Unpacking the Green Innovation Slowdown*

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Abstract

The slowdown in green innovation since the 2010s is well documented, but the firm types driving this pattern remain unclear. Using U.S. patent data, we develop a decomposition framework showing that green innovation is concentrated among firms filing ICT-related green patents, and that the post-2010 slowdown is driven largely by these firms. Motivated by these observations, we build a general equilibrium model in which heterogeneous firms choose technology class, innovation, entry, and exit. ICT firms have a comparative advantage in green innovation because they face lower green innovation costs. The model implies that ICT-targeted green innovation subsidies are more efficient when dirty innovation is relatively costly, whereas universal green innovation subsidies are preferred when dirty innovation costs are low—because concentrating subsidies on firms with a comparative advantage in green innovation is more efficient when the resulting loss in dirty-technology productivity is small. Once pollution externalities are incorporated into welfare, however, ICT-targeted green innovation subsidies dominate universal green innovation subsidies. Estimation results suggest that a slower decline in ICT green innovation costs is the primary driver of the slowdown in green transition and innovation, with falling energy prices and shifts in the productivity distribution of ICT entrants playing secondary roles. Finally, comparing a universal green subsidy with an ICT-targeted green subsidy—each calibrated to match the observed green-subsidy-to-R&D ratio—we find that ICT targeting delivers a higher consumption-to-subsidy multiplier throughout the post-2010 period.

Keywords: Climate change; green innovation; firm heterogeneity; general purpose technology.

JEL classification: D24, E22, O16, O30, O40.

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1 Introduction

Transitioning away from carbon-intensive technologies requires sustained growth in green innovation to offset the established dominance of dirty technologies in energy production and use. Although climate policies have traditionally focused on the reduction of emissions through carbon pricing and regulation, an emerging consensus recognizes that redirecting technological change towards clean alternatives is equally critical (Acemoglu et al., 2012, 2016a). A distinctive and increasingly important feature of many emerging clean technologies is their deep integration with information and communication technologies (ICT). Consider the stark contrast between traditional environmental innovations and their ICT-integrated counterparts: conventional solar panels versus AI-optimized photovoltaic systems that dynamically adjust to weather patterns and electricity demand; standard wind turbines versus digital "twins" that simulate performance in thousands of scenarios to maximize energy capture; basic battery storage versus smart grid systems that use machine learning to balance intermittent renewable supply with consumption patterns across millions of households. These examples suggest that ICT-related capabilities are central to the next wave of green innovation and have led policymakers to emphasize a "twin transition" between digitalization and decarbonization (e.g., Centre, 2022; Suraj, 2024).

Recent evidence documents a puzzling slowdown in green innovation since the early 2010s (Acemoglu et al., 2023; Popp et al., 2022), despite growing climate concerns and policy support. Although the shale gas revolution and declining energy prices have been identified as contributing factors, we lack understanding of which firms drive these trends and how policy should respond in the presence of firm heterogeneity. Do ICT-intensive firms—those whose green innovations leverage digital capabilities—behave differently from traditional green innovators in their response to energy prices and policy interventions? Should public subsidies target ICT-green firms specifically or spread broadly across all green innovators?

The distinction of ICT-green firms and green firms is increasingly reflected—though without rigorous economic justification—in emerging policy design. The Green Deal of the European Union explicitly advocates for a "twin transition," with Horizon Europe dedicating targeted calls

like "Digital solutions for the integration of energy systems", which require applicants to integrate AI, ICT, or digital twins into clean energy projects. Similarly, the European Innovation Council Accelerator evaluates "Green Deal" proposals using scoring criteria that award premium points for "use of advanced digital technologies," creating de facto preferential access for ICT-green firms. In the United States, ARPA-E's DIFFERENTIATE program exclusively funds AI-driven grid optimization projects, while the DOE's GRIDIS initiative prioritizes proposals combining machine learning with grid-scale storage—effectively channeling resources toward firms with dual ICT-green capabilities. China's 14th Five-Year Plan for Green Development similarly directs subsidies toward projects "smart manufacturing" and "AI-enabled resource efficiency". Critically, these policies lack an economic framework to determine when such targeting maximizes welfare versus when broad-based support is superior. Without understanding how energy prices, innovation costs, and externalities interact with firm heterogeneity, governments risk an inefficient allocation of scarce innovation resources—a gap that this paper directly addresses.

In this paper, we demonstrate that this firm heterogeneity is central to understanding the slowdown in green innovation and designing effective climate policy. Using U.S. patent data from 1999-2017, we find that ICT firms account for approximately 61% of all green patents[1]. Moreover, the post-2010 stagnation in green innovation is overwhelmingly driven by these ICT-green firms. This finding challenges conventional wisdom that treats green innovation as homogeneous across firm types.

Motivated by these empirical patterns, we build a general equilibrium model with heterogeneous firms that endogenously choose technology classes and innovation investments. The model incorporates a crucial feature: ICT firms face lower costs of green innovation, reflecting their comparative advantage in integrating digital capabilities with environmental applications. Using this framework, we quantify how declining energy prices since the 2010s differentially affected innovation incentives across firm types and evaluate the efficiency of different subsidy policies.

Our analysis reveals important policy trade-offs that depend critically on innovation costs

[1]The share increases from 42% in 2006 to 61% in 2014. Using an alternative definition of ICT firm in robustness check, the share is stable around 60%.

and the strength of pollution externalities. When the cost of doing dirty innovation is relatively high, green subsidies targeted to ICT firms generate a higher consumption-to-subsidy multiplier than a universal subsidy, whereas when this cost is relatively low, such targeting yields a lower multiplier than a universal subsidy. When welfare incorporates pollution externalities, green subsidies targeted to ICT firms are more efficient than universal green subsidies, reflecting ICT firms' comparative advantage in green innovation and the lower opportunity cost of moving away from dirty production. Estimating the model, we find that declining energy prices since the 2010s have weakened incentives for green innovation, especially among ICT-related firms. We then compare a universal green innovation subsidy with a subsidy targeted only to ICT firms, calibrating both to match the observed ratio of green subsidies to R&D (about 0.056%). For all estimated energy prices since the 2010s, targeting ICT firms yields a higher consumption-to-subsidy multiplier than a universal subsidy.

Related Literature. This paper contributes to three strands of literature. First, we add new empirical facts and extend the models of directed technical change (e.g., Acemoglu et al., 2012, 2016a, 2023; Fried, 2018) by incorporating firm heterogeneity along the ICT dimension, connecting micro-level innovation decisions to aggregate results through rich general equilibrium channels. Unlike previous work that operates at the sector level or assumes symmetric firms, our framework captures how differential innovation capabilities shape the green transition.[2]

Second, we bridge the literature on ICT as a general purpose technology with environmental innovation (e.g., Acemoglu et al., 2016b; Andres et al., 2022; Fouquet, 2017; Napolitano et al., 2020). This literature emphasizes that general purpose technologies like ICT and AI diffuse across sectors and reshape innovation networks, with clean technologies often acting as downstream recipients of ICT spillovers. Andres et al. (2022) argue that dirty and green technologies differ in their absorptive capacities and intrinsic connections to ICT and AI, suggesting that firms with stronger ICT capabilities can invest more in green technologies. We provide empirical evidence and a quantitative framework to assess the economic importance of ICT firms. Our model formalizes

[2]Recent contributions such as Aghion et al. (2025) study the green transition along supply chains and Acemoglu et al. (2016a) and Aghion et al. (2024) allow heterogeneous innovation responses but rely on quality ladder structures that do not differentiate ICT intensive and non-ICT-intensive firms.

how ICT capabilities create heterogeneous opportunity costs of doing green innovation across firms—a mechanism often overlooked in climate policy evaluation.

Third, our work speaks to the design of climate policies under firm heterogeneity. A large literature compares carbon taxes, broad green subsidies, and regulatory tools (e.g., Acemoglu et al., 2016a; De Haas and Popov, 2023; Hambel et al., 2024; HSU et al., 2023; Schuldt and Lessmann, 2023), often without distinguishing among firms by innovation capability. More recent work (e.g., Aghion et al., 2024; Dörr, 2022) highlights the importance of firm-level financial constraints and advocates for targeting subsidies to small, financially constrained green innovators. Our analysis complements this by asking a different policy question: how should a given green innovation subsidy budget be distributed between ICT intensive firms and the broader firm population, conditional on the energy price and the dynamics of the energy environment? In doing so, we provide a quantitative framework that links the ongoing policy debate on the "twin" digital and green transitions (e.g., Centre, 2022; Suraj, 2024) to the design of firm-specific green innovation subsidies.

Roadmap. The rest of this paper proceeds as follows. Section 2 presents our empirical decomposition of green innovation trends. Section 3 develops the theoretical model and analyzes the mechanism of the model. Section 4 estimates the model and evaluates policy counterfactuals. Section 5 concludes.

2 Data and Empirical Facts

In this section, we conduct a decomposition to provide a clearer insight into the green innovation trend. Specifically, we decompose the green patents by the type of firms, being an ICT or non-ICT firm. Then we decompose according to the firm innovation behavior, of being a frequent or infrequent innovator. Lastly, we decompose by firm size.

2.1 Data and Variables

2.1.1 Data Description

Our patent data are collected from the USPTO and cover the period from 1999 to 2017. The year is defined as the patent's application year. Each patent is assigned a unique IPC (International Patent Classification) code, which indicates the technology class to which it belongs.

Our firm data are drawn from ORBIS Historical, which contains information such as the BvD ID code, the number of employees, and other details of the balance sheet. We use ORBIS Intellectual Property to merge the patent data with the firm data. Each patent in ORBIS Intellectual Property is associated with a BvD ID code, which represents the applicant's identity in the U.S. patent application system. For this paper, we focus on U.S. firms and therefore patents applied by U.S. firms.

Moreover, we focus on a subset of industries that conduct green innovation according to patent codes: Transportation, Electricity, and Mechanical Engineering except weapons and blasting, consistent with other studies such as Andres et al. (2022).

2.1.2 Patent Categorization

In this section, we define green patents and ICT patents. To identify green patents, we adopt the IPC green inventory to see whether a patent category falls into that inventory. The inventory is introduced by the World Intellectual Property Organization to identify environmentally friendly technologies since 2010. The inventory involves the following main classes of technology: Alternative energy production; Transportation; Energy conservation; Waste management; Agriculture and forestry.

2.1.3 Firm Classification

First, we divide green firms into two categories year by year: $h \in H = \{I, N\}$, where I represents a firm that owns an ICT-green patent above some threshold, N represents a firm that owns any

green patent but owns an ICT-green patent below some threshold. In the main context, we take the threshold to be three ICT-green patents and implement the robustness check in the Appendix A.4. The robustness check defines an ICT-green firm as one that had a share of ICT-green patents more than 10% of all green patents throughout the sample. The results are also robust using other absolute values of the threshold, which can be provided upon request.

Second, we classify firms in each of the above categories into three sub-categories according to their innovation behavior: the frequent innovator, the entrant innovator, and the exiting innovator, denoted $\{S, E, X\}$. The dynamic changing margin of the last two types of innovator is called the change of infrequent innovators. We can further decompose the firms into small and large firms $G = \{s, l\}$.

2.2 Decomposing Green Patent Change

To investigate the dynamic changes of patents among different groups of firms, we follow Melitz and Polanec (2015) to perform a dynamic decomposition. The number of new green patents in a year is:

$$\Phi_t = \sum_{g \in G_t} \sum_{h \in H_t} \sum_{\phi_t} \phi_t^{S,g,h} + \sum_{g \in G_t} \sum_{h \in H_t} \sum_{\phi_t} \phi_t^{X,g,h}, \tag{2.1}$$

where ϕ is a green patent that a firm has applied for in a year. It is decomposed to green patents by incumbent innovators (the first term) and innovators that about to exit in the next period (the second term), and further decomposed according to types of firms in H and G respectively. Another way of split is:

$$\Phi_t = \sum_{g \in G_t} \sum_{h \in H_t} \sum_{\phi_t} \phi_t^{S,g,h} + \sum_{g \in G_t} \sum_{h \in H_t} \sum_{\phi_t} \phi_t^{E,g,h}, \tag{2.2}$$

where it is decomposed to green patents by incumbent innovators (the first term) and innovators that just enter the innovation in this period but absent from records in the last period (the second term). For convenience, we denote $\Phi_t^{S,g,h} = \sum_{\phi_t} \phi_t^{S,g,h}$ as the incumbent firm's total patents in year t that lie in firm group (g, h), and denote $\Phi_t^{X,g,h}$ as an exiting firms' total patents and $\Phi_t^{E,g,h}$ as an entrant firms' total patents. The decomposition is to take the difference of the second split in the

next period with the first split in the current period:

$$\Delta\Phi_t = \sum_{g\in G_t}\sum_{h\in H_t}(\Phi_t^{S,g,h} - \Phi_{t-1}^{S,g,h}) + \sum_{g\in G_t}\sum_{h\in H_t}(\Phi_t^{E,g,h} - \Phi_{t-1}^{X,g,h}), \tag{2.3}$$

where the first term is the change of green patents by an invariant incumbent group, the second term is the change by firms that enter and exit the green innovation.

2.3 Facts

Based on the above data analysis, we provide two key facts about green innovation patterns with important implications for the distinction between ICT-green and non-ICT green firms.

Fact 1. ICT-green firms increasingly dominate green innovation. The green innovation slowdown is concentrated among ICT-green firms. Figure 2.2 shows that the change in green patents is mainly driven by firms that apply for the ICT-green patent. However, over time, all types of firms share similar trend and stagnate and shirk around the year of 2010.[3][4]

Although prior literature has documented an aggregate slowdown in green patenting since 2010 (Popp et al. (2022)), our decomposition reveals that this trend is driven almost entirely by firms filing ICT-green patents.

Fact 2. Most green innovation is done by frequent and large ICT-green firms. The innovation slowdown is also driven by these firms. Using dynamic decomposition, we compare the contribution of frequent innovators with the contribution of infrequent innovators to the change of green patents over time. We find that the aggregate pattern is driven mainly by frequent innovators.

We also divide the innovators by firm size according to their total sales per year. From Figure 2.3, we observe that since 2010, large firms have stagnated in green patenting.

[3]The share of ICT-green patents as well as the number of it is increasing over time. Though, the fraction of green firms that own an ICT green patent is only around 20 percent.

[4]Figure A.1 shows that the ICT-related green patents are increasing rapidly up to 2010 and become stagnated.

Our patent level evidence is consistent with literature, as shown in Figure 2.1, the share of green patents has declined since 2010; the number of green patents also stagnates since 2010. We also present patent-level evidence of ICT-green patents, as shown in Appendix A.2. The number and share of ICT-green patents have increased since the early 2000s but stagnated around 2010 as well.

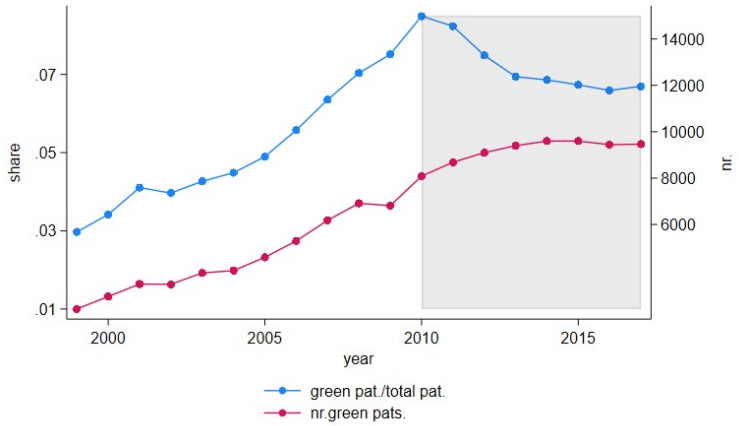


Figure 2.1: Green innovation trends

Note: Green patents are defined according to IPC green inventory. Patents' year are at the time applied, it includes patents granted and owned by a company with a BvD ID code.

2.4 Robustness

Technology-specific green innovation decomposition. Motivated by Popp et al. (2022), we classify green patents into several groups by technology class (or energy source) to see if the baseline facts hold in all technology classes (or energy sources).

We divide the patents into the following categories: bio-fuel, hydro and thermal; wind and solar; transportation; waste management; energy storage. We find that wind and solar, transportation, and energy storage account for most of the growth in green innovation and exhibit a pattern similar to that of the full sample. The results are shown in Appendix A.3.

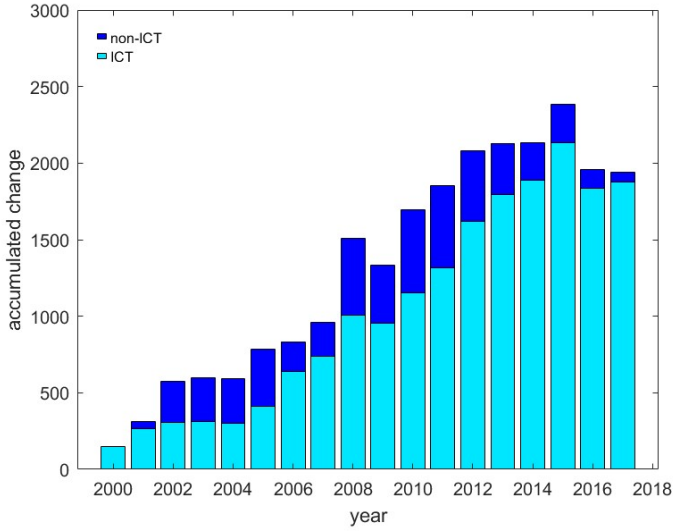


Figure 2.2: Accumulated green pats. change by firm type

Note: ICT represents firms that own ICT-green patent above some threshold as defined in section 2.1.3. Non-ICT represents other green firms.

