Credit-driven economy: A unified framework of segregated boom, asset inflation and monetary policy

Hitoshi Hayakawa

Department of Economics, Hokkaido University

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Abstract

This paper establishes a new dynamic monetary (non-stochastic) general equilibrium framework, which is dubbed as "credit-driven" economy. The framework intends to demonstrate the view of Schumpeter(1934) on the role of credit creation in enhancing development. The core structure of the framework is based on the model suggested by Ray(2010) that aims to address "unequal growth" in the course of development by adopting hierarchical preferences in framing two separate production sectors (e.g., Agriculture and Manufacture industries).

The aim of this article is to give insights particularly on the following three issues in a unified manner; 1: economic development and inequality, 2: asset bubble and 3: monetary policy. In the perspective of economic development and inequality, the analysis demonstrates the emergence of asset bubble as an extension of a segregated boom that is argued in the work of Ray(2010). In the view of theoretical foundation on asset bubble, this article departs from the rational bubble theory in two respects: firstly, to establish a distinct concept of asset bubble from fiat money by pursuing the ultimate source of bubble in pecuniary externality of asset market, and secondly, to address inequality exacerbating nature of asset bubble (i.e., regressive transfer) while showing an efficiency harming mechanism through bubble-riding (e.g., financial management techniques, or "Zai-tech"). In giving a theoretical foundation on the functioning of monetary policy in macro perspective, the framework of this article departs form the core structure of New Keynesian model in three respects. Firstly, nominal rigidity is introduced not as mere friction but as serving a stabilizing role of the whole economy against a potential process of instability and destruction. Secondly, credit supply is explicitly examined considering the incentive of banks rather than essentially assuming away credit resorting to the idea of cashless economy. Thirdly, the article addresses a market based monetary policy where the policy rates are controlled while market interest rates are only indirectly affected. On this basis, this article successfully addresses high credit growth in bringing asset bubble, while the New Keynesian model has essential limitation to this regard. Furthermore, on the issue of implication of bank credit in monetary policy, this article poses a novel concept of "equilibrium excess credit," while the existent research mostly focuses on the mechanisms of potential under-supply of bank

credit. In this perspective, this article shows conditions for inevitable consequences of "efficiency-first" monetary policy in bringing persistent excess credit. It is also shown that persistent excess credit above certain level can bring asset bubble in the form of asset inflation, thus establishing the role of "efficiency-first" monetary policy in bringing asset inflation.

1 Introduction

At the background of severe economic consequences brought by 1980s Japan's asset bubble and 2000s U.S. housing bubble, there emerged wide attention on the essential mechanism of asset bubbles as well as the role of monetary policy in arising asset bubble. While there is an established strand of research that examines these issues using non-monetary growth models, this article intends to accentuate monetary perspective in grasping asset bubble as well as unravel the underlying development dynamics. In this purpose, this article invents a monetary general equilibrium model wherein bank credit has essential role in enhancing development. This article refers to the model as a credit-driven development model.

A traditional discussion on the role of bank credit in enhancing development is provided in Schumpeter and Opie (1934). In more recent years, there is a strand of literature that empirically examines this issue, particularly in the frame of financial deepening.[1] However, there is few study that provides a consistent formal model in demonstrating the discussion. In this respect, the base contribution of this article is to provide a general equilibrium framework that allows to formulate the role of bank credit in enhancing development as discussed in Schumpeter and Opie (1934). For our purpose, this article uses the framework to demonstrate credit channel of monetary policy and discusses the emergence of asset bubble and the role of monetary policy in this context. In this perspective, a critical observation in recent bubble episodes is expansion in bank credit.[2] On this point, New Keynesian framework that is established as the current standard framework of monetary policy faces apparent limitation as it essentially omits bank credit. The framework cannot argue any necessary relationship between credit expansion and the formation of asset bubble as well as the role of monetary policy in affecting credit supply.[3] This article provides a formal analysis on these issues by explicitly incorporating banks in providing credit as well as explicitly demonstrating the route through which monetary policy affects bank credit.

This article constructs a monetary framework based on the non-monetary development framework—*dual economy* framework—proposed in Ray (2010). In addressing the issue

[1]Early contribution to the literature of financial deepening is made by King and Levine (1993) and Beck, Levine, and Loayza (2000).

[2]Credit expansion in recent bubbles are documented, for example, in Jorda, Schularick, and Taylor (2015).

[3]In this regard, Gali (2018) points out the limitation of existent New Keynesian analyses in addressing the role of high credit growth in bringing asset bubble.

of inequality in the course of development, Ray (2010) outlines a closed two-sector model wherein the sectors are characterized with hierarchical preferences.[4] Economic development is captured through inter-sector labor shift such that workers (households) shift from the satiation sector (primary sector) to the non-satiation sector (secondary sector). Departing from the non-monetary setting, the framework proposed in this article explicitly addresses the role of bank credit in promoting such inter-sector labor shift. While capturing such desirable expansion in bank credit, this article demonstrates the occurrence of excess expansion in bank credit, and also demonstrates the emergence of asset bubble as a consequence of rather extreme expansion in bank credit. In this respect, key observation on the implied dynamics is that expansion in bank credit (to the secondary sector) has labor-shifting effect only when there already exists excess labor supply in the primary sector. In fact, when there is no excess labor in the primary sector, additional credit supply (to the secondary sector) no more promotes inter-sector labor shift, instead it becomes the source of inter-sector income (wage) inequality, and consequently brings inequality in consumption and wealth (i.e., holding assets). It is further demonstrated that asset bubble arises at certain critical point in the course of widening inequality in asset holding that is brought under continuing excess bank credit.

