Limited Participation, Unemployment, and Inflation

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Abstract

This paper explores the extent to which limited participation in the labor market – sustained unemployment and labor force withdrawal – reconciles the absence of a long-run relation between unemployment and inflation in the U.S. since 2008. I provide preliminary evidence on a potential role for limited participation. Then I build a tractable general equm framework of labor participation and money based on frictions in labor and goods markets. The equilibrium generically exists and multiple equalibria can occur. A novel feature is that cyclical selection affects both intensive and extensive margins of labor supply: during bad times, job matches have to be unusually productive to warrant job creation, reducing the "liquidity" in the labor market; it also induces more workers in bad matches to quit, contracting the labor force. Another novel feature is that movements in and out of the labor force makes unemployment less reponsive to changes in inflation. I apply the framework to numerical illustrations and depart from the literature in identifying three key parameters: marginal product, the utility of leisure, and workers' bargaining power. Results show that labor market shocks that affect participation dominate monetary policy shock in explaining the absence of a long-run association between unemployment and inflation. Markup and workers's bargaining power have negligible effects.

keywords: search, non-participation, labor, money demand

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"There is no long-run trade-off between unemployment and inflation," Milton Friedman (1968).

"It was the Fed that killed the Phillips Curve," James Bullard (2020).

1 Introduction

Since 2008, the U.S labor market has experienced a large and persistent drop in labor supply—characterized by both labor force withdrawal and long-term unemployment — phenomena I refer to as 'limited participation'. Meanwhile, the long-run relation between unemployment and inflation in the U.S. since the WWII had been positive until 2007, thereafter a relation has been absent.

In this paper, I explore the extent to which limited participation explains the long-run unemployment and inflation disconnect since the Great Recession (2007-08). I begin by presenting facts on the long-run relation between unemployment and inflation from 1950 to 2024, and on large and persistent drop in labor supply. To formalize my conjecture, I build a tractable general equm framework of unemployment and money based on frictions in labor and goods markets, abstracting from nominal misperceptions and rigidities. Describing labor market transitions and goods market transactions that use money is a job for search theory. In particular, each period two decentralized markets – labor and goods – convene sequentially before a centralized settlement market. In the labor market, workers are in one of the three states at any point in time: employed, unemployed, and out of the labor force. Unemployed workers make sequential decisions in the following sense. After meeting a vacant job, he decides whether to accept the job depending on the realized match quality. If a meeting does not occur, or if it occurs and the realized match-quality is unacceptable, then he decides whether to drop out from the labor force or to search again. Employed workers decide whether to drop out the labor force if they do not get a job destruction shock. After the labor market, workers at any labor market state act as buyers and participate in the goods market. Productive firms are sellers. Buyers and sellers engage in bilateral exchange, using fiat money when monitoring over a transaction is absent. A period concludes with a settlement market that clear all payments.

In this framework, high match quality workers are less prone to quit. I show that labor market equilibrium generically exist, and it consists of three types of equilibria – high-quit, low-quit, and no-quit equa – depending on participation decisions, regardless of whether monetary equilibrium exists. When monetary equilibrium(a) exists, uniqueness of the labor market depends on the relative strength of unemployment welfare, anonymity, and inflation. Without a monetary equilibrium, the

labor market always contains multiple equilibria. When multiple equa emerge, two observationally similar workers can end up with very different outcomes, with one quitting the labor market (high-quit) and the other staying active (no-quit). On one hand, a sluggish goods market prompts firms to reduce job openings, and unemployed workers can quit the market. Wage also falls because firms generate less profit from sales. This reduces the value of employed workers. Even workers with high match-quality will quit at low leisure. This constitutes the Type H (high quit) equm. On the other hand, relatively high welfare benefits make searching in the labor market appealing. This keeps unemployed workers searching for jobs instead of quitting. When the unemployed does not quit, no employed workers quit. This constitutes the Type F (no-quit) equm.

The framework produces two previously unexplored features. One feature is cyclical selection, which affects the intensive as well as the extensive margin of labor supply. When aggregate productivity is low, say, job matches have to be unusually productive to warrant job creation. This reduces the speed of finding a job, or the "liquidity" in the labor market. This can create sustained low job-finding opportunities. At the same time, workers in bad matches quit more often, contracting the labor force. On the whole, cyclical selection fosters limited participation.

The other novel feature is that nonparticipation weakens the connection between the labor market and the goods market. Increasingly easy adjustments in and out of the labor force makes unemployment less responsive to changes in inflation. When workers can quit the market, the pool of the unemployed does not expand too much during bad times. During good times, firms can be reluctant to create much vacancy in light of the possibility of quitting, so job finding probability may not adjust too quickly.

I apply the above framework to numerical evaluations. The numerical exercises are not meant to be exhaustive. However, they are based on consistent microdata sources, study periods, and sample selections. Further, I depart from the literature regarding how to identify three key parameters. The first parameter is the utility of leisure when unemployed. Identification of this utility has been problematic in the literature. There is also no consensus as to what its value it ought to be, it can range from a negative to a positive number even for practitioners using the same dataset. And the method to obtain the parameter value differs widely among studies. I propose an alternative approach here. I use McFadden's random utility framework to estimate utility. Using observed choice to reveal preference is intuitive, yet rarely done in applied job search studies.

The second parameter is marginal product of labor. The literature typically replace it in numerical exercises with average product of labor, which is what the Bureau of Labor Statistics

publishes. To make progress, I use marginal product of labor based on results from recent estimates on production functions, e.g. Demirer (2020). The third parameter is the bargaining power in the labor market. It is typically set so that the bargaining power is the same as the matching elasticity, assuming Hosios rule (1990). But this is a strong assumption. My approach comes from the observation that with linear utility, Nash bargaining yields the same solution as Kalai's proportional bargaining. Kalai perceives workers' surplus to be a fraction of the total surplus, where the fraction is workers' bargaining power. Using this concept, I proxy workers' bargaining strength as the labor share, equaling labor compensation divided by gross value added.[1]

Results show that comparing the model with a two-state framework, the predicted unemployment is amplified in the two-state framework and unemployment exhibits a positive relation with inflation. This result suggests that when limited participation is abundant, using a two-state framework in analysis draws biased inferences.

I consider a number of counterfactual exercises: money demand, monetary policy, markups, workers' bargaining power, and limited participation. Money demand has fallen rapidly during the last few decades. The rapid rise of fintech has shifted the economy from cash-based to digital-based. The share of cash payments fell from 84 per cent in 1984 to 18 per cent in 2022. This reduces money demand, which in turn may diminish the impact of monetary policy on the labor market. Since the 1990s, the U.S economy has seen growing markup and a fall in labor share, which I consider as affecting workers' bargaining strength.

Limited participation encapsulates sustained unemployment and nonparticipation. In the framework, it is affected by meeting frictions, leisure, and welfare benefits. In particular, market frictions govern transitions to and from labor market states, while substantial welfare benefit extensions during the GR induced (i) the unemployed to stay as unemployed rather than taking a real job or quit the market, and (ii) non-participants to become job seekers. These phenomena amplify long-term unemployment despite low inflation during the Great Recession. Preference for leisure matters as technological change has rapidly bolstered the productivity of leisure, which may foster nonparticipation and may help reduce the unemployment response to monetary policy. Some examples include the explosion of online social media platforms (e.g. yahoo, instagram, tiktoks, facebook, X, etc.) and gaming and entertainment (e.g. stock prices of Gamestop, Electronic Arts, Xbox, and Netflix have exploded in recent years), etc.

Results show that while monetary policy and limited participation are comparable in their respective extent in explaining the non-relation between unemployment and inflation, labor market

[1] Some recent studies on labor share are Gouin-Bonenfant (2022) and Elsby et al. (2013).

shocks that promote limited participation dominate interest rate movement as monetary policy shock. The fall in money demand, markup, and workers' bargaining power have negligible impacts.

A recent study that also aims to explain the recent absence of long-run relation between unemployment and inflation is Ratner and Sim (2022). The authors posit that labor policies that have eroded workers' bargaining power may explain the non-relation of unemployment and inflation. While Milton Friedman (1968) is among the first who argues that in the long run, inflation should have little or no relation to unemployment. Here, the non-relation occurs for different reasons. Another related study is Berenstein et al. (2011), which sketches out a framework to address the positive relation between inflation and unemployment during 1955 to 2005.

One result in this paper is that the model produces countercyclical selection: when aggregate productivity is low, job matches have to be unusually productive to warrant job creation. This result complements that in Dorny (2018) who studies **cyclicality of wages and match quality using data from Germany. But the framework proposed here also produces procyclical selection. Selection can also be pro-cyclical in a sense that more low-quality matches dissolve in ba**d times. This prediction cannot occur in Dorny's or conventional frameworks where workers are always in the labor force.

This paper also relates to the literature on long-term unemployment. Many of which studies focus on the supply side effect of the Great Recession. Kroft et. al (2014) offer one of the most comprehensive views on long-term unemployment. They find that UI benefits and duration dependence are much more important than compositional factors in explaining long-term unemployment. Focusing on young men, Aquiar et. al (2022) propose leisure technology (e.g. video games) to explain the fall in their labor supply. In contrast, Davis et al (2013) explain long-term unemployment by positing employers control opportunities by limiting job openings. In this paper, employers' 'reluctance' to engage in hiring can be reflected by having unacceptable match quality that deters a job match to occur.

Modeling workers' non-participation using the search theoretic approach is first done by Burdett, Kiefer, Mortensen, and Neumann (1984). More recent examples include Alverez and Shimer (2011), Shimer (2011, 2013), and Wasmer and Weill (2021). Similar to Shimer, this study endogenizes participation choice in the labor market. In contrast to Shimer, this study focuses on low-frequency data, the framework allows job matches to differ ex post, the equm to admit multiplicity, and selection cyclical. It also strives to connect with goods market to address the issue of unemployment and inflation. While all these studies are about the labor market, Williamson (2006)

studies exogenous limited participation of buyers in the goods market using the New Monetarists's approach.

The literature on money and credit is large and is generally divided into two camps. One perceives money and credit as substitutes, e.g. Kocherlakota (1998), Sanches and Williamson (2010), Townsend (1989), Corbae and Ritter (2003), and Cavalcanti and Wallace (1999). The other camp perceives money and credit being complementary to each other, e.g. Shi (1996), Berentsen et al. (2007), Ferraris (2010), and Geromichalos and Herrenbrueck (2011). This paper views money and credit as substitutable payment instruments.

Section 2 describes motivational facts on unemployment, inflation, and limited participation. I also present preliminary evidence on the role of limited participation. Section 3 lays out the framework. Equilibrium and its characterization are contained in Section 4. Section 5 are numerical illustrations. Conclusion is in Section 6.

2 Motivational Facts

This section presents unemployment and inflation facts, as well as facts on limited participation.

Unemployment and inflation data cover the period from 1950 to 2024. Inflation data come from the Minneapolis Federal Reserve Bank. Post-Millennium unemployment data come from the annual American Commuinty Surveys (ACS) 2001-2024. This data source contains substantially larger sample size (at least 10 times) and response rates than other U.S. survey data such as the March CPS. For example, in a given year, the ACS final sample size for the youth exceeds 300,000 observations, and the 1990 census exceeds 2.5 million observations. Pre-Millenium unemployment data come from the Bureau of Labor Statistics as weighted labor force population tabulated by age groups, from which I calculated the unemployment rates. I further utilize the decennial census 1950 to 2020 to gain sample size accuracy.

Sample selection criteria are that respondents being non-institutionalized, whose age is between 21 and 64, and are not self-employed. For the most part, the qualitative results in the paper are not overtly sensitive to the inclusion criteria used. All statistics are weighted by the person weight given by the data source (except the tabulated data) to reproduce the census population.

Figure 1 plots uemployment rate and inflation rate using raw annual data and HP filtered trends (smoothing parameter set to 100). The two objects appear to comove for about three decades since 1970, and then shifted to opposite direction afterwards. To see the relation differently, Figure 2 plots unemployment rate vs inflation rate. Raw data show no correlation, while filtered data raise

the correlation slightly. Note that the regression lines in Figure 2 have slope being statistically insignificant with p-values close to 1.0. Figure 3 plots the relation using data prior to 2008. All plots deliver a postive long-run relation between inflation and unemployment. Table 1 presents the correlation values for the plots in Figures 2 and 3. The difference in how the two objects relate between 1950-2024 and 1950-2007 remains valid even nominal interest rate instead of inflation rate is used.

