Unpacking Green Innovation Slowdown in the Digital Age: Firm Heterogeneity and Policy Trade-offs*

Yeju Chen†
University of Hong Kong

Xiaomei Sui‡
University of Hong Kong

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March 1, 2026

Abstract

The slowdown in green innovation since the 2010s has been documented. However, it remains unclear which types of firms are reshaping the green transition. Using U.S. patent data, we provide a decomposition framework and demonstrate that green innovation is primarily conducted by firms applying for ICT-related green patents, and that the slowdown in green innovation is mainly driven by these firms. Motivated by these observations, we build a general equilibrium model in which heterogeneous firms face different green innovation costs and make decisions regarding technology class choice, innovation, entry, and exit. The estimated model indicates that declining energy prices since the 2010s have weakened incentives for green innovation, particularly among ICT-related firms. We then compare a universal green innovation subsidy with a subsidy targeted only to ICT firms, calibrating both to match the observed ratio of green subsidies to R&D (about 0.056%). Under 2010 energy prices, when the dirty to green innovation cost is relatively high, targeting ICT firms yields a consumption-to-subsidy multiplier that is 0.8 higher than under a universal subsidy, whereas it reduces consumption when the dirty to green innovation cost is relatively low. When welfare is evaluated including pollution externalities, lower energy prices make subsidies targeted to ICT green firms increasingly more efficient than universal green subsidies, reflecting ICT firms' comparative advantage in green innovation and the lower opportunity cost of shifting away from dirty production.

Keywords: Climate change; green innovation; firm heterogeneity; general purpose technology.

JEL classification: D24, E22, O16, O30, O40.

*All errors are our own.

†Contact: yejuchen123@connect.hku.hk, Faculty of Business and Economics, The University of Hong Kong, Pokfulam Road, Hong Kong.

‡Contact: xsui@hku.hk, Faculty of Business and Economics, The University of Hong Kong, Pokfulam Road, Hong Kong.

1 Introduction

Transitioning away from carbon-intensive technologies requires sustained growth in green innovation to offset the entrenched dominance of dirty technologies in energy production and use. While climate policies have traditionally focused on emissions reduction through carbon pricing and regulation, an emerging consensus recognizes that redirecting technological change toward clean alternatives is equally critical (Acemoglu et al., 2012, 2016a). A distinctive and increasingly important feature of many emerging clean technologies is their deep integration with information and communication technologies (ICT). Consider the stark contrast between traditional environmental innovations and their ICT-integrated counterparts: conventional solar panels versus AI-optimized photovoltaic systems that dynamically adjust to weather patterns and electricity demand; standard wind turbines versus digital "twins" that simulate performance under thousands of scenarios to maximize energy capture; basic battery storage versus smart grid systems that use machine learning to balance intermittent renewable supply with consumption patterns across millions of households. These examples suggest that ICT-related capabilities may be central to the next wave of green innovation and have led policymakers to emphasize a "twin transition" between digitalization and decarbonization (e.g., Centre, 2022; Suraj, 2024).

Recent evidence documents a puzzling slowdown in green innovation since the early 2010s (Acemoglu et al., 2023; Popp et al., 2022), despite growing climate concerns and policy support. While the shale gas revolution and declining energy prices have been identified as contributing factors, we lack understanding of which firms drive these trends and how policy should respond to firm heterogeneity. Do ICT-intensive firms—those whose green innovations leverage digital capabilities—behave differently from traditional green innovators in their response to energy prices and policy interventions? Should public subsidies target ICT-green firms specifically or spread broadly across all green innovators?

The distinction of ICT-green firms and green firms is increasingly reflected—though without rigorous economic justification—in emerging policy design. The European Union's Green Deal explicitly champions a "twin transition," with Horizon Europe dedicating targeted calls like "Dig-

ital solutions for energy systems integration", which require applicants to integrate AI, IoT, or digital twins into clean energy projects. Similarly, the European Innovation Council Accelerator evaluates "Green Deal" proposals using scoring criteria that award premium points for "use of advanced digital technologies," creating de facto preferential access for ICT-green firms. In the United States, ARPA-E's DIFFERENTIATE program exclusively funds AI-driven grid optimization projects, while the DOE's GRIDIS initiative prioritizes proposals combining machine learning with grid-scale storage—effectively channeling resources toward firms with dual ICT-green capabilities. China's 14th Five-Year Plan for Green Development similarly directs subsidies toward "smart manufacturing" and "AI-enabled resource efficiency" projects. Critically, these policies lack an economic framework to determine when such targeting maximizes welfare versus when broad-based support is superior. Without understanding how energy prices, innovation costs, and externalities interact with firm heterogeneity, governments risk inefficient allocation of scarce innovation resources—a gap this paper directly addresses.

In this paper, we demonstrate that this firm heterogeneity is central to understanding the slowdown in green innovation and designing effective climate policy. Using U.S. patent data from 1999-2017, we develop a novel decomposition framework that reveals ICT-connected firms account for approximately 80% of all green patents despite representing only 20% of green-innovating firms. Moreover, the post-2010 stagnation in green innovation is overwhelmingly driven by these ICT-green firms. This finding challenges conventional wisdom that treats green innovation as homogeneous across firm types.

Motivated by these empirical patterns, we build a general equilibrium model with heterogeneous firms that endogenously choose technology classes and innovation investments. The model incorporates a crucial feature: ICT firms face lower costs of green innovation, reflecting their comparative advantage in integrating digital capabilities with environmental applications. Using this framework, we quantify how declining energy prices since the 2010s differentially affected innovation incentives across firm types and evaluate the efficiency of alternative subsidy policies.

Our analysis reveals important policy trade-offs that depend critically on energy prices and

the strength of pollution externalities. When energy prices are high (as in our 2010 baseline calibration), targeting green innovation subsidies exclusively to ICT firms yields a consumption-to-subsidy multiplier 36% higher than universal subsidies. However, when energy prices fall, universal subsidies can become more effective by expanding the pool of potential green adopters. When accounting for environmental externalities, ICT-targeted subsidies increasingly dominate as energy prices decline, reflecting these firms' superior ability to accelerate the transition away from carbon-intensive production.

Related Literature. This paper contributes to three strands of literature. First, we add new empirical facts and extend the models of directed technical change (e.g., Acemoglu et al., 2012, 2016a, 2023; Fried, 2018) by incorporating firm heterogeneity along the ICT dimension, connecting micro-level innovation decisions to aggregate results through rich general equilibrium channels. Unlike previous work that operates at the sector level or assumes symmetric firms, our framework captures how differential innovation capabilities shape the green transition.[1]

Second, we bridge the literature on ICT as a general purpose technology with environmental innovation (e.g., Acemoglu et al., 2016b; Andres et al., 2022; Fouquet, 2017; Napolitano et al., 2020). This literature emphasizes that GPTs like ICT and AI diffuse across sectors and reshape innovation networks, with clean technologies often acting as downstream recipients of ICT spillovers. Andres et al. (2022) argue that dirty and green technologies differ in their absorptive capacities and intrinsic connections to ICT and AI, suggesting that firms with stronger ICT capabilities may endogenously invest more in green technologies. We provide empirical evidence and a quantitative framework to assess the economic significance of ICT firms. Our model formalizes how ICT capabilities create heterogeneous innovation costs across firms—a mechanism often overlooked in climate policy evaluation.

Third, our work speaks to the design of climate policies under firm heterogeneity. A large literature compares carbon taxes, broad green subsidies, and regulatory tools (e.g., Acemoglu et al.,

[1]Recent contributions such as Aghion et al. (2025) study green transition along supply chains, and Acemoglu et al. (2016a) and Aghion et al. (2024) allow for heterogeneous innovation responses but rely on quality ladder structures that do not differentiate ICT intensive and non-ICT intensive firms.

2016a; De Haas and Popov, 2023; Hambel et al., 2024; HSU et al., 2023; Schuldt and Lessmann, 2023), often without distinguishing among firms by innovation capability. More recent work (e.g., Aghion et al., 2024; Dörr, 2022) highlights the importance of firm level financial constraints and advocates for targeting subsidies to small, financially constrained green innovators. Our analysis complements this by asking a different policy question: how should a given green innovation subsidy budget be distributed between ICT intensive firms and the broader firm population, conditional on the energy price and the dynamics of the energy environment? In doing so, we provide a quantitative framework that links the ongoing policy debate on the "twin" digital and green transitions (e.g., Centre, 2022; Suraj, 2024) to the design of firm-specific green innovation subsidies.

Roadmap. The rest of this paper proceeds as follows. Section 2 presents our empirical decomposition of green innovation trends. Section 3 develops the theoretical model incorporating ICT-based firm heterogeneity. Section 4 estimates the model and evaluates policy counterfactuals. Section 5 concludes.

2 Data and Empirical Facts

In this section, we conduct decomposition to provide a clearer insight into the green innovation trend. Specifically, we decompose the green patents by the type of firms, being an ICT or non-ICT firm. Then we decompose according to firm innovation behavior, of being a frequent or infrequent innovator. Lastly, we also investigate the trends by firm size.

2.1 Data and Variables

2.1.1 Data description

Our patent data is collected from the USPTO and covers the period from 1999 to 2017. The year is defined as the patent's application year. Each patent is assigned a unique IPC (International Patent Classification) code, which indicates the technology class it belongs to.

Our firm data is drawn from ORBIS Historical, which contains information such as the BvD ID code, number of employees, and other balance sheet details. We use ORBIS Intellectual Property to merge the patent data with the firm data. Each patent in ORBIS Intellectual Property is associated with a BvD ID code, which represents the applicant's identity in the U.S. patent application system.

We focus on a subset of industries that conduct green innovation: Transportation, Electricity, and Mechanical Engineering except weapons and blasting, which are commonly adopted industries by such as Andres et al. (2022).

2.1.2 Patent categorization

In this section, we define the green patents and ICT patents. To identify the green patents, we adopt the IPC green inventory to see whether a patent category falls in that inventory. The inventory is introduced by World Intellectual Property Organization to identify environmental friendly technologies since 2010. The inventory involves of the following main classes of technology: Alternative energy production; Transportation; Energy conservation; Waste management; Agriculture and forestry.

2.2 Decomposing Green Patent Change

First, we divide green firms into three categories year by year: $h \in H = \{I, N, M\}$, where I represents a firm owns ICT green patent only and owns at least one ICT green patent, N represents a firm owns non-ICT green patent only and owns at least one non-ICT green patent, and M represents a firm that owns at least one ICT green patent and at least one non-ICT green patent.

Second, we classify firms in each of the above category into three sub-categories according to their innovation behavior: the frequent innovator, the entrant innovator, and the exiting innovator, denoted as $\{S, E, X\}$. The dynamic changing margin of the last two types of innovators is called change by infrequent innovators.

Inside the frequent innovators, we can further decompose the firms into small and large sized

firms $G = \{s, l\}$. Firstly, we can preliminarily split the number of green patents in each year to three types of firms. The results is shown in Figure 2.3.

To investigate the dynamics among different groups of innovators, we apply Melitz and Polanec (2015), the dynamic Olley-Pakes decomposition. A way of representing the flow of green patent is:

$$\Phi = \sum_{g \in G} \sum_{h \in H} \sum_{\phi} \phi^{S,g,h} + \sum_{g \in G} \sum_{h \in H} \sum_{\phi} \phi^{X,g,h}, \tag{2.1}$$

where ϕ is a green patent owned by a firm. It is decomposed to green patents by incumbent innovators and innovators that about to exit in the next period, for the three types of firms respectively. Another way of split is:

$$\Phi = \sum_{g \in G} \sum_{h \in H} \sum_{\phi} \phi^{S,g,h} + \sum_{g \in G} \sum_{h \in H} \sum_{\phi} \phi^{E,g,h}, \tag{2.2}$$

where it is decomposed to green patents by incumbent innovators and innovators that just enter the innovation in this period. The decomposition is to take the difference of the second split in the next period with the first split in the current period:

$$\Delta\Phi = \sum_{g \in G} \sum_{h \in H} (\Phi'^{S,g,h} - \Phi^{S,g,h}) + \sum_{g \in G} \sum_{h \in H} (\Phi'^{E,g,h} - \Phi^{X,g,h}), \tag{2.3}$$

where the first term is the change of green patents by an invariant incumbent group, the second term is the change by firms that enter and exit the green innovation. The decomposition by innovator type is shown in Figure 2.4. The decomposition further by firm size for frequent innovators are shown in Figure 2.5.

2.3 Facts

Based on the decomposition, we provide four key facts about green innovation patterns with important implications for the distinction between ICT-green and non-ICT green firms.

Fact 1. The green innovation slowdown is concentrated among ICT-green firms. While prior literature has documented an aggregate slowdown in green patenting since 2010 (Popp et al. (2022)), our decomposition reveals this trend is driven almost entirely by firms filing ICT-green patents. As shown in Figure 2.1, the share of green patents declines since 2010; the number of green patents also becomes stagnated since 2010.

Fact 2. ICT-green firms dominate high-impact green innovation. The share of ICT-green patents as well as the number of it is increasing over time. Though, the fraction of green firms that own an ICT green patent is only around 20 percent.

Figure 2.2 shows that the ICT related green patents is increasing quickly up to 2010 and the become stagnated. Figure 2.3 demonstrates the change of green patents is mainly driven by firms that owns ICT green patent (the combination of ICT green only and mixed firms). But from perspective of time series, any type of the firms share similar trend, where they become stagnated around the year of 2010.

Fact 3. The change of green patents is consistent across different types of firms. Though firms that owns an ICT green patent mainly account for green innovation, and this feature is enhanced over time. By the dynamic decomposition, we can compare the contribution of frequent innovators with the net contribution of infrequent innovators. We find that the aggregate pattern is mainly driven by frequent innovators. Both of frequent and infrequent innovators across three types of firms share similar time trend, despite the fact that firms owning ICT green patent have different reliance on frequent innovators with those owning non ICT green patent only.

Fact 4. The stagnation trend of green innovation is consistent among frequent and infrequent innovators, as well as large versus small sized firms, across different types of firms. We further divide the frequent innovators of the three types of firms by firm size. From Figure 2.5, we observe that since 2010, the large sized firm becomes stagnated in green patenting. The small sized firms were undertaking a relatively much higher responsibility at the early 2000s, but drops

dramatically after then. This indicates that despite the sharing trend of stagnation, the large sized firms become dominator in the green innovation.

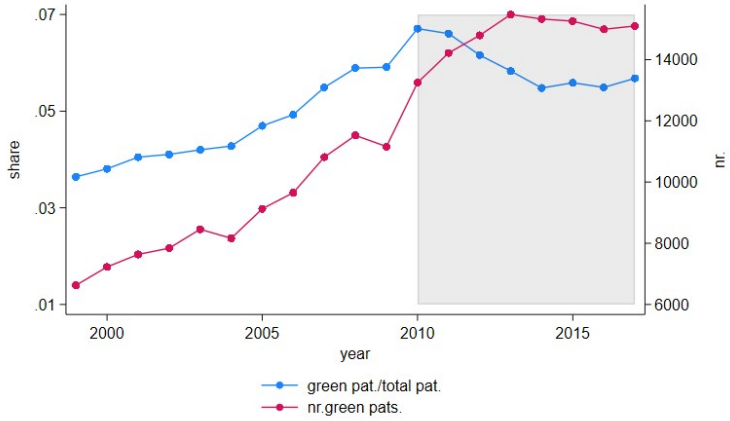


Figure 2.1: Green innovation trends

Note: Green patents are defined according to IPC green inventory. Patents' year are at the time applied, it includes patents granted and owned by a company with a BvD ID code.

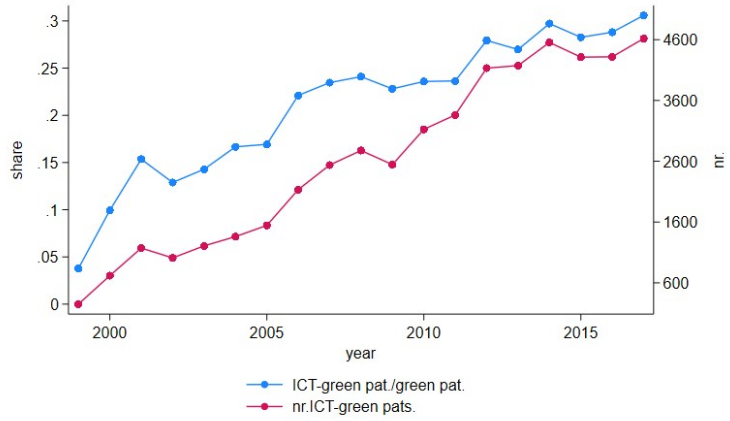


Figure 2.2: ICT green innovation trends

Note: The ICT green patent is defined as patents fallen both into the IPC green inventory and the ICT patent category.