On the basis of the featured functioning of bank credit in promoting development and the consequences of excess bank credit, this article examines the role of monetary policy focusing on the effects through credit channel. In this context, it is highlighted that monetary policy essentially faces a trade-off between attaining efficiency in labor distribution and maintaining inter-sector income equality, and the emergence of asset bubble is demonstrated as a consequence of "efficiency-first" monetary policy that primarily pursues efficiency while equality is pursued as the secondary objective.

Given efficiency-first monetary policy, the analysis addresses the implication of technological improvements in eventual emergence of asset inflation. The analysis frames general technological improvements with the concepts of "growth" (productivity improvements of secondary sector, no labor shift inducing) and "development" (productivity improvements of primary sector, labor shift inducing). It is shown that once-and-for-all or even finite sequence of "growth" tend to have only temporal effect in generating excess credit, and thus, "growth" is not placed as an ultimate source of asset inflation. Meanwhile, once-and-for-all "development" brings persistent excess credit. Thus, "development" is identified as an ultimate source of asset inflation. The analysis further reveals subtle effects of sequential "development": the effect of the initial "development" in inducing persistent excess credit could be attenuated by the subsequent "development." The result implies that a temporal large scale of development with subsequent stagnation is a typical underlying

[4]Hierarchical preferences are adopted to investigate development issues also in Eswaran and Kotwal (1993) in a similar way. In the literature, the two sectors are largely framed as "Agriculture" (primary) sector and "Manufacture" (secondary) sector in examining the process of "industrialization". This article stands on a more flexible view such that the secondary sector could represent some emerging sector in each context while the primary sector represents the whole existent sectors. For example, in applying to recent industrial context, the secondary sector could be framed as "AI" industry while the other existent industries including automobile or agriculture as a whole constitutes the primary sector.

situation of eventual emergence of asset inflation. Furthermore, the analysis articulates non-linear effects of once-and-for-all "growth" that simultaneously arises once-and-for-all "development" in that it could both amplify or attenuate the persistently observed excess credit. These results establish foundation in discussing the implication of technological improvements in eventual occurrence of asset inflation, while also could bring insights on specific observations in our history; for example, as to why there emerged episodes of asset inflation in several countries between 1980s-2000s while no remarkable episode was observed between 1940s-1980s.

This article has major contribution to each of the following three strands of literature: 1. Economic development and inequality, 2. Asset bubble, and 3. monetary policy.

Contributions: 1. Economic development and inequality

Firstly, concerning the issue of development and inequality, Matsuyama (2002) shows a trickle-down mechanism such that the occurrence of income inequality promotes demand to the goods in higher sectors and consequently enhance development. Against this, Ray (2010) argues the occurrence of segregated boom such that higher wage in higher sectors will not spill over to the lower sector particularly when there exists real friction in labor shift (e.g., skill learning cost). This article reframes the setting of Ray (2010) as a monetary economy to find a distinctive monetary mechanism that serves to restrain inter-sector labor shift. On this basis, while Ray (2010) demonstrates no inter-sector spillover effect of a boom, this article adds to find certain negative spillover effect of a boom that is captured through the occurrence of asset bubble. Also, while Ray (2010) focuses on the implications on consumption inequality, this article highlights implications on wealth inequality. Particularly, the emergence of asset bubble is demonstrated as a consequence of exacerbating wealth inequality in this context. To give a sweeping view on these points, this article argues that credit expansion under restrained inter-sector labor shift serves to generate consumption boom in the higher sector as well as exacerbating inter-sector wealth inequality, and then asset bubble is emerged at a critical point in the process of widening wealth inequality. In this perspective, a notable contribution is to demonstrate the emergence of asset bubble as an extension of a segregated boom argued in the work of Ray (2010).

Contributions: 2. Asset bubble

Secondly, concerning the theory of asset bubble, there exists an extensive literature of rational bubble that follows the seminal papers of Samuelson (1958) and Tirole (1985).[5] This article points out two critical limitations of the rational bubble framework. For one, in the conceptual perspective, there is limitation in discriminating bubble phenomenon from monetary phenomenon. For the other, in the perspective of relevance to recent bubble episodes, there is limitation in demonstrating the emergence of asset bubble in a form of asset inflation—rising asset prices during stable good prices. These two limitations

[5]For the literature of rational bubble, see the surveys provided in Hirano and Toda (2024), Miao (2014) and Barlevy (2007).

have the same root in the essential approach of rational bubble. Rational bubble framework demonstrates asset "bubble"—the emergence of extra value beyond fundamental value—in a way that some valueless asset gains value through market trades. There, the assumed essential situation that allows bubble to emerge is lack of medium of exchange (in OLG or infinite horizon growth models), and bubble value emerges on some asset when it serves to circulate as a medium of exchange and thus gains corresponding market value. In this perspective, rational bubble argument is essentially not differentiated from arguing implications of the (partial) existence of money that serves as a medium of exchange, and also asset (nominal) prices are not well evaluated in terms of money (that is distinct from bubble asset). Concerning these issues, Hirano and Toda (2025) examines bubble emergence on inherently valuable assets instead of valueless assets, and thus intends to discriminate bubble from fiat money. Although the article successfully presents a new perspective on bubble phenomenon as an extension of rational bubble arguments, their framework still rests on real economy without medium of exchange and the discrimination concerning bubble and money as a medium of exchange is not well addressed. This article presents a general equilibrium model that explicitly incorporates money that serves as the medium of exchange, and successfully identifies asset bubble as a distinct phenomenon from money as well as demonstrates the emergence of asset bubble in a form asset inflation.