3 The Model

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. As shown in Figure 3.1, there exist two types of technologies in the economy, i.e., dirty and green, denoted as $m \in \{d, g\}$. At the beginning of each period, potential entrants decide whether or not to enter the market. If entering, they become ICT or non-ICT firms at an exogenous rate.[5] After becoming incumbent firms, they choose the dirty or green technology class, and then make production decisions using the chosen technology. They then decide on the optimal level of directed innovation in the technology class they chose. Finally, they decide whether to remain in the market or exit. Innovation costs, entry costs, and transition costs in the technology-class are all borne by labor. The details are specified below.

[5]In the extension of the model in section B.4, firms can endogenously choose their type.

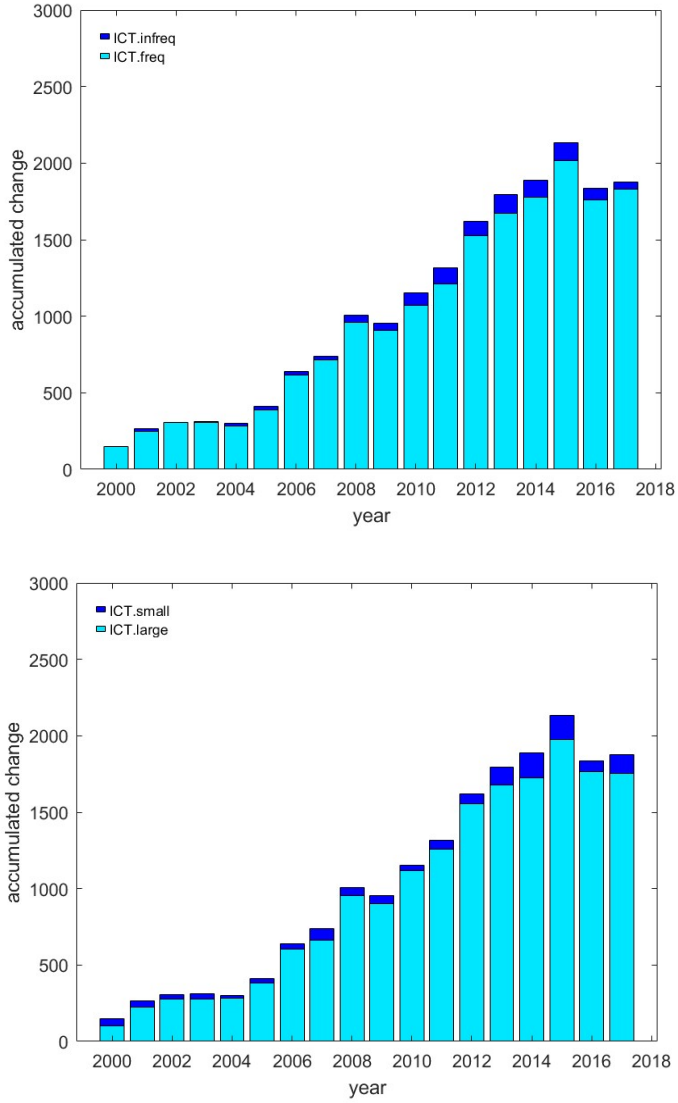


Figure 2.3: Accumulated green pats. change by innovator type

Note: Frequent innovators are firms that apply for green patents in consecutive years, whereas infrequent innovators are firms that did not apply for any green patents in the previous year. Large firms are those with employment above the median, and small firms are those below the median.

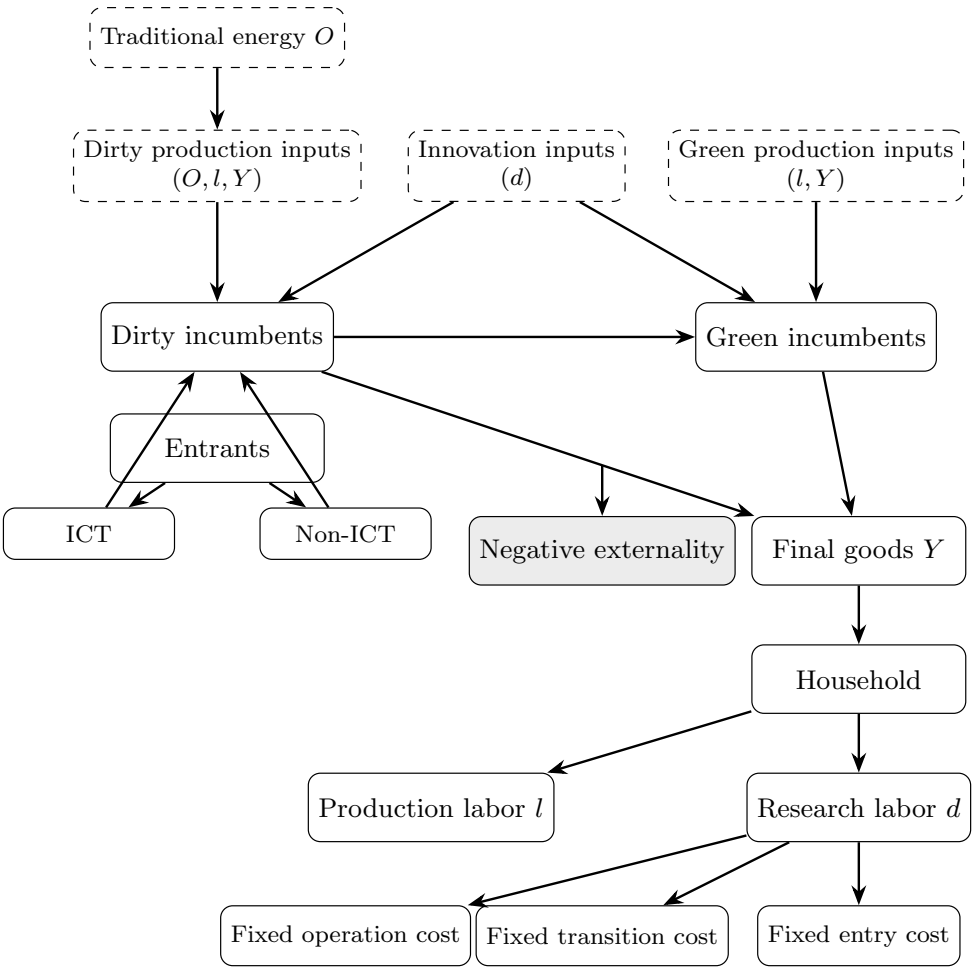


Figure 3.1: Model Setup

3.1 Model Setup

Household and environmental externality The representative household maximizes her lifetime utility:

$$\max \sum_{t=0}^{\infty} \beta^t u(C_t, E_t) \tag{3.1}$$

$$s.t. \quad W_t + P_t C_t \leq W_{t-1}(1 + r_t) + w_t L, \tag{3.2}$$

where $u(C_t, E_t) = C_t - \Xi E_t$, C_t is the aggregate consumption of final goods, E_t is the aggregate dirty output (the functional form of it is shown as (3.39)), and Ξ denotes the relative strength of the externality of the dirty output in affecting the utility of the household. β denotes the discount

factor. P_t is the price of the final goods. W_t is the wealth of households in period t (value of firms in period t), and r_t is the rate of return on owning firms. The household supplies labor L inelastically in each period that is given exogenously. The utility maximization problem leads to the standard Euler equation:

$$\beta = \frac{1}{1+r_t}. \tag{3.3}$$

Final goods technology Perfectly competitive firms produce final goods Y_t with technology

$$Y_t = [\int_{m=d} y_t(a,m)^{\frac{\sigma-1}{\sigma}}\, d\mu_t(a,m) + \int_{m=g} y_t(a,m)^{\frac{\sigma-1}{\sigma}}\, d\mu_t(a,m)]^{\frac{\sigma}{\sigma-1}}, \tag{3.4}$$

where $y_t(a,m)$ denotes the output of an intermediate firm with productivity $a = \{a_d, a_g\}$ that chooses technology class $m \in \{d,g\}$, d denotes dirty technology, g denotes green technology, $\{a_d, a_g\}$ denotes the productivity when using the respective technology and $\sigma > 1$ is the elasticity of substitution across intermediate goods, $\mu_t(a,m)$ denotes the distribution over two-dimensional productivity and technology class.

Final goods producers buy $y_t(a,m)$ amounts of intermediate output at price $p_t(a,m)$, and sell the final goods at price P_t. The profit maximization problem gives the demand function for intermediate goods:

$$y_t(a,m) = (\frac{p_t(a,m)}{P_t})^{-\sigma} Y_t, \tag{3.5}$$

where the price of final goods is defined as: $P_t = [\int_{m\in\{d,g\}} p_t(a,m)^{1-\sigma} d\mu_t(a,m)]^{\frac{1}{1-\sigma}}$.

Intermediate firm operation At the end of each period, the incumbent intermediate firms decide whether to remain in the market by choosing $\kappa(a,m) \in \{0,1\}$ such that

$$V_t^{0,S}(a,m) = \max_{\kappa(a,m)\in\{0,1\}} [0, V_t^S(a,m)], \tag{3.6}$$

where

$$V_t^S(a,m) = \max\, [U_t^S(a,m;m), U_t^S(a,m;-m)], \tag{3.7}$$

$U_t^S(a, m; m)$ denotes the value of staying in the market and maintaining the same technology class as in the previous period, while $U_t^S(a, m; -m)$ denotes the value of staying in the market and switching to the other technology class relative to the previous period. The indicator $\kappa(a, m) = 0$ represents exiting the market, and $\kappa(a, m) = 1$ represents continuing to operate.

Intermediate firm production The intermediate firms are in a monopolistically competitive market. They can adopt dirty and green technologies that they have technology specific productivity.

The dirty firms use labor, oil and final goods to produce a dirty variety. The price of the oil after tax τ is

$$P^* = P_o(1 + \tau), \tag{3.8}$$

which can also be interpreted as the price of dirty energy.

The green firms use only labor and final goods to produce a green variety. The production function of the firm using dirty technology is

$$y_d(a) = exp(a_d)^{\frac{1}{\sigma-1}}[Y^D(a)^\eta \cdot (O(a))^{1-\eta}]^{1-\alpha} \cdot (l(a))^\alpha, \tag{3.9}$$

where α is the labor share, η represents the weight of final goods input relative to oil, $\exp(a_d)^{\frac{1}{\sigma-1}}$ is the scaled firm productivity, Y^D is the input of final goods, O is the energy input and l is the labor input. η governs the share of final goods inputs of dirty technology, this captures an economy wide productivity feedback that higher aggregate productivity implies lower input cost for both dirty and green technology. And we assume that the economy is more biased towards green technology, which is consistent with the assumption at Aghion et al. (2025).

Similarly, the production function of the firm using clean technology is:

$$y_g(a) = exp(a_g)^{\frac{1}{\sigma-1}} \cdot Y^D(a)^{1-\alpha} \cdot (l(a))^\alpha \tag{3.10}$$

Because the market clears for all intermediate goods, we have $y_d(a) = y(a, d), y_g(a) = y(a, g)$.

The firm that uses dirty technology solves the following:

$$\max_{p(a),l(a)O(a),Y^D(a)} \quad p(a)y_d(a) - wl(a) - PY^D(a) - P_oO(a) \tag{3.11}$$

$$s.t. \quad y_d(a) = (\frac{p(a)}{P})^{-\sigma}Y, \tag{3.12}$$

The firm that uses green technology solves the following:

$$\max_{p(a),l(a),Y^D(a)} \quad p(a)y_g(a) - wl(a) - PY^D(a) \tag{3.13}$$

$$s.t. \quad y_g(a) = (\frac{p(a)}{P})^{-\sigma}Y \tag{3.14}$$

By profit maximization, we can solve the optimal inputs, the price, and the total output selected by a typical dirty firm, which are presented in B.1. The optimal profit for a dirty firm is the following:

$$\pi_d(a) = C_d\Pi \exp(a_d)(w^\alpha P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma} \tag{3.15}$$

where $C_d = (\frac{1+\tau}{(1-\eta)(1-\alpha)})^{(1-\sigma)(1-\alpha)(1-\eta)} \cdot (\frac{1}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{(1-\alpha)\eta})^{(1-\sigma)(1-\alpha)\eta}$. The optimal profit for a green firm is the following:

$$\pi_g(a) = C_g\Pi \exp(a_g)(w^\alpha P^{1-\alpha})^{1-\sigma} \tag{3.16}$$

where $C_g = (\frac{1}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{1-\alpha})^{(1-\sigma)(1-\alpha)}$. We denote $\Pi =: (\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^\sigma Y$ as the common part of variable profits of using dirty and green technology. We denote the dirty and green variable profits as $\Pi_d = C_d\Pi(w^\alpha P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}$ and $\Pi_g = C_g\Pi(w^\alpha P^{1-\alpha})^{1-\sigma}$ respectively.

Intermediate firm technology class choice and innovation The firms with status quo technology class $m \in \{d, c\}$ solve the following problem (entrants enter as dirty firms only, incumbents inherit the technology from the last period):

$$V_t^S(a, m) = \max_{x(a,m)\in\{m,-m\}} U_t^S(a, m; x(a, m)), \tag{3.17}$$

where $x = -m$ represents the transition, $x = m$ represents there is no transition, $S \in \{u, n\}$ represents the firm being an ICT or non-ICT firm, which is determined randomly upon entry.

Conditional on staying in the market and the technology class chosen, incumbents make innovation efforts to maximize their expected values:

$$\begin{aligned} U_t^S(a, m; x(a, m)) = &\max_e \ \pi_x(a_x) - n_f - \mathbf{1}\{x \neq m\} \cdot c \cdot V_t^S(a, m; -m) - \Phi_x^S(e) \\ &+ \frac{1}{1 + r_t}\gamma[e \cdot (V_{t+1}(a_x exp(J), a_{-x}, x) + (1 - e)(V_{t+1}(\frac{a_x}{\exp(J)}, a_{-x}, x))], \end{aligned} \tag{3.18}$$

where $U_t^S(a, m, m')$ is the firm's interim value function before innovation but after choosing the technological class, which is the expected value of entering each class of technology. We denote $\pi_m(a_m)$ as the optimal profit per period of the firm given its state productivity, and n_f is the fixed cost of operation. $e \in [0, 1]$ is the costly innovation effort chosen by firms. If innovation is successful, firms increase innovation by step size J, if not, they lag by the same size. The parameterization of innovation costs is to follow Atkeson and Burstein (2010), we assume that ICT and non-ICT firms have differentiated green innovation cost

$$\Phi_g^u(e) = h_g^u \exp(a_g) \exp(\xi_g^u e), \tag{3.19}$$

and

$$\Phi_g^n(e) = h_g^n \exp(a_d) \exp(\xi_g^n e_g), \tag{3.20}$$

while they have homogeneous dirty innovation cost

$$\Phi_d(e) = h_d \exp(a_d) \exp(\xi_d e_d). \tag{3.21}$$

The cost function is captured by two parameters: one is the scalar term h and another is the curvature term ξ. It is also proportional to the exponential of firm productivity, which represents an increasing cost of innovation with technology specific firm size.

Entrants Entrants decide whether or not to enter the market, and we impose the free entry condition so that this market is competitive. We assume that firms only enter by using dirty technology:

$$n_e = \frac{1}{1+r_t} \int [\theta V^u(a,d) dG_e(a) + (1-\theta) V^n(a,d) dG_e(a)] \tag{3.22}$$

where θ is the ex ante probability of being an ICT firm for entrants. The free entry condition is that entrants' ex ante expected value of entering is zero. The entrants draw their productivity according to an exogenous distribution $G_e(a) = G_e(\bar{a_d}, a_g)$, where the dirty productivity upon entry is degenerate. The conditional distribution on green productivity grid is downward sloping, which means that there are fewer firms as productivity is higher.

Entrants enter as using dirty technology, and they are free to choose to switch to green technology by paying a transition cost. We assume that green technology is an absorbing state: once they enter, they can exit the market but cannot switch to the dirty technology. We consider the green capital of firms as irreversible investment as mentioned in Lanteri (2018) that the investment cannot be easily reinvested.

Distribution evolution Given the distribution from the previous period $\Psi(a_m, a_{-m}, m)$, $\Psi(a_m, a_{-m}, -m)$, the distribution after technology class choice is $\tilde{\Psi}(a_m, a_{-m}, m)$ evolves as follows:

$$\tilde{\Psi}(a_m, a_{-m}, m) = x(a_m, a_{-m}, -m)\Psi(a_m, a_{-m}, -m) + \Psi(a_m, a_{-m}, m) \tag{3.23}$$

and after innovation, the updated distribution $\Psi'(a_m, a_{-m}, m)$ is:

$$\begin{aligned}\Psi'(a_m, a_{-m}, m) =& G_e(a'_m, a'_{-m}) + (1-\delta)e_m(\frac{a_m}{J}, a_{-m})\tilde{\Psi}(\frac{a_m}{J}, a_{-m}, m) \\ &+ (1-\delta)(1-e_m(a_m J, a_{-m}))\tilde{\Psi}(a_m J, a_{-m}, m)\end{aligned} \tag{3.24}$$

Aggregate productivity Define the aggregate productivity of green and dirty technology of ICT and non-ICT firms as

$$\begin{aligned}A_d^n = \int \exp(a_d) d\mu_d^n(a_d),\ A_d^u = \int \exp(a_d) d\mu_d^u(a_d), \\ A_g^n = \int \exp(a_g) d\mu_g(a_g),\ A_g^u = \int \exp(a_g) d\mu_g^u(a_g).\end{aligned} \tag{3.25}$$

where $A_d^n, A_d^u, A_g^n, A_g^u$ represents the aggregate productivity of non-ICT dirty firms, ICT dirty firms, non-ICT green firms and ICT green firms, respectively. The aggregate productivity of each technology class is to sum up that of the ICT and non-ICT firms together such that

$$A_d = A_d^n + A_d^u,\ A_g = A_g^n + A_g^u. \tag{3.26}$$

The aggregate productivity for the economy is

$$A = A_d + A_g. \tag{3.27}$$

For future convenience, we normalize the distribution of firms by entrants' mass, denoted as

$$\tilde{\mu}_d = \frac{\mu_d}{M^e},\ \tilde{\mu}_g = \frac{\mu_g}{M^e}. \tag{3.28}$$

The aggregated productivity computed according to the normalized distribution is denoted as $\tilde{A_d}, \tilde{A_g}, \tilde{A}$.

