When to Push? Market Forces, Government Intervention, and the Optimal Timing of Industrial Policy*

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Abstract

When is industrial policy effective? We develop a theory in which effectiveness hinges on timing. Market evolution—driven by factor price changes, technological progress, or demand shifts—gradually enhances a technology's private competitiveness, yet diffusion is stalled by adoption externalities where productivity takes off only after aggregate adoption exceeds a critical mass. A planner can use subsidies to push adoption across this threshold but faces a dynamic tradeoff: intervening too early incurs high market distortion costs, while intervening too late forgoes large externality gains. The model characterizes a precise window for welfare-improving intervention, pinned down by closing the wedge between private and social Euler equations. We apply this framework to China's agricultural mechanization (1979–2020), where capital-intensive tractors gradually displaced labor-intensive draft animals and mechanization rose from roughly 20% to 70%. We show that market forces—specifically a declining capital-to-labor price ratio—primarily drove the initial transition. However, the government's targeted subsidies introduced in 2004 accelerated adoption near the critical threshold, increasing mechanization by 20 percentage points and yielding sizable welfare gains relative to the laissez-faire benchmark, though still falling short of the social optimum. Counterfactual analysis suggests that either earlier (premature) or later (delayed) intervention would have resulted in significant welfare losses. Our findings underscore that effective industrial policy does not override market forces but strategically amplifies their natural momentum.

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1 Introduction

In the midst of intensifying global technology competition and the urgent demands of the green transition, industrial policy has returned to the forefront of economic strategy. From the U.S. CHIPS and Science Act to China's strategic emerging industries initiatives, governments are aggressively intervening to steer technological trajectories. Yet despite this policy renaissance, academic evaluation of such interventions remains challenging. Empirically, it is notoriously difficult to disentangle the effects of government policy from underlying market forces—making it hard to identify the causal effect of intervention (Pack and Saggi, 2006; Juhász, Lane, and Rodrik, 2024). Theoretically, while the rationales for intervention—such as Marshallian externalities and coordination failures—are well established (see, e.g., Harrison and Rodríguez-Clare, 2010, for a survey) and indeed frequently invoked to justify policy in practice, the stark variance in outcomes—where some policies succeed while others falter—reveals the need to identify the more specific conditions under which intervention is welfare-improving rather than distortionary. As Rodrik (2004) argues, the hard problem is not whether to intervene but how and when.

This paper focuses on one critical but underexplored dimension: timing. We ask not just whether the government should intervene, but when. We develop a theory of the optimal timing of industrial policy in which the effectiveness of intervention hinges on the interplay between market-driven evolution and latent market failures. We apply this framework to China's agricultural mechanization (1979–2020), a setting where market forces and government intervention can be cleanly isolated, to quantify the welfare implications of well-timed versus mistimed intervention.

Our theoretical framework is deliberately general, modeling the adoption of a new technology that displaces an incumbent one—a dynamic applicable to contexts ranging from electric vehicles replacing combustion engines to renewable energy supplanting fossil fuels, and from digital platforms to industrial automation. The economy is driven by two forces. First, market evolution gradually improves the private competitiveness of the new technology. This force can originate from various sources: it may stem from technological breakthroughs (e.g., rising battery energy density), evolving consumer preferences (e.g., environmental awareness), or changes in factor prices (e.g., rising labor costs). Second, the new technology exhibits an adoption externality: sectoral productivity is endogenous to the aggregate scale of adoption, reflecting learning-by-doing, technological spillovers, or network effects. Specifically, productivity takes off only after aggregate adoption exceeds a critical mass. These two forces together imply a dynamic tension: a social planner can use subsidies to push adoption across this threshold, but doing so incurs market distortionary costs by misallocating resources.

Our central insight is that optimal intervention is confined to a precise policy window. When market fundamentals are weak—when the new technology remains far from private competitiveness—

the subsidy required to bridge the gap is prohibitively large, generating distortion costs that outweigh the gains from the externality. Conversely, when fundamentals are sufficiently strong, the market will cross the threshold autonomously, rendering intervention unnecessary. Welfare-maximizing policy occurs only in an intermediate range where the market is "almost ready"—where fundamentals are ripe enough that a moderate push yields a steep rise in productivity, yet private agents acting individually fail to make the leap because they do not internalize the spillover. Crucially, if market fundamentals eventually stabilize within this intermediate range rather than improving further, the decentralized economy may remain permanently trapped in a low-adoption steady state, whereas the planner ensures a transition to the high-productivity regime. In such cases, intervention determines not merely the *speed of convergence* but the *ultimate long-run outcome*, preventing the economy from stalling in an inefficient equilibrium.

More concretely, our model identifies two distinct regions as market fundamentals evolve. The first is a "coordination region", where multiple equilibria exist: both a high-adoption outcome (with high productivity) and a low-adoption outcome (with low productivity) are self-fulfilling. In this region, consistent with the classic Big Push logic of Murphy, Shleifer, and Vishny (1989), policy can coordinate expectations onto the superior equilibrium. The second—and our novel result—is a "pushed region" that lies adjacent to the coordination region, where market fundamentals have improved but have not yet reached the point at which multiplicity arises. Here, the equilibrium is unique: only the low-adoption outcome survives under decentralized decision-making. Yet this unique equilibrium is inefficient, because private agents fail to internalize the adoption externality. In this region, the planner finds it optimal to intervene—not to resolve indeterminacy, but to correct a dynamic inefficiency by pushing adoption past the productivity threshold. The welfare gain from doing so exceeds the distortion cost precisely because fundamentals have evolved far enough to make a moderate subsidy sufficient. This two-region structure implies that the case for industrial policy extends beyond the classic multiplicity setting: even when the equilibrium is unique, well-timed intervention can be welfare-improving.

We apply this general framework to data from China's agricultural mechanization (1979–2020), where the mechanization rate surged from roughly 20% to 70%. This setting offers a rare opportunity to cleanly isolate market forces from government intervention. For the first 25 years (1979–2003), mechanization was driven primarily by market fundamentals—specifically, a declining capital-to-labor price ratio that increased the cost-effectiveness of capital-intensive tractors relative to labor-intensive draft animals. This phase unfolded without major policy support, aligning with theories that emphasize endowment structure and factor prices as drivers of technological change (Ju, Lin, and Wang, 2015), as well as with the historical experience of the U.S. (Manuelli and Seshadri, 2014).

However, in 2004, the Chinese government introduced a major subsidy program targeting agri-

cultural machinery purchases. Using structural estimation, we show that this intervention roughly coincided with the "pushed region" identified by our theory. The subsidy did not initiate the transition but accelerated adoption near the critical threshold for widespread externalities. Our estimates indicate that the subsidy boosted mechanization by approximately 20 percentage points and generated social benefits that exceeded the subsidy costs by 60%-85%.

Our counterfactual analysis reveals that timing was pivotal. Introducing the same subsidy magnitude ten years earlier—when market fundamentals remained weak—would have resulted in net welfare losses due to severe resource misallocation. Conversely, delaying intervention by ten years would have forfeited roughly 40% of the achievable welfare gains, as the window to activate externalities would have narrowed as the market approached autonomous threshold-crossing. These findings demonstrate the quantitative importance of synchronizing intervention with market momentum—a lesson that extends to contemporary debates on electric vehicles, green energy, and other emerging technologies.

Literature review. Our paper contributes to two strands of literature: the macroeconomic theory of industrial policy, and the macro-development literature on structural transformation.

On the theory of industrial policy, we extend the canonical "Big Push" framework by formalizing the timing dimension of industrial policy. The idea that coordinated investment can break economies out of low-development traps originates with Rosenstein-Rodan (1943) and was formalized by Murphy, Shleifer, and Vishny (1989), who showed how simultaneous industrialization can resolve coordination failures by enabling transitions between multiple equilibria (see also Krugman, 1991; Matsuyama, 1991, 1992). Azariadis and Drazen (1990) highlight that multiplicity in terms of multiple steady states requires a coordinated push to escape low-development traps. Recent work of Aghion et al. (2025) studies industrial policy in a big-push production network with multiple steady states, extending the network framework of Liu (2019). Our model advances this literature along two dimensions. First, by embedding these mechanisms within an economy where fundamentals evolve continuously, we show how the coordination region—studied in prior work—arises endogenously at a specific stage of market development. Second, and more importantly, we identify the "pushed region" where the equilibrium is unique yet inefficient, demonstrating that the case for industrial policy need not rely on multiplicity. This distinction implies that effective policy requires not just the presence of externalities, but precise timing relative to the state of market fundamentals.[1,2]

[1] Acemoglu et al. (2016) model path-dependent innovation in directed technical change. Because past innovation makes future innovation in the same sector cheaper, a temporary research subsidy can permanently redirect the economy toward clean technology. For a recent theoretical evaluation of industrial policy welfare in a static setting with external economies of scale, see Bartelme et al. (2019). Buera et al. (2021) quantitatively evaluate the welfare effect of a big push in distorted economies.

[2] A growing empirical literature evaluates Big Push dynamics, including Kline and Moretti (2014) on the long-run effects of the Tennessee Valley Authority, Juhász (2018) on temporary protection during the Napoleonic Blockade,

Our paper contributes to the literature on structural transformation and the role of the state. Ju, Lin, and Wang (2015) propose a theory of endowment-driven structural change, emphasizing that technology adoption follows comparative advantage determined by factor endowments.[3] More broadly, supply-side theories of structural change—such as Acemoglu and Guerrieri (2008), who highlight sectoral differences in capital intensities interacting with capital deepening, and Ngai and Pissarides (2007), who emphasize sectoral differences in productivity growth—show how production-side forces drive sectoral reallocation. Empirical work is supportive. In the context of agriculture, Manuelli and Seshadri (2014) find that U.S. tractor diffusion reflected optimal responses to changes in factor prices rather than adoption frictions,[4] and Boppart et al. (2024) document that capital accumulation drives the transition to intensive agriculture. Our model incorporates this factor-driven force but shows that production externalities create a timing window where decentralized transformation is inefficiently slow, justifying a temporary, "facilitating" role for the state.[5] In this regard, our work complements but differs from Itskhoki and Moll (2019) and Liu and Xu (2025). Those studies characterize how optimal policies should evolve over the development path— reversing signs between early and late stages to manage dynamic pecuniary or production externalities. In contrast, our paper focuses on the interplay between market-driven evolution and latent market failures, emphasizing the timing dimension of industrial policy: when governments should defer to markets versus when strategic acceleration is warranted.

The remainder of the paper is organized as follows. Section 2 develops the static model to elucidate the core mechanism. Section 3 extends the analysis to a dynamic infinite-horizon framework with durable capital. Section 4 applies this framework to agricultural mechanization in China. Section 5 concludes.

2 Static Model

This section develops a parsimonious static framework to elucidate the paper's core mechanism. The model characterizes the tension between market-driven evolution and latent market failures, establishing the theoretical basis for the optimal timing of industrial policy.

and Choi and Levchenko (2025) and Lane (2025) on the long-term comparative advantage shifts resulting from South Korea's heavy industry drive, among others.

[3]See Lin (2012) for the new structural economics framework and Lin, Liu, and Zhang (2023) for some evidence.

[4]For related empirical evidence on how agricultural technology adoption drives structural transformation, see Foster and Rosenzweig (1995) on learning-by-doing in agricultural technology, Bustos, Caprettini, and Ponticelli (2016) on labor-saving agricultural innovation and structural change in Brazil, and Caunedo and Keller (2021) on agricultural capital obsolescence. See also Gollin, Parente, and Rogerson (2007) for the broader macroeconomic role of agricultural productivity in development.

[5]Ju, Lin, and Wang (2011), extending Ju, Lin, and Wang (2015), study industrial upgrading with Marshallian externalities in a three-sector growth model. Their focus is on coordination failures in selecting the correct target industry based on comparative advantage, whereas our paper emphasizes the optimal timing of intervention and applies it to agricultural mechanization in China.

2.1 Environment

Consider an economy with a single service, M, generated by two distinct technologies: a labor-intensive traditional technology (Sector 1) and a capital-intensive new technology (Sector 2). The service is produced by competitive producers and consumed by a representative user. The two technologies are perfect substitutes, such that aggregate service is $M = M_1 + M_2$.

Market Fundamentals. Let Ω denote an aggregate state vector summarizing the market conditions relevant for new technology adoption. Specifically, we assume $\Omega \equiv (\theta, R, w)$, where θ captures demand-side shifts (e.g., evolving consumer preferences for ESG products), R is the rental price of capital (factor price), and w is the wage (factor price).

Demand and Supply. The representative user demands service M according to the demand function

$$M = D(P_M; \theta),$$

where P_M is the service price. Demand is downward sloping ($\partial D/\partial P_M < 0$). We assume $\partial D/\partial \theta > 0$, meaning that demand shifts out as θ increases.

The traditional sector (Sector 1) supplies M_1 according to

$$M_1 = \tilde{S}_1(P_M; w),$$

which is upward sloping ($\partial \tilde{S}_1/\partial P_M > 0$). We assume $\partial \tilde{S}_1/\partial w < 0$. As is standard, the inverse supply function $P_M = \tilde{S}_1^{-1}(M_1; w)$ (corresponding to marginal cost) is homogeneous of degree 1 in the factor price w.

The new-technology sector (Sector 2) supplies M_2 according to

$$M_2 = \tilde{S}_2(P_M; A_2, R),$$

where A_2 denotes the *sectoral-level* total factor productivity (TFP). Supply is upward sloping ($\partial \tilde{S}_2/\partial P_M > 0$), with $\partial \tilde{S}_2/\partial A_2 > 0$ and $\partial \tilde{S}_2/\partial R < 0$. The inverse supply function $P_M = \tilde{S}_2^{-1}(M_2; A_2, R)$ is homogeneous of degree 1 in the factor price R.

Externality. The new-technology sector exhibits a positive externality. The sectoral-level productivity A_2 is endogenous to the *aggregate scale* of adoption, M_2, reflecting learning-by-doing (Arrow, 1962), technological spillovers (Griliches, 1957), or network effects (Katz and Shapiro,

1985). We specify this relationship as

$$A_2 = \Lambda(M_2), \quad \text{with } \Lambda'(\cdot) \geq 0.$$

This setting also captures demand externalities in the spirit of Murphy, Shleifer and Vishny (1989): an increase in aggregate demand leads to an increase in aggregate productivity. Following the literature, we further specify Λ as an "S"-shaped function, namely,

$$\Lambda\left(M_2\right) = A_L + \frac{A_H - A_L}{1 + \exp(-\kappa\left(M_2 - M_2^c\right))}, \tag{1}$$

where $A_H > A_L > 0$, $\kappa > 0$, and M_2^c is a threshold. As $\kappa \to +\infty$, Λ converges to a step function[6]

$$\Lambda\left(M_2\right) = \begin{cases} A_H & \text{if } M_2 \geq M_2^c \\ A_L & \text{if } M_2 < M_2^c \end{cases}. \tag{2}$$

2.2 Characterization of Equilibrium

Market Equilibrium. A decentralized competitive equilibrium is defined by an allocation $\{M_1, M_2, M\}$, a price P_M, and productivity A_2 such that markets clear and the externality constraint is satisfied:

$$\begin{cases} P_M = D^{-1}(M;\theta) & \text{(aggregate demand for } M\text{)} \\ P_M = \tilde{S}_1^{-1}(M_1;w) & \text{(supply of S1)} \\ P_M = \tilde{S}_2^{-1}(M_2;A_2,R) & \text{(supply of S2)} \\ M_1 + M_2 = M & \text{(market clearing)} \\ A_2 = \Lambda(M_2) & \text{(externality)} \end{cases} \tag{3}$$

Individual producers take A_2 as given, even though it is determined by aggregate M_2. The equilibrium is characterized by the equalization of private marginal benefits and private marginal costs across sectors:

$$\underbrace{D^{-1}\left(M;\theta\right)}_{\text{marginal benefit}} = \underbrace{\tilde{S}_1^{-1}\left(M_1;w\right)}_{\text{marginal cost (S1)}} = \underbrace{\tilde{S}_2^{-1}\left(M_2;A_2,R\right)}_{\text{marginal cost (S2)}}. \tag{4}$$

Planner's Problem. A social planner seeks to maximize total surplus, defined as the gross utility of consumption net of the real resource costs of production:

$$\begin{aligned} \max_{(M_1,M_2,M)} \mathcal{W} \;=\; & \int_0^M D^{-1}(m;\theta)\,dm - \left(\int_0^{M_1} \tilde{S}_1^{-1}(m;w)\,dm + \int_0^{M_2} \tilde{S}_2^{-1}(m;A_2,R)\,dm\right) \\ & \text{subject to } M_1 + M_2 = M \text{ and } A_2 = \Lambda\left(M_2\right), \end{aligned} \tag{5}$$

[6] Without loss of generality, we assume that when $M_2 = M_2^c$, $\Lambda\left(M_2\right) = A_H$. We can instead assume that when $M_2 = M_2^c$, $\Lambda\left(M_2\right)$ can be any value belonging to $[A_L, A_H]$, which would not alter the model results generically.

by noting that $D^{-1}(\cdot)$ represents the marginal utility while $\tilde{S}_1^{-1}(\cdot)$ and $\tilde{S}_2^{-1}(\cdot)$ represent the marginal costs. The planner internalizes the dependence of A_2 on M_2. The first-order conditions imply

$$\underbrace{D^{-1}(M;\theta)}_{\text{marginal benefit}} = \underbrace{\tilde{S}_1^{-1}(M_1;w)}_{\text{marginal cost (S1)}} = \underbrace{\tilde{S}_2^{-1}(M_2;A_2,R) + \overbrace{\frac{\partial A_2}{\partial M_2}\int_0^{M_2}\frac{\partial \tilde{S}_2^{-1}(m;A_2,R)}{\partial A_2}dm}^{\textbf{externality }(-)}}_{\textbf{social marginal cost (S2)}}. \quad (6)$$

Compared with the decentralized equilibrium, the social marginal cost of the new technology is lower than the private marginal cost due to the presence of externality.

In what follows, we treat $\Omega \equiv (\theta, R, w)$ as exogenous and analyze the equilibrium under different values of Ω. A sequence of static equilibria under varying Ω can be interpreted as a dynamic equilibrium under an exogenous process for Ω. To deliver sharp analytical results, we first analyze the knife-edge case where $\kappa \to \infty$, and then turn to the smooth case where $\kappa < \infty$. Under $\kappa \to \infty$, the first-order conditions for the planner, (6), are not binding.

2.3 Market Equilibrium

We solve for the market equilibrium, and examine the effect of a decline in R holding w and θ fixed. Lemma 1 follows.

Lemma 1 *Consider the case where $\kappa \to \infty$ (i.e., the step function for $\Lambda(\cdot)$ given in (2)). There exist two thresholds, R^* and R^{**} with $R^* > R^{**}$,* such that:

i) When $R > R^*$, there exists a unique equilibrium ($A_2 = A_L$ and $M_2 < M_2^c$);

ii) When $R^* \geq R > R^{**}$, multiple (two) equilibria exist; one is the bad equilibrium ($A_2 = A_L$) and the other is the good equilibrium ($A_2 = A_H$).

