Equipment, Structures, and the Limits of Investment-Specific Technological Change*

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Abstract

The declining relative price of equipment has long been viewed as a primary driver of long-run growth through investment-specific technological change (ISTC). We document that this view overlooks a critical countervailing force: the rising relative price of structures. We establish four facts: (i) high-income countries face sharply rising structures prices despite falling equipment prices; (ii) investment rates in equipment and structures respond negatively to their own prices; (iii) equipment prices predict income growth more strongly than structures prices; and (iv) U.S. structures price increases are broad-based across categories. Using KLEMS data, we further show that about half of the rise in construction-sector relative prices since 1996 is explained by declining TFP. We develop a two-capital endogenous growth model where structures enter both production and R&D along with equipment. Calibrating to the U.S. economy, we find that ISTC contributes 0.82 percentage points to annual per-capita growth in net terms, reflecting an equipment boost of 1.32 percentage points partially offset by a structural drag of 0.50 percentage points. This structural drag operates through three channels: reduced capital deepening in production, higher R&D facility costs, and tighter economy-wide resources for innovation. Counterfactual analysis shows that the U.S. growth shortfall relative to other high-income economies is driven almost entirely by the structures margin. Our findings help explain why rapid equipment ISTC has not translated into faster aggregate growth.

Keywords: Equipment, Structures, Investment-Specific Technological Change, Endogenous Growth
JEL Codes: O30, O40, E22

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1 Introduction

The decline in the relative price of equipment is one of the most robust features of the postwar U.S. economy. Since the seminal analysis of Greenwood et al. (1997), economists have viewed this trend as the signature of investment-specific technological change (ISTC) and a primary driver of long-run growth. By attributing productivity gains to the production of new, more efficient capital goods, this framework accounts for the major part of aggregate growth. Implicit in this consensus, however, is the assumption that the price of equipment is a sufficient statistic for the cost of capital, or that other investment goods share similar efficiency gains.

But treating all capital as the same misses an important difference in how production actually works. Capital formation involves two very different inputs: equipment to do the work, and structures to house that work. While the cost of equipment decreased sharply, the cost of building the physical structures has not dropped in the same way globally. This raises a key question: if machines get cheaper but the buildings needed to hold them get more expensive, does the cost of structures eventually drag down potential growth?

We argue that the divergence in capital prices creates a drag that significantly dampens the aggregate gains from investment-specific technological change. Utilizing data from the Penn World Table (PWT), the International Comparison Program (ICP), and the U.S. Bureau of Economic Analysis (BEA), we document four stylized facts.

First, we document a striking divergence in price trends based on income levels. While high-income countries benefit from rapidly falling equipment prices, they face sharply rising structures prices relative to low-income countries. In the United States, the relative price of structures has risen particularly steeply, outpacing the general upward trend observed in other high-income nations. Second, we find that real investment rates in both sectors respond negatively to their own price levels. Third, we show that lower equipment prices are strongly associated with higher output per capita and total factor productivity (TFP), whereas structures prices show a much weaker or insignificant association with these development indicators. Finally, focusing on the U.S., we observe that the post-1970 decline in equipment price was driven largely by information technology, whereas the post-1990 rise in structures price was broad-based across all categories, suggesting an economy-wide shift in the cost of producing structures.

To shed light on why the relative price of structures has risen, we decompose construction-sector output prices using KLEMS data. At the aggregate level, TFP growth largely absorbs input-cost pressure. In construction, by contrast, weak TFP amplifies rather than

offsets rising costs. From 1996 to 2020, about half of the cumulative increase in the construction sector's relative output price is explained by declining total factor productivity. This pattern extends beyond the United States. We document similar productivity shortfalls in the construction sectors of Japan, the United Kingdom, Germany, and France.

To quantify the implications of this imbalance, we develop an endogenous growth model with two capital stocks. Unlike standard models where capital is aggregated or structures are assumed to be technologically neutral, our framework requires both equipment and structures for final production and R&D. In other words, our model introduces two types of capital (equipment and structures) in the "lab equipment" formulation of endogenous growth Rivera-Batiz and Romer (1991) and quantify the consequences of the ISTC both in equipment and structures. We characterize the balanced growth path and derive "growth multipliers" that weight the contribution of each capital type.

We find that the rising relative price of structures acts as a drag on growth, both by reducing capital deepening and by restricting the resource available for innovation. The structural drag operates through two distinct channels. First, the direct input channel arises because structures (such as laboratories and offices) are physical inputs in knowledge production. Slower efficiency growth in this sector directly raises the cost of R&D facilities. Second, the indirect resource channel operates through the final-goods constraint. Sluggish productivity in the structures sector dampens aggregate output growth and capital accumulation, thereby tightening the resources available for innovation.

We calibrate the model to the U.S. economy. Quantitatively, our model since 1970 yields an average per-capita output growth of 1.46 percentage points annually.[1] Of this, ISTC contributes 0.82 percentage points in net terms, reflecting a substantial equipment boost of 1.32 percentage points partially offset by a structural drag of 0.50 percentage points, while the remaining 0.64 percentage points arise from the population component. This "structural drag" operates through three channels: reduced capital deepening in production (0.38 percentage points), higher R&D facility costs (0.06 percentage points), and tighter economy-wide resources for innovation (0.06 percentage points).

A natural concern is that the secular rise in the relative price of structures may partly reflect deflator mismeasurement rather than a true decline in construction productivity. We address this using estimates from Garcia and Molloy (2025), who quantify the potential bias through three approaches, yielding estimates ranging from near zero to an upper bound of 0.8 percentage points per year. Even under this most generous estimate, the correction reduces the baseline structural drag from 0.50 to approximately 0.10 percentage

[1] The actual annual average growth rate was 1.69%.

points, attenuating but not eliminating its quantitative importance.

We perform a counterfactual analysis to assess the global implications of these trends. We feed the historical relative price series of representative high-income (HIC) and low-income economies (LIC) into our calibrated model. We find that the counterfactual U.S. economy with the representative high-income economies' price series generates higher ISTC-induced growth than the United States. The gap driven almost entirely by the severe "structural drag" in the U.S. sector. Of the 0.35 percentage points gap in ISTC-induced GDP per capita growth rate between the HIC counterfactual (1.17 percentage points) and the U.S. baseline (0.82 percentage points), 82.9% (0.29 percentage points) is explained by the structures margin.[2] Conversely, relative to the counterfactual U.S. economy with the representative low-income economies' price series, the United States still achieves higher growth. While low-income countries benefit from the absence of a structural drag, this stability is insufficient to outweigh the substantial "equipment boost" enjoyed by the U.S.. Relative to the LIC counterfactual, the U.S. 0.10 percentage points advantage in GDP per capita growth rate reflects a 0.56 percentage points equipment advantage that is offset by a 0.46 percentage points larger structural drag.

Finally, we extend the model to allow for CES aggregation of equipment and structures in both production and R&D. The magnitude of the structural drag depends on the elasticity of substitution between the two capital types. If equipment and structures are complements, firms cannot easily substitute away from structures when their relative cost rises, amplifying the drag. If they are substitutes, firms can shift toward cheaper equipment, attenuating the drag.[3] Our Cobb-Douglas baseline which imposes unit elasticity represents a middle ground between these cases.

Our work connects to four areas of research. First, we build upon the foundational literature on investment-specific technological change (ISTC). Greenwood et al. (1997) established the relative price of equipment as a sufficient statistic for technological progress in the investment sector, attributing the bulk of postwar growth to the decline in this price. We depart from this framework by explicitly modeling the heterogeneity between equipment and structures. While we confirm their finding regarding the efficiency gains in equipment, we document that the rising relative price of structures acts as a significant offsetting force.[4]

[2]Because the population component is held fixed across scenarios, this gap also equals the difference in total per-capita growth rates (1.81 versus 1.46 percentage points).

[3]This resembles Baumol's cost disease (Baumol, 1967), where the extent of aggregate productivity slowdown depends on how easily the economy can substitute away from sectors with low productivity growth toward sectors with high productivity growth.

[4]The investment-specific technological change (ISTC) has also been used as an explanation for a wide

Second, our framework builds on the "lab-equipment" formulation of endogenous growth (Rivera-Batiz and Romer, 1991), in which capital goods are direct inputs into R&D. In this class of models, ISTC unambiguously lowers the user cost of research capital, so the collapsing price of equipment should have fueled an explosion in idea production. We demonstrate why this has not occurred: the rising relative price of structures constitutes a countervailing force that raises R&D facility costs and tightens the resources available for innovation.[5]

Third, we contribute to the literature on relative prices and development accounting. Regarding relative prices, Jones (1994) documents a negative relationship between the relative price of capital and the rate of economic growth. We confirm this relationship using extended data and build a model that replicates it. Regarding development accounting, Hsieh and Klenow (2007) argue that low real investment rates in poor countries stem from the high relative price of investment goods. We refine this view by decomposing the investment bundle. We document that the dynamics of capital prices differ systematically by development level. Low-income countries have experienced a slower decline in equipment prices but also a slower increase in structures price compared to advanced economies. This implies a dual effect on global inequality. While the equipment channel favors divergence as rich nations benefit more from falling technology costs, the structure channel acts as a leveling force as economies at the technological frontier are disproportionately penalized.

Finally, we provide a macroeconomic transmission mechanism for recent micro-founded evidence on construction sector productivity. Goolsbee and Syverson (2023) document a secular decline in physical value-added per worker in the U.S. structures industry and call it "strange and awful path". Using KLEMS data, we show that this productivity decline is the primary driver of rising construction-sector relative prices, both in the United States and in other major economies. Our model then demonstrates that this decline constitutes a binding constraint on the aggregate economy, effectively bottling up the potential gains from rapid decline in equipment prices.

range of macro trends, including the increasing skill premium (Krusell et al., 2000) and the falling labor share (Karabarbounis and Neiman, 2014).

[5]Our approach is also related to Foerster et al. (2022), who quantify the role of particular sectors in trend GDP growth. While they employ a model with capital accumulation and network structure, we address this question within an endogenous growth framework.

2 Data and Stylized Facts

2.1 Data

Our primary panel dataset for cross-country analysis is constructed by merging two sources: the International Comparison Program (ICP) and the Penn World Table (PWT). We work at the country-year level and focus on the 1950–2019 period, which provides the broadest coverage of the disaggregated investment price series required for our analysis. For the analysis of the United States, we use the Fixed Assets Account tables from the Bureau of Economic Analysis (BEA). The definition of capital and the coverage of each dataset used in our analysis are discussed in Appendix A.

The core explanatory variables are the prices of investment goods by asset type. We obtain these from the World Bank's ICP, which reports Purchasing Power Parities (PPPs) for detailed expenditure categories. Rather than relying on aggregate investment price indices, we extract PPP series for two distinct capital categories: equipment[6] and structures.[7] This disaggregation is central to our analysis because it allows us to trace heterogeneous movements in relative prices across capital types and to link them to differential ISTC. In the empirical simulations, we proxy γ_E and γ_S with the long-run log growth rates of the corresponding ICP relative-price series.

Measures of aggregate economic performance and productivity are taken from PWT version 11.0 (Feenstra et al., 2015). We use output-side real GDP at current PPPs (`cgdpo`) as our main measure of aggregate output and the TFP series (`ctfp`) to characterize productive efficiency. To account for cross-country differences in labor quality, we include the human capital index (`hc`). We complement this measure with the PWT Labor Detail file, which provides more granular information on educational attainment. In particular, we use average years of schooling (`yr_sch`) primarily from Barro and Lee (2013). Finally, we use total population (`pop`) to control for scale.

2.2 Stylized Facts

Fact 1: High-income countries benefit from falling equipment prices but face sharply rising structures price relative to low-income countries.

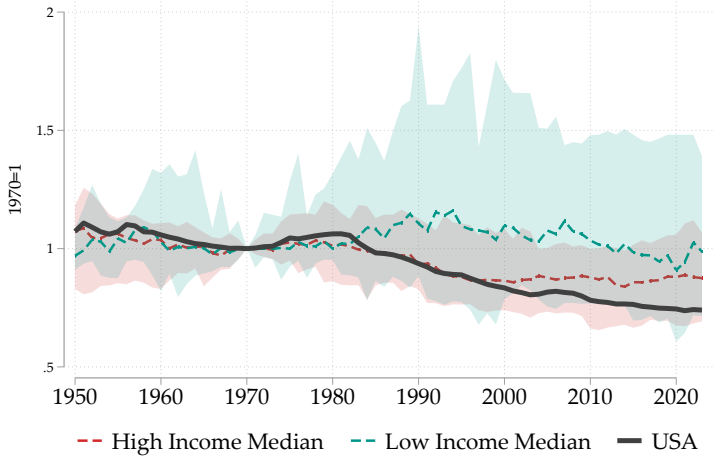
[6] We use the term 'equipment' consistently throughout. In the underlying data, related categories appear under varying labels (e.g., 'Machine and Equipment', 'equipment'); we harmonize these into 'equipment'.

[7] We mainly focus on equipment and structures, which together account for about 80% of total investment and over 90% of capital stock. As shown in Appendix B.1.1, their investment shares are stable over time and across regions.

We begin by examining trends in the relative prices of equipment and structures, utilizing data from the PWT. All price series are deflated by the GDP price index and normalized to 1 in 1970. We divide countries into three groups based on their 1970 GDP per capita: the top and bottom terciles are defined as the High Income Group and Low Income Group, respectively.

As shown in Figure 1, the relative price of aggregate investment declines after 1970 for the United States (solid black line) and for both income groups (dashed medians). The low-income median (green dashed line) is above the high-income median (red dashed line) for much of the post-1980 period, indicating a modest but visible divergence in central tendencies. In addition, the low-income group displays substantially greater cross-sectional dispersion, as reflected in the wider shaded band.

Figure 1: **Relative Investment Price Index Trend (1950-2020)**



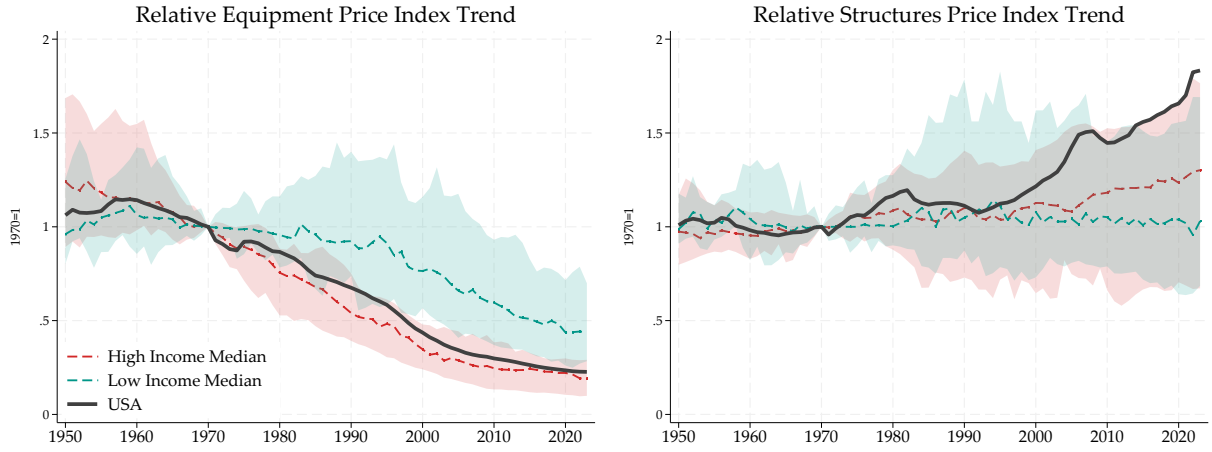
Notes: The relative price is calculated by dividing the investment price index by the GDP price index, normalized to 1 in 1970. High and low income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

However, disaggregating investment into equipment and structures reveals striking differences.[8] The left panel of Figure 2 plots the evolution of equipment prices. The U.S. experienced the most rapid decline in equipment prices, even relative to other high-income economies. Notably, in contrast to the aggregate investment price, the median equipment price for the High Income Group falls significantly faster than that of the Low Income Group, which aligns with Lian et al. (2020).

[8]PWT Capital Detail reports four different types of capital: Structures, Machinery and Equipment, Transport Equipment, and Other. The other two capital trends and detailed capital measures are discussed in Appendix A.

The right panel of Figure 2 shows the evolution of relative prices for structures. Here, we observe a trend distinct from that of equipment. In this category, the United States exhibits a particularly steep increase, outpacing the general upward trend observed across the High Income Group. In sharp contrast, the Low Income Group displays a modest downward trend over the same period. Trends for the remaining asset categories (transport equipment and other assets) are reported in Appendix B.1.2. Finally, we obtain similar patterns when using capital-stock-based price indices for both sectors, as reported in Appendix B.1.3.

Figure 2: **Relative Investment Price Index Trend by Sector (1950-2020)**



Notes: The relative price for each sector is calculated by dividing each sector's investment price index by the GDP price index, normalized to 1 in 1970. High and low income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. Dashed lines indicate the median for each group, and the solid black line denotes the United States. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

In addition to the PWT, we also examine price trends using the ICP and confirm that the patterns we document in the PWT are also present in the ICP data. Following Hsieh and Klenow (2007), we normalize U.S. GDP per capita (measured in PPP terms) to one in each ICP survey year, and we likewise normalize each country's relative equipment price and relative structures price by the corresponding U.S. values to enable cross-country comparisons. Examining changes from 1996 to 2021, we find that countries' equipment prices relative to the United States increase on average, whereas relative structures prices decline steadily. Over the same period, the correlation between relative equipment prices and GDP per capita decreases from 0.02 to -0.08, while the correlation between relative structures price and GDP per capita increases from 0.26 to 0.66. Appendix B.1.4 reports the corresponding plots as well as the estimated coefficient changes in the cross-sectional regressions across survey years.

Fact 2: Real investment rates in both sectors respond negatively to their relative prices.

To investigate the distinct roles of equipment and structures in economic development, we exploit disaggregated price and expenditure data from the ICP spanning the period from 1996 to 2021. The ICP data allows us to identify the specific price levels and investment volumes for each asset sector across countries. For cross-country comparability, we measure the price level of each asset type by dividing its Purchasing Power Parity (PPP) based price by the exchange rate (relative to USD). In addition, we calculate the investment rate using the volume of investment in each sector and GDP measured in real PPP terms.

We explicitly employ the ICP data to capture the cross-country dispersion in absolute price levels across sectors, as these level differences are essential for identifying the distinct impact of asset-specific costs on investment rates in contrast to previous studies that rely on time-series price indices.[9] This is crucial because it preserves the cross-sectional variation in absolute price levels, allowing us to estimate the sensitivity of investment to the actual cost of capital assets rather than mere relative changes.

To identify the impact of asset-specific price levels on investment rates, we estimate a pooled regression model with country-year fixed effects as follows:

$$\ln\left(\frac{I_{it}^k}{Y_{it}}\right) = \beta_{\text{Equip}}^k \ln(P_{it}^{\text{Equip}}) + \beta_{\text{Str}}^k \ln(P_{it}^{\text{Str}}) + \mathbf{X}_{it}'\Gamma^k + \alpha_i^k + \delta_t^k + \epsilon_{it}^k, \quad k \in \{\text{Equip}, \text{Str}\} \tag{1}$$

where I_{it}^k/Y_{it} denotes the investment rate for asset type $k \in \{\text{Equip}, \text{Str}\}$. That is, the investment rate is defined as the ratio of the PPP-adjusted real investment volume in sector k to the PPP-adjusted real GDP. The key explanatory variables, $\ln(P_{it}^{\text{Equip}})$ and $\ln(P_{it}^{\text{Str}})$, denote the log price levels of equipment and structures in PPP, respectively, and normalized by the exchange rate. The vector $\mathbf{X}_{it}$ includes country-specific controls such as human

[9] The dataset most widely used in the literature for cross country analysis is the PWT. Its main advantage is that it provides PPP prices for investment and consumption, which makes it possible to study, in both cross section and panel settings, how relative investment prices affect growth (Jones, 1994; Restuccia and Urrutia, 2001). However, for the central purpose of this paper, to examine how equipment and structures differentially affect investment rates and growth, the PWT is not sufficient. As noted above, although the PWT reports disaggregated capital prices, these are expressed in local currency units and normalized so that a given reference year equals one in each country. This normalization generates a unit inconsistency that complicates cross country comparisons and therefore allows us to identify only trends in price changes. We therefore use the ICP data, despite its more limited time coverage. In addition, while ICP prices come from direct price surveys of standardized items conducted simultaneously across countries, PWT between benchmark years is interpolated using national accounts deflators. Hsieh and Klenow (2007) is one example of a study that uses disaggregated ICP data on investment (durable and structure, durable is matched with equipment in our setting) and consumption goods (tradable and nontradable) for cross country analysis.

capital and population structure.[10] α_i^k and δ_t^k represent country and year fixed effects, respectively.

The coefficients β_{Equip}^k and β_{Str}^k are the parameters of interest, capturing the price elasticities of investment rate of sector k. Specifically, the own-price coefficient measures ($\beta_{\text{Equip}}^{\text{Equip}}$ and $\beta_{\text{Str}}^{\text{Str}}$) the direct impact of capital costs on accumulation within the same sector; a negative sign implies that higher costs hinder investment. Meanwhile, the cross-price coefficients ($\beta_{\text{Equip}}^{\text{Str}}$ and $\beta_{\text{Str}}^{\text{Equip}}$) shed light on the inter-sectoral relationship. For instance, a negative cross-price elasticity would suggest that equipment and structures are complements in the production function, whereas a positive sign would imply substitutability.

We include a vector of controls $\mathbf{X}_{it}$: the human capital index and years of schooling to ensure that estimated price effects do not confound the impact of labor quality improvements; the logarithm of population to control for scale effects; and the 5-year lagged level of GDP per worker ($\ln y_{it-5}$) to account for conditional convergence dynamics. Finally, we include country fixed effects (α_i) to absorb time-invariant country-specific factors, such as geography and institutional heritage, and year fixed effects (δ_t) to control for global common shocks. Standard errors are clustered at the country level.

Table 1 presents the regression results for equation (1). The estimates highlight two distinct channels through which prices drive investment allocation.

First, we find evidence of negative own-price elasticities in both sectors. As shown in column (1), a 1% increase in the relative price of equipment leads to a 0.47% decrease in the equipment investment rate. Similarly, column (2) indicates that a 1% increase in the price of structures reduces the structures investment rate by approximately 0.63%. These results confirm that higher capital costs directly dampen accumulation within the respective sector. This result is consistent with Restuccia and Urrutia (2001), who find that the relative price of investment is negatively correlated with investment rates in a cross section of countries using data on 125 countries over 1960–1985.

Second, the cross-price elasticities reveal a significant substitution pattern between asset classes. The coefficient of the structures price on equipment investment is positive and significant (0.26), and likewise, the coefficient of the equipment price on structures investment is positive (0.37). This positive relationship implies that equipment and structures function as substitutes in the aggregate production function. Consequently, when the cost of one capital type rises, economies tend to optimize their input mix by reallocat-

[10] Since the PWT human capital index is constructed in part from schooling data and relies on interpolation, the two measures may be mechanically correlated. Accordingly, we interpret their coefficients only as nuisance controls and verify in robustness checks that our key price elasticities are essentially unchanged when controlling for either schooling or the human-capital index.

Table 1: **The Effect of Capital Prices on Sectoral Investment Rates**

	Equipment Investment Rate (1)	Structures Investment Rate (2)
Relative prices		
Log Equipment Price	-0.469***	0.369**
	(0.128)	(0.152)
Log Structures Price	0.255***	-0.632***
	(0.086)	(0.066)
Controls		
Years of Schooling	0.737**	0.808**
	(0.336)	(0.353)
Human Capital Index	-3.248**	-4.258***
	(1.641)	(1.622)
Log Population	-0.560*	0.723***
	(0.323)	(0.262)
Lagged Log GDP per Worker	-0.088	-0.182
	(0.139)	(0.150)
Constant	0.364	-0.668
	(1.816)	(1.605)
Observations	629	629
R^2	0.708	0.682
Country FE	✓	✓
Year FE	✓	✓

Notes: Standard errors are reported in parentheses and are clustered at the country level. All specifications include country and year fixed effects. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

ing resources toward the relatively cheaper capital type, rather than reducing aggregate investment uniformly.

Fact 3: Lower equipment prices are strongly associated with higher output per capita, whereas structures prices show a much weaker association.

Having established the relationship between capital prices and sectoral investment rates, we next assess whether prices are transmitted directly to income levels. To this end, we estimate equation (1) after replacing the dependent variable with GDP per capita from the PWT. We then extend the baseline specification to a set of dynamic regressions that evaluate predictive content at different horizons. Specifically, we test whether current capital prices (P_{it}) predict contemporaneous and future income, and we additionally verify that they do not predict pre-determined past income. The latter serves as a placebo test for reverse causality and spurious correlation. We estimate the following specification for

horizons $h \in \{-5, 0, 5\}$:

$$\ln(y_{i,t+h}) = \beta^h_{\text{Equip}} \ln(P^{\text{Equip}}_{it}) + \beta^h_{\text{Str}} \ln(P^{\text{Str}}_{it}) + \mathbf{X}'_{it}\Gamma^h + \lambda^h \ln(y_{i,t-\tau}) + \alpha^h_i + \delta^h_t + \epsilon^h_{it} \tag{2}$$

where τ denotes the lag length for the lagged GDP per capita. As a baseline, we estimate the effect of the current price measure on output at each horizon h (including the contemporaneous case $h = 0$), and we check robustness to using different lag lengths for the initial-income control, $\tau \in \{6, 10\}$.

Table 2: **The Effect of Capital Prices on GDP per Capita**

	Control: Lagged GDP ($t-6$)			Control: Lagged GDP ($t-10$)		
	$\ln(y_{i,t-5})$	$\ln(y_{i,t})$	$\ln(y_{i,t+5})$	$\ln(y_{i,t-5})$	$\ln(y_{i,t})$	$\ln(y_{i,t+5})$
	(1)	(2)	(3)	(4)	(5)	(6)
Relative prices						
Log Equipment Price	-0.045*	-0.279***	-0.235**	-0.179**	-0.318***	-0.223*
	(0.023)	(0.097)	(0.113)	(0.077)	(0.114)	(0.117)
Log Structures Price	-0.008	-0.064	0.019	-0.010	-0.104*	-0.031
	(0.014)	(0.048)	(0.062)	(0.036)	(0.055)	(0.060)
Controls						
Years of Schooling	0.064	0.310*	0.498**	0.404**	0.370*	0.405
	(0.051)	(0.165)	(0.247)	(0.159)	(0.207)	(0.262)
Human Capital Index	-0.347	-1.448*	-2.326*	-1.816**	-1.613*	-1.871
	(0.237)	(0.773)	(1.231)	(0.725)	(0.968)	(1.325)
Log Population	-0.023	-0.120	-0.161	0.037	0.038	-0.040
	(0.032)	(0.113)	(0.135)	(0.093)	(0.125)	(0.137)
$\ln(y_{i,t-6})$	0.909***	0.386***	0.013			
	(0.040)	(0.048)	(0.068)			
$\ln(y_{i,t-10})$				0.524***	0.089	-0.082
				(0.046)	(0.072)	(0.083)
Constant	0.128	-0.595	-1.012	-0.341	-2.134**	-2.092
	(0.271)	(0.828)	(1.329)	(0.693)	(0.957)	(1.459)
Observations	632	632	494	617	617	479
R^2	0.997	0.973	0.968	0.980	0.971	0.971
Country FE	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓

Notes: Standard errors are reported in parentheses and are clustered at the country level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Columns (1)–(3) control for lagged GDP per capita at $t-6$ ($\ln(y_{i,t-6})$), while columns (4)–(6) control for lagged GDP capita at $t-10$ ($\ln(y_{i,t-10})$).

Table 2 presents the estimated elasticities of GDP per worker with respect to capital prices across different time horizons. The results reveal two critical patterns. First, the timing of the correlation supports a interpretation running from equipment prices to economic development. As shown in Columns (1) through (3), the magnitude of the coefficient on equipment prices is heavily dependent on the time horizon. The current equipment price is a strong and statistically significant predictor of current income (Column 2, $\beta = -0.28$) and future income (Column 3, $\beta = -0.24$). However, its association with

past income (Column 1, $\beta = -0.05$) is substantially smaller in magnitude. This asymmetry where prices strongly predict future growth but barely "predict" the past serves as a placebo test. Instead, favorable equipment prices appear to precede and propel subsequent economic takeoff.

Second, the results highlight a heterogeneity between capital asset types. While low equipment prices are consistently associated with higher income levels across specifications, the coefficients on structures prices are largely insignificant or economically small. For example, in the concurrent specification (Column 2), the elasticity of equipment prices is nearly four times greater than that of structures price. This indicates that while high structure costs may depress investment volume (as established in Fact 2), they do not constitute the primary binding constraint on aggregate productivity and long-term development compared to the affordability of equipment.

