Learning, Cash Flow, and Financing in the Intangible Economy

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Abstract

We study how the rise of intangible assets reshapes the role of cash flow in corporate finance. Over time, cash flow has become less relevant for investment but increasingly important for cash accumulation and external financing, particularly among high-tech firms. To explain these patterns, we develop a dynamic structural model with financial frictions and Bayesian learning, where firms infer long-term productivity from realized cash flow and soft signals. The model disentangles the financing and informational roles of cash flow, rationalizing the diminished investment—cash flow sensitivity and the growing precautionary motive for cash. Methodologically, we combine simulated method of moments with the Shapley value decomposition to quantify the contribution of learning, showing that information frictions account for much of the observed time-series and cross-sectional variations. Our findings provide new insights into how firms adapt investment and financing strategies in an economy increasingly dominated by intangibles.

Keywords: Intangible capital; Cash flow; External financing; Learning; Structural estimation

JEL classification: G32, G31, E22, D83

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1 Introduction

The surge in intangible assets over the past few decades has profoundly reshaped corporate behavior, redefining how firms finance and allocate capital. This transformation raises important questions about the effectiveness of traditional finance models, particularly regarding the role of cash flow in driving investment, savings, and external financing. Understanding these dynamics is essential, as the growing prominence of intangible assets fundamentally redefines the financial strategies firms adopt in an increasingly knowledge-driven economy.

Over the past four decades, the role of cash flow in corporate finance has reversed—from investment to boosting cash accumulation and reducing reliance on external financing.[1] At the cross-sectional level, we find that the shift is particularly pronounced in high-tech industries. This new stylized fact challenges the conventional view of cash flow as a signal of firm value and a determinant of investment, and calls for a reassessment of its role in shaping corporate financial policies.

We develop a learning-based model with financial frictions to explain this reversal and provides a new microfoundation for the corporate savings puzzle. In our framework, firms learn about their unobserved long-term productivity through realized cash flow and a soft signal containing intangible information about future productivity. Because intangible information shapes firms' beliefs in ways that need not rely on observed cash flow, a divergence arises between learning driven productivity and cash flow realizations. This divergence, in turn, alters the informational role of cash flow and fundamentally changes firms' investment and financing policies. The model rationalizes the diminished role of cash flow in affecting investment and its increasing importance in cash management and external financing, thus offering a new perspective on how firms adapt financial policies in an economy increasingly driven by intangibles.

This paper introduces a novel Shapley–SMM decomposition for attribution analysis. In general, it is difficult to quantify how the model mechanism is reflected in the data. However, this methodological innovation, combined with structural estimation, clearly isolates the contribution of information frictions and quantifies the importance of the learning channel. More specifically, it allows us to measure how much of the observed reversal in cash flow is driven by changes in the information environment.

We begin by documenting how firms' investment, cash savings, and equity issuance respond to cash flow and Tobin's Q in both time series and cross section. First, to trace the evolving role

[1] These patterns hold even after controlling for Tobin's Q, challenging traditional finance models.

of cash flow, we analyze its impact on investment, cash savings, and equity issuance over three sub-periods: 1975–1994, 1995–2004, and 2005–2024. Investment has become much less dependent on internal cash flow, with sensitivity declining by about 75%, while reliance on Tobin's Q has risen by roughly 50%. Conversely, cash savings have become highly dependent on cash flow, with sensitivity rising from near zero to 0.062, underscoring a shift toward precautionary savings. Equity issuance has become increasingly aligned with growth opportunities (Tobin's Q) rather than cash flow, with the issuance-–cash flow sensitivity flipping from positive to negative. Second, we conduct a cross-sectional analysis comparing non-tech and tech industries. The results show that tech firms' investment exhibits lower sensitivity to cash flow but roughly 80% higher sensitivity to Tobin's Q compared with non-tech firms. In contrast, tech firms save significantly more cash out from cash flow, with the sensitivity of cash savings to cash flow over ten times higher than that of non-tech firms. Tech firms also exhibit a stronger negative sensitivity of equity issuance to cash flow, indicating a substitution between internal and external financing. These findings are robust to alternative measures of investment, Tobin's Q, and cash flow proposed by Peters and Taylor (2017).

Motivated by the empirical evidence on heterogeneous firm responses to productivity shocks and the evolving role of cash flow, we develop a dynamic structural model to rationalize the interplay between cash flow dynamics and firm behavior. Our model integrates firms' investment, saving, and financing decisions with a Bayesian learning process about long-term productivity, capturing the impact of information uncertainty on corporate policies. It is cast in a standard infinite-horizon, discrete-time stochastic framework, where managers maximize firm value under costly external financing.

The central innovation of our model lies in disentangling the informational and financing roles of cash flow through a two-factor productivity structure—comprising persistent and transitory shocks—augmented by a Bayesian learning mechanism. This mechanism enables firms to infer long-term productivity from noisy cash flow and soft signals. Unlike standard dynamic corporate finance models, such as Bolton et al. (2011), which typically assume fully observable productivity, our framework captures the impact of information frictions on firm behavior. By integrating capital adjustment costs, costly equity issuance, and information informativeness, the model generates a precautionary demand for cash to mitigate mismatches between investment opportunities and internal funds. This approach provides a robust theoretical foundation for the observed cross-sectional and time-series variation in firms' cash holdings and investment, shedding light on how learning about productivity shapes corporate real and financial decisions.

Our analysis presents these insights by examining how firms respond to different types of signals and how the quality of information influences their behavior. We show the dynamic effects of two types of signals: the soft signal, which provides intangible information about future productivity but do not affect current cash flow; and the cash-flow signal, which reflects observable firm performance metrics and generate immediate cash flow. A positive soft signal shock leads firms to increase investment based on optimistic expectations about future productivity, but without additional cash flow, they must draw down cash reserves or issue costly equity to finance these investments. In contrast, the cash-flow signal boost both expectations and cash flow, allowing firms to increase investment and build cash reserves while reducing reliance on equity issuance. However, the short-lived nature of cash flow gains from the cash-flow signal can lead firms to over-invest relative to available liquidity, potentially depleting cash reserves over time. These findings highlight how information frictions shape firms' investment and financing decisions, with the soft signal amplifying the impact of costly equity financing and the cash flow signal revealing a tension between short-term liquidity and long-term investment opportunities.

To rigorously examine these dynamics, we estimate the model using the simulated method of moments (SMM), which allows us to discipline key parameters by matching empirical moments with those generated from a dynamic model of firm behavior. This approach is particularly well suited to our framework, as the degree of information informativeness is a latent parameter that cannot be directly observed in the data. Using the data moments of investments, savings, and external financing, we identify this parameter and capture its time series and cross-sectional patterns.

Quantitatively, the baseline estimation achieves a close fit to the data, successfully reproducing the levels and volatilities of investment and equity issuance, as well as the level of cash holdings and the sensitivities of corporate deicisions to Tobin's Q and cash flow. Subsample analyses provide further validation. Over time, the rising informativeness of the soft signal helps account for the increase in cash holdings and the shift in savings-related and investment-related sensitivities. Across industries, firms that rely more on intangible assets display stronger dependence on soft signals, consistent with their larger cash holdings, greater investment volatility, and distinct financing patterns. A decomposition exercise confirms that changes in the information environment alone explain a sizable portion of the documented time-series trends and cross-sectional differences.

To quantify the role of the learning channel in our model, we employ a Shapley value decomposition suggested by Kwon et al. (2024). By combining this approach with the simulated method of moments, we systematically quantify the contributions of individual parameters, enabling us to directly measure the impact of the learning channel on corporate finance outcomes. This inte-

gration not only provides a transparent and rigorous measure of the learning channel's effect but also constitutes a methodological innovation in structural estimation. Applying this method, we demonstrate that the learning channel accounts for the majority of the shifts in cash savings, investment, and issuance sensitivities, highlighting the central role of information frictions in shaping corporate policies in the intangible economy.

Related Literature

Our paper contributes to several strands of literature. First, it falls within the extensive literature on the debate over Q and cash flow. Q theory was first proposed by Tobin (1969) and was significantly advanced by Hayashi (1982). A key implication of the theory is that (marginal) Q should serve as a sufficient statistic for investment decisions. Empirically, Fazzari et al. (1988) and others document a significant sensitivity of investment to cash flow even after controlling for Q, interpreting this as evidence of financial constraints. In contrast, Gomes (2001), Abel and Eberly (2011), and Abel (2018) develop formal theoretical models of investment decisions and argue that the observed sensitivity may instead reflect information in cash flow about firms' investment opportunities that is not captured by standard measures of Q. Supporting this view, Erickson and Whited (2000) provide empirical evidence that the investment–cash flow sensitivity largely disappears once Q is measured accurately. However, Chen et al. (2012) find that the investment–cash flow sensitivity varies significantly across firms and over time, even after controlling for financial conditions and measurement errors of Q—a phenomenon unexplained by the standard Q model. Our paper addresses this gap by incorporating a two-factor productivity structure–comprising persistent and transitory shocks—and a Bayesian learning mechanism, which generates an endogenous dynamic relationship between cash flow and Q.

The controversy has continued to spill over into the field of cash policy. Hennessy et al. (2007) and Bolton et al. (2011) extend Q-theoretic model by including financial frictions to study cash accumulation and its interaction with investment policies. Using a model related to Hennessy et al. (2007), Riddick and Whited (2009) show that savings and cash flow can be negatively related after controlling for Q, because firms draw on cash reserves to invest when receiving a positive productivity shock, while Almeida et al. (2004) find a positive sensitivity of savings to cash flow in financially constrained firms. Our paper integrates both financial frictions and information frictions into the Q theory of investment. In doing so, we distinguish between the financing role and the informational role of cash flow in determining investment and savings within a unified

framework, and we quantify the extent to which each channel contributes to the structural change in the role of cash flow.

Second, our paper contributes to the literature on learning and uncertainty in corporate decision-making. Prior studies, such as Pástor and Pietro (2003), show that uncertainty about firm profitability drives stock price volatility as investors learn from observable signals. Alti and Tetlock (2014) further explore how biased beliefs about growth prospects impact asset prices in frictionless markets. Andrei et al. (2019) find that firms' learning about productivity affects investment dynamics. These studies primarily focus on beliefs and equity valuation. Both Alti (2003) and Andrei et al. (2019) suggests the importance of information on firm decisions. However, they focus on the role of information in frictionless market. Our model extends this literature by incorporating financial frictions. We explicitly model how learning shapes both real and financial decisions under costly external financing.

Third, our paper contributes to the recent literature on the impact of growing intangibility on financing. The rise of intangibles and their positive properties have a rich implication in stock returns (Eisfeldt and Papanikolaou, 2013), corporate valuation (Crouzet and Eberly, 2023) and the measurement of Q (Peters and Taylor, 2017). These studies highlight how intangibles reframe corporate financing in the 21st century, yet they do not fully explore how it associates with information frictions and the role of cash flow dynamics. Literature also highlights that the growing intangibility leads to financial constraints, and precautionary cash holdings (Begenau and Palazzo, 2021; Falato et al., 2022). In line with previous findings, we show that intangibility leads to more cash holdings. In addition, we suggest that cash serves as a financing buffer, providing flexibility under greater uncertainty and hedge against the information-driven divergence between Q and cash flow. This framework explains the secular rise in cash holdings and shifts in the sensitivities of savings to Q and savings to cash flow, providing a unified perspective on corporate financing in an intangible-dominated economy.

The rest of the paper is organized as follows. Section 2 provides empirical facts about the shifts in the role of cash flow. Section 3 proposes a dynamic stochastic model. Section 4 outlines the estimation methodology and implications from simulations. Section 5 provides more quantitative analysis to the assumption and implication of the model.

2 Empirical Facts

The central message of our theory is that the information loadings of cash flow have changed, and therefore significantly affect firm decisions such as investment, cash savings, and equity issuance. In this section, we begin by reviewing the role of cash flow as a crucial financing source for investment, savings, and equity issuance over the last few decades. Next, we conduct a cross-sectional analysis to demonstrate the financing characteristics of various industries.

2.1 Data

Our main sample covers the period between 1975 and 2024 and includes all Compustat firms except regulated utilities (Standard Industrial Classification codes 4900-4999), financial firms (6000-6999), and firms categorized as public service, international affairs, or non-operating establishments (9000+). We restrict the sample to firms listed on NYSE, Amex, Nasdaq that have been public for at least one year after their IPO.[2] Following Peters and Taylor (2017), we also drop any observations with negative or missing total assets or sales. We also require that the firm's physical capital (PP&E) and total assets be at least \$5 million (in 1990 dollars), and sales be at least \$1 million (in 1990 dollars). Finally, to minimize the impact of sample attrition on the stability of the data process, we require that firms only enter our sample if they appear for at least three years in the data (this is the minimum number of years required for firms to enter our baseline regression models). All variables are winsorized at the 1% level to remove outliers.

2.2 The time series reversal of cash flow's role

To capture these shifts, we divide our sample into two sub-periods: the "Early period" (1975–1994) and the "Late period" (2005–2024).[3] Our dependent variables are the investment-to-capital ratio, the savings-to-capital ratio, and the equity-issuance-to-capital ratio, while the main independent variable is the cash-flow-to-capital ratio. We also include Tobin's Q, given its well-documented role in shaping investment policy. Both Tobin's Q and the cash-flow-to-capital ratio are standardized at the firm level to absorb heterogeneity across firms and to evaluate their relative importance.

[2]The literature suggests that IPO has a large impact on the firm's valuations and fundamentals (Ibbotson, 1975; Fama and French, 2004; Bates et al., 2009)

[3]For brevity, we focus on the early and late periods, which highlight the sharpest contrast in patterns.

The results are reported in Figure 1.[4]

[Figure 1 is about here]

Investment decisions have become less related to cash flow and more related with Tobin's Q. The first row of Figure 1 reports a significant decline in investment–cash flow sensitivity, with the coefficient dropping from 0.037 in the early period to 0.009 in the late period. Conversely, investment–Q sensitivity increases from 0.017 to 0.026. These findings suggest a shift toward Q-driven capital allocation, where investment decisions increasingly reflect forward-looking market valuations rather than internal financing constraints.

Cash Savings exhibit a contrasting trend, with firms increasingly retaining cash flow. The second row shows the cash flow coefficient rising from -0.003 to 0.062, while the Q coefficient declines from -0.010 to -0.080. This indicates that firms now direct a larger share of cash flow toward precautionary savings, and high-Q firms are less inclined to accumulate cash reserves, likely reflecting greater confidence in market opportunities.

Equity Issuance has shifted to serve as a substitute for internal cash flow. The third row documents a sharp increase in the Q coefficient from 0.011 to 0.046, alongside a reversal in the cash flow coefficient from 0.011 to –0.036. This suggests that firms with higher valuations increasingly rely on equity markets for financing and reduce dependence on internal cash flow, highlighting a substitution effect between internal and external capital sources.

Overall, the results show a clear reallocation in the role of cash flow over time: they have become less important in driving investment, more important in supporting savings, and more negatively related to equity issuance. Meanwhile, Tobin's Q has gained prominence in shaping investment and financial decisions.

2.3 Cross-sectional heterogeneity

This subsection analyzes how cash flow and Tobin's Q shape financing decisions, comparing the "Non-tech" industry and the "Tech indsutry". The classification of non-tech and tech industries follows Loughran and Ritter (2004).[5] The dependent variables are the investment-to-capital ratio,

[4]We also control for firm size due to standard economies-of-scale considerations in resource management, but our results are robust to excluding this control. See variable definitions in Table B1.