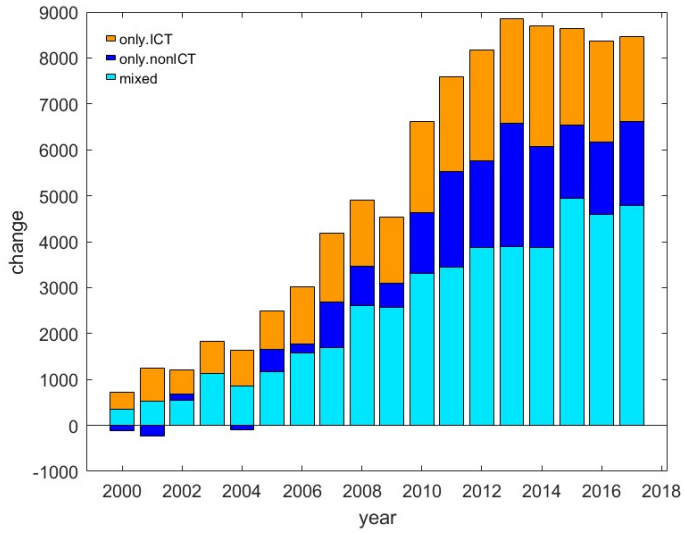


Figure 2.3: Accumulated green pats. change by firm type

Note: only.ICT represents firms have green-ICT patents but have no other green patents; only.non-ICT represents firms have green non-ICT patents only but have no other green patents; mixed represents firms have both ICT-green and non-ICT green patents.

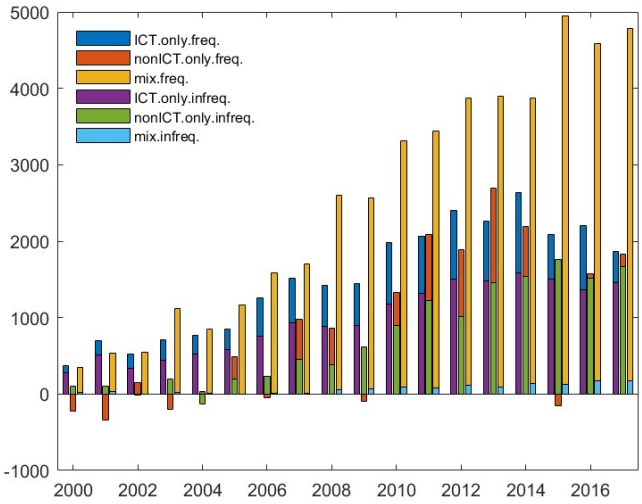


Figure 2.4: Accumulated green pats. change by innovator type

Note: Frequent innovator represents firms applying green patents for consecutive years; infrequent innovator represents firms have missing application records last year.

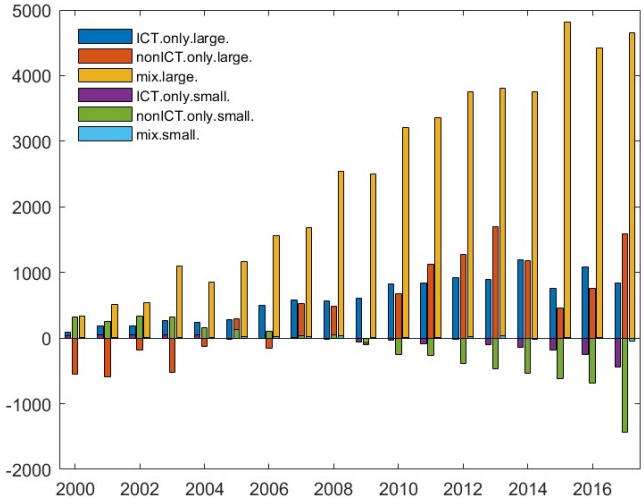


Figure 2.5: Accumulated green pats. change by firm size

Note: Large sized firms have number of employees above median; small sized firms have number of employees below median.

2.4 Robustness

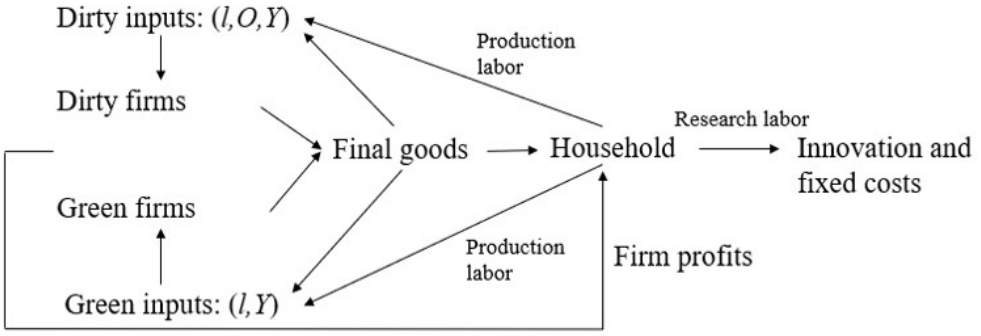
Technology-specific green innovation decomposition. Motivated by Popp et al. (2022), we classify green patents into several groups by technology class (or energy source) to see if the baseline facts hold in all technology classes (or energy sources).

We divide the patents into the following categories: bio-fuel, hydro and thermal; wind and solar; transportation; waste management; energy storage. We find that wind and solar, transportation, and energy storage account for most of the growth in green innovation and exhibit the same pattern as in the full sample. The results are shown in Appendix B.1.

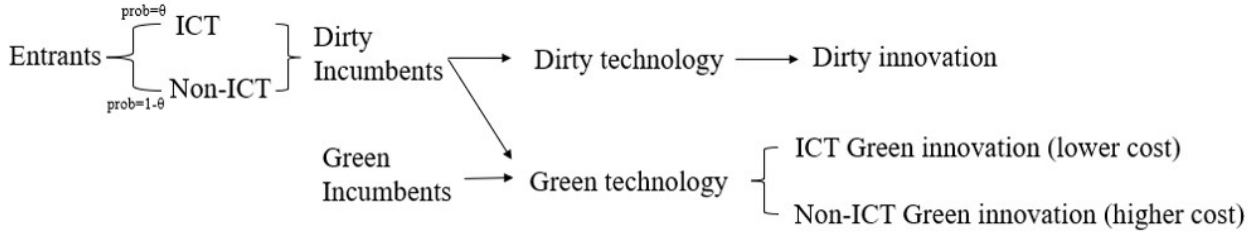
3 The Model

Time is discrete and indexed by $t = 1, 2, ...$ As shown in Figure 3.1, there exists two types of technologies in the economy, i.e., dirty and green, denoted as $m \in \{d, g\}$. At the beginning of each

period, potential entrants decide whether to enter and, if they do, draw an ICT or non-ICT type. Incumbent firms simultaneously decide whether to remain in the market or exit. Next, incumbent firms choose their technology class and then produce using the chosen technology. They then decide on the optimal level of directed innovation for that technology class. Innovation costs, entry costs, and technology-class transition costs are all borne by labor. The details are specified in section 3.1.



(a) production



(b) innovation

Figure 3.1: Model Setup

3.1 Model Setup

Household and environmental externality The representative household maximizes her lifetime utility:

$$\max \sum_{t=0}^{\infty} \beta^t u(C_t, E_t) \tag{3.1}$$

$$s.t. \quad W_t + P_t C_t \leq W_{t-1}(1 + r_t) + w_t L, \tag{3.2}$$

where $u(C_t, E_t) = C_t - \Xi E_t$, C_t is the aggregate consumption of final goods, E_t is the aggregate dirty output (the functional form of it is shown as (3.39)), and Ξ denotes the relative strength of the externality from dirty output in affecting household utility. β denotes the discount factor. P_t is the price of final goods. W_t is the wealth of households in period t (value of firms in period t), and r_t is the rate of return on owning firms. The household supplies labor $L \equiv 1$ inelastically in each period. The utility maximization problem leads to the standard Euler equation:

$$\beta = \frac{1}{1 + r_t}. \tag{3.3}$$

Final goods technology Perfectly competitive firms produce final goods Y_t with the technology

$$Y_t = [\int_{m=d} y_t(a,m)^{\frac{\sigma-1}{\sigma}} \, d\mu_t(a,m) + \int_{m=g} y_t(a,m)^{\frac{\sigma-1}{\sigma}} \, d\mu_t(a,m)]^{\frac{\sigma}{\sigma-1}}, \tag{3.4}$$

where $y_t(a,m)$ denotes the output of an intermediate firm with productivity a that chooses technology class $m \in \{d, g\}$, d denotes dirty technology, g denotes green technology, and $\sigma > 1$ is the elasticity of substitution across intermediate goods.

Final goods producers buy $y_t(a,m)$ amounts of intermediate output at price $p_t(a,m)$, and sell the final goods at price P_t. The profit maximization problem gives the demand function

$$y_t(a,m) = (\frac{p_t(a,m)}{P_t})^{-\sigma} Y_t, \tag{3.5}$$

where the final goods price $P_t = [\int_{m \in \{d,g\}} p_t(a,m)^{1-\sigma} d\mu_t(a,m)]^{\frac{1}{1-\sigma}}$.

Intermediate firm operation At the beginning of each period, the incumbent intermediate firms decide whether to remain in the market by choosing $\kappa(a,m) \in \{0,1\}$ such that

$$V_t^{0,S}(a,m) = \max_{\kappa(a,m)\in\{0,1\}} [0, V_t^S(a,m)], \tag{3.6}$$

where

$$V_t^S(a,m) = \max\ [U_t^S(a,m;m), U_t^S(a,m;-m)], \tag{3.7}$$

$U_t^S(a,m;m)$ denotes the value of staying in the market and keeping the same technology class as in the previous period, while $U_t^S(a,m;-m)$ denotes the value of staying in the market and switching to the other technology class relative to the previous period. The indicator $\kappa(a,m) = 0$ represents exiting the market, and $\kappa(a,m) = 1$ represents continuing to operate.

Intermediate firm production The intermediate firms are in a monopolistically competitive market. They can adopt dirty and green technologies that they have technology specific productivity, denoted as $a = \{a_d, a_g\}$.

The dirty firms use labor, oil and final goods to produce a dirty variety. The price of the oil after tax τ is

$$P^* = P_o(1+\tau), \tag{3.8}$$

which can also be interpreted as the price of dirty energy.

The green firms use only labor and final goods to produce a green variety. The production function of the firm using dirty technology is

$$y_d(a) = exp(a_d)^{\frac{1}{\sigma-1}} [Y^D(a)^\eta \cdot (O(a))^{1-\eta}]^{1-\alpha} \cdot (l(a))^\alpha, \tag{3.9}$$

where σ is the elasticity of substitution across varieties, α is the labor share, η represents the weight of final goods input relative to oil,
$exp(a_d)^{\frac{1}{\sigma-1}}$ is the scaled firm productivity, Y^D is the input of final goods, O is the energy input and

l is the labor input. η governs the share of final goods inputs of dirty technology, this captures an economy wide productivity feedback that higher aggregate productivity implies lower input cost for both dirty and green technology. And we assume that the economy is more biased to green technology which is coherent with the assumption at Aghion et al. (2025).

Similarly, the production function of the firm using clean technology is:

$$y_g(a) = exp(a_g)^{\frac{1}{\sigma-1}} \cdot Y^D(a)^{1-\alpha} \cdot (l(a))^{\alpha} \tag{3.10}$$

Because that market clears for all intermediate goods, we have $y_d(a) = y(a,d), y_g(a) = y(a,g)$. The firm that uses the dirty technology solves:

$$\max_{p(a),l(a)O(a),Y^D(a)} \quad p(a)y_d(a) - wl(a) - PY^D(a) - P_oO(a) \tag{3.11}$$

$$s.t. \quad y_d(a) = (\frac{p(a)}{P})^{-\sigma}Y, \tag{3.12}$$

The firm that uses green technology solves:

$$\max_{p(a),l(a),Y^D(a)} \quad p(a)y_g(a) - wl(a) - PY^D(a) \tag{3.13}$$

$$s.t. \quad y_g(a) = (\frac{p(a)}{P})^{-\sigma}Y \tag{3.14}$$

By profit maximizing, we can solve out the optimal inputs, price, and total output selected by a typical dirty firm, which are presented in the C.1. The optimal profit is:

$$\pi_d(a) = C_d\Pi \exp(a_d)(w^{\alpha}P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma} \tag{3.15}$$

where $C_d = (\frac{1+\tau}{(1-\eta)(1-\alpha)})^{(1-\sigma)(1-\alpha)(1-\eta)} \cdot (\frac{1}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{(1-\alpha)\eta})^{(1-\sigma)(1-\alpha)\eta}$. The optimal profit for a

green firm is:

$$\pi_g(a) = C_g \Pi \exp(a_g)(w^\alpha P^{1-\alpha})^{1-\sigma} \tag{3.16}$$

where $C_g = (\frac{1}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{1-\alpha})^{(1-\sigma)(1-\alpha)}$. And $\Pi =: (\frac{\sigma}{\sigma-1})^{1-\sigma} \frac{1}{\sigma} P^\sigma Y$ is the common part of variable profits of using dirty and green technology. We denote the dirty and green variable profits as $\Pi_d = C_d \Pi (w^\alpha P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}$ and $\Pi_g = C_g \Pi (w^\alpha P^{1-\alpha})^{1-\sigma}$.

Intermediate firm technology class choice and innovation The incumbent firms with status quo technology class $m \in \{d, c\}$ solve the following problem:

$$V_t^S(a, m) = \max_{x(a,m) \in \{m, -m\}} U_t^S(a, m; x(a, m)), \tag{3.17}$$

Where $x = -m$ represents transition, $x = m$ represents not transition.

Conditional on staying in the market and choosing the technology class, incumbent intermediate firms make innovation efforts to maximize their present value:

$$\begin{aligned} U_t^S(a, m; x(a, m)) = \max_e \; & \pi_x(a_x) - n_f - \mathbf{1}\{x \neq m\} \cdot c \cdot V_t^S(a, m; -m) - \Phi_x^S(e) \\ & + \frac{1}{1 + r_t} \gamma [e \cdot (V_{t+1}(a_x exp(J), a_{-x}, x) + (1 - e)(V_{t+1}(\frac{a_x}{\exp(J)}, a_{-x}, x))], \end{aligned} \tag{3.18}$$

where $U_t^S(a, m, m')$ is the firm's interim objective function before innovation but after choosing the technological class, which is the expected value of entering each class of technology. n_f is the fixed cost of operation. We denote $\pi_m(a_m)$ as the maximum per period payoff of the firm given its state productivity. $s \in \{u, n\}$ represents the firm being an ICT or non-ICT firm, which is determined randomly upon entry. If innovation is successful, the firms increase the innovation by step size J, if not, they lag by the same size. The parameterization of innovation costs is to follow Atkeson and Burstein (2010), we assume that ICT and non-ICT firms have differentiated green innovation cost

$$\Phi_g^u(e) = h_g^u \exp(a_g) \exp(\xi_g^u e), \tag{3.19}$$

and

$$\Phi_g^n(e) = h_g^n \exp(a_d) \exp(\xi_g^n e_g), \tag{3.20}$$

and they have homogeneous dirty innovation cost

$$\Phi_d(e) = h_d \exp(a_d) \exp(\xi_d e_d). \tag{3.21}$$

Π_d, Π_g are aggregates (variable profits) specific to each technology class as introduced above. We are assuming that technology that is more profitable face higher innovation costs by introducing the terms.

Entrants Entrants decide whether to enter the market, and we impose the free entry condition. We assume that firms only enter by using the dirty technology:

$$n_e = \frac{1}{1+r_t} \int [\theta V^u(a,d) dG_e(a) + (1-\theta) V^n(a,d) dG_e(a)] \tag{3.22}$$

where θ is the ex ante probability of being an ICT firm for entrants. The free entry condition is that their expected value of enter is zero, indicating that the entry market is competitive. We assume that the entrants draw their productivity from an exogenous distribution $G_e(a) = G_e(\bar{a_d}, a_g)$, where we assume that the dirty productivity upon entry is degenerated and green productivity follows uniform distribution over one part of productivity grid.

Entrants enter as using dirty technology, and they are free to choose switch to green technology by paying a transition cost c. We assume that green technology is an absorbing state that once they enter, they can only choose to exit the market but not return to use the dirty technology. We consider the green capital of firms as irreversible investment as mentioned in Lanteri (2018) that these investment cannot be easily reinvested. Moreover, the entrants are bounded by a free entry condition.

Distribution evolution The distribution after technology class choice is:

$$\tilde{\Psi}(a_m, a_{-m}, m) = x(a_m, a_{-m}, -m)\Psi(a_m, a_{-m}, -m) + \Psi(a_m, a_{-m}, m) \tag{3.23}$$

and after innovation is:

$$\begin{aligned}\Psi'(a_m, a_{-m}, m) =& G_e(a'_m, a'_{-m}) + (1-\delta)e_m(\frac{a_m}{J}, a_{-m})\tilde{\Psi}(\frac{a_m}{J}, a_{-m}, m) \\ &+ (1-\delta)(1-e_m(a_m J, a_{-m}))\tilde{\Psi}(a_m J, a_{-m}, m)\end{aligned} \tag{3.24}$$

Aggregate productivity Define the aggregate productivity of green and dirty technology of ICT and non-ICT firms as

$$\begin{aligned}A_d^n = \int \exp(a_d) d\mu_d^n(a_d),\ A_d^u = \int \exp(a_d) d\mu_d^u(a_d), \\ A_g^n = \int \exp(a_g) d\mu_g(a_g),\ A_g^u = \int \exp(a_g) d\mu_g^u(a_g).\end{aligned} \tag{3.25}$$

The aggregate productivity of each technology class is to sum up that of the ICT and non-ICT firms together such that

$$A_d = A_d^n + A_d^u,\ A_g = A_g^n + A_g^u. \tag{3.26}$$

The aggregate productivity for the economy is

$$A = A_d + A_g. \tag{3.27}$$

For future convenience, we normalize the distribution of firms by entrants' mass, denoted as

$$\tilde{\mu}_d = \frac{\mu_d}{M^e},\ \tilde{\mu}_g = \frac{\mu_g}{M^e}. \tag{3.28}$$

The aggregated productivity computed according to the normalized distribution is denoted as $\tilde{A_d}, \tilde{A_g}, \tilde{A}$.

