Speculative Risk Attitude versus Pure Risk Attitude: Estimation from Cryptomarket Decisions

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Abstract

Limited research has been conducted on the joint estimation of different kinds of risk attitudes. This paper develops and estimates a structural model of investors' speculative and pure risk attitudes, by using rich transaction-level data from the cryptomarket. The model considers the unique features of the cryptomarket, where investors can allocate wealth across various cryptos and purchase on-chain insurance to protect their cryptos against risks such as hacking. We show that the speculative risk attitude is the weighted average of the curvature of the reference-dependent utility function in both gain and loss regions, while the pure risk attitude captures the curvature in the loss region only. Empirically, major investors are both speculative risk averse and pure risk averse. Importantly, we find that cross-domain consistency is conditional: speculative and pure risk attitudes move together for investors with the same risk orientation across domains, i.e., investors who are either risk averse in both speculative and pure risk domains or risk seeking in both domains, but move in opposite directions for investors with different risk orientations. This evidence indicates that cross-domain consistency in risk attitudes depends not only on the nature of the risk domains, as emphasized in prior literature, but also on investor-specific attributes.

Keywords: Risk attitude, Cryptomarket, Portfolio choice, Insurance, Consistency of risk attitudes.

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1 Introduction

Standard models in macroeconomics, finance, public finance, and labor economics typically adopt the classical expected utility framework: individuals (or households) are assumed to have a single concave utility function over wealth, leading to domain-invariant risk preferences. Following this assumption, researchers commonly calibrate their models using external estimates of risk-aversion parameters derived from specific decision-making domains (Einav et al., 2012). At the opposite end of the spectrum lies a vast literature in psychology and behavioral economics, which argues that individuals display little consistency in their risk-taking across different domains. In other words, risk preferences may be highly domain-dependent, varying significantly depending on the decision environment.

The existing literature on cross-domain consistency of risk attitudes typically estimates risk attitudes separately by domain (Barseghyan et al., 2011, 2016; Einav et al., 2012; Wolf and Pohlman, 1983). However, it is important to estimate domain-specific risk attitudes jointly rather than in isolation. The reason is that decisions across domains often interact, so separate estimation can introduce endogeneity and yield biased measures of multiple risk attitudes. Specifically, we follow Halek and Eisenhauer (2001), Powell and Ansic (1997), and Williams (1966) and distinguish between two domains: (i) speculative risk, which entails both gains and losses (e.g., investment returns), and (ii) pure risk, which entails losses only (e.g., hacks). Speculative risk attitude—primarily revealed through portfolio allocation—and pure risk attitude—primarily revealed through insurance demand—should be jointly determined. Larger positions in risky cryptos raise the marginal benefit of purchasing hack insurance, while the availability and pricing of coverage increase the attractiveness of holding cryptos. Consequently, estimating portfolio choice alone will generally confound speculative risk attitude with pure risk attitude, because portfolio decisions already embed expectations about insurance protection. Conversely, estimating insurance demand alone will also be confounded, since insurance purchases reflect investors' willingness to take speculative risk.

Our paper addresses this limitation by introducing a model that jointly estimate the speculative risk attitude and pure risk attitude. Specifically, we derive the expressions of speculative risk attitude and pure risk attitude in the framework of reference-dependent utility, with speculative risk attitude being a weighted average of the curvature of the reference-dependent utility function in both gain

and loss regions, while pure risk attitude captures the curvature in the loss region only. Then we construct a structural model that simultaneously estimates investors' speculative risk attitude and pure risk attitude by modeling their insurance choices and investment decisions. We use data from the cryptomarket to estimate the speculative risk attitude and pure risk attitude.

Our estimates indicate that most investors are both speculative risk averse and pure risk averse. We further find that the relationship between speculative and pure risk attitudes depends on whether investors exhibit the same risk orientation across domains. For investors with the same risk orientation across domains—those who are either risk averse in both speculative and pure risk domains or risk seeking in both domains—pure risk attitude is positively correlated with speculative risk attitude: the more (less) speculative risk averse an investor is, the more (less) pure risk averse she is. In contrast, for investors with different risk orientations across domains—those who are speculative risk averse but pure risk seeking, or speculative risk seeking but pure risk averse—the relationship is negative: the more (less) speculative risk averse an investor is, the less (more) pure risk averse she is. These findings indicate that the relationship between speculative and pure risk attitudes varies systematically across investors with different risk orientations, rather than exhibiting a uniform pattern in all cases. Moreover, we anticipate that heterogeneous responses to negative events provide a plausible explanation for why the consistency between speculative and pure risk attitudes varies with investors' risk orientations.

We contribute to the literature on cross-domain consistency in risk attitudes in two ways. First, as described above, we propose a methodology that jointly estimates two domain-specific risk attitudes. Second, we provide new empirical evidence on the cross-domain consistency in risk attitudes. The existing literature on the consistency of risk attitudes across domains has produced mixed results. Wolf and Pohlman (1983) conclude that an investor's degree of risk aversion may depend on the specific context in which their choices are made, and Barseghyan et al. (2011) reject stable risk preferences. By contrast, Einav et al. (2012) find a significant domain-general component to risk attitudes, and Barseghyan et al. (2016) do not reject stability. Our estimates suggest that cross-domain consistency in risk attitudes depends not only on domain characteristics emphasized in prior work (Barseghyan et al., 2011; Wolf and Pohlman, 1983), but also on investor-specific attributes. This suggests that the mixed findings in prior literature may stem from overlooking investor heterogeneity.

We also contribute to the literature on risk attitude or preference measures. Our model combines two approaches to estimating risk attitudes from the real data rather than experimental data. One is insurance choice, similar to Barseghyan et al. (2013), Cohen and Einav (2007), and Halek and Eisenhauer (2001) who estimate risk attitude from the choice of insurance deductible or coverage. The other one is portfolio decision, similar to Paravisini et al. (2017) who estimate risk attitude from the holdings of risky and riskless assets, and similar to incentivized experiments (Cohn et al., 2015; Holt and Laury, 2002) since investors in our model face choices similar to those faced by experimental subjects. However, few studies have jointly estimated speculative risk attitudes and pure risk attitudes, **potentially due to a lack of comprehensive data**. Even when both types of risk attitudes are estimated, existing studies do not integrate their measurement into a single framework (Einav et al., 2012; Halek and Eisenhauer, 2001), **potentially due to a lack of unified methodology**. In our research, we address the two gaps by using the rich data on cryptomarket containing both portfolio and insurance information, and introducing a novel model that jointly estimates both speculative risk attitude and pure risk attitude by embedding investors'portfolio and insurance decisions within a unified framework.

2 Background

Ethereum is a blockchain platform that allows developers to build decentralized applications (dApps) using smart contracts. Ethereum's native crypto is called Ether (ETH), and it's used to pay for transaction fees and computational services on the Ethereum network. For example, Bitcoin's blockchain tracks only currency transactions, while Ethereum's blockchain tracks ETH transfers and executes programs like lending platforms (e.g., Aave) or NFT marketplaces (e.g., OpenSea).

[Figure 1 about here.]

Figure 1 shows the complete process of investing and insuring in Ethereum. Investors can buy and hold cryptos like Ether (ETH) or other cryptos on the Ethereum network from exchanges such as Coinbase, Binance, Kraken, or decentralized exchanges (DEXs) like Uniswap and SushiSwap. Once bought, investors hold their cryptos in digital accounts (software or hardware). Popular accounts include MetaMask (browser extension), Trust Wallet (mobile), and Ledger or Trezor (hardware accounts). These accounts provide a secure way to store ETH and other Ethereum-based cryptos.

For example, an investor can purchase ETH on Coinbase using a bank transfer and then transfer the ETH to the MetaMask wallet for safekeeping.

After buying and holding cryptos in their accounts, investors can deposit some of their cryptos into Ethereum-based **decentralized finance (DeFi) protocols**. These DeFi protocols allow investors to earn rewards, provide liquidity, or participate in lending and borrowing activities. To deposit, the investor typically interacts with smart contracts through DeFi applications. DeFi protocols allow users to deposit crypto into liquidity pools or yield farms. Some popular protocols include: Compound (for lending and borrowing), Aave (for decentralized lending), Uniswap (for providing liquidity in decentralized exchanges) and Curve Finance (for stablecoin trading) etc. For example, an investor with 5 ETH might deposit their ETH into the Compound protocol to earn interest on it. The investor would connect their MetaMask wallet to Compound, choose ETH as the asset to lend, and approve the transaction.

Once deposit is made, the cryptos in DeFi protocols are at risk of being hacked or exploited. For example, a hacker might find a vulnerability in a smart contract and drain the funds from the protocol. To protect against these risks, investors can also purchase insurance to protect their crypto assets from risks like hacking, smart contract failures, or other vulnerabilities. One of the most largest insurance provider in Ethereum is Nexus Mutual, a decentralized insurance protocol that allows users to purchase cover for their crypto assets. Nexus Mutual is a decentralized autonomous organization (DAO) that uses smart contracts to pool funds from members and pay out claims when necessary. To purchase insurance, the investor connects their wallet (e.g., MetaMask) to Nexus Mutual, selects the protocol they want to insure (e.g., Compound), and chooses the amount of coverage. The insurance is purchased by paying a premium in ETH or the platform's native crypto (e.g., NXM for Nexus Mutual).