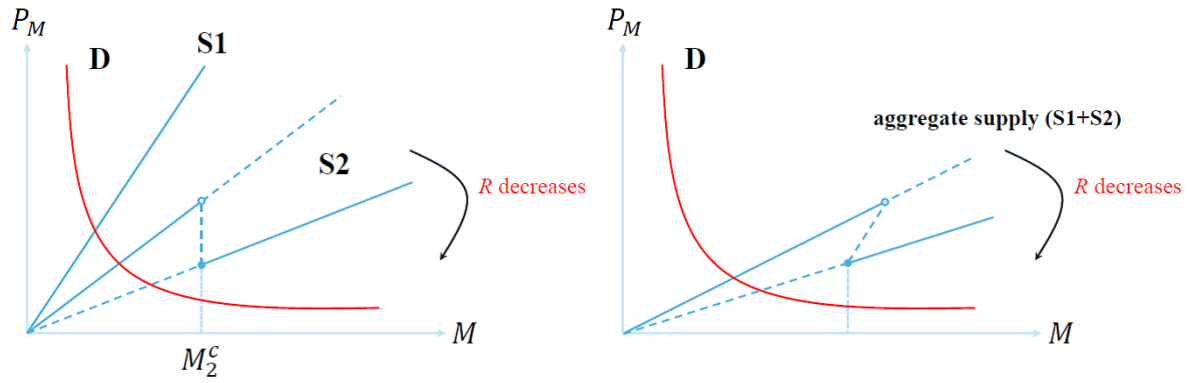
iii) When $R \leq R^{**}$, there exists a unique equilibrium ($A_2 = A_H$ and $M_2 > M_2^c$);

***Proof**: See the Appendix.*

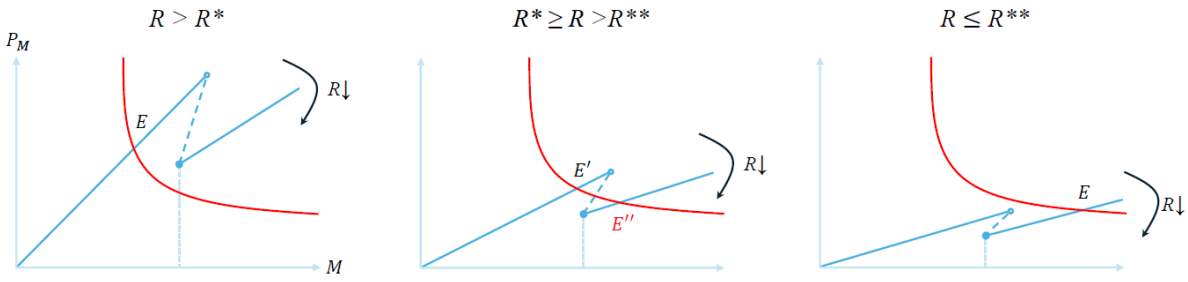
Lemma 1 highlights that $R = R^*$ is the tipping point at which equilibrium multiplicity emerges as R gradually falls from a high to a low level. At this threshold, the high-productivity regime of the new technology (i.e., $A_2 = A_H$) becomes feasible, but it need not be uniquely selected in equilibrium. The cutoff $R = R^{**}$ marks the point where the bad equilibrium ceases to exist and only the good equilibrium prevails.

The intuition behind multiplicity in Lemma 1 mirrors Murphy, Shleifer, and Vishny (1989). When $R^* \geq R > R^{**}$, two self-fulfilling allocations coexist. In one equilibrium, the service price P_M is relatively low, which depresses the traditional sector's incentives to supply. The resulting demand

"gap" must then be filled by greater output from the new sector. Higher adoption raises A_2 through the externality, lowering the new sector's marginal cost and supporting the low price P_M—rendering the outcome (low-price, low-M_1, high-M_2) self-fulfilling. In the other equilibrium, P_M is relatively high, which sustains a larger supply from the traditional sector; the new sector remains small, the externality is not activated, and productivity stays low, validating the outcome (high-price, high-M_1, low-M_2). As in Murphy, Shleifer, and Vishny (1989), the government can implement a "big push" that coordinates private agents' investment and adoption (industrialization) decisions and expands the new-sector activities simultaneously to a scale that activates the externality, thereby achieving the Pareto-efficient high-adoption equilibrium. However, as noted in the introduction, the main focus of our paper is not on equilibrium multiplicity. Figure 1 illustrates Lemma 1.



(a) Aggregate demand curve and aggregate supply curve



(b) Market equilibrium without government subsidy

Figure 1: Market equilibrium

2.4 Planner Solution

Under the *step function* of $A_2 = \Lambda(M_2)$, it is easy to verify that when $R > R^*$, the optimal allocation (M_1, M_2, M) for the planner in Program (5) reduces to one of two candidates: (a) the decentralized equilibrium allocation given in Lemma 1, or (b) a corner allocation, with M_2 being

pushed to the productivity threshold M_2^c, which solves

$$\begin{cases} D^{-1}(M;\theta) = \tilde{S}_1^{-1}(M_1;w) \\ M_1 + M_2 = M \\ M_2 = M_2^c \end{cases} \tag{7}$$

Let $\mathcal{W}_{\text{mkt}}$ and $\mathcal{W}_{\text{push}}$ denote welfare under (a) and (b), respectively, and define

$$\triangle\mathcal{W} = \mathcal{W}_{\text{push}} - \mathcal{W}_{\text{mkt}}.$$

Lemma 2 follows.

Lemma 2 *Consider the case where $\kappa \to \infty$ (i.e., the step function for $\Lambda(\cdot)$ given in (2)). There exists a unique threshold $\hat{R} > R^*$ such that $\triangle\mathcal{W}\left(\hat{R}\right) = 0$ (i.e., satisfying the single-crossing property).*

i) When $R > \hat{R}$, $\triangle\mathcal{W}(R) < 0$, so the planner adopts the market allocation.

ii) When $R^* < R < \hat{R}$, $\triangle\mathcal{W}(R) > 0$, so the planner chooses the pushed allocation that solves (7).

***Proof**: See the Appendix.*

When $R > \hat{R}$, the planner opts for the decentralized outcome. When $R^* < R < \hat{R}$, however, the planner pushes M_2 up to the threshold M_2^c to unlock the positive externality. Figure 2 illustrates the M_2 and the welfare as functions of R under the two candidates of allocations.

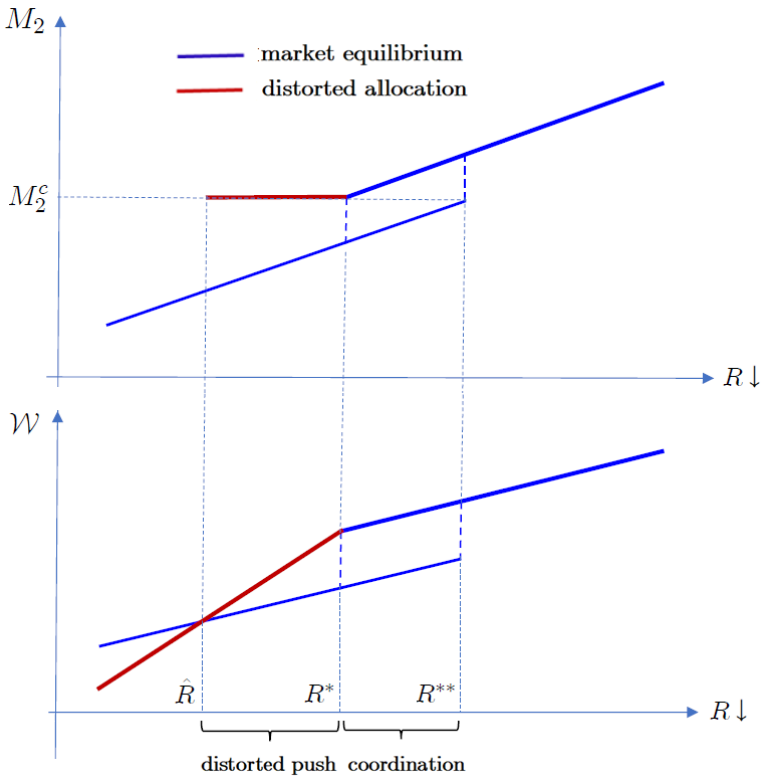


Figure 2: Decentralized market equilibrium allocation versus pushed allocation

Figure 3 illustrates the mechanism behind Lemma 2, by showing the planner's welfare tradeoff between the two options at different levels of R. To keep the graph uncluttered, only the S2 supply is shown (the tradeoff is similar when both S1 and S2 are included). The basic intuition is as follows. Pushing M_2 to M_2^c triggers the productivity jump from A_L to A_H, delivering an externality-driven reduction in the new technology's marginal costs. This push, however, creates a distortion because, at the pushed allocation, *the marginal cost exceeds the marginal benefit*. When R is far above R^*, achieving M_2^c requires a large push, so the resulting distortion is large and outweighs the externality benefit, implying that no intervention is optimal. When R is close to R^*, only a small push is needed; the distortion is small and the externality gain dominates; in the limit as $R \to R^{*+}$, the push becomes a pure coordination exercise without any distortion. In other words, the planner should start to push at a point $R = \hat{R}$, where $\hat{R}$ is neither too high nor too close to R^* from above.

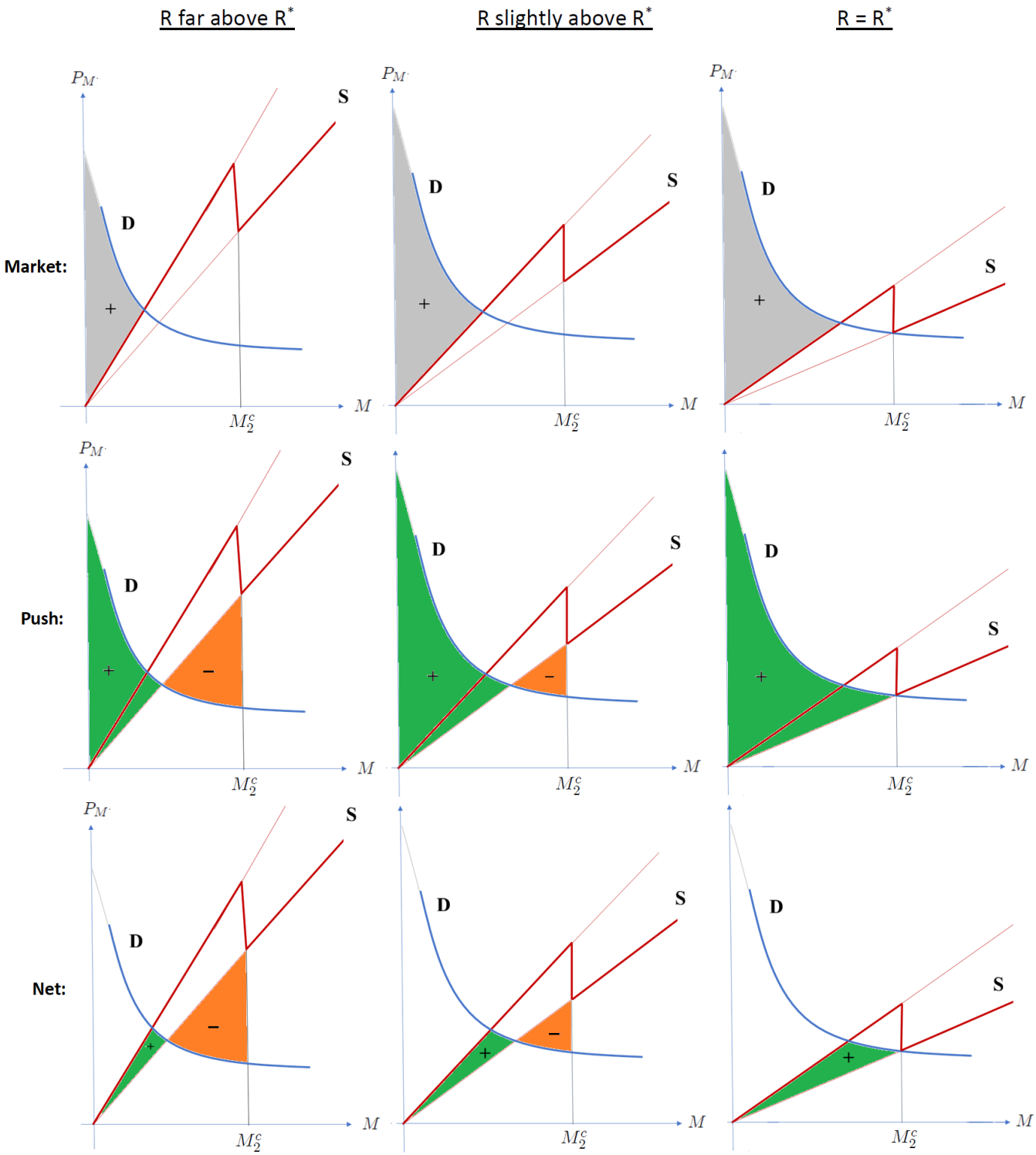


Figure 3: The welfare tradeoff for the planner when using the market equilibrium allocation versus the pushed allocation

2.5 Implementation and Optimal Intervention

The planner can implement the efficient allocation and thereby maximize welfare via the decentralized market using a subsidy/tax. Suppose the government subsidizes the supply of the new technology at rate $\tau \geq 0$. Specifically, suppose the service produced by the new technology is sold at price $\tilde{P}_2$, so consumers pay the effective price $(1-\tau)\tilde{P}_2$, with the government covering the remainder $\tau\tilde{P}_2$. Under this intervention scheme, no-arbitrage implies $P_M = \tilde{P}_1 = (1-\tau)\tilde{P}_2$, where $\tilde{P}_1$ is the selling price of the service produced by the traditional technology. The equilibrium system becomes:

$$\begin{cases} P_M = D^{-1}(M;\theta) & \text{(aggregate demand for } M\text{)} \\ P_M = \tilde{S}_1^{-1}(M_1;w) & \text{(supply of S1)} \\ \frac{P_M}{1-\tau} = \tilde{S}_2^{-1}(M_2;A_2,R) & \text{(supply of S2)} \\ M_1 + M_2 = M & \text{(market clearing)} \\ A_2 = \Lambda(M_2) & \text{(externality)} \end{cases} \tag{8}$$

Under this intervention scheme, the welfare is still given in (5), that is:

$$\mathcal{W} = \int_0^M D^{-1}(m;\theta)\,dm - \left(\int_0^{M_1} \tilde{S}_1^{-1}(m;w)\,dm + \int_0^{M_2} \tilde{S}_2^{-1}(m;A_2,R)\,dm\right). \tag{9}$$

Note that this targeted intervention is effectively a transfer within the economy; it alters the equilibrium allocation (M_1, M_2, M), but does not change the fundamental form of the real resource welfare function given in (5).[7] This welfare function is determined by the demand and supply functions—the underlying real marginal benefits and costs.

Alternatively, the government could subsidize the input (capital) price R for Sector 2. In this case, the supply function of S2 in the equilibrium system given in (3) changes to $P_M = \tilde{S}_2^{-1}(M_2;A_2,(1-\tau)\,R)$, and the welfare function that explicitly accounts for the government's fiscal outlay becomes $\mathcal{W} = \int_0^M D^{-1}(m;\theta)\,dm - \left(\int_0^{M_1} \tilde{S}_1^{-1}(m;w)\,dm + \int_0^{M_2} \tilde{S}_2^{-1}(m;A_2,(1-\tau)\,R)\,dm\right) - B$, where $B = \int_0^{M_2} \tilde{S}_2^{-1}(m;A_2,R)\,dm - \int_0^{M_2} \tilde{S}_2^{-1}(m;A_2,(1-\tau)\,R)\,dm$ is the government's fiscal outlay. Substituting B back into the objective function perfectly recovers the original welfare function in (9). A similar equivalence holds if the government subsidizes to shift the aggregate demand. Proposition 1 follows.

Proposition 1 *Consider the case where $\kappa \to \infty$ (i.e., the step function for $\Lambda(\cdot)$ given in (2)).*

[7] In fact, the welfare function in Equation (9) can be rewritten as:

$$\mathcal{W} = \underbrace{\int_0^M \left(D^{-1}(m;\theta) - P_M\right) dm}_{\text{consumer surplus}} + \underbrace{\int_0^{M_1} \left(P_M - \tilde{S}_1^{-1}(m;w)\right) dm}_{\text{profit of S1}} + \underbrace{\int_0^{M_2} \left(\frac{P_M}{1-\tau} - \tilde{S}_2^{-1}(m;A_2,R)\right) dm}_{\text{profit of S2}} - B$$

where $B = \frac{\tau P_M}{1-\tau} M_2$ is the government's fiscal outlay.

There exists a unique threshold $\hat{R} > R^*$ (given in Lemma 2) such that:

i) When $R \geq \hat{R}$, the optimal policy is no intervention ($\tau = 0$).

ii) When $\hat{R} > R > R^*$, the planner implements the efficient allocation by setting the subsidy rate $\tau = 1 - \frac{R^*}{R}$.[8]

***Proof**: See the Appendix.*

Figure 4 shows how a subsidy can implement the planner's optimal allocation. The intuition behind the optimal subsidy rate being $\tau = 1 - \frac{R^*}{R}$ is as follows. To give the new-technology sector incentives to supply more, the government can either raise the output price or lower the input price. In the absence of additional market frictions, these two approaches are equivalent. As shown in Lemma 1, if the input (capital) price decreases from R to R^*, the quantity of output is $M_2 = M_2^c$. Therefore, to implement this quantity via a subsidy on the output price, the effective output price must increase from P_M to $\frac{P_M}{1-\tau}$ such that $\frac{P_M}{1-\tau}/R = P_M/R^*$ (see (8) where $P_M = \tilde{S}_2^{-1}(M_2; A_2, R)$ is homogeneous of degree 1 in R), which yields $\tau = 1 - \frac{R^*}{R}$.

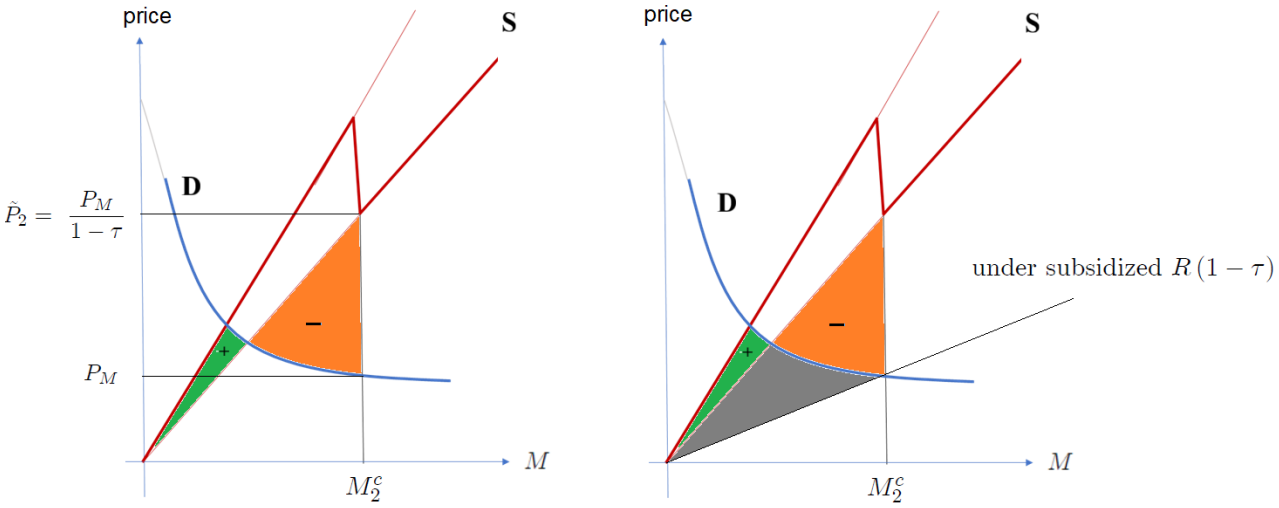


Figure 4: Implementing the planner's allocation (Left: a subsidy to the output price of the new technology; Right: a subsidy to the input cost of the new technology)

Effects of w and θ. The results in Lemmas 1–2 and Proposition 1 hold when the aggregate state vector $\Omega \equiv (\theta, R, w)$ affects either the aggregate demand or one of the sectoral supplies (S1 and S2). So far we have examined the effect of a decline in R holding (w, θ) fixed. The mirror cases—an increase in w holding (R, θ) fixed or an increase in θ holding (R, w) fixed—have the same qualitative implications. Here we only formally state the result for the case of an increase in w.