3.2 The Equilibrium

Equilibrium definition The stationary competitive equilibrium is a collection of forward looking decision rules $\{e_d^S, e_g^S, x^S, \kappa^S\}^{S\in\{u,n\}}$, production decisions $\{p_d, l_d, Y_d^D, O_d; p_g, l_g, Y_g^D\}^{S\in\{u,n\}}$, value functions $\{V_t^S, V_t^{0,S}\}^{S\in\{u,n\}}$, the stationary distributions over productivity $\{\mu^S(a_d, a_g)\}^{S\in\{u,n\}}$ and aggregates $\{\Pi, P, Y, M_e, C, Y^D, L_p, L_r\}$, such that for a given oil price P_o:

1. Households maximize utility such that equation (3.3) is satisfied;
2. Firms maximize per period profit according to problem (3.11) and problem (3.13);
3. Firms maximize value function in each decision stage through the Bellman equations of equation (3.18), (3.17) and (3.6);
4. The distribution over productivity (a_m, a_{-m}), where $m \in \{d, g\}$, is stationary;
5. Labor market clears, i.e., equation (3.35) holds;
6. Output market clears, i.e., equation (3.34) holds;
7. Free entry condition holds, i.e., equation (4.4) holds.

The equilibrium ratio of production labor and research labor in the economy:

$$\chi \equiv \frac{L_p}{L_r} = \frac{\Pi\alpha(\sigma-1)A^{\sigma-1}}{L_r}, \tag{3.29}$$

where

$$\begin{aligned} L_r = &\int \Phi_d(a_d) e_d^2(a,m) d\mu(a_d) + \int \Phi_g(a_g) e_g^2(a,m) d\mu_g(a_g) \\ &+ \int_{m\neq m_{-1}} c \cdot V_g(a,m) d\mu_g(a_g) + n_f \cdot M_I + n_e M_e \end{aligned} \tag{3.30}$$

includes all costs other than production side. n_f is the entry cost, n_e is the fixed cost, and c is the transition cost, where we assume firms entry as dirty firms at first place. M_e, M_I are mass of entrants and incumbents respectively.

Computing equilibrium aggregates The unknowns are $\{\Pi, P, Y, M_e, C, Y^D, L_p, L_r\}$. According to computation in the appendix, the aggregates are pinned down through the following system of equations:

By definition:

$$\Pi \equiv (\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^{\sigma}Y. \tag{3.31}$$

Free entry condition for the downstream firms:

$$n_e = \beta \int V^d(a_d, a_g)dG_e(a). \tag{3.32}$$

The expression of aggregate final price:

$$P = \frac{\sigma P^{\eta(1-\alpha)}}{(\sigma-1)w^{-\alpha}}[(\frac{1}{P_o^{-(1-\eta)(1-\alpha)}})^{1-\sigma}C_dA_d + (\frac{1}{P^{-(1-\eta)(1-\alpha)}})^{1-\sigma}C_gA_g]^{\frac{1}{1-\sigma}}. \tag{3.33}$$

The expression for consumption:

$$C = Y - Y^D. \tag{3.34}$$

And above labor market clearing condition:

$$L_p + L_r = L. \tag{3.35}$$

The ratio of production and research labor:

$$\chi = \frac{L_p}{L_r} = \frac{\Pi\alpha(\sigma-1)A^{\sigma-1}}{L_r}. \tag{3.36}$$

By equalizing the two equations of Y, we get the expression of M_e:

$$M_e = \frac{L_p}{\alpha\Pi(\sigma-1)\tilde{A}^{\sigma-1}}. \tag{3.37}$$

The input amount is:

$$Y^D = \frac{(1-\alpha)\Pi}{P(\sigma-1)}\left[\frac{\eta C_d A_d}{P^{(\sigma-1)(1-\alpha)\eta} p_o^{(\sigma-1)(1-\alpha)(1-\eta)}} + \frac{C_g A_g}{P^{(\sigma-1)(1-\alpha)}}\right]. \tag{3.38}$$

Besides, we can compute the aggregate dirty output which serves the externality term:

$$E = (\sigma-1)\Pi_d (A_d^n + A_d^u))^{\frac{\sigma}{\sigma-1}} \tag{3.39}$$

3.3 Model Mechanism

In this section, we first discuss the firms' decision rules in the stationary equilibrium. We then decompose the equilibrium conditions of aggregate variables to the first order to investigate how general equilibrium effects work. Lastly, we analyze model mechanism by varying innovation costs and energy price and compare steady state changes.

Incumbent innovation incentive and technology class choice Before innovation, firms make decisions on technology class selection. Since we assume that only dirty firms can switch, the cut-off decision rules of dirty firms are shown in Figure 3.2. When dirty firm's green productivity is low, there exists a cut-off of dirty productivity below which dirty firms exit the market. Because now transition is less profitable even than exiting the market, no firms switch to green technology. When dirty firm's green productivity is high enough, there exists a cut-off of dirty productivity below which dirty firms switch to green technology. This is shown as the colored part of the figure. The cut-off is non-decreasing in the green productivity of dirty firms. Compare the left panel with the right panel, the results imply that dirty ICT firms have higher cut-offs which means they are easier to transit given the same green productivity. The technology class selection reflects an extensive margin difference of green transition between ICT and non-ICT firms.

Then we present the innovation decision rules by ICT and non-ICT firms. The technology class selection rules imply that near the cut-off, ICT dirty firms near the cut-off devote less dirty

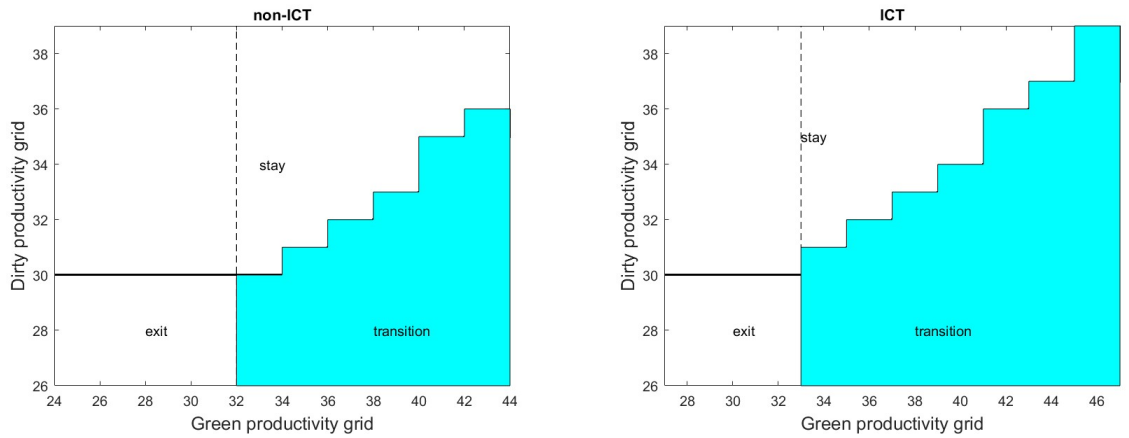


Figure 3.2: Technology class selection and exit of dirty firms

innovation effort, as they have stronger incentive to wait for the failure of dirty innovation and switch, compared to non-ICT dirty firms. This is reflected in the left panel of Figure 3.3. The green innovation decision rules are shown in the right panel of Figure 3.3. When green productivity is low enough, firm exit the market and has no incentive to devote positive innovation. As ICT firms have lower green innovation cost compared to non-ICT firms, (which will be implied from data in the next section) the green innovation intensity is higher.

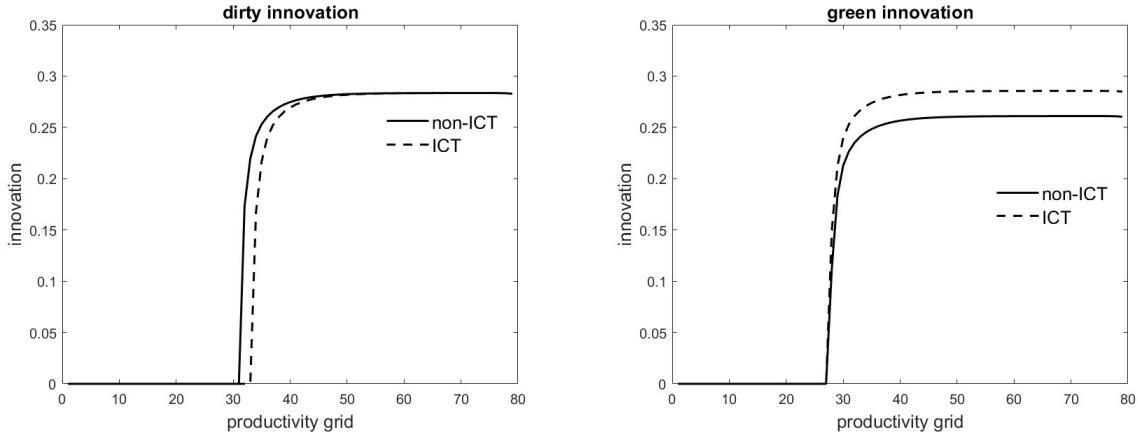


Figure 3.3: Optimal innovation incentive

First order decomposition of incumbent production profits For the discussion of the model mechanism, it is important to understand how aggregate variables are connected and how they contribute to the general equilibrium effects. The variable production profits of intermediate

firms govern firms' entry and exit decisions, technology class choices, and innovation decisions. Therefore, we study how these profits are determined.

Recall that the variable production profit for dirty firms is

$$\Pi_d = \Pi C_d (P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}, \tag{3.40}$$

and the variable production profit of green firms is

$$\Pi_g = \Pi C_g P^{(1-\alpha)(1-\sigma)}. \tag{3.41}$$

In a first step, we need to understand what determines the common part of variable profit, Π. The aggregated production labor implies that $L_p = \alpha \frac{\sigma-1}{\sigma} PY$, combined with the definition of Π, it gives:

$$\Pi = (L - L_r) \frac{1}{\alpha(\sigma-1)} \frac{1}{(P\frac{\sigma-1}{\sigma})^{1-\sigma}}. \tag{3.42}$$

and we decompose common variable profit Π as follows:

$$\Delta \log \Pi = \Delta \log(L - L_r) + (\sigma - 1)\Delta \log P. \tag{3.43}$$

This directly implies that the change of common variable profit Π is determined by the change of the labor reallocation between production and research labor as well as the change of final good price.

We can further decompose the final goods price P:

$$\begin{aligned} \Delta \log P = & \underbrace{\frac{1}{(1-\sigma)(1-\omega_d)} [\Delta \log M_e + \iota_d \Delta \log \tilde{A}_d + (1-\iota_d)\Delta \log \tilde{A}_g]}_{\text{innovation and entry}} \\ & + \underbrace{\frac{(1-\eta)(1-\alpha)}{1-\omega_d} \Delta \log P_o}_{\text{resource price effect}}, \end{aligned} \tag{3.44}$$

where $\iota_d = \frac{C_d A_d (P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}}{P^{1-\sigma}}$ represents ratio of aggregate dirty profits, and $\omega_d = \iota_d \eta(1-\alpha) + (1-\iota_d)(1-\alpha)$ is an weighed average on dirty price share and green price share. The factor price directly contributes to aggregate price or productivity as shown in the last term. On the other hand, it could potentially contribute to sorting of firms to green technology, affect innovation intensity and affect entry decisions, which are reflected in the first three terms.

For convenience, we further decompose the variable profits of using the dirty technology:

$$\begin{aligned}\Delta \log(\Pi_d) =& \underbrace{\Delta \log(L - L_r)}_{\text{labor reallocation}} - \underbrace{\Lambda_d \Delta \log M_e}_{\text{entry}} - \underbrace{\Lambda_d(1-\iota_d)\Delta \log A_g - \iota_d \Lambda_d \Delta \log A_d}_{\text{technology sorting}} \\ &+ \underbrace{(\sigma-1)(1-\eta)(1-\alpha)(\Lambda_d - 1)\Delta \log P_o}_{\text{resource price effect}}\end{aligned} \tag{3.45}$$

where $\Lambda_d = \frac{1-(1-\alpha)\eta}{1-\omega_d}$. Analogously, we have the decomposition for variable profits of using the green technology:

$$\begin{aligned}\Delta \log(\Pi_g) =& \Delta \log(L - L_r) - \Lambda_g \Delta \log M_e - \Lambda_g \iota_d \Delta \log A_d - \Lambda_g (1-\iota_d)\Delta \log A_g \\ &+ (\sigma-1)(1-\eta)(1-\alpha)\Lambda_g \Delta \log P_o\end{aligned} \tag{3.46}$$

where $\Lambda_g = \frac{\alpha}{1-\omega_d}$. The relative variable profit across technologies can be shown as the following:

$$\begin{aligned}\Delta \log(\Pi_d) - \Delta \log(\Pi_g) =& (\Lambda_g - \Lambda_d)[\Delta \log M_e + \iota_d \Delta \log A_d + (1-\iota_d)\Delta \log A_g] \\ &+ (\sigma-1)(1-\eta)(1-\alpha)(\Lambda_d - \Lambda_g - 1)\Delta \log P_o.\end{aligned} \tag{3.47}$$

Hence, the relative variable profit across technologies does not rely on labor reallocation effect to a first order approximation. It relies on innovation and entry effect, as well as the factor price effect. We provide an analytical result in Appendix C.3 on how our model setup establishes the connection between energy price and firm decisions as well as aggregate variables. The entrants' mass contributes negatively to relative variable profit of dirty to green, implying that more competitive entry market favors green technology. Both aggregate green productivity and

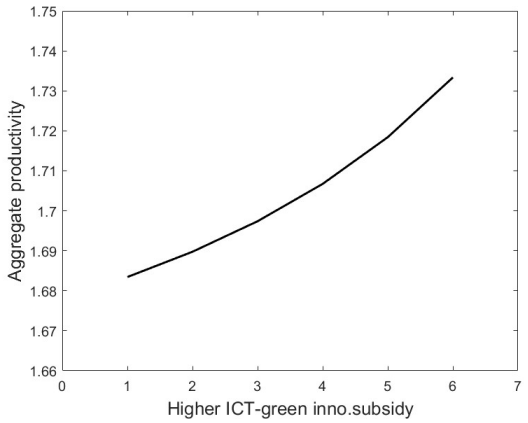
aggregate dirty productivity contribute negatively to relative variable profit of dirty to green. The results is consistent with our assumption that green technology rely more on final goods input. As the aggregate productivity increases, the economy naturally favor the green technology more. Due to the parameter restriction demonstrated below where $\eta > 0.5$, we know that the energy price negatively contributed to relative variable profit of dirty to green that higher input cost for dirty will weaken its profit of production.

3.3.1 Role of Key Parameters

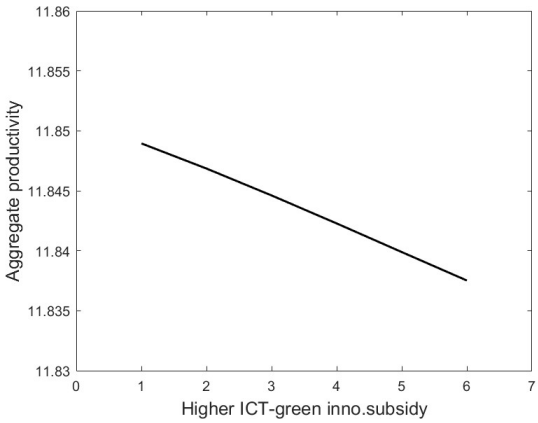
Role of the different innovation cost of ICT and non-ICT firms In this subsection, we study how the change of green innovation costs works through the model mechanisms. We compute steady states that reduces the green innovation cost of ICT and non-ICT firms respectively. We consider several general equilibrium effects including opportunity cost of directed innovation within group, input price effects through aggregate price, and labor reallocation effects. When doing the analysis, we should bear in mind that changing innovation cost works either affect firms' innovation decisions directly, or further works on aggregate variables which affect firms' production decisions and innovation decisions.

Several general equilibrium mechanism could work in the process. The first mechanism is straightforward that lower green innovation cost for ICT firms will sort the dirty firms to green firms, which potentially weakens the positive effect on aggregate productivity. Moreover, this could affect the entrants' mass by changing the expected value upon entry which also contribute the aggregate productivity. Secondly, labor reallocates between production and research that affects market size and output as well. Thirdly, the overall effect on aggregate productivity determines the direction of the change of the aggregate price, which contributes to the relative profit of using dirty or green technology and further affects green innovation of firms.