3 Model

We first derive the expression of speculative risk attitude and pure risk attitude in the framework of reference-dependent utility, and then we set up the optimization problem in which an investor makes two decisions: (1) how to allocate wealth across various cryptos, and (2) how much insurance coverage to purchase for the cryptos held in DeFi protocols, in order to mitigate risks such as hacking. Finally, we show how to estimate the two risk attitudes.

3.1 Speculative Risk Attitude and Pure Risk Attitude

Definition 1 (Speculative risk) *We say that ε is a* speculative risk *if it can generate both gains and losses with positive probability, that is,*

$$\Pr(\varepsilon > 0) > 0 \quad \textit{and} \quad \Pr(\varepsilon < 0) > 0.$$

In words, a speculative risk has strictly positive probability mass on both the gain domain and the loss domain (possibly also on $\varepsilon = 0$).

Definition 2 (Pure risk) *We say that ε is a* pure risk *if it never generates a gain, but generates a loss with positive probability. Formally,*

$$\Pr(\varepsilon > 0) = 0 \quad \textit{and} \quad \Pr(\varepsilon < 0) > 0.$$

Equivalently, $\varepsilon \leq 0$ almost surely, and the event $\{\varepsilon < 0\}$ has strictly positive probability.

For later use, it is convenient to introduce the sets

$$\mathcal{S} := \big\{\varepsilon : \Pr(\varepsilon > 0) > 0,\ \Pr(\varepsilon < 0) > 0\big\}, \qquad \mathcal{P} := \big\{\varepsilon : \Pr(\varepsilon > 0) = 0,\ \Pr(\varepsilon < 0) > 0\big\}. \quad (1)$$

We then simply say that a random variable ε is a speculative risk if $\varepsilon \in \mathcal{S}$, and a pure risk if $\varepsilon \in \mathcal{P}$.

We use the reference-dependent utility (Kőszegi and Rabin, 2007; Sugden, 2003; von Gaudecker et al., 2011) throughout. The reference-dependent utility function is defined as

$$v(x) = \begin{cases} u_1(x), & x \geq W_{rp}, \\ u_2(x), & x < W_{rp}, \end{cases} \quad (2)$$

where W_{rp} is the reference point and we assume that $u_1(\cdot)$ and $u_2(\cdot)$ are twice continuously differentiable and $u_1'(W_{rp}) = u_2'(W_{rp})$.

Next, we derive expressions for speculative risk attitude (SRA) and pure risk attitude (PRA) within the reference-dependent utility framework, using the standard Arrow–Pratt local approximation. We start from the reference wealth level W_{rp} and introduce a small risk ε, so that ex post

wealth is

$$W_{rp} + \varepsilon, \tag{3}$$

where ε is a random variable. The certainty equivalent of a small sure loss $\pi > 0$ is defined implicitly by

$$v(W_{rp} - \pi) = \mathbb{E}\big[v(W_{rp} + \varepsilon)\big]. \tag{4}$$

Then we state the following propositions.

Proposition 1 (Speculative risk attitude) *Consider the speculative risk $\varepsilon_s \in \mathcal{S}$. Let $w := \frac{\mathbb{E}\left[(\varepsilon_s^+)^2\right]}{\mathbb{E}\left[\varepsilon_s^2\right]}$, where $\varepsilon_s^+ = \max\{\varepsilon_s, 0\}$. Under the reference-dependent utility function $v(x) = u_1(x)$ for $x \geq W_{rp}$ and $v(x) = u_2(x)$ for $x < W_{rp}$, the speculative risk attitude is*

$$\textit{SRA} = w \cdot \left[-\frac{u_1''(W_{rp})}{u_1'(W_{rp})}\right] + (1-w) \cdot \left[-\frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right].$$

Proof. The result follows by matching a first-order expansion of the left-hand side and a second-order expansion of the right-hand side around W_{rp} in the certainty-equivalent equation. See Appendix A. □

Intuitively, SRA is a weighted average of local curvature in the gain and loss domains. The weight w is the share of the total second moment of ε_s contributed by its upside (gain) realizations.

Proposition 2 (Pure risk attitude) *Consider the pure risk $\varepsilon_p \in \mathcal{P}$. The pure risk attitude is*

$$\textit{PRA} = -\frac{u_2''(W_{rp})}{u_2'(W_{rp})}.$$

Proof. The argument parallels Proposition 1. See Appendix A. □

PRA depends only on curvature in the loss domain, reflecting that pure risk generates losses (relative to the reference point) but no gains.

3.2 The Model

Next, we set up the optimization problem in which an investor makes two decisions together: (1) how to allocate wealth across various cryptos, and (2) how much insurance coverage to purchase for the cryptos held in DeFi protocols, in order to mitigate risks such as hacking.

In the cryptomarket, an investor holds J cryptos, indexed as $(j = 1, \cdots, J)$ with return of R_j. Out of these, K cryptos $(k = 1, \cdots, K)$ can be insured (represented by set $\mathcal{I}$), while N cryptos $(n = 1, \cdots, N)$ cannot (represented by set $\mathcal{U}$). It follows that $J = K + N$. The insurable cryptos are those deposited in protocols that can be insured, while the uninsurable ones are either deposited in protocols that cannot be insured or are held in the investor's own wallet. For instance, an investor might hold 5 ETH and 10 DAI, with 3 ETH deposited in Compound (denoted as ETH_Compound), leaving 2 ETH (denoted as ETH_wallet) and 10 DAI (denoted as DAI_wallet) in her wallet. In this case, the investor can purchase insurance for the 3 ETH_Compound because cryptos in Compound can be insured by insurance providers like Nexus Mutual, but not for the 2 ETH_wallet or 10 DAI_wallet. Additionally, we consider the investor's portfolio as consisting of three cryptos: ETH_Compound, ETH_wallet, and DAI_wallet ($J = 3$, $K = 2$, and $N = 1$). While ETH_Compound and ETH_wallet share the same return and volatility in the crypto market, ETH_Compound faces risks such as hacking, while ETH_wallet does not. We only consider the insurable cryptos in our model, i.e., $N = 0$ and $J = K$, since the investor cannot purchase insurance for uninsurable cryptos. The optimization problem for the investor of the two decisions is as follows:

$$\begin{aligned} \max_{\{W_j, V_j\}_{j=1}^{J}} \quad & \mathbb{E}_R \mathbb{E}_I \, v\left(\sum_{j=1}^{J} \left[W_j - m_j V_j + \big(V_j - (W_j - m_j V_j) \big) I_j \right] R_j \right) := \mathbb{E}_R \mathbb{E}_I \, v(\overline{W}). \\ s.t. \quad & V_j \leq W_j - m_j V_j \quad \forall j, \qquad \sum_{j=1}^{J} W_j = W. \end{aligned} \tag{5}$$

where J is the number of risky assets (cryptos), R_j is the random return of asset j, I_j is the indicator variable of loss occurrence for asset j (valued 0 or 1 with probabilities $1-p_j$ and p_j respectively), and m_j is the insurance premium rate for asset j. The total initial wealth is W. The decision variables are W_j, the amount invested in asset j, and V_j[1], the amount of insurance purchased for asset j. The final wealth $\overline{W}$ depends on both the returns R_j and the loss indicators I_j. The investor's wealth is subject to the constraint that it must be allocated among the cryptos. $V_j \leq W_j - m_j V_j$ means that

[1]In decentralized insurance platforms, the insurance coverage is denominated in the crypto, meaning that the coverage amount fluctuates with the value of the crypto.

the coverage amount cannot exceed the investment amount after paying the insurance premium. We assume a full loss, i.e., if an accident happens, the loss L_j equals the crypto value $W_j - m_j V_j$.

We now derive the first-order conditions.

$$W_k: \quad \mathbb{E}_R \mathbb{E}_I \left[v'(\overline{W}) \, (1 - I_k) R_k \right] - \gamma = 0, \tag{6}$$

$$V_k: \quad \mathbb{E}_R \mathbb{E}_I \left[v'(\overline{W}) \, (I_k - m_k + I_k m_k) R_k \right] = 0. \tag{7}$$

The term γ is the Lagrange multiplier associated with the budget constraint. After (i) taking a first-order Taylor expansion for Eq.6 and Eq.7 around the reference point W_{rp}; (ii) plugging the expansion for SRA and PRA (Prop.1 and Prop.2) into the expanded first-order conditions, and (iii) rearranging the terms, we can express both first-order conditions as functions of SRA and PRA. The full derivation is in Appendix B. The two first-order conditions can be expressed as the following system of equations:

$$\begin{cases} \mathbb{E}_R \mathbb{E}_I \left[(1 - I_k) R_k \right] = \text{SRA} \frac{a_{1,k}}{w} + \text{PRA} \left[b_{1,k} - \frac{a_{1,k}}{w} (1 - w) \right] + \frac{\eta}{u_1'}, \\ \mathbb{E}_R \mathbb{E}_I \left[(I_k - m_k + I_k m_k) R_k \right] = \text{SRA} \frac{a_{2,k}}{w} + \text{PRA} \left[b_{2,k} - \frac{a_{2,k}}{w} (1 - w) \right]. \end{cases} \tag{8}$$

Here, $a_{1k}, a_{2k}, b_{1k}, b_{2k}$ are coefficients derived from Eq.6 and Eq.7. The term $w = \frac{\mathbb{E}(\overline{W} - W_{rp})^{+2}}{\mathbb{E}(\overline{W} - W_{rp})^2}$ captures the proportion of upside risk in the total risk of final wealth $\overline{W}$.