Table 1. Correlation between Unemployment and Inflation

	1950-2024	1950-2007	1970-2024	1970-2007
raw data	0.0089	0.2241	-0.0850	0.1805
trend data (HP=100)	0.2103	0.5729	-0.0164	0.4311
trend data (HP=1000)	0.2898	0.5689	-0.1455	0.4357
Five year average	0.2415	0.5470	0.1340	0.5663

Note: trend unemployment and inflation are HP filtered with the smoothing parameter=100

Alternatively, plotting the output gap vs inflation rate from 1955 to 2013, Williamson finds a pattern (Figure 4) that resembles closely to the top left panel in Figure 2.[2] The significance of Figure 4 is that the non-relation is evident before inflation targeting officially kicked in in 2012.[3] Figures 2 to 4 suggest that market shifts and shocks after 2007 may exert strong negative forces to nullify the long-run positive relation between inflation rate and unemployment rate in the distant past.

One major post-Millennium shock is the Great Recession 2007-08. The pandemic started in 2020 in the U.S. again wreaked havoc in the labor market. These two shocks brought large and persistent drop in labor supply, which can be distinguished into two salient features.

One feature is long-term unemployment, defined in the U.S as unemployment duration of at least 26 weeks. Figure 5 plots the share of long-term unemployed workers (out of all unemployed workers) from 1995 to 2024. The years soon after the Great Recession saw almost half of the unemployed being long-term unemployed, which peaked again during the pandemic, making the post-Great Recession long-term unemployment share about 29 per cent on average, significantly higher than the pre-Great Recession (1995-2007) average of 16 per cent.

[2] https://newmonetarism.blogspot.com/2013/12/phillips-curves-and-fisher-relations.html

[3] Such non-relation is not only occurring in the U.S., it also occurs in the U.K. In the Data Appendix, a plot between the two variables using U.K data is shown.

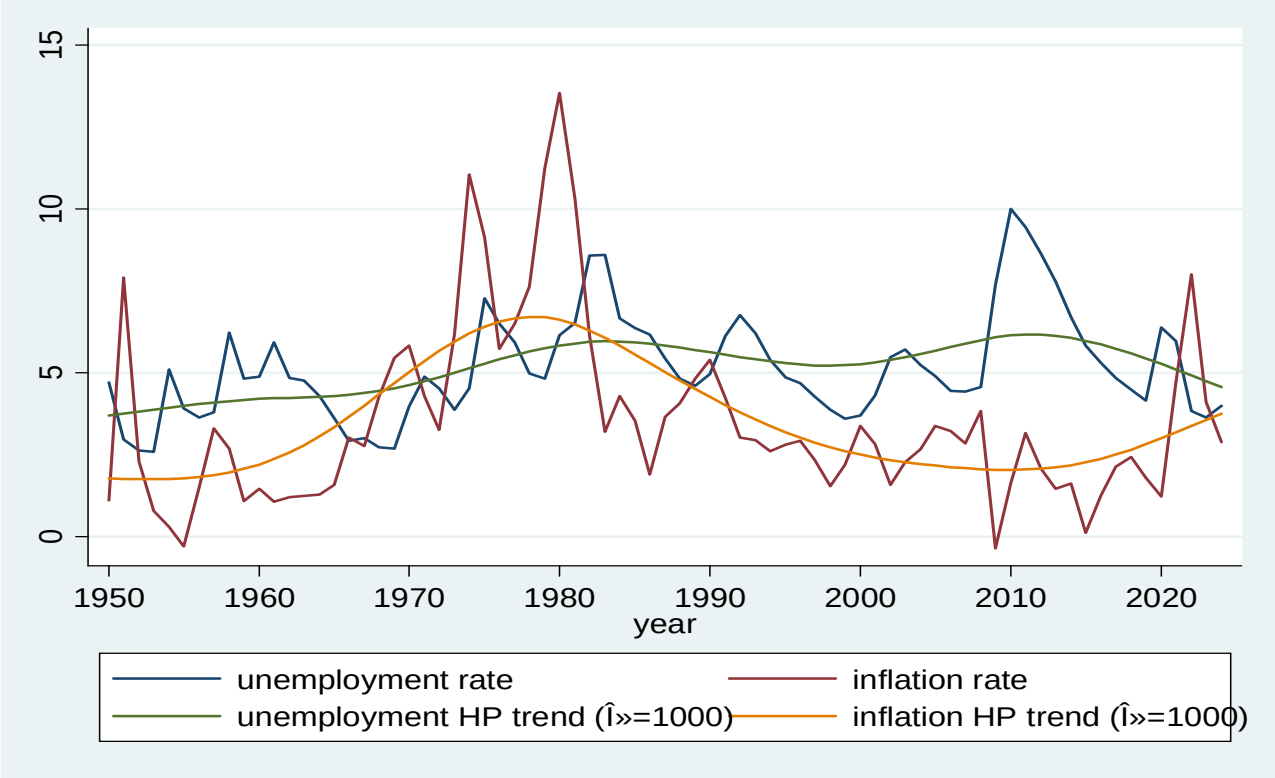


Figure 1: Unemployment and Inflation in the U.S, 1950-2024

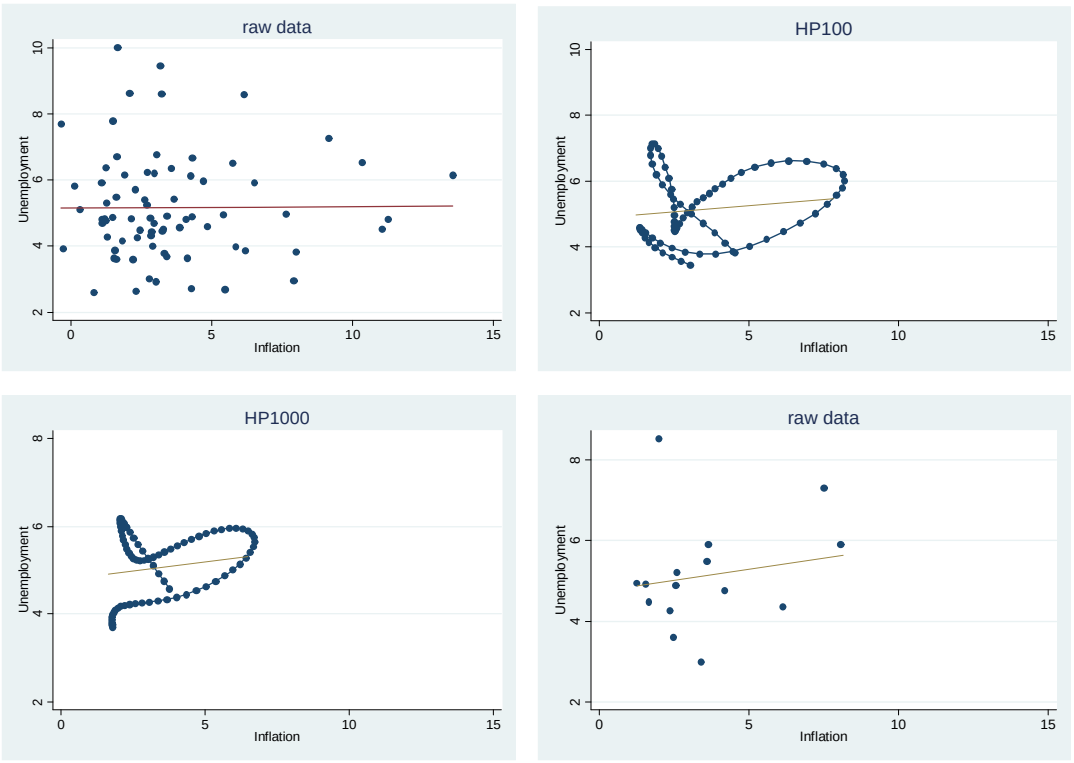


Figure 2: Long-Run Relation Between Unemployment and Inflation in the U.S 1950-2024

To get a sense of persistence, I use the census data to get the fraction who did not work over the previous year (unemployed or out of the labor force). This way minimizes snapshot or stop-gap phenomenon in cross-sectional data. Figure 6 shows that between 2000 and 2021, non-employment has grown 25 per cent, from 15.82 per cent in 2000 to 19.71 per cent in 2021. This increase does not appear to be driven only by the pandemic because from 2000 to 2015, the share is 31.3 per cent. Among the pool of inactive people, the survey question further delineates whether no work was only for the previous year or for the previous one to five years. On average, about two third of the responses indicates the latter.

The other salient feature is labor force withdrawal. Figure 7 plots movements to and from nonparticipation. I use the March CPS data file and set 9/2008 as the normalizing month. In general, there have been large changes in transitions to and from non-participation since late 2008. In particular, transition to non-participation (the blue line) has grown steadily since late 90s, picking up the pace after 2009. Indeed, the labor force partipation rate (not shown) has been trending downward since 2009-10.

I call the above two features limited participation, and conjecture that it may play a role in

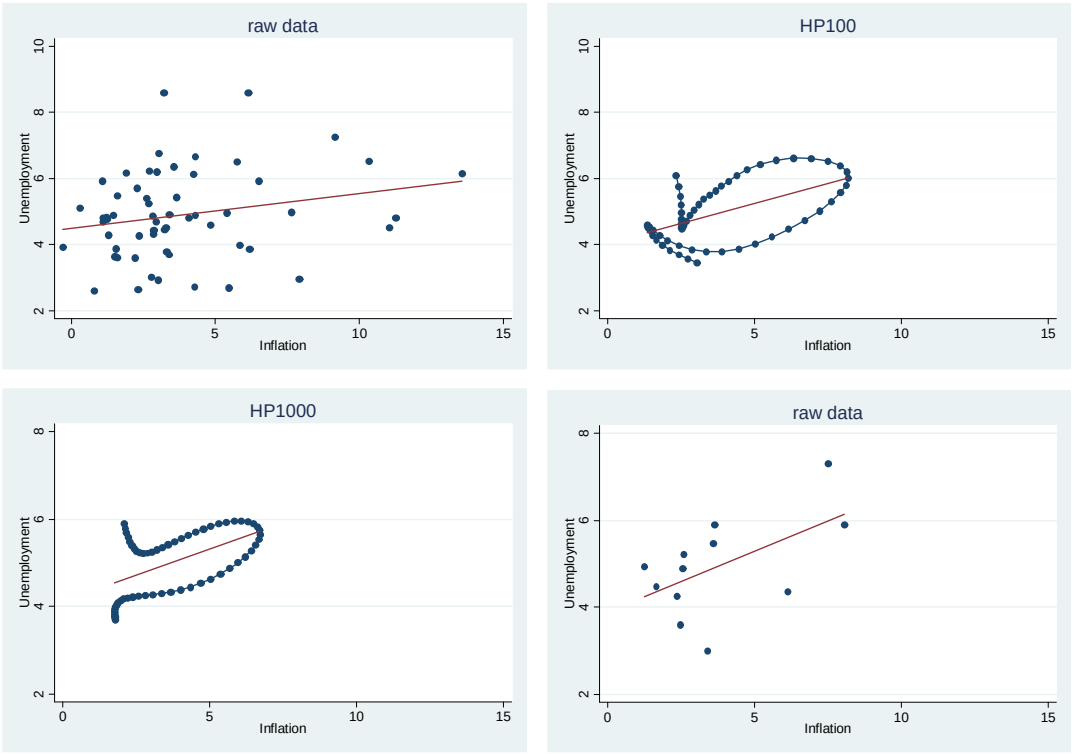


Figure 3: Long-Run Relation Between Unemployment and Inflation in the U.S 1950-2007

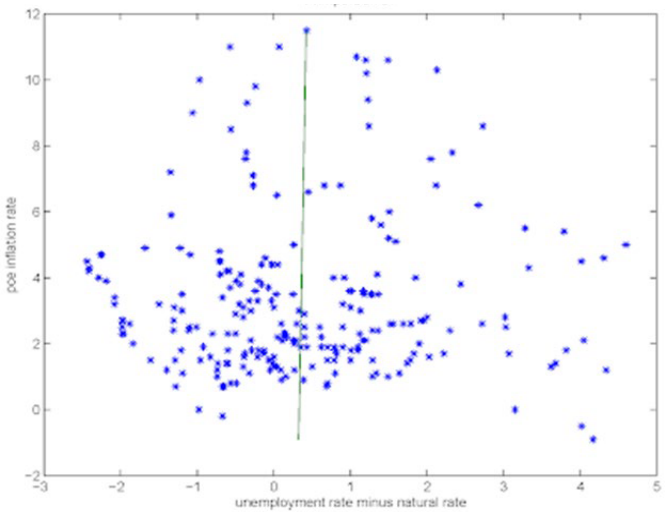


Figure 4: Williamson (U.S. 1955-2013)