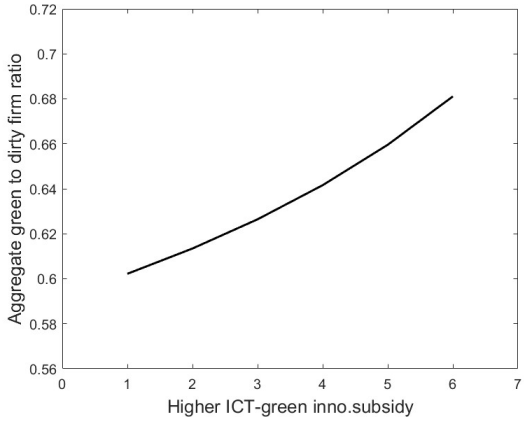
As shown in the bottom Figure 3.4, lowering ICT green innovation costs promotes green transition. The difference between the left and right is that the left has relatively lower opportunity cost of selecting green technology (lower relative green to dirty innovation cost). To see this in



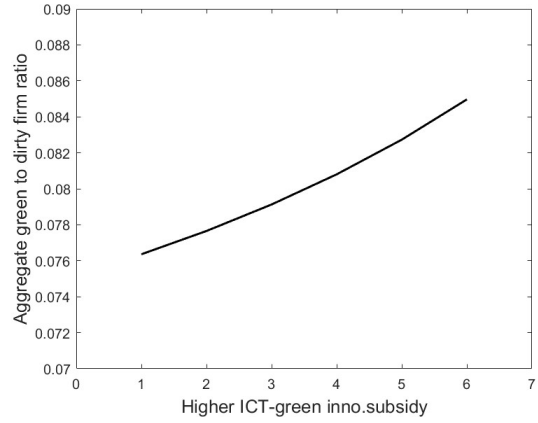
(a) Higher dirty inno.cost: A



(b) Lower dirty inno.cost: A



(c) Higher dirty inno.cost: green firm fraction

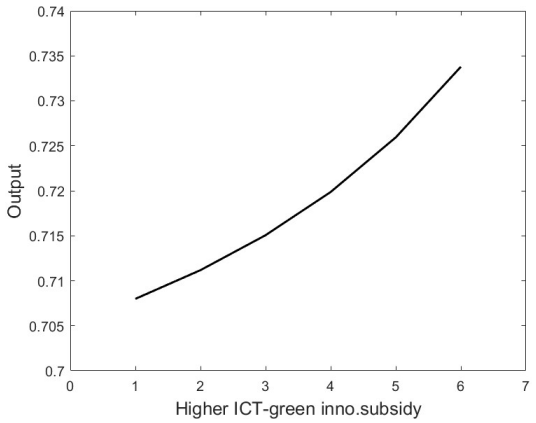


(d) Lower dirty inno.cost: green firm fraction

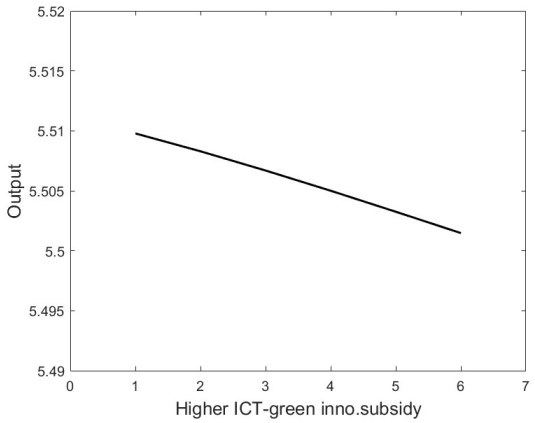
Figure 3.4: General equilibrium effects of innovation cost reduction

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

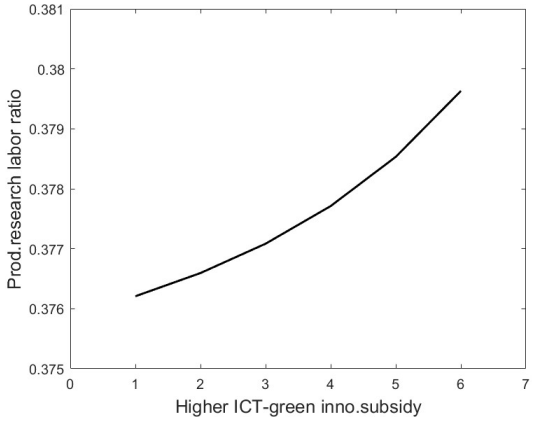
a more clear way, Figure 3.6 demonstrates that firms are doing more green innovation and less dirty innovation in both cases. As a result, in ovation cost). To see this in a more clear way, Figure 3.7, the distribution of ICT firms' green productivity shifts upwards while dirty distribution shifts downwards. However, the outcome on aggregate productivity is different, as in Figure 3.4. In low green opportunity cost case, the aggregate productivity is increasing as ICT-green innovation subsidy is higher, while in high green opportunity cost case it is decreasing as green innovation subsidy is higher. The relative innovation cost of dirty and green technology is a key that determines the relative strength of opportunity cost green innovation and affects the aggregate effect of innovation subsidies.



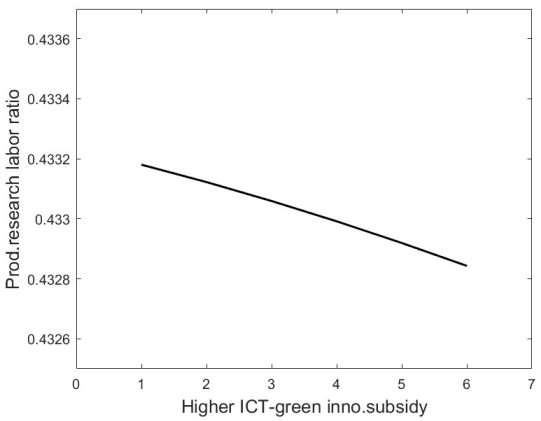
(a) Higher dirty inno.cost: Y



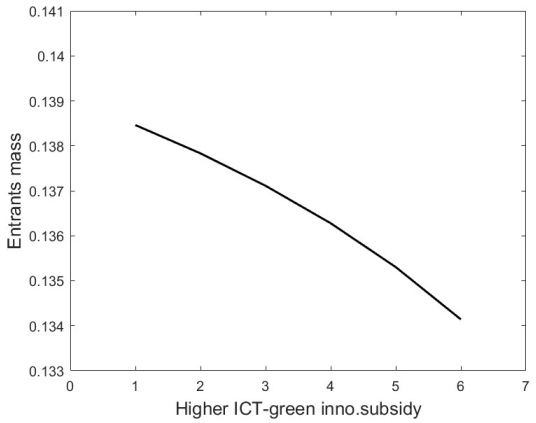
(b) Lower dirty inno.cost: Y



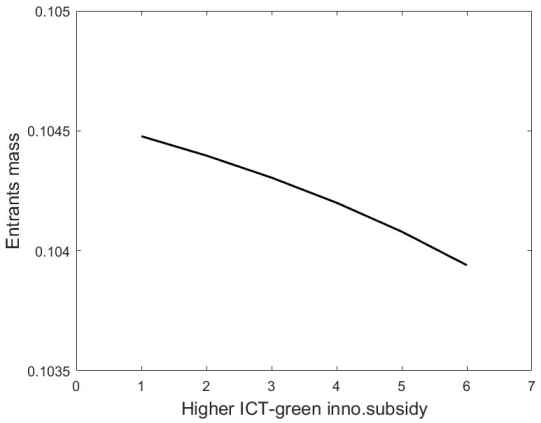
(c) Higher dirty inno.cost: χ



(d) Lower dirty inno.cost: χ



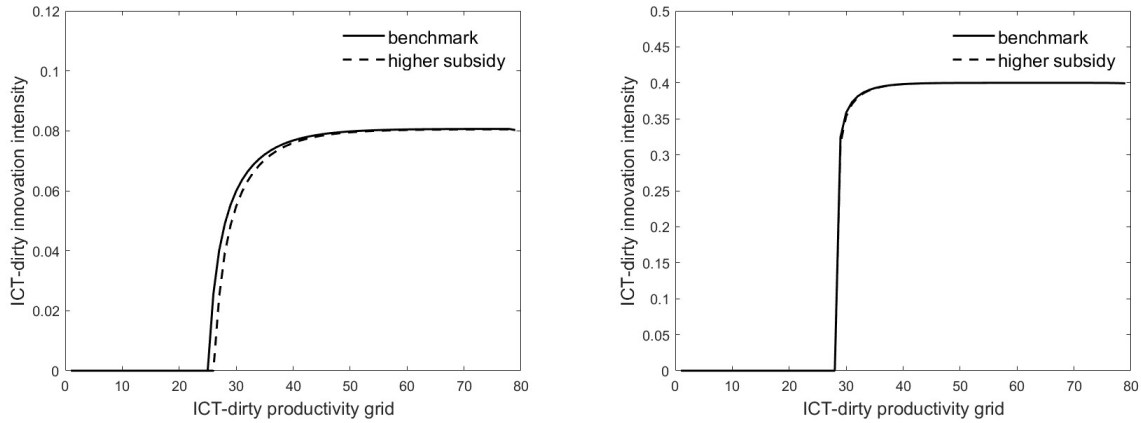
(e) Higher dirty inno.cost: M_e



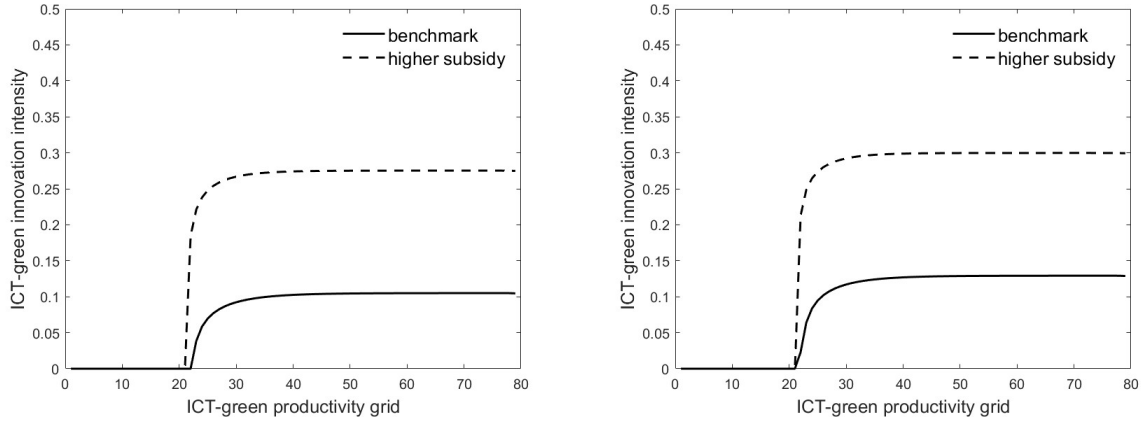
(f) Lower dirty inno.cost: M_e

Figure 3.5: General equilibrium effects of innovation cost reduction

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.



(a) Higher dirty inno.cost: e_d^u (b) Lower dirty inno.cost: e_d^u

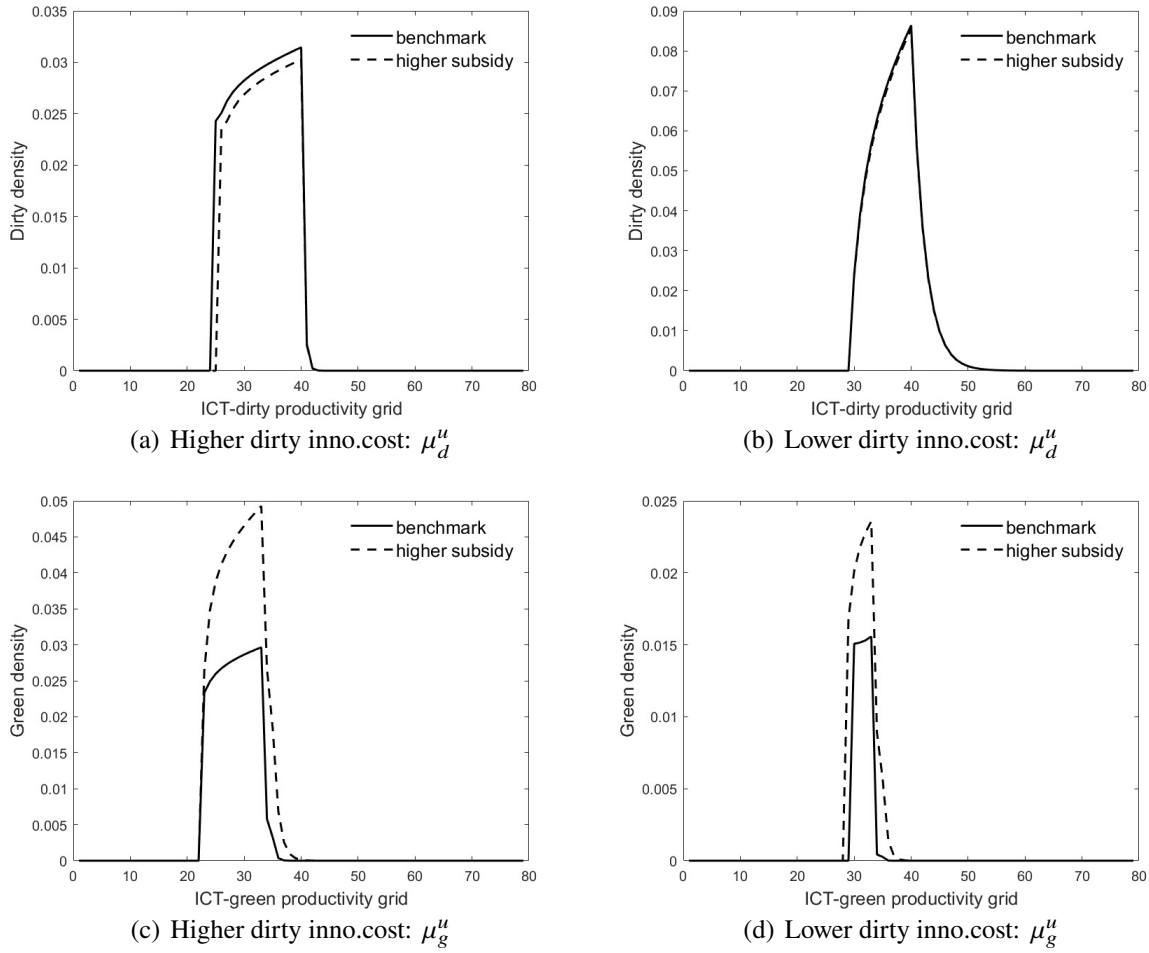


(c) Higher dirty inno.cost: e_g^u (d) Lower dirty inno.cost: e_g^u

Figure 3.6: General equilibrium effects of innovation cost reduction

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

From Figure 3.5, we observe that labor reallocates is in the same direction with productivity, it includes intra technology reallocation between production and research labor and labor reallocation across technologies. Lastly, firm entry decisions are determined by the expected profit of entry that is affected by market size, expected value of innovation and labor allocation, etc. The overall effect implies that mass of entrants drop as green innovation cost drops. From the expression of variable profit of green technology of equation (3.46), these general equilibrium including productivity redistribution across technologies, labor reallocation and entry decisions all affect the green innovation across groups of firms, weakening or strengthening the original effect by innovation cost changes.



(a) Higher dirty inno.cost: μ_d^u (b) Lower dirty inno.cost: μ_d^u

(c) Higher dirty inno.cost: μ_g^u (d) Lower dirty inno.cost: μ_g^u

Figure 3.7: General equilibrium effects of innovation cost reduction

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

Role of the share of final goods as intermediate inputs In this section, we study the interplay between final goods input share of dirty technology (that share of green technology is 1-labor share) and green innovation cost reduction. The reason is that the effect of innovation cost would drive change of aggregate price and productivity, which enhance or weaken the original effect by changing the input price for firms. To begin with, we should notice that the effect is dependent on the relative value of wage and equilibrium aggregate price, that is real wage $\frac{w}{P}$ in the economy. If $\frac{w}{P} < 1$, labor is cheaper compared to final goods input. Given the same level of energy price, higher dirty final goods input share implies using green technology as production have lower opportunity cost. If $\frac{w}{P} > 1$, higher dirty final goods input share implies using green technology as production

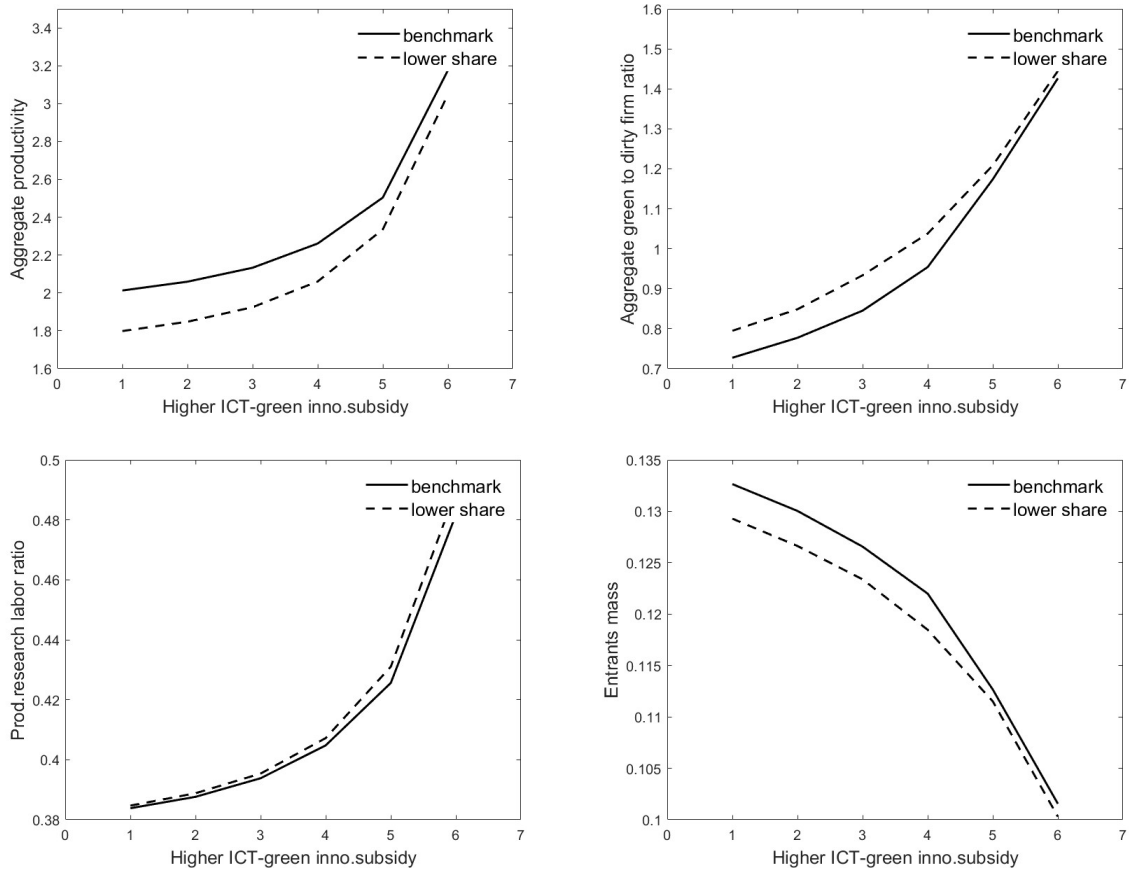


Figure 3.8: General equilibrium effects of final goods input share: higher dirty inno.cost

have higher opportunity cost.

