Drilling in the Drought? The Industrial Organization of Groundwater *

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February 19, 2026

*We are grateful to Abhijit Banerji, Alain Ayong le Kama, Kelsey Jack, Larry Karp, Matt Kochen, Gilbert Metcalf and Leo Simon for very helpful comments as well as seminar and workshop participants in Annecy, Berkeley, Connecticut, IIITDelhi, INRA Montpellier, MIT Sloan, Resources for the Future, TERI University, UC Riverside, Yale, Conference on Environment and Development Economics in Clermont-Ferrand, India Growth and Development Economics Conference (ISI Delhi), Indira Gandhi Institute for Development Research (IGIDR), Mumbai; Montreal Resource and Environmental Economics Workshop, Peking University Workshop on Environmental Economics and the Shanghai University of Finance and Economics (SUFE) Conference on Global Environmental Challenges: The Role of China.

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Abstract

Agricultural production in developing countries like India and China relies on small farms that trade water for irrigation. Most empirical research on water depletion does not take into account the role of water markets when evaluating impacts of policy. We develop a model in which households farm small parcels of land. The fixed costs of well drilling prevent each farmer from owning a well. This simple framework endogenously generates markets for groundwater which are commonly observed in many poor economies. The model allows for endogenous entry decisions and provides a flexible framework that can take account of the role of markets when analyzing the effects of policies. We show that if water is abundant, then equilibrium with free entry results in Bertrand competition (or when fixed costs are high, in a mix of monopolistic and Bertrand pricing) with well owners charging higher prices to buyers located closer to them. Unless fixed costs are high, there is over-capitalization, with too many wells. However, aggregate water use is always lower than it would be if all farms were large enough to afford their own wells. When water becomes scarce, as is often the case in hard-rock aquifers, or if electricity is rationed, we find that there may actually be a socially perverse boom in well-drilling, exacerbating the over-capitalization. Corrective policies include taxing the fixed costs of well-drilling to reduce entry and small subsidies to pumping costs that reduce static inefficiency.

Keywords: Bertrand pricing, groundwater, depletion, irrigation, market structure, Salop circle, spatial competition, transport costs, water scarcity

JEL Codes: D43,L13,Q25

1 Introduction

Groundwater resources are under threat globally, mainly because of rising demand for irrigation. Thanks to the widespread availability of electricity, farmers in many poor countries take two or three crops per year, and irrigate their crops by pumping groundwater in the dry season (Ryan and Sudarshan, 2022). Cheap, often subsidized electricity has made irrigation cost-effective and profitable.

There are four stylized facts that the textbook economic model of irrigation does not explain adequately. First, relative to Western nations, farm sizes are small in many countries (e.g., China, India and Bangladesh) so that individual farmers do not own tubewells for irrigation.[1] Second, in many developing country settings, irrigation water is traded among farmers. Many farms depend on other farmers who own a tubewell for their water supply. Third, there is considerable evidence that tubewell irrigation has exploded in recent decades. In India alone, the number of tubewells and dugwells has increased from nearly zero to 22 million in the period 1986-2020, according to its Minor Irrigation Census.[2] And finally, large tracts of farmland in developing countries are being abandoned or left fallow as aquifers deplete and approach exhaustion.

We explain the above stylized facts by developing a simple static model that allows for endogenous entry in the market for tubewells and price interactions among neighboring farmers. The model accounts for the fixed costs of digging new wells, pumping costs and the transport cost of shipping water to farms further away. If groundwater is abundant, the spatial equilibrium is characterized by either Bertrand competition or a combination of monopolistic pricing and Bertrand competition, where each water seller charges a monopoly price to its immediate neighbors but a Bertrand price to those

[1] For example, the mean plot size in China is 0.6 ha, and in India it is 1.3 ha. In the US, mean farm size is 94 ha.

[2] https://mowr.nic.in/irrigationcensus/

farther away. This occurs because for farms located farther away, there is price competition from other well owners located closer to them.

We have two main results. With linear demand for water, when compared to a single large farm, aggregate water use is always lower in the small farm setting. Furthermore, unless the fixed costs of installing a new tubewell are very high, there is over-capitalization, i.e., there are more tubewells in equilibrium than when farm sizes are large enough to be served by their own wells so that water trading is not needed. Prices are higher than marginal cost due to imperfect competition but free entry drives prices lower. The net result is higher than optimal prices and lower aggregate water use. The implication is that the water markets we observe in populous countries like China and India, where farms are small, are likely using less aggregate water than if large farms were to allocate irrigation water. The broader implication is that if structural transformation were to occur in the agriculture sector in these countries, this will exacerbate the water depletion problem (Foster and Rosenzweig, 2022).

How is this spatial equilibrium impacted when there is depletion in the underlying groundwater aquifer? Our second main result is that if the aquifer is depleted to the extent that the water stock is insufficient to meet demand, then this can lead to a perverse boom in well drilling due to a rise in water prices. This cannot happen when farm sizes are large so that well owners and farmers are the same entity. However, if the water stock is sufficiently small, then prices can rise no further than the monopoly price. This results in fewer wells since the scarcity of water is no longer compensated by higher prices. With further reductions in the available stock of groundwater, the area under irrigation declines and wells continue to exit the market, leaving parts of the command area without irrigation. This phenomenon leads to an inverted U-shaped curve of well density with respect to the stock of water available in the underlying aquifer.

Our framework suggests that when fixed costs of well drilling are low, the

Bertrand equilibrium leads to over entry and policies that tax well-drilling may increase welfare. Static inefficiencies in water pricing can be corrected through small subsidies to the pumping cost; however significant depletion externalities may call for taxing pumping cost as well.

The economic literature on groundwater pumping has focused on the dynamics of depletion and market failures arising from common property access to water aquifers (e.g., see Gisser (1983), Holland and Moore (2003) and Gaudet et al. (2002)). Other studies have examined the institutional aspects of groundwater irrigation (e.g., see Ostrom (1990), Libecap (2010)). Our innovation in this paper relative to previous work is to explicitly introduce the spatial characteristics of groundwater extraction and use — relevant in developing countries where farm sizes are small. The paper extends the theory of spatial product differentiation, e.g., see Mas-Colell et al. (1995) for an overview and Salop (1979), Borenstein (1985) and Norman and Thisse (1996)) in a novel setting.

A broader contribution of the paper is to develop a conceptual model to understand the economic behavior of a large number of small farmers in the developing world who depend on groundwater to produce food crops, and participate as buyers and sellers in water markets; see Chakravorty et al. (2023) for a recent example from Bangladesh.[3] We show how depletion may lead to the digging of more wells, and ultimately a shrinking of the irrigated area. On a larger scale, groundwater management policies may critically affect the viability of smallholder agriculture, which is responsible for producing a major share of food production in the developing world.

Section 2 develops a spatial model of a groundwater market. Section 3

[3]A typical small farmer needs a pump for about 200-250 hours a year (Shah, 2010). However, once the investment is made, a pump can run for about 3,000 plus hours a year, and could supply water to other farmers as well. Another reason for the prevalence of water trading is fragmentation of land-holdings. Banerji et al. (2012) in their study of water trading in western Uttar Pradesh, find that each farmer in the village owned an average of 3.2 plots, in two or more non-contiguous fragments. Only 10% of the farmers in their study did not buy or sell water.

studies the effect of depletion. Section 4 compares water allocation in small vs large farms. Section 5 concludes the paper.

2 A Model of Wells in Space

We consider a simple setting where a continuum of identical farms is distributed uniformly on top of an aquifer along a Salop circle of unit circumference. For analytical convenience, we ignore intra-farm distances and assume all farms to be points in space. The demand for water at any farm is assumed to be a decreasing function $D(p)$ where p is the price of water.

Suppose there are n wells spaced evenly on the circle (see Figure 1). Let us assume for now that the wells are fixed in space. Later, we consider the entry and exit of wells. Each well sells water to farms located on either side along the circle. Let the average cost of pumping a unit of water from the aquifer vertically to the surface be a constant $c > 0$ and the unit cost of transporting water over distance x along the circumference of the circle be $t(x)$ with $t > 0$.[4]

Note that the marginal cost of pumping water to a farm located at distance x is thus given by $c + tx$. The presence of transport costs introduces differentiation among buyers because it costs more to deliver water to buyers located farther away.[5]

Let us temporarily assume that the monopoly price of water at the wellhead (where the seller is located) is at least $c + \frac{t}{n}$.[6] That is, $p_m(0) \geq c + \frac{t}{n}$,

[4]Irrigation water is generally conveyed through earthen ditches or pipes, which suffer from significant friction losses. Hydrologists assume a standard loss ratio due to conveyance of water from tubewells to be about 15% for every 100 meters traveled. Portable, plastic pipes are popular in both India and China, and they can convey water to about 300 meters. Open and lined ditches are often used in developing countries, and they suffer from significant losses from evaporation, seepage and percolation, which may also be thought of as transport costs. They are proportional to volume and distance traveled.

[5]We consider zero and non-linear transport costs later in the paper.

[6]We will see in the next section that this will hold for only some configurations of parameter values.