Robustness Checks The results are robust to alternative specifications of the conditional convergence term. In the baseline estimation, we fixed the timing of the lagged income control (e.g., at $t-6$), which implies a varying time interval between the outcome and the control across different forecast horizons. To ensure that our findings are not driven by these varying lag lengths, we re-estimate the equation (2) in Appendix B.2.1 by maintaining a constant year gap (e.g., 5 years) between the dependent variable and the lagged GDP per capita:

$$\ln(y_{i,t+h}) = \beta^{h,\tau}_{\text{Equip}} \ln(P^{\text{Equip}}_{it}) + \beta^{h,\tau}_{\text{Str}} \ln(P^{\text{Str}}_{it}) + \mathbf{X}'_{it}\Gamma^{h,\tau} + \lambda^{h,\tau} \ln(y_{i,t+h-\tau})^h + \alpha^{h,\tau}_i + \delta^{h,\tau}_t + \epsilon^h_{it}$$

As shown in Table B.I, the dynamic asymmetry remains robust: equipment prices strongly predict current and future income levels but show little association with past income, confirming that the results are not an artifact of the convergence specification.

In addition, we verify the robustness of our results by replacing the point-in-time observations with smoothed measures based on centered moving averages. This approach is intended to mitigate measurement error and short-run business cycle fluctuations inherent in the PWT series. Specifically, for each selected benchmark year t, we construct the outcome variables and controls as a five-year centered average:

$$\bar{X}_{it} = \frac{X_{i,t-2} + X_{i,t-1} + X_{i,t} + X_{i,t+1} + X_{i,t+2}}{5}.$$

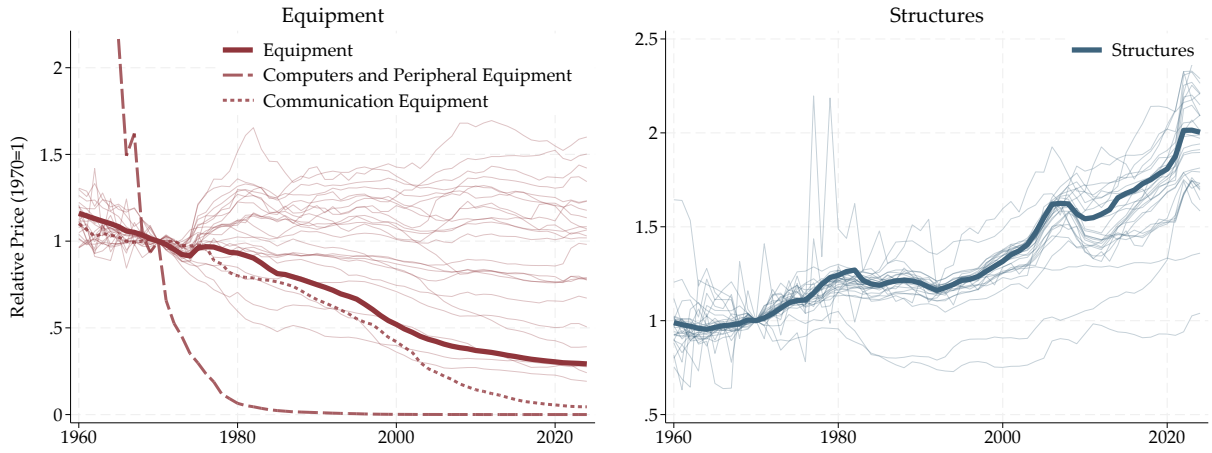
We implement this procedure for the ICP benchmark years (1996, 2005, 2011, 2017, and 2021) and additionally retain the 1990 and 2001 benchmarks to construct the necessary

lag structures. The smoothed outcomes include the logarithm of GDP per capita and the (log) TFP index, while the baseline covariates consist of the human capital index, years of schooling, and log population. We find that using these moving-average outcomes yields estimates that are qualitatively similar to the baseline, indicating that our results are not driven by transitory noise in the level measures. The detailed results are reported in Appendix B.2.2.

Finally, in addition to GDP per capita, we examine to what extent capital prices in each sector are related to Total Factor Productivity (TFP) and find that lower equipment prices are associated with higher contemporaneous TFP, while structures prices are positively related to TFP, highlighting strong heterogeneity in how different capital types affect productive efficiency. Results are reported in Appendix B.2.3.

Fact 4: In the U.S., the post-1970 decline in equipment prices was largely driven by computers and related items, whereas the post-1990 rise in structures price was broad-based across nearly all types of structures.

Figure 3: **Relative Investment Price Index Trend by Sector (1950-2020)**



Notes: The figure plots trends in relative investment prices by BEA asset type. For each asset category, an implicit price index is recovered from BEA data as nominal investment divided by the corresponding chain-type quantity index. The resulting series is deflated using the GDP Deflator and then normalized to 1 in 1970 for each category. The sample is restricted to 1960–2020. Thin lines show disaggregated categories, while thicker highlighted lines emphasize the aggregate equipment series and selected subcomponents (information processing equipment, computers and peripheral equipment, and communication equipment). Details of the asset classes are provided in Table A.I. The structure for petroleum and natural gas is omitted from the right panel. *Sources:* U.S. Bureau of Economic Analysis (BEA), NIPA Table 2.7 and Table 2.8; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

We now narrow our focus to the United States and document the evolution of investment prices at the level of BEA asset types.[11] Figure 3 highlights two contrasting

[11]BEA provides detailed asset-type price indexes, whereas the PWT Capital Detail provides only four categories: equipment, structures, transportation, and other assets. In Appendix A, we describe the cover-

patterns in relative investment prices across BEA asset categories. First, the relative price of equipment exhibits a sharp and persistent decline after 1970. This decline is not uniform across equipment categories; rather, it is concentrated in information-processing assets. In particular, computers and peripheral equipment display the steepest and most sustained price declines, while communication equipment shows similarly large reductions. Together, these two categories account for about 20% of total equipment investment, and the broader category of information-processing equipment represents roughly from 35 to 40%. The Table A.I in Appendix reports each asset's relative price and its 2020 investment share within the corresponding capital segment, measured as the asset's investment divided by total sectoral investment. This concentration is consistent with rapid improvements in embodied technology and the associated quality-adjusted price declines emphasized in the investment-specific technological change literature (Jones, 1994; Byrne and Pinto, 2015; Lian et al., 2020).

Second, in sharp contrast, the relative price of structures rises markedly after 1990. Importantly, this increase is not driven by a single type of structure: the disaggregated structure series move upward together, indicating a broad-based rise across nearly all structure categories. This pattern suggests that post-1990 structures price dynamics are better characterized as an economy-wide shift affecting the structure component of investment rather than a compositional change within structures. These results are robust to using BEA price indices constructed from the current-cost net stock (capital stock) series for both sectors, reported in Appendix B.3. In addition, the contribution of each component to the total change in each sector's relative price, including the intellectual property products sector, is analyzed in the Appendix C.

2.3 Why Have Structural Prices Increased? Construction Productivity

A rapid increase in the price of structures after 1990 is consistent with a decline in construction productivity in the U.S. during the same period. Goolsbee and Syverson (2023) document that, while U.S. aggregate productivity (measured in value-added per worker) doubled between 1970 and 2020, it simultaneously fell by 40 percent in the construction sector. We find that the Bureau of Labor Statistics (BLS)'s productivity growth estimates for the construction sector are negative on average, −0.55%, per year over the 1987 to 2024 period, while productivity growth averaged 0.89% per year for the private business

age of each dataset in detail. In addition, the Figure A.1 shows that the relative prices of equipment and structures reported by the BEA are broadly consistent with those in the PWT, which supports the use of both datasets.

sector.[12]

Several factors may have contributed to stagnated construction productivity. For example, Goolsbee and Syverson (2023) showed that construction firms' abilities to turn materials into output has deteriorated and there is little reallocation from low productivity places to high-productivity places. D'Amico et al. (2024) find that the post-1970 productivity decline coincides with increases in proxies for land-use regulation. Local land-use controls limit the size of building projects, reducing both scale economies and incentives to invest in innovation.[13]

To assess the extent to which the rise in construction sector prices reflects productivity decline, we use the U.S. data from the EU KLEMS dataset to decompose the relative output price of construction into input cost and TFP components. EU KLEMS also offers harmonized cross-country coverage under a common industrial classification, making it well suited to a comparative analysis of construction-sector price dynamics within a unified framework.

We decompose the construction sector output price using the Törnqvist approach within a duality framework.[14] This decomposition follows from the duality between the production function and the cost function under constant returns to scale and competitive factor markets. The primal identity states that output growth equals the share-weighted sum of input growth and TFP growth. Its dual counterpart expresses output price growth as the share weighted sum of input price growth minus TFP growth. Accordingly, under constant returns to scale and competitive factor markets, the gross-output price of sector i, P_i, satisfies the following unit-cost identity:

$$\Delta \ln P_i = \bar{s}_{L,i} \Delta \ln P_{L,i} + \bar{s}_{K,i} \Delta \ln P_{K,i} + \bar{s}_{II,i} \Delta \ln P_{II,i} - \Delta \ln A_i, \tag{3}$$

[12]The annual average productivity growth from 1987 to 2024 is 1.18% for Agriculture, 1.51% for Mining, 0.67% for Utilities, 1.15% for Durable manufacturing sector, 0.09% for Nondurable manufacturing sector, 1.10% for Trade, 0.58% for Transportation and warehousing, 1.01% for Information, -0.03% for Finance, insurance, real estate, and leasing, and 0.23% for Services.

[13]Productivity problem also appears in the public sector. Brooks and Liscow (2023) find that spending per mile in the Interstate Highway System increased more than threefold from the 1960s to the 1980s.

[14]Additional details on the Törnqvist approach are provided in Appendix D.1. Since the Törnqvist index can be theoretically motivated within a duality framework, we adopt the duality approach for two main reasons. First, our question concerns why construction prices have risen faster than the general price level, which is inherently a price-based question. The dual formulation maps directly into this issue by attributing relative price movements to identifiable cost drivers, such as labor, capital, and intermediate inputs, as well as to productivity. Second, industry-level output quantities are often measured with greater error or constructed residually by deflating nominal values, whereas input price indexes and cost shares are generally measured more reliably in the national accounts. For this reason, the dual residual provides a more robust estimate of sectoral TFP for our purposes.

where $\bar{s}_{j,i}$ denotes the Törnqvist two-period average cost share of input j in gross output, $P_{j,i}$ denotes the price of input j, and $\Delta \ln A_i$ denotes TFP growth in sector i. Subtracting $\Delta \ln P_{agg}$ from both sides yields the following expression

$$\Delta \ln \left(\frac{P_i}{P_{agg}} \right) = \text{rel}_{L,i} + \text{rel}_{K,i} + \text{rel}_{II,i} - \Delta \ln A_i,$$

where each relative input cost term is defined as

$$\text{rel}_{j,i} = \bar{s}_{j,i} \left(\Delta \ln P_{j,i} - \Delta \ln P_{agg} \right).$$

Accordingly, each relative cost term measures the contribution of sector specific input price movements, relative to the aggregate price level, to the sector's relative output price change.[15]

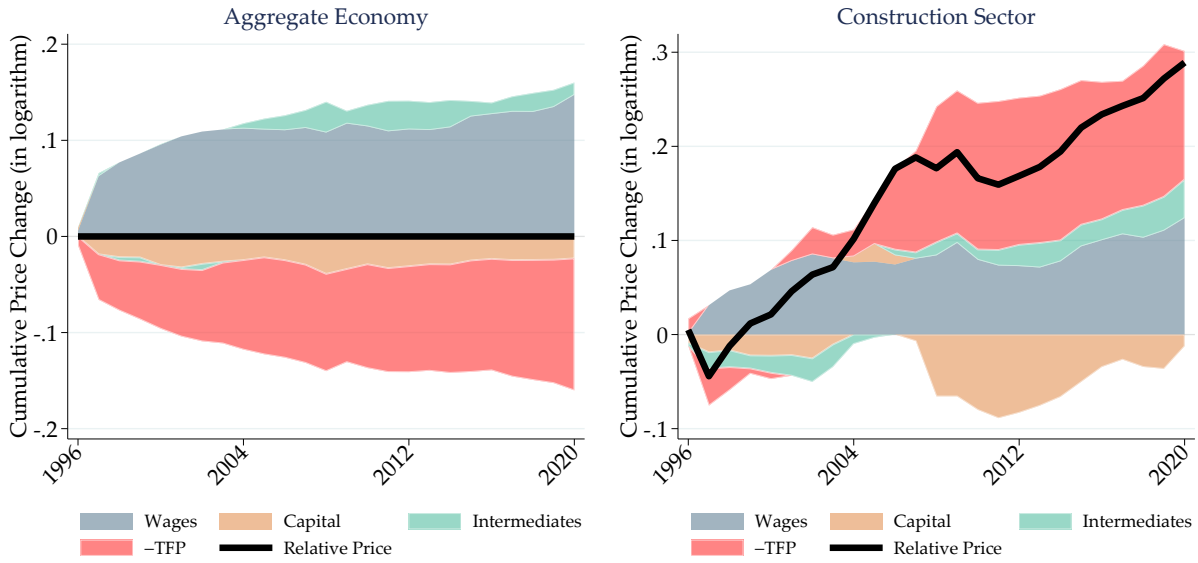
Figure 4 presents a decomposition of the cumulative change in output prices, $\sum \Delta \ln(P_i/P_{agg})$, since 1997 into relative input-cost components and TFP for both the aggregate economy and the construction sector. The aggregate level decomposition is included for comparison with the construction sector. Note that, by construction, the decomposition in the aggregate output-price panel, $\sum \Delta \ln(P_{agg}/P_{agg})$, sums to zero.

The aggregate panel on the left provides a useful benchmark. Although labor costs make a positive contribution at the aggregate level, this upward pressure is largely offset by TFP growth and movements in capital costs. The construction sector presents a clear contrast. In the construction sector, the positive contribution of labor costs is not accompanied by an offsetting productivity component. Instead, the contribution of negative TFP is itself strongly positive, indicating that weak productivity growth amplified rather than absorbed cost pressures. Intermediate input costs also push relative prices upward, while the capital component is negative over part of the sample and therefore provides only limited relief. This pattern is also consistent with the value-added growth decomposition reported in Appendix D.2. At the aggregate level, value added growth is driven largely by TFP growth, whereas in construction TFP acts as a drag on value added growth. As further shown in Appendix D.3, a similar pattern appears in the euro area, especially in the largest economies, as well as in Japan.

Taken together, these results suggest that the rise in relative prices in the construction

[15] Labor contribution (rel_L) reflects sector specific labor cost pressure relative to the aggregate price level, capital contribution (rel_K) reflects relative capital service cost pressure, and intermediate input contribution (rel_{II}) reflects relative price pressure from purchased materials and services. Positive values indicate that the corresponding input cost rises faster than the aggregate benchmark and therefore pushes the sector's relative output price upward.

Figure 4: Decomposition of Relative Prices in the U.S.: Aggregate vs. Construction Sector



Notes. The figure presents a cumulative decomposition of sectoral gross-output prices relative to the aggregate economy in the United States within the Törnqvist dual framework. The left panel reports the aggregate economy, while the right panels present selected sectors, including construction. Each shaded area denotes the cumulative log-point contribution of a given channel to the change in a sector's gross-output price relative to the aggregate economy since the beginning of the sample. The productivity component, $(-\Delta \ln A)$, is defined as the negative of the dual total factor productivity residual. A negative value indicates that productivity growth offsets input-cost pressures and lowers the relative price. Accordingly, the −TFP area captures the contribution of productivity shortfalls rather than productivity gains. The black line, $\sum \Delta \ln(P_i/P_{agg})$, traces the cumulative change in the sector's gross-output price relative to the aggregate economy. *Sources:* EU KLEMS 2024 and authors' calculations.

sector cannot be understood simply as an increase in input costs. Rather, the key distinction from the aggregate economy is the absence of offsetting productivity growth in construction. In this sense, the figure provides direct support for the view that the increase in construction prices after 1990 is closely linked to the productivity slowdown in the U.S. construction sector (Goolsbee and Syverson, 2023), which may in turn affect the productivity of structures capital.

3 The Model

Motivated by these findings, we develop an endogenous growth framework that disaggregates capital into equipment and structures. Building on Romer (1990), we adopt the Rivera-Batiz and Romer (1991) "lab-equipment" specification, in which capital is not only essential for final-goods production but also enters as an input in the R&D sector. Within this framework, we characterize the balanced growth path and show that an increase in the relative price of structures exerts a drag on long-run growth through two channels: it

distorts capital accumulation and it tightens the resource constraint facing innovation.

3.1 Capital Accumulation and ISTC

The economy features two distinct capital stocks: equipment, denoted by K_E, and structures, denoted by K_S.

Let $I_E(t)$ and $I_S(t)$ denote investment expenditures in equipment and structures, respectively, measured in units of the final good (the *numeraire*). Following Greenwood et al. (1997), we introduce ISTC via $q_E(t)$ and $q_S(t)$, which capture the efficiency with which one unit of the final good is transformed into physical capital. The relative price of each capital good is therefore the inverse of its investment efficiency:

$$P_E(t) \equiv \frac{1}{q_E(t)}, \qquad P_S(t) \equiv \frac{1}{q_S(t)}. \tag{4}$$

Capital stocks evolve according to

$$\begin{aligned} \dot{K}_E(t) &= q_E(t)\, I_E(t) - \delta_E K_E(t), \\ \dot{K}_S(t) &= q_S(t)\, I_S(t) - \delta_S K_S(t), \end{aligned}$$

where δ_E and δ_S denote the depreciation rates for equipment and structures, respectively. Investment efficiency evolves exogenously at constant rates γ_E and γ_S:

$$\frac{\dot{q}_E(t)}{q_E(t)} = \gamma_E, \qquad \frac{\dot{q}_S(t)}{q_S(t)} = \gamma_S. \tag{5}$$

Equipment capital K_E is allocated between final-good production (K_E^Y) and R&D (K_E^R); structures K_S are allocated analogously. Market-clearing conditions for capital therefore requires

$$K_E(t) = K_E^Y(t) + K_E^R(t), \qquad K_S(t) = K_S^Y(t) + K_S^R(t).$$

3.2 Firms

3.2.1 Final-Goods Sector

A representative competitive firm produces the final good using equipment (K_E^Y), structures (K_S^Y), and unskilled labor (L_u). The production technology is Cobb-Douglas

$$Y(t) = A(t)\left(K_E^Y(t)\right)^{\alpha_E}\left(K_S^Y(t)\right)^{\alpha_S}\left(L_u(t)\right)^{1-\alpha_E-\alpha_S}. \tag{6}$$

Taking prices as given, the firm chooses inputs to maximize profits

$$\max_{K_E^Y, K_S^Y, L_u}\left\{Y(t) - R_E(t)K_E^Y(t) - R_S(t)K_S^Y(t) - w_u(t)L_u(t)\right\}.$$

The first-order conditions imply factor prices equal marginal revenue products:

$$R_E(t) = \alpha_E\frac{Y(t)}{K_E^Y(t)}, \qquad R_S(t) = \alpha_S\frac{Y(t)}{K_S^Y(t)}, \qquad w_u(t) = \left(1-\alpha_E-\alpha_S\right)\frac{Y(t)}{L_u(t)}.$$

3.2.2 R&D Sector

The R&D sector produces new designs that drive the evolution of total factor productivity $A(t)$. The sector operates competitively and uses equipment capital (K_E^R), structures (K_S^R), and skilled labor (L_s). Knowledge production is given by

$$\dot{A}(t) = \kappa A(t)^{\phi}\left(K_E^R(t)\right)^{\eta_E}\left(K_S^R(t)\right)^{\eta_S}\left(L_s(t)\right)^{1-\eta_E-\eta_S}. \tag{7}$$

Let $A(t)$ denote the aggregate stock of knowledge and let $p_A(t)$ denote the (final good denominated) shadow value of a marginal increase in $A(t)$ for the representative household. In the baseline model, $A(t)$ does not generate direct payment stream in the household budget constraint and accordingly, $p_A(t)$ is a costate variable associated with $A(t)$. This interpretation is in the spirit of Jones and Williams (1998), who characterize the return to knowledge using the economy's production possibilities and interpret $p_A(t)$ as the shadow value of the knowledge stock rather than the price of a traded claim.[16]

Since $A(t)$ augments production, its technological marginal product in the final-goods

[16] In case of introducing an market for knowledge, the appropriate dividend in the no-arbitrage condition is the profit flow associated with a marginal patent, $\pi_A(t)$. Formally, the asset-pricing equation becomes $r(t)p_A(t) = \pi_A(t) + \dot{p}_A(t)$, where $\pi_A(t)$ is the profit flow accruing to the owner of a marginal design under an explicit licensing or monopolistic-competition microfoundation such as Romer (1990) and Jones (1995).

technology is

$$\frac{\partial Y(t)}{\partial A(t)} = \frac{Y(t)}{A(t)}. \tag{8}$$

This object is a technological marginal product. It enters the costate dynamics for $p_A(t)$, and it need not correspond to any payment made by firms to an owner of $A(t)$ in competitive equilibrium. The shadow value $p_A(t)$ evolves according to the costate equation

$$\dot{p}_A(t) = r(t)\, p_A(t) - \frac{\partial Y(t)}{\partial A(t)} = r(t)\, p_A(t) - \frac{Y(t)}{A(t)}. \tag{9}$$

In the competitive equilibrium described here, factor payments exhaust output in the final-goods sector. The term $Y(t)/A(t)$ should therefore be interpreted as the marginal product of $A(t)$ that appears in the evolution of the shadow value $p_A(t)$, not as a dividend on a traded knowledge asset.

In equilibrium, R&D decisions are disciplined by $p_A(t)$, the shadow value of knowledge; we use $p_A(t)$ as a device to summarize the marginal value of $A(t)$. The representative R&D firm maximizes current profits:

$$\max_{K_E^R, K_S^R, L_s} \left\{ p_A(t)\dot{A}(t) - R_E(t)K_E^R(t) - R_S(t)K_S^R(t) - w_s(t)L_s(t) \right\}.$$

The corresponding first-order conditions are

$$R_E(t) = \eta_E \frac{p_A(t)\dot{A}(t)}{K_E^R(t)}, \qquad R_S(t) = \eta_S \frac{p_A(t)\dot{A}(t)}{K_S^R(t)}, \qquad w_s(t) = \big(1 - \eta_E - \eta_S\big) \frac{p_A(t)\dot{A}(t)}{L_s(t)}.$$

3.3 Households

3.3.1 Population and Labor Supply

Population $N(t)$ grows at a constant exogenous rate n:

$$\dot{N}(t) = nN(t).$$

We normalize the labor endowment per capita to unity, such that total labor supply equals population size, $L(t) = N(t)$. A fixed fraction $s \in (0,1)$ of the workforce supplies skilled labor to the R&D sector, while the remaining fraction $(1 - s)$ supplies unskilled labor to the final-goods sector:

$$L_s(t) = sN(t), \qquad L_u(t) = (1 - s)N(t).$$

In equilibrium, firms' labor demand is pinned down by wages and equals the sectoral labor supplies above.

3.3.2 Preferences

The representative household maximizes discounted lifetime utility from per-capita consumption $c(t) \equiv C(t)/N(t)$:

$$U_0 = \int_0^\infty e^{-\rho t} N(t) \, \frac{c(t)^{1-\sigma}}{1-\sigma} \, dt,$$

where $\rho > 0$ is the subjective discount rate and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution.

3.3.3 Budget Constraint and Euler Equation

Households own the economy's capital stock and rent it to firms, and they supply labor. Let $\Pi(t)$ denote aggregate firm profits rebated lump-sum to households. The aggregate budget constraint is

$$C(t) + I_E(t) + I_S(t) = w_s(t) L_s(t) + w_u(t) L_u(t) + r_E(t) K_E(t) + r_S(t) K_S(t) + \Pi(t).$$

The household's optimization implies the standard Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\big(r(t) - \rho\big),$$

where $r(t)$ is the real interest rate.

3.3.4 Optimal Savings and Arbitrage

Households allocate savings between financial assets yielding $r(t) = r_E(t) = r_S(t)$ and physical capital. No-arbitrage requires that the return on each capital type, including rental income, capital gains or losses, and depreciation, equals $r(t)$. Using $P_k(t) = 1/q_k(t)$

and $\dot{P}_k(t)/P_k(t) = -\gamma_k$, the no arbitrage conditions are

$$r(t) = \frac{R_E(t)}{P_E(t)} + \frac{\dot{P}_E(t)}{P_E(t)} - \delta_E = q_E(t)R_E(t) - \gamma_E - \delta_E, \tag{10}$$

$$r(t) = \frac{R_S(t)}{P_S(t)} + \frac{\dot{P}_S(t)}{P_S(t)} - \delta_S = q_S(t)R_S(t) - \gamma_S - \delta_S. \tag{11}$$

In equilibrium, the marginal product of each capital type in final-good production equals its user cost. Combining FOCs, we get

$$\alpha_E \frac{Y(t)}{K_E^Y(t)} = P_E(t)\big(r(t) + \delta_E + \gamma_E\big),$$
$$\alpha_S \frac{Y(t)}{K_S^Y(t)} = P_S(t)\big(r(t) + \delta_S + \gamma_S\big).$$

3.4 Balanced Growth Path (BGP)

To ensure the existence of a stable BGP, we impose the standard semi-endogenous restriction (see Appendix E for the full derivation):

$$\Delta \equiv (1-\phi)(1-\alpha_E-\alpha_S) - (\eta_E + \eta_S) > 0. \tag{12}$$

On a BGP, aggregate quantities $(Y, C, I_E, I_S, K_E, K_S)$ grow at constant rates, and sectoral allocation shares remain constant. In particular, capital growth rates satisfy

$$g_{K_E} = g_Y + \gamma_E, \qquad g_{K_S} = g_Y + \gamma_S,$$

and, given the fixed labor allocation rule, labor in both sectors grows at the population rate,

$$\frac{\dot{L}_s}{L_s} = \frac{\dot{L}_u}{L_u} = n.$$

Log-differentiating the final-goods and innovation technologies yields the BGP system:

$$g_Y = g_A + \alpha_E(g_Y + \gamma_E) + \alpha_S(g_Y + \gamma_S) + (1 - \alpha_E - \alpha_S)n, \tag{13}$$

$$(1-\phi)g_A = (\eta_E + \eta_S)g_Y + \eta_E\gamma_E + \eta_S\gamma_S + (1 - \eta_E - \eta_S)n. \tag{14}$$

Solving (13) for g_Y delivers a decomposition:

$$g_Y = n + \underbrace{\frac{1}{1-\alpha_E-\alpha_S} g_A}_{\text{TFP contribution}} + \underbrace{\frac{\alpha_E \gamma_E + \alpha_S \gamma_S}{1-\alpha_E-\alpha_S}}_{\text{ISTC-induced deepening}}. \tag{15}$$

Substituting (15) into (14) yields the endogenous TFP growth rate:

$$g_A = \frac{(1-\alpha_E-\alpha_S)n + \big(\eta_E(1-\alpha_S) + \eta_S\alpha_E\big)\gamma_E + \big(\eta_S(1-\alpha_E) + \eta_E\alpha_S\big)\gamma_S}{(1-\phi)(1-\alpha_E-\alpha_S) - (\eta_E+\eta_S)}. \tag{16}$$

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to match U.S. long-run moments and parameter estimates from the literature, using transparent mappings from data to model objects. In particular, we construct the exogenous trends that pin down the BGP by targeting population growth and ISTC.

The population growth rate n is set to the average growth rate of total population in the PWT from 1970 to 2020. We proxy sectoral ISTC with relative-price trends in PWT: the equipment ISTC rate γ_E is set to the average annual decline in the relative price of equipment, while the structures ISTC rate γ_S is set analogously using the relative price of structures. Over our sample, from 1970 to 2020, this implies around $\gamma_E = 0.03$ and $\gamma_S = -0.01$, consistent with a rapidly falling relative price of equipment and a rising relative price of structures.

On the production side, we calibrate the Cobb-Douglas factor elasticities by combining information on the aggregate labor income share and the composition of the investment. Using the PWT Labor Detail report, we set the labor income share to 0.62, which implies an aggregate capital share of 0.38, reported in 2015. We then split this aggregate capital share between equipment and structures using the U.S. average share of equipment within the total investment from the PWT Capital Detail data. Specifically, the equipment share is 0.39, so that the output elasticity of equipment is calibrated to $\alpha_E = 0.15$, while the output elasticity of structures is calibrated to $\alpha_S = 0.23$. We use this investment composition as a simple empirical proxy to split the aggregate capital income share across asset types.[17]

[17] Along the balanced growth path, investment-flow composition can be derived as $\frac{I_E/I_S}{\alpha_E/\alpha_S} = \frac{g_Y+\delta_E}{g_Y+\delta_S} \cdot \frac{r+\delta_S}{r+\delta_E}$.