[5]The classification excludes biotech firms for the main analysis. For robustness, we also apply the Fama and French five-industry classification, including biotech firms, in Table C2 in Appendix C.

savings-to-capital ratio, and equity-issuance-to-capital ratio, with the cash-flow-to-capital ratio as the primary independent variable of interest, controlling for Tobin's Q and firm size. Regression results are reported in Figure 2.

[Figure 2 about here]

Investment in tech firms are more closely tied to growth opportunities than in non-tech firms. The first row of Figure 2 shows that the cash-flow coefficient is positive for both groups but higher for non-tech firms (0.024) than for tech firms (0.018). Conversely, Tobin's Q has a stronger influence on tech firms, with a coefficient of 0.040 compared to 0.022 for non-tech firms. These findings indicate that, while cash flow remains relevant, tech firms' investment decisions are more driven by market-based growth signals captured by Tobin's Q.

Cash savings show a stronger sensitivity to cash flow in tech firms compared to non-tech firms. The second row reports a cash-flow coefficient of 0.094 for tech firms, substantially higher than 0.007 for non-tech firms, suggesting that tech firms prioritize precautionary savings, likely due to volatile investment opportunities. Meanwhile, Tobin's Q has a more negative coefficient for tech firms (-0.101) than for non-tech firms (-0.022), indicating that high-Q tech firms are less likely to accumulate cash reserves and more inclined to pursue growth-oriented investments.

Equity issuance in tech firms is more strongly driven by growth opportunities. The third row shows that the cash-flow coefficient is statistically insignificant for both groups (-0.006 for tech firms, 0.002 for non-tech firms). However, Tobin's Q has a larger effect on tech firms, with a coefficient of 0.047 compared to 0.021 for non-tech firms. This suggests that tech firms leverage equity markets to capitalize on high-valuation opportunities, relying less on internal cash flow.

Overall, our results reveal distinct financing patterns between tech and non-tech firms. While cash flow influences financing decisions across both groups, tech firms demonstrate a greater sensitivity to Tobin's Q, rely more heavily on equity issuance to fund investment opportunities driven by market valuations, and channel more cash flow into liquidity assets.

2.4 Robustness

Alternative measures of Tobin's Q and cash flow

Peters and Taylor (2017) argue that firms increasingly invest more in intangible assets rather than tangible assets. This raises concerns that investment, Tobin's Q, and cash flow may be

mismeasured, potentially biasing our results. Specifically, adding intangible investments (R&D and part of SG&A) back into cash flow may significantly alter the estimated relationships between investment and cash flow as well as between cash savings and cash flow. To address this issue, we adopt alternative measures of investment, Tobin's Q, and cash flow developed by Peters and Taylor (2017). Table C1 in Appendix C shows that our main results remain robust under these alternative measures. In particular, we find that, relative to the medium and late periods, marginal dollars of total cash flow in the early period are more likely to be allocated to investment rather than to cash holdings. The difference in investment—cash flow sensitivity between the late and early periods (late minus early) is –0.021, while the corresponding difference in the sensitivity of savings to cash flow is 0.003; both estimates are statistically significant.

Alternative classifications

To provide a more comprehensive cross-sectional analysis, we employ the Fama and French five-industry classification to compare the manufacturing, consumer, high-tech, and health industries. High-tech and health firms are intensive users of intangible capital, whereas manufacturing and consumer firms rely less on intangibles. In our sample, the average intangible intensity for the manufacturing, consumer, high-tech, and health industries is 34.7%, 50.6%, 59.9%, 66.5%, respectively. Table C2 in Appendix C presents regression results comparing manufacturing and health firms. Consistent with our baseline findings, relative to health firms, marginal dollars of total cash flow for manufacturing firms are more likely to be allocated to investment rather than to cash holdings. The difference in investment-–cash flow sensitivity between health and manufacturing firms (health minus manufacturing) is –0.008, while the corresponding difference in the sensitivity of savings to cash flow is 0.01; both estimates are statistically significant.

Overall, the results highlight economically meaningful and statistically significant shifts in the role of cash flow over time and across industries. Moreover, the effects related to Tobin's Q are economically larger and more statistically significant in the late period and in industries with high intangible intensity. The results remain robust to alternative measures and classifications.

3 The Model

To rationalize the empirical findings on the role of cash flow dynamics and their implications for firm behavior, we develop a dynamic structural model that integrates firms' investment, saving, and financing decisions with a learning mechanism about long-term productivity. Motivated by

evidence of heterogeneous firm responses to productivity shocks and the evolving role of cash flow, our model captures the interplay between financial frictions and information frictions within a unified discrete-time, infinite-horizon framework. Key features of the model include a production function with decreasing returns to scale, dual productivity shocks (transitory and persistent), convex capital adjustment costs, costly equity financing, and a Bayesian learning process that allows firms to infer long-term productivity from noisy cash flow and soft signals.

The model explains how financially constrained firms **strategically manage cash savings to hedge against information-driven divergence between investment opportunities and cash flow**, and respond to varying degrees of information uncertainty, thereby providing a theoretical foundation for the observed cross-sectional and time-series patterns in corporate decisions.

3.1 Production

A firm produces output Π_t in period t using capital K_t according to the production function:

$$\Pi_t = Z_t K_t^{\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where Z_t represents firm-specific productivity, and α governs the degree of decreasing returns to scale.[6] Productivity Z_t comprises a persistent long-term component and a transitory component, evolving as:

$$z_{t+1} = \mu_t + \sigma_z \varepsilon_{z,t+1}, \tag{2}$$

where $z_{t+1} = \ln(Z_{t+1})$, $\varepsilon_{z,t+1} \sim \mathcal{N}(0,1)$ is an i.i.d. standard normal shock, and σ_z denotes the volatility of transitory productivity shocks. The long-term productivity component, μ_t, reflects the expected mean of next period's productivity and follows an AR(1) process:

$$\mu_{t+1} = (1-\rho_\mu)\bar{\mu} + \rho_\mu \mu_t + \sigma_\mu \varepsilon_{\mu,t+1}, \tag{3}$$

where $\varepsilon_{\mu,t+1} \sim \mathcal{N}(0,1)$ is an i.i.d. standard normal shock, $\rho_\mu \in (0,1)$ captures the persistence, σ_μ measures the volatility of persistent shocks, and $\bar{\mu}$ is the stationary mean of long-term productivity, calibrated to normalize the long-run average capital stock to unity.[7]

[6] For brevity, we suppress the firm index i unless necessary.

[7] Specifically, we pick $\bar{\mu}$ so that $\mathbb{E}[K_t] = 1$ in the frictionless case.

Capital accumulates according to the standard law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{4}$$

where I_t is investment and δ is the capital depreciation rate. Investment incurs convex adjustment costs, specified as:

$$G(I_t, K_t) = \xi_t \frac{gI_t^2}{2K_t}, \quad g > 0, \tag{5}$$

where g determines the curvature of the adjustment cost, and ξ_t is an idiosyncratic shock to investment efficiency, with $\ln(\xi_t) \sim \mathcal{N}\left(-\frac{1}{2}\sigma_\xi^2, \sigma_\xi^2\right)$. The shock ξ_t captures exogenous variation in the efficiency of transforming investment goods into installed capital.

3.2 Learning

The firm cannot directly observe long-term productivity but can learn about it using two sources of information: realized cash flow and a soft signal. Realized cash flow is a noisy indicator of firm's long-term productivity. The soft signal summarizes intangible aspects of productivity that complements the information from cash flow. The soft signal s_t is specified as:

$$s_{t+1} = \Phi\varepsilon_{\mu,t+1} + \sqrt{1-\Phi^2}\varepsilon_{s,t+1}. \tag{6}$$

This soft signal contains the information of the true long-term productivity shock, $\varepsilon_{\mu,t+1}$, and a noise term $\varepsilon_{s,t+1}$. Note that all the disturbances, $\varepsilon_{z,t+1}$, $\varepsilon_{\mu,t+1}$ and $\varepsilon_{s,t+1}$ are the standard Gaussian shocks and uncorrelated with each other. The parameter $\Phi \in (0,1)$ measures the informativeness of the soft signal: a higher value of Φ indicates that the soft signal is more informative. If $\Phi \to 1$, the soft signal fully reveals the history of long-term productivity shocks.

The firm forms its beliefs about unobserved long-term productivity as an econometrician who filters it from observed state variables. Using standard filtering theory, the distribution of μ_t conditional on information set $\mathcal{F}_t = \{[z_\tau, s_\tau]\}_{\tau=0}^{\tau=t}$ at time t is Gaussian with the posterior mean $\hat{\mu}_{t|t}$ and the posterior variance ζ_t,

$$\mu_t|\mathcal{F}_t \sim N(\hat{\mu}_{t|t}, \zeta_t). \tag{7}$$

We obtain the following proposition and its corollary from standard filtering theory, with the proof provided in Appendix A.1.

Proposition 1. *The filter $\hat{\mu}_{t|t}$ evolves according to*

$$\hat{\mu}_{t+1|t+1} = (1-\rho_\mu)\bar{\mu} + \rho_\mu \hat{\mu}_{t|t} + \frac{\rho_\mu \zeta_t}{\sqrt{\zeta_t + \sigma_z^2}} \hat{\varepsilon}_{z,t+1} + \Phi \sigma_\mu s_{t+1}, \tag{8}$$

where $\hat{\varepsilon}_{z,t+1} = \frac{1}{\sqrt{\zeta_t+\sigma_z^2}}\left(z_{t+1} - \hat{\mu}_{t|t}\right)$ is the cash-flow signal, which is standardized surprise component of productivity and s_{t+1} is the soft signal, following a standard normal distribution. The posterior variance process follows the locally deterministic process:

$$\zeta_{t+1} = \frac{\rho_\mu^2 \sigma_z^2 \zeta_t}{\zeta_t + \sigma_z^2} + (1-\Phi^2)\sigma_\mu^2. \tag{9}$$

Since we build a time-homogeneous setup for infinite-horizon models, we can use the long-run stationary Kalman filter in which $\zeta_{t+1} = \zeta_t \to \bar{\zeta}$. Thus, $\bar{\zeta}$ is the positive root of Equation (9):

$$\bar{\zeta} = \frac{\sigma_\mu^2(1-\Phi^2) - (1-\rho_\mu^2)\sigma_z^2 + \sqrt{[(1-\rho_\mu^2)\sigma_z^2 - \sigma_\mu^2(1-\Phi^2)]^2 + 4\sigma_\mu^2\sigma_z^2(1-\Phi^2)}}{2}. \tag{10}$$

Corollary 1. *At steady state, the posterior variance $\bar{\zeta}$ is strictly decreasing in Φ,*

$$\frac{\partial \bar{\zeta}}{\partial \Phi} < 0. \tag{11}$$

The firm learns about long-term productivity from two observable signals: the cash-flow signal ($\hat{\varepsilon}_{z,t+1}$) and the soft signal ($s_{t+1}$). Long-term productivity determines the level of cash flow in the future, and its disturbance is contemporaneously correlated with that of the soft signal. Hence, although unobservable, long-term productivity can be inferred from the time-series of cash-flow and soft signals.

More importantly, the informativeness of the soft signal affects the sensitivity of the posterior mean to these signals in opposite directions. The sensitivity of posterior mean to the standardized cash-flow signal, $\frac{\rho_\mu \bar{\zeta}}{\sqrt{\bar{\zeta}+\sigma_z^2}}$, decreases in Φ. This follows directly from Corollary 1, which shows that the posterior variance declines as Φ increases. Conversely, the sensitivity of the posterior mean to the soft signal increases in Φ, indicating that the firm relies more on the soft signal to form its belief when the soft signal is more informative.

3.3 Financing

The final part of the model introduces external financing costs. We assume all external financing is equity. To preserve tractability, we abstract from explicit asymmetric-information or agency-based formulations and instead follow a reduced-form approach, as in Gomes (2001), Hennessy and Whited (2007), Bolton et al. (2011), and Alfaro et al. (2024).

The firm's payout before financing costs is defined as after-tax operating cash flow net of investment, cash accumulation, and capital adjustment costs:

$$E_t = (1-\tau)(Z_t K_t^{\alpha} + rC_t - f) + \tau\delta K_t - I_t - G_t - (C_{t+1} - C_t). \tag{12}$$

Here, τ is the corporate income tax rate, f denotes fixed operating costs, and rC_t is the return on cash holdings. I_t and G_t are investment in physical capital and the associated adjustment costs, respectively. The last term $C_{t+1} - C_t$ represents the investment in cash holdings at period t. Cash holdings are constrained to be non-negative, $C_{t+1} \geq 0$

When $E_t > 0$, the firm makes a payout; when $E_t < 0$, it raises external equity. Following Gao et al. (2021), issuance costs are given by:

$$\Psi(E_t) = \mathbf{1}_{E_t<0}(a_0 - a_1 E_t), \tag{13}$$

where $\mathbf{1}_{E_t<0}$ is an indicator function that equals one if $E_t < 0$, and zero otherwise, a_0 is the fixed financing cost, and a_1 is the linear financing cost.

The effective cash flow distributed to shareholders is given by

$$D_t = E_t - \Psi(E_t). \tag{14}$$

3.4 The Firm's Problem

We define $V(K_t, C_t, z_t, \hat{\mu}_{t|t}, \xi_t)$ as the cum-dividend market value. The firm solves the maximization problem by making capital investment, and cash holdings decisions:

$$V(K_t, C_t, z_t, \hat{\mu}_{t|t}, \xi_t) = \max_{K_{t+1}, C_{t+1}} D_t + \mathbb{E}\left[\frac{1}{1+r} V(K_{t+1}, C_{t+1}, z_{t+1}, \hat{\mu}_{t+1|t+1}, \xi_{t+1}) | \mathcal{F}_t\right] \tag{15}$$

subject to the non-negativity constraint on cash holdings:

$$C_{t+1} \geq 0 \tag{16}$$

The firm's dynamic problem balances the trade-off between immediate payouts and continuation value, while accounting for financial frictions and information uncertainty. Financial frictions give rise to equity issuance costs, whereas information frictions motivate learning about long-term productivity and shape expectations for future states. The interaction between financing costs and learning jointly determines the firm's optimal investment and cash policies.

3.4.1 Optimal Investment Policy, Marginal Q, and Learning

To derive the optimal investment policy, we differentiate the Bellman equation (15) with respect to K_{t+1}, yielding the following first-order condition:

$$\left(1 + \xi_t \frac{gI_t}{K_t}\right)\left(1 + \mathbf{1}_{E_t<0} a_1\right) = Q_t, \tag{17}$$

where Q_t is the Lagrange multiplier associated with the capital accumulation constraint (4). This condition equates the marginal cost of investment (left-hand side) to its marginal value (right-hand side). The marginal cost comprises three components: (i) the opportunity cost of forgone payouts, (ii) the marginal adjustment cost scaled by the realized investment efficiency shock ξ_t, and (iii) the markup from external financing costs if the firm issues equity ($E_t < 0$).