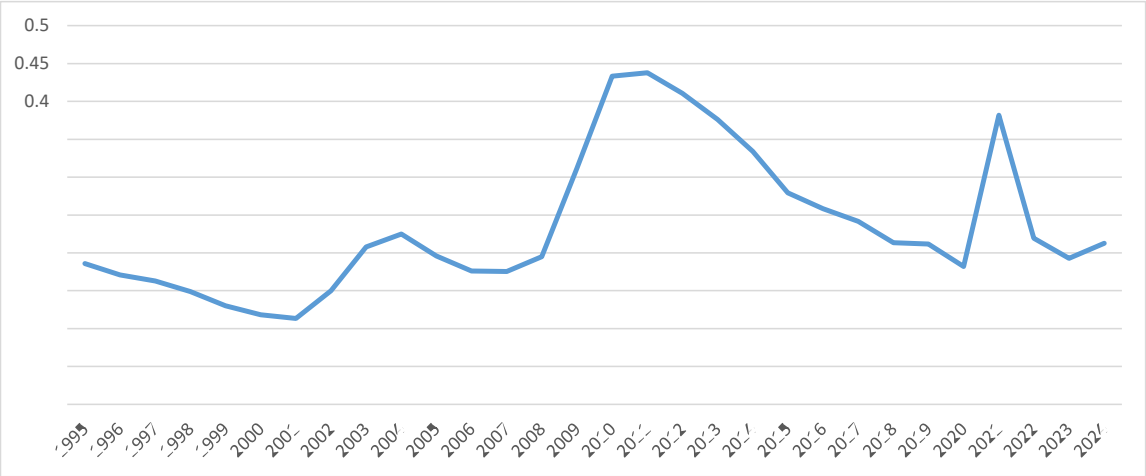


Figure 5: Share of Long-Term Unemployment in the U.S, 1995-2024

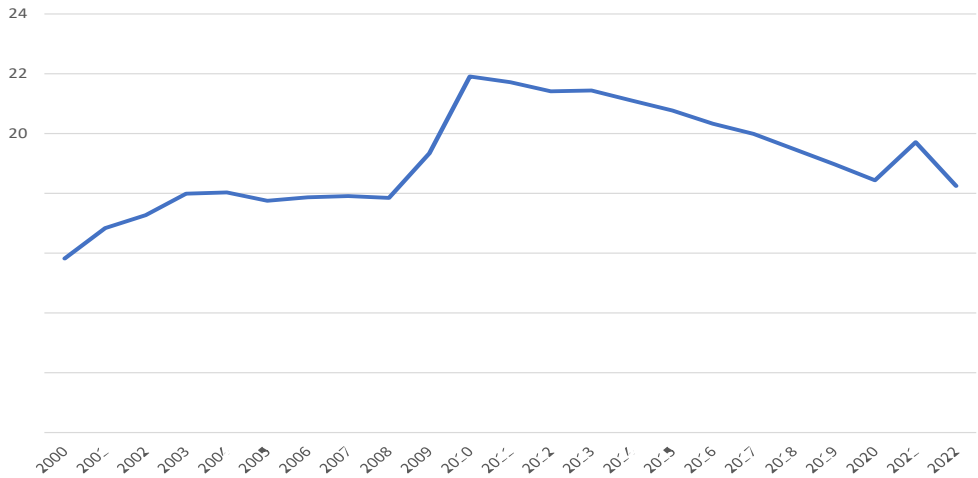


Figure 6: Share of Population Not Working Last Year

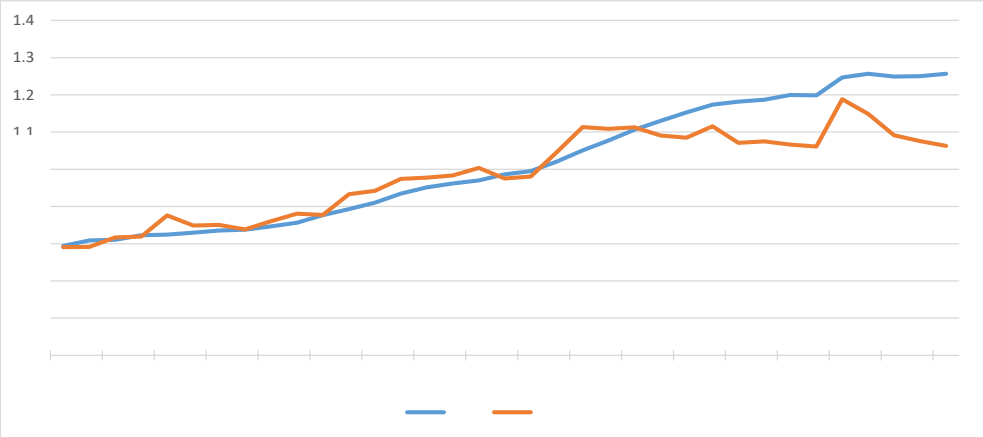


Figure 7: Movements To and From Nonparticipation, 1990-2024

making the labor market lack responsiveness to inflation in recent years.

3 Model

Time is discrete and continues forever. In each period three markets convene sequentially: a frictional decentralized labor market (LM), followed by a fricitonal decentralized goods market (GM), and then a frictionless settlement market (SM). There are two types of agents: firms and workers. The set of workers is $[0, 1]$. Workers differ in their labor market states e: employed ($e = 1$), unemployed ($e = 0$), or out of the labor force ($e = 2$). Let n_e represents the measure of a type e agent with $n_0 + n_1 + n_2 = 1$. The set of firms is arbitrarily large.

In the decentralized markets, agents meet pairwise and at random. In the LM, consider a CRS matching function $M(n_0, v)$, where v is the measure of vacancies. Let $\tau_1 = v/n_0$ denote market tightness. Assume that M is strictly increasing in τ_1, with $M(0, 1) = 0$ and $M(\infty, 1) = 1$, and $M(\tau_1, 1)/\tau_1$ is strictly decreasing with $M(0, 1)/0 = 1$ and $M(\infty, 1)/\infty = 0$. The probability workers meet firms is $M(n_0, v)/n_0 = M(1, \tau_1) = \lambda\tau_1$, and the probability firms meet workers $M(n_0, v)/v = M(\tau_1, 1)/\tau_1 = \lambda$.

Note, however, that a meeting does not result in a match: workers need to weigh in on match quality $q \in [0, 1]$. Workers consumate a job match opportunity when the realization of a random draw of match quality from distribution $F(q)$ exceeds some minimum acceptable quality. Match quality distribution $F(.)$ is twice differentiable and log-concave. In addition to determining whether a job opportunity is acceptable, workers in state $e = 0, 1$ decide whether to leave the labor market. The probability that a worker quits the labor market state $e \neq 2$ is ρ_e. There is no recall or on the

job search.

In the GM, a matching function is given by a CRS matching function $\mathcal{N}(B,S)$, where B and S are the measures of buyers and sellers in the goods market. Properties of the matching function $\mathcal{N}$ is similar to those in $\mathcal{M}$. Let $\tau_2 = B/S$ be market tightness in this market. The probability that a buyer meets a seller is $\mathcal{N}(B,S)/B = \mathcal{N}(\tau_2,1)/\tau_2 = \alpha$, and the probability a seller meets a buyer is $\mathcal{N}(B,S)/S = \mathcal{N}(\tau_2,1) = \alpha\tau_2$. A meeting in the GM results in a match. Two types of payment instruments can be used. With probability μ meeting is not monitored and money is used; with probability $(1-\mu)$ non-cash instruments are used; μ is exogenous.

In the SM, agents consume, enjoy utility from consumption and leisure, get paid, and pay tax and loans. There is no strategic default. They also decide how much money to carry to the next period. Real balances are given by $z \in [0,\infty)$. I adopt the standard convention for measuring real balances: when an agent bring m dollars to the SM, $z = m/p$, where p is the current price level. Meanwhile, the agent takes $\widehat{z} = \widehat{m}/p$ out of that market for use in the next period. The price level in the next SM is $\widehat{\mathtt{p}}$, so the real value of money is $\widehat{z}\widehat{\rho}$, where $\widehat{\rho} = p/\widehat{\mathtt{p}}$ converts $\widehat{z}$ into units of the numeraire good x in SM. Between the SM and the next LM, there is a discount factor $\beta = 1/(1+r)$, where r is the rate of time preference.

Although wage does not surface until the SM, the bargaining process occurs in the LM. Wage is determined by generalized Nash bargaining. Unmatched workers and firms bargain over their respective surplus with workers' threat point given by unemployment and that for firms' by posting vacancy. In the GM, the terms of trade are also determined by Nash bargaining.

The value functions for workers and firms are denoted by $W_e^j(q,z)$ and $J_e^j(q)$, where $e = 0,1,2$ refers to employment states, and $j = 1,2,3$ refers to the submarket within a period, respectively.

3.1 Workers

3.1.1 LM Payoffs and Participation

At any point in time in the labor force, workers decide whether to leave the market. Unemployed workers further decide whether a job opportunity is acceptable. The Bellman equations for workers at each labor market state is

$$W_0^1(0,z) = \lambda\tau_1 \int_{q^*} W_1^2(q,z)dF(q) + [1-\lambda\tau_1+\lambda\tau_1 F(q^*)][\rho_0 W_2^2(0,z) + (1-\rho_0)W_0^2(0,z)] \quad (1)$$

$$W_1^1(q,z) = \delta W_0^2(0,z) + (1-\delta)[\rho_1 W_2^2(0,z) + (1-\rho_1)W_1^2(q,z)] \quad (2)$$

$$W_2^1(0,z) = \psi W_0^2(0,z) + (1-\psi)W_2^2(0,z) \quad (3)$$

Equation (1) is the value for unemployed workers; it is made up of two components. The first component is that conditional on meeting a vacant job with probability $\lambda\tau_1$, a worker accepts the job if the realization of a random draw of match quality exceeds reservation match quality q^*. Reservation match quality is the quality at which a worker is indifferent between employed and unemployed, and will accept a job. In other words, if workers' surplus of a job match is $S_w(q) = W_1^2(q,z) - W_0^2(0,z)$, then $S_w(q^*) \equiv 0$.

The second component in (1) captures the event when a worker does not meet a vacant job, or if the realized match quality is below q^*, then the worker decides whether to leave the labor force with probability ρ_0.

Equations (2) and (3) are the values for the employed and the non-participants respectively, and have similar but simpler interpretation than (1). The employed encounters job destruction with probability $\delta > 0$; otherwise, he decides whether to leave the market with probability ρ_1; while non-participants return to the labor force looking for jobs with an exogenous probability $\psi > 0$.

Now consider the decision rules (ρ_0, ρ_1). Workers leave the labor market state if the 'quit' surplus, S_{qe}, is positive, where quit surplus for the unemployed and the employed are respectively $S_{q0} = W_2^2(0,z) - W_0^2(0,z)$ and $S_{q1}(q) = W_2^2(0,z) - W_1^2(q,z)$. More precisely,

$$\rho_0 = \begin{cases} 1 & \text{if } S_{q0} > 0 \\ [0,1] & \text{if } S_{q0} = 0 \\ 0 & \text{if } S_{q0} < 0 \end{cases}, \; \rho_1 = \begin{cases} 1 & \text{if } S_{q1}(q) > 0 \\ [0,1] & \text{if } S_{q1}(q) = 0 \\ 0 & \text{if } S_{q1}(q) < 0 \end{cases} \tag{4}$$

are the best response conditions (BR).

Note that the quit decision for the employed depends on match quality. Figure 7 shows the quit surplus for the employed, $S_{q1}(q)$, as a function of q. Observe that the surplus is falling in q, so the employed are less prone to leave the market the higher the match quality. Further, evaluate the quit surplus at q^*, one gets $W_2^2(0,z) - W_1^2(q^*,z) = W_2^2(0,z) - W_0^2(0,z)$, which is the quit surplus for unemployed workers, i.e. $S_{q1}(q^*) = S_{q0}$. This equality implies that the employed quits the market iff the unemployed at $q = q^*$ does. Thus, if $\rho_0 = 0$, then $\rho_1 = 0 \; \forall q \geq q^*$. However, if $\rho_0 = 1$, then the employed may not quit for all $q \geq q^*$: $\rho_1 = 1$ if $q < \widehat{q}$ and $\rho_1 = 0$ if $q \geq \widehat{q}$, where $\widehat{q} > q^*$ is the 'quit quality,' defined as $S_{q1}(\widehat{q}) \equiv 0$.

3.1.2 GM Payoffs

In the GM, g and d are quantity and dollar exchanged when using money, while $\widetilde{g}$ and $\widetilde{d}$ are quantity and credit exchanged using non-cash payments. Buyers obtain utility from goods $v(g)$ or $v(\widetilde{g})$ with $v(0) = 0$, $v' > 0$ and $v" < 0$.