This article also provides a distinctive perspective concerning welfare effects of asset bubble. Since the essential role of bubble asset in the rational bubble framework is to serve as a medium of exchange, the primary welfare effect of bubble is naturally grasped as a positive effect in improving (dynamic) efficiency through enhancing trades. Although negative welfare effects of bubble are also argued within the rational bubble framework, those are brought as secondary effects.[6] In contrast, the essential source of asset bubble shown in this article is pecuniary externality of asset market. In fact, asset bubble is sustained by real gains (in terms of purchasing power) of asset market participants but the gains are sourced from loss of non-participants. Particularly, such transfers of purchasing power as a whole is shown to form a regressive transfer. In this perspective, the primary welfare effect of asset bubble is negative in exacerbating inequality while bringing no necessary consequence on efficiency. On this basis, this article also demonstrates negative effect of asset bubble in worsening efficiency, particularly in terms of decreased production. The critical assumption there is to assume constraint in managerial resource in operating firms such that putting larger resource to financial strategy tends to harm efficiency in production. In this setting, it is shown that profit maximizing firms earn larger profit through riding asset bubble while the resultant decrease in production efficiency has a macro consequence in decreasing production and consumption. In fact, concerning the episode of 1980s Japanese asset bubble, there is an anecdotal story that firms devoted too much effort in financial management in improving profits while neglecting the core

[6]See Barlevy (2018) for the welfare effects of asset bubbles discussed within the rational bubble framework. The author points out the gap between those theoretical implications and practitioners' view.

business.[7] While existent theoretical studies have little attention, this article provides a formal analysis on this issue. Taken together, this article finds asset bubble whose nature is inequality exacerbating while also demonstrates an efficiency harming mechanism through certain bubble riding behavior of firms.

Contributions: 3. Monetary policy

Thirdly, concerning the theory of monetary policy, New Keynesian framework[8] successfully demonstrates monetary non-neutrality under certain setting of nominal rigidity; however, since the framework largely abstracts monetary circulation and accordingly assumes away the role of banking system in supplying credit, it has severe limitation particularly in addressing potential relationship between credit expansion and the occurrence of asset bubble. In contrast, credit-driven development framework proposed in this article successfully demonstrates monetary non-neutrality by adopting a different setting of nominal rigidities, while it explicitly incorporates the role of banks in supplying credit; and on this basis, it successfully reveals the relationship between credit expansion and the occurrence of asset bubble and thus allows to discuss the role of monetary policy in this context.

Specifically, this article points out three limitations in the approach of New Keynesian framework: 1. ad-hoc assumption on nominal rigidity, 2. ignorance of credit channel of monetary policy and 3. lack of substantial mechanism regarding the central bank control on market interest rates. Concerning the first point, New Keynesian framework addresses nominal rigidity on good prices based on the assumption of some friction faced by firms in changing nominal prices.The essential implication of the setting is that removing such friction would improve efficiency. In contrast, this article argues certain stabilizing role of nominal price rigidities in the perspective of unstabilizing nature of monetary economy.[9] Concretely, this article introduces both upward and downward rigidities on nominal prices of primary good, assuming different origins for the two rigidities. The downward rigidity is assumed to be sustained through weak coalition of the producers in the primary sector; the producers collectively dispose or reduce produced goods when and only when excess production realizes. Meanwhile, the upward rigidity is assumed to be set by the government in the form of price cap. In the model economy, if prices of primary good (as well as prices of secondary good) would have faced no nominal rigidities, a shock of productivity improvement in the primary sector could kick in a swelling fluctuation in labor shift across sectors and eventually cause a disastrous breakdown of the economic structure—lost labor in the secondary sector. In this context, the assumed upward and

[7] This kind of stance in operating firms is dubbed "Zai-tech" in Japanese, which is an abbreviation of "financial management techniques".

[8] The core structure of the New Keynesian framework is presented, for example, in Woodford (2003) and Gali (2015).

[9] In a classical study, Wicksell (1898) discusses unstable nature of monetary economy in the frame of disequilibrium cumulative process. Although this perspective is almost neglected in the standard macroeconomic framework, the work of Iwai (1981) formulates the idea using a model of monopolistic competition. This article characterizes the unstable nature of monetary economy in a distinct framework.

downward nominal rigidities serve to stabilize economy through restraining such an excess fluctuation of labor shift. But the other side of the gain in stability is the loss in efficiency through restraining labor shift even when it is socially desirable in enhancing development. In this perspective, monetary expansion (to the secondary sector) has a role in promoting inter-sector labor shift and thus enhances development. The essential social role of monetary policy is grasped as to appropriately realize such monetary expansion through increasing credit supply.

Concerning the second point, there is a strand of empirical studies that supports the existence of credit channel of monetary policy.[10] In the theoretical perspective, Stiglitz and Greenwald (2003) provides macroeconomic analysis on equilibrium credit rationing in view of agency problem, and argues the role of monetary policy through increasing credit supply.Meanwhile, Bernanke and Gertler (1989) focuses on the role of collateral in constraining credit supply and argues the role of monetary policy in this context. Although these studies successfully provide mechanisms of insufficient credit supply and accordingly grasp the role of monetary policy in attenuating the inefficient loss in credit supply, their frameworks are largely silent on the mechanism of excess credit, thus would give little clue in understanding an increase in credit supply during asset inflation as well as the role of monetary policy in generating asset inflation. Key contribution of this article in examining these issues is to invent a framework of equilibrium excess credit. The emergence of asset inflation as well as the role of monetary policy in generating asset inflation are consistently analyzed in this perspective.