Applying the envelope theorem to the value function with respect to K_t and iterating forward, we obtain:

$$Q_t = \sum_{j=1}^{\infty} \frac{(1-\delta)^{j-1}}{(1+r)^j} \mathbb{E}\left[\left((1-\tau)\alpha e^{z_{t+j}} K_{t+j}^{\alpha-1} + \xi_{t+j}\frac{gI_{t+j}^2}{2K_{t+j}^2} + \tau\delta\right)\left(1 + \mathbf{1}_{E_{t+j}<0} a_1\right)|\mathcal{F}_t\right], \tag{18}$$

Q_t equals to the discounted present value of the marginal revenue product of capital, net of marginal adjustment costs, plus the tax shield term from depreciation. In the absence of information and financial frictions, Q_t reduces to the case in the standard Q theory of investment (Abel, 1980, 1982; Hayashi, 1982). Learning about long-term productivity introduces some key

changes. First, Q_t is mainly driven by the filter ($\hat{\mu}_t$), not the true long-term productivity (μ_t).[8] Second, a higher signal informativeness Φ raises the volatility of the filter as shown in Corollary 2 and thus leads to a more volatile Q_t.

The evolution of the filter in Equation (8) allows us to compute the stationary solution for the conditional variance of $\hat{\mu}_{t+1|t+1}$, which measures the intensity of learning and updating its beliefs about long-term productivity. We characterize the variance of the filter in the following corollary.

Corollary 2. *The conditional variance of the filter is strictly increasing in* Φ*,*

$$Var(\hat{\mu}_{t+1|t+1}|\mathcal{F}_t) = \sigma_\mu^2 - (1-\rho_\mu^2)\bar{\zeta}, \quad \text{and} \quad \frac{\partial Var(\hat{\mu}_{t+1|t+1}|\mathcal{F}_t)}{\partial \Phi} > 0. \tag{19}$$

Corollary 2 shows that learning with a more informative soft signal strictly increases the variance of the filter. In particular, when $\Phi = 1$, the firm perfectly observes the long-term productivity, and the conditional variance of $\hat{\mu}_{t+1|t+1}$ equals to σ_μ^2. It should be noted that learning affects the conditional variance of the filter and the posterior variance in opposite ways: learning with more informative soft signals increases the variance of the filter, $Var(\mu_{t+1|t+1}|\mathcal{F}_t)$, but decreases the posterior variance, $Var(\mu_t|\mathcal{F}_t)$. The former one is a measure of time variation in firm's beliefs. The latter is a measure of uncertainty about μ_t conditional on the information set $\mathcal{F}_t$. Learning with a more informative soft signal decreases the degree of uncertainty about μ_t, but makes belief more volatile as the firm is more aggresively updated from one moment to the other.

Rearranging Equation (17) gives the investment-capital ratio:

$$\frac{I_t}{K_t} = \frac{1}{\xi_t g}\left(\frac{Q_t}{1+\mathbf{1}_{E_t<0}a_1} - 1\right). \tag{20}$$

Learning has several testable implications on investment. On the one hand, higher Φ amplifies the variance of Q_t, thereby increasing the volatility of investment. On the other hand, a more volatile Q_t increases the sensitivity of investment to Q_t, which is $\frac{1}{g}\sqrt{Var(Q_t|\mathcal{F}_{t-1})}$ if financing costs are zero. However, financial frictions moderate this effect: higher financing costs dampen the sensitivity of investment to Q_t.

[8] See from $\mathbb{E}[e^{z_{t+j}}|\mathcal{F}_t] = \mathbb{E}\left[e^{\hat{\mu}_{t+j-1|t+j-1}+\sqrt{\bar{\zeta}+\sigma^2}\hat{\varepsilon}_{z,t+j}}|\mathcal{F}_t\right] = e^{\rho_\mu^{j-1}\hat{\mu}_{t|t}+\sum_{k=1}^{k=j-1}\rho_\mu^{k-1}(1-\rho_\mu)\bar{\mu}+\frac{1}{2}\left[\frac{1-\rho_\mu^{2(j-1)}}{1-\rho_\mu^2}\sigma_\mu^2+\sigma_z^2+\rho^{2(j-1)}\bar{\zeta}\right]}$.

3.4.2 Optimal cash policy and learning

We derive the optimal cash policy by analyzing the first-order condition with respect to cash holdings C_{t+1}. Differentiating the value function (Equation (15)) with respect to C_{t+1}, we obtain:

$$1 + \mathbf{1}_{E_t<0} a_1 = \beta \mathbb{E}\left[\frac{\partial V_{t+1}}{\partial C_{t+1}} | \mathcal{F}_t\right] + \gamma_t^c, \tag{21}$$

where $\gamma_t^c \geq 0$ is the Lagrange multiplier associated with the non-negativity constraint $C_{t+1} \geq 0$, and $\mathbf{1}_{E_t<0} a_1$ captures the marginal cost of equity issuance when external financing is required ($E_t < 0$).

The envelope condition is:

$$\frac{\partial V_t(x_t)}{\partial C_t} = (1 + r(1-\tau))(1 + \mathbf{1}_{E_t<0} a_1). \tag{22}$$

Substituting Equation (22) into Equation (21), we get:

$$1 + \mathbf{1}_{E_t<0} a_1 = \beta \mathbb{E}\left[(1 + r(1-\tau))(1 + \mathbf{1}_{E_{t+1}<0} a_1) | \mathcal{F}_t\right] + \gamma_t^c. \tag{23}$$

Firms with positive cash holdings balances the marginal cost of holding cash (forgone dividends plus associated financing costs) against its marginal benefit, which includes the after-tax interest return and reduced future equity issuance costs.

Learning characterizes firms' cash policies through two distinct channels: a volatility channel and a divergence channel.

Volatility channel. As suggested by Corollary 2, the variance of the filter rises as the soft signal becomes more informative. The greater volatility in belif increases the probability of larger investment in the future. To finance such investments, firms are more likely to issue equity, which raises the marginal value of cash by increasing the term $\mathbb{E}[\mathbf{1}_{E_{t+1}<0} | \mathcal{F}_t]$. As a result, firms hold more cash to mitigate the expected increase in financing costs.

Divergence channel. The divergence between investment opportunities and cash flow increases firms' financing needs, because firms are more likely to encounter situations when investment demand is high but internal funds are insufficient. Consequently, firms increase cash holdings, which serve as an unconditional hedge against this mismatch. In our model, informativeness Φ plays a central role in determining the correlation between investment opportunities and cash flow.

As shown below, higher informativeness reduces this correlation, thereby widening the gap between investment opportunities and internal cash flow.

Corollary 3. *The conditional correlation between the filter ($\hat{\mu}$) and productivity (z) is strictly decreasing in* Φ

$$Corr(\hat{\mu}_{t+1|t+1}, z_{t+1}|\mathcal{F}_t) = \frac{\rho_\mu \bar{\zeta}}{\sqrt{(\bar{\zeta} + \sigma_z^2)[\sigma_\mu^2 - (1-\rho_\mu^2)\bar{\zeta}]}}, \quad \text{and} \quad \frac{\partial Corr(\hat{\mu}_{t+1|t+1}, z_{t+1}|\mathcal{F}_t)}{\partial \Phi} < 0. \tag{24}$$

According to Corollary 3, the correlation between the filter and productivity decreases with informativeness Φ. Intuitively, as Φ increases, firms place more weight on soft signals that influence investment opportunities but not current cash flow, thereby widening the divergence between investment opportunities and cash flow. When Φ is low, firms rely primarily on realized cash flow ($\hat{\varepsilon}_{z,t+1}$) to learn about μ_{t+1}, leading to a strong correlation between cash flow and investment opportunities. This divergence raises the probability that firms will require external financing, $\mathbb{E}[\mathbf{1}_{E_{t+1}<0}|\mathcal{F}_t]$, thereby reinforcing the precautionary motive. Thus, an increase in Φ raises the marginal value of cash holdings by amplifying the divergence between investment opportunities and cash flow.

In summary, a more informative signal (Φ) leads firms to update their beliefs about long-term productivity more frequently, thereby increasing investment volatility. This increases the probability of states requiring external financing, making cash more valuable as a buffer against costly equity issuance. Additionally, an increase in Φ widens the divergence between investment opportunities and cash flow, strengthening the precautionary motive. Firms hold more cash holdings as they anticipate potential mismatches between internal funds and investment needs.

3.5 Dynamic Responses to Signal Shocks

Our model integrates a two-factor productivity structure, comprising persistent shocks to long-term productivity (ε_μ) and transitory shocks to short-term productivity (ε_z). The underlying shocks (ε_μ, ε_z, and ε_s) are unobservable, precluding direct measurement of their magnitudes. Employing a Bayesian updating framework, we estimate the posterior mean of these shocks, quantifying their contributions to observed signals—namely, noisy cash-flow signals (z) and soft signals (s). Our analysis examines how these signals shape firms' beliefs about long-term productivity ($\hat{\mu}$) and influence their investment and financing decisions in the presence of costly equity issuance.

To illustrate the learning mechanisms of the two-factor productivity structure, we analyze model-implied impulse response functions (IRFs) in response to exogenous signal shocks, as presented in Figure 3. Specifically, we focus on soft signal shocks (s) and cash-flow signal shocks ($\hat{\varepsilon}_{z,t+1}$), the latter representing the surprise component of productivity.

[Figure 3 about here]

Response to Soft Signal Shocks

Panel A of Figure 3 presents two subfigures illustrating the IRFs of key quantities following a one-standard-deviation positive shock to the soft signal (s) at $t = 1$, over a 20-period horizon. In the left subfigure, we plot the deviations of firms' beliefs about long-term productivity ($\hat{\mu}$) and realized cash flow (z) from their steady-state values. As derived in Equation (8), a positive shock to s prompts an immediate upward revision of $\hat{\mu}$ at $t = 1$. However, as specified in Equation (2), the translation of updated beliefs into realized cash flow (z) is subject to a time lag. Consequently, while beliefs adjust instantaneously, true productivity is gradually revealed through subsequent cash flow, reflecting the model's dynamic structure and the informational role of soft signals.

In the right subfigure, we examine the response of investment, cash savings, and equity issuance. Consistent with theoretical predictions, a positive shock to s raises expected future productivity, leading to an immediate increase in investment at $t = 1$. Since soft signals do not augment current cash flow (z), firms finance their investment by drawing down cash savings and increasing equity issuance, underscoring the frictions imposed by costly equity financing.

Response to Cash-Flow Signal Shocks

Panel B of Figure 3 displays the effects of a one-standard-deviation positive shock to the cash-flow signal ($\hat{\varepsilon}_{z,t+1}$) at $t = 1$, which is derived from observable firm performance metrics. Unlike soft signals, cash-flow signals simultaneously affect beliefs about future productivity and generate immediate cash flow. In the left subfigure, we plot the deviations of $\hat{\mu}$ and cash flow (z) from their steady-state values. The cash-flow signal shock increases both $\hat{\mu}$ and z at $t = 1$, with a larger effect on z, while only a portion of the signal is incorporated into long-term productivity beliefs through the Bayesian learning process.

In the right subfigure, we examine the responses of investment, cash savings, and equity issuance. A positive cash-flow signal shock raises expected future productivity, leading to an im-

mediate increase in investment at $t = 1$. The contemporaneous increase in cash flow from the cash-flow signal shock allows firms to raise cash savings and reduce equity issuance, thereby mitigating financial constraints. However, the IRFs reveal a dynamic precautionary savings process: the effect of the cash-flow signal shock on z dissipates rapidly, with a sharp decline by $t = 2$, as the surprise component of productivity is largely driven by transitory shocks.[9] By contrast, $\hat{\mu}$ exhibits greater persistence, declining more gradually. This differential persistence can cause cash flow to decline faster than investment, potentially resulting in negative cash savings as firms prioritize investment based on persistent optimistic beliefs.

Implications for Firm Value and Cash Flow

The distinct effects of soft and cash-flow signal shocks arise from their impacts on $\hat{\mu}$ and z, which drive Tobin's q and current cash flow, respectively. Tobin's q, reflecting firm value and heavily influenced by expected future productivity ($\hat{\mu}$), rises immediately following both shocks. However, soft signal shocks, lacking immediate cash flow effects, lead to a greater reliance on costly equity financing, constraining investment and depleting cash savings. Cash-flow signal shocks, by providing cash flow support (z), allow firms to increase cash savings and reduce equity issuance while sustaining investment. The rapid decline in z following cash-flow signal shocks, contrasted with persistent investment opportunities, can lead to negative cash savings, highlighting the tension between short-term liquidity and long-term investment incentives. These differences underscore the critical role of cash flow in alleviating financial frictions, whereas Tobin's q predominantly drives investment decisions. In this context, soft signal shocks tend to amplify financial constraints by creating a divergence between investment opportunities and cash flow, while cash-flow signal shocks help alleviate these constraints, illustrating how the interaction between informational and financial frictions shapes investment and liquidity dynamics.

4 Structural Estimation

Our model does not have an analytical closed form, and thus parameters cannot be inferred from conventional regression techniques. The estimation of parameters is achieved by simulated method of moments (SMM) which minimizes a distance criterion between key moments from the

[9] See Equation (2) for the specification of the productivity process, where transitory shocks dominate the short-term dynamics of z.

actual data and the simulated data. We simultaneously estimate as many of the key structural parameters of the model as is computationally feasible, and calibrate the remainder from the literature. In addition, we employ the Shapley value decomposition to quantify the importance of various channels in a structural model.

4.1 Simulated Method of Moments (SMM)

SMM proceeds as follows - a set of actual data moments Ψ_A is selected for the model to match. For an arbitrary vector θ from the parameter space Θ, the dynamic program is solved and associated policy functions are generated. These policy functions are used to create a simulated data panel. We create $\kappa = 100$ artificial panels to reduce randomness in model sampling. For each panel, we simulate 3000 firms for 500 periods, and keep the last 50, which corresponds to the time span of our main sample. The simulated moments $\Psi_S(\theta)$ are then calculated on the remaining simulated data panel, along with an associated criterion function $\Gamma(\theta)$, where $\Gamma(\theta) = [\Psi^A - \Psi^S(\theta)]' W [\Psi^A - \Psi^S(\theta)]$, which is a weighted distance between the actual moments Ψ_A and the simulated moments $\Psi_S(\theta)$. The parameter $\hat{\theta}$ is the derived by searching over the parameter space to find the parameter vector which minimizes the criterion function:

$$\hat{\theta} = \arg\min_{\theta \in \Theta} [\Psi_A - \Psi_S(\theta)]' W [\Psi_A - \Psi_S(\theta)], \tag{25}$$

Where W is a symmetric and positive-definite weighting matrix. Given the potential for discontinuities in the model and the discretization of the state space, we use an annealing algorithm for the parameter search. Different initial values of θ are selected to achieve global minimum.

The efficiency choice of W is the inverse of the variance-covariance matrix of $[\Psi_A - \Psi_S(\theta)]$. The intuition is that moments measured with greater noise or uncertainty should receive less weight in estimation, while more precisely estimated moments should receive greater weight. Deine Ω to be the variance-covariance matrix of the data moments. Ω is calculated by block bootstrap with replacement on the actual data. The asymptotic variannce-covariance matrix for parameter estimates $\hat{\theta}$ is given by $\left(1 + \frac{1}{\kappa}\right) \left[\frac{\partial \Psi^S(\hat{\theta})}{\partial \hat{\theta}}' W \frac{\partial \Psi^S(\hat{\theta})}{\partial \hat{\theta}} \right]^{-1}$, where $\frac{\partial \Psi^S(\hat{\theta})}{\partial \hat{\theta}}$ is the derivative of simulated moments with respec to the parameter vector (so this wil be a $m \times n$ matrix for m moments and n parameters).[10]

[10] The factor $\frac{1}{\kappa}$ serves to correct for randomness in the simulated data, and its effect becomes negligible when κ is sufficiently large.