3.2 The Equilibrium

Equilibrium definition The stationary competitive equilibrium is a collection of forward looking decision rules $\{e_d^S, e_g^S, x^S, \kappa^S\}^{S\in\{u,n\}}$, production decisions $\{p_d, l_d, Y_d^D, O_d; p_g, l_g, Y_g^D\}^{S\in\{u,n\}}$, value functions $\{V_t^S, V_t^{0,S}\}^{S\in\{u,n\}}$, the stationary distributions over productivity $\{\mu^S(a_d, a_g)\}^{S\in\{u,n\}}$ and aggregates $\{\Pi, P, Y, M_e, C, Y^D, L_p, L_r\}$, such that for a given oil price P_o, wage rate w, labor supply L and parameters:

1. Representative households maximize their life-time utility such that equation (3.3) is satisfied;
2. Firms maximize profit per period according to profit maximization (3.11) and (3.13);
3. Firms maximize the value function in each decision stage through the Bellman equations of (3.18), (3.17) and (3.6);
4. The distribution μ_d, μ_g over productivity is stationary;
5. The labor market clears, i.e., equation (3.35) holds;
6. The output market clears, i.e., equation (3.34) holds;
7. The free entry condition holds, i.e., equation (B.33) holds.

The equilibrium ratio of production labor and research labor in the economy:

$$\chi \equiv \frac{L_p}{L_r} = \frac{\Pi\alpha(\sigma-1)A^{\sigma-1}}{L_r}, \tag{3.29}$$

where

$$\begin{aligned} L_r = &\int \Phi_d(a_d) e_d^2(a,m) d\mu(a_d) + \int \Phi_g(a_g) e_g^2(a,m) d\mu_g(a_g) \\ &+ \int_{m\neq g} c \cdot V_g(a,m) d\mu_g(a_g) + n_f \cdot M_I + n_e M_e \end{aligned} \tag{3.30}$$

includes all costs other than production side. n_f is the entry cost, n_e is the fixed cost, and c is the transition cost. M_e, M_I are the masses of entrants and incumbents, respectively.

Computing equilibrium aggregates The unknowns are $\{\Pi, P, Y, M_e, C, Y^D, L_p, L_r\}$. According to computation in the appendix, the aggregates are pinned down through the following system of equations:

By definition:

$$\Pi \equiv (\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^{\sigma}Y. \tag{3.31}$$

Free entry condition for the downstream firms:

$$n_e = \beta \int V^d(a_d, a_g)dG_e(a). \tag{3.32}$$

The expression of aggregate final price:

$$P = \frac{\sigma P^{\eta(1-\alpha)}}{(\sigma-1)w^{-\alpha}}[(\frac{1}{P_o^{-(1-\eta)(1-\alpha)}})^{1-\sigma}C_dA_d + (\frac{1}{P^{-(1-\eta)(1-\alpha)}})^{1-\sigma}C_gA_g]^{\frac{1}{1-\sigma}}. \tag{3.33}$$

The expression for consumption:

$$C = Y - Y^D. \tag{3.34}$$

And above labor market clearing condition:

$$L_p + L_r = L. \tag{3.35}$$

The ratio of production and research labor:

$$\chi = \frac{L_p}{L_r} = \frac{\Pi\alpha(\sigma-1)A^{\sigma-1}}{L_r}. \tag{3.36}$$

By equalizing the two equations of Y, we get the expression of M_e:

$$M_e = \frac{L_p}{\alpha\Pi(\sigma-1)\tilde{A}^{\sigma-1}}. \tag{3.37}$$

The input amount is

$$Y^D = \frac{(1-\alpha)\Pi}{P(\sigma-1)}\left[\frac{\eta C_d A_d}{P^{(\sigma-1)(1-\alpha)\eta} p_o^{(\sigma-1)(1-\alpha)(1-\eta)}} + \frac{C_g A_g}{P^{(\sigma-1)(1-\alpha)}}\right]. \tag{3.38}$$

In addition, we can compute the aggregate dirty output which serves the externality term:

$$E = (\sigma-1)\Pi_d (A_d^n + A_d^u))^{\frac{\sigma}{\sigma-1}} \tag{3.39}$$

3.3 Model Mechanism

In this section, we first discuss the firms' decision rules in stationary equilibrium. We then decompose the equilibrium conditions of aggregate variables to the first order to investigate how general equilibrium effects work. Lastly, we analyze model mechanism by varying innovation costs, energy price, energy input share, and entrants' distribution, to compare steady states.

3.3.1 Decision Rules and General Equilibrium Effects

Incumbent innovation incentive and technology class choice Before innovation, firms make decisions about the selection of technology classes. Since we assume that only dirty firms can switch, the cut-off decision rules of dirty firms are shown in Figure 3.2. When dirty firm's green productivity is low, there exists a cut-off of dirty productivity below which dirty firms exit the market. Because now the transition is less profitable even than exiting the market, no firms switch to green technology. When dirty firm's green productivity is high enough, there exists a cut-off of dirty productivity below which dirty firms switch to green technology. This is shown as the colored part of the figure. The cut-off is non-decreasing in the green productivity of dirty firms. Comparing the left panel with the right panel, the results imply that dirty ICT firms have higher cut-offs, which means they are easier to transit given the same green productivity. The technology class selection reflects an extensive margin difference of green transition between ICT and non-ICT firms.

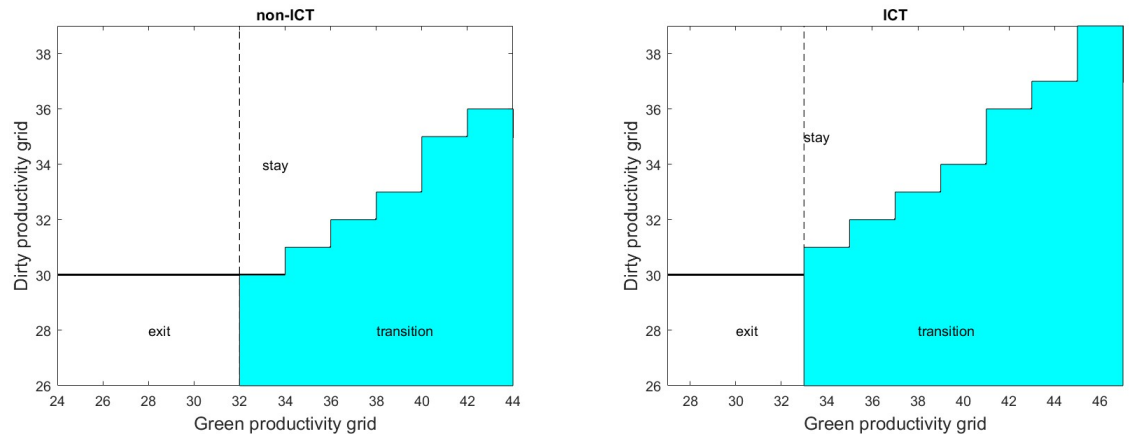


Figure 3.2: Technology class selection and exit of dirty firms

Then we present the innovation decision rules by ICT and non-ICT firms. The technology class selection rules imply that near the cut-off, ICT dirty firms near the cut-off devote less dirty innovation effort, as they have stronger incentive to wait for the failure of dirty innovation and switch, compared to non-ICT dirty firms. This is reflected in the left panel of Figure 3.3. The green innovation decision rules are shown in the right panel of Figure 3.3. When green productivity is low enough, the firm exits the market and has no incentive to devote positive innovation. As ICT firms have lower green innovation cost compared to non-ICT firms (which will be implied from data in the next section), the green innovation intensity is higher.

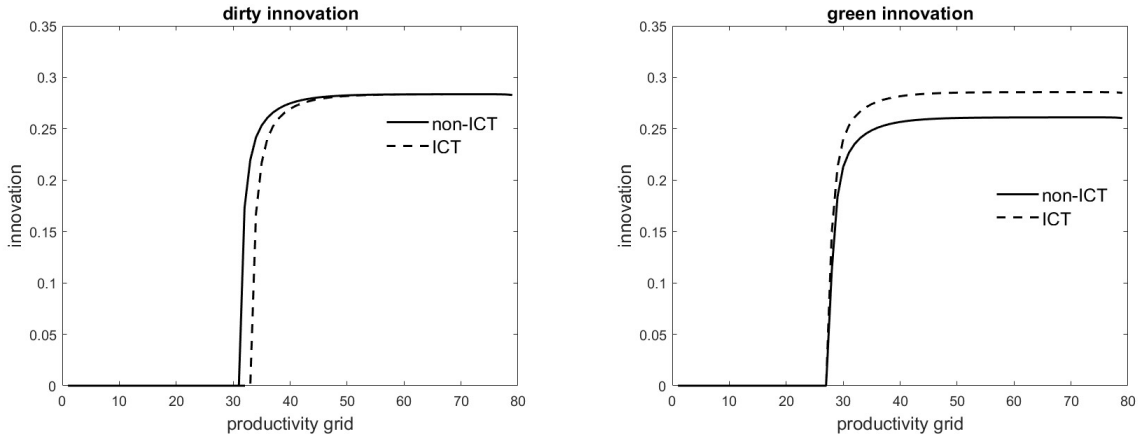


Figure 3.3: Optimal innovation incentive

First order decomposition of incumbent production profits For the discussion of the model mechanism, it is important to understand how the aggregate variables are connected and how they contribute to the general equilibrium effects. The variable production profits of intermediate firms govern the entry and exit decisions of the firms, the choice of the technology class, and the innovation decisions. Therefore, we study how these profits are determined.

Recall that the variable production profit for dirty firms is

$$\Pi_d = \Pi C_d (P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}, \tag{3.40}$$

and the variable production profit of green firms is

$$\Pi_g = \Pi C_g P^{(1-\alpha)(1-\sigma)}. \tag{3.41}$$

In a first step, we need to understand what determines the common part of variable profit, Π. The aggregated production labor implies that $L_p = \alpha \frac{\sigma-1}{\sigma} PY$, combined with the definition of Π, gives:

$$\Pi = (L - L_r) \frac{1}{\alpha(\sigma-1)} \frac{1}{(P\frac{\sigma-1}{\sigma})^{1-\sigma}}. \tag{3.42}$$

and we decompose the common variable profit Π as follows:

$$\Delta \log \Pi = \Delta \log(L - L_r) + (\sigma - 1) \Delta \log P. \tag{3.43}$$

This directly implies that the change in the common variable profit Π is determined by the change in the reallocation of labor between production and research labor as well as the change in the final price of the good.

We can further decompose the final goods price P:

$$\begin{aligned}\Delta \log P = &\underbrace{\frac{1}{(1-\sigma)(1-\omega_d)}[\Delta \log M_e + \iota_d \Delta \log \tilde{A}_d + (1-\iota_d)\Delta \log \tilde{A}_g]}_{\text{innovation and entry}} \\ &+ \underbrace{\frac{(1-\eta)(1-\alpha)}{1-\omega_d}\Delta \log P_o}_{\text{resource price effect}},\end{aligned} \tag{3.44}$$

where $\iota_d = \frac{C_d A_d (P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}}{P^{1-\sigma}}$ represents the ratio of the aggregate dirty profits, and $\omega_d = \iota_d \eta(1-\alpha) + (1-\iota_d)(1-\alpha)$ is a weighed average of the dirty price share and the green price share. The factor price directly contributes to the aggregate price or productivity as shown in the last term. On the other hand, it could potentially contribute to sorting of firms to green technology, affect innovation intensity, and affect entry decisions, which are reflected in the first three terms.

For convenience, we further decompose the variable profits of using the dirty technology:

$$\begin{aligned}\Delta \log(\Pi_d) = &\underbrace{\Delta \log(L - L_r)}_{\text{labor reallocation}} - \underbrace{\Lambda_d \Delta \log M_e}_{\text{entry}} - \underbrace{\Lambda_d(1-\iota_d)\Delta \log A_g - \iota_d \Lambda_d \Delta \log A_d}_{\text{technology sorting}} \\ &+ \underbrace{(\sigma-1)(1-\eta)(1-\alpha)(\Lambda_d - 1)\Delta \log P_o}_{\text{resource price effect}}\end{aligned} \tag{3.45}$$

where $\Lambda_d = \frac{1-(1-\alpha)\eta}{1-\omega_d}$. Analogously, we have the decomposition for variable profits of using green technology:

$$\begin{aligned}\Delta \log(\Pi_g) = &\Delta \log(L - L_r) - \Lambda_g \Delta \log M_e - \Lambda_g \iota_d \Delta \log A_d - \Lambda_g (1-\iota_d)\Delta \log A_g \\ &+ (\sigma-1)(1-\eta)(1-\alpha)\Lambda_g \Delta \log P_o\end{aligned} \tag{3.46}$$

where $\Lambda_g = \frac{\alpha}{1-\omega_d}$. The relative variable profit across technologies can be shown as the following:

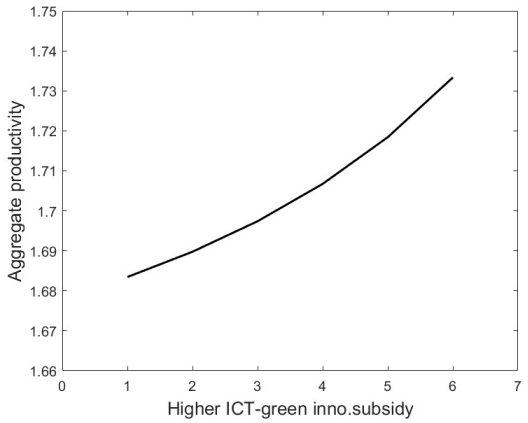
$$\begin{aligned}\Delta \log(\Pi_d) - \Delta \log(\Pi_g) = &(\Lambda_g - \Lambda_d)[\Delta \log M_e + \iota_d \Delta \log A_d + (1-\iota_d)\Delta \log A_g] \\ &+ (\sigma-1)(1-\eta)(1-\alpha)(\Lambda_d - \Lambda_g - 1)\Delta \log P_o.\end{aligned} \tag{3.47}$$

Hence, the relative variable profit across technologies does not rely on labor reallocation effect to a first order approximation. It relies on innovation and the entry effect, as well as the factor price effect. We provide an analytical result in Appendix B.3 to see how our model setup establishes the connection between energy price and firm decisions as well as aggregate variables. The entrants' mass contributes negatively to relative variable profit of dirty to green, implying that more competitive entry market favors green technology. Both aggregate green productivity and aggregate dirty productivity contribute negatively to the relative variable profit of dirty to green. The results are consistent with our assumption that green technology is more dependent on the input of final goods. As aggregate productivity increases, the economy naturally favors green technology more. The energy price negatively contributed to the relative variable profit of dirty to green because a higher input cost for dirty will weaken its production profit and encourages firms to switch to green technology.

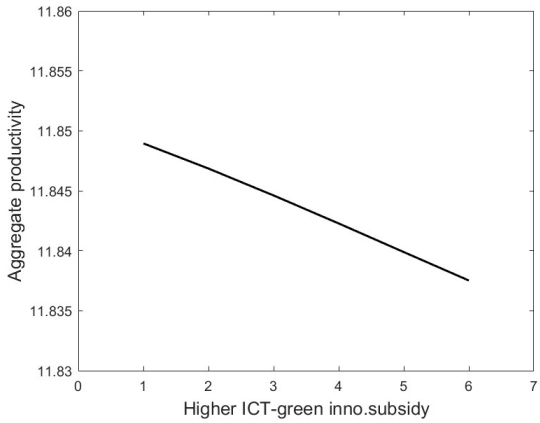
3.3.2 Difference between ICT firms and non-ICT firms

Role of the different innovation cost of ICT and non-ICT firms In this subsection, we study the model mechanisms through the change of green innovation costs. We consider several general equilibrium effects including the opportunity cost of directed innovation within the group, the effects of input price through the aggregate price, and the effects of labor reallocation. When doing the analysis, we should bear in mind that changing innovation cost works on either affecting firms' innovation decisions directly, or further works on aggregate variables which affect firms' production decisions and innovation decisions.