Concerning the third point, New Keynesian framework assumes that the central bank can directly control market lending rates. To demonstrate a central banking system that pursues monetary policy within a market system, a more suitable approach would be to present situations of indirect control on market lending rates. Also, as observed in developed economies, central banks are endowed power to control policy rates (such as interbank loan rates and deposit rates of corresponding central bank accounts), while have little power in directly setting banks' lending rates. On this recognition, this article addresses the feature of indirect control on market lending rates. This paper presents the feature of indirect control in the perspective of credit channel of monetary policy. To put it simply, a change in policy rates can affect credit supply, while credit supply serves to form market lending rates. To address the former relationship, the current framework rests on a cost view such that policy rates form the marginal cost in making lending. On this basis, the current framework further reveals an equilibrium relationship between market lending rates and credit supply. Specifically, under a setting of lag in production and the associated overlapping sales structure, expected sales of a currently borrowing firm is shown to be affected by future credit supply. Combined with the setting of competitive firms in which firms' repayments are aligned with their sales, future credit supply affects current lending rates. Thus, this article successfully provides a unified picture of the indirect control on market lending rates by a central bank which resorts to a market oriented method of policy rate control. In addition, this article discusses a subtle issue

[10]See Kashyap and Stein (2000).

on which rates could be used as policy rates. In particular, there is wide attention among practitioners on the role of the rates of deposit facilities in pursuing monetary policy in the midst of accumulated reserve holding that are commonly faced by recent developed economies. On these issues, this article suggests implementation of policy rates under two regimes; one is called interbank settlement regime while the other is required reserve regime. There, the methods of successful implementation of policy rates are discussed for the situations of both excess and insufficient reserve in a consistent manner.

The rest of the article is constructed as follows. Section 2 provides the overview of the framework and the main results, added with some remark on relevant economic indicators in conducting monetary policy. Section 3 introduces the partial framework that allows analyses given money/credit supply, and Section 4 provides the relevant basic results concerning excess credit and asset bubble. Section 6 introduces the full framework, and Section 7 provides the analyses on monetary policy. Section 8 articulates the implementation of monetary policy, while Section 9 provides an extension concerning the channel of monetary policy. Section 10 provides the concluding remarks. Appendix includes relevant results and proofs.

2 Overview of the framework and main results

2.1 Framework of monetary policy

The essential structure of the framework can be grasped with four relationships: two of them capture the essential functioning of credit-driven economy in a simplified form and the rest two describe the fundamental conditions in conducting monetary policy. More concretely, the former two relationships in combination indicates the existence of optimal credit supply, and the latter two conditions in combination form the equilibrium relationship between policy rates and credit supply. Formally, the essential functioning of credit-driven economy is captured with:[11] [12]

$$\textbf{(Efficiency property)} \quad \frac{dY_t}{dL_t} > 0 \text{ if } L_t < L_t^*, \text{ and } \frac{dY_t}{dL_t} \leq 0 \text{ otherwise.} \tag{1}$$

$$\textbf{(Inequality property)} \quad \frac{dWI_t}{dL_t} = 0 \text{ if } L_t < L_t^*, \text{ and } \frac{dWI_t}{dL_t} > 0 \text{ otherwise.} \tag{2}$$

[11] For relationship (1), decrease in output ($\frac{dY_t}{dL_t} < 0$) is brought by the emergence of asset inflation. It is shown that larger credit supply tends to cause larger scale of asset inflation, which then causes larger loss in production through bubble riding by firms. Note that asset inflation in general is shown to arise as continuous multiple equilibria including varying degree of asset inflation together with no asset inflation. On this basis, relationship (1) is demonstrated particularly under the equilibrium with maximum asset inflation whenever it exists.

[12] As the purpose of the current section is to illustrate the existence of optimal credit supply, relationship (2) does not refer to wealth inequality or consumption inequality that would be crucial in evaluating welfare, while these issues are elaborated in the analysis section.

where Y_t is the indicator of output, WI_t is the indicator of income inequality, L_t is the aggregate credit supply and L_t^* shows a certain level of credit supply. The efficiency property indicates that L_t^* is the maximum credit supply that successfully promotes production. Meanwhile, the inequality property indicates that L_t^* is the maximum credit supply that does not exacerbate income inequality. In combination, L_t^* is placed as the optimal credit supply in attaining efficiency in production while bringing no exacerbating effect on income inequality. Note that any dynamic issue in allocating resources for production is omitted in the underlying setting; thus, the efficiency in production is grasped on the basis of each period. Given this, the existence of optimal credit supply in each period (i.e., L_t^*) indicates no existence of essential trade-off between attaining efficiency in allocating resource for production and attaining income equality, while the functioning and limitation of monetary policy in attaining the optimal credit supply are the central issues under this framework.