Corollary 1 ***(R-w duality and comparative statics)*** *i) In Proposition 1, $\hat{R}$ is an increasing*

[8] When $R \leq R^*$, the market achieves the efficient equilibrium ($A_2 = A_H$) through possible coordination without subsidy intervention, so $\tau = 0$.

function of w. ii) A rise in w (holding R and θ fixed) has a similar effect as a fall in R (holding w and θ fixed): there exist two thresholds on w (analogous to the two thresholds $\hat{R}$ and R^* on R) that characterize the optimal intervention window.

***Proof**: See the Appendix.*

Government intervention effectively increases demand for new-technology production and crowds out demand for traditional-technology production. When w rises, the distortionary cost of intervention falls, so the government has a stronger incentive to intervene and will start doing so at a higher level of R. This explains part i). Part ii) formalizes that raising w or lowering R has identical policy implications.

Case of a smooth function of $\Lambda(\cdot)$. We now study the case of a smooth function for $\Lambda(\cdot)$ given in (1), where κ is a finite number. The key insight carries over. The question we study essentially becomes how the planner should choose the optimal subsidy τ to maximize the aggregate welfare $\mathcal{W}$ given in (9), subject to the market constraints given in (8).[9] Under the smooth function of $\Lambda(\cdot)$, the first-order derivative is well defined everywhere in τ, allowing us to write

$$\begin{aligned}\frac{\partial \mathcal{W}(\tau;R)}{\partial \tau} &= \underbrace{\left[\frac{\partial M}{\partial \tau} D^{-1}(M;\theta) - \frac{\partial M_1}{\partial \tau}\tilde{S}_1^{-1}(M_1;w) - \frac{\partial M_2}{\partial \tau}\tilde{S}_2^{-1}(M_2;A_2,R)\right]}_{\text{deadweight loss due to market distortion}} \\ &\quad + \underbrace{\frac{\partial M_2}{\partial \tau}\frac{\partial A_2}{\partial M_2}\int_0^{M_2}\frac{\partial \tilde{S}_2^{-1}(m;A_2,R)}{\partial A_2}dm}_{\text{productivity gain from externality}}.\end{aligned}$$

Due to the S-shaped nature of the function $\Lambda(M_2)$, $\frac{\partial A_2}{\partial M_2} = \frac{d\Lambda(M_2)}{dM_2}$ is positive but small when M_2 is far below M_2^c. We assume that implementing a government subsidy scheme incurs *a small fixed regulatory cost*, such as administrative expenses. This cost is fixed and positive, regardless of the size of the subsidy rate. This means that the government faces a participation condition: the welfare gain from the subsidy scheme must exceed the fixed regulatory cost. With this additional assumption, the result that there exists a threshold $\hat{R}$ at which intervention should begin still holds. Proposition 2 follows.

Proposition 2 *Consider the case where κ is finite (i.e., a smooth function for $\Lambda(\cdot)$) and assume that implementing a government subsidy scheme incurs **a small fixed regulatory cost** c_0. There exists a unique threshold, $\hat{R}$, above which it is not optimal for the government to intervene with subsidy, and below which intervention is optimal.*

***Proof**: See the Appendix.*

[9] It is easy to show that the planner's choosing τ is equivalent to choosing M_2.

Figure 5 illustrates the case of a smooth function of $\Lambda(\cdot)$ using a numerical example. Observe that the optimal subsidy τ^* exhibits a *discontinuous jump* under a sufficiently high κ (see the left and middle panels of Figure 5). In fact, based on the planner problem in (5), we can express welfare $\mathcal{W}$ as a function of M_2. This function is continuous and displays *two local maxima* when κ is sufficiently large: one to the left of the threshold $M_2 = M_2^c$, and the other to the right. As R decreases to a certain point, the global maximum of $\mathcal{W}$ switches abruptly from the left peak to the right peak. This regime change corresponds to a discrete jump in the optimal allocation M_2 and, equivalently, in the optimal subsidy τ^*, illustrated in Figure 6.

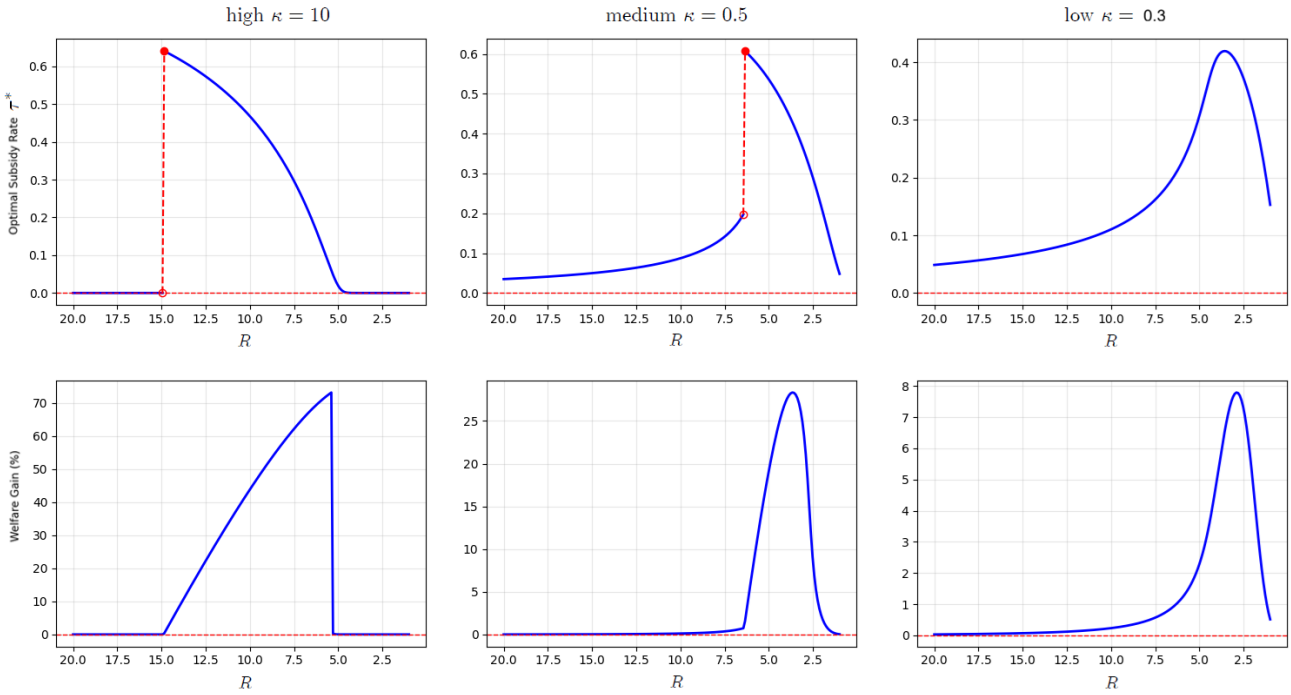


Figure 5: Optimal subsidy and welfare under a smooth function of $\Lambda(\cdot)$

Note: This figure provides a numerical example for the case of a smooth function of $\Lambda(\cdot)$, where we specify $P_M = 20/M$, $P_M = 2wM_1$ with $w = 1$, $P_M = 2RA_2^{-2}M_2$, $A_H = 4.5$, $A_L = 1.5$, and $M_2^c = 5$. The upper panel shows the optimal subsidy τ^*, required to maximize welfare $W(\tau; R)$, as a function of R under different levels of κ. The lower panel shows the corresponding welfare gain, calculated as $\frac{W(\tau=\tau^*;R)-W(\tau=0;R)}{W(\tau=0;R)}$ (without considering the fixed regulatory cost), where $W(\tau = 0; R)$ corresponds to the good market equilibrium benchmark if multiple market equilibria exist.

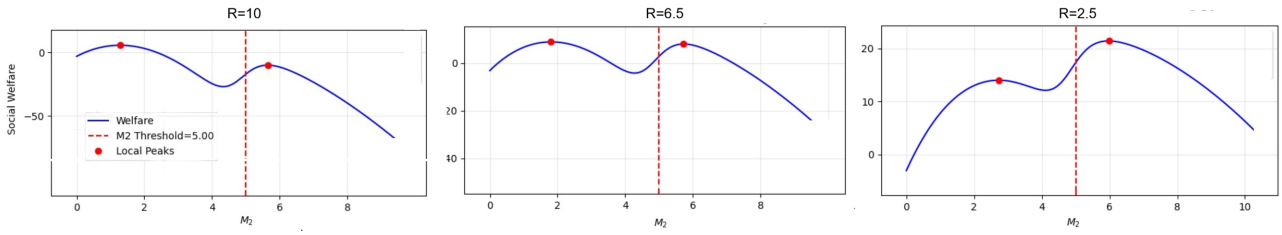


Figure 6: Two-peak function of $\mathcal{W}(M_2)$ under a sufficiently high κ

3 Dynamic Model

While the static model elucidates the core timing mechanism, real-world technology adoption often involves *durable capital goods* (e.g., machinery, infrastructure, software platforms) rather than one-period inputs. This section extends our framework to incorporate this feature, establishing the optimal timing of intervention in a dynamic environment. The dynamic model can admit multiple competitive equilibrium paths and multiple steady states, with new implications.

3.1 Environment

Time is discrete and infinite: $t = 0, 1, 2, \ldots$. A homogeneous service M_t is produced using two perfectly substitutable technologies. Sector 1 uses a non-durable flow input, while Sector 2 uses a durable capital stock. All markets are competitive.

3.1.1 Service Demand and Production

A representative downstream user demands services according to the demand function

$$M_t = D(P_{M,t}), \qquad D'(\cdot) < 0,$$

where $P_{M,t}$ is the service price.[10] This demand function, corresponding to the marginal utility/profit, is derived endogenously in the appendix.

Aggregate service is the sum of the two technology-specific services:

$$M_t = M_{1,t} + M_{2,t}. \tag{10}$$

Services are produced from intermediate inputs $Y_{1,t}$ and $Y_{2,t}$ via

$$M_{1,t} = A_1 Y_{1,t}, \qquad M_{2,t} = A_{2,t} Y_{2,t}, \tag{11}$$

[10] We do not consider the demand shift here, so normalize θ in the static model to 1 here.

where $A_1 > 0$ is constant productivity of the incumbent technology and $A_{2,t}$ is endogenously determined productivity of the new technology. The profit maximization of service producers, $\max_{Y_{1,t},Y_{2,t}} P_{M,t}M_t - P_{1,t}Y_{1,t} - P_{2,t}Y_{2,t}$, implies

$$P_{1,t} = P_{M,t}A_1, \qquad P_{2,t} = P_{M,t}A_{2,t}, \tag{12}$$

where $P_{1,t}$ and $P_{2,t}$ are the prices of the intermediate inputs $Y_{1,t}$ and $Y_{2,t}$, respectively.

3.1.2 Traditional sector (flow input)

A representative producer in the traditional sector (Sector 1) uses a labor-intensive technology to produce the intermediate goods $Y_{1,t}$, supplying it according to an upward-sloping schedule:

$$Y_{1,t} = S_1(P_{1,t}; w_t), \ \ \frac{\partial S_1}{\partial P_{1,t}} > 0, \ \ \frac{\partial S_1}{\partial w_t} < 0, \tag{13}$$

where $P_{1,t}$ is the price of $Y_{1,t}$ and w_t is wage. This supply function, corresponding to the marginal cost of production, is derived endogenously in the appendix. We assume that w_t is an exogenous process and increases over time with $\lim_{t\to+\infty} w_t = \bar{w} < \infty$.

3.1.3 New sector (durable capital)

Sector 2 produces the intermediate good $Y_{2,t}$ using durable capital capacity. Let $K_{2,t}$ denote the beginning-of-period stock of durables and let $I_{2,t}$ denote new investment installed within period t. With zero time-to-build, investment is immediately productive, so

$$Y_{2,t} = K_{2,t} + I_{2,t}. \tag{14}$$

The law of motion for the durable stock is

$$K_{2,t+1} = (1 - \delta_2)(K_{2,t} + I_{2,t}), \tag{15}$$

where $\delta_2 \in (0, 1)$ is the depreciation rate. The initial state $K_{2,0}$ is given.

Asset pricing. Competitive owners of Sector-2 capital buy investment at the asset price $Q_{2,t}$ and sell the intermediate goods $Y_{2,t}$ at the flow price $P_{2,t}$. Let $V_t(K_{2,t})$ be the value of a representative capital owner. The investment problem is

$$V_t(K_{2,t}) = \max_{I_{2,t}\geq 0} \{P_{2,t}(K_{2,t} + I_{2,t}) - Q_{2,t}I_{2,t} + \beta_t \mathbb{E}_t[V_{t+1}(K_{2,t+1})]\}$$

subject to (15), where β_t is the (exogenous) intertemporal discount rate. The above Bellman equation yields the standard no-arbitrage condition, which links the asset price to the flow payoff ("dividend") and the expected future resale value:

$$Q_{2,t} = P_{2,t} + \beta_t (1 - \delta_2) \mathbb{E}_t [Q_{2,t+1}]. \tag{16}$$

Investment supply. Investment goods are produced by a capital-intensive technology. The aggregate supply ffunction is

$$I_{2,t} = S_I(Q_{2,t}; R_t, z_t) \tag{17}$$

with

$$\frac{\partial S_I}{\partial Q_{2,t}} > 0, \qquad \frac{\partial S_I}{\partial R_t} < 0, \qquad \frac{\partial S_I}{\partial z_t} > 0. \tag{18}$$

where R_t is the rental price of capital and z_t is productivity in upstream investment-goods production. This production function corresponds to the marginal cost of production; a microfoundation is that a representative upstream firm with a convex cost function $C(I_{2,t}; R_t, z_t)$ so that $Q_{2,t} = C_I(I_{2,t}; R_t, z_t)$ (equivalently $I_{2,t} = S_I(Q_{2,t}; R_t, z_t)$). We assume that R_t is an exogenous process and decreases over time with $\lim_{t \to +\infty} R_t = \underline{R} > 0$.

3.1.4 Externality

We assume that the adoption of the new technology exhibits a positive externality. Specifically, productivity of the new technology depends on the contemporaneous scale of adoption:

$$A_{2,t} = \Lambda(M_{2,t}), \qquad \Lambda'(\cdot) \geq 0. \tag{19}$$

We still specify that $\Lambda(\cdot)$ has an S-shaped form as in (1). Individual agents take the path $\{A_{2,t}\}_{t \geq 0}$ as given.

We may also consider an additional externality: a positive externality in manufacturing investment goods, i.e., z_t increases with the aggregate $I_{2,t}$. As this externality has a similar effect in spirit as the externality on $A_{2,t}$, we abstract it away to simplify the analysis.

3.2 Charaterzation of Dynamic Equilibrium

3.2.1 Competitive Equilibrium (CE)

A *Dynamic Competitive Rational Expectations Equilibrium* (DCREE) is a sequence of quantities $\{M_t, M_{1,t}, M_{2,t}, Y_{1,t}, Y_{2,t}, K_{2,t}, I_{2,t}\}_{t \geq 0}$ and prices $\{P_{M,t}, P_{1,t}, P_{2,t}, Q_{2,t}\}_{t \geq 0}$ given the exogenous processes $\{w_t, R_t, \beta_t, z_t\}_{t \geq 0}$ such that, for all t:

1. *Market clearing*: $M_t = D(P_{M,t})$ and $M_t = M_{1,t} + M_{2,t}$.

2. *Service production*: $M_{1,t} = A_1 Y_{1,t}$ and $M_{2,t} = A_{2,t} Y_{2,t}$.

3. *Intermediate input supply*: $Y_{1,t} = S_1(P_{1,t}; w_t)$; $I_{2,t} = S_I(Q_{2,t}; R_t, z_t)$; $Y_{2,t} = K_{2,t} + I_{2,t}$.

4. *Durable capital accumulation*: $K_{2,t+1} = (1 - \delta_2)(K_{2,t} + I_{2,t})$.

5. *Pricing*: $P_{1,t} = P_{M,t} A_1$ and $P_{2,t} = P_{M,t} A_{2,t}$.

6. *No-arbitrage*: $Q_{2,t} = P_{2,t} + \beta_t (1 - \delta_2) \mathbb{E}_t [Q_{2,t+1}]$.

7. *Externality consistency*: $A_{2,t} = \Lambda(M_{2,t})$.

In CE, individual agents take the path $\{A_{2,t}\}_{t \geq 0}$ as given. Because there is no uncertainty in the model, the rational-expectations equilibrium coincides with a *perfect-foresight competitive equilibrium*: given the exogenous processes, agents correctly anticipate all future endogenous variables.

3.2.2 Planner Problem (SP)

A social planner seeks to maximize total surplus, defined as the gross utility of consumption net of the real resource costs of production. Let $W_t(K_{2,t})$ denote the planner's value function. The planner chooses $(Y_{1,t}, I_{2,t})$ to solve

$$W_t(K_{2,t}) = \max_{Y_{1,t}, I_{2,t} \geq 0} \left\{ \int_0^{M_{1,t}+M_{2,t}} D^{-1}(m) dm - \left(\begin{array}{l} \int_0^{Y_{1,t}} S_1^{-1}(y; w_t)\, dy \\ + \int_0^{I_{2,t}} S_I^{-1}(x; R_t, z_t)\, dx \end{array} \right) + \beta_t \mathbb{E}_t \left(W_{t+1}(K_{2,t+1}) \right) \right\} \tag{20}$$

subject to

$$\begin{aligned} & M_{1,t} = A_1 Y_{1,t} \\ & M_{2,t} = A_{2,t} (K_{2,t} + I_{2,t}),\ A_{2,t} = \Lambda (M_{2,t}) \\ & K_{2,t+1} = (1 - \delta_2) (K_{2,t} + I_{2,t}). \end{aligned}$$

The planner uses two inputs with marginal costs $S_1^{-1}(\cdot)$ and $S_I^{-1}(\cdot)$ to generate the flow of one output with marginal utility $D^{-1}(\cdot)$.

Wedge between SP and CE. We find the planner's Euler equation:

$$\underbrace{S_I^{-1}(I_{2,t}; R_t, z_t)}_{=Q_{2,t}} = \frac{1}{1 - \underbrace{\Lambda'(M_{2,t})(K_{2,t} + I_{2,t})}_{\textbf{externality}}} \underbrace{A_{2,t} D^{-1}(M_{1,t} + M_{2,t})}_{=P_{2,t}} + \beta_t (1 - \delta_2) \underbrace{S_I^{-1}(I_{2,t+1}; R_{t+1}, z_{t+1})}_{=Q_{2,t+1}}. \tag{21}$$

The planner's Euler equation differs from the competitive Euler equation by an externality term that captures the marginal social benefit of raising adoption (because higher $M_{2,t}$ increases $A_{2,t}$).