Given current setting, the case fall into $\frac{w}{P} > 1$ that final goods are cheaper than labor. As shown in the upper two panels in Figure 3.8, either lower share or lower innovation cost implies higher green productivity distribution. Lower share also implies higher degree of increment when lowering green innovation cost. This is because as in the middle panels, aggregate productivity is increasing and price is decreasing. Lower dirty final goods input share implies dirty technology benefits less from input price decreasing, leading to a faster green transition. As in the bottom panels, lower share implies higher labor reallocation to production as innovation cost changes. It also implies higher entrants mass due to lower productivity and lower market size in this case.

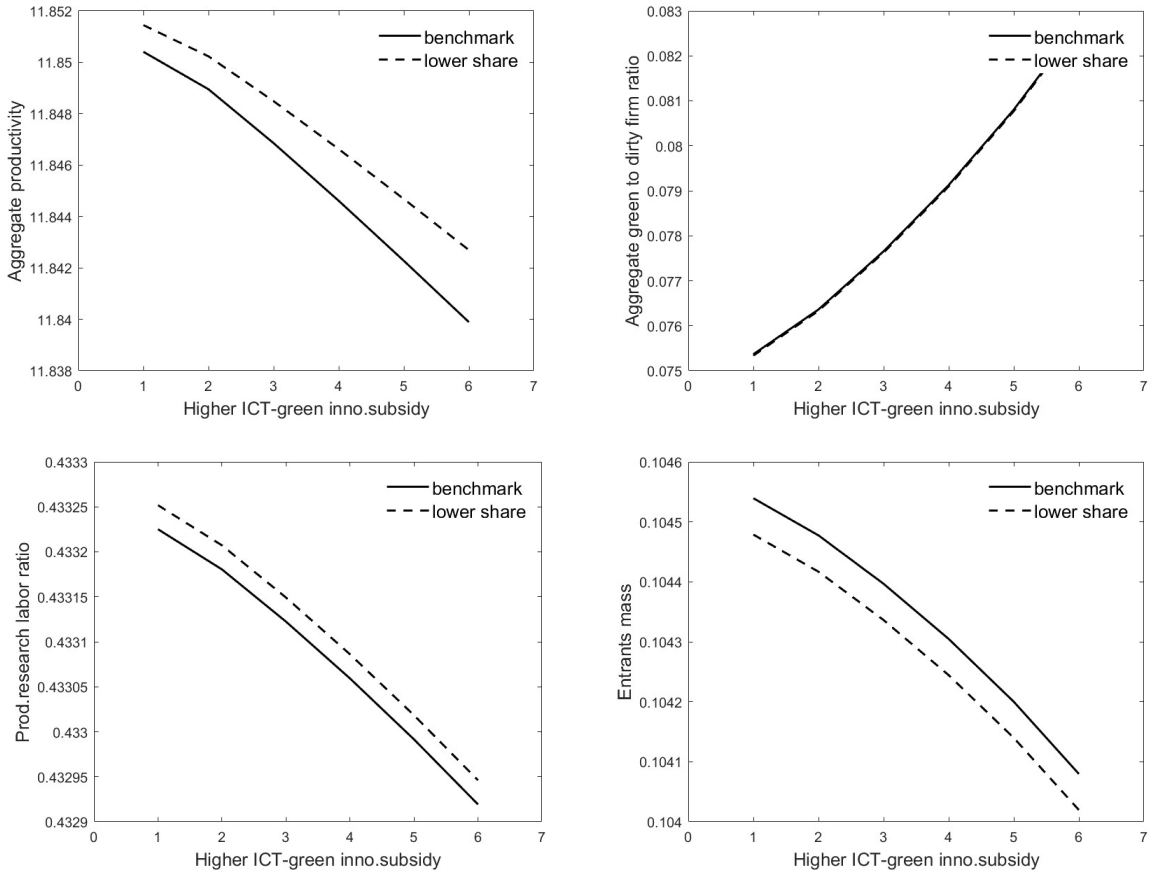


Figure 3.9: General equilibrium effects of final goods input share: lower dirty inno.cost

Role of entrant productivity distribution In this section, we analyze how the change of entrants' distributions affects the equilibrium outcomes. In a benchmark case, we assume that the ICT and non-ICT entrants have the same distribution that the probability mass declines with potential green productivity as they enter as dirty firms. The we move the probability mass of low green productivity ICT firms to high green productivity ICT firms such that the distribution becomes first order stochastically dominates the previous one.

From the upper four panels of Figure 3.10, the ICT green firms' productivity distribution shifts rightwards, implying more green transition. The overall effect on aggregate productivity is also positive. When the opportunity cost of green technology is high, however, the positive effect generated by higher entrants' green productivity is weakened much as we can see all the slopes are lower than the benchmark. The reason is that higher loss of dirty productivity mitigates the positive

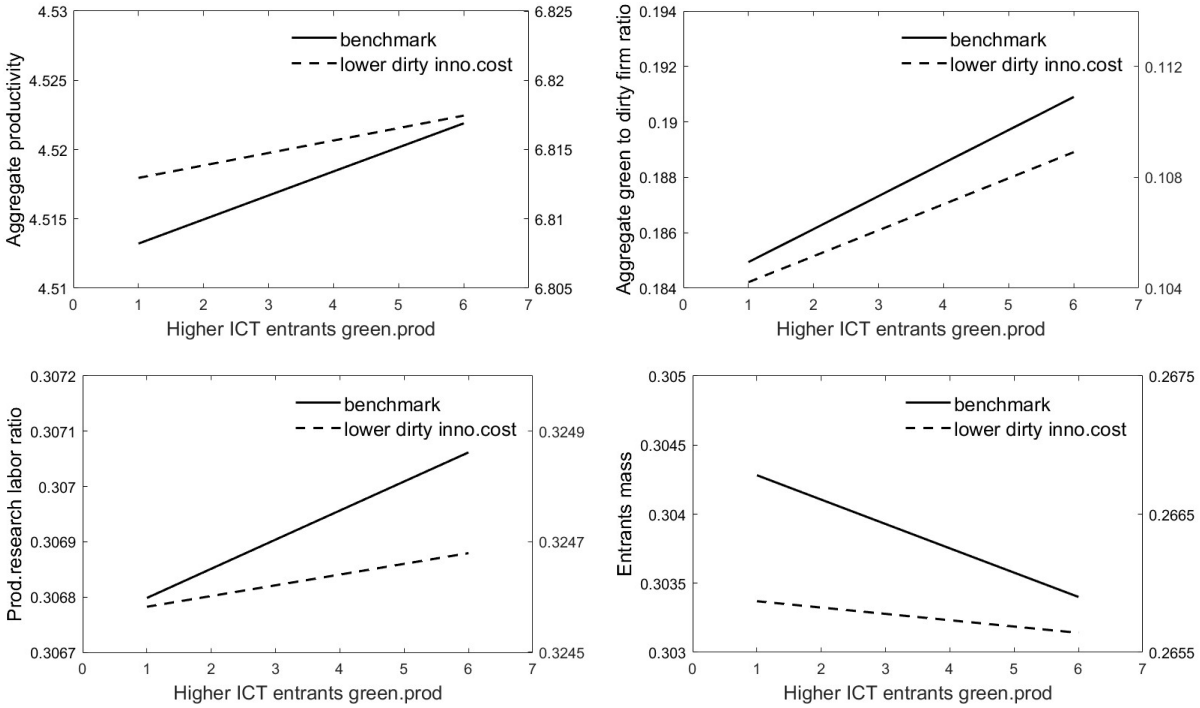


Figure 3.10: General equilibrium effects of ICT entrants green productivity change

Note: Each grid of X-axis is to reshape the ICT entrants green productivity distribution which first order stochastic dominates the previous one.

effect originated from entrants' improvement.

Role of the energy price change In this part, we further explore how does the energy prices interplay with the above green innovation cost change. The first feature is that increasing of energy price reduces aggregate productivity. Although it encourages green transition of firms, it causes loss of dirty productivity by increasing its production cost as in the first panel of Figure 3.11 which shows that the overall effect on aggregate productivity is decreasing. Furthermore, it shows that when opportunity cost of green innovation is higher, the energy price has stronger effect on productivity decreasing but weaker effect on green transition. The results demonstrate that higher green innovation subsidy and higher energy price complements in green transition.

The bottom panels of Figure 3.11 shows the pattern of labor reallocation and entrants' mass respectively. Higher energy price change implies higher labor reallocation towards production and

lower entrants' mass but in opposite direction with lower green opportunity cost.

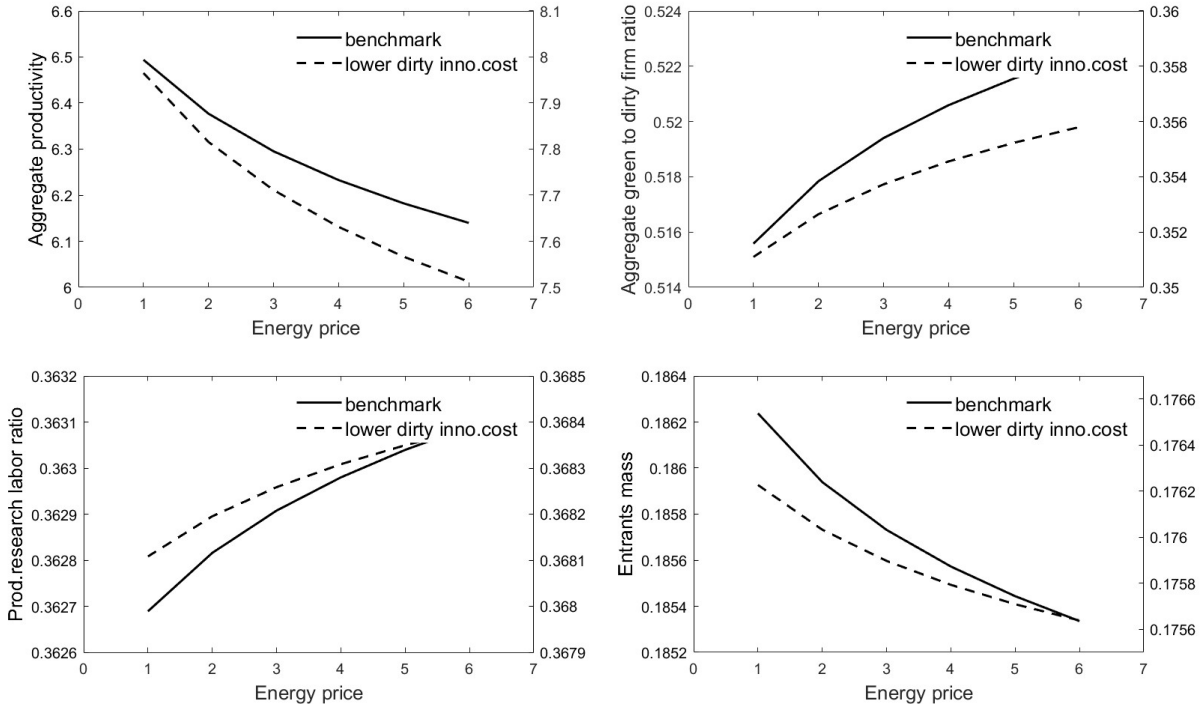


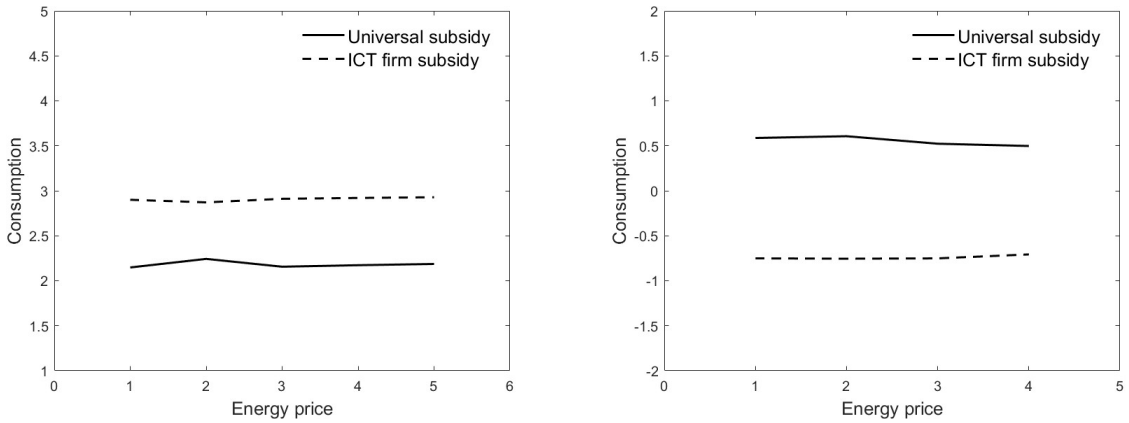
Figure 3.11: General equilibrium effects when energy price change

3.3.2 Green Innovation Subsidy: Universal or ICT Only

According to U.S. Energy Information Administration, the public R&D related to clean energy in U.S. in 2016 is 347 million USD, which accounts for 0.019‰ of total GDP (or around 0.056% of total R&D spending). According to National Center for Science and Engineering statistics, the public R&D related to clean energy in 2023 is 2000 million, with a share of GDP 0.074 ‰. Our analysis is to match the ratio of green subsidy to R&D in 2016.

First of all, we allocate the green innovation subsidy proportional to firm size which does not distinguish ICT or non-ICT firms. Then we concentrate all green subsidies to ICT firms. The equivalence means that the aggregate innovation subsidy are equalized in the two different policies.

As the analysis above shows that energy price also affect aggregate productivity and green transition, we would like to compare the policy efficiency under energy price changes. The result of



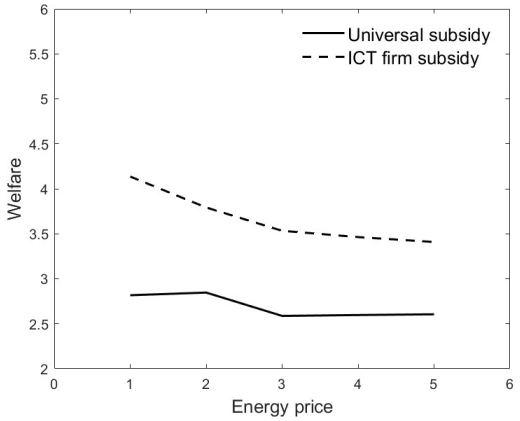
(a) Higher dirty inno.cost: C (b) Lower dirty inno.cost: C

Figure 3.12: Consumption change under subsidies

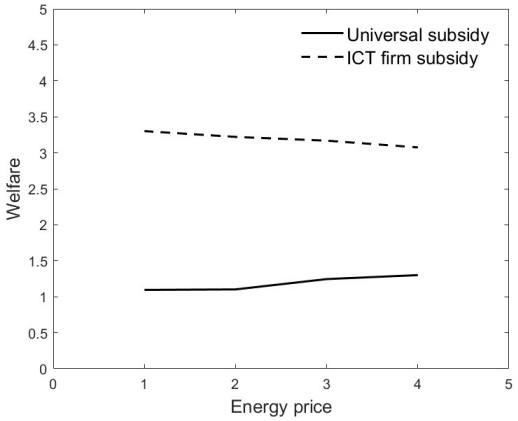
Note: The Y-axis is the relative change compared to benchmark without any subsidy. The universal subsidy matches green subsidy ratio to total R&D in the data.

consumption is shown in the left panel of Figure 3.12. The Y-axis is the relative change compared to the benchmark without any subsidy. When the dirty to green innovation cost is high, both subsidies increase consumption as in the left panel and subsidizing ICT green firms are more efficient. When the dirty to green innovation cost is low, the universal green subsidy generates positive effect on consumption while the ICT green subsidy has negative effect. The pattern is rather stable along the change of energy price. The result is consistent with above analysis that it is more efficient to concentrate the subsidy to firms that have comparative advantage in green innovation when the opportunity cost of dirty productivity loss is low.