The essential economic mechanism behind the efficiency property is that additional credit supply has a role in extracting labor from the satiation sector toward the non-satiation sector only when there exists redundant labor in the satiation sector. This role of credit supply is highlighted under sufficiently slow adjustment of labor distribution that is demonstrated under certain nominal price rigidities. The inequality property is brought thorough two competing factors. One factor is higher income elasticity of non-satiation goods compared with satiation goods, which tends to increase inter-sector income inequality against an increase in overall credit supply given fixed labor distribution (i.e., no inter-sector labor shift). On this basis, the competing factor is inter-sector labor shift toward non-satiation sector, which has an mitigating effect on the inter-sector income equality. In particular, the framework considers the situation in which the latter effect is sufficiently strong by assuming that labor shift is flexibly made under given credit supply (i.e., there is no exogenous hurdle or friction that hinders labor shift). In this context, against marginal increase in credit supply, the latter effect successfully suppresses the former effect as long as credit supply is below the threshold level (i.e., L_t^*), while the latter effect disappears once credit supply has attained the threshold level. Explicit analysis on these mechanisms is an issue of a later section.[13]

[13]Note that the functioning of credit driven economy demonstrated with relationships (1) and (2) are shown to hold innocuously under Cobb-Douglas utility function (Observation 1), but certain twist needs be added under more general preferences (e.g.., CES utility). For those cases, following relationships serve to identify the optimal credit supply, as elaborated in the analysis section.

$$\{Y_t \leq Y_t^{max}, WI_t > WI^{min}\} \text{ or } \{Y_t < Y_t^{max}, WI_t = WI^{min}\} \quad \text{if } L_t < L_t^*. \tag{3}$$

$$Y_t = Y^{max} \text{ and } WI_t = WI^{min} \quad \text{if } L_t = L_t^*. \tag{4}$$

$$\frac{dY_t}{dL_t} \leq 0 \text{ and } \frac{dWI_t}{dL_t} > 0 \quad \text{if } L_t > L_t^*. \tag{5}$$

Here, Y_t^{max} indicates the maximum possible output and WI^{min} indicates the minimum value for the index of income inequality (i.e., maximum equality). Note that the existence of optimal credit supply is ensured and the optimal level does not depend on the specific functional form of the utility function within our focus (Lemma 3).

Now, the fundamental conditions for monetary policy are formally stated as below:[14]

(Loan market clearing condition)

$$1 + i_t^L = \frac{L_t + M_t^e}{L_t}\Theta_{t,t} + \frac{E_t[L_{t+1}] + E_t[M_{t+1}^e]}{L_t}E_t[\Theta_{t,t+1}]. \tag{6}$$

(Bank incentive condition)

$$i_t^L = \xi\Big(\frac{L_t + M_t^e}{L_t}\Theta_{t,t}i_t^R + \frac{E_t[L_{t+1}] + E_t[M_{t+1}^e]}{L_t}E_t[\Theta_{t,t+1}i_{t+1}^R]\Big). \tag{7}$$

Concerning the notations, there are two types of interest rates: i_t^L denotes bank net lending rate (e.g., against firms) while i_t^R and i_{t+1}^R are the policy rates controlled by the central bank. The inter-temporal nature is characterized with the variables of $\Theta_{t,t}$ and $\Theta_{t,t+1}$, where $\Theta_{t,t}$ shows the share of the current borrowing firm's sale in the current period (i.e., period t) while $\Theta_{t,t+1}$ shows that in the following period (i.e., period $t+1$). M_t^e and M_{t+1}^e show the equity finance made in period t and $t+1$ respectively. The relevance of expectation is indicated with the expectation operator $E_t[.]$, for which rational expectation formation is assumed for the main analysis. Lastly, ξ shows the marginal reserve demand in credit supply.

The loan market clearing condition shown in Equation (6) is easier to interpret by multiplying both sides of the equation with L_t, which becomes as below.

$$L_t(1 + i_t^L) = \Theta_{t,t}(L_t + M_t^e) + E_t[\Theta_{t,t+1}(L_{t+1} + M_{t+1}^e)]. \tag{8}$$

The condition in the form of Equation (8) states that the loan repayment (Left side of the equation) equals the (expected) revenue of the currently borrowing firms (Right side of the equation). The revenue in the right side is then broken down to the current revenue ($\Theta_{t,t}(L_t + M_t^e)$) and the expected future revenue ($E_t[\Theta_{t,t+1}(L_{t+1} + M_{t+1}^e)]$). Note that the current framework adopts a simple setting of inter-temporal loan such that the repayment for a loan in the current period is made in the next period. Also, the framework assumes away any (meaningful) monetary saving by the households (through assuming short-sightedness) so that the current credit supply (i.e., L_t) added with equity finance (i.e., M_t^e) equals the aggregate sales of nonsatiation sector in the current period, and so does for the next period. On this basis, we adopt a setting of partial lag structure between production and sale, which forms $\Theta_{t,t}$ and $\Theta_{t,t+1}$. Concretely, we introduce $\theta \in (0,1)$ as an exogenous parameter that denotes the lag in production such that $1-\theta$ ratio of the current production is sold in the current period and the rest θ ratio is sold in the following period. By letting X_t denote the relevant aggregate production in period t, the overlapping nature brought by the lag in production is explicitly captured as below.

$$\Theta_{t,t} = \frac{(1-\theta)X_t}{Y_t}, \text{ where } Y_t = \theta X_{t-1} + (1-\theta)X_t. \tag{9}$$

$$E_t[\Theta_{t,t+1}] = \frac{\theta X_t}{E_t[Y_{t+1}]}, \text{ where } E_t[Y_{t+1}] = \theta X_t + (1-\theta)E_t[X_{t+1}]. \tag{10}$$

[14] Section 9 provides an extension in forming the relationships (6) and (7) by incorporating housing loans. Notably, while relationships (6) and (7) imply only indirect effects of policy rates in affecting credit supply, the extended equilibrium relationships also demonstrate direct effects of policy rates.

To see the relevance of the lag structure in forming bank lending rate, suppose that there is no lag between production and sale as $\theta = 0$, then we have $\Theta_{t,t} = 1$ and $\Theta_{t,t+1} = 0$; and thus, bank lending rate becomes always zero. When there is a lag (i.e., $\theta > 0$), bank lending rate in current period depends not only on the current credit supply but also on the (expected) credit supply in the following period. Thus, the lag structure has an essential role in addressing potential variation in bank lending rate.