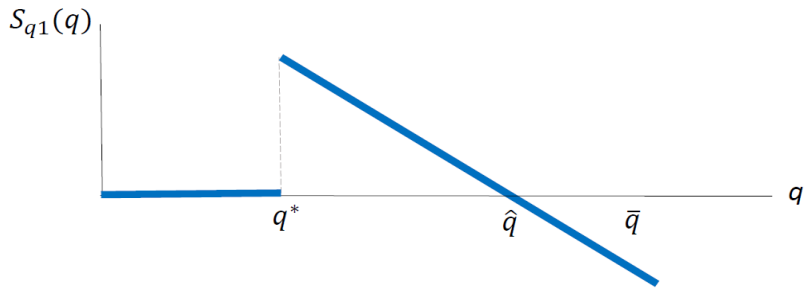


Figure 8: Quit Surplus (employed workers)

The Bellman equation for buyers of employment status e is

$$W_e^2(q,z) = \alpha\{\mu[\upsilon(g) + W_e^3(q,\rho(z-d),0)] + (1-\mu)[\upsilon(\widetilde{g}) + \nu + W_e^3(q,\rho z,\widetilde{d})]\} + (1-\alpha)W_e^3(q,\rho z,0)$$

where α is the probability of trade, μ is the probability that a random match is anonymous, and ν is the utility (or cost if $\nu < 0$) of using credit incurring debt $\widetilde{d}$. To reduce clusters, I set $\mu = 1$ in what follows to focus on the characteristics of limited participation when money is used, and $\mu \in (0,1)$ in the numerical exercises. With $\mu = 1$, the value function $W_e^3(q,\rho z,0)$ can be simplified as $W_e^3(q,\rho z)$.

Using the linearity of W_e^3 (see equation (6)), the Bellman equation becomes

$$W_e^2(q,z) = \alpha[\upsilon(g) - \rho d] + W_e^3(q,0) \ + \rho z$$

3.1.3 SM

In the SM, workers solve

$$\begin{aligned} W_e^3(q,z) &= \max_{x,\widehat{z}} x + I(e=0)\ell_0 + I(e=2)\ell_2 + \beta W_e^1(q,\widehat{z}) \\ \text{s.t. } x &= I(e=1)w(q) + I(e=0)b + \triangle - T + z - \widehat{z} \end{aligned} \tag{5}$$

where x is consumption, ℓ_e type e workers' utility of leisure with ℓ_1 normalized to one, $w(q)$ wage, b income from welfare if unemployed, $\triangle$ income from dividends, informal sectors, or returns from social capital in general (all in units of numeraires), and T a lump-sum tax. Between the SM and the next LM, there is a discount factor $\beta = 1/(1+r)$, where r is the rate of time preference. Define $I_e = I(e=1)w(q) + I(e=0)(b+\ell_0) + I(e=2)\ell_2 + \triangle - T$ and eliminate x from the budget equation, equation (5) becomes

$$W_e^3(q,z) = I_e + z + \max_{\widehat{z}}\{-\widehat{z} + \beta W_e^1(q,\widehat{z})\} \tag{6}$$

Equation (6) shows that W_e^3 is linear in I_e and z, and the choice of $\widehat{z}$ is independent of them as well. Even though $\widehat{z}$ may seem to depend on e and q from W_e^1, as will be shown below, $\widehat{z}$ will be independent of e and q. This implies that every worker exits the SM carrying the same amount of $\widehat{z}$.

To summarize the three markets, I first substitute the GM equations into the LM, and use the linearity in the SM to get

$$\begin{aligned}
W_0^1(0,z) &= \alpha[\upsilon(g) - \rho d] \ + \rho z + \lambda\tau_1 E[W_1^3(q,0)] \\
&\quad + [1 - \lambda\tau_1 \overline{F}(q^*)][\rho_0 W_2^3(0,0) + (1-\rho_0) W_0^3(0,0)] \\
W_1^1(q,z) &= \alpha[\upsilon(g) - \rho d] \ + \rho z + \delta W_0^3(0,0) \\
&\quad + (1-\delta)[\rho_1 W_2^3(0,0) + (1-\rho_1) W_1^3(q,0)] \\
W_2^1(0,z) &= \alpha[\upsilon(g) - \rho d] \ + \rho z + \psi W_0^3(0,0) + (1-\psi) W_2^3(0,0)
\end{aligned}$$

where $\overline{F}(q^*) \equiv 1 - F(q^*)$.

These equations have a general pattern:

$$W_e^1(q,z) = \alpha[\upsilon(g) - \rho d] \ + \rho z + \mathbb{E} W_e^3(q,0)]$$

where the expectation $\mathbb{E}$ is wrt employment transitions, which may depend on workers' participation decisions. Inserting $W_e^1(q,z)$ into (6), the SM problem becomes

$$W_e^3(q,z) = I_e + z + \max_{\widehat{z}} \{-\widehat{z} + \beta\alpha[\upsilon(g) - \rho d] \ + \beta\rho\widehat{z}\} + \beta \mathbb{E} W_{\widehat{e}}^3(q,0) \tag{7}$$

Equation (7) indicates that GM terms of trade (g,d) are related to $\widehat{z}$,but $\widehat{z}$ does not depend on employment status, I_e, nor z; every worker takes the same amount of money to the GM.

3.2 Firms

Firms make participation decision in the SM and do not carry money out of SM. In the LM, the Bellman equations are

$$J_0^1 = \lambda \int_{q^*} J_1^2(q) dF(q) + [1 - \lambda\overline{F}(q^*)] J_0^2 \tag{8}$$

$$J_1^1(q) = \delta J_0^2 + (1-\delta)[\rho_1 J_0^2 + (1-\rho_1) J_1^2(q)] \tag{9}$$

with $\lambda = \mathcal{M}(u,v)/v = \mathcal{M}(\tau_1, 1)/\tau_1$. Equation (8) indicates that if unmatched firms meet a worker with acceptable match quality, a match is formed; otherwise if unmatched firms do not meet a worker or meet a worker with unacceptable match quality, firms remain unmatched. Equation (9) is the Bellman equation for a productive firm. It indicates that with probability δ a job match terminates exogenously. With complimentary probability, a job match may also terminate if the worker decides to quit; otherwise it remains productive.

After production, firms take the output to trade in the GM. Output (marginal product) is given by pq, where p represents productivity measured in units of the numeraire good, and is complementary to match quality q. With probability $\alpha\tau_2$ a firm meets a buyer, sells g units of output, and incurs cost $c(g)$, with $c' > 0$ and $c'' \geq 0$. If no meeting occurs, firms take pq to the SM. Thus,

$$J_1^2(q) = \alpha\tau_2 J_1^3[pq - c(g), \rho d, q] + (1 - \alpha\tau_2) J_1^3(pq, 0, q)$$

where $\alpha\tau_2 = \mathcal{N}(B,S)/S = \mathcal{N}(\tau_2, 1)$. Only firms in production have positive payoffs. Firms with vacancy have no value in the GM or SM, so $J_0^2 = J_0^3 = 0$.

In the SM, firms with inventory x, real balances z, wage cost $w(q)$, and match quality q has expected payoff given by

$$J_1^3(x, z, q) = x + z - w(q) + \beta J_1^1(q)$$

Unmatched firms decide to enter the LM with a vacancy next period with payment $k > 0$ in units of x in the SM:

$$\begin{aligned} J_0^3 &= \max\{0, -k + \beta J_0^1\} \\ &= \max\{0, -k + \beta\lambda \int_{q^*} J_1^2(q) dF(q) + [1 - \lambda \overline{F}(q^*)] J_0^2\} \end{aligned} \tag{10}$$

where $J_0^3 = J_0^2 = 0$ by free entry. Consequently, firm's surplus of match in the LM is $S_f(q) = J_1^2(q) - J_0^2 = J_1^2(q)$; and $k = \beta\lambda \int_{q^*} S_f(q) dF(q)$.

3.3 Steady State

Let $\mathbf{n} = (n_0, n_1, n_2)$. The Appendix describes the derivation. Here is the result. The steady state $\mathbf{n}$ is given by:

$$n_0 = \frac{\psi\{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]\}}{\mathcal{D}} \tag{11}$$

$$n_1 = \frac{\psi\lambda\tau_1 \overline{F}(q^*)}{\mathcal{D}} \tag{12}$$

$$n_2 = 1 - n_0 - n_1 \tag{13}$$

where $\mathcal{D} = \{[1-\lambda\tau_1\overline{F}(q^*)]\rho_0 + \lambda\tau_1\overline{F}(q^*) + \psi\}\{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]\} - (\delta - \psi)\lambda\tau_1\overline{F}(q^*) > 0$. For example, in a two-state framework where $\rho_0 = 0$ (implying $\rho_1 = 0$), $n_0 = \delta/[\delta + \lambda\tau_1\overline{F}(q^*)]$, $n_1 = 1 - n_0$, and $n_2 = 0$, as conventional results suggested. It is straightforward to show that the employment rate (or equivalently, measure) is higher in a two-state framework. This has direct implication on market spillover. See Section 4.3.

3.4 Government

Besides firms and workers, there is a government here. Government consumes G, pays unemployment benefits b, levies tax T, and prints money at rate π, or π is inflation in the steady state. Government's budget constraint is $G + bu = T + \pi M/p$ that holds every period. Due to the Fisher equation $1 + i = (1+\pi)/\beta$, in steady state, one can equivalently describe monetary policy in terms of nominal interest rate i or π.

4 Equilibrium

Let $\mathbf{W} = (W_0^1, W_1^1, W_2^1, W_0^2, W_1^2, W_2^2)$ and $\mathbf{J} = (J_0^1, J_1^1, J_0^2, J_1^2)$.

Definition. *A steady state equilibium is a nonnegative list* $(\mathbf{W}, \mathbf{J}, \rho_0, \rho_1, \mathbf{n}, v, q^*, \widehat{q}, x, \widehat{z})$ *such that:* $\mathbf{W}$ *and* $\mathbf{J}$ *satisfy (1)-(10),* (ρ_0, ρ_1) satisfy the best response conditions (4), $\mathbf{n}$ *satisfies steady state (11)- (13),* v *satisfies free entry* $J_0^3 = 0$, q^* *satisfies* $S_w(q^*) = 0$, $\widehat{q}$ *satisfies* $S_{q1}(\widehat{q}) = 0$, $\widehat{z}$ *satisfies the choice of money holding (7), and the SM clears.*

In what follows, I solve the equm and then characterize it. To solve the equm, I first take the terms of trade in the goods market, g, as given, then solve for (q^*, $\widehat{q}$, v, $\mathbf{n}$). Next, taking (q^*, $\widehat{q}$, v, $\mathbf{n}$) as given, I solve for g. Numerical solution involves iterations until contracting mapping is achieved.

4.1 Labor Market

4.1.1 Reseration Match Quality and Quit Quality

Recall the surplus of a job match for unemployed workers is $S_w(q) = W_1^2(q,z) - W_0^2(0,z)$ and that for firms is $S_f(q) = J_1^2(q) - J_0^2$. After simplification, the surpluses are respectively

$$\begin{aligned}
[1-\beta(1-\delta)(1-\rho_1)]S_w(q) &= \alpha[v(g) - \rho d] + \rho z + I_1 - \widehat{z} \qquad (14)\\
&\quad +\beta(1-\delta)\rho_1[W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - (1-\beta)W_0^2(0,\widehat{z})\\
[1-\beta(1-\delta)(1-\rho_1)]S_f(q) &= pq + \alpha\tau_2[\rho d - c(g)] - w(q) - (1-\beta)J_0^2(q) \qquad (15)
\end{aligned}$$

Total surplus is $S(q) = S_w(q) + S_f(q)$.

An unemployed worker and a vacant firm maximize joint surplus of a job match with θ be workers' bargaining power. Bargaining solution gives the familiar expression

$$\theta S_f(q) S'_w(q) = (1-\theta) S_w(q) S'_f(q)$$

Unemployed workers accept a job when $q \geq q^*$, where q^* is given by $S_w(q^*) = 0$. From the bargaining solution, $S_w(q^*) = 0$ implies $S_f(q^*) = S(q^*) = 0$. Thus, setting total surplus at $q = q^*$ to zero and simplifying yields

$$\begin{aligned} pq^* &= (1-\beta) W_0^2(0,\widehat{z}) - \beta(1-\delta)\rho_1 [W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - \triangle + T + \widehat{z} \\ &\quad -\alpha\tau_2[\rho d - c(g)] - \alpha[v(g) - \rho d] - \rho z \end{aligned} \tag{16}$$

Reservation match quality given in (16) indicates that q^* increases with workers' outside option W_0^2 and falls with W_2^2.

