Measuring Assortative Matching: A Unified Axiomatic Framework

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Abstract

Assortative matching in marriage and partnership markets is central to inequality and inter-generational mobility, yet its measurement remains contested because existing indices respond differently to changes in group size and population composition. Recent work by Chiappori et al. (2025) proposed axioms for aggregate measures and concluded that both the trace and the odds ratio satisfy them. This paper develops an axiomatic framework for measuring assortative matching at both the type-specific level—previously absent from the literature—and the aggregate level. We uniquely characterize a simple and interpretable index, the Average Homogamy Share (AHS_i) for type i, which is defined for each group i as the average of its male-side and female-side homogamy shares. We then derive aggregate measures from the AHS and propose refined axioms, under which the trace index approximates our preferred index when gender imbalances are small, while the odds ratio is ruled out as theoretically inconsistent. Applying our framework to U.S. data from 1962 to 2024, we document a decline in assortative matching among low-educated groups and an increase among highly educated individuals, while racial assortative matching remains high-at around 85%-despite an overall decline. The type-specific measure yields results similar to the homogamy rate when gender imbalances are small, but differences become pronounced for highly selective groups, such as postgraduate education, where assortative matching may have the greatest implications for inequality and elite-group formation. We further decompose assortative matching by gender, distinguish between random and positive (or intentional) matching, analyze cross-type matching patterns, and extend AHS to multi-dimensional settings through population decomposability. By combining axiomatic rigor with rich empirical evidence, this paper refines the theoretical foundations of assortative matching and clarifies which measures are appropriate for economic analysis.

Keywords: assortative matching, marital homogamy, marital sorting, axiomatic analysis, IPUMS, education, race

JEL codes: I24; I32; J12

1 Introduction

The recent rise in economic inequality has renewed interest in inter-household inequality and marital sorting. Partner selection plays a central role in shaping aggregate outcomes such as income inequality, social mobility, and equality of opportunity (Bingley et al., 2022, Heblich et al., 2021, Reardon and Bischoff, 2011). Assortative matching (AM)-the extent to which individuals form partnerships with others who share similar characteristics-is therefore a key determinant of inequality. Yet measuring AM is inherently challenging. Changes in group sizes— driven by educational expansion, demographic shifts, or migration—can mechanically increase the number of homogamous couples even when the underlying matching behavior remains unchanged. Distinguishing genuine changes in assortative behavior from shifts in population composition is thus central to understanding the evolution of inequality.

This distinction carries substantial implications for empirical work and is crucial for interpreting trends in AM correctly. Existing measures of assortative matching respond differently to changes in marginal distributions (Eika et al., 2019, Chiappori et al., 2020a, 2025, Greenwood et al., 2014, Lichter and Qian, 2019, Liu and Lu, 2006). As a result, empirical conclusions about trends of assortative matching often depend on the metric employed. For example, Chiappori et al. (2025) propose axioms for aggregate measures and show that both the trace and the odds ratio satisfy them, yet these measures yield contradictory assessments of how AM has evolved.

To address this challenge, we develop a unified axiomatic framework for measuring assortative matching that integrates type-specific and aggregate perspectives. We begin by establishing axioms for type-specific measures that ensure two core properties: (i) matching intensity is isolated from population-scale effects (homogeneity of degree zero), and (ii) gender imbalances are properly accounted for (marginal scale asymmetry). Within this framework, we propose the Average Homogamy Share for type i (AHS_i), defined as the simple average of the male and female homogamy shares. We show that AHS_i satisfies all axioms and provide a unique characterization of the measure. By contrast, widely used alternatives, such as the likelihood ratio, violate homogeneity of degree zero and monotonicity, thereby conflating behavioral changes in matching with compositional shifts.

We then derive aggregate measures from AHS_i and refine the axiomatic analysis of aggregate assortative matching, building on Chiappori et al. (2025). Specifically, we introduce a marginal sensitivity axiom: holding constant the total number of homogamous matches and total population size, achieving a given diagonal mass should be considered more demanding in a population with gender-imbalanced marginals than in one with balanced marginals.This requirement formalizes the intuition that matching under tighter feasibility constraints reflects stronger assortative forces. Our preferred aggregate measure satisfies marginal sensitivity along with all other axioms. In contrast, the trace and the odds ratio fail marginal sensitivity, but the trace aligns with our measure when gender imbalances are small.

The choice between AHS and the trace ultimately depends on the research question. If the objective is to describe the level of inequality generated by assortative matching at a point in time, aggregate measures such as the trace may suffice, as they summarize the overall concentration of homogamous couples. However, such measures abstract from feasibility constraints induced by gender imbalances. By contrast, AHS incorporates these constraints and therefore reveals the underlying behavioral process: achieving a

given number of homogamous matches is more demanding when one gender is undersupplied. Consequently, AHS increases not only with rising homogamy but also with stronger assortative behaviour relative to the constraints faced.

The odds ratio performs worse on both fronts. Beyond being limited to two groups, it collapses distinct matching patterns, assigning the same value to full homogamy for both genders and to full homogamy for only one gender, making it a poor measure of either matching intensity or inequality.

Finally, we empirically compare our new measure with existing indices, showing how the empirical patterns reveal the limitations of alternative measures and underscore the internal consistency and robustness of our approach. Using USA marriage data from 1962 to 2024, we first study educational assortative matching, which is less endogenous than income or occupation and strongly predicts socioeconomic status. We find that AM has declined among low-educated groups and increased among highly educated individuals, with aggregate trends sensitive to education categorization. We then analyze racial assortative matching across four groups: White, Black/African American, Hispanic, and Asian/Other. Although racial AM has declined overall, it remains high at around 85%. We also uncover a U-shaped relationship between group size and AM: small groups exhibit strong assortative matching, while homogamy among large groups rises with group size. This result is consistent with small groups being more affected by segregation, and random matching increasing mechanically with group size.

Once the random-matching component is accounted for, intentional assortative matching increases in aggregate terms, driven by the growing segregation pressures faced by the low-educated and lower tolerance among the white population toward other racial groups. Decomposition by gender reveals the importance of structural constraints, especially among postgraduates, where gender imbalances generate substantial involuntary cross-type matching. Additional analyses show that cross-type matching is more likely among individuals with similar types of education and that non-white individuals are more likely to partner with white individuals than with members of other minority groups. Joint decompositions by race and education reveal that low-educated non-white individuals face the strongest segregation pressures and are the most vulnerable to social isolation. Together, these extensions underscore the flexibility, interpretability, and empirical power of the AHS measure.

The remainder of the paper is organized as follows. Section 2 introduces our type-specific measure, presents a set of axioms, and evaluates existing type-specific indices against these axioms. Section 3 defines aggregate measures derived from our type-specific measure, proposes axioms for aggregate measures, and examines the compliance of commonly used measures. Section 4 presents the empirical analysis. Section 5 explores extensions and applications using our measure exclusively. Section 6 concludes.

2 Type-specific measures

In this section, we define a new type-specific measure, present a set of axioms that any type-specific measure should satisfy, and evaluate which existing type-specific measures satisfy these axioms. We also uniquely characterize our measure by defining an additional property.

Let us first define the conceptual framework. We use the education dimension in the explanation, but it can be applied to other individual characteristics, such as race

or income decile. We consider three categories because they allow for the most general framework. Extending it to more categories or reducing it to two is trivial once the three-category case is set.

Consider three types of education: type 1 corresponds to college (C), type 2 to some college (SC), and type 3 to high school (HS). Figure 1 presents the corresponding contingency table, denoted as matrix M. Let m_{ij} denote the mass of couples in which the men have a type of education i and the women have a type of education j. Here m_{ij} is the (i,j)th entry of the matrix M. For instance, $m_{12} = b$ represents the mass of couples in which men have a college education (type 1) and women have some college education (type 2). Consequently, the share of men who have education type i is $\sum_{j=1}^{3} m_{ij}$ and the share of women who have education type i is $\sum_{j=1}^{3} m_{ji}$. The total number of couples is:

$$|M| = \sum_{i=1}^{3}\sum_{j=1}^{3} m_{ij} = a + b + c + d + v + w + x + y + z. \tag{1}$$

For any i, the diagonal entry m_{ii} represents the mass of couples in which both the man and the woman have education level i. We assume that for every type i, both $\sum_j m_{ij}$ and $\sum_j m_{ji}$ are strictly positive; that is, there exist men and women with education level i in the population.

The two-education setup in Chiappori et al. (2025) is subsumed in our model and can be obtained by taking arbitrary values of a, b, c, d and setting $v = w = 0 = x = y = z$.

2.1 Definition of our type-specific measure (AHS_i)

In this section, we introduce a new measure of assortative matching *specific to each education type i*. For any education type $i \in \{1,2,3\}$, assortative matching for type i or average homogamy share for type i is AHS_i^M, where

$$AHS_i^M = \frac{1}{2}\left(\frac{m_{ii}}{\sum_j m_{ij}} + \frac{m_{ii}}{\sum_j m_{ji}}\right) \tag{2}$$

Henceforth, we denote AHS_i^M by AHS_i, specifying the matrix only when required. The measure AHS_i is defined as the average of two shares: the proportion of men with education type i who are in a homogamous couple, and the proportion of women with education type i who are in a homogamous couple. It is worth emphasizing that this measure is defined for each education type, enabling a detailed analysis of assortative matching for each type. It complements aggregate measures, which may obscure important variation by education type and fail to identify type-specific patterns.

AHS_i ranges between 0 and 1 since $0 \leq m_{ii} \leq \sum_j m_{ij}$ and $m_{ii} \leq \sum_j m_{ji}$. Therefore, the maximum value of AHS_i for any type i is 1. The specific value of AHS_i measures the level of assortative matching in education type i, which can be interpreted as the average share of homogamy in the economy.

One can determine whether the population exhibits positive or negative assortative matching by comparing the observed level of assortative matching to the expected level of

assortative matching under random matching. For any matrix M, let the corresponding matrix that represents the expected random matching be M^{rm}. The (i,j)th element of this matrix is the expected number of couples (i,j) if matching were random.

$$m_{ij}^{rm} = \frac{(\sum_k m_{ik})(\sum_k m_{kj})}{|M|}$$

This is the number of random matches between men of type i and women of type j. The measure of random assortative matching for type i is denoted by AHS_i^{rm}, where,

$$AHS_i^{rm} = \frac{1}{2}\left(\frac{m_{ii}^{rm}}{\sum_j m_{ij}} + \frac{m_{ii}^{rm}}{\sum_j m_{ji}}\right) \tag{3}$$

The above measure of random matching can be used to determine when a population exhibits positive assortative matching (PAM) or negative assortative matching (NAM).

Definition 1. *For any type i, a matrix M exhibits positive assortative matching when $AHS_i \geq AHS_i^{rm}$, and negative assortative matching when $AHS_i < AHS_i^{rm}$.*

The following lemma states an equivalent condition for PAM and NAM.

Lemma 1. *A matrix M exhibits PAM for type i if*

$$m_{ii} \geq \frac{(\sum_j m_{ij})(\sum_j m_{ji})}{|M|}.$$

A matrix M exhibits NAM for type i if

$$m_{ii} < \frac{(\sum_j m_{ij})(\sum_j m_{ji})}{|M|}.$$

Proof. The proof follows directly from the definitions above. □

In words, if the mass of observed homogamous couples of type i is greater (smaller) than the expected mass of homogamous couples of type i, then there is positive (negative) assortative matching. Moreover, we can measure the extent of positive assortative matching exhibited under our measure as follows. For any population M, the level of positive assortative matching for any type i is $PAHS_i$ where

$$PAHS_i = AHS_i - AHS_i^{rm}. \tag{4}$$

To understand the maximum level of assortative matching that is attainable for any population, we define the maximum feasible level of assortative matching (also discussed in Liu and Lu (2006) and Kirkeboen et al. (2021)). Suppose the mass of women with education i is lower than the mass of men with education type i, that is, $\sum_j m_{ji} < \sum_j m_{ij}$. Then, the maximum feasible level of assortative matching for i occurs when all women with education i marry men with education type i. In this case, a mass of $\sum_j m_{ij} - \sum_j m_{ji}$ men have to marry women of a different education type, simply because all women of

education type i are already matched. Thus, the maximum feasible level of AHS_i is computed as follows.

$$AHS_i^{max} = \frac{1}{2}\min\left\{1 + \frac{\sum_j m_{ji}}{\sum_j m_{ij}}, 1 + \frac{\sum_j m_{ij}}{\sum_j m_{ji}}\right\} \tag{5}$$

For the specific example above,

$$AHS_i^{max} = \frac{1}{2}\left(1 + \frac{\sum_j m_{ji}}{\sum_j m_{ij}}\right) < 1$$

Note that $AHS_i^{max} \leq 1$ and equality holds only if $\sum_j m_{ij} = \sum_j m_{ji}$. Hence, equality holds in the case of full homogamy, that is, when there are no cross-type matches. Moreover, $1 - AHS_i^{max}$ measures the share of cross-type matches that are *involuntary* or *structural*. The maximum feasible level of assortative matching embodies a key axiom that any measure of assortative matching should satisfy. We elaborate on this and related properties in the following section.

2.2 Axioms for a type-specific measure

In this section, we define a set of axioms that any type-specific measure of assortative matching should satisfy.

Axiom S1: Measure definition. For any matrix M and any type i, a measure of assortative matching for type i is $\tilde{I}_i$, which is a function of the mass of homogamous couples m_{ii}, the total mass of men of type i $\left(\sum_j m_{ij}\right)$, and total mass of women of type i $\left(\sum_j m_{ji}\right)$.

Formally, a type-specific measure of assortative matching is a function $\tilde{I}_i : \mathbb{R}^3_+ \setminus \{\mathbf{0}\} \to \mathbb{R}_+$, where the arguments correspond to $\left(m_{ii}, \sum_j m_{ij}, \sum_j m_{ji}\right)$.

Axiom S2: Homogeneity of degree zero. The measure $\tilde{I}_i$ is homogeneous of degree zero in all three arguments. That is, for any $\lambda > 0$,

$$\tilde{I}_i\left(\lambda m_{ii}, \lambda\sum_j m_{ij}, \lambda\sum_j m_{ji}\right) = \tilde{I}_i\left(m_{ii}, \sum_j m_{ij}, \sum_j m_{ji}\right)$$

This axiom requires that if the three arguments increase proportionally, then the assortative matching for i remains unaffected. This axiom ensures that the measure isolates the intensity of assortative matching from population-scale effects.

Lemma 2. *Axiom S2 has two implications:*

1. *The measure $\tilde{I}_i$ can always be expressed as a function of the share of homogamous couples of type i, the share of men of type i, and the share of women of type i, that is,*

$$\tilde{I}_i(m_{ii}, \sum_j m_{ij}, \sum_j m_{ji}) = \tilde{I}_i\left(\frac{m_{ii}}{|M|}, \frac{\sum_j m_{ij}}{|M|}, \frac{\sum_j m_{ji}}{|M|}\right).$$

2. *The measure $\tilde{I}_i$ can be expressed as I_i, a function of the share of homogamous couples among women and men of type i, that is,*

$$I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right).$$

Proof. The first claim is obtained directly by assuming $\lambda = 1/|M|$ in axiom S2. For the second claim, let $\lambda = 1/m_{ii}$. Then, by the homogeneity of degree zero, $\tilde{I}_i$ can be written as I_i such that:

$$\tilde{I}_i\left(\lambda m_{ii}, \lambda \sum_j m_{ij}, \lambda \sum_j m_{ji}\right) = \tilde{I}_i\left(1, \frac{\sum_j m_{ij}}{m_{ii}}, \frac{\sum_j m_{ji}}{m_{ii}}\right) = I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right)$$

□

By Lemma 2, a measure of assortative matching for type i can be expressed as a function of the share of men and women of type i who exhibit homogamy. From this point on, all axioms will be formulated using the I_i index.

Axiom S3: Side invariance. For any matrix M and any type i, the measure of assortative matching is invariant to the labeling of men and women. That is, if M' is obtained by interchanging the i-th row and the i-th column of M, then:

$$I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) = I_i\left(\frac{m_{ii}}{\sum_j m_{ji}}, \frac{m_{ii}}{\sum_j m_{ij}}\right)$$

This axiom ensures that the measure treats men and women symmetrically, reflecting the idea that the direction of matching (male-to-female or female-to-male) should not affect the level of assortative matching.

Axiom S4: Monotonicity. For any matrix M, a measure $I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right)$ is strictly increasing in each argument, keeping the other argument fixed.

The axiom requires that if the share of homogamous couples for men or women of type i increases, then the level of assortative matching for type i increases. In other words, the measure increases with diagonal elements and decreases with off-diagonal elements. This also gives the following result.

Lemma 3. *The monotonicity axiom implies two properties:*

1. ***full homogamy*** *- for any matrix M and any type i such that $m_{ii} = \sum_j m_{ij} = \sum_j m_{ji}$, there exists no matrix M' that exhibits higher assortative matching for i.*
2. ***partial homogamy*** *- for any matrix M and any i that does not satisfy $m_{ii} = \sum_j m_{ij} = \sum_j m_{ji}$, there exists M' that exhibits higher assortative matching for i.*

Proof. For matrices M and M', $m'_{ii}/\sum_j m'_{ij} \leq 1 = m_{ii}/\sum_j m_{ij}$ and $m'_{ii}/\sum_j m'_{ji} \leq 1 = m_{ii}/\sum_j m_{ji}$ where one or both inequalities are strict. By monotonicity, it follows that $I_i(M') < I_i(M)$, thereby proving the first implication. For the second implication, note that when $m_{ii} = \sum_j m_{ij} = \sum_j m_{ji}$ does not hold, then without loss of generality, $m_{ii} < \sum_j m_{ij}$. By monotonicity, decreasing $\sum_j m_{ij}$, keeping all else fixed, increases assortative matching, thereby proving the second implication. □

Intuitively, Lemma 3 holds as when some mass of men or women of type i match with other individuals of type $j \neq i$, then the level of assortative matching for i increases by decreasing the non-homogamous matches, in other words, by decreasing m_{ij} and keeping all else equal.

Axiom S5: Boundedness. For any matrix M and any type i, a measure of assortative matching satisfies the following bounds:

$$\min\left\{\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right\} \leq I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) \leq \max\left\{\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right\}.$$

This axiom requires the index for type i to lie between the smaller and the larger of the homogamy shares observed for men and women of that type. These two shares provide natural empirical bounds and an index outside this interval would ascribe to type i a level of homogamy that neither men nor women of that type exhibit, thereby violating internal consistency. Boundedness thus keeps the index disciplined by the data and ensures a transparent interpretation.

Lemma 4. *The boundedness axiom implies that the measure I_i is bounded between 0 and 1. Moreover, by boundedness, $I_i(M) = 0$ if and only if $m_{ii} = 0$ and, by boundedness and monotonicity, $I_i(1,1) = 1$ if and only if $m_{ii} = \sum_j m_{ij} = \sum_j m_{ji}$.*

Proof. Since the homogamous share of men or women lies between 0 and 1 and I_i is bounded by the minimum and maximum of the shares, it follows that I_i lies between 0 and 1.

When $m_{ii} = 0$, both homogamy rates are equal to zero, so boundedness implies $0 \leq I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) \leq 0$, thereby proving $I_i = 0$. This is also the necessary condition for the index to take value 0, as when $m_{ii} > 0$, the minimum of the homogamy shares is

positive, and so is the index. Instead, when $m_{ii} = \sum_j m_{ij} = \sum_j m_{ji}$, all men and women of type i are homogamous and, $1 \leq I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) \leq 1$. Thus, $I_i(1,1) = 1$. Instead, if one of the shares is strictly less than 1, then by monotonicity the index must be less than $I_i(1,1)$, that is, less than 1. □

Axiom S6: Bisymmetry. For any non-negative values x, y, u, v, a measure I_i satisfies the bisymmetry condition:

$$I_i\left(I_i\left(x,y\right), I_i\left(u,v\right)\right) = I_i\left(I_i\left(x,v\right), I_i\left(u,y\right)\right).$$

This is a technical axiom. The bisymmetry axiom requires an index to yield the same outcome regardless of how information from multiple subgroups of type i is combined. Specifically, the measure must be invariant to whether male and female homogamy shares are first combined within each subgroup and then compared across subgroups, or first compared across subgroups within each gender and subsequently combined. This condition ensures that the overall assessment of homogamy for type i is not affected by the order of the combination between subgroups.

Lemma 5. *Let $I_i : \mathbb{R}^2_+ \setminus \{\mathbf{0}\} \to \mathbb{R}_+$ be a type-specific measure of assortative matching. The measure satisfies side invariance, monotonicity, boundedness, and bisymmetry if and only if I_i is a quasi-arithmetic mean.*

Proof. The proof follows from Corollary 10 in Burai et al. (2021). □

Axiom S7: Homogeneity of degree one. A measure I_i is homogeneous of degree one in both arguments. That is, for any scalar $\lambda > 0$,

$$I_i\left(\frac{\lambda m_{ii}}{\sum_j m_{ij}}, \frac{\lambda m_{ii}}{\sum_j m_{ji}}\right) = \lambda I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right)$$

This axiom implies that if both shares of homogamous couples are scaled by λ, the measure of assortative matching scales proportionally. For example, suppose that the share of homogamous couples for men and women of type i is twice that of the other population. In that case, the measure of assortative matching in the former population is twice as large.

Lemma 6. *Let $I_i : \mathbb{R}^2_+ \setminus \{\mathbf{0}\} \to \mathbb{R}_+$ be a type-specific measure that is the quasi-arithmetic mean of its arguments. Then I_i is a power mean if and only if it satisfies homogeneity of degree one.*

Proof. The proof follows from Aczél and Oser (2006). □

This Lemma states that any measure that satisfies side invariance (S3), monotonicity (S4), boundedness (S5), bisymmetry (S6), and homogeneity of degree one (S7) is a

generalized or power mean. That is,

$$I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) = \left(\frac{1}{2}\right)^{1/p}\left(\left(\frac{m_{ii}}{\sum_j m_{ij}}\right)^p + \left(\frac{m_{ii}}{\sum_j m_{ji}}\right)^p\right)^{1/p},$$

where p is a non-zero real number. We highlight three specific power means of interest.

- For $p = 1$, the index is the arithmetic mean and gives our proposed measure, the average homogamy share (AHS_i).
- For $p = -1$, the index is the harmonic mean and gives the harmonic homogamy share measure (HHS_i). This index is also called the homogamy rate.
- As $p \to 0$, the index is the geometric mean and gives the geometric homogamy share measure (GHS_i).

Next, we introduce an axiom that allows an asymmetric influence of the size of the gender group on the measure.

Axiom S8: Marginal scale asymmetry. For any matrix M such that $\sum_j m_{ij} < \sum_j m_{ji}$ and any $\lambda > 1$, assortative matching for i is lower for M' than for M''; where M' is obtained by scaling $\sum_j m_{ij}$ by λ, M'' is obtained by scaling $\sum_j m_{ji}$ by λ and keeping m_{ii} constant for both, that is,

$$I_i\left(\frac{m_{ii}}{\lambda\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) < I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\lambda\sum_j m_{ji}}\right).$$

Intuitively, this axiom requires that a measure of assortative matching be more responsive to changes in the size of the smaller side of the market. Suppose that for type i, men are fewer than women. Homogamous matching is then constrained by the scarcity of men: the number of same-type couples cannot exceed their number. Holding m_{ii} fixed, increasing the smaller group relaxes this binding constraint and raises the maximum feasible level of homogamy, whereas increasing the larger group does not. Consequently, if the number of homogamous couples remains unchanged, assortative matching should decline more when the smaller group expands than when the larger group does.

Consider the corner case with $m_{ii} = \sum_j m_{ij}$ and $\sum_j m_{ij} < \sum_j m_{ji}$. An increase in the mass of women should affect assortative matching of i much less than an increase in the mass of men, as the latter forms the smaller group and restricts the maximal feasible assortative matching. In other words, the effect of scaling $\sum_j m_{ij}$ or $\sum_j m_{ji}$ should not be symmetric; the smaller group should have a larger effect on the measure of assortative matching.

2.3 Comparing type-specific measures

Table 1 summarizes which of the proposed axioms are satisfied by each type-specific measure, and which are violated. The comparison includes the commonly used likelihood

ratio, LR_i, defined in Eika et al. (2019), as well as the three power means introduced above. All proofs are provided in the Appendix.

The LR_i fails to satisfy the homogeneity of degree zero, monotonicity, boundedness, and homogeneity of degree one axioms. Violating homogeneity of degree zero means that the measure is excessively sensitive to changes in group scale. Failure of monotonicity implies that a society with a higher share of homogamous couples may nonetheless display a lower value of LR_i than a society with a lower share of homogamous couples. Finally, without boundedness and homogeneity of degree one, the interpretation of the index becomes less transparent.

The three power mean measures satisfy axioms S1 to S7. Among them, only AHS_i satisfies the marginal scale asymmetry axiom.[1]

	LR_i	AHS_i	GHS_i	HHS_i
S1: Measure definition	no	yes	yes	yes
S2: Homogeneity of degree zero	no	yes	yes	yes
S3: Side invariance	yes	yes	yes	yes
S4: Monotonicity	no	yes	yes	yes
S5: Boundedness	no	yes	yes	yes
S6: Bisymmetry	–	yes	yes	yes
S7: Homogeneity of degree one	no	yes	yes	yes
S8: Marginal scale asymmetry	yes	yes	no	no

Table 1: Overview of axioms satisfied by each type-specific measure.

In what follows, we propose a property that gives a full characterization of AHS_i.

2.4 Characterization property for AHS_i

Below is the property that uniquely characterizes AHS_i.