The bank incentive condition shown in Equation (7) states that marginal revenue of lending (Left side of the equation) equals marginal cost (Right side of the equation). To note, this article posits the view that the bank incentive condition is not a natural property but somehow implemented by the central bank so as to control overall bank credit. As elaborated in Section 8, two typical ways of implementation—called settlement regime and reserve requirement regime—are examined. In both ways, the central bank lets a bank demand additional reserve (i.e, central bank money) when the bank increases lending. In generating reserve demand in such a manner, one way is to take advantage of interbank settlements, while the other way is to utilize reserve requirements. To implement the former interbank settlement regime, the average reserve holding level for each bank needs be set sufficiently small so that the reserve demand formed through interbank settlements causes shortage of own holding reserve and the reserve are supplied under the policy rate (i.e., interbank overnight lending rate). In addition, the deposit rate of central bank accounts (i.e., the interest rate of deposit facility) would need be set as zero when interbank obligations tend to be formed symmetrically across banks such that each bank would face both shortage of reserve and excess reserve in the course of the settlements.[15] Meanwhile, the required reserve regime can be implemented when banks already hold sufficiently large reserve in coping with the reserve demand formed through interbank settlements. It is consistently implemented by setting the rate for excess reserve as the policy rate.[16]

Under the perspective of interbank settlement regime, Equation (7) is interpreted such that the marginal cost is concerning the cost of financing reserve for settling the associated interbank obligations, and the cost is broken down to the current cost ($\xi\Theta_{t,t}i_t^R$) and the future cost ($\xi\frac{E_t[L_{t+1}]}{L_t}E_t[\Theta_{t,t+1}i_{t+1}^R]$). Concerning the formulation of the costs, the critical assumption is that when a bank increases lending, it owes larger interbank obligation, for which the marginal increase in bank lending is indicated by ξ.[17] Meanwhile, under the

[15]Note that if interbank obligations tend to be formed asymmetrically, the rate of deposit facility could be consistently set at some positive level. As an illustration, suppose that banks are divided in two groups such that one group of banks (Urban banks) actively supply funds to the emerging industry, while the other group of banks (Rural banks) hold large household deposits and not engage in active lending and thus largely serves as lenders in interbank loan markets. In such a situation, Urban banks would consistently face shortage of reserve and thus the positive rate of deposit facility would not much matter in their decisions in making lending.

[16]The reserve requirement regime would fit with the recent situation in developed economies that face large excess reserve (or could be referred to as excess liquidity) formed through quantitative easing conducted after the 2008 financial turmoil.

[17]Although implicit in the analysis, the nature of ξ could be understood with two base factors ξ^a and ξ^b such that $\xi = \xi^a\xi^b$. Here, ξ^a concerns the underlying distribution of bank accounts of firms and households. In an extreme case, $\xi^a = 0$ when only one bank exists, since there never occurs interbank

perspective of required reserve regime, ξ is interpreted as the ratio of required reserve. The marginal cost is formed as the opportunity cost in making lending.[18] [19]

The fundamental conditions introduced in Equations (6) and (7) are combined to derive the following relationship.

(Balancing equation of inter-temporal policy rates)

$$\frac{L_t + M_t^e}{L_t}\Theta_{t,t}\big(1 - \xi i_t^R\big) + \frac{E_t[L_{t+1}] + E_t[M_{t+1}^e]}{L_t}E_t[\Theta_{t,t+1}]\big(1 - \xi E_t[i_{t+1}^R]\big) = 1. \tag{11}$$

The problem of optimal monetary policy is formally to realize the optimal credit supply (i.e., $L_t = L_t^*$ for every $t = 1, 2, ...$) given the inter-period constraints shown in Equation (11) and the initial situation.

2.2 Results on excess credit and asset bubble

Results on excess credit and asset bubble are shown in Section 5 under the partial framework that deals with limited dynamics given money/credit supply, omitting the explicit analysis on the incentive of banks in supplying credit. On the basis of the results on optimal money supply (shown in Lemmas 2, 3, 4 and 5), it is shown (in Proposition 1) that sustained excess credit brings in asset accumulation dynamics such that (secondary) asset is accumulated to the households in the secondary sector, to which the sustained

settlements. In general, when firm or household payments are made across banks (i.e., payer's account and receivers' account belong to different banks), they form interbank obligation. In this context, ξ^a captures the ratio of the payments that form interbank obligation. Given this, ξ^b concerns the required reserve in settling those interbank obligation. In reality, interbank obligations are eventually settled in the corresponding interbank settlement system for which reserve acts as the settlement fund (e.g, Fedwire in U.S. and BOJ-NET in Japan). In the course of settlements, some obligations can be net with each other (settled without requiring transfer of reserve). In this context, ξ^b captures the ratio of reserve required for settling given interbank obligation. There is a strand of literature that conducts microstructure analyses on the formation of ξ^b examining the roles of institutional settings such as central clearing counterparty and liquidity saving mechanism. See, for example, Hayakawa (2025) on the relevant literature.

[18] Observe that additional lending by a bank increases deposits of the bank, and thus increases required reserve. Lost interest earning by the increase in required reserve forms the opportunity cost in making larger lending.

