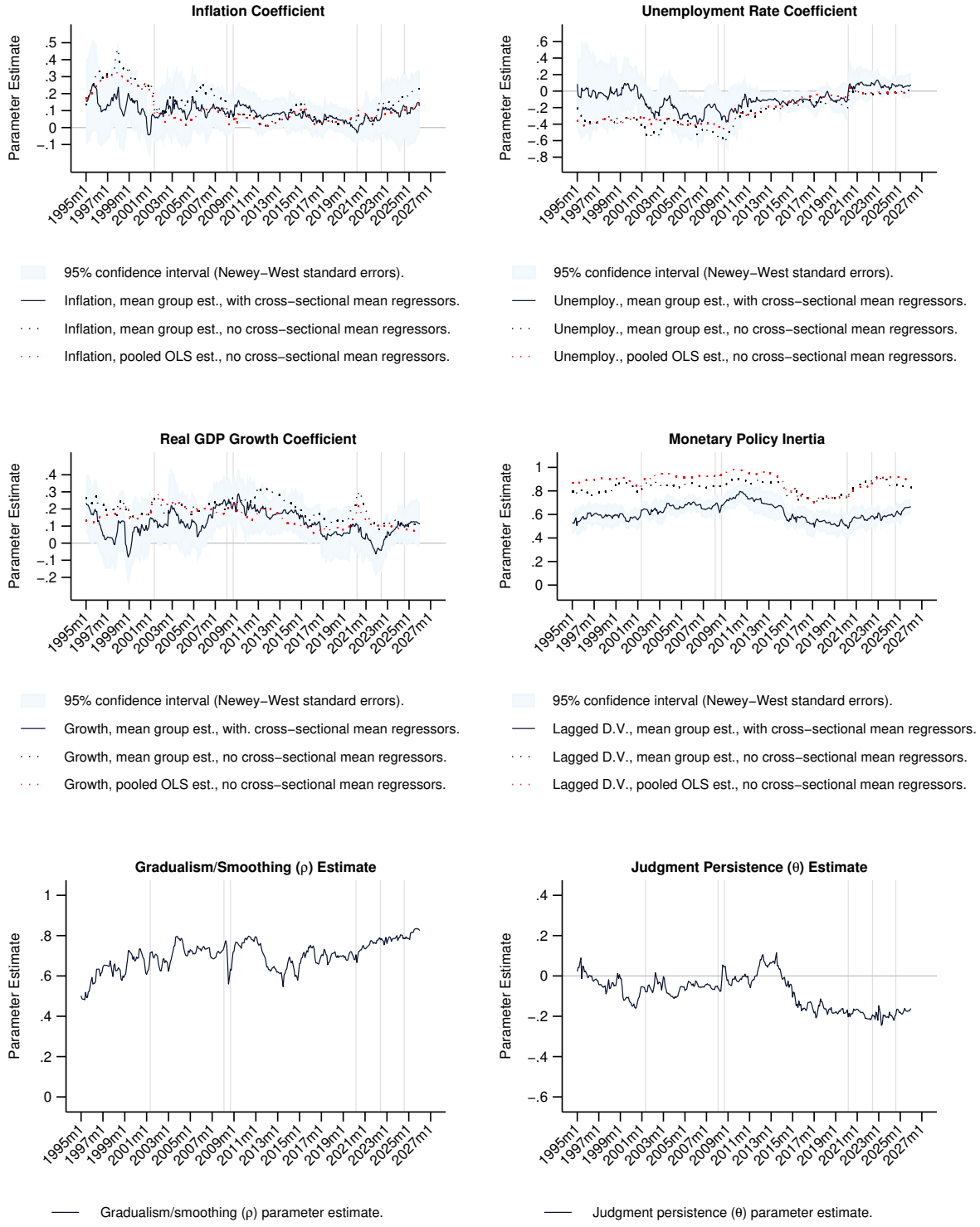
3.1 Expected FOMC Reaction Function Estimates

Figure 2: **Expected FOMC Reaction Function Coefficient Estimates (3-month forecast horizon)**



Note 1: The thin, grey, vertical lines denote the case studies described below.
Note 2: Results available for months 1995m1 to 2026m1. Forecasts made on second or third Monday of each month.

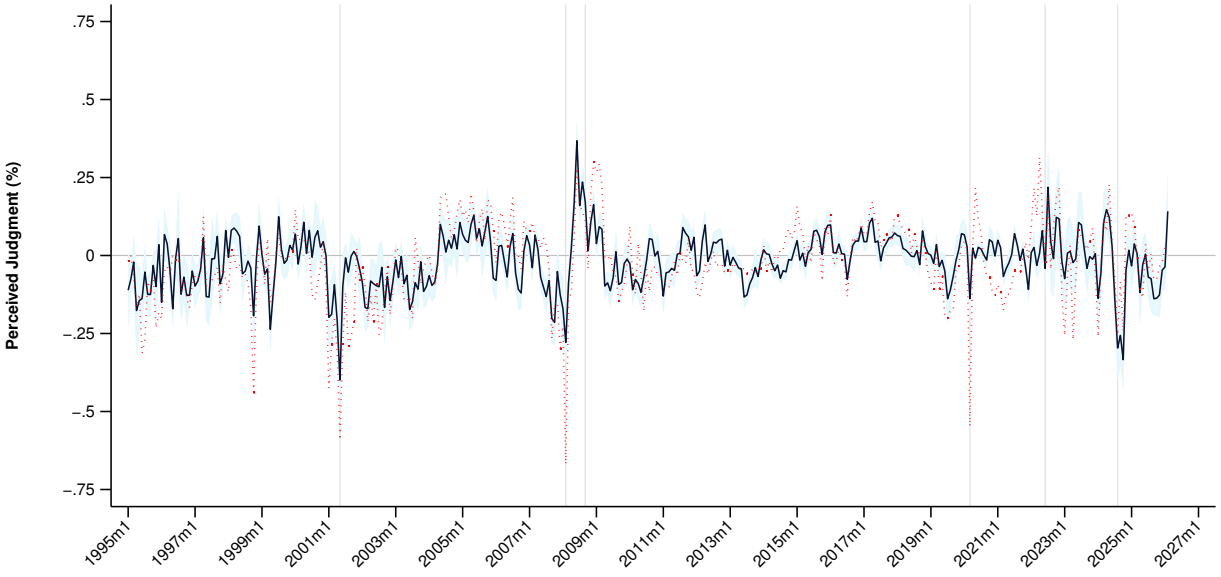
Figure 3: **Expected FOMC Reaction Function Coefficient Estimates (12-month forecast horizon)**



Note 1: The thin, grey, vertical lines denote the case studies described below.
Note 2: Results available for months 1995m1 to 2026m1. Forecasts made on second or third Monday of each month.

3.2 Expected FOMC Reaction Function-Implied Judgment Shocks

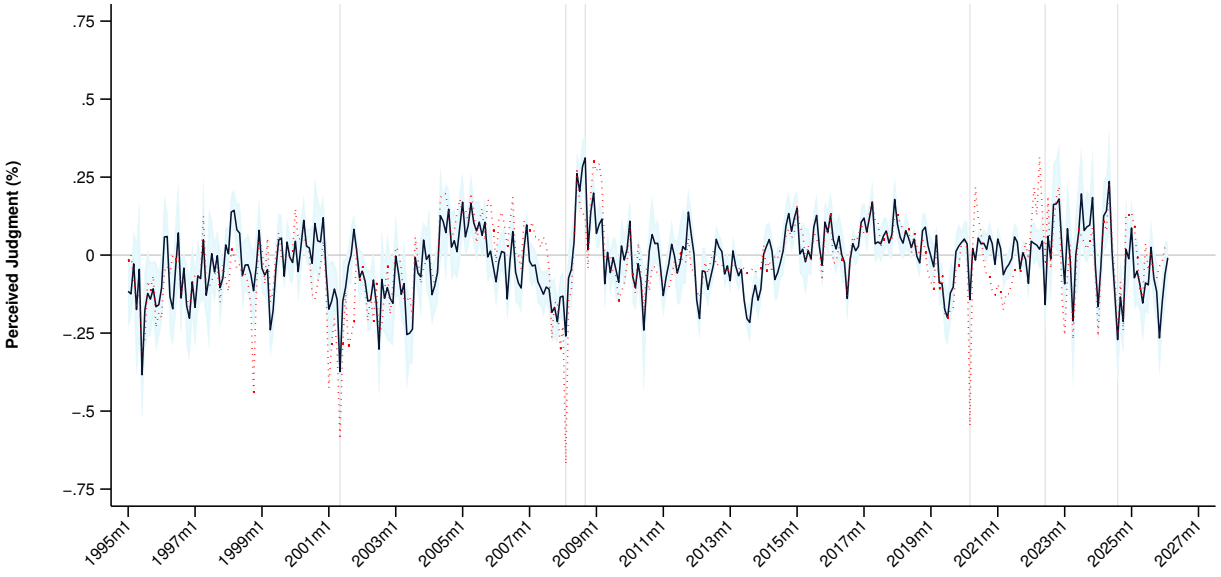
Figure 4: **FOMC Judgment Shock Estimates (3-month Forecast Horizon)**



95% confidence intervals.
Expected reaction function−implied judgment shock. Mean group dynamic common correlated effects estimator.
Expected reaction function−implied judgment shock. Pooled OLS estimator with no cross−sectional mean regressors.

Sample period: 1995m1 to 2026m2. The thin, grey, vertical lines denote the case studies described below.

Figure 5: **FOMC Judgment Shock Estimates (12-month Forecast Horizon)**



95% confidence intervals.
Expected reaction function−implied judgment shock. Mean group dynamic common correlated effects estimator.
Expected reaction function−implied judgment shock. Pooled OLS estimator with no cross−sectional mean regressors.

Sample period: 1995m1 to 2026m2. The thin, grey, vertical lines denote the case studies described below.

3.3 Why are the Estimated Inflation Coefficients Less Than One?

First, the dynamic nature of the monetary policy rule implies that the macro response coefficient estimates in Figures 2 and 3 are **short-run** in nature—that is, they are proportional to $(1-\rho_{it})$.[20] Now, since the estimates of expected $\bar{\rho}_{it}$ in Figures 2 and 3 oscillate around 0.8 most of the time, that implies that the long-run coefficients are approximately 5 times the magnitude of the short-run expected coefficient estimates at a given time. So, the long-run coefficients are, sometimes, likely close to or even above one.

The idea that The Fed should raise nominal rates more than one-for-one with inflation dates back to Taylor (1993), and reflected the need to establish inflation-fighting credibility. However, the expected macro response coefficient estimates in Figures 2 and 3, which are significantly lower than one, corroborate several points made by Nakamura, Riblier, and Steinsson (2025): "While the Taylor rule (generally) fits well during the Greenspan period, it (generally) fits poorly in the early 1980s and after the early 2000s."

Nakamura, Riblier, and Steinsson (2025) offer three reasons why central banks may optimally vary the nominal interest rate less than one-for-one with inflation in certain circumstances.[21] First, in response to cost-push and sectoral shocks, optimal policy calls for a substantial amount of forward guidance, which weakens the contemporaneous relationship between the nominal interest rate and inflation (i.e. central banks respond in a shock-contingent manner). Second, a combination of inflationary cost push shocks and negative demand shocks calls for a muted response of interest rates (i.e. shocks to the Phillips curve and the dynamic IS equation may be correlated). Third, "long and variable" lags imply that there may be little monetary policy can do in response to transitory shocks.

Two additional, related reasons include anchored forecaster inflation expectations due to the success of inflation-targeting. That is, when inflation spends most of the time (in the sample data anyway) around 2%, inflation news may become less important with respect to forecaster-expected reaction functions. As well, the effective lower bound (ELB) limits The Fed's ability to respond to below-target inflation, which could also drive down the expected inflation response coefficient during periods in which the policy rate is forecasted to be at the ELB.[22]

3.4 A Parsimonious Coibion and Gorodnichenko (2012) Test

Coibion and Gorodnichenko (2012) ask why are target interest rate changes so persistent? They acknowledge that inertia in central banks' reaction functions is now standard in theoretical and empirical monetary economics, but argue that the source of this inertia is controversial. The authors propose a number of tests to discriminate between competing explanations of inertia, and conclude that the evidence strongly favors the interest-smoothing explanation over the notion of persistent monetary policy shocks.

The perceived reaction function framework and the corresponding estimator in this paper provide a structural test of the competing hypotheses from Coibion and Gorodnichenko (2012). Figures 2 and 3 provide estimates of the gradualism/smoothing ($\widehat{\rho}_{it}$) and judgment persistence ($\widehat{\theta}_{it}$) structural parameters respectively over time, providing a straightforward way to see the extent to which each channel is dominating. As we seek to explain positive persistence, we can see that, consistent with Coibion and Gorodnichenko (2012), gradualism/smoothing explains that positive persistence the great majority of the time, with pos-

[20] As Coibion and Gorodnichenko (2012), Bauer, Pflueger, and Sunderam (2024), and others have also observed.
[21] This assumes anchored long-run inflation expectations.
[22] See Debortoli, Galí, and Gambetti (2020) for an analysis of the empirical (ir) relevance of the ELB.

itively persistent judgment dominating only occasionally—particularly during the post-financial crisis ELB/forward guidance period, which accords with economic intuition. Interesting, smoothing still dominated in the wake of the pandemic, which is also consistent with the idea that central banks were behind the curve on inflation, responding perhaps too slowly to inflationary pressures.

Ricco and Savini (2025) and Miranda-Agrippino and Williams (2026).

3.5 Case Studies

February 2008. We estimate that there was a large, negative judgment shock at both the three-month (Figure 4) and twelve-month (Figure 5) forecast horizons. The expected reaction function coefficient estimates were fairly stable around this time (Figures 2 and 3). Amidst a weakening economic outlook, increasing downside risks to growth, strains in short-term funding markets, deteriorating financial market conditions, tightening credit conditions, deepening housing contraction, and a softening in labor markets, the FOMC cut the federal funds rate at an unscheduled meeting on January 22, 2008 by 75 basis points—the largest single-day reduction in over two decades. Immediately after the first cut, futures contracts implied a 72 percent chance of another 50 basis points rate cut at the scheduled January 30 meeting (source).