Property 1: Additive homogeneity. For any $x, y, c \geq 0$, a measure I_i is additively homogeneous in both arguments, that is,

$$I_i(x + c, y + c) = I_i(x, y) + c.$$

Thus, if the share of homogamous men and women of type i increases by c, the measure of assortative matching also increases by c. This property ensures interpretability and, linearity allows the measure to reflect changes in homogamy in a transparent and proportional way. This makes comparisons across groups and time meaningful. The average homogamy share is the only power mean that satisfies this property.

Lemma 7. *Consider a type-specific measure that is a power mean. A measure satisfies additive homogeneity if and only if it is the average homogamy share measure.*

Proof. The proof can be found in Bullen (2013), and is also a standard result in the theory of means. □

[1] All power means with $p > 0$ satisfy the marginal scale asymmetry axiom, whereas none of the remaining power means do. For $p < 0$ (including HHS_i), the inequality reverses. As $p \to 0$ (corresponding to GHS_i), the expression holds with equality.

The above result implies that a measure that satisfies the axioms $S1-S7$ and Property 1, also satisfies $S8$. Furthermore, AHS_i satisfies another desirable property: population decomposability.

Property 2: Population decomposability. For any decomposition of a matrix M into matrices M_1 and M_2, the assortative matching index for type i satisfies:

$$I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) = \alpha_1 I_i\left(\frac{m^1_{ii}}{\sum_j m^1_{ij}}, \frac{m^1_{ii}}{\sum_j m^1_{ji}}\right) + \alpha_2 I_i\left(\frac{m^2_{ii}}{\sum_j m^2_{ij}}, \frac{m^2_{ii}}{\sum_j m^2_{ji}}\right),$$

where α_k is a function of the sums $(\sum_j m^k_{ij},\ \sum_j m^k_{ji})$, and $(\sum_j m_{ij}, \sum_j m_{ji})$. Further, if a matrix M is a decomposition of K matrices, then,

$$I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) = \sum_{k=1}^{K} \alpha_k I_i\left(\frac{m^k_{ii}}{\sum_j m^k_{ij}}, \frac{m^k_{ii}}{\sum_j m^k_{ji}}\right).$$

This property requires that for any population decomposition, the level of assortative matching can be expressed as a weighted sum of assortative matching levels across all subpopulations. The average homogamy share, the $Trace$, and the geometric homogamy share satisfy this property, with corresponding weights α_k given by

$$\frac{am(\sum_j m^k_{ij}, \sum_j m^k_{ji})}{am(\sum_j m_{ij}, \sum_j m_{ji})}, \frac{hm(\sum_j m^k_{ij}, \sum_j m^k_{ji})}{hm(\sum_j m_{ij}, \sum_j m_{ji})}, \frac{gm(\sum_j m^k_{ij}, \sum_j m^k_{ji})}{gm(\sum_j m_{ij}, \sum_j m_{ji})},$$

respectively. Here, $am(.,.)$, $hm(.,.)$ and $gm(.,.)$ denote the arithmetic, harmonic, and geometric mean, respectively. In contrast, the likelihood ratio does not satisfy this property, since LR_i fails axiom S1. Specifically, LR_i depends on the number of homogamous couples of other types $j \neq i$.[2]

3 Aggregate Measures

In this section, we derive two aggregate measures from the AHS_i measure, propose a set of axioms for aggregate measures of assortative matching, and evaluate which existing aggregate measures satisfy these axioms.

3.1 Aggregate measures based on AHS_i

We propose two aggregate measures of assortative matching, based on the type-specific AHS_i index. In the literature, a common approach is to compute a weighted sum of the index across types. For instance, Chiappori et al. (2020b), Greenwood et al. (2014), and Eika et al. (2019) use weighted averages of likelihood ratios to construct aggregate

[2]Note that LR_i would satisfy population decomposability if the weights were allowed to depend on the relative population sizes $|M_i|$ and $|M|$. Using weights $\alpha_i = \frac{(\sum m_{ij})(\sum m_{ji})}{|M|} \Big/ \frac{(\sum m_{ij}+\sum m'_{ij})(\sum m_{ji}+\sum m'_{ji})}{|M+M'|}$, LR_i satisfies the axiom.

measures of assortative matching. The core idea is to aggregate the type-specific indices using weights derived from the marginal distributions. Formally, an aggregated index of assortative matching, denoted by $I(M)$, is defined as:

$$I(M) = \sum_i \phi_i I_i, \tag{6}$$

where I_i is the type-specific index and ϕ_i is the weight assigned to type i. These weights adjust for changes in the marginal distributions over time and across regions.

We consider two alternative weighting schemes. First, Greenwood et al. (2014) use as weights the expected mass of homogamous pairs under random matching:

$$\phi_i^G = \frac{(\sum_j m_{ij})(\sum_j m_{ji})}{\sum_i (\sum_j m_{ij} \sum_j m_{ji})}.$$

Using these weights, the Average Homogamy Share, denoted by AHS_G is:

$$AHS_G = \frac{\sum_i m_{ii}(\sum_j m_{ij} + \sum_j m_{ji})}{2\sum_i (\sum_j m_{ij} \sum_j m_{ji})}. \tag{7}$$

Chiappori et al. (2020b) propose using the marginal distributions of men and women for the weights. Since there are two distributions, they take the average of the two weighted indices. This is equivalent to weighting by the average share of individuals within each type in the population, and the weight for any type i is:

$$\phi_i^C = \frac{\sum_j m_{ij} + \sum_j m_{ji}}{2|M|}.$$

Using these weights, the Average Homogamy Share, denoted by AHS_C, is:

$$\begin{aligned} AHS_C &= \sum_i \left(\frac{\sum_j m_{ij} + \sum_j m_{ji}}{2|M|} \right) AHS_i, \\ &= \sum_i \frac{\left(\frac{\sum_j m_{ij} + \sum_j m_{ji}}{2} \right)^2}{(\sum_j m_{ij})(\sum_j m_{ji})} \frac{m_{ii}}{|M|}, \\ &= \sum_i \left(\frac{am_i}{gm_i} \right)^2 \frac{m_{ii}}{|M|}, \end{aligned} \tag{8}$$

where am_i and gm_i are the arithmetic and geometric mean of the number of men and women of type i, respectively.

3.2 Axioms for aggregate measures

In this section, we propose a set of axioms that any aggregate measure of assortative matching should satisfy. These are based on the framework introduced by Chiappori et al. (2025). We refine and extend their axioms.

Invariance Axioms

We retain the invariance axioms proposed by Chiappori et al. (2025), which ensure that the measure is unaffected by certain transformations of the matching matrix.

Axiom A1: Scale invariance. For any matrix M and scalar $\lambda > 0$, the measure satisfies $I(\lambda M) = I(M)$.

Scale invariance requires that an index should depend only on relative frequencies, not on the absolute size of the matrix. Intuitively, two populations that differ only in size but share the same type distribution and the same matching patterns should exhibit the same level of assortative matching and therefore receive the same index value. This axiom is the analog of axiom $S2$.

Axiom A2: Side invariance. For any matrix M and its transpose M', the measure satisfies $I(M) = I(M')$.

This axiom requires that a measure should be invariant to the labeling of men and women. It is similar to axiom $S3$, but it applies to the whole matrix.

Axiom A3: Type invariance. Permuting types does not affect the level of assortative matching.

This axiom ensures symmetry across types: no type is intrinsically special.

Monotonicity axiom

We generalize the marginal monotonicity axiom of Chiappori et al. (2025) below and discuss it in detail in the appendix.

Axiom A4: Marginal Monotonicity. For any positive matrices M and M' with the same marginal distributions, that is, for all i, $\sum_j m_{ij} = \sum_j m'_{ij}$ and $\sum_j m_{ji} = \sum_j m'_{ji}$, if $m_{ii} > m'_{ii}$ for some i and $m_{jj} \geq m'_{jj}$ for all $j \neq i$, then $I(M) > I(M')$.

Increasing the number of homogamous matches of any type—without reducing the diagonal mass of other types—should strictly increase the index. The axiom reflects the idea that, given fixed marginal constraints, shifting additional mass onto the diagonal unambiguously increases assortativity.

A stricter monotonicity axiom would require that the addition of homogamous couples always increases assortative matching (diagonal monotonicity), while the addition of non-homogamous couples always decreases it (off-diagonal monotonicity), independently of any induced changes in population size or composition. Such requirements are generally inappropriate for aggregate measures constructed as weighted averages of type-specific homogamy. In these measures, changes in matches affect both the level of homogamy within each type and the relative population shares used as weights. As a result, diagonal and off-diagonal monotonicity need not hold. To illustrate, consider a two-type population in which the number of homogamous couples of type 1 increases. This raises both the type-specific index I_1 and its population share ϕ_1, while reducing the share of type

2. The aggregate index is:

$$I = \phi_1 I_1 + (1 - \phi_1) I_2.$$

If I_2 is sufficiently large, the decline in its weight may more than offset the increase in $\phi_1 I_1$, causing the aggregate index I to fall. This outcome is not problematic: the aggregate measure is designed to capture both within-type homogamy and the relative size of each type. An increase in homogamy for one type necessarily reshapes population shares, which in turn affects overall assortative matching. A similar argument applies to off-diagonal monotonicity. Consider a population with three types and suppose that the number of mixed couples m_{12} increases. Then I_1 and I_2 decline, while their population shares increase. At the same time, the index I_3 remains unchanged, but its weight falls. If I_3 is sufficiently small, the reallocation of weight toward types 1 and 2 may increase the aggregate index I. This result is also not problematic: a reduction in homogamy for some types should be evaluated relative to homogamy levels elsewhere in the population, rather than in isolation.

Homogamy Axioms

We refine and extend the homogamy axioms proposed in Chiappori et al. (2025).

Axiom A5: Maximal homogamy. If all the off-diagonal elements of a matrix M are zero, then for any matrix M', $I(M') \leq I(M)$. Moreover, if any off-diagonal element of M' is positive, then $I(M')$ is not maximal.

The first part of this axiom is equivalent to the full homogamy axiom in Chiappori et al. (2025). The second part states that if a population has a non-homogamous couple, then the index is not maximal. Taken together, this axiom requires that maximal assortative matching is attained if and only if all matches are homogamous. This axiom differs from the maximum homogamy axiom in Chiappori et al. (2025), where maximal assortative matching may occur even when there are some heterogamous matches, provided that they are confined to one triangle of the matrix.

The existence of cross-type couples, due to any reason (structural or otherwise), has potential implications for society in terms of inequality, social mobility, and other outcomes. Therefore, distinguishing between a population with zero cross matches and a population with some positive cross matches is important. Axiom A5 accounts for all non-homogamous couples and requires that the presence of any off-diagonal match should preclude maximal homogamy.

Axiom A6: Minimal homogamy. For any population M with no homogamous couples, that is, $m_{ii} = 0$ for all i, $I(M') \geq I(M)$, for any M'. Moreover, if any diagonal element of M' is positive, then $I(M') > I(M)$ holds.

This axiom is analogous to Axiom A5 and requires that minimal homogamy is achieved if and only if a population has no homogamous couples.

Marginal Sensitivity Axiom

We introduce a new axiom, which requires that a measure should be sensitive to marginal imbalances.

Axiom A7: Marginal sensitivity. Let M and M' be two matrices such that

$$\sum_i m_{ii} = \sum_i m'_{ii} \quad \text{and} \quad |M| = |M'|.$$

If M satisfies $\sum_j m_{ij} = \sum_j m_{ji}$ for all i, and M' satisfies $\sum_j m'_{ij} \neq \sum_j m'_{ji}$ for at least one i, then

$$I(M') > I(M).$$

Intuitively, holding the total number of homogamous matches and population size fixed, marginal imbalances across sides of the market make homogamy harder to achieve, as opportunities for homogamous matching are more limited. An index should reflect this difficulty and assign a higher value to a matrix that achieves the same diagonal mass under marginal imbalances. From hereon, we shall refer to any matrix with equal marginals for each type as *balanced*, and any matrix where the marginals of at least one type are not equal, as *imbalanced*.

3.3 Comparing aggregate measures

Table 2 summarizes the axioms satisfied by different aggregate measures. In addition to the measures examined by Chiappori et al. (2025)—namely the odds ratio, the trace, and the aggregate likelihood ratio—we also examine our aggregate measures, AHS_C and AHS_G. The proofs are in the appendix. Before presenting the results, we characterize the $Trace$ measure as an aggregation of a type-specific measure.

Lemma 8. *The Trace is the aggregation of the type-specific harmonic homogamy share, using the marginal distributions as weights.*

Proof. Consider the aggregate measure as described in the lemma above, and denote it by HHS_C. Then,

$$HHS_C = \sum_i \left(\frac{\sum_j m_{ij} + \sum_j m_{ji}}{2|M|} \right) HHS_i = \frac{\sum_i m_{ii}}{|M|}.$$

The above is the $Trace$ measure as seen in Chiappori et al. (2025), thereby proving the lemma. □

	Invariance			Monotonicity	Homogamy		Marginal
	Scale	Side	Type	Marginal	Maximal	Minimal	Sensitivity
AHS_C	yes	yes	yes	yes	yes	yes	yes
AHS_G	yes	yes	yes	yes	yes	yes	no
$Trace$	yes	yes	yes	yes	yes	yes	no
OR	yes	yes	yes	yes	no	no	no
LR_C	yes	yes	yes	yes	yes	yes	no
LR_G	yes	yes	yes	yes	no	yes	no

Table 2: Overview of axioms satisfied by each aggregate measure.

The invariance and monotonicity axioms hold for all aggregate measures under study, including the AHS. Our homogamy axioms differ from Chiappori et al. (2025). The

LR_G and OR measures do not satisfy full homogamy and the odds ratio does not satisfy our minimal homogamy.

The measures AHS_G, $Trace$, OR, LR_C, and LR_G do not satisfy the marginal sensitivity axiom. All indices (except the $Trace$ and OR) satisfy this axiom for 2×2 matrices, but violate it once the number of types exceeds two. At the same time, the axiom itself is not restrictive. For any $p > 0$, one may aggregate the quasi-arithmetic mean of order p using the Chiappori weights; the resulting aggregate measure satisfies the marginal sensitivity axiom.

An important takeaway of the marginal sensitivity axiom is the comparison between AHS_C and $Trace$, as seen below.

Corollary 1. *The $Trace$ is a lower bound for AHS_C. For any balanced population the two measures are equal; for any unbalanced population, AHS_C is strictly larger.*

Therefore, the $Trace$ underestimates assortative matching as it does not discount for imbalances in the population. The choice between using AHS_C or $Trace$ depends on the research objective. The $Trace$ provides a compact summary of the overall concentration of homogamous couples and captures broad patterns in the matching structure, making it suitable to characterize the level of inequality in a population at a given point in time. However, it does not account for the feasibility constraints that naturally arise from gender imbalances in a population. AHS_C explicitly incorporates these constraints and is capable of distinguishing between mere changes in the concentration of homogamous couples versus changes in the strength of assortative behavior. This makes AHS_C particularly valuable in contexts where population imbalances are substantial or where analyzing the behavioral intensity of assortative matching, driven by segregation and preferences, is the objective.

4 Empirical analysis

4.1 Data and methodology

In this section, we examine the evolution of educational and racial assortative matching in the United States of America (USA). To highlight the empirical relevance of our axioms, we compare our proposed measure of assortative matching with existing alternatives, drawing on data from the public-use March Current Population Survey (CPS) from 1962 to 2024. Our analysis focuses on heterosexual couples (married or cohabiting) in which at least one partner is between the ages of 35 and 44. We choose to begin our analysis at 35 years old, a point at which most people, regardless of their type of education, have typically completed their union-formation process, and partnerships tend to stabilize. We cap the age range at 44 to mitigate potential endogeneity concerns arising from union dissolution (Kalmijn, 1991, Lichter and Qian, 2019).

We compute educational assortative matching. We follow Chiappori et al. (2025) by defining five types of education: high school dropouts (< 12 years of schooling), high school graduates (12 years of schooling), individuals with some college ($13 - 15$ years of schooling), college graduates (16 years of schooling), and postgraduate graduates (> 16 years of schooling). In the survey, the question about educational attainment changed in 1992, which explains the break in the data between 1991 and 1992.

We also measure racial assortative matching using a four-group classification for race: White, Hispanic, African American, and Asian/Other. Due to the unavailability of the Hispanic identifier, the racial analysis is limited to data from 1971 onward.

4.2 Educational assortative matching

In this section, we assess the educational assortative matching by computing the type-specific and aggregate AHS measures and comparing them with the most relevant existing measures. First, we focus on the type-specific measures directly comparable to the type-specific Average Homogamy Share, that is, the likelihood ratio measure and the homogamy rate. Although LR_i fails to satisfy several of the axioms we propose, the comparison is instructive, particularly in illustrating the significance of the homogeneity of degree zero property. We then turn to aggregate measures. We compare our measure to the trace and the odds ratio, both aggregate measures that conform to the axioms in Chiappori et al. (2025). To do so, we first compare the trace and several aggregate measures using five types of education. Later, we evaluate the odds ratio, which requires a two-category classification of education.

Type-specific LR_i, AHS_i, and HHS_i

We first calculate the LR_i for each type of education. Our Figure 3A replicates Figure 3A in Eika et al. (2019), extending the time horizon from 2013 to 2024 and using five types of education. According to this figure, AM has increased dramatically among the lowest education group, whereas it has decreased among college and postgraduate degree holders.

Figure 3B displays the relationship between the LR_i measure and group size for each education category. A strong negative correlation emerges: larger groups exhibit systematically lower levels of AM according to LR_i. This finding contradicts the common intuition that larger groups should facilitate matching with similar partners, mainly due to higher random matching. The strong negative correlation likely stems from the over-dependence of LR_i on group size, which is captured by the fact that the LR_i measure does not satisfy the homogeneity of degree zero property. As a result, changes in AM as captured by this metric are heavily driven by shifts in group sizes. The observed increase in AM among the lowest education group, for instance, is largely a consequence of the declining share of individuals in that category.

The AHS_i measure for each type of education is shown in Figure 4. Figure 4A shows the evolution of AHS_i over time, while Figure 4B depicts its relationship with group size. The results differ from those based on LR_i. Assortative matching is the highest among the lowest education group in 1962, and we observe a strong convergence in AHS_i values between educational types over the study period. The assortative matching among the lowest education group was the highest, at approximately 70%, in 1962, steadily declined until 1990, and then stabilized around 55%. Among high school graduates, the AHS_i initially increased and then declined, with values ranging between 50 and 60%. The assortative matching among those with some college education exhibited the lowest level in 1962 (22%), and rose from the 1970s to 1990, reaching 32%.[3] Since then, it has remained relatively stable at around 45%. For college graduates, assortative matching remained roughly constant at 30% until 1980, after which it increased, reaching 45% by

[3]The sharp increase in 1991 is due to a change in the education variable in the CPS.

2024. Postgraduate degree holders experienced a rise in assortative matching until 1985 (reaching nearly 50%), followed by a decline to 38% in 2000, and a subsequent increase back to 50% in 2024.

Figure 4B shows the relationship between the AHS_i measure and group size. Unlike LR_i, there is no specific relationship between AHS_i and group size. Since the AHS_i measure satisfies the property of homogeneity of degree zero, a proportional increase of the number of homogamous couples, and of the number of men and women of a group, would result in a horizontal line. However, we observe that AHS_i increases with group size when the group size is above some threshold, suggesting that the number of homogamous couples increases in higher proportion relative to group size. This finding reflects the intuition that larger groups provide more opportunities to encounter partners with similar traits, which in turn promotes assortative matching. Nonetheless, the relationship does not hold for small group sizes and even becomes negative in some cases, such as for those without a high school degree. This may reflect factors such as segregation or discrimination affecting smaller groups. In certain educational categories, such as no high school degree, some college, and college degree, the relationship even resembles a U-shape. NEW: This is the result of two forces affecting AM. On the one hand, random matching increases mechanically with group size. On the other hand, if social exclusion or segregation depends negatively on group size, small groups will show high AM. The trade-off between these two forces explains the observed U-shape relationship. Overall, these findings highlight that the evolution of AHS_i is not mechanically driven by changes in group size, as was observed with the LR_i measure.

Figure 5 replicates the same relationships for the Harmonic Homogamy Share, which corresponds to the share of homogamous couples, also known as the homogamy rate. The results are very similar to the AHS_i measure when the gender groups are balanced. However, significant differences can occur when gender groups are imbalanced. The largest difference is for the postgraduate degree group, before 1992. According to the HHS_i measure, the assortative matching increased from 20% to close to 50%, while according to the AHS_i measure, it started at much higher levels and the increase was less severe. This difference arises because the number of women with a postgraduate degree was substantially lower than that of men until 1992 (see Figure 2). Since AHS_i satisfies the marginal scale asymmetry axiom, it is more sensitive to changes in the smaller group, women. As the share of homogamous women changes gradually over time, the AHS_i measure also changes gradually. By contrast, HHS_i satisfies the opposite of marginal scale asymmetry and is therefore more responsive to changes in the larger group, men. As the share of homogamous men changes sharply, the HHS_i measure also changes sharply.

Aggregate LR, AHS, and $Trace$

We now analyze aggregate assortative matching in education, by comparing the aggregate measures based on LR_i, AHS_i, and HHS_i. We construct five aggregate measures: two weighted averages of LR_i, two weighted averages of AHS_i, and the $Trace$, which is a weighted average of HHS_i (see Lemma 8). As described in Section 3.1, two alternative weighting schemes are used. The first weights by the population share of individuals with education type i, yielding the aggregate measures LR_C and AHS_C. The $Trace$ corresponds to HHS_C. The second weights by the random matching probability share, producing LR_G and AHS_G.

The evolution of the aggregate measures of educational assortative matching is shown

in Figure 6. The aggregate likelihood ratio measures show a rising trend in assortative matching. In contrast, both the $Trace$ and the AHS measures indicate a decline in assortative matching from 1962 to 1990, and a relatively stable pattern thereafter. Among the measures considered, the AHS_C aligns most closely with the $Trace$. Given its consistency with theoretical properties, we adopt AHS_C as our preferred measure of aggregate assortative matching.

Odds Ratio

Another relevant measure that satisfies the invariance, monotonicity, and homogamy axioms proposed by Chiappori et al. (2025) is the odds ratio. However, a key limitation of the odds ratio is that it requires the categorization of individuals into two groups. As Gihleb and Lang (2020) and others show, the level of assortative matching is sensitive to group categorization. Restricting the types of education to two groups imposes broad education categories, which may obscure important heterogeneity.

The approach of Chiappori et al. (2025) is to consider five education groups and compute the odds ratio of several 2×2 submatrices. For example, consider the contingency table in Figure 1 and compute the odds ratio for the submatrix $[a, b; c, d]$ that includes only individuals with some college and college graduates. The odds ratio is given by ad/bc. However, this index excludes individuals who matched across other types of education, such as those with a high school education, thereby potentially overstating the degree of assortative matching. Alternatively, they measure the odds ratio of the entire population using a two-education categorization. For example, one could consider college and some college in one group and high school in the other. Then the odds ratio would be

$$\frac{(a+b+c+d)z}{(v+w)(x+y)},$$

which captures the extent to which individuals match within their education group. This approach may mask finer patterns of sorting.

We adopt the latter approach and compare the odds ratio with the AHS measure of assortative matching by computing AM across all possible binary groupings of education categories. The results are shown in Figures 7 to 10. The first finding is that grouping matters: the evolution of aggregate assortative matching differs markedly between education groupings. When we split the sample between postgraduates and the rest, the odds ratio first strongly decreases, then increases to decrease again after 2000 (Figure 7), while when we split the sample between high school dropouts and the rest, the odds ratio declines until 1990 and remains constant thereafter (Figure 10). Moreover, AHS_C, AHS_G, and the $Trace$ yield consistent patterns that, in some cases, are contrary to those implied by the odds ratio and likelihood ratio measures (see, for example, Figure 10). Finally, note that the likelihood ratio can produce conflicting results depending on the weighting scheme used (see Figures 9 and 10).

Sensitivity to group categorization

Following Eika et al. (2019), we aggregate college and postgraduates into a single category, yielding four education groups: high school dropouts, high school graduates, some college, and college graduates. Using this classification, we compute both the $Trace$ and AHS measures of assortative matching. Figure 11A displays the education-specific AHS_i and HHS_i, and Figure 11B the corresponding aggregate measures. Under this four-group

classification, assortative matching among those with at least a college degree is high and increasing, approaching 75% by 2024. At the aggregate level, assortative matching declines until the 1990s and rises thereafter, driven by increasing assortative matching within the highest education group. By contrast, under the five-category classification, aggregate assortative matching remains relatively stable from the 1990s to 2024 (see Figure 6). This discrepancy highlights the sensitivity of these aggregate measures to the definition of education categories. Moreover, as shown in Chiappori et al. (2025), measures that satisfy some form of marginal monotonicity are generally not invariant to categorization. Taken together, these results suggest that complementing aggregate indices with type-specific measures is essential for capturing the heterogeneous evolution of assortative matching across population groups.

4.3 Racial assortative matching

In this section, we examine the evolution of racial assortative matching in the USA. We consider four racial groups in our sample: Whites, Hispanics, African Americans, and Asians/Others. Figure 12 shows the changing racial composition of the population between 1971 and 2024. Although White individuals are the large majority throughout the period, their share has declined significantly from above 85% in 1971 to around 60% in 2024. The Hispanic population has increased the most, with a share that goes from 5% to around 20%. The Asians/Others group has grown from around 2% in 1971 to above 10% of the sample in 2024. Finally, the African Americans group represents about 7% of the population, with a significant imbalance towards men than women.