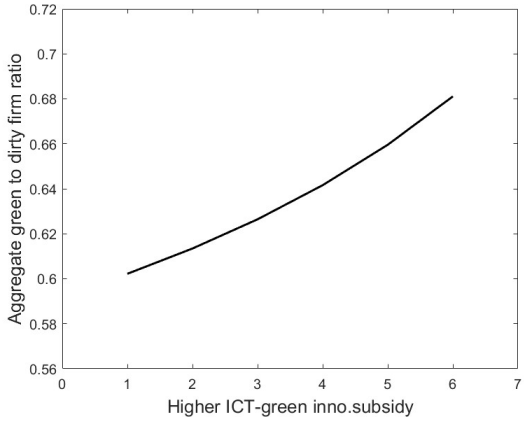
Several general equilibrium mechanisms could work in the process. The first mechanism is straightforward, that the lower cost of green innovation for ICT firms will sort the dirty firms into green firms, which potentially weakens the positive effect on aggregate productivity. In addition, this could affect the mass of entrants by changing the expected value at the time of entry, which also contributes to aggregate productivity. Secondly, labor reallocates between production and research, which affects market size and output as well. Third, the overall effect on aggregate productivity



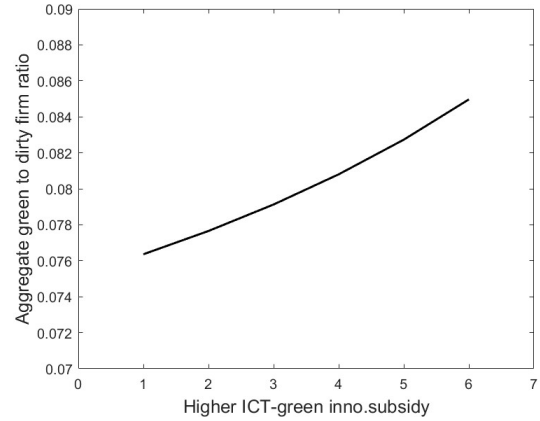
(a) high dirty innovation cost: A



(b) low dirty innovation cost: A



(c) high dirty innovation cost: green firm fraction



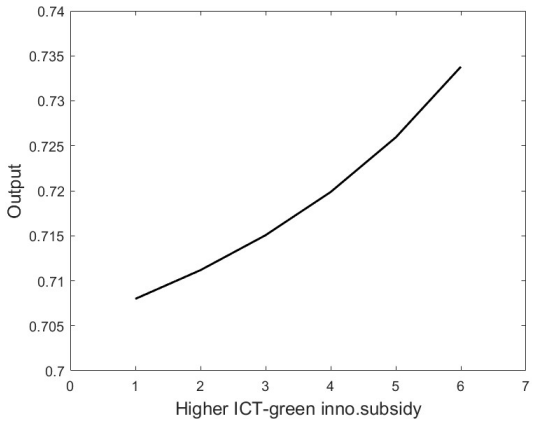
(d) low dirty innovation cost: green firm fraction

Figure 3.4: General equilibrium effects of innovation cost reduction

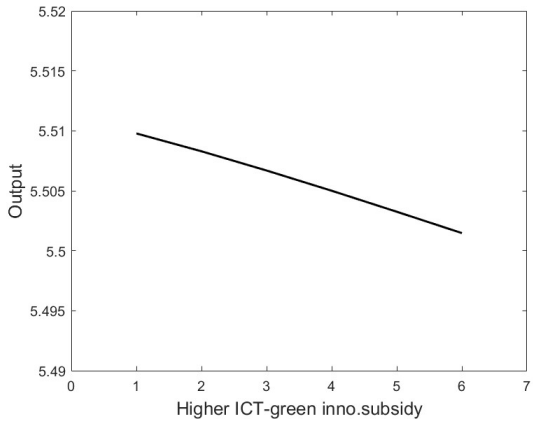
Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

determines the direction of the change of the aggregate price, which changes the relative profit of using dirty and green technology and further affects green innovation and the transition of firms.

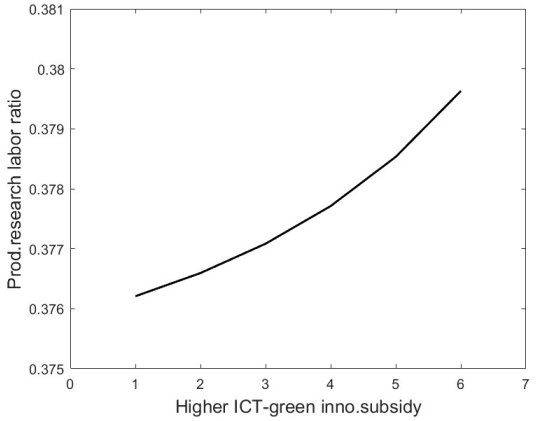
As shown in the bottom of Figure 3.4, lowering the costs of ICT green innovation promotes the green transition. The difference between the left and right is that the left has a relatively lower opportunity cost of selecting green technology (lower relative cost of green to dirty innovation). To see this clearly, Figure 3.6 shows that firms are doing more green innovation and less dirty innovation in both cases. Furthermore, as shown in Figure 3.7, the green productivity distribution of ICT firms shifts upward while the dirty distribution shifts downward. However, the result in aggregate productivity is different. Now come back to the upper panels of Figure 3.4. In the low



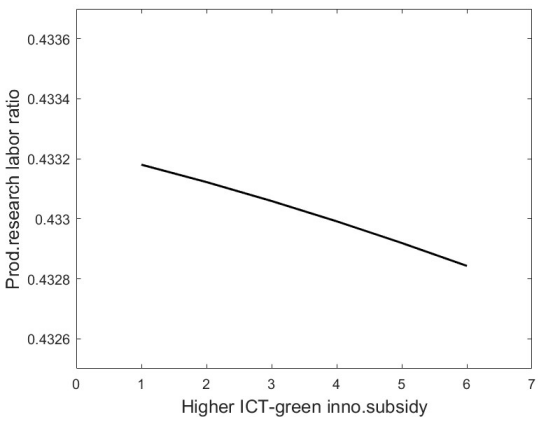
(a) high dirty innovation cost: Y



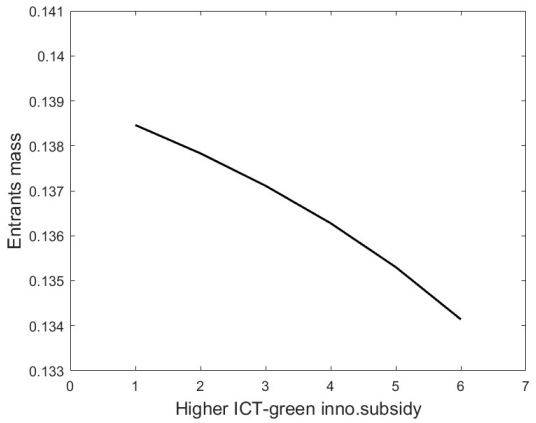
(b) low dirty innovation cost: Y



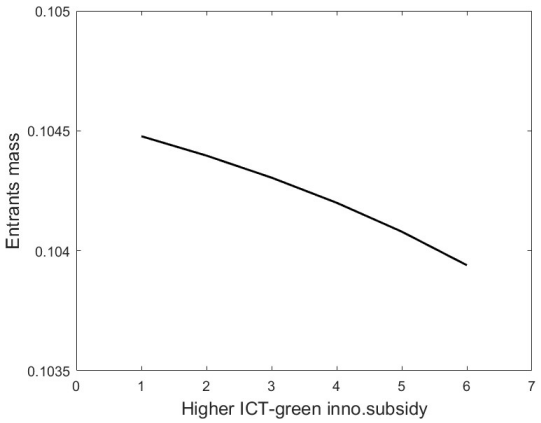
(c) high dirty innovation cost: χ



(d) low dirty innovation cost: χ



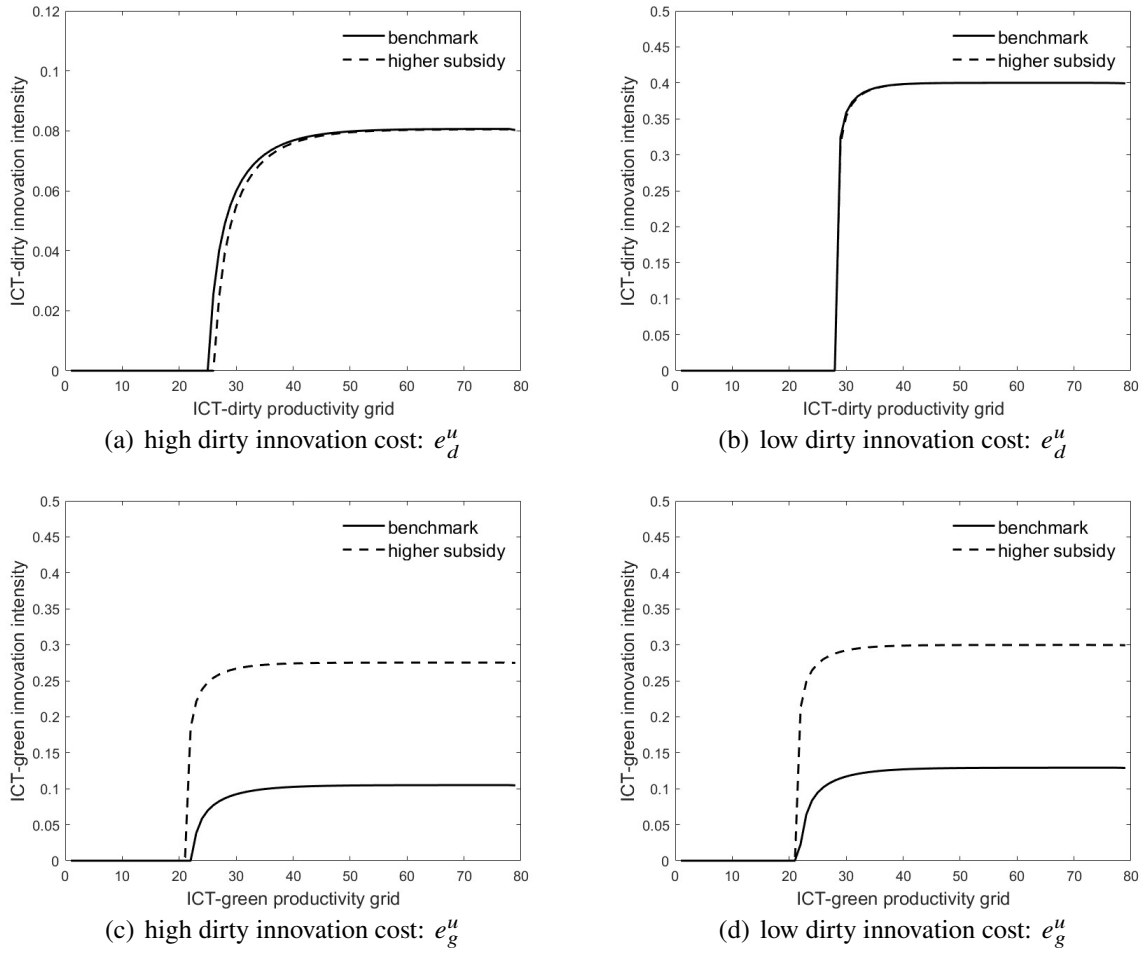
(e) high dirty innovation cost: M_e



(f) low dirty innovation cost: M_e

Figure 3.5: General equilibrium effects of innovation cost reduction

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.



(a) high dirty innovation cost: e_d^u (b) low dirty innovation cost: e_d^u

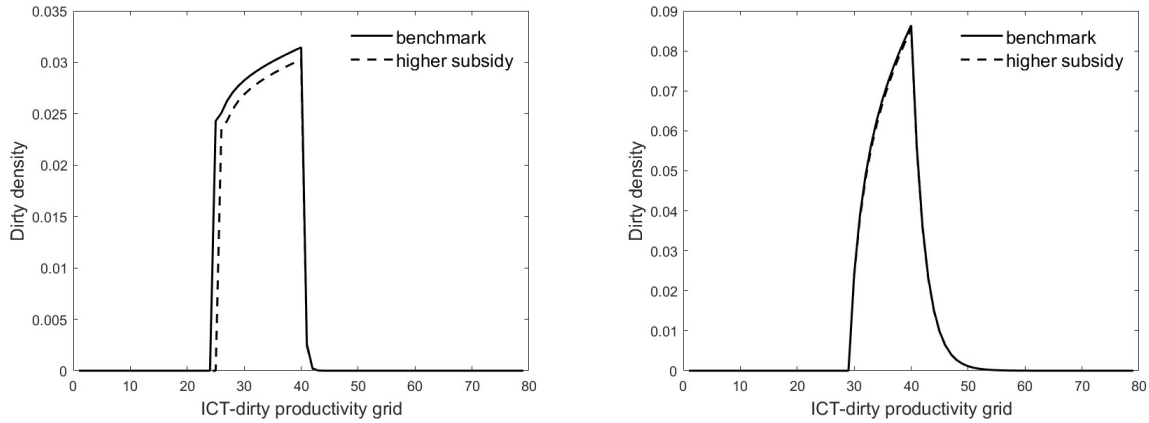
(c) high dirty innovation cost: e_g^u (d) low dirty innovation cost: e_g^u

Figure 3.6: General equilibrium effects of innovation cost reduction

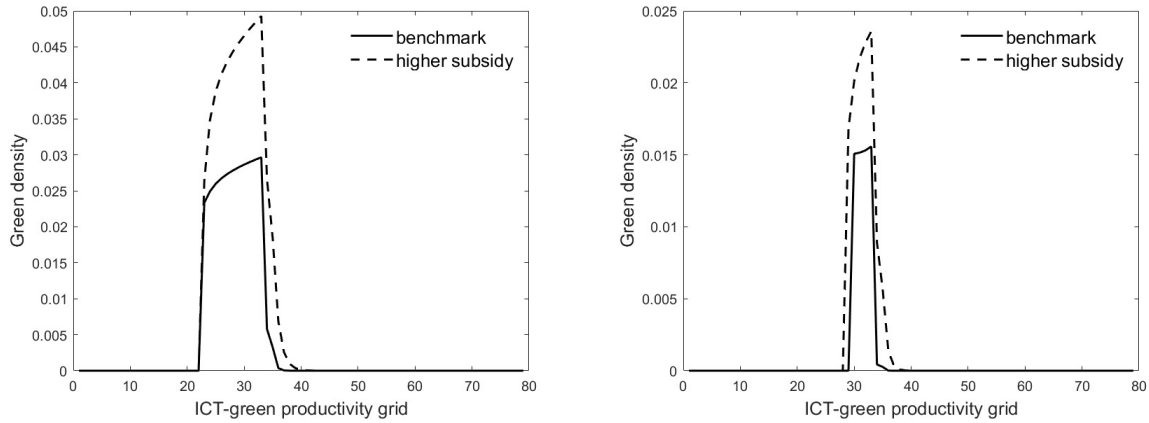
Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

green opportunity cost case, aggregate productivity increases as the ICT-green innovation subsidy is higher, while in the high green opportunity cost case it decreases as the green innovation subsidy is higher. The relative innovation cost of dirty and green technology is a key that determines the relative strength of the opportunity cost of green innovation and affects the aggregate effect of innovation subsidies.

From Figure 3.5, we observe that labor reallocation is in the same direction with the change in productivity, including intra-technology reallocation between production and research labor and labor reallocation across technologies. Lastly, firm entry decisions are determined by the expected



(a) high dirty innovation cost: μ_d^u (b) low dirty innovation cost: μ_d^u



(c) high dirty innovation cost: μ_g^u (d) low dirty innovation cost: μ_g^u

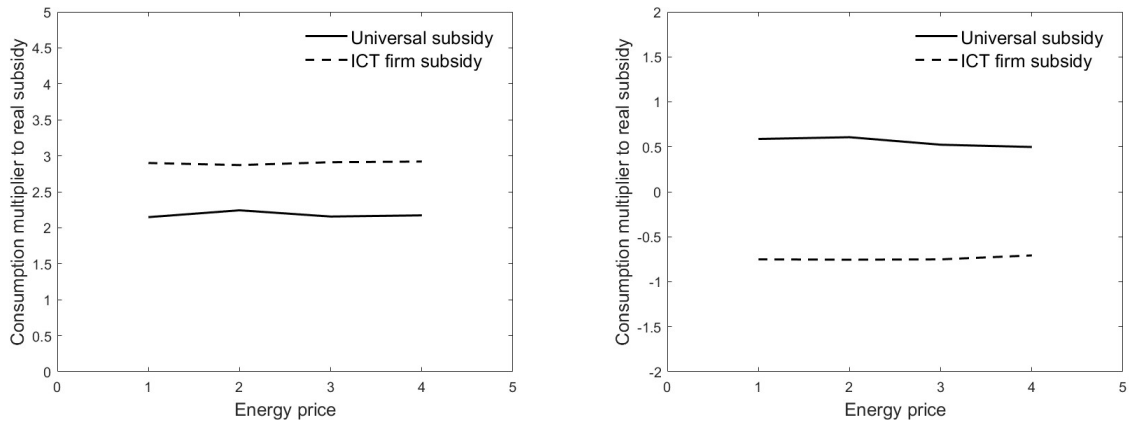
Figure 3.7: General equilibrium effects of innovation cost reduction

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

profit of entrants that is affected by market size, expected value of innovation and labor allocation, etc. The overall effect implies that the mass of entrants drops as the cost of green innovation drops. From the expression of the variable profit of green technology in equation (3.46), these general equilibrium including productivity redistribution across technologies, labor reallocation, and entry decisions all affect the green innovation and transition across different groups of firms, weakening or strengthening the original effect by innovation cost changes.

Green Innovation Subsidy: Universal or ICT Only According to the US Energy Information Administration, the public R&D related to clean energy in the US in 2016 is 347 million USD,

which represents 0.0019% of total GDP (or around 0.056% of total R&D spending). According to National Center for Science and Engineering statistics, the public R&D related to clean energy in 2023 is 2000 million, with a share of GDP 0.0074%. Our analysis is to match the ratio of green subsidy to R&D in 2016.



(a) high dirty innovation cost: C (b) low dirty innovation cost: C

Figure 3.8: Consumption change under subsidies

Note: The multiplier computed in the Y-axis is: $\frac{C^i - C^0}{S^i/P}$, which is the ratio of consumption increment to real subsidies. The subsidy amount of each policy is to target green subsidy to R&D ratio in the data, which is 0.056%.