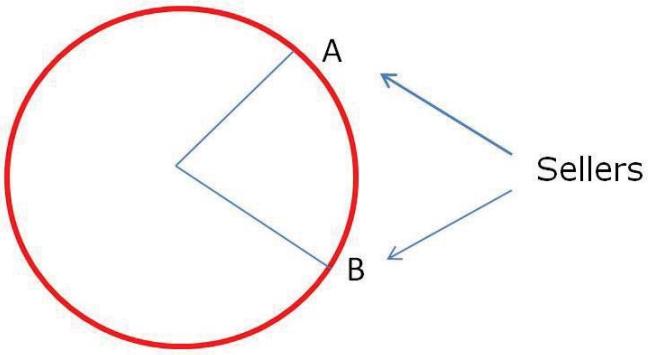


Figure 1: Buyers are located on a continuum along the Salop circle of unit circumference; here sellers are located at points A and B.

where $p_m(x)$ denotes the price charged by a monopoly seller of water to a farmer located at distance x.[7] As we will see below, this assumption is sufficient to rule out monopoly behavior and leads to Bertrand competition at each location. It says that the monopoly price is "high enough" and will hold if the unit cost of pumping c and the unit transport cost of water t are sufficiently low.

What is the price of water in this model with a fixed number of water sellers? See Figure 2 which shows two sellers with wells located at A and B. Consider the buyer at C located at distance x from the seller at well-head A, as shown. A's marginal cost of sending water to C is given by $c + tx$ (the dashed line) which is strictly less than the marginal cost to seller B of sending water to buyer C, given by $c + t\left(\frac{1}{n} - x\right)$, the solid line. With n sellers and unit circumference, the distance between each seller is $\frac{1}{n}$. Therefore, seller A can capture the entire demand of buyer C by selling at just under seller B's marginal cost. The price seller A charges at location C is $c + t\left(\frac{1}{n} - x\right)$. Consumers located closer to well A are charged higher prices, with the closest consumer at A paying $c + \frac{t}{n}$ which is less than the monopoly price at the

[7]Note that p_m is a function of distance x and increases with the marginal cost of water.

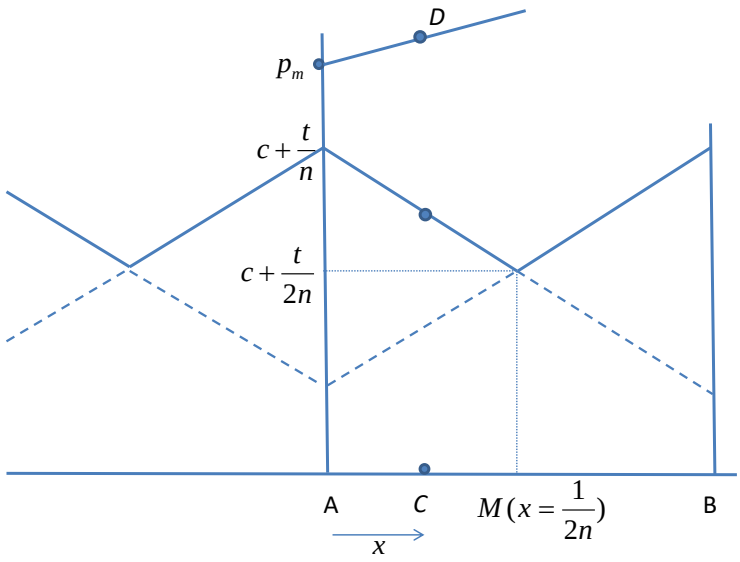


Figure 2: Price and marginal cost paths for seller A. Point M is the location of the buyer midpoint between adjacent sellers.

wellhead, by assumption. Since the monopoly price at x (see point D) is greater than the monopoly price at the well-head, it is greater than $c + t\left(\frac{1}{n} - x\right)$. Therefore, this price is profit maximizing for A: lower prices at this location x will lower profits since they will be farther away from the monopoly price. Thus, we can write the Bertrand price as a function of distance from the nearest seller x and the number of sellers n as

$$p(n, x) = c + t\left(\frac{1}{n} - x\right), \tag{1}$$

which is decreasing in x. We can therefore state the following:

If the monopoly price is large enough, the price of water charged to the buyer decreases with her distance from the well.

Intuitively, each seller will split the market with her neighbors on either side. On each side, she will supply to buyers located from zero to distance $\frac{1}{2n}$. Even though it costs more to transport water to farms further away, the more distant buyers pay a lower price than those nearby. The reason is Bertrand competition between a seller and her immediate neighbor. Each seller is unable to charge a price higher than what it costs to the buyer to receive water from the nearest competitor. The price of water is simply equal to the nearest neighbor's marginal cost in order to cut that neighbor out of the market. Although the market is completely segmented, with each farm buying water exclusively from the nearest well, it is a series of Bertrand duopolies.[8]

The total profit to a seller when there are n of them is given by

$$\pi(n) = 2\int_0^{\frac{1}{2n}} \left[p(n,x) - (c+tx)\right] D(p(n,x))dx.$$

Substituting for $p(n,x)$ from (1), we get the profit under the Bertrand regime (in which all sellers face Bertrand prices) as

$$\pi(n) = 2\int_0^{\frac{1}{2n}} t(\frac{1}{n} - 2x)D(p(n,x))dx. \tag{2}$$

From (2) note that the mark-up $t(\frac{1}{n} - 2x)$ is *independent* of the cost of pumping water to the surface.[9] This is because the cost is the same for the Bertrand competitor and the mark-up depends on the *difference* in marginal costs of the two sellers at each location.

[8]That distant buyers may pay a lower price, has been observed by Norman and Thisse (1996) in a model of spatial competition in which firms price discriminate among uniformly located buyers with inelastic demand.

[9]If the unit pumping cost c is large enough, then our assumption of a sufficiently high monopoly price may fail, in which case firm profits may not be independent of pumping costs. We discuss this case later in the paper.

Increased entry implies lower water prices. Conversely, equation (1) shows that a decrease in the number of sellers n raises the price of water at every location. Since by assumption, this price is less than the monopoly price at every location, the decrease in n must increase the profit at every location because prices approach the monopoly price. Since a lower n expands a seller's market (see condition (2)), it must result in an increase in every seller's profit. Profits $\pi(n)$ decrease in n. As the number of firms increases, both the maximum and minimum prices converge to the unit pumping cost c.[10]

3 Equilibrium Entry of Sellers

Until now, the number of sellers was fixed. We now allow for entry and exit of wells in the market. A seller will enter the market only if her profit exceeds the fixed cost of digging a well and installing the pump, which we aggregate and denote by $F > 0$.[11] With free entry, firms will enter the market until profits are driven to zero. That is, the *equilibrium well density* let us call it n_e, must satisfy[12]

$$\pi(n_e) = F. \tag{3}$$

We will continue with a general demand function which is continuous and downward sloping with choke price $\bar{p}$ at which demand for water is zero.

[10] The well-head price is $c + \frac{t}{n}$ and the minimum price mid-point between two sellers is $c + \frac{t}{2n}$, see equation (1): both converge to c as n goes to infinity.

[11] The cost of digging a well may depend on the type of soil, for example, alluvial soils such as in the Indo-Gangetic Plain in northern India may be easier to drill, relative to the hard rock aquifers that characterize the south of the country: the fixed cost may depend on the depth of the water table and capacity of the pump, issues we discuss later. The cost may be higher in areas where permits are required to install a pump or an electricity connection is needed, that is, when electricity is available.

[12] Since we have normalized the circumference of the Salop circle to unity, the number of firms can be interpreted as the well density. This allows us to treat n as a continuous variable.

It is important to understand how the market structure evolves beginning with very few wells along the circle. Define $\underline{n}$ as the minimum number of wells such that the entire circle is irrigated. This implies that for the farmer in the middle between two adjacent wells, the choke price $\bar{p}$ should not be lower than the price of water at that location, or $\bar{p} \geq c + \frac{t}{2n}$. Since $\underline{n}$ is the smallest such number, $\underline{n} = \frac{t}{2(\bar{p}-c)}$.

When there are very few wells so that the well density approaches zero, the monopoly price at the well-head is much lower than the Bertrand price of the competitor $c + \frac{t}{n}$ so the well owner charges the monopoly price all the way until the monopoly price $p_m(x)$ hits the choke price. Beyond this point, there is no irrigation until the area served by another well is reached. As new wells enter, they occupy the unirrigated land, but the profit from each well remains constant until n equals $\underline{n}$ at which point all farms are irrigated.

We note that this configuration provides the maximum possible profit $\pi(\underline{n})$ for sellers (for a given demand curve and cost parameters c and t), when they are unconstrained by competitors. If the fixed cost $F > \pi(\underline{n})$, there will be no irrigation. If it falls below this level, this will spark entry. This is seen in Fig.3.

When n exceeds $\underline{n}$ but not by much, then farms near the midpoint between two wells face Bertrand prices while those closer to the nearest well face monopoly prices as depicted in Figure 4. Profits fall as n increases beyond $\underline{n}$ because each seller's market shrinks and also because sellers receive the Bertrand price rather than the monopoly price at some of the farms that they serve. Moreover, as competitors are closer to each other for higher n, the Bertrand prices received at any given farm also fall with n.