And, indeed The FOMC cut another 50 basis points. Later, during testimony to Congress, Chairman Bernanke noted that "*The FOMC has responded aggressively to the weaker outlook for economic activity, having reduced its target for the federal funds rate by 225 basis points since last summer*." "*The FOMC will be carefully evaluating incoming information bearing on the economic outlook and will act in a timely manner as needed to support growth and to provide adequate insurance against downside risks*."

March-April 2020, Early-Pandemic Shocks. In March 2020 (2020m3), we observe negative judgment shocks following a 50-basis-point FFR decrease (Figures 4, 5). In early April (2020m4), we observe large, downward forecast adjustments, and sharp upward adjustment to the FOMC's perceived unemployment responsiveness (Figures 2, 3). Due to these negative judgment shocks, the expected FOMC rate path over the next year or so was lower than that implied by the systematic component of the forecaster-expected reaction function.

These patterns are partly attributable to FOMC monetary policy. First, at an unscheduled meeting on March 3, 2020 (2020m3), the FFR was unexpectedly lowered by 50 basis points to the effective lower bound (ELB), helping explain the negative judgment shocks observed in early March. Commensurately, forecasters lowered their three-month rate forecasts to the ELB, and so could not go lower; twelve-month rate forecasts also fell to the ELB, but dropped from a higher starting point. Second, at another unscheduled meeting on March 15, 2020 (2020m4), strong FOMC forward guidance was provided.[23] These downward interest rate forecast changes are also partly attributable to the differences in perceived FOMC responsiveness to unemployment forecasts across the two forecast horizons (3-month horizon: from -0.04 in February 2020 (2020m2), to 0.08 in March 2020 (2020m3), to 0.19 in April 2020 (2020m4); 12-month horizon: from -0.15 in February 2020, to -0.03 in March 2020, to 0.06 in April 2020). This coefficient heterogeneity demonstrates the utility in estimating the model separately across the two forecast horizons.[24] Additional case studies can be found in the online $\mathcal{A}$ppendix.

[23] "*The Committee expects to maintain this target range until it is confident that the economy has weathered recent events and is on track to achieve its maximum employment and price stability goals*." (Source).

[24] In moving from the seven-year rolling window ending March 2020 to that ending April 2020, the data from April 2013 drop out. Fortunately, the data and coefficient estimates at this time are highly stable; the monetary shock estimates corresponding to this period were each near-zero, at about 0.01.

4 Macroeconomic Effects

In this section, we use our shock measures as instrumental variables to estimate impulse response functions by OLS in a local projections framework (LP-IV as in Jordà, Schularick, and Taylor (2015)). We favour LP over VAR to contain extrapolation bias, as LP dominate VAR with respect to bias. Given the inherent bias-variance tradeoff, this comes at the cost of higher variance—(much) larger standard errors (compared to the common, mis-specified, small-to-moderate-lag VARs). The two sets of estimates do coincide, however, as increasingly many lags are (appropriately) added to VARs (Montiel Olea, Plagborg-Møller, Qian, and Wolf (2024)).

For comparison to the relevant literature, the system of equations is based on Gertler and Karadi (2015).[25]

$$y_{n,t+h} = \mu_n^h + \psi_n^h r_t + \sum_{\ell=0}^{3} \mathbf{y}_{\mathbf{t}-\ell}^{'} \kappa_{\mathbf{l}}^{\mathbf{h}} + \xi_{nt}^h \tag{52}$$

where $\mathbf{y_t}$ is a vector of n (n=1, 2, ...) endogenous monthly (t) variables: the natural logarithm (ln) of the consumer price index, ln of industrial production, the one-year Treasury yield, and the (updated) Gilchrist and Zakrajšek (2012) Excess Bond Premium, respectively.[26] r_t is the policy variable (the one-year Treasury yield). The lag length is three. Whereas the bias and standard deviation of VAR estimates are heavily dependent on the choice of lag length, this choice matters much less for the Local Projections estimator (which was tested and confirmed). Let $\mathbf{z}_\mathbf{t}^{'} = \left[\widehat{\varepsilon_t^{h=3}}, \widehat{\varepsilon_t^{h=12}}\right]$ be the vector of instrumental variables. The conditions for valid estimation of Eq. 52 are:[27]

1. $\mathbb{E}\left[\xi_{1t}^{h,\perp} \mathbf{z}_\mathbf{t}^{\perp}\right] = \alpha > \alpha^*$ (instrument **relevance** for monetary shock, $\xi_{1t}^{h,\perp}$).
2. $\mathbb{E}\left[\xi_{nt}^{h,\perp} \mathbf{z}_\mathbf{t}^{\perp}\right] = 0$ (**contemporaneous exogeneity** of instrument to other shocks, $\xi_{nt}^{h,\perp}, n \neq 1$).
3. $\mathbb{E}\left[\xi_{n,t+j}^{h,\perp} \mathbf{z}_\mathbf{t}^{\perp}\right] = 0 \quad \forall j{\neq}0$ (exogeneity of instrument to **leads and lags** of all shocks).

where the $\perp$ superscript denotes orthogonalization with respect to control variables, α^* is a vector of critical values, such as those derived in Montiel Olea and Pflueger (2013), When Eq. 52 is estimated by two-stage least squares, as we do in §4.3, the unit effect normalization is imposed automatically (Stock and Watson (2018), p. 924). Condition 3 is testable, and indeed holds, as neither the $\widehat{\varepsilon_t^h}$ nor the shocks from the literature are forecastable using leads or lags of y_{nt}.

4.1 Instrument Relevance

In this section, we provide weak instrument test statistics based on our shock measures and those from the literature. When using monetary instruments to estimate the effects of monetary policy, researchers should, at a minimum, provide Montiel Olea and Pflueger (2013) test statistics and critical values, which are robust to serial correlation, heteroskedasticity, and clustering (*better still* would be to use the Lewis

[25] Bauer and Swanson (2023) also based their model on Gertler and Karadi (2015), although they use the two-year Treasury yield. We favour the one-year yield as this endogenous monetary policy variable is more closely aligned with the structure of all candidate monetary policy instruments. The results are not sensitive to this choice.

[26] We could include more controls in an effort to increase efficiency (by soaking up residual variation in the outcome). Yet, once we control for the variables that are *strong* predictors of either the outcome or the impulse variable, LP estimation is relatively robust to dynamic misspecification (Montiel Olea, Plagborg-Møller, Qian, and Wolf (2025)), so we favour preserving degrees of freedom, and comparability to the literature.

[27] This is an augmented version of the list from Stock and Watson (2018), which is discussed on p. 924.

and Mertens (2025) test[28]). We provide three main results here. First, we find evidence that the forecast-derived perceived monetary policy shocks are not weak instruments (Figure 6).[29] Second, we find clear evidence that high-frequency monetary policy shocks are weak instruments (Figure 6). Third, narrative-based monetary policy shocks (particularly the state-of-the-art Aruoba and Drechsel (2025) variant) appear to lie somewhere in between (Figure 6).

We suggest this is because monetary policy expectations evolve gradually and continuously. Bianchi, Ludvigson, and Ma (2025) "*finds that much of the causal impact of monetary policy on markets occurs outside of tight windows around policy announcements*." Using monthly forecast data that allows for this slow-moving expectation formation process to be reflected in the data allows us to capture more of both the endogenous and exogenous movements in monetary policy expectations.

The weak instruments tests are based on our Local Projections Instrumental Variables estimator (Eq. 52). During estimation, we take steps to adjust for potential bias from weak instruments. Eq. 52 is estimated by limited information maximum likelihood (LIML). LIML is a linear combination of OLS and Two-Stage Least Squares (TSLS) estimates. The weights are chosen so that they (approximately) eliminate any TSLS bias. With weak instruments, LIML has better small-sample properties than TSLS. The just-identified estimates below are approximately (median-) unbiased (or less biased) when using weak instruments.

In the monetary policy shock literature, most authors appear to use the Stock and Yogo (2005) tests for weak instruments, if any. Yet, these tests assume conditionally homoskedastic and serially uncorrelated model errors. Hence, the Stock and Yogo (2005) tests are not robust to heteroskedasticity, serial correlation, and/or clustering, but Montiel Olea and Pflueger (2013) develops tests that are.

That is, most studies test *the null hypothesis of weak instruments* by comparing the commonly-reported yet not heteroskedasticity-robust Cragg-Donald Wald F statistic or the heteroskedasticity-robust Kleibergen-Paap Wald F statistic to the Stock, Wright, and Yogo (2002), Stock and Yogo (2005) critical values (e.g. 8.68, the 5% critical value for testing the null hypothesis that the TSLS bias exceeds 10% of the OLS bias). Despite the popularity of this approach, there is no theoretical justification for its use (Andrews, Stock, and Sun (2019), p. 738). This is because these statistics are not computed using the appropriate estimator of the covariance matrix. That is, the Stock et al. (2002); Stock and Yogo (2005) critical values are based on the assumption of conditionally homoskedastic, and serially uncorrelated model errors. Montiel Olea and Pflueger (2013) show that even the heteroskedasticity-robust (Kleibergen-Paap) F statistic may be too high when instruments are weak.

By contrast, the Montiel Olea and Pflueger (2013) effective F statistic and critical values reported below in Figure 6 are based on Newey and West (1987) errors. This more robust *effective F statistic* is a scaled, corrected version of the non-robust first-stage F statistic; it depends on the robust covariance matrix estimate. The robust *critical values* depend on an estimate of the covariance matrix of the OLS reduced form regression coefficients, and on the covariance matrix of the reduced form errors, which can be estimated using standard procedures (Montiel Olea and Pflueger (2013), p. 358).

[28]This test generalizes the tests of Stock and Yogo (2005) and Sanderson and Windmeijer (2016) to be robust to heteroskedasticity and autocorrelation, and extends the Montiel Olea and Pflueger (2013) robust test for models with a single endogenous regressor to multiple endogenous regressors.

[29]Nor, interestingly, are the residuals from Bauer, Pflueger, and Sunderam (2024) rolling regressions, even if they are unlikely to be exogenous.

The weak instruments test we need to perform differs depending on whether the model is overidentified (i.e. using the two forecaster-perceived reaction function shocks, or combining the Bauer and Swanson (2023) shock, Bauer, Pflueger, and Sunderam (2024) residual, and the Aruoba and Drechsel (2025) shock), or just identified (i.e. using each shock measure one at a time). **In the overidentified case**, we must use the effective F statistic (solid light blue line, Figure 6) and compare this to the Stock and Yogo (2005) critical values (e.g. 8.68, or the rule of thumb of 10). **In the just identified case**, we must again use the effective F statistic,[30] but in this case, compare it to the Montiel Olea and Pflueger (2013) critical values (two dashed lines, Figure 6), which are LIML-specific.[31] When we use the Montiel Olea and Pflueger (2013) effective F statistic and critical values, we are testing the null hypothesis that the estimator's approximate asymptotic bias exceeds a fraction "τ" of a "worst-case" benchmark.[32]

Test Results. Figure 6 shows us that we reject the null hypothesis that the perceived reaction function-implied monetary policy shocks (overidentified model) are weak instruments most of the time, but that the decision is always borderline. So, while our impulse response function estimates in §4.3 are robust to weak instruments problems, inference would still be improved by stronger instruments.

The other three sets of test statistics in Figure 6 pertain to just-identified models. When we use the Bauer, Pflueger, and Sunderam (2024) residuals as an instrument, we always reject the null hypothesis that the TSLS approximate asymptotic bias exceeds 5% of the "worst-case" benchmark with a size of 5%.

For the Bauer and Swanson (2023) shocks, we cannot reject the null (even that bias exceeds 30%). So, when using this instrument, this implies that the distribution of the LP-IV estimator is not centered at ψ_n^h and inference based on conventional IV standard errors is unreliable (Stock and Watson (2018), p. 929), implying the need for weak-instrument-robust inference. That instruments based on high-frequency market surprises may be weak instruments was suggested in Eberly, Stock, and Wright (2020) p. 26. Figure 6 provides further evidence of this. The Aruoba and Drechsel (2025) shocks lie somewhere in between as shown in Figure 6.

Implications for Inference. Should researchers wish to proceed using high-frequency shocks as instruments, then a large and expanding literature shows that special care must be taken when using such potentially weak instruments, particularly with respect to inference (see Stock and Wright (2000), Stock, Wright, and Yogo (2002), Dufour and Taamouti (2005), Mikusheva (2010), Andrews, Stock, and Sun (2019), Ganics, Inoue, and Rossi (2021), Keane and Neal (2023), Barnichon and Mesters (2024), Lewis and Mertens (2025)).

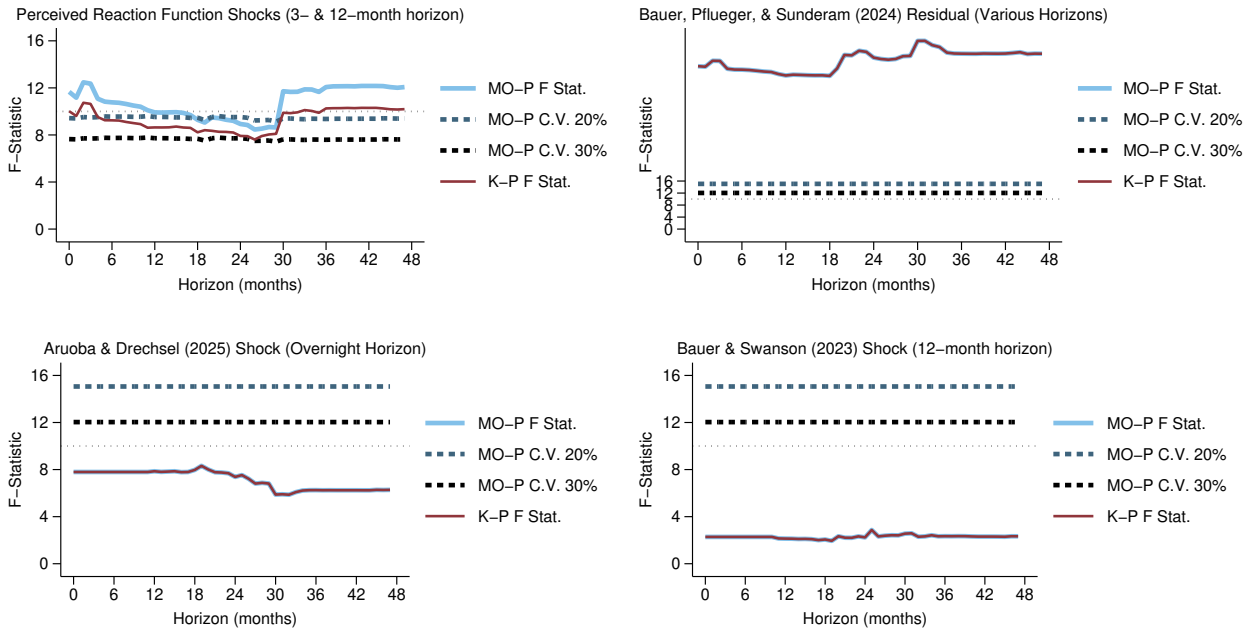
[30] If just identified, the Kleibergen-Paap Wald F statistic equals the effective F statistic.