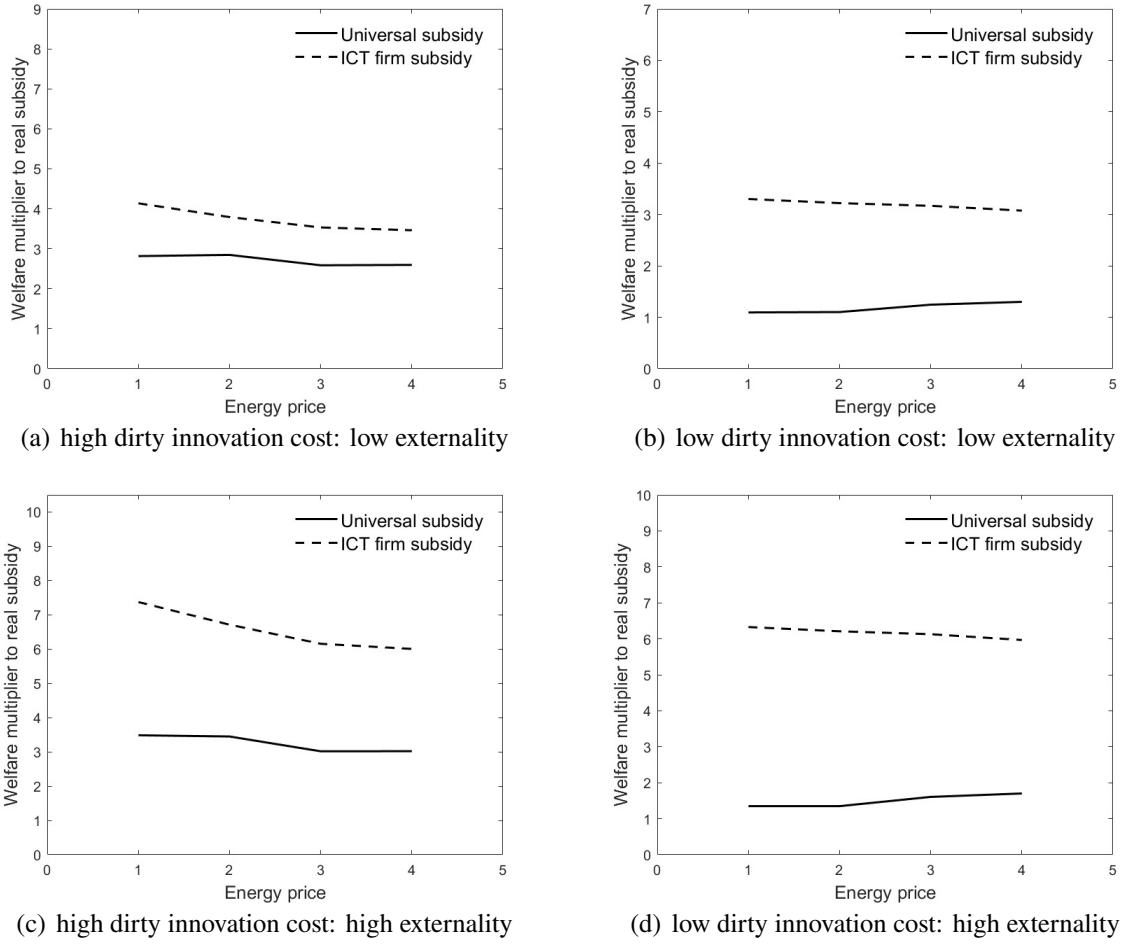
First of all, we allocate the green innovation subsidy proportional to firm size which does not distinguish ICT or non-ICT firms. Then we concentrate all green subsidies on ICT firms. Both types of subsidies target data of 0.056% of total R&D spending in the economy.

As the energy price potentially affects aggregate productivity and the green transition, we compare the efficiency of policy under changes in the energy price. The standards we select is to compare the policy efficiency to the case without any subsidy. The efficiency of policy i is measured by a multiplier:

$$\text{efficiency}_i = \frac{W^i - W^0}{S^i/P},$$

where W^i is welfare under policy i, as described in 3.1:

$$W = C - \Xi E.$$



(a) high dirty innovation cost: low externality

(b) low dirty innovation cost: low externality

(c) high dirty innovation cost: high externality

(d) low dirty innovation cost: high externality

Figure 3.9: Welfare change under subsidies: higher vs low externality

Note: The multiplier computed in the Y-axis is: $\frac{W^i - W^0}{S^i/P}$, which is the ratio of welfare increment to real subsidies. The subsidy amount of each policy is to target green subsidy to R&D ratio in the data, which is 0.056%.

W^0 is welfare under no policies. S^i/P is the amount of subsidy in terms of real goods.

The consumption result is shown in the left panel of Figure 3.8. The Y-axis is the relative change compared to the benchmark without any subsidies. When the cost of dirty to green innovation is high, both subsidies increase consumption as in the left panel, and subsidizing green ICT firms are more efficient. When the cost of dirty to green innovation is low, the universal green subsidy generates a positive effect on consumption while the ICT green subsidy has a negative effect. The pattern is rather stable along the change of the energy price. The result is consistent with the above analysis that it is more efficient to concentrate subsidies on firms with a comparative advantage in

green innovation when the resulting loss in dirty-technology productivity is low.

We also compute the welfare multiplier rather than the consumption multiplier in Figure 3.9. As in the first panel, in which case the cost of dirty to green innovation is relatively high, when the price of energy increases, the welfare increment generated by the ICT green subsidy decreases significantly. This implies that although the energy price and the green subsidy complement the green transition, the marginal effect of the ICT green subsidy is low when the energy price is already high. In addition, a higher price of energy drives down aggregate productivity, which also mitigates the effect of subsidy. When the cost of dirty to green innovation is relatively low, the decreasing effect is much weaker. This is because a lower relatively lower cost of dirty innovation weakens the effect of energy price on the green transition. The bottom panels are used to set a higher value of the externality parameter. In both cases, the welfare increase is higher compared with the low externality case and the subsidized ICT green innovation is significantly improved. But in case the cost of dirty to green innovation is relatively high, the sensitivity of the efficiency of the policy to the energy price is also increased.

3.3.3 Role of Other Mechanisms

Role of the share of final goods as intermediate inputs In this section, we study the interplay between the final goods input share of dirty technology (that share of green technology is 1-labor share) and the reduction in the cost of green innovation. The reason is that the effect of innovation cost would drive the change in aggregate price and productivity, which would enhance or weaken the original effect by changing the input price for firms. To begin with, we should notice that the effect is dependent on the relative value of the wage and the equilibrium aggregate price, that is, the real wage $\frac{w}{P}$ in the economy. If $\frac{w}{P} < 1$, labor is cheaper compared to the input of the final goods. Given the same level of energy price, a higher share of the input of dirty final goods implies the use of green technology, as production has a lower opportunity cost. If $\frac{w}{P} > 1$, a higher input share of dirty final goods implies using green technology as production has a higher opportunity cost.

Given the current setting, the case falls into $\frac{w}{P} > 1$ that the final goods are cheaper than labor.

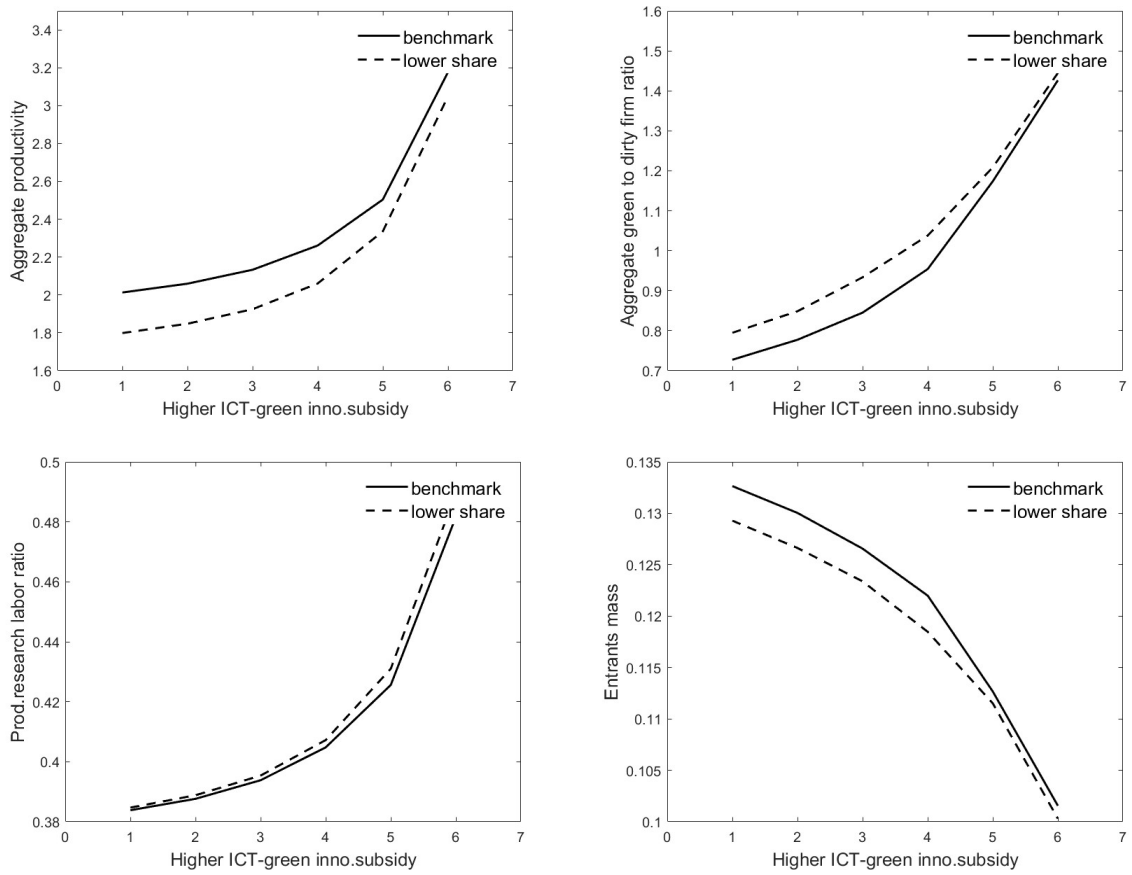


Figure 3.10: General equilibrium effects of final goods input share: high dirty innovation cost

As shown in the upper two panels in Figure 3.10, a lower share or a lower innovation cost implies a higher distribution of green productivity. The lower share also implies a higher degree of increment when reducing the cost of green innovation. This is because as in the middle panels, aggregate productivity is increasing and price is decreasing. The lower input share of dirty final goods implies that dirty technology benefits less from the decrease in input price, leading to a faster green transition. As in the lower panels, a lower share implies a higher reallocation of labor to production as the innovation cost changes. It also implies a higher entrant mass due to lower productivity and a lower market size in this case.

Role of entrant productivity distribution In this section, we analyze how the change in entrant distributions affects the equilibrium outcomes. In a benchmark case, we assume that the ICT and non-ICT entrants have the same distribution that the probability mass declines with potential green

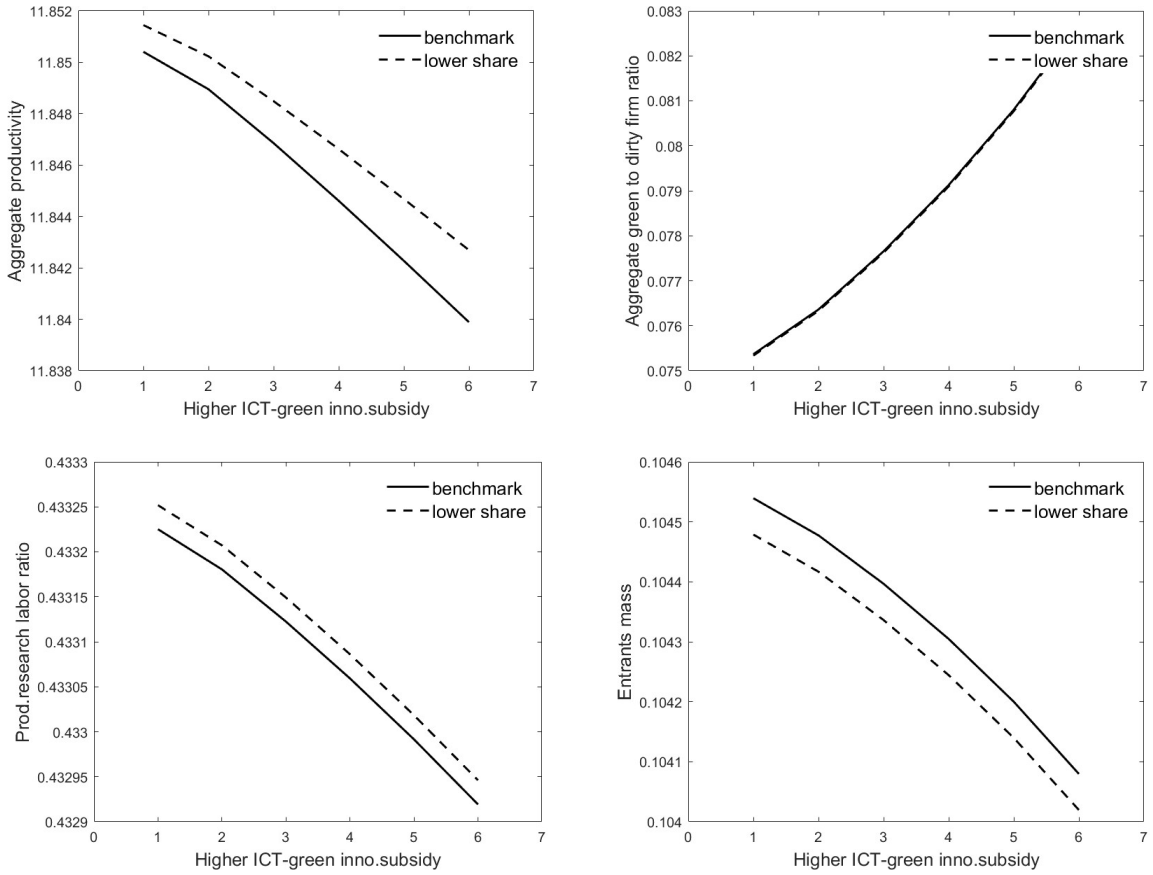


Figure 3.11: General equilibrium effects of final goods input share: low dirty innovation cost

productivity as they enter as dirty firms. We move the probability mass of low green productivity ICT firms to high green productivity ICT firms such that the distribution becomes first order stochastically dominates the previous one.

From the upper four panels of Figure 3.12, the productivity distribution of green ICT firms shifts rightward, implying a more green transition. The overall effect on aggregate productivity is also positive. However, when the opportunity cost of green technology is high, the positive effect generated by higher entrants' green productivity is weakened much as we can see that all the slopes are lower than the benchmark. The reason is that a higher loss of dirty productivity mitigates the positive effect originating from entrants' improvement.

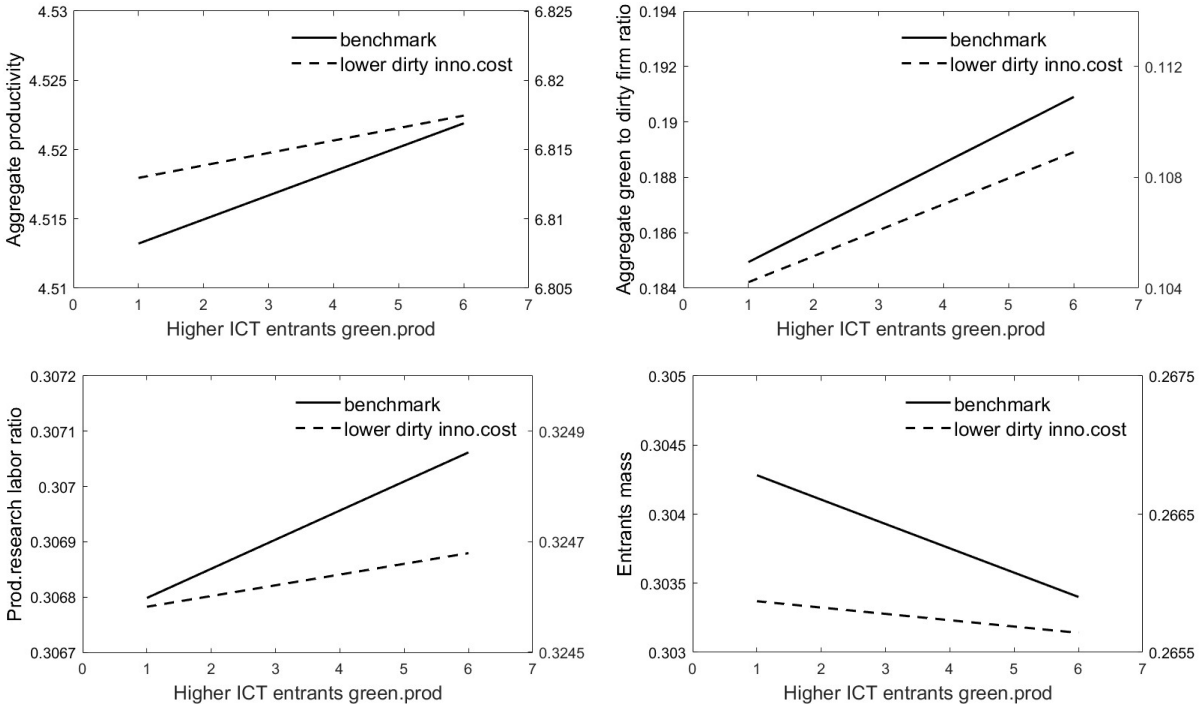


Figure 3.12: General equilibrium effects of ICT entrants green productivity change

Note: Each grid of X-axis is to reshape the ICT entrants green productivity distribution which first order stochastic dominates the previous one.

Role of the energy price change In this part, we further explore how energy prices interact with the above change in green innovation costs. The first feature is that increasing the energy price reduces aggregate productivity. Although it encourages the green transition of firms, it causes a loss of dirty productivity by increasing its production cost as in the first panel of Figure 3.13, which shows that the overall effect on aggregate productivity is decreasing. Furthermore, it shows that when the opportunity cost of green innovation is higher, the energy price has a stronger effect on the decreasing productivity but a weaker effect on the green transition. The results demonstrate that a higher green innovation subsidy and a higher energy price complement the green transition.

The bottom panels of Figure 3.13 show the pattern of labor reallocation and the mass of the entrants, respectively. Higher energy price change implies higher labor reallocation towards production and lower entrants' mass but in opposite direction with lower green opportunity cost.