Turning to the innovation block, we set the intertemporal knowledge-spillover parameter to $\phi = -2.0$ following Bloom et al. (2020). We discipline the Cobb-Douglas input elasticities in the R&D production function using factor-share evidence from the National Center for Science and Engineering Statistics (NCSES) Business Enterprise Research and Development (BERD) data (Égert, 2016; Moretti et al., 2025). Using the BERD surveyed in 2015, we set the labor share in R&D to 0.67, which implies an R&D capital share of 0.33. We allocate this R&D capital share across equipment and structures using the observed equipment share in R&D capital spending, 0.79, which implies that equipment accounts for $\eta_E = 0.26$ of the R&D technology, while structures accounts for the remaining $\eta_S = 0.07$.[18] Table 3 reports the full set of calibrated parameters used in the numerical analysis.

4.2 Model Performance

We first evaluate the calibrated model's ability to replicate the growth trajectory of the United States from 1970 to 2020. Subsequently, we analyze the quantitative impact of the structural drag on long-run growth. Figure 5 compares the simulated series with empirical data.

Targeted Moments: Sectoral Prices The top panels of Figure 5 show the relative prices of equipment ($1/q_E$) and structures ($1/q_S$). These series are the primary exogenous drivers of the model, with γ_E and γ_S calibrated to match the empirical trends. The simulation closely tracks the steady decline in equipment costs and the secular rise in structures price, reflecting the ISTC imbalance observed in the U.S. data.

Untargeted Moments: GDP per capita and TFP The bottom panels present the results for untargeted moments: GDP per capita (Y/L) and the TFP stock (A). The empirical benchmarks are sourced from the PWT and the Bureau of Labor Statistics (BLS), respec-

Under our calibration ($\delta_E = 0.13$, $\delta_S = 0.03$, $g_Y = 0.0244$, $r = 0.03$), this ratio is approximately 1.06, so the implied I_E/I_S is close to α_E/α_S. In the United States, the equipment share is about 38.8% in 2015, which pins down α_E at 0.1469. Figure B.2 shows that the equipment share is stable across countries despite variation in income, implying that the allocation of investment between equipment and structures is relatively stable across countries as well. Finally, Figure B.3 shows the trend in the share of equipment investment out of total investment in equipment and structures. The share remains stable, except for a fluctuation during the 2008 Great Financial Crisis.

[18] Analogously, we use the observed composition of R&D capital spending as an external proxy to allocate the aggregate R&D capital share across equipment and structures. Figure A.2 illustrates that the composition of R&D capital expenditures is stable across countries, with equipment accounting for roughly 80% of total equipment-plus-structures spending over a wide range of income levels.

Table 3: **Calibration of Parameters for the U.S. Economy**

Symbol	Description	Value	Source
	Demographics & Labor		
n	Population growth rate	0.0098	PWT (1970–2020)
	ISTC		
γ_E	Inverse relative price growth of Equipment (TFP)	0.0290	PWT (1970–2020)
γ_S	Inverse relative price growth of Structures (TFP)	−0.0101	PWT (1970–2020)
δ_E	Depreciation Rate of Equipment	0.130	BEA (1970–2020)
δ_S	Depreciation Rate of Structures	0.030	BEA (1970–2020)
	Production Technology		
α_E	Share of Equipment in Output	0.1469	PWT (2015)
α_S	Share of Structures in Output	0.2312	PWT (2015)
	R&D Sector Parameters		
ϕ	Knowledge spillover parameter	−2.0	Bloom et al. (2020)
η_E	Share of Equipment in R&D	0.2607	BERD (2015)
η_S	Share of Structures in R&D	0.0699	BERD (2015)

Notes: The model is calibrated at annual frequency. Table summarizes the baseline parameterization for the U.S. economy. Together, these values discipline the balanced-growth implications used in the quantitative exercises.

tively. The calibrated model closely matches the observed growth paths of GDP per capita and the TFP stock.

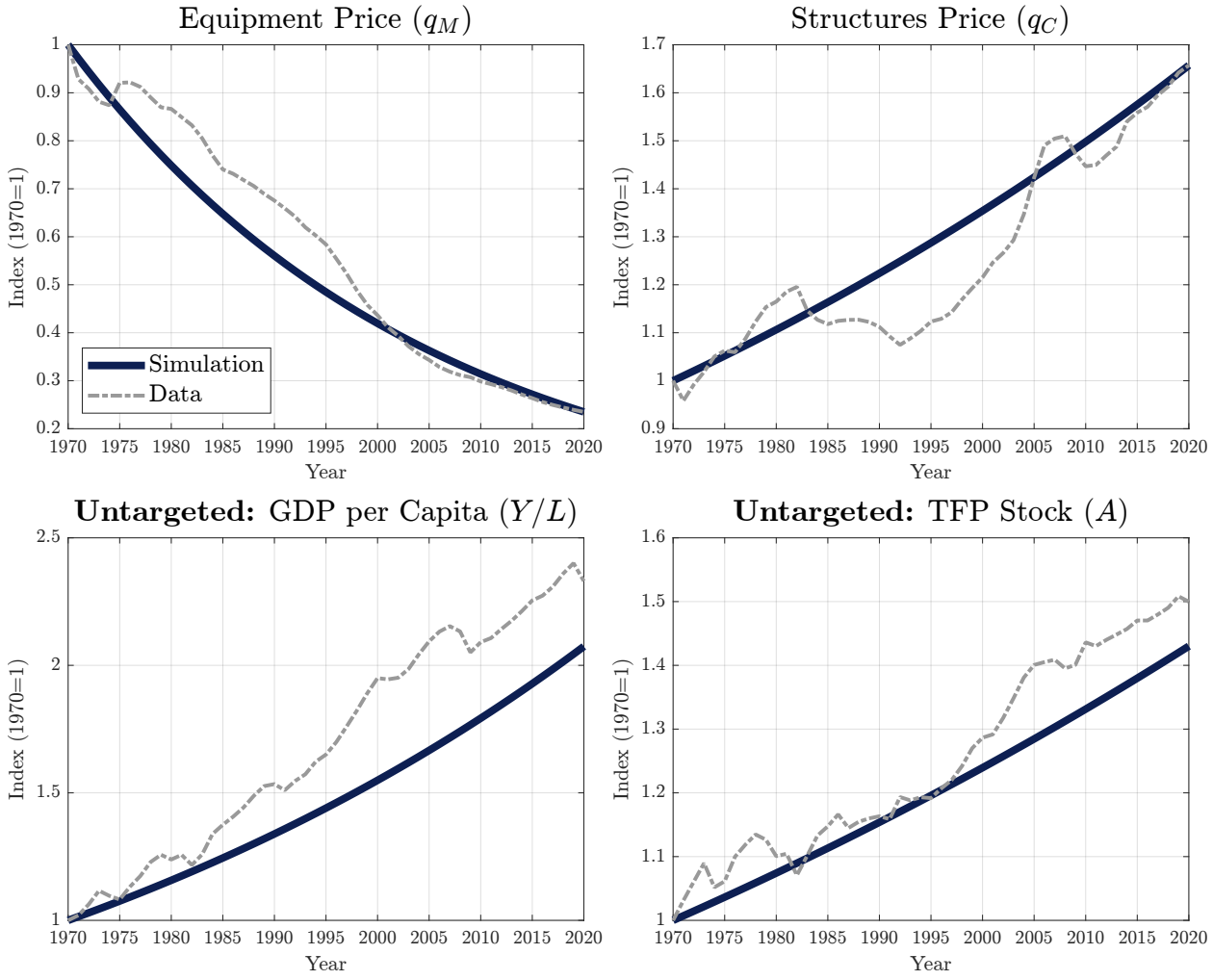
4.2.1 Sensitivity Analysis

We conducted a sensitivity analysis to assess the robustness of our model across different parameter settings, particularly ϕ. The candidates for ϕ values are drawn from Jones (2022).[19] Table 4 summarizes the steady-state growth rates of technology, g_A, and output per capita, $g_{Y/L}$, calculated for each ϕ.

As shown in Table 4, the growth rates of both technology and output per capita are highly sensitive to the value of ϕ, which represents the returns to the existing stock of

[19]We impose a technical restriction on the parameter range to ensure the model's structural integrity. Specifically, following Jones (2022), we can match ϕ with β in Jones (2022) by $\phi \equiv 1 - \beta$. In Jones (2022) specification $\dot{A}/A = A^{-\beta}$, β measures the strength of diminishing research opportunities: when $\beta > 0$, a higher technology level A reduces the proportional growth rate. In our specification $\dot{A} = A^{\phi}$, ϕ governs the curvature (returns to scale) of knowledge production and $\phi < 1$ implies diminishing returns. Therefore, the higher β is, the lower the ϕ is. We note that the case where $\beta = 0.2$ (which implies $\phi = 0.8$) is excluded from our analysis as it violates the fundamental regularity condition of our model, $\Delta > 0$ (defined in equation 12).

Figure 5: **Baseline Simulation: Sectoral Prices and Macroeconomic Aggregates**



Notes: The top panels show the targeted moments: the relative prices of equipment ($1/q_E$) and structures ($1/q_S$), which the simulation is calibrated to match. The bottom panels show untargeted moments: GDP per capita (Y/N) and the TFP stock (A). The sectoral relative price series and GDP per capita are taken from the Penn World Table (PWT), while the TFP stock is obtained from the Bureau of Labor Statistics (BLS) (Private Business Sector: Total Factor Productivity (`MFPPBS` in FRED)). All series are normalized to 1 in 1970.

knowledge. When ϕ increases from -4 toward 0, we observe a significant acceleration in growth. For instance, the technology growth rate (g_A) at $\phi = 0$ is nearly ten times higher than that at $\phi = -4$.

4.2.2 Benchmark Comparisons

Our model introduces two new margins relative to standard growth frameworks: capital heterogeneity (equipment versus structures) and the use of capital in the R&D sector. To assess the quantitative importance of these margins, we compare our baseline specifica-

Table 4: **Growth Rates Under Various Values of ϕ**

ϕ	g_A	$g_{Y/L}$
0	0.0377	0.0637
−1	0.0120	0.0224
−2 (benchmark)	0.0072	0.0146
−4	0.0040	0.0094

Notes: This table reports the implied balanced-growth rates of TFP, g_A, and labor-productivity, $g_{Y/L}$, as the knowledge-spillover parameter ϕ varies. More negative values of ϕ reduce the strength of effective spillovers (equivalently, increase the degree of diminishing returns in research), lowering both g_A and $g_{Y/L}$. The candidates for ϕ are taken from Jones (2022).

tion to commonly used benchmarks, including a neoclassical one-capital growth model and a Romer-type model. We show that our model delivers GDP per capita growth rates that are closer to the U.S. data than these alternative specifications. Additional details and model comparisons are provided in Appendix F.

4.3 Structural Drag Effect on Growth

To quantify the long-run impact of structural drag, we decompose balanced-growth per-capita output growth. Combining (15) with the closed-form expression for g_A in (16), and defining per-capita output growth as $g_y \equiv g_Y - n$, yields the linear representation

$$g_y = \mu_n^y \cdot n + \mathcal{M}_E \cdot \gamma_E + \mathcal{M}_S \cdot \gamma_S. \tag{17}$$

The coefficients in (17) are interpretable multipliers that map observed trend components into long-run per-capita growth. Grouping terms by n, γ_E, and γ_S implies

$$\mu_n^y \equiv \frac{1}{\Delta}, \qquad \mathcal{M}_E \equiv \frac{\alpha_E}{\Psi} + \frac{\eta_E(1-\alpha_S)+\eta_S\alpha_E}{\Psi\,\Delta}, \qquad \mathcal{M}_S \equiv \frac{\alpha_S}{\Psi} + \frac{\eta_S(1-\alpha_E)+\eta_E\alpha_S}{\Psi\,\Delta}, \tag{18}$$

where $\Psi \equiv 1-\alpha_E-\alpha_S$ and $\Delta \equiv (1-\phi)\Psi - (\eta_E+\eta_S)$ as in (12). A finite and stable BGP requires $\Delta > 0$.

Per-capita output growth can be decomposed into three parts. The population growth multiplier $\mu_n^y \equiv 1/\Delta$ captures a pure R&D scale (induced-innovation) effect (Romer, 1990; Jones, 1995). The equipment ISTC term, $\mathcal{M}_E\gamma_E$, captures the equipment contribution. When $\gamma_E > 0$, equipment efficiency growth raises per-capita output growth through capital deepening and, in the two-capital model, through induced innovation. The structure term, $\mathcal{M}_S\gamma_S$, captures the structures contribution. When structures ISTC is sluggish or

negative, as in the U.S. case, this term turns negative and depresses long-run per-capita growth.

Each multiplier $\mathcal{M}_k$ has a transparent two-part structure. The terms α_k/Ψ capture the static capital-deepening channel through the final-goods technology. The remaining terms, proportional to $1/(\Psi\Delta)$, capture the induced-innovation channel that operates through the equilibrium idea-growth response embedded in (16).

Focusing on the structures contribution, we can rewrite $\mathcal{M}_S$ as the sum of three interpretable components: (i) capital deepening in final-goods production, (ii) the direct cost of structures used as R&D facilities, and (iii) an indirect resource-allocation channel that operates through the economy-wide constraint on innovation:

$$\mathcal{M}_S = \underbrace{\frac{\alpha_S}{\Psi}}_{\text{Static Deepening}} + \underbrace{\underbrace{\frac{\eta_S(1-\alpha_E)}{\Psi\Delta}}_{\text{Direct Input Channel}} + \underbrace{\frac{\eta_E\alpha_S}{\Psi\Delta}}_{\text{Indirect Resource Channel}}}_{\text{R\&D-related Structural Drag}}. \tag{19}$$

A decline in structures ISTC, i.e., a lower γ_S, reduces growth not only by lowering output and weakening incentives for capital accumulation in the final-goods sector (the static-deepening component), but also by increasing R&D-related structural drag through two mechanisms: it raises the effective user cost of structures required for R&D activity (the direct input channel) and, through the general-equilibrium resource constraint, tightens the economy-wide availability of resources for innovation (the indirect resource channel). Equation (19) quantifies the contribution of each channel to the overall drag. In particular, the term $\eta_S(1-\alpha_E)/(\Psi\Delta)$ captures the direct input channel because it is proportional to η_S and therefore arises only when structures enter the R&D technology as an input (e.g., facilities). The term $\eta_E\alpha_S/(\Psi\Delta)$ captures the indirect resource channel because it loads on α_S: changes in structures affect aggregate production incentives and, via the final-goods resource constraint, the equilibrium scale of research inputs.

Under the U.S. calibration, we decompose balanced-growth per-capita output growth by applying (17) together with the structures-channel split in (19). Table 5 reports the resulting contributions in percentage points (pp) per year.

Per-capita output growth is $g_y = 1.46$ pp per year. This growth reflects contributions from the population term (0.64 pp) and, in particular, equipment-specific ISTC (1.32 pp). By contrast, structures reduce growth by 0.50 pp, which is about 38% of the equipment boost in magnitude.

As shown in Table 6, the structural drag decomposes into a static-deepening compo-

Table 5: **U.S. Per-Capita Output Growth g_y Decomposition**

Component	Growth rate (pp/year)	Share of g_y (%)
Population ($\mu_n^y n$)	0.64	43.8
Equipment Boost ($\mathcal{M}_E \gamma_E$)	1.32	90.4
Structural Drag ($\mathcal{M}_S \gamma_S$)	-0.50	-34.2
Total g_y	**1.46**	**100.0**

Notes: This table decomposes U.S. per-capita output growth, g_y, into three additive components. The population term, $\mu_n^y n$, captures the contribution of population growth scaled by the model-implied multiplier. The equipment component, $\mathcal{M}_E \gamma_E$, measures the positive contribution from equipment-specific investment technology improvement, while the structures component, $\mathcal{M}_S \gamma_S$, captures the effect of structures-specific price changes, which is negative in the U.S. calibration. "Share of g_y" reports each component divided by total g_y; because the structures term is negative, shares can exceed 100 percent in absolute value.

Table 6: **U.S. Structural Drag: Channel Decomposition**

Component	Growth rate (pp/year)	Share of Structural Drag (%)	Share of R&D Drag (%)
Structural drag $\mathcal{M}_S \gamma_S$	-0.50	100	—
Static Deepening $\frac{\alpha_S}{\Psi} \gamma_S$	–0.38	74.9	—
R&D-related Structural Drag	-0.12	25.3	100
Direct Channel $\frac{\eta_S (1-\alpha_E)}{\Psi \Delta} \gamma_S$	-0.06	12.5	49.6
Indirect Channel $\frac{\eta_E \alpha_S}{\Psi \Delta} \gamma_S$	-0.06	12.7	50.4

Notes: Shares in the third column are computed relative to the total structures contribution, $\mathcal{M}_S \gamma_S$. Shares in the last column are computed relative to the R&D-related structural drag component. The *direct input channel* corresponds to the term $\eta_S (1 - \alpha_E)/(\Psi \Delta) \gamma_S$ and captures the increase in the effective user cost of structures used as an input into R&D activity (e.g., research facilities) when structures ISTC is weak. The *indirect resource channel* corresponds to the term $\eta_E \alpha_S / (\Psi \Delta) \gamma_S$ and captures the general-equilibrium effect whereby weak structures efficiency growth tightens the final-goods resource constraint and reduces the equilibrium scale of research inputs. Negative growth rates indicate a drag on per-capita output growth.

nent of −0.38 pp and R&D-related structural drag of −0.12 pp. This R&D-related structural drag is split almost evenly across the two channels implied by equation (19): the direct input channel accounts for 49.7% (−0.06 pp) and the indirect resource channel accounts for 50.3% (−0.06 pp). The near-equal split indicates that both mechanisms contribute materially to overall structural drag.

4.4 Measurement Error Adjustment

While the decline in the construction sector productivity may constitute one possible driver as discussed in Section 2.3, concerns remain that the secular increase in the relative price of structures may, at least in part, reflect deflator mismeasurement rather than

a genuine decline in construction productivity.[20] Even though Goolsbee and Syverson (2023) conclude that the poor performance of construction sector is not just a figment of measurement error, structures price index from the PWT may have missed quality improvement of structures.[21]

Garcia and Molloy (2025) assess the extent to which growth in construction costs could be biased upward by improvements in unobserved structure quality.[22] We borrow their estimated range of bias and revisit the size of structural drag. Specifically, Garcia and Molloy (2025) quantify the bias using three different approaches. First, they use detailed industry construction cost data from R.S. Means to estimate the change in construction costs for specific housing types. This approach yields close to zero bias. Second, they obtain an assessment of structure quality from property tax assessors, a rating of structure quality from the resident of the home, and an estimate of energy efficiency. This approach yields 0.50 percentage points per year bias. Third, they apply an econometric technique for assessing the magnitude of unobserved variable bias (Oster, 2019). This approach yields the upper bound of 0.8 percentage points per year bias.

We impose a conservative upper bound of 0.8 percentage point per year on the bias in aggregate construction productivity growth. In our calibration, structures ISTC is proxied by the relative-price trend, which implies 1 percentage point per year. Under this upper bound, at most 80% of the implied structures-price growth can be attributed to measurement error, leaving the remaining roughly 20% to reflect a genuine decline in productivity.

In terms of growth contributions, the baseline structural drag is $\mathcal{M}_S\gamma_S = 0.50$ pp. The upper-bound correction reduces this drag by 0.40 pp, leaving a residual structural drag of about 0.10 pp. Thus, measurement error can attenuate, but cannot eliminate, the quantitative importance of structural drag.

4.5 Counterfactual Analysis: Quantifying Structural Drag

To quantify the extent to which U.S. long-run growth has been hindered by the rising relative cost of structures, we conduct a counterfactual exercise benchmarking the United

[20] Several authors have documented potential biases in the International Comparison Program; see, e.g., Deaton and Heston (2010) and Argente et al. (2023).

[21] The analysis in Goolsbee and Syverson (2023) control for quality changes by accounting for changes in housing size, but this adjustment alone could be insufficient, as it does not account for other aspects of quality.

[22] Their conclusion is that even under generous assumptions, their estimates of the magnitude of the bias are not large enough to alter the view that construction-sector productivity growth has been weak.

States against two reference groups: the median High-Income and median Low-Income economies, as classified in Section 2.2.

Table 7 summarizes the observed ISTC profiles. While equipment ISTC, γ_E, remains a robust driver of growth across all groups, structures ISTC ,γ_S, exhibits a stark divergence. Notably, the U.S. displays substantially more adverse structures ISTC (−1.01%) than the High-Income median (−0.42%), indicating a distinctive "structural drag" in the U.S. economy.

Table 7: **Observed ISTC Rates by Region (1970–2020)**

Region	Equipment (γ_E)	Structures (γ_S)	Gap ($\gamma_E - \gamma_S$)
Low Income	1.66%	−0.08%	1.74pp
High Income	3.02%	−0.42%	3.44pp
The United States	2.90%	−1.01%	3.91pp

Notes: ISTC rates are measured as the negative of the average growth rate of relative investment prices for equipment and structures over 1970–2020. The last column reports the implied ISTC gap between equipment and structures. Median LIC/HIC trends are taken from Figure 2.

To translate these differences into aggregate performance, we map the empirical ISTC rates into the calibrated growth-accounting framework in (18). Specifically, we feed the region-specific ISTC pairs from Table 7 into the U.S.-calibrated model as alternative shocks, holding the multipliers fixed at their U.S. values ($\mathcal{M}_E = 0.46$, $\mathcal{M}_S = 0.50$). Table 8 reports the implied growth contributions from ISTC.

Table 8: **Counterfactual U.S. Growth under Alternative ISTC Shocks**

Shock Scenario	Equipment Boost	Structural drag	g_y^{ISTC}
LIC Scenario	0.76pp	−0.04pp	0.72pp
HIC Scenario	1.38pp	−0.21pp	1.17pp
The United States	1.32pp	−0.50pp	0.82pp

Notes: Each row reports the implied per-capita output growth contribution from ISTC, g_y^{ISTC}, under alternative relative-price (ISTC) shock profiles. The "Equipment Boost" and "Structural drag" columns decompose g_y^{ISTC} into the contributions associated with equipment and structures ISTC, respectively. LIC (HIC) scenarios apply the observed low-income (high-income) ISTC rates to the U.S. calibration, while the last row reports the baseline U.S. benchmark.

These counterfactuals deliver two key insights. First, relative to the High-Income median, the U.S. growth shortfall is driven almost entirely by the structures margin. The equipment contribution is nearly identical (1.32pp vs. 1.38pp), but the U.S. experiences a much more severe structural drag (−0.50pp vs. −0.21pp). Second, relative to the Low-

Income median, the U.S. outcome reflects offsetting forces: a larger equipment contribution (1.32pp vs. 0.76pp) is partly offset by a substantially more negative structures contribution.

To further characterize this drag, Table 9 decomposes the total structures contribution, $\mathcal{M}_S\gamma_S$, into (i) static capital deepening and (ii) an R&D-related structural drag component that aggregates the direct input and indirect resource channels in equation (19).

Table 9: **Decomposition: U.S. Baseline vs. LIC Counterfactual**

Component	U.S. Baseline (pp)	LIC Counterfactual (pp)	Δ pp
Structural Drag ($\mathcal{M}_S\gamma_S$)	-0.502	-0.040	+0.462
Static Deepening $\frac{\alpha_S}{\Psi}\gamma_S$	-0.375	-0.030	+0.345
R&D-related Structural Drag	-0.127	-0.010	+0.117

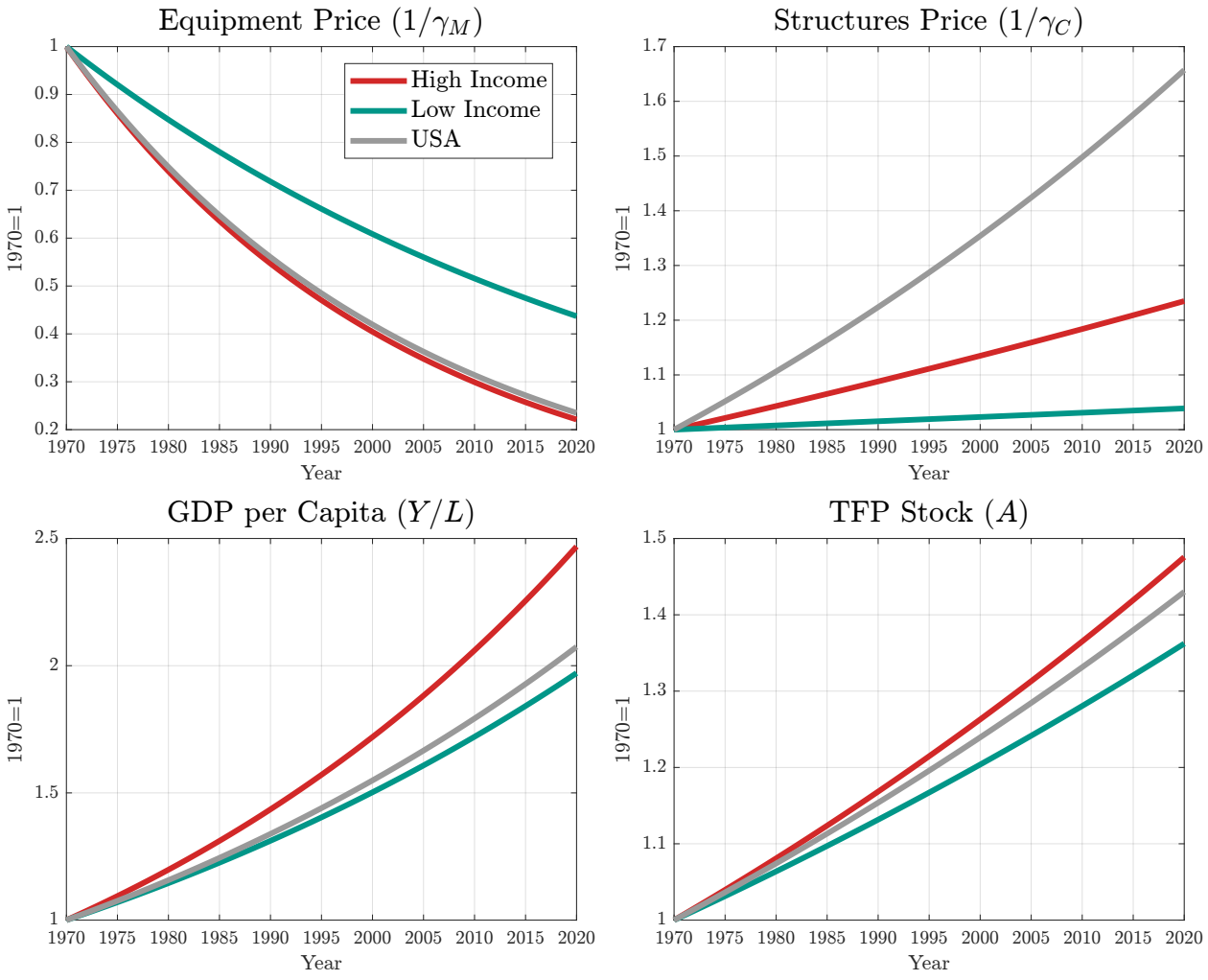
Notes: This table contrasts the structures-related contribution to U.S. per-capita output growth in the baseline calibration with a counterfactual that replaces the U.S. structures ISTC rate with the LIC value. The last column reports the change in percentage points for each component. The total structures term $\mathcal{M}_S\gamma_S$ is decomposed into (i) static capital deepening and (ii) an R&D-related structural drag component, which combines the direct input and indirect resource channels from equation (19).

If the U.S. had instead faced the LIC structures ISTC rate ($\gamma_S = -0.08\%$), per-capita growth would have been higher by 0.46 pp per year. Of this improvement, 0.12 pp comes from a smaller R&D-related structural drag component. In the baseline calibration, weak structures ISTC raises the effective cost of research infrastructure and, through general-equilibrium resource reallocation, reduces the equilibrium scale of research inputs. Under the LIC scenario, these R&D-related wedges are substantially attenuated, indicating that sluggish structures ISTC can act as a quantitatively meaningful constraint on sustained productivity growth.

Figure 6 visualizes these dynamics. The U.S. (gray) and High-Income (green) scenarios share similar equipment paths, but the divergence in structures ISTC causes the U.S. GDP per capita to fall behind. While the U.S. remains above the Low-Income (red) level due to its equipment efficiency, structural drag remains the primary obstacle preventing the U.S. from reaching the growth potential implied by the High-Income benchmark.