[31] See Andrews, Stock, and Sun (2019), p. 739.

[32] In Figure 6, one can compare Montiel Olea and Pflueger (2013) critical values for *the null hypothesis that the TSLS approximate asymptotic bias exceeds 10% of the "worst-case" benchmark with a size of 5%* with Stock et al. (2002) critical values for *the null hypothesis that the TSLS bias exceeds 10% of the OLS bias* (for now, the dashed grey line equal to ten).

Figure 6: **Weak Instruments Test Statistics**

Each graph corresponds to a different set of instruments.



MO–P C.V.: Montiel Olea–Pflueger critical values.
Confidence level = 5%, tau = 20% or tau = 30%.
Dotted, grey line: Stock & Yogo (2005) F–Statistic threshold of 10, which is a standard threshold used in the literature.
Sample period (monthly data): 1990m1 to 2026m1.

4.2 Instrument Exogeneity

In this section, we test whether the instruments are exogenous using tests of over-identifying restrictions. The evidence suggests that neither the two forecaster-perceived reaction function shocks, nor the combination of the Bauer and Swanson (2023) shocks, Bauer, Pflueger, and Sunderam (2024) residuals, and the Aruoba and Drechsel (2025) shocks show much sign of endogeneity.

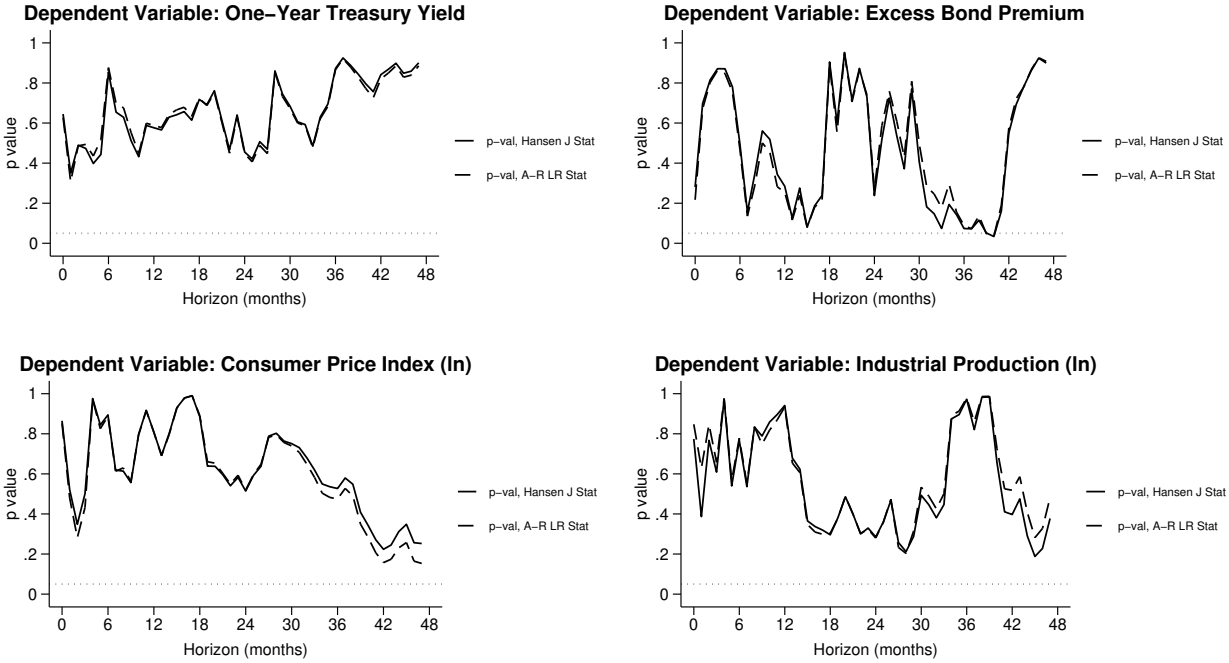
In Figures 7 and 8 below, we plot two sets of statistics.

First, for each horizon, we plot the p values from Hansen J Tests, which test the null hypothesis that all instruments are exogenous (conditional on being relevant)—that they are uncorrelated with the error term and correctly excluded from the structural equation. The Hansen J Test is robust to heteroskedasticity but can be size distorted under weak identification.

Second, for each horizon, we plot the p values from Anderson-Rubin Tests, which (jointly) test the null hypothesis that the instruments are exogenous and that the structural coefficients are identified (whether the instruments are exogenous at a hypothesized value of the structural parameter). The Anderson-Rubin Test maintains proper size under weak identification.

Figure 7: **Tests of Overidentifying Restrictions**

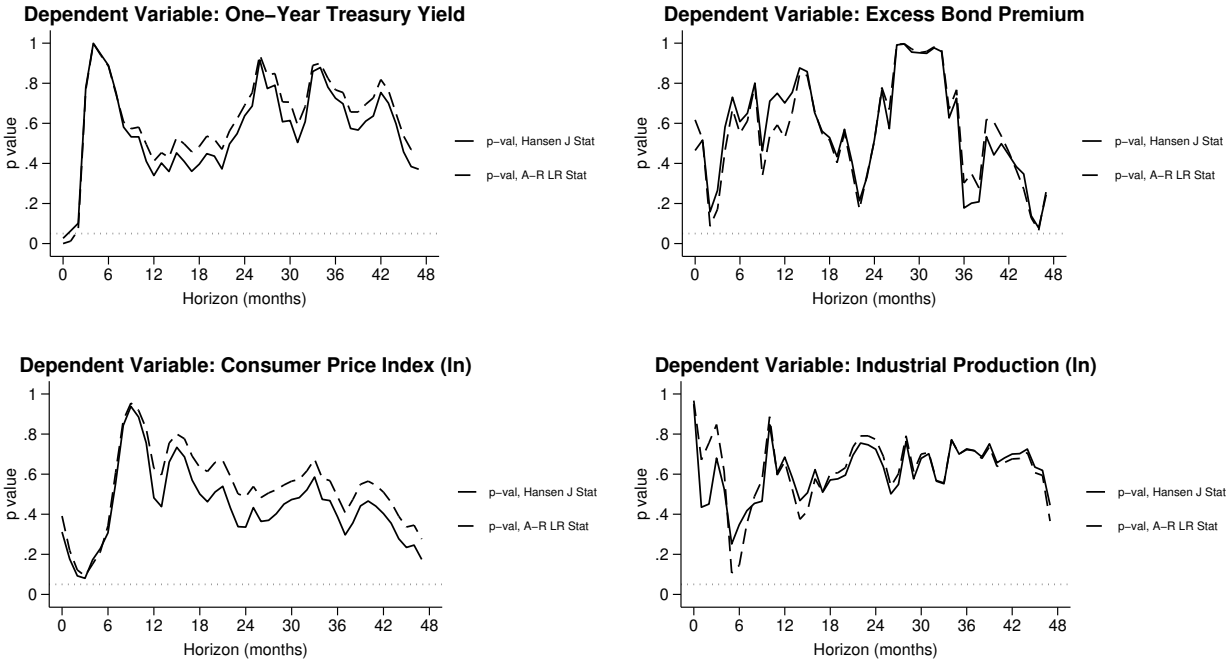
Test statistics based on forecaster-perceived monetary policy shocks instruments.



Dotted, straight line denotes p=0.05. Sample period (monthly data): 1990m1 to 2026m1.
Hansen J−test: robust to heteroskedasticity. Anderson−Rubin test: robust to weak instruments.

Figure 8: **Tests of Overidentifying Restrictions**

Test statistics using a set of Bauer and Swanson (2023), Bauer, Pflueger, and Sunderam (2024), Aruoba and Drechsel (2025) instruments.



Dotted, straight line denotes p=0.05. Sample period (monthly data): 1990m1 to 2026m1.
Hansen J−test: robust to heteroskedasticity. Anderson−Rubin test: robust to weak instruments.

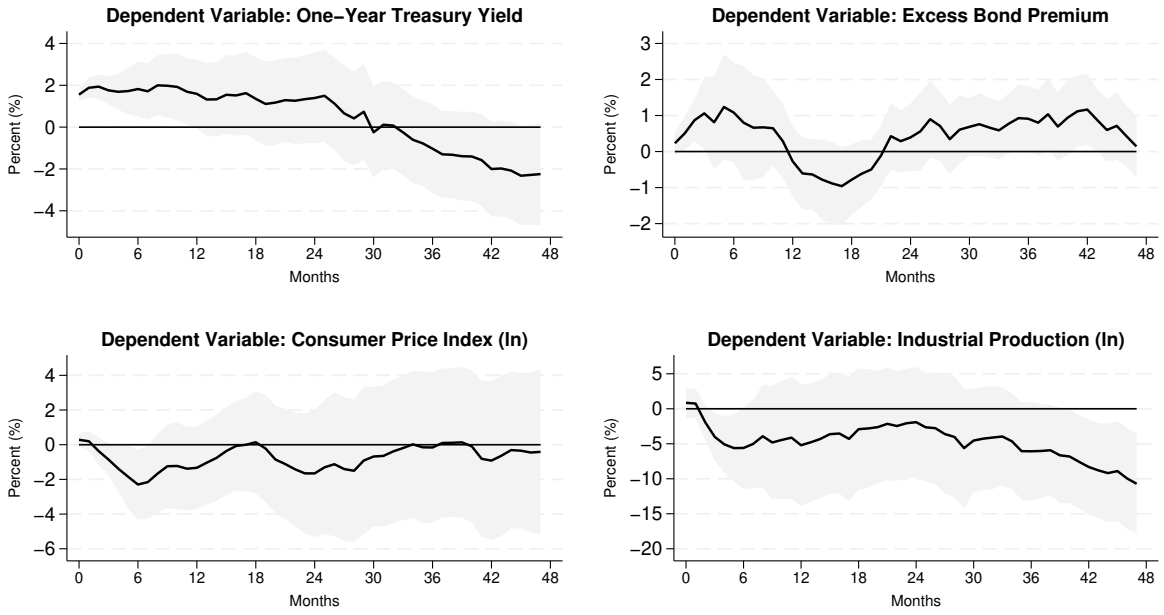
4.3 Impulse Response Functions

Below, we plot the impulse response function estimates associated with each instrument. The estimates created using the forecaster-perceived reaction function-implied monetary policy shocks (Figure 9) are broadly consistent with New Keynesian theory. First, the one-year Treasury yield rises in response to a monetary policy shock. Second, prices tend to fall, although with wide confidence intervals. Third, industrial production tends to fall. The relationship with the Excess Bond Premium is less clear.

The estimates created using the Bauer, Pflueger, and Sunderam (2024) residuals (Figure 10) are broadly similar to those in Figure 9, although with narrower confidence intervals attributable to the stronger instrument. Conversely, the estimates created using the Bauer and Swanson (2023) shocks (Figure 11) have rather wide confidence intervals given the weakness of the instrument, so it is difficult to draw conclusions. The estimates created using the Aruoba and Drechsel (2025) shocks (Figure 12) are also broadly consistent with New Keynesian theory.

Figure 9: **Impulse Response Functions**

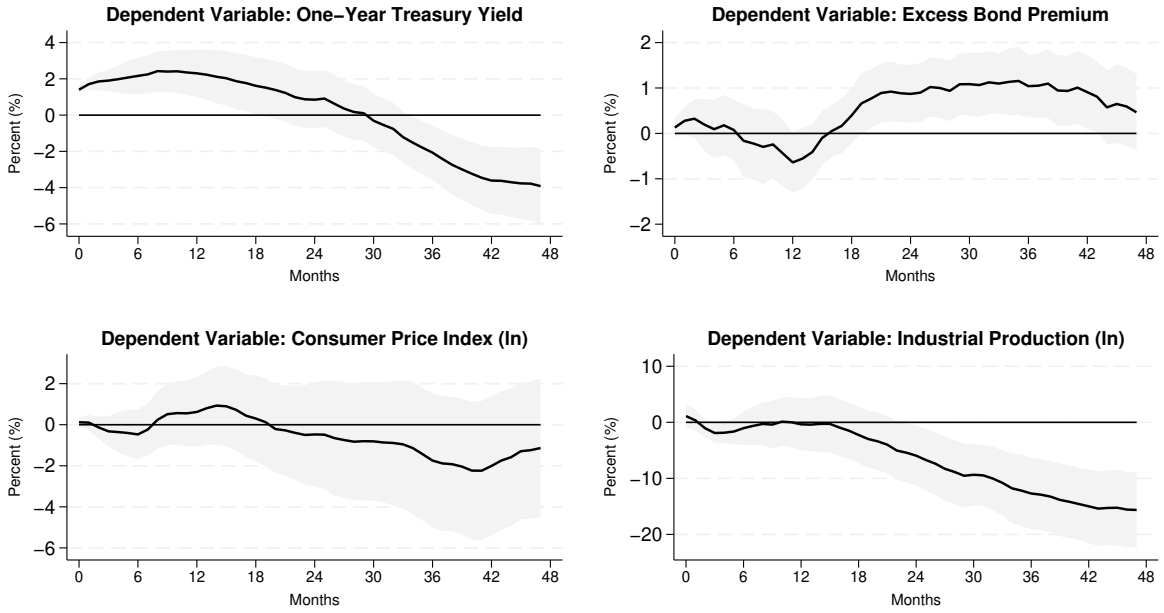
Instruments: perceived monetary policy shocks instruments (based on expected reaction function estimates using forecast data.