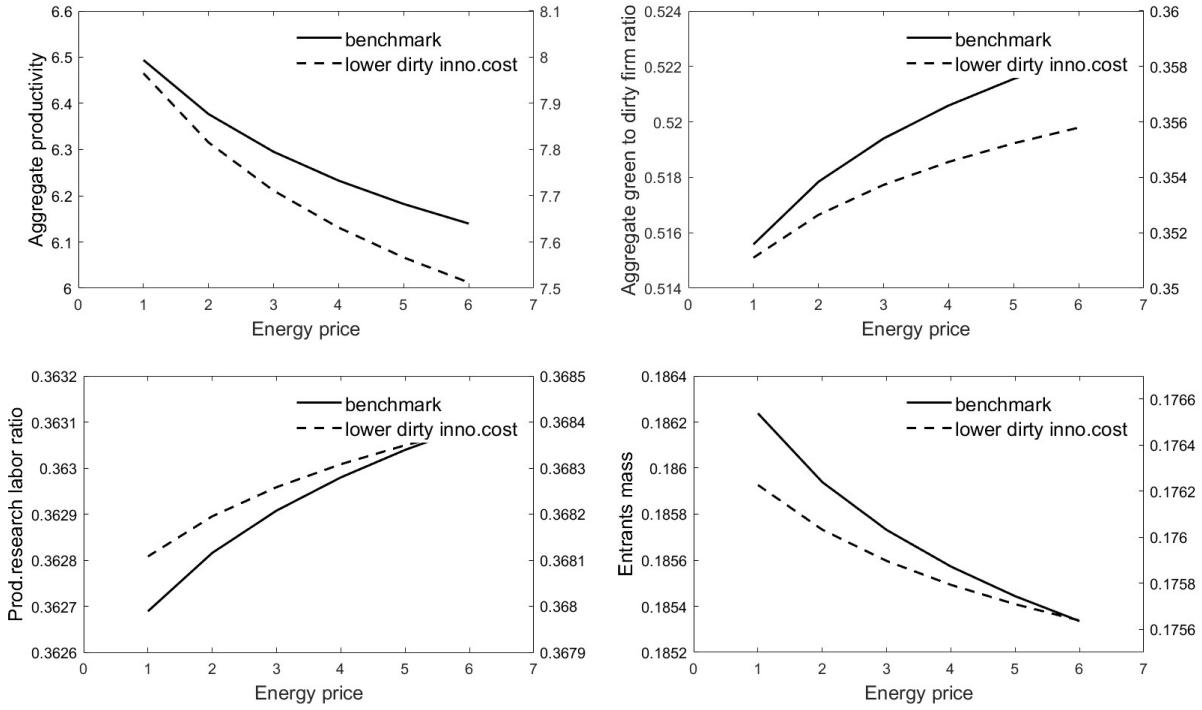


Figure 3.13: General equilibrium effects when energy price change

4 Quantitative Analysis

Before implementing any policy analysis, we calibrate 16 parameters, where half of them are internally estimated using U.S. patent data from the USPTO matched with firm data from Compustat. We set the dirty innovation cost scale and curvature parameter h_d, ξ_d as a benchmark and also set the curvature of ICT green innovation and non-ICT green innovation ξ_g, ξ_g^u. We focus on disciplining the scalar parameter of ICT green innovation and non-ICT green innovation h_g, h_g^u. The other internally estimated parameters are the innovation step size J, entry cost n_e, fixed cost of operation n_f ,transition cost c, energy price P_o and the curvature of ICT-green entrants productivity distribution k^u .

Next, we implement a green innovation subsidy schedule for all firms proportional to their firm size. We then compute another kind of green subsidy policy which targets on ICT firms only. The result implies that the ICT targeted green subsidy policy always dominate the universal green subsidy policy, though the degree of dominance depends on energy price, ICT entrant's distribution

and ICT-green innovation cost.

4.1 Parameterization

Table 4.1: Model parameterization

Parameter	Notation	Value	Targeted Moments	Model	Data
External					
labor.share	α	0.65	Matthew et al. (2022)		
final.goods.share	η	0.9	Acemoglu et al. (2016a)		
exit.prob	γ	0.0055	Atkeson and Burstein (2010)		
discount factor	β	0.95	common.estimates		
elas.of.subs	σ	1.5	common estimates		
share.ICT.firm	θ	0.515	share.ICT.innovator		
dirty.cost	h_d	3.24	Assigned.value		
dirty.curvature	ξ_d	30	Assigned.value		
green.curvature	ξ_g^n	30	Assigned.value		
ICT.green.curvature	ξ_g^u	30	Assigned.value		
Internal					
green.cost	h_g^n	2.08	non-ICT.green.to.dirty.sales.ratio	0.73	0.72
ICT.green.cost	h_g^u	1.23	ICT.non-ICT.green.sales.ratio	1.72	1.72
ICT.entrant.distri.slope	k^u	0.728	green.firm.ratio	0.58	0.58
transition.cost	c	0.05	non-ICT.green.dirty.firm.ratio	0.40	0.39
energy.price	P_o	0.02	ratio.sd.log-sales.non-ICT.green/dirty	1.15	1.15
inno.step	J	0.485	std.firm.log-sales	1.14	1.16
entry.fixed.costs	n_e	2.118	entrant.ratio	0.02	0.02
operat.fixed.costs	n_f	0.136	firm.exit.rate	0.09	0.10

Notes: We consider different values of the preference parameter for externality Ξ, which delivers different policy implications.

In this section, we estimate the parameters listed in Table 4.2 using simulated methods of moments (SMM). There are 16 parameters, 8 of them are externally estimated and 8 of them are internally estimated.

4.1.1 External Estimation

The labor share of income is to target recent data of firms in U.S., the value is estimated to be around 0.6 by Matthew et al. (2022). The share of oil input when using dirty technology is $(1-\alpha)(1-\eta)$, the value of which is 0.04 implied by Acemoglu et al. (2016a). Therefore, given the labor share, the value of η is 0.9. Our model assumes an exogenous exit rate γ, we adopt the value used in Atkeson and Burstein (2010), to match the average exit rate of large firms in the United States. The discount factor β is computed by the inverse of the annual interest rate. The elasticity of substitution σ among the output varieties is a common estimator in the literature. Upon entry, firms are randomly assigned to be an ICT firm of probability θ, we use the ratio of ICT innovators to total innovators in the patent data as its value. We set the dirty innovation cost scale and curvature parameter h_d, ξ_d, the curvatures of green innovation and non-ICT green innovation cost ξ_g, ξ_g^u as given.

4.1.2 Internal Estimation

For internal estimation, we separate the parameters into two groups. The first group contains 5 parameters h_g, h_g^u, k^u, c, P_o. The second group contains 3 parameters n_e, n_f, J .

The first group includes the scalars of the cost of green innovation of ICT and non-ICT firms h_g^u, h_g. Other three parameters are The transition cost c, the shape of ICT entrants distribution k^u and the energy price P_o are disciplined from green firm ratio. We jointly discipline these parameters from the moments: 1.non-ICT green to dirty firms' sales ratio; 2.ICT green to non-ICT green firms' sales ratio; 3.green firm ratio 4. non-ICT green to dirty firm ratio; 5. ratio of log-sales standard deviation of non-ICT green firms to dirty firms.

The second group includes three parameters. The innovation step size J is estimated to match the standard deviation of firm log-sales. n_e affects the entry mass from free entry decisions, and we match it to real entrants' rate. n_f affects firms that hit the margin of exit or maintain operations and we use the firm exit rate to match this moment.

We then jointly minimize the loss function within each group as follows:

$$min \sum_{k=1}^{N} \frac{|model_k - data_k|}{(|model_k| + |data_k|)^{\frac{1}{2}}} \tag{4.1}$$

The data we target is the data in 2006. This year serves as a benchmark during of our main interested sample period from 2006 to 2014, that witness a first emerging but then stagnation of green and ICT-green related indicators of green innovation. The estimated parameter values and matched moments are listed in Table 4.2.

Table 4.2: Mapping to the data

Model	Data
$\theta \int e^u(a) dG_e(a)$	firm.start.inno.ICT.green.pat.from.current.period
$(1-\theta) \int e^n(a) dG_e(a)$	firm.start.inno.non-ICT.green.pat.from.current.period
$\int e^u(a) d\mu^u(a)$	firm.also.inno.ICT.green.pat.last.period
$\int e^n(a) d\mu^n(a)$	firm.also.inno.non-ICT.green.pat.last.period
$\int (1 - e^u(a^u)) d\mu^u(a^u)$	firm.quit.inno.ICT.green.pat.next.period
$\int (1 - e^n(a^n)) d\mu^n(a^n)$	firm.quit.inno.non-ICT.green.pat.next.period

4.2 Model Validation

In this section, we use calibrated parameters to conduct an external validity check. We are going to implement three variations exogenously and jointly: the energy price, the entrant's distribution of productivity, and the ICT-green innovation cost, and to match the three moments jointly: the ICT to non-ICT green firms' sales ratio, the green firm ratio and the green to dirty firms' sales ratio. Then we compare the paths of parameters implied by the change of model moments and data moments to verify how each element can explain the change of data moments.

The reason why we focus on these three parameters is to follow the hints from literature. Acemoglu et al. (2023) trace part of the green slowdown to the shale gas revolution: the large and persistent decline in the cost of fossil energy - lowered the relative profitability of green technologies and discouraged innovation in clean alternatives. This motivates us to model changing energy

prices. Atkeson and Patrick (2007) and Aghion et al. (2009) explain the growth of the productivity of specific technologies of incumbents with the productivity of the entrants. Although they do not directly apply to the context of green innovation, it is obvious that it is our interest to vary entrant distribution of green productivity especially that of ICT firms to see how it drives the green transition. Acemoglu et al. (2016a) and Aghion et al. (2024) model that the competition between green and dirty technologies depends on the creative destruction rate between them, which is based on the relative cost of innovation. Our paper also emphasizes the importance of this mechanism. Specifically, we are interested in varying the cost of ICT-green innovation in this paper to study the green transition.

4.2.1 Change of Energy Prices

The data implies that the path of energy price is shown in Figure 4.1, it is an inverted U-shaped path. It firstly increases and falls after 2011. This variable implies a first increasing incentive for firms to adopt green technology and a stagnated stage after then. Literature such as Popp et al. (2022), Acemoglu et al. (2023) and Aghion et al. (2024) consider energy price as a key factor determines the opportunity cost of green innovation and also provide data facts on energy price change due to shale gas technology development. Our model not only confirms it but incorporates other explanations including ICT entrants productivity distribution and ICT-green innovation cost discussed below. Though joint targeted, a closely related moment is the number of green firms to number of all firms in the economy. The time series is shown as in Figure 4.1: the quick emerging of the share of green firms since the 2006 up to 2009 or 2010, and then witness a stagnation and drop until 2014. The energy price change can preliminarily explain the change in the sorting of firms to dirty and green technology, but unable to tell the differences between ICT and non-ICT firms.

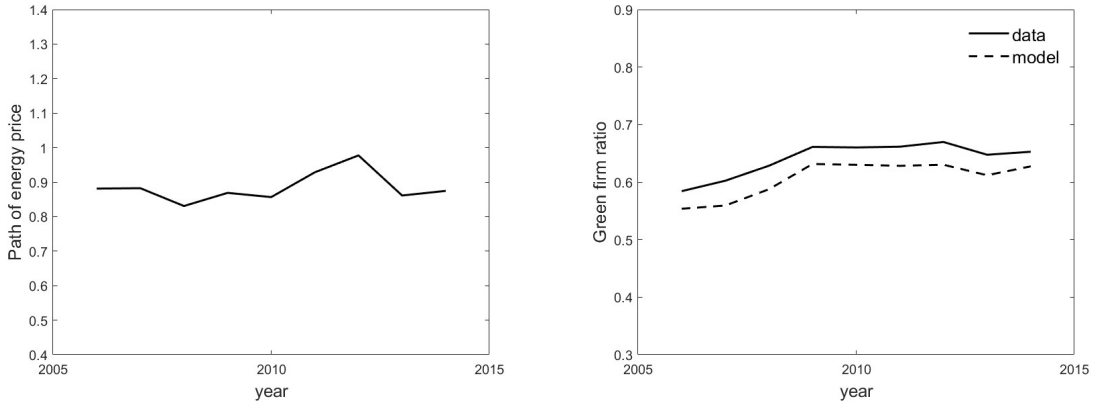


Figure 4.1: Vary energy price: trends of green firm ratio

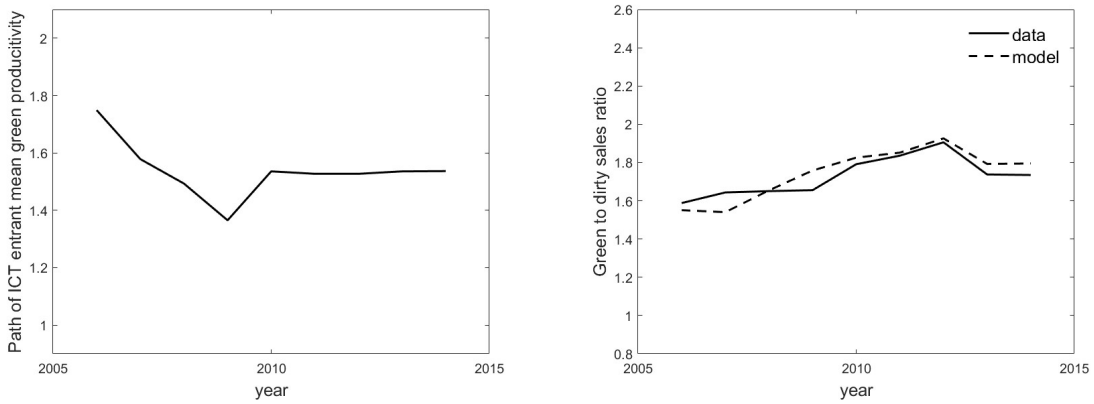


Figure 4.2: Vary ICT entrants distribution: trends of green sales share

4.2.2 Change of ICT Entrants Productivity Distribution

The data implies the path of green productivity distribution of ICT entrants, the mean of the distribution is shown in Figure 4.2. The pattern shows that ICT entrants productivity decreases first and then increases until the stagnation since 2010. The explanation is that entrants distribution is more powerful to generate change in extensive margin green transition rather than incremental green innovation that are reflected from sales ratio. Hence, it first implies lower entrant distribution in face of a widening gap between green firm ratio and green sales ratio. But then the pattern reverses which implies a higher and stagnated entrants productivity distribution. The moments are shown in the right panel of Figure 4.2, as the sales share increases gradually but slows down around 2010 and drops. The emergence of it lags behind the green firm ratio change.

4.2.3 Change of ICT Green Innovation Cost

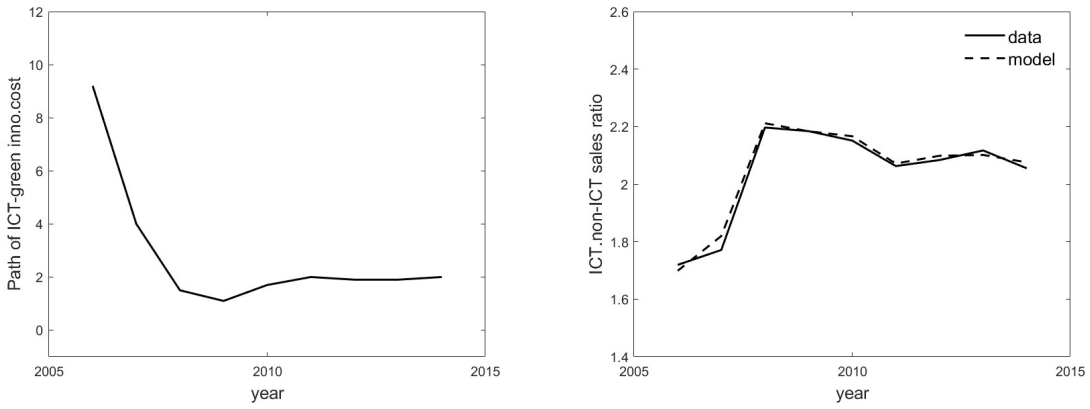


Figure 4.3: Vary ICT-green innovation cost: trends of ICT to non-ICT green sales ratio

The data implies the path of ICT green innovation cost, as shown in Figure 4.3. The change of the path is more sharply than energy price and entrants distribution changes. The green innovation cost for ICT firms drops dramatically since 2006 but the trends end in 2009. It reverses back slightly in 2010 and stagnated since then. A closely related moment is the ICT to non-ICT sales ratio, which is listed in the right panel of Figure 4.3. The moment also reflects a stagnation of ICT green firms' sales ratio around 2010. The validation implies that the drop and stagnation of ICT green innovation cost is a key in determining the green transition for ICT firms and also for all firms.

4.3 Analysis of Subsidy Policies

In this section, we compare two policies based on the parameters calibrated from the data. The first policy is to launch a universal green innovation subsidy over all firms. The second policy is to make the green innovation subsidy target ICT firms only.

The benchmark case is the parameters calibrated according to data in 2006, which is shown in the calibration part. The compared year is 2010 when green innovation and ICT-green innovation reached the peak from data observations. The parameter we vary is from the above subsection accordingly: the energy price; the ICT entrants green productivity distribution, and the ICT-green

innovation cost. The value of these parameters is also from the above subsection that matches the moments in 2010. The results are presented in Table 4.3.

Table 4.3: Policy analysis: welfare multiplier of innovation subsidies

Exogenous ICT choice	year	$\Xi = 0$	$\Xi = 10$	$\Xi = 50$
Benchmark				
Policy1	2006	3.59	3.40	2.64
Policy2	2006	13.35	12.20	7.61
Energy price				
Policy1	2010	3.45	3.42	3.30
Policy2	2010	12.75	12.52	11.59
Entrant distribution				
Policy1	2010	3.41	3.26	2.62
Policy2	2010	15.40	14.00	8.39
Innovation cost				
Policy1	2010	3.49	3.34	2.71
Policy2	2010	17.14	15.97	11.29

Notes: The multiplier computed in the table is: $\frac{W^i - W^0}{S^i/P}$, which is the ratio of welfare increment to real subsidies. The subsidy amount of each policy is to target green subsidy to R&D ratio in the data, which is 0.056%.