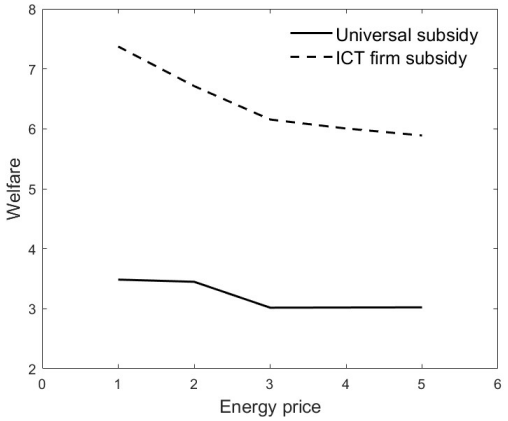
We also compute the welfare rather than consumption in Figure 3.13. As in the first panel in which case dirty to green innovation cost is relatively high, when energy price goes up, the welfare increment generated by ICT green subsidy decreases significantly. This implies that although energy price and green subsidy complements in green transition, the marginal effect of ICT green subsidy is low when energy price is already high. Besides, higher energy price drives down aggregate productivity which also mitigates the effect of subsidy. When dirty to green innovation cost is relatively low, the decreasing effect is much weaker. This is because lower relatively lower dirty innovation cost weakens the effect of energy price on green transition. The bottom panels are



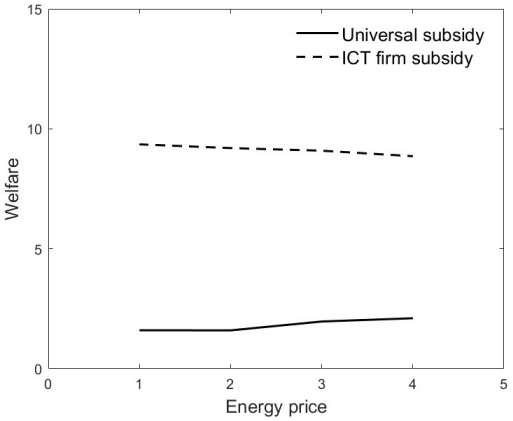
(a) Higher dirty inno.cost: low externality



(b) Lower dirty inno.cost: low externality



(c) Higher dirty inno.cost: high externality



(d) Lower dirty inno.cost: high externality

Figure 3.13: Welfare change under subsidies: higher vs low externality

Note: The Y-axis is the relative change compared to benchmark without any subsidy. The universal subsidy matches green subsidy ratio to total R&D in the data.

to set a higher externality parameter value. In both cases, the welfare increment is higher compared with low externality case and subsidize ICT green innovation is significantly more improved. But in case when dirty to green innovation cost is relatively high, the policy efficiency's sensitivity to energy price is also increased.

4 Quantitative Analysis

Before implementing any policy analysis, we calibrate 16 parameters, where half of them are internally estimated using U.S. patent data from USPTO matched with firm data from Compustat. We set the innovation cost scale and curvature h_d, ξ_d as a benchmark, while discipline that of ICT green innovation and non-ICT green innovation $h_g, \xi_g, h_g^u, \xi_g^u$ from information of green patent and green firm ratios. The other internally estimated parameters are the innovation step size J, entry cost n_e, fixed cost of operation n_f and transition cost c.

Next, we implement a green innovation subsidy schedule for all firms proportional to their firm size. We then compute the amount equivalent policy which target only on ICT firms. The result implies that subsidy should diverted differently as energy price changes.

4.1 Parameterization

In this section, we estimate the parameters listed in Table 4.3 using simulated methods of moments (SMM). There are 16 parameters, 8 of them are externally estimated and 8 of them are internally estimated.

External estimation The labor share of income is to target recent data of firms in U.S., the value is estimated to be around 0.6 by Matthew et al. (2022). The share of oil inputs when using the dirty technology is $(1-\alpha)(1-\eta)$, the value of which is 0.04 implied by Acemoglu et al. (2016a). Therefore, given labor share, the value of η is 0.9. Our model assumes an exogenous exit rate γ, we adopt the value used in Atkeson and Burstein (2010), to match the average exit rate of large firms in U.S. The discount factor β is computed by the inverse of annul interest rate. The elasticity of substitutes σ among output varieties is a common estimator in literature. Upon entry, firms are randomly assigned to be an ICT firm of probability θ, we use the ratio of ICT innovators to total innovators in the patent data as its value.The innovation cost of dirty firms is specified as $c(e) = h_d e^{\xi_d e}$, as our paper focus on the relative level of green to dirty innovation and firms, we

Table 4.1: Model parameterization

Parameter	Notation	Value	Targeted Moments	Model	Data
External					
labor.share	α	0.65	Matthew et al. (2022)		
final.goods.share	η	0.9	Acemoglu et al. (2016a)		
exit.prob	γ	0.0055	firm.exit.rate		
discount factor	β	0.95	common.estimates		
elas.of.subs	σ	1.5	common estimates		
share.ICT.firm	θ	0.515	share.ICT.innovator		
dirty.cost	h_d	3.24	Assigned.value		
dirty.curvature	ξ_d	30	Assigned.value		
green.curvature	ξ_g^n	30	non-ICT.green.to.dirty.firm		
ICT.green.curvature	ξ_g^u	30	ICT.green.to.dirty.firm		
Internal					
green.cost	h_g^n	2.08	non-ICT.green.to.dirty.sales.ratio	0.73	0.72
ICT.green.cost	h_g^u	1.23	ICT.non-ICT.green.sales.ratio	1.72	1.72
ICT.entrant.distri.slope	k^u	0.728	green.firm.ratio	0.58	0.58
transition.cost	c	0.05	non-ICT.green.dirty.firm.ratio	0.40	0.39
energy.price	P_o	0.02	sd.sales.non-ICT.green/dirty	1.15	1.15
inno.step	J	0.485	std.firm.sales.growth.rate	1.14	1.16
entry.fixed.costs	n_e	2.118	entrant.ratio	0.02	0.02
operat.fixed.costs	n_f	0.136	firm.exit.rate	0.09	0.10

Notes: We consider different values of the preference parameter for externality Ξ, which delivers different policy implications.

adopt the value in Atkeson and Burstein (2010) of normal firms as the benchmark for dirty firms.

Internal estimation The green innovation cost of ICT and non-ICT firms are proportional to $h_g^u e^{\xi_g^u e}$, $h_g e^{\xi_g e_g}$ respectively, which are governed by the scalar h_g^u, h_g and the curvature ξ_g^u, ξ_g. Both of the parameters will determine the marginal cost of innovation, and thus the distribution of firms. Here, we targe four moments from data: the green patent to dirty patent ratio of ICT firms, the green patent to dirty patent ratio of non-ICT firms, the green to dirty firms ratio of ICT firms, and the green to dirty firms ratio of non-ICT firms. The transition cost will affect the incentive of firms for choosing the technology class. Hence, we use the ratio of green firms of ICT firms to non-ICT firms as a targeted moment. The innovation step size J is the target the standard deviation of firm sales growth rate in our data. n_e affects the entry mass from free entry decisions and we match it to

real entrants' rate. n_f affect firms which hit the margin of exit or keep operation. Since only small firms make such decisions, we use small firm exit rate to match this moment.

For the internal estimation, we separate the parameters into two groups. The first group contains 6 parameters $h_g, h_g^u; \xi_g, \xi_g^u; c, P_o$, which are mainly disciplined from patent data. The second group contains 3 parameters n_e, n_f, J which mainly driven by firms' foundamental information. We then jointly minimize the loss function within each group as follows:

$$min \sum_{k=1}^{N} \frac{|model_k - data_k|}{(|model_k| + |data_k|)^{\frac{1}{2}}} \tag{4.1}$$

The estimated parameter values and matched moments are listed in Table 4.3.

Table 4.2: Mapping to the data

Model	Data
$\theta \int e^u(a) dG_e(a)$	firm.start.inno.ICT.green.pat.from.current.period
$(1-\theta) \int e^n(a) dG_e(a)$	firm.start.inno.non-ICT.green.pat.from.current.period
$\int e^u(a) d\mu^u(a)$	firm.also.inno.ICT.green.pat.last.period
$\int e^n(a) d\mu^n(a)$	firm.also.inno.non-ICT.green.pat.last.period
$\int (1 - e^u(a^u)) d\mu^u(a^u)$	firm.quit.inno.ICT.green.pat.next.period
$\int (1 - e^n(a^n)) d\mu^n(a^n)$	firm.quit.inno.non-ICT.green.pat.next.period

4.2 Model Validation

In this section, we use calibrated parameters to conduct external validity check. We are going to implement three variations exogenously: the energy price, the entrant's distribution of productivity and the innovation cost. Then we compare the paths implied by the change of model moments and data moments to verify the extent how each elements can explain the change of data.

Acemoglu et al. (2023) trace part of green slowdown to the shale gas revolution: the large and persistent decline in the cost of fossil-based energy lowered the relative profitability of green technologies and discouraged innovation in clean alternatives. This motivates us to model changing energy prices. Atkeson and Patrick (2007) and Aghion et al. (2009) explain productivity growth of

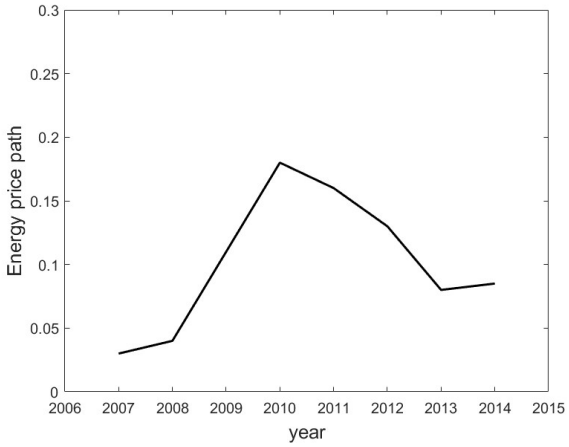


Figure 4.1: Path of energy price

specific technologies of incumbents with entrants' productivity. Though they do not directly apply to the context of green innovation, it is obvious that it is our interest to vary entrant distribution of green productivity especially that of ICT firms to see how does it drive the green transition. Acemoglu et al. (2016a) and Aghion et al. (2024) model that the competition between green and dirty technology depends on creative destruction rate between them, which relies on relative innovation cost. Our paper also emphasize the importance of this mechanism. But we are more interested in varying the ICT-green innovation cost to study the green transition.

4.2.1 Change of energy prices

As we implement the energy price change path as shown in Figure 4.1, it is an inverted U-shape path. We then take data moments of share of green patents for ICT and all firms from 2007 to 2014. This time series shows a sharp feature: the quick emerging of green innovation since 2000s up to 2010 and then a dramatic drop since 2010. When energy price increases, ICT green patent share and total green patent share in the economy is increasing up to 2010. After then, due to the exogenous energy price drop, the green patent share both for ICT firms and the economy is dropping. The energy price change preliminarily can explain the change of common trend of green innovation across different groups of firms but do not explain differences across different groups of firms.

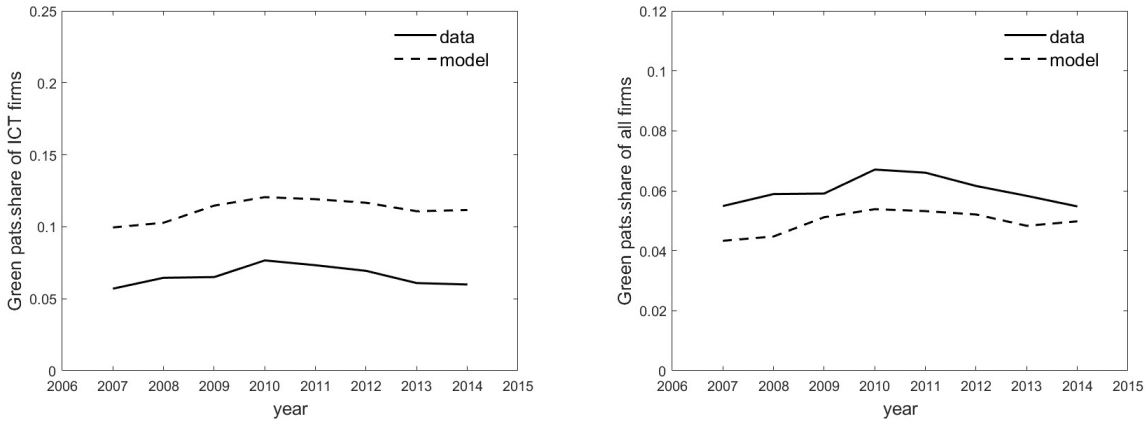


Figure 4.2: Trends of share of green patents

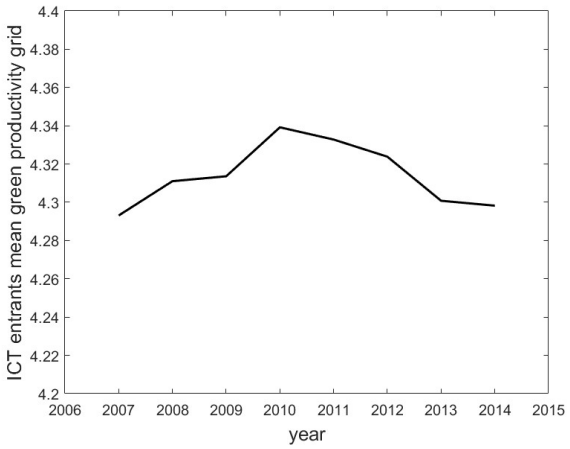


Figure 4.3: Path of ICT entrants' mean green productivity

4.2.2 Change of ICT entrants productivity distribution

In this subsection, we implement the ICT entrants' green productivity distribution change path, the mean of the distribution is shown in Figure 4.3. It is also an inverted U-shape path. Now the time series match the trends of green patent share of ICT firms closely from 2007 to 2014. However, it is difficult to match the overall change of green patents in the economy. This implies that ICT related green productivity change can partly account for the overall change of green innovation only. The ICT entrants' green productivity distribution change can explain the change of specific trend of green innovation of ICT related firms.

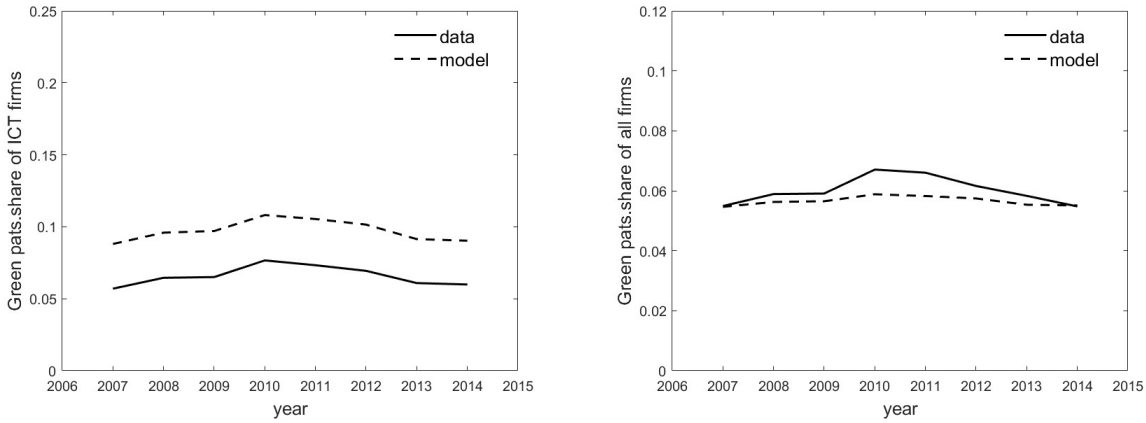


Figure 4.4: Trends of share of green patents: by entrants' productivity

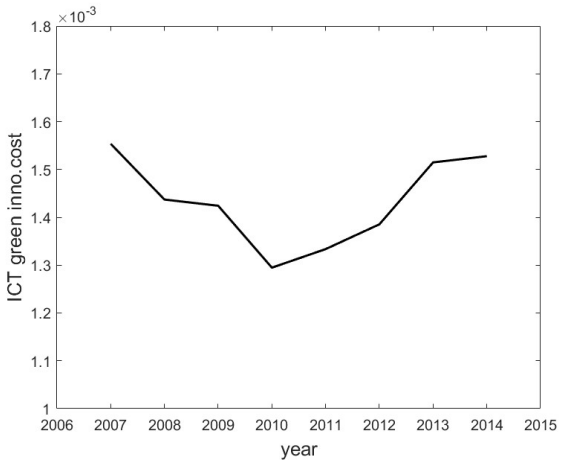


Figure 4.5: Path of ICT green inno.cost

4.2.3 Change of ICT green innovation cost

In this subsection, we implement the ICT related firm green innovation change path, the scalar of green innovation cost function is shown in Figure 4.5. It is an U-shape path. The model moments can match the trends of green patent share of ICT firms closely from 2007 to 2014, similar as the entrants' productivity change.It is also difficult to match the overall change of green patents in the economy. We could summarize that the change of entrants' productivity or green innovation cost can explain mostly firm group specific green innovation change, while energy price can explain the common part of green innovation change of different groups of firms.