4.2 Shapley Value Decomposition

The Shapley value (Shapley, 1953) is a solution concept designed to fairly allocate total gains or costs among a group of collaborating players (or parameters). It provides a natural approach to attributing the relative importance of each parameter in explaining variation in simulation outcomes.

To formalize this, let θ denote a $n \times 1$ vector of parameters. For any subset $p \subseteq N = \{1, 2, \cdots, n\}$, let θ_p denote the subvector of θ with indices in p. Consider a calibration shift from a benchmark case θ^b to a counterfactual case θ^c, which changes the simulation outcome from $\Psi_S(\theta^b)$ to $\Psi_S(\theta^c)$. The Shapley value (vector) for the i-th parameter is given by:

$$\varphi_i = \sum_{s \in \mathcal{A}_i} \frac{|s|!(n-|s|-1)!}{n!} \left[\Psi_S\left(G(s \cup i) \cup \theta_i^c\right) - \Psi_S\left(G(s \cup i) \cup \theta_i^b\right)\right], \tag{26}$$

where $G(s \cup i) = \theta^b_{N \setminus \{s \cup i\}} \cup \theta^c_s$, $\mathcal{A}_i$ denotes the set of all possible partitions of $N \setminus i$ and $|s|$ denotes the cardinality of s. The Shapley value considers all $n!$ possible orders in which parameters change from θ^b to θ^c. The coefficient $|s|!(n-|s|-1)!/n!$ represents the probability that the order of changing parameter i immediately follows the order of changing parameters with indices given by s in a uniformly random ordering of all parameters. The term $\left[\Psi_S\left(G(s \cup i) \cup \theta_i^c\right) - \Psi_S\left(G(s \cup i) \cup \theta_i^b\right)\right]$ captures the marginal effect of changing parameter i from θ_i^b to θ_i^c, given that the parameters with indices in s have already been changed to their counterfactual values θ_s^c, while the others remain at their benchmark values θ_s^b. Consequently, φ_i represents the expected marginal contributions of parameter i to $\Psi_S(\theta^c) - \Psi_S\left(\theta^b\right)$ when one of the $n!$ orderings of parameters is chosen at random with equal probability.

The Shapley value is theoretically well-grounded, supported by four key axiomatic properties.[11] The additivity axiom ensures that the marginal contribution of a group of parameters is simply the sum of their individual Shapley values. The dummy axiom guarantees that the Shapley value of a parameter is zero if changing that parameter has no impact. The symmetry axiom asserts that if two parameters produce identical marginal effects, their Shapley values will be the same. Finally, the efficiency axiom ensures that the sum of all Shapley values equals the total change in the outcome, $\Psi_S(\theta^c) - \Psi_S\left(\theta^b\right)$. Therefore, the contribution weight of the i-th parameter is

[11] The traditional ceteris paribus approach assesses the contribution of a parameter of interest by varying it while fixing all others. This approach is widely used and provides intuitive insights, but it has certain limitations when parameters interact with each other. In particular, the relative contributions of individual parameters do not always sum to one, because interaction (or synergy) effects may arise.

given by $\frac{\varphi_i}{\sum_{j=1}^{n}\varphi_j}$. Moreover, the Shapley value is the unique valuation that satisfies these desired properties.

4.3 Model Parameterization

In principle every parameter could be estimated, but in practice the size of the estimated parameter space is limited by computational constraints. We therefore focus on following eight key parameters of the model: the fixed operating cost (f), the fixed financing cost (a_0), the linear financing cost (a_1), the standard deviation of the shor-run shock (σ_z), the standard deviation of the long-run shock, the persistence of the long-run shock (ρ_μ), the standard deviation of the investment efficiency shock (σ_ξ), and the informativeness of the soft signal (Φ). The other parameters are calibrated based on values in the data and literature.

4.3.1 Predefined Parameters

We begin by describing the calibrated parameters, which are broadly consistent with values reported in the existing literature. The calibration frequency is annual. The interest rate is set at 2 percent, which is close to the average one-year T-bill rate over our sample period. The depreciation rate is set at $\delta = 0.12$, consistent with the average ratio of accounting depreciation to the capital stock in our data. Following Hennessy and Whited (2007), we set the curvature of the production function, α, to be 0.6. The quadratic capital adjustment cost is set at 0.7, follwing Nikolov and Whited (2014) and Gao et al. (2021). Finally, we set the corporate tax rate at 10%, following Eisfeldt and Muir (2016). While the statutory corporate income tax rate is higher, we adopt a lower rate in the model to reduce the tax cost of holding cash, given that the model abstracts from other drivers of liquidity and financing decisions. This adjustment allows the model to generate sufficient liquidity accumulation, comparable to that observed in the data.

4.3.2 Identification

Identification of the remaining parameters requires that the moments respond to changes in parameter values. We find that the fixed operating cost is helpful in matching the average cash-to-asset ratio. The fixed and linear financing costs are important for matching both the mean and the volatility of the issuance-to-asset ratio. The standard deviation of the investment efficiency shock helps to match the volatility of the investment rate. The persistence of the long-run shock helps

to match the autocorrelation of the cash flow-to-asset ratio.

The identification of remaining parameters (σ_z, σ_μ and Φ) warrants further discussions. First, the volatility of the cash flow-to-asset ratio is primarily determined by σ_z and σ_μ. Second, both σ_μ and Φ make the beta of savings on Q more negative. An increase in either parameter raises the volatility of Tobins' Q: a higher σ_μ raises the variance of underlying shocks, while a higher Φ intensifies learning. The resulting higher variance in Tobins'Q enhances its explanatory power for savings decisions, leading to a more negative savings–Q sensitivity. However, this overlapping effect may necessitate an additional moment for precise identification.

A crucial distinction between σ_μ and Φ lies in their opposing effects on the informational role of cash flow: a higher σ_μ strengthens it while a higher Φ weakens it. A stronger informational role of cash flow reduces the sensitivity of savings to cash flow, as a larger fraction of positive cash flow shocks is allocated to investment. Consequently, the volatility of the cash flow-to-asset ratio, together with the beta of savings on Q and the beta of savings on cash flow, jointly identify σ_z, σ_μ and Φ. Furthermore, since issuance plays a significant role in both the model and the data, we also utilize the betas of issuance on Q and on cash flow to strengthen identification.

4.4 Comparative Statics

To highlight the role of the learning channel, we examine the model's comparative statics with respect to the informativeness of learning, Φ. Since the learning mechanism constitues the primary channel in our framework, this exercise illustrates directly how learning influences firm dynamics through the role of cash flow and Q. We also compare this learning channel with a standard volatility channel, captured by the relative magnitudes of fundamental shocks, σ_z, and noise shocks, σ_u. The variance decomposition between σ_z and σ_u also affects the alignment between cash flow and Q, as higher noise reduces the information content of market valuations while amplifying the signaling role of internal cash flow.

4.4.1 Learning channel

Table 1 reports firm characteristics and the sensitivities of decisions to cash flow and Q as the informativeness of the learning channel, Φ, gradually increases from 0.1 to 0.9. First, average cash holdings rises monotonically with Φ, from 0.102 at $\Phi = 0.10$ to 0.168 at $\Phi = 0.9$. This pattern reflects stronger precautionary savings under a more informative environment. Second, the

volatility of investment increases significantly from 0.083 at $\Phi = 0.10$ to 0.102 at $\Phi = 0.9$, consistent with the idea that firms with more informative signals update their beliefs more frequently and adjust capital investment more aggressively in response to new information. However, Issuance moments change a little. Although volatility of issuance exhibits an upward trend as Φ increases, the average issuance almost remains unchanged. This reflect the fact that firms are more likely to trigger very large equity issuance but decreases the frequency of small equity issuance by holding more cash in a more informative environment.

[Table 1 is about here]

Regarding the role of cash flow and Q, changes in sensitivities reveal two distinct sets of patterns. First, the relationship between Q and firm decisions strengthens with informativeness. The sensitivity of investment to Q rises from 0.010 at $\Phi = 0.1$ to 0.078 at $\Phi = 0.9$, the sensitivity of issuance to Q grows from almost zero to 0.014. At the same time, the sensitivity of savings to Q declines from -0.004 to -0.077, reflecting a more substitutable relationship between cash accumulation and investment when firms face more informative signals.

Second, the relationship between cash flow and firm decisions weakens with informativeness. The loading of investment on cash flow falls from 0.033 at $\Phi = 0.1$ to -0.006 at $\Phi = 0.9$, and the correlation between issuance and cash flow remains close to zero throughout. In contrast, the sensitivity of savings to cash flow rises from 0.010 to 0.057, indicating that as informativeness improves, firms treat surprised cash flow more as a short-term liquidity shock rather than a proxy for investment opportunities.

Taken together, these results highlight the central role of the learning channel: More informative learning reduces firms' reliance on cash flow for investment, increases their reliance on external financing, and amplifies precautionary savings motives.

4.4.2 Volatility channel

In Appendix Table C5, we report firm characteristics and sensitivities as we vary the ratio of long-run volatility (σ_μ) to short-run volatility (σ_z). Specifically, we move from a short-run–dominant environment ($\sigma_z = 0.248, \sigma_\mu = 0.180$) toward a long-run–dominant environment ($\sigma_z = 0.163, \sigma_\mu = 0.230$).

[Table C5 is about here]

First, the rationale behind this grouping is to hold the overall volatility of cash flow constant while varying only the relative magnitudes of σ_z and σ_μ. This design ensures that changes in the relationship between cash flow and Q are not driven by differences in the aggregate level of volatility, but by the distinct contributions of long-term and short-term productivity.

Second, the volatility of firm decisions rises systematically as long-run volatility becomes more important. The standard deviation of investment increases from 0.081 to 0.094, and the volatility of equity issuance rises from 0.052 to 0.067. This mirrors the learning channel, where stronger informativeness also amplifies investment volatility, though here the effect arises mechanically from larger long-run shocks rather than belief updating.

Third, the sensitivities of corporate decisions to Q and cash flow reveal important contrasts with the learning channel. On the Q side, investment–Q sensitivity strengthens (0.040 to 0.061), and issuance–Q sensitivity rises (0.007 to 0.012), much like in the learning channel. On the cash flow side, however, the patterns diverge. In the volatility channel, investment—cash flow sensitivity remains essentially flat (0.006 to 0.005), and savings—cash flow sensitivity slightly declines (0.042 to 0.038). By contrast, in the learning channel, investment—cash flow sensitivity sharply declines and savings—cash flow sensitivity rises with informativeness. Thus, while both channels strengthen the role of Q, only the learning channel reallocates more cash flow from investment toward savings.

Taken together, the volatility channel and the learning channel both amplify the role of Q and raise investment volatility, but they diverge in how they reshape the role of cash flow. The volatility channel leaves cash flow sensitivities largely stable, while the learning channel fundamentally reallocates cash flow from investment to savings. Beyond comparative statics, we further evaluate the relative contributions of the learning and volatility channels in our following quantitative analysis.

5 Quantitative Analysis

In this section, we estimate the model parameters and assess the quantitative importance of our proposed mechanism. We begin by demonstrating that our baseline estimation provides a close fit between the model and the data. Next, we estimate parameters for subsamples to test if the model can explain the time-series structural change and cross-sectional heterogeneity in the data. Finally, we employ an attribution technique to provide a systematic counterfactual analysis. The results show that the evolving role of cash flow and the rise in cash holdings can be largely explained by

the learning channel.

5.1 Baseline estimation

Table 2 presents the main results of the structural estimation for the full sample. Panel A reports structural parameter estimates and their corresponding standard errors. Panel B reports data and model moments. The model demonstrates a strong fit to the data, capturing key financial behaviors and supporting theories of information's role in corporate finance.

[Table 2 is about here]

Panel A of Table 2 shows that all model parameters are both economically plausible and statistically significant. The fixed and linear financing costs, a_0 and a_1, are estimated at 0.004 and 0.046, respectively. Consistent with Hennessy and Whited (2007) and Gao et al. (2021), these values imply that the fixed cost of equity issuance is about 0.4% of the steady-state capital stock, and the linear equity issuance cost is approximately 4.6% of issuance proceeds. The persistence of the long-run shock, ρ_μ, is 0.64, which lies within the range of the estimates reported by Nikolov and Whited (2014). The standard deviation of the short-run shock (σ_z) and the long-run shock (σ_μ) are 0.175 and 0.225, respectively, which together generate an unconditional variance of productivity of about 0.12. This value is close to 0.14 implied by the cash flow process in Nikolov and Whited (2014). The fixed operating cost, f, is 0.109, corresponds to approximately 11% of the steady-state capital stock. The informativeness of the soft signal, Φ, is 0.566. The associated signal precision is given by, $\frac{\Phi}{\Phi+\sqrt{1-\Phi^2}} \approx 0.41$, which is close to 0.496 reported by Alti and Tetlock (2014). We examine the economic importance of these parameters in the next section.

Panel B of Table 2 reveals that the baseline model provides a close fit to the data. For instance, the average equity issuance-to-asset ratio (0.009 in both the data and the model) and the cash-to-asset ratio (0.127 in both) are nearly identical. The standard deviation of investment rate (0.101 in the data vs. 0.092 in the model), equity issuance-to-asset ratio (0.069 vs. 0.064), and cash flow-to-asset ratio (0.082 vs. 0.081) closely align, capturing volatility in investment, financing, and productivity. The beta of savings on Tobin's Q (-0.048 vs. -0.050) and the beta of savings on cash flow (0.040 vs. 0.038) precisely match the data. The only notable discrepancy arises in the beta of issuance on Tobin's Q, which is about one-third of its empirical value. However, this deviation is not economically important since our identification is robust to excluding this moment. Panel B also reports model-implied non-target moments, which are not expected to perfectly replicate the

data but serve to validate model implications. We show that the beta of investment on Tobin's Q (0.058 vs. 0.027) and the beta of investment on cash flow (-0.014 vs. -0.011) are qualitatively similar.

Overall, the model is exceptionally fit to data moments, validating its empirical relevance. The parameter estimates reinforce alignment with literature, confirming the model's ability to replicate stylized facts in corporate finance.

5.2 Subsample Estimation

This section offers additional empirical validation of the learning channel through both time-series and cross-sectional analyses. We employ the simulated method of moments to estimate model parameters across two historical periods (1975–1994 and 2005–2024) and two industry groups (Non-tech and Tech), thereby assessing the empirical relevance and robustness of the learning channel.

5.2.1 Time-Series Subsample Analysis

To investigate the evolving role of the learning channel in explaining key corporate finance moments—namely cash holdings, investment, and equity issuance—we first perform a time-series subsample analysis. We estimate the model over two distinct periods—1975–1990 (early) and 2005–2024 (late)—using the simulated method of moments (SMM). The parameter estimates, reported in Panel A of Table 3, reveal pronounced temporal shifts. Notably, the informativeness of learning (Φ) rises from 0.364 in the early period to 0.724 in the late period, with both estimates statistically significant at conventional levels. This pattern suggests that firms increasingly rely on soft signals to update beliefs about future productivity, thereby diminishing the informational role of cash flow. In addition, the standard deviation of the short-run shock (σ_z) increases from 0.103 to 0.218, and the long-run shock (σ_μ) increases from 0.152 to 0.193, indicating heightened cash-flow volatility.