Taken together, these results show that the efficiency gap ($\gamma_E - \gamma_S$) is a useful summary measure of structural drags, but it does not fully characterize cross-group differences in outcomes. When comparing the United States to other High-Income economies, the shortfall is accounted for mainly by the structures margin, consistent with a sizable "structural drag." By contrast, relative to Low-Income economies, differences in equipment ISTC remain quantitatively important, so aggregate performance reflects both an equipment boost and a structural drag. In the model, the structural drag operates through

Figure 6: **Transitional Paths under Alternative ISTC Regimes, 1970–2020**



Notes: The figure plots model-implied time paths when the economy is calibrated to the U.S. parameterization (including the U.S. multipliers) and is subjected to alternative ISTC shocks. The High-Income and Low-Income shock pairs (γ_E, γ_S) are taken from Table 7, while all other parameters are held fixed at their U.S.-calibrated values. The gray line corresponds to the U.S. shock pair, and the green and red lines apply the High-Income and Low-Income shock pairs, respectively. The top-left panel reports the equipment relative investment price index P_E and the top-right panel reports the structures relative investment price index P_S, both normalized to 1970=1. The bottom-left panel reports GDP per capita, and the bottom-right panel reports TFP (knowledge stock) A, each normalized to 1970=1.

two channels: (i) it dampens capital deepening in final-goods production and (ii) it raises the effective cost of research infrastructure, thereby reducing the rate of innovation.

4.6 Extension: CES Technology Case

The benchmark drag weight, ω_{drag}, is derived under Cobb–Douglas aggregation of equipment and structures in both the final-goods and R&D sectors. Allowing CES aggregation

makes the effective drag state-dependent through the relative user cost $x \equiv u_E/u_S$ and can amplify the drag when the two capital types are complements (elasticity below one). In this section we consider a CES extension and document the implied state-dependent drag at a given state x (Pseudo-CES case),while the fully endogenous CES economy (including the transition dynamics of $x(t)$ and the induced share drift) is derived and proved in Appendix G.1.[23]

In each sector $j \in \{Y, R\}$, capital services are aggregated as

$$Q_j(K_E^j, K_S^j) = \left(\theta_j^{1/\sigma_j}(K_E^j)^{\rho_j} + (1-\theta_j)^{1/\sigma_j}(K_S^j)^{\rho_j}\right)^{1/\rho_j}, \qquad \rho_j \equiv \frac{\sigma_j - 1}{\sigma_j}, \quad \sigma_j > 0,\ \theta_j \in (0,1). \tag{20}$$

Output and knowledge accumulation are

$$Y(t) = A(t)\, Q_Y(t)^{\alpha_Y} L_u(t)^{1-\alpha_Y}, \qquad \alpha_Y \in (0,1), \tag{21}$$

and

$$\dot{A}(t) = \kappa A(t)^{\phi} Q_R(t)^{\alpha_R} L_s(t)^{1-\alpha_R}, \qquad \alpha_R \in (0,1),\ \kappa > 0,\ \phi < 1. \tag{22}$$

At a given state $x \equiv u_E/u_S$, the innovation-growth rate admits the representation

$$g_A = \frac{(1-\alpha_Y)n + \alpha_R \gamma_E - \alpha_R\, \Omega(x)\, (\gamma_E - \gamma_S)}{(1-\phi)(1-\alpha_Y) - \alpha_R}, \tag{23}$$

where

$$\Omega(x) = \alpha_Y\, s_{S,Y}(x) + (1-\alpha_Y)\, s_{S,R}(x), \qquad s_{S,j}(x) \equiv 1 - \frac{1}{1 + \frac{1-\theta_j}{\theta_j} x^{\sigma_j - 1}}. \tag{24}$$

Define the state-dependent drag weight

$$\omega_{\text{drag}}(x) \equiv \alpha_R\, \Omega(x).$$

User costs satisfy $u_k = P_k(r + \delta_k + \gamma_k)$, so

$$x = \frac{u_E}{u_S} = \frac{P_E}{P_S} \cdot \frac{r + \delta_E + \gamma_E}{r + \delta_S + \gamma_S}.$$

[23] When $\sigma_Y \neq 1$ or $\sigma_R \neq 1$ and $\gamma_E \neq \gamma_S$, the relative user cost typically drifts over time, so constant-growth balanced-growth-path expressions for (g_A, g_Y) need not apply globally. The expressions below are therefore best interpreted as instantaneous (conditional) growth-rate representations at a given state x. In addition, under complementarity with a locally declining x, allowing x to move raises structures shares and increases wedge loading locally, so holding x fixed provides a conservative local lower bound conditional on the same current x. Formal proofs are in Appendix G.3.

The structural drag multiplier on the growth of per capita can be derived as follows.

$$g_y = \mu_n^y \, n + \mathcal{M}_E(x)\gamma_E + \mathcal{M}_S(x)\gamma_S, \qquad \mu_n^y = \frac{1}{\Delta}, \tag{25}$$

where state dependence enters only through the CES shares embedded in $s_{S,Y}(x)$ and $s_{S,R}(x)$. The closed form is

$$\mathcal{M}_S(x) = \underbrace{\frac{\alpha_Y}{1-\alpha_Y}\, s_{S,Y}(x)}_{\text{static deepening}} + \underbrace{\frac{\alpha_R}{\Delta_{ces}}\, s_{S,R}(x)}_{\text{direct R\&D input}} + \underbrace{\frac{\alpha_R}{\Delta_{ces}}\,\frac{\alpha_Y}{1-\alpha_Y}\, s_{S,Y}(x)}_{\text{indirect resource channel}}. \tag{26}$$

where $\Delta_{ces} \equiv (1-\phi)(1-\alpha_Y) - \alpha_R$. The three terms in (26) correspond, respectively, to the direct production-side effect of structures, the direct role of structures as an R&D input, and the feedback from production-side unit costs into innovation through the resource channel, scaled by the same denominator Δ that governs the innovation-growth fixed point.

Special case: Cobb–Douglas. When $\sigma_Y = \sigma_R = 1$, CES collapses to Cobb–Douglas and the sectoral expenditure shares become constants, equal to the Cobb–Douglas weights. Let

$$\alpha_E \equiv \alpha_Y\theta_Y, \quad \alpha_S \equiv \alpha_Y(1-\theta_Y), \qquad \eta_E \equiv \alpha_R\theta_R, \quad \eta_S \equiv \alpha_R(1-\theta_R),$$

so that $\alpha_Y = \alpha_E + \alpha_S$ and $\alpha_R = \eta_E + \eta_S$. Then

$$\Omega^{CD} = \alpha_Y(1-\theta_Y) + (1-\alpha_Y)(1-\theta_R) = \alpha_S + (1-\alpha_E-\alpha_S)\frac{\eta_S}{\eta_E+\eta_S},$$

and hence

$$\omega_{\text{drag}}^{CD} \equiv \alpha_R\Omega^{CD} = (\eta_E+\eta_S)\alpha_S + (1-\alpha_E-\alpha_S)\eta_S = \eta_E\alpha_S + \eta_S(1-\alpha_E),$$

which matches the benchmark expression. The complete Cobb–Douglas reduction (including the limiting argument from CES and the step-by-step algebra) is shown in Appendix G.4.

When does CES amplify structural drag. Recall the state-dependent CES drag weight

$$\omega_{\text{drag}}(x) \equiv \alpha_R\,\Omega(x), \qquad \Omega(x) \equiv \alpha_Y\, s_{S,Y}(x) + (1-\alpha_Y)\, s_{S,R}(x),$$

with CES structures shares

$$s_{S,j}(x) \equiv \frac{\left(\frac{1-\theta_j}{\theta_j}\right)x^{\sigma_j-1}}{1+\left(\frac{1-\theta_j}{\theta_j}\right)x^{\sigma_j-1}}, \qquad j \in \{Y,R\}.$$

The Cobb–Douglas benchmark corresponds to $\sigma_Y = \sigma_R = 1$, in which case

$$s_{S,Y}^{CD} = 1-\theta_Y, \qquad s_{S,R}^{CD} = 1-\theta_R,$$

so

$$\Omega^{CD} \equiv \alpha_Y(1-\theta_Y) + (1-\alpha_Y)(1-\theta_R), \qquad \omega_{\text{drag}}^{CD} \equiv \alpha_R\,\Omega^{CD}.$$

Subtracting the benchmark from the CES object yields the exact difference decomposition

$$\omega_{\text{drag}}(x) - \omega_{\text{drag}}^{CD} = \alpha_R\alpha_Y\Big(s_{S,Y}(x) - (1-\theta_Y)\Big) + \alpha_R(1-\alpha_Y)\Big(s_{S,R}(x) - (1-\theta_R)\Big). \tag{27}$$

Hence CES amplifies structural drag relative to Cobb–Douglas if and only if the relevant CES structures shares exceed their Cobb–Douglas counterparts in the sectors entering $\Omega(\cdot)$.

A convenient sign characterization follows directly from the CES share map. For each sector $j \in \{Y,R\}$,

$$s_{S,j}(x) > (1-\theta_j) \qquad \Longleftrightarrow \qquad (\sigma_j - 1)\log x > 0.$$

Therefore, in the empirically relevant region $x < 1$ (equipment cheaper in user-cost terms, so $\log x < 0$),

$$\sigma_j < 1 \;\Rightarrow\; s_{S,j}(x) > (1-\theta_j) \;\Rightarrow\; \omega_{\text{drag}}(x) > \omega_{\text{drag}}^{CD},$$

While Cobb–Douglas technology imposes fixed factor weights and thus pins down the benchmark drag with time-invariant shares, CES aggregation changes the mapping in a minimal but economically meaningful way. At any given state $x \equiv u_E/u_S$, the CES cost shares $s_{S,Y}(x)$ and $s_{S,R}(x)$ govern how strongly the economy relies on structures in final production and in R&D capital services. Because structures typically have the slower trend component γ_S, a higher structures share mechanically increases the loading on the slow-moving component inside the relevant unit-cost wedge and therefore strengthens the drag generated by the ISTC gap.

Even when contemporaneous adjustment wedges and transition dynamics are shut

down, the same primitives (n, γ_E, γ_S) can imply different instantaneous growth implications depending on the current relative user-cost state x and the elasticities (σ_Y, σ_R). In the empirically relevant region $x < 1$ (equipment relatively cheap), complementarity ($\sigma_Y < 1$ and/or $\sigma_R < 1$) limits substitution away from structures, so $s_{S,j}(x)$ tends to exceed its Cobb–Douglas counterpart $1 - \theta_j$. The economy therefore behaves as if it were more structure-intensive than in the benchmark, which raises wedge loading on γ_S and amplifies structural drag.

The fully endogenous CES economy adds an additional channel—share drift over time as $x(t)$ moves—but the Pseudo-CES case already delivers the key static implication: CES makes the drag weight and growth multipliers state-dependent through $(s_{S,Y}(x), s_{S,R}(x))$, whereas Cobb–Douglas fixes these objects by assumption.

5 Concluding Remarks

This paper challenges the prevailing view that investment-specific technological change in equipment is a sufficient statistic for understanding long-run growth. By disaggregating capital into equipment and structures, we uncover a fundamental tension in the growth process. While falling equipment prices have indeed lowered the cost of producing output and ideas, rising structures price have simultaneously imposed a binding constraint on capital accumulation and innovation.

Quantitatively, our model since 1970 yields an average per-capita output growth of 1.46 percentage points annually. Of this, ISTC contributes 0.82 percentage points in net terms, reflecting a substantial equipment boost of 1.32 percentage points partially offset by a structural drag of 0.50 percentage points, while the remaining 0.64 percentage points arises from the population component. Of this "structural drag," about three-quarters operates through reduced capital deepening in production, while the remaining quarter reflects the R&D-related structural drag. It is split nearly equally between the direct input channel (raising R&D facility costs) and the indirect resource channel (tightening economy-wide resources for innovation).

These findings carry important implications for growth policy. Efforts to accelerate technological progress in equipment-producing sectors may yield diminishing returns if not accompanied by policies addressing productivity stagnation in the structures sector. Understanding why structures productivity has followed what Goolsbee and Syverson (2023) call a "strange and awful path" remains a critical question for future research. Our framework provides a lens through which to evaluate the aggregate implications

of any improvements in construction sector productivity and to assess whether policy interventions targeting the construction sector could meaningfully relax the constraint that structures impose on long-run growth.

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APPENDIX

A Details on Capital Price and Expenditure Measurement

Our primary analysis examines how heterogeneity in capital relates to development outcomes using three main data sources: the International Comparison Program (ICP) and the Penn World Table (PWT) version 11.0, particularly the "Capital Detail" module. For the research and development (R&D) sector, we use the National Center for Science and Engineering Statistics (NCSES) Business Enterprise Research and Development (BERD) survey for U.S. data, complemented by gross domestic expenditure on R&D (GERD) for cross-country comparisons.

We use the PWT capital detail data to characterize time-series trends in capital prices, with a focus on equipment and structures. We use ICP price data primarily for cross-sectional regressions that relate sectoral price levels to GDP per capita. We use BERD to measure and decompose R&D-related capital expenditures by sector, and we use GERD for robustness checks.

This section clarifies the interpretation of each variable and describes data coverage in detail.

A.1 The International Comparison Program (ICP)

We focus on the ICP price measures for two capital categories, Machine and Equipment, `item_1501100`) and structures (structures, `item_1501200`). In the ICP classification, these correspond to the following expenditure items under gross fixed capital formation.

- **Machine and Equipment (`item_1501100`):** This corresponds to `1501100 equipment AND EQUIPMENT` under `1501000 GROSS FIXED CAPITAL FORMATION` in the ICP classification. It covers investment in:
  - `1501110 Metal products and equipment`, (including `1501111 Fabricated metal products, except equipment and equipment`)
  - `1501112 Electrical and optical equipment`
  - `1501115 General purpose equipment`
  - `1501116 Special purpose equipment`
  - `1501120 Transport equipment`

- **Structures (**item_1501200**):** This corresponds to 1501200 structures under 1501000 GROSS FIXED CAPITAL FORMATION in the ICP classification. It covers:
  - 1501210 Residential buildings
  - 1501220 Non-residential buildings
  - 1501230 Civil engineering works

Other capital formation components in the ICP classification. In addition to 1501100 and 1501200, the ICP capital formation block includes the following components. We report them here for completeness.

- **Other products (**item_1501300**):** 1501300 OTHER PRODUCTS within 1501000 GROSS FIXED CAPITAL FORMATION.
- **Changes in inventories (**item_1502000**):** 1502000 CHANGES IN INVENTORIES within 1500000 GROSS CAPITAL FORMATION.
- **Acquisitions less disposals of valuables (**item_1503000**):** 1503000 ACQUISITIONS LESS DISPOSALS OF VALUABLES within 1500000 GROSS CAPITAL FORMATION.

A.2 Penn World Table (PWT) Capital Detail

PWT capital measures (for example, capital services and relative prices of capital) are constructed from investment data by asset.[24] Because investment is originally estimated at a more disaggregated level, the PWT capital detail file provides a convenient aggregation into four asset groups: *machine and equipment*, *structures*, *transport equipment*, and *other assets*.

- **Machine and Equipment:** including computers, communication equipment, and other equipment.
- **Structures:** including residential and non-residential structures.
- **Transport equipment:** including transport equipment reported as a separate asset group.
- **Other assets:** including software, other intellectual property products, and cultivated assets.

[24] https://www.rug.nl/ggdc/docs/user_guide_to_pwt90_data_files.pdf

A.3 U.S. Bureau of Economic Analysis (BEA) Fixed Assets and Industry Accounts

For the U.S. focused analyses in the main text, we use detailed data from the U.S. BEA, particularly for the analyses underlying Facts 4. We rely on two main BEA data systems: the Fixed Assets Accounts and the GDP-by-Industry Accounts. Together, these sources allow us to measure asset specific investment prices, capital stocks, industry level exposure to structures, the outcomes.

Asset Categories. The BEA asset classification[25] allows us to distinguish sharply between equipment and equipment and structures. In the analysis, we define:

- **Equipment**[26] This category includes information-processing equipment (computers and peripheral equipment, communication equipment), industrial equipment, transportation equipment, and other equipment assets.
- **Structures**[27] This category includes residential and nonresidential structures (and associated nonbuilding structures). Importantly, the post-1990 increase in relative structures price documented in the main text is broad based across these subcategories rather than driven by a single component.

BEA Fixed Assets Accounts Based on the above defintion, the BEA Fixed Assets Accounts provide detailed information on investment flows, capital stocks, and implicit price indices by asset type and by industry. We use both investment-based and stock-based measures to characterize the evolution of relative prices and capital composition.

- **Investment in Private Fixed Assets by Asset Type.** We use National Income and Product Accounts (NIPA) Tables 2.7 and 2.8, which report nominal investment and

[25] BEA NIPA Handbook, Ch. 6 (Dec 2024 update) defines PFI fixed assets as structures, equipment, and intellectual property products. https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-06.pdf

[26] BEA's capitalization guidance treats an item as *equipment* if it (1) has a useful life of more than 1 year, (2) is *not* an integral part of a structure and is *not* included in the value of that structure (example: an elevator), and (3) would normally be charged to a capital account in business accounting records.

[27] BEA includes in *structures* the structures of new nonresidential and residential buildings, improvements to buildings, and certain types of equipment (e.g., plumbing and heating systems and elevators) that are considered an integral part of the structure; it also includes nonbuilding structures (e.g., pipelines, railroad tracks, power lines and plants, dams and levees), mobile structures, manufactured homes, well drilling and exploration, mine development, brokers' commissions/ownership transfer costs, and net purchases of existing structures from governments.

chain-type quantity indexes for private fixed assets by detailed asset category. For each asset type, we construct an implicit investment price index as the ratio of nominal investment to the corresponding chain-type quantity index.

- **Capital Stock by Asset Type.** To verify that the observed investment price dynamics extend to the capital stock itself, we also use BEA Fixed Assets Tables 3.1ESI (Current-Cost Net Stock of Private Fixed Assets) and 3.1S (Current-Cost Net Stock of Private Structures).

BEA Fixed Assets Accounts by Industry. To study cross-industry heterogeneity in exposure to structures, we use BEA Fixed Assets Tables that report investment and capital stocks by industry.

- **Investment by Industry.** Structures intensity at the industry level is measured using BEA Table 3.7ESI (Investment in Private Fixed Assets by Industry) and Table 3.7S (Investment in Private Structures by Industry). For each industry, we compute the structures investment share in 1990 as structures investment divided by total private fixed investment.

- **Capital Stock by Industry.** As an alternative measure of exposure, we compute the structures share in the capital stock using Table 3.1ESI (Current-Cost Net Stock of Private Fixed Assets, Equipment, Software, and Intellectual Property Products by Industry) and Table 3.1S (Current-Cost Net Stock of Private Structures by Industry). This stock-based measure captures longer-run technological and cost exposure beyond contemporaneous investment flows.

BEA GDP-by-Industry Accounts. To evaluate the real economic consequences of differential exposure to structures, we combine the Fixed Assets data with industry-level outcomes from the BEA GDP-by-Industry Accounts.

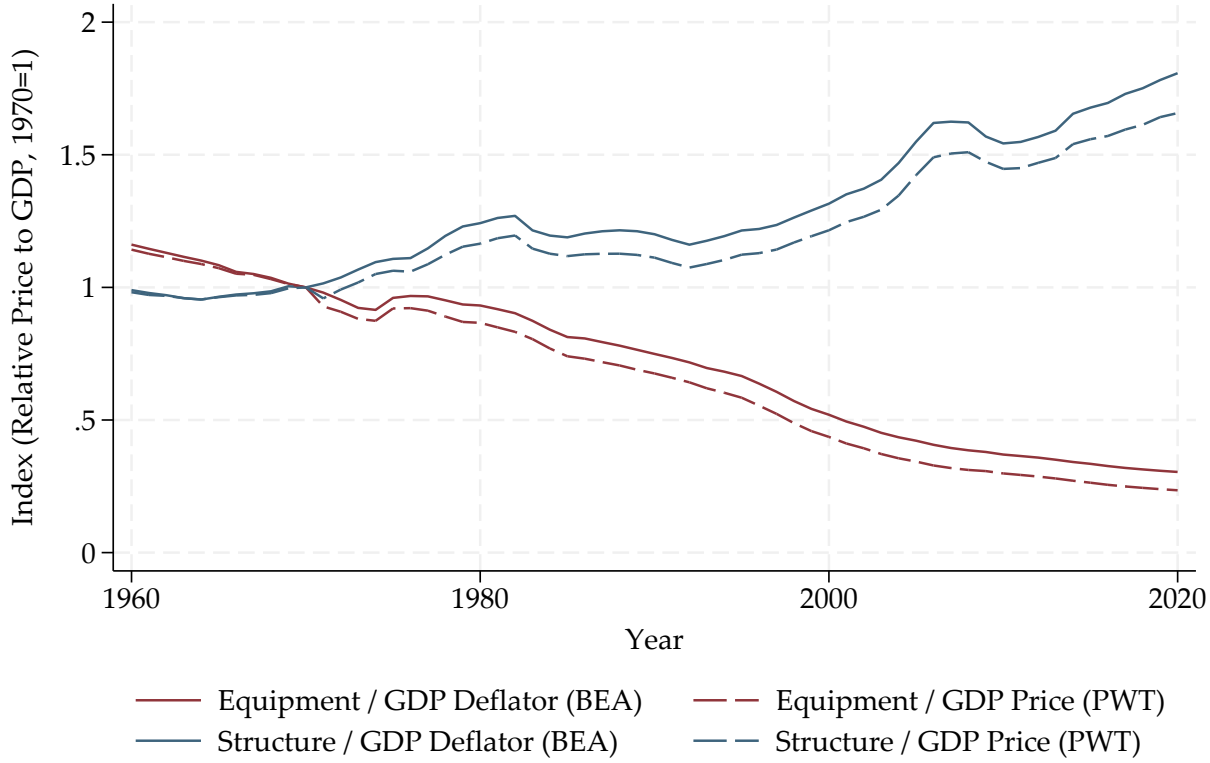
- **Value Added.** Industry value-added shares are obtained from the BEA GDP-by-Industry accounts and are defined as each industry's nominal value added divided by aggregate private-sector value added.

- **Employment.** Employment shares are computed using BEA data on full-time and part-time employment by industry. An industry's employment share is defined as its employment divided by total private employment.

Industry Coverage. The industry-level analysis focuses on major private-sector industries for which consistent investment, capital stock, value added, and employment data are available over time. These include agriculture, mining, utilities, structures, manufacturing (with detailed subsectors), wholesale and retail trade, transportation and warehousing, information, and selected service industries. Excluding a small number of service sectors with limited fixed-asset exposure does not affect the qualitative results.

Comparison with Penn World Table Capital To ensure that our two main datasets, the BEA and the PWT Capital Detail, exhibit consistent trends, we compare the relative prices (relative to the GDP price) of equipment and structures across the two sources.

Figure A.1: Relative Price Trend of Two Sector Investment from BEA and PWT



Notes: The figure compares U.S. asset price indices for equipment and structures constructed from the BEA and the PWT Capital Detail over 1960–2020, with all series normalized to 1970=1. For the BEA series, an asset-type price index is constructed as the ratio of nominal investment to real investment, where real investment is obtained by converting the BEA quantity index (2017=100) using the 2017-dollar level. The BEA relative price is computed by dividing the normalized asset price index by the GDP deflator (GDPDEF). For the PWT series, asset price indices for machinery/equipment and structures are normalized to 1970=1; the GDP price in national accounts is computed as $p_{gdp} = v_{gdp}/q_{gdp}$, normalized to 1970=1; and the PWT relative price is defined as the normalized asset price index divided by the normalized p_{gdp}. Solid lines denote BEA series and long-dashed lines denote PWT series; equipment and structures are distinguished by color as indicated in the legend. *Sources:* Bureau of Economic Analysis (Fixed Assets); Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

Figure A.1 compares the U.S. relative prices of equipment and structures from the BEA and the PWT Capital Detail over 1960–2020. For the BEA, we normalize the price indexes

for these two asset types by the GDP deflator. For the PWT Capital Detail, we normalize the corresponding price indexes by the GDP price in the national accounts, constructed by dividing nominal GDP in local currency units by real GDP.

BEA Asset Composition of Sectoral Investment and Relative Prices Table A.I reports, for the year 2020, how total investment in each sector is allocated across detailed BEA asset categories within the broad groups of Equipment and Structures. The table also reports relative prices in 2020, where each asset's real price in 1970 is normalized to 1.

Table A.I: Investment Shares by BEA Asset Category and Relative Prices in 2020

Asset Class	Share	Relative Price	Asset Class	Share	Relative Price
Equipment	100%	0.304	Structures	100%	1.808
Communication Equipment	10.961%	0.056	Housing (1 To 4 Unit)	21.473%	1.719
Computers And Peripheral Equipment	10.916%	0.000	Improvements	19.777%	1.545
Medical Equipment And Instruments	9.573%	0.426	Brokers' Commision	12.897%	1.866
Light Trucks	9.447%	0.763	Office	6.173%	1.907
General Industrial Machinery	8.257%	1.062	Electric	5.661%	1.715
Other Nonresidential Equipment	6.454%	0.765	Housing (5 Or More)	5.540%	2.135
Electrical Transmissions	4.273%	0.760	Manufacturing	5.001%	1.735
Furniture And Fixtures	4.092%	1.008	Petroleum And Natural Gas	3.972%	4.399
Other Trucks, Buses, And Truck Trailers	3.795%	1.206	Warehouses	2.854%	1.869
Special Industry Machinery	3.786%	1.219	Lodging	2.282%	1.769
Aircraft	3.597%	1.524	Communication	1.595%	0.929
Nonmedical Instruments	3.074%	0.795	Hospitals	1.535%	1.524
Construction Machinery	3.065%	1.368	Other Power	1.488%	2.042
Agricultural Machinery	2.948%	1.306	Educational And Vocational	1.205%	2.020
Service Industry Machinery	2.875%	0.857	Amusement And Recreation	1.097%	1.682
Metalworking Machinery	2.794%	1.045	Medical Buildings	1.010%	1.526
Mining And Oilfield Machinery	2.299%	1.549	Other Commercial	0.996%	1.729
Fabricated Metal Products	1.803%	1.117	Manufactured Homes	0.841%	1.324
Residential Equipment	1.550%	0.279	Land	0.761%	1.333
Engines And Turbines	0.874%	1.094	Multimerchandise Shopping	0.734%	1.768
Electrical Equipment	0.820%	0.655	Mining	0.606%	1.797
Railroad Equipment	0.703%	1.203	Food And Beverage Establishments	0.592%	1.830
Photocopy And Related Equipment	0.658%	0.228	Farm	0.565%	1.753
Ships And Boats	0.550%	1.252	Special Care	0.411%	1.532
Autos	0.451%	0.502	Air	0.303%	1.685
Office And Accounting Equipment	0.388%	0.289	Other Residential	0.289%	2.021
			Religious	0.236%	1.866
			Other	0.121%	1.516

Notes: Share denotes the share of 2020 sectoral investment allocated to each detailed BEA asset category within the corresponding broad group (Equipment or Structures). Relative Price is the ratio of the asset's real price in 2020 to its real price in 1970, where real prices are nominal prices deflated by the GDP deflator and the 1970 level is normalized to 1. Within each broad group, categories are ordered in descending order of Share.

A.4 NCSES Business Enterprise Research and Development (BERD): Capital Expenditures Classification

We utilize the National Center for Science and Engineering Statistics (NCSES) Business Enterprise Research and Development (BERD) survey to calibrate the model.

BERD defines capital expenditures as payments for assets that typically have a useful life of more than one year and are recorded on a company's balance sheet. Long-lived assets used in R&D operations are not counted as current R&D expense; instead, any depreciation recorded for those assets would be included in R&D expense.[28]

For the BERD survey year 2015 instrument, the relevant items request *domestic* capital expenditures for R&D operations (Question 2.32) and a required breakdown of those domestic R&D capital expenditures into

- **Structures**: Include the costs of purchased or improved buildings and other facilities such as signal towers or windmills that are fixed to the land
- **Equipment**
- **Capitalized software**
- **All other expenditures**: Include the costs of purchased patents or other intangible assets

in Question 2.33.[29] The instrument also notes that assets acquired through merger and acquisition activities should be excluded, and that capital expenditures benefiting both R&D and non-R&D operations should be allocated on a reasonable basis.

A.5 Gross Domestic Expenditure on R&D (GERD)

Gross domestic expenditure on research and development (GERD) measures the total intramural spending on R&D performed within a country during a given reference year. GERD is constructed as the aggregate of R&D expenditures reported by all performing sectors under the OECD reporting framework, which distinguishes both performing sectors and sources of funds. In this paper, we focus on the business enterprise performing sector and exclude government, higher education, private non profit, and foreign funded performing sectors from our baseline construction.

The capital type is classified based on The Frascati Manual[30] and variables we use are as follows.