Type-specific LR_i, AHS_i and HHS_i

When studying racial assortative matching, the relationship between group size and assortative matching is not necessarily monotonic. In majority groups, individuals are more likely to find a partner of the same race simply due to the large group size, which facilitates random matching. However, in smaller communities, strong cultural norms or ethnic preferences may foster high within-group affinity, resulting in elevated levels of assortative matching ((Fisman et al., 2008)). At the same time, as minority groups grow in size, their social integration and acceptance may increase, potentially reducing the out-group aversion and facilitating inter-racial unions (Jarvis et al., 2023). These dynamics suggest that the relationship between group size and AM can be complex and non-linear. In this context, the homogeneity of degree zero property becomes crucial for accurately identifying the effect of group size on assortative matching. Figure 13B shows how the likelihood ratio (LR_i) fails to capture this non-monotonic relationship. Specifically, the LR_i measure consistently implies that larger groups exhibit lower levels of assortative matching.

Our AHS_i measure identifies different patterns in racial assortative matching. As shown in Figure 14B, the relationship between group size and AM varies significantly across racial groups. For the white population, assortative matching increases with group size, consistent with the idea that, for large groups, an increase in the pool of potential same-race partners facilitates homogamy. In contrast, the relationship is negative for Hispanics, suggesting that as a small group grows, inter-racial unions become more common, possibly due to greater integration and reduced social barriers. For African Americans and Asians/Other groups, the relationship is less clear, indicating more complex dynamics

are at play. Figure 14A shows that the White and African Americans groups consistently exhibit the highest levels of AM, remaining above 80% throughout the period, despite a gradual decline. The Hispanic group maintains a relatively stable level of AM, with a slight decrease in the last decade. For the Asians/Others racial group, AM declined until around 2005, then rebounded and stabilized in recent years. Overall, the AHS_i measure highlights persistently high levels of assortative matching across all racial groups. Figure 15A displays the evolution of AM according to the HHS_i measure, which gives very similar results to the AHS_i measure.

Aggregate AHS, LR, and $Trace$

Figure 16 shows the evolution of aggregate measures of racial AM. The measures based on the LR follow the left scale. Depending on the weights used, the aggregate measure of AM increases (LR_G) or decreases (LR_C) during the study period. In contrast, both the AHS and the $Trace$ measures show a consistent downward trend in racial assortative matching. According to these measures, the share of homogamous couples declined from around 98% in 1971 to around 85% in 2024. This implies a gradual increase in interracial unions over the past five decades.

Odds Ratio

To compare the AHS measure with the odds ratio, we construct a series of binary racial groups. We begin by comparing Whites to all other racial groups. Next, we expand the reference group by including Hispanics alongside Whites. Finally, we add African Americans to this group, leaving Asians/Others as the comparison group. Figures 17 to 19 report the results for each of these two-group classifications. This approach allows for a direct comparison between our preferred measure of AM (AHS_C) and the odds ratio. In all cases, the level of AM remains high and shows a downward trend over the study period according to both the AHS_C and $Trace$ measures. In contrast, the odds ratio and likelihood ratio measures often yield inconsistent or contradictory trends[4], reinforcing the value of using theoretically grounded metrics like AHS.

Singles

Thus far, our analysis has focused exclusively on couples, omitting individuals who remain single. Following the approach of Zhang (2024), we extend our framework to include singles. Incorporating singles naturally lowers the level of assortative matching, as it accounts for unmatched individuals. Figure 20 compares our preferred AHS measure with and without singles. The inclusion of singles reveals particularly notable effects among Hispanic and African American populations, where the prevalence of singlehood has increased substantially over the study period from around 3% to 20% and 30%, respectively.[5] Adjusting by singles offers a more comprehensive perspective of matching behavior and highlights an important demographic shift that may shape future trends of AM, especially as singlehood continues to rise in certain groups.

[4]In Figure 18, for instance, AHS and Trace show a decreasing trend of racial AM, while OR shows a different pattern with a pick in AM in 1978 and a rather constant level of AM since 2010. The LR shows a different trend depending on the weighting scheme used.

[5]In comparison, singlehood among whites and Asians remains below 15% by 2024.

5 Extensions and applications using AHS

In this section, we illustrate several applications of the AHS measure. Its flexibility allows for the decomposition of assortative matching across genders, the measurement of cross-type matching, and the separation of voluntary and involuntary cross-type matches. In addition, the population decomposability property facilitates the study of assortative matching along multiple dimensions. We also examine the role of compositional changes in shaping the evolution of overall assortative matching.

5.1 Gender decomposition

An advantage of the AHS_i measure is that AM can be decomposed by gender. Let us define the gender gap of AM as

$$\frac{m_{ii}}{\sum_j m_{ji}} - \frac{m_{ii}}{\sum_j m_{ij}} = \frac{m_{ii}(\sum_j m_{ij} - \sum_j m_{ji})}{\sum_j m_{ij} \sum_j m_{ji}}.$$

It increases with gender imbalance $(\sum_j m_{ij} - \sum_j m_{ji})$ and homogamy (m_{ii}). Both elements make it difficult for the overrepresented gender to find a partner of the same type, as it means that there are fewer such candidates available. By definition, the underrepresented gender always has a higher AM level than the overrepresented gender. The intuition being that it is easier to find a partner with the same characteristics for the underrepresented gender than for the overrepresented gender.

Figure 21 shows gender-specific trends in educational assortative matching. Among high school dropouts, women consistently exhibit higher rates of homogamy than men throughout the sample period. The gender imbalance is small in this group, as shown in Figure 2; however, its high AM generates a significant gender gap. For high school graduates, men had higher levels of assortative matching until around 2000, after which the pattern reversed in favor of women. In this case, both the significant gender imbalance and the AM level explain the large gender gap in AM. Among people with some college education, gender differences in AM were negligible until the 1990s. Between 1990 and 2020, men had higher AM levels than women, but the gap closed again around 2020. The gender imbalance drives this trend. For college and postgraduate degree holders, women initially had higher levels of assortative matching than men. However, this trend reversed around 2000 for college graduates and around 2010 for postgraduates. For these education groups, gender imbalances are significant and the AM level is sizable, which translates into large gender gaps in AM.

Moreover, when one gender is underrepresented, some cross-type pairings are driven by structural constraints, namely the unavailability of suitable partners, while others reflect voluntary choices. Figure 22 displays the decomposition of non-assortative matching into voluntary and involuntary cross-type pairings. The sum of AM and voluntary cross-type matching gives the maximum feasible level of assortative matching (see equation 8), given the gender distribution between education types in each year. The voluntary cross-type pairings thus measured are represented by the red area. The involuntary cross-type pairings occur due to the scarcity of the opposite gender, are measured as residual pairings that are neither assortative nor voluntary, and are represented by the blue area. Among education groups, postgraduates have been most affected by gender imbalance, as the level of involuntary cross-type pairings show.

Figure 21 also shows the difference between AHS_i and HHS_i for each education group. When the gender imbalance is large and the homogamy within the group is large, leading to high involuntary cross-type matching, the measures AHS_i and HHS_i differ significantly. This can also be seen theoretically; when two gender groups are equal, then the measures AHS_i and HHS_i are equal. Instead, under gender imbalance, $HHS_i < AHS_i$ holds. AHS_i is a better measure as it reflects the role of gender imbalance in AM. Figure 23 shows how the difference between AHS_i and HHS_i increases with the share of involuntary cross-type matching.

Figure 24 shows the evolution of racial AM by gender. Although the gender gap in AM is negligible for whites, there are gender differences in AM among Hispanics, African Americans, and Asians/Others. In the case of Hispanics, men were more likely to marry Hispanic women, but gender differences in AM have decreased over time; for African Americans, women are more likely to marry within the group than men, and these differences have increased over time; while for Asians and others, men are more likely to match within their race than women, and gender differences in AM persist over time. These results are driven by imbalances in gender composition within racial groups. This is the case of African Americans and Asians/Others, for which there are some gender imbalances (see Figure 12). The magnitude of the gender gap also depends on the level of homogamy in the group. In general, the levels of AM by race are high, amplifying the effect of gender imbalances. Figure 25 shows the decomposition of AM, voluntary, and involuntary cross-type matching. As already shown before, there are large levels of AM by race. Involuntary cross-race matching, caused by gender imbalances, is limited, and only Hispanics and Asians/Others show a significant portion of voluntary cross-racial matching.

5.2 Positive assortative matching

The AHS captures the overall level of assortative matching in a society. To distinguish matching driven by preferences, segregation or cultural norms from matching driven by random encounters, we construct a counterfactual matching matrix in which individuals pair randomly (see Section 2.1).

The difference in assortative matching between the observed matrix and the random matrix reflects the extent of *intentional or positive assortative matching*, which we interpret as stemming from social, cultural or structural factors such as group segregation, social acceptance or individual preferences.

For any type-specific measure I_i, the corresponding positive assortative matching is $PI_i = I_i - I_i^{rm}$. Positive assortative matching for type i under measure AHS_i is then $PAHS_i = AHS_i - AHS_i^{rm}$, where AHS_i^{rm} defined in equation 3. Simplifying,

$$PAHS_i = \frac{1}{2}\left(\frac{m_{ii} - m_{ii}^{rm}}{\sum_j m_{ij}} + \frac{m_{ii} - m_{ii}^{rm}}{\sum_j m_{ji}}.\right)$$

Similarly, positive assortative matching under measure HHS_i is $PHHS_i$ where,

$$PHHS_i = 2\left(\frac{m_{ii} - m_{ii}^{rm}}{\sum_j m_{ij} + \sum_j m_{ji}}\right)$$

Taken together,

$$PAHS_i - PHHS_i = \left(\frac{m_{ii} - m_{ii}^{rm}}{m_{ii}}\right)[AHS_i - HHS_i]. \tag{9}$$

It follows that $PAHS_i \geq PHHS_i$ and $PAHS_i - PHHS_i \leq AHS_i - HHS_i$ when $m_{ii} > m_{ii}^{rm}$, which is the empirically relevant case.

The evolution of $PAHS_i$ by education group is shown by bold lines in Figure 26A and its relationship with group size is shown in Figure 26B. Positive assortative matching has increased steadily among individuals without a high school diploma. Since 1980, it has also risen—though more modestly—for high school graduates and individuals with some college education. In contrast, intentional assortative matching among college and postgraduate degree holders has remained largely stable since 1990. These patterns point to increasing segregation of lower-education groups, potentially contributing to rising social exclusion.

The dashed lines in Figure 26 correspond to $PHHS_i$ and allow for a comparison with $PAHS_i$. While the two measures yield similar patterns for most education groups, with $PAHS_i \geq PHHS_i$, notable differences emerge for individuals with postgraduate degrees. In particular, $PAHS_i$ increases at a smaller rate than $PHHS_i$ for this group.

Figure 27 compares aggregate assortative matching, as measured by AHS and the Trace, with aggregate positive assortative matching, measured by $PAHS$ and $PHHS$, respectively. The two comparisons yield similar conclusions; we therefore focus on the $AHS-PAHS$ comparison. While AHS declines from 55% to 47% between 1960 and 1990 and then stabilizes, $PAHS$ increases steadily from 21% to 25% over the full period. Two implications follow. First, roughly half of the total assortative matching can be attributed to random matching. Second, although aggregate assortative matching appears to decline over time, intentional assortative matching is substantial and increasing—driven primarily by rising positive assortative matching among lower-education groups. In this sense, intentional homogamy has strengthened over time.

Intentional assortative matching maybe more salient in the context of racial matching, where preferences, cultural norms, and segregation play a central role. Figure 28 shows the evolution of $PAHS_i$ across racial groups and its relationship with group size.[6] Although total assortative matching among White individuals is very high (around 90%), most of it is accounted for by random matching. Their intentional assortative matching remains below 30%, albeit with a rising trend over time. In contrast, smaller racial groups display substantially higher levels of intentional assortative matching—ranging from 60% to 90%—though these levels decline over the sample period. Among these groups, African Americans exhibit the highest level of intentional assortative matching. Since Whites constitute the largest racial group, the aggregate $PAHS$ (Figure 29) exhibits an upward trend. While total racial assortative matching AHS declines over time, $PAHS$ doubles—from 20% to 40%—driven primarily by changes in the matching behavior of White individuals.

5.3 Cross-type matching

A comprehensive assessment of matching patterns requires considering not only exact homogamy but also the structure of cross-type matches. Intuitively, couples composed of

[6] We do not report $PHHS_i$ because it closely tracks $PAHS_i$ in this setting.

individuals with very different characteristics should be less common than those formed by individuals with similar traits. To formalize this idea, we extend the Average Homogamy Share (AHS_i) to non-homogamous couples, in order to quantify cross-type matching. We refer to this extended measure as the Average Share (AS_{ij}), defined as:

$$AS_{i,j} = \frac{1}{2}\left(\frac{m_{ij}}{\sum_k m_{ik}} + \frac{m_{ij}}{\sum_k m_{ki}}\right) \tag{10}$$

This measure captures the average share of individuals of type i (and j) who match with type j (and i), thereby characterizing the full matching structure of the population. Figure 30 presents, for each education type, the evolution of the average share of individuals married to every education group. For example, the upper-left graph shows that most high school dropouts tend to marry within their group (violet area) or with high school graduates (yellow area). Although the share of dropouts marrying individuals with some college education has increased over time, it remains relatively low. At the other end of the distribution, individuals with postgraduate degrees are more likely to marry someone with at least a college degree. While around 20% of postgraduates were married to high school graduates at the beginning of the sample period, this share declines significantly by 2020.

Figure 31 presents the corresponding patterns for race. Cross-group matching is considerably less frequent than in the case of education. Moreover, most interracial couples are composed of white individuals and members of other racial groups.

Since education is ordinal, it is useful to consider a measure that accounts for both exact matches and matches between similar (though not identical) types. As shown in Figure 30, individuals are more likely to partner with others who share their own or adjacent educational types, while the likelihood of matching declines as the distance between types increases.

We therefore incorporate this broader notion of similarity by extending the Average Homogamy Share (AHS) to include near-type matches. We propose the following measure, which we call the Average Matching Share (AMS):

$$AMS = \sum_i \sum_j w(i,j)\phi_{i,j} AS_{i,j} \tag{11}$$

where education categories are ordered, $\phi_{i,j} = \frac{\sum_k m_{ik} + \sum_k m_{kj}}{2|M|}$, and $w(i,j)$ is the distance-based weight. We assume $w(i,i) = 1$, $w(i,j) = 0.5$ if $|i-j| = 1$, and $w(i,j) = 0$ otherwise.[7] Choi and Kim (2025) transform the trace and aggregate LR to a continuous measure of matching using a similarity-weighted matching matrix, which can be asymmetric and dynamic. In our case, we assume a simple static and symmetric weight structure.

Figure 32 presents the evolution of the Average Matching Share (AMS) for educational assortative matching, alongside the AHS and $Trace$ measures. Unlike AHS and the Trace, which are bounded between zero and one, AMS can exceed one because it assigns positive weight to near-type matches and the weights do not sum to one. A decline in AMS reflects a reduction in either strictly homogamous matches or matches between adjacent educational categories. Over the sample period, AMS exhibits a downward trend. After 1990, this decline is mainly driven by a decrease in the number of near-type

[7] Note that this measure does not satisfy type invariance.

matches, as AHS remains relatively stable.[8]

5.4 Population decomposability

Our type-specific measure satisfies the population decomposability property (see Section 2.4), which allows us to assess the degree of assortative matching within distinct population subgroups. This feature allows identification of how matching patterns vary across these groups, rather than treating the population as homogeneous.

We present two population decompositions. First, we divide couples into two categories: homogamous white couples and all other couples. The latter group includes homogamous non-white couples and mixed-race couples; thus, it contains both homogamous and non-homogamous unions. Within each of these two groups, individuals are further classified by educational attainment: high school dropouts, high school graduates, and individuals with some college or more. For each subgroup, we compute the corresponding AHS_i. In other words, we compute the level of assortative matching among high school dropouts in white-homogamous couples, among high school dropouts in the non-white/mixed group, and so on for each education category.

The results in Figure 33 reveal that educational AM among high school dropouts is substantially higher in the non-white/mixed group than among white-homogamous couples. Differences across groups are smaller for the other education categories. This suggests that the low-educated non-white individuals and the low-educated whites who form mixed-race unions may face stronger segregation pressures than low-educated whites in white-homogamous couples.

In a second analysis, we examine racial assortative matching within four educational groups. We classify couples as (i) educationally homogamous couples where both members are high school dropouts, (ii) where both are high school graduates, and (iii) where both have been in college for at least some years, and (iv) educationally non-homogamous couples. Within each group, we compute the AHS_i for white and non-white individuals (including Hispanics, African Americans, Asians, and others). As shown in Figure 34, racial AM among whites has declined similarly across all education groups during the study period. Among non-whites, however, the pattern varies by education. Racial AM among non-whites in homogamous high school dropout couples remains close to 1 throughout, indicating persistent racial separation. By contrast, for non-white couples with at least some college education, AM dropped sharply until 1980 and remained relatively low thereafter, suggesting greater interracial mixing for this group. The remaining education groups fall between these two cases.

Taken together, both decompositions indicate that non-white individuals with low levels of education may be particularly vulnerable to social segregation. These findings highlight how race and education jointly shape inequality in assortative matching patterns.

5.5 Isolating the compositional effect on AM

Our measure allows us to isolate the impact of compositional changes in group shares on aggregate assortative matching. To do so, we compute AHS while keeping the educational-type weights fixed in a given year. Figure 35A shows the evolution of AHS using the education shares in 1962 as weights, while Figure 35B uses those in 2024.

[8]Note that the jump in 1991 can be caused by the redesign of the education question in the CPS.

The expansion of education between 1962 and 1990 - marked by a sharp decline in the share of high school dropouts and a substantial increase in college attainment (see Figure 2) - contributes significantly to the observed decline in AM over this period. Had the educational composition remained constant, AM would have been approximately 4 percentage points higher. This reflects the fact that AM levels are higher among high school dropouts and graduates than among those with higher education, as shown in Figure 4.

When 2024 weights are applied throughout the sample (Figure 35B), AM instead rises until 1990. In this case, the increasing weight placed on higher education groups—whose AM levels were rising during this period—generates an upward trend in aggregate AM.

Figure 36 presents the analogous exercise for racial assortative matching. Panel A fixes weights at the 1971 racial composition, while Panel B uses 2024 weights. The results suggest that only a small portion of the decline in racial AM can be attributed to compositional changes across racial groups.

6 Conclusion

Assortative matching is central to the study of inequality and mobility, yet its measurement remains theoretically unsettled. Existing indices often respond differently to marginal distributions and gender imbalances, producing conflicting narratives about the evolution of sorting. This paper develops a unified axiomatic framework that clarifies what a coherent measure of assortative matching must satisfy—both at the type-specific and aggregate levels—and shows that commonly used measures fail key economic requirements.

At the type-specific level, we introduce a set of intuitive axioms and show that the Average Homogamy Share (AHS_i) satisfies them. Further, we uniquely characterize the AHS_i measure. Amongst the axioms, homogeneity of degree zero ensures that a measure isolates matching intensity from population scale effects, preventing group size changes from mechanically driving assortativity. Moreover, the marginal scale asymmetry requires that a measure responds more strongly to changes in the smaller side of the market, reflecting the fact that feasible homogamy is constrained by the scarce group. Widely used alternatives—most notably the likelihood ratio—fail homogeneity of degree zero and monotonicity, and therefore confound changes in matching behavior with compositional shifts.

At the aggregate level, we refine and strengthen the axioms proposed in Chiappori et al. (2025). We introduce a marginal sensitivity axiom: holding the total number of homogamous matches and total population constant, a population with imbalanced marginals should be more assortative than under balanced marginals. Under this requirement, none of the existing aggregate measures are admissible. Our preferred aggregate measure, AHS_C, satisfies this axiom and also axioms of invariance, monotonicity and homogamy. The trace emerges as a special case, satisfying all axioms under balanced marginals but systematically understating assortative matching when imbalances are present.

Empirically, U.S. data from 1962 to 2024 yields three central findings. First, educational assortative matching declined until around 1990 and, depending on the categorization used, has since stabilized (five types of education) or increased (four types of education). AM among low-educated individuals decreased until 1990, although it re-

mains above 50%. Among highly educated groups, AM has increased substantially during the whole period. By 2020, among those with at least a college degree, more than 70% are partnered with someone with at least a college degree. Second, racial assortative matching remains markedly higher than educational matching, declining from roughly 98% in 1971 to 85% in 2024, yet still reflecting strong within-group sorting. Third, once random matching is accounted for, intentional (positive) assortative matching has increased in aggregate terms, particularly among lower-education groups and among whites in racial matching. The flexibility of AHS_i allows for further insight that aggregate measures obscure. We document how gender imbalances generate structural (involuntary) cross-type matching, particularly among postgraduate degree holders; how positive assortative matching evolves differently from total homogamy; how cross-type matching patterns reflect educational proximity; and how race and education interact in shaping segregation. Across decompositions, a consistent pattern emerges: low-educated non-white individuals appear particularly vulnerable to social isolation.

Empirically, U.S. data from 1962 to 2024 yields three central findings. First, educational assortative matching declined until around 1990 and has since stabilized with convergence across education groups. Low-educated individuals continue to exhibit the highest homogamy rates, but assortative matching among highly educated groups has increased substantially. Second, racial assortative matching remains markedly higher than educational matching, declining from roughly 98% in 1971 to 85% in 2024, yet still reflecting strong within-group sorting. Third, once random matching is accounted for, intentional (positive) assortative matching has increased in aggregate terms, particularly among lower-education groups and among whites in racial matching. The flexibility of AHSi allows further insights that aggregate measures obscure. We document how gender imbalances generate structural (involuntary) cross-type matching, particularly among postgraduate degree holders; how positive assortative matching evolves differently from total homogamy; how cross-type matching patterns reflect educational proximity; and how race and education interact in shaping segregation. Across decompositions, a consistent pattern emerges: low-educated non-white individuals appear particularly vulnerable to social isolation.

The broader lesson is that measurement is not neutral. Different indices can generate opposite conclusions about trends in assortative matching because they embed different responses to marginal distributions and market constraints. By grounding measurement in transparent and economically interpretable axioms, this paper separates changes in matching behavior from changes in population composition and feasibility constraints. This yields a framework that is internally consistent, decomposable, and robust to shifts in group size. Future work can build on this framework to connect assortative matching more directly to inequality dynamics, residential segregation, and intergenerational mobility.

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Appendix A: Axiom compliance of type-specific measures

In this section, we provide proof of the results in Table 1. First, we define each measure.

The Average Homogamy Share is our main measure and has been defined as follows.

$$AHS_i = \frac{1}{2}\left(\frac{m_{ii}}{\sum\limits_j m_{ij}} + \frac{m_{ii}}{\sum\limits_j m_{ji}}\right) \tag{12}$$

The likelihood ratio (Eika et al., 2019) for any type i is LR_i and is defined as follows.

$$LR_i = \frac{m_{ii}/|M|}{(\sum\limits_j m_{ij}/|M|)(\sum\limits_j m_{ji}/|M|)} = \frac{m_{ii}|M|}{(\sum\limits_j m_{ij})(\sum\limits_j m_{ji})} \tag{13}$$

The geometric homogamy share for any type i is GHS_i and is defined as follows.

$$GHS_i = \frac{m_{ii}}{\left(\sum\limits_j m_{ij}\right)^{1/2}\left(\sum\limits_j m_{ji}\right)^{1/2}} \tag{14}$$

The harmonic homogamy share for any type i, which is equivalent to the homogamy rate, is HHS_i and is defined as follows.

$$HHS_i = \frac{2m_{ii}}{\sum\limits_j m_{ij} + \sum\limits_j m_{ji}} \tag{15}$$

Axiom S1: Measure definition

This axiom states that a measure should depend on the mass of homogamous couples of type i (m_{ii}), the total mass of men of type i ($\sum_j m_{ij}$), and the total mass of women of type i ($\sum_j m_{ji}$). AHS_i, GHS_i, and HHS_i satisfy this axiom, as can be directly seen from the definitions above. LR_i does not fulfill this axiom, as it also depends on the total number of couples $|M|$.

Axiom S2: Homogeneity of degree zero

This axiom requires that scaling m_{ii}, $\sum\limits_j m_{ij}$ and $\sum\limits_j m_{ji}$ by λ leaves the measure unchanged. While AHS_i, GHS_i, and HHS_i satisfy this property, LR_i does not.

For any matrix M, scale m_{ii}, $\sum\limits_{j=1}^{3} m_{ij}$ and $\sum\limits_{j=1}^{3} m_{ji}$ by λ. The effect on AHS_i is as follows.

$$AHS_i^{\lambda} = \frac{1}{2}\left(\frac{\lambda m_{ii}}{\lambda\sum\limits_{j=1}^{3} m_{ij}} + \frac{\lambda m_{ii}}{\lambda\sum\limits_{j=1}^{3} m_{ji}}\right) = \frac{1}{2}\left(\frac{m_{ii}}{\sum\limits_{j=1}^{3} m_{ij}} + \frac{m_{ii}}{\sum\limits_{j=1}^{3} m_{ji}}\right) = AHS_i$$

Thus, our measure is homogeneous of degree zero. Similarly, it can be proven that GHS_i and HHS_i also satisfy this axiom. To study the LR_i measure, let us consider the 2-type case.

$$LR_i^{\lambda} = \frac{\lambda a(\lambda a + \lambda b + \lambda c + d)}{\lambda(a+b)\lambda(a+c)} = \frac{a(a+b+c+d/\lambda)}{(a+b)(a+c)} \neq LR_i$$

Since d is not multiplied by λ, therefore, $LR_i^{\lambda} \neq LR_i$. Also, it follows from the last expression that LR_i^{λ} is decreasing in λ, for any population with arbitrarily number of types. Thus, the likelihood ratio is not homogeneous of degree zero and scaling by λ decreases the measure.