[Table 3 about here]

The quantitative results, presented in Panel B of Table 3, demonstrate the model's ability to capture key moments and their evolution over time. Overall, the model provides a good fit across subsamples, with data and model moments largely indistinguishable both statistically and

economically. In the data, the average cash-to-asset ratio rises significantly from 0.092 in the early period to 0.161 in the late period, reflecting a stronger precautionary motive. The model predicts an increase from 0.101 to 0.154, accounting for 77% of the observed rise in cash holdings. The model also reproduces comparable magnitudes for the observed volatility trends. Beyond volatility, the model captures shifts in the sensitivities of savings and issuance to key financial variables, including Tobin's Q and cash flow. Specifically, the model replicates the more negative sensitivity of savings to Tobin's Q (data: –0.010 to –0.080; model: –0.007 to –0.073) and the increase in the sensitivity of savings to cash flow (data: –0.003 to 0.062; model: 0.008 to 0.045). It also captures the shifts in the sensitivity of issuance to Tobin's Q (data: 0.011 to 0.046; model: 0.002 to 0.011) and to cash flow (data: 0.011 to –0.036; model: –0.003 to –0.009). Moreover, the model successfully matches trends in non-target moments, such as the sensitivity of investment to Tobin's Q (data: 0.017 to 0.026; model: 0.009 to 0.074) and to cash flow (data: 0.037 to 0.009; model: 0.023 to 0.006).

These observed shifts can be largely attributed to the learning channel, as the informativeness of the soft signal (Φ) increases from 0.364 to 0.724 over time. A higher Φ amplifies the intensity of learning. As a result, Tobin's Q becomes more informative and is updated more frequently, thereby driving corporate decisions and accounting for the stronger Q sensitivity observed in the data. Simultaneously, the divergence channel reduces the correlation between cash flow and investment opportunities and weakens the informational role of cash flow, thereby explaining the increased sensitivity of savings to cash flow and the decreased sensitivity of investment to cash flow. Moreover, this divergence channel further increases the likelihood of binding financing constraints ($\mathbb{E}_t[\mathbf{1}_{E_{t+1}<0}]$) and raises the marginal value of cash holdings, thereby accounting for the observed upward trend in cash holdings.

5.2.2 Cross-sectional Subsample Analysis

We further explore the learning channel across industries, comparing non-tech frms with tech firms. The definition of non-tech and tech firms are defined in Section 2. Table 4, Panel A reports the model parameter estimates. The informativeness (Φ) is substantially higher for tech firms (0.858) than for non-tech firms (0.225), underscoring greater reliance on soft signals in tech industries. Additionally, tech firms face higher volatility in short-run ($\sigma_z = 0.228$ for tech firms vs. 0.053 for non-tech firms) and long-run shocks ($\sigma_\mu = 0.337$ vs. 0.261). These patterns reflect a more volatile and less predictable environment for tech firms, amplifying the value of informative signals.

[Table 4 about here]

Quantitative results, shown in Panel B of Table 4, confirm the model's ability to replicate cross-sectional patterns. In the data, the difference in the average cash-to-asset ratio between tech and non-tech firms is about 0.079 (0.182 for tech firms and 0.103 for non-tech firms), and the difference for the simulated data is about 0.088 (0.199 for tech firms and 0.110 for non-tech firms). In addition, tech firms exhibit greater volatility in investment, issuance and cash flow than non-tech firms both in the data and the model. For example, the difference in investment volatility between tech and non-tech firms is 0.024 in the data and 0.038 in the model. The model also successfully matches the pattern in both targeted and non-targeted sensitivities. For example, the model replicates the pattern for the sensitivity of savings to Tobin's Q (data: -0.022 for non-tech firms and -0.101 for tech firms; model: -0.009 and -0.111) and the sensitivity of savings to cash flow (data: 0.007 and 0.094; model: 0.010 and 0.076). The model also capture the trend in the sensitivity of investment to Tobin's Q (data: 0.022 for non-tech firms and 0.040 for tech firms; model: 0.020 and 0.090) and the sensitivity of investment to cash flow (data: 0.024 and 0.018; model: 0.017 and -0.006). The results hight the information role of cash flow for non-tech firms but the financing role of cash flow for tech firms.

Collectively, our time-series and cross-sectional analyses demonstrate that the learning channel, governed by signal informativeness (Φ), is pivotal in shaping firms' real and financial decisions, particularly in environments marked by high uncertainty and intangibility.

5.3 Attribution and Counterfactual Analysis

So far, our model performs well in explaining time-series changes and cross-sectional heterogeneity in several key moments, and the estimated parameter Φ varies significantly across subsamples. However, we still do not know to what extent the changes in key moments, like the increase in average cash-to-asset ratio, can be explained by the changes in Φ. In this section, we employ an SMM based Shapley value decomposition, as introduced in Section 4.2, to systematically quantify the contribution of the learning channel.

5.3.1 Attribution Analysis for Time-series Subsamples

Table 5 presents the attribution analysis across subsamples (early v.s. late). The first eight columns show the percentage contributions of each estimated parameter to the changes in the simulated

moments of interest between the early and late periods. The last column displays the differences in simulated moments between the two periods. The results indicate that the learning channel makes a substantial contribution to variations in key moments over time. For instance, the rise in informativeness (Φ) from 0.364 in the early period to 0.724 in the late period explains over half of the rise in the mean cash-to-asset ratio (55.23%), contributing 0.029 to the total simulated increase of 0.053. It also raises the standard deviation of the investment rate by 0.006, as firms update their beliefs about long-term productivity more frequently. With respect to sensitivities, the parameter Φ plays the dominant role in determining investment- and savings-related sensitivities, with contribution shares ranging from 32.65% to 89.71%.[12] Specifically, it increases the sensitivity of savings to cash flow by 0.021, accounting for 55.57% of the change in the simulated moment, while it reduces the sensitivity of investment to cash flow by 0.015, accounting for 89.71% of the change in the simulated moment.

In addition to the learning channel, changes in the cash flow process also play a central role in shaping volatility dynamics. The standard deviations of both short-run and long-run shocks have risen substantially over time, accounting for 86.85% of the increase in cash flow volatility, with σ_z and σ_μ contributing 42.83% and 44.02%, respectively. Among these, σ_μ plays a particularly strong role, raising the standard deviations of investment and issuance rates by 0.011 and 0.015, respectively. By contrast, the main factor behind the decline in investment rate volatility is the reduction in investment efficiency shocks, which alone explains 267.49% of the change and corresponds to a 0.029 decrease in investment volatility.

[Table 5 is about here]

Overall, the attribution analysis highlights the central role of the learning channel in explaining the time-series patterns of key firm behaviors. The rise in informativeness (Φ) not only accounts for more than half of the increase in cash holdings but also drives changes in investment volatility and the sensitivities of investment and savings to cash flow.

[12] Surprisingly, operating costs play a nontrivial role in shaping issuance-related sensitivities. This pattern reflects a size-related measurement issue that firm size mechanically reduces Tobin's Q. This measurement error is difficult to eliminate entirely, even after controlling for firm size. However, increases in operating costs reduce the value of smaller firms more than that of larger firms, thereby mitigating the size-related measurement error and making Tobin's Q more informative about investment opportunities.

5.3.2 Attribution Analysis for Cross-sectional Subsamples

Table 6 presents the attribution analysis for non-tech and tech firms. The parameter Φ again plays a central role in driving higher cash holdings and investment volatility, as well as shaping savings- and investment-related sensitivities, with contribution shares ranging from 53.59% to 124.55%. For example, Φ accounts for 64.18% of the change in the mean of cash-to-asset ratio. Given that the simulated difference in the average cash-to-asset ratio is 0.088, the increase in Φ from 0.225 to 0.858 translates into a 0.056 rise in this ratio. Moreover, Φ raises the sensitivity of savings to cash flow by 0.051 (76.79% of the total change), while reducing the sensitivity of investment to cash flow by 0.029 (124.55% of the total change).

While the learning channel remains important, the cash flow process contributes substantially to volatility moments across non-tech and tech firms. In particular, increases in σ_μ and σ_z raise the standard deviation of cash flow by 0.03, accounting for approximately 80% of the total change. Beyond cash flow volatility, the increase in σ_μ significantly increases the volatility of investment and issuance by 0.017 and 0.016, respectively, corresponding to 43.76% and 51.63% of the overall change.

In summary, the results suggest that both time-series and cross-sectional patterns are largely driven by the learning channel. Specifically, early-period and non-tech firms mainly rely on the informational role of cash flow, whereas later-period and tech firms increasingly utilize soft signals and the financing role of cash flow. This structural shift helps explain corporate behavior in the intangible economy, manifested in higher cash holdings, greater sensitivity of savings to cash flow, and lower sensitivity of investment to cash flow. Methodologically, the use of Shapley value decomposition proves highly effective. By allocating the contributions of each structural parameter in a comprehensive and precise manner, this approach offers a powerful tool to identify the dominant economic forces underlying long-term corporate dynamics.

5.3.3 Counterfactual Analysis

In addition to the Shapley value decomposition, we conduct a series of traditional counterfactual exercises to quantify the contributions of different channels to the reversal in the role of cash flow. In Appendix, Table C3 compares the early and late subsamples, while Table C4 contrasts non-tech and tech firms. Across both dimensions, counterfactual experiments show that changes in the informativeness of the soft signal, Φ, account for the majority of the shifts in firms' behavior.

Specifically, when we replace the early-period parameters with the late-period value of Φ, the model reproduces roughly half of the increase in cash holdings, two-thirds of the decline in the investment—cash flow sensitivity, and one-third of the increase in the savings–cash flow sensitivity. By contrast, changes in parameters of financing costs (a_0, a_1) or cash flow process (ρ_μ, σ_z, σ_μ) generate only modest effects. A similar pattern emerges in the cross-sectional comparison: replacing the non-tech firm's learning parameter with that of tech firms explains over half of the increase in cash holdings and the rise in savings—cash flow sensitivity, whereas financing and volatility channels contribute little.

Taken together, it is the learning channel that dominates in explaining both the time-series and cross-sectional variation in the role of cash flow and its impact on corporate decisions, while shifts in the magnitudes of volatility and financing channels affect certain margins, as echoed by the results from the attribution analysis.

6 Conclusion

We investigate the evolving role of cash flow in shaping firms' investment, savings, and financing decisions, focusing on the impact of the rising dominance of intangible assets. Our empirical analysis documents a significant decline in the sensitivity of investment to cash flow, dropping by approximately 75% over the period from 1975–1994 to 2005–2024, alongside a 50% increase in reliance on Tobin's Q. Concurrently, cash savings exhibit a dramatic increase in sensitivity to cash flow from near zero to 0.062, while equity issuance shifts from being driven by cash flow to aligning with growth opportunities signaled by Tobin's Q. These patterns are particularly pronounced in tech firms, which display lower investment–cash flow sensitivity, higher sensitivity to Tobin's Q, and a stronger propensity to channel cash flow into savings.

To rationalize these findings, we develop a dynamic structural model that integrates financial frictions and a Bayesian learning mechanism to capture the interplay between cash flow dynamics, information uncertainty, and firm behavior. The model introduces a two-factor productivity structure, comprising persistent and transitory shocks, allowing firms to learn about long-term productivity from noisy cash flow and soft signals. This framework explains the diminished role of cash flow in investment decisions, the increased reliance on Tobin's Q, and the heightened importance of cash flow for precautionary savings and reduced dependence on external financing. By incorporating convex capital adjustment costs and costly equity issuance, the model generates a

precautionary demand for cash, providing a unified explanation for the observed time-series and cross-sectional variations in corporate policies.

Through estimation using the simulated method of moments (SMM), we show that the model quantitatively reproduces the empirical levels, volatilities, and sensitivities of investment, cash holdings, and equity issuance. Subsample analyses further validate the model, demonstrating that the rising informativeness of signals accounts for the secular increase in cash holdings and the shift in sensitivities toward Tobin's Q. A novel application of Shapley value decomposition, combined with SMM, quantifies the contribution of the learning channel, confirming that information frictions play a critical role in driving the observed trends. Across industries, firms with higher reliance on intangible assets exhibit greater dependence on external signals, consistent with their elevated cash ratios and investment volatility.

Our findings extend the literature on Q theory, cash policy, and learning in corporate finance. By integrating financial and information frictions into a unified framework, we provide a new perspective on how firms adapt their financial strategies in an intangible-dominated economy. The model highlights the dual role of cash flow as both a financing tool and an information signal, offering insights into the mechanisms driving corporate behavior. An avenue for future research is to explore the implications of these dynamics in a general equilibrium setting, where information and financial frictions could interact with aggregate economic conditions to further shape firm behavior and market outcomes.

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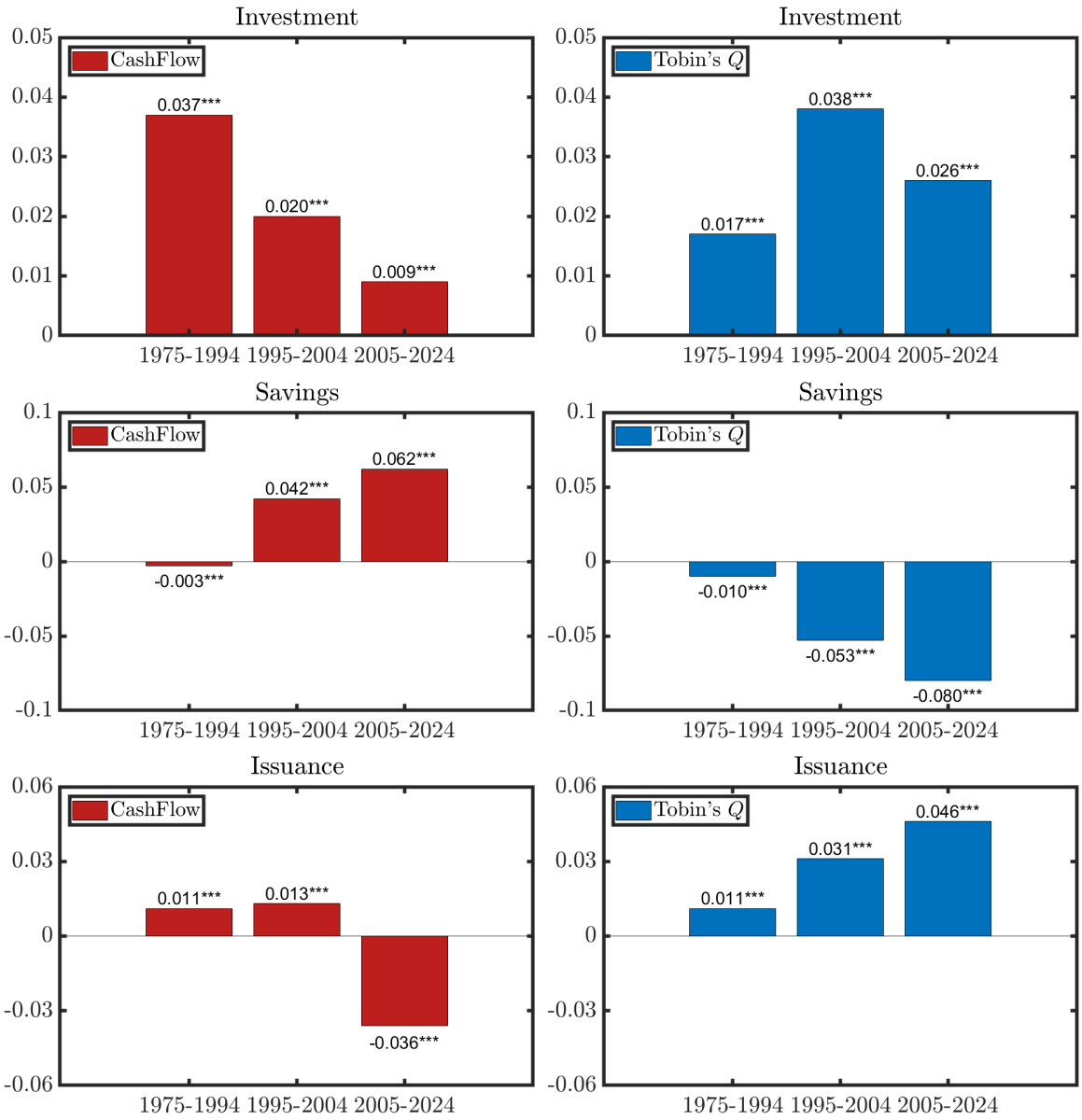


Figure 1: Sub-Period Comparison of Cash Flow and Tobin's Q Coefficients

This figure plots the coefficients estimated from panel regressions of investment, savings, and equity issuance on cash flow, Tobin's Q, and firm size across three sub-periods 1975–1994, 1995–2004, and 2005–2024. All specifications include firm fixed effects. The dependent variables are investment scaled by lagged capital, changes in the ratio of cash to capital, and equity issuance scaled by lagged capital. The key regressors—cash flow and Tobin's Q—are standardized at the firm level. Variable definitions are provided in Table B1 in Appendix B. Standard errors are clustered at the firm level. *, **, *** represent significance at the 10%, 5%, and 1% levels, respectively.