- **Land and Buildings**: Include land acquired for R&D use (such as testing grounds and sites for laboratories or pilot plants) and buildings constructed or purchased for

[28] https://ncses.nsf.gov/surveys/business-enterprise-research-development

[29] https://www2.census.gov/programs-surveys/brdis/information/qbyqbrdi.pdf

[30] https://www.oecd.org/content/dam/oecd/en/publications/reports/2015/10/frascati-manual-2015_g1g57dcb/9789264239012-en.pdf

R&D use, including major improvements, modifications, and repairs. (From Section 4.48 to 4.50)

- **Machinery and equipment**: Include major capitalized machinery and equipment acquired for use in the performance of R&D. (Section 4.51)

identify equipment expenditures.

A practical issue for empirical work is that land and buildings are frequently reported jointly, and separate identification of land is not consistently available. While land is conceptually part of the land and buildings category, when the category is reported as a single total, the reported value is typically dominated by buildings and related construction and improvements rather than land acquisition. For this reason, we use land and buildings.

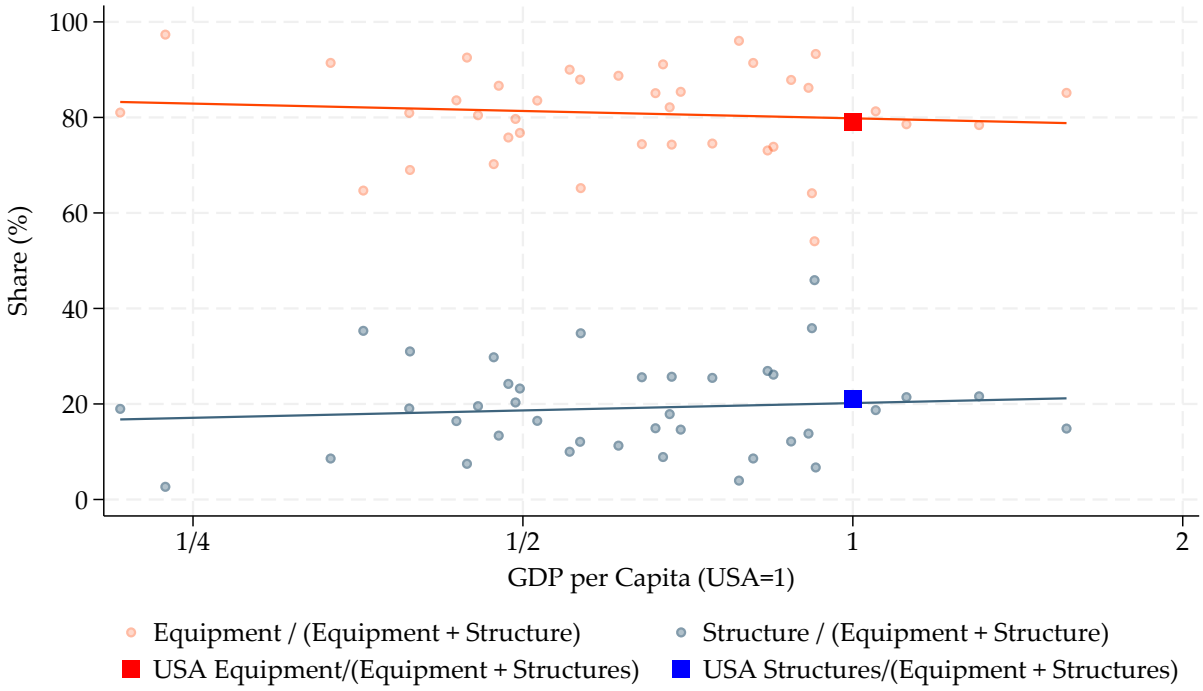
We calculate the allocation of capital expenditures between equipment (machinery and equipment) and structures (land and buildings) using GERD data for 45 countries (2014-2019 averages) and compare these patterns with U.S. sectoral evidence from BERD, yielding a total sample of 46 countries.

Figure A.2 plots the share of structures in total equipment-plus-structures investment and the share for equipment, against relative GDP per capita in 2017. (normalized to 1 for the United States). Across the income distribution, the equipment share in R&D capital expenditure is close to 80%, consistent with the 79% estimate for the United States from BERD. Correspondingly, the structures share is close to 20%. Overall, these results indicate that the equipment intensity of R&D capital spending is broadly stable across countries.

A.6 EU KLEMS Database

The EU KLEMS database provides harmonized growth and productivity accounts for major countries, including the U.S. It offers detailed industry-level data within a consistent accounting framework across countries. In this paper, we use the 2024 vintages that starts to span from 1997. From the national accounts module, we draw gross output at current prices (`GO_CP`), intermediate inputs (`II_CP`), value added (`VA_CP`), labor compensation (`COMP`), and the corresponding price indexes (`GO_PI`, `II_PI`, and `VA_PI`) for each NACE/ISIC sector. From the growth accounts module, we use quantity indexes for labor input (`LAB_QI`) and capital services (`CAP_QI`), from which we derive implicit factor price indexes as the ratio of nominal compensation to the corresponding quantity index. These

Figure A.2: Equipment and Structures Investment Allocation in R&D Sector



Notes: The orange series reports the equipment share and the blue series reports the residual share, i.e., the structures share. For countries other than the United States, we use GERD and define the equipment share as machine and equipment divided by (land and buildings + machine and equipment), with the structures share defined as land and buildings divided by (land and buildings + machine and equipment). For the United States, we use BERD and define the equipment share as equipment divided by (structures + equipment), with the structures share defined as structures divided by (structures + equipment). GDP per capita (in PPP terms) on the horizontal axis is expressed relative to the United States so that the U.S. equals one. *Sources:* BERD; GERD; PWT 11.0; and authors' calculations.

variables allow us to construct industry-level cost shares and input price changes for each country-year, which constitute the building blocks of the dual price decomposition.

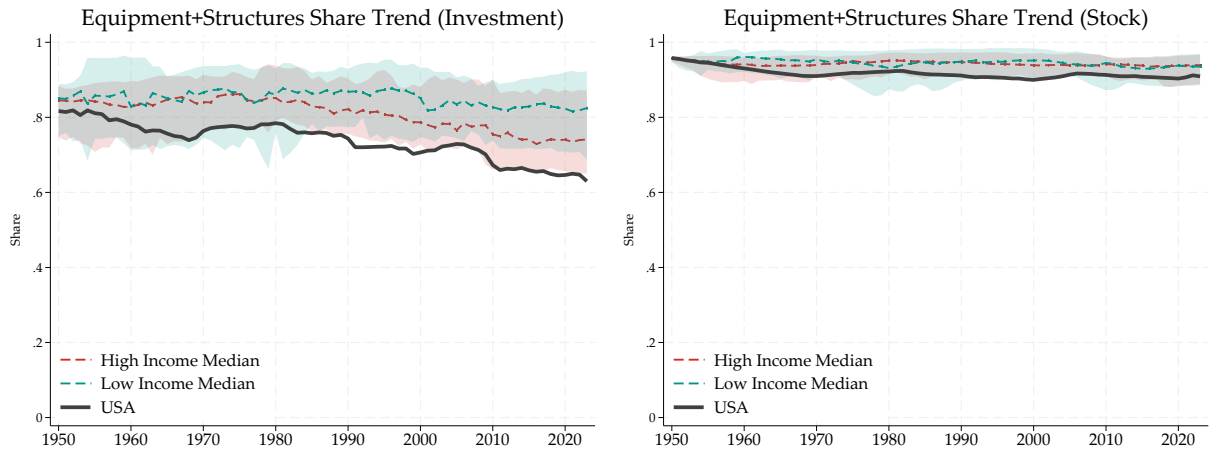
B Robustness Checks on Stylized Facts

B.1 Additional Trends on Fact 1

B.1.1 Investment Share Trend

The components we emphasize in the main text, which are equipment and structures, constitute a large and stable fraction of aggregate investment and capital. Figure B.1 shows that the combined equipment and structures share is high and broadly stable over time in both flows and stocks.

Figure B.1: **Shares of Structure and Equipment in Total Investment**



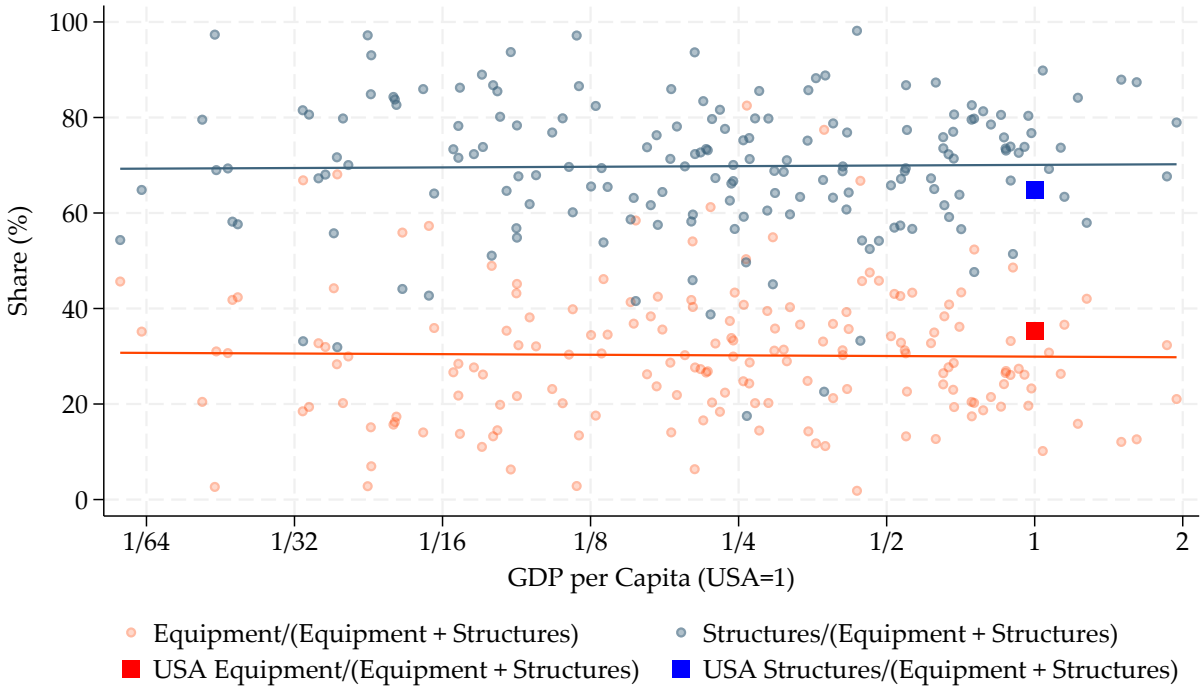
Notes: The figure plots time series of investment shares constructed from Penn World Table capital-detail categories. The share is computed as the ratio of a component to the sum of four components: Structure, Equipment, Transport Equipment, and Other assets. The left panel is based on investment (*Ic*) and the right panel is measured based on captial stock (*Nc*). High and low income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. Dashed lines indicate the median for each group, and the solid black line denotes the United States. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

In the left panel (investment), the group medians remain in a relatively narrow band throughout the sample, indicating that cross-country differences in the evolution of disaggregated relative prices documented in the main text are not driven by large shifts in compositional weights between equipment and structures. In the right panel (capital stock), the combined share is even more stable, with medians close to unity and limited dispersion across groups, consistent with equipment and structures accounting for the overwhelming majority of the capital stock over time.

Additionally, we plot how the equipment investment and structures investments vary across countries. Figure B.2 plots, for 2021, each country's equipment and structures shares of total investment against relative GDP per capita (USA = 1), with the United

States highlighted and fitted trend lines shown.

Figure B.2: Equipment and Structures Investment Allocation



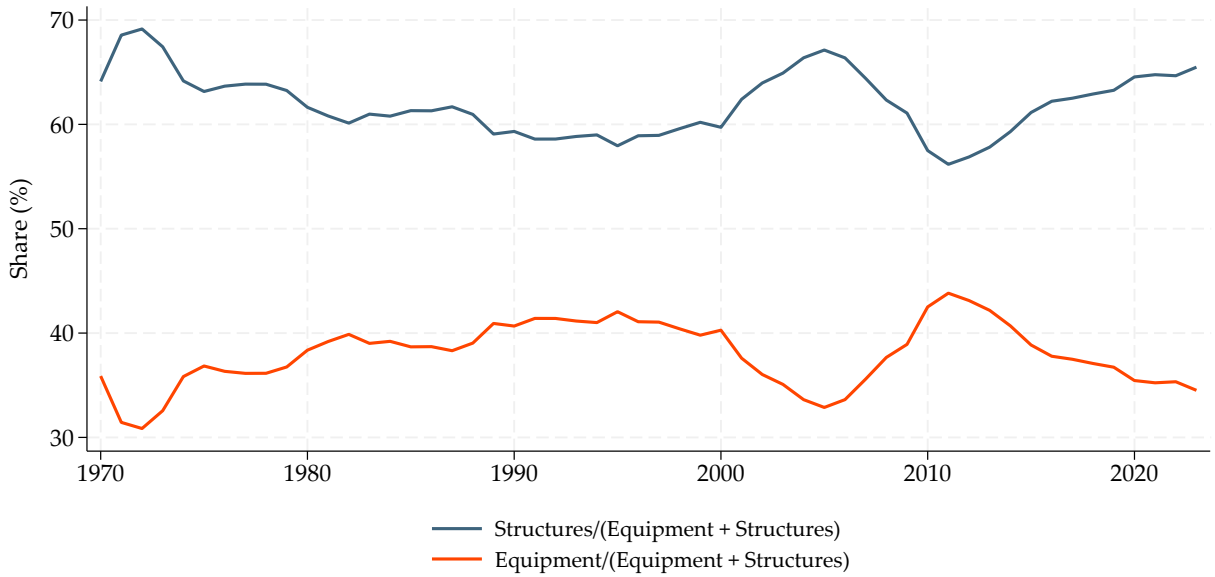
Notes: The figure plots, for year 2021, each country's investment allocation between equipment and structures against relative GDP per capita (USA=1). Circles show country observations; the U.S. observation is highlighted. Solid lines are separate linear fits of the equipment share and the structures share on relative GDP per capita. The equipment share is defined as $E/(E+S)$ and the structures share as $S/(E+S)$, where E and S correspond to equipment and structures components of investment, respectively. GDP per capita (in PPP terms) is expressed relative to the United States so that the U.S. equals one. *Source*: PWT 11.0 Capital Detail

Finally, we plot the time series for the share of equipment investment in total investment in equipment and structures in Figure B.3. This series is used to calibrate the model parameter that pins down the output share of equipment relative to structures. As the figure shows, the equipment share increased by roughly 5 percentage points from the 1970s to the 1980s and remained broadly stable for the subsequent two decades. The series displays a transitory fluctuation around the 2008 Great Financial Crisis, with a mean of approximately 40%.

B.1.2 Relative Price Trend of Other Capital Sectors

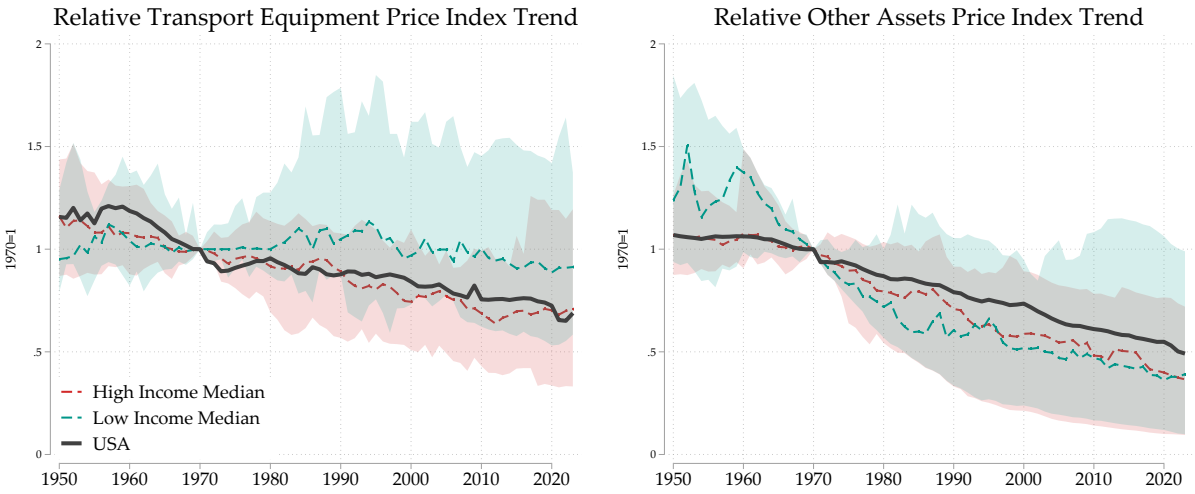
PWT's equipment measure does not include transport capital, whereas the ICP data indicate that transport equipment is at least partially incorporated. In addition, the category of other assets may also be subject to compositional mixing. Motivated by these concerns, we replicate the same exercise using price data for transport equipment and other assets.

Figure B.3: Equipment and Structures Investment Allocation Time Series in the United States



Notes: The figure plots the allocation of investment between equipment and structures in the United States from 1970 to 2024. The equipment share is defined as $E/(E+S)$ and the structures share as $S/(E+S)$, where E and S denote the equipment and structures components of investment, respectively. *Source*: PWT 11.0 Capital Detail

Figure B.4: **Relative Investment Price Index Trend of Other Capitals**



Notes: The relative price for each sector is calculated by dividing each sector's investment price index by the GDP price index, normalized to 1 in 1970. High and low income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. Dashed lines indicate the median for each group, and the solid black line denotes the United States. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

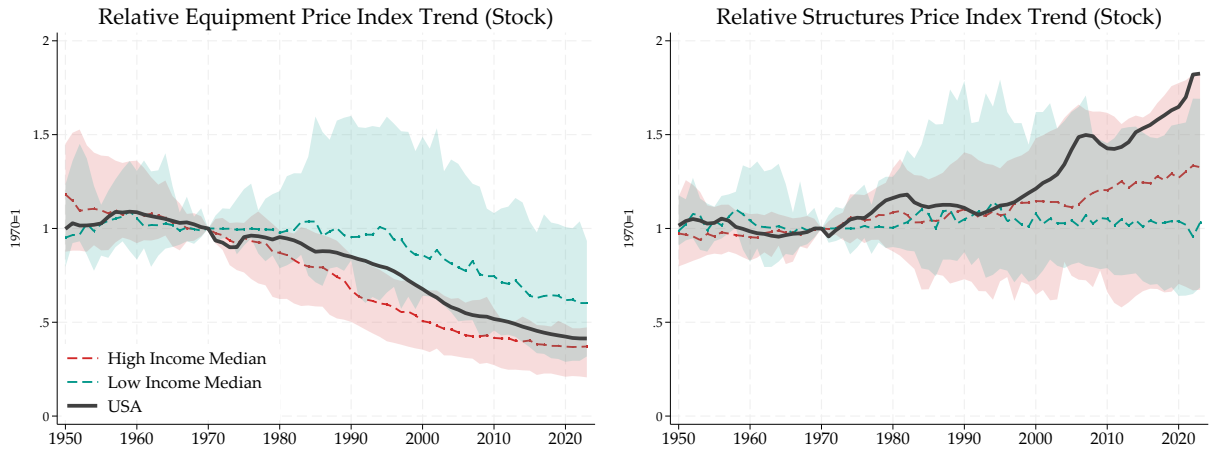
Figure B.4 shows that for these two capital categories it is difficult to identify heterogeneity across the United States, the high-income group, and the low-income group that

is as pronounced as what we observe for equipment or structures. This finding mitigates our concern that the cross-group patterns documented for equipment and structures are driven mechanically by category definitions or compositional mixing in the underlying price data. If the inclusion of transport equipment in ICP's equipment, or the potential mixture embedded in other assets, were a first-order source of the observed heterogeneity, we would expect to see similarly large and systematic differences across income groups for transport equipment and other assets themselves. Instead, the absence of comparable divergence suggests that our main results for equipment and structures are unlikely to be an artifact of these classification issues.

B.1.3 Relative Price Trend based on Stock

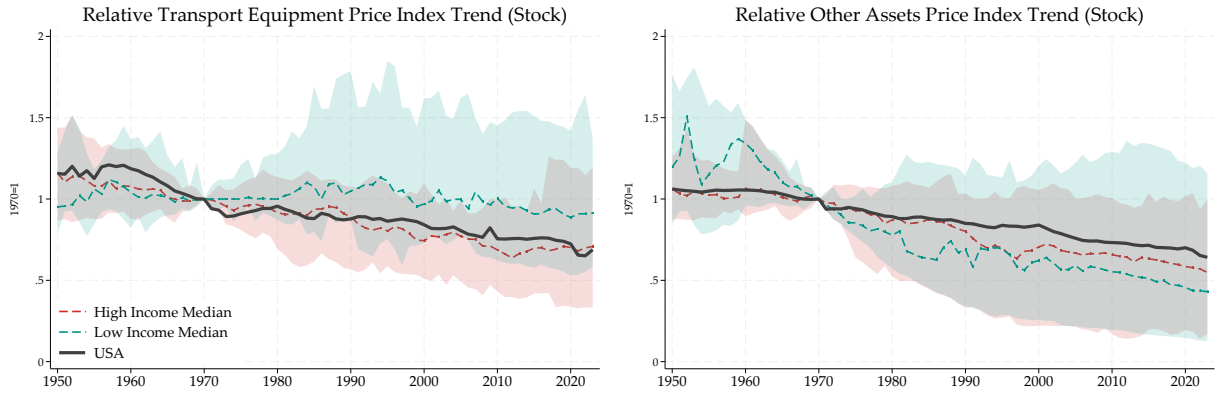
PWT provides price indices for capital based on capital stocks (Capital Stock Price Indices; series codes begin with `Np`). The stylized facts we report for other capital-asset trends remain unchanged when we use these stock-based price indices. Figure B.5 shows that equipment prices decline more rapidly in high-income economies, including the United States, than in low-income economies. By contrast, for structures, the capital stock price index rises in high-income economies, especially the United States, while it remains relatively stable in low-income economies. The other assets, transport equipment and other asset, show the same pattern found in the previous Appendix, as in Figure B.6.

Figure B.5: **Relative Capital Stock Price Index Trend**



Notes: The relative price for each sector is calculated by dividing each sector's capital stock price index by the GDP price index, normalized to 1 in 1970. High and low income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. Dashed lines indicate the median for each group, and the solid black line denotes the United States. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

Figure B.6: **Relative Capital Stock Price Index Trend of Other Capitals**



Notes: The relative price for each sector is calculated by dividing each sector's capital stock price index by the GDP price index, normalized to 1 in 1970. High and low income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. Dashed lines indicate the median for each group, and the solid black line denotes the United States. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

B.1.4 Price Trends from ICP

We construct ICP-based cross-country relative prices and how we estimate the evolving cross-sectional relationship between relative prices and income across ICP survey years, and it reports the corresponding figures (Hsieh and Klenow, 2007).

We use ICP survey-year data on PPP prices for (i) equipment, (ii) structures, and (iii) GDP, together with the associated market exchange rate information to express PPP price measures in a common currency basis. We restrict the sample to countries with non-missing information for the relevant items in each survey year and drop aggregate regions and non-sovereign groupings. We focus on the ICP benchmark years 1996, 2005, 2011, 2017, and 2021 for consistent cross-year comparisons. Following Hsieh and Klenow (2007), we normalize each variable by the United States within each survey year to facilitate cross-country comparisons. Let $P^k_{i,t}$ denote the PPP price of item $k \in \{\text{equipment, structures}\}$ for country i in survey year t, and let $y_{i,t}$ denote PPP GDP per capita. We construct U.S.-normalized relative prices and income as

$$\tilde{P}^k_{i,t} = \ln\left(\frac{P^k_{i,t}}{P^k_{\text{USA},t}}\right), \qquad \tilde{y}_{i,t} = \ln\left(\frac{y_{i,t}}{y_{\text{USA},t}}\right).$$

For visualization, we plot these ln-based variables on axes labeled in base-2 log units (so that a value of 1 corresponds to twice the U.S. level). The underlying variables remain in natural logs; only the axis scaling and tick labels use base-2 units to aid interpretation.

The resulting scatter plots report $\tilde{P}^k_{i,t}$ on the vertical axis and $\tilde{y}_{i,t}$ on the horizontal axis, both normalized to zero for the United States. In addition, for each survey year t, we estimate the income gradient of each relative price using a no-intercept specification:

$$\tilde{P}^k_{i,t} = \beta^k_t \, \tilde{y}_{i,t} + \varepsilon^k_{i,t},$$

where β^k_t captures how relative prices covary with relative income in year t.

Figure B.7 presents ICP-based cross-country evidence on relative prices and their income gradients for equipment and structures across survey years (Hsieh and Klenow, 2007). The top row plots, for each ICP benchmark year (1996, 2005, 2011, 2017, and 2021), the cross-country relationship between U.S.-normalized PPP prices and U.S.-normalized PPP GDP per capita. The fitted lines for equipment are relatively stable across years, although the correlation with GDP per capita turns negative over time, whereas the fitted lines for structures become increasingly steep, indicating a strengthening cross-sectional association between structures price and income in later survey waves. The bottom row summarizes the corresponding year-by-year regression slopes and their 95% confidence intervals.

B.2 Additional Results on Fact 3

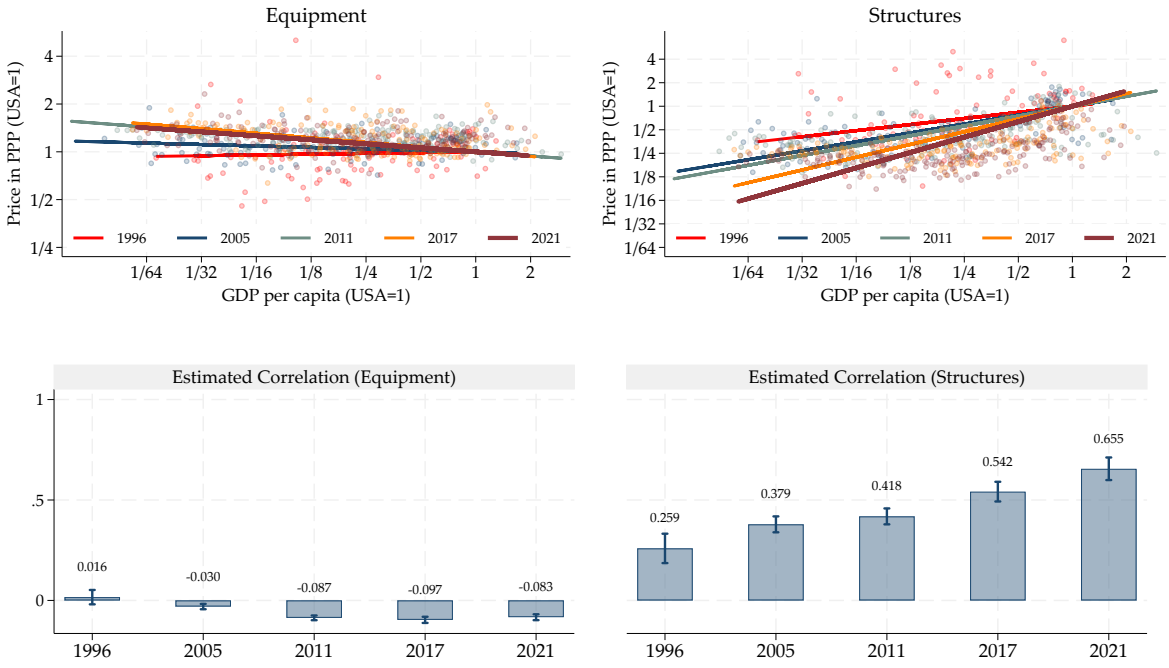
B.2.1 Results on GDP per capita

In our baseline estimation, we controlled for initial conditions using a fixed lagged income term (e.g., $y_{i,t-6}$). While this approach standardizes the control variable across specifications, it implies that the time horizon between the outcome variable $y_{i,t+h}$ and the control variable varies with the forecast horizon h. To ensure that our results are not driven by these varying lag lengths, we re-estimate the local projections by imposing a constant time gap, denoted by τ, between the dependent variable and the lagged control.

$$\ln(y_{i,t+h}) = \beta^{h,\tau}_{\text{Equip}} \ln(P^{\text{Equip}}_{it}) + \beta^{h,\tau}_{\text{Str}} \ln(P^{\text{Str}}_{it}) + \mathbf{X}'_{it}\Gamma^{h,\tau} + \lambda^{h,\tau} \ln(y_{i,t+h-\tau}) + \alpha^{h,\tau}_i + \delta^{h,\tau}_t + \epsilon^h_{it} \quad \text{(B.1)}$$

Table B.I presents the estimation results. Columns (1)-(3) utilize a 10-year gap ($\tau = 10$), while Columns (4)-(6) utilize a 6-year gap ($\tau = 6$). The results confirm the dynamic asymmetry observed in the baseline model. The coefficient on equipment prices is negative across horizons, but statistical significance varies across specifications. While the point estimates for current and future income remain negative in this specification, there is a strong correlation with the past GDP per capita and the equipment prices.