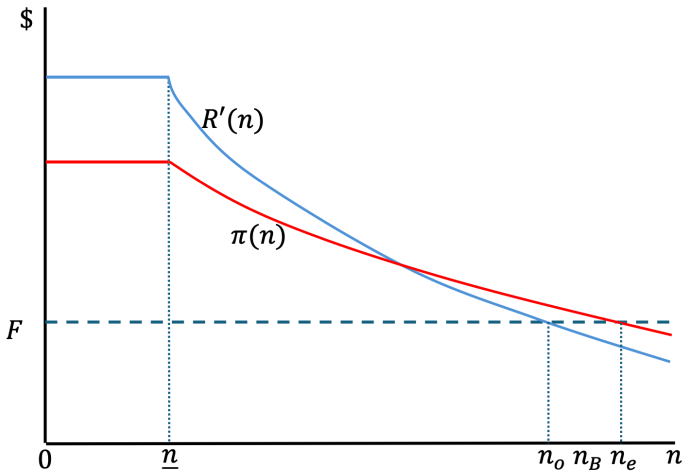


Figure 3: For very low well densities, both $R'(n)$ and $\pi(n)$ are constant since there is still fallow land available. Beyond $\underline{n}$, there is no more land available, and wells compete with their neighbors. Both curves decline and eventually cross.

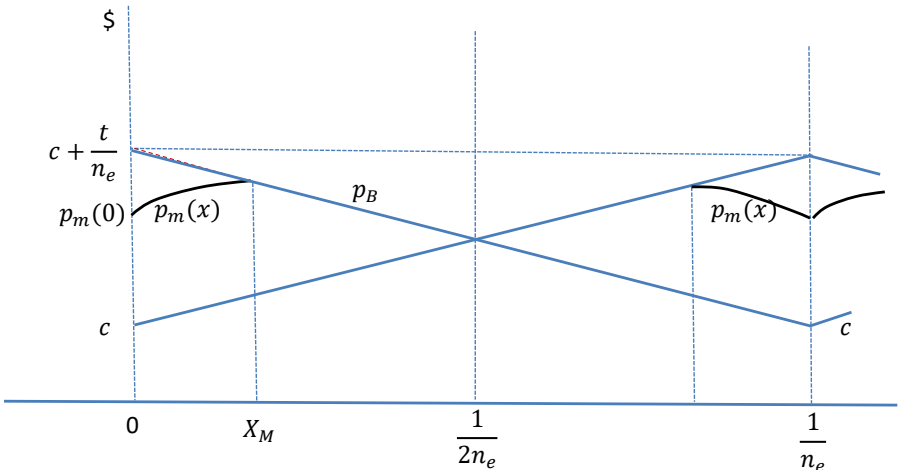


Figure 4: The mixed regime with a monopoly segment near the well head followed by a Bertrand segment. Wells are sufficiently further apart that the monopoly price at the well-head is lower than the Bertrand price, until location X_M.

For sufficiently high values of n, the monopoly pricing segment disappears since $c + \frac{t}{n}$ falls below $p_m(0)$, and all farms face Bertrand prices. We saw earlier that in this Bertrand regime, profits are decreasing in n. Putting this together with the argument above, we conclude that $\pi(n)$ is constant for $n \leq \underline{n}$, and strictly decreasing thereafter as depicted in Fig.3. Thus the equilibrium well density n_e given by $\pi(n_e) = F$ is strictly decreasing in F for $F < \pi(\underline{n})$. When $F > \pi(\underline{n})$, there is no irrigation in equilibrium.

We now derive, in the case of linear demand, an explicit expression for the minimum value of well density such that the pricing regime is pure Bertrand with no monopoly segment. The monopolist maximizes profits $\pi_m(x) = pq - (c + tx)q$ which, upon substituting the linear demand curve $q = a - bp$ and differentiating, yields the monopoly price $p_m(x) = \frac{1}{2}[\frac{a}{b} + (c + tx)]$. Hence, the monopoly price at the well head is $p_m(0) = \frac{1}{2}(\frac{a}{b} + c)$. This price must not be lower than the Bertrand price $c + \frac{t}{n}$. Equating them we obtain $n_B = \frac{2t}{(\frac{a}{b} - c)}$. That is, when well density exceeds n_B the regime is pure Bertrand. We note that $n_B = 4\underline{n}$ for this linear demand case.

4 Water use relative to the large farm case

In this section we compare the model of water markets developed above that is suitable when farm sizes are small to the situation that would prevail if farm sizes are large enough that it would generally not be economical to transport water to neighboring farms. This is the case, for example, in the western United States. Here, the well owner and the farmer are the same entity. The goal is to see whether imperfect competition in the small farm case leads to more or less aggregate water use than in the large farm case.

Since the aquifers are open access resources in both cases, this has led to their depletion that is a major concern both in Asia and in the United States. A question of central importance, therefore, is whether the groundwater markets generated by the presence of small farms lead to more or less depletion

than would occur if farm sizes were large and there were no groundwater markets. In other words, we want to know if the presence of water markets reduces or increases groundwater depletion.

We model the large farm as one with a single farmer who must choose the number of wells n on her land that consists of the entire Salop circle. The farmer's net revenue or surplus net of pumping and water transport cost is given by

$$R(n) = 2n \int_0^{\frac{1}{2n}} \int_0^{D(c+tx)} \left[D^{-1}(q) - (c+tx)\right] dqdx. \tag{4}$$

We note that for $n < \underline{n}$, the net revenue per well $\frac{R(n)}{n}$ is constant and strictly greater than $\pi(n)$ because the well-owning farmer captures the entire surplus at every farm up to a distance of $c + \frac{t}{2\underline{n}}$ from the well while the seller in the small farm case gets only the monopoly profit. As n increases beyond $\underline{n}$, the revenue per well $\frac{R(n)}{n}$ falls continuously since the area served by each well starts to decline. Thus, the marginal revenue $R'(n) = \frac{R(n)}{n}$ for $n < \underline{n}$ and decreases thereafter as well. Finally, we note that $R'(n)$ tends to $\frac{R(\underline{n})}{\underline{n}}$ as $n \to \underline{n}$ from the right.[13] $R'(n)$ is also plotted in Figure 3 above.

The large farm will irrigate if and only if $R(n) - F \geq 0$ or $F \leq \frac{R(n)}{n}$. The large farmer chooses well density by solving

$$\max_n R(n) - nF \tag{5}$$

which yields the first order condition $R'(n_o) = F$, where the subscript o represents the optimal value, i.e., for the single owner case.

Figure 3 plots the two functions $R'(n)$ and $\pi(n)$ along with the equilibrium well density in the small farm case n_e and the well density that is optimal for a large farmer n_o for a particular value of F.[14]

[13] $R'(n) = \frac{R(n)}{n} - \frac{1}{n}\int_0^{D(c+\frac{t}{2n})} \left[D^{-1}(q) - (c+\frac{t}{2n})\right] dq$. But $c + \frac{t}{2n} \to \overline{p}$ as $n \to \underline{n}$ from the right, so the second term vanishes at $\underline{n}$.

[14] The well density is optimal for the large farmer, but will not be socially optimal if

We now show that when water demand is linear, then the aggregate water use in the small farm case is always lower than in the case of a large farm with a single decision maker. We proceed in two steps. We first show that aggregate water use is lower in the small farm case than in the single farm case iff $n_e < 3n_o$. That is, when equilibrium well density is less than three times optimal well density, aggregate water use in the former is always lower. Next we assume that $n_e = 3n_o$ and derive a contradiction. Thus n_e must always be lower than $3n_o$.

See Figure 5. It shows the prices for the Bertrand equilibrium in the small farm case and for the single farm decision maker when well density in the small farm model is exactly equal to three times that of the large farm, i.e., $n_e = 3n_o$. The distance between adjacent wells on the large farm is $\frac{1}{n_o}$ and the distance between wells in the small farm is $\frac{1}{n_e}$. Between the two neighboring wells for the large farm, there are three wells for the small farm, as shown. The blue lines depict water prices in the large farm case and the red lines for the Bertrand case. Note that there are areas where the prices are exactly equal, hence water use will also be equal. They cancel out. However, Area A and C are regions where the Bertrand price is higher than in the large farm case, and area B is the opposite, where the large farm price is higher. Because of linear demand, the slope of the demand curve is the same at all prices, hence, the price differences are proportional to the quantity difference at any location. Since all the lines have an absolute slope of $|t|$ the aggregate area of the two triangles A+C exactly equals area B. Therefore aggregate water use over all the wells in the circle is equal as well. Now if n_e is lower, prices must go up as wells are further apart and so water use must fall. Thus, water use must be lower in the small farm case when $n_e < 3n_o$. This completes the first part of the proof. See Appendix for the remaining part of the proof which shows that $n_e < 3n_o$.

there is a depletion externality.