90% confidence intervals.
HAC−robust standard errors.
Sample period (monthly data): 1990m1 to 2026m1.

Figure 10: **Impulse Response Functions**

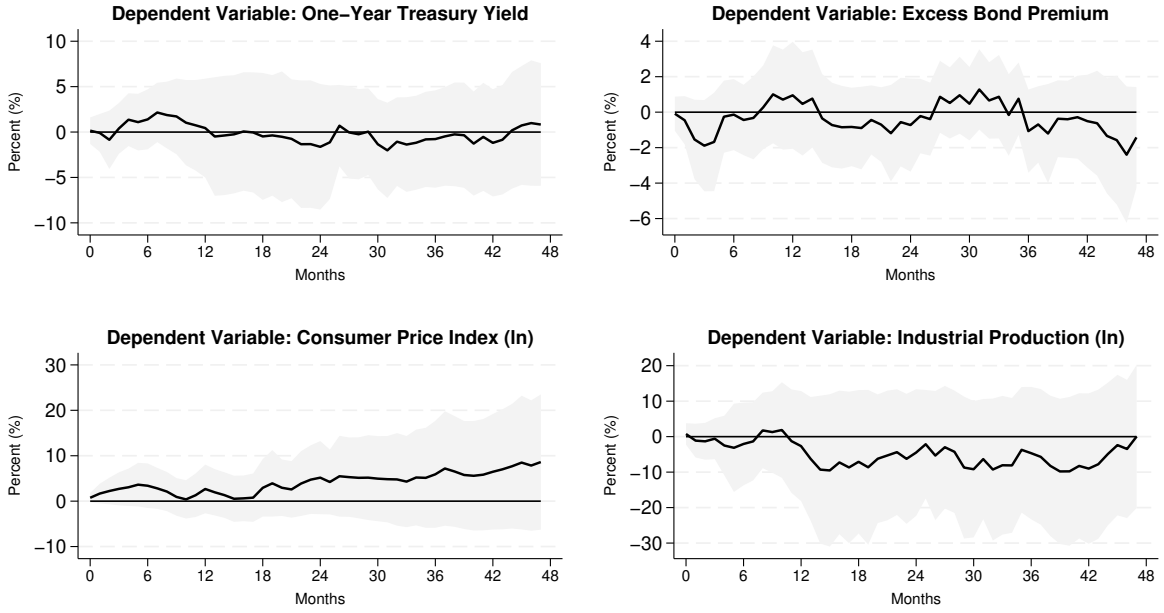
"Instrument": Bauer, Pflueger, and Sunderam (2024) residuals.



90% confidence intervals.
HAC−robust standard errors.
Sample period (monthly data): 1990m1 to 2026m1.

Figure 11: **Impulse Response Functions**

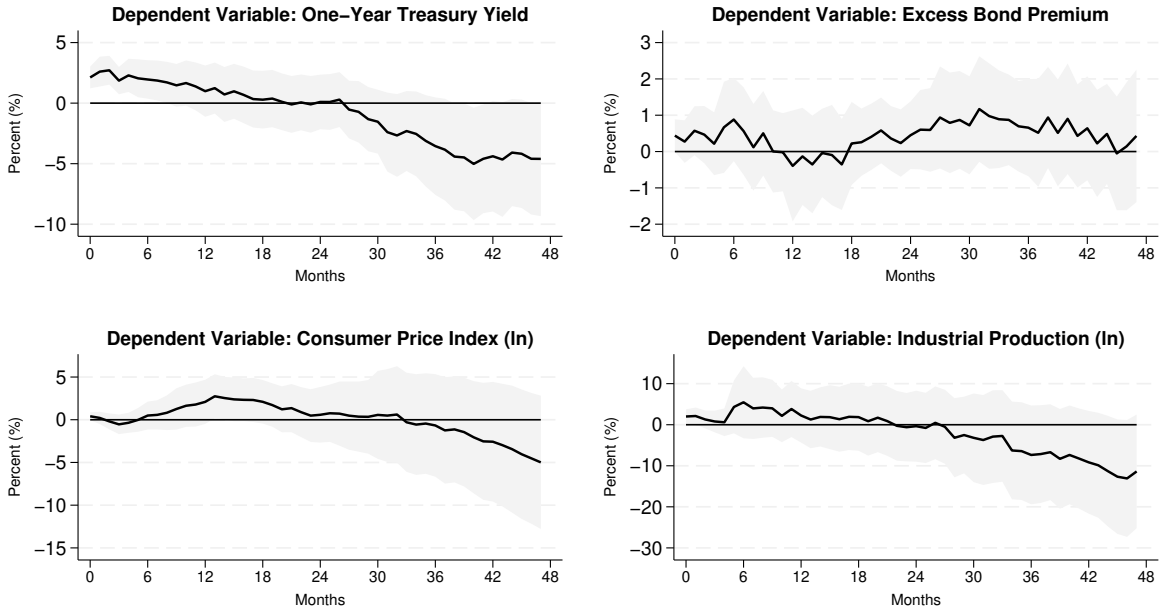
Instrument: Bauer and Swanson (2023) shock measure.



90% confidence intervals.
HAC−robust standard errors.
Sample period (monthly data): 1990m1 to 2026m1.

Figure 12: **Impulse Response Functions**

Instrument: Aruoba and Drechsel (2025) shock measure.



90% confidence intervals.
HAC−robust standard errors.
Sample period (monthly data): 1990m1 to 2026m1.

5 Conclusion

This paper has proposed a number of ideas. First, the reaction function framework incorporates central bank judgment and multidimensional parameter heterogeneity. It also delineates smoothing, neutral rate, information, macro response, and FOMC judgment channels.

Second, we have proposed an identification strategy and econometric framework based on Dynamic Common Correlated Effects for the structural estimation of forecaster-expected central bank reaction functions using forecast data. We hope this may be particularly useful as our framework and code can easily be adapted to other countries—not least because *Consensus Economics* offers standardized data for over 100 countries.

Third, a consequence of our framework is that we can extract numerous monetary shock measures—one for each forecast horizon. We propose to use these measures as instruments with a Local Projections Instrumental Variables estimator to measure the effect of monetary policy on the economy. In doing so, we test the validity of the instrument set. The evidence suggests that the instruments are neither endogenous nor weak. We also provide impulse response function estimates that are consistent with New Keynesian models (with shifting steady states). We draw some broader conclusions for the empirical monetary economics literature.

Monetary policy instrument construction approach I: high-frequency market-based surprises. These are shocks to monetary policy expectations (as perceived by markets), and are a function of the underlying, true FOMC shocks. Instruments constructed using this methodology tend to be weak. As such, all results based on such instruments must be read through the lens of weak instruments (estimates bi-

ased towards OLS with understated standard errors). Fortunately, more recent efforts to incorporate more monetary policy events hold promise (for example the the *U.S. Monetary Policy Event-Study Database* (USMPD) of Acosta, Ajello, Bauer, Loria, and Miranda-Agrippino (2025)).

Monetary policy instrument construction approach II: narrative shocks. These represent underlying, true FOMC shocks. It is likely that the original versions of these instruments (Romer and Romer (2004)) did not fully disentangle endogenous components from exogenous components (Aruoba and Drechsel (2025)). Newer versions of these instruments (Aruoba and Drechsel (2025)) seem to have addressed this problem, perhaps even extracting too much information—not leaving enough exogenous variation—potentially leading to a weak instruments problem. In principle, the approach could straightforwardly be adjusted accordingly to solve any such problem.

Monetary policy instrument construction approach III: forecaster-perceived monetary policy shocks. These are shocks to monetary policy expectations (as perceived by forecasters), and are a function of the underlying, true FOMC shocks. They are extracted by estimating the reaction function implied by short-term interest rate forecasts, and the shocks these imply. These measures appear to balance relevance and exogeneity better than its predecessors, but in principle, all three approaches should be able to accomplish such balance.

Testing monetary policy instruments for relevance and exogeneity. *Relevance*. To test this, either the Montiel Olea and Pflueger (2013) or the Lewis and Mertens (2025) robust tests for weak instruments for 2SLS should be used. *Exogeneity*. It is useful to use numerous monetary policy instruments, not least because we can test the exogeneity of the instrument set. Doing so suggests that all three methodologies listed above can lead to exogenous instruments.

In future work, we plan to relax some of the assumptions made in this paper. First, can we increase the frequency of this analysis to increase its monetary policy practicality by applying the method Ghysels and Wright (2009). Second, what if parameters follow a random walk instead of a random coefficients model? Third, what if coefficients are not known at the time forecasts are made? Fourth, can we improve our Local Projections estimates by, for example, using Smooth or Bayesian Local Projections ? Finally, we plan to adapt our approach to The Bank of Canada, Bank of England, European Central Bank, and to publish these estimates.

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A Reaction Function Framework

A.1 The Federal Open Market Committee Reaction Function

We begin by defining the information set (Ω) of each FOMC ($\sim$) participant, $\tilde{i}$, at time t_n, $\tilde{\Omega}_{\tilde{i}t_n}$.[33] Now, because our focus will eventually be the *forecaster-expected* reaction function, and because financial markets, forecasters, journalists, and the broader public generally focus on the **median FOMC projection**, we abstract from each FOMC member, $\tilde{i}$, and focus on the median FOMC projection. Because each point projection is anonymous and not linked to its other matching point projections, forecasters must try to "*infer a reaction function from committee-wide median economic and interest rate projections that often come from different individuals*." (Goldman Sachs Economics Research, July 1, 2025).

Yet, even if the projections from each FOMC member were linked, "*this information would likely be useful to investors—**knowing the reaction function of the median participant inferred from their linked projections** would have helped to predict monetary policy surprises in the past*." (ibid, emphasis added). In other words, forecasters would still focus on the median participant. So, for simplicity, and without any substantial loss of generality, we will treat all median projections as part of the same set of projections, i.e. having followed the projection process below. As such, we do not need the $\tilde{i}$ subscript.

Next, we define function $\tilde{f}$, a function that maps the information set, $\tilde{\Omega}_{t_n}$ to a conditional expectation, $\tilde{\mathbb{E}}_{t_n}\left[\,.\;\middle|\;\tilde{\Omega}_{t_n}\right]$.[34] This conditional expectation represents an FOMC macroeconomic projection at time t_n.

$$\tilde{f} : \tilde{\Omega}_{t_n} \rightarrow \tilde{\mathbb{E}}_{t_n}\left[\;\mathbf{x_{t_n+h_{t_n}}}\;\middle|\;\tilde{\Omega}_{t_n}\;\right] \tag{53}$$

$\mathbf{x_{t_n+h_{t_n}}}$ denotes a vector of k=4 macroeconomic variables (real GDP growth, unemployment rate, inflation rate, core inflation rate), at some point in the future, $t+h_{t_n}$, where $h_{t_n} \in \{h^0_{t_n}, h^1_{t_n}, h^2_{t_n}, h^3_{t_n}, h^4_{t_n}\}$ denotes a fixed forecast horizon. In March, June, September, and December, The FOMC projects (only) these macroeconomic variables for the current year ($h^0_{t_n}$), one year from now ($h^1_{t_n}$), two years from now ($h^2_{t_n}$), three years from now ($h^3_{t_n}$) (in September and December only), and longer run ($h^4_{t_n}$). As such, the forecast horizon, h_{t_n}, is t_n-dependent because each horizon becomes shorter as the calendar year progresses.

The median FOMC member's information set, $\tilde{\Omega}_{t_n}$, is composed of available macroeconomic data at t_n, the path of the policy rate, i.e. the federal funds rate (FFR), path up to t_n, past FOMC projections, past FOMC judgment, financial market data, market intelligence, staff judgment, and news.[35]

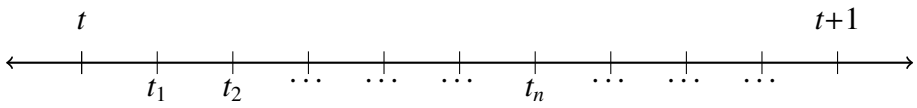
Time. To make sense of evolving information sets (filtrations), shocks, and what is known at the time a projection is made, we must be very careful with the sequence of events. Throughout the paper, t=$\{1, 2, \ldots, T\}$, $t \in \mathbb{N}^+$ is a standard indexing set, which denotes monthly periods, as all data are monthly. Further, let the set t_n=$\{t_1, t_2, \ldots\}$, $n \in \mathbb{N}^+$ denote arbitrarily many sub-periods. The gaps between each t_n correspond to small increments of time, over which information is accumulated, information sets are updated, and new expectations are formed. This would usually happen during the FOMC policy deliberation process, but in principle, it could happen at any time as new information arrives continuously (and forecasters & markets know this). The notation that follows is intricate but necessary—at least until §A.2, at which point we simplify the notation considerably.

[33] Throughout the paper, $\sim$ denotes objects attributable to The FOMC.

[34] The conditional expectation $\tilde{\mathbb{E}}_{t_n}\left[\,.\;\middle|\;\tilde{\Omega}_{t_n}\right]$ is defined as an $\tilde{\Omega}_{t_n}$–measurable function.

[35] We define $\tilde{\Omega}_{t_n}$ as a filtration or sigma-algebra to keep the framework general. This maintains flexibility over possible informational states, which allows us to consider counterfactuals (alternative scenarios) within the model.