Axiom S3: Side invariance

This axiom requires that a measure remains unchanged when the share of homogamous men and women are exchanged. All the above measures satisfy this axiom, as they are symmetric in $m_{ii}/\sum_j m_{ij}$ and $m_{ii}/\sum_j m_{ji}$.

Axiom S4: Monotonicity

The measure I_i should be increasing in each argument, keeping the other argument fixed.

It is straightforward to check that AHS_i, GHS_i, and HHS_i satisfy this axiom. However, LR_i does not. Consider a population M with parameters a, b, c, d and population M' with parameters $\lambda a, b, \lambda c, d$, for some $\lambda > 1$. Then, the share of homogamous women is $a/(a+c)$ in both but the share of homogamous men is $\lambda a/(\lambda a + b)$, which is higher than $a/(a+b)$. Then for $ad > bc$,

$$\begin{aligned} LR_i(\lambda a, b, \lambda c, d) &= \frac{a}{a+c}\frac{\lambda a + b + \lambda c + d}{\lambda a + b} \\ &= \frac{a}{a+c}\left(1 + \frac{\lambda c + d}{\lambda a + b}\right) \\ &< \frac{a}{a+c}\left(1 + \frac{c+d}{a+b}\right) = LR_i(a, b, c, d), \end{aligned}$$

Thus, monotonicity is not always satisfied by LR_i.

Lemma 3 gives two implications of monotonicity. The measures AHS_i, GHS_i and HHS_i satisfy full and incomplete homogamy.

LR_i fails full homogamy. For instance, consider a population with two types of education with entries $a > 0$, $b = c = 0$, $d > 0$, then $LR_i = 1 + d/a$. Increasing d further increases LR_i, thereby showing that LR_i does not satisfy full homogamy. LR_i satisfies incomplete homogamy. When $m_{ii} = \sum_{j=1}^{3} m_{ij} = \sum_{j=1}^{3} m_{ji}$ does not hold, then without loss of generality, $m_{ii} < \sum_{j=1}^{3} m_{ij}$. By decreasing $\sum_{j=1}^{3} m_{ij} - m_{ii}$ and keeping all else fixed, LR_i increases, thereby proving the result.

Axiom S5: Boundedness

This axiom imposes bounds on any measure of assortative matching, specifically, a measure should be between 0 and 1. AHS_i, GHS_i, and HHS_i satisfy this axiom but LR_i does not. For instance, consider a population with two types and entries $a > 0$ and $b = c = 0$. Then, as $d \to \infty$, $LR_i \to \infty$, thereby showing that the likelihood ratio is unbounded.

Axiom S6: Bisymmetry

This axiom is satisfied by AHS_i, GHS_i, and HHS_i, which are defined over a function with two arguments.

$$I_i\left(I_i\left(x,y\right),I_i\left(u,v\right)\right)=I_i\left(I_i\left(x,v\right),I_i\left(u,y\right)\right).$$

The AHS_i satisfies it:

$$\left(\frac{x+y}{2}+\frac{u+v}{2}\right)/2=\left(\frac{x+v}{2}+\frac{u+y}{2}\right)/2$$

The GHS_i satisfies it:

$$[(xy)^{1/2}]^{1/2}[(uv)^{1/2}]^{1/2}=[(xv)^{1/2}]^{1/2}[(uy)^{1/2}]^{1/2}$$

The HHS_i satisfies it. First, note that $HHS_i(x,y)=\frac{2xy}{x+y}$.

$$HHS_i(HHS_i(x,y),HHS_i(u,v))=\frac{2\frac{2xy}{x+y}\frac{2uv}{u+v}}{\frac{2xy}{x+y}+\frac{2uv}{u+v}}=4\frac{xyuv}{uxy+vxy+xuv+yuv}$$
$$HHS_i(HHS_i(x,v),HHS_i(u,y))=\frac{2\frac{2xv}{x+v}\frac{2uy}{u+y}}{\frac{2xv}{x+v}+\frac{2uy}{u+y}}=4\frac{xvuy}{xvu+xvy+uyx+uyv}$$

This axiom does not apply to LR_i.

Axiom S7: Homogeneity of degree one

Scaling $m_{ii}/\sum_i m_{ij}$ and $m_{ii}/\sum_i m_{ji}$ by a factor λ should increase the level of assortative matching by λ. This property is satisfied by AHS_i, GHS_i, and HHS_i, but not by LR_i. Suppose that m_{ii} increases by λ. The LR_i depends proportionally on m_{ii} and the total population, which are multiplied together, and the population size is affected by m_{ii}. Therefore, LR_i increases by more than λ.

Axiom S8: Marginal scale asymmetry

Both AHS_i and LR_i satisfy this axiom. But GHS_i and HHS_i do not.

Consider the LR_i measure.

$$LR_i\left(\frac{m_{ii}}{\lambda\sum_j m_{ij}},\frac{m_{ii}}{\sum_j m_{ji}}\right)=\frac{m_{ii}\left(\lambda\sum_j m_{ij}+\sum_{k\neq i}\sum_j m_{kj}\right)}{(\lambda\sum_j m_{ij})(\sum_j m_{ji})}$$
$$<\frac{m_{ii}\left(\lambda\sum_j m_{ji}+\sum_{k\neq i}\sum_j m_{jk}\right)}{(\sum_j m_{ij})(\lambda\sum_j m_{ji})}=LR_i\left(\frac{m_{ii}}{\sum_j m_{ij}},\frac{m_{ii}}{\lambda\sum_j m_{ji}}\right),$$

where the inequality follows as $\sum_j m_{ij} < \sum_j m_{ji}$ and $\lambda > 1$. Thus, LR_i satisfies the axiom. Now consider any power mean with power p. Then,

$$I_i\left(\frac{m_{ii}}{\lambda\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) = \left(\frac{1}{2}\right)^{1/p}\left(\left(\frac{m_{ii}}{\lambda\sum_j m_{ij}}\right)^p + \left(\frac{m_{ii}}{\sum_j m_{ji}}\right)^p\right)^{1/p} \tag{16}$$

$$I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\lambda\sum_j m_{ji}}\right) = \left(\frac{1}{2}\right)^{1/p}\left(\left(\frac{m_{ii}}{\sum_j m_{ij}}\right)^p + \left(\frac{m_{ii}}{\lambda\sum_j m_{ji}}\right)^p\right)^{1/p} \tag{17}$$

For $p > 0$, (16) > (17) if

$$\left[1 - \frac{1}{\lambda^p}\right]\left(\left(\frac{m_{ii}}{\sum_j m_{ij}}\right)^p - \left(\frac{m_{ii}}{\sum_j m_{ji}}\right)^p\right) < 0. \tag{18}$$

Since $\lambda > 1$ and $p > 0$, $1 - \frac{1}{\lambda^p} > 0$ holds. Further, since $\sum_j m_{ji} > \sum_j m_{ij}$, $\left(\frac{m_{ii}}{\sum_j m_{ij}}\right)^p - \left(\frac{m_{ii}}{\sum_j m_{ji}}\right)^p < 0$ also holds, thereby proving that any power mean with positive power $p > 0$ satisfies the axiom.

For $p < 0$, (16) > (17) only if equation (18) were positive. However, for $p < 0$ and $\sum_j m_{ij} < \sum_j m_{ji}$, the expression in (18) is negative as $\left(\frac{m_{ii}}{\sum_j m_{ij}}\right)^p - \left(\frac{m_{ii}}{\sum_j m_{ji}}\right)^p > 0$ but $1 - \frac{1}{\lambda^p} < 0$, for $p < 0$. Thus, for any negative power mean, the measure is more affected when the larger group is scaled, and the opposite of the marginal scale asymmetry axiom holds.

For $p = 0$, the axiom is not satisfied as

$$I_i\left(\frac{m_{ii}}{\lambda\sum_j m_{ij}}, \frac{m_{ii}}{\sum_j m_{ji}}\right) = \sqrt{\left(\frac{m_{ii}}{\lambda\sum_j m_{ij}}\right)\left(\frac{m_{ii}}{\sum_j m_{ji}}\right)}$$

$$= \sqrt{\left(\frac{m_{ii}}{\sum_j m_{ij}}\right)\left(\frac{m_{ii}}{\lambda\sum_j m_{ji}}\right)} = I_i\left(\frac{m_{ii}}{\sum_j m_{ij}}, \frac{m_{ii}}{\lambda\sum_j m_{ji}}\right)$$

Property 1: Additive homogeneity

We now consider the class of power means and show that Property 1 is satisfied only by AHS_i.

First, observe that

$$AHS_i(x + c, y + c) = \frac{x + c + y + c}{2} = \frac{x + y}{2} + c = AHS_i(x, y) + c,$$

so AHS_i satisfies additive homogeneity.

For uniqueness, let M_p denote the power mean of order p, i.e. for $p \neq 0$,

$$M_p(u,v) = \left(\frac{u^p + v^p}{2}\right)^{1/p},$$

(and $M_0(u,v) = \sqrt{uv}$). Clearly, M_0 does not satisfy additive homogeneity. Suppose that, for some $p \neq 0$, the measure M_p satisfies

$$M_p(x+c, y+c) = M_p(x,y) + c \quad \text{for all } x, y, c \geq 0.$$

Fix $x \neq y$ and define

$$f(c) := M_p(x+c, y+c) - c.$$

Property 1 implies that $f(c) \equiv M_p(x,y)$ is constant in c, hence

$$f''(c) = 0 \quad \text{for all } c > 0.$$

For $p \neq 0$, a direct differentiation yields

$$f''(c) = \frac{d^2}{dc^2} M_p(x+c, y+c) = \frac{(p-1)(x-y)^2 (x+c)^p (y+c)^p}{2^{1/p}(x+c)^2(y+c)^2\big((x+c)^p + (y+c)^p\big)^{2-1/p}}.$$

All factors in this expression are strictly positive for $x \neq y$ and $c > 0$, except the factor $(p-1)$. Therefore, $f''(c) = 0$ for all $c > 0$ implies $p = 1$.

Thus, among power means, only the arithmetic mean ($p = 1$) satisfies additive homogeneity. Since AHS_i is exactly the arithmetic mean of the two homogamy shares of type i, it is the unique power mean satisfying Property 1.

Property 2: Population decomposability

We first present the argument for $K = 2$ and then note the generalization. Let $M = M_1 + M_2$. By construction,

$$m_{ii} = m^1_{ii} + m^2_{ii}, \quad \sum_j m_{ij} = \sum_j m^1_{ij} + \sum_j m^2_{ij}, \quad \sum_j m_{ji} = \sum_j m^1_{ji} + \sum_j m^2_{ji}.$$

Define

$$AHS^k_i \;=\; \frac{1}{2}\left(\frac{m^k_{ii}}{\sum_j m^k_{ij}} + \frac{m^k_{ii}}{\sum_j m^k_{ji}}\right), \qquad \alpha_k \;=\; \frac{1}{2}\left(\frac{\sum_j m^k_{ij}}{\sum_{\ell=1}^2 \sum_j m^\ell_{ij}} + \frac{\sum_j m^k_{ji}}{\sum_{\ell=1}^2 \sum_j m^\ell_{ji}}\right).$$

Then, for $k \in \{1,2\}$,

$$\begin{aligned}
\alpha_k AHS^k_i \quad &= \frac{1}{4}\left(\frac{\sum_j m^k_{ij}}{\sum_\ell \sum_j m^\ell_{ij}} + \frac{\sum_j m^k_{ji}}{\sum_\ell \sum_j m^\ell_{ji}}\right)\left(\frac{m^k_{ii}}{\sum_j m^k_{ij}} + \frac{m^k_{ii}}{\sum_j m^k_{ji}}\right) \\
&= \frac{1}{4}\left(\frac{\sum_j m^k_{ij}}{\sum_\ell \sum_j m^\ell_{ij}} \cdot \frac{m^k_{ii}}{\sum_j m^k_{ij}} + \frac{\sum_j m^k_{ij}}{\sum_\ell \sum_j m^\ell_{ij}} \cdot \frac{m^k_{ii}}{\sum_j m^k_{ji}}\right) \\
&+ \frac{1}{4}\left(\frac{\sum_j m^k_{ji}}{\sum_\ell \sum_j m^\ell_{ji}} \cdot \frac{m^k_{ii}}{\sum_j m^k_{ij}} + \frac{\sum_j m^k_{ji}}{\sum_\ell \sum_j m^\ell_{ji}} \cdot \frac{m^k_{ii}}{\sum_j m^k_{ji}}\right) \\
&= \frac{m^k_{ii}}{4}\left(\frac{1}{\sum_\ell \sum_j m^\ell_{ij}} + \frac{\sum_j m^k_{ij}}{\sum_\ell \sum_j m^\ell_{ij}} \cdot \frac{1}{\sum_j m^k_{ji}} + \frac{\sum_j m^k_{ji}}{\sum_\ell \sum_j m^\ell_{ji}} \cdot \frac{1}{\sum_j m^k_{ij}} + \frac{1}{\sum_\ell \sum_j m^\ell_{ji}}\right).
\end{aligned}$$

Summing this identity over $k = 1, 2$ and using

$$m_{ii} = m_{ii}^1 + m_{ii}^2, \qquad \sum_j m_{ij} = \sum_{\ell=1}^{2} \sum_j m_{ij}^\ell, \qquad \sum_j m_{ji} = \sum_{\ell=1}^{2} \sum_j m_{ji}^\ell,$$

we obtain term-by-term:

$$\sum_{k=1}^{2} \frac{m_{ii}^k}{4} \cdot \frac{1}{\sum_\ell \sum_j m_{ij}^\ell} = \frac{m_{ii}}{4 \sum_j m_{ij}}, \qquad \sum_{k=1}^{2} \frac{m_{ii}^k}{4} \cdot \frac{1}{\sum_\ell \sum_j m_{ji}^\ell} = \frac{m_{ii}}{4 \sum_j m_{ji}},$$

and the cross terms aggregate as

$$\sum_{k=1}^{2} \frac{m_{ii}^k}{4} \frac{\sum_j m_{ij}^k}{\sum_\ell \sum_j m_{ij}^\ell} \cdot \frac{1}{\sum_j m_{ji}^k} = \frac{m_{ii}}{4} \cdot \frac{1}{\sum_j m_{ji}}, \qquad \sum_{k=1}^{2} \frac{m_{ii}^k}{4} \frac{\sum_j m_{ji}^k}{\sum_\ell \sum_j m_{ji}^\ell} \cdot \frac{1}{\sum_j m_{ij}^k} = \frac{m_{ii}}{4} \cdot \frac{1}{\sum_j m_{ij}}.$$

Adding the four contributions gives

$$\sum_{k=1}^{2} \alpha_k AHS_i^k = \frac{m_{ii}}{2} \left(\frac{1}{\sum_j m_{ij}} + \frac{1}{\sum_j m_{ji}} \right) = \frac{1}{2} \left(\frac{m_{ii}}{\sum_j m_{ij}} + \frac{m_{ii}}{\sum_j m_{ji}} \right) = AHS_i.$$

Thus $AHS_i = \alpha_1 AHS_i^1 + \alpha_2 AHS_i^2$.

For general $K \geq 2$, the same computation holds verbatim with sums over $k = 1, \ldots, K$ in the definitions of α_k and in the denominators. Alternatively, the K-way result follows by iterating the two-way case.

It can be similarly proven that HHS_i and GHS_i satisfy this property.

Appendix B: Comparing aggregate measures

Below are the proofs for the results in Table 2. First, we define each measure.

$$AHS = \sum_i \phi_i AHS_i,$$

where AHS_i is the type-specific measure:

$$AHS_i = \frac{1}{2} \left(\frac{m_{ii}}{\sum\limits_j m_{ij}} + \frac{m_{ii}}{\sum\limits_j m_{ji}} \right),$$

and ϕ_i can take two values:

$$\begin{aligned} \phi_i^G &= \frac{(\sum_j m_{ij})(\sum_j m_{ji})}{\sum_i (\sum_j m_{ij} \sum_j m_{ji})}, \\ \phi_i^C &= \frac{\sum_j m_{ij} + \sum_j m_{ji}}{2|M|}. \end{aligned}$$

Similarly, we define LR_C and LR_G as a weighted average of the type-specific LR_i, where,

$$LR_i = \frac{m_{ii}|M|}{\sum_j m_{ij} \sum_j m_{ji}}.$$

The $Trace$ is equal to HHS_C, where,

$$Trace = HHS_C = \frac{\sum_i m_{ii}}{|M|}.$$

The odds ratio is only defined on a 2-type matrix:

$$\text{Odds ratio} = \begin{cases} \dfrac{ad}{bc}, & \text{if } b \neq 0 \text{ and } c \neq 0, \\ +\infty, & \text{if } bc = 0. \end{cases}$$

Invariance axioms

All invariance axioms are trivially satisfied by all the measures.

Scale invariance: the type-specific measures AHS_i, LR_i and HHS_i and the shares ϕ_i^G and ϕ_i^C are unaffected by the scaling of the whole matrix. The odds ratio is also unaffected.

$$\begin{aligned}
AHS_i^{\lambda M} &= \frac{1}{2}\left(\frac{\lambda m_{ii}}{\sum_j \lambda m_{ij}} + \frac{\lambda m_{ii}}{\sum_j \lambda m_{ji}}\right) = \frac{1}{2}\left(\frac{m_{ii}}{\sum_j m_{ij}} + \frac{m_{ii}}{\sum_j m_{ji}}\right) = AHS_i, \\
LR_i^{\lambda M} &= \frac{\lambda m_{ii}\lambda|M|}{\sum_j \lambda m_{ij}\sum_j \lambda m_{ji}} = \frac{m_{ii}|M|}{\sum_j m_{ij}\sum_j m_{ji}} = LR_i, \\
HHS_i^{\lambda M} &= \frac{\lambda m_{ii}}{\sum_j \lambda m_{ij} + \sum_j \lambda m_{ji}} = \frac{m_{ii}}{\sum_j m_{ij} + \sum_j m_{ji}} = HHS_i, \\
\phi_i^{G,\lambda M} &= \frac{(\sum_j \lambda m_{ij})(\sum_j \lambda m_{ji})}{\sum_i(\sum_j \lambda m_{ij}\sum_j \lambda m_{ji})} = \frac{(\sum_j m_{ij})(\sum_j m_{ji})}{\sum_i(\sum_j m_{ij}\sum_j m_{ji})} = \phi_i^G, \\
\phi_i^{C,\lambda M} &= \frac{\sum_j \lambda m_{ij} + \sum_j \lambda m_{ji}}{2\lambda|M|} = \frac{\sum_j m_{ij} + \sum_j m_{ji}}{2|M|} = \phi_i^C, \\
\text{Odds ratio}^{\lambda M} &= (\lambda a)(\lambda d)/(\lambda b)(\lambda c) = ad/bc = \text{Odds ratio}.
\end{aligned}$$

Side invariance: For the AHS, LR and $Trace$ indices, side invariance of the aggregate measure follows directly from side invariance of the underlying type-specific indices together with side-invariant weights. The odds ratio also satisfies side invariance.

Type invariance: The aggregate AHS, LR, and $Trace$ measures are invariant to permutations of types, since the underlying type-specific measures and weights are themselves type invariant. The odds ratio likewise remains unaffected by switching types.

Marginal monotonicity axiom

In Chiappori et al. (2025), the marginal monotonicity axiom is defined for 2×2 matrices. In this setting, when two matrices M and M' share the same marginal distributions, an increase in homogamy for one type necessarily implies an increase in homogamy for the other type.

This implication does not generally extend to higher-dimensional matrices. For $n \times n$ matrices with $n \geq 3$, even when two matrices have identical marginal distributions, an increase in the number of homogamous couples for one type in M relative to M' does not

guarantee an increase in homogamy for all other types. The example below illustrates this distinction.

The two-type case. Consider matrices $M = (a, b; c, d)$ and $M' = (a', b'; c', d')$ with identical marginal distributions, that is,

$$a + c = a' + c', \quad a + b = a' + b', \quad d + b = d' + b', \quad d + c = d' + c'.$$

Marginal monotonicity as defined in Chiappori et al. (2025) implies that $I(M) > I(M')$ if and only if $a > a'$. This condition is equivalent to $b < b'$, $c < c'$, and $d > d'$. Hence, in the two-type case, an increase in homogamy for one type necessarily implies an increase in homogamy for the other type as well.

The three-type case. This equivalence no longer holds when there are three types. Let

$$M = \begin{bmatrix} a & b & v \\ c & d & w \\ x & y & z \end{bmatrix}, \qquad M' = \begin{bmatrix} a' & b' & v' \\ c' & d' & w' \\ x' & y' & z' \end{bmatrix},$$

with identical marginal distributions. In this case, $a > a'$ does not necessarily imply that $b > b'$, $c > c'$, or $d > d'$.

For instance, consider the perturbation

$$\begin{bmatrix} a + \varepsilon & b - \varepsilon & v \\ c - \varepsilon & d & w + \varepsilon \\ x & y + \varepsilon & z - \varepsilon \end{bmatrix}.$$

Here, the increase in a is offset by a decrease in z, along with other compensating changes. Whether assortative matching increases or decreases depends on the relative weights assigned to different types in the index.

Therefore, to extend the marginal monotonicity axiom to general $n \times n$ matrices, it is necessary to restrict comparisons to matrices with the same marginal distributions and weakly higher homogamy for all types, with a strict increase for at least one type. For example, the matrix M above can be meaningfully compared with

$$\begin{bmatrix} a + \varepsilon & b - \varepsilon & v \\ c - \varepsilon & d + \varepsilon & w \\ x & y & z \end{bmatrix}.$$

Consequently, the marginal monotonicity axiom is as follows: for any positive matrices M and M' with identical marginal distributions, if $m_{ii} > m'_{ii}$ for some i and $m_{jj} \geq m'_{jj}$ for all $j \neq i$, then $I(M) > I(M')$.

This axiom is satisfied by all measures. The weights remain unchanged as the marginal distributions, and hence also the total population, remain unchanged. The value of AHS_i, LR_i and HHS_i increases as m_{ii} increases, and the marginal distributions remain unchanged ($\sum_i m_{ij}, \sum_i m_{ji}, |M|$). For any $j \neq i$, AHS_j, LR_j and HHS_j do not decrease; taken together, this proves that the aggregate of the three measures satisfy marginal monotonicity. The odds ratio also satisfies marginal monotonicity as $(a + \epsilon)(d + \epsilon)/(b - \epsilon)(c - \epsilon) > (ad)(bc)$ for any $\epsilon > 0$.

Homogamy Axioms

Maximal homogamy

Maximal homogamy requires that when $\sum_j m_{ij} = \sum_j m_{ji} = m_{ii}$ for all i, the maximal level of homogamy is attained. Moreover, if there is some non-homogamous couple, then homogamy is not maximal. The AHS measures satisfy this axiom: under full homogamy, $AHS_i = 1$ for all i, and therefore the corresponding aggregate measures also equal 1. More generally, whenever each type-specific measure satisfies full homogamy (for that type), the aggregate measures also satisfy maximal homogamy. Similarly, if $m_{ij} \neq 0$ for some $i \neq j$, then $AHS_i < 1$ and $AHS_j < 1$; hence, their weighted average is strictly less than 1, and the aggregate measure does not attain its maximum.

For the likelihood ratio, consider first the aggregate measure,

$$LR_G = \sum_i \frac{m_{ii}|M|}{\sum_i (\sum_j m_{ij})(\sum_j m_{ji})}.$$

We prove that it does not satisfy maximal homogamy with a numerical example. Consider the matrices $M = (40, 0, 0, 30)$ and $M' = (67, 1, 1, 1)$. By maximal homogamy, $LR_G(M) > LR_G(M')$ should hold. However, $LR_G(M) \approx 4.08 < LR_G(M') \approx 18.51$, thereby showing that LR_G does not satisfy maximal homogamy. We prove that LR_G satisfies the second part of the axiom by showing that LR_G is unbounded and, hence, there is no maximal element. Take the 2×2 case. Let $M = (x, \epsilon, \epsilon, |M| - x - 2\epsilon)$, with $\epsilon > 0$. Then,

$$LR_G \approx \frac{|M|}{x} + \frac{|M|}{|M| - x},$$

and $LR_G \to \infty$ as $x \to 0^+$. Thus, LR_G is unbounded and no matrix exhibits maximal assortative matching, thereby proving that LR_G satisfies the second part of maximal homogamy. We can extend this argument to the $n \times n$ case.

Next consider the aggregate measure,

$$LR_C = \sum_{i=1}^{n} \frac{\left(\sum_j m_{ij} + \sum_j m_{ji}\right) m_{ii}}{2\left(\sum_j m_{ij}\right)\left(\sum_j m_{ji}\right)}.$$

Without loss of generality, assume that $\sum_j m_{ji} \leq \sum_j m_{ij}$. Then $(\sum_j m_{ij} + \sum_j m_{ji})/2 \leq \sum_j m_{ij}$. Since $m_{ii} \leq \sum_j m_{ji}$ always holds, each summand is bounded above by 1. Hence, $LR_C \leq \sum_j 1 = n$ for any $n \times n$ matrix, implying that LR_C is bounded above by n.