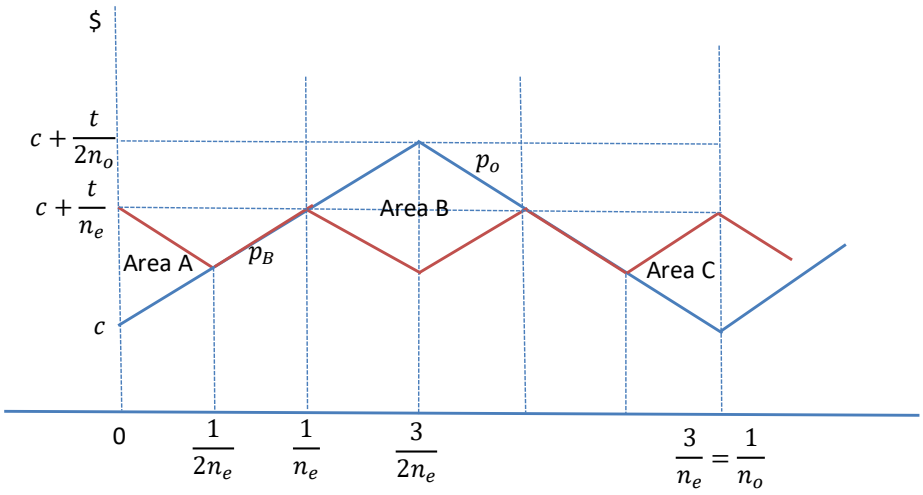


Figure 5: Bertrand and optimal prices p_B and p_o under linear demand when $n_e = 3n_o$. Aggregate water use is equal, i.e., Area B = Areas A+C.

We have computed the $\pi(n)$ curve assuming only the Bertrand solution in (14), that $n_e > n_B$ when $n_e = 3n_o$. If, instead, $n_e < n_B$ and the regime is mixed, then profits will be smaller than the notional Bertrand profit for such a value of n_e. Hence the true $\pi(n)$ lies below the notional $\pi(n)$ curve that we computed assuming Bertrand pricing. The implication that $n_o < \underline{n}$ then follows *a fortiori*, leading to the same contradiction.

We now appeal to the continuity of $\pi(n)$ and $R'(n)$ and the fact that $n_e < n_o$ for n sufficiently close to $\underline{n}$ to rule out $n_e > 3n_o$ for any F. This completes the proof of greater water use in the small farm case.

We now show that with linear demand, $\pi(n)$ crosses $R'(n)$ from below at a value of well density between $\underline{n}$ and n_B. This ensures that overcapitalization will occur unless fixed costs F are very high.

Note from Figure 3 that for small values of well density immediately to the right of $\underline{n}$, $\pi(n)$ is lower than $R'(n)$. If they cross, then at that value

equation (17) must equal equation (2). Equating the two, we obtain

$$n = \frac{4bt}{3(a - bc)} < \frac{2bt}{a - bc}. \tag{6}$$

The latter is the value of well density at the pure Bertrand equilibrium. This means that at the well density where marginal returns to a well equal firm profits, the regime is part monopoly and part Bertrand.

Again, we computed the $\pi(n)$ curve assuming only the Bertrand solution. When the regime is mixed, the true $\pi(n)$ lies below below the notional $\pi(n)$ curve that we computed assuming Bertrand pricing. Thus, overcapitalization occurs starting from well densities lower than

$$n = \frac{4bt}{3(a - bc)}.$$

We summarize this discussion as

Proposition 1 *When the demand for water is linear and downward sloping, then for any configuration of cost and demand parameters, there is less water used in equilibrium when farm size is small than when it is large. Furthermore, unless fixed costs are very high, there is over-capitalization in the small farm case, with too many wells.*

The above results suggest that when fixed costs are low, the policy maker can reduce entry by simply taxing the cost of entry, that is, taxing well drilling. This is clearly seen in Fig.3 where over entry occurs for fixed costs below the intersection of $R'(n)$ and $\pi(n)$. If fixed costs are high so that entry is lower than the optimal level, then a subsidy for well drilling will correct the inefficiency. Note also from (2) that imperfect competition induces static inefficiency: farms closer to the well head pay a mark-up over marginal cost and farms in the middle pay prices that equal marginal cost at their location. Thus, a small subsidy to pumping cost can reduce prices at the head while lowering prices at the mid-point between adjacent wells. This subsidy may

increase welfare for small values of the subsidy. However, if there is a dynamic cost of depletion in the aquifer, the social marginal cost at each location may be higher than the pumping cost, so that a tax on the pumping cost in addition to one on the fixed cost would be needed.

5 Entry with a limited water stock

In this section, we ask how the gradual depletion of the water stock may impact the equilibrium number of firms. We assume that wells on the Salop circle draw water from an aquifer which gets a constant annual recharge $R > 0$. For simplicity assume that the recharge occurs instantaneously at the end of each period. We continue to assume that demand is sufficiently inelastic as before.

We first consider the setting when pumping costs do not change as the aggregate water stock in the aquifer decreases with use. This situation is plausible when for example, farmers pay a flat fee for electricity. Recall the static Bertrand equilibrium under sufficiently inelastic demand. Let the aggregate water consumed by all the farms on the circle in a given time period be defined by W_B where the subscript B denotes the Bertrand equilibrium with no restriction on how much water is pumped up from the aquifer. Also define $\bar{W}$ be the aggregate stock water in the aquifer any time period. If the water used every period W_B consistently exceeds the annual recharge R, i.e., $W_B > R$, then at some point, the bottom of the aquifer will be reached. Eventually the total volume of water in the aquifer $\bar{W}$ will be less than the unconstrained stock allocated in the Bertrand equilibrium, i.e., $\bar{W} < W_B$.[15] Since firms are identical, this constrained stock of water $\bar{W}$ must be shared symmetrically among the incumbent firms.

Since water demand will now exceed the available stock at the Bertrand

[15] There is plenty of evidence to suggest that water tables in many agriculturally intensive regions of China and India have been falling steadily (Livingston, 2009).

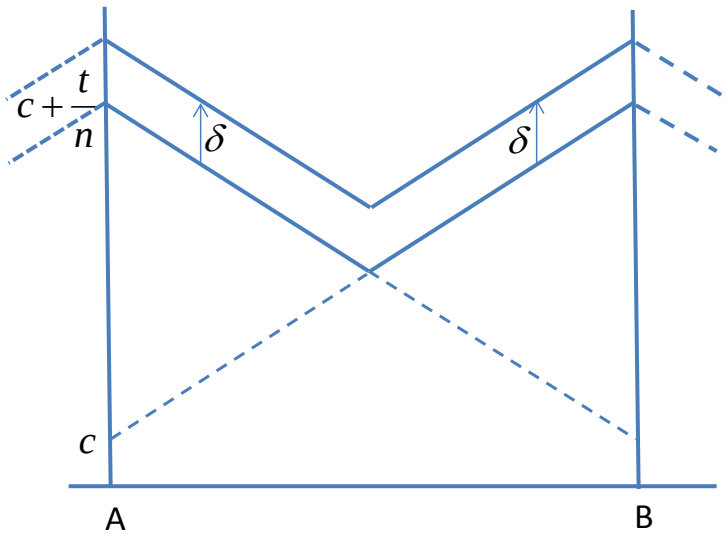


Figure 6: Bertrand equilibrium with scarce water: prices rise uniformly to equate aggregate demand and supply. The lower the aggregate stock of water, the higher is the shift parameter δ.

equilibrium, prices must jump by some $\delta > 0$ in order to equate water demand with water supply, as shown in Figure 6) The price increase must be uniform for all farms served by a given seller, so that the new price lines have the same slope t as before.[16]

At these higher prices, gross profits must be higher than in the Bertrand equilibrium, so there must be new entry. When n increases, this will reduce the price $t(\frac{1}{n}-x)+\delta$ at every farm, once again resulting in an excess demand

[16] As long as the price is below the monopoly price. We defer consideration of the case when the monopoly price is reached to later in this section. This is because the marginal unit of water sold by a seller at each location x must fetch the same mark-up δ over marginal cost $c+tx$. otherwise the seller could re-allocate water across farms and increase total profit. Hence the price jump must be the same for all sellers. If any seller had a greater δ than her neighbor, she would be undercut by her neighbor who could increase his margin on a unit of water by lowering the price. This argument shows that the mark-up δ over $c+t(\frac{1}{n}-x)$ cannot be greater for larger x. This is because the seller gets a greater mark-up for farmers closer to him, so he wants to allocate more water to farms closer to the well-head. But the slope of the price line cannot be steeper than t otherwise farmers near the well-head will buy water from those farther away since they will get a lower price net of transport cost. Arbitrage ensures that the slope can not be greater than t.

for water. There must, therefore be a further jump in prices, and the process will repeat. This process converges to a unique equilibrium follows:

Proposition 2 *For each level of the water stock $\bar{W}$, there exists a unique equilibrium well density $n(\bar{W})$. There exists a stock of water $W_M < W_B$ at which $n(\bar{W})$ is maximized. To the left of W_M equilibrium well density is increasing in $\bar{W}$ and not all buyers get water, while to the right of W_M, well density is decreasing and all buyers are served.*

Proof in Appendix.

Let us define the aggregate quantity of water demanded as a function of the price at the midpoint L between two sellers, $W(p(L))$. Since the slopes of the price paths are defined by the transport cost parameter t, the higher is $p(L)$, the lower is the aggregate stock of water consumed over the entire circle. Therefore $W'(p(L)) < 0$. For each given aggregate stock of water $\bar{W}$, there is a unique $p(L)$ and since costs are constant, there is a unique profit function and equilibrium number of firms $n^*(\bar{W})$ defined by $\pi(n^*, \bar{W}) = 0$.