Figure 13: **Monthly Period (t) and Sub-Period (t_n) Timeline**



Before the FOMC releases its *Summary of Economic Projections* (SEP), each FOMC participant maps their realized information set ($\tilde{\omega}_{t_n}$) to a vector of point forecasts. For ease of exposition, here FOMC participants start their recursive projection process at t_1:

$$\tilde{f} : \tilde{\omega}_{t_1} \in \tilde{\Omega}_{t_1} \rightarrow \tilde{\mathbb{E}}_{t_1}\left[\mathbf{x}_{\mathbf{t_1}+\mathbf{h}^{\mathbf{0}}_{\mathbf{t_1}}} \;\middle|\; \tilde{\Omega}_{t_1} = \tilde{\omega}_{t_1} \right] \in \mathbb{R}^k \tag{54}$$

This is a recursive process in which the FOMC participant adds these macroeconomic point forecasts to their information set, yielding $\tilde{\Omega}_{t_2}$, where $t_2 > t_1$ and $\tilde{\Omega}_{t_1} \subset \tilde{\Omega}_{t_2}$.[36] Next, as monetary policy is forward-looking (because it operates with a lag), each FOMC participant maps these point macroeconomic projections to a future FFR (i.e. an FFR projection) using $\tilde{g}$, a **monetary policy reaction function**:

$$\tilde{g} : \mathbb{R}^k \rightarrow \mathbb{R} \tag{55}$$

$$\tilde{g} : \tilde{\mathbb{E}}_{t_1}\left[\mathbf{x}_{\mathbf{t_1}+\mathbf{h}^{\mathbf{0}}_{\mathbf{t_1}}} \;\middle|\; \tilde{\Omega}_{t_1} = \tilde{\omega}_{t_1} \right] \rightarrow \tilde{\mathbb{E}}_{t_2}\left[\tilde{r}_{t_2+h^0_{t_2}} \;\middle|\; \tilde{\Omega}_{t_2} = \tilde{\omega}_{t_2} \right] \in \mathbb{R} \tag{56}$$

Or, equivalently:

$$\tilde{g}\left(\tilde{f}\left(\tilde{\omega}_{t_1}\right)\right) = \tilde{g}\left(\tilde{\mathbb{E}}_{t_1}\left[\mathbf{x}_{\mathbf{t_1}+\mathbf{h}^{\mathbf{0}}_{\mathbf{t_1}}} \;\middle|\; \tilde{\Omega}_{t_1} = \tilde{\omega}_{t_1}\right]\right) = \tilde{\mathbb{E}}_{t_2}\left[\tilde{r}_{t_2+h^0_{t_2}} \;\middle|\; \tilde{\Omega}_{t_2} = \tilde{\omega}_{t_2}\right] \in \mathbb{R} \tag{57}$$

Next, before submitting this FFR point projection to The Federal Reserve Board of Governors, the FOMC participant considers whether to add judgment $\tilde{\lambda}_{t_n}$, to the projection.[37] That is, the final projection from Eq. 56 may ultimately differ somewhat from the point projection published as part of the SEP because the FFR projection may be adjusted by **FOMC-expected judgment**, $\tilde{\mathbb{E}}_{t_n}\left[\tilde{\lambda}_{t_n+h_{t_n}} \;\middle|\; \tilde{\Omega}_{t_n} = \tilde{\omega}_{t_n} \right]$.

Given the sources of judgment (discussed in §2), we hypothesize that the law of motion for judgment is a persistent process:

$$\tilde{\mathbb{E}}_{t_n}\left[\tilde{\lambda}_{t_n+h_{t_n}} \;\middle|\; \tilde{\Omega}_{t_n} = \tilde{\omega}_{t_n} \right] = \tilde{\theta}_{t_n}\tilde{\mathbb{E}}_{t_{n-2}}\left[\tilde{\lambda}_{t_{n-2}+h_{t_{n-2}}} \;\middle|\; \tilde{\Omega}_{t_{n-2}} = \tilde{\omega}_{t_{n-2}} \right] + \tilde{\varepsilon}^{h_{t_{n-1}}}_{t_{n-1}}, \qquad \tilde{\varepsilon}^{h_{t_n}}_{t_n} \overset{iid}{\sim} \left(0, \sigma^2_{\tilde{\varepsilon}}\right) \tag{58}$$

$\tilde{\varepsilon}^{h_{t_n}}_{t_n}$ is a shock to FOMC-expected judgment. When h_{t_n}=0, then $\tilde{\mathbb{E}}_{t_2}\left[\tilde{r}_{t_2+h^0_{t_2}} \;\middle|\; \tilde{\Omega}_{t_2} = \tilde{\omega}_{t_2}\right]$ simply corresponds to the policy rate, and so $\tilde{\varepsilon}^{h_{t_n}}_{t_n}$ is a **traditional monetary policy shock**. When h_{t_n}>0, then $\tilde{\varepsilon}^{h_{t_n}}_{t_n}$ is a **monetary policy rate path projection shock**, which plausibly has comparable influence on markets and the economy to the traditional monetary policy shock. Although not formally stated as such, estimating perceptions of $\tilde{\varepsilon}^{h_{t_n}}_{t_n}$ has been the focus of the literature that estimates perceived monetary shocks based on high-frequency interest rate futures price changes (i.e. from Kuttner (2001) to Bauer and Swanson (2023)). This further underscores the importance of clarifying the definition of $\tilde{\varepsilon}^{h_{t_n}}_{t_n}$.

[36] More formally, we add to the increasing filtration $\{\tilde{\Omega}_{t_n}\}$ that reflects growing information over time.

[37] The FOMC participant could have similarly added judgment to their macroeconomic projections of $\mathbf{x}_{\mathbf{t_n}}$ earlier as well, but that is not our focus here.

So, FOMC-expected judgment, or, $\tilde{\mathbb{E}}_{t_n}\left[\ \tilde{\lambda}_{t_n+h_{t_n}}\ \middle|\ \tilde{\Omega}_{t_n}=\tilde{\omega}_{t_n}\ \right] := \tilde{\lambda}_{t_n}^{h_{t_n}}$, for short, is known when the FFR projection is finalized ($\tilde{\Omega}_{t_4}$–measurable s.t. $\tilde{\Omega}_{t_2}\subset\tilde{\Omega}_{t_3}\subset\tilde{\Omega}_{t_4}$).

$$\tilde{\mathbb{E}}_{t_4}\left[\ \tilde{r}_{t_4+h^0_{t_4}}\ \middle|\ \tilde{\Omega}_{t_4}=\tilde{\omega}_{t_4}\ \right]=\tilde{\mathbb{E}}_{t_2}\left[\ \tilde{r}_{t_2+h^0_{t_2}}\ \middle|\ \tilde{\Omega}_{t_2}=\tilde{\omega}_{t_2}\ \right]+\tilde{\mathbb{E}}_{t_3}\left[\ \tilde{\lambda}_{t_3+h^0_{t_3}}\ \middle|\ \tilde{\Omega}_{t_3}=\tilde{\omega}_{t_3}\ \right] \tag{59}$$

Next, the projection produced using Eq. 59 is added to $\tilde{\Omega}_{t_5}$, and the FOMC participant then begins producing longer-horizon ($h^1_{t_n} > h^0_{t_n}$) projections.

$$\tilde{f}:\tilde{\omega}_{t_5}\in\tilde{\Omega}_{t_5}\rightarrow\tilde{\mathbb{E}}_{t_5}\left[\ \mathbf{x}_{\mathbf{t_5}+\mathbf{h}^{\mathbf{1}}_{\mathbf{t_5}}}\ \middle|\ \tilde{\Omega}_{t_5}=\tilde{\omega}_{t_5}\ \right]\in\mathbb{R}^k \tag{60}$$

The FOMC participant then adds these longer-horizon ($h^1_{t_5}$) macroeconomic projections to their updated information set, $\tilde{\omega}_{t_6}$ ($\tilde{\Omega}_{t_5}\subset\tilde{\Omega}_{t_6}$). These projections are then mapped to a more-distant FFR projection:

$$\tilde{g}:\tilde{\mathbb{E}}_{t_5}\left[\ \mathbf{x}_{\mathbf{t_5}+\mathbf{h}^{\mathbf{1}}_{\mathbf{t_5}}}\ \middle|\ \tilde{\Omega}_{t_5}=\tilde{\omega}_{t_5}\ \right]\rightarrow\tilde{\mathbb{E}}_{t_6}\left[\ \tilde{r}_{t_6+h^1_{t_6}}\ \middle|\ \tilde{\Omega}_{t_6}=\tilde{\omega}_{t_6}\ \right]\in\mathbb{R} \tag{61}$$

Once again, FOMC-expected judgment, $\tilde{\mathbb{E}}_{t_7}\left[\ \tilde{\lambda}_{t_7+h^1_{t_7}}\ \middle|\ \tilde{\Omega}_{t_7}=\tilde{\omega}_{t_7}\ \right]$ may then be added.

$$\tilde{\mathbb{E}}_{t_8}\left[\ \tilde{r}_{t_8+h^1_{t_8}}\ \middle|\ \tilde{\Omega}_{t_8}=\tilde{\omega}_{t_8}\ \right]=\tilde{\mathbb{E}}_{t_6}\left[\ \tilde{r}_{t_6+h^1_{t_6}}\ \middle|\ \tilde{\Omega}_{t_6}=\tilde{\omega}_{t_6}\ \right]+\tilde{\mathbb{E}}_{t_7}\left[\ \tilde{\lambda}_{t_7+h^1_{t_7}}\ \middle|\ \tilde{\Omega}_{t_7}=\tilde{\omega}_{t_7}\ \right] \tag{62}$$

The FOMC participant iterates the process until arriving at the *Long-Run Projection* of The FFR and each macroeconomic variable included in each SEP. Turning to **functional form**, following the detailed studies of the functional form of The Fed's monetary policy rule by Rudebusch (2002), Rudebusch (2006), Coibion and Gorodnichenko (2012) and Stein and Sunderam (2018), as well as historical Tealbooks. To simplify notation, **we will suppress the explicit conditioning event and write conditional expectations as** $\tilde{\mathbb{E}}_{t_n}[\cdot]$, where the subscript t_n denotes conditioning on the realized information set $\tilde{\Omega}_{t_n}=\tilde{\omega}_{t_n}$.

Assumption 7. *We assume the forward-looking monetary policy reaction function takes the form:*

$$\begin{aligned}&\tilde{\mathbb{E}}_{t_n}\left[\ \tilde{r}_{t_n+h_{t_n}}\ \right]=\tilde{\mathbb{E}}_{t_{n-1}}\left[\ \tilde{\lambda}_{t_{n-1}+h_{t_{n-1}}}\ \right]+\\&\tilde{\mathbb{E}}_{t_{n-2}}\left[\ \tilde{\rho}^{h_{t_{n-2}}}_{t_{n-2}}(L)\tilde{r}_{t_{n-2}+h_{t_{n-2}}}+\tilde{\phi}^{\pi^*}_{t_{n-2}}\pi^*_{t_n}+\tilde{\phi}^{r^*}_{t_{n-2}}r^*_{t_{n-2}}+\left(\mathbf{x}_{\mathbf{t_{n-2}}+\mathbf{h}_{\mathbf{t_{n-2}}}}-\mathbf{x}^*_{\mathbf{t_{n-2}}}\right)'\tilde{\boldsymbol{\beta}}^{\mathbf{h}_{\mathbf{t_{n-2}}}}_{\mathbf{t_{n-2}}}\ \right]\end{aligned} \tag{63}$$

$\tilde{\rho}^{h_{t_n}}_{t_n}(L)$ is a lag polynomial such that $\tilde{\rho}^{h_{t_n}}_{t_n}(L)\tilde{r}_{t_n+h_{t_n}}=\sum_{\ell=1}^{L}\tilde{\rho}^{\ell}_{t_n}\tilde{r}_{t_n+h_{t_n}-l}$. $\tilde{\rho}^{\ell}_{t_n}$ is a monetary policy gradualism parameter.[38] Accordingly, $\tilde{\phi}^{\pi^*}_{t_n}\propto\left(1-\sum_{\ell=1}^{L}\tilde{\rho}^{\ell}_{t_n}\right)$, $\tilde{\phi}^{r^*}_{t_n}\propto\left(1-\sum_{\ell=1}^{L}\tilde{\rho}^{\ell}_{t_n}\right)$ (as in Bauer et al. (2024)).

$\pi^*_{t_n}\in\mathbb{R}$ is the inflation target, which is set at two percent, but which could, in theory, be time-varying. $r^*_{t_n}\in\mathbb{R}$ is the time-varying central bank estimate of the real neutral policy rate. $\mathbf{x}^*_{\mathbf{t_n}}\in\mathbb{R}^k$ is a vector that matches $\mathbf{x}_{\mathbf{t_n}+\mathbf{h}_{\mathbf{t_n}}}$, which includes the inflation target, and the time-varying, median-FOMC-projected, long-run real GDP growth and unemployment rates, respectively—each in the appropriate place.

[38] Using Bayesian Information Criterion, Coibion and Gorodnichenko (2012) find that a higher-order autoregressive process best captures historical Fed reaction functions. Woodford (2003) shows that the optimal interest rate rule in a New Keynesian model has AR(2) interest rate inertia. In this general form of the reaction function, we are agnostic about the lag order, L.