3.3 Equilibrium Solution

We analytically characterize the dynamic paths for CE and SP in the limiting case of a step-function externality ($\kappa \to \infty$), which delivers a clear and transparent insight into the comparison between the CE and SP paths. The equilibrium for the general case with finite κ is solved numerically in the quantitative exercise of the next section.

Proposition 3 (Dynamic equilibrium) *Consider the case where $\kappa \to \infty$ (i.e., the step function of $\Lambda(\cdot)$ given in (2)). Assume the exogenous processes $\{w_t, R_t\}_{t\geq 0}$ converge to constants:*

$$\lim_{t\to\infty} w_t = \bar{w}, \qquad \lim_{t\to\infty} R_t = \underline{R} > 0,$$

and $\beta_t = \beta > 0$, $z_t = z > 0$ for all t. Let the initial capital stock $K_{2,0}$ be sufficiently low (specified in the proof).

*(**Transitional path**) The SP admits a unique transitional path, while the CE admits a continuum of equilibrium paths bounded by a "best" path (earliest reaching $M_{2,t} = M_2^c$) and a "worst" path (latest reaching $M_{2,t} = M_2^c$). Even the best CE path is less efficient than the SP, reaching the threshold $M_{2,t} = M_2^c$ later than the SP.*

*(**Steady states**) If $\underline{R}$ is sufficiently small, both the CE and the SP converge to a unique common high steady state ($A_2 = A_H$). If $\underline{R}$ is in an intermediate range, the SP converges to a unique high steady state ($A_2 = A_H$); the best CE exhibits two steady states ($A_2 = A_H$ and $A_2 = A_L$), while the worst CE converges to a unique low steady state ($A_2 = A_L$).*

***Proof**: See the Appendix.*

Figure 7 illustrates the dynamic equilibrium paths. The nature of the inefficiency depends on the level of $\underline{R}$. When $\underline{R}$ is sufficiently low, the inefficiency is purely transitional: the decentralized economy eventually reaches the high-productivity steady state, but does so at a lower speed relative to the social optimum. In contrast, when $\underline{R}$ lies in the intermediate range, the inefficiency extends to the long-run outcome: self-fulfilling expectations can trap the decentralized economy in the low-productivity steady state, whereas the social planner always achieves the high-productivity outcome.

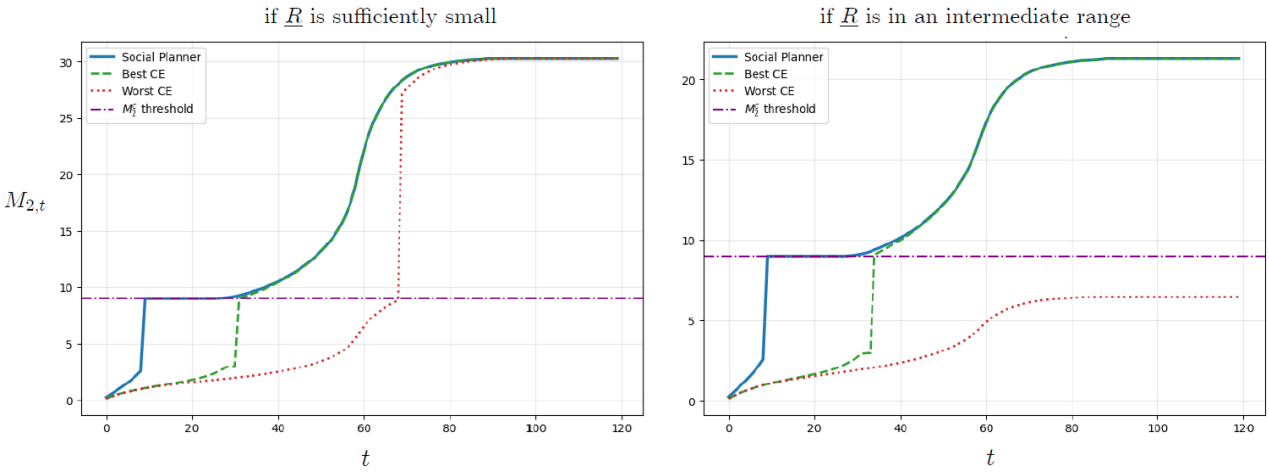


Figure 7: Dynamic equilibrium paths of CE and SP

More concretely, the competitive equilibrium (CE) is inefficient relative to the social planner (SP) along two dimensions, paralleling the static model. First, the CE is subject to coordination failures and equilibrium multiplicity. If $\underline{R}$ is in an intermediate range, the best CE and the worst CE can converge to different steady states; in this case, decentralized expectations can lead to a bad CE path, trapping the economy in a low-adoption, low-productivity steady state. A temporary intervention that coordinates expectations onto the best CE path can therefore has a long-run effect, lifting the economy to the high steady state. If $\underline{R}$ is sufficiently small, the long-run steady state is unique (high); however, a continuum of transitional CE paths implies that inefficient, delayed adoption trajectories can still arise.

Second, even absent coordination failures and equilibrium multiplicity (i.e., assuming the best CE is selected), the best CE transition is still less efficient than the SP because it crosses the threshold $M_{2,t} = M_2^c$ later than is socially optimal. This inefficiency arises because private agents do not internalize the positive adoption externality. The social planner, by contrast, optimally pushes adoption to reach the threshold M_2^c earlier and unlock the productivity gain A_H—a phase that corresponds to the corner solution "flat" segment shown in Figure 7.

3.4 Implementation and Optimal Intervention

We now study two policy instruments that can implement the SP allocation: a subsidy on the intermediate input price $P_{2,t}$ and a subsidy on the durable capital price $Q_{2,t}$.

Subsidizing intermediate inputs. Suppose the government provides a subsidy at rate τ_t on purchasing the intermediate input $Y_{2,t}$; that is, service providers only need to pay the price

$(1-\tau_t)\,P_{2,t}$. The profit maximization problem for service providers then becomes

$$\max_{Y_{1,t},\,Y_{2,t}} P_{M,t}M_t - P_{1,t}Y_{1,t} - (1-\tau_t)\,P_{2,t}Y_{2,t} - T_t,$$

where $T_t = \tau_t P_{2,t} Y_{2,t}$ is a lump-sum tax levied on service providers at time t, and the government budget constraint is balanced period by period. This implies $(1-\tau_t)\,P_{2,t} = P_{M,t}A_{2,t}$; that is, the effective price received by capital owners is

$$P_{2,t} = \frac{P_{M,t}A_{2,t}}{1-\tau_t}.$$

The CE Euler equation remains $Q_{2,t} = P_{2,t} + \beta_t\,(1-\delta_2)\,\mathbb{E}_t\,[Q_{2,t+1}]$. Substituting $P_{2,t} = P_{M,t}A_{2,t}/\,(1-\tau_t)$ yields

$$Q_{2,t} = \frac{P_{M,t}A_{2,t}}{1-\tau_t} + \beta_t\,(1-\delta_2)\,\mathbb{E}_t\,[Q_{2,t+1}]\,.$$

Comparing this with the SP Euler equation (21), we see that, to implement the SP allocation, the subsidy must satisfy

$$\tau_t = \Lambda^{'}\,(M_{2,t})\,(K_{2,t}+I_{2,t})\Big|_{\mathrm{SP}}\,.$$

In the step-function limit ($\kappa \to \infty$), the implementation becomes particularly transparent. As we show in the Appendix, the SP Euler equation (21) is not always binding. There exist two time thresholds, t_1^* and t_2^*, such that:

- For $t < t_1^*$, the SP Euler equation binds and the optimal subsidy is $\tau_t = 0$.
- For $t \in (t_1^*, t_2^*)$, the SP Euler equation is slack; the SP chooses $I_{2,t}$ as a corner solution that holds $M_{2,t} = M_2^c$. In this phase, a unique positive subsidy $\tau_t > 0$ is required.
- For $t > t_2^*$, the SP Euler equation binds again and $\tau_t = 0$.

Consequently, the optimal policy entails a unique intervention window $t \in (t_1^*, t_2^*)$.

Subsidizing capital purchases. An alternative implementation is to subsidize the purchase of new durable capital. Suppose the government subsidizes purchases of new durable capital at rate s_t; capital owners pay only $(1-s_t)Q_{2,t}$ per unit of new capital. By no arbitrage, the resale price of existing capital must also be adjusted by the same subsidy rate, so that the effective price of capital in period t is $(1-s_t)Q_{2,t}$ for both new and existing units.

Under this intervention scheme, the capital owner's Bellman equation becomes

$$V_t(K_{2,t}) = \max_{I_{2,t}\geq 0}\left\{P_{2,t}(K_{2,t}+I_{2,t}) - (1-s_t)Q_{2,t}I_{2,t} - T_t + \beta_t\mathbb{E}_t[V_{t+1}(K_{2,t+1})]\right\},$$

where $T_t = s_t Q_{2,t} I_{2,t}$ is a lump-sum tax that finances the subsidy. The resulting Euler equation is

$$(1 - s_t)\, Q_{2,t} = \; P_{2,t} + (1 - s_{t+1})\, \beta_t\, (1 - \delta_2)\, Q_{2,t+1}.$$

To implement the SP allocation given by (21), the subsidy sequence $\{\tau_t\}$ must satisfy, for every t,

$$(1 - s_t)\, S_I^{-1}\left(I_{2,t}; R_t, z_t\right) - \; \left[A_{2,t} D^{-1}(M_{1,t} + M_{2,t}) + (1 - s_{t+1})\, \beta_t\, (1 - \delta_2)\, S_I^{-1}\left(I_{2,t+1}; R_{t+1}, z_{t+1}\right)\right]\Big|_{\text{SP path}} = 0. \tag{22}$$

Again, under the step-function externality, the SP Euler equation is slack during the corner phase $t \in (t_1^*, t_2^*)$. As we show in the Appendix, the optimal sequence $\{s_t\}$ is hump-shaped:

- For $t < t_1^*$ (when the SP Euler equation binds), the subsidy is strictly positive and increasing over time.
- For $t \in (t_1^*, t_2^*)$ (when the SP Euler equation is slack), it remains positive and relatively high.
- For $t > t_2^*$ (when the SP Euler equation binds again), the subsidy drops to zero.

Although the SP Euler equation is binding for $t < t_1^*$, the subsidy is strictly positive and increasing over time during this phase. Intuitively, investment today contributes to the future capital stock that will trigger the externality; therefore, a positive subsidy is justified even at this stage. However, depreciation ($\delta_2 > 0$) attenuates the contribution of early investment, with a "discounted accumulation" effect. It is therefore inefficient to invest too much too early; the optimal subsidy starts small and increases gradually as the economy approaches t_1^*.

If a small fixed regulatory cost $c_0 > 0$ is incurred for each period of active subsidization, the government will initiate the subsidy only when the incremental gain in welfare exceeds this cost. Consequently, an optimal intervention window exists.

Proposition 4 *Consider the case where $\kappa \to \infty$ (i.e., the step function of $\Lambda(\cdot)$ given in (2)).*

i) Subsidizing the intermediate input price $P_{2,t}$ can implement the SP allocation. The optimal policy entails a unique intervention window $t \in (t_1^*, t_2^*)$.

ii) Subsidizing the durable capital price $Q_{2,t}$ can also implement the SP allocation. Assuming that implementing a government subsidy scheme incurs ***a small fixed regulatory cost*** *c_0 per period of active subsidization, the optimal policy entails a unique intervention window $t \in (t_{start}, t_2^*)$, where $t_{start} \in (0, t_1^*)$.*

Proof*: See the Appendix.*

4 Quantitative Application

This section applies the dynamic framework to China's agricultural mechanization from 1979 to 2020. Our quantitative analysis pursues two main objectives. First, we assess whether the calibrated model can replicate the key empirical regularities of the transition, specifically the observed evolution of tractor and draft animal stocks over this period. Second, we use the model to decompose the relative contributions of market forces—primarily the declining capital-to-labor price ratio—and government intervention (the 2004 subsidy program) to the observed path of mechanization. We quantify the welfare effects of the actual policy and conduct counterfactual experiments to evaluate alternative intervention timings.

4.1 Institutional Background

China's transition from animal-powered to mechanized agriculture between 1979 and 2020 provides a compelling setting for studying technology adoption in the presence of externalities. The transition unfolded in two broad phases: a first phase (1979–2003) in which market forces drove a gradual and incomplete shift away from draft animals, and a second phase (2004–2020) in which central-government subsidy intervention accelerated adoption and reshaped the domestic machinery industry. Figure 8 summarizes the key time series; Figure 9 presents suggestive evidence on productivity and resilience outcomes.

Rural reform, fragmentation, and the persistence of draft animals. The Household Responsibility System (HRS; 家庭年产承包责任制), introduced in 1979, dismantled collective farming and redistributed land-use rights to individual households (Lin, 1992). While the HRS dramatically improved agricultural incentives, it fragmented arable land into small, scattered plots, limiting the scope for indivisible machinery (Chen and Lan, 2020). Draft animals—divisible, suited to small plots, and low in upfront cost—became the natural technology choice. As Figure 8 shows, the total number of tractors increased steadily throughout the 1980s, but this growth was driven almost entirely by small tractors; the stock of medium and large tractors stagnated or even declined during this period, reflecting the limited suitability of large machinery for fragmented smallholder farms (China National Bureau of Statistics, various years). The opportunity cost of capital remained high during this period (Bai et al., 2006), making it unattractive to tie up scarce capital in farm equipment when returns in industry and commerce were far greater. Over the subsequent decade, the relative price environment shifted gradually: quality-adjusted machinery costs declined due to productivity improvements in the domestic agricultural machinery industry, while rural wages began to rise as off-farm employment opportunities expanded (Zhang, 2021; National Bureau of Statistics, 2004). Draft animal stocks peaked around the mid-1990s and began a secular decline, the diffusion of large tractors remained constrained by fragmented plots and the absence of thick

service networks.

The early 2000s and the rationale for intervention. Around China's WTO accession in 2001, expanding urban labor demand reinforced a wave of rural-to-urban migration already underway, raising the shadow wage in agriculture and reducing the labor available for draft animal husbandry. Simultaneously, the Grain-for-Green Program (退耕还林工程), launched in 1999, converted substantial marginal cropland to forest and grassland. These forces, together with broader structural adjustment, contributed to a sustained decline in grain output from roughly 512 million tons in 1998 to approximately 431 million tons in 2003 (China National Bureau of Statistics, 2004), heightening food security concerns among policymakers.

The 2004 subsidy and its design features. The central government responded in 2004 with a landmark farm machinery purchase subsidy, formally established under the Law of the People's Republic of China on Promotion of Agricultural Mechanization (中华人民共和国农业机械化促进法, promulgated June 25, 2004) and implemented through the Interim Measures for the Administration of Special Funds for Agricultural Machinery Purchase Subsidies (农业机械购置补贴专项资金使用管理暂行办法), jointly issued by the Ministry of Finance and Ministry of Agriculture. The policy was initially piloted in 66 grain-producing counties across 16 provinces in 2004, and was progressively expanded to cover 500 counties in 2005, 1,080 counties in 2006, 1,500 counties in 2007, and finally all agricultural counties by 2008 (Zhang, 2021). The government document explicitly states the dual objectives of the policy: supporting agricultural production and fostering agricultural machinery industrial upgrading. The subsidy rate was typically capped at 30 percent of the retail price. The effective subsidy rate shown in Figure 8 is calculated as total national subsidy expenditure divided by total national agricultural machinery purchase expenditure, following Zhang (2021). A critical design feature was the subsidy catalog (补贴目录), which restricted eligibility to machinery meeting specified quality, safety, and environmental standards. Notably, from the policy's inception, the catalog focused primarily on medium and large agricultural machinery; for example, the 2005 catalog explicitly included "large and medium-sized tractors" as a key category, while small tractors were often excluded or received lower priority (Zhang, 2021). This channeled competitive incentives toward firms capable of meeting formal standards and scaling production.

Post-subsidy patterns: production and use-side externalities. On the production side, expanded subsidized demand allowed domestic manufacturers to achieve greater scale. Industry data show that the number of agricultural machinery enterprises above designated size (规模以上企业) soared after 2004 (Zhang et al., 2018). The catalog system created incentives for quality improvement and R&D; empirical studies document significant increases in both profitability and patenting activity among agricultural machinery enterprises after 2004 (Zhang et al., 2018; Zhou et al., 2016). On the use side, rising machinery density generated important adoption complementarities. Thicker markets for repair, maintenance, operator training, and cross-regional custom harvesting services

(跨区作业) reduced the effective cost of machine use for all farmers. Land consolidation through farmland transfer markets, facilitated by mechanization, further improved machine efficiency (Guan et al., 2025). Zhang (2019) estimate that the subsidy policy contributed 40.41 percent to the growth ofgricultural machinery stocks and 18.27 percent to the growth of agricultural mechanization levels over 2008-2015.

Figure 9 shows patterns consistent with these mechanisms. Grain output per unit of agricultural labor rose markedly after the mid-2000s, reflecting automation and labor efficiency gains. The share of disaster-affected sown area declined, suggesting that timely mechanized operations improved crop resilience through deeper plowing and rapid harvesting. By 2017, China's agricultural mechanization rate—defined as the area operated by machinery divided by total planting area—had reached 66.26 percent, nearly double the 2004 level of 34.32 percent (Ministry of Agriculture and Rural Affairs in China, 2018). These patterns, while suggestive rather than causal, motivate the structural identification strategy developed in subsequent sections.