[19] Note that the required reserve is essentially calculated on the basis of outstanding deposit . In this perspective, Equation (7) is consistently interpreted by assuming that the lending to each firm is repaid both in the current period and the following period, and the current sales of the firms that have produced in the previous period flow out from the current lending banks. Furthermore, assume that the calculation of required reserves in each period is made after the sales have determined and before the repayments of the loan are made. Then, the increase in the outstanding deposit in the current period that are formed through current lending (i.e., L_t) equals the current sales of the borrowing firms (i.e., $\Theta_{t,t}L_t$), as the remaining amount (i.e., $(1-\Theta_{t,t})L_t$) is assumed to flow out of the corresponding bank. Also, the increase in the outstanding deposit in the following period equals the sales of the borrowing firms in the following period (i.e., $E_t[\Theta_{t,t+1}L_{t+1}]$) since the previous period sales (i.e., $\Theta_{t,t}L_t$) have already been repaid at the end of the current period.

excess credit is to give relatively higher purchasing power. It is further shown that sustained excess credit that is sufficiently severe (i.e., excess credit is above certain threshold) will eventually cause extreme wealth inequality such that all (secondary) assets are accumulated to the households in the secondary sector. It is then shown (in Proposition 2) that such extreme wealth inequality allows any speculative trade to emerge, and further shown (in Proposition 3) that those speculative trades must form asset bubble. In addition, bubble riding by firms is also shown to occur when the sustained excess credit is even severer than that required for any asset bubble to occur.

Concerning the effect of asset bubble on good prices, it is shown (in Proposition 4) that the emergence of asset bubble has an effect of raising secondary good price, while the subsequently sustained asset bubble of the same scale will not have any additional effect on secondary good price (nor primary good price). This result underlies the result on the formation of asset inflation (Proposition 9). Furthermore, the effect of the emergence of bubble on raising the price of secondary good is shown to be decomposed into two effects: direct and indirect effects. The direct effect is brought by "bubble money" that pours into asset market and then used to purchase secondary good, while the indirect effect is brought by the additional money flow to purchase secondary good in intending to maintain consumption against the inflow of "bubble money." This result demonstrates leveraging effect of asset bubble in raising good prices.

Concerning welfare effects of asset bubble, it is shown (in Proposition 5) that asset bubble has efficiency loss in terms of production only when bubble riding arises. Given the result, fundamental welfare effects of asset bubble is shown (in Proposition 6) to arise on the domain of inequality. Particularly, it is shown that "riches" (households in secondary sector) who have accumulated secondary asset to the maximum level are never affected by the occurrence of asset bubble in terms of consumption nor holding assets. Meanwhile, asset bubble tends to realize transfer of purchasing power between "poors" (households in primary sector) such that "poors" who do not participate to asset market lose while "poors" who participate to asset market gain. This demonstrates that the primary welfare effect of asset bubble is to bring regressive transfer, thus inequality exacerbating.

2.3 Results on monetary policy

Results on monetary policy are shown in Section 7 under the fully dynamic framework. For the purpose of this article, the fundamental issue is whether monetary policy could keep realizing optimal credit supply in the course of technological improvements (i.e., productivity improvements with respect to both primary and secondary sectors). Given generical impossibility of optimal monetary policy (shown in Lemma 8), subsequent analyses focus on the implications of "efficiency-first" monetary policy that primarily pursues efficiency in labor allocation while income equality is pursued as the secondary objective.

Particularly to examine the effect of efficiency-first monetary policy in bringing or sustaining excess credit, specific types of productivity improvement sequence are analyzed. Firstly, against "one-time growth," which refers to (expected) once-and-for-all productiv-

ity improvement in secondary sector, it is shown (in Proposition 7) that efficiency-first monetary policy brings temporal excess credit at the time of the productivity improvement, but it is shown that the excess credit is vanished immediately after the period and never arises. Thus, the result indicates that one-time growth is not a source of persistent excess credit. Meanwhile, against "one-time development," which refers to (expected) once-and-for-all productivity improvement in primary sector, it is shown (in Proposition 8) that efficiency-first monetary policy always brings persistent excess credit that is arisen immediately after the improvement period. On this basis, it is further shown (in Proposition 9) that asset bubble in a form of asset inflation occurs under the arisen excess credit when the productivity improvement is sufficiently large and accordingly excess credit is sufficiently large.

The analysis proceeds to examine whether the arisen persistent excess credit could be affected by subsequent "growth" or "development." Firstly, it is shown (in Proposition 10) that subsequent sequence of growth (referred to as subsequent "sequential growth") has no effect on the level of persistent excess credit. In contrast, it is shown (in Proposition 11) that under the situations of subsequent "sequential development," the initially arisen excess credit is possibly attenuated by a steady development. These results indicate that a typical situation that brings persistent excess credit is the abrupt occurrence of large scale development combined with subsequent stagnation in development. Furthermore, it is shown (in Proposition 12) in the context of simultaneous one-time growth and development, the effect of the initial development in arising excess credit can be amplified or attenuated by the occurrence of growth in the same period. Particularly, sufficiently small simultaneous growth serves to attenuate the excess credit, while sufficiently large simultaneous growth serves to amplify the excess credit.