As an aside, if we simply set h_{t_n}=0, $\tilde{\lambda}^{h}_{t_n}$=0, and the parameters to be constant, then Eq. 63 nests a standard monetary (Taylor) rule—where $\tilde{\mathbb{E}}_{t_n}\left[\mathbf{x_{t_n}}\right]$ is a nowcast. For clarity, we use t instead of t_n here:

$$\tilde{\mathbb{E}}_t\left[\ \tilde{r}_t\ \right] = \tilde{\mathbb{E}}_t\left[\ \tilde{\rho}(L)\tilde{r}_t + \tilde{\phi}^{\pi^*}\pi^*_t + \tilde{\phi}^{r^*}r^*_t + \left(\mathbf{x_t} - \mathbf{x^*_t}\right)'\tilde{\beta}\ \right] \Rightarrow \tag{64}$$

$$\tilde{r}_t = \tilde{\rho}(L)\tilde{r}_t + \tilde{\phi}^{\pi^*}\pi^*_t + \tilde{\phi}^{r^*}r^*_t + \left(\mathbf{x_t} - \mathbf{x^*_t}\right)'\tilde{\beta} \tag{65}$$

where, invariably, a monetary policy shock is added, which implicitly, must follow the projection timing sequence and definitions outlined above:

$$\tilde{r}_t = \tilde{\rho}(L)\tilde{r}_t + \tilde{\phi}^{\pi^*}\pi^*_t + \tilde{\phi}^{r^*}r^*_t + \left(\mathbf{x_t} - \mathbf{x^*_t}\right)'\tilde{\beta} + \tilde{\varepsilon}_t \tag{66}$$

Assumption 8. *Coefficients are known given* $\tilde{\Omega}_{t_n}$ *($\tilde{\Omega}_{t_n}$–measurable).*

This implies that we can pull the coefficients outside the conditional expectation operator. Combining Assumptions 7, 8, and the linear property of the conditional expectation operator implies:

$$\begin{aligned}\tilde{\mathbb{E}}_{t_n}\left[\ \tilde{r}_{t_n+h_{t_n}}\ \right] = \tilde{\rho}^{h_{t_n}}_{t_n}(L)\tilde{\mathbb{E}}_{t_{n-2}}\left[\tilde{r}_{t_{n-2}+h_{t_{n-2}}}\ \right] + \tilde{\phi}^{\pi^*}_{t_n}\tilde{\mathbb{E}}_{t_{n-2}}\left[\ \pi^*\ \right] + \tilde{\phi}^{r^*}_{t_n}\tilde{\mathbb{E}}_{t_{n-2}}\left[\ r^*_{t_{n-2}}\ \right] + \\ \tilde{\mathbb{E}}_{t_{n-2}}\left[\ \left(\mathbf{x_{t_{n-2}+h_{t_{n-2}}}} - \mathbf{x^*_{t_{n-2}}}\right)'\ \right]\tilde{\beta}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} + \tilde{\mathbb{E}}_{t_{n-1}}\left[\ \lambda^{h_{t_{n-1}}}_{t_{n-1}}\ \right]\end{aligned} \tag{67}$$

Assumption 9. *The coefficients in Equation 67 follow a random coefficients model.*

Bauer, Pflueger, and Sunderam (2024) instead assume a random walk model, which implies the need to use Kalman Filtering. Their Kalman Filtering estimation results are very similar to those from their baseline approach, and their baseline approach is similar to our approach below. More specifically, $\tilde{\beta}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} \in \mathbb{R}^k$ is a vector of time-varying (and also projection-horizon-specific), macro-response coefficients:

$$\tilde{\beta}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} = \tilde{\beta} + \tilde{\upsilon}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} \tag{68}$$

where $\tilde{\upsilon}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} \in \mathbb{R}^k$ is a vector of macro-response coefficient shocks, where $\tilde{\upsilon}^{h_{t_n}}_{t_n} \sim i.i.d.\left(0, \sigma^{\mathbf{2}}_{\tilde{\upsilon}}\right)$.[39] The source of $\tilde{\upsilon}^{h_{t_n}}_{t_n}$ could be a new committee participant, changing preferences, uncertainty, or inconsistency.

Forward guidance (FG) provides additional information about the FOMC's current reaction function(s) to the public *beyond* the monetary policy committee's macroeconomic projections. FG is broader than just FOMC-expected judgment, $\tilde{\lambda}^{h_{t_n}}_{t_n}$, only. To understand how FG works in practice, in theory, and to make policy recommendations, it is crucial that we account for its multi-faceted nature, rather than conflating distinct signals. FG takes one of the following four forms:

1. A **quantitative policy rate projection**, $\tilde{\mathbb{E}}_{t_n}\left[\tilde{r}_{t_n+h_{t_n}}\right]$ (left-hand side of Eq. 63), which is time-contingent FG, i.e., the FOMC Fed Funds Rate (FFR) projections released quarterly in The Summary of Economic Projections (SEP).

[39] Time-varying coefficients depart from many models in this literature, such as the noisy information model of Miranda-Agrippino and Ricco (2021), but are consistent with Bauer and Swanson (2023), Bauer, Pflueger, and Sunderam (2024), and Hack, Istrefi, and Meier (2023).

2. **Qualitative judgment** $\left(\tilde{\mathbb{E}}_{t_n}\left[\ \tilde{\lambda}_{t_n+h_{t_n}}\ \right]\right)$ delivered as qualitative FG, e.g., "*The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period.*" (source).[40]

3. An updated description of the **response to macroeconomic projections** in the reaction function:

$$\Delta_n \tilde{\beta}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} = \tilde{\beta}_{\mathbf{t_n}} - \tilde{\beta}_{\mathbf{t_{n-1}}} = \left(\tilde{\beta} + \tilde{\upsilon}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}}\right) - \left(\tilde{\beta} + \tilde{\upsilon}^{\mathbf{h_{t_{n-1}}}}_{\mathbf{t_{n-1}}}\right) = \Delta_n \tilde{\upsilon}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} \tag{69}$$

   (from Eq. 63 and 68), which is state-contingent FG, e.g. FG indicating a stronger response to inflation post-pandemic, such as "*The Committee is strongly committed to returning inflation to its 2 percent objective.*" (source).[41]

4. New information about the **functional form of the reaction *function***, $\tilde{g}^{h_{t_n}}_{t_n}$ e.g. COVID information being de facto added to the reaction function: "*The Committee will continue to monitor the implications of incoming information for the economic outlook, including **information related to public health**, as well as global developments and muted inflation pressures, and will use its tools and act as appropriate to support the economy.*" (source).[42]

The information channel is separate and is represented by either quantitative or qualitative updates to the central bank's macroeconomic projections (*excluding* policy rate projections), particularly $\Delta_n \tilde{\mathbb{E}}_{t_n}\left[\mathbf{x_{t_n+h_{t_n}}}\right] = \tilde{\mathbb{E}}_{t_n}\left[\mathbf{x_{t_n+h_{t_n}}}\right] - \tilde{\mathbb{E}}_{t_{n-1}}\left[\mathbf{x_{t_{n-1}+h_{t_{n-1}}}}\right]$. The concept of the information channel dates back to Romer and Romer (2000) at least.

> "*The key idea is that information the Federal Reserve has about the economy that is not known to market participants is likely to be reflected in the Federal Reserve's internal forecasts.*" "*[The paper] demonstrates that the Federal Reserve has considerable **information about inflation** beyond what is known to commercial forecasters. It also shows that monetary-policy actions provide signals of the Federal Reserve's information and that commercial forecasters modify their forecasts in response to those signals.*" (Romer and Romer (2000), p.429).

The idea has since been carried forward, "*Following Romer and Romer (2000), we refer to the effect of FOMC announcements on private-sector views of non-monetary economic fundamentals as "Fed information effects.*"" Nakamura and Steinsson (2018), p.1286.[43] Several papers have adjusted interest rate-based measures for information effects. Romer and Romer (2004) use Federal Reserve Greenbook forecasts to control for information effects, as do Miranda-Agrippino and Ricco (2021).

$\Delta_n \tilde{\mathbb{E}}_{t_n}\left[\mathbf{x_{t_n+h_{t_n}}}\right]$ is driven by: new macroeconomic data, new information about how the projections are formed, FOMC judgment added to the macro projections.[44] Although new macroeconomic data

[40] $\tilde{\lambda}^{h_{t_n}}_{t_n}$ may be either qualitative or time-contingent FG, and less likely, state-dependent FG; it may be either Delphic or Odyssean FG (Sutherland (2023) elaborates on types of FG).

[41] So, here, $\Delta_n \tilde{\upsilon}^{\mathbf{h_{t_n}}}_{\mathbf{t_n}} > 0$. The update could, in principle, be quantitative also, but monetary policy committees, to date, do not provide real-time quantitative monetary policy reaction functions.

[42] We **exclude** information about the central bank's macroeconomic projections. This is a subtle yet crucial distinction, and one that must be made to differentiate between state-dependent FG and information effects.

[43] Information effects are studied in, e.g., Bauer and Swanson (2023), Hoesch, Rossi, and Sekhposyan (2023).

[44] Byrne, Goodhead, McMahon, and Parle (2023) delve even deeper into the information channel and provide a complementary framework for conceptualizing and analyzing economic assessment updates.

are public information, the central bank does not always explain which of these three factors drove $\Delta_n \tilde{\mathbb{E}}_{t_n}\left[\mathbf{x_{t_n+h_{t_n}}}\right]$, and by how much. So, the public usually cannot disentangle which of the three channels drove $\Delta_n \tilde{\mathbb{E}}_{t_n}\left[\mathbf{x_{t_n+h_{t_n}}}\right]$. Further, some central banks, such as The European Central Bank, Bank of England, and Reserve Bank of Australia use financial market-implied policy rate paths to produce their macroeconomic projections, so the information and signal content of those projections is different.

Ideally, we would estimate Eq. 63 directly. Essentially, Aruoba and Drechsel (2025) present a natural-language approach for doing something similar with h_{t_n}=0, and assuming constant parameters. Yet, at present, we do not have enough data to do so for h_{t_n}>0. We would if we have complete *Summary of Economic Projections* panel data attributable to each FOMC participant, but there are substantial gaps.[45] We *might* if we chose to pool data over forecast horizon h (as in González-Astudillo and Tanvir (2023)), but that would not be consistent with our framework. Instead, we adopt a strategy that affords us more data: we estimate the forecaster-expected reaction function using monthly forecaster data.

A.2 The Expected Central Bank Reaction Function

A crucial distinction is the true versus *perceived* central bank reaction function. It is the perceived reaction function from which most of the monetary shocks series in the literature are derived. That is, most papers derive or use monetary policy shocks from high-frequency shifts in the expected path of monetary policy as implied by interest rate futures and swaps. As such, we could also derive monetary policy shocks from shifts in the expected path of monetary policy as judged by professional forecasters, or even the public. The key reason to use perceived shocks is that we observe monetary expectations data at higher frequencies (monthly, daily, continuously) than FOMC decisions (eight times per year).

Following a large literature on the systematic violations of full information rational expectations shown by professional forecasters (e.g. Coibion and Gorodnichenko (2015), Bordalo, Gennaioli, Ma, and Shleifer (2020), Bianchi, Ilut, and Saijo (2024)), forecaster expectation formation is not assumed to be consistent with rational expectations. Further, if the forecasts were rational, they should have the same time series properties as the underlying data being forecasted, but they do not ($\mathcal{A}$). So, we assume that forecasters use a simple, linear model, which makes the parameters of interest identifiable.

Time. To begin, we take expectations of Eq. 63 with respect to the information set of a forecaster, $\ddot{\Omega}_{it}$, at the next monthly forecasting submission deadline, t+1, which by definition (Figure 13), always occurs after t_n. $\ddot{\mathbb{E}}_{it}\left[. \;\middle|\; \ddot{\Omega}_{it}\right]$ is individual i's ($i = 1, ..., N$) expectation conditional on their information set, $\ddot{\Omega}_{it}$.[46] We use the double dots to refer to a forecaster expectation. So, the forecaster-expected central bank reaction function is:[47]

$$\begin{aligned}&\ddot{\mathbb{E}}_{i,t+1}\left[\;\tilde{\mathbb{E}}_{t_n}\left[\;\tilde{r}_{t_n+h_{t_n}}\;\right]\;\middle|\;\ddot{\Omega}_{i,t+1}\right] = \ddot{\mathbb{E}}_{i,t+1}\left[\;\tilde{\mathbb{E}}_{t_{n-1}}\left[\;\tilde{\lambda}_{t_{n-1}+h_{t_{n-1}}}\;\right]\;\middle|\;\ddot{\Omega}_{i,t+1}\;\right]+\\ &\ddot{\mathbb{E}}_{i,t+1}\left[\;\tilde{\mathbb{E}}_{t_{n-2}}\left[\;\tilde{\rho}^{h_{t_n}}_{t_{n-2}}(L)\tilde{r}_{t_{n-2}+h_{t_{n-2}}}+\tilde{\phi}^{\pi^*}_{t_{n-2}}\pi^*_t+\tilde{\phi}^{r^*}_{t_{n-2}}r^*_{t_{n-2}}+\left(\mathbf{x_{t_{n-2}+h_{t_{n-2}}}}-\mathbf{x^*_{t_{n-2}}}\right)'\tilde{\boldsymbol{\beta}}^{\mathbf{h_{t_{n-2}}}}_{\mathbf{t_{n-2}}}\;\right]\;\middle|\;\ddot{\Omega}_{i,t+1}\;\right]\end{aligned} \tag{70}$$

[45] Such data are available from 2007 to 2012, and from 2016 to 2020 only (Arai (2023)).

[46] If the function that forms $\ddot{\Omega}_{i,t+1}$ were $f^{\ddot{\Omega}}: \tilde{\Omega}_t \to \ddot{\Omega}_{i,t+1}$, then we could simplify the conditioning set in Eq. 71 and write $\ddot{\mathbb{E}}_{i,t+1}\left[\tilde{r}_{t+h} \;\middle|\; \ddot{\Omega}_{i,t+1}\right]$ by the Law of Iterated Expectations, but it is more likely that $f^{\ddot{\Omega}}: \tilde{\Omega}_t \bigcup \Omega^i_t \to \ddot{\Omega}_{i,t+1}$, where Ω^i_t is a set of private signals. Caballero and Simsek (2022) suggests that central banks partially accommodate market views, so we can further postulate $f^{\tilde{\Omega}}: \ddot{\Omega}_{i,t+1} \bigcup \Omega^c_t \to \tilde{\Omega}_t$, where Ω^c_t is a set of private central bank signals.

[47] We treat the function $\tilde{\mathbb{E}}_{t_n}\left[\;.\;\middle|\tilde{\Omega}_t\right]$ as a measurable random variable with respect to the outer sigma-algebra $\sigma\left(\ddot{\Omega}_{i,t+1}\right)$—consistent with the information asymmetry across agents.

Now, The FOMC releases its projections every three months, but in principle, an FOMC participant could update their projection at any time t_n (formally or informally). For example, if a major U.S. trade agreement were to be agreed some time after the latest SEP was released, then the FOMC participant may begin adjusting their growth projections in response (at least mentally, if not formally by updating a projections model). Forecasters know this.