When the aggregate quantity of water is unconstrained at W_B, we know that the profit function is downward sloping and the equilibrium number of firms is unique. As the aggregate water available declines below W_B, the price line must rise, as shown in Figure 6. For a given $\bar{W}$, the constrained Bertrand allocation must satisfy the following two conditions:

$$2n \int_0^{\frac{1}{2n}} D(p)dx = \bar{W} \tag{7}$$

and

$$\pi(n) = F. \tag{8}$$

The first condition can be re-written as

$$2n \int_0^{\frac{1}{2n}} q(c + t(\frac{1}{n} - x) + \delta)dx = \bar{W}. \tag{9}$$

where the parameter $\delta > 0$ adjusts such that the aggregate water use equals $\bar{W}$, the total constrained stock of water. These two equations can be solved for the two unknowns n and δ, given the parameters c, t, F and $\bar{W}$.[17] This constrained Bertrand equilibrium will hold for "small" declines in the water stock for which the price rise will not be large enough for the price at the wellhead to exceed the monopoly price.[18]

For a fixed number of firms n, a lower aggregate stock of water raises profits. Figure 7 shows how the aggregate water constraint shifts up the profit per firm. Since the profit function is downward-sloping, the new Bertrand equilibrium with higher prices has a higher number of firms.[19] Thus $n_e(W_B) < n_e(\bar{W})$, as shown in the proof of Proposition 2.

We have already shown that at the unconstrained Bertrand equilibrium, the equilibrium number of firms is higher than the optimal, i.e., $n_o(W_B) < n_e(W_B)$. The optimal number of firms must be lower when aggregate water is constrained at $\bar{W}$ than when it is unconstrained, i.e., $n_o(\bar{W}) < n_o(W_B)$.[20] Collecting all the inequalities, we have $n_o(\bar{W}) < n_o(W_B) < n_e(W_B) < n_e(\bar{W})$.

How far can prices rise with rising scarcity in the aquifer? With rising prices, profits increase because the price approaches the monopoly price, inviting new entry of tubewells. However, new entry cannot lead to lowering

[17] Prices rise by δ at each location, including at $\frac{n}{2}$.

[18] The consumer at the well-head pays the highest price, and if her price is lower than the monopoly price, all others are.

[19] These curves can not cross, because that would mean that there are two different stocks of water that yield the same profit which violates the existence of the unique price equilibrium. We assume a continuous n - with discrete values of n, we may obtain a weak inequality.

[20] With a lower stock of water, fixed costs of supply per unit of water can be reduced by decreasing the number of firms.

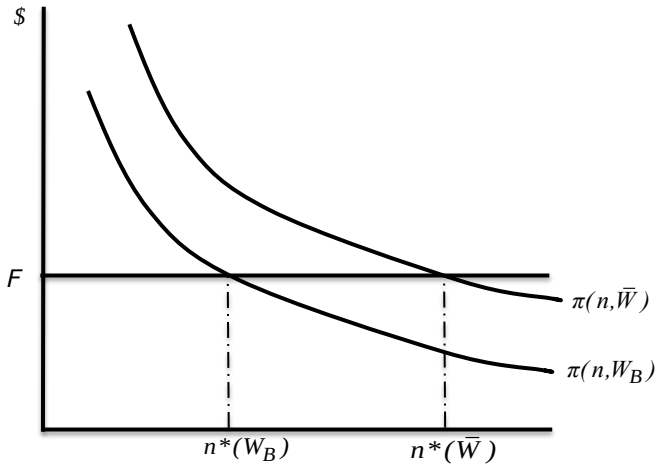


Figure 7: Firm's profit function shifts up with water scarcity, resulting in a higher equilibrium number of firms.

of prices because aggregate water use is constrained. But it can lead to market segmentation - each tubewell supplies a smaller area at higher prices and equilibrium profits equal zero. Ultimately prices reach monopoly prices beginning at the well-head where the price is highest, as shown in Figure 8.[21]

Monopolization extends outwards from the well-head until aggregate water supply equals aggregate demand along the circle. The monopoly price increases in x and then declines with slope t along the Bertrand price line to $\frac{1}{2n}$. However, given free entry, firm profits must always equal zero. Thus the monopolization is accompanied by new entry and a reduction in area served by each firm. Aggregate industry profits and the number of firms in equilibrium are maximized when water prices equal monopoly prices at every location along the circle. Bertrand prices are completely eliminated. All consumers are served.[22] Define this stock of water as W_M, shown in Figure **??**.

As the aggregate stock of water declines further from W_M, each firm

[21]Monopoly prices rise with marginal cost.

[22]Note that the number of firms n should be interpreted as well density: n can be any real number. If we restricted it to integers, then at monopoly prices, aggregate water may not equal aggregate demand and there may be a small area between wells without water.

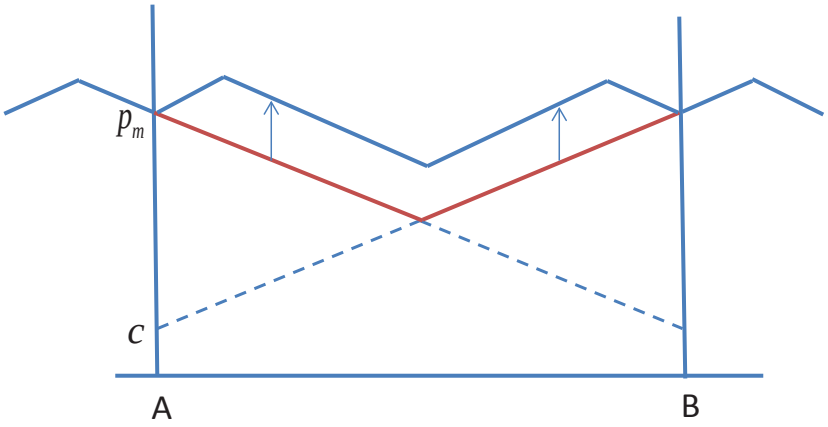


Figure 8: Increased water scarcity leads to monopoly pricing starting at the well-head: monopoly prices rise with marginal cost.

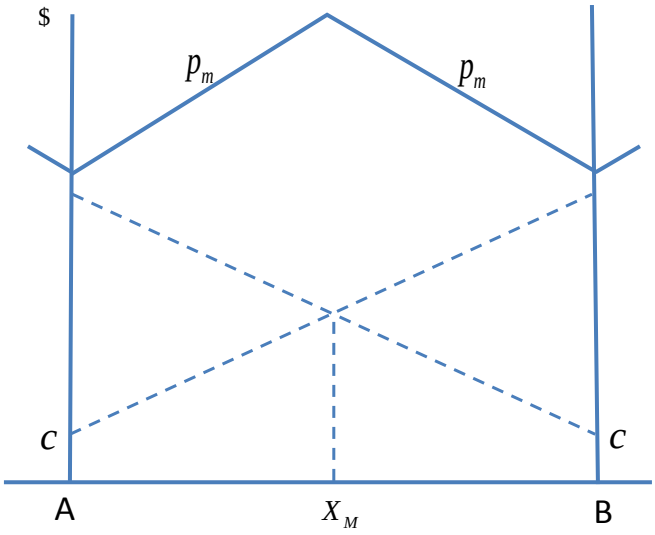


Figure 9: At the aggregate stock of water W_M, all consumers are served at their monopoly price. Each seller serves buyers until distance $X_M = \frac{1}{2n_M}$. Each firm makes zero net profits.

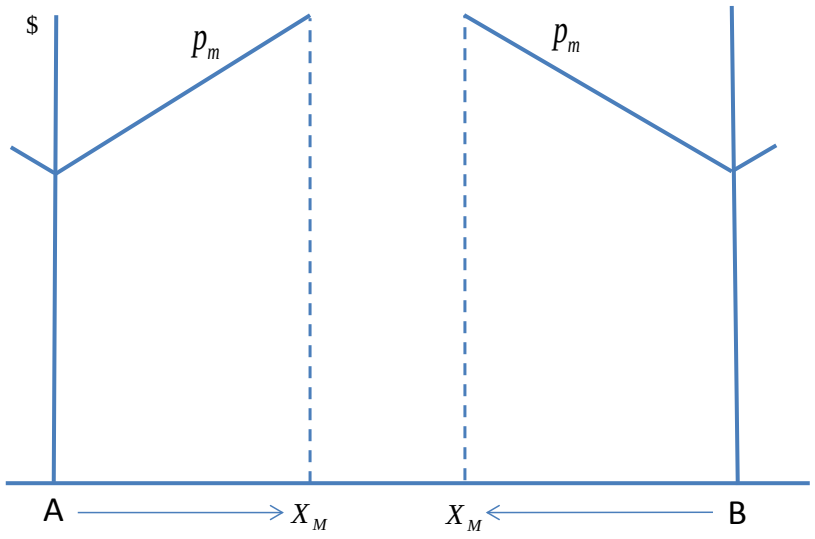


Figure 10: The aggregate stock of water is below W_M. Some firms exit leaving gaps in service in between wells, as shown. Each seller is still a local monopoly serving X_M consumers on either side, with zero net profits.

continues to be a local monopoly and supply to consumers until distance X_M on either side. But due to the lower stock of water, there emerge gaps in service between firms, as shown in Figure 10. The lower the stock $\bar{W}$, the lower the number of sellers remain, and fewer the number of consumers served between wells. At the end only one firm remains standing. When the water level goes below this $\bar{W}_1$, no well can cover its fixed cost. The terminal stock of water is $\bar{W}_1$, as shown in Figure 11. At a stock lower than this level, there is no service.