Figure B.7: **ICP cross-country relative prices and income gradients by survey year**



Notes: Each dot is a country. GDP per capita (in PPP terms) on the horizontal axis is expressed relative to the United States so that the U.S. equals one. The vertical axis reports the corresponding log relative PPP price (equipment or structures), also normalized so that the United States equals zero. The fitted lines in the top row are survey-year specific cross-sectional regressions with no intercept. The bottom row reports the estimated slopes and 95% confidence intervals for equipment and structures by survey year.

B.2.2 Smoothed Data using Centered Moving Averages

In this section, we check the robustness of our results by replacing the point-in-time outcomes with smoothed measures based on centered moving averages. Specifically, for each selected benchmark year t, we construct the variables as the five-year centered average:

$$\bar{X}_{it} = \frac{X_{i,t-2} + X_{i,t-1} + X_{i,t} + X_{i,t+1} + X_{i,t+2}}{5}$$

This procedure mitigates measurement error and short-run fluctuations. We utilize data from the ICP benchmark years (1990, 1996, 2005, 2011, 2017, 2021) to construct these averages.

Table B.II presents the baseline relationship between relative price and investment rates. Table B.III reports the dynamic specifications for GDP per capita.

Table B.I: **The Effect of Capital Prices on GDP per Capita (Stable Lag Distance)**

	Control Lag Gap ($\tau = 10$)			Control Lag Gap ($\tau = 6$)		
	$\ln(y_{i,t-5})$ (1)	$\ln(y_{i,t})$ (2)	$\ln(y_{i,t+5})$ (3)	$\ln(y_{i,t-5})$ (4)	$\ln(y_{i,t})$ (5)	$\ln(y_{i,t+5})$ (6)
Relative prices						
Log Equipment Price	-0.157*	-0.145	-0.214*	-0.203**	-0.279***	-0.047
	(0.092)	(0.107)	(0.112)	(0.081)	(0.097)	(0.092)
Log structures price	-0.048	-0.156***	0.024	-0.017	-0.064	0.040
	(0.042)	(0.059)	(0.061)	(0.039)	(0.048)	(0.047)
Controls						
Years of Schooling	0.498**	0.266	0.462*	0.419**	0.310*	0.160
	(0.215)	(0.208)	(0.242)	(0.177)	(0.165)	(0.231)
Human Capital Index	-2.118**	-0.779	-2.187*	-1.838**	-1.448*	-0.869
	(1.024)	(0.950)	(1.203)	(0.807)	(0.773)	(1.162)
Log Population	-0.005	0.179	-0.175	0.108	-0.120	-0.163
	(0.140)	(0.142)	(0.134)	(0.099)	(0.113)	(0.100)
Lagged Dependent Variables						
Lagged GDP ($t-15$)	0.018					
	(0.054)					
Lagged GDP ($t-10$)		0.108				
		(0.086)				
Lagged GDP ($t-5$)			0.101			
			(0.072)			
Lagged GDP ($t-11$)				0.458***		
				(0.054)		
Lagged GDP ($t-6$)					0.386***	
					(0.048)	
Lagged GDP ($t-1$)						0.477***
						(0.071)
Constant	-2.121*	-3.780***	-0.717	-0.818	-0.595	-0.350
	(1.092)	(0.943)	(1.286)	(0.763)	(0.828)	(1.088)
Observations	617	617	494	617	632	494
R^2	0.972	0.966	0.969	0.978	0.973	0.974
Country FE	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓

Notes: Standard errors are reported in parentheses and are clustered at the country level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The dependent variable is Log GDP per capita (ln y). Columns (1)–(3) control for lagged GDP per capita with a 10-year gap ($\tau = 10$), while columns (4)–(6) control for lagged GDP per capita with a 6-year gap ($\tau = 6$).

B.2.3 Results on TFP Dynamics

We further investigate whether the observed relative price effects on GDP per capita are driven by productivity channels. To test this mechanism, we estimate the same dynamic panel specification as Equation (B.2), but we replace the dependent variable (and its corresponding lag) with Total Factor Productivity (TFP), denoted as $A_{i,t}$.

$$\ln(A_{i,t+h}) = \beta^{h,\tau}_{\text{Equip}} \ln(P^{\text{Equip}}_{it}) + \beta^{h,\tau}_{\text{Str}} \ln(P^{\text{Str}}_{it}) + \mathbf{X}'_{it}\Gamma^{h,\tau} + \lambda^{h,\tau} \ln(A_{i,t+h-\tau}) + \alpha^{h,\tau}_i + \delta^{h,\tau}_t + \epsilon^h_{it} \quad \text{(B.2)}$$

Table B.IV reports the TFP results under the stable-lag specification. The estimates of β_{Equip} are negative and statistically significant in several columns (e.g., Columns 2, 4, and

Table B.II: **Effect of Relative Prices on Investment Rates (Smoothed Data)**

	Log Equipment Inv. Rate (1)	Log Str. Inv. Rate (2)
Log Equipment Price	-0.714***	0.700***
	(0.272)	(0.212)
Log Structures price	0.143	-0.714***
	(0.094)	(0.080)
Years of Schooling	0.734*	0.456
	(0.430)	(0.382)
Human Capital	-3.389*	-2.388
	(1.969)	(1.769)
Log Population	-0.488	0.249
	(0.437)	(0.295)
Lagged Dep. Var.	-0.028	-0.136
	(0.157)	(0.195)
Constant	0.665	-1.351
	(2.287)	(1.602)
Observations	534	534
R^2	0.724	0.682

Notes: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.
Note on Lags/Forwards: All variables are constructed using 5-year centered moving averages.

Table B.III: **Dynamic Effects on GDP per Capita (Smoothed Data)**

	Specification A (No Lag Control)			Specification B (With Lag Control)		
	Past $L.\ln(y)$ (1)	Current $\ln(y)$ (2)	Future $F.\ln(y)$ (3)	Past $L.\ln(y)$ (4)	Current $\ln(y)$ (5)	Future $F.\ln(y)$ (6)
Log Equipment Price	-0.147*	-0.292***	-0.231**	-0.027	-0.209***	0.002
	(0.081)	(0.100)	(0.106)	(0.078)	(0.078)	(0.081)
Log Structures price	-0.082**	-0.076	0.035	0.048	-0.029	0.051
	(0.036)	(0.047)	(0.059)	(0.035)	(0.037)	(0.041)
Years of Schooling	0.536***	0.559***	0.548**	0.323***	0.256*	0.166
	(0.195)	(0.210)	(0.265)	(0.160)	(0.139)	(0.240)
Human Capital	-2.264**	-2.458**	-2.550*	-1.516***	-1.181*	-0.922
	(0.935)	(0.993)	(1.323)	(0.740)	(0.642)	(1.222)
Log Population	-0.046	-0.114	-0.168	-0.176	-0.088	-0.174**
	(0.114)	(0.132)	(0.134)	(0.126)	(0.096)	(0.084)
Lagged Dep. Var.	No	No	No	Yes ($L2$)	Yes (L)	Yes (F)
Observations	632	632	494	537	632	494
R^2	0.973	0.973	0.972	0.988	0.982	0.980

Notes: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.
Note on Lags/Forwards: Since the data is constructed using benchmark years, one unit of lag ($L.$) or forward ($F.$) corresponds to the adjacent 5-year centered average period. For example, in Column (4), the dependent variable is the previous period's income, and we control for the income two periods prior ($L2.$).

Table B.IV: **The Effect of Capital Prices on Total Factor Productivity (TFP)**

	Control Lag Gap (τ = 10)			Control Lag Gap (τ = 6)		
	$\ln(A_{i,t-5})$ (1)	$\ln(A_{i,t})$ (2)	$\ln(A_{i,t+5})$ (3)	$\ln(A_{i,t-5})$ (4)	$\ln(A_{i,t})$ (5)	$\ln(A_{i,t+5})$ (6)
Relative prices						
Log Equipment Price	-0.102	-0.163**	-0.072	-0.111**	-0.133**	-0.018
	(0.072)	(0.066)	(0.068)	(0.051)	(0.058)	(0.055)
Log Structures price	-0.015	0.085**	-0.019	0.034	0.075**	-0.022
	(0.034)	(0.040)	(0.037)	(0.028)	(0.035)	(0.028)
Controls						
Years of Schooling	0.157	0.066	0.086	0.118	0.018	0.033
	(0.114)	(0.103)	(0.113)	(0.090)	(0.088)	(0.098)
Human Capital Index	-0.604	-0.245	-0.361	-0.442	-0.047	-0.154
	(0.561)	(0.495)	(0.556)	(0.436)	(0.426)	(0.462)
Lagged Dependent Variables						
Lagged TFP ($t-15$)	0.249*					
	(0.135)					
Lagged TFP ($t-10$)		0.194*				
		(0.102)				
Lagged TFP ($t-5$)			-0.031			
			(0.125)			
Lagged TFP ($t-11$)				0.633***		
				(0.115)		
Lagged TFP ($t-6$)					0.339***	
					(0.092)	
Lagged TFP ($t-1$)						0.341***
						(0.111)
Constant	0.770	0.724	0.911	0.459	0.499	0.548
	(0.588)	(0.462)	(0.586)	(0.444)	(0.398)	(0.470)
Observations	507	522	407	522	522	421
R^2	0.801	0.853	0.850	0.857	0.870	0.860
Country FE	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓

Notes: Standard errors are reported in parentheses and are clustered at the country level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The dependent variable is Log TFP ($\ln A$). Columns (1)–(3) control for lagged TFP with a 10-year gap ($\tau = 10$), while columns (4)–(6) control for lagged TFP with a 6-year gap ($\tau = 6$).

5), while the future-TFP estimate with $\tau = 6$ is not statistically significant (Column 6).

Table B.V reports the estimated elasticities of TFP with respect to capital prices across different time horizons using the baseline horizon specification employed in the main GDP per capita analysis. These results broadly mirror the patterns found for aggregate output, reinforcing the specific role of equipment and equipment. First, the results confirm that lower equipment prices are significantly associated with higher current TFP. In the concurrent specification (Column 2), the coefficient for equipment prices is negative and statistically significant ($\beta = -0.133$), indicating that a 10% decrease in the relative price of equipment is associated with an approximately 1.3% increase in TFP. Crucially, the placebo test in Column (1) supports a causal interpretation: current equip-

Table B.V: **The Effect of Capital Prices on Total Factor Productivity (TFP)**

	Control: Lagged TFP ($t-6$)			Control: Lagged TFP ($t-10$)		
	$\ln(A_{i,t-5})$	$\ln(A_{i,t})$	$\ln(A_{i,t+5})$	$\ln(A_{i,t-5})$	$\ln(A_{i,t})$	$\ln(A_{i,t+5})$
	(1)	(2)	(3)	(4)	(5)	(6)
Relative prices						
Log Equipment Price	-0.008	-0.133**	-0.078	-0.096*	-0.163**	-0.069
	(0.019)	(0.058)	(0.069)	(0.049)	(0.066)	(0.065)
Log Structures price	-0.006	0.075**	-0.019	0.034	0.085**	-0.025
	(0.013)	(0.035)	(0.037)	(0.025)	(0.040)	(0.038)
Controls						
Years of Schooling	-0.035	0.018	0.093	0.110	0.066	0.079
	(0.035)	(0.088)	(0.116)	(0.083)	(0.103)	(0.113)
Human Capital Index	0.150	-0.047	-0.385	-0.451	-0.245	-0.338
	(0.172)	(0.426)	(0.571)	(0.397)	(0.495)	(0.548)
Lagged Log TFP ($t-6$)	1.023***	0.339***	-0.090			
	(0.051)	(0.092)	(0.136)			
Lagged Log TFP ($t-10$)				0.686***	0.194*	-0.083
				(0.132)	(0.102)	(0.076)
Constant	-0.103	0.499	0.956	0.514	0.724	0.941*
	(0.187)	(0.398)	(0.598)	(0.408)	(0.462)	(0.559)
Observations	522	522	407	522	522	407
R^2	0.981	0.870	0.852	0.866	0.853	0.851
Country FE	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓

Notes: Standard errors are reported in parentheses and are clustered at the country level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The dependent variable is log TFP at different time horizons ($t-5, t, t+5$).

ment prices have a near-zero and statistically insignificant relationship with past TFP ($\beta = -0.008$). This suggests that the correlation is not driven by persistent country characteristics or reverse causality where high-TFP countries simply have historically cheaper capital. Rather, the improvement in efficiency materializes contemporaneously with the access to affordable equipment. Second, the heterogeneity between capital types is even more pronounced for TFP than for GDP. While equipment prices show a consistent negative elasticity (lower prices boost efficiency), structures price exhibit a positive and significant relationship with TFP in the concurrent specification ($\beta = 0.075$). This contrast underscores that while structures are a consequence of development, affordable equipment is a driver of productive efficiency (Jones, 1994).

Finally, Table B.VI shows robust results under smoothed data.

B.3 Additional results on Fact 4

We conduct a parallel exercise using BEA data on current-cost net capital stocks in order to assess whether the patterns observed for investment prices extend to capital stock prices.

Table B.VI: **Dynamic Effects on TFP (Smoothed Data)**

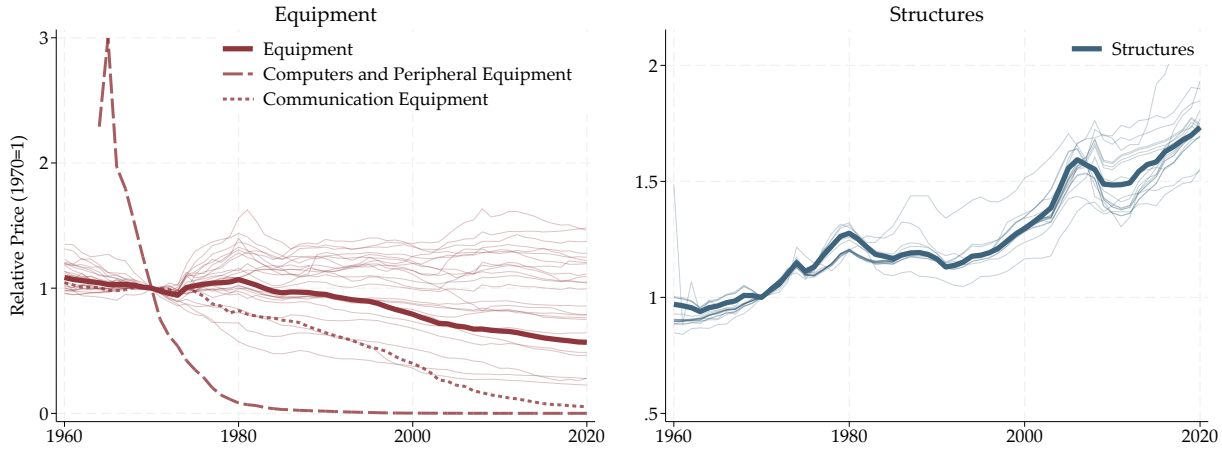
	Past *L*.TFP (1)	Current TFP (2)	Future *F*.TFP (3)
Log Equipment Price	-0.104	-0.169**	-0.081
	(0.075)	(0.070)	(0.067)
Log Structures price	-0.007	0.088**	0.008
	(0.031)	(0.040)	(0.039)
Years of Schooling	0.142	0.078	0.137
	(0.140)	(0.114)	(0.121)
Human Capital	-0.635	-0.324	-0.605
	(0.676)	(0.548)	(0.592)
Log Population	0.092	0.063	-0.075
	(0.130)	(0.079)	(0.092)
Constant	0.935	0.807	1.299*
	(0.841)	(0.606)	(0.708)
Observations	522	536	421
R^2	0.800	0.843	0.816

Notes: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.
Note on Lags/Forwards: All variables are 5-year centered moving averages. One unit of lag (*L*.) or forward (*F*.) corresponds to the previous or next 5-year average period.

Figure B.8 plots relative net capital stock price indices by BEA asset type over the period 1960–2020. The structures closely mirror that of the investment price indices: for each asset category, an implicit stock price index is recovered from nominal net capital stock values divided by the corresponding chain-type quantity indices, deflated by the CPI, and normalized to unity in 1970.

The resulting trends closely mirror those for investment prices. Equipment prices fall sharply after 1970, driven primarily by information-processing assets such as computers and peripheral equipment and communication equipment, while structures price rise broadly after 1990 across disaggregated categories. Overall, the close alignment between investment- and stock-price trends suggests that these divergent dynamics are not unique to investment flows but are also reflected in the evolution of the capital stock itself.

Figure B.8: **Relative Net Stock Price Index Trend by Sector (1950-2020)**



Notes: The figure plots trends in relative net capital stock prices by BEA asset type. For each category, we construct an implicit net-stock price index as nominal current-cost net capital stock divided by the corresponding chain-type quantity index for net stocks. The resulting series is deflated by GDP Deflator and normalized to 1 in 1970 for each category. The sample is restricted to 1960–2020. Thin lines show disaggregated categories, while thicker highlighted lines emphasize the aggregate equipment series and selected subcomponents (computers and peripheral equipment, and communication equipment). *Sources:* U.S. Bureau of Economic Analysis (BEA), BEA Fixed Assets Tables 3.1ESI and 3.1S; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

C Contribution of Sub-asset to Sectoral Price changes

Although sub-asset categories within a given sector broadly comove with the sectoral price trend, there is heterogeneity in both the timing and the magnitude of price adjustment. To identify which components account for the sector-level movements, we decompose each sectoral price index into the cumulative contributions of its constituent subcategories using a Törnqvist framework (see Appendix D.1 for details). This is because the sectoral price index is constructed as a Fisher chain index, it cannot in general be written as a contemporaneous expenditure-share-weighted average of component prices:

$$\frac{\text{Nominal Expenditure}_{s,t}}{\text{Quantity Index}_{s,t}} \neq \sum_{i \in s} \frac{\text{Nominal Expenditure}_{i,t}}{\text{Nominal Expenditure}_{s,t}} \frac{\text{Nominal Expenditure}_{i,t}}{\text{Quantity Index}_{i,t}}, \quad \text{(C.1)}$$

where s denotes a broad asset sector and i indexes the subcategories within that sector. Instead, we use the Törnqvist identity:

$$\Delta \ln P^T_{s,t} = \sum_{i \in s} \bar{w}_{i,t} \, \Delta \ln p_{i,t}, \quad \text{(C.2)}$$

where

$$\bar{w}_{i,t} \equiv \frac{w_{i,t} + w_{i,t-1}}{2}, \qquad \Delta \ln p_{i,t} \equiv \ln p_{i,t} - \ln p_{i,t-1}. \quad \text{(C.3)}$$

Here, $\bar{w}_{i,t}$ is the average nominal expenditure share of component i between periods $t-1$ and t. We then define the cumulative contribution of component i from the base year 0 to year T as

$$C^T_{i,s} \equiv \sum_{t=1}^{T} \bar{w}_{i,t} \, \Delta \ln p_{i,t}. \quad \text{(C.4)}$$

By construction,

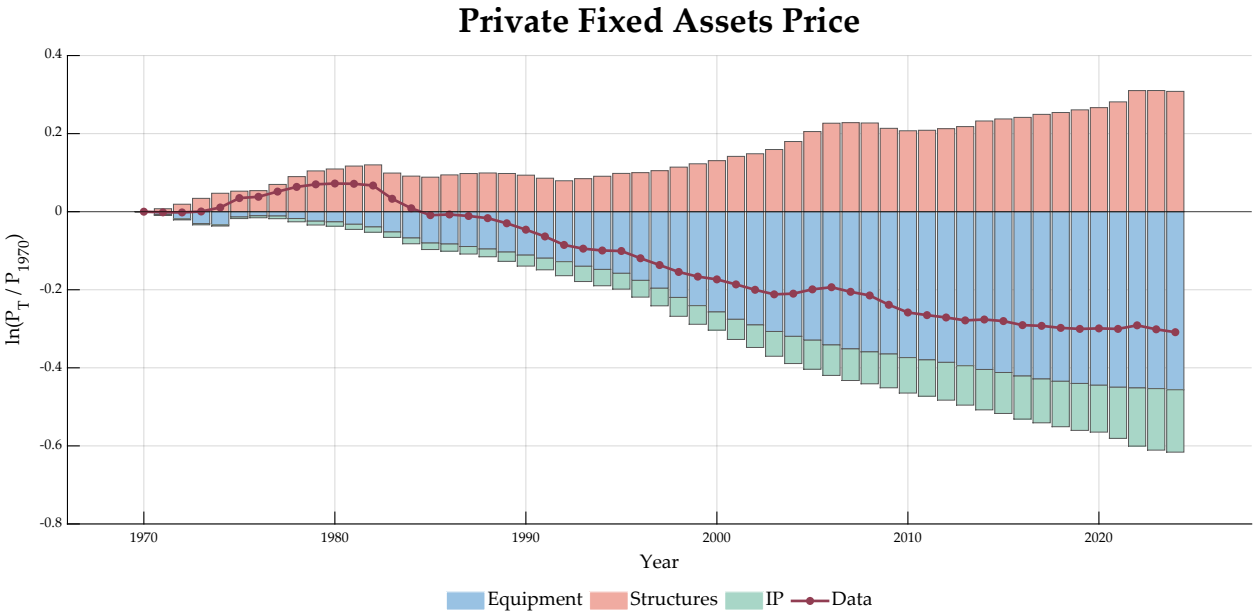
$$\sum_{i \in s} C^T_{i,s} = \sum_{t=1}^{T} \Delta \ln P^T_{s,t} \approx \ln P^{\text{chain}}_{s,T} - \ln P^{\text{chain}}_{s,0}. \quad \text{(C.5)}$$

The decomposition captures both own-price movements and changes in expenditure shares. Accordingly, the cumulative contribution should not be interpreted as a pure within-component effect. Rather, it summarizes the joint effect of price change and compositional reallocation across subcategories.

We begin with the aggregate decomposition across equipment, structures, and intellectual property products. Figure C.1 shows that the long-run decline in the price of private fixed assets is driven primarily by equipment and intellectual property products. Both categories make increasingly negative contributions after the early 1980s, whereas

structures contribute positively over most of the sample and partly offset the downward pressure from the other two sectors. The close alignment between the stacked contributions and the data line indicates that the Törnqvist decomposition provides a good approximation to the observed chain-price movement.

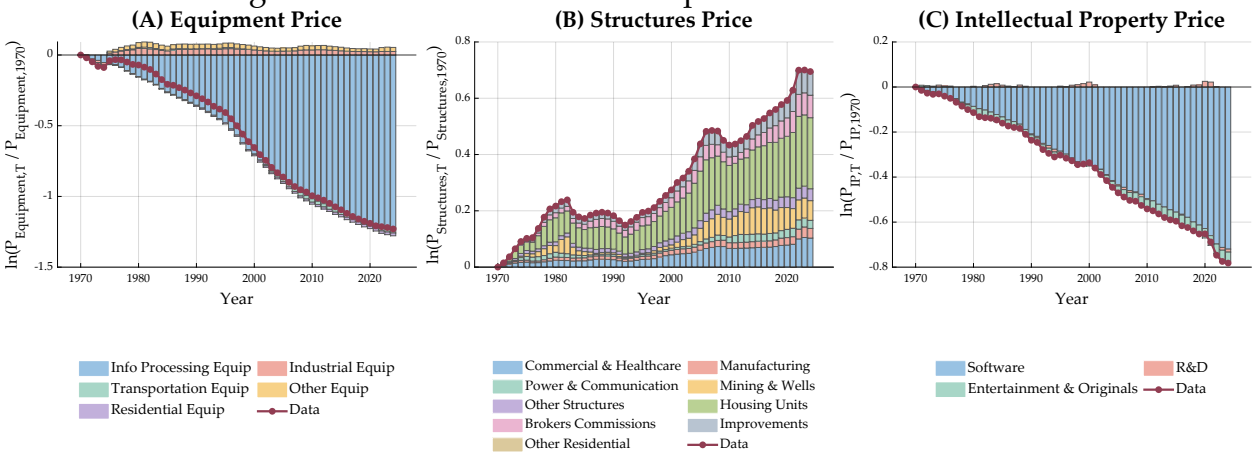
Figure C.1: Decomposition of Private Fixed Asset Investment Price



Notes: The bars report the cumulative Törnqvist contributions of equipment, structures, and intellectual property products, while the line reports the cumulative log change in the private fixed asset price index, normalized to zero in 1970. *Sources:* U.S. Bureau of Economic Analysis (BEA), BEA Fixed Assets Tables 2.7 and 2.8; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

We next examine heterogeneity within each major asset sector. Figure C.2 reveals that the decline in equipment prices is overwhelmingly explained by information processing equipment. By contrast, industrial equipment, transportation equipment, residential equipment, and other equipment make only modest contributions to the long-run downward trend. Within structures, the increase in the sectoral price index is broad-based, with especially large contributions from housing units and commercial and healthcare structures, along with additional positive contributions from power and communication, mining and wells, brokers' commissions, and other residential-related categories. For intellectual property products, the decline is dominated by software, whereas research and development and entertainment originals contribute comparatively little to the long-run movement of the sectoral index.

Figure C.2: Within-sector Decomposition of Relative Price



Notes: The bars report the cumulative Törnqvist contributions of each sub-asset type to its sector-level price. Panel (A) reports the contribution of equipment subcategories, Panel (B) reports the contribution of structure subcategories, and Panel (C) reports the contribution of intellectual property subcategories. Tables 2.7 and 2.8; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

D Robustness Checks on Construction Sector Productivity Decline

This section shows how we decompose price and value added using the Törnqvist decomposition, following Diewert (1976), and presents additional evidence that reinforces the interpretation that the rise in structure capital prices is closely linked to weak productivity performance in the construction sector.

D.1 Price Decomposition

The cost-function dual of the production function provides a complementary decomposition that operates on prices rather than quantities. Under constant returns to scale and competitive factor markets, zero economic profit implies that the output price equals unit cost. Differentiating this identity yields, at the gross-output level,

$$\Delta \ln P_{i,t} = \bar{s}_{L,i,t}\,\Delta \ln P^{L}_{i,t} + \bar{s}_{K,i,t}\,\Delta \ln P^{K}_{i,t} + \bar{s}_{II,i,t}\,\Delta \ln P^{II}_{i,t} - \Delta \ln A_{i,t}, \tag{D.1}$$

where P^{L}, P^{K}, and P^{II} are the implicit price indexes of labor, capital services, and intermediate inputs. Shares $s_{j,t}$ is the observed nominal cost share of factor j in period t, and $\bar{s}_{j,t}$ is defined as $\bar{s}_{j,t} = \frac{1}{2}\left(s_{j,t} + s_{j,t-1}\right)$. These weights are symmetric across the two periods, the resulting index is free of the end-point bias. TFP enters with a negative sign because productivity gains reduce unit cost and hence the output price. To study why a sector's price

rises faster or slower than the economy-wide average, we express everything relative to the aggregate gross-output price P_{agg} (taken from the total-economy sector):

$$\Delta \ln\left(\frac{P_{i,t}}{P_{agg,t}}\right) = \underbrace{\bar{s}_{L,i,t}\left(\Delta \ln P^{L}_{i,t} - \Delta \ln P_{agg,t}\right) + \bar{s}_{K,i,t}\left(\Delta \ln P^{K}_{i,t} - \Delta \ln P_{agg,t}\right) + \bar{s}_{II,i,t}\left(\Delta \ln P^{II}_{i,t} - \Delta \ln P_{agg,t}\right)}_{\text{Cost Pressure}} - \Delta \ln A_{i,t}. \tag{D.2}$$

The left-hand side is the change in the sector's relative output price. The right-hand side decomposes it into cost pressure, the share-weighted excess growth of each input price over the aggregate price level, and the negative of TFP growth. A sector with stagnant productivity ($\Delta \ln A \approx 0$) and rising relative input costs will see its relative price drift upward over time.