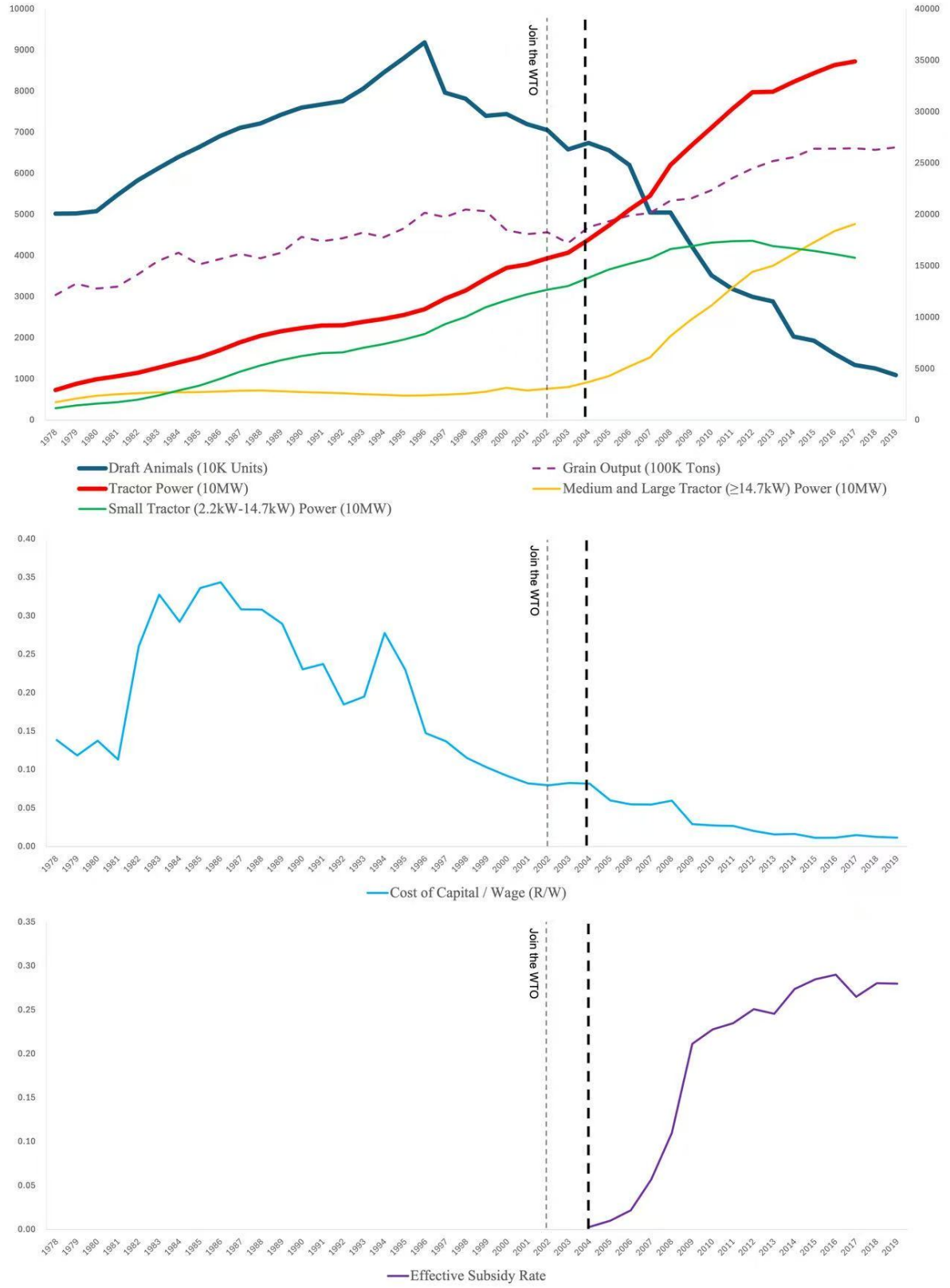


Figure 8: Key time series of China's agricultural mechanization

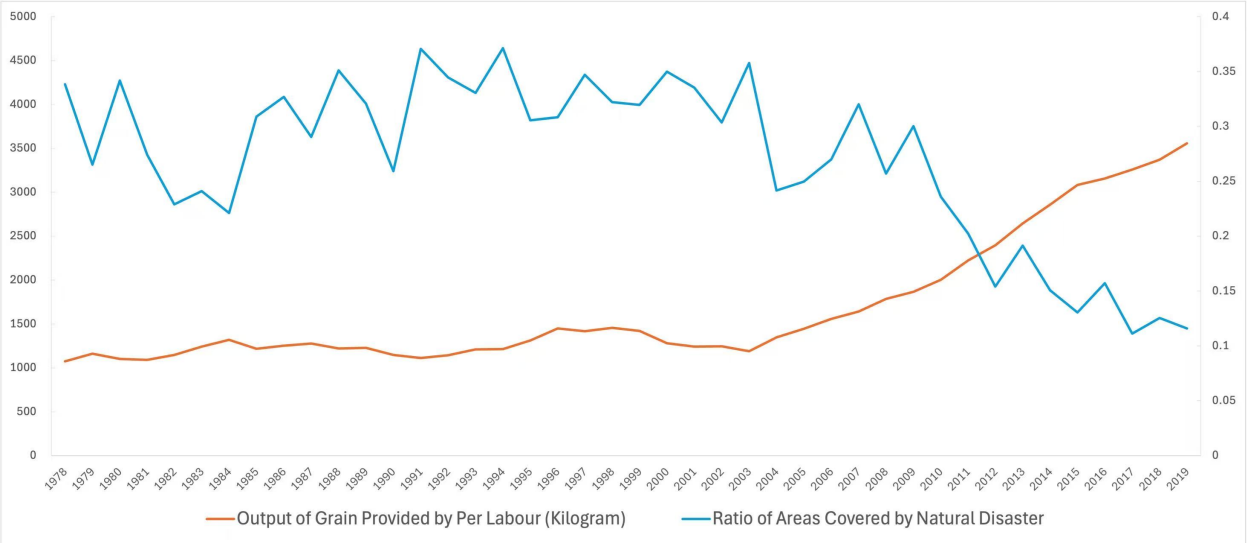


Figure 9: Productivity and resilience in agriculture

4.2 Calibration

This subsection describes the calibration strategy and parameter identification. We specify constant-elasticity functional forms that maintain a transparent mapping between empirical moments and model parameters while preserving tractability.

4.2.1 Functional forms and data

Our dynamic model implies constant-elasticity specifications for tractor supply, animal-powered services supply, and demand for plowing services.

The inverse supply curve for new tractors corresponds to the investment supply function $I_{2,t} = S_I(Q_{2,t}; R_t, z_t)$ in the model. Recall that $I_{2,t}$ denotes new investment in tractors and $Q_{2,t}$ is the asset price (quality-adjusted tractor price). We specify

$$Q_{2,t} = b\, R_t (1 - s_t)\, I_{2,t}^{\gamma}, \tag{23}$$

where R_t captures time variation in the capital cost (i.e., the upstream cost shifter in tractor production) and s_t is the subsidy rate. The parameter γ governs the inverse supply elasticity, and b is a scale parameter. We treat R_t as a time shifter disciplined by the quality-adjusted tractor price series and normalize its long-run level to $\underline{R} = 1$.

The inverse supply curve for animal-powered services corresponds to the traditional sector

supply $Y_{1,t} = S_1(P_{1,t}; w_t)$ in the model. We specify

$$P_{1,t} = c\, Y_{1,t}^{\alpha}, \tag{24}$$

where $Y_{1,t}$ denotes the stock of draft animals providing services and $P_{1,t}$ is the price of animal services. The parameter α governs the inverse supply elasticity, and c is a scale parameter.

The inverse demand curve for plowing services corresponds to $M_t = D(P_{M,t})$ in the model:

$$P_{M,t} = a\, M_t^{-\phi}, \tag{25}$$

where M_t denotes total plowing services and $P_{M,t}$ is the corresponding price. The parameter ϕ governs the inverse demand elasticity. In the data, prices and quantities are directly observed for both supply relationships, allowing the supply elasticities γ and α to be estimated from observed variation. By contrast, aggregate plowing services M_t and its price $P_{M,t}$ are not directly observable. We therefore identify the demand-side parameter ϕ using model-implied elasticities conditional on the estimated supply behavior.

A central feature of the model is the learning externality associated with tractor adoption. Recall that the new-technology sector produces intermediate services $Y_{2,t}$ with productivity $A_{2,t}$, so that mechanical plowing services are $M_{2,t} = A_{2,t}Y_{2,t}$. In our context, $Y_{2,t} = K_{2,t} + I_{2,t}$ measures the flow of mechanical capacity (the beginning-of-period tractor stock plus new investment). We assume that agricultural output using mechanical services follows

$$\mathcal{Y}_t = A_{2,t} M_{2,t}^{\eta} \varepsilon_t, \tag{26}$$

where $\mathcal{Y}_t$ denotes agricultural output, $M_{2,t}$ measures mechanical plowing services, and η governs the private returns to mechanical services. The term ε_t captures the contribution of other inputs, such as fertilizer and complementary factors.

We parameterize mechanical productivity $A_{2,t}$ as an S-shaped learning function of the scale of mechanization:

$$A_{2,t} = \Lambda(M_{2,t}) \equiv A_L + \frac{A_H - A_L}{1 + \exp[-\kappa\,(M_{2,t} - M_2^c)]}, \tag{27}$$

which allows productivity to increase slowly at low levels of mechanization, accelerate during diffusion, and eventually converge to an upper bound. This specification corresponds directly to the externality function introduced in equation (1) of the theoretical model.

4.2.2 Calibration strategy and parameters

We group parameters into (i) externality parameters that govern the endogenous evolution of mechanical productivity and (ii) remaining structural parameters that determine preferences, elasticities, depreciation, and long-run outcomes. Within group (ii), some parameters are fixed externally using standard values or accounting identities, while others are calibrated internally to match steady-state or long-run moments.

Externality parameters Mechanical productivity evolves through learning-by-doing externalities. To discipline this mechanism, we first recover a time series for mechanical productivity from the data without imposing a functional form on its law of motion.

We estimate equation (26) to identify η and recover $A_{2,t}$ as a residual after accounting for observed inputs. This decomposition separates scale effects (captured by η) from productivity growth (captured by $A_{2,t}$) and provides an empirical measure of the learning component.

We then estimate the parameters of the learning function in (27). Specifically, the lower and upper bounds (A_L, A_H), the curvature parameter κ, and the inflection point M_2^c are estimated by nonlinear least squares to best fit the recovered productivity series $\{A_{2,t}\}$. Appendix C provides estimation details.

Other parameters The remaining parameters govern discounting, demand and supply elasticities, depreciation, and scale. We either fix them externally using standard values and accounting identities or calibrate them internally to match long-run moments.

Externally fixed parameters. We set the discount factor to $\beta = 0.96$, consistent with the discount rate β_t in the dynamic model (Section 2). We pin down the inverse demand elasticity ϕ using observed factor shares (land rents and labor income), which discipline the curvature of the demand schedule. We estimate the depreciation rate of tractor capital δ_2 from tractor-stock dynamics, where δ_2 corresponds to the depreciation rate in equation (15).

We normalize two scale parameters. Because the unit of plowing services is arbitrary, we set $a = 1$ in (25). Likewise, we set $c = 1$ for the scale of animal-service supply in (24).

Internally calibrated parameters. The inverse supply elasticities—γ for tractors (equation 23) and α for animal services (equation 24)—are estimated from the observed relationships between tractor prices and new investment, and between animal-service prices and animal stocks, respectively. Subsequently, the scale parameter b in (23) is calibrated to target a long-run mechanization level where the steady-state tractor stock is 1.5 times the 2013 observed stock.

Parameter values Overall, externality parameters are disciplined by productivity dynamics, while the remaining parameters are fixed externally or calibrated to long-run targets. Table 1 reports all parameter values by economic role and identification method.

Table 1: Calibrated and estimated parameter values

Parameter	Value	Description	Source / Target
Externality parameters			
A_L	0.956	Lower bound of mechanical productivity	NLS estimation
A_H	1.585	Upper bound of mechanical productivity	NLS estimation
κ	4.396	Curvature of learning function $\Lambda(M_{2,t})$	NLS estimation
M_2^c	1.886	Inflection point of $\Lambda(M_{2,t})$	NLS estimation
η	0.410	Returns to scale in mechanical services	Estimated from Eq. (26)
Other parameters: externally fixed			
β	0.960	Discount factor	Standard annual value
ϕ	0.590	Inverse demand elasticity	Factor share
δ_2	0.070	Depreciation rate of tractors	Tractor stock dynamics
c	1.000	Scale of animal service supply	Normalization
a	1.000	Scale of plowing service demand	Normalization
$\underline{R}$	1.000	Long-run capital cost (normalized lower bound)	Normalization
Other parameters: internally calibrated			
γ	0.750	Inverse supply elasticity of tractors	Estimated from $\ln Q_{2,t}$–$\ln I_{2,t}$
α	0.828	Inverse supply elasticity of animal services	Estimated from $\ln P_{1,t}$–$\ln Y_{1,t}$
b	0.915	Scale of tractor supply curve	Matches $K_{2,ss} = 1.5K_{2,2013}$

Note: This table reports parameter values used in the benchmark economy. Externality parameters govern the evolution of mechanical productivity through the learning function $\Lambda(M_{2,t})$ and are estimated from productivity dynamics using nonlinear least squares. Externally fixed parameters are taken from standard values or normalization choices. Internally calibrated parameters are chosen to match observed elasticities and long-run moments, including the steady-state level of tractor capital. All parameters are annual.

The inflection point $M_2^c = 1.886$ implies that mechanical productivity accelerates once tractor intensity per unit of land rises by about 89% relative to its 1978 level. The curvature $\kappa = 4.396$ implies a sharp increase around this region, with an implied productivity gain of up to $A_H/A_L - 1 \approx 66\%$.

The inverse demand elasticity $\phi = 0.59$ corresponds to a demand elasticity of about $1/\phi \approx 1.69$, implying relatively inelastic demand for plowing services.

The inverse supply elasticities $\gamma = 0.750$ and $\alpha = 0.828$ imply that tractor supply is more elastic than animal-powered supply. A natural interpretation is that mechanical capacity may

expand more quickly through investment and utilization, whereas animal services are constrained by biological adjustment.

4.3 Model Fit

We simulate transition dynamics starting from the observed tractor capital in 1978, denoted $K_{2,0}$ in the model. The economy evolves under perfect foresight, taking the sequences of capital costs R_t and subsidy rates s_t as exogenous and measured from the data.

Figure 10 plots tractor prices in the data. The raw retail price (dashed) trends upward, reflecting quality improvements. Appendix B constructs a quality-adjusted series (solid), which declines by about 70% from 1978 to 2013. This quality-adjusted price series disciplines the time path of R_t in the quantitative model.

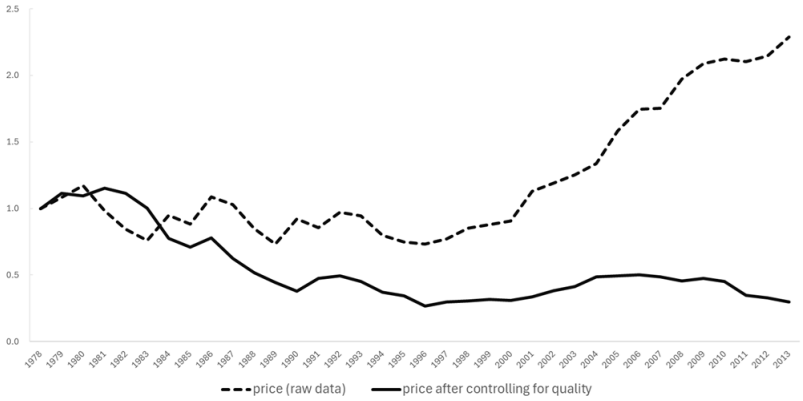


Figure 10: Real and quality-adjusted tractor prices (1978=1)

Note: This figure plots the time series of real tractor prices, normalized to one in 1978, for the period 1978–2013. The dashed line shows the raw retail price series, while the solid line reports the quality-adjusted price constructed by adjusting for observed improvements in tractor quality (see Appendix B for details). The adjusted series reflects the effective cost of tractor capital faced by producers and is used to discipline the time path of the capital cost R_t in the quantitative model.

Figure 11 reports the subsidy rate s_t, which is zero from 1978 to 2002 and rises to about 27% during 2003–2013. This corresponds to the policy variable s_t in equations (8) and (23).

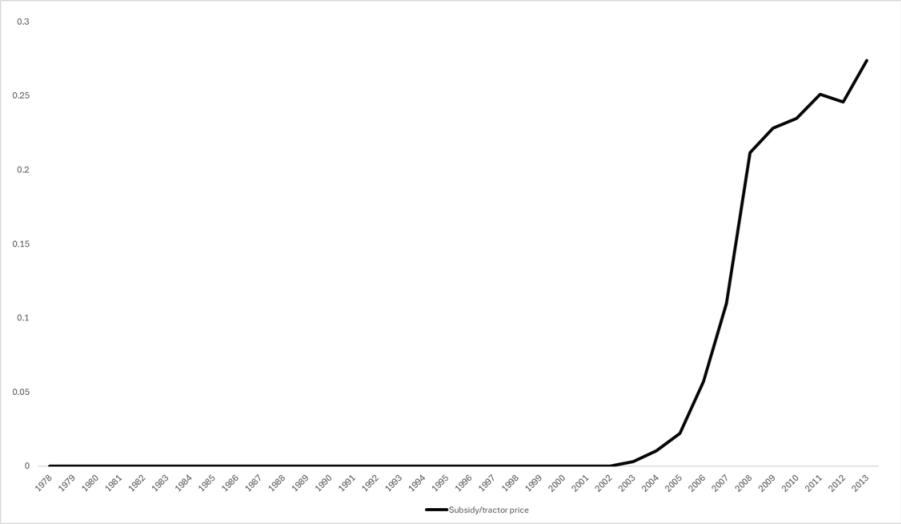


Figure 11: Tractor subsidy rate

Note: This figure plots the subsidy rate applied to tractor purchases over the period 1978–2013. The subsidy rate is defined as the fraction of the purchase price covered by government subsidies and corresponds to the policy variable s_t used in the quantitative model.

In simulations, we hold R_t and s_t at their 2013 levels for ten additional years and then let them converge to long-run values: $R_t \to \underline{R} = 1$ and $s_t \to 0$ after 2024. Figure 12 summarizes the resulting paths for R_t, s_t, and the effective price $R_t(1 - s_t)$.

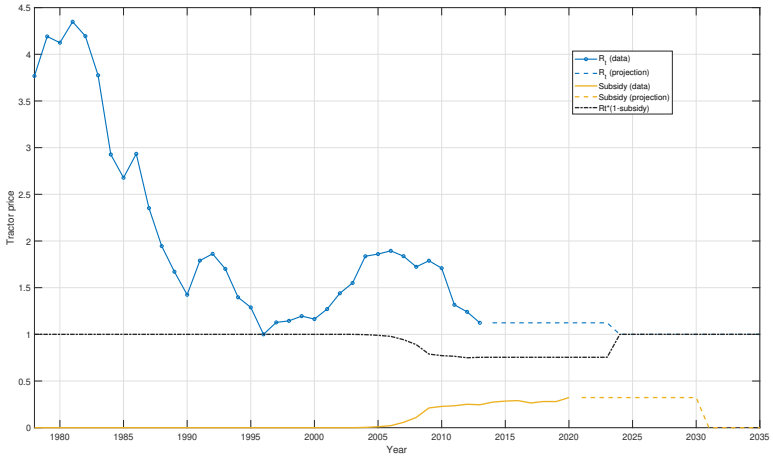


Figure 12: Capital cost, subsidy rate, and effective tractor cost in the model

Note: This figure plots the time paths of the capital cost R_t, the subsidy rate s_t, and the effective tractor cost $(1 - s_t)R_t$ used in the quantitative model. Solid lines correspond to observed data for the period 1978–2013, while dashed lines denote projected values assumed in the transition after 2013. The effective tractor cost summarizes the combined effects of changes in capital costs and subsidy policy on tractor investment incentives.

Given these exogenous paths, we evaluate untargeted moments that are central to the mechanism. We focus on (i) the evolution of mechanical productivity $A_{2,t}$ governed by the learning function $\Lambda(\cdot)$ in equation (27) and (ii) the implied tractor-capital response $K_{2,t}$ over time, which follows the law of motion in equation (15). These moments are informative because the learning function governs how scale translates into productivity growth, while the supply side determines how prices and subsidies map into investment and capital accumulation.

Figure 13 compares the estimated learning function $\Lambda(M_{2,t})$ with the productivity series recovered from equation (26). The close alignment indicates that the estimated $\Lambda(\cdot)$ captures the relationship between mechanization scale and productivity in the data.