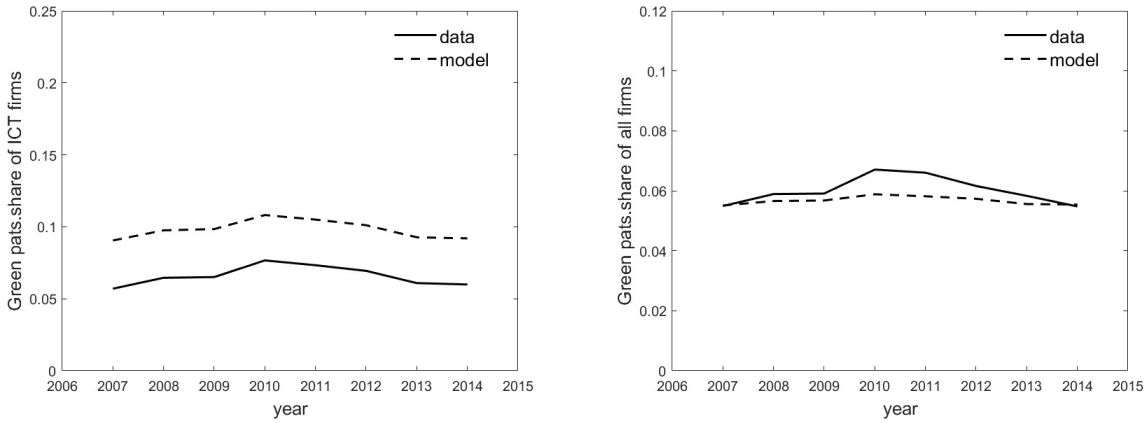


Figure 4.6: Trends of share of green patents: by ICT green inno.cost

4.3 Analysis of Subsidy Policies

In progress.

Table 4.3: Policy analysis

Exogenous ICT choice	Consumption ($\Xi = 0$)	Low externality	High externality
Energy price			
Entrant distribution			
Innovation cost			
Endogenous ICT choice	Consumption ($\Xi = 0$)	Low externality	High externality
Energy price			
Entrant distribution			
Innovation cost			

Notes:

4.4 Model extensions

4.4.1 Endogenous choice of being ICT entrants

In this section, we endogenize the decision of being an ICT firm or a normal firm upon entry. Entrants still draw from a distribution of productivity $G_e(\bar{a_d}, a_g)$, but can choose the investment of being an ICT firm. Let e denote the investment rate. The entrants problem becomes:

$$U_t^E(a_d, a_g) = \max_e \; e \cdot V_t^u(a) + (1-e)V_t^n(a) - \Phi^E(a, e), \tag{4.2}$$

where the investment cost function takes the same format as the innovation costs:

$$\Phi^E(a, e) = h^E \exp(a_g) \exp(\xi^E e), \tag{4.3}$$

Note that since they have the same dirty productivity upon entry, we assume the investment cost is proportional to the exponential of green productivity upon entry. We have the following entrants' investment rates versus the green productivity as in Figure 4.7.

As the green productivity increases, the marginal benefit of being an ICT firm increases faster than investment cost, causing more firms selecting into ICT. When ICT green innovation cost decreases, it provides firms with higher incentive of being an ICT firm as well.

The distribution of ICT-green entrants is therefore $G_e(\cdot, a_g) \cdot e(a_g)$ and the free entry condition becomes:

$$n_e = \frac{1}{1+r_t} \int [e(a_g)V^u(a,d)dG_e(a) + (1-e(a_g))V^n(a,d)dG_e(a)] \tag{4.4}$$

This extension allows us to have richer general equilibrium effects and implications. ICT-green innovation cost reduction not only incentivize ICT entrants innovate more intensively in this direction but also attracts a higher fraction of firms into ICT. Then higher fraction of ICT firms will amplify the effects of innovation cost reduction.

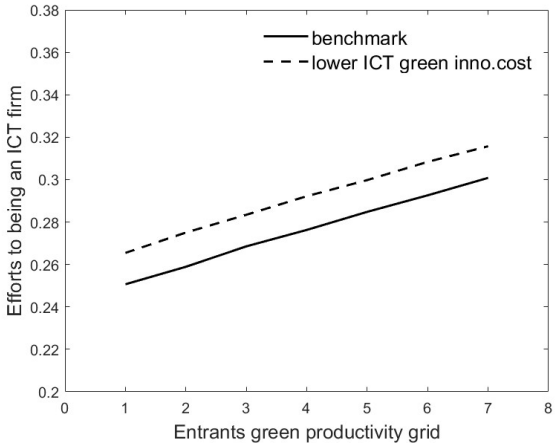
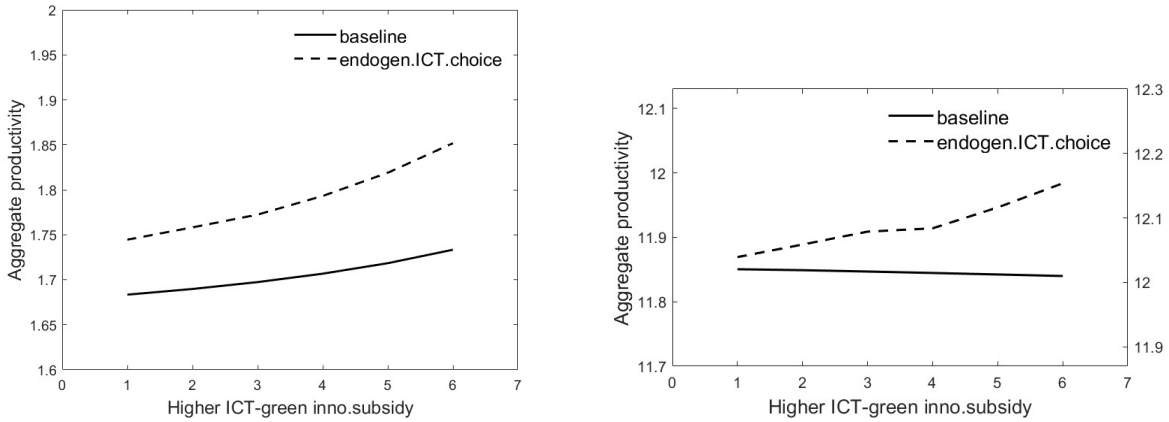


Figure 4.7: Entrants effort of being an ICT firm given dirty productivity



(a) Higher dirty inno.cost: A (b) Lower dirty inno.cost: A

Figure 4.8: ICT-green innovation cost reduction: endogenize ICT choice

Note: Each grid of X-axis is to increase the green innovation subsidy such that innovation cost faced by firms are half.

We implement the same exercise as in the mechanism part where we increase the ICT-green innovation subsidy gradually. The results are shown in Figure 4.8. When dirty innovation cost is relatively high, the opportunity cost of green transition is low such that productivity is increasing over ICT-green innovation subsidy. In the endogenous choice of being an ICT firm, both the level and slope becomes higher, implying the selection effect of firms upon entry. When dirty innovation cost is relatively low, in the exogenous case, the productivity is decreasing as opportunity cost of green transition is high. In the endogenous case, since the ICT firm dominates non-ICT firm

in green innovation (the same in dirty innovation), higher fraction of entrants being an ICT firm promotes the productivity which is faster than the decline in the baseline case.

5 Conclusion

The paper examines the slowdown in green innovation since the 2010s. Using U.S. patent data, we develop a decomposition framework and show that green innovation is disproportionately conducted by firms applying for ICT-related green patents, and that the aggregate slowdown in green innovation is largely attributable to these ICT-related firms. This motivates a general equilibrium model in which heterogeneous firms face different green innovation costs and make endogenous decisions about technology class choice, innovation, entry, and exit. Estimating the model, we find that the decline in energy prices since the 2010s has weakened incentives for green innovation, with especially pronounced effects on ICT-related firms. Because these firms bear distinct green innovation costs and respond differently to changes in energy prices, their behavior plays a central role in reshaping the green transition. We use the model to compare a green innovation subsidy that is applied uniformly across all firms with one targeted solely at ICT firms, and find the effectiveness of policies highly depend on the relative green innovation cost of ICT-related firms as well as the energy price level.

Our focus on ICT-related firms is particularly relevant in light of rapid advances in artificial intelligence and the associated surge in energy demand from digital infrastructure. As AI and other ICT-intensive technologies become increasingly central to production and innovation, these firms emerge as both key drivers of green technological change and major contributors to rising energy use. The analysis implies that policymakers designing green innovation subsidies must account not only for heterogeneity in firms' green innovation costs and their exposure to energy prices, but also for the growing importance and energy intensity of ICT and AI-related activities.

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Online Appendix for "Unpacking the Green Innovation Slowdown in the Digital Age: Firm Heterogeneity and Policy Trade-offs"

A Data Appendix

- Transportation: IPC: $B60 - B68$;
- Electricity: IPC: H;
- Mechanical Engineering except weapons and blasting: IPC: F except $F41, F42, F99$.
- PETROLEUM, GAS OR COKE INDUSTRIES: $C10$
- CHEMICAL OR PHYSICAL PROCESSES: $B01J$

B Empirical Appendix

B.1 Decomposition of Green Patents in Different Technology Fields

In this section, we further decompose the green patents according to several subcategories based on the feature of technology. We first divide the energy production sector into: wind and solar, and: bio-fuel, hydro and thermal. The first two types of energy production is usually featured with intermittent supplying and requires coordination for supply and demand side with ICT and AI technique. The other categories including energy storage, transportation and waste management.

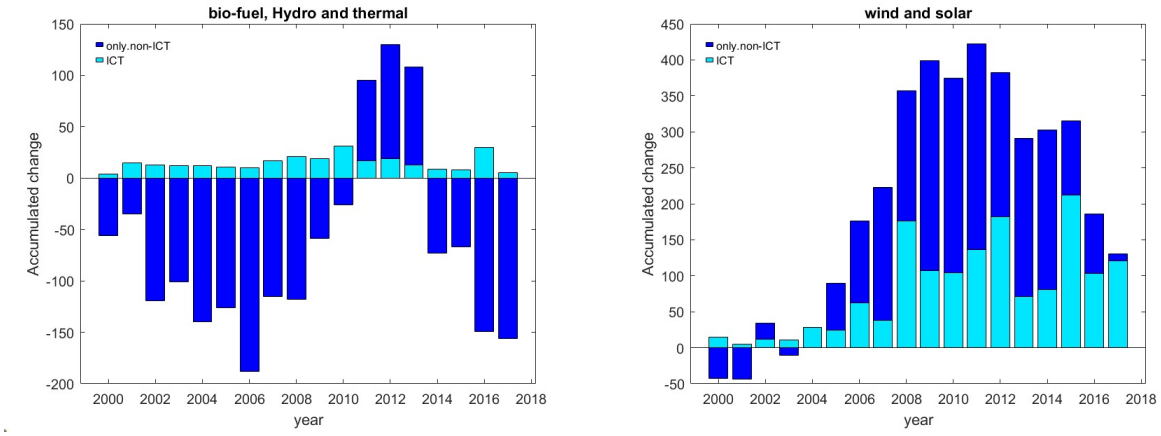


Figure B.1: Accumulated green patents change in energy production

Note: We combine the firms doing only and any ICT related green patents as ICT in the appendix figures. The category is identified according to the description of IPC green inventory.

B.2 Robustness check

C Model Appendix

C.1 Decision Rules of Intermediate Firms

From the FOCs of optimal production decisions, we know the proportions of inputs by dirty firms:

$$\frac{O(a)}{l(a)} = \frac{(1-\eta)(1-\alpha)}{\alpha} \frac{w}{P_o(1+\tau)} \tag{C.1}$$

and:

$$\frac{Y^D(a)}{l(a)} = \frac{(1-\alpha)\eta}{\alpha} \frac{w}{P} \tag{C.2}$$

Analogously, the firm that uses the green technology have input share:

$$\frac{Y(a)}{l(a)} = \frac{1-\alpha}{\alpha} \frac{w}{P}. \tag{C.3}$$

The profit maximization problem of dirty firms yield the expression of prices and inputs, the

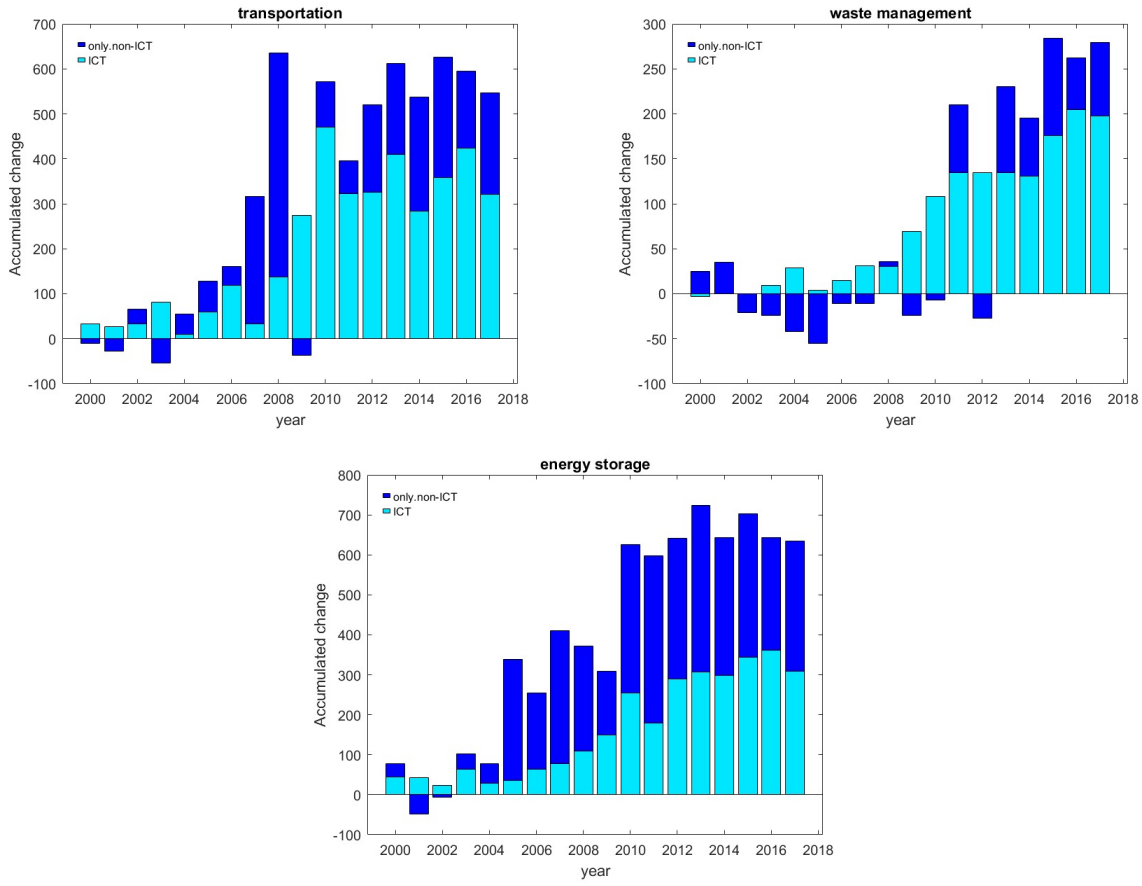


Figure B.2: Accumulated green patents change in other fields

Note: We combine the firms doing only and any ICT related green patents as ICT in the appendix figures. The category is identified according to the description of IPC green inventory.

expression of final goods demand is:

$$Y_d^{D*}(a) = \frac{P^{\sigma} C (\frac{\sigma}{\sigma-1})^{-\sigma} \exp(a_d) \alpha^{\alpha(\sigma-1)} (1-\alpha)^{1+(1-\alpha)(\sigma-1)} \eta^{1+\eta(1-\alpha)(\sigma-1)} (1-\eta)^{(1-\eta)(1-\alpha)(\sigma-1)}}{w^{\alpha(\sigma-1)} (P_o(1+\tau))^{(1-\eta)(1-\alpha)(\sigma-1)} P^{1+(\sigma-1)\eta(1-\alpha)}} \tag{C.4}$$

The expression of labor demand by dirty firms is:

$$l_d^*(a) = \frac{P^{\sigma} C (\frac{\sigma}{\sigma-1})^{-\sigma} \exp(a_d) \alpha^{1+\alpha(\sigma-1)} (1-\alpha)^{(1-\alpha)(\sigma-1)} \eta^{\eta(1-\alpha)(\sigma-1)} (1-\eta)^{(1-\eta)(1-\alpha)(\sigma-1)}}{w^{1+\alpha(\sigma-1)} (P_o(1+\tau))^{(1-\eta)(1-\alpha)(\sigma-1)} P^{(\sigma-1)\eta(1-\alpha)}} \tag{C.5}$$

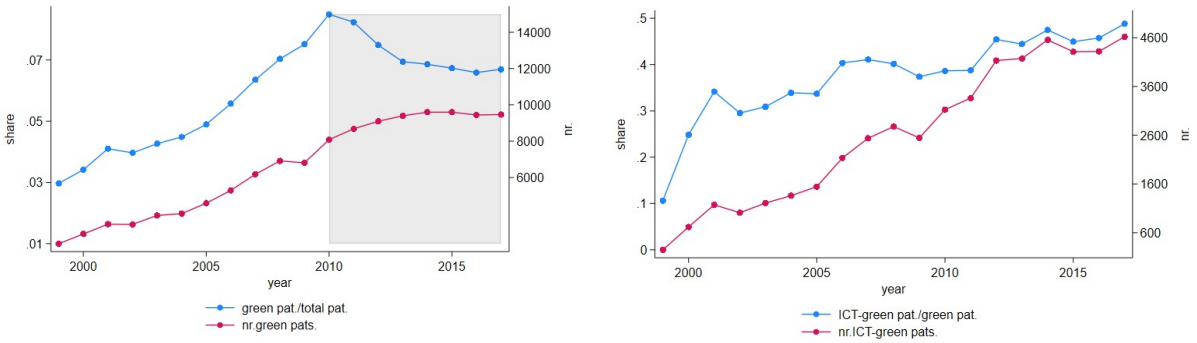


Figure B.3: Accumulated green patents change: targeted sectors

Note: Now we focus only on sectors as listed in A plus the ICT sector.