Quit quality $\widehat{q}$ is given by $S_{q1}(\widehat{q}) = W_1^2(\widehat{q}, z) - W_2^2(0, z) = 0$. Simplify and rearrange terms to get

$$\begin{aligned} p\widehat{q} &= (1-\beta) W_2^2(0,\widehat{z}) + \beta\delta [W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - \triangle + T + \widehat{z} \\ &\quad -\alpha\tau_2[\rho d - c(g)] - \alpha[v(g) - \rho d] - \rho z \end{aligned} \tag{17}$$

Compare (16) and (17), it is straightforward to show that $q^* \leq \widehat{q}$. So the model is internally consistent with the best response conditions (4).

Next, substitute out the value functions in each of the match quality expressions and simplify (see Appendix for details), the results are summed up in Proposition 1

Proposition 1 *There exist a unique reservation match quality q^* and a unique quit quality $\widehat{q}$ that satisfy*

$$\begin{aligned} pq^* &= \frac{[1-\beta(1-\psi)]\Gamma(q^*,\widehat{q}|\rho_1) + \beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0\ell_2}{D} - \beta(1-\delta)\rho_1\frac{[\ell_2 - \Gamma(q^*,\widehat{q}|\rho_1)]}{D} \\ &\quad -\alpha\tau_2[\rho d - c(g)] \\ p\widehat{q} &= \ell_2 + \beta(\delta-\psi)\frac{[\ell_2 - \Gamma(q^*,\widehat{q}|\rho_1)]}{D} - \alpha\tau_2[\rho d - c(g)] \end{aligned}$$

where $\Gamma(q^*,\widehat{q}|\rho_1) \equiv b + \ell_0 + \beta\lambda\tau_1\theta p\left[\int_{q^*}^{\widehat{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)(1-\rho_1)}dx + \int_{\widehat{q}}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)}dx\right]$ *and* $D \equiv 1-\beta(1-\psi) + \beta[1-\lambda\tau_1\overline{F}(q^*)]\rho_0$.

The solution for reservation match quality q^* consists of three terms. The first is the weighted

average of the value of unemployment Γ and utility from non-market activities ℓ_2 if the unemployed quit the market. The second term is the expected gains from quitting the market while employed. The last term is the expected surplus of frms in the GM. High expected gains in the GM prompt agents to get productive by accepting job matches more easily. In frameworks without workers' participation concerns ($\rho_0 = \rho_1 = 0$), but with ex-post heterogeneity and the goods market, reservation match quality is reduced to $pq^* = b + \ell_0 + \beta\lambda\tau_1\theta p \int_{q^*}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)} dx - \alpha\tau_2[\rho d - c(g)]$.

The solution for quit quality $\widehat{q}$ also contains three terms: it compensates for non-market productivity ℓ_2 and the discounted quit surplus in case of job destruction with probability δ, net of the quit surplus from the possibility of returning to the workforce with probability ψ. The last term is firms' expected surplus in the goods market. It comes from wage, which affects workers' quit strategy and which is affected by gains in the goods market (as will be seen in (23)): a more robust goods market raises wage, because firms generate more profit from sales. A higher wage raises the value of employed workers on the job, $W_1^2(\widehat{q}, z)$. Workers become more inclined to stay at work, reducing the threshold to quit.

There is a relation between $q^*$and $\widehat{q}$. Quit quality can alternatively be expressed as

$$p\widehat{q} = pq^* + [1 - \beta(1-\delta)(1-\rho_1)]\frac{[\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)]}{D} \tag{18}$$

Equation (18) indicates that quit quality $\widehat{q}$ may or may not exceed q^* depending on unemployed workers' surplus from quitting the market: $W_2^2 - W_0^2 = [\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)]/D$. When $\rho_0 = 0$, the surplus $W_2^2 - W_0^2$ is negative, implying $\widehat{q} < q^*$ and $\rho_1 = 0$. When $\rho_0 = 1$, $\widehat{q} > q^*$, at least some employed chooses to quit with probability $\rho_1 > 0$.

Note that the solution for $\widehat{q}$ admits the possibility that $\widehat{q} > \overline{q}$. This can happen when ℓ_2 is sufficiently higher than some threshold level $\overline{\ell}_2 = Dp(\overline{q} - q^*) + \Gamma(q^*, \widehat{q}|1)$. When this is the case, all employed workers choose to quit (conditional on shocks).

Proposition 1 shows a novel market spillover between the markets: firms' GM surplus reduces labor market selection $q^*$and $\widehat{q}$; it serves as a substitution effect: workers are better off in the labor market when times are better in the goods market, in the sense that high expected gain is in the GM prompts agents to get productive; agents accept job opportunities easily and stay there longer so as to remain productive.

4.1.2 Vacancy

To get v, start with the bargaining solution:

$$\begin{aligned} w(q) &= \theta\{pq + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\{(1-\beta)W_0^2(0,\widehat{z}) \\ &\quad -\beta(1-\delta)\rho_1[W_2^2(0,\widehat{z}) - W_0^2(0,\widehat{z})] - \alpha[v(g) - \rho d] - \rho z - \triangle + T + \widehat{z}\} \end{aligned} \tag{19}$$

In general, wage is a weighted average between productivity pq and firms' surplus in the GM, and workers' outside option. With limited participation, workers' outside option contains their' value when unemployed W_0^2, net of (i) their surplus from quitting while employed, and (ii) their surplus from the GM. While a high value of unemployment raises the wage, leaving the market while employed or engaging in exchange in the GM reduces it because these two components (or expected gains) reduce reservation quality.

Insert (19) back into the surplus functions (14) and (15) and simplify to get

$$S_w(q) = \frac{\theta(pq - pq^*)}{1-\beta(1-\delta)(1-\rho_1)} \tag{20}$$

$$S_f(q) = \frac{(1-\theta)(pq - pq^*)}{1-\beta(1-\delta)(1-\rho_1)} \tag{21}$$

From Section 2.2, firms' entry implies $k = \beta\lambda\int_{q^*} S_f(q)dF(q)$. Using (21) and performing integration by parts, the job creation condition becomes

$$k = \beta\lambda(\tau_1)(1-\theta)p\left[\int_{q^*}^{\widehat{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)(1-\rho_1)}dx + \int_{\widehat{q}}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)}dx\right] \tag{22}$$

This condition indicates that the measure of vacancy v, which is imbedded in τ_1, can be pinned down when the marginal cost of entry (left side) is exhausted by the expected marginal surplus from a job match.

The solution for vacancy v given by (22) and the threshold qualities q^* and $\widehat{q}$ given in Proposition 1 need to be solved simultaneously, along with steady state $\mathbf{n}$ and money holding $\widehat{z}$. These endogenous objects depend on workers' participation strategy; thus, they vary by types of equilibria.

A note on wage to wrap up this section. Simplify the bargaining solution (19) to obtain

$$w(q) = \theta\{pq + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\{pq^* + \alpha\tau_2[\rho d - c(g)]\} \tag{23}$$

wage is a function of match quality. It is a weighted average between the highest workers can get from firms – productivity and the expected surplus from the GM – and the lowest workers can get in order to supply their labor. Wage is increasing in q. Reservation compensation w^* is obtained

by setting $q = q^*$ in (23), and simplifying to get

$$w^* = pq^* + \alpha\tau_2[\rho d - c(g)] \tag{24}$$

Proposition 2 *Researvation compensation is the sum of reservation productivity in the labor market and firm's expected surplus from the goods market.*

A novel result is that w^* is higher than reservation productivity pq^*, which is a result in conventional framework when only labor market is considered. Here, there are spillovers across markets so that firms gain not only productivity from the input market, but also the expected surplus from selling outputs in the goods market.

Reservation compensation is affected by workers' participation decisions via q^*. When $(\rho_0, \rho_1) = (0, 0)$, w^* is reduced to $\Gamma(q^*, \widehat{q}|0)] = b + \ell_0 + \beta\lambda\tau_1\theta p \int_{q^*}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)} dx$, or the value of unemployment as in conventional frameworks.

Similarly, quit compensation $\widehat{w}$ is obtained by setting $q = \widehat{q}$ in (23) and rearranging terms to get

$$\widehat{w} = w^* + [1 - \beta(1-\delta)(1-\rho_1)]\frac{[\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)]}{D}$$

It is straightforward to show that $\widehat{w} > p\widehat{q}$.

4.2 Goods Market

Recall the meeting probability is $\alpha = N(B, S)/B$. In equilibrium, all workers participate in the GM, and $B = 1$. Also, every worker is identical to firms because they all bring the same amount of money. However, firms participate in the goods market only if production takes place, which requires $e = 1$. Thus, $\alpha = \mathcal{N}(1, n_1)$, i.e. more employment facilitates trade. Recall $n_1 = 1 - n_0 - n_2$, and participation decisions $(\rho_0,\ \rho_1)$ affect the measure of employed workers. Thus, the second spillover is that participation decisions in the LM affect the speed of meeting in the GM.

Next, I obtain the solution for $\widehat{z}$. The terms of trade (g, d) are determined by the generalized Nash bargaining with η being buyers' bargaining power:

$$\max_{g,d} [v(g) - \rho d]^{\eta} [\rho d - c(g)]^{1-\eta}$$

subject to $d \leq z$ and $c(g) \leq y$, i.e. buyers cannot spend more than the amount of cash z they brought into the market and firms cannot sell more than they produced. Assuming that $c(g) \leq y$

is not binding, the solution for the terms of trade is $d = z$ and $g = h^{-1}(\rho z)$, where

$$h(g) \equiv \frac{\eta c(g)v' + (1-\eta)v(g)c'}{\eta v' + (1-\eta)c'}$$

Inserting this solution to workers' problem in the SM, the choice of $\widehat{z}$ from (7) can be expressed as

$$\max_{\widehat{z}\geq 0}\{-\widehat{z} + \beta\alpha v[h^{-1}(\rho\widehat{z})] \ + \beta(1-\alpha)\rho\widehat{z}\}$$

The solution satisfies

$$\frac{1}{\beta\rho} = \alpha\frac{v'}{h'} + 1 - \alpha$$

Substituting out $1/(\beta\rho) = 1 + i$ and $\alpha = \mathcal{N}(1, n_1)$ gives

$$\frac{i}{\mathcal{N}(1,n_1)} = \frac{v'}{h'} - 1 \tag{25}$$

If v'/h' is monotone, (25) gives a unique solution for $g > 0$, with $\partial g/\partial n_1 < 0$.

4.3 Characterization

In general, equm exists and may not be unique. There can be two types of equa in the goods market: monetary or non-monetary equm. Regardless of which type of monetary equm, three different types of participation equa in the labor market always exist. Since participation equa is novel, I begin by illustrating different types of equa in the labor market assuming monetary equm, before describing the equm with non-monetary equm.

From Proposition 1, it is easy to see that $(\rho_0, \rho_1) = (0, 1)$ or $(\rho_0, \rho_1) = (1, 0)$ can be ruled out. When $\rho_0 = 0$, $\widehat{q} < q^*$, i.e. no employed workers quit, so ρ_1 cannot equal one. When $\rho_0 = 1$, $\widehat{q} > q^*$, and at least some employed workers quit, so ρ_1 cannot be zero.

There are three types of equilibria that differ as follows:

1. A *Type F* equm has $\rho_0 = 0$ and $\rho_1 = 0$.
2. A *Type L* equm has $\rho_0 = 1$ and $\rho_1 = 1$ if $q^* < \widehat{q} < \overline{q}$.
3. A *Type H* equm has $\rho_0 = 1$ and $\rho_1 = 1$ if $\overline{q} \leq \widehat{q}$.

In *Type F* equm, unemployed and employed workers fully participate in the LM, F stands for 'full participation'. In *Type H* equm - 'H' for high non-participation - unemployed and employed workers quit with probaility 1. In *Type L* equm, 'L' for low non-participation, the unemployed quit with probability 1 while there can be non-participation by the employed when $q^* < q < \widehat{q}$ and participation when $q > \widehat{q}$.

For each type of equm, steady state measures $\mathbf{n}$ and v vary, and they affect the GM tightness, hence, the demand for z. Define the following critical value of ℓ_2,

$$\overline{\Gamma}_H = \frac{p\overline{q} + \alpha\tau_{2H}[\rho d - c(g)] + \beta(\delta - \psi)\Gamma(q_H^*, \overline{q}|1)}{1 - \beta(1-\delta) + \beta[1 - \lambda_H \tau_{1H} \overline{F}(q_H^*)]}$$

Proposition 2 presents the existence conditions.