Now consider a matrix in which all off-diagonal elements are zero. Then $LR_C = k$, where k are the number of non-zero diagonal elements. Further, $k = n$ (as we assume that each type i has some men and women), so $LR_C = n$. Since this is the maximum attainable value, we have shown that any diagonal matrix is maximal, and LR_C satisfies the first part of maximal homogamy.

Next, suppose that a matrix M has one off-diagonal element $m_{kl} \neq 0$. Then,

$$\begin{aligned} LR_C^M &= \frac{\left(\sum_j m_{kj} + \sum_j m_{jk}\right) m_{kk}}{2\left(\sum_j m_{ij}\right)\left(\sum_j m_{ji}\right)} + \sum_{i \neq k} \frac{\left(\sum_j m_{ij} + \sum_j m_{ji}\right) m_{ii}}{2\left(\sum_j m_{ij}\right)\left(\sum_j m_{ji}\right)} \\ &< 1 + (n-1) = n, \end{aligned}$$

where the first term is less than 1 as $m_{kl} > 0$, while each of the remaining $n-1$ terms is bounded above by 1. Hence, M is not maximal, and LR_C also satisfies the second part of maximal homogamy.

The odds ratio satisfies the first part of maximal homogamy as it goes to infinity, when both off-diagonal elements b and c are zero. However, it violates the second part, since the odds ratio goes to infinity when either b or c is equal to zero.

The trace satisfies maximal homogamy as it takes its maximum value 1 when all off-diagonal elements are zero. If any off-diagonal element is positive, then $\sum_i m_{ii} < |M|$, and the trace is strictly less than 1.

Minimal homogamy

Minimal homogamy requires that when all diagonal elements are zero, the minimal level of homogamy is attained. Moreover, if any diagonal element is positive, homogamy is not minimal.

AHS_C, AHS_G, LR_C, LR_G and $Trace$ satisfy this axiom as they are aggregates of type-specific measures, and AHS_i, LR_i, and HHS_i equal zero when $m_{ii} = 0$ and positive if $m_{ii} > 0$. Therefore, the weighted sum is zero only when $m_{ii} = 0$ for all i.

The odds ratio does not satisfy this axiom as the matrix $M = [a, b, c, 0]$ gives $OR = 0$ even if one element of the diagonal is not zero.

Marginal sensitivity axiom

The AHS_C measure satisfies marginal sensitivity. Consider a balanced matrix M and an unbalanced matrix M' with the same diagonal sum and same total mass. By 8,

$$AHS_C(M') = \sum_i \left(\frac{am_i}{gm_i}\right)^2 \frac{m'_{ii}}{|M'|},$$

where am_i and gm_i are the arithmetic and geometric mean of the number of men and women of type i in M', respectively. Since M' is imbalanced, by the AM-GM inequality, the below inequality is obtained.

$$AHS_C(M') = \sum_i \left(\frac{am_i}{gm_i}\right)^2 \frac{m'_{ii}}{|M'|} > \sum_i \frac{m'_{ii}}{|M'|} = \frac{\sum_i m'_{ii}}{|M'|}.$$

Further, since M is balanced, therefore, the corresponding arithmetic and geometric mean for each type are equal and, $AHS_C(M) = \sum_i m_{ii}/|M|$. Further, by equality of the diagonal sums and total mass, it follows that,

$$AHS_C(M') > \frac{\sum_i m'_{ii}}{|M'|} = \sum_i \frac{m_{ii}}{|M|} = AHS_C(M).$$

Thus, AHS_C satisfies the marginal sensitivity axiom.

The AHS_G measure does not satisfy marginal sensitivity. Recall that,

$$AHS_G(M) = \frac{\sum_i m_{ii}\left(\sum_j m_{ij} + \sum_j m_{ji}\right)}{2\sum_i \left(\sum_j m_{ij}\right)\left(\sum_j m_{ji}\right)}.$$

Consider the matrices,

$$M = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}, \qquad M' = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

Matrix M is balanced, while M' is imbalanced. Moreover,

$$|M| = |M''| = 15, \qquad \sum_i m_{ii} = \sum_i m''_{ii} = 3.$$

Nevertheless,

$$AHS_G(M) = \frac{1}{5} = 0.2 \quad \text{and} \quad AHS_G(M'') = \frac{15}{76} \approx 0.197,$$

with $AHS_G(M'') < AHS_G(M)$. Thus, imbalance does not necessarily increase the value of AHS_G, violating the marginal sensitivity axiom.

The $Trace$ does not satisfy the marginal sensitivity axiom. Since the measure only depends on the sum of diagonals and the total mass, $Trace(M) = Trace(M')$ and the imbalanced matrix M' does not exhibit higher assortative matching, as compared to the balanced matrix M, under the $Trace$ measure; thereby showing that the measure violates the marginal sensitivity axiom.

The odds ratio fails the marginal sensitivity axiom. Consider the balanced matrix

$$M = \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix}$$

and the imbalanced matrix

$$M' = \begin{pmatrix} 6.5 & 4 \\ 6 & 3.5 \end{pmatrix}.$$

Both matrices have sum of diagonals equal to 10 and $|M| = |M'| = 20$. However,

$$OR(M) = \frac{ad}{bc} = \frac{(6)(4)}{(5)(5)} = 0.96,$$

and

$$OR(M') = \frac{a'd'}{b'c'} = \frac{(6.5)(3.5)}{(4)(6)} \approx 0.948.$$

Therefore, $OR(M') < OR(M)$, thereby showing that the odds ratio does not satisfy the marginal sensitivity axiom.

The LR_C measure does not satisfy the marginal sensitivity axiom. Recall that,

$$LR_C = \sum_{i=1}^{n} \frac{\left(\sum_j m_{ij} + \sum_j m_{ji}\right) m_{ii}}{2\left(\sum_j m_{ij}\right)\left(\sum_j m_{ji}\right)}.$$

Consider the balanced matrix

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

and the imbalanced matrix

$$M' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}.$$

Both matrices satisfy $\sum_i m_{ii} = \sum_i m'_{ii} = 3$ and $|M| = |M'| = 13$. However,

$$LR_C(M) = \frac{11}{15} \approx 0.733, \qquad LR_C(M'') = \frac{29}{40} \approx 0.725,$$

with $LR_C(M') < LR_C(M)$. Thus, imbalance does not necessarily increase LR_C when total mass and diagonal sums are held fixed. This shows that LR_C violates the marginal sensitivity axiom.

The LR_G measure does not satisfy the marginal sensitivity axiom. Recall that,

$$LR_G(M) \;=\; \sum_i \frac{m_{ii}|M|}{\sum_k \left(\sum_j m_{kj}\right)\left(\sum_j m_{jk}\right)} \;=\; \frac{\left(\sum_i m_{ii}\right)|M|}{\sum_k \left(\sum_j m_{kj}\right)\left(\sum_j m_{jk}\right)}.$$

Consider the balanced matrix

$$M = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

and the imbalanced matrix

$$M'' = \begin{pmatrix} 1 & 5 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

The matrices satisfy $\sum_i m_{ii} = \sum_i m''_{ii} = 3$ and $|M| = |M''| = 15$. However,

$$LR_G(M) = \frac{3}{5}, \qquad LR_G(M'') = \frac{45}{79},$$

with $LR_G(M'') < LR_G(M)$. Thus, imbalance does not necessarily increase LR_G when total mass and diagonal sums are held fixed. This shows that LR_G does not satisfy the marginal sensitivity axiom.

Proof of Corollary 1:

Proof. For any balanced population M,

$$AHS_C(M) = Trace(M)$$

as $AHS_i(M) = HHS_i(M)$ for all i, since the marginals for each type i are equal. Instead, for any unbalanced population M', with the same diagonal sum and total mass, we know from Table 2 that AHS_C satisfies marginal sensitivity and $Trace$ does not. In particular,

$$AHS_C(M') > AHS_C(M),$$
$$Trace(M') = Trace(M),$$

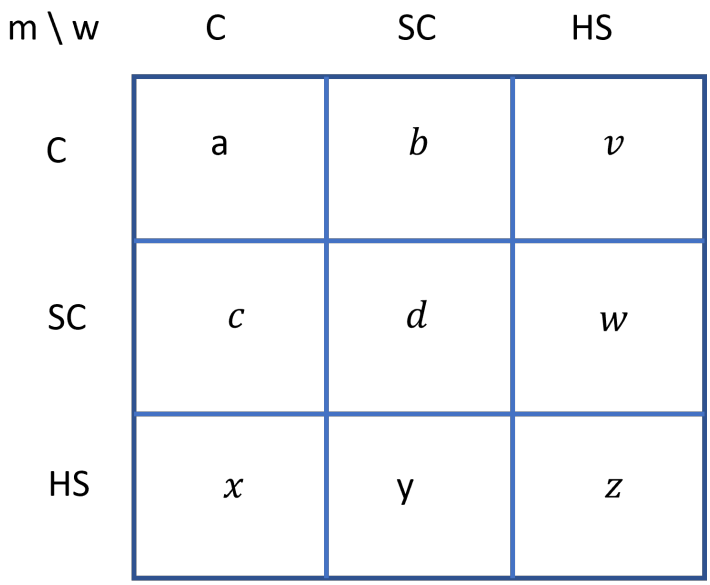
as seen in the marginal sensitivity axiom subsection. Combining the above inferences gives,

$$AHS_C(M') > AHS_C(M) = Trace(M) = Trace(M').$$

Therefore, $AHS_C(M') > Trace(M')$ holds, thereby completing the proof of the corollary. □

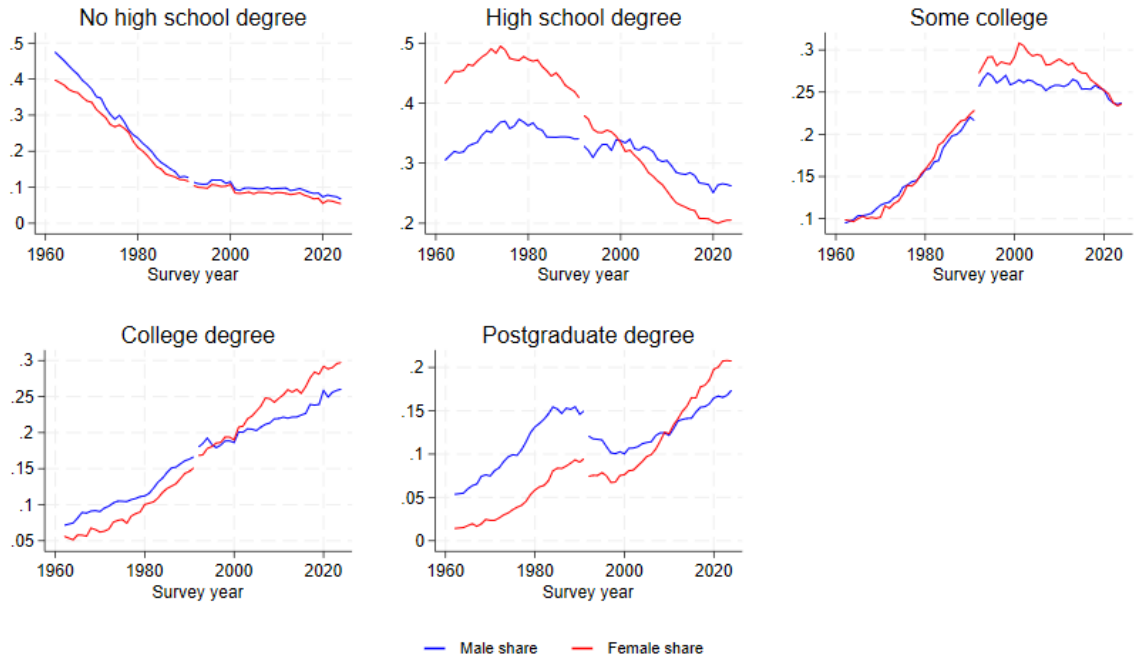
Tables and Figures

Figure 1: Contingency table.



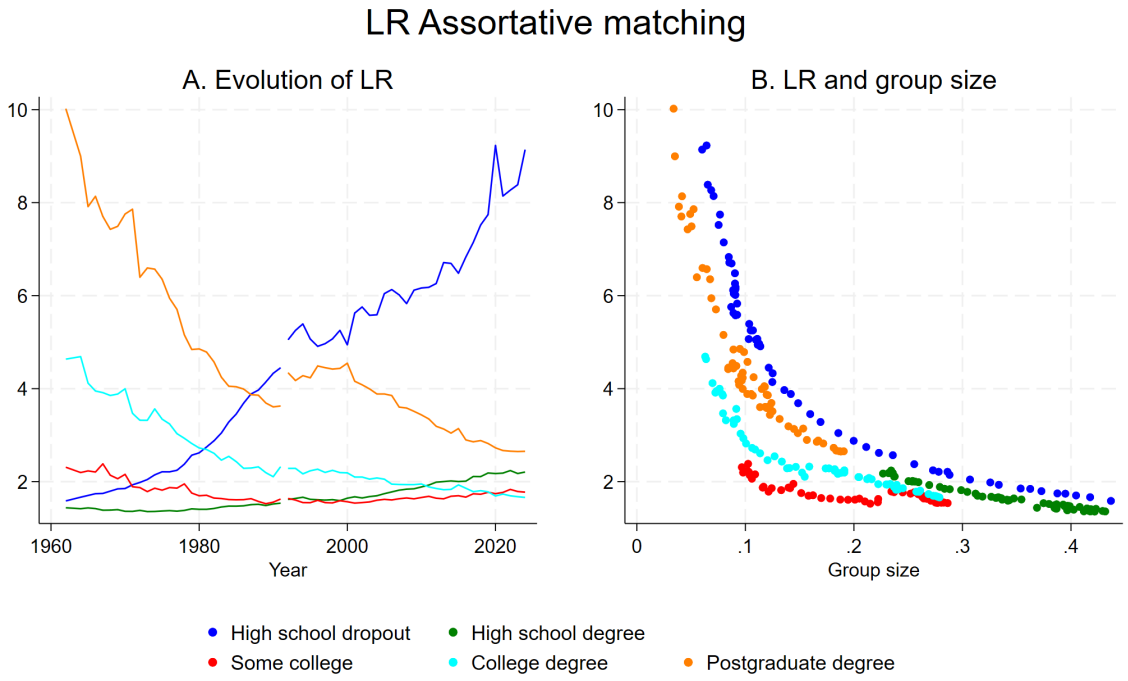
Note: Each cell in this matrix represents the number of couples with men type i and women type j, where types are C, SC and HS.

Figure 2: Evolution of educational achievement in the USA



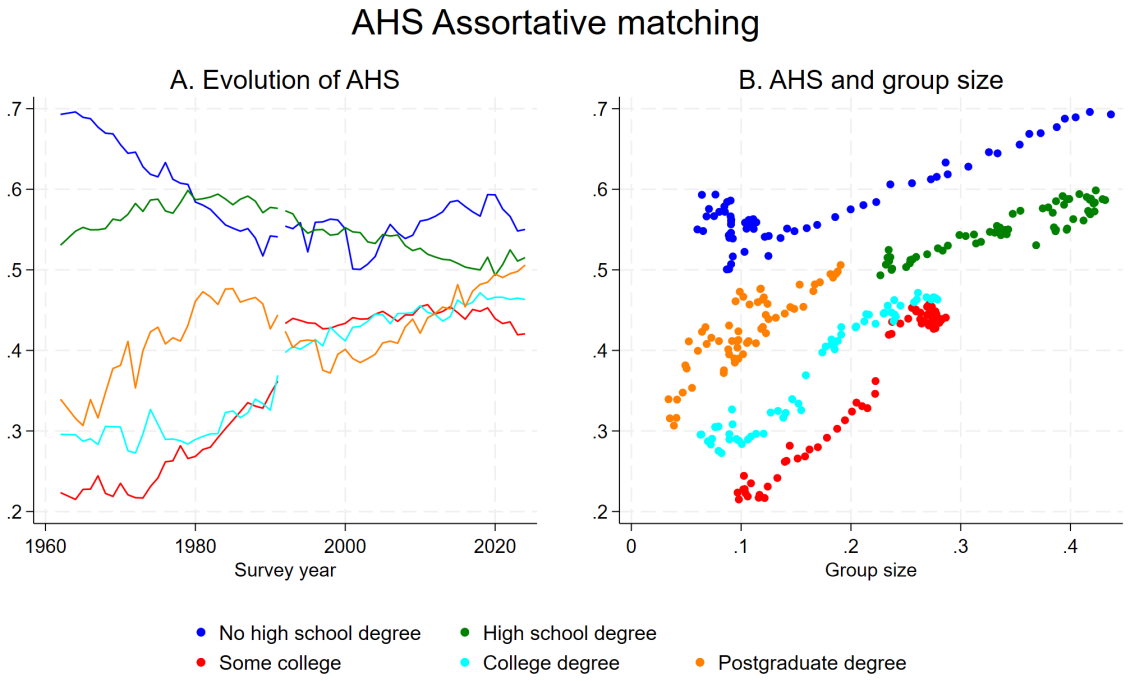
Note: This figure illustrates the evolution of the share of men and women with each educational level in the sample under study. There is a break in 1992 because the education question in the survey changed. Each year, the sample includes all heterosexual couples (married or cohabiting) in which at least one partner is aged 35-44.

Figure 3: LR_i measure of educational assortative matching in the US



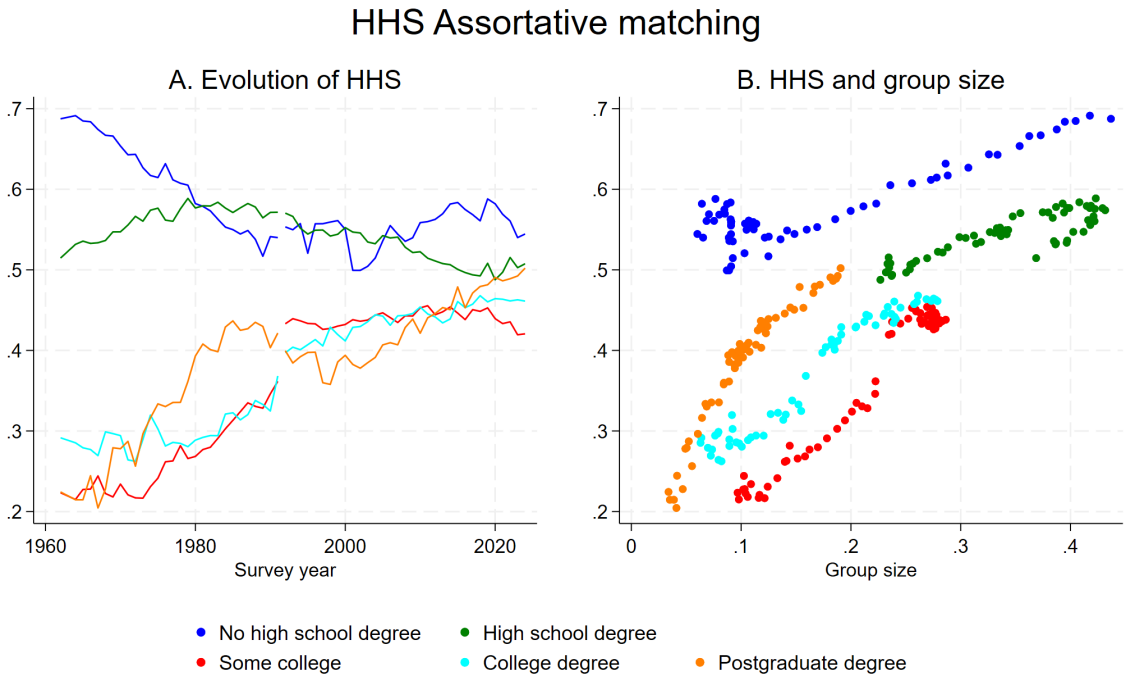
Note: Figure A illustrates the evolution of LR_i assortative matching for each education type in the US. This graph replicates Figure 3A in Eika et al. (2019) using five types of education and extending the period from 2013 to 2024. Figure B shows the relationship between LR_i assortative matching and group size.

Figure 4: AHS_i measure of educational assortative matching in the US



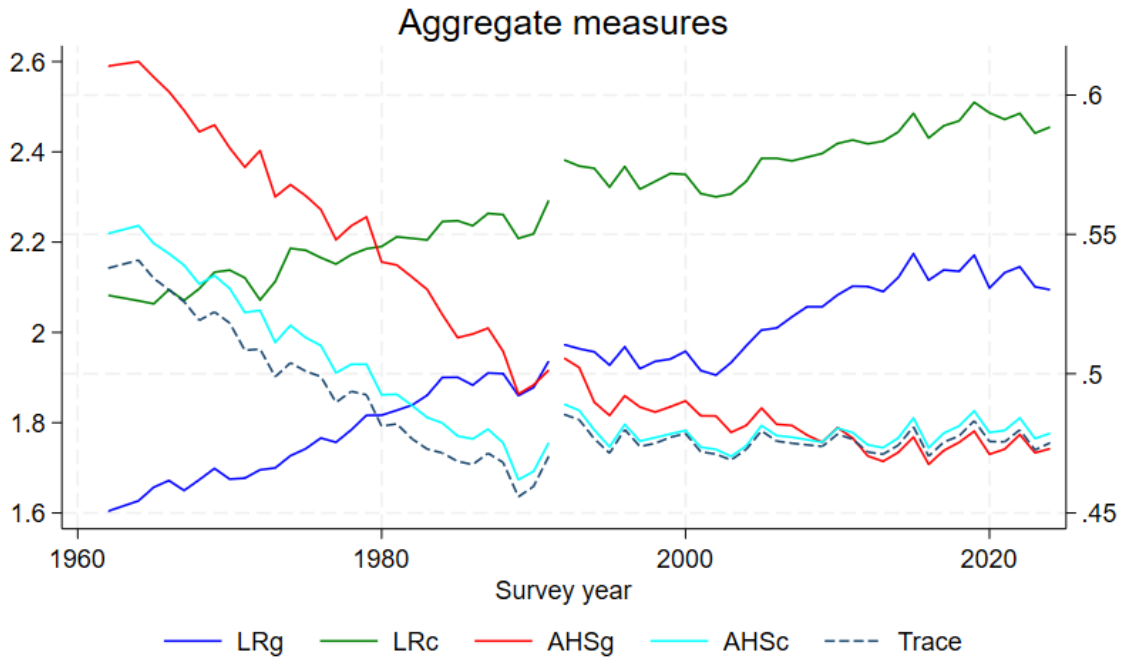
Note: Figure A illustrates the evolution of AHS_i assortative matching for each type of education in the US. Figure B shows the relationship between AHS_i assortative matching and group size.

Figure 5: HHS_i measure of educational assortative matching in the US



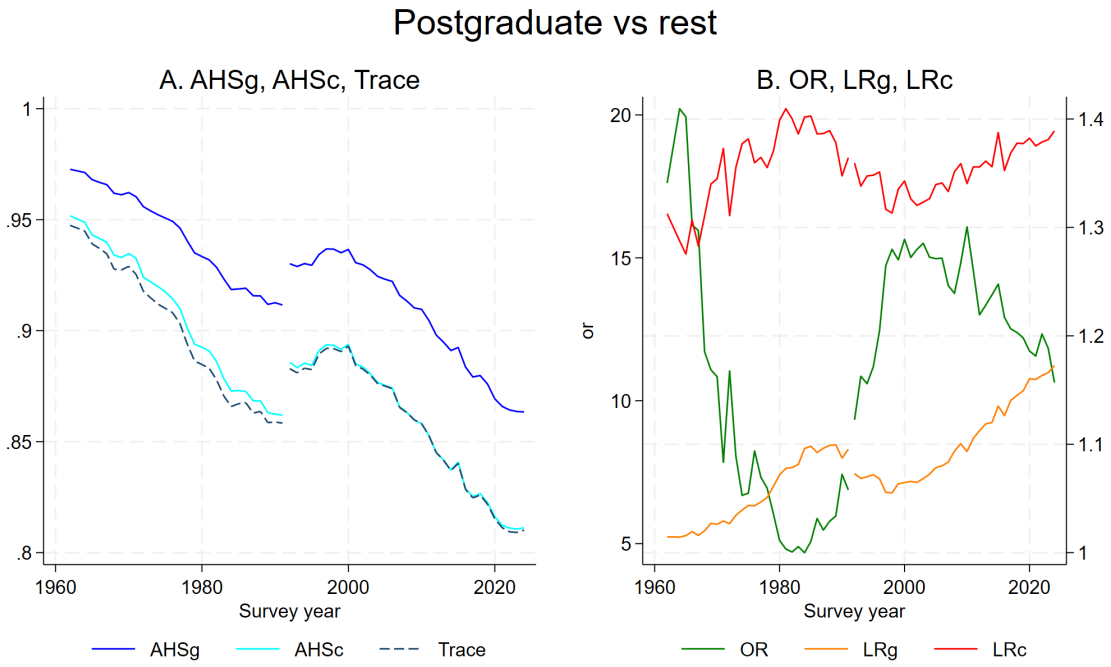
Note: Figure A illustrates the evolution of HHS_i assortative matching for each type of education in the US. Figure B shows the relationship between HHS_i assortative matching and group size.