The expression of fossil fuel demand by dirty firms is:

$$o^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma} \exp(a_d)\alpha^{\alpha(\sigma-1)}(1-\alpha)^{1+(1-\alpha)(\sigma-1)}\eta^{\eta(1-\alpha)(\sigma-1)}(1-\eta)^{1+(1-\eta)(1-\alpha)(\sigma-1)}}{w^{\alpha(\sigma-1)}(P_o(1+\tau))^{1+(1-\eta)(1-\alpha)(\sigma-1)}P^{(\sigma-1)\eta(1-\alpha)}} \tag{C.6}$$

The expression of total output selected by dirty firms is:

$$y_d^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}(\exp(a_d))^{\frac{\sigma}{\sigma-1}}\alpha^{\alpha\sigma}(1-\alpha)^{(1-\alpha)\sigma}(1-\eta)^{(1-\eta)(1-\alpha)\sigma}\eta^{\eta(1-\alpha)\sigma}}{w^{\alpha\sigma}(P_o+\tau)^{(1-\eta)(1-\alpha)\sigma}P^{\sigma\eta(1-\alpha)}} \tag{C.7}$$

The expression of output price by dirty firms is:

$$p_d^*(a) = \frac{\frac{\sigma}{\sigma-1}(\exp(a_d))^{-\frac{1}{\sigma-1}}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}(1-\eta)^{(\eta-1)(1-\alpha)}\eta^{-\eta(1-\alpha)}}{w^{-\alpha}(P_o(1+\tau))^{-(1-\eta)(1-\alpha)}P^{-\eta(1-\alpha)}} \tag{C.8}$$

Similarly, we solve the profit maximization problem of green firms. The expression of final goods demand is:

$$Y_g^{D*}(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma}\exp(a_g)\alpha^{\alpha(\sigma-1)}(1-\alpha)^{1+(1-\alpha)(\sigma-1)}}{w^{\alpha(\sigma-1)}P^{1+(\sigma-1)(1-\alpha)}} \tag{C.9}$$

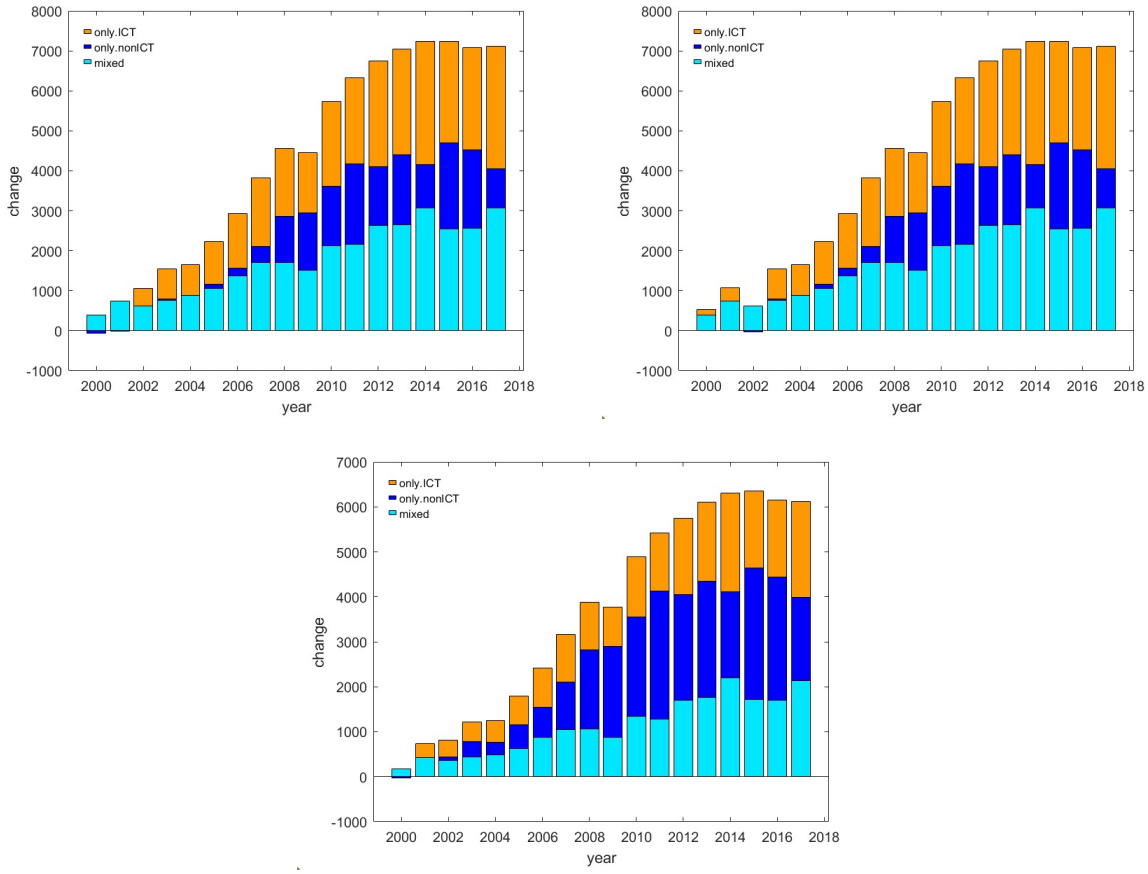


Figure B.4: Accumulated green patents change: targeted sectors and alternative measure of ICT-green firm

Note: Upper left panel shows the results that we focus only on sectors as listed in Appendix A plus the ICT sector; Right panel shows the results that ICT-green firm are consistently defined as long as with a record of such patent during sample period; Bottom panel shows the results that we set the threshold to be an ICT-green firm with at least 3 ICT-green pats.

The expression of labor demand by green firms is:

$$l_g^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma} \exp(a_g) \alpha^{1+\alpha(\sigma-1)} (1-\alpha)^{(1-\alpha)(\sigma-1)}}{w^{1+\alpha(\sigma-1)} P^{(\sigma-1)(1-\alpha)}} \tag{C.10}$$

The expression of total output selected by green firms is:

$$y_g^*(a) = \frac{P^\sigma C(\frac{\sigma}{\sigma-1})^{-\sigma} (\exp(a_g))^{\frac{\sigma}{\sigma-1}} \alpha^{\alpha\sigma} (1-\alpha)^{(1-\alpha)\sigma}}{w^{\alpha\sigma} P^{\sigma(1-\alpha)}} \tag{C.11}$$

The expression of output price by green firms is:

$$p_g^*(a) = \frac{\frac{\sigma}{\sigma-1}(\exp(a_g))^{-\frac{1}{\sigma-1}}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}}{w^{-\alpha}P^{-(1-\alpha)}} \tag{C.12}$$

C.2 Deriving Aggregate Variables

Final goods price and aggregate productivity According to definition in the model part, the aggregate final price is:

$$\begin{aligned} P &= [\int_{\omega\in\Omega_j} p(\omega)^{1-\sigma}d\omega]^{\frac{1}{1-\sigma}} \\ &= [\int_{a\in A_d} p_d(a)^{1-\sigma}d\mu(a) + \int_{a\in A_g} p_g(a)^{1-\sigma}d\mu(a)]^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{(\sigma-1)w^{-\alpha}}[(P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}C_dA_d + P^{(1-\alpha)(1-\sigma)}C_gA_g]^{\frac{1}{1-\sigma}} \end{aligned} \tag{C.13}$$

where we denote for convenience that:

$$C_d(\tau) = (\frac{1+\tau}{(1-\eta)(1-\alpha)})^{(1-\sigma)(1-\alpha)(1-\eta)} \cdot (\frac{w}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{(1-\alpha)\eta})^{(1-\sigma)(1-\alpha)\eta} \tag{C.14}$$

and:

$$C_g = (\frac{w}{\alpha})^{(1-\sigma)\alpha} \cdot (\frac{1}{1-\alpha})^{(1-\sigma)(1-\alpha)} \tag{C.15}$$

The aggregate productivity is defined as:

$$A = [(P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}C_dA_d + P^{(1-\alpha)(1-\sigma)}C_gA_g]^{\frac{1}{\sigma-1}} \tag{C.16}$$

The productivity index $\tilde{A}$ without considering entrants' mass is $A = M_e^{\frac{1}{\sigma-1}}\tilde{A}$.

Market size By definition, the common part of aggregate variable profit of dirty and green technology, which captures a market size effect, is given bt:

$$\Pi =: (\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^{\sigma}Y \tag{C.17}$$

Production labor demand The aggregate dirty profit is:

$$\begin{aligned}\tilde{\Pi}_d = \int P^{\sigma}C(\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}\exp(a_d)\cdot(\frac{p_o(1+\tau)}{(1-\eta)(1-\alpha)})^{(1-\sigma)(1-\alpha)(1-\eta)} \\ \cdot(\frac{w}{\alpha})^{(1-\sigma)\alpha}\cdot(\frac{P}{(1-\alpha)\eta})^{(1-\sigma)(1-\alpha)\eta}d\mu(a) \\ = C_dP^{\sigma}CA_d(\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^{(1-\sigma)(1-\alpha)\eta}p_o^{(1-\sigma)(1-\alpha)(1-\eta)}\end{aligned} \tag{C.18}$$

Note that $wL_p^d = \tilde{\Pi}_d(\sigma-1)\alpha$. And the aggregate green profit is:

$$\tilde{\Pi}_g = C_gP^{\sigma}CA_g\cdot(\frac{\sigma}{\sigma-1})^{1-\sigma}\frac{1}{\sigma}P^{(1-\sigma)(1-\alpha)} \tag{C.19}$$

Note that $wL_p^g = \tilde{\Pi}_g(\sigma-1)\alpha$. We normalize the wage rate to be unity.

Research labor and production labor The ratio of the production labor to research labor in the downstream is:

$$\chi = \frac{L_p}{L_r} = \frac{\Pi_d(\sigma-1)[(\alpha+\frac{(1-\alpha)(1-\eta)}{a_oP_o})C_dA_d(P^{\eta(1-\alpha)}P_o^{(1-\eta)(1-\alpha)})^{1-\sigma}+\alpha C_gA_gP^{(1-\alpha)(1-\sigma)}]}{\int\Phi_d(a_d)e_d^2(a,m)d\mu(a)+\int\Phi_g(a_g)e_g^2(a,m)d\mu(a)+c\cdot M_c+n_f\cdot(M_d+M_g)+n_e(M_e+M_e^g)} \tag{C.20}$$

The labor market clearing condition is:

$$L_p + L_r = L \tag{C.21}$$

which implies $L_r = \frac{L}{1+\chi}, L_p = \frac{\chi L}{1+\chi}$.

Entrant mass By labor market clearing condition:

$$Y = \frac{L - L_r}{P^{\sigma}(\frac{\sigma}{\sigma-1})^{-\sigma} w^{\alpha(1-\sigma)-1} \cdot \alpha[C_d A_d (P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma} + C_g A_g P^{(1-\alpha)(1-\sigma)}]} \tag{C.22}$$

Plug the expression of Y in market size expression, we have:

$$M_e = \frac{L - L_r}{\alpha\Pi(\sigma-1)[C_d \tilde{A}_d (P^{\eta(1-\alpha)} P_o^{(1-\eta)(1-\alpha)})^{1-\sigma} + C_g \tilde{A}_g P^{(1-\alpha)(1-\sigma)}]} \tag{C.23}$$

By definition of $\tilde{A}$, it can be expressed as:

$$M_e = \frac{L_p}{\alpha\Pi(\sigma-1)\tilde{A}^{\sigma-1}} \tag{C.24}$$

Final goods input and consumption The input amount is:

$$\begin{aligned} Y^D &= \int_{a\in A_d} Y_d^D(a)d\mu(a) + \int_{a\in A_g} Y_g^D(a)d\mu(a) \\ &= \frac{(1-\alpha)\Pi}{P(\sigma-1)}[\frac{\eta C_d A_d}{P^{(\sigma-1)(1-\alpha)\eta} p_o^{(\sigma-1)(1-\alpha)(1-\eta)}} + \frac{C_g A_g}{P^{(\sigma-1)(1-\alpha)}}] \end{aligned} \tag{C.25}$$

and the consumption is:

$$C = Y - Y^D \tag{C.26}$$

The aggregate dirty output E is:

$$\begin{aligned} E &= C_d P^{\sigma} Y P^{\eta(1-\alpha)(1-\sigma)} P_o^{(1-\eta)(1-\alpha)(1-\sigma)} (A_d^n + A_d^u))^{\frac{\sigma}{\sigma-1}} (\frac{\sigma}{\sigma-1})^{-\sigma} \\ &= (\sigma-1)\Pi_d (A_d^n + A_d^u))^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{C.27}$$

C.3 Analytical Results

Exogenous sorting to green transition We need to separate the general equilibrium effect through the aggregate price adjustment, from the effect from firm decision changes. To understand how

the resource price works on firm decisions, we consider a case where firms using dirty technology are randomly sorted to green technology with an exogenous probability v. Hence, firms only make decisions on entry and innovation within technology class and without considering transition.

Proposition C.1 *When no labor is used ($\alpha \to 0$) and under exogenous sorting to green technology, when the resource input price changes, firm variable profits and all decisions have no change.*

Proof Assume that the resource input price changes from P_o to $\tilde{P}_o$. Because sorting is exogenous, dirty firms have the following value function in original technology class selection stage:

$$V_t^S(a,m) = vU_t^S(a,d;g) + (1-v)U_t^S(a,d;d) \tag{C.28}$$

Before it, the firms decide whether to continue the operation:

$$V_t^{0,S}(a,m) = max\ [0, V_t^S(a,m)] \tag{C.29}$$

Firstly, we argue that $\tilde{P} = P\frac{\tilde{P}_o}{P_o}$ is a solution to the fixed point of P. By the fixed point equation:

$$\tilde{P}^{1-\sigma} = [\frac{\sigma}{(\sigma-1)w^{-\alpha}}]^{1-\sigma}[\tilde{P_o}^{(1-\eta)(1-\sigma)}\tilde{P}^{\eta(1-\sigma)}C_dA_d + \tilde{P}^{1-\sigma}C_gA_g] \tag{C.30}$$

Obviously, it is the solution of the original problem given all other aggregate variables unchanged. Let $\tilde{\Pi} = \Pi(\frac{\tilde{p_o}}{p_o})^{(1-\eta)(1-\sigma)}$. This ensures that the variable profits of dirty technology is unchanged. By the solution of $\tilde{P}$, the variable profit of green technology also remains unchanged. By uniqueness of the solution, $\tilde{\Pi}$ is the new solution to the new dynamic problem. Hence, firms decisions are unchanged in the process. Q.E.D □

The input of labor leads to some general equilibrium effect on aggregate price, which makes the across technology adjustment not completely shut down even in the case of exogenous sorting. However, once we ignore this effect, we find that the endogenized process of technology class selection breaks down the invariance clearly. For whichever value of $\tilde{\Pi}$ chosen in the new optimization

problem, dirty firms are separated into two groups according to their own selection: transition or non transition. This would generate a kink point of the value function of dirty firms, which makes the variable profit of either group change. Once this happens, the relative variable profit across technology also changes. Hence, variable profit dispersions make firms sorted more or less to green technology which changes the aggregates through directed innovation decisions further.

D Quantitative Appendix

D.1 Computation Algorithm

1. Guess the initial value of the aggregate price $P^{(i)}$.

2. With the guessed price:

   (a) Guess the initial value of market size $\Pi^{(i,j)}$.

   (b) Given the guessed values, compute the variable profit for dirty and green technology. Through the dynamic programming of downstream firms, solve out the corresponding innovation decision rules e_d, e_g, the technology class choice rule m', the value functions V_d, V_g and the distribution conditional on technology class $\tilde{\mu}_d(a), \tilde{\mu}_g(a)$.

   (c) Compute the equilibrium Π^* through free entry condition. Repeat step $2.(a) \sim 2.(c)$ until $\Pi^{(i,j)} = \Pi^*$.

   (d) With the solved Π^*, compute equilibrium decision rules, value functions and the distributions. Compute the equilibrium price P, aggregate productivity A_g, A_d. Compute out the implied aggregate price P^* through the definition.

3. Repeat step $1 \sim 2$ until $P^* = P^{(i)}$.

4. Compute the equilibrium decision rules, distributions and aggregate variables based on the converged value of P^* and Π^*.