6 The large farm when water is scarce

Finally we study how well density for a single decision maker behaves with a declining stock of water in the aquifer. When the water supply constraint is binding, then the farmer can no longer supply all her land with water up to the point where the marginal value of water $D^{-1}(q)$ equals its marginal

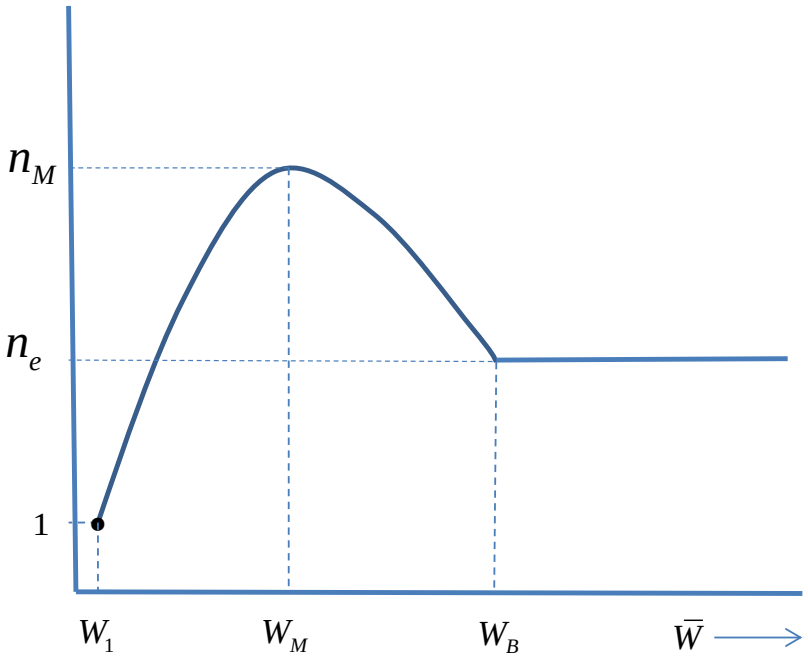


Figure 11: The number of firms in equilibrium as a function of the aggregate stock of water $\bar{W}$. At Bertrand equilibrium, W_B units of water are used. When the stock is below this level, prices rise leading to new entry. The number of firms peaks upon complete monopolization then falls as the stock of water depletes.

cost $c + tx$. She will, therefore, supply a quantity q of water to each part of her land such that the wedge δ between the marginal value $D^{-1}(q)$ and the marginal cost $c + tx$ is the same everywhere. The water supply constraint is thus given by

$$W(n, \delta) \equiv 2n \int_0^{\frac{1}{2n}} D(c + tx + \delta)dx = \overline{W}. \tag{10}$$

This condition implicitly defines δ as an increasing function of n since water demand is increasing in n and decreasing in δ. The single farmer's surplus is given by

$$R(n, \delta) \equiv 2n \int_0^{\frac{1}{2n}} \int_0^{D(c+tx+\delta)} \left[D^{-1}(q) - (c + tx)\right] dqdx. \tag{11}$$

where R represents surplus, with the additional argument δ that represents the water constraint. Like before, the farmer will maximize

$$R(n, \delta) - nF \tag{12}$$

subject to (10).[23]

The farmer's iso-profit curves in (n, δ) space are given by

$$R(n, \delta) - nF = \overline{\Pi}. \tag{13}$$

In the appendix, we show that these iso-profit curves have an inverted-U shape and are orthogonal to the n-axis. Clearly, lower iso-profit curves correspond to higher profits, and higher water-constraint curves correspond to lower water stocks. The solution to the unconstrained problem is $(n_0, 0)$ while the solutions to the constrained problems are given by the locus of tangencies of the water constraints with iso-profit curves. In the appendix,

[23]The surplus $S(n)$ in the unconstrained maximization problem (5) is obtained by setting $\delta = 0$ in $S(n, \delta)$.

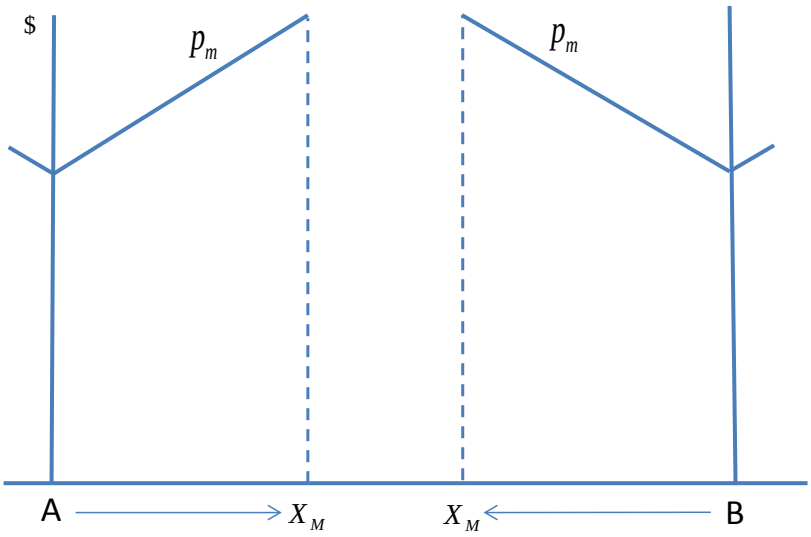


Figure 12: The aggregate stock of water is below W_M. Some firms exit leaving gaps in service in between wells, as shown. Each seller is still a local monopoly serving X_M consumers on either side, with zero net profits.

we prove that this locus is downward-sloping. Therefore, when the water constraint is binding, well density declines monotonically as the water stock falls. This is in sharp contrast to what happens when farm size is small and water scarcity leads to an increase in prices and triggers a race to drill.

Proposition 3 *With large farm sizes, well density must decrease monotonically as the water stock declines.*

Proof in Appendix.

7 Concluding Remarks

This paper addresses the issue of groundwater depletion, which is already a serious problem in the developing world. The main point of departure from the voluminous literature on the economics of groundwater is the explicit

recognition that farm sizes are small in developing countries and market interactions between these small farms need to be explicitly modeled in order to understand how market structure affects depletion. Previous studies do not address the issue of trading in groundwater, a phenomenon increasingly observed in countries with dense rural populations and small farm sizes.

We have shown that groundwater markets result in less depletion than what would occur if farm sizes were large. However, they also result in too many wells unless the fixed costs of tubewell installation are very high. Furthermore, this over-capitalization can get even worse when there is a limited stock of water as can happen in fragmented and depleted hard rock aquifers.

Our framework, although simple and highly stylized, provides a range of predictions that can be tested empirically. Low fixed costs in well drilling and low energy costs of pumping will lead to a higher well density. Higher costs of water transport and losses in conveyance will increase well density.

An important contribution of the paper is in providing an empirical setting for further analysis of the textbook Bertrand pricing model in situations where sellers face an aggregate stock constraint. To the best of our knowledge, this theory has not been developed in the industrial organization literature. Future research can also focus on a more explicit dynamic formulation of water scarcity in the context we describe and its effects on pricing. We have restricted ourselves to developing a static model of competition between tubewells, which generates insights on the effects of water depletion. However, we have abstained from explicit consideration of discount rates and other parameters that may impact dynamic extraction.

8 Appendix

Proof that the ratio $\frac{n_e}{n_o}$ must always be less than 3:
Assume that $n_e = 3n_o$. Note that at the equilibrium values of well density n_e and n_o, $\pi(n_e) = F = R'(n_o)$. Substituting $q = a - bp$ in the profit function given by (2) we get

$$\pi(n) = 2\int_0^{\frac{1}{2n}} \left[\frac{t}{n}\left(a - b(c + \tfrac{t}{n} - tx)\right)\right] dx \tag{14}$$

which after some simplification yields

$$\pi(n) = \frac{t}{2n^2}\left[a - bc - \frac{5}{6}\frac{bt}{n}\right]. \tag{15}$$

Letting $a - bc = A$, we get

$$\pi(n_e) = \frac{t}{2n_e^2}\left(A - \frac{5bt}{6n_e}\right) = F \tag{16}$$

Next we take the $R(n)$ equation from (4) and differentiate to get

$$\begin{aligned} R'(n) &= 2\int_0^{\frac{1}{2n}} \int_0^{D(c+tx)} \left[D^{-1}(q) - (c + tx)\right] dq\, dx \\ &\quad - 2\left(\frac{1}{2n}\right)\int_0^{D\left(c+\frac{t}{2n}\right)} \left[D^{-1}(q) - \left(c + \frac{t}{2n}\right)\right] dq \\ &= 2\int_0^{\frac{1}{2n}} \left[\int_0^{D(c+tx)} \left[D^{-1}(q) - (c + tx)\right] dq \right. \\ &\qquad \left. - \int_0^{D\left(c+\frac{t}{2n}\right)} \left[D^{-1}(q) - \left(c + \frac{t}{2n}\right)\right] dq\right] dx. \end{aligned} \tag{17}$$

Substituting the linear demand equation, equating it to F and simplifying we obtain

$$R'(n_o) = \frac{t}{2n_o^2}\left(\frac{A}{2} - \frac{bt}{6n_o}\right) = F \tag{18}$$

Substituting $3n_o$ for n_e in (16), using our assumption that $n_e = 3n_o$ and (18) we get

$$\frac{t}{18n_o^2}\left(A - \frac{5bt}{18n_o}\right) = \frac{t}{2n_o^2}\left(\frac{A}{2} - \frac{bt}{6n_o}\right).$$

Simplifying, we get

$$n_o = \frac{22bt}{63A} = \frac{44}{63A}\underline{n} < \underline{n}. \tag{19}$$

This is a contradiction since the existence of n_e implies that $F \leq \pi(\underline{n})$ which implies that $n_o > \underline{n}$.