Figure 6: Aggregate measures of educational assortative matching in the US



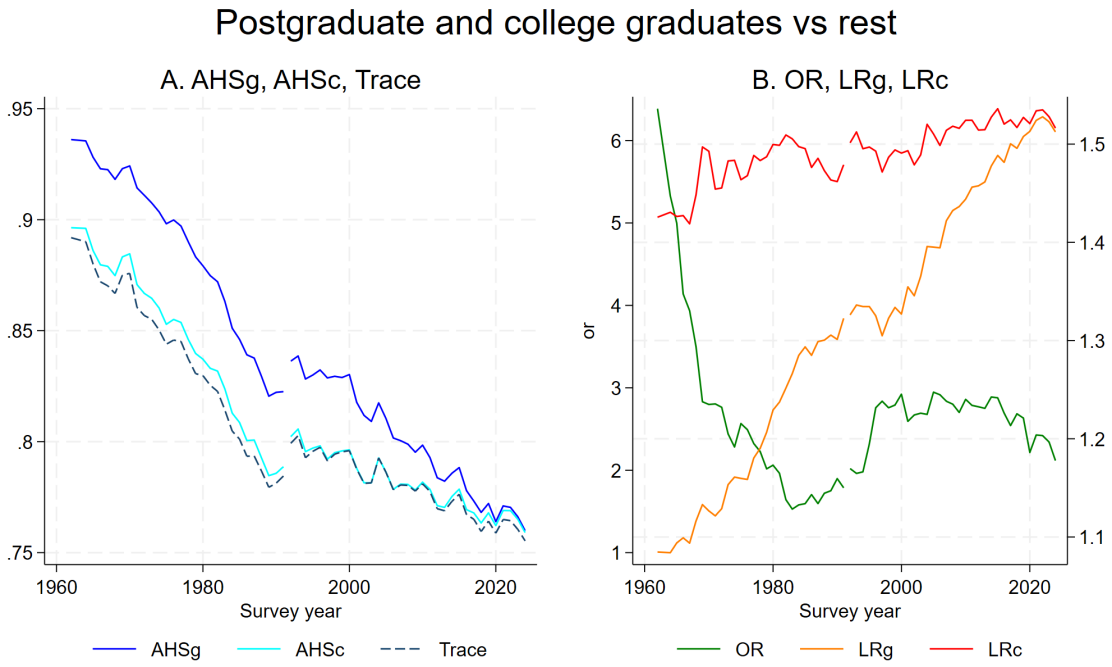
Note: This figure shows the aggregate measures of educational AM using five education categories: less than high school, high school, some college, college degree, and postgraduate. The left y-axis shows the LR measures, while the right y-axis shows the AHS and Trace measures. For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of education type in the population as weights.

Figure 7: Odds ratio versus AHS. Two groups: postgraduates vs the rest



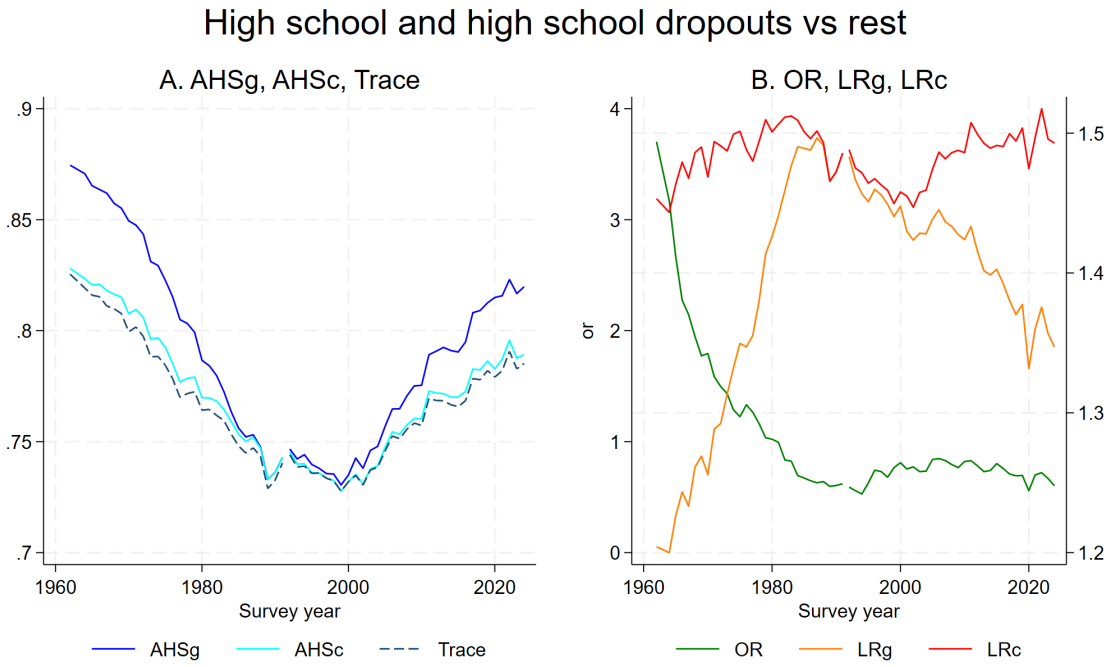
Note: Figure A shows the evolution of AHS and Trace measures. Figure B displays the LR measures (right-axis) and the OR measure (left-axis). For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of education type in the population as weights.

Figure 8: Odds ratio versus AHS. Two groups: postgraduates and college graduates vs the rest



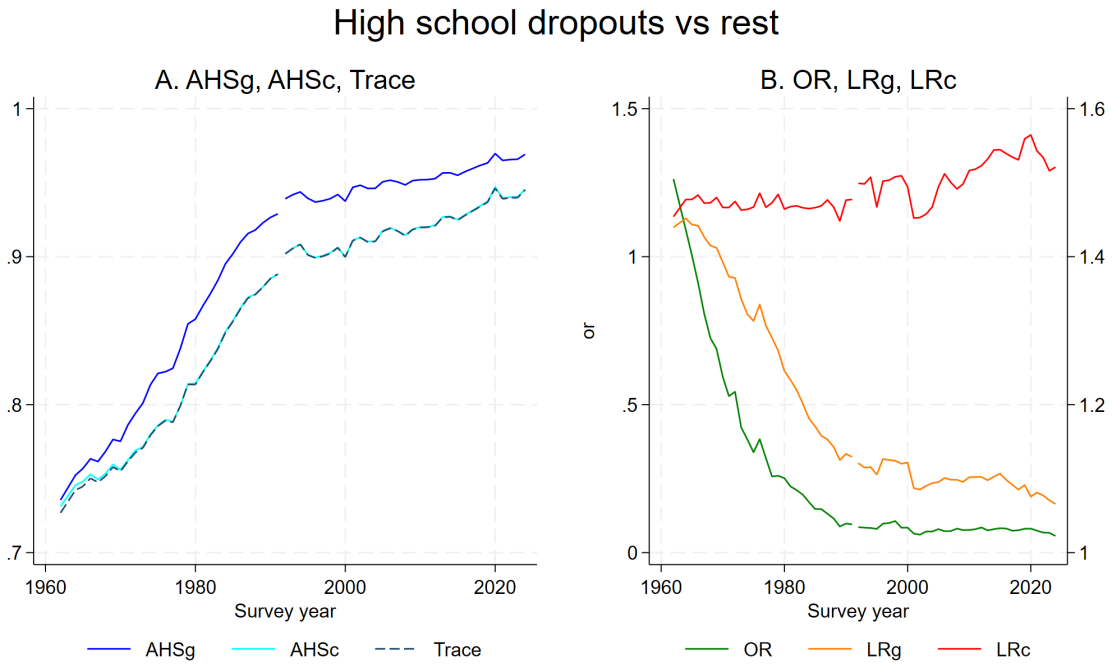
Note: Figure A shows the evolution of AHS and Trace measures. Figure B displays the LR measures (right-axis) and the OR measure (left-axis). For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of education type in the population as weights.

Figure 9: Odds ratio versus AHS. Two groups: At most high school vs the rest



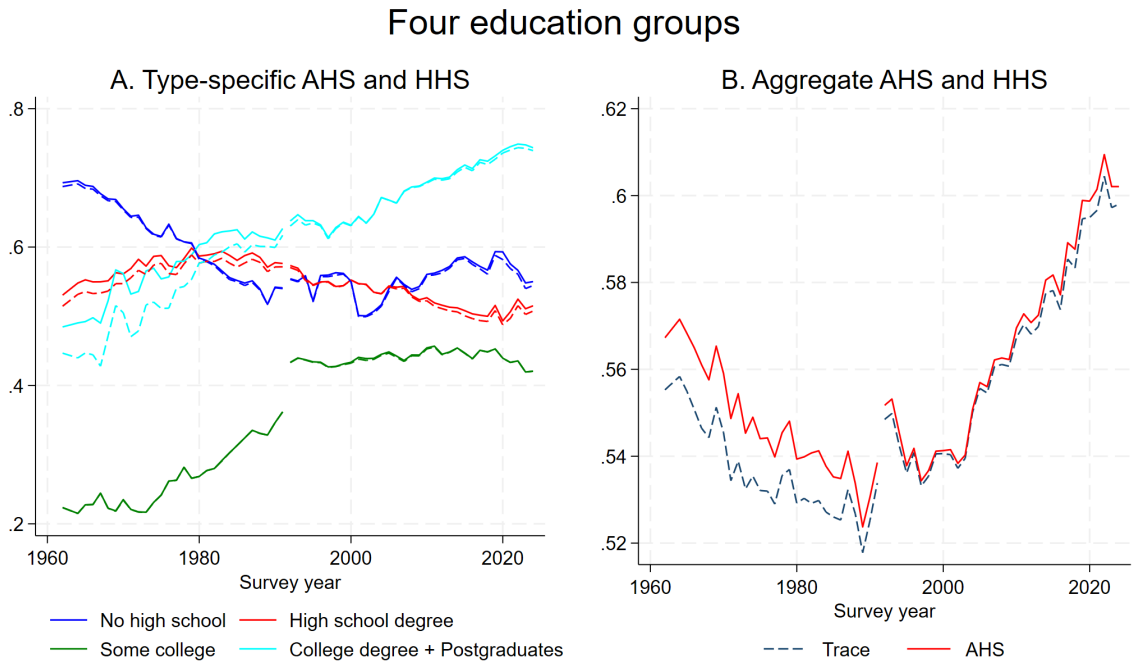
Note: Figure A shows the evolution of AHS and Trace measures. Figure B displays the LR measures (right-axis) and the OR measure (left-axis). For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of education type in the population as weights.

Figure 10: Odds ratio versus AHS. Two groups: High school dropouts vs the rest



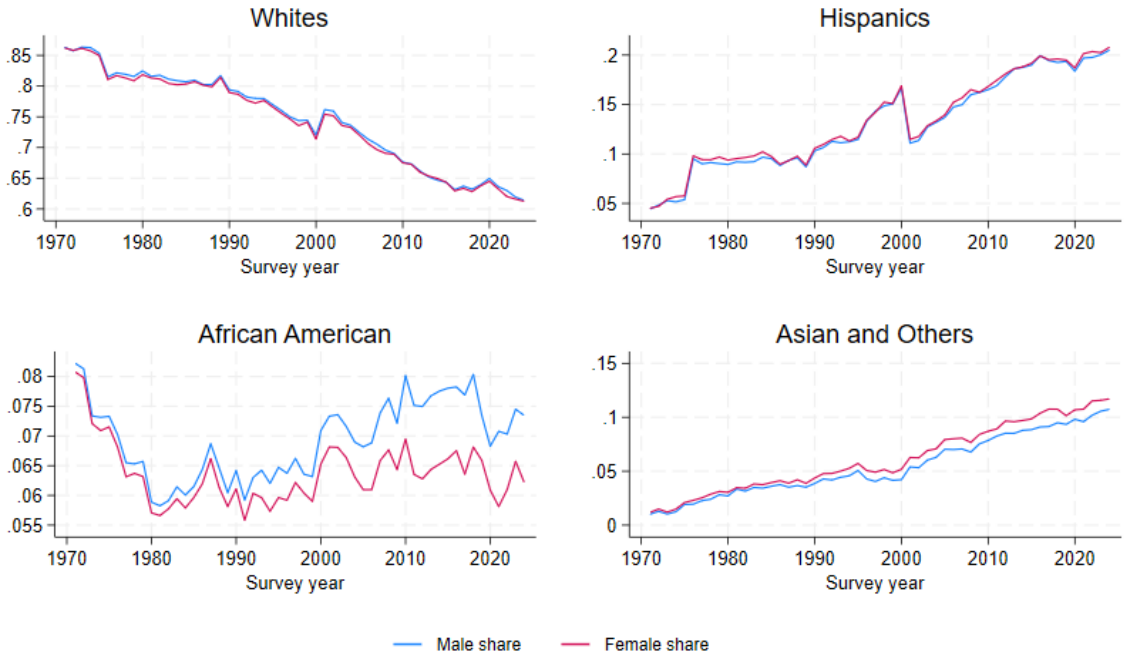
Note: Figure A shows the evolution of AHS and Trace measures. Figure B displays the LR measures (right-axis) and the OR measure (left-axis). For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of education type in the population as weights.

Figure 11: Assortative matching with four categories of education



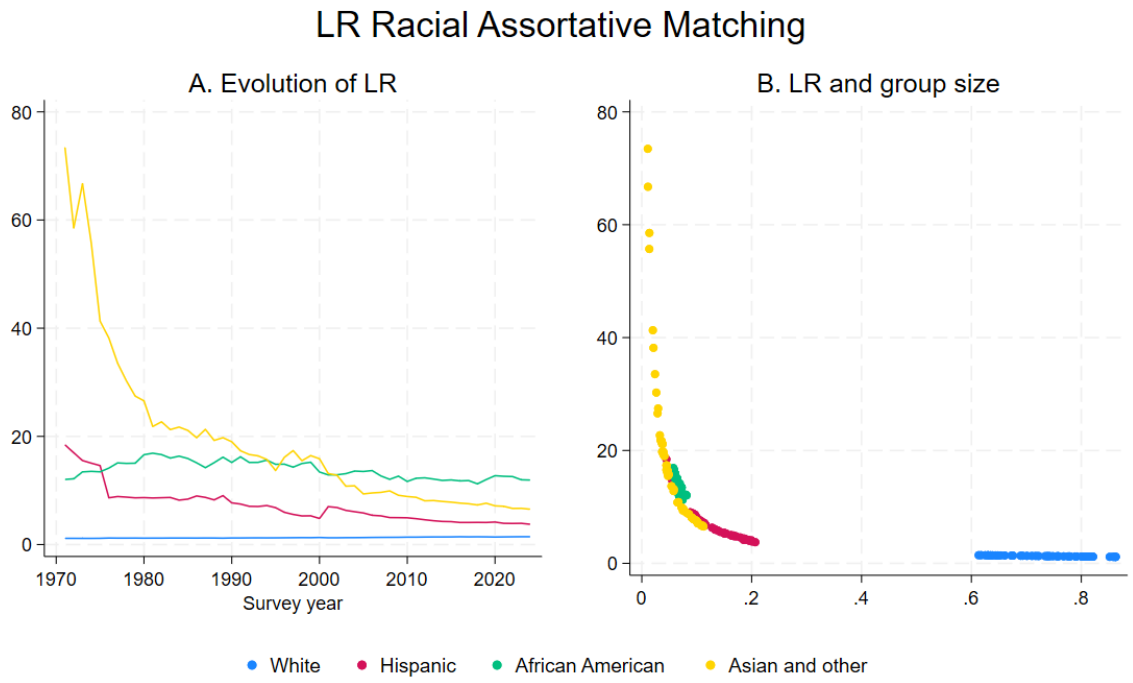
Note: Figure A shows the evolution of the education-specific AHS_i (continuous lines) and HHS_i (dashed lines), and Figure B illustrates the evolution of the aggregate AHS_C and $Trace$ measures of assortative matching when considering four types of education: less than high school, high school, some college, and college graduates.

Figure 12: Evolution of racial shares in the USA



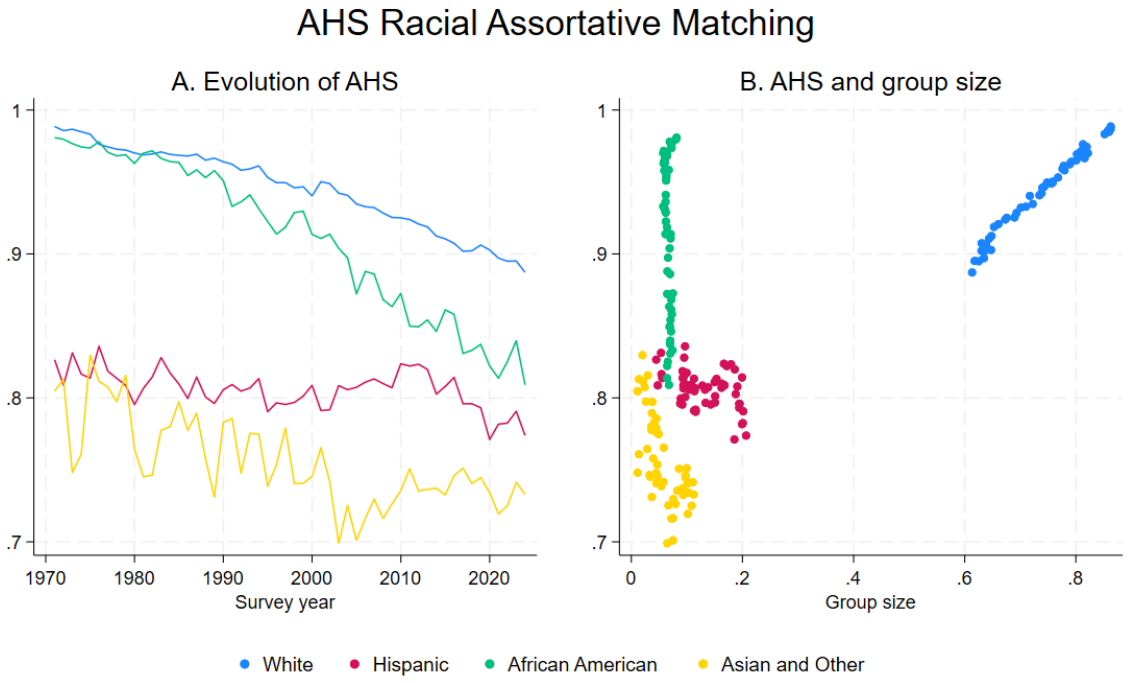
Note: This figure illustrates the evolution of race distribution in the sample under study. Each year, the sample includes all heterosexual couples (married or cohabiting) in which at least one partner is aged 35-44.

Figure 13: LR_i measure of racial assortative matching in the US



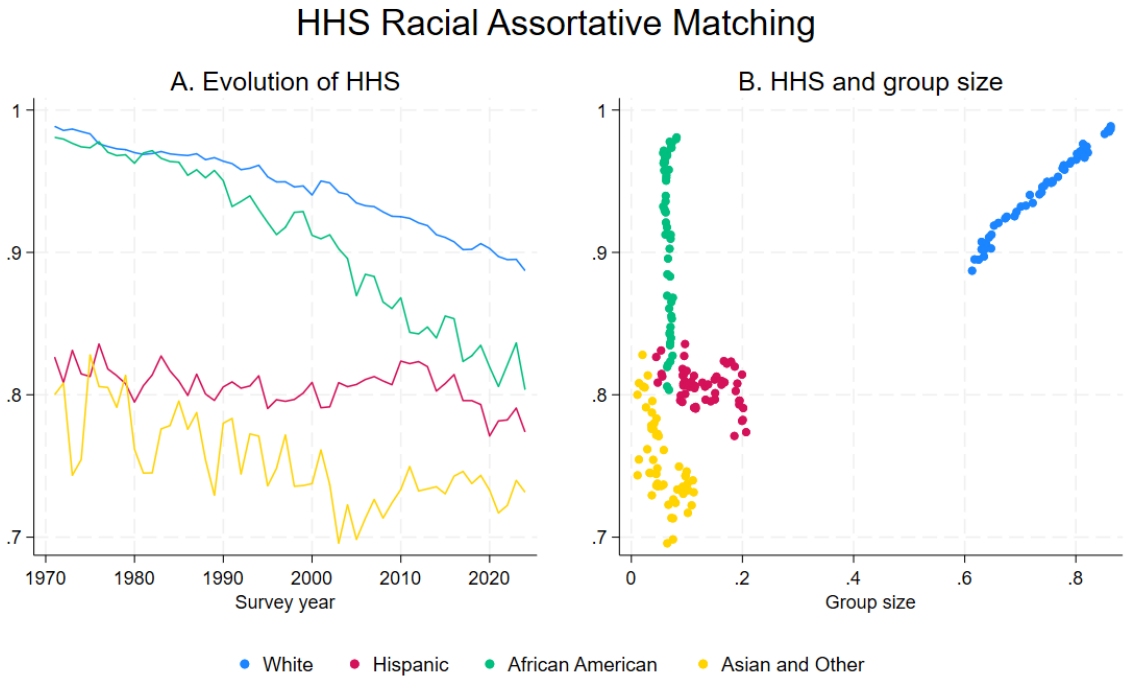
Note: Figure A illustrates the evolution of LR_i assortative matching for each race group in the USA. Figure B shows the relationship between LR_i and group size.

Figure 14: AHS_i measure of racial assortative matching in the US



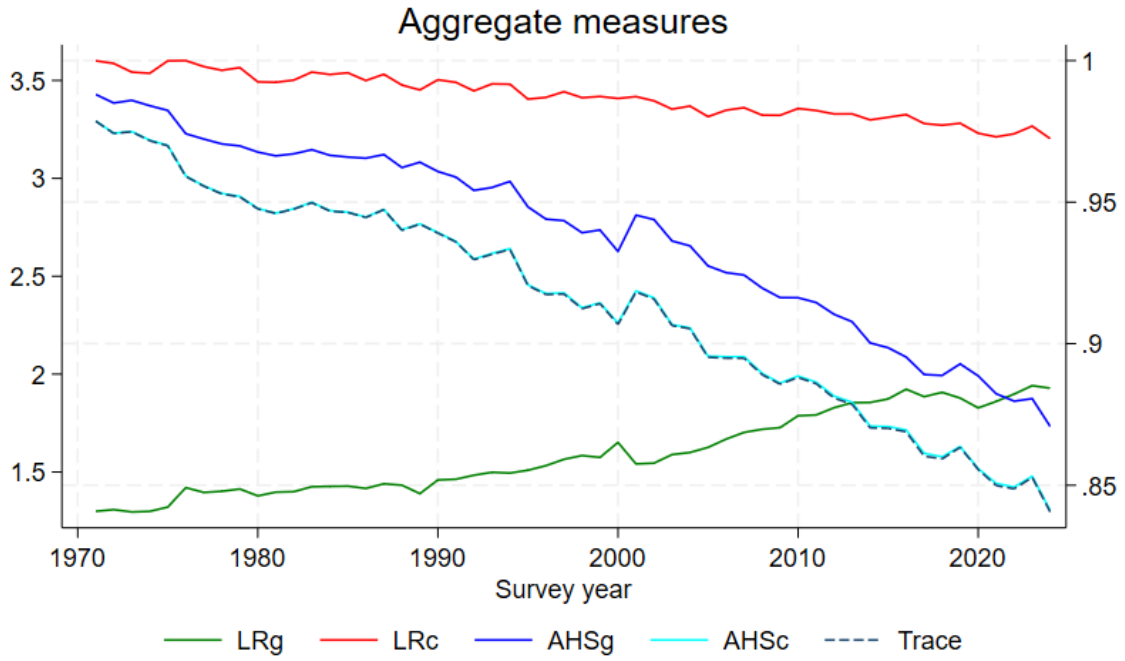
Note: Figure A illustrates the evolution of AHS_i assortative matching for each race group in the USA. Figure B shows the relationship between AHS_i assortative matching and group size.

Figure 15: HHS_i measure of racial assortative matching in the US



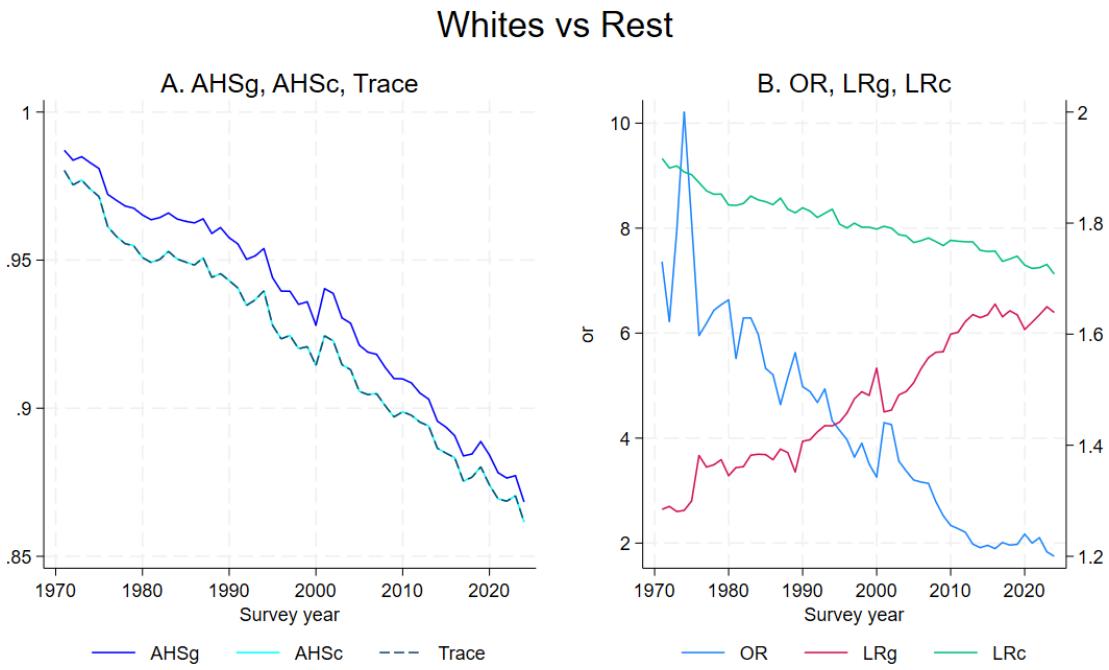
Note: Figure A illustrates the evolution of HHS_i assortative matching for each race group in the USA. Figure B shows the relationship between HHS_i assortative matching and group size.