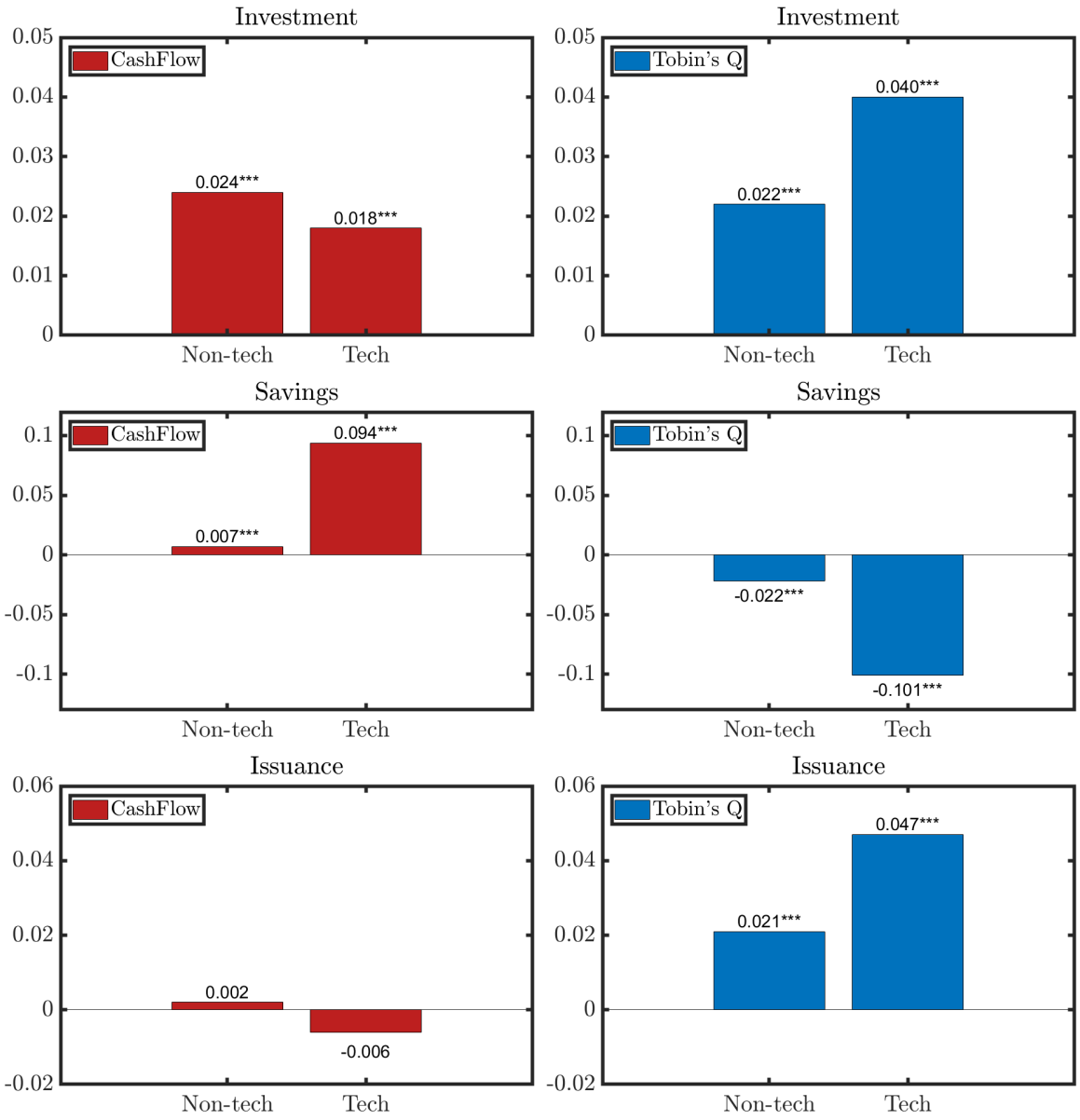
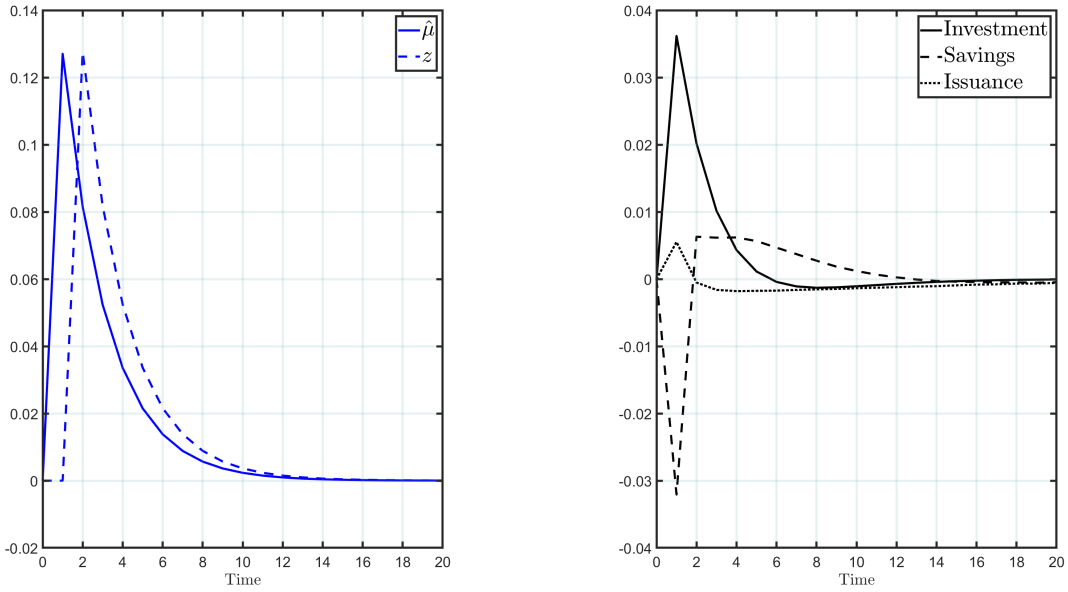
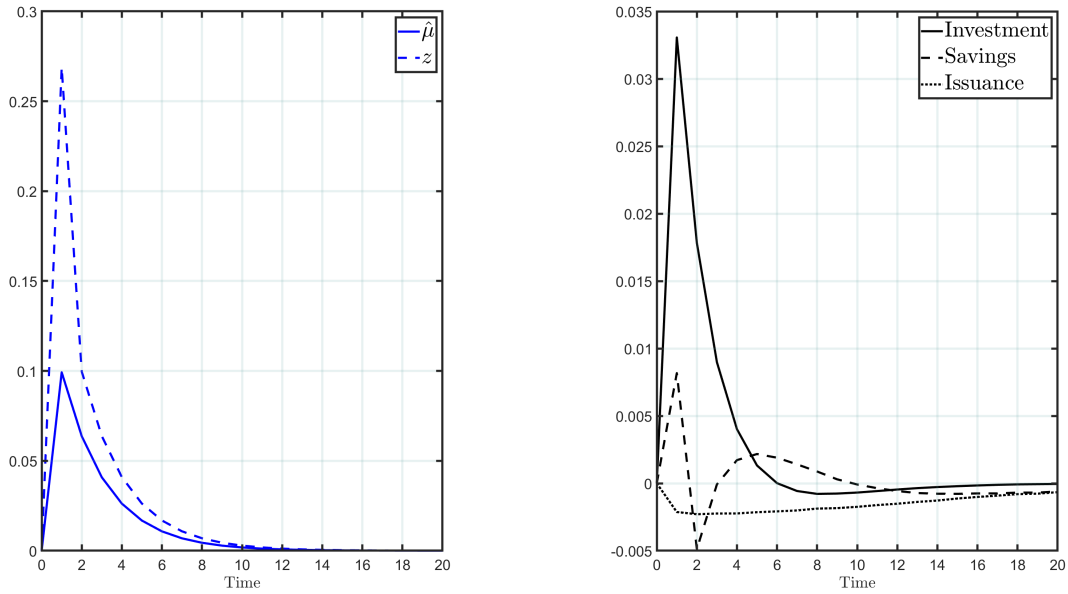


Figure 2: Cross-Section Comparison of Cash Flow and Tobin's Q Coefficients

This figure plots the coefficients estimated from panel regressions of investment, savings, and equity issuance on cash flow, Tobin's Q, and firm size across the "Non-tech" industry and the "Tech industry". All specifications include firm fixed effects. The classification of non-tech and tech industries follows Loughran and Ritter (2004). The dependent variables are investment scaled by lagged capital, changes in the ratio of cash to capital, and equity issuance scaled by lagged capital. The key regressors—cash flow and Tobin's Q—are standardized at the firm level. Variable definitions are provided in Table B1 in Appendix B. Standard errors are clustered at the firm level. *, **, *** represent significance at the 10%, 5%, and 1% levels, respectively.



(a) Panel A. Impulse Responses to Soft Signal Shocks (s)



(b) Panel B. Impulse Response to Cash-Flow Signal Shocks ($\hat{\varepsilon}_z$)

Figure 3: Model-Implied Impulse Response Analysis

This figure plots model-implied impulse responses of quantities and beliefs starting from the steady state to one-standard-deviation positive shocks to the soft signal (s) and the cash-flow signal ($\hat{\varepsilon}_z$). Panel A shows the responses to a one-standard-deviation positive shock to the soft signal (s), with the left subfigure illustrating the effects on firms' beliefs about long-term productivity ($\hat{\mu}$) and realized productivity (z), and the right subfigure displaying the responses of investment, cash savings, and equity issuance. Panel B shows the responses to a one-standard-deviation positive shock to the cash-flow signal, with the left subfigure illustrating the effects on $\hat{\mu}$ and z, and and the right subfigure displaying the responses of investment, cash savings, and equity issuance.

Table 1: Comparative Statics across Informativeness

This table reports comparative statics for model moments at different levels of informativeness of the soft signal (Φ). Means, standard deviations (STDs), betas, and the autocorrelation are computed as averages across 100 simulations of 3,000 firms and 50 periods.

Informativeness (Φ):	0.1	0.3	0.5	0.7	0.9
Means of					
Equity Issuance/Asset	0.009	0.009	0.009	0.009	0.008
Cash Holdings/Asset	0.102	0.110	0.121	0.142	0.168
STDs of					
Investment Rate	0.083	0.087	0.091	0.096	0.102
Equity Issuance/Asset	0.061	0.065	0.065	0.065	0.064
Cash Flow/Asset	0.079	0.081	0.081	0.082	0.081
Betas of					
Savings on Tobin's Q	-0.004	-0.024	-0.044	-0.063	-0.077
Savings on Cash Flow/Capital	0.010	0.020	0.035	0.046	0.057
Issuance on Tobin's Q	-0.001	0.006	0.011	0.013	0.014
Issuance on Cash Flow/Capital	-0.005	-0.009	-0.011	-0.011	-0.010
Investment on Tobin's Q	0.010	0.037	0.053	0.067	0.078
Investment on Cash Flow/Capital	0.033	0.018	0.008	0.001	-0.006
Autocorrelation of					
Cash Flow/Asset	0.441	0.472	0.474	0.475	0.453

Table 2: Simulated Moments Estimation for the Full Sample

This table presents parameters and model moments estimated via the simulated method of moments (SMM) for nonfinancial, unregulated firms in Compustat over the period 1975–2024. Panel A reports the estimated structural parameters, with standard errors in parentheses, focusing on eight key parameters: f (fixed operating cost), a_0 (fixed financing cost), a_1 (linear financing cost), σ_z (standard deviation of short-run shock), σ_μ (standard deviation of long-run shock), ρ_μ (persistence of long-run shock), σ_ξ (standard deviation of investment efficiency shock), and Φ (informativeness of the soft signal). The remaining parameters are calibrated using data and prior literature. Panel B compares target and non-target data moments with model moments. Model moments are computed as averages across 100 simulations of 3,000 firms over 50 periods.

Panel A: Paramter Estimates

f	a_0	a_1	σ_z	σ_μ	ρ_μ	σ_ξ	Φ
0.109	0.004	0.046	0.175	0.225	0.641	0.218	0.566
(0.003)	(0.000)	(0.007)	(0.017)	(0.018)	(0.038)	(0.007)	(0.096)

Panel B: Moments

	Data	Model
Target Moments		
Means of		
Equity Issuance/Asset	0.009	0.009
Cash Holdings/Asset	0.127	0.127
STDs of		
Investment Rate	0.101	0.092
Equity Issuance/Asset	0.069	0.064
Cash Flow/Asset	0.082	0.081
Betas of		
Savings on Tobin's Q	-0.048	-0.050
Savings on Cash Flow/Capital	0.040	0.038
Issuance on Tobin's Q	0.038	0.012
Issuance on Cash Flow/Capital	-0.014	-0.011
Autocorrelation of		
Cash Flow/Asset	0.473	0.474
Non-Target Moments		
Beta of Investment on Tobin's Q	0.027	0.058
Beta of Investment on Cash Flow/Capital	0.021	0.005

Table 3: Simulated Moments Estimation for Subsapmles (Early v.s. Late)

This table presents parameters and model moments estimated via the simulated method of moments (SMM) for nonfinancial, unregulated firms in Compustat, with separate estimates for the "Early period" (1975–1994) and the "Late period" (2005–2024). Panel A reports the estimated structural parameters, with standard errors in parentheses, focusing on eight key parameters: f (fixed operating cost), a_0 (fixed financing cost), a_1 (linear financing cost), σ_z (standard deviation of short-run shock), σ_μ (standard deviation of long-run shock), ρ_μ (persistence of long-run shock), σ_ξ (standard deviation of investment efficiency shock), and Φ (informativeness of the soft signal). The remaining parameters are calibrated using data and prior literature. Panel B compares target and non-target data moments with model moments for the early and late periods. Model moments are computed as averages across 100 simulations of 3,000 firms over 50 periods.