Further, forecasters may also update their forecasts at any time, but must submit them to *Consensus Economics* and *Blue Chip*, respectively, once per month. Now, an FOMC projection at t_n could happen immediately before the next private forecast submission deadline at t+1—essentially coinciding with t+1. In that case, a FOMC projection at t_n would represent the latest state of the projection at t+1. We take the stance that, while forming their own forecasts, forecasters indeed have a view on the latest state of The FOMC SEP given recent news.

As such, we are interested in precisely the case in which t_{n-2}, t_{n-1}, t_n, and t+1 occur very close together in time. So, we can now abstract from the t_n sub-periods and their notation, and instead focus on monthly forecasting periods t with the knowledge that the projection formation process still allows for a causal effect of FOMC projections on private forecasts, and that our reaction function framework appropriately nests information sets and is consistent with measure theory such that it can be mapped to our econometric model.[48] And so we can simplify the notation in Eq. 70 as follows:

$$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ \tilde{r}_{t+h_t}\ \right]\ \right] = \ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ \tilde{\rho}_t^{h_t}(L)\tilde{r}_{t+h_t} + \tilde{\phi}_t^{\pi^*}\pi_t^* + \tilde{\phi}_t^{r^*}r_t^* + \left(\mathbf{x_{t+h_t}} - \mathbf{x_t^*}\right)'\tilde{\boldsymbol{\beta}}_{\mathbf{t}}^{\mathbf{h_t}}\ \right]\ \right] + \ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ \tilde{\lambda}_{t+h_t}\ \right]\ \right] \tag{71}$$

Forecasters are assumed to form their projections and to update their information sets in a manner consistent with the FOMC (Equations 53 to 68), which will be important to bear in mind as we consider forecaster-expected FOMC judgment later on (Eq. 74).

Assumption 10. *The coefficients are known given* $\ddot{\Omega}_{it}$ *($\ddot{\Omega}_{it}$–measurable). This implies that:*

$$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ \left(\mathbf{x_{t+h_t}} - \mathbf{x_t^*}\right)'\ \right]\ \tilde{\boldsymbol{\beta}}_{\mathbf{t}}^{\mathbf{h_t}}\ \right] = \ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ \left(\mathbf{x_{t+h_t}} - \mathbf{x_t^*}\right)'\ \right]\ \right]\ \ddot{\boldsymbol{\beta}}_{\mathbf{it}}^{\mathbf{h_t}} \tag{72}$$

This implies that we can pull the coefficients outside the conditional expectation operator. This means that each forecaster knows the coefficients of their own perceived, forward-looking reaction function. To simplify subsequent derivations somewhat, we use the following notation for forecaster's coefficients:

$$\ddot{\boldsymbol{\beta}}_{\mathbf{it}}^{\mathbf{h_t}} := \ddot{\mathbb{E}}_{it}\left[\ \tilde{\boldsymbol{\beta}}_{\mathbf{t}}^{\mathbf{h_t}}\ \right] \tag{73}$$

By the linearity of the expectation operator, individuals can form a separate expectation for each component of Eq. 71. We consider each in turn below.

> "*Do market participants accept at face value what the FOMC declares about future policy, or do they form their own inferences about likely FOMC policy from other clues in the statements? More important, do their forecasts of future funds rate change because the statement changes their beliefs about the FOMC's reaction function, or because it changes their forecast of economic conditions that are expected to determine FOMC policy change, as a result of inferences they make from the statement about information available to the FOMC*?" (Michael Woodford in Campbell et al. (2012), p.66).

[48] This framework could also apply to market participants, who may also update their expectations at any point.

The framework above, below (§A.3), and its accompanying identification strategy (§2), allow us to differentiate between not only these channels, but more. That is, we can identify whether forecasters' "*forecasts of the future funds rate change because the statement changes their beliefs about the FOMC's reaction function*," (the perceived parameters): ρ_{it}^{ℓ}, $\ddot{\beta}_{\mathbf{it}}^{\mathbf{h}}$, and ε_{it}^{h}, "*or because it changes their forecast of economic conditions*" (the information channel): $\ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ \left(\mathbf{x_{t+h}} - \mathbf{x_t^*}\right)'\ \right]\ \right]$.

$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\rho}_t^{h_t}(L)\ \tilde{\mathbb{E}}_t\left[\ \tilde{r}_{t+h_t}\ \right]\ \right]$ is **forecaster-expected monetary policy *gradualism***. That is, it is the monetary policy gradualism that is expected to prevail at time $t+h_t$. As above, we use the following notation for brevity: $\rho_{it}^{h_t}(L) := \ddot{\mathbb{E}}_{it}\left[\ \tilde{\rho}_t^{h_t}(L)\ \right]$.

$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ r_{t+h_t}^*\ \right]\ \right]$ is the **forecaster-expected FOMC real neutral rate projection**. If we treat the "Longer-run" FFR projection from The SEP as $\tilde{\mathbb{E}}_t\left[\ r_{t+h_t}^*\ \right]$, then forecasters receive updates quarterly.

$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\mathbb{E}}_t\left[\ \left(\mathbf{x_{t+h_t}} - \mathbf{x_t^*}\right)'\ \right]\ \right]$ is a **vector** (bolded term) containing the forecaster-expected set of FOMC macroeconomic projections. Because The FOMC provides macroeconomic projections on a quarterly basis in the SEP, these figures are known at those times. At all other times, forecasters must process information to form private views about how FOMC participants may have revised their projections. $\pi_{\mathbf{t+h_t}}^*$ is a vector also of length k that includes median long-run real GDP growth from the SEP, median long-run unemployment rate from the SEP, and the inflation target (or median long-run inflation growth from the SEP as they coincide).

$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\beta}_{\mathbf{t}}^{\mathbf{h_t}}\ \right] := \ddot{\beta}_{\mathbf{it}}^{\mathbf{h_t}}$ is the vector of forecaster-expected central bank macroeconomic projection response coefficients. Estimates of these coefficients are almost certainly heterogeneous because so little quantifiable information about them is provided by The FOMC. The FOMC occasionally provides implicit information about these coefficients' values in the form of qualitative or state-contingent forward guidance. Yet, central banks seldom articulate their reaction functions precisely enough to provide estimates of these coefficients.[49] Dispersion in forecaster-expected macro response coefficients is hypothesized to be a principal driver of the observed heterogeneity in forecasters' perceived policy rate projections.

Forecaster-expected FOMC judgment, our main focus, by Eq. 58, is:

$$\ddot{\mathbb{E}}_{i,t+1}\left[\ \tilde{\mathbb{E}}_{t_n}\left[\ \tilde{\lambda}_{t_n+h_{t_n}}\ \right]\ \right] = \ddot{\mathbb{E}}_{i,t+1}\left[\ \tilde{\mathbb{E}}_{t_n}\left[\ \tilde{\theta}_{t_n}\tilde{\mathbb{E}}_{t_{n-2}}\left[\ \tilde{\lambda}_{t_{n-2}+h_{t_{n-2}}}\ \right] + \tilde{\varepsilon}_{t_{n-1}}^{h_{t_{n-1}}}\ \right]\ \right] \tag{74}$$

Using our shorthand notation, $\tilde{\lambda}_{t_n}^{h_{t_n}} := \tilde{\mathbb{E}}_{t_n}\left[\ \tilde{\lambda}_{t_n+h_{t_n}}\ \right]$, and assumptions thus far:

$$\ddot{\mathbb{E}}_{i,t+1}\left[\ \tilde{\lambda}_{t_n}^{h_{t_n}}\ \right] = \ddot{\mathbb{E}}_{i,t+1}\left[\ \tilde{\theta}_{t_n}\tilde{\lambda}_{t_{n-2}}^{h_{t_{n-2}}} + \tilde{\varepsilon}_{t_{n-1}}^{h_{t_{n-1}}}\ \right] \tag{75}$$

But, as above, we want to aggregate to a monthly version to match our data and better understand what

[49] There are exceptions. The Bank of Canada specified the monetary policy rule from its main DSGE model used to generate its economic projections (§1.3) along with corresponding parameter estimates (§2.2.2) here. To provide information about the historical pattern of its monetary policy reaction function, The Norges Bank included estimates of a simple interest rate rule in its December 2021 Monetary Policy Report here.

we will ultimately estimate:[50]

$$\tilde{\lambda}^{h}_{t_n} = \tilde{\theta}_{t_n}\tilde{\lambda}^{h}_{t_{n-2}} + \tilde{\varepsilon}^{h}_{t_{n-1}} \tag{76}$$

$$= \tilde{\theta}_{t_n}\left(\tilde{\theta}_{t_{n-2}}\tilde{\lambda}^{h}_{t_{n-4}} + \tilde{\varepsilon}^{h}_{t_{n-3}}\right) + \tilde{\varepsilon}^{h}_{t_{n-1}} \tag{77}$$

$$= \tilde{\theta}_{t_n}\tilde{\theta}_{t_{n-2}}\tilde{\lambda}^{h}_{t_{n-4}} + \tilde{\theta}_{t_n}\tilde{\varepsilon}^{h}_{t_{n-3}} + \tilde{\varepsilon}^{h}_{t_{n-1}} \tag{78}$$

$$\vdots$$

$$= \bar{\theta}_{t_n}\tilde{\lambda}^{h}_{t_{n-2R}} + \sum_{r=1}^{R}\tilde{\theta}^{r-1}\tilde{\varepsilon}^{h}_{t_{n-(2r-1)}} \quad \text{where} \quad \bar{\theta}_{t_n} := \tilde{\theta}_{t_n}\cdot\tilde{\theta}_{t_{n-2}}\cdot\ldots \tag{79}$$

where R denotes the total number of sub-periods within a given month. This yields the monthly-level law of motion:

$$\tilde{\lambda}^{h_t}_{t} = \bar{\theta}_t\tilde{\lambda}^{h_{t-1}}_{t-1} + \bar{\varepsilon}^{h_t}_{t}, \quad \text{where} \quad \bar{\theta}_t := \tilde{\theta}_{t_n}\cdot\tilde{\theta}_{t_{n-2}}\cdot\ldots, \quad \bar{\varepsilon}^{h_t}_{t} := \sum_{r=1}^{R}\tilde{\theta}^{r-1}\tilde{\varepsilon}^{h}_{t_{n-(2r-1)}} \tag{80}$$

Now we take expectations with respect to $\ddot{\Omega}_{it}$ such that t and t_n (from Eq. 80) coincide:

$$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\lambda}^{h_t}_{t}\ \right] = \ddot{\mathbb{E}}_{it}\left[\ \bar{\theta}_t\tilde{\lambda}^{h_{t-1}}_{t-1} + \bar{\varepsilon}^{h_t}_{t}\ \right] \Rightarrow \tag{81}$$

$$\ddot{\mathbb{E}}_{it}\left[\ \tilde{\lambda}^{h_t}_{t}\ \right] = \theta_{it}\ddot{\mathbb{E}}_{it}\left[\ \tilde{\lambda}^{h_{t-1}}_{t-1}\ \right] + \ddot{\mathbb{E}}_{it}\left[\ \bar{\varepsilon}^{h_t}_{t}\ \right] \tag{82}$$

And, finally, employing shorthand notation once again $\left(\ddot{\lambda}^{h_t}_{it} := \ddot{\mathbb{E}}_{it}\left[\ \tilde{\lambda}^{h_t}_{t}\ \right]\right)$:

$$\ddot{\lambda}^{h_t}_{it} = \theta_{it}\ddot{\lambda}^{h_{t-1}}_{i,t-1} + \varepsilon^{h_t}_{it} \tag{83}$$

The interpretation of which is that forecaster-expected FOMC judgment is the sum of the perceived judgment persistence times past forecaster-expected FOMC judgment, and the cumulated forecaster-expected FOMC judgment shocks that occurred in the sub-periods ($(t-1)_n$) between month $t-1$ and month t. $\varepsilon^{h_t}_{it}$ **a cumulated variant of the monetary policy shocks that are extracted from high-frequency changes in market prices**, as in Kuttner (2001) and the many improvements since then. Indeed, it is $\varepsilon^{h_t}_{it}$ that most interests us in this paper. **The perceived reaction function-implied monetary policy shocks that we will extract from the data, test, and use are**:

$$\varepsilon^{h_t}_{it} := \ddot{\mathbb{E}}_{it}\left[\ \bar{\varepsilon}^{h_t}_{t}\ \right] = \ddot{\mathbb{E}}_{it}\left[\ \sum_{r=1}^{R}\tilde{\theta}^{r-1}\tilde{\varepsilon}^{h}_{t_{n-(2r-1)}} \ \middle|\ \tilde{\Omega}_{it} = \tilde{\omega}_{it}\ \right] \tag{84}$$

Interpretation of the Instrument. The estimated forecaster-perceived monetary policy shocks, $\varepsilon^{h_t}_{it}$, are a measurable function of the underlying, true FOMC judgment shocks, $\tilde{\varepsilon}^{h}_{t_{n-(2j-1)}}$, filtered through the forecaster's (realized) information set, $\tilde{\Omega}_{it} = \tilde{\omega}_{it}$. Specifically, the perceived shocks are the conditional expectation of a weighted sum of the true shocks, where the weights reflect the perceived persistence of judgment.

Econometrically, $\varepsilon^{h_t}_{it}$ can be interpreted as an instrument that satisfies the relevance condition by construction, as it is a non-degenerate function of the underlying FOMC FFR projection shocks. Under the maintained assumption that forecaster-perceived FOMC judgment shocks are conditionally exogenous

[50] For simplicity, we use h in this derivation instead of h_{t_n} as we will ultimately aggregate to the monthly level, t.

with respect to other determinants of private interest rate expectations, the instrument satisfies the exclusion restriction. That is, its only effect on the private interest rate forecast operates through the perceived FOMC projection shocks. As such, it recovers a projected version of the structural monetary policy shocks that are visible and influential to private forecasters.

More formally, the instrument is an *informational projection* of the structural FOMC judgment shocks onto the forecaster's sigma-algebra, $\ddot{\sigma}(\tilde{\Omega}_{it})$. This projection maps the full-information shock process —measurable with respect to a richer, higher-dimensional sigma-algebra due to its inclusion of more sources of information relevant to the structural shocks and finer temporal resolution (t_n rather than t), such as internal deliberations, confidential briefings, model outputs, and real-time data updates—onto a coarser, lower-dimensional sub-σ-algebra that reflects the forecaster's limited access to those FOMC-relevant dimensions. The projection filters out the unobservable components of the structural shocks and retains only those features that are measurable with respect to the forecaster's information set, that is, the components forecasters can potentially observe or infer and that influence FOMC participants' FFR projections.