The two equations above imply that both speculative risk attitude and pure risk attitude can be identified from the two decisions: portfolio allocation and insurance decision. This is because both the change of W_j and the change of V_j affect the influence of speculative risk (R_j) and pure risk (I_j) on the final wealth $\overline{W}$. Therefore, by observing how investors adjust their portfolio allocation and insurance decisions in response to changes in speculative risk and pure risk, we can identify their speculative risk attitude and pure risk attitude.

3.3 System Estimation

We add error terms to the system (8) to estimate the two risk attitudes SRA and PRA.

$$\begin{cases} \mathbb{E}_R\mathbb{E}_I\big[(1-I_k)R_k\big] = \text{SRA}\frac{a_{1,k}}{w} + \text{PRA}\left[b_{1,k} - \frac{a_{1,k}}{w}(1-w)\right] + \frac{\eta}{u_1'} + \mu_{1k}, \\ \mathbb{E}_R\mathbb{E}_I\big[(I_k-m_k)R_k\big] = \text{SRA}\frac{a_{2,k}}{w} + \text{PRA}\left[b_{2,k} - \frac{a_{2,k}}{w}(1-w)\right] + \mu_{2k}. \end{cases} \tag{9}$$

where μ_1 and μ_2 are the error terms. We treat as a single choice the full set of an investor's crypto allocations and insurance decisions made within a month. Under this timing convention, η/u_1' can be interpreted as a constant term: it is investor- and choice-specific, yet common across cryptos within a given monthly choice set (Paravisini et al., 2017). We estimate the system in (9) using system estimation to obtain the estimates of SRA and PRA. Specifically, for each investor-month, we estimate the two equations jointly using all insurable cryptos held by the investor in that month. Consequently, both risk attitudes are identified at the investor-month level.[2] For all moments required for estimation, we use 30-day sample analogues to approximate their population counterparts.

Additionally, we set W_{rp} as the initial wealth W at the beginning of each month for each investor. This is because, in our context, the reference point being the initial wealth at the start of the decision-making period is a reasonable assumption. It reflects the idea that pure risk is associated with potential losses relative to one's current financial status[3].

4 Data, Sample and Estimation Methodology

4.1 Data Description

We focus on the cryptomarket primarily due to the transparency of on-chain data, which enables us to **obtain both portfolio data and insurance data tied to the same account**. This is a significant advantage over previous studies, which were limited to either portfolio data of risky assets alone (e.g., Paravisini et al., 2017) or insurance data alone (e.g., Barseghyan et al., 2013; Cohen and Einav, 2007; Halek and Eisenhauer, 2001). These limitations likely explain why prior literature has

[2]With daily data, estimated risk attitudes would vary implausibly fast, which is at odds with economic intuition. In 2024, Ethereum experienced 66 attacks (about 5.5 per month). In addition, over our sample period, there were on average 1.38 trading days per month with yield fluctuations exceeding 10%. Monthly aggregation therefore provides a more plausible frequency at which risk attitudes can adjust.

[3]If W_{rp} is set as the expected final wealth $\mathbb{E}_R\mathbb{E}_I[\overline{W}]$, then the pure risk is associated with not only losses but also gains, which violates the definition of pure risk.

only been able to estimate speculative or pure risk attitudes alone. In contrast, on-chain data allows for the joint estimation of the two risk attitudes. However, blockchain data also presents a challenge: anonymity. A single investor may control multiple accounts, and due to the anonymous nature of blockchain, it is not possible to link those accounts back to the same person [4].

We select Ethereum as the blockchain of interest because it dominates decentralized finance (DeFi) activities. According to DefiLlama[5], as of May 12, 2025, Ethereum accounts for 53.22% of the Total Value Locked (TVL) across all chains, amounting to $62.217 billion. In comparison, the second- and third-largest chains, Solana and Bitcoin, have TVL of $9.401 billion (8.04%) and $6.493 billion (5.55%) respectively. Moreover, during our sample period (2019-07-24 to 2024-02-13), Ethereum's TVL share peaked at 96.78% on January 15, 2021. Therefore, Ethereum provides a comprehensive representation of DeFi activities.

We consider the users purchasing insurance covers from Nexus Mutual. Nexus Mutual is a decentralized blockchain-based insurance platform founded in 2018. Operating globally, Nexus Mutual has become one of the largest on-chain insurance providers in the decentralized finance (DeFi) ecosystem. Its business model focuses on smart contract risk protection and is representative of emerging decentralized insurance markets. Among all insurance providers on Ethereum, Nexus Mutual is the most prominent. According to DefiLlama[6], as of May 12, 2025, Nexus Mutual holds $159.58 million in TVL, accounting for the vast majority of the total $177.96 million across all Ethereum-based insurance protocols. The second-largest provider, Ease.org, holds only $6.48 million. This dominance makes Nexus Mutual the most suitable and representative platform for studying decentralized insurance behavior.

Above all, given the representativeness of Ethereum and Nexus Mutual, we obtain portfolio data from Ethereum and insurance data from Nexus Mutual. The two datasets are linked using the investor address, which serves as a unique identifier.

Ethereum Portfolio Dataset

Wallet portfolios are reconstructed through historical on-chain transaction data. Historical on-

[4]In Appendix C, we demonstrate that, by applying multiple heuristic algorithms, only 104 accounts are likely to belong to the same investor,

[5]DefiLlama is a leading open-source analytics platform in the DeFi space. It aggregates data from over 5,000 DeFi protocols across more than 300 blockchain networks, providing users with comprehensive insights into Total Value Locked (TVL), yields, stablecoins, and other key metrics. The data is sourced from https://defillama.com/chains

[6]https://defillama.com/protocols/Insurance/Ethereum

chain transaction data are sourced from Etherscan[7], covering all Ethereum-based crypto transfers associated with users in the insurance sample. We collect **complete historical on-chain crypto transactions** data from Etherscan, including the timestamp, transaction hash, transaction method, sender, receiver, amount, and crypto (e.g., ETH, DAI, USDC, etc.) of users who purchased insurance covers from Nexus Mutual. An example of a transaction is shown in Table 1.

Then we use the transaction data to reconstruct the portfolio of each user. The portfolio is constructed by tracking the flow of cryptos in and out of the investor over time. For example, if a user receives 10 ETH on March 1, 2021, and sends 3 ETH on March 30, 2021, the portfolio would reflect that the user held 10 ETH from March 1 to March 29 and 7 ETH thereafter until subsequent transactions. More details regarding the construction of portfolios (i.e., the calculation of variables W_{ijt} shows in the section 3.3) are in Appendix D. The portfolio is updated with each transaction, allowing us to track the user's holdings over time.

[Table 1 about here.]

Nexus Mutual Insurance Dataset

The insurance data are obtained from Nexus Mutual. Nexus Mutual mainly offers Protocol Cover which insures the crypto assets held in a single protocol deployed on a blockchain network against risks arising from various loss events caused by smart contract code errors or defects, as well as unexpected severe economic events such as oracle manipulation or failure, significant liquidation failures, or governance takeovers leading to crypto asset losses.

The original insurance data includes details such as the policy ID, coverage amount (denominated in ETH or DAI), premium (paid by ETH, DAI, or NXM), coverage period, insured protocol name. The dataset also contains information about the insured protocol. In addition, the dataset contains claim amounts and claim submission dates.

Coinmarket Crypto Price Dataset

Additionally, to calculate the U.S. dollar value of crypto holdings, as well as returns and volatilities of cryptos for estimating risk attitudes, we require the prices of all cryptos on Ethereum. We

[7]Etherscan is a free and widely used blockchain explorer for the Ethereum network. It allows us to search and view detailed information about Ethereum addresses, transactions, smart contracts, crypto holdings, and gas fees. By entering a wallet address or transaction hash, we can track activity, verify transfers, and explore smart contract interactions.

obtain daily closing prices for each crypto from CoinMarketCap (Liu et al., 2022) for the full sample period from July 24, 2019, to February 13, 2024. CoinMarketCap is a widely used crypto market data platform that provides comprehensive and reliable price information across a broad range of digital assets.

4.2 Sample

Insurance policies and investor portfolios are linked at the investor level through investor address matching. The original dataset includes 8336 policies of 2421 investors from 2019 to 2024. Given that Protocol insurance has a high percentage (89%), our focus is solely on Protocol insurance [8].