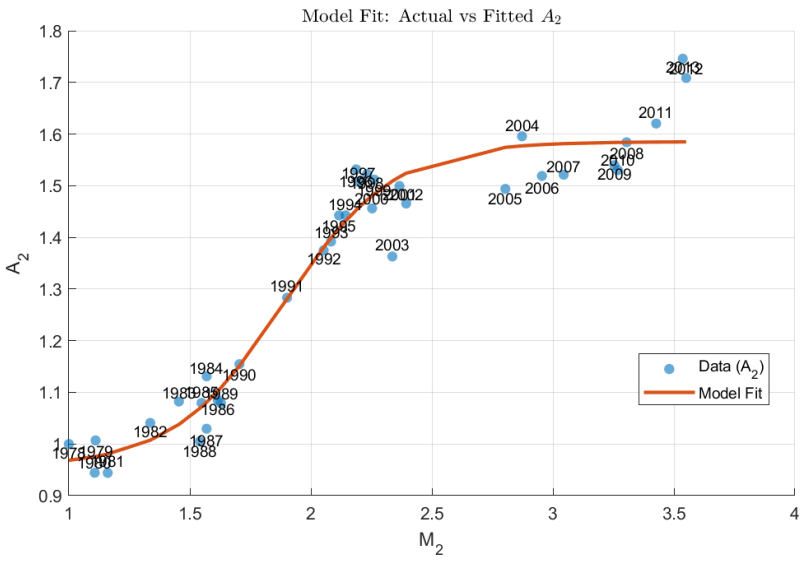


Figure 13: Fit of the mechanical productivity function $\Lambda(M_{2,t})$

Note: This figure compares the mechanical productivity implied by the data with the productivity generated by the estimated learning function $\Lambda(M_{2,t})$ in the model. The data-implied series is constructed from equation (26), while the model-implied series reflects the fitted relationship between productivity and the scale of mechanization.

Figure 14 and 15 compare tractor-capital and animal plowing service dynamics in the data with the model-implied path under observed R_t and s_t. The model matches both the timing and magnitude of these two sequences, supporting the estimated supply elasticity and policy transmission mechanism.

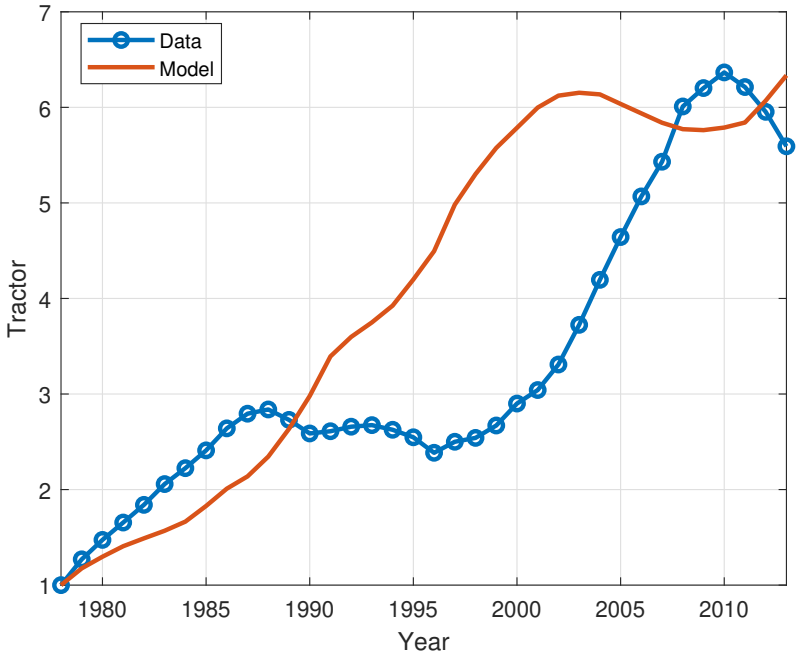


Figure 14: Fit of tractor capital dynamics

Note: This figure compares the evolution of tractor capital in the data with the model-implied path generated under the observed sequences of capital costs R_t and subsidy rates s_t.

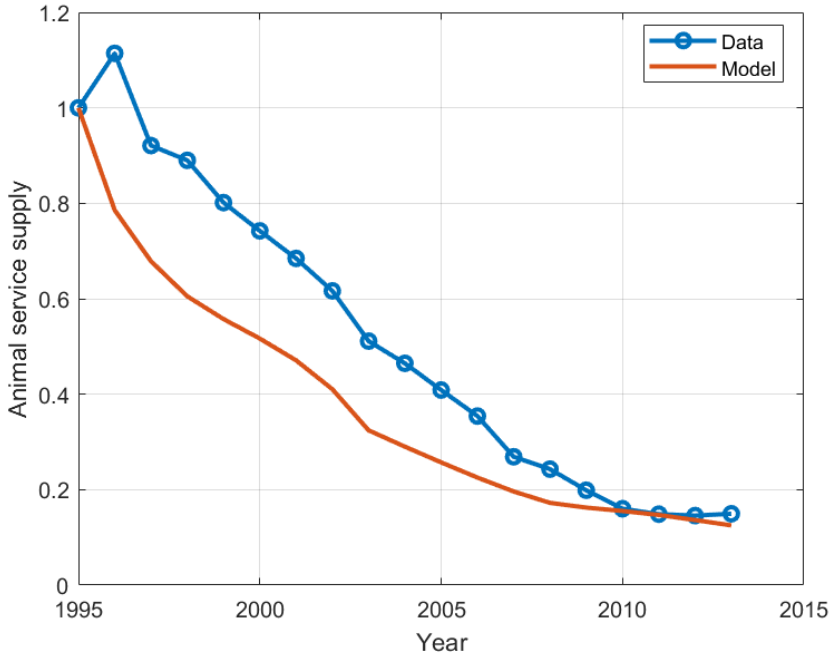


Figure 15: Fit of Animal plowing service dynamics

Note: This figure compares the evolution of animal service in the data with the model-implied path generated under the observed sequences of capital costs R_t and subsidy rates s_t.

4.4 Counterfactual Analysis

We use counterfactual experiments to quantify the roles of subsidies and learning externalities in the transition to mechanization. The experiments leverage two features of the dynamic model developed in Section 2. First, subsidies affect adoption both directly—through the user cost in equation (23)—and indirectly by accelerating learning through $\Lambda(\cdot)$. Second, learning dynamics can generate multiple competitive equilibria as characterized in Proposition 3, creating coordination failures in early investment. By varying policy and comparing decentralized (CE) and planner (SP) allocations, we quantify effects on both the level and the trajectory of mechanization.

Decentralized equilibrium versus the social planner. We begin by comparing the competitive equilibrium (CE) with the social planner (SP) allocation to quantify the importance of the learning externality. In the decentralized economy, producers do not internalize the effect of current investment on future mechanical productivity $A_{2,t}$, leading to insufficient adoption early in the transition. This corresponds to the wedge between the CE and SP Euler equations identified in equation (21). The social planner internalizes this effect and adopts earlier.

The first row of Panel A of Table 2 reports welfare at $t = 1978$, normalized by the planner

allocation. Under the observed subsidy path, welfare equals 0.98 in the best CE and 0.96 in the worst CE. The welfare gap is modest because the benchmark calibration features a unique steady state (as discussed in Proposition 3), so equilibrium selection primarily affects transition dynamics rather than long-run mechanization outcomes.

Eliminating tractor subsidies. We set $s_t \equiv 0$ for all t to isolate the role of subsidies in lowering the effective user cost. Without subsidies, tractor accumulation slows and the transition to mechanical services is delayed, weakening productivity growth through the learning function $\Lambda(\cdot)$.

The second row of Panel A of Table 2 shows that welfare falls to 0.96 (best CE) and 0.94 (worst CE) when subsidies are removed. The decline reflects transition dynamics rather than differences in long-run outcomes. Without subsidies, tractor adoption is slower, which delays learning and keeps productivity lower during the transition. Because the benchmark economy features a unique steady state, the long-run level of mechanization is unchanged; welfare losses therefore arise from slower productivity growth along the adjustment path.

Multiple equilibria and coordination failures. To highlight coordination failures, we consider an environment with multiple steady states. We let $\kappa \to \infty$ so that $\Lambda(\cdot)$ becomes a threshold function, corresponding to the step function analyzed in equation (2) and Proposition 3. Holding A_L fixed, we re-estimate M_2^c and A_H to fit the observed productivity series and recalibrate b so that the best CE implies a steady-state tractor stock 50% above the 2013 level.[11] All other parameters remain unchanged. In this economy, there are two steady states: the low steady state features a tractor stock only 18% above the 2013 level, while the high steady state features a stock 50% above.

Under this specification, strong learning externalities generate both a low- and a high-mechanization steady state, making equilibrium selection quantitatively important. Panel B of Table 2 reports the associated welfare outcomes. Under the observed subsidy policy, welfare equals 0.83 in the best CE and 0.75 in the worst CE. Eliminating subsidies further lowers welfare slightly, to 0.82 and 0.74, respectively.

[11] The new estimates are $M_2^c = 3.77$, $A_H = 1.90$, and $b = 0.99$.

Table 2: Welfare under alternative policy and equilibrium scenarios

Welfare (SP=1)	Best CE	Worst CE
	Panel A: Unique steady state	
Observed subsidies	0.98	0.96
No subsidies	0.96	0.94
	Panel B: Multiple steady states	
Observed subsidies	0.83	0.75
No subsidies	0.82	0.74

Note: This table reports welfare evaluated at the initial period ($t = 1978$) under alternative policy environments and equilibrium selections. Welfare is normalized by the social planner allocation (SP=1). Panel A corresponds to the benchmark calibration with a unique steady state, while Panel B considers an alternative specification with multiple steady states generated by stronger learning externalities (threshold externality with $\kappa \to \infty$ as in equation 2). "Observed subsidies" denotes the subsidy path observed in the data, and "No subsidies" refers to a counterfactual economy with $s_t = 0$ for all t. Best CE and Worst CE denote competitive equilibria associated with high- and low-mechanization paths, respectively, as characterized in Proposition 3.

These results highlight that once learning externalities are sufficiently strong, equilibrium selection becomes quantitatively important. Failure to coordinate on early investment can permanently trap the economy in a low-mechanization equilibrium with substantially lower welfare. This echoes the multiplicity result in Proposition 3 and the role of policy in coordinating expectations discussed in Proposition 4.

Summary. Taken together, the counterfactual experiments emphasize two mechanisms. First, subsidies accelerate mechanization by reducing early-stage investment frictions and strengthening learning dynamics through $\Lambda(\cdot)$. Second, when technological externalities are sufficiently strong, policy plays a coordination role by helping the economy transition toward the high-productivity equilibrium. These effects are particularly pronounced when the economy is close to the threshold separating low- and high-mechanization regimes, consistent with the optimal timing of intervention characterized in Propositions 1 and 4.

Overall, the counterfactual analysis provides a quantitative decomposition of the forces driving agricultural mechanization and clarifies the channels through which policy intervention can accelerate structural transformation. The results confirm the theoretical predictions: market forces (declining R_t) drove initial adoption, but government subsidies (positive s_t starting in 2004) were crucial for pushing mechanization across the critical threshold M_2^c, unlocking productivity gains

and generating sizable welfare improvements.

5 Conclusion

This paper addresses a central question in the resurgence of state interventionism: not whether governments should intervene to foster structural change, but when. By integrating the logic of endowment-driven growth with the mechanics of Marshallian production externalities, we formalize a temporary, facilitating role for the state.

Our theoretical contribution highlights that the success of the "Big Push" is inherently dynamic. We identify a specific "intervention window" governed by the evolution of market fundamentals. When factor prices (such as wages relative to capital rental rates) are unfavorable, the distortionary costs of forcing technology adoption outweigh the benefits of potential spillovers. Conversely, when fundamentals are fully mature, the market coordinates on the efficient equilibrium unaided. It is in the intermediate regime—where market forces have primed the economy but private agents remain trapped by the neglect of dynamic externalities—that policy becomes welfare-improving. Here, the planner does not fight the market but strategically accelerates it, closing the wedge between the private and social Euler equations to pull the economy across the productivity threshold.

We quantify this mechanism through the lens of China's agricultural mechanization. The transition from draft animals to tractors was not solely a result of state planning nor purely a response to labor scarcity. Our structural estimation reveals that while rising wages and falling capital prices created the necessary conditions for mechanization, they were insufficient to generate the rapid diffusion observed after 2004. The government's targeted subsidies, introduced precisely as the economy approached the critical adoption threshold, acted as the optimal catalyst. Our counterfactual analysis confirms that timing was paramount: an earlier "push"would have resulted in substantial deadweight losses due to premature capital accumulation, while a delayed intervention would have left significant productivity gains on the table.

These findings may also have implications for contemporary industrial policy in sectors characterized by learning-by-doing and network effects, such as green energy and artificial intelligence. Effective policy requires a diagnosis of the underlying market readiness. Governments should avoid the temptation to "pick winners" before the endowment structure supports them. Instead, the optimal role of the state is to monitor the evolution of comparative advantage and intervene decisively—but temporarily—when the market is on the verge of a tipping point. In this view, the visible hand of the state is most effective when it pushes in the same direction as the invisible hand of the market, simply ensuring the economy arrives at its destination sooner.

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Appendix

A Proofs

Proof of Lemma 1: First, we find R^*. At $R = R^*$, the "good" equilibrium ($A_2 = A_H$) just becomes feasible at the threshold $M_2 = M_2^c$. So, at $R = R^*$, the following conditions are satisfied:

$$D^{-1}(M_1 + M_2; \theta) = \tilde{S}_1^{-1}(M_1; w) = \tilde{S}_2^{-1}(M_2; A_H, R)$$
$$M_2 = M_2^c$$

We solve the system of equations above with three equations and three unknowns (M_1, M_2, R) and find a unique R^*.

Second, we find R^{**}. We look for $R = R^{**}$ where the "bad" equilibrium ($A_2 = A_L$) is no longer sustainable because the new technology is so cost-effective that even with low productivity A_L, the market chooses $M_2 \geq M_2^c$. That is, at $R = R^{**}$, the following conditions are satisfied:

$$D^{-1}(M_1 + M_2; \theta) = \tilde{S}_1^{-1}(M_1; w) = \tilde{S}_2^{-1}(M_2; A_L, R)$$
$$M_2 = M_2^c$$

We solve the above equation system with three equations and three unknows (M_1, M_2, R) and find a unique R^{**}.

Since $A_H > A_L$, it is easy to verify $R^* > R^{**}$.

Proof of Lemma 2: Recall the welfare function definition:

$$\mathcal{W} = \int_0^M D^{-1}(m; \theta)dm - \int_0^{M_1} \tilde{S}_1^{-1}(m; w)dm - \int_0^{M_2} \tilde{S}_2^{-1}(m; A_2, R)dm.$$

Satisfying the standard property, the marginal cost function $\tilde{S}_2^{-1}(m; A_2, R)$ is homogeneous of degree 1 in the factor price R (i.e., $\tilde{S}_2^{-1} = R \cdot g(m, A_2)$). This implies $R\frac{\partial \tilde{S}_2^{-1}}{\partial R} = \tilde{S}_2^{-1}$.

Using the Envelope Theorem, we have

$$\frac{\partial \mathcal{W}_{\text{mkt}}}{\partial R} = -\int_0^{M_2^{\text{mkt}}} \frac{\partial \tilde{S}_2^{-1}(m; A_L, R)}{\partial R} dm = -\frac{1}{R}\int_0^{M_2^{\text{mkt}}} \tilde{S}_2^{-1}(m; A_L, R)dm.$$

Similarly, for the pushed allocation where M_2 is fixed at M_2^c, the derivative is:

$$\frac{\partial \mathcal{W}_{\text{push}}}{\partial R} = -\int_0^{M_2^c} \frac{\partial \tilde{S}_2^{-1}(m; A_H, R)}{\partial R} dm = -\frac{1}{R}\int_0^{M_2^c} \tilde{S}_2^{-1}(m; A_H, R) dm.$$

Thus, the derivative of the welfare difference is:

$$\frac{\partial \Delta\mathcal{W}(R)}{\partial R} = \frac{1}{R}\left[\int_0^{M_2^{\text{mkt}}} \tilde{S}_2^{-1}(m; A_L, R) dm - \int_0^{M_2^c} \tilde{S}_2^{-1}(m; A_H, R) dm\right]. \tag{A.1}$$

We determine the sign of this derivative at the crossing point. Specifically, we evaluate it at the specific R where $\Delta\mathcal{W}(R) = 0$. The condition $\Delta\mathcal{W}(R) = 0$ implies $\mathcal{W}_{\text{push}} = \mathcal{W}_{\text{mkt}}$. Expanding the welfare terms:

$$\begin{aligned}
&\int_0^{M^{\text{push}}} D^{-1}(m) dm - \int_0^{M_1^{\text{push}}} \tilde{S}_1^{-1}(m) dm - \int_0^{M_2^c} \tilde{S}_2^{-1}(m; A_H, R) dm \\
=&\int_0^{M^{\text{mkt}}} D^{-1}(m) dm - \int_0^{M_1^{\text{mkt}}} \tilde{S}_1^{-1}(m) dm - \int_0^{M_2^{\text{mkt}}} \tilde{S}_2^{-1}(m; A_L, R) dm.
\end{aligned}$$

Rearranging to isolate the difference in Sector 2 costs (which appears in Eq. A.1):

$$\begin{aligned}
&\int_0^{M_2^{\text{mkt}}} \tilde{S}_2^{-1}(m; A_L, R) dm - \int_0^{M_2^c} \tilde{S}_2^{-1}(m; A_H, R) dm \\
=&\left(\int_0^{M_1^{\text{push}}} \tilde{S}_1^{-1}(m) dm - \int_0^{M_1^{\text{mkt}}} \tilde{S}_1^{-1}(m) dm\right) - \left(\int_0^{M^{\text{push}}} D^{-1}(m) dm - \int_0^{M^{\text{mkt}}} D^{-1}(m) dm\right) \\
=&\int_{M_1^{\text{mkt}}}^{M_1^{\text{push}}} \tilde{S}_1^{-1}(m; w) dm - \int_{M^{\text{mkt}}}^{M^{\text{push}}} D^{-1}(m; \theta) dm.
\end{aligned} \tag{A.2}$$

Substituting (A.2) back into (A.1), we obtain:

$$R\frac{\partial \Delta\mathcal{W}(R)}{\partial R}\bigg|_{\Delta\mathcal{W}=0} = \int_{M_1^{\text{mkt}}}^{M_1^{\text{push}}} \tilde{S}_1^{-1}(m; w) dm - \int_{M^{\text{mkt}}}^{M^{\text{push}}} D^{-1}(m; \theta) dm. \tag{A.3}$$

We now analyze the signs of the terms in (A.3). By $D^{-1}(M; \theta) = \tilde{S}_1^{-1}(M_1; w)$, we must have $M_1^{\text{push}} < M_1^{\text{mkt}}$ (crowding out of traditional sector) and $M^{\text{push}} > M^{\text{mkt}}$ (expansion of aggregate demand) (in fact, proving by contradiction, otherwise, $D^{-1}(M; \theta) = \tilde{S}_1^{-1}(M_1; w)$ cannot hold both at the market equilibrium and at the pushed allocation). Hence, the RHS of (A.3) is a negative number minus a positive number, which is negative. Therefore,

$$R\frac{\partial \Delta\mathcal{W}(R)}{\partial R}\bigg|_{\Delta\mathcal{W}=0} < 0.$$

This implies that $\Delta\mathcal{W}(R)$ is strictly decreasing at any point where it crosses zero. Since $\Delta\mathcal{W}(R)$

is continuous, it can cross zero at most once.