Proposition 3 *The existence conditions for the three types of equilibria in the labor market are:*

(a) a Type F equm exists iff $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$

(b) a Type L equm exists iff $\Gamma(q_L^*, \widehat{q}_L|1) < \ell_2 < \overline{\Gamma}_H$

(c) a Type H equm exists iff $\ell_2 \geq \overline{\Gamma}_H$

Proof. *Type H* equm requires $\rho_0 = 1$ and $\rho_1 = 1$ for all q, where $\widehat{q}_H \geq \overline{q}$. Setting $\widehat{q}_H = \overline{q}$ in the expression for $\widehat{q}_H$ from Proposition 1, and rearranging terms, $\rho_1 = 1$ iff $\ell_2 \geq \overline{\Gamma}_H$. For $\rho_0 = 1$, the best response condition is $W_2^2 > W_0^2 \implies \ell_2 > \Gamma(q_H^*, \overline{q}|1)$. Next, find $\max\{\overline{\Gamma}_H, \Gamma(q_H^*, \overline{q}|1)\}$. Evaluate $\widehat{q}_H = \overline{q}$ using (18) to get $p\overline{q} = pq^* + [\ell_2 - \Gamma(q_H^*, \overline{q}|1)]/D$. Rearrange terms to get

$$\ell_2 = (p\overline{q} - pq_H^*)D + \Gamma(q_H^*, \overline{q}|1)$$

Observe $\widehat{q}_H \geq \overline{q} \iff \ell_2 \geq (p\overline{q} - pq_H^*)D + \Gamma(q_H^*, \overline{q}|1) = \overline{\Gamma}_H > \Gamma(q_H^*, \overline{q}|1)$ because $\overline{q} > pq_H^*$. Hence, $\max\{\overline{\Gamma}_H, \Gamma(q_H^*, \overline{q}|1)\} = \overline{\Gamma}_H$. The best response for $\rho_0 = 1$ is not binding.

For *Type L* equm, it occurs when $\rho_0 = 1$ and $\rho_1 = 1$ for $q_L^* < q < \widehat{q}_L < \overline{q}$. The best response condition for $\rho_0 = 1$ is $W_2^2 > W_0^2 \implies \ell_2 > \Gamma(q_L^*, \widehat{q}_L|1)$. For $\rho_1 = 1$, the upper-bound $\widehat{q}_L < \overline{q}$ is satisfied when $\ell_2 < \overline{\Gamma}_H$. For the lower bound, it requires $W_2^2 > W_1^2(q)$ for $q > q_L^*$. Simplify and evaluate $q = q_L^*$ to get $pq_L^* < \ell_2 + \beta(\delta - \psi)\frac{[\ell_2 - \Gamma(q_L^*, \widehat{q}_L|1)]}{D} - \alpha\tau_2[\rho d - c(g)]$. Substitute out q_L^* using Proposition 1 and simplify to get $\ell_2 > \Gamma(q_L^*, \widehat{q}_L|1)$. Thus, *Type L* equm exists iff $\Gamma(q_L^*, \widehat{q}_L|1) < \ell_2 < \overline{\Gamma}_H$.

For *Type F* equm, the best response condition for $\rho_0 = 0$ is $W_2^2 < W_0^2$. Simplify to get $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$. The best response condition for $\rho_1 = 0$ is $W_2^2 < W_1^2(q)$, for all $q > q_L^*$. Simplify to get $pq_L^* > \ell_2 + \beta(\delta - \psi)\frac{[\ell_2 - \Gamma(q_L^*, \widehat{q}_L|1)}{D} - \alpha\tau_2[\rho d - c(g)]$. Substitute out q_L^* using Proposition 1 and simplify to get $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$. Thus, *Type F* equm exists iff $\ell_2 < \Gamma(q_L^*, \widehat{q}_L|1)$. ■

I now show where different types of equa exist by partitioning the (p, ℓ_2) plane into regions that constitute equilibria. I start by comparing the boundaries of various equa given in Proposition 3, namely, $\overline{\Gamma}_H$ and $\Gamma(q_L^*, \widehat{q}_L|1)$. While it can easily be shown that both slopes are positive and the slope of $\overline{\Gamma}_H$ is steeper than that of $\Gamma(q_L^*, \widehat{q}_L|1)$, the relative size of the intercepts is unclear. It can

be shown that the intercept of $\overline{\Gamma}_H$ is higher than that of $\Gamma(q_L^*, \widehat{q}_L|1)$ iff

$$\alpha\tau_{2H}[\rho d - c(g_H)] > (b + \ell_0)D \tag{26}$$

The LHS is the expected net gain for firms in the GM and the RHS is the utility when unemployed. Note that both LM and GM policies affect (26): low inflation or low unemployment benefit favors (26).

The left panel of Figure 9 shows that when the condition given in (26) is satisfied, all three types of equa exist and is unique. Moreover, higher non-market productivity ℓ_2 induces non-participation. Starting with a low ℓ_2, expected gains from being in the labor LM exceeds ℓ_2 and no one quits, rationalizes the *Type F* equm. As ℓ_2 increases, unemployed workers prefer to quit, and low match-quality employed workers do the same; as long as ℓ_2 is not high enough to entice high match-quality workers to quit, then *Type L* equm occurs; otherwise (when the expected gains from employment for high match-quality workers are sufficiently low) all employed workers are willing to quit.

When the inequality in (26) is not satisfied, the line $\overline{\Gamma}_H$ shifts down relative to $\Gamma(q_L^*, \widehat{q}_L|1)$, as shown in the right panel of Figure 9. In this case, a larger parameter space supports both Type H and Type F equa. *Type L* equm is squeezed to the right as a high $p > \widehat{p}$ is needed to retain high match-quality workers in the LM. When $p < \widehat{p}$, employed workers with all match-quality would quit despite a low $\ell_2 > \overline{\Gamma}_H$. Also, *Type H* equm coexists with *Type F* equm.

These results suggest that starting with a low b or low inflation, equm in the LM is unique. As b or inflation increases, a larger parameter space supports the *Type F* equm as the unemployed find profitable to keep searching for jobs. It also supports *Type H* equm as higher inflation reduces gain in the GM. For low p firms in particular, wage shrinks, inducing more workers with high quality matches to quit the market. The result on unemployment depends on the net transition to and from non-participation and employment.

When there is no monetary equm, it is straightforward to show that (26) is violated. The three types of participation equa will be described by the right panel of Figure 9.

The above results lead to the following Proposition.

Proposition 4 *Given existence in Proposition 3, (1) in a monetary equilibrium(a), the labor market is unique iff (26) is satisfied, otherwise, Type F and Type H equilibrium coexist, and (2) in a non-monetary equilibrium, the labor market contains multiple equilibria.*

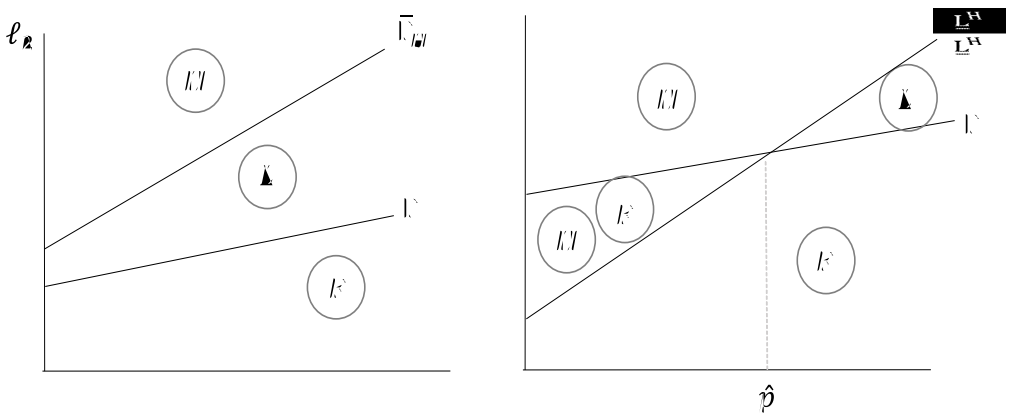


Figure 9: Types of Limited Participation Equilibria

4.4 Discussions

The framework generates spillovers between the two decentralized markets producing two previously unexplored features. The first feature is cyclical selection, which affects both the “intensive" and the “extensive" margin of labor supply. When aggregate productivity is low, say, job matches have to be unusually productive to warrant job creation. This reduces the speed of finding a job, or the “liquidity" in the labor market. At the same time, workers in bad matches quit more often. Thus, cyclical selection promotes limited participation. To wit, sustained low job-finding opportunities emerge either from severe meeting friction because demand for labor is low, or from counter-cyclical selection. While labor force withdrawal arising from pro-cyclical selection contracts the labor force.

The second novel feature is that nonparticipation weakens the spillover from the labor market to the goods market. Without participation consideration, the measure of employment is normalized by the labor force; with participation, it is normalized by the population. Thus, the employment rate is smaller with participation. As employment rate governs the speed at which buyers and sellers meet, admitting non-participation leads to fewer meetings between buyers and sellers, i.e. the goods market becomes less ‘liquid’ and fewer goods are sold. This effectively weakens the spillover from the labor market to the goods market.

The spillover from the goods market to the labor market may also be weakened. During bad times, unemployment may not increase too quickly because workers can quit the market. During good times, high expected gain in the goods market prompts agents to get productive: the unemployed are more eager to accept job opportunities, i.e. reservation match quality is lowered. However, unemployment may not fall too quickly because foreseeing a chance that quits can occur, firms adjust its outlook slightly downward. Firms do not create too many jobs, so job finding prob-

ability does not adjust too quickly. Hence, unemployment may not be reduced too quickly in good times when quit to nonparticipation is plausible. Section 5.2 demonstrates results of a two-state labor market as a comparison to those of a three-state labor market.

Another new result is that multiplicity in the labor market can occur. It happens when the goods market is doing sufficiently poorly or when unemployment benefit is sufficiently high. When either condition happens, two observationally similar workers can end up with very different outcomes, with one quitting the labor market and the other staying active. On one hand, a sluggish goods market prompts firms to reduce job openings, and unemployed workers can quit the market. Wage also falls because firms generate less profit from sales. This reduces the value of employed workers. Even workers with high match-quality will quit at low ℓ_2. This constitutes the Type H equm. On the other hand, a relatively high b makes searching in the labor market appealing. This keeps unemployed workers searching for jobs instead of quitting. When the unemployed does not quit, no employed workers quit. This constitutes the Type F equm.

5 A Numerical Illustration

In this section, I apply the framework proposed in Section 3 to illustrate whether limited participation has any impact on the long-run relation between unemployment and inflation from 1970-2024. Illustrations herein are not meant to be exhaustive.

I start with 1970 instead of 1950 because most micro information are unavailable prior to 1970. Starting with 1970, however, appears to have negligible effect on the nonrelation between unemployment and inflation. Figure 10 shows that data from 1970-2024 produces a relation that resembles closely to that using data from 1950 as shown in Figures 2 and 3. This fact indicates that adding or removing more distant data does not change the non-relation, because the issue of the non-relation appears to do with more recent data, particularly, since the Great Recession. In what follows, I begin with a description of my numerical strategy.

5.1 Numerical Strategy

Estimation is done in two steps. In step 1, I assign value to parameters that are independent from model solutions. In step 2, I estimate the rest of the parameters by matching moments. The only other study with numerical illustrations on unemployment and inflation using a search and matching framework is Berentsen et al. (2011). Tables 3 and 4 compare identification of model

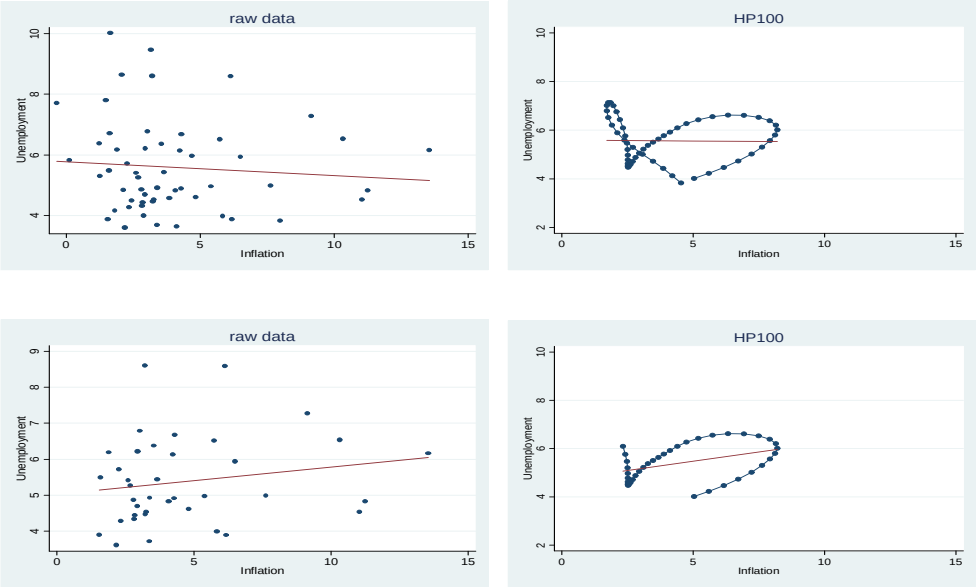


Figure 10: Long-Run Relation Between Unemployment and Inflation in the U.S 1970-2024

parameters between their study and this study.