Panel A: Paramter Estimates

	f	a_0	a_1	σ_z	σ_μ	ρ_μ	σ_ξ	Φ
Early	0.096	0.007	0.049	0.103	0.152	0.683	0.277	0.364
	(0.004)	(0.004)	(0.031)	(0.024)	(0.026)	(0.075)	(0.015)	(0.053)
Late	0.111	0.005	0.071	0.218	0.193	0.700	0.153	0.724
	(0.004)	(0.001)	(0.015)	(0.020)	(0.031)	(0.072)	(0.013)	(0.126)

Panel B: Moments

	Early		Late		Late−Early	
	Data	Model	Data	Model	Data	Model
Target Moments						
Means of						
Equity Issuance/Asset	0.010	0.002	0.006	0.007	-0.004	0.004
Cash Holdings/Asset	0.092	0.101	0.161	0.154	0.068	0.053
STDs of						
Investment Rate	0.095	0.091	0.076	0.080	-0.019	-0.011
Equity Issuance/Asset	0.042	0.033	0.061	0.063	0.019	0.030
Cash Flow/Asset	0.053	0.056	0.085	0.082	0.031	0.026
Betas of						
Savings on Tobin's Q	-0.010	-0.007	-0.080	-0.073	-0.070	-0.065
Savings on Cash Flow/Capital	-0.003	0.008	0.062	0.045	0.065	0.037
Issuance on Tobin's Q	0.011	0.002	0.046	0.011	0.035	0.009
Issuance on Cash Flow/Capital	0.011	-0.003	-0.036	-0.009	-0.046	-0.006
Autocorrelation of						
Cash Flow/Asset	0.552	0.567	0.439	0.436	-0.113	-0.131
Non-Target Moments						
Beta of Investment on Tobin's Q	0.017	0.009	0.026	0.074	0.009	0.065
Beta of Investment on Cash Flow/Capital	0.037	0.023	0.009	0.006	-0.028	-0.017

Table 4: Simulated Moments Estimation for Subsapmles (Non-tech v.s. Tech)

This table presents parameters and model moments estimated via the simulated method of moments (SMM) for nonfinancial, unregulated firms in Compustat, with separate estimates for the "Non-tech industry" and the "Tech industry". The classification of non-tech and tech industries follows Loughran and Ritter (2004). Panel A reports the estimated structural parameters, with standard errors in parentheses, focusing on eight key parameters: f (fixed operating cost), a_0 (fixed financing cost), a_1 (linear financing cost), σ_z (standard deviation of short-run shock), σ_μ (standard deviation of long-run shock), ρ_μ (persistence of long-run shock), σ_ξ (standard deviation of investment efficiency shock), and Φ (informativeness of the soft signal). The remaining parameters are calibrated using data and prior literature. Panel B compares target and non-target data moments with model moments for the non-tech industry and the tech industry. Model moments are computed as averages across 100 simulations of 3,000 firms over 50 periods.

Panel A: Paramter Estimates

	f	a_0	a_1	σ_z	σ_μ	ρ_μ	σ_ξ	Φ
Non-tech	0.096	0.012	0.045	0.053	0.261	0.514	0.250	0.225
	(0.002)	(0.006)	(0.007)	(0.026)	(0.009)	(0.012)	(0.006)	(0.053)
Tech	0.116	0.002	0.049	0.228	0.337	0.584	0.221	0.858
	(0.007)	(0.001)	(0.008)	(0.073)	(0.063)	(0.114)	(0.051)	(0.174)

Panel B: Moments

	Non-Tech		Tech		Tech−Non-Tech	
	Data	Model	Data	Model	Data	Model
Target Moments						
Means of						
Equity Issuance/Asset	0.009	0.004	0.010	0.013	0.001	0.009
Cash Holdings/Asset	0.103	0.110	0.182	0.199	0.079	0.088
STDs of						
Investment Rate	0.093	0.088	0.117	0.126	0.024	0.038
Equity Issuance/Asset	0.050	0.044	0.086	0.075	0.036	0.032
Cash Flow/Asset	0.069	0.072	0.103	0.111	0.035	0.038
Betas of						
Savings on Tobin's Q	-0.022	-0.009	-0.101	-0.111	-0.079	-0.103
Savings on Cash Flow/Capital	0.007	0.010	0.094	0.076	0.087	0.066
Issuance on Tobin's Q	0.021	0.004	0.047	0.021	0.026	0.018
Issuance on Cash Flow/Capital	0.002	-0.005	-0.006	-0.014	-0.008	-0.008
Autocorrelation of						
Cash Flow/Asset	0.491	0.492	0.414	0.415	-0.077	-0.077
Non-Target Moments						
Beta of Investment on Tobin's Q	0.022	0.020	0.040	0.110	0.018	0.090
Beta of Investment on Cash Flow/Capital	0.024	0.017	0.018	-0.006	-0.005	-0.023

Table 5: Attribution Analysis for Subsamples (Early v.s. Late) using Shapley Value Decomposition

This table reports the contribution weights (in percentages) of each estimated parameter to changes in both target and non-target model moments between the "Early period" (1975–1994) and the "Late period" (2005–2024), based on Shapley value decomposition. The first eight columns presents the contribution weights (in percentages) of eight estimated parameters: f (fixed operating cost), a_0 (fixed financing cost), a_1 (linear financing cost), σ_z (standard deviation of short-run shock), σ_μ (standard deviation of long-run shock), ρ_μ (persistence of long-run shock), σ_ξ (standard deviation of investment efficiency shock) and Φ (informativeness of the soft signal). The last column shows the changes in simulated moments of interest when using the estimated parameters for the early period (θ^{Early}) versus the late period (θ^{Late}), i.e., $\Psi_s(\theta^{Late}) - \Psi_s(\theta^{Early})$. These parameters are estimated via simulated method of moments (SMM), with their specific values reported in Table 3. Model moments are computed as averages across 100 simulations of 3,000 firms over 50 periods. The contribution weights (in percentages) are defined as each parameter's Shapley value divided by the sum of Shapley values for all eight estimated parameters. Shapley values are computed using Equation (26), with $\theta^b = \theta^{Early}$ and $\theta^c = \theta^{Late}$.

	f	a_0	a_1	σ_z	σ_μ	ρ_μ	σ_ξ	Φ	Model (Late−Early)
Target Moments									
Means of									
Equity Issuance/Asset	76.82%	0.28%	-26.81%	7.32%	41.10%	8.41%	0.36%	-7.49%	0.004
Cash Holdings/Asset	36.50%	-11.87%	19.13%	19.42%	30.74%	2.96%	-52.11%	55.23%	0.053
STDs of									
Investment Rate	-15.32%	3.48%	-1.12%	19.11%	-103.18%	-18.89%	267.49%	-51.57%	-0.011
Equity Issuance/Asset	79.37%	-14.78%	-20.18%	-1.99%	49.99%	12.93%	-4.60%	-0.74%	0.030
Cash Flow/Asset	-2.08%	0.95%	-1.23%	42.83%	44.02%	5.20%	8.23%	2.08%	0.026
Betas of									
Savings on Tobin's Q	25.86%	-1.42%	3.02%	-5.17%	17.49%	1.56%	17.43%	41.23%	-0.065
Savings on Cash Flow/Capital	11.95%	0.75%	2.10%	20.92%	8.72%	-2.19%	2.17%	55.57%	0.037
Issuance on Tobin's Q	66.03%	-0.85%	-16.65%	-27.27%	35.59%	8.37%	4.13%	30.67%	0.009
Issuance on Cash Flow/Capital	81.51%	-1.08%	-20.27%	-17.85%	45.32%	10.55%	-2.69%	4.52%	-0.006
Autocorrelation of									
Cash Flow/Asset	-6.38%	2.08%	-2.03%	165.90%	-43.81%	-14.81%	1.20%	-2.16%	-0.131
Non-Target Moments									
Beta of Investment on Tobin's Q	29.85%	-0.01%	-0.95%	-8.14%	21.88%	3.17%	21.55%	32.65%	0.065
Beta of Investment on Cash Flow/Capital	18.01%	1.40%	-3.37%	-11.40%	-4.67%	-5.53%	15.85%	89.71%	-0.017

Table 6: Attribution Analysis for Subsamples (Non-tech v.s. Tech) using Shapley Value Decomposition

This table reports the contribution weights (in percentages) of each estimated parameter to changes in both target and non-target model moments between the "Non-tech industry" and the "tech industry", based on Shapley value decomposition. The classification of non-tech and tech industries follows Loughran and Ritter (2004). The first eight columns presents the contribution weights (in percentages) of eight estimated parameters: f (fixed operating cost), a_0 (fixed financing cost), a_1 (linear financing cost), σ_z (standard deviation of short-run shock), σ_μ (standard deviation of long-run shock), ρ_μ (persistence of long-run shock), σ_ξ (standard deviation of investment efficiency shock) and Φ (informativeness of the soft signal). The last column shows the changes in simulated moments of interest when using the estimated parameters for the non-tech subsample ($\theta^{non-tech}$) versus the tech subsample (θ^{tech}), i.e., $\Psi_s(\theta^{non-tech}) - \Psi_s(\theta^{tech})$. These parameters are estimated via simulated method of moments (SMM), with their specific values reported in Table 4. Model moments are computed as averages across 100 simulations of 3,000 firms over 50 periods. The contribution weights (in percentages) are defined as each parameter's Shapley value divided by the sum of Shapley values for all eight estimated parameters. Shapley values are computed using Equation (26), with $\theta^b = \theta^{non-tech}$ and $\theta^c = \theta^{tech}$.

	f	a_0	a_1	σ_z	σ_μ	ρ_μ	σ_ξ	Φ	Model (Tech−Non-tech)
Target Moments									
Means of									
Equity Issuance/Asset	46.58%	7.18%	-5.19%	3.10%	30.07%	16.26%	-0.20%	2.19%	0.009
Cash Holdings/Asset	35.52%	-42.25%	4.20%	14.21%	25.62%	5.30%	-6.80%	64.18%	0.088
STDs of									
Investment Rate	5.00%	-3.24%	-0.03%	-9.97%	43.76%	24.82%	-19.67%	59.32%	0.038
Equity Issuance/Asset	82.18%	-66.50%	-5.22%	-1.35%	51.62%	34.09%	-0.58%	5.75%	0.032
Cash Flow/Asset	-3.96%	6.84%	-0.54%	28.71%	49.83%	12.64%	2.00%	4.49%	0.038
Betas of									
Savings on Tobin's Q	18.54%	-5.60%	0.78%	-7.34%	14.37%	5.18%	2.17%	71.90%	-0.103
Savings on Cash Flow/Capital	10.48%	0.20%	0.69%	5.58%	10.54%	-4.33%	0.05%	76.79%	0.066
Issuance on Tobin's Q	34.02%	1.94%	-3.00%	-15.65%	18.20%	15.53%	-0.93%	49.89%	0.018
Issuance on Cash Flow/Capital	54.56%	-1.06%	-5.37%	-14.66%	30.46%	23.47%	-2.51%	15.11%	-0.008
Autocorrelation of									
Cash Flow/Asset	-5.82%	9.77%	-0.29%	213.79%	-25.43%	-86.01%	0.55%	-6.56%	-0.077
Non-Target Moments									
Beta of Investment on Tobin's Q	25.14%	0.27%	-0.24%	-8.66%	18.85%	8.34%	2.70%	53.59%	0.090
Beta of Investment on Cash Flow/Capital	4.49%	4.76%	-0.99%	-19.48%	-0.57%	-13.42%	0.65%	124.55%	-0.023

Appendix A Proofs

Appendix A.1 Proof of Proposition 1

We can write the dynamics of the observable variables as the following measurement equation:

$$Y_{t+1} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{P} \mu_t + \underbrace{\begin{bmatrix} \sigma_z & 0 & 0 \\ 0 & \sqrt{1-\Phi^2} & \Phi \end{bmatrix}}_{R} \varepsilon_{t+1}, \tag{27}$$

Where Y_{t+1} is a 2×1 column vector $[\mathbf{z_{t+1}}; \mathbf{s_{t+1}}]^{\mathrm{T}}$, including two observable signals at time $t+1$ (cash flow z_{t+1} and a soft signal s_{t+1}), and ε_{t+1} is a 3×1 column vector $[\varepsilon_{\mathbf{z},\mathbf{t+1}}; \varepsilon_{\mathbf{s},\mathbf{t+1}}; \varepsilon_{\mu,\mathbf{t+1}}]^{\mathrm{T}}$, including short-term productivity shocks $\varepsilon_{z,t+1}$, a noise term $\varepsilon_{s,t+1}$, and long-term productivity shocks $\varepsilon_{\mu,t+1}$.

In this framework, μ_t is unobservable, following a transition process:

$$\mu_{t+1} = \underbrace{(1-\rho_\mu)\bar{\mu}}_{p} + \underbrace{\rho_\mu}_{q} \mu_t + \underbrace{\begin{pmatrix} 0 & 0 & \sigma_\mu \end{pmatrix}}_{r} \varepsilon_{t+1}. \tag{28}$$

Following standard filtering theory as in Harvey (1990), we can assume

$$\hat{\mu}_{t+1|t+1} = \hat{\mu}_{t+1|t} + G_{t+1}(Y_{t+1} - \hat{Y}_{t+1|t}), \tag{29}$$

Where G_{t+1} is called a Kalman gain, which measures how much $\mu_{t+1|t+1}$ is updated based on the error in predicting Y_{t+1}. Alternatively, Equation (29) can be rewritten as:

$$\hat{\mu}_{t+1|t+1} = (1-\rho_\mu)\bar{\mu} + \rho_\mu \hat{\mu}_{t|t} + G_{t+1}(Y_{t+1} - \hat{Y}_{t+1|t}). \tag{30}$$

The optimal Kalman gain G_{t+1} minimizes the sum of variance of the estimation errors:

$$\zeta_{t+1} = \min_{G_{t+1}} \mathbb{E}[(\mu_{t+1} - \hat{\mu}_{t+1|t+1})^2 | \mathcal{F}_{t+1}] \tag{31}$$

The solution is given by:

$$G_{t+1} = (q\zeta_t P^{\mathrm{T}} + rR^{\mathrm{T}})(P\zeta_t P^{\mathrm{T}} + RR^{\mathrm{T}})^{-1}. \tag{32}$$

Then, we substitute Equation (32) into Equation (30) and obtain

$$\hat{\mu}_{t+1|t+1} = (1-\rho_\mu)\bar{\mu} + \rho_\mu \hat{\mu}_{t|t} + \frac{\rho_\mu \zeta_t}{\zeta_t + \sigma_z^2}(z_{t+1} - \hat{\mu}_{t|t}) + \Phi\sigma_\mu s_{t+1}. \tag{33}$$

The law of motion for the posterior variance is then derived as:

$$\zeta_{t+1} = \frac{\rho_\mu^2 \sigma_z^2 \zeta_t}{\zeta_t + \sigma_z^2} + (1-\Phi^2)\sigma_\mu^2 \tag{34}$$

Appendix A.2 Proof of Corollary 1, 2 and 3

Appendix A.2.1 Corollary 1

We have

$$\frac{\partial \bar{\zeta}}{\partial \Phi} = \frac{1}{2}\frac{\partial (A + \sqrt{B})}{\partial \Phi}, \tag{35}$$

Where $A = \sigma_\mu^2(1-\Phi^2) - (1-\rho_\mu^2)\sigma_z^2$, and $B = [(1-\rho_\mu^2)\sigma_z^2 - \sigma_\mu^2(1-\Phi^2)]^2 + 4\sigma_\mu^2\sigma_z^2(1-\Phi^2)$.