Next, we examine the boundary behavior by considering the extreme values of R. On the one side, when R is sufficiently small and close to R^*,$\Delta\mathcal{W}(R)$ must be positive. This is straightforward. As $R \to R^{*+}$ (from above), the market equilibrium corresponds to the "bad" equilibrium (A_L) at the tipping point, while the pushed allocation corresponds to the "good" equilibrium (A_H) at the same R. Since the good equilibrium is Pareto superior, $\Delta\mathcal{W}(R^*) > 0$. On the other side, when R is sufficiently high, $\Delta\mathcal{W}(R)$ must be negative. In fact,

$$\Delta\mathcal{W}(R) = \int_{M^{\text{mkt}}}^{M^{\text{push}}} D^{-1}(m;\theta)dm - \int_{M_1^{\text{mkt}}}^{M_1^{\text{push}}} \tilde{S}_1^{-1}(m;w)dm - \begin{pmatrix} \int_0^{M_2^c} \tilde{S}_2^{-1}(m;A_H,R)dm \\ -\int_0^{M_2^{\text{mkt}}} \tilde{S}_2^{-1}(m;A_L,R)dm \end{pmatrix}$$

When $R \to +\infty$, $M_2^{\text{mkt}} \to 0$ and the welfare under the market equilibrium approaches that of the traditional economy. The first two terms on the RHS are finite, and the term $\int_0^{M_2^{\text{mkt}}} \tilde{S}_2^{-1}(m;A_L,R)dm \to 0$ by noting that $\tilde{S}_2^{-1}(m;A_L,R) = P_M$ is bounded in equilibrium. So, when $R \to +\infty$, $\Delta\mathcal{W}(R) \to -\infty$.

By the Intermediate Value Theorem and the single-crossing property established above, there exists a unique threshold $\hat{R} > R^*$ such that $\Delta\mathcal{W}(\hat{R}) = 0$. For $R > \hat{R}$, $\Delta\mathcal{W} < 0$ (market is optimal); for $R^* < R < \hat{R}$, $\Delta\mathcal{W} > 0$ (push is optimal). Q.E.D.

Proof of Proposition 1: We determine the optimal subsidy τ to implement the efficient allocation. The planner's optimal allocation depends on R:

- If $R > \hat{R}$, the optimal allocation is the market allocation (M_2^{mkt}, A_L). This is implemented by $\tau = 0$.
- If $R^* < R < \hat{R}$, the optimal allocation is the pushed allocation $M_2 = M_2^c$ with $A_2 = A_H$.

To implement $M_2 = M_2^c$ when the market fundamental is R, the government sets a subsidy τ such that the private agents choose M_2^c. The equilibrium condition for Sector 2 with subsidy is:

$$\frac{P_M}{1-\tau} = \tilde{S}_2^{-1}(M_2^c;A_H,R)$$

From the proof of Lemma 1, we defined R^* as the rental price where the market naturally chooses M_2^c under productivity A_H. That is:

$$P_M = \tilde{S}_2^{-1}(M_2^c;A_H,R^*)$$

Note that P_M is determined by equalizing the demand and the total supply of Sector 1 and M_2^c,

so P_M is the same in both equations. Therefore, we require:

$$(1-\tau)\tilde{S}_2^{-1}(M_2^c; A_H, R) = \tilde{S}_2^{-1}(M_2^c; A_H, R^*)$$

As the marginal cost function $\tilde{S}_2^{-1}(m; A_2, R)$ is homogeneous of degree 1 in the factor price R (i.e., $\tilde{S}_2^{-1}(M_2; A_2, R) = R \cdot g(M_2, A_2)$), the condition becomes $(1-\tau)R \cdot g(M_2^c, A_H) = R^* \cdot g(M_2^c, A_H)$, which implies

$$(1-\tau)R = R^* \implies \tau = 1 - \frac{R^*}{R}.$$

Since $R > R^*$, $\tau > 0$.

Proof of Corollary 1: We analyze the comparative statics with respect to wage w.

$\hat{R}$ **is increasing in** w: Recall $\Delta\mathcal{W}(R, w) = \mathcal{W}_{\text{push}}(R, w) - \mathcal{W}_{\text{mkt}}(R, w)$. The threshold $\hat{R}$ is defined by $\Delta\mathcal{W}(\hat{R}, w) = 0$. By the Implicit Function Theorem:

$$\frac{d\hat{R}}{dw} = -\frac{\partial\Delta\mathcal{W}/\partial w}{\partial\Delta\mathcal{W}/\partial R}$$

We already know $\frac{\partial\Delta\mathcal{W}(R,w)}{\partial R}\Big|_{R=\hat{R}} < 0$. We only need to prove $\frac{\partial\Delta\mathcal{W}(R,w)}{\partial w} > 0$, which would imply $\frac{d\hat{R}}{dw} > 0$. Note that $\frac{\partial\Delta\mathcal{W}}{\partial w} = \frac{\partial\mathcal{W}_{\text{push}}}{\partial w} - \frac{\partial\mathcal{W}_{\text{mkt}}}{\partial w}$. By the envelope theorem, $\frac{\partial\mathcal{W}}{\partial w} = -\int_0^{M_1} \frac{\partial\tilde{S}_1^{-1}}{\partial w} dm$. Thus,

$$\frac{\partial\Delta\mathcal{W}}{\partial w} = -\left(\int_0^{M_1^{\text{push}}} \frac{\partial\tilde{S}_1^{-1}}{\partial w} dm - \int_0^{M_1^{\text{mkt}}} \frac{\partial\tilde{S}_1^{-1}}{\partial w} dm\right)$$

We have proved $M_1^{\text{push}} < M_1^{\text{mkt}}$ (crowding out of traditional sector) in the proof of Lemma 2. Consequently, $\frac{\partial\Delta\mathcal{W}}{\partial w} > 0$. Combining signs, $\frac{d\hat{R}}{dw} = -\frac{(+)}{(-)} > 0$. Higher wages increase the threshold $\hat{R}$, meaning intervention is justified at higher capital prices (earlier in time).

Thresholds on w: Analogous to the derivation for R, we can define w^* and $\hat{w}$. Since Sector 1 is labor intensive, an increase in w shifts the Sector 1 supply curve up, raising P_M. This encourages substitution toward Sector 2. There exists a w^* such that for $w > w^*$, the market naturally adopts $M_2 \geq M_2^c$ (with a possible coordination). There exists a $\hat{w} < w^*$ such that for $w \in (\hat{w}, w^*)$, intervention is optimal and for $w < \hat{w}$ intervention is not optimal. This mirrors the logic for R, with the direction reversed (high w favors adoption, whereas low R favors adoption).

Proof of Proposition 2: Consider the smooth case where κ is finite. The welfare function is $\mathcal{W}(\tau; R)$. The government chooses τ to maximize $\mathcal{W}$ subject to the market equilibrium constraints. Let $\tau^*(R)$ be the optimal subsidy ignoring fixed costs. The welfare gain from intervention is $G(R) = \mathcal{W}(\tau^*(R); R) - \mathcal{W}(0; R)$. Due to the S-shaped externality $\Lambda(M_2)$, the marginal benefit of

adoption increases sharply near the inflection point. Even with a smooth function, if the transition is steep enough (finite but large κ), the welfare landscape resembles the step function case. Specifically, there is a range of R where the market equilibrium remains at low adoption ($A_2 \approx A_L$) but a subsidy can push adoption to the high productivity region ($A_2 \approx A_H$). The gain $G(R)$ is small when R is large enough. With a fixed regulatory cost $c_0 > 0$, intervention is optimal if and only if $G(R) \geq c_0$. So there exists a cutoff $\hat{R}_{smooth}$ such that $G(\hat{R}_{smooth}) = c_0$. For $R > \hat{R}_{smooth}$, the net gain is negative, so no intervention. For $R < \hat{R}_{smooth}$ (but R above the point where market naturally succeeds), intervention is optimal.

Proof of Proposition 3: We treat the step-function externality:

$$\Lambda(M_{2,t}) = \begin{cases} A_H, & M_{2,t} \geq M_2^c, \\ A_L, & M_{2,t} < M_2^c, \end{cases} \qquad A_H > A_L > 0. \tag{A.4}$$

The exogenous sequences $\{w_t\}$ (increasing to $\bar{w}$), $\{R_t\}$ (decreasing to $\underline{R}$), constant $\beta \in (0,1)$ and $z > 0$ are given. Initial capital $K_{2,0}$ is so low that $M_{2,0} < M_2^c$.

1. Transitional path

Define the static problem

$$u(M_{2,t}; w_t) = \max_{Y_{1,t} \geq 0} \left\{ \int_0^{A_1 Y_{1,t} + M_{2,t}} D^{-1}(m)\, dm - \int_0^{Y_{1,t}} S_1^{-1}(y; w_t)\, dy \right\},$$

which implies $\partial u(M_{2,t}; w_t)/\partial M_{2,t} = D^{-1}(A_1 Y_{1,t} + M_{2,t}) = P_{M,t}$ by the envelope theorem. Then we rewrite the SP program as

$$\begin{aligned} W_t(K_{2,t}) = \max_{I_{2,t} \geq 0} & \left\{ u(M_{2,t}; w_t) - \int_0^{I_{2,t}} S_I^{-1}(x; R_t, z_t)\, dx + \beta \mathbb{E}_t[W_{t+1}(K_{2,t+1})] \right\} \\ \text{s.t.} \quad & M_{2,t} = A_{2,t}(K_{2,t} + I_{2,t}), \;\; A_{2,t} = \Lambda(M_{2,t}), \\ & K_{2,t+1} = (1 - \delta_2)(K_{2,t} + I_{2,t}). \end{aligned}$$

where $\Lambda(M_{2,t})$ is given in (A.4).

Write $A_{2,t}$ and $M_{2,t}$ as functions of $I_{2,t}$. Given $K_{2,t} = K$, define the two cutoffs:

$$\underline{I}(K) \equiv \max\left\{0, \frac{M_2^c}{A_H} - K\right\}, \qquad \overline{I}(K) \equiv \max\left\{0, \frac{M_2^c}{A_L} - K\right\}. \tag{A.5}$$

Then the fixed-point restrictions $M_{2,t} = A_{2,t}(K + I_{2,t})$ and $A_{2,t} = \Lambda(M_{2,t})$ imply

$$A_{2,t} = \begin{cases} A_H, & I_{2,t} \geq \overline{I}(K), \\ A_H \text{ or } A_L, & I_{2,t} \in [\underline{I}(K), \overline{I}(K)), \\ A_L, & I_{2,t} < \underline{I}(K), \end{cases} \qquad M_{2,t} = \begin{cases} A_H(K + I_{2,t}), & I_{2,t} \geq \overline{I}(K), \\ A_H(K + I_{2,t}) \text{ or } A_L(K + I_{2,t}), & I_{2,t} \in [\underline{I}(K), \overline{I}(K)), \\ A_L(K + I_{2,t}), & I_{2,t} < \underline{I}(K). \end{cases}$$

Part 1: solve SP

We solve the SP in two steps.

Step 1: Constant $\{w_t, R_t\}_{t \geq 0}$. We find the SP path with the value function $W(K)$:

$$W(K) = \max\{W_1(K), W_2(K)\}, \tag{A.6}$$

where $W_1(K)$ and $W_2(K)$ satisfy

$$W_2(K) = \max_{I_2 \geq \underline{I}(K)} \left\{ u(A_H(K + I_2)) - \int_0^{I_2} S_I^{-1}(x)\, dx + \beta W((1 - \delta_2)(K + I_2)) \right\}, \tag{A.7}$$

$$W_1(K) = \max_{0 \leq I_2 < \underline{I}(K)} \left\{ u(A_L(K + I_2)) - \int_0^{I_2} S_I^{-1}(x)\, dx + \beta W((1 - \delta_2)(K + I_2)) \right\}. \tag{A.8}$$

By construction, if $I_2 \geq \underline{I}(K)$ then A_H is consistent, and the planner selects the A_H branch whenever it is feasible. Note that the optimization problem in (A.7) may involve the corner solution $I_2 = \underline{I}(K) = \frac{M_2^c}{A_H} - K$. We can verify that the value function $W(K)$ has the following properties. There exist two thresholds K_1^{SP} and K_2^{SP}, with $K_1^{SP} < K_2^{SP}$, such that: (i) when $K > K_2^{SP}$, $W(K) = W_2(K)$ and the optimal choice satisfies $I_2 > \underline{I}(K)$; (ii) when $K \in [K_1^{SP}, K_2^{SP}]$, the optimal choice is the corner solution $I_2 = \underline{I}(K)$ and also $W(K) = W_2(K)$; and (iii) when $K < K_1^{SP}$, $W(K) = W_1(K)$ and the optimal choice satisfies $I_2 < \underline{I}(K) < \overline{I}(K)$. Moreover, K_1^{SP} is defined by $W_1(K_1^{SP}) = W_2(K_1^{SP})$, and K_2^{SP} is the point at which the lower bound in (A.7) ceases to bind (so the optimizer switches from $I_2 = \underline{I}(K)$ to $I_2 > \underline{I}(K)$).

Step 2: Time-varying R_t, w_t. Similar to Step 1, we can find a unique SP path under the sufficient condition that R_t declines slowly enough and w_t increases slowly enough over time.

Part 2: solve CE

We can verify that CE solves the same problem as the SP, but takes $A_{2,t}$ as exogenous. We solve the CE in two steps.

Step 1: Constant $\{w_t, R_t\}_{t \geq 0}$.

There exists a continuum of CE paths. Consider a self-fulfilling CE path with value function

$$V(K) = \begin{cases} V_1(K), & K < K^{CE}, \\ V_2(K), & K \geq K^{CE}, \end{cases} \tag{A.9}$$

where K^{CE} is a threshold defined later and $V_1(K)$ and $V_2(K)$ satisfy

$$V_2(K) = \max_{I_2 \geq 0} \left\{ u\left(A_H(K+I_2)\right) - \int_0^{I_2} S_I^{-1}(x)\, dx + \beta V\left((1-\delta_2)(K+I_2)\right) \right\}, \quad \text{for } K \geq K^{CE}, \tag{A.10}$$

$$V_1(K) = \max_{I_2 \geq 0} \left\{ u(A_L(K+I_2)) - \int_0^{I_2} S_I^{-1}(x)\, dx + \beta V((1-\delta_2)(K+I_2)) \right\}, \quad \text{for } K < K^{CE}. \tag{A.11}$$

The following *self-fulfilling conditions* must be met. Let $I_H^*(K)$ denote an optimizer of (A.10) and $I_L^*(K)$ an optimizer of (A.11). The threshold K^{CE} is self-fulfilling if

$$\forall K \geq K^{CE}: \quad I_H^*(K) \geq \underline{I}(K), \qquad \forall K < K^{CE}: \quad I_L^*(K) < \overline{I}(K). \tag{A.12}$$

We find two thresholds of K:

$$K_1^{CE} \text{ such that } \left[u'(A_H(K+I_2))\, A_H - S_I^{-1}(I_2) + \beta\,(1-\delta_2)\, V'((1-\delta_2)(K+I_2))\right]\Big|_{K=K_1^{CE},\ I_2=\frac{M_2^c}{A_H}-K} = 0 \tag{A.13}$$

$$K_2^{CE} \text{ such that } \left[u'(A_L(K+I_2))\, A_L - S_I^{-1}(I_2) + \beta\,(1-\delta_2)\, V'((1-\delta_2)(K+I_2))\right]\Big|_{K=K_2^{CE},\ I_2=\frac{M_2^c}{A_L}-K} = 0. \tag{A.14}$$

That is, K_1^{CE} and K_2^{CE} make the optimal choice of I_2 such that $M_{2,t} = M_2^c$ under the two value functions $V_2(K)$ and $V_1(K)$, respectively. Any $K^{CE} \in [K_1^{CE}, K_2^{CE}]$ with the value function (A.9) forms an equilibrium. Hence there is a continuum of CE paths indexed by K^{CE}.

Finally, it is easy to verify that $K_1^{CE} = K_2^{SP}$: on the high branch the SP selects A_H whenever it is feasible, and conditional on $A = A_H$ both SP and CE solve the same control problem; hence the boundary at which the high-branch corner ceases to bind coincides.

Step 2: Time-varying R_t, w_t. Similar to Step 1, we can find a continuum of CE paths under the sufficient condition that R_t declines slowly enough and w_t increases slowly enough over time.

2. Steady states

Based on $M_t = D(P_{M,t})$, $M_t = M_{1,t} + M_{2,t}$, $M_{1,t} = A_1 Y_{1,t}$, and $Y_{1,t} = S_1(P_{1,t}; w_t)$, we can find the residual inverse demand schedule for M_2, denoted $\widetilde{D}^{-1}(\cdot)$, hence $M_{2,t} = \widetilde{D}^{-1}(M_{2,t})$. We characterize steady states under long-run constant fundamentals ($w_t = \bar{w}, R_t = \underline{R}, \beta_t = \beta, z_t = z$).

A steady state satisfies $K_{2,t+1} = K_{2,t} = K_2$, implying:

$$I_2 = \frac{\delta_2}{1-\delta_2} K_2, \qquad Y_2 = K_2 + I_2 = \frac{K_2}{1-\delta_2}, \qquad M_2 = A_2 Y_2 = A_2 \frac{K_2}{1-\delta_2}.$$

The externality condition (A.4) remains. The steady-state Euler equation is $[1-\beta(1-\delta_2)]\, S_I^{-1}(I_2; \underline{R}) = P_M A_2$. Substituting $P_M = \widetilde{D}^{-1}(M_2)$ and $M_2 = A_2 I_2/\delta_2$, we have

$$[1-\beta(1-\delta_2)]\, S_I^{-1}(I_2; \underline{R}) = \widetilde{D}^{-1}\left(A_2 \frac{I_2}{\delta_2}\right) A_2, \tag{A.15}$$

which gives I_2 as a function of A_2, written as $I_2 = f(A_2; \underline{R})$. For each technology regime $A_2 \in \{A_L, A_H\}$, we solve (A.15) and find two candidates of steady state, denoted by $\left(A_L, I_{2,L}^*(\underline{R})\right)$ and $\left(A_H, I_{2,H}^*(\underline{R})\right)$ for a given $\underline{R}$. We have $I_{2,H}^*(\underline{R}) > I_{2,L}^*(\underline{R})$ and both are decreasing in $\underline{R}$.