■

Proof of Proposition 2:

We begin by defining W_M. First, we define x_M to be the farthest buyer served by a seller whose gross profit exactly covers its fixed cost when it charges the monopoly price at every farm. Next, let $n_M \equiv 1/2x_M$ so that n_M is the number of sellers that would result in every buyer getting water from the nearest seller if each seller charged the monopoly price. Now define W_M to be the water stock $\bar{W}$ at which, if n_M sellers serve a length x_M on either side of their wells and charge the monopoly price at every farm, then water demanded equals the available water stock (see Figure **??**).

Let $\bar{W} \in [W_M, W_B)$. We now show that there exists a unique equilibrium in which all farms get water. Informally, the idea is to follow a hypothetical process of convergence in prices and the number of firms n that would occur

if the water supply were to suddenly contract from a level higher than (or equal to) W_B to $\bar{W}$. Let n_e be the equilibrium number of sellers in a Bertrand equilibrium with no water supply constraint. Define

$$p(x,n,\delta) = \begin{cases} c + t\left(\frac{1}{n} - x\right) + \delta & \text{if this LHS } \leq p^m(x), \text{ the monopoly price at } x, \\ p^m(x) & \text{otherwise.} \end{cases} \tag{20}$$

Now water demand

$$W(p(x,n,\delta)) = 2n \int_0^{1/2n} D(p(x,n,\delta))dx \tag{21}$$

is a strictly decreasing and continuous function of δ with $W(p(x,n_e,0)) = W_B$ and $W(p(x,n_e,\delta)) < W_M$ for high enough δ high enough.[24] Therefore, there exists a unique $\delta_1 > 0$ such that

$$W(p(x,n_e,\delta_1)) = \bar{W}. \tag{22}$$

We may think of $p(x,n_e,\delta_1)$ as the jump in prices that would occur when the aggregate water stock contracted to $\bar{W}$ from something greater than W_B. Since $p(x,n,\delta_1) > p(x,n,0)$, meaning that the strict inequality holds for a range of x of positive measure, and the weak inequality holds for all x, it follows that each seller's profit when there are n_e sellers and prices are given by $p(x,n_e,\delta_1)$, is given by

$$\pi(n_0, p(x,n_e,\delta_1)) > \pi(n_e, p(x,n_e,0)) = F. \tag{23}$$

Therefore, there exists $n_1 > n_e$ such that

$$\pi(n_1, p(x,n_1,\delta_1)) = F, \tag{24}$$

[24]For large enough δ, the price at every location will equal the monopoly price, raising profits above F. Therefore, $n_e < n_M$, so that prices are higher for $x > x_M$ when there are n_e rather than n_M sellers. So $W(p(x,n_e,\delta)) < W_M$.

assuming water supply was unconstrained. Since $n_1 > n_e$ but $\pi(n_1, p(x, n_1, \delta_1)) = F = \pi(n_e, p(x, n_e, 0))$, so that sellers have smaller markets but make the same profits, it must be the case that they are charging higher prices: $p(x, n_1, \delta_1) > p(x, n_e, 0)$ for all $x \in [0, 1/2n_1]$. By an argument similar to that made in the footnote above, we conclude that water demand with n_1 sellers charging prices $p(x, n_1, \delta_1)$ satisfies the inequality

$$W_1 < W_B. \tag{25}$$

Since prices are lower after entry resulted in n_1 sellers, we also know that $\bar{W} < W_1$.

We can repeat this process to construct a monotone decreasing sequence of water demands $\{W_i\}_{i=1}^{\infty}$ that is bounded below by $\bar{W}$. This sequence must, therefore, have a limit that is not less than $\bar{W}$. In fact, the limit must equal $\bar{W}$, otherwise the associated sequence $\{\delta_i\}_{i=1}^{\infty}$ would converge to a positive constant, contradicting the fact that prices cannot rise above the monopoly price. Thus, the number of sellers and price functions at the limits of the associated sequences of n and p constitute an equilibrium. An equilibrium, therefore, exists.

To see that it is unique, suppose n^* is an equilibrium well density for some $\bar{W}$. If $n_1^* > n^*$ is also an equilibrium, profits have to be the same for both, which is only possible when the price function for n_1^* is higher than that for n^*. However, as seen earlier, water demand with n_1^* sellers would be lower than $\bar{W}$, so that n_1^* cannot be an equilibrium well density.

Now let $\bar{W} < W_M$. If $\bar{W}$ is less than the water demand when there is a single seller serving x_M farms on each side (and breaking even at the monopoly price), then there can be no water supply and the equilibrium number of wells must be zero. If $\bar{W}$ is between this level and W_M, then it is clear that the equilibrium n must be less than n_M by exactly enough to equate water demand with water supply when every seller serves x_M farms

on either side and charges the monopoly price at all farms. In this range of $\bar{W}$, not all farms get water. ■

Proof of Proposition 3:

Define net surplus as

$$S(n,\lambda) = R(n,\lambda) - nF \tag{26}$$

where $R(n,\lambda)$ is gross surplus. Totally differentiating and reorganizing we get

$$\frac{d\lambda}{dn} = -\frac{\partial S/\partial}{\partial S/\partial\lambda} = \frac{-(\partial R/\partial n - F)}{\partial R/\partial\lambda} \tag{27}$$

where

$$R(n,\lambda) = 2n\int_0^{1/2n}\int_0^{D(c+tx+\lambda)} \left(D^{-1}(q) - (c+tx)\right) dqdx.$$

Since $D'(\cdot) < 0$, $\partial R/\partial\lambda < 0$. Therefore $sgn(\frac{d\lambda}{dn}) = sgn(\frac{\partial R}{\partial n} - F)$. Differentiating $R(n,\lambda)$ with respect to n gives

$$\begin{aligned}\frac{\partial R}{\partial n}(n,\lambda) &= 2\int_0^{\frac{1}{2n}}\int_0^{D(c+tx+\lambda)} \left[D^{-1}(q) - (c+tx)\right] dqdx \\ &\quad - 2n\left(\frac{1}{2n^2}\right)\int_0^{D\left(c+\frac{t}{2n}+\lambda\right)} \left[D^{-1}(q) - \left(c+\frac{t}{2n}\right)\right] dq \\ &= 2\int_0^{\frac{1}{2n}}\left[\int_0^{D(c+tx+\lambda)} \left[D^{-1}(q) - (c+tx)\right] dq \right. \\ &\quad \left. - \int_0^{D\left(c+\frac{t}{2n}+\lambda\right)} \left[D^{-1}(q) - \left(c+\frac{t}{2n}\right)\right] dq\right] dx\end{aligned} \tag{28}$$

See Fig.13. The RHS of the above equation equals the area in the top

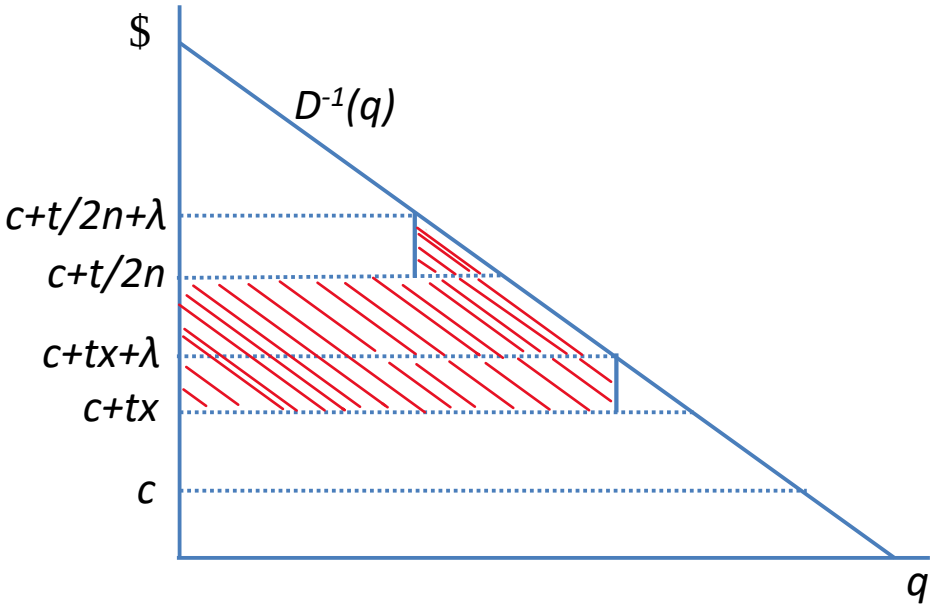


Figure 13: Figure shows the areas in the $\frac{\partial R}{\partial n}(n, \lambda)$ expression.

left of the triangle that is not shaded. The difference between the two inner integrals is the difference between the areas of the two triangles in the figure. This is clearly positive, decreasing in n, and approaches zero as n tends to infinity. It follows that the first-order condition defines a maximum.