Step 1 I identify the transition probability from not in the labor force to unemployment, ψ, by the average empirical transition probability. Similarly, I identify the job destruction probability, δ, by the average empirical transition probability from employment to unemployment. After many attempts, I find that incorporating piecewise constants of ψ and δ gives better fits than one average value over the study period. This can be explained by the fact that the labor market characteristics are fairly different particularly before and after the Great Recession.

I model utility in the labor market, ℓ, as the residual values obtained using McFadden's random utility framework. In particular, the three labor market states are cast as multinomial logit framework with the base state being employment. I use microdata IPUMS decennial census (for their more representativeness than the CPS) from 1970 to 2020 for estimation to get ℓ_j, $j \neq 1$. The covariates are standard demographic characteristics, including age, sex, race, schooling levels, marital status, labor market experience, and occupational and industry groups.

General productivity, p, represents marginal product of labor. However, the majority of data sources publish average output instead of marginal output, and the majority of practitioners in applied search and matching studies use average product as marginal product. I tap on recent advances in production function estimation to make progress here. Using data from 1961-2014,

Demirer (2020) finds that estimates of labor elasticity using the more sophisticated method by Ackerberg et al. (2015) are similar to those assuming a Cobb-Douglas production function. With Cobb-Douglas production function, it is simple to obtain marginal product from average product. So, given data on average output in the nonfarm business sector from the Bureau of Labor Statistics and a labor elasticity estimate 0.48 as industry average from the results in Demirer (2020), I calculate marginal product each year and use it in my estimation.

Firms' labor market entry cost is captured by k. The literature consistently shows an estimate of k to be close to zero. I set $k = 0.0001$.

In terms of the bargaining power in the labor market, observe that linear utility yields a Nash solution to be the same as Kalai's proportional bargaining solution. Kalai perceives workers' surplus to be a fraction of the total surplus, where the fraction is workers' bargaining power θ. Using this concept, I proxy workers' bargaining strength as the labor share, equaling labor compensation divided by gross value added. The average labor share over 1970-2024 is 0.60.[4]

I identify the probability of cash payments μ by the empirical fraction of cash usage in transactions. The data are from the Survey and Diary of Consumer Payment Choice (CPC) 2008-2023 collected by the Atlanta Fed and 1984 and 1986 Surveys of Consumer Transactions collected by the Federal board. The values for μ are 0.84, 0.37, 0.31, and 0.18 for 1984, 2008, 2016, and 2022 respectively. I find that using piecewise constant for μ fits the data considerably better than using an average value.

Meeting technology in the goods market is uniformly random so that $\mathcal{N}(B, S) = BS/(B + S)$. The measure of buyers is the population and is normalized to one. The measure of sellers is the measure of employment, endogenously determined in the model. Distribution cost is specified as $c(g) = g^{\gamma}$. When $\gamma = 1$, the MRT between the numeraire good and the GM good is 1, which is a harmless assumption, and is adopted here.

The discount rate β is the difference betwen the rate on Aaa bonds and realized inflation, which is 0.999.

Step 2: Matching Moments After specifying the utility in the goods market as $v(g; a) = Ag^{1-a}/(1 - a)$, I group the rest of the parameters into two categories: (q^*, F, b) and $(\sigma,\ A, a,\ \eta)$. The parameters in the first category (q^*, F, b) affect selection probability. From equation (24), reservation match quality can be expressed as a function of reservation wage. Following the first-

[4] The labor share in the U.S. has declined since 1990. Source: https://www.bls.gov/opub/mlr/2017/article/estimating-the-us-labor-share.htm#:~:text=The%20labor%20share%20is%20the,produced%20over%20the%20same%20period

generation job search econometric research, I let the minimum order statistics of residual wage to be the reservation wage, and then use equation (24) to get the reservation match quality. It is convenient to map the structural parameters in terms of derived parameters, so that I invert equation (16) to obtain b.[5] Next, I use the residual log wage distribution and equation (23) to obtain a distribution for q. Note that the transformation from q to w involves shifts and scale, so the distribution of q is *not* the same as the log-wage distribution, in contrast to the literature that often equates the two distributions. Since q^* and F vary with equm outcomes, they are jointly obtained with the procedure below.

In the second category, the parameter σ is associated with the meeting probability, which is given by $\lambda\tau_1\overline{F}(q^*)$. Given $q^*$and F, σ can be identified from the transition probability from unemployment to employment with a specification of the meeting technology: $M(n_0, v; \sigma) = n_0 v/(n_0^\sigma + v^\sigma)^{1/\sigma}$. The average value of the transition probability is 0.256.

The parameters A and a are associated with the utility in the goods market. The two parameters can be identified by money demand and its elasticity respectively. In particular, money demand in the model is

$$\frac{M}{pY} = \frac{h(g)}{n_1\{\alpha\tau_2[h(g) - c(g)] + y\}}$$

and the interest elasticity of money demand is

$$\zeta = \frac{\partial M/p}{\partial i}\frac{i}{M/p}.$$

The parameter A can be identified with average $M/(pY)$ and a by the empirical elasticity, using $M1$ as the measure of money. According to FRED data, between 1970Q1 to 2024 Q4, the average velocity of M1 is 6.95, and its inverse ($= M/(pY)$) 0.14. Some examples of interest rate elasticity is between 0.20 (1867–2007) and 0.30 (post-2008 financial innovation) respectively from Keating and Smith (2011) and Hendrickson and Salter (2021). During the pandemic era, it is near zero. A value of 0.3 means that for a 10 per cent increase in interest rates, money demand would fall by about 3 per cent. I set it to 0.24.

Buyers's bargaining power in the goods market is captured by η. It can be set so that the markup in the goods market matches the markup from retail data. Dopper et al. (2025) report

[5] Instead of assuming a number as done in many applied job search studies, I estimate b from the framework. One reason is that in most states, UI systems replace 30 to 50 per cent of prior weekly earnings or a maximum cap, whichever is lower. In 2019, only four states had UI rates over 40 per cent. Further, many unemployed workers do not get UI benefits, e.g. quitters or entrants. In addition, UI payouts has a limited horizon of 26 weeks. Also, UI is only one of the sources of welfare benefits for the unemployed, particularly since the Great Recession. And the response rate for unemployment benefits from public survey data is typically close to zero. All these complications make b difficult to be estimated.

average markup of 45 per cent in 2006 and 60 per cent in 2019. Using similar data, Atalay et al. (2023) analyzed markup for 33,000 products across 72 distinct retail product markets from 2006 to 2018 and found that the average weighted median markup across all product markets in their sample is 40 per cent. While De Loecker et al. (2020) and Nikiforos et al. (2024) obtained about 45 per cent (1970-2020) and 70 per cent (2019-2022) respectively based on Compustat data. I take markup to be 52 per cent.

Define $\boldsymbol{\theta} \equiv (\sigma,\ A, a,\ \eta)$. Denote the vector of statistics in the data by Λ and the vector of the statistics in the model evaluated at $\boldsymbol{\theta}$ by $\boldsymbol{\Lambda}(\boldsymbol{\theta})$. For each value of $\boldsymbol{\theta} \in \mathbb{R}^4_+$, I solve the model numerically and compute the model implied statistics $\boldsymbol{\Lambda}(\boldsymbol{\theta})$. I then minimize the following function

$$\min_{\boldsymbol{\theta}\in\mathbb{R}^4_+} (\boldsymbol{\Lambda}(\boldsymbol{\theta}) - \Lambda)'(\boldsymbol{\Lambda}(\boldsymbol{\theta}) - \Lambda)$$

5.2 Results

Columns 1 and 2 of Table 5 contain the statistics in the data and in the model. The fit is not bad. It is not perfect because the equations mapping the first three structural parameters in $\boldsymbol{\theta}$ to the statistics are nonlinear, so that an exact solution need not exist. In addition, the table presents model predictions on the long-run correlation between unemployment rate and inflation rate. Overall, the fit is reasonably good.

Table 5. Data Moments, Parameter Estimates, and Correlation

	1970-2024			1970-2007
	Data	3-state model	2-state model	3-state model
Meeting elasticity (labor market)	0.26	0.274	0.202	0.310
Money demand	0.14	0.152	0.159	0.169
Interest rate elasticity	0.24	0.371	0.398	0.516
Retail sector's markup	0.52	0.448	0.402	0.422
Correlation between unemployment and inflation	-0.016	0.102	0.250	0.499

Model Validation To assess whether the proposed three-state framework is more suitable for analyzing the recent unemployment-inflation non-relation than conventional two-state frameworks, I reestimate the model by setting $\rho_0 = \rho_1 = 0$ and $n_2 = 0$. Column 3 of Table 5 shows that the model does not predict the transition probability (from unemployment to employment) very

well, while the rest of the moments is little affected. The latter finding demonstrates that the two markets – labor and goods market – are not very connected, and potential mis-specification of the model (in the labor market) cannot be revealed from the last three moments alone. The two-state framework does not predict the transition probability well because in a two-state setting, the measure of the employed workers is larger than that in a three-state framework. To match the transition probability from unemployment to employment without inflow into nonparticipation requires more meetings between workers and vacancies. This distorts the estimate of σ (larger than it should be).

The two-state model yields a much higher correlation coefficient. During the early 2000 when inflation started to be low, there was an increase in the inflow to nonparticipation. This makes unemployment and inflation positively related. Without considering non-participation, the fall in the unemployment would be attributed to faster job finding, which coincidentally matches up with low inflation. Without considering the influx of non-participants back to the labor force, as in 2021, an increase in unemployment would be attributed to more sluggish job finding, which appears to be associated with a spike in inflation in 2021. As a result, unemployment and inflation appear to comove.

This exercise demonstrates that at times when movements into and out of the labor force are large, as shown in Figure 6, ignoring non-participation leads to bias inferences. For the rest of the paper, I use the three-state model developed in this paper.

Counterfactual Given $\boldsymbol{\theta}$, I compute the model equilibrium annually. To focus on long-run behavior and eliminating business-cycle fluctuations, all predicted unemployment outcomes are HP filtered with a smoothing parameter equaling 100. I report results using correlation coefficient because it is a more precise measure of association than eyeballing trends.

In addition, I re-estimating the model for 1970 to 2007 (Column 4 of Table 5). The idea is to obtain a set of parameters that enables me to perform counterfactual calculations based on changes in actual parameter values before and after 2008, rather than base on arbitrary percentage change in parameter values.

Table 6 contains the results on changes in the correlation coefficients. All results and data pass through a 100 HP-filter to eliminate business cycle fluctuations. The first two rows of Table 6 recorded the baseline predicted correlation for 1970 to 2007 and for 1970 to 2024 respectively. The difference in correlation value is 0.398. Then, I assess various plausible factors that the model can do to explain the drop in correlation.

Table 6. Counterfactual Results: Correlation between unemployment and inflation

	Correlation	Difference from 1970-2007
Baseline		
1970-2007	0.4998	
1970-2024	0.1018	0.3980
Counterfactuals		
Cash payment	0.1560	0.3438
Interest rate	0.1882	0.3116
Limited participation	0.1935	0.3063
Markup	0.1225	0.3773
Workers' bargaining power	0.1018	0.3980

Money Demand

The rapid rise of fintech has shifted the economy from cash-based to digital-based in recent decades. The fall in money demand may diminish the impact of monetary policy on the labor market. Using the 1970-2007 average estimate for $\mu = 0.60$ (share of transactions using cash payments) and keeping the rest of the parameter estimates from 1970 to 2024, I compute the unemployment rates. If money demand were at the pre-Great Recession level, then unemployment would be more responsive to inflation. The correlation coefficient increases from 0.1018 to 0.156, and the difference in correlation coefficient is 0.3438 (0.4998-0.1560=0.3438). In other words, when money demand rose from 0.425 to 0.6, or a 30 per cent increase, goods market and labor market would be more closely related, making spillover more effective. This explains the correlation difference by about 13.62 per cent.

Interest Rate

Berentsen et al. (2011) consider interest rate movement as a reflection of monetary policy. Inflation targeting officially started in 2012, but the Fed implicitly began targeting since the late 1990s. I study the impact of monetary policy, I keep all the baseline parameter values, but use the empirical autocorrelation and standard deviation of the nominal rate between 1970 and 2005 (which are 0.989 and 0.006 (quarterly) respectively), and recompute unemployment rates. The correlation coefficient increases from 0.1018 to 0.1882, and the difference in correlation coefficient is 0.3116. Thus, if monetary policy followed similar movement as during 1970 to 2007, the difference in correlation from the 1970 to 2007 counterpart would be 0.3116, explaining 21.7 per cent of the correlation difference.