We perform the four following steps to clean the dataset: (1) Some cryptos aren't provided prices by CoinMarketCap due to low liquidity. Thus we exclude all cryptos that we do not have prices. (2) We only keep the insurable cryptos. The insurable cryptos are those deposited in protocols that can be insured by Nexus Mutual, while the uninsurable cryptos are either deposited in protocols that cannot be insured by Nexus Mutual or are held in the investors own wallet. (3) We restrict attention in our main analysis to the investors that have at least three cryptos in their portfolio during the cover period. This is because we need to estimate the speculative risk attitude, pure risk attitude and an intercept term, a total of three parameters. This processing is the same as Paravisini et al. (2017). These processings result in a final sample of 9573206 investor–day–crypto observations, covering all cryptos held by all investors across all days from 2019-07-24 to 2024-02-13.

The detailed definitions of all variables used in the system estimation are shown in Table 2.

[Table 2 about here.]

Table 3 reports summary statistics for the variables defined in Table 2. Each observation in Table 3 corresponds to investor i's holding of crypto j in day t. On average, an investor holds 274 cryptos in a month[9], so we use roughly 274 observations to estimate both speculative and pure risk attitudes for that investor in that month. In total, we obtain 31401 investor-month estimates of each risk attitude.

[Table 3 about here.]

[8] Although Protocol and Custody insurance investor for a high percentage (98%), we still focus solely on protocol insurance since some custody providers may already offer their own insurance to customers.

[9] For example, if an investor holds ETH and DAI from April 1 to April 15, 2023, and holds USDT from April 16 to April 30, 2023, we count this as $2 \times 15 + 1 \times 15 = 45$ crypto–day observations in April 2023.

4.3 Estimation Methodology

Given that on average we use 274 observations (a relatively small sample) to estimate the two risk attitudes, we use Bayesian Structural Equation Model (bayesSEM) to perform the system estimation of the two risk attitudes. In the context of small samples, Bayesian methods are known to offer better small-sample performance than frequentist methods such as maximum likelihood estimation (e.g.,McNeish, 2016; Zellner, 1998; Baldwin and Fellingham, 2013; Depaoli, 2013; Stenling etc, 2015). The theoretical research of a Bayesian approach to estimate simultaneous equation models has long attracted attention (Drèze, 1976; Kai, 1998; Toker and Özbay, 2023). Empirically, a number of papers have applied Bayesian system estimation as well (Balachander and Ghosh, 2013; Ding and Li, 2019; Murrar et al., 2024; Yang et al., 2006, 2010). For the choice of prior distributions, we adopt weakly informative priors (Balachander and Ghosh, 2013; Barseghyan et al., 2013; Gelman et al., 2008; Murrar et al., 2024), specifying normal distributions centered at zero.

5 Estimates and Consistency of Risk attitudes

5.1 Estimates and Distributions

We estimate the pure and speculative risk attitudes for each investor in each month. The summary statistics of two risk attitudes are shown in Table 4. The number of two risk attitudes is 31401 respectively. We winsorize the risk attitudes at the 1% and 99% levels.

[Table 4 about here.]

A positive speculative risk attitude indicates that the investor is risk averse, and the larger the value, the more risk averse the investor becomes. Conversely, a negative speculative risk attitude indicates that the investor is risk seeking, and the smaller the value, the more risk seeking the investor is. This interpretation also applies to pure risk attitudes.

[Figure 2 about here.]

Figure 2 shows that the both the distribution of speculative risk attitudes and pure risk attitudes are right-skewed. This suggests that a majority of investors are risk averse in both speculative and pure risk attitudes, matching the findings of pure risk attitudes (Cohen and Einav, 2007) and speculative risk attitudes (Bliss and Panigirtzoglou, 2004; Choi et al., 2007).

5.2 Consistency of Speculative and Pure Risk Attitudes

We use Eq.10 to examine the consistency of speculative and pure risk attitudes. The regression is conducted at the investor-month level, with investor fixed effects and year-month fixed effects included to control for unobserved heterogeneity across investors and time.

$$\text{SRA}_{it} = \alpha + \beta \text{PRA}_{it} + \gamma X_{it} + \varepsilon_{it}, \tag{10}$$

where SRA_{it} is the speculative risk attitude for investor i at month t, PRA_{it} is the pure risk attitude, and X_{it} includes control variables. The control variables are defined as follows. Monthly trading days: the number of days in a month that the investor trades. Monthly trading counts: the number of trades made by the investor in a month. Monthly crypto counts: the number of cryptos held by the investor in a month. Monthly received amounts: the net value of cryptos received by the investor in a month. Premium asset: a category variable indicating by which asset to pay the premium, such as ETH, DAI, or NXM. Cover period: the duration of the insurance coverage in months. Dollar value: the total dollar value of the insurance coverages held by the investor in a month. ETH value: the total ETH value of the insurance coverages held by the investor in a month. The regression results are shown in Table 5.

[Table 5 about here.]

Overall, the relationship between speculative risk attitude and pure risk attitude is positive and statistically significant in the full sample. As shown in column (1) of Table 5, the estimated correlation coefficient is 0.3757***, indicating that investors who are more speculative risk averse also tend to be more pure risk averse. This finding suggests a degree of cross-domain consistency in risk attitudes and is consistent with the domain-consistency hypothesis proposed in prior literature (Barseghyan et al., 2016; Einav et al., 2012).

However, this average relationship conceals substantial heterogeneity. When investors are partitioned into four groups according to whether they are risk averse or risk seeking in each domain, the relationship between speculative risk attitude and pure risk attitude differs markedly, as illustrated in Figure 3 and columns (2)–(5) of Table 5. For investors who are speculative risk averse and pure risk averse (column 2), as well as those who are speculative risk seeking and pure risk seeking

(column 5), the relationship between the two attitudes is positive, with correlation coefficients of 0.6971*** and 0.3729***, respectively. In contrast, for investors who are speculative risk averse but pure risk seeking (column 3), or speculative risk seeking but pure risk averse (column 4), the relationship is negative, with correlation coefficients of −1.3020*** and −0.1248***, respectively.

[Figure 3 about here.]

Taken together, these results indicate that cross-domain consistency in risk attitudes depends critically on investor-specific attributes—specifically, whether an investor is risk averse or risk seeking in each domain. When investors exhibit the same risk orientations (i.e., speculative risk averse and pure risk averse investors, speculative risk seeking and pure risk seeking investors) across domains, speculative risk attitude and pure risk attitude move in the same direction. When risk orientations differ across domains, the two attitudes move in opposite directions. Overall, the consistency of risk attitudes across domains is not uniform but instead reflects heterogeneity in investor attributes.

5.3 Potential Explanation of the Consistency of Risk Attitudes

We next expect that negative events may help explain why the consistency between speculative and pure risk attitudes differs across investor attributes. Our hypothesis is that investors with different risk orientations respond differently to the same adverse shocks, leading to distinct adjustment patterns in speculative risk attitude and pure risk attitude. To investigate this mechanism, we estimate the following regressions:

$$\text{SRA}_{it} = \alpha + \beta \text{Negative Events}_t + \delta X_{it} + \varepsilon_{it}, \tag{11}$$

$$\text{PRA}_{it} = \alpha + \beta \text{Negative Events}_t + \delta X_{it} + \mu_{it}, \tag{12}$$

where X_{it} denotes a vector of control variables. We consider two types of negative events. Speculative negative events capture adverse market-wide shocks and are proxied by the maximum drawdown of the ETH index over the previous six months, defined as the largest peak-to-trough decline in prices. Pure negative events capture protocol-specific losses and are proxied by the number of insurance claims submitted and accepted, which reflect realized hacking incidents or protocol failures.

We anticipate that, among investors with the same risk orientation across domains—those who

are both speculative risk averse (or both speculative risk seeking) and pure risk averse (or pure risk seeking)—negative events will reinforce their existing preferences, leading to higher (or lower) risk aversion in both speculative and pure risk domains. By contrast, for investors whose risk orientations differ across domains, negative events are expected to have asymmetric effects. For investors who are speculative risk averse but pure risk seeking, negative events should increase speculative risk aversion while reducing pure risk aversion. Symmetrically, for investors who are speculative risk seeking but pure risk averse, negative events should increase pure risk aversion while encouraging more risk seeking in the speculative domain.

Negative events can affect risk attitudes differently because different investors interpret bad outcomes in different ways. For pure risk, events such as hacks or protocol failures often make losses more salient and increase caution, leading many investors to become more pure-risk averse. However, some investors—especially those who are already pure-risk seeking—may interpret a recent loss as "bad luck" that is unlikely to repeat soon. This belief, consistent with the gambler's fallacy (Chen et al., 2016; Clotfelter and Cook, 1993; Rabin and Vayanos, 2010), can reduce their perceived loss probability and thus lower pure risk aversion. A similar mechanism applies to speculative risk. Negative market shocks increase uncertainty, prompting speculative risk-averse investors to reduce risk taking and become more risk averse. In contrast, speculative risk-seeking investors may increase risk taking in an attempt to recover losses or exploit perceived opportunities, again consistent with gambler's-fallacy reasoning. As a result, the same negative events can increase or decrease risk aversion depending on investor attributes, explaining why speculative and pure risk attitudes move together for some investors but in opposite directions for others.