Figure 16: Racial assortative matching in the USA



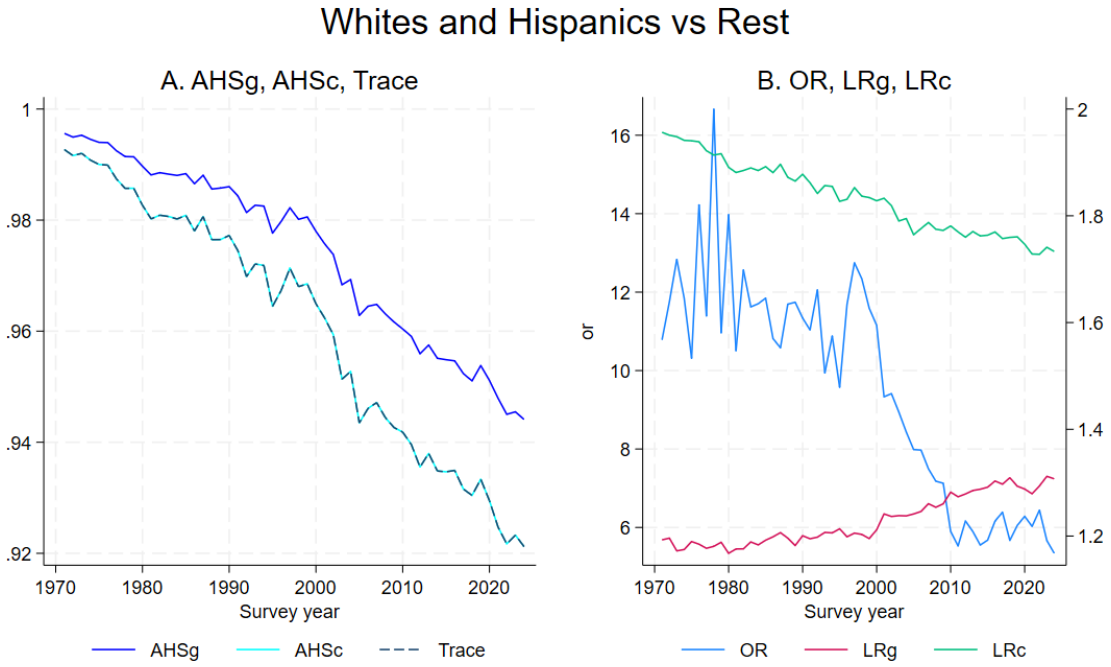
Note: The figure illustrates the evolution of aggregate racial assortative matching in the USA using different measures. The left y-axis shows the LR measures, while the right y-axis shows the AHS and Trace measures. For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of the racial group in the population as weights.

Figure 17: Odds ratio versus AHS. Two groups: Whites vs Rest



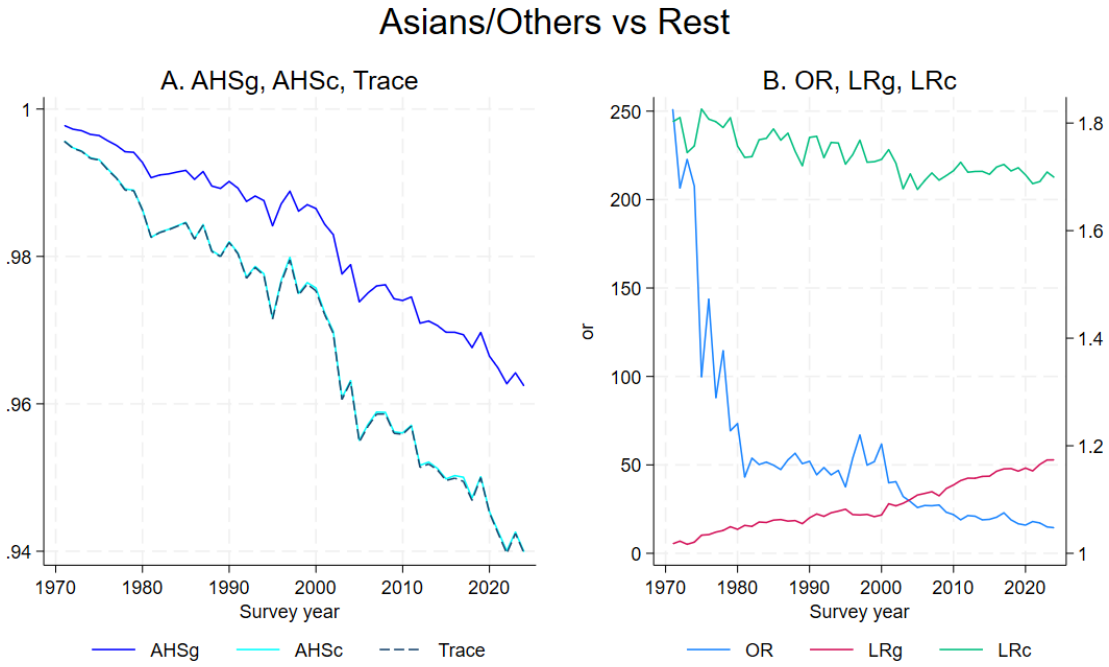
Note: Figure A shows the evolution of AHS and Trace measures. Figure B displays the LR measures (right-axis) and the OR measure (left-axis). For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of the racial group in the population as weights.

Figure 18: Odds ratio versus AHS. Two groups: Whites and Hispanics vs Rest



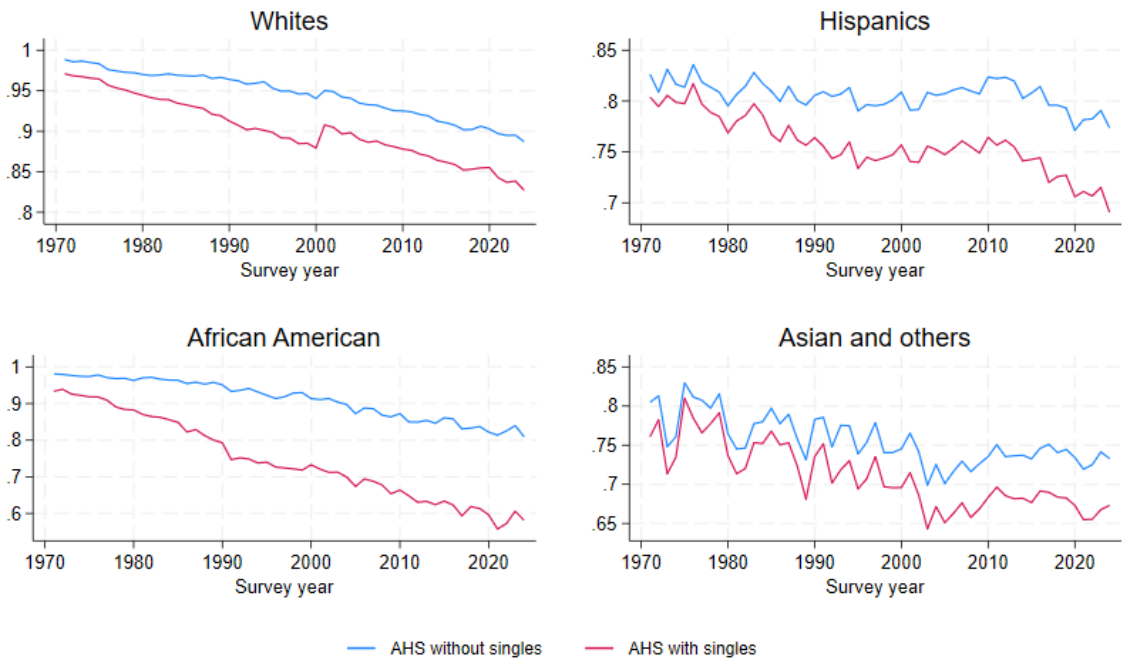
Note: Figure A shows the evolution of AHS and Trace measures. Figure B displays the LR measures (right-axis) and the OR measure (left-axis). For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of the racial group in the population as weights.

Figure 19: Odds ratio versus AHS. Two groups: Asian group vs the rest



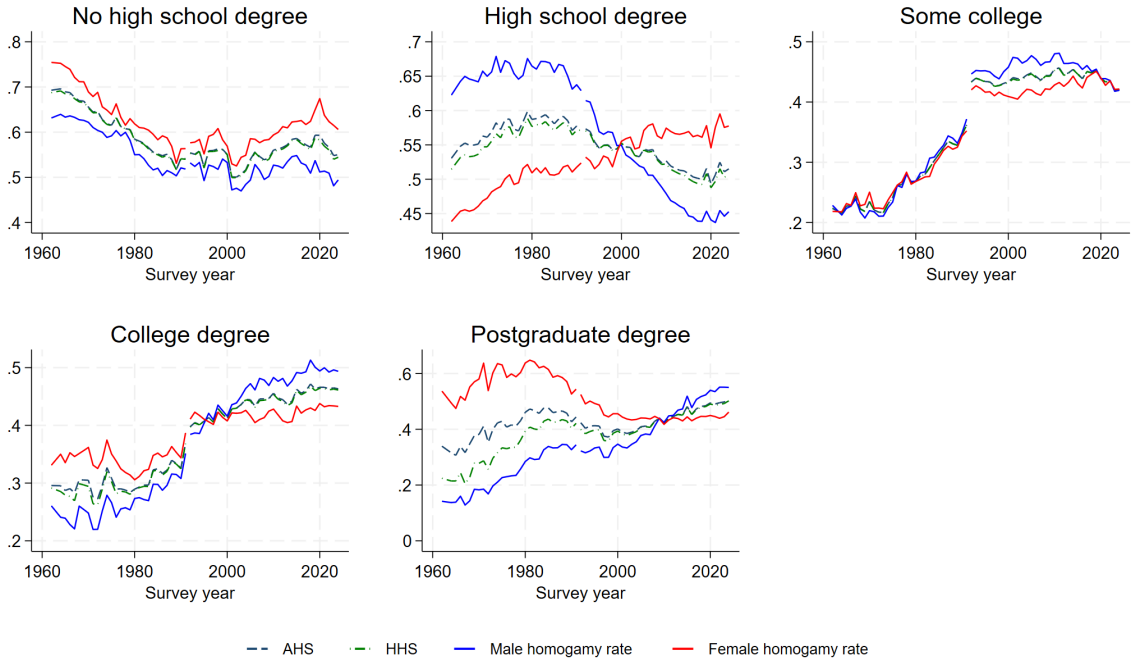
Note: Figure A shows the evolution of AHS and Trace measures. Figure B displays the LR measures (right-axis) and the OR measure (left-axis). For LR and AHS, we aggregate using two alternative weightings: I_G uses the share of expected random matches as weights, and I_C uses the share of the racial group in the population as weights.

Figure 20: Adding singles



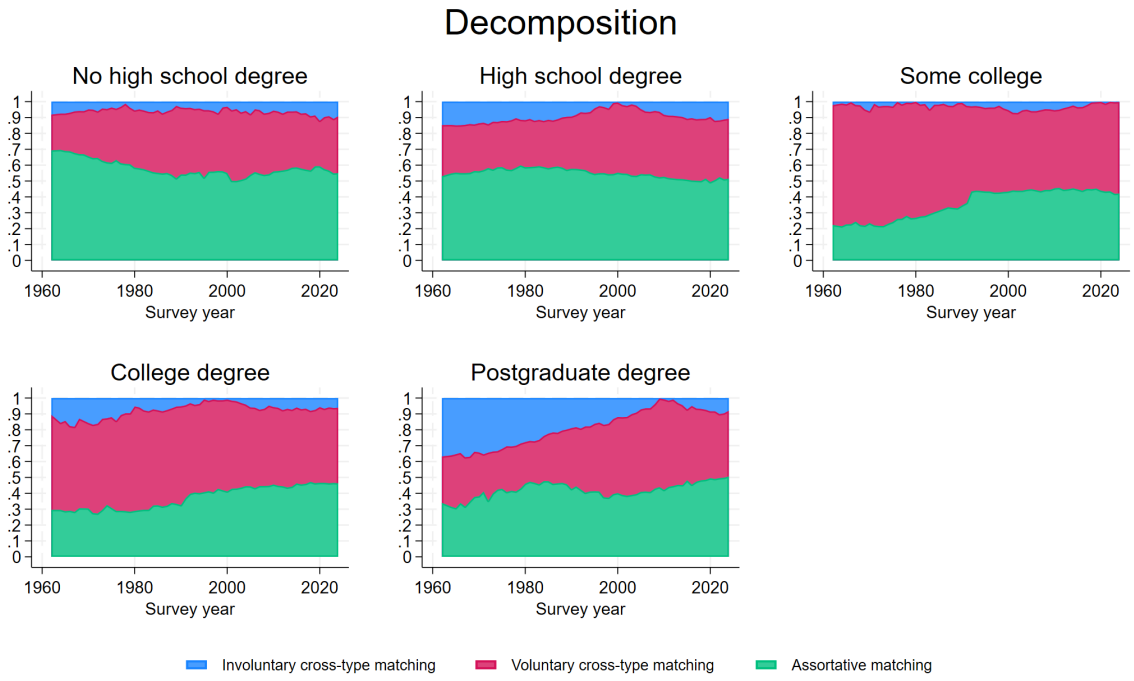
Note: We plot the evolution of AHS with and without singles. We aggregate using the share of racial groups in the population as weights.

Figure 21: Gender decomposition of AHS_i



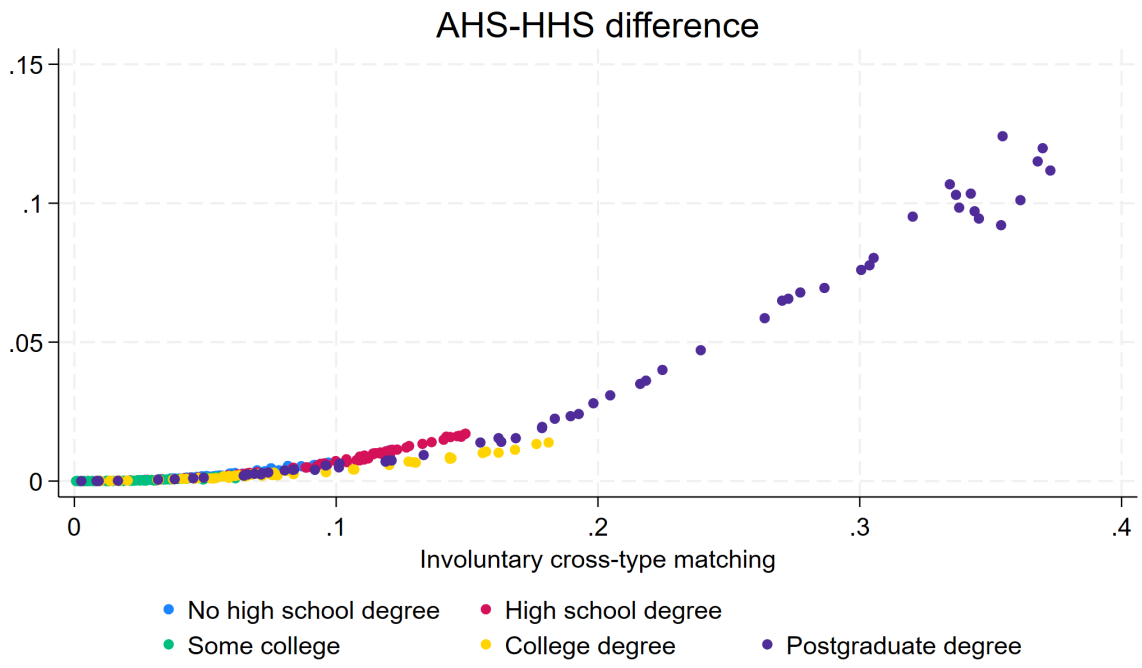
Note: The figure illustrates, for each education level, the homogamy rate by gender, as well as the AHS_i and HHS_i measures.

Figure 22: Decomposition of voluntary and involuntary cross-type matching



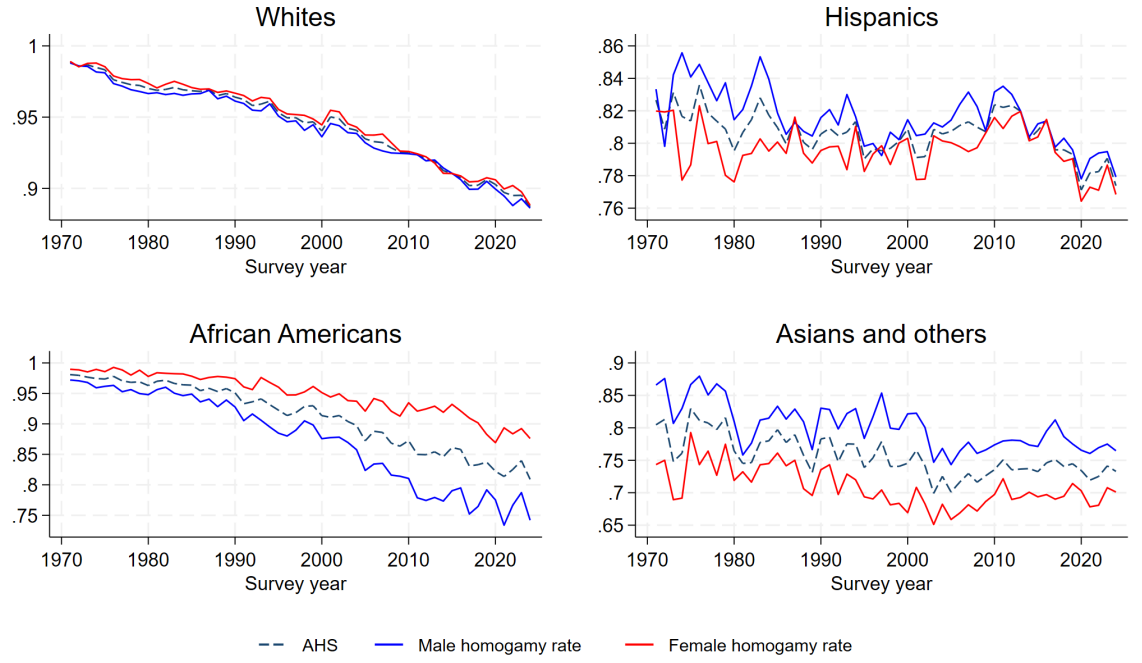
Note: The figure illustrates the decomposition of non-assortative matching into voluntary and involuntary cross-type matching. All measures are computed as the average of the gender shares, similar to AHS_i.

Figure 23: $AHS_i - HHS_i$ of education assortative matching in the US



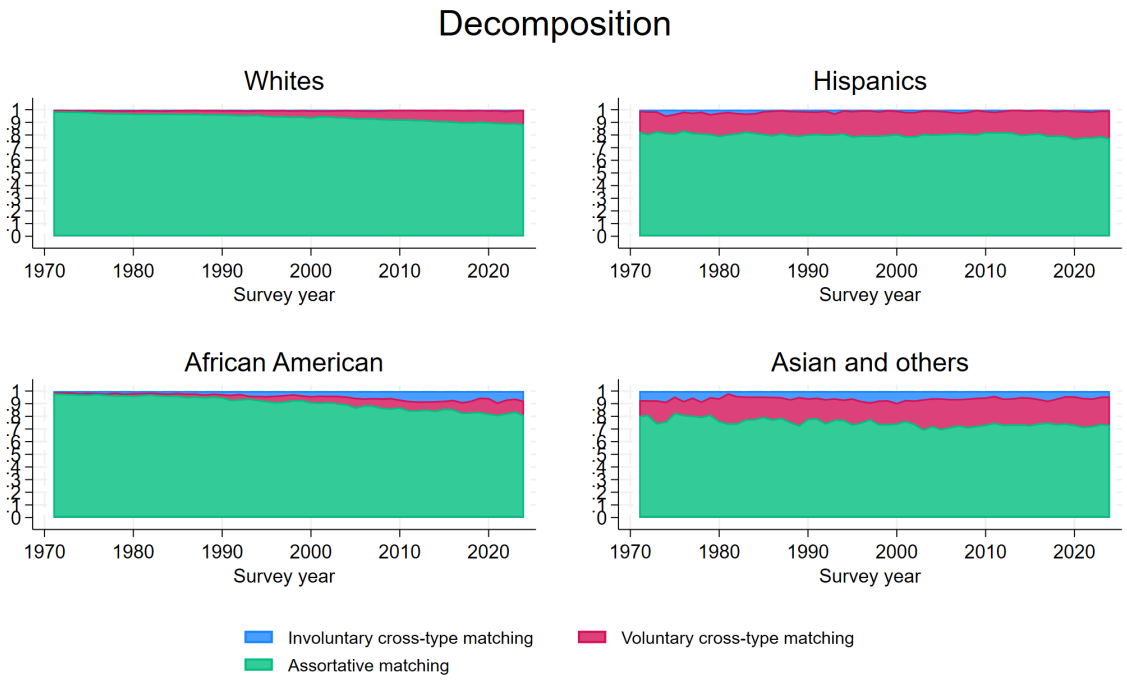
Note: The figure illustrates the difference between AHS_i and HHS_i for each education group versus the involuntary cross-type matching.

Figure 24: AHS_i measure of racial assortative matching in the US by gender



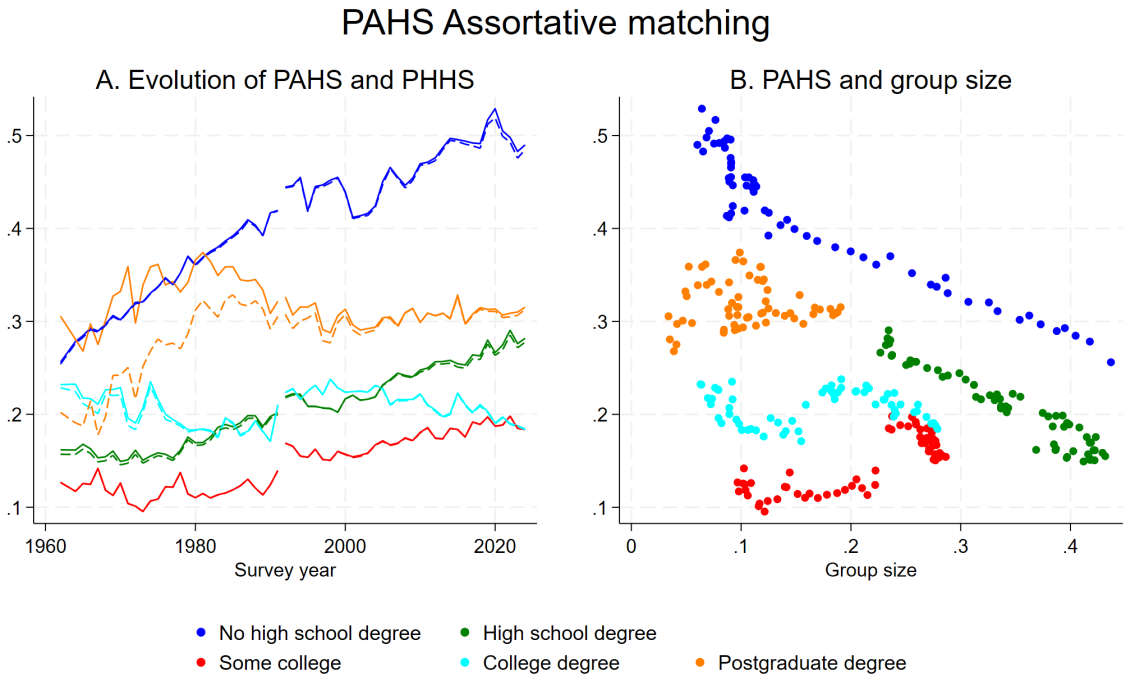
Note: The figure illustrates the AHS_i measure for each race group by gender.

Figure 25: Decomposition of voluntary and involuntary cross-type matching



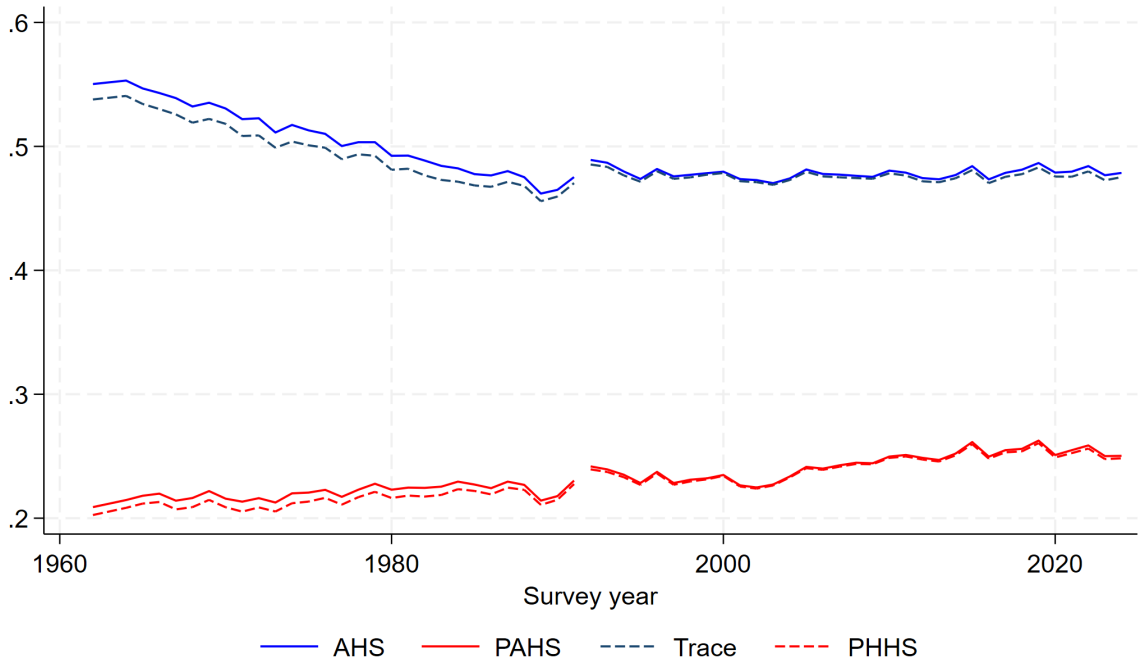
Note: The figure illustrates the decomposition of couples by assortative matching, as well as voluntary and involuntary cross-type matching. We measure it analogously to AHS_i.