Limited Participation

Three channels affect participation: labor market frictions, welfare benefits, and preference for leisure. By labor market frictions I mean frictional parameters that govern matching, job destruction, and entry: σ, δ, and ψ. Welfare benefits and leisure are respectively given by b and ℓ_2. I use the 1970 to 2007 estimates for these five parameter values and keep the rest of the parameter values from the 1970-2024 estimation, and recompute results. Table 5 shows that were the labor market participation pattern similar the time prior to the Great Recession, unemployment and inflation would yield a correlation coefficient of 0.1935, explaining about 23.04 per cent of the correlation difference.

Markup

As shown in Section 5.1, markup has increased steadily in the last few decades. Dopper et al. (2025) report an average markup of 45 per cent in 2006. De Loecker et al. (2020) obtained a markup estimate of about 45 per cent (1970-2020) based on ComputeStat data. I set markup to be 45 per cent between 1970 and 2007. Using this estimate and retaining the rest of the parameter values in the baseline framework, I recompute unemployment rates and obtain a correlation coefficient of 0.1225. So, as markup fell from 52 to 45 per cent, all else equal, the relation between unemployment and inflation would change little, explaining about 5.2 per cent of the correlation difference.

Workers' Bargaining Power

To draw comparisons with Ratner and Sim (2022), I probe into the effect of workers' bargaining power. Recall that in Section 5.1, labor share has been falling particularly since 1990. Between 1970 and 2007, the average labor share is 0.62. Had workers' bargaining strength held at 0.62, the correlation would remain the same. This result is expected because the change in labor share is small, from 0.60 to 0.62.

To conclude, monetary policy is much more important than the fall in money demand, and is comparable as limited participation in supporting the long-run non-relation. Further, the results indicate that limited participation as labor market shock dominates interest rate movement as monetary policy shock. Though markup affects goods exchanged in the retail market, which then affects money demand, it has relatively small effect on the long-run relation. Workers' bargaining power has no effect.

6 Conclusion

In this paper, I postulate that limited participation for workers in the U.S labor market since the Great Recession can potentially negate the positive long-run unemployment and inflation relation

observed prior to the Great Recession. My analysis is based on a framework in which the labor market and the goods market are decentralized and not dichotomized, job match is ex-post heterogeneous, unemployed workers decide on the acceptability of a job match, and both unemployed and employed workers make non-participation decisions. The framework is abstract from changes in demography, fiscal policies, or market environments such as housing or banking markets that are likely to impact the labor market but are well beyond the scope of the current study. Nonetheless, the framework proposed here is tractable, at least for steady states; and is amenable to quantitative analysis.

This paper's finding of no long-run association between unemployment and inflation over the period 1970–2024, driven primarily by a sustained contraction in labor supply following the Great Recession, echoes the central insight of Milton Friedman's natural rate hypothesis (1968). Though the underlying mechanism departs from Friedman's emphasis on adaptive expectations and the eventual futility of demand-side policies aimed at permanently lowering unemployment below its natural rate, both perspectives converge on the same fundamental conclusion: unemployment ultimately gravitates toward a level independent of the steady-state inflation rate.

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Appendix

Equate in and out flow for measures of workers at each state $i = 0, 1, 2$ to get:

$$\begin{aligned}
\lambda\tau_1\overline{F}(q^*)n_0 + [1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 n_0 &= \delta n_1 + \psi n_2 && (27)\\
\{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]\}n_1 &= \lambda\tau_1 F(q^*)n_0 && (28)\\
\psi n_2 &= (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]n_1 + [1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 n_0 && (29)
\end{aligned}$$

with the exhaustive condition $n_0 + n_1 + n_2 = 1$. Use the exhaustive condition and any of the two equations from (27) to (29), one can solve for $\mathbf{n}$. For example, substitute out n_2 in (27) and rearrange terms to get

$$\{[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 + \lambda\tau_1\overline{F}(q^*) + \psi\}n_0 = (\delta - \psi)n_1 + \psi$$

Then use (28) to substitute out n_1 in (27), and rearrange terms to get

$$\{[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 + \lambda\tau_1\overline{F}(q^*) + \psi\}n_0 - \psi = (\delta - \psi)\frac{\lambda\tau_1\overline{F}(q^*)}{\delta + (1-\delta)\rho_1[F(\widehat{q}) - F(q^*)]}n_0$$

Solve for n_0, and use it to recover n_1 and n_2 to get the results in the main text.

Proof of Proposition 1.

1. Reservation quality

From (16), q^* depends on $W_0^2(0, \widehat{z})$ and $W_2^2(0, \widehat{z}) - W_0^2(0, \widehat{z})$. To obtain $W_2^2 - W_0^2$, rearrange terms to get

$$\begin{aligned}
W_2^2 &= \frac{\ell_2 + \triangle + \beta\psi(W_2^2 - W_0^2)}{1 - \beta}\\
W_0^2 &= \frac{b + \ell_0 + \triangle - \beta c + \beta\lambda\tau_1 \int_{q^*}^{\overline{q}} W_1^2(q) - W_0^2 dF(q) + \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0(W_2^2 - W_0^2)}{1 - \beta}
\end{aligned}$$

Take the difference and simplify to obtain

$$W_2^2 - W_0^2 = \frac{\ell_2 - (b + \ell_0) + \beta c - \beta\lambda\tau_1 \int_{q^*}^{\overline{q}} W_1^2(q) - W_0^2 dF(q)}{1 - \beta(1-\psi) - \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0} \tag{30}$$

The term inside the integral is workers' surplus $S_w(q) = W_1^2(q) - W_0^2$. Inserting (19) into the surplus equation (14) and substitute out terms using (16),

$$S_w(q) = \frac{\theta(pq - pq^*)}{1 - \beta(1-\delta)(1 - \theta\rho_1)}$$

Substitute out $W_1^2(q) - W_0^2$ in $W_2^2 - W_0^2$ using $S_w(q)$ and perform integration by parts,

$$W_2^2 - W_0^2 = \frac{\ell_2 - (b + \ell_0) + \beta c - \beta\lambda\tau_1\theta p \left[\int_{q^*}^{\widehat{q}} \frac{1-F(x)}{1-\beta(1-\delta)(1-\theta\rho_1)} dx + \int_{\widehat{q}}^{\overline{q}} \frac{1-F(x)}{1-\beta(1-\delta)} dx\right]}{1 - \beta(1-\psi) - \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0} \tag{31}$$

Next, rearrange terms in W_0^2 to get

$$(1-\beta)W_0^2 = \frac{[1 - \beta(1-\psi)]\Gamma(q^*, \widehat{q}|\rho_1) + \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0(\ell_2 + \triangle)}{1 - \beta(1-\psi) - \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0} \tag{32}$$

where $\Gamma(q^*, \widehat{q}|\rho_1) \equiv b + \ell_0 - \beta c + \beta\lambda\tau_1\theta p \left[\int_{q^*}^{\widehat{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)(1-\theta\rho_1)} dx + \int_{\widehat{q}}^{\overline{q}} \frac{[1-F(x)]}{1-\beta(1-\delta)} dx\right]$.

Insert (31) and (32) into (16) and simplify to get

$$\begin{aligned} pq^* &= \frac{1 - \beta(1-\psi) + \beta(1-\delta)\rho_1}{D}\Gamma(q^*, \widehat{q}) + \frac{[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0 - (1-\delta)\rho_1}{D}\beta\ell_2 \\ &\quad -\alpha\tau_2[\rho d - c(g)] \end{aligned}$$

where $D \equiv 1 - \beta(1-\psi) - \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0$. The left side increases in q^* and the right side falls with it. The above solution can be re-expressed as

$$\begin{aligned} Dpq^* &= [1 - \beta(1-\psi)]\Gamma(q^*, \widehat{q}|\rho_1) + \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0\ell_2 - \beta(1-\delta)\rho_1[\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)] \\ &\quad -\alpha\tau_2[\rho d - c(g)]D \end{aligned}$$

2. quit quality

Recall $\widehat{q}$ is defined by $W_2^2 \equiv W_1^2(\widehat{q})$. Observe that

$$W_2^2 - W_1^2(q) = [W_2^2 - W_0^2] - [W_1^2(q) - W_0^2]$$

where $W_2^2 - W_0^2$ is given by (31) and $W_1^2(q) - W_0^2 = S_w(q)$ given in (20). Insert these components into the right side of the equation and simplify to get

$$[1 - \beta(1-\delta)(1-\rho_1)][\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)] = (p\widehat{q} - pq^*)D$$

Rearrange terms to get $p\widehat{q} = pq^* + [1 - \beta(1-\delta)(1-\rho_1)][\ell_2 - \Gamma(q^*, \widehat{q}|\rho_1)]/D$. When $\rho_0 = 1$, $W_2^2 - W_0^2 > 0 \Rightarrow \ell_2 - \Gamma(q^*, \widehat{q}|\rho_1) > 0$. If $D > 0$, $q^* < \widehat{q}$; otherwise $q^* > \widehat{q}$.■

Wage

At $q = q^*$, (19) becomes

$$\begin{aligned} w(q^*) &= \theta\{pq^* + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\{(1-\beta)W_0^2 \\ &\quad -\beta(W_2^2 - W_0^2)[\delta\rho_0 + (1-\delta)\rho_1] \\ &\quad -\alpha[v(g) - \rho d] - \rho z - \triangle + T + \widehat{z}\} \end{aligned}$$

Substitute out $W_2^2 - W_0^2$ and W_0^2 given in (31) and (32) and define

$$\mathtt{A} = \beta[1 - \lambda\tau_1\overline{F}(q^*)]\rho_0\frac{\ell_2}{D} + [1 - \beta(1-\psi)]\frac{\Gamma}{D} - \beta[\delta\rho_0 + (1-\delta)\rho_1]\frac{(\ell_2 - \Gamma)}{D}$$

Reservation wage becomes

$$w(q^*) = \theta\{pq^* + \alpha\tau_2[\rho d - c(g)]\} + (1-\theta)\mathtt{A}$$

Note that from Proposition 1, $pq^* = \mathtt{A} - \alpha\tau_2[\rho d - c(g)]$, insert this into $w(q^*)$ to obtain $w(q^*) = pq^* + \alpha\tau_2[\rho d - c(g)]$.■

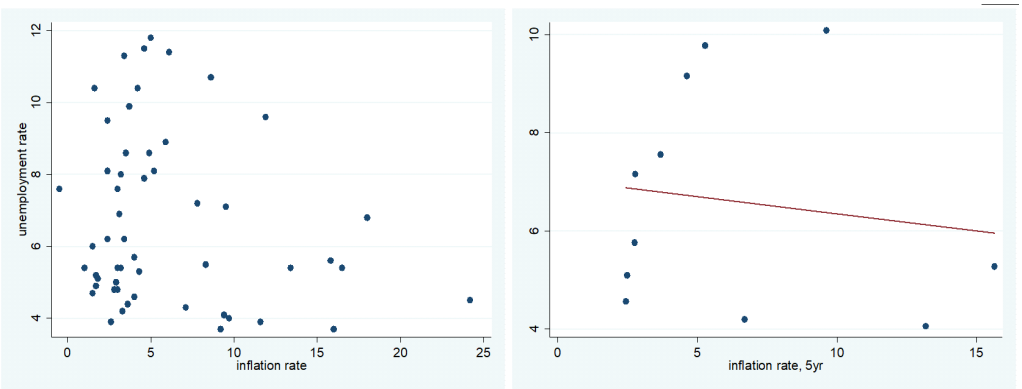


Figure 11: U.K-ONS, 1971-2023, civilian aged 16+

Data Appendix

The data used to plot Figures 1 and 2 come from the American Commuinty Surveys (ACS) 2001-2022, as well as the decennial census 1990, 2000, 2010, and 2020. This data source contains much larger sample size (at least 10 times) than other U.S. survey data such as the March CPS. For example, in a given year, the ACS final sample size for the youth exceeds 300,000 observations, and the 1990 census exceeds 2.5 million observations. Further, the response rates for hours and weeks worked are 100 per cent. To fill the data gap during 1991-1999, I use the CPS data.

My sample contains people who are non-institutionalized, whose age is between 21 and 64 and are not self-employed. For the most part, the qualitative results in the paper are insensitive to the inclusion criteria used.

Note 1: All statistics are weighted by the person weight given in the sample to reproduce the census population.

UK Data

Data source: https://www.ons.gov.uk. For unemployment data: data contain civilians aged 16 and above.