Considering the step externality rule (A.4) and using the result $M_2 = \frac{A_2 I_2}{\delta_2}$ in steady state, we define the investment cutoffs

$$I_H^c \equiv \frac{\delta_2 M_2^c}{A_H}, \qquad I_L^c \equiv \frac{\delta_2 M_2^c}{A_L}. \tag{A.16}$$

Hence, the externality rule (A.4) can be rewritten as

$$g(I_2) = \begin{cases} \{A_L\}, & I_2 < I_H^c, \\ \{A_L, A_H\}, & I_H^c \le I_2 < I_L^c, \\ \{A_H\}, & I_2 \ge I_L^c. \end{cases} \tag{A.17}$$

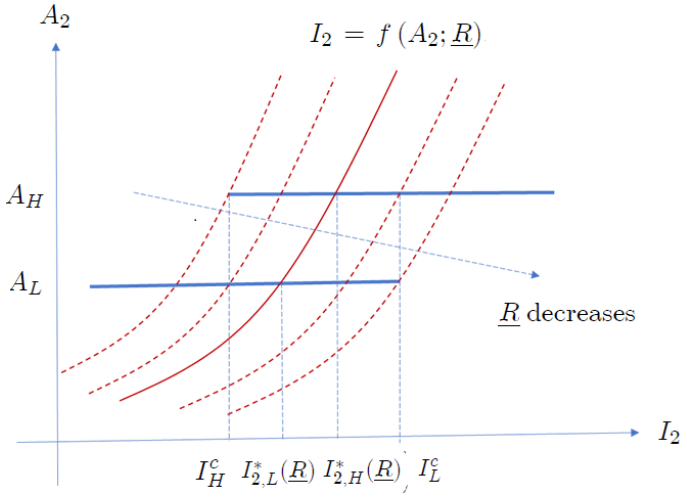


Figure A.1: Functions $I_2 = f(A_2; \underline{R})$ and $A_2 = g(I_2)$

The two candidates of steady state $\left(A_L, I_{2,L}^*(\underline{R})\right)$ and $\left(A_H, I_{2,H}^*(\underline{R})\right)$ must be consistent with

the rule of (A.17). Based on this, we find two thresholds $\underline{R}^*$ and $\underline{R}^{**}$with $\underline{R}^* > \underline{R}^{**}$ such that

$$I^*_{2,L}(\underline{R}) = I^c_L\big|_{\underline{R}=\underline{R}^{**}} \quad \text{and} \quad I^*_{2,H}(\underline{R}) = I^c_H\big|_{\underline{R}=\underline{R}^{*}}$$

When $\underline{R} \leq \underline{R}^{**}$, there exists a unique steady state with $A_2 = A_H$; when $\underline{R}^* \geq \underline{R} > \underline{R}^{**}$, there are two steady states with $A_2 = A_H$ or A_L; when $\underline{R} > \underline{R}^*$, there exists a unique steady state with $A_2 = A_L$. See Figure A.1.

Moreover, we can find other two thresholds $\underline{R}^{*\text{best}}$ and $\underline{R}^{*\text{worst}}$ with $\underline{R}^{*\text{best}} < \underline{R}^{*\text{worst}}$ such that

$$I^*_{2,H}(\underline{R}) = I^c_L\big|_{\underline{R}=\underline{R}^{*\text{best}}} \quad \text{and} \quad I^*_{2,L}(\underline{R}) = I^c_H\big|_{\underline{R}=\underline{R}^{*\text{worst}}}$$

It is easy to verify that $\underline{R}^* > \underline{R}^{*\text{worst}} > \underline{R}^{*\text{best}} > \underline{R}^{**}$. Hence, i) if $\underline{R} < \underline{R}^{**}$, both the best CE and the worst CE admit a unique steady state with $A_2 = A_H$; ii) if $\underline{R} \in \left(\underline{R}^{**}, \underline{R}^{*\text{best}}\right)$, the best CE has a unique steady state with $A_2 = A_H$, whereas the worst CE admits two steady states; iii) if $\underline{R} \in \left(\underline{R}^{*\text{best}}, \underline{R}^{*\text{worst}}\right)$, both the best CE and the worst CE exhibit two steady states; iv) if $\underline{R} \in \left(\underline{R}^{*\text{worst}}, \underline{R}^{*}\right)$, the worst CE has a unique steady state with $A_2 = A_L$, whereas the best CE admits two steady states; and v) if $\underline{R} > \underline{R}^*$, both the best CE and the worst CE admit a unique steady state with $A_2 = A_L$.

Proof of Proposition 4: We prove each policy instrument separately.

1. Subsidizing intermediate inputs

Suppose the government provides a subsidy for purchasing the intermediate input $Y_{2,t}$, with a subsidy rate τ_t; that is, service providers only need to pay the price $(1-\tau_t)\,P_{2,t}$. The profit maximization problem for service providers becomes

$$\max_{Y_{1,t},\, Y_{2,t}} P_{M,t}M_t - P_{1,t}Y_{1,t} - (1-\tau_t)\,P_{2,t}Y_{2,t} - T_t$$

where $T_t = \tau_t P_{2,t}Y_{2,t}$ is a lump-sum tax to service providers at time t, and the government budget constraint is balanced period by period. This implies $(1-\tau_t)\,P_{2,t} = P_{M,t}A_{2,t}$; that is, the effective price received by capital owners is

$$P_{2,t} = \frac{P_{M,t}A_{2,t}}{1-\tau_t}.$$

It is easy to verify that with subsidy the CE Euler equation is still $Q_{2,t} = P_{2,t} + \frac{1-\delta_2}{1+r_t}\mathbb{E}_t\left[Q_{2,t+1}\right]$. Substituting $P_{2,t} = P_{M,t}A_{2,t}/\left(1-\tau_t\right)$ yields

$$Q_{2,t} = \frac{P_{M,t}A_{2,t}}{1-\tau_t} + \frac{1-\delta_2}{1+r_t}\mathbb{E}_t\left[Q_{2,t+1}\right].$$

The SP Euler equation is given in (21), that is,

$$\underbrace{S_I^{-1}\left(I_{2,t};R_t,z_t\right)}_{=Q_{2,t}} = \underbrace{\frac{1}{1-\Lambda^{'}\left(M_{2,t}\right)\left(K_{2,t}+I_{2,t}\right)}}_{\textbf{externality}}\cdot\underbrace{D^{-1}(M_{1,t}+M_{2,t})}_{=P_{M,t}}A_{2,t}+\frac{1-\delta_2}{1+r_t}\underbrace{S_I^{-1}\left(I_{2,t+1};R_{t+1},z_{t+1}\right)}_{=Q_{2,t+1}}.$$

Therefore, to implement SP, the subsidy should satisfy $1-\tau_t = 1-\Lambda^{'}\left(M_{2,t}\right)\left(K_{2,t}+I_{2,t}\right)$; that is, the state-dependent subsidy is

$$\tau(K) = \Lambda^{'}\left(M_2\right)\left(K_2+I_2\right)\Big|_{\text{SP}}.$$

Now we consider that $\Lambda\left(\cdot\right)$ is a step function, in which case the SP Euler equation is not always binding. For SP, when $K\in[K_1^{SP},K_2^{SP}]$, the optimal choice is the corner solution $I_2=\underline{I}(K)$. The first-order condition is not binding:

$$\left[u'(A_H(K+I_2))\,A_H - S_I^{-1}(I_2)+\frac{1-\delta_2}{1+r}V'((1-\delta_2)(K+I_2))\right]\Big|_{I_2=\underline{I}(K)} < 0\ ,$$

where $\underline{I}(K)\equiv\max\left\{0,\frac{M_2^c}{A_H}-K\right\}$. That is, for $K\in[K_1^{SP},K_2^{SP})$, individual agents have an incentive to downward deviate away from the social planner's choice $I_2=\underline{I}(K)$.

In order to let the decentralized CE implement SP, we need to have subsidy τ such that for $K\in[K_1^{SP},K_2^{SP})$,

$$\left[\underbrace{u'(A_H(K+I_2))}_{=P_M}\frac{A_H}{1-\tau} - S_I^{-1}(I_2)+\frac{1-\delta_2}{1+r}V'((1-\delta_2)(K+I_2))\right]\Big|_{I_2=\underline{I}(K)} = 0.$$

Therefore, there exists a unique $\tau\left(K\right)$ for $K\in[K_1^{SP},K_2^{SP})$ such that

$$\tau(K) = 1-\left.\frac{u'(M_2^c)\,A_H}{S_I^{-1}(\underline{I}(K))-\frac{1-\delta_2}{1+r}V'((1-\delta_2)\frac{M_2^c}{A_H})}\right|_{\text{SP}}.$$

Considering $K_1^{CE}=K_2^{SP}$ given in the proof on transitional path, when $K<K_1^{SP}$, the SP and the CE have the same Euler equation as in (A.14); when $K\geq K_2^{SP}$, the SP and the CE have the same Euler equation as in (A.13). Hence, in both cases, $\tau(K)=0$. Overall, there exists an optimal intervention window.

2. Subsidizing capital purchases

The government subsidizes purchases of new durable capital at rate s_t; capital owners pay only $(1-s_t)Q_{2,t}$ per unit of new capital. By no arbitrage, the resale price of existing capital must also

be adjusted by the same subsidy rate, so that the effective price of capital in period t is $(1-s_t)Q_{2,t}$ for both new and existing units.

With subsidy, the capital owner's Bellman equation becomes

$$V_t(K_{2,t}) = \max_{I_{2,t}\geq 0}\left\{P_{2,t}(K_{2,t}+I_{2,t}) - (1-s_t)Q_{2,t}I_{2,t} - T_t + \frac{1}{1+r_t}\mathbb{E}_t[V_{t+1}(K_{2,t+1})]\right\},$$

where $T_t = s_t Q_{2,t} I_{2,t}$ is a lump-sum tax that finances the subsidy. The resulting no-arbitrage Euler equation is

$$(1-s_t)\,Q_{2,t} = \; P_{2,t} + (1-s_{t+1})\,\frac{1-\delta_2}{1+r_t}Q_{2,t+1}. \tag{A.18}$$

The planner's Euler equation is

$$\underbrace{S_I^{-1}\left(I_{2,t};R_t,z_t\right)}_{=Q_{2,t}} \;=\; \underbrace{\frac{1}{1-\Lambda'\left(M_{2,t}\right)\left(K_{2,t}+I_{2,t}\right)}}_{\textbf{externality}}\underbrace{A_{2,t}D^{-1}(M_{1,t}+M_{2,t})}_{=P_{2,t}} + \frac{1-\delta_2}{1+r_t}\underbrace{S_I^{-1}\left(I_{2,t+1};R_{t+1},z_{t+1}\right)}_{=Q_{t+1}} \tag{A.19}$$

To implement the SP allocation, the subsidy sequence must satisfy, for every t,

$$(1-s_t)\,S_I^{-1}\left(I_{2,t};R_t,z_t\right) - \left[A_{2,t}D^{-1}(M_{1,t}+M_{2,t}) + (1-s_{t+1})\,\frac{1-\delta_2}{1+r_t}S_I^{-1}\left(I_{2,t+1};R_{t+1},z_{t+1}\right)\right]\Big|_{\text{SP path}} = 0. \tag{A.20}$$

Now we consider the step function of $\Lambda(\cdot)$. Based on the proof of transitional path above, there exist two thresholds of time, t_1^* and t_2^*: when $t > t_2^*$ (with $M_{2,t} > M_2^c$), the Euler equation for SP is binding:

$$S_I^{-1}\left(I_{2,t};R_t,z_t\right) = A_H D^{-1}(M_{1,t}+M_{2,t}) + \frac{1-\delta_2}{1+r_t}S_I^{-1}\left(I_{2,t+1};R_{t+1},z_{t+1}\right); \tag{A.21}$$

when $t < t_1^*$ (with$M_{2,t} < M_2^c$), the Euler equation for SP is binding:

$$S_I^{-1}\left(I_{2,t};R_t,z_t\right) = A_L D^{-1}(M_{1,t}+M_{2,t}) + \frac{1-\delta_2}{1+r_t}S_I^{-1}\left(I_{2,t+1};R_{t+1},z_{t+1}\right); \tag{A.22}$$

when $t \in (t_1^*, t_2^*)$ (with corner solution $M_{2,t} = M_2^c$), the Euler equation for SP is not binding:

$$S_I^{-1}\left(I_{2,t};R_t,z_t\right) > A_H D^{-1}(M_{1,t}+M_{2,t}) + \frac{1-\delta_2}{1+r_t}S_I^{-1}\left(I_{2,t+1};R_{t+1},z_{t+1}\right); \tag{A.23}$$

We find the subsidy sequence by proceeding backward in time.

1) **Phase 3 ($t > t_2^*$).** Here $M_{2,t} > M_2^c$, $A_{2,t} = A_H$, and $\Lambda'(M_{2,t}) = 0$; the SP Euler equation A.21) holds with equality. Setting $s_t = 0$ for all $t > t_2^*$ makes (A.18) coincide with (A.19).

2) **Phase 2 ($t_1^* \leq t \leq t_2^*$).** The SP is at the corner solution $M_{2,t} = M_2^c$ with $A_{2,t} = A_H$, and its

Euler inequality in (A.23) is strict. Given s_{t+1} (which is zero for $t = t_2^*$), we solve (A.20) for s_t:

$$1 - s_t = \left. \frac{A_H D^{-1}(M_{1,t} + M_2^c) + (1 - s_{t+1})\frac{1-\delta_2}{1+r_t} S_I^{-1}(I_{2,t+1}; R_{t+1}, z_{t+1})}{S_I^{-1}(I_{2,t}; R_t, z_t)} \right|_{\text{SP path}}. \tag{A.24}$$

Because (A.23) holds, the numerator is strictly smaller than the denominator, so $s_t > 0$.

3) **Phase 1 ($t < t_1^*$).** Here $M_{2,t} < M_2^c$ and $A_{2,t} = A_L$; the SP Euler equation (A.22) binds. Substituting (A.22) into (A.20) gives the recursion

$$s_t = s_{t+1} \cdot \frac{1 - \delta_2}{1 + r_t} \cdot \frac{S_I^{-1}(I_{2,t+1}; R_{t+1}, z_{t+1})}{S_I^{-1}(I_{2,t}; R_t, z_t)}. \tag{A.27}$$

Since $s_{t_1^*} > 0$ and $\frac{1-\delta_2}{1+r_t} < 1$, (A.27) implies $s_t > 0$ for all $t < t_1^*$. Moreover, under the sufficient condition that R_t declines slowly enough over time, s_t decreases as we move backward in time (i.e., $s_t < s_{t+1}$ for $t < t_1^*$). This follows from the fact that $\frac{1-\delta_2}{1+r_t} < 1$. Intuitively, investment accumulates capital and adds to the capital stock that will trigger the externality, so $s_t > 0$. However, because of depreciation ($\delta_2 > 0$), a unit of capital purchased in period t will have only fraction $1 - \delta_2$ remaining when period $t + 1$ arrives. That is, early investment has a diminished effect on the capital stock, exhibiting a "discounted accumulation" effect. This makes it inefficient to invest too much too early. Consequently, the subsidy should be smaller in earlier periods and increase as time approaches t_1^*.

The resulting sequence $\{s_t\}$ is hump-shaped: positive and increasing in Phase 1, positive and relatively high in Phase 2, and zero in Phase 3 for $t > t_2^*$. If a fixed regulatory cost $c_0 > 0$ is incurred when the subsidy scheme is active, the government will intervene only when the incremental welfare gain exceeds c_0; consequently there exists an optimal intervention window that begins at some $t_{\text{start}} \in (0, t_1^*)$ and ends at t_2^*.

B Tractor price

We follow Manuelli and Seshadri (2014) and decompose the price of a new tractor into a quality component and an industry productivity component. Specifically, the price of a vintage-t tractor is

$$p_{kt} = \frac{v(\mathbf{x}_t)}{\gamma_{ct}}, \tag{B.1}$$

where $v(\mathbf{x}_t)$ maps observed characteristics $\mathbf{x}_t$ into service flow and is interpreted as quality. The term γ_{ct} captures productivity in the tractor industry at time t and shifts the quality-adjusted

price. Under this decomposition, higher quality raises prices, while higher industry productivity lowers the price per unit of quality.

To estimate quality-adjusted prices, we specify a hedonic pricing equation for model-m tractors produced at time t:

$$p_{mkt} = e^{-d_t} \prod_{j=1}^{N} \left(x_{jt}^{m}\right)^{\lambda_j} e^{\epsilon_{mt}}, \tag{B.2}$$

where p_{mkt} is the real price (CPI-deflated) of model m at time t, $\mathbf{x}_t^m = (x_{1t}^m, \ldots, x_{Nt}^m)$ is a vector of characteristics, d_t is a time effect that captures industry productivity, and ϵ_{mt} is an idiosyncratic pricing error.

Taking logs yields a linear specification:

$$\ln p_{mkt} = -d_t + \sum_{j=1}^{N} \lambda_j \ln x_{jt}^{m} + \epsilon_{mt}. \tag{B.3}$$

In this regression, the time fixed effects $-d_t$ summarize changes in the quality-adjusted price level over time, while the characteristics term $\sum_j \lambda_j \ln x_{jt}^m$ captures measured quality.

We assemble tractor price data for China over 1978–2013 for two categories (small and medium/large) and deflate nominal prices using the CPI. Because detailed historical characteristics are limited in China, we proxy the time profile of technical progress using U.S. tractor data for 1920–1955 from White (2000) and assume that China' s characteristic frontier during 1978–2013 follows the same trend. Our characteristics vector includes horsepower, RPM, speed, and a dummy for tractor model. We estimate the hedonic equation by pooled OLS with time fixed effects.

Let $\hat{p}_{mkt}$ and $\hat{d}_t$ denote fitted values from the hedonic regression. We compute the average price of vintage-t tractors as a sales-share-weighted average,

$$\bar{p}_{kt} = \sum_{m} s_{mkt} \hat{p}_{mkt}, \tag{B.4}$$

where $m \in \{\text{small, medium/large}\}$ indexes the tractor model. s_{mkt} is model m's share in total tractor sales at time t. We then recover tractor-industry productivity as

$$\hat{\gamma}_{ct} = e^{\hat{d}_t}. \tag{B.5}$$

Finally, we compute an average quality index for vintage-t tractors as

$$\bar{v}(\mathbf{x}_t) = \bar{p}_{kt} \hat{\gamma}_{ct}. \tag{B.6}$$

This object summarizes measured quality net of changes in the quality-adjusted price level.

Figure 10 reports the resulting series. The dashed line plots the estimated vintage price $\bar{p}_{kt}$, while the solid line plots the quality-adjusted price $\hat{\gamma}_{ct}^{-1}$ implied by the decomposition above.

C Estimating Returns to Scale

We assume agricultural output is produced with a Cobb–Douglas technology:

$$\mathcal{Y}_t = A_{2t} L_t^{\alpha} M_{2t}^{\beta} S_t^{\gamma} H_{1t}^{\mu} H_{2t}^{\varepsilon}, \tag{C.1}$$

where $\mathcal{Y}_t$ is agricultural output, L_t is labor input, $M_{2t} = K_t + X_t$ is tractor services, S_t is land, and H_{1t} and H_{2t} denote seeds and fertilizer. The term A_{2t} captures tractor productivity.

Our data are measured per unit of land, so we normalize inputs and output by S_t. Using observed factor shares, we set $\alpha = 0.30$, $\eta = 0.41$, $\gamma = 0.10$, $\mu = 0.05$, and $\varepsilon = 0.14$. After this step, the remaining objects to be recovered are A_t and M_{2t}.

We construct tractor services by aggregating quality-adjusted tractor vintages:

$$M_{2t} = \sum_{\tau=1978}^{t} \tilde{m}_{kt}(\tau) \frac{v(\mathbf{x}_\tau)^{\theta}}{\gamma_{c\tau}}, \tag{C.2}$$

where $\tilde{m}_{kt}(\tau)$ is the number of vintage-τ tractors operated at time t, $v(\mathbf{x}_\tau)$ is vintage quality, and $\gamma_{c\tau}$ is tractor-industry productivity at vintage date τ. We set $\theta = 0.1$.

We model the operating stock of vintage-τ tractors as

$$\tilde{m}_{kt}(\tau) \equiv m_{k\tau}(1 - \delta_k)^{t-\tau}, \tag{C.3}$$

where $m_{k\tau}$ is new purchases of vintage τ and δ_k is the depreciation rate. We set $\delta_k = 0.07$ to fit the observed average tractor depreciation rate.

Given δ_k and the observed tractor stock at each date, we invert the stock-accumulation equation to recover the vintage purchase series $\{m_{k\tau}\}_{\tau \le t}$ and hence M_{2t}. We then back out A_{2t} as the residual implied by the production function and report the resulting series in Figure 13.