Figure 26: $PAHS_i$ measure of educational assortative matching in the US



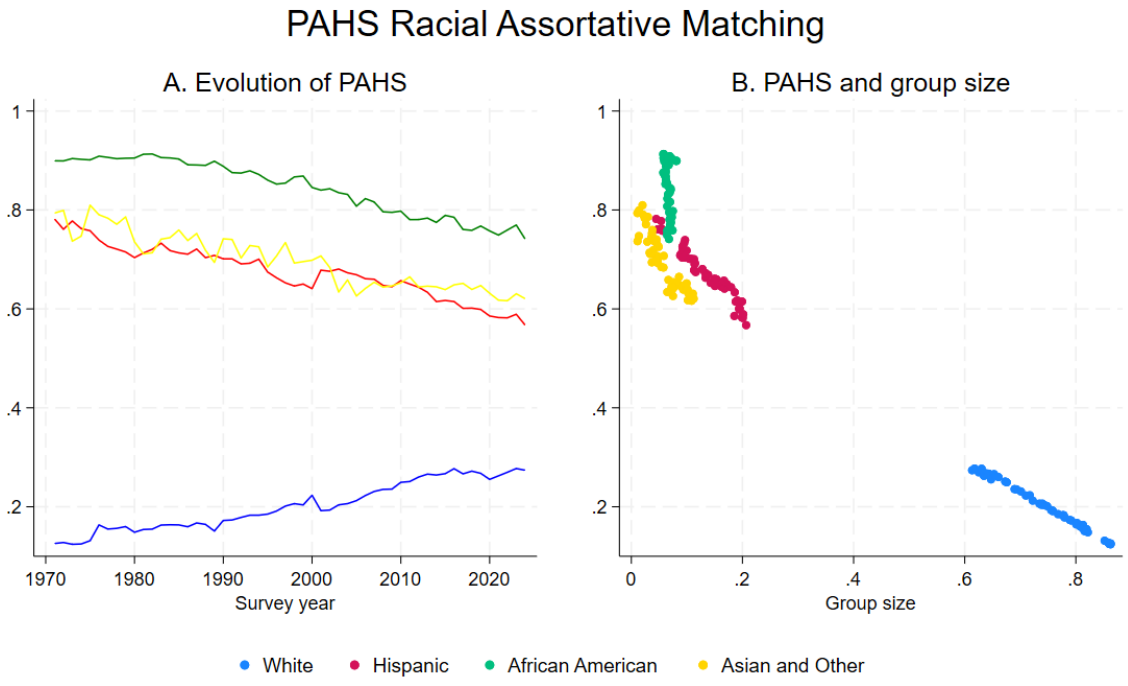
Note: Figure A illustrates the education-specific $PAHS_i$ measure (continuous lines) and the $PHHS_i$ measure (dashed lines). Figure B shows the relationship between $PAHS_i$ and group size.

Figure 27: $PAHS$ measure of educational assortative matching in the US



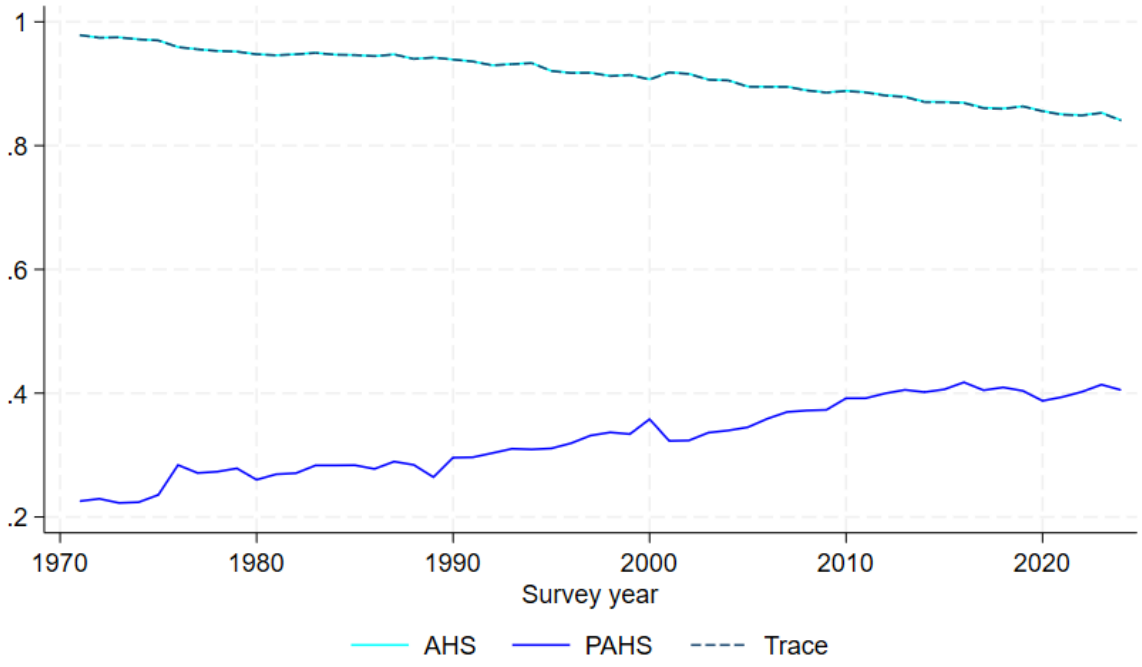
Note: The figure illustrates the evolution of aggregate education AM, measured as AHS_C and $Trace$, and the evolution of aggregate education positive AM, using $PAHS_C$ and $PHHS_C$ measures.

Figure 28: $PAHS_i$ measure of racial assortative matching in the US



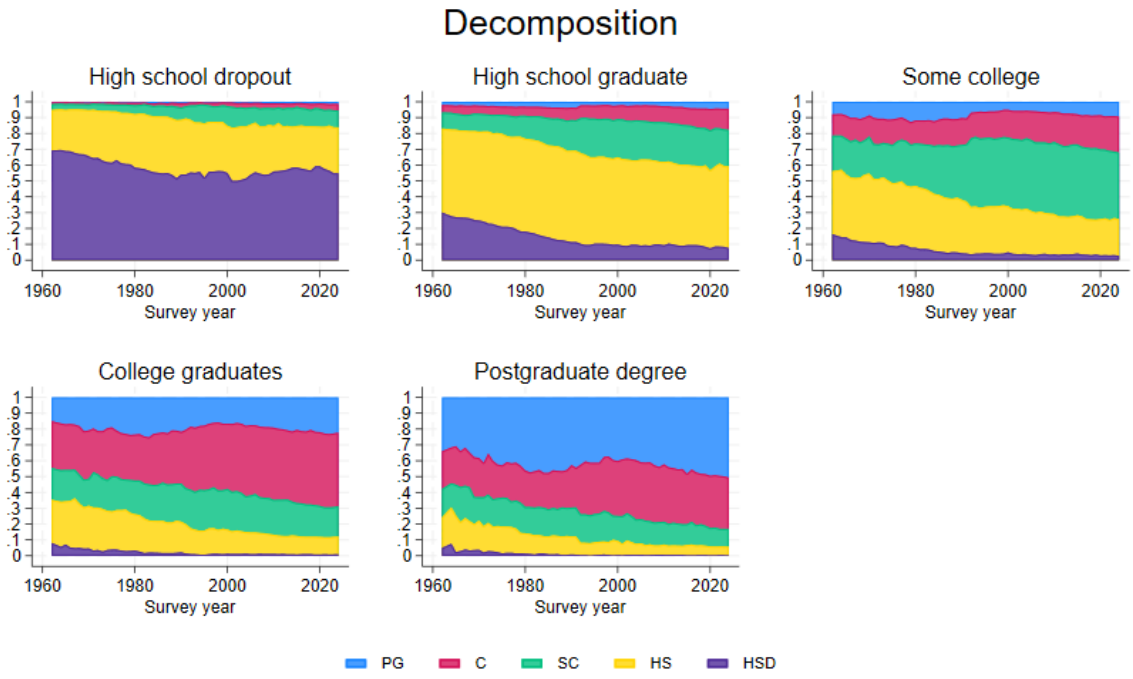
Note: Figure A illustrates the evolution of the race-specific $PAHS_i$ measure. Figure B shows the relationship between $PAHS_i$ and group size.

Figure 29: $PAHS$ measure of racial assortative matching in the US



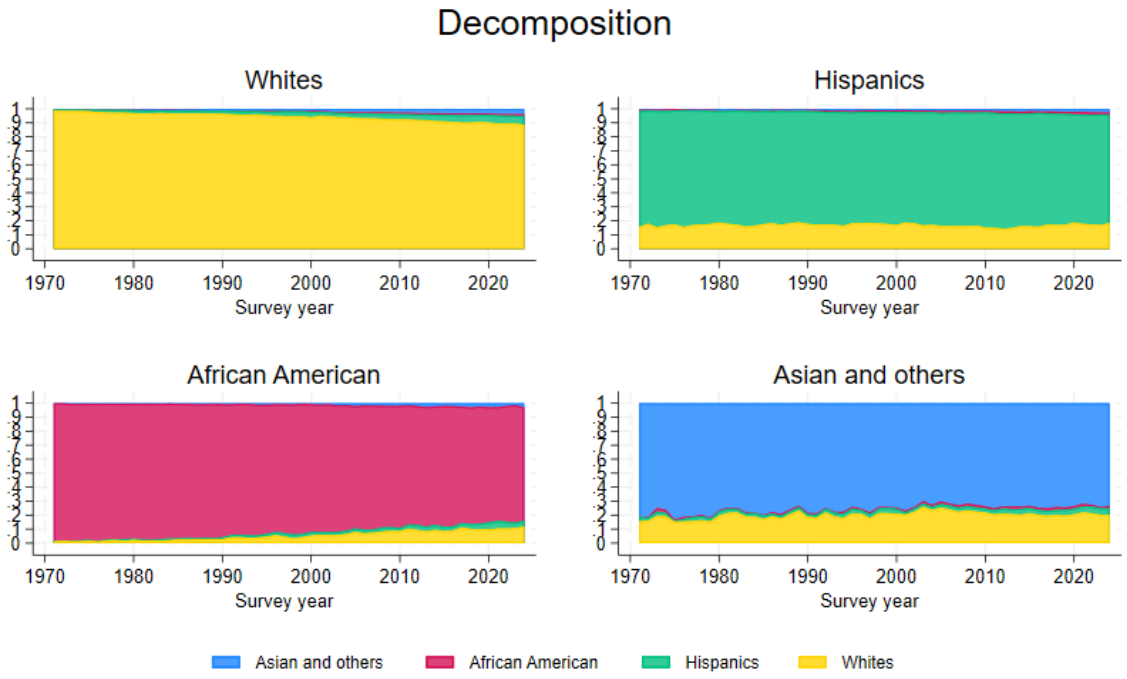
Note: The figure illustrates the aggregate racial $PAHS_C$ measure compared to the AHS_C and the $Trace$.

Figure 30: Full matching pattern by education



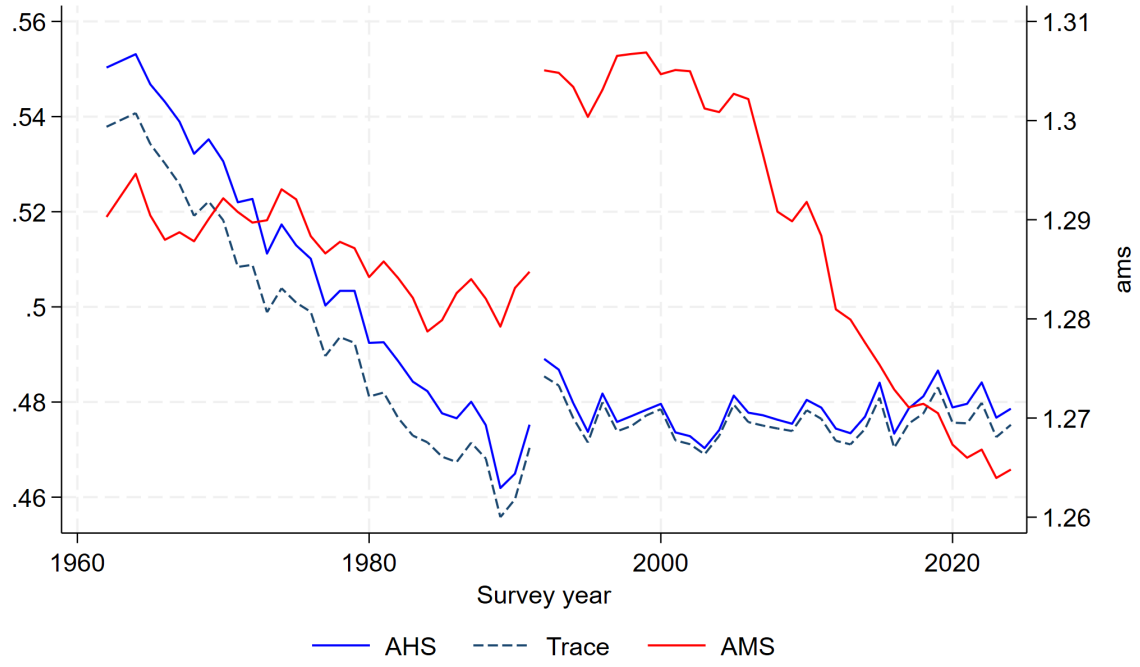
Note: The figure illustrates, for each education type, the average share of couples according to their education mix. We measure it analogously to the AHS_i assortative matching, that is, the average gender share. PG (Postgraduate); C (College degree); SC (Some college); HS (High school); HSD (High school dropout).

Figure 31: Full matching pattern by race



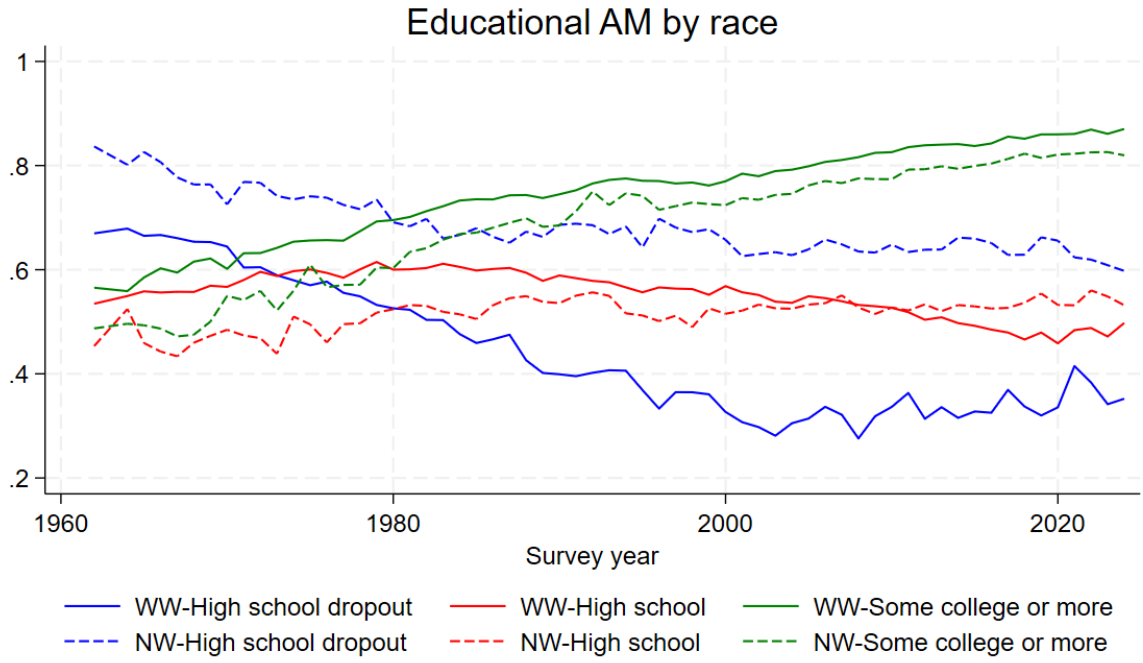
Note: The figure illustrates, for each race, the average share of couples according to their race mix. We measure it analogously to the AHS_i assortative matching, that is, the average gender share.

Figure 32: Average matching share



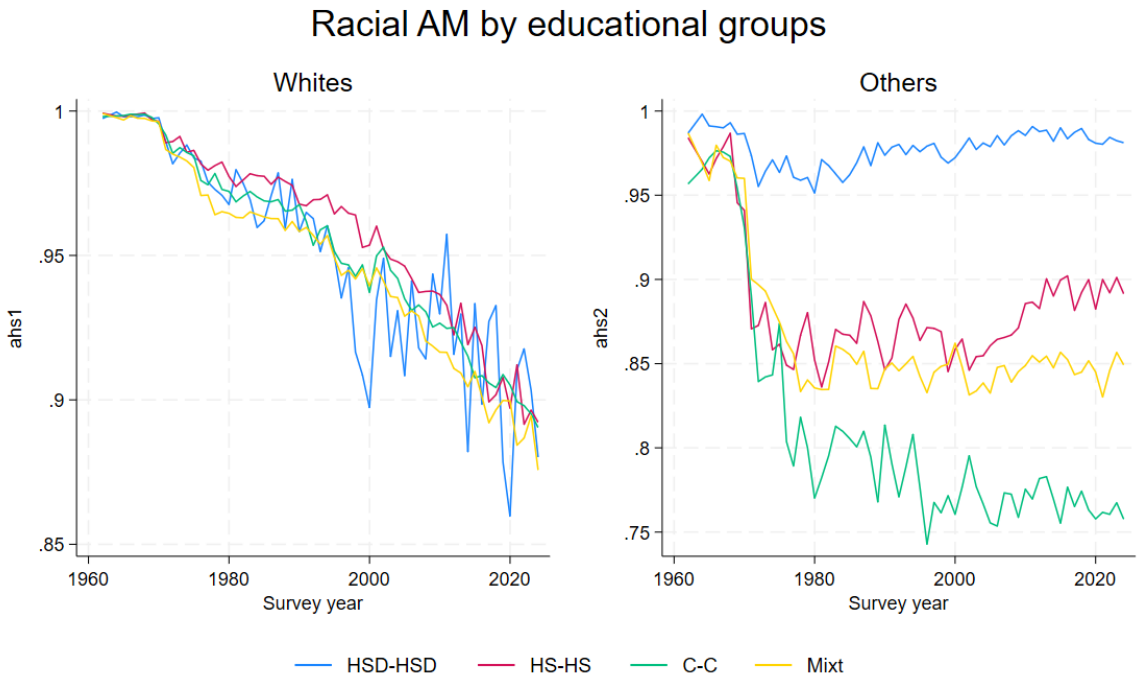
Note: The figure illustrates the average matching share, which includes the subdiagonal and superdiagonal of the matrix as homogamous matches with a lower weight (right axis).

Figure 33: Educational AM across racial groups



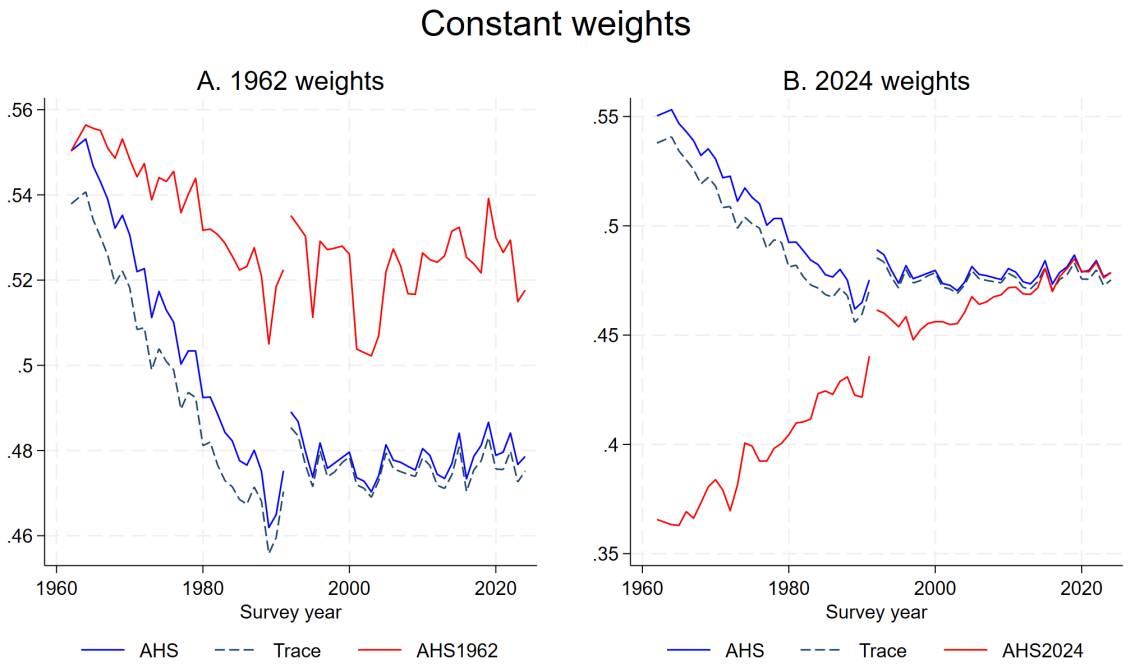
Note: The figure illustrates the AHS_i of each education type by two racial subpopulations: white homogamous couples (WW) and mixed or non-white couples (NW).

Figure 34: Racial AM across educational groups



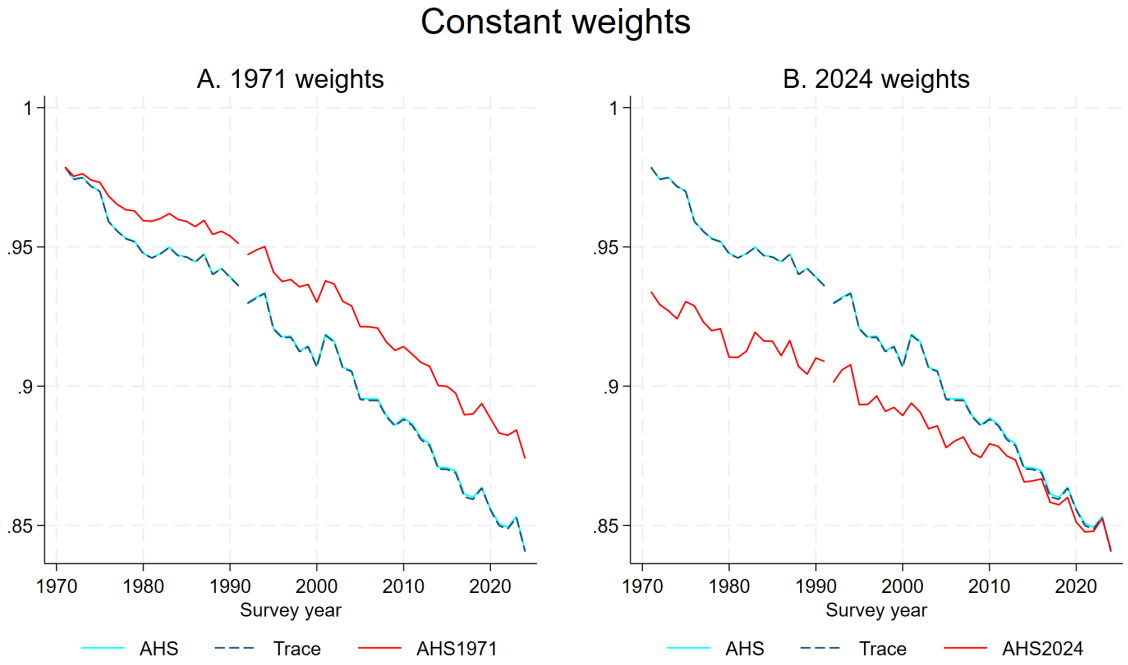
Note: The figure illustrates the AHS_i considering two racial groups (Whites versus the rest) by four subpopulations: educational homogamous couples with high school dropouts (HSD-HSD), some high school (HS-HS), and some years of college or more (C-C), respectively, and non-homogamous couples in terms of education (Mixt).

Figure 35: Educational AM with constant population weights



Note: The figure illustrates the AHS_C computed with constant weights. Figure A's weights are the education shares from 1962, and Figure B's weights are from 2024.

Figure 36: Racial AM with constant population weights



Note: The figure illustrates the AHS_C computed with constant weights. Figure A's weights are the race shares from 1971, and Figure B's weights are from 2024.