A.3 Mapping The Expected Reaction Function to the Forecast Data

Assumption 11. ***Factor Loadings***. *The factor loadings* $\mathbf{\Gamma_i}$ *are independently and identically distributed across i, and of the common factors,* $\mathbf{f_t}$*, for all i and t, with mean* $\mathbf{\Gamma}$.

Assumption 12. ***Weakly Exogenous Regressors***. $\mathbf{x_{it}^H}$ *is a vector of weakly exogenous regressors.*

A section in the online appendix will cover why the regressors are weakly exogenous. In the meantime, we refer the reader to Carvalho, Nechio, and Tristao (2021), Sutherland (2023), and Bauer, Pflueger, and Sunderam (2024), which we elaborate on below..

First, Carvalho, Nechio, and Tristao (2021) argue in favor of OLS estimation of policy rules. The authors show analytically in a three-equation New Keynesian model that the asymptotic OLS estimation bias is proportional to the fraction of the variance of regressors due to monetary policy shocks. Yet, they argue that since there is ample evidence that such shocks explain only a small fraction of the variance of the regressors typically included in estimation of monetary policy rules (and those included in this paper), their analytical finding suggests that any endogeneity bias is likely to be small.

Second, Sutherland (2023) addresses the potential endogeneity concerns due to the simultaneity between interest rate forecasts and macroeconomic forecasts, such as inflation or growth. Yet, because monetary policy is assumed to operate with a lag, and especially when the interest rate forecast horizon actually exceeds the inflation and growth forecast horizons (as is the case with the interest rate forecasts at the 12-month forecast horizon), the paper argues that there should be very little risk of endogeneity due to this simultaneity, which applies here for the same reasons.[51]

Third, Bauer, Pflueger, and Sunderam (2024) build on Carvalho, Nechio, and Tristao (2021) by using a model that captures the potential endogeneity bias described by Carvalho, Nechio, and Tristao (2021) and show that such bias is unlikely to affect the their main results.

[51]The corollary is that there may be omitted variable bias due to omitted long-term inflation expectations, but we observe that, in the data, the (lower frequency, quarterly) longer-term inflation expectations do not vary much.

Assumption 13. ***Individual-Specific Errors***. *The individual specific errors, ε_{it}, which are forecaster-perceived monetary policy rate path shocks are independently distributed of* $\mathbf{v}^{\mathbf{h}}_{\mathbf{jt}'}$ $\quad \forall i, j, t, t'$. *The vector of errors,* $\varepsilon_{\mathbf{t}} = \{\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt}\}$ *is spatially correlated.*

Lemma A.1.

First, consider the term of interest, $\Delta\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right]$, in levels, $\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right]$, and expand using Eq. 8 while applying the substitutions discussed above:

$$\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] = \rho_{it}^{h-1}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-2}\right]\right] + \mathbf{x}_{\mathbf{it}}^{\mathbf{H-1}\prime}\beta_{\mathbf{it}}^{\mathbf{h-1}} + \phi_{it}^{r^*} r_t^* + \phi_{it}^{\pi^*}\pi_t^* + \upsilon_{\mathbf{it}}^{\mathbf{H-1}\prime}\ddot{\beta}_{\mathbf{it}}^{\mathbf{h-1}} + \lambda_{it}^{h-1} \tag{85}$$

Now, to match the equation of interest, take first differences (and apply the product rule to the first two terms on the right-hand side):

$$\begin{aligned}\Delta\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] = \rho_{it}^{h-1}\Delta\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-2}\right]\right] + \Delta\rho_{it}^{h-1}\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-2}\right]\right] + \Delta\mathbf{x}_{\mathbf{it}}^{\mathbf{H-1}\prime}\beta_{\mathbf{it}}^{\mathbf{h-1}} + \mathbf{x}_{\mathbf{it}}^{\mathbf{H-1}\prime}\Delta\beta_{\mathbf{it}}^{\mathbf{h-1}} + \\ \Delta\left(\phi_{it}^{r^*} r_t^*\right) + \Delta\left(\phi_{it}^{\pi^*}\pi_t^*\right) + \Delta\left(\upsilon_{\mathbf{it}}^{\mathbf{H-1}\prime}\ddot{\beta}_{\mathbf{it}}^{\mathbf{h-1}}\right) + \Delta\lambda_{it}^{h-1}\end{aligned} \tag{86}$$

To simplify the derivation, we define:

$$R_{it}^{h-1} := \ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right] \tag{87}$$

Then the differenced equation above becomes:

$$\Delta R_{it}^{h-1} = \rho_{it}^{h-1}\Delta R_{it}^{h-2} + \Delta\rho_{it}^{h-1}R_{it}^{h-2} + \mathcal{S}_{it}^{h-1} + \Delta\lambda_{it}^{h-1} \tag{88}$$

Where, for compactness, we have defined the structural-shock block:

$$\mathcal{S}_{it}^{h-1} := \Delta\mathbf{x}_{\mathbf{it}}^{\mathbf{H-1}\prime}\beta_{\mathbf{it}}^{\mathbf{h-1}} + \mathbf{x}_{\mathbf{it}}^{\mathbf{H-1}\prime}\Delta\beta_{\mathbf{it}}^{\mathbf{h-1}} + \Delta\left(\phi_{it}^{r^*} r_t^*\right) + \Delta\left(\phi_{it}^{\pi^*}\pi_t^*\right) + \Delta\left(\upsilon_{\mathbf{it}}^{\mathbf{H-1}\prime}\ddot{\beta}_{\mathbf{it}}^{\mathbf{h-1}}\right) \tag{89}$$

If we apply the same structure one horizon back:

$$\Delta R_{it}^{h-2} = \rho_{it}^{h-2}\Delta R_{it}^{h-3} + \Delta\rho_{it}^{h-2}R_{it}^{h-3} + \mathcal{S}_{it}^{h-2} + \Delta\lambda_{it}^{h-2} \tag{90}$$

And then substitute ΔR_{it}^{h-2} into the first recursion:

$$\Delta R_{it}^{h-1} = \rho_{it}^{h-1}\left[\rho_{it}^{h-2}\Delta R_{it}^{h-3} + \Delta\rho_{it}^{h-2}R_{it}^{h-3} + \mathcal{S}_{it}^{h-2} + \Delta\lambda_{it}^{h-2}\right] + \Delta\rho_{it}^{h-1}R_{it}^{h-2} + \mathcal{S}_{it}^{h-1} + \Delta\lambda_{it}^{h-1} \tag{91}$$

Proceeding recursively j steps yields:[52]

$$\Delta R_{it}^{h-1} = \sum_{j=0}^{h-1}\left(\prod_{k=0}^{j-1}\rho_{it}^{h-1-k}\right)\left(\mathcal{S}_{it}^{h-1-j} + \Delta\lambda_{it}^{h-1-j}\right) + \sum_{j=0}^{h-1}\left(\prod_{k=0}^{j-1}\rho_{it}^{h-1-k}\right)\Delta\rho_{it}^{h-1-j}R_{it}^{h-2-j} \tag{92}$$

Now we also substitute for the R_{it}^{h-2-j} term, where in general:[53]

$$R_{it}^{m} = \rho_{it}^{m}R_{it}^{m-1} + \phi_{it}^{r^*} r_t^* + \phi_{it}^{\pi^*}\pi_t^* + \mathbf{x}_{\mathbf{it}}^{\mathbf{m+h_x}\prime}\beta_{\mathbf{it}}^{\mathbf{m}} + \upsilon_{\mathbf{it}}^{\mathbf{m+h_x}\prime}\ddot{\beta}_{\mathbf{it}}^{\mathbf{m}} + \lambda_{it}^{m} \tag{93}$$

When we recursively substitute out R_{it}^{m}, we get:

$$R_{it}^{m} = \sum_{\ell=0}^{m}\left(\prod_{q=0}^{\ell-1}\rho_{it}^{m-q}\right)\left(\phi_{it}^{r^*} r_t^* + \phi_{it}^{\pi^*}\pi_t^* + \mathbf{x}_{\mathbf{it}}^{\mathbf{m-\ell+h_x}\prime}\beta_{\mathbf{it}}^{\mathbf{m-\ell}} + \upsilon_{\mathbf{it}}^{\mathbf{m-\ell+h_x}\prime}\ddot{\beta}_{\mathbf{it}}^{\mathbf{m-\ell}} + \lambda_{it}^{m-\ell}\right) \tag{94}$$

[52] Note: when j=0, the product is empty i.e. equals 1.

[53] Recall that H=h+h_x.

Which we now substitute into the Eq. above:

$$\Delta R_{it}^{h-1} = \sum_{j=0}^{h-1}\left(\prod_{k=0}^{j-1}\rho_{it}^{h-1-k}\right)\left(\mathcal{S}_{it}^{h-1-j}+\Delta\lambda_{it}^{h-1-j}\right)+\sum_{j=0}^{h-1}\left(\prod_{k=0}^{j-1}\rho_{it}^{h-1-k}\right)\Delta\rho_{it}^{h-1-j}\left(\sum_{\ell=0}^{h-2-j}\left(\prod_{q=0}^{\ell-1}\rho_{it}^{h-2-j-q}\right)\right.$$
$$\left.\left(\phi_{it}^{r^*}r_t^*+\phi_{it}^{\pi^*}\pi_t^*+\mathbf{x_{it}^{h-2-j-\ell+h_x}}'\beta_{\mathbf{it}}^{\mathbf{h-2-j-\ell}}+\upsilon_{\mathbf{it}}^{\mathbf{h-2-j-\ell+h_x}}{}'\ddot{\beta}_{\mathbf{it}}^{\mathbf{h-2-j-\ell}}+\lambda_{it}^{h-2-j-\ell}\right)\right. \quad (95)$$

We substitute in Eq. 19:

$$\Delta R_{it}^{h-1} = \sum_{j=0}^{h-1}\left(\prod_{k=0}^{j-1}\rho_{it}^{h-1-k}\right)\left(\mathcal{S}_{it}^{h-1-j}+\Delta\lambda_{it}^{h-1-j}\right)+\sum_{j=0}^{h-1}\left(\prod_{k=0}^{j-1}\rho_{it}^{h-1-k}\right)\Delta\rho_{it}^{h-1-j}\left(\sum_{\ell=0}^{h-2-j}\left(\prod_{q=0}^{\ell-1}\rho_{it}^{h-2-j-q}\right)\right.$$
$$\left.\left(\phi_{it}^{r^*}r_t^*+\phi_{it}^{\pi^*}\pi_t^*+\left(\mathbf{c}_{xi}+\mathbf{\Gamma_{it}^{h-2-j-\ell+h_x}f_t}+\mathbf{v_{it}^{h-2-j-\ell+h_x}}\right)'\beta_{\mathbf{it}}^{\mathbf{h-2-j-\ell}}+\upsilon_{\mathbf{it}}^{\mathbf{h-2-j-\ell+h_x}}{}'\ddot{\beta}_{\mathbf{it}}^{\mathbf{h-2-j-\ell}}+\lambda_{it}^{h-2-j-\ell}\right)\right. \quad (96)$$

We can now re-write this more compactly in order to substitute Eq. 97 below back into Eq. 23:

$$\Delta\ddot{\mathbb{E}}_{it}\left[\tilde{\mathbb{E}}_t\left[\tilde{r}_{t+h-1}\right]\right]=\Delta R_{it}^{h-1}=\varphi_{it}^{r^*}r_t^*+\varphi_{it}^{\pi^*}\pi_t^*+\underbrace{\mathbf{\Psi_{it}^{h}f_t}+\mathbf{\check{\Psi}_{it}^{h}\Delta f_t}}_{\text{Macro shocks}}+\underbrace{\Xi_{it}^{h}}_{\substack{\text{Composite}\\ \text{Error}}} \quad (97)$$

Interpretation.

- $\mathbf{\Psi_{it}^{h}}$ collects all the persistence-weighted (ρ terms) loadings on the common macro factors ($\mathbf{f_t}$) arising from $\mathbf{x_{it}^{H}}$ (through β, $\Delta\phi$, $\mathbf{\Gamma}$, $\mathbf{\Delta}\beta$ terms).
- $\mathbf{\check{\Psi}_{it}^{h}}$ collects the persistence-weighted (ρ terms) loadings (ϕ, $\mathbf{\Gamma}$, β terms) on common macro factor innovations ($\mathbf{\Delta f_t}$); these arise through both factor innovations and slow parameter drift and are expected to be small when coefficients evolve smoothly.
- $\varphi_{it}^{r^*}$ and $\varphi_{it}^{\pi^*}$ collect persistence-weighted loadings on r_t^* and π_t^*, respectively.
- Ξ_{it}^{h} is a composite error term collecting the remaining components of the expansion. These terms arise from interactions between persistence changes, parameter drift, discretionary adjustments, and idiosyncratic belief shocks.

Importantly, the elements entering Ξ_{it}^{h} **are products of quantities that are either bounded or small**:

1. persistence parameters $\rho_{it} \in (0, 1)$ (bounded).
2. changes in coefficients ($\Delta\rho_{it}$, $\Delta\beta_{it}$, $\Delta\phi_{it}$), which are assumed to evolve smoothly over time (small).
3. forecaster-specific deviations υ_{it} and judgment adjustments $\Delta\lambda_{it}$, which are idiosyncratic and not systematically related to the common macro factors (small).

Consequently, Ξ_{it}^{h} consists primarily of higher-order interaction terms (e.g., products of coefficient changes, persistence changes interacting with residual components, and similar cross-products). Under smooth parameter evolution and bounded persistence, these terms are second-order in magnitude relative to the macro-shock components and therefore can be treated as a small composite disturbance.

This simplification allows the expression to be substituted back into the main equations without introducing unnecessary algebraic clutter, while preserving the first-order macro drivers of forecast revisions.