D.2 Value-Added Growth Decomposition

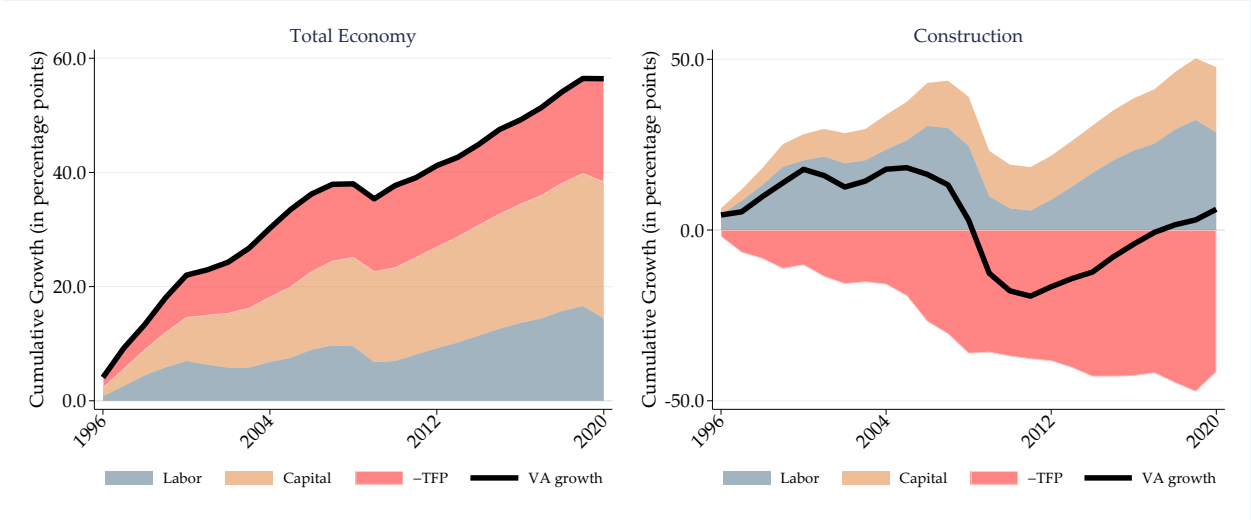
We apply the Törnqvist framework to decompose real value-added growth into factor contributions. Under the assumption that the value-added function exhibits constant returns to scale and that factors are paid their marginal products, the growth-accounting identity is

$$\Delta \ln VA_{i,t} = \bar{s}_{L,i,t}\, \Delta \ln L_{i,t} + \bar{s}_{K,i,t}\, \Delta \ln K_{i,t} + \Delta \ln A_{i,t}, \tag{D.3}$$

where i indexes sectors, $\bar{s}_L$ and $\bar{s}_K = 1 - \bar{s}_L$ are the Törnqvist-averaged shares of labor compensation and capital income in value added, L and K are composition-adjusted indexes of labor and capital services, and $\Delta \ln A$ is TFP growth obtained as the residual, which is Solow residual. The EU KLEMS database publishes each term in this identity as a precomputed contribution, so the decomposition holds exactly by construction.

Figure D.1 reports a cumulative value-added growth decomposition for the total economy and the construction sector in the United States. In the left panel that decomposes the aggregate level economy, value-added growth is supported by both factor accumulation and a sizable productivity contribution. In construction, by contrast, the productivity component is negative over most of the sample and acts as a persistent drag on growth, offsetting much of the positive contribution from labor and capital. This contrast suggests that the construction slowdown is not simply a consequence of weak factor accumulation, but reflects a broader deterioration in sectoral efficiency.

Figure D.1: Cumulative Value-Added Growth Decomposition in the United States: Total Economy vs. Construction



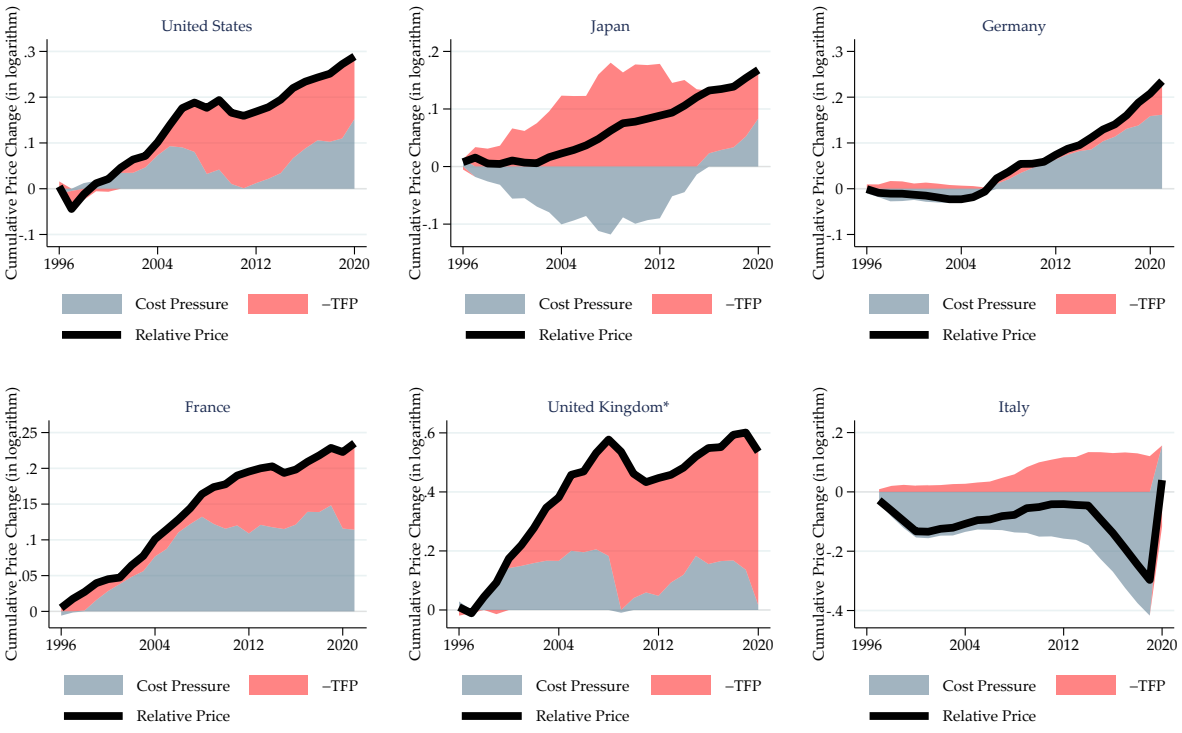
Notes. The figure presents a cumulative decomposition of value-added growth in the United States into labor, capital, and productivity components. The left panel reports the total economy, and the right panel reports the construction sector. The black line traces cumulative value-added growth since the beginning of the sample. The shaded areas show the cumulative contributions of labor input growth, capital input growth, and the productivity component. In the plotted figure, the productivity component is labeled −TFP for consistency with the price decomposition below, but in this value-added decomposition negative values indicate that productivity acts as a drag on growth. The figure shows that aggregate value-added growth is supported substantially by productivity, whereas in construction productivity makes a negative contribution for much of the sample and offsets gains from labor and capital accumulation. *Sources:* EU KLEMS 2024 and authors' calculations.

D.3 Cross-country Evidence

Figure D.2 extends the analysis to a cross-country comparison of construction-sector relative prices.

The broad rise in construction prices relative to the aggregate price level is visible in the United States, Japan, Germany, France, and the United Kingdom, although the decomposition differs across countries. In the United States, Japan, and the United Kingdom, weak productivity performance accounts for a large share of the increase in relative prices. Germany and France also exhibit rising construction relative prices, but in these cases aggregate cost pressure appears quantitatively more important. Italy is more volatile and stands out as a partial exception, with little net increase by the end of the sample. Overall, the cross-country evidence suggests that the U.S. pattern is not unique. Productivity shortfalls play a central role in several major economies, although their importance varies across countries.

Figure D.2: Cross-Country Decomposition of Construction Relative Prices: Cost Pressure vs. Productivity



Relative to each country's aggregate price (* = VA-based)

Notes. The figure presents a cumulative decomposition of the construction sector's relative output price into two components: cost pressure and −TFP. A positive −TFP contribution indicates upward price pressure arising from weak productivity growth. Cost pressure combines the contribution of all input costs relative to the aggregate price level. For the United States, Japan, Germany, France, and Italy, the decomposition is based on gross output. For the United Kingdom, it is based on value added because gross-output price indexes are unavailable in the 2024 EU KLEMS release. *Sources:* EU KLEMS 2024 and authors' calculations.

E Detailed Derivations of the Balanced Growth Path

In this appendix, we provide a step-by-step derivation of the BGP equilibrium when both equipment and structures are used as inputs in R&D. We define the BGP as a trajectory where aggregate variables $(Y, C, I_E, I_S, K_E, K_S)$ grow at constant rates and sectoral allocations (e.g., K_E^Y/K_E, K_S^Y/K_S, K_E^R/K_E, K_S^R/K_S) remain constant over time.

Step 1: Capital Accumulation and Capital Growth Rates

The laws of motion are

$$\dot{K}_E(t) = q_E(t)I_E(t) - \delta_E K_E(t), \tag{E.1}$$

$$\dot{K}_S(t) = q_S(t)I_S(t) - \delta_S K_S(t), \tag{E.2}$$

with exogenous ISTC growth

$$\frac{\dot{q}_E(t)}{q_E(t)} = \gamma_E, \qquad \frac{\dot{q}_S(t)}{q_S(t)} = \gamma_S.$$

Dividing by stocks,

$$g_{K_E}(t) \equiv \frac{\dot{K}_E(t)}{K_E(t)} = \frac{q_E(t)I_E(t)}{K_E(t)} - \delta_E, \tag{E.3}$$

$$g_{K_S}(t) \equiv \frac{\dot{K}_S(t)}{K_S(t)} = \frac{q_S(t)I_S(t)}{K_S(t)} - \delta_S. \tag{E.4}$$

On a BGP, g_{K_E} and g_{K_S} are constant. With constant investment shares, $g_{I_E} = g_{I_S} = g_Y$. Hence the ratio $\frac{q_k I_k}{K_k}$ must be constant, implying

$$g_{q_k} + g_{I_k} = g_{K_k}, \qquad k \in \{M, C\}.$$

Using $g_{q_E} = \gamma_E$ and $g_{q_S} = \gamma_S$ yields

$$g_{K_E} = g_Y + \gamma_E, \tag{E.5}$$

$$g_{K_S} = g_Y + \gamma_S. \tag{E.6}$$

Step 2: Final-Goods Production and the g_Y–g_A Relationship

The final-goods technology is

$$Y(t) = A(t)\big(K_E^Y(t)\big)^{\alpha_E}\big(K_S^Y(t)\big)^{\alpha_S}\big(L_u(t)\big)^{1-\alpha_E-\alpha_S}.$$

Taking logs and differentiating,

$$g_Y = g_A + \alpha_E g_{K_E^Y} + \alpha_S g_{K_S^Y} + (1-\alpha_E-\alpha_S)n.$$

On the BGP, the sectoral allocation shares are constant, so $g_{K_E^Y} = g_{K_E}$ and $g_{K_S^Y} = g_{K_S}$. Substituting (E.5)–(E.6),

$$g_Y = g_A + \alpha_E(g_Y + \gamma_E) + \alpha_S(g_Y + \gamma_S) + (1-\alpha_E-\alpha_S)n.$$

Collecting g_Y terms,

$$(1-\alpha_E-\alpha_S)g_Y = g_A + \alpha_E\gamma_E + \alpha_S\gamma_S + (1-\alpha_E-\alpha_S)n,$$

so

$$g_Y = \frac{g_A}{1-\alpha_E-\alpha_S} + \frac{\alpha_E\gamma_E + \alpha_S\gamma_S}{1-\alpha_E-\alpha_S} + n. \tag{E.7}$$

Step 3: Innovation Production and the Second Relationship

R&D produces new designs according to

$$\dot{A}(t) = \kappa A(t)^{\phi}\big(K_E^R(t)\big)^{\eta_E}\big(K_S^R(t)\big)^{\eta_S}\big(L_s(t)\big)^{1-\eta_E-\eta_S}.$$

Dividing by $A(t)$,

$$g_A \equiv \frac{\dot{A}(t)}{A(t)} = \kappa A(t)^{\phi-1}\big(K_E^R(t)\big)^{\eta_E}\big(K_S^R(t)\big)^{\eta_S}\big(L_s(t)\big)^{1-\eta_E-\eta_S}.$$

For g_A to be constant on the BGP, the growth rate of the RHS must be zero:

$$0 = (\phi-1)g_A + \eta_E g_{K_E^R} + \eta_S g_{K_S^R} + (1-\eta_E-\eta_S)n.$$

Equivalently,

$$(1-\phi)g_A = \eta_E g_{K_E^R} + \eta_S g_{K_S^R} + (1-\eta_E-\eta_S)n. \tag{E.8}$$

On the BGP, $g_{K_E^R} = g_{K_E}$ and $g_{K_S^R} = g_{K_S}$, so using (E.5)–(E.6),

$$(1-\phi)g_A = (\eta_E + \eta_S)g_Y + \eta_E\gamma_E + \eta_S\gamma_S + (1-\eta_E-\eta_S)n. \tag{E.9}$$

Step 4: Solving for (g_A, g_Y)

Let $\boldsymbol{\alpha} \equiv 1-\alpha_E-\alpha_S$ and $\boldsymbol{\eta} \equiv \eta_E+\eta_S$. From (E.7),

$$g_Y = \frac{g_A}{\boldsymbol{\alpha}} + \frac{\alpha_E\gamma_E + \alpha_S\gamma_S}{\boldsymbol{\alpha}} + n.$$

Substitute into (E.9):

$$\begin{aligned}(1-\phi)g_A &= \boldsymbol{\eta}\left[\frac{g_A}{\boldsymbol{\alpha}} + \frac{\alpha_E\gamma_E + \alpha_S\gamma_S}{\boldsymbol{\alpha}} + n\right] + \eta_E\gamma_E + \eta_S\gamma_S + (1-\boldsymbol{\eta})n \\ &= \frac{\boldsymbol{\eta}}{\boldsymbol{\alpha}}g_A + \frac{\boldsymbol{\eta}}{\boldsymbol{\alpha}}(\alpha_E\gamma_E + \alpha_S\gamma_S) + n + \eta_E\gamma_E + \eta_S\gamma_S.\end{aligned}$$

Collecting g_A terms,

$$g_A\left[(1-\phi) - \frac{\boldsymbol{\eta}}{\boldsymbol{\alpha}}\right] = n + \frac{\boldsymbol{\eta}}{\boldsymbol{\alpha}}(\alpha_E\gamma_E + \alpha_S\gamma_S) + \eta_E\gamma_E + \eta_S\gamma_S.$$

Multiplying by $\boldsymbol{\alpha}$,

$$g_A\big[(1-\phi)\boldsymbol{\alpha} - \boldsymbol{\eta}\big] = \boldsymbol{\alpha}n + \boldsymbol{\eta}(\alpha_E\gamma_E + \alpha_S\gamma_S) + \boldsymbol{\alpha}(\eta_E\gamma_E + \eta_S\gamma_S).$$

Therefore,

$$g_A = \frac{\boldsymbol{\alpha}n + \boldsymbol{\eta}(\alpha_E\gamma_E + \alpha_S\gamma_S) + \boldsymbol{\alpha}(\eta_E\gamma_E + \eta_S\gamma_S)}{(1-\phi)\boldsymbol{\alpha} - \boldsymbol{\eta}}. \tag{E.10}$$

An equivalent expanded expression is

$$g_A = \frac{(1-\alpha_E-\alpha_S)n + \big(\eta_E(1-\alpha_S) + \eta_S\alpha_E\big)\gamma_E + \big(\eta_S(1-\alpha_E) + \eta_E\alpha_S\big)\gamma_S}{(1-\phi)(1-\alpha_E-\alpha_S) - (\eta_E+\eta_S)}.$$

Given g_A, output growth follows from (E.7):

$$g_Y = \frac{g_A}{1-\alpha_E-\alpha_S} + \frac{\alpha_E\gamma_E + \alpha_S\gamma_S}{1-\alpha_E-\alpha_S} + n. \tag{E.11}$$

A finite BGP requires $(1-\phi)(1-\alpha_E-\alpha_S) > (\eta_E+\eta_S)$.

E.1 Consistency Checks: Factor Prices

1. Rental rate of equipment in final goods. From $R_E = \alpha_E Y / K_E^Y$,

$$g_{R_E} = g_Y - g_{K_E^Y} = g_Y - (g_Y + \gamma_E) = -\gamma_E.$$

2. Rental rate of structures in final goods. From $R_S = \alpha_S Y / K_S^Y$,

$$g_{R_S} = g_Y - g_{K_S^Y} = g_Y - (g_Y + \gamma_S) = -\gamma_S.$$

3. Unskilled wage. From $w_u = (1 - \alpha_E - \alpha_S) Y / L_u$ with $g_{L_u} = n$,

$$g_{w_u} = g_Y - n.$$

4. Skilled wage and design value. From the R&D FOC,

$$w_s = (1 - \eta_E - \eta_S) \frac{p_A \dot{A}}{L_s}.$$

Since $g_{\dot{A}} = g_A$ on the BGP and $g_{L_s} = n$,

$$g_{w_s} = g_{p_A} + g_A - n.$$

If the wage premium is constant on the BGP (i.e., w_s / w_u constant), then $g_{w_s} = g_{w_u}$, implying

$$g_{p_A} = g_Y - g_A,$$

5. Consistency of R&D capital demands (both capitals). From the R&D FOCs,

$$R_E = \eta_E \frac{p_A \dot{A}}{K_E^R}, \qquad R_S = \eta_S \frac{p_A \dot{A}}{K_S^R}.$$

Thus,

$$g_{R_E}^{R\&D} = g_{p_A} + g_A - g_{K_E^R} = (g_Y - g_A) + g_A - (g_Y + \gamma_E) = -\gamma_E,$$
$$g_{R_S}^{R\&D} = g_{p_A} + g_A - g_{K_S^R} = (g_Y - g_A) + g_A - (g_Y + \gamma_S) = -\gamma_S,$$

which matches the growth rates implied by the final-goods sector, so both capital markets clear consistently on the BGP.

E.2 Consistency Checks: Household No-Arbitrage and Interest Rate Consistency

No-arbitrage for each capital type implies

$$r(t) = \frac{R_E(t)}{P_E(t)} + \frac{\dot{P}_E(t)}{P_E(t)} - \delta_E = q_E(t)R_E(t) - \gamma_E - \delta_E,$$
$$r(t) = \frac{R_S(t)}{P_S(t)} + \frac{\dot{P}_S(t)}{P_S(t)} - \delta_S = q_S(t)R_S(t) - \gamma_S - \delta_S.$$

On the BGP, $g_{q_E R_E} = \gamma_E + g_{R_E} = \gamma_E - \gamma_E = 0$ and $g_{q_S R_S} = \gamma_S + g_{R_S} = \gamma_S - \gamma_S = 0$, so $r(t)$ is constant.

Finally, since $C(t)$ grows at rate g_Y on the BGP, per-capita consumption grows at $g_S = g_Y - n$. The Euler equation implies the constant interest rate

$$r = \rho + \sigma(g_Y - n).$$

F Comparison with Standard Growth Models

To highlight the implications of capital heterogeneity, we compare our model to standard neoclassical benchmarks, in particular a *one-capital model* that aggregates equipment and structures into a single capital stock K.

F.1 BGP Comparison

Formally, the one-capital environment can be interpreted as imposing a common ISTC process across capital types, $\gamma_S = \gamma_E \equiv \gamma$, aggregating capital shares $\alpha_{\text{one}} = \alpha_E + \alpha_S$, and collapsing R&D capital intensities to a single parameter $\eta_{\text{one}} \equiv \eta_E + \eta_S$.

Under this restriction, the BGP system implies the familiar semi-endogenous growth expression:

$$g_A^{\text{one}} = \frac{(1-\alpha_{\text{one}})n + \eta_{\text{one}}\gamma}{(1-\phi)(1-\alpha_{\text{one}}) - \eta_{\text{one}}}. \tag{F.1}$$

Output growth satisfies

$$g_Y^{\text{one}} = n + \frac{g_A^{\text{one}} + \alpha_{\text{one}}\gamma}{1-\alpha_{\text{one}}}. \tag{F.2}$$

Table F.I summarizes the analytic solutions for (g_A, g_Y) in the two models.

Table F.I: **Analytical Comparison of BGP Growth Rates**

Model	**One-Capital Model**	**Two-Capital Model**
TFP Growth (g_A)	$\frac{(1-\alpha_{\text{one}})n + \eta_{\text{one}}\gamma}{(1-\phi)(1-\alpha_{\text{one}}) - \eta_{\text{one}}}$	$\frac{(1-\alpha_E-\alpha_S)n + (\eta_E(1-\alpha_S)+\eta_S\alpha_E)\gamma_E + (\eta_S(1-\alpha_E)+\eta_E\alpha_S)\gamma_S}{(1-\phi)(1-\alpha_E-\alpha_S) - (\eta_E+\eta_S)}$
GDP per capita Growth ($g_{Y/L}$)	$\frac{g_A^{\text{one}} + \alpha_{\text{one}}\gamma}{1-\alpha_{\text{one}}}$	$\frac{g_A + \alpha_E\gamma_E + \alpha_S\gamma_S}{1-\alpha_E-\alpha_S}$

Notes: This table compares closed-form balanced-growth-path growth rates across the one-capital and two-capital models. The expressions show how population growth n and exogenous knowledge-growth terms (γ or γ_E, γ_S) map into g_A and $g_{Y/L}$ through spillover elasticities (the η's) and factor shares (the α's), with $(1-\phi)$ governing the strength of scale effects.

Structural Drag in One-Capital Model By comparison with the one-capital model, the structural-drag mechanism becomes more transparent. Focusing on the denominator of the TFP growth rate in the two-capital model in Table F.I,

$$(1-\alpha_E-\alpha_S)\cdot n + (\eta_E+\eta_S)\cdot\gamma_E - (\eta_S(1-\alpha_E)+\eta_E\alpha_S)\cdot(\gamma_E-\gamma_S), \tag{F.3}$$

which makes explicit that $(\gamma_E - \gamma_S) > 0$ generates a negative "structural drag" term with weight

$$\omega_{\text{drag}} \equiv \eta_S(1 - \alpha_E) + \eta_E \alpha_S. \tag{F.4}$$

Relative to the one-capital benchmark (or the knife-edge case with $\gamma_S = \gamma_E$), this term lowers the equilibrium innovation rate by limiting the extent to which rapid equipment ISTC translates into sustained idea growth. When structures enter the R&D technology and $\gamma_S < \gamma_E$ empirically, long-run innovation is depressed through R&D-related structural drag.

This divergence between the two models is driven by heterogeneity in ISTC across capital types. In the one-capital model, aggregation eliminates any relative-price wedge associated with structures: all investment benefits from the same efficiency growth rate γ, and R&D uses a single composite capital input. By contrast, in the two-capital model with $\gamma_E > \gamma_S$, sluggish structures ISTC lowers the equilibrium innovation rate via a structural drag component that is proportional to the ISTC gap $(\gamma_E - \gamma_S)$, with coefficient ω_{drag} (defined in equation (F.4)); that is, the drag enters as $-\omega_{\text{drag}}(\gamma_E - \gamma_S)/\Delta$ where $\Delta \equiv (1 - \phi)\Psi - (\eta_E + \eta_S)$ and $\Psi \equiv 1 - \alpha_E - \alpha_S$ as in the main text. This effect operates through (i) a direct increase in the effective user cost of structures used in R&D and (ii) an indirect tightening of the economy-wide resource constraint, which attenuates the extent to which rapid equipment ISTC translates into sustained idea growth. Consequently, the one-capital model suffers from aggregation bias: it imputes the fast efficiency gains of equipment to the entire capital stock and overlooks the bottleneck created by slowly improving structures, thereby overstating long-run innovation and output growth relative to the two-capital economy.

F.2 Model Comparison

Our framework introduces two departures from the standard one-capital model: capital heterogeneity (equipment versus structures) and the use of capital in the R&D sector. To isolate the contribution of each margin, we compare (i) a one-capital model with an R&D sector, in which equipment and structures are aggregated into a single composite capital input, and (ii) a no-R&D specification in which productivity growth is exogenous.

In the one-capital-with-R&D specification, final-goods production takes the form $Y = AK^{\alpha}L^{1-\alpha}$, where $\alpha = \alpha_E + \alpha_S = 0.38$ denotes the aggregate capital share. The R&D technology similarly aggregates equipment and structures into a single composite capital input with elasticity $\eta = \eta_E + \eta_S = 0.33$. In the no-R&D specification, TFP growth is assumed to

be exogenous and set to $g_A = n/(1-\phi) = 0.33\%$, following Romer (1990).

A critical choice in implementing the one-capital model is which price series to use to discipline ISTC. Following the literature, we consider two alternative ISTC processes for the one-capital model: (i) the equipment price decline, which is widely used to measure investment-specific technological change and its contribution to growth (Greenwood et al., 1997; Cummins and Violante, 2002); and (ii) the aggregate investment price trend, which is estimated around 0.59% per year, which captures the composition-weighted average of equipment and structures prices, consistent with the definition and aggregate implications (Justiniano et al., 2011).

Table F.II: **Growth Rates: One-Capital vs. Two-Capital Models**

Model Specification	**Without R&D**	**With R&D**
One-Capital Model		
Using Equipment ISTC (γ_E)	2.29%	3.40%
Using Aggregate Investment ISTC (γ_I)	0.88%	1.20%
Two-Capital Model		
Using Equipment and Structures ISTC (γ_E and γ_S)	0.83%	**1.46%**
Data		
U.S. GDP per capita growth (1970–2020)	1.69%	

Notes: This table compares steady-state growth rates of GDP per capita across different model specifications. The one-capital model aggregates equipment and structures into a single capital input, with TFP growth in the no-R&D case set to $n/(1-\phi) = 0.33\%$. Two alternative ISTC calibrations are considered: equipment price trends ($\gamma_E = 0.0290$) and aggregate investment price trends ($\gamma_I = 0.0059$). The two-capital baseline model distinguishes between equipment and structures, calibrated to $\gamma_E = 0.0290$ and $\gamma_S = -0.0101$. The empirical benchmark is the average annual growth rate of real GDP per capita in the United States from 1970 to 2020, taken from the Penn World Table.

Table F.II reports the implied balanced-growth rates under each specification and shows that the choice of price series materially affects the model's predictions. When the one-capital model is calibrated to the equipment price trend (γ_E), it substantially overpredicts GDP per capita growth, yielding $g_{Y/L} = 2.29\%$ without R&D and $g_{Y/L} = 3.40\%$ with endogenous innovation, both far above the observed U.S. rate of 1.69% over 1970–2020. Conversely, when calibrated to the aggregate investment price trend (γ_I), the one-capital model underpredicts growth, delivering $g_{Y/L} = 0.88\%$ without R&D and $g_{Y/L} = 1.20\%$ with R&D.

In contrast, our baseline two-capital model with endogenous R&D delivers $g_{Y/L} = 1.46\%$, close to the observed U.S. growth rate. This improvement relies on capital heterogeneity: distinguishing equipment and structures allows the model to translate their

sharply different relative price trends into separate accumulation margins, rather than forcing them through a single investment-specific technological change (ISTC) process. Endogenous R&D is also essential for matching long-run growth because it provides a mechanism that links investment incentives to innovation-driven TFP growth. Consistent with this logic, the two-capital model without R&D predicts only $g_{Y/L} = 0.83\%$, well below the data, indicating that capital heterogeneity alone cannot account for U.S. growth in the absence of an R&D sector with capital.

G Detailed Derivation on CES Extension Case

This appendix proceeds in three blocks. First, it develops the fully endogenous CES economy and derives an drag-weight identity and multiplier decompositions as functions of the relative user-cost state. Second, it constructs the held-fixed evaluation and derives the closed-form multipliers used in the main text, together with a local lower-bound argument and a complementarity comparative static. Third, it establishes the Cobb–Douglas reduction, proves the Cobb–Douglas drag weight, and derives the Cobb–Douglas multiplier representation in benchmark shares.

G.1 General CES economy

G.1.1 Environment

Labor allocation. Let population and labor supply $N(t)$ split into production and R&D with time-varying fractions $\ell_u(t)$ and $\ell_s(t)$:

$$L_u(t) = \ell_u(t)N(t), \qquad L_s(t) = \ell_s(t)N(t), \qquad \ell_u(t) + \ell_s(t) = 1.$$

Output and knowledge technologies. Final output uses TFP $A(t)$, sector-Y capital services $Q_Y(t)$, and production labor $L_u(t)$. Knowledge accumulates through an R&D technology using the knowledge stock, sector-R capital services $Q_R(t)$, and R&D labor $L_s(t)$:

$$Y(t) = A(t)\, Q_Y(t)^{\alpha_Y} L_u(t)^{1-\alpha_Y}, \qquad \dot{A}(t) = \kappa\, A(t)^{\phi}\, Q_R(t)^{\alpha_R} L_s(t)^{1-\alpha_R},$$

with $\alpha_Y, \alpha_R \in (0,1)$ and $\phi < 1$.