6 Conclusion

We develop a model of crypto insurance demand and crypto portfolio allocation to jointly estimate the speculative risk attitude and pure risk attitude. Working within a reference-dependent utility framework, we obtain the expressions for both risk attitudes. SRA is a weighted average of local utility curvature in the gain and loss domains, where the weight is the ratio of the truncated second moment on the gain domain to the overall second moment. In contrast, PRA depends only on curvature in the loss domain, reflecting that pure risk generates losses only. We then derive a system of estimating equations that identifies SRA and PRA from the observed crypto portfolio

allocation and insurance decision. Leveraging detailed on-chain data that link portfolio holdings to insurance purchases at the account level, we estimate both speculative and pure risk attitudes at the investor-month frequency using all insurable cryptos held by the investor in that month.

The estimates reveal heterogeneity in both speculative and pure risk attitudes among investors in the cryptocurrency market. On average, investors exhibit risk aversion in both domains, consistent with prior findings in the literature. However, the relationship between speculative and pure risk attitudes varies systematically across investors. For investors with the same risk orientation across domains, the two attitudes are positively correlated, whereas for investors with different risk orientations, the correlation is negative. This pattern indicates that cross-domain consistency in risk attitudes depends not only on domain characteristics, as emphasized in prior work (Barseghyan et al., 2011; Einav et al., 2012; Wolf and Pohlman, 1983), but also on investor-specific attributes—particularly whether an investor is risk averse or risk seeking in each domain. We also anticipate that the consistency of speculative and pure risk attitudes can be explained by the different responses to negative events for investors with different attributes.

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Appendix A

The certainty equivalent of a small loss $\pi > 0$ is defined implicitly by

$$v(W_{rp} - \pi) = \mathbb{E}\big[v(W_{rp} + \varepsilon)\big]. \tag{13}$$

We first consider speculative risk $\varepsilon_s \in \mathcal{S}$. The left-hand side is expanded at the reference point W_{rp} as

$$\text{LHS} = v(W_{rp} - \pi) \approx u_1(W_{rp}) - \pi\, u_2'(W_{rp}). \tag{14}$$

The right-hand side is

$$\text{RHS} = \mathbb{E}\, v(W_{rp}+\varepsilon_s) \approx \mathbb{E}\Big[u_1(W_{rp})+u_1'(W_{rp})\,\varepsilon_s^+ - u_2'(W_{rp})\,\varepsilon_s^- + \tfrac{1}{2}u_1''(W_{rp})(\varepsilon_s^+)^2 + \tfrac{1}{2}u_2''(W_{rp})(\varepsilon_s^-)^2\Big]. \tag{15}$$

By equating LHS and RHS, we obtain

$$\begin{aligned} -\pi = {} & \frac{u_1'(W_{rp})}{u_2'(W_{rp})}\mathbb{E}\varepsilon_s^+ - \mathbb{E}\varepsilon_s^- \\ & + \frac{1}{2}\,\mathbb{E}\varepsilon_s^2 \left[w \cdot \frac{u_1''(W_{rp})}{u_2'(W_{rp})} + (1-w)\cdot \frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right], \end{aligned} \tag{16}$$

where the weight w assigned to the gain region is

$$w = \frac{\mathbb{E}\,(\varepsilon_s^+)^2}{\mathbb{E}\,(\varepsilon_s^-)^2 + \mathbb{E}\,(\varepsilon_s^+)^2} = \frac{\mathbb{E}\,(\varepsilon_s^+)^2}{\mathbb{E}\,(\varepsilon_s^-)^2 + \mathbb{E}\,(\varepsilon_s^+)^2} = \frac{\mathbb{E}\,(\varepsilon_s^+)^2}{\mathbb{E}\varepsilon_s^2}. \tag{17}$$

With the assumption that $u_1'(W_{rp}) = u_2'(W_{rp})$ we can rewrite the certainty equivalent equation as

$$-\pi = \mathbb{E}\varepsilon_s^+ - \mathbb{E}\varepsilon_s^- + \frac{1}{2}\mathbb{E}\varepsilon_s^2 \left[w \cdot \frac{u_1''(W_{rp})}{u_1'(W_{rp})} + (1-w)\cdot \frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right]. \tag{18}$$

Thus, similar to Arrow-Pratt risk aversion coefficient, we define **Speculative risk attitude (SRA)** as the coefficient of the variance term:

$$\text{SRA} := \frac{\mathbb{E}(\varepsilon_s^+)^2}{\mathbb{E}\varepsilon_s^2} \cdot \left[-\frac{u_1''(W_{rp})}{u_1'(W_{rp})}\right] \;+\; \frac{\mathbb{E}(\varepsilon_s^-)^2}{\mathbb{E}\varepsilon_s^2} \cdot \left[-\frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right]. \tag{19}$$

Similarly, we now consider pure risk $\varepsilon_p \in \mathcal{P}$. The left-hand side certainty equivalent is

$$\text{LHS} = u_1(W_{rp}) - \pi\, u_2'(W_{rp}). \tag{20}$$

The right-hand side is the expected value of the reference-dependent utility utility function:

$$\text{RHS} = \mathbb{E}\, v(W_{rp} + \varepsilon_p). \tag{21}$$

Using a local second-order Taylor expansion:

$$\begin{aligned} \text{RHS} &\approx \mathbb{E}\Big[u_1(W_{rp}) - u_2'(W_{rp})\, \mathbb{E}\varepsilon_p^- + \tfrac{1}{2} u_2''(W_{rp})\, \mathbb{E}(\varepsilon_p^-)^2\Big] \\ &= u_1(W_{rp}) - u_2'(W_{rp}) \mathbb{E}\varepsilon_p^- + \tfrac{1}{2} u_2''(W_{rp}) \mathbb{E}(\varepsilon_p^-)^2, \end{aligned} \tag{22}$$

because for pure risk $-\varepsilon_p^- = \varepsilon_p$. Equating LHS and RHS gives

$$-\pi = \mathbb{E}\varepsilon_p + \tfrac{1}{2} \mathbb{E}\varepsilon_p^2\, \frac{u_2''(W_{rp})}{u_2'(W_{rp})}. \tag{23}$$

Thus **pure risk attitude (PRA)** is defined as

$$\text{PRA} = -\frac{u_2''(W_{rp})}{u_2'(W_{rp})}. \tag{24}$$

Appendix B

To approximate $v'(\overline{W})$, we take a first order linearization on $v'(\overline{W})$ around the reference point W_{rp}:

$$\begin{aligned} v'(\overline{W}) = &\left[u_1'(W_{rp}) + u_1''(W_{rp})\,(\overline{W} - W_{rp})\right] \mathbf{1}\big(\overline{W} - W_{rp} > 0\big) \\ &+ \left[u_2'(W_{rp}) + u_2''(W_{rp})\,(\overline{W} - W_{rp})\right] \mathbf{1}\big(\overline{W} - W_{rp} < 0\big). \end{aligned} \tag{25}$$

Thus, substituting the linearized $v'(\overline{W})$ into the first-order condition (6), we have the following approximated first-order conditions. W_{rp}, we obtain

$$\begin{aligned}\mathbb{E}_R\mathbb{E}_I\Big\{\Big[&u_1'(W_{rp})\,\mathbf{1}(\overline{W}-W_{rp}>0)+u_1''(W_{rp})(\overline{W}-W_{rp})^+\\&+u_2'(W_{rp})\,\mathbf{1}(\overline{W}-W_{rp}<0)-u_2''(W_{rp})(\overline{W}-W_{rp})^-\Big](1-I_k)R_k\Big\}-\gamma=0.\end{aligned}\tag{26}$$

We can rewrite this as

$$\begin{aligned}&u_1'(W_{rp})\mathbb{E}_R\mathbb{E}_I\big[\mathbf{1}(\overline{W}-W_{rp}>0)(1-I_k)R_k\big]+u_1''(W_{rp})\mathbb{E}_R\mathbb{E}_I\big[(\overline{W}-W_{rp})^+(1-I_k)R_k\big]\\&+u_2'(W_{rp})\mathbb{E}_R\mathbb{E}_I\big[\mathbf{1}(\overline{W}-W_{rp}<0)(1-I_k)R_k\big]-u_2''(W_{rp})\mathbb{E}_R\mathbb{E}_I\big[(\overline{W}-W_{rp})^-(1-I_k)R_k\big]-\gamma=0.\end{aligned}\tag{27}$$

With $u_1'(W_{rp})=u_2'(W_{rp})$, factoring out $u_1'(W_{rp})$ yields

$$\mathbb{E}_R\mathbb{E}_I\big[(1-I_k)R_k\big]=a_{1,k}\left(-\frac{u_1''(W_{rp})}{u_1'(W_{rp})}\right)+b_{1,k}\left(-\frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right)+\frac{\gamma}{u_1'},\tag{28}$$

where

$$a_{1,k}=\mathbb{E}_R\mathbb{E}_I\big[(\overline{W}-W_{rp})^+(1-I_k)R_k\big],\qquad b_{1,k}=-\mathbb{E}_R\mathbb{E}_I\big[(\overline{W}-W_{rp})^-(1-I_k)R_k\big].\tag{29}$$

For the second first-order condition (7), we similarly substitute the linearized $v'(\overline{W})$ to obtain:

$$\begin{aligned}\mathbb{E}_R\mathbb{E}_I\Big\{\Big[&u_1'(W_{rp})\mathbf{1}(\overline{W}-W_{rp}>0)+u_1''(W_{rp})(\overline{W}-W_{rp})^+\\&+u_2'(W_{rp})\mathbf{1}(\overline{W}-W_{rp}<0)-u_2''(W_{rp})(\overline{W}-W_{rp})^-\Big](I_k-m_k+I_km_k)R_k\Big\}=0.\end{aligned}\tag{30}$$