It is straightforward to verify that, under valid paramter values,[13]

$$\frac{\partial (A + \sqrt{B})}{\partial \Phi} = -2\sigma_\mu^2 \Phi \left(\frac{A + \sqrt{B} + 2\sigma_z^2}{\sqrt{B}} \right) < 0 \tag{36}$$

Hence, we conclude that

$$\frac{\partial \bar{\zeta}}{\partial \Phi} < 0, \tag{37}$$

which yields Corollary 1.

Appendix A.2.2 Corollary 2

The law of motion of the filter can be rewritten as

$$\hat{\mu}_{t+1|t+1} = (1-\rho_\mu)\bar{\mu} + \rho_\mu \hat{\mu}_{t|t} + \frac{\rho_\mu \bar{\zeta}}{\bar{\zeta} + \sigma_z^2}(\mu_t - \hat{\mu}_{t|t} + \sigma_z \varepsilon_{z,t+1}) + \Phi\sigma_\mu(\sqrt{1-\Phi^2}\varepsilon_{s,t+1} + \Phi\varepsilon_{\mu,t+1}). \tag{38}$$

[13] $0 < \rho_\mu < 1$, $\sigma_\mu > 0$, $\sigma_z > 0$ and $0 < \Phi < 1$

From the perspective of μ_t, the unconditional variance of z_{t+1} is

$$Var[z_{t+1}] = \frac{\sigma_\mu^2}{1-\rho_\mu^2} + \sigma_z^2. \tag{39}$$

From the perspective of $\hat{\mu}_{t|t}$, using the decomposition $z_{t+1} = \mu_t - \hat{\mu}_{t|t} + \hat{\mu}_{t|t} + \sigma_z \varepsilon_{z,t+1}$, we obtain

$$Var[z_{t+1}] = \frac{Var(\hat{\mu}_{t+1|t+1}|\mathcal{F}_t)}{1-\rho_\mu^2} + \bar{\zeta} + \sigma_z^2. \tag{40}$$

Comparing the two expressions, we deduce

$$Var(\hat{\mu}_{t+1|t+1}|\mathcal{F}_t) = \sigma_\mu^2 - (1-\rho_\mu^2)\bar{\zeta}. \tag{41}$$

Finally, with the help of Corollary 1, we conclude that the conditional variance of the filter is strictly increasing in Φ:

$$\frac{\partial Var(\hat{\mu}_{t+1|t+1}|\mathcal{F}_t)}{\partial \Phi} > 0. \tag{42}$$

Appendix A.2.3 Corollary 3

From the law of motion of the filter, we have

$$Corr(\hat{\mu}_{t+1|t+1}, z_{t+1}|\mathcal{F}_t) = \frac{Cov(\hat{\mu}_{t+1|t+1}, z_{t+1}|\mathcal{F}_t)}{\sqrt{Var(\hat{\mu}_{t+1|t+1}|\mathcal{F}_t)Var(z_{t+1}|\mathcal{F}_t)}} = \frac{\rho_\mu \bar{\zeta}}{\sqrt{(\bar{\zeta}+\sigma_z^2)[\sigma_\mu^2 - (1-\rho_\mu^2)\bar{\zeta}]}} \tag{43}$$

With the help of Corollary 1, we conclude that the conditional correlation between the filter and productivity is strictly decreasing in Φ:

$$\frac{\partial Corr(\hat{\mu}_{t+1|t+1}, z_{t+1}|\mathcal{F}_t)}{\partial \Phi} < 0. \tag{44}$$

Appendix B Variable Construction

Table B1: Definition of Variables

CashFlow	The sum of income before extraordinary items (IB), and depreciation and amortization (DP), divided by lagged capital (L.PPEGT).	Compustat
Savings	Cash-to-capital ratio (CHE/PPEGT) minus lagged Cash-to-capital ratio (L.CHE/L.PPEGT).	Compustat
Investment	Capital expenditure (CAPX), divided by lagged capital (L.PPEGT).	Compustat
Issuance	Sale of common and preferred stock (SSTK), divided by lagged capital (L.PPEGT).	Compustat
Tobin's Q	Firm's market value (PRCC_F*CSHO+DLTT+DLC-ACT), divided by capital (PPEGT).	Compustat
Size	Natual logarithm of capital (PPEGT).	Compustat
Tot Investment	The sum of capital expenditure (CAPX) and intangible investment (R&D+0.3*SG&A), divided by the total capital constructed by Peters and Taylor (2017).	Compustat
Tot CashFlow	The sum of income before extraordinary items (IB), depreciation and amortization (DP), and intangible investment (R&D+0.3*SG&A), divided by the total capital constructed by Peters and Taylor (2017).	Compustat
Tot Q	Firm's market value (PRCC_F*CSHO+DLTT+DLC-ACT), divided by the total capital constructed by Peters and Taylor (2017).	Compustat

Appendix C Robustness

Table C1: Robustness: Sub-period Comparison of Cash Flow and Tobin's Q Coefficients

The table presents the coefficients estimated from panel regressions of total investment, savings, and equity issuance on total cash flow, total Q, and firm size across three sub-periods 1975–1994, 1995–2004, and 2005–2024. All specifications include firm fixed effects. The dependent variables are total investment scaled by lagged total capital, changes in the ratio of cash to total capital, and equity issuance scaled by lagged total capital. The key regressors—total cash flow and total Q—are standardized at the firm level. Variable definitions are provided in Table B1 in Appendix B. Standard errors are clustered at the firm level. *, **, *** represent significance at the 10%, 5%, and 1% levels, respectively.

	Panel A: Dep Var = Tot investment/Total capital			
	1975–1994	1995–2004	2005–2024	Diff (3)–(1)
	(1)	(2)	(3)	
Tot CashFlow	0.040***	0.032***	0.019***	-0.021***
	(0.001)	(0.001)	(0.001)	(0.001)
Tot Q	0.012***	0.026***	0.015***	0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
Controls	Yes	Yes	Yes	
Observations	35,007	22,693	40,573	
R-squared	0.274	0.246	0.221	
	Panel B: Dep Var = ΔCash/Total capital			
	(1)	(2)	(3)	
Tot CashFlow	-0.003***	0.004***	0.000	0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
Tot Q	-0.004***	-0.013***	-0.013***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)
Controls	Yes	Yes	Yes	
Observations	35,378	22,977	38,626	
R-squared	0.009	0.012	0.012	
	Panel C: Dep Var = Issuance/Total capital			
	(1)	(2)	(3)	
Tot CashFlow	0.006***	0.009***	-0.001	-0.007***
	(0.001)	(0.001)	(0.001)	(0.001)
Tot Q	0.005***	0.010***	0.006***	0.001
	(0.000)	(0.001)	(0.001)	(0.001)
Controls	Yes	Yes	Yes	
Observations	34,495	22,426	39,680	
R-squared	0.017	0.029	0.012	

Table C2: Robustness: Cross-section Comparison of Cash Flow and Tobin's Q Coefficients

The table presents the coefficients estimated from panel regressions of total investment, savings, and equity issuance on total cash flow, total Q, and firm size across non-tech and tech subsamples. All specifications include firm fixed effects. The classification of non-tech and tech industries follows Loughran and Ritter (2004). The dependent variables are total investment scaled by lagged total capital, changes in the ratio of cash to total capital, and equity issuance scaled by lagged total capital. The key regressors—total cash flow and total Q—are standardized at the firm level. Variable definitions are provided in Table B1 in Appendix B. Standard errors are clustered at the firm level. *, **, *** represent significance at the 10%, 5%, and 1% levels, respectively.

	Panel A: Dep Var = Tot investment/Total capital				
	Manufacturing	Consumer	High-tech	Health	Diff: (4)-(1)
	(1)	(2)	(3)	(4)	
Tot CashFlow	0.034***	0.037***	0.043***	0.026***	-0.008***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
Tot Q	0.012***	0.011***	0.027***	0.030***	0.017***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
Controls	Yes	Yes	Yes	Yes	
Observations	28,514	23,657	24,267	9,746	
R-squared	0.263	0.396	0.376	0.287	
	Panel B: Dep Var = ΔCash/Total capital				
	(1)	(2)	(3)	(4)	
Tot CashFlow	-0.000	-0.004***	0.005***	0.010***	0.010***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
Tot Q	-0.005***	-0.005***	-0.018***	-0.017***	-0.012***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
Controls	Yes	Yes	Yes	Yes	
Observations	28,279	23,377	23,960	9,557	
R-squared	0.004	0.008	0.012	0.011	
	Panel C: Dep Var = Issuance/Total capital				
	(1)	(2)	(3)	(4)	
Tot CashFlow	0.004***	0.006***	0.008***	-0.009***	-0.013***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
Tot Q	0.005***	0.005***	0.013***	0.019***	0.014***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
Observations	27,937	23,221	23,873	9,592	
R-squared	0.020	0.024	0.032	0.040	

Table C3: Counterfactual Analysis for Subsamples (Early v.s. Late)

The table reports target and non-target model moments for time-series quantitative counterfactual analysis, involving a comparison of data and model moments between the "Early period" (1975–1994) and the "Late period" (2005–2024). Model moments are computed as averages across 100 simulations of 3,000 firms and 50 periods. The first column presents model moments for a counterfactual exercise in which the informativeness of the soft signal (Φ) is set to its estimated value for the late period, while all other parameters remain at their estimated values for the early period. The second column presents model moments for a counterfactual exercise in which parameters of financing costs (a_0 and a_1) are set to their estimated values for the late period, while all other parameters remain at their estimated values for the early period. The third column presents model moments for a counterfactual exercise in which parameters of cash flow process (ρ_μ, σ_z and σ_μ) are set to their estimated values for the late period, while all other parameters remain at their estimated values for the early period.

	$\{\Phi\}$=Late	$\{a_0, a_1\}$= Late	$\{\rho_\mu, \sigma_z, \sigma_\mu\}$= Late
Target Moments			
Means of			
Equity Issuance/Asset	0.002	0.002	0.005
Cash Holdings/Asset	0.131	0.101	0.121
STDs of			
Investment Rate	0.095	0.091	0.101
Equity Issuance/Asset	0.032	0.027	0.054
Cash Flow/Asset	0.057	0.057	0.081
Betas of			
Savings on Tobin's Q	-0.030	-0.007	-0.014
Savings on Cash Flow/Capital	0.031	0.008	0.019
Issuance on Tobin's Q	0.003	0.002	0.004
Issuance on Cash Flow/Capital	-0.003	-0.003	-0.006
Autocorrelation of			
Cash Flow/Asset	0.567	0.567	0.425
Non-Target Moments			
Beta of Investment on Tobin's Q	0.030	0.009	0.019
Beta of Investment on Cash Flow/Capital	0.001	0.023	0.025
Change in Cash—Data (Late vs Early):	0.068	0.068	0.068
Change due to Alternatives (Counterfactual):	0.030	0.000	0.020
Change in Investment-Cash Flow sensitivity—Data (Late vs Early):	-0.028	-0.028	-0.028
Change due to Alternatives (Counterfactual):	-0.021	0.001	0.002
Change in Savings-Cash Flow sensitivity—Data (Late vs Early):	0.065	0.065	0.065
Change due to Alternatives (Counterfactual):	0.023	0.000	0.011

Table C4: Counterfactual Analysis for Subsamples (Non-tech v.s. Tech)

The table reports target and non-target model moments for cross-section quantitative counterfactual analysis, involving a comparison of data and model moments between the "Non-tech industry" and the "Tech industry". The classification of non-tech and tech industries follows Loughran and Ritter (2004). Model moments are computed as averages across 100 simulations of 3,000 firms and 50 periods. The first column presents model moments for a counterfactual exercise in which the informativeness of the soft signal (Φ) is set to its estimated value for the tech industry, while all other parameters remain at their estimated values for the Non-tech industry. The second column presents model moments for a counterfactual exercise in which parameters of financing costs (a_0 and a_1) are set to their estimated values for the tech industry, while all other parameters remain at their estimated values for the Non-tech industry. The third column presents model moments for a counterfactual exercise in which parameters of cash flow process (ρ_μ, σ_z and σ_μ) are set to their estimated values for the tech industry, while all other parameters remain at their estimated values for the Non-tech industry.

	$\{\Phi\}$=Tech	$\{a_0, a_1\}$= Tech	$\{\rho_\mu, \sigma_z, \sigma_\mu\}$= Tech
Target Moments			
Means of			
Equity Issuance/Asset	0.003	0.004	0.008
Cash Holdings/Asset	0.167	0.091	0.148
STDs of			
Investment Rate	0.105	0.087	0.109
Equity Issuance/Asset	0.040	0.032	0.078
Cash Flow/Asset	0.075	0.073	0.109
Betas of			
Savings on Tobin's Q	-0.064	-0.008	-0.014
Savings on Cash Flow/Capital	0.060	0.012	0.014
Issuance on Tobin's Q	0.004	0.003	0.007
Issuance on Cash Flow/Capital	-0.003	-0.005	-0.010
Autocorrelation of			
Cash Flow/Asset	0.501	0.485	0.419
Non-Target Moments			
Beta of Investment on Tobin's Q	0.055	0.020	0.033
Beta of Investment on Cash Flow/Capital	-0.012	0.016	0.027
Change in Cash—Data (Tech vs Non-tech):	0.079	0.079	0.079
Change due to Alternatives (Counterfactual):	0.056	-0.020	0.038
Change in Investment-Cash Flow sensitivity—Data (Tech vs Non-tech):	-0.005	-0.005	-0.005
Change due to Alternatives (Counterfactual):	-0.029	-0.001	0.010
Change in Savings-Cash Flow sensitivity—Data (Tech vs Non-tech):	0.087	0.087	0.087
Change due to Alternatives (Counterfactual):	0.050	0.002	0.003

Table C5: Comparative Statics across the Ratio of Long-run Volatility to Short-run Volatility

This table reports comparative statics for model moments at different levels of the ratio of long-run volatility (σ_μ) to short-run volatility (σ_z), while keeping the unconditional variance of productivity constant. Means, standard deviations (STDs), betas, and the autocorrelation are computed as averages across 100 simulations of 3,000 firms and 50 periods.

Short-run volatility (σ_z):	0.248	0.232	0.212	0.190	0.163
Long-run volatility (σ_μ):	0.180	0.193	0.205	0.218	0.230
Means of					
Equity Issuance/Asset	0.007	0.007	0.008	0.008	0.009
Cash Holdings/Asset	0.128	0.128	0.128	0.126	0.129
STDs of					
Investment rate	0.081	0.084	0.087	0.090	0.094
Equity Issuance/Asset	0.052	0.055	0.059	0.062	0.067
Cash Flow/Asset	0.079	0.079	0.080	0.081	0.082
Betas of					
Savings on Tobin's Q	-0.038	-0.041	-0.045	-0.047	-0.053
Savings on Cash Flow/Capital	0.042	0.040	0.040	0.039	0.038
Issuance on Tobin's Q	0.007	0.008	0.009	0.011	0.012
Issuance on Cash Flow/Capital	-0.009	-0.009	-0.010	-0.011	-0.011
Investment on Tobin's Q	0.040	0.045	0.050	0.054	0.061
Investment on Cash Flow/Capital	0.006	0.006	0.006	0.006	0.005
Autocorrelation of					
Cash Flow/Asset	0.295	0.342	0.392	0.442	0.497