CES capital services. For each sector $j \in \{Y, R\}$, capital services aggregate equipment and structures:

$$Q_j(K_E^j, K_S^j) = \left(\theta_j^{1/\sigma_j}(K_E^j)^{\rho_j} + (1-\theta_j)^{1/\sigma_j}(K_S^j)^{\rho_j}\right)^{1/\rho_j}, \qquad \rho_j \equiv \frac{\sigma_j - 1}{\sigma_j},$$

with $\sigma_j > 0$ and $\theta_j \in (0,1)$.

Investment rates and capital-output ratios. Define investment rates and capital-output ratios:

$$i_E(t) \equiv \frac{I_E(t)}{Y(t)}, \quad i_S(t) \equiv \frac{I_S(t)}{Y(t)}, \qquad \kappa_E(t) \equiv \frac{K_E(t)}{Y(t)}, \quad \kappa_S(t) \equiv \frac{K_S(t)}{Y(t)}.$$

Let $q_E(t)$ and $q_S(t)$ denote investment efficiency, and let δ_E, δ_S denote depreciation. Then

$$\dot{\kappa}_E(t) = q_E(t) i_E(t) - \big(\delta_E + g_Y(t)\big)\kappa_E(t), \tag{G.1}$$

$$\dot{\kappa}_S(t) = q_S(t) i_S(t) - \big(\delta_S + g_Y(t)\big)\kappa_S(t), \tag{G.2}$$

where $g_Y(t) \equiv \dot{Y}(t)/Y(t)$.

Capital split across sectors. Allow each aggregate capital stock to split across final production and R&D:

$$K_E(t) = K_E^Y(t) + K_E^R(t), \qquad K_S(t) = K_S^Y(t) + K_S^R(t).$$

Along an interior allocation, introduce R&D reallocation shares

$$\chi_E(t) \equiv \frac{K_E^R(t)}{K_E(t)} \in (0,1), \qquad \chi_S(t) \equiv \frac{K_S^R(t)}{K_S(t)} \in (0,1),$$

so sectoral inputs satisfy

$$K_E^Y = (1-\chi_E)K_E, \quad K_S^Y = (1-\chi_S)K_S, \quad K_E^R = \chi_E K_E, \quad K_S^R = \chi_S K_S. \tag{G.3}$$

User costs and the relative user-cost state. For $k \in \{E, S\}$, let

$$u_k(t) \equiv P_k(t)\big(r(t) + \delta_k + \gamma_k\big), \qquad x(t) \equiv \frac{u_E(t)}{u_S(t)}.$$

Log-differentiation gives the exact identity

$$\frac{\dot{x}(t)}{x(t)} = -(\gamma_E - \gamma_S) + \frac{\dot{r}(t)}{r(t) + \delta_E + \gamma_E} - \frac{\dot{r}(t)}{r(t) + \delta_S + \gamma_S}.$$

Sectoral expenditure shares and their drift. Define sector-j cost shares on equipment and structures:

$$s_{E,j}(x) \equiv \frac{u_E K_E^j}{u_E K_E^j + u_S K_S^j}, \qquad s_{S,j}(x) \equiv 1 - s_{E,j}(x), \qquad j \in \{Y, R\}.$$

CES cost minimization implies the closed-form share maps

$$s_{E,j}(x) = \frac{1}{1 + \left(\frac{1-\theta_j}{\theta_j}\right) x^{\sigma_j - 1}}, \qquad s_{S,j}(x) = \frac{\left(\frac{1-\theta_j}{\theta_j}\right) x^{\sigma_j - 1}}{1 + \left(\frac{1-\theta_j}{\theta_j}\right) x^{\sigma_j - 1}}. \tag{G.4}$$

Differentiate $s_{S,j}(x)$ with respect to $\log x$ by writing $z_j(x) \equiv \left(\frac{1-\theta_j}{\theta_j}\right) x^{\sigma_j - 1}$ so that $s_{S,j}(x) = z_j(x)/(1 + z_j(x))$ and $\partial z_j / \partial \log x = (\sigma_j - 1) z_j$. Then

$$\frac{\partial s_{S,j}(x)}{\partial \log x} = \frac{1}{(1 + z_j(x))^2} (\sigma_j - 1) z_j(x) = (\sigma_j - 1)\, s_{E,j}(x)\, s_{S,j}(x). \tag{G.5}$$

Along any interior trajectory, the chain rule yields

$$\dot{s}_{S,j}(x(t)) = (\sigma_j - 1)\, s_{E,j}(x(t))\, s_{S,j}(x(t)) \cdot \frac{\dot{x}(t)}{x(t)}. \tag{G.6}$$

Growth accounting identities. Let $g_A(t) \equiv \dot{A}(t)/A(t)$ and $n(t) \equiv \dot{N}(t)/N(t)$. Log- differentiation of $Y = A Q_Y^{\alpha_Y} L_u^{1-\alpha_Y}$ gives

$$g_Y = g_A + \alpha_Y g_{Q_Y} + (1 - \alpha_Y) \left(n + \frac{\dot{\ell}_u}{\ell_u} \right). \tag{G.7}$$

Under cost minimization for $Q_j(\cdot)$,

$$g_{Q_j} = s_{E,j}(x)\, g_{K_E^j} + s_{S,j}(x)\, g_{K_S^j}.$$

Use (G.3) to obtain

$$g_{K_E^Y} = g_{K_E} + \frac{d}{dt} \log(1 - \chi_E) = g_{K_E} - \frac{\dot{\chi}_E}{1 - \chi_E}, \qquad g_{K_E^R} = g_{K_E} + \frac{d}{dt} \log(\chi_E) = g_{K_E} + \frac{\dot{\chi}_E}{\chi_E},$$

and analogously for structures with χ_S. Finally, the aggregate capital accumulation law $\dot{K}_k = q_k I_k - \delta_k K_k$ implies

$$g_{K_k} = \frac{\dot{K}_k}{K_k} = q_k \frac{I_k}{K_k} - \delta_k = q_k \frac{i_k}{\kappa_k} - \delta_k, \qquad k \in \{E, S\}.$$

Substitution yields the exact interior identities

$$g_{Q_Y} = s_{E,Y}(x)\left(-\frac{\dot{\chi}_E}{1-\chi_E} + q_E\frac{i_E}{\kappa_E} - \delta_E\right) + s_{S,Y}(x)\left(-\frac{\dot{\chi}_S}{1-\chi_S} + q_S\frac{i_S}{\kappa_S} - \delta_S\right), \tag{G.8}$$

$$g_{Q_R} = s_{E,R}(x)\left(\frac{\dot{\chi}_E}{\chi_E} + q_E\frac{i_E}{\kappa_E} - \delta_E\right) + s_{S,R}(x)\left(\frac{\dot{\chi}_S}{\chi_S} + q_S\frac{i_S}{\kappa_S} - \delta_S\right). \tag{G.9}$$

Innovation-growth drift. From $\dot{A} = \kappa A^{\phi} Q_R^{\alpha_R} L_s^{1-\alpha_R}$, log-differentiation gives

$$\frac{\dot{g}_A}{g_A} = (\phi - 1)g_A + \alpha_R g_{Q_R} + (1-\alpha_R)\left(n + \frac{\dot{\ell}_s}{\ell_s}\right).$$

G.1.2 Drag weight and multiplier decomposition in the general CES economy

For each sector $j \in \{Y, R\}$, define the sectoral effective ISTC rate as the user-cost-share-weighted average of the two type-specific ISTC components:

$$\gamma_j^{\text{istc}}(x) \equiv s_{E,j}(x)\gamma_E + s_{S,j}(x)\gamma_S = \gamma_E - s_{S,j}(x)\,(\gamma_E - \gamma_S).$$

This definition isolates the role of the ISTC gap $(\gamma_E - \gamma_S)$ through the structures share $s_{S,j}(x)$, which is pinned down by the CES share map evaluated at the relative user-cost state $x = u_E/u_S$.

To summarize how the ISTC gap loads into the economy-wide wedge that is relevant for the growth system, introduce the wedge-loading index

$$\Omega_{\text{wedge}}(x) \equiv \alpha_Y\, s_{S,Y}(x) + (1-\alpha_Y)\, s_{S,R}(x),$$

and define the associated drag weight

$$\omega_{\text{drag}}(x) \equiv \alpha_R\, \Omega_{\text{wedge}}(x).$$

A direct substitution of $\gamma_Y^{\text{istc}}(x)$ and $\gamma_R^{\text{istc}}(x)$ into the weighted ISTC wedge $\alpha_Y\gamma_Y^{\text{istc}}(x) + (1-\alpha_Y)\gamma_R^{\text{istc}}(x)$ yields the exact identity

$$\alpha_Y\gamma_Y^{\text{istc}}(x) + (1-\alpha_Y)\gamma_R^{\text{istc}}(x) = \gamma_E - \Omega_{\text{wedge}}(x)\,(\gamma_E - \gamma_S). \tag{G.10}$$

Equivalently, the same object can be written in a linear multiplier form,

$$\alpha_Y\gamma_Y^{\text{istc}}(x) + (1-\alpha_Y)\gamma_R^{\text{istc}}(x) = \big(1 - \Omega_{\text{wedge}}(x)\big)\gamma_E + \Omega_{\text{wedge}}(x)\gamma_S,$$

so $\Omega_{\text{wedge}}(x)$ is the exact structures multiplier inside the economy-wide ISTC wedge, and $\omega_{\text{drag}}(x) = \alpha_R \Omega_{\text{wedge}}(x)$ is the corresponding drag weight once this wedge enters the knowledge-growth block with elasticity α_R.

G.1.3 General-economy multiplier representation for g_y with g_A substituted

Let per-capita output growth be $g_y \equiv g_Y - n$. Along any interior path, the production-side growth accounting delivers the exact identity

$$g_y = \frac{g_A}{1-\alpha_Y} + \frac{\alpha_Y}{1-\alpha_Y}\gamma_Y^{\text{istc}}(x) - \frac{\alpha_Y}{1-\alpha_Y}\dot{r}\,\mathcal{H}_Y(r,x) + \frac{\dot{\ell}_u}{\ell_u}, \tag{G.11}$$

where

$$\gamma_Y^{\text{istc}}(x) = s_{E,Y}(x)\gamma_E + s_{S,Y}(x)\gamma_S, \qquad \mathcal{H}_Y(r,x) \equiv \frac{s_{E,Y}(x)}{r+\delta_E+\gamma_E} + \frac{s_{S,Y}(x)}{r+\delta_S+\gamma_S}.$$

Thus, conditional on g_A and on contemporaneous adjustment terms $(\dot{r}, \dot{\ell}_u)$, the ISTC component enters g_y through the sector-Y cost shares.

To substitute out g_A without imposing held-fixed restrictions, use the innovation-growth drift,

$$\frac{\dot{g}_A}{g_A} = (\phi-1)g_A + \alpha_R g_{Q_R} + (1-\alpha_R)\left(n + \frac{\dot{\ell}_s}{\ell_s}\right),$$

which can be rearranged into the exact identity

$$g_A = \frac{\alpha_R g_{Q_R} + (1-\alpha_R)\left(n + \frac{\dot{\ell}_s}{\ell_s}\right) - \frac{\dot{g}_A}{g_A}}{1-\phi}. \tag{G.12}$$

Here g_{Q_R} is itself an exact object along any interior path, given by

$$g_{Q_R} = s_{E,R}(x)\left(\frac{\dot{\chi}_E}{\chi_E} + q_E\frac{i_E}{\kappa_E} - \delta_E\right) + s_{S,R}(x)\left(\frac{\dot{\chi}_S}{\chi_S} + q_S\frac{i_S}{\kappa_S} - \delta_S\right). \tag{G.13}$$

Substituting (G.12) into (G.11) yields an exact representation of g_y in which the innovation term is expressed in observables and adjustment wedges:

$$\begin{aligned} g_y = {} & \frac{1}{(1-\alpha_Y)(1-\phi)}\left[\alpha_R g_{Q_R} + (1-\alpha_R)\left(n + \frac{\dot{\ell}_s}{\ell_s}\right) - \frac{\dot{g}_A}{g_A}\right] \\ & + \frac{\alpha_Y}{1-\alpha_Y}\gamma_Y^{\text{istc}}(x) - \frac{\alpha_Y}{1-\alpha_Y}\dot{r}\,\mathcal{H}_Y(r,x) + \frac{\dot{\ell}_u}{\ell_u}. \end{aligned} \tag{G.14}$$

Finally, expanding $\gamma_Y^{\text{istc}}(x) = s_{E,Y}(x)\gamma_E + s_{S,Y}(x)\gamma_S$ gives the conditional ISTC multipliers that operate through the production sector:

$$\frac{\alpha_Y}{1-\alpha_Y}\gamma_Y^{\text{istc}}(x) = \mathcal{M}_E^Y(x)\gamma_E + \mathcal{M}_S^Y(x)\gamma_S, \qquad \mathcal{M}_E^Y(x) \equiv \frac{\alpha_Y}{1-\alpha_Y}s_{E,Y}(x), \quad \mathcal{M}_S^Y(x) \equiv \frac{\alpha_Y}{1-\alpha_Y}s_{S,Y}(x).$$

Equivalently, the production-side ISTC-gap component inside g_y can be isolated as

$$\mathcal{M}_E^Y(x)\gamma_E + \mathcal{M}_S^Y(x)\gamma_S = \frac{\alpha_Y}{1-\alpha_Y}\gamma_E - \frac{\alpha_Y}{1-\alpha_Y}s_{S,Y}(x)\,(\gamma_E - \gamma_S),$$

so the instantaneous drag contribution in the general economy loads the gap $(\gamma_E - \gamma_S)$ through the state-dependent structures share $s_{S,Y}(x)$.

It is important to note that (G.14) is exact but it is not a closed-form multiplier system in (n, γ_E, γ_S) alone, because the right-hand side still contains contemporaneous adjustment wedges and endogenous objects, notably g_{Q_R} (which depends on $(i_k, \kappa_k, \chi_k, \dot{\chi}_k)$), $\dot{g}_A/g_A$, $\dot{r}$, and labor reallocation terms.

G.2 Pseudo CES Case

G.2.1 Restrictions for the Pseudo CES Case and closed-form evaluation

Fix an evaluation instant t and impose the pseudo CES restrictions

$$\dot{r}(t) = 0, \qquad \dot{g}_A(t) = 0, \qquad \frac{\dot{\ell}_u(t)}{\ell_u(t)} = 0, \qquad \frac{\dot{\ell}_s(t)}{\ell_s(t)} = 0, \qquad \dot{x}(t) = 0, \qquad \dot{\chi}_E(t) = 0, \qquad \dot{\chi}_S(t) = 0,$$

while evaluating the CES cost shares at the prevailing state $x(t)$. These restrictions shut down contemporaneous adjustment wedges (interest-rate movements, labor reallocation, drift in x, and sectoral reallocation of the existing capital stocks) but preserve the dependence of the instantaneous mapping on the current state through $s_{E,j}(x)$ and $s_{S,j}(x)$.

Under these restrictions, the general per-capita growth identity (G.11) collapses to the two-block system

$$g_y = \frac{g_A}{1-\alpha_Y} + \frac{\alpha_Y}{1-\alpha_Y}\gamma_Y^{\text{istc}}(x), \tag{G.15}$$

together with the stationary innovation-growth condition that will be used to solve for g_A at the evaluation point.

For notational convenience, define the auxiliary index

$$\Omega_{\text{static}}(x) \equiv \frac{\alpha_Y}{1-\alpha_Y}s_{S,Y}(x) + s_{S,R}(x), \qquad \Omega_{\text{wedge}}(x) = (1-\alpha_Y)\Omega_{\text{static}}(x).$$

This index collects the two structures-share terms that matter for the pseudo CES mapping, with the first term capturing the production-side (resource) channel and the second term capturing the direct R&D-input channel.

Let

$$\Delta_{ces} \equiv (1-\phi)(1-\alpha_Y) - \alpha_R, \qquad \Delta_{ces} > 0,$$

so that the instantaneous fixed-point mapping for growth is well-defined. Stationarity of g_A under the pseudo CES restrictions implies

$$(1-\phi)g_A = \alpha_R g_{Q_R} + (1-\alpha_R)n.$$

The pseudo CES closure then evaluates the contemporaneous contribution of R&D sector capital in the same unit-cost form used to define the ISTC wedge, namely

$$g_{Q_R} = g_Y + \gamma_R^{\text{istc}}(x).$$

Using $g_Y = g_y + n$ and substituting (G.15) for g_y delivers

$$g_{Q_R} = \left(\frac{g_A}{1-\alpha_Y} + \frac{\alpha_Y}{1-\alpha_Y}\gamma_Y^{\text{istc}}(x)\right) + n + \gamma_R^{\text{istc}}(x).$$

Insert this expression into $(1-\phi)g_A = \alpha_R g_{Q_R} + (1-\alpha_R)n$, multiply by $(1-\alpha_Y)$, and collect g_A terms to obtain

$$\big((1-\phi)(1-\alpha_Y) - \alpha_R\big)g_A = (1-\alpha_Y)n + \alpha_R\Big(\alpha_Y\gamma_Y^{\text{istc}}(x) + (1-\alpha_Y)\gamma_R^{\text{istc}}(x)\Big).$$

Dividing by Δ_{ces} and using the wedge identity (G.10) to rewrite the ISTC bracket yields the closed-form expression

$$g_A = \frac{(1-\alpha_Y)n + \alpha_R\gamma_E - \alpha_R(1-\alpha_Y)\Omega_{\text{static}}(x)\,(\gamma_E - \gamma_S)}{\Delta_{ces}}. \tag{G.16}$$

G.2.2 Pseudo CES multiplier representation for g_y

Substituting (G.16) into (G.15) gives the pseudo CES multiplier representation

$$g_y = \mu_n^y\, n + \mathcal{M}_E(x)\gamma_E + \mathcal{M}_S(x)\gamma_S, \qquad \mu_n^y = \frac{1}{\Delta_{ces}}, \tag{G.17}$$

where state dependence enters only through the CES shares embedded in $s_{S,Y}(x)$ and $s_{S,R}(x)$.

The structural drag multipliers can be written in closed form as

$$\mathcal{M}_S(x) = \underbrace{\frac{\alpha_Y}{1-\alpha_Y}\, s_{S,Y}(x)}_{\text{static deepening}} + \underbrace{\frac{\alpha_R}{\Delta_{ces}}\, s_{S,R}(x)}_{\text{direct R\&D input}} + \underbrace{\frac{\alpha_R}{\Delta_{ces}}\frac{\alpha_Y}{1-\alpha_Y}\, s_{S,Y}(x)}_{\text{indirect resource channel}}. \tag{G.18}$$

The three terms in (G.18) correspond, respectively, to the direct production-side effect of structures, the direct role of structures as an R&D input, and the feedback from production-side unit costs into innovation through the resource channel, scaled by the same denominator Δ_{ces} that governs the innovation-growth fixed point.

G.3 Why the pseudo CES object is a conservative local lower bound

The pseudo CES drag term is most transparent in the innovation block (G.16). The ISTC-gap component there is

$$-\frac{\alpha_R}{\Delta_{ces}}\,\Omega_{\text{wedge}}(x)\,(\gamma_E-\gamma_S) = -\frac{1}{\Delta_{ces}}\,\omega_{\text{drag}}(x)\,(\gamma_E-\gamma_S),$$

so, at a given state x, the magnitude of drag scales with the drag weight $\omega_{\text{drag}}(x) = \alpha_R\Omega_{\text{wedge}}(x)$.

To sign how the drag weight moves with the state under complementarity, differentiate $\Omega_{\text{wedge}}(x)$ with respect to $\log x$ using (G.5):

$$\frac{\partial\Omega_{\text{wedge}}(x)}{\partial\log x} = \alpha_Y(\sigma_Y-1)s_{E,Y}(x)s_{S,Y}(x) + (1-\alpha_Y)(\sigma_R-1)s_{E,R}(x)s_{S,R}(x).$$

If $\sigma_Y < 1$ and $\sigma_R < 1$, each term is strictly negative at any interior x because $(\sigma_j-1) < 0$ and $s_{E,j}(x)s_{S,j}(x) > 0$. Hence $\Omega_{\text{wedge}}(x)$, and therefore $\omega_{\text{drag}}(x)$, is strictly decreasing in $\log x$ under complementarity.

Now suppose $\gamma_E > \gamma_S$ and that the equilibrium path satisfies $\dot{x}/x < 0$ over a short interval after the evaluation instant. Then (G.6) implies $\dot{s}_{S,j}(x(t)) > 0$ when $\sigma_j < 1$, so $\Omega_{\text{wedge}}(x(t))$ and $\omega_{\text{drag}}(x(t))$ increase over time along that interval. Consequently, for small $\Delta t > 0$,

$$\omega_{\text{drag}}(x(t+\Delta t)) \geq \omega_{\text{drag}}(x(t)),$$

and the associated ISTC-gap drag term at $t+\Delta t$ is weakly more negative than the pseudo CES drag evaluated at t. In this precise local sense, freezing the state x at its current value yields a conservative lower bound on the magnitude of drag when equipment-biased

ISTC pushes x downward under complementarity.

G.4 Cobb–Douglas Case

When $\sigma_Y = \sigma_R = 1$, each CES capital-services aggregator becomes Cobb–Douglas up to a normalization constant, and all sectoral cost shares become state-invariant. This section records the Cobb–Douglas limit, then derives the implied drag weight and the closed-form multiplier representation for g_y in benchmark-share notation

Fix $j \in \{Y, R\}$ and write $\rho = \rho_j$, so that $1/\sigma_j = 1 - \rho$. Then

$$Q_j(K_E^j, K_S^j) = \left(\theta_j^{1-\rho}(K_E^j)^\rho + (1-\theta_j)^{1-\rho}(K_S^j)^\rho\right)^{1/\rho}.$$

Let

$$F(\rho) \equiv \theta_j^{1-\rho}(K_E^j)^\rho + (1-\theta_j)^{1-\rho}(K_S^j)^\rho,$$

so that $\log Q_j(\rho) = \rho^{-1}\log F(\rho)$ and $F(0) = 1$. By l'Hôpital's rule,

$$\lim_{\rho\to 0}\log Q_j(\rho) = \left.\frac{F'(\rho)}{F(\rho)}\right|_{\rho=0} = \theta_j\big(\log K_E^j - \log\theta_j\big) + (1-\theta_j)\big(\log K_S^j - \log(1-\theta_j)\big).$$

Exponentiating yields

$$\lim_{\sigma_j\to 1} Q_j(K_E^j, K_S^j) = \frac{(K_E^j)^{\theta_j}(K_S^j)^{1-\theta_j}}{\theta_j^{\theta_j}(1-\theta_j)^{1-\theta_j}}.$$

The multiplicative constant is independent of (K_E^j, K_S^j), so it does not affect cost shares, factor demands, or growth rates. Consistently, setting $\sigma_j = 1$ in the CES share maps (G.4) implies constant sectoral shares

$$s_{E,j}(x) = \theta_j, \qquad s_{S,j}(x) = 1-\theta_j, \qquad j \in \{Y, R\}.$$

G.4.1 Cobb–Douglas drag weight and benchmark-share mapping

Define benchmark shares

$$\alpha_E \equiv \alpha_Y\theta_Y, \quad \alpha_S \equiv \alpha_Y(1-\theta_Y), \qquad \eta_E \equiv \alpha_R\theta_R, \quad \eta_S \equiv \alpha_R(1-\theta_R),$$

so that $\alpha_Y = \alpha_E + \alpha_S$, $\alpha_R = \eta_E + \eta_S$, and

$$\theta_Y = \frac{\alpha_E}{\alpha_E+\alpha_S} = \frac{\alpha_E}{\alpha_Y}, \qquad \theta_R = \frac{\eta_E}{\eta_E+\eta_S} = \frac{\eta_E}{\alpha_R}.$$

Evaluating the CES wedge-loading index at $\sigma_Y = \sigma_R = 1$ gives

$$\Omega^{CD} = \alpha_Y(1-\theta_Y) + (1-\alpha_Y)(1-\theta_R).$$

Using $1-\theta_Y = \alpha_S/\alpha_Y$ and $1-\theta_R = \eta_S/\alpha_R$ yields

$$\Omega^{CD} = \alpha_S + (1-\alpha_Y)\frac{\eta_S}{\alpha_R} = \alpha_S + \big(1-\alpha_E-\alpha_S\big)\frac{\eta_S}{\eta_E+\eta_S}.$$

Multiplying by $\alpha_R = \eta_E + \eta_S$ delivers the Cobb–Douglas drag weight

$$\omega^{CD}_{\text{drag}} \equiv \alpha_R\Omega^{CD} = (\eta_E+\eta_S)\alpha_S + (1-\alpha_E-\alpha_S)\eta_S = \eta_E\alpha_S + \eta_S(1-\alpha_E),$$

which matches the benchmark expression in the main text.

G.4.2 Cobb–Douglas multiplier representation for g_y in benchmark shares

Under Cobb–Douglas, $s_{S,Y} = 1-\theta_Y$ and $s_{S,R} = 1-\theta_R$ are constants, so the pseudo CES held-fixed multiplier representation (G.17) becomes

$$g_y = \mu^y_n\, n + \mathcal{M}^{CD}_E\gamma_E + \mathcal{M}^{CD}_S\gamma_S, \qquad \mu^y_n = \frac{1}{\Delta}.$$

Substituting $s_{S,Y} = 1-\theta_Y$ and $s_{S,R} = 1-\theta_R$ into (G.18) yields

$$\mathcal{M}^{CD}_S = \frac{\alpha_Y}{1-\alpha_Y}(1-\theta_Y) + \frac{\alpha_R}{\Delta}(1-\theta_R) + \frac{\alpha_R}{\Delta}\frac{\alpha_Y}{1-\alpha_Y}(1-\theta_Y).$$

Now substitute the benchmark mapping $1-\theta_Y = \alpha_S/\alpha_Y$ and $1-\theta_R = \eta_S/\alpha_R$:

$$\mathcal{M}^{CD}_S = \frac{\alpha_S}{1-\alpha_Y} + \frac{\eta_S}{\Delta} + \frac{\alpha_R}{\Delta}\cdot\frac{\alpha_S}{1-\alpha_Y} = \frac{\alpha_S}{1-\alpha_Y} + \frac{\eta_S}{\Delta} + \frac{\eta_E+\eta_S}{\Delta}\cdot\frac{\alpha_S}{1-\alpha_Y}.$$

Using the main-text definitions

$$\Psi \equiv 1-\alpha_E-\alpha_S = 1-\alpha_Y, \qquad \Delta \equiv (1-\phi)\Psi - (\eta_E+\eta_S),$$

the same expression can be written as

$$\mathcal{M}^{CD}_S = \frac{\alpha_S}{\Psi} + \frac{\eta_S}{\Delta} + \frac{\eta_E+\eta_S}{\Delta}\cdot\frac{\alpha_S}{\Psi}.$$

Therefore the Cobb–Douglas case delivers the same benchmark-share multiplier form as in the main text, with identical (Ψ, Δ) and with $\mathcal{M}^{CD}_S$ given above.

G.5 Why stronger complementarity strengthens structural drag?

Comparative static of shares in σ_j. Fix $x > 0$ and write $z_j(x) \equiv \left(\frac{1-\theta_j}{\theta_j}\right)x^{\sigma_j-1}$ so that $s_{S,j}(x) = z_j/(1+z_j)$. Then

$$\frac{\partial s_{S,j}(x)}{\partial \sigma_j} = \frac{1}{(1+z_j(x))^2}\frac{\partial z_j(x)}{\partial \sigma_j} = \frac{z_j(x)\log x}{(1+z_j(x))^2} = s_{E,j}(x)s_{S,j}(x)\log x.$$

Hence, if $x < 1$ then $\log x < 0$ and $\partial s_{S,j}(x)/\partial \sigma_j < 0$. Thus a smaller σ_j (stronger complementarity) raises $s_{S,j}(x)$ in both sectors whenever $x < 1$.

Implication for drag weights. Because $\Omega_{\text{wedge}}(x)$ and $\Omega_{\text{static}}(x)$ are increasing in each $s_{S,j}(x)$, the same condition $x < 1$ implies

$$\frac{\partial \Omega_{\text{wedge}}(x)}{\partial \sigma_Y} < 0, \qquad \frac{\partial \Omega_{\text{wedge}}(x)}{\partial \sigma_R} < 0,$$

and therefore

$$\frac{\partial \omega_{\text{drag}}(x)}{\partial \sigma_Y} < 0, \qquad \frac{\partial \omega_{\text{drag}}(x)}{\partial \sigma_R} < 0.$$

So stronger complementarity (smaller σ) raises the drag weight in the general CES wedge identity (G.10) through larger structures shares.

Implication for Pseudo-CES multipliers. In the Pseudo-CES, $\mathcal{M}_S(x)$ in (G.18) is increasing in $s_{S,Y}(x)$ and $s_{S,R}(x)$ term by term. Thus, under $x < 1$, stronger complementarity raises $\mathcal{M}_S(x)$ and increases the magnitude of the ISTC-gap drag in g_y through the same share channel.