Again factoring out $u_1'(W_{rp})$, we have

$$\mathbb{E}_R\mathbb{E}_I\big[(I_k-m_k+I_km_k)R_k\big]=a_{2,k}\left(-\frac{u_1''(W_{rp})}{u_1'(W_{rp})}\right)+b_{2,k}\left(-\frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right),\tag{31}$$

where

$$a_{2,k}=\mathbb{E}_R\mathbb{E}_I\big[(\overline{W}-W_{rp})^+(I_k-m_k+I_km_k)R_k\big],\qquad b_{2,k}=-\mathbb{E}_R\mathbb{E}_I\big[(\overline{W}-W_{rp})^-(I_k-m_k+I_km_k)R_k\big].\tag{32}$$

Next, we show the specific expressions of SRA and PRA in our setup. First, we derive the specific expressions of speculative risk ε_s and pure risk ε_p in our setup. From the target function (Eq.5) with $V_j = 0 \; \forall j$, we can write and then decomposite the total risk ε as

$$\begin{aligned} \varepsilon &= \sum_{j=1}^{J} W_j(R_j(1-I_j)-1) \\ &= \sum_{j=1}^{J} W_j(R_j-1) - \sum_{j=1}^{J} W_j I_j R_j \\ &= \varepsilon_s + \varepsilon_p. \end{aligned} \tag{33}$$

Thus, the total risk ε is decomposed into ε_s and ε_p as[10]:

$$\begin{cases} \varepsilon_s = \sum_{j=1}^{J} W_j(R_j-1), \\ \varepsilon_p = -\sum_{j=1}^{J} W_j I_j R_j. \end{cases} \tag{34}$$

Then we can obtain the specific expressions of SRA and PRA. Recall that the expression of the speculative risk attitudes (Prop.1) and pure risk attitudes (Prop.2) as

$$\text{SRA} = w \cdot \left(-\frac{u_1''(W_{rp})}{u_1'(W_{rp})}\right) + (1-w)\left(-\frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right), \tag{35}$$

$$\text{PRA} = -\frac{u_2''(W_{rp})}{u_2'(W_{rp})}, \tag{36}$$

and the weighting term w is obtained by substituting the expression of ε_s in equation (34) into equation (17):

$$w = \frac{\mathbb{E}\left[\left(\sum_{j=1}^{J} W_j(R_j-1)\right)_+^2\right]}{\mathbb{E}\left[\left(\sum_{j=1}^{J} W_j(R_j-1)\right)^2\right]}. \tag{37}$$

Finally, we show the system of equations (8) that can be used to estimate the two risk attitudes.

[10]The ε can also be decomposed as $\sum_{j=1}^{J} W_j(R_j-1)(1-I_j) - \sum_{j=1}^{J} W_j I_j$. However, in this case, the pure risk doesn't reflect the on-chain reality that the losses are denominated in cryptos so the losses are affected by the returns of cryptos. Therefore, we choose the decomposition in equation (33).

Let the following linear combinations capture the two curvatures and two risk attitudes:

$$a_{1,k}\left(-\frac{u_1''(W_{rp})}{u_1'(W_{rp})}\right)+b_{1,k}\left(-\frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right)=c_1\,\text{SRA}+d_1\,\text{PRA}, \tag{38}$$

$$a_{2,k}\left(-\frac{u_1''(W_{rp})}{u_1'(W_{rp})}\right)+b_{2,k}\left(-\frac{u_2''(W_{rp})}{u_2'(W_{rp})}\right)=c_2\,\text{SRA}+d_2\,\text{PRA}. \tag{39}$$

We have

$$a_{1,k}=c_1\cdot w, \qquad a_{2,k}=c_2\cdot w\cdot . \tag{40}$$

Therefore, equations (28) and (31) can be transformed to:

$$\mathbb{E}_R\mathbb{E}_I\big[(1-I_k)R_k\big]=\text{SRA}\frac{a_{1,k}}{w}+\text{PRA}\left[b_{1,k}-\frac{a_{1,k}}{w}(1-w)\right]+\frac{\eta}{u_1'}, \tag{41}$$

and similarly

$$\mathbb{E}_R\mathbb{E}_I\big[(I_k-m_k+I_km_k)R_k\big]=\text{SRA}\frac{a_{2,k}}{w}+\text{PRA}\left[b_{2,k}-\frac{a_{2,k}}{w}(1-w)\right]. \tag{42}$$

Appendix C

The account merging process leverages two complementary heuristic algorithms to consolidate interconnected accounts within a sample of 2,192 accounts. The first funder algorithm operates by tracing the initial funding source of each account, grouping those sharing a common seed transaction into unified entities. This method is particularly effective for identifying coordinated networks like sybil attacks, where multiple accounts originate from a single funding root. However, its limitation lies in potential under-merging when seed funds derive from decentralized or external sources. The most frequent mutual transfers algorithm adopts a different approach, merging accounts exhibiting the highest volume or value of bidirectional transactions. By calculating pairwise transfer frequencies and amounts across the dataset, it clusters accounts exceeding a predefined interaction threshold—such as the top 5 most frequently transferred counterparties. This excels at revealing tightly coupled relationships like merchant-customer ecosystems.

Applying these heuristics to the sample enabled the consolidation of 104 accounts into 48 distinct accounts 54% reduction that underscores significant interconnectivity. All the results in the

paper are robust if we implement the account merging process.

Appendix D

As mentioned in the Sample Section, we focus exclusively on insurable cryptos within portfolios, specifically those deposited in DeFi protocols covered by Nexus Mutual insurance. Section 2.1 details crypto deposition mechanisms with protocol examples. The calculation methodology of insurable cryptos'investment follows these steps:

1. We filter transactions containing Deposit or Withdraw methods (hereafter "deposit transaction" or "withdraw transaction"), discarding all others. This ensures we only track cryptos deposited into or withdrawn from protocols. Deposited cryptos must additionally belong to Nexus Mutual-insured protocols. A typical example is as follows.

- Transaction Hash: 0x5970...d277
- Timestamp: 2021-01-08 20:30:34
- Protocol: UniswapV2
- Method: add Liquidity ETH
- Deposited cryptos: ETH + GEM (0x30b1...0602), the corresponding amount is -0.042969 and -600, respectively
- Received LP crypto: 0xed0b...3b41, the corresponding amount is +4.340638

This transaction, with hash 0x5970...d277, occurred on 2021-01-08 20:30:34. Its method is add_liquidity_eth, and the protocol is UniswapV2, which indicates that ETH is being provided as liquidity to UniswapV2. To maintain the pool's liquidity balance, UniswapV2 also requires investors to provide other tokens—in this case, GEM (crypto hash 0x30b1...0602). Meanwhile, the investor receives an LP crypto (crypto hash 0xed0b...b41) from UniswapV2, which represents the investor's share of the liquidity pool. In this transaction, the deposited cryptos are GEM and ETH in UniswapV2. Moreover, since UniswapV2 is insured by Nexus Mutual, these deposited cryptos are considered insurable.

2. After identifying all the deposited transactions of an investor, we calculate the investment of deposited cryptos by summing the crypto amounts across those transactions. For example, if an investor deposits 1 ETH and 600 GEM into UniswapV2 on 2021-01-08, and then withdraws 0.3 ETH and 200 GEM on 2022-03-04, the investment of deposited cryptos is 1 ETH + 600 GEM on 2021-01-08, and 0.7 ETH + 400 GEM on 2022-03-04. For technical reasons, we treat any crypto balance valued in U.S. dollars below 0.0001 as zero. This is because crypto transactions often use extremely long decimal strings—for example:

- 0xe4be814b1b0b8578d506aa...: -137481.341569685660611541
- 0xb9689d2f5a7734518532f9...: -54990.735358867987152205
- 0x37300525ea1befb50ae259...: +192472.076928553647763746

In theory these three transactions sum exactly to zero, but in Python—due to the limitations of floating-point precision—their sum comes out as –2.1827872842550278e-11. Additionally, we may encounter dust balances. A dust balance is a tiny fraction of a crypto left in a blockchain or exchange account, typically caused by things like transaction fees and rounding discrepancies, or by data-source/display precision limits.

3. We calculate investment separately for each protocol. An example for UniswapV2 is shown before. However, an investor may deposit cryptos into multiple protocols, such as UniswapV2 and Compound. The calculation of deposited cryptos in other protocols follows exactly the same procedure as described in point 2.

4. Finally, we select the deposited cryptos which are associated with protocols insured by Nexus Mutual. Examples of such protocols include Compound, Aave, UniswapV2, etc.

In addition, we exclude short-term crypto holdings from our analysis. Specifically, if a crypto outflow occurs within 14 days of an inflow, we adjust these transactions accordingly. For example, suppose 3 ETH are acquired on Day 1, but within 14 days, 1 ETH is subsequently withdrawn. In this case, we treat the initial 3 ETH as partially short-term and adjust the Day 1 balance to reflect only the long-term holding, resulting in a net holding of 3 – 1 = 2 ETH. We consider short-term crypto holdings to fall outside the scope of long-term investment and therefore exclude them from the calculation of investment.