Firstly, we find that no matter in 2006 or 2010, targeting on ICT firms only will generate much higher efficiency of green innovation subsidy policies. This is because now dirty opportunity cost is low and ICT firms have lower green innovation cost than non-ICT firms, incentive them to engage in high productive green technology is relatively easy.

Secondly, the dominance pattern is robust against change in externality parameter Ξ. Furthermore, the efficiency of both of the policies drops as externality increases, implying a diminishing return when implementing such innovation subsidy policies with environmental externality.

Lastly, both lower ICT-green innovation cost and higher ICT entrants' green productivity complement ICT targeted green subsidy, as the multiplier increases compared to the benchmark

year. But it substitutes the universal subsidy, as the gap between subsidizing two types of firms is widened when ICT firms have even higher advantage in green innovation. The effect of energy price is ambiguous: it weakens both of policies when externality is low but benefits them when externality is high. This is because a higher energy price deteriorates aggregate productivity that mitigates the positive effect of innovation subsidies. But it attracts more firms to adopt green technology which help decrease externality further.

4.4 Model Extensions

In Appendix B.4, we consider several model extensions to evaluate the robustness of policy implications.

5 Conclusion

The paper examines the slowdown in green innovation since the 2010s. Using U.S. patent data, we develop a decomposition framework and show that green innovation is disproportionately conducted by firms applying for ICT-related green patents and that the aggregate slowdown in green innovation is largely attributable to these ICT firms. This motivates a general equilibrium model in which heterogeneous firms face different green innovation costs and make endogenous decisions about technology class choice, innovation, entry, and exit. Estimating the model, we find that stagnation of ICT green innovation cost decline is a key element determines the stagnation of green transition for ICT firms as well as for all firms. Though other factors such as the decline in energy prices and stagnation of ICT entrants productivity distribution also weakened incentives for green innovation and transition, with especially pronounced effects on ICT firms. Because these firms bear distinct green innovation costs and respond differently to changes in energy prices, their behavior plays a central role in reshaping the green transition. We use the model to compare a green innovation subsidy that is applied uniformly across all firms with one targeted solely at ICT firms, and find that the effectiveness of ICT targeted green subsidy policies dominates the universal one.

Our focus on ICT firms is particularly relevant in light of rapid advances in artificial intelligence and the associated surge in energy demand from digital infrastructure. As AI and other ICT-intensive technologies become increasingly central to production and innovation, these firms emerge as key drivers of green technological change and major contributors to increasing energy use. The analysis implies that policymakers designing green innovation subsidies must account not only for heterogeneity in firms' green innovation costs and their exposure to energy prices, but also for the growing importance and energy intensity of ICT and AI-related activities.

References

D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The Environment and Directed Technical Change. *The American economic review*, 102(1):131–166, 2012.

D. Acemoglu, U. Akcigit, and W. Hanley, D.and Kerr. Transition to Clean Technology. *Journal of Political Economy*, 124(1):52–104, 2016a.

D. Acemoglu, P. Aghion, Lint Barrage, and D. Hémous. Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution. In *NBER Working Paper Series*, 2023.

Daron Acemoglu, Ufuk Akcigit, and William R. Kerr. Innovation network. *Proceedings of the National Academy of Sciences*, 113(41):11483–11488, 2016b.

P. Aghion, B. Blundell, Griffith R., P. Howitt, and S. Prantl. The effects of entry on incumbent innovation and productivity. *The review of economics and statistics,*, 91(1):20–32, 2009.

P. Aghion, A. Bergeaud, M. De Ridder, and J. Van Reenen. Lost in transition: Financial barriers to green growth. *working paper,*, 2024.

Philippe Aghion, Barrage Lint, Donald Eric, Hémous David, and Liu Ernest. Transition to Green Technology along the Supply Chain. *NBER working paper,*, 33934, 2025.

P. Andres, E. Dugoua, and M. Dumas. Directed technological change and general purpose technologies: can AI accelerate clean energy innovation? *Online,*, 2022.

A. Atkeson and T. Burstein, A. Innovation, Firm Dynamics, and International Trade. *The Journal*

of political economy,, 118(3):433–484, 2010.

Andrew Atkeson and Kehoe Patrick, J. Modeling the Transition to a New Economy: Lessons from Two Technological Revolutions. *The American economic review*, 97(1):64–88, 2007.

Joint Research Centre. The twin green & digital transition: How sustainable digital technologies could enable a carbon-neutral EU by 2050. *Joint Research Centre Report,*, 2022.

R. De Haas and A. Popov. Finance and Green Growth. *The Economic Journal,*, 133(650):637–668, 2023.

J. O. Dörr. Mapping Technologies to Business Models: An Application to Clean Technologies and Entrepreneurship. *Online,*, pages 22–057, 2022.

R. Fouquet. Make low-carbon energy an integral part of the knowledge economy. *Nature,*, 551 (7682):S141–S141, 2017.

S. Fried. Climate Policy and Innovation: A Quantitative Macroeconomic Analysis. *American Economic Journal. Macroeconomics*, 10(1):90–118, 2018.

Christoph Hambel, Kraft Holger, and Ploeg Frederick, van der. ASSET DIVERSIFICATION VERSUS CLIMATE ACTION. *International economic review*, 2024.

P. HSU, K. LI, and C. TSOU. The Pollution Premium. *The Journal of Finance,*, 78(3):1343–1392, 2023.

A. Lanteri. The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle. *The American economic review,*, 108(9):2383–2419, 2018.

Smith Matthew, Yagan Danny, Zidar Owen, M., and Zwick Eric. Rise of Pass-Throughs Understates Labor's Share of Income. *NBER working paper,*, No.29400, 2022.

Marc J. Melitz and Sašo Polanec. Dynamic olley-pakes productivity decomposition with entry and exit. *The RAND Journal of Economics*, 46(2):362–375, 2015.

L. Napolitano, E. Evangelou, E. Pugliese, P. Zeppini, and G. Room. Technology networks: the autocatalytic origins of innovation. *Royal Society Open Science,*, 5(6):172445–172445, 2020.

David Popp, Pless Jacquelyn, Haščič Ivan, and Johnstone Nick. Innovation and Entrepreneurship in the Energy Sector. *NBER working paper,*, No.27145, 2022.

H. Schuldt and K. Lessmann. Financing the low-carbon transition: the impact of financial frictions

on clean investment. *Macroeconomic Dynamics,*, 27(7):1932–1971, 2023.

Srinivasan Suraj. The dual transition The path to a digital and sustainable economy. *Quaterly review, Capgemini Research Institute,*, 2024.

Online Appendix for "Unpacking the Green Innovation Slowdown" by Yeju Chen and Xiaomei Sui

A Empirical Appendix

A.1 List of Green-Innovation-Related Technologies and Corresponding IPC Codes

We divide the industries according to patent codes to the second sub-level of category. We focus on patents that are relevant to dirty and green technology dichotomy. The categorization is as follows:

- Transportation: IPC: $B60$ – $B68$;
- Electricity: IPC: H;
- Mechanical Engineering except weapons and blasting: IPC: F except $F41, F42, F99$;
- Petroleum , Gas or Coke: $C10$;
- Chemical or Physical processes: $B01J$.

The list of green patents are drawn from green patent inventory from WIPO (World Intellectual Property Organization) official website: IPC green inventory.

The list of ICT relevant patent is categorized according to patent codes:

- Computing, calculating or counting: G06;
- Controlling or regulating: G05;
- Information and communication technology: G16;
- Information storage: G11;
- Electric elements: H01.

A.2 Trends of ICT-green Patents

Figure A.1 provides additional trends of the ICT-green patents.

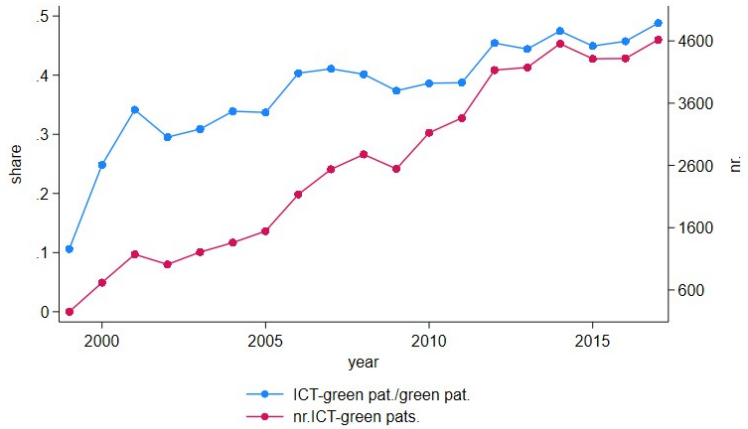


Figure A.1: ICT green innovation trends

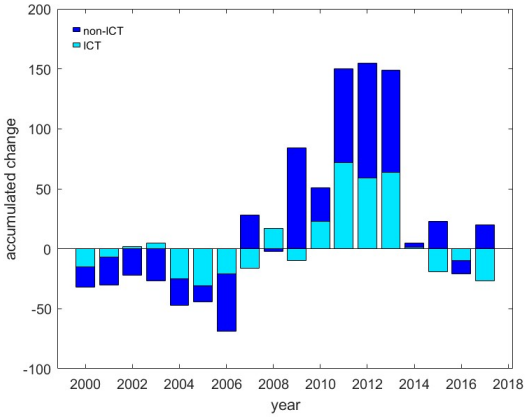
Note: The ICT green patent is defined as patents fallen both into the IPC green inventory and the ICT patent category.

A.3 Decomposition of Green Patents in Different Technology Fields

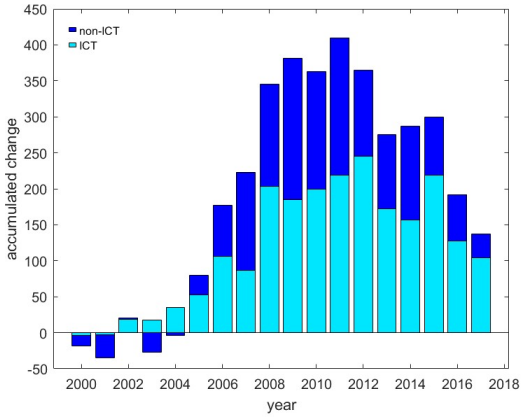
In this section, we further decompose the green patents according to several subcategories based on the feature of technology. We first divide the energy production sector into: wind and solar, and: bio-fuel, hydro and thermal. The first two types of energy production is usually featured with intermittent supplying and requires coordination for supply and demand side with ICT and AI technique. The other categories including energy storage, transportation and waste management. Figures A.2 and A.3 display the results. They suggest that in most technology fields, an increasing share of green innovation is conducted by ICT firms.

A.4 Robustness Check

In Figure A.4, we show our empirical findings are robust to alternative measures of ICT-green firms.



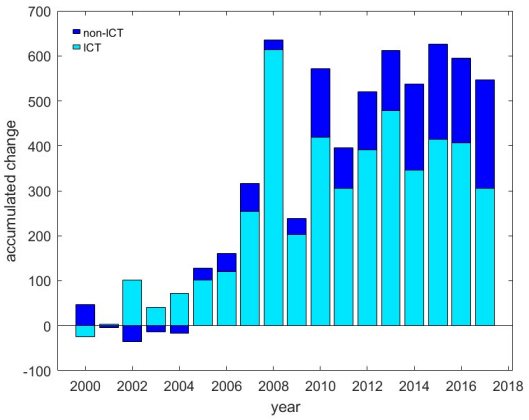
(a) bio-fuel, hydro and thermal



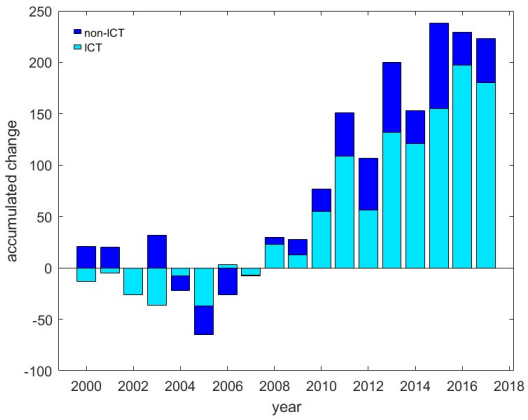
(b) wind and solar

Figure A.2: Accumulated green patents change in energy production

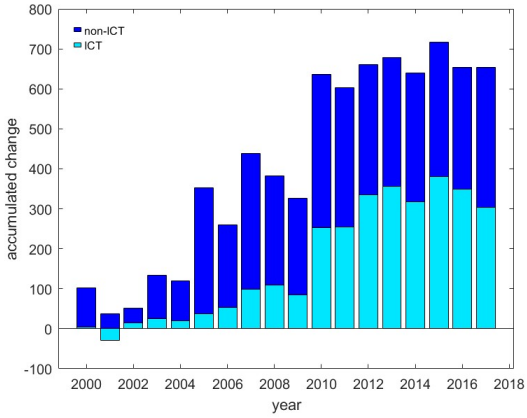
Note: We combine the firms doing only and any ICT-related green patents as ICT in the appendix figures. The category is identified according to the description of IPC green inventory.



(a) transportation



(b) waste management



(c) energy storage

Figure A.3: Accumulated green patents change in other fields

Note: We combine the firms doing only and any ICT-related green patents as ICT in the appendix figures. The category is identified according to the description of IPC green inventory.

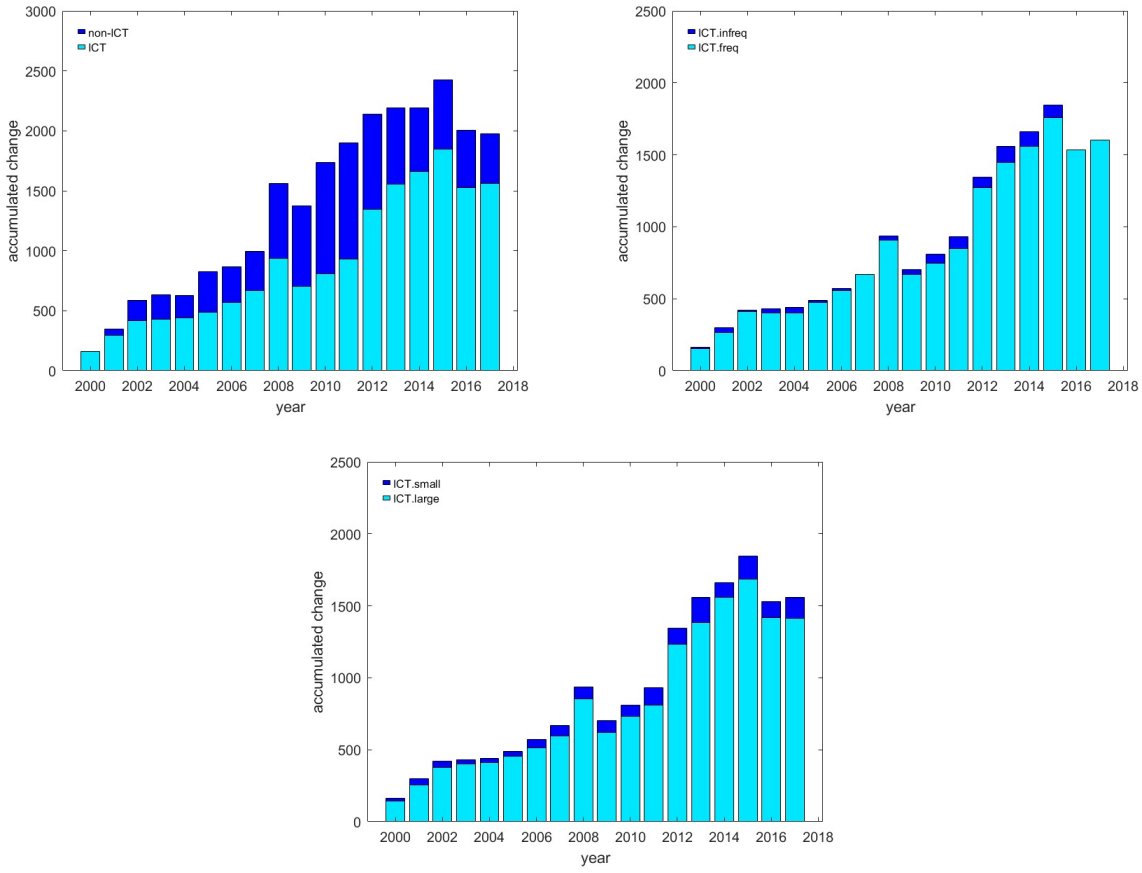


Figure A.4: Alternative measure of ICT-green firm

Note: In this robustness check, we define an ICT-green firm as that had a share of ICT-green patent over 10% of all green patents in at least on year.