For a given value of λ, the first integral inside the square bracket is always larger than the second so that $\frac{\partial R}{\partial n}(n,\lambda) > 0$. Note that as n increases, $\frac{t}{2n}$ decreases, so the term inside the second integral becomes larger, and the upper bound of this integral also increases. Hence the value of the second integral increases. Therefore the term inside the square brackets decreases and $\frac{1}{}$ decreases as well. Thus $\frac{\partial R}{\partial n}(n,\lambda)$ is strictly decreasing in n. As n goes to infinity, $\frac{\partial R}{\partial n}$ goes to zero.

When $\lambda = 0$, $\frac{\partial R}{\partial n} - F > 0$ otherwise there would be no drilling. Since $\partial R/\partial\lambda < 0$, by continuity, there must be some value of λ such that $\frac{\partial R}{\partial n}(0,\lambda) - F > 0$. As $n \to 0+$, for fixed λ, $\frac{\partial R}{\partial n} - F$ approaches a finite upper bound.

In Fig.14, we plot iso-profit curves $S(n,\lambda) = K$ as shown. Lower curves denote higher profits since the value of λ is smaller. From (27), $sgn(\frac{d\lambda}{dn}) = sgn(\frac{\partial R}{\partial n} - F)$ which is positive for small n and negative for large n, as shown in the figure. Differentiating (8) with respect to λ we get

$$\frac{\partial R}{\partial \lambda} = 2n \int_0^{\frac{1}{2n}} [D'(c + tx + \lambda)\lambda]\, dx \tag{29}$$

Note that $\frac{\partial R}{\partial \lambda} \to 0$ as λ goes to zero. From (27), this implies that $\frac{d\lambda}{dn} = \infty$ as λ goes to zero, as shown in the figure.

The water supply constraint is given by (10). W is strictly increasing in n since more wells implies that each location on the circle is closer to a well, hence $c+tx$ is lower, thus water used $D(c+tx+\lambda)$ is higher. Higher λ implies less water used, hence the locus of $W(n,\lambda) = \overline{W}$ is upward sloping as shown in figure. Contours in the north-west direction imply a tighter constraint or lower $\bar{W}$. In the figure, n_o denotes the number of firms with an unconstrained water stock.

Differentiating (10) with respect to n we get

$$\begin{aligned}\frac{\partial W}{\partial n} &= 2\int_0^{\frac{1}{2n}} D(c+tx+\lambda)\,dx - 2n\frac{1}{2n^2}\Big(c+\frac{t}{2n}+\lambda\Big) \\ &= 2\int_0^{\frac{1}{2n}} \Big[D(c+tx+\lambda) - D\Big(c+\frac{t}{2n}+\lambda\Big)\Big]\,dx > 0\end{aligned} \tag{30}$$

If the water supply constraint is binding, then the optimal values of n and λ must be at the point of tangency of the two curves $S(n,\lambda)$ and $W(n,\lambda)$ as shown in Fig. Using (27) this must satisfy the following equality:

$$\frac{d\lambda}{dn} = \frac{-(\frac{\partial R}{\partial n} - F)}{\frac{\partial R}{\partial \lambda}} = \frac{-\frac{\partial W}{\partial n}}{\frac{\partial W}{\partial \lambda}}. \tag{31}$$

which can be re written as

$$\frac{2\displaystyle\int_0^{\frac{1}{2n}}\Bigg[\int_0^{D(c+tx+\lambda)}\big(D^{-1}(q)-(c+tx)\big)\,dq - \int_0^{D\left(c+\frac{t}{2n}+\lambda\right)}\Big(D^{-1}(q)-\Big(c+\frac{t}{2n}\Big)\Big)\,dq\Bigg]dx - F}{2n\displaystyle\int_0^{\frac{1}{2n}} D'(c+tx+\lambda)\,dx}$$

$$= \frac{2\displaystyle\int_0^{\frac{1}{2n}}\Big[D(c+tx+\lambda) - D\Big(c+\frac{t}{2n}+\lambda\Big)\Big]\,dx}{2n\displaystyle\int_0^{\frac{1}{2n}} D'(c+tx+\lambda)\,dx}.$$

We cancel the two denominator terms, and rearrange to get

$$f(n(\bar{W}),\lambda(\bar{W})) = 0 \tag{32}$$

where

$$f(n(\bar{W}),\lambda(\bar{W})) = 2\int_0^{\frac{1}{2n}}\Bigg[\int_0^{D(c+tx+\lambda)}\left[D^{-1}(q)-(c+tx)\right]dq - \int_0^{D\left(c+\frac{t}{2n}+\lambda\right)}\left[D^{-1}(q)-\left(c+\frac{t}{2n}\right)\right]dq\Bigg]dx - F - 2\lambda\int_0^{\frac{1}{2n}}\left[D(c+tx+\lambda)-D(c+\tfrac{t}{2n}+\lambda)\right]dx = 0. \tag{33}$$

From (10), since demand is downward sloping, $\lambda'(\bar{W}) < 0$, i.e., a tighter water constraint increases the scarcity premium λ. Therefore we want to show that remaining terms on the right hand side $\partial f/\partial\lambda$ and $\partial f/\partial n$ are of the same sign. That will establish that $n'(\bar{W}) > 0$. This implies

$$\frac{\partial f}{\partial\lambda} = 2\int_0^{\frac{1}{2n}}\Big[D'(c+tx+\lambda)\big(c+tx+\lambda-(c+tx)\big) - D'\left(c+\frac{t}{2n}+\lambda\right)\left(c+\frac{t}{2n}+\lambda-(c+tx)\right)\Big]dx. \tag{34}$$

since the first two terms and the last term cancel. Repeating for $\partial f/\partial n$, we have

$$\frac{\partial f}{\partial n} = \frac{-1}{n^2}\int_0^{D\left(c+\frac{t}{2n}+\lambda\right)}\left[D^{-1}(q)-\left(c+\frac{t}{2n}\right)\right]dq + 2\int_0^{\frac{1}{2n}}\left[-D'\left(c+\frac{t}{2n}+\lambda\right)\left(\frac{t}{2n^2}\right)\right]\left[D(c+\tfrac{t}{2n}+\lambda)-\big(c+\tfrac{t}{2n}\big)\right]dx - 2\lambda\int_0^{\frac{1}{2n}}D'\left(c+\frac{t}{2n}+\lambda\right)\left(\frac{t}{2n^2}\right)dx. \tag{35}$$

Note that the first and the fourth terms cancel out, the second term

cancels with the last term, and the third term is negative. This proves that $n'(\bar{W}) < 0$. That is, when the available stock of water $\bar{W}$ declines, the number of wells increases.

Since $W(n, \lambda) = \bar{W}$ is strictly increasing, the point of tangency is shown in Figure 14 for a given $\bar{W}_1$. For any lower stock of water $\bar{W}_2$ the new $S(n, \lambda)$ curve must be as shown. Hence the equilibrium number of wells is n_2 which is lower than n_1. ■

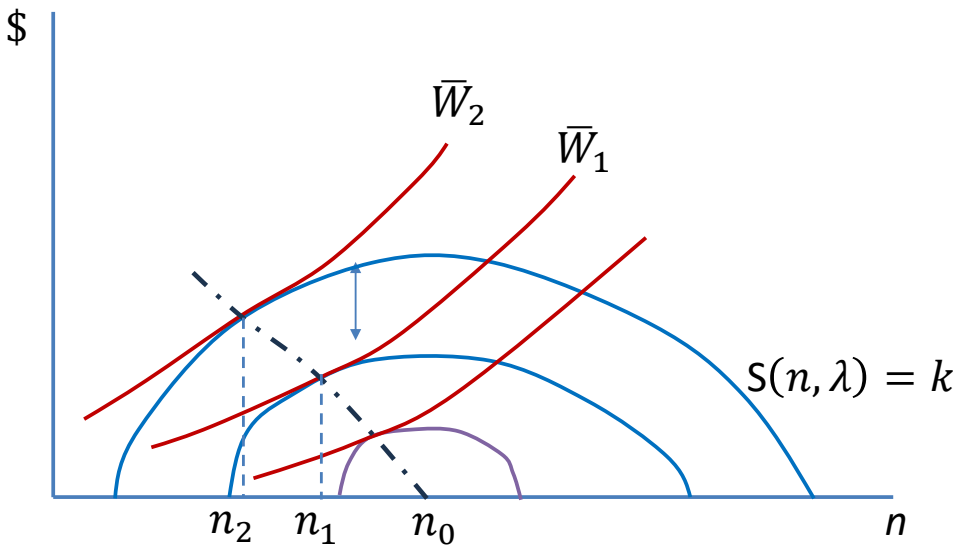


Figure 14: For a large farm, the iso-profit contours $S(n, \lambda)$ show surplus as a function of well density and the shadow price of water. When there is no water constraint, the efficient well density is n_o as before. Well density with scarce water is given by the tangency of the water constraint $W(n, \lambda)$ with the isoprofit contour.

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