Figures

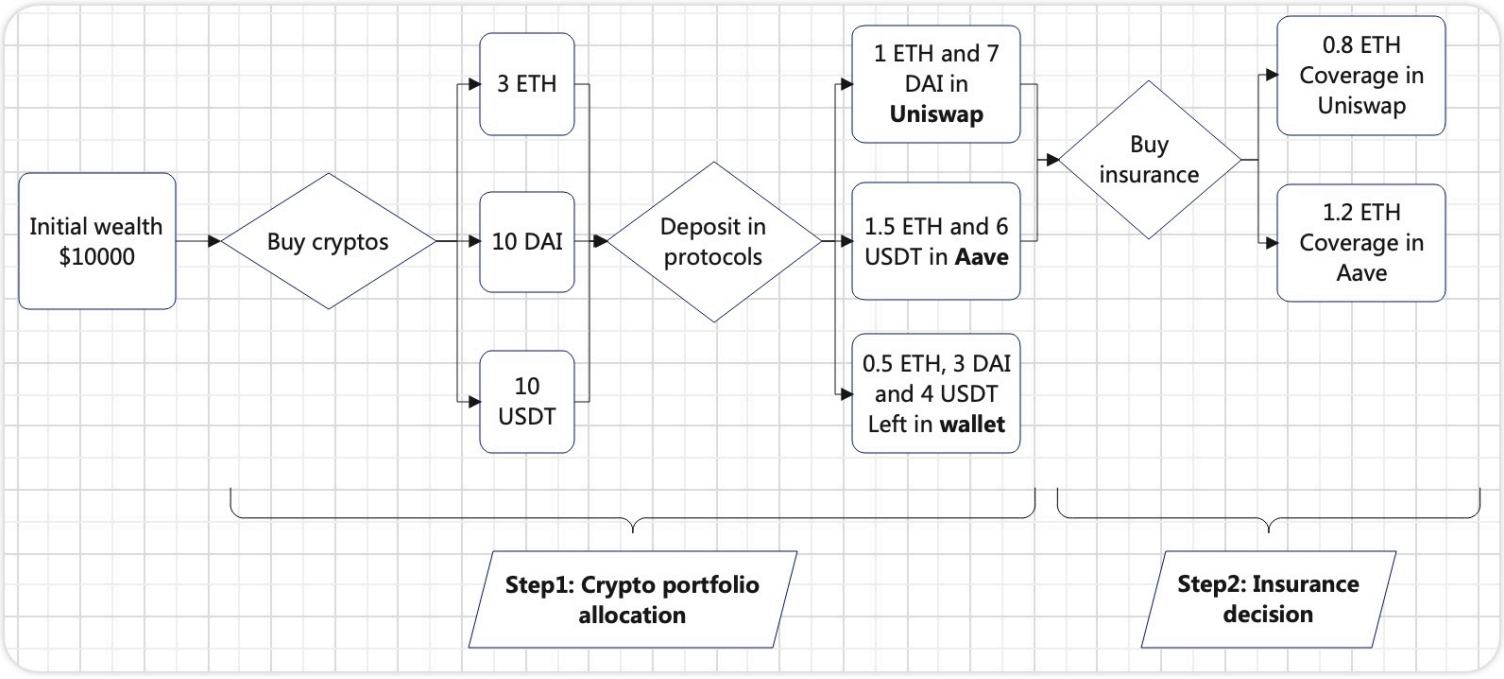


Figure 1: Invest and insure in Ethereum

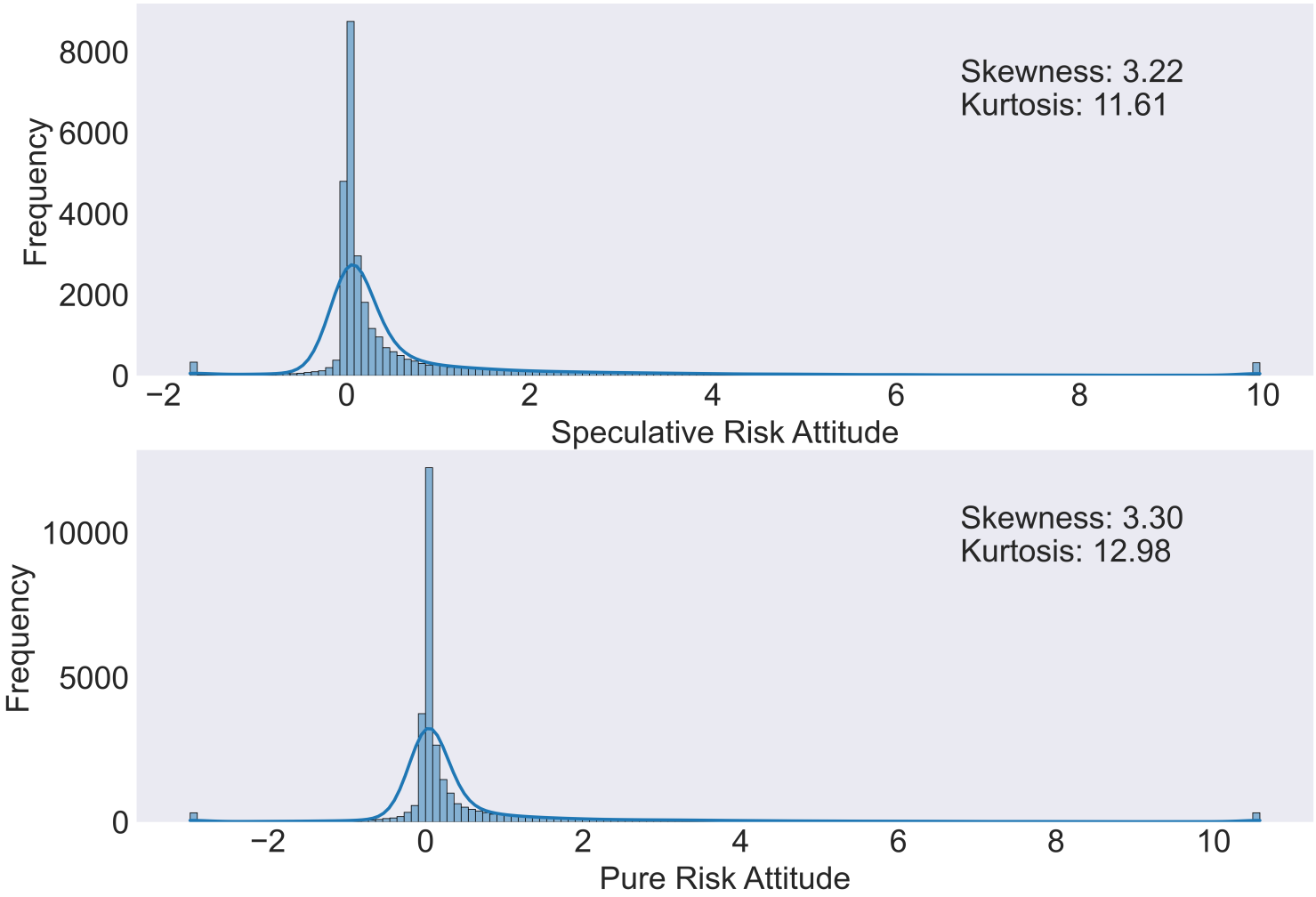


Figure 2: Distribution of speculative and pure risk attitudes

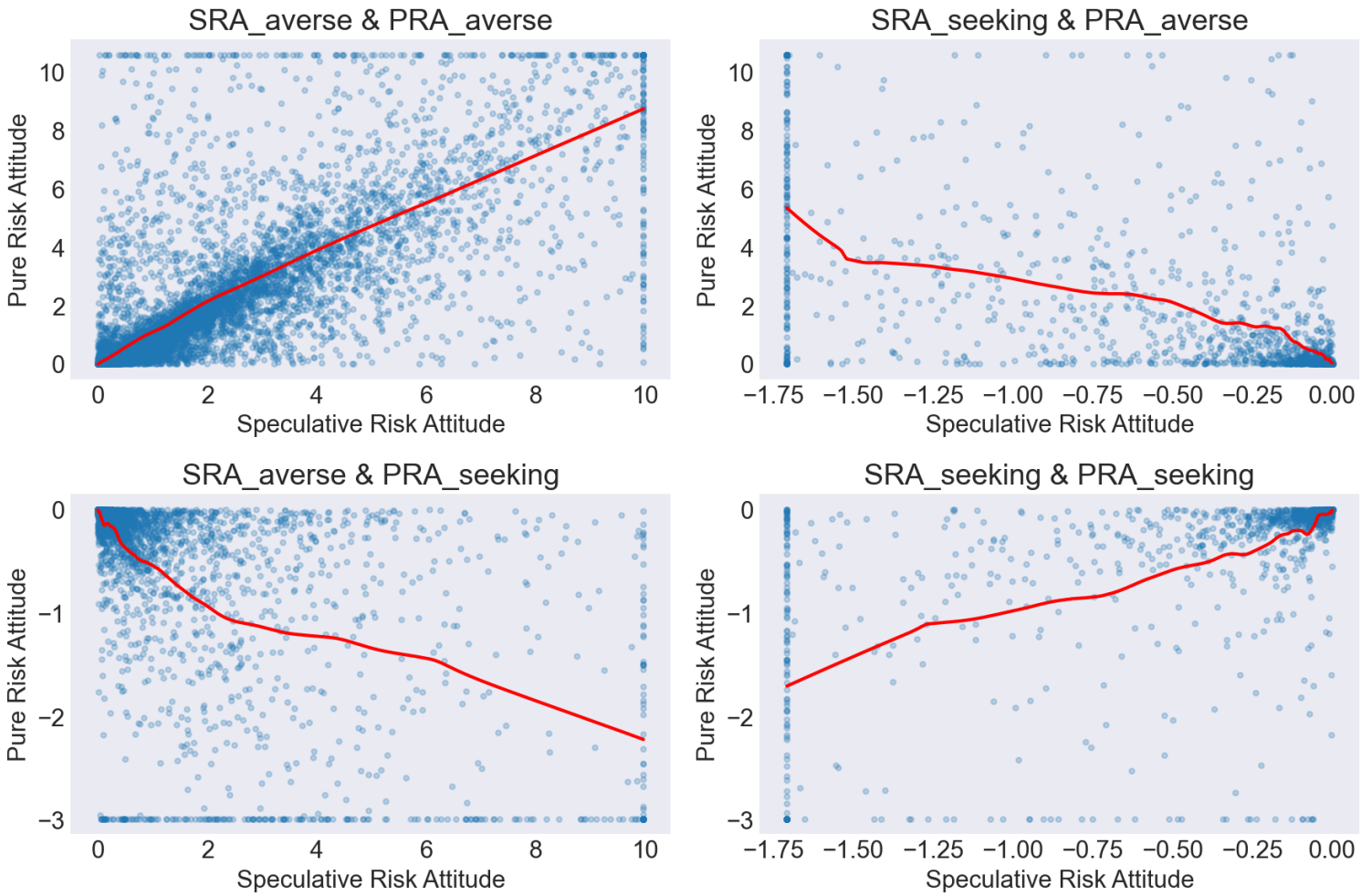


Figure 3: Correlation of speculative and pure risk attitudes

Tables

Table 1: An example of a transaction

Field	Value	Description
tx_hash	0x1db3e8a38f8677bfbcd5653df232f5f7 76c7263093493bf23415a4a38f74e51c	Unique identifier of the transaction
time_stamp	Mar-29-2021 03:33:26 PM UTC	Timestamp of when the transaction occurred
tx_method	Deposit	The method of the transaction, such as depositing crypto
sender_address	0x0000CE08fa224696 A819877070BF378e8B131ACF	Address of the sender, similar to a bank account number
receiver_address	0x8743cc30727e9E460 A5E69E217893f42DFad1650	Address of the receiver, similar to a bank account number
amount	400	The amount of crypto involved in the transaction
crypto_name	Dai	The name of the crypto being transacted (in this case, a stablecoin)
crypto_address	0x6B175474E89094C4 4Da98b954EedeAC495271d0F	The specific address of the crypto involved

Table 2: Variable definitions

Variable	Definition
W_{ijt}	Investment of insurable crypto j for investor i in day t. Simply put, the investment value is based on the investor's transaction history. We include more details of the calculation of W_{ijt} in Appendix D.
R_{jt}	Gross return of crypto j in day t, which is calculated by $\frac{\text{close price in day } t}{\text{close price in day } t-1}$.
V_{ijt}	Insurance coverage of crypto j for investor i in day t from Nexus Mutual. For cryptos stored in the same protocol, these cryptos share the insurance coverage. The coverage for each crypto is calculated based on the average coverage relative to the investment of each crypto. For example, if Compound holds 1 ETH and 1 DAI, with values of 50 USD and 10 USD respectively, and the total insurance coverage for Compound is 30 USD, then the coverage for ETH would be 25 USD, and the coverage for DAI would be 5 USD.
m_{ijt}	Premium rate applicable to cryptocurrency j held by investor i at time t. We construct m_{ijt} as follows. If investor i purchases insurance for j, then m_{ijt} is observed and computed as the premium paid divided by the associated coverage amount. If investor i does not purchase insurance for j at time t, m_{ijt} is unobserved; in this case, we impute it using the premium rate from the temporally closest policy written for the same protocol in which j is deposited. This assignment is reasonable in the context of Nexus Mutual because pricing and underwriting are primarily protocol-level and driven by the available capacity of the insurance pool for that protocol, which varies with market-wide supply and demand over time, rather than by any assessment of the individual policyholder's financial circumstances.
I_{ijt}	The loss indicator I_{ijt} equals one if crypto j held by investor i experiences a loss on day t. We follow the logic of Credibility Theory and integrate realized hack events from SlowMist and DefiLlama with simulated individual losses calibrated from premium rates provided by Nexus Mutual. Formally, I_{ijt} equals one if either a realized market hack occurs or a simulated individual loss event is triggered.