B Model Appendix

B.1 Decision Rules of Intermediate Firms

From the FOCs of optimal production decisions, we know the proportions of inputs by dirty firms:

$$\frac{O(a)}{l(a)} = \frac{(1-\eta)(1-\alpha)}{\alpha}\frac{w}{P_o(1+\tau)} \tag{B.1}$$

and:

$$\frac{Y^D(a)}{l(a)} = \frac{(1-\alpha)\eta}{\alpha}\frac{w}{P} \tag{B.2}$$

Analogously, the firm that uses the green technology have input share:

$$\frac{Y(a)}{l(a)} = \frac{1-\alpha}{\alpha}\frac{w}{P}. \tag{B.3}$$

The profit maximization problem of dirty firms yield the expression of prices and inputs, the expression of final goods demand is:

$$Y_d^{D*}(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}\exp(a_d)\alpha^{\alpha(\sigma-1)}(1-\alpha)^{1+(1-\alpha)(\sigma-1)}\eta^{1+\eta(1-\alpha)(\sigma-1)}(1-\eta)^{(1-\eta)(1-\alpha)(\sigma-1)}}{w^{\alpha(\sigma-1)}(P_o(1+\tau))^{(1-\eta)(1-\alpha)(\sigma-1)}P^{1+(\sigma-1)\eta(1-\alpha)}} \tag{B.4}$$

The expression of labor demand by dirty firms is:

$$l_d^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}\exp(a_d)\alpha^{1+\alpha(\sigma-1)}(1-\alpha)^{(1-\alpha)(\sigma-1)}\eta^{\eta(1-\alpha)(\sigma-1)}(1-\eta)^{(1-\eta)(1-\alpha)(\sigma-1)}}{w^{1+\alpha(\sigma-1)}(P_o(1+\tau))^{(1-\eta)(1-\alpha)(\sigma-1)}P^{(\sigma-1)\eta(1-\alpha)}} \tag{B.5}$$

The expression of fossil fuel demand by dirty firms is:

$$o^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}\exp(a_d)\alpha^{\alpha(\sigma-1)}(1-\alpha)^{1+(1-\alpha)(\sigma-1)}\eta^{\eta(1-\alpha)(\sigma-1)}(1-\eta)^{1+(1-\eta)(1-\alpha)(\sigma-1)}}{w^{\alpha(\sigma-1)}(P_o(1+\tau))^{1+(1-\eta)(1-\alpha)(\sigma-1)}P^{(\sigma-1)\eta(1-\alpha)}} \tag{B.6}$$

The expression of total output selected by dirty firms is:

$$y_d^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}(\exp(a_d))^{\frac{\sigma}{\sigma-1}}\alpha^{\alpha\sigma}(1-\alpha)^{(1-\alpha)\sigma}(1-\eta)^{(1-\eta)(1-\alpha)\sigma}\eta^{\eta(1-\alpha)\sigma}}{w^{\alpha\sigma}(P_o+\tau)^{(1-\eta)(1-\alpha)\sigma}P^{\sigma\eta(1-\alpha)}} \tag{B.7}$$

The expression of output price by dirty firms is:

$$p_d^*(a) = \frac{\frac{\sigma}{\sigma-1}(\exp(a_d))^{-\frac{1}{\sigma-1}}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}(1-\eta)^{(\eta-1)(1-\alpha)}\eta^{-\eta(1-\alpha)}}{w^{-\alpha}(P_o(1+\tau))^{-(1-\eta)(1-\alpha)}P^{-\eta(1-\alpha)}} \tag{B.8}$$

Similarly, we solve the profit maximization problem of green firms. The expression of final goods demand is:

$$Y_g^{D*}(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}\exp(a_g)\alpha^{\alpha(\sigma-1)}(1-\alpha)^{1+(1-\alpha)(\sigma-1)}}{w^{\alpha(\sigma-1)}P^{1+(\sigma-1)(1-\alpha)}} \tag{B.9}$$

The expression of labor demand by green firms is:

$$l_g^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}\exp(a_g)\alpha^{1+\alpha(\sigma-1)}(1-\alpha)^{(1-\alpha)(\sigma-1)}}{w^{1+\alpha(\sigma-1)}P^{(\sigma-1)(1-\alpha)}} \tag{B.10}$$

The expression of total output selected by green firms is:

$$y_g^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}(\exp(a_g))^{\frac{\sigma}{\sigma-1}}\alpha^{\alpha\sigma}(1-\alpha)^{(1-\alpha)\sigma}}{w^{\alpha\sigma}P^{\sigma(1-\alpha)}} \tag{B.11}$$

The expression of output price by green firms is:

$$p_g^*(a) = \frac{\frac{\sigma}{\sigma-1}(\exp(a_g))^{-\frac{1}{\sigma-1}}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}}{w^{-\alpha}P^{-(1-\alpha)}} \tag{B.12}$$

B.2 Deriving Aggregate Variables

Final goods price and aggregate productivity According to definition in the model part, the aggregate final price is:

$$\begin{aligned} P &= [\int_{\omega\in\Omega_j} p(\omega)^{1-\sigma} d\omega]^{\frac{1}{1-\sigma}} \\ &= [\int_{a\in A_d} p_d(a)^{1-\sigma} d\mu(a) + \int_{a\in A_g} p_g(a)^{1-\sigma} d\mu(a)]^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{(\sigma-1)w^{-\alpha}} [(P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma} C_d A_d + P^{(1-\alpha)(1-\sigma)} C_g A_g]^{\frac{1}{1-\sigma}} \end{aligned} \tag{B.13}$$

where we denote for convenience that:

$$C_d(\tau) = (\frac{1+\tau}{(1-\eta)(1-\alpha)})^{(1-\sigma)(1-\alpha)(1-\eta)} \cdot (\frac{w}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{(1-\alpha)\eta})^{(1-\sigma)(1-\alpha)\eta} \tag{B.14}$$

and:

$$C_g = (\frac{w}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{1-\alpha})^{(1-\sigma)(1-\alpha)} \tag{B.15}$$

The aggregate productivity is defined as:

$$A = [(P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma} C_d A_d + P^{(1-\alpha)(1-\sigma)} C_g A_g]^{\frac{1}{\sigma-1}} \tag{B.16}$$

The productivity index $\tilde{A}$ without considering entrants' mass is $A = M_e^{\frac{1}{\sigma-1}} \tilde{A}$.

Market size By definition, the common part of aggregate variable profit of dirty and green technology, which captures a market size effect, is given bt:

$$\Pi =: (\frac{\sigma}{\sigma-1})^{1-\sigma} \frac{1}{\sigma} P^{\sigma} Y \tag{B.17}$$

Production labor demand The aggregate dirty profit is:

$$\begin{aligned}\tilde{\Pi}_d = \int P^\sigma C(\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}\exp(a_d)\cdot(\frac{p_o(1+\tau)}{(1-\eta)(1-\alpha)})^{(1-\sigma)(1-\alpha)(1-\eta)} \\ \cdot(\frac{w}{\alpha})^{(1-\sigma)\alpha}\cdot(\frac{P}{(1-\alpha)\eta})^{(1-\sigma)(1-\alpha)\eta}d\mu(a) \\ = C_d P^\sigma C A_d(\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^{(1-\sigma)(1-\alpha)\eta}p_o^{(1-\sigma)(1-\alpha)(1-\eta)}\end{aligned} \tag{B.18}$$

Note that $wL_p^d = \tilde{\Pi}_d(\sigma-1)\alpha$. And the aggregate green profit is:

$$\tilde{\Pi}_g = C_g P^\sigma C A_g\cdot(\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^{(1-\sigma)(1-\alpha)} \tag{B.19}$$

Note that $wL_p^g = \tilde{\Pi}_g(\sigma-1)\alpha$. We normalize the wage rate to be unity.

Research labor and production labor The ratio of the production labor to research labor in the downstream is:

$$\chi = \frac{L_p}{L_r} = \frac{\Pi_d(\sigma-1)[(\alpha+\frac{(1-\alpha)(1-\eta)}{a_o P_o})C_d A_d(P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}+\alpha C_g A_g P^{(1-\alpha)(1-\sigma)}]}{\int\Phi_d(a_d)e_d^2(a,m)d\mu(a)+\int\Phi_g(a_g)e_g^2(a,m)d\mu(a)+c\cdot M_c+n_f\cdot(M_d+M_g)+n_e(M_e+M_e^g)} \tag{B.20}$$

The labor market clearing condition is:

$$L_p + L_r = L \tag{B.21}$$

which implies $L_r = \frac{L}{1+\chi}, L_p = \frac{\chi L}{1+\chi}$.

Entrant mass By labor market clearing condition:

$$Y = \frac{L-L_r}{P^\sigma(\frac{\sigma}{\sigma-1})^{-\sigma}w^{\alpha(1-\sigma)-1}\cdot\alpha[C_d A_d(P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}+C_g A_g P^{(1-\alpha)(1-\sigma)}]} \tag{B.22}$$

Plug the expression of Y in market size expression, we have:

$$M_e = \frac{L - L_r}{\alpha\Pi(\sigma-1)[C_d\tilde{A}_d(P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma} + C_g\tilde{A}_g P^{(1-\alpha)(1-\sigma)}]} \tag{B.23}$$

By definition of $\tilde{A}$, it can be expressed as:

$$M_e = \frac{L_p}{\alpha\Pi(\sigma-1)\tilde{A}^{\sigma-1}} \tag{B.24}$$

Final goods input and consumption The input amount is:

$$\begin{aligned} Y^D &= \int_{a\in A_d} Y_d^D(a)d\mu(a) + \int_{a\in A_g} Y_g^D(a)d\mu(a) \\ &= \frac{(1-\alpha)\Pi}{P(\sigma-1)}[\frac{\eta C_d A_d}{P^{(\sigma-1)(1-\alpha)\eta}p_o^{(\sigma-1)(1-\alpha)(1-\eta)}} + \frac{C_g A_g}{P^{(\sigma-1)(1-\alpha)}}] \end{aligned} \tag{B.25}$$

and the consumption is:

$$C = Y - Y^D \tag{B.26}$$

The aggregate dirty output E is:

$$\begin{aligned} E &= C_d P^\sigma Y P^{\eta(1-\alpha)(1-\sigma)} P_o^{(1-\eta)(1-\alpha)(1-\sigma)} (A_d^n + A_d^u))^{\frac{\sigma}{\sigma-1}} (\frac{\sigma}{\sigma-1})^{-\sigma} \\ &= (\sigma-1)\Pi_d (A_d^n + A_d^u))^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{B.27}$$

B.3 Analytical Results

Exogenous sorting to green transition We need to separate the general equilibrium effect through the aggregate price adjustment, from the effect from firm decision changes. To understand how the resource price works on firm decisions, we consider a case where firms using dirty technology are randomly sorted to green technology with an exogenous probability v. Hence, firms only make decisions on entry and innovation within technology class and without considering transition.

Proposition B.1 *When no labor is used ($\alpha \to 0$) and under exogenous sorting to green technology,*

when the resource input price changes, firm variable profits and all decisions have no change.

Proof Assume that the resource input price changes from P_o to $\tilde{P}_o$. Because sorting is exogenous, dirty firms have the following value function in original technology class selection stage:

$$V_t^S(a,m) = vU_t^S(a,d;g) + (1-v)U_t^S(a,d;d) \tag{B.28}$$

Before it, the firms decide whether to continue the operation:

$$V_t^{0,S}(a,m) = max\ [0, V_t^S(a,m)] \tag{B.29}$$

Firstly, we argue that $\tilde{P} = P\frac{\tilde{P}_o}{P_o}$ is a solution to the fixed point of P. By the fixed point equation:

$$\tilde{P}^{1-\sigma} = [\frac{\sigma}{(\sigma-1)w^{-\alpha}}]^{1-\sigma}[\tilde{P_o}^{(1-\eta)(1-\sigma)}\tilde{P}^{\eta(1-\sigma)}C_dA_d + \tilde{P}^{1-\sigma}C_gA_g] \tag{B.30}$$

Obviously, it is the solution of the original problem given all other aggregate variables unchanged. Let $\tilde{\Pi} = \Pi(\frac{\tilde{p_o}}{p_o})^{(1-\eta)(1-\sigma)}$. This ensures that the variable profits of dirty technology is unchanged. By the solution of $\tilde{P}$, the variable profit of green technology also remains unchanged. By uniqueness of the solution, $\tilde{\Pi}$ is the new solution to the new dynamic problem. Hence, firms decisions are unchanged in the process. Q.E.D □

The input of labor leads to some general equilibrium effect on aggregate price, which makes the across technology adjustment not completely shut down even in the case of exogenous sorting. However, once we ignore this effect, we find that the endogenized process of technology class selection breaks down the invariance clearly. For whichever value of $\tilde{\Pi}$ chosen in the new optimization problem, dirty firms are separated into two groups according to their own selection: transition or non transition. This would generate a kink point of the value function of dirty firms, which makes the variable profit of either group change. Once this happens, the relative variable profit across technology also changes. Hence, variable profit dispersions make firms sorted more or less to green technology which changes the aggregates through directed innovation decisions further.

B.4 Model Extensions

B.4.1 Endogenous Choice of Being ICT Entrants

In this section, we endogenize the decision of being an ICT firm or a normal firm upon entry. Entrants still draw from a distribution of productivity $G_e(\bar{a_d}, a_g)$, but can choose the investment of being an ICT firm. Let e denote the investment rate. The entrants problem becomes:

$$U_t^E(a_d, a_g) = \max_e \; e \cdot V_t^u(a) + (1-e)V_t^n(a) - \Phi^E(a, e), \tag{B.31}$$

where the investment cost function takes the same format as the innovation costs:

$$\Phi^E(a, e) = h^E \exp(a_g) \exp(\xi^E e), \tag{B.32}$$

Note that since they have the same dirty productivity upon entry, we assume that the investment cost is proportional to the exponential of green productivity upon entry. We have the following entrant investment rates versus green productivity as in Figure B.1.

As green productivity increases, the marginal benefit of being an ICT firm increases faster than investment cost, causing more firms to choose ICT. When the cost of green innovation of ICT decreases, it provides firms with a higher incentive to be an ICT firm as well.

The distribution of ICT-green entrants is therefore $G_e(\cdot, a_g) \cdot e(a_g)$ and the free entry condition becomes:

$$n_e = \frac{1}{1+r_t} \int [e(a_g)V^u(a,d)dG_e(a) + (1-e(a_g))V^n(a,d)dG_e(a)] \tag{B.33}$$

This extension allows us to have richer general equilibrium effects and implications. The reduction in ICT-green innovation cost not only incentivizes ICT entrants to innovate more intensively in this direction but also attracts a larger fraction of firms into ICT. The higher fraction of ICT firms will then amplify the effects of innovation cost reduction.

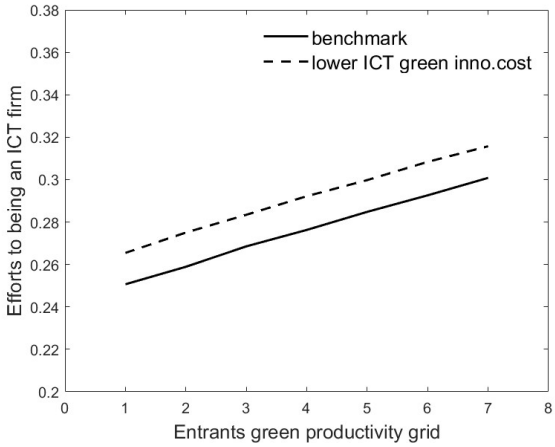
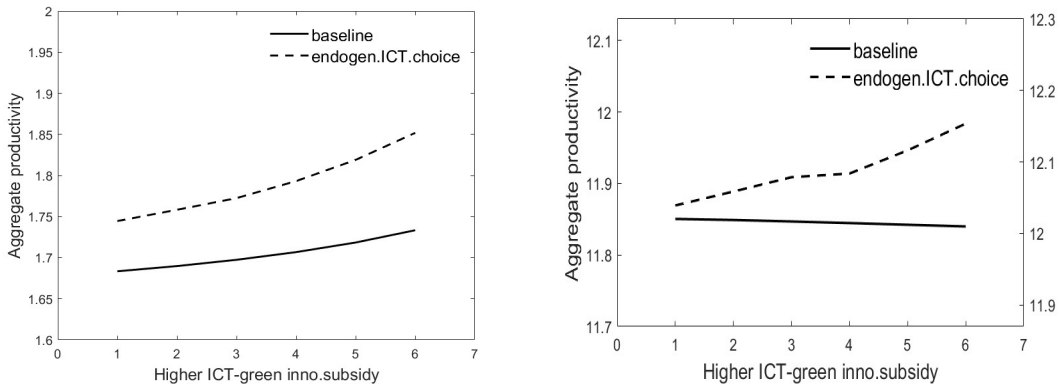


Figure B.1: Entrants effort of being an ICT firm given dirty productivity



(a) high dirty innovation cost: A (b) low dirty innovation cost: A

Figure B.2: ICT-green innovation cost reduction: endogenize ICT choice

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

We implement the same exercise as in the mechanism part, where we increase the ICT-green innovation subsidy gradually. The results are shown in Figure B.2. When the cost of dirty innovation is relatively high, the opportunity cost of the green transition is low such that productivity is increasing over the ICT-green innovation subsidy. In the endogenous choice of being an ICT firm, both the level and slope become higher, implying the selection effect of firms upon entry. When the cost of dirty innovation is relatively low, in the exogenous case, the productivity is decreasing as the opportunity cost of the green transition is high. In the endogenous case, since the ICT firm dominates non-ICT firm in green innovation (the same in dirty innovation), a higher fraction of

entrants being an ICT firm promotes the productivity, which is faster than the decline in the baseline case.

B.4.2 Entrants Choose to Become Green Firms

C Quantitative Appendix

C.1 Computation Algorithm

1. Guess the initial value of the aggregate price $P^{(i)}$.

2. With the guessed price:

   (a) Guess the initial value of market size $\Pi^{(i,j)}$.

   (b) Given the guessed values, compute the variable profit for dirty and green technology. Through the dynamic programming of downstream firms, solve out the corresponding innovation decision rules e_d, e_g, the technology class choice rule m', the value functions V_d, V_g and the distribution conditional on technology class $\tilde{\mu}_d(a), \tilde{\mu}_g(a)$.

   (c) Compute the equilibrium Π^* through free entry condition. Repeat step $2.(a) \sim 2.(c)$ until $\Pi^{(i,j)} = \Pi^*$.

   (d) With the solved Π^*, compute equilibrium decision rules, value functions and the distributions. Compute the equilibrium price P, aggregate productivity A_g, A_d. Compute out the implied aggregate price P^* through the definition.

3. Repeat step $1 \sim 2$ until $P^* = P^{(i)}$.

4. Compute the equilibrium decision rules, distributions and aggregate variables based on the converged value of P^* and Π^*.