Notes: We rescaled the variables W_{ijt} and V_{ijt} by dividing them by \$10,000. This rescaling is done to ensure that the estimated risk attitudes are not excessively small, as the original variables can take on very large values.

Table 3: Summary statistics of variables

	mean	std	min	25%	50%	75%	max
Unit of observation: Investor-month-crypto ($N = 9575206$)							
Investment	18.2266	280.9736	0.0000	0.0285	0.2721	2.4352	62198.7393
Gross return	1.0001	0.0426	0.4719	0.9857	1.0000	1.0129	2.1135
Coverage	0.9672	24.2030	0.0000	0.0000	0.0000	0.0000	6117.0065
Premium rate	0.0371	0.0597	0.0130	0.0260	0.0260	0.0260	0.6056
Accident	0.0017	0.0406	0.0000	0.0000	0.0000	0.0000	1.0000

Notes: All the variables are defined in Table 2. The units for Investment and Coverage are in \$10,000. Gross return is defined as the rate of return plus 1. There are extremely large values for gross return because the price of some cryptos jumps from zero to a positive value within two trading days, resulting in very high returns—such as Dogecoin (DOGE). For example, in January 2021, DOGE surged approximately 10,000%, jumping from around \$0.007 to a peak near \$0.73 within months. Therefore, the gross return for all the cryptos has been winsorized at the 5% upper and lower percentiles.

Table 4: Summary statistics of SRA and PRA

	mean	std	min	25%	50%	75%	max
Unit of observation: Investor-month-crypto ($N = 31401$)							
SRA	0.7384	1.7446	-1.7069	0.0099	0.0919	0.5677	9.9692
PRA	0.6300	1.8315	-2.9865	0.0014	0.0419	0.3611	10.5877

Notes: SRA is the abbreviation of speculative risk attitude, and similarly hereinafter. PRA is the abbreviation of pure risk attitude.

Table 5: The consistency of SRA and PRA

	(1) Full sample	(2) SRA_averse PRA_averse	(3) SRA_averse PRA_seeking	(4) SRA_seeking PRA_averse	(5) SRA_seeking PRA_seeking
			Dependent variable: SRA		
PRA	0.3757***	0.6971***	-1.3020***	-0.1248***	0.3729***
	(0.0241)	(0.0224)	(0.0881)	(0.0155)	(0.0351)
trade_days	0.0006	-0.0004	-0.0005	0.0020	0.0007
	(0.0023)	(0.0018)	(0.0056)	(0.0058)	(0.0023)
trade_counts	0.0000	-0.0001	0.0001	0.0006*	-0.0000
	(0.0001)	(0.0000)	(0.0002)	(0.0004)	(0.0000)
token_counts	-0.0004	-0.0003	-0.0026	-0.0008	0.0015
	(0.0009)	(0.0005)	(0.0020)	(0.0042)	(0.0010)
received_amounts	-0.0002	-0.0002	-0.0026	-0.0028	0.0015
	(0.0007)	(0.0006)	(0.0020)	(0.0021)	(0.0010)
premium_asset_num	-0.0002	-0.0090	0.0116	0.0238	0.0006
	(0.0130)	(0.0118)	(0.0334)	(0.0174)	(0.0133)
cover_period	-0.0000	0.0000	-0.0000	0.0001	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
dollar_value	0.0002	-0.0022	0.0128	-0.0032	0.0043
	(0.0036)	(0.0031)	(0.0109)	(0.0067)	(0.0037)
_cons	0.5031***	0.3374***	0.4380*	-0.4580***	-0.2205**
	(0.1013)	(0.0960)	(0.2656)	(0.1443)	(0.1065)
N	31360	23456	4303	1151	1375
R2	0.565	0.805	0.669	0.671	0.783
Controls	Yes	Yes	Yes	Yes	Yes
Investor_FE	Yes	Yes	Yes	Yes	Yes
Year_month_FE	Yes	Yes	Yes	Yes	Yes

Notes: Robust standard errors clustered at the investor-level are provided in parentheses. *, **, and *** denote the significant differences from 0 at the 10%, 5%, and 1% levels, respectively. The table presents regression results examining the relationship between speculative and pure risk attitudes.