2.4 Remark: relevant economic indicators for monetary policy

Aside from the formal results shown in the analysis section, the framework has some practical implications concerning the relevance of economic indicators in conducting monetary policy. In addition to the indicators for the level of credit supply, we can come up with two primary relevant indicators. One is the indicator of the gap between labor demand and supply. The indicator would tell whether the current credit supply is insufficient or not (but not tell whether it is already excessive or not), based on the mechanism outlined in the previous part in this section. Another primary relevant indicator is the indicator of the inequality in inter-sector monetary distribution. Such indicator would tell whether the current credit supply is already excessive or not (but not tell whether it is insufficient or not). In reality, the latter indicator would be formed by inter-sector wage inequality when the wage is sufficiently elastic on the firm's sale, while it would be formed by firm's profits or shareholders' earning when they are more elastic to the sale. When the primary relevant indicators are not perfectly available in estimating the optimal credit supply, some other indicators would need be combined. In this perspective, the secondary relevant indicator would be inflation trend, which could tell when the credit supply has

shifted from an insufficient level to an excessive level. In fact, marginal inflation in credit supply would get larger once credit supply has shifted from an insufficient level to an excessive level as long as the relevant productivity (including quality) is appropriately adjusted.

In the perspective of the current framework, one of the thirdly relevant indicators which would be almost useless, is the (trend of) real interest rates. Observe that real interest rate is not a directly relevant factor in determining the economic activities in the current framework as it excludes the motive of consumption smoothing (through assuming away any meaningful monetary saving).In this perspective, estimating the real interest rate would have meaning only in indirectly estimating the (trend of) inflation rate. Relatedly, the concept of neutral real interest rate has no meaningful role in the current framework.

3 Partial framework

We consider a closed monetary economy in which one currency is used as the means of payments. There are three types of subjects; households, firms and banks. Periods are discrete and infinite as $t = 0, 1, 2, ...$ There are two types of goods and two types of (land) assets. The types of goods are referred to as good 1 and 2, or interchangeably as primary and secondary goods. In the same manner, the types of assets are referred to as asset 1 and 2, or primary and secondary assets. Goods depreciate at the end of the period of production, while assets cannot be produced and never depreciate. We let A_0^1 and A_0^2 denote the initial aggregate endowments of primary and secondary assets, respectively.

Primary and secondary goods are produced in different sectors and let sector j refer to production sector of good j. In each sector, there is a pool of infinite firms with the homogeneous production technology of the corresponding sector. Let J^j denote the set of firms in sector $j = 1, 2$, and let $J = J^1 \cup J^2$. Production technology of primary sector is linear to labor input as $F_t^1(n_t^1) = \alpha_t^1 n_t^1$, where α_t^1 denotes productivity and n_t^1 denotes labor input in period t. For primary sector, we consider no lag in production so that all produced goods are ready for sale in the period of labor input. Production of secondary good requires labor input as well as utilization of secondary asset in the form of Leontief type as $F_t^2(n_t^2, a_t^2) = \alpha_t^2 \min(n_t^2, \kappa a_t^2)$, where α_t^2 denotes productivity, n_t^2 and a_t^2 respectively denote the inputs of labor and secondary asset in period t, while $\kappa > 0$ is the parameter of asset intensity in production. We let $0 < \theta < 1$ indicate the lag for production of secondary good such that $1 - \theta$ ratio of production is made and ready for sale in the current period, while the rest θ ratio of production is made and ready for sale only in the next period.[20]

There is a continuous unit mass population of households who live infinitely. Let $I = [0, 1]$ denote the set of households. Each household $i \in I$ is endowed one unit of

[20]Concerning the lag structure, note that we effectively allow separation of (labor and asset) inputs and production so that production is made in the following period without any input in the period. This is rather a technical setting to discuss positive (lending) interest rate in the context of banks' credit supply.

labor in each period, which is indivisible across sectors. Each household is assumed to be short-sighted such that utility depends only on the current period consumption and asset holding. We consider a homogeneous utility function which expresses hierarchical preferences while encompassing homothetic feature across consumption of goods and holding of assets. Specifically, let x_t^j and a_t^l denote consumption of good $j = 1, 2$ and holding of asset $l = 1, 2$ of a household, then utility of the household is expressed with two functions f and c as below.[21][22]

$$u((x_t^j)_{j=1,2}, (a_t^l)_{l=1,2}) = f(x, a) - c(\triangle x_t^1, \triangle x_t^2, \triangle a_t^1), \tag{12}$$

where x and a respectively denote consolidated consumption and asset holding as below.

$$x : = \begin{cases} \min\{x_t^1, \bar{x}^1\} & \text{if } x_t^1 < \bar{x}^1 \text{ or } a_t^1 < \bar{a}^1 \\ \bar{x}^1 + x_t^2 & \text{if } x_t^1 \geq \bar{x}^1,\ a_t^1 \geq \bar{a}^1 \text{ and } x_t^2 < \bar{x}^2 \\ \bar{x}^1 + \bar{x}^2 & \text{otherwise.} \end{cases}$$

$$a : = \begin{cases} \min\{a_t^1, \bar{a}^1\} & \text{if } a_t^1 < \bar{a}^1 \text{ or } x_t^1 < \bar{x}^1 \\ \bar{a}^1 + a_t^2 & \text{otherwise.} \end{cases}$$

Here, $\bar{x}^j$ and $\bar{a}^1$ respectively show the satiation levels of consumption of good j and the holding of primary asset. We let $f : \mathbb{R}^2_{0+} \to \mathbb{R}$ be strictly increasing, twice differentiable, quasi-concave and homothetic (i.e., Marginal rate of substitution is homogeneous of degree zero).[23] Meanwhile, $c : \mathbb{R}^3_{0+} \to \mathbb{R}_{0+}$ represents a cost function against excess consumption $\triangle x_t^j := \max(x_t^j - \bar{x}^j, 0)$ for good $j = 1, 2$ and excess holding $\triangle a_t^1 := max(a_t^1 - \bar{a}^1, 0)$ of asset 1. We let the function be strictly increasing and $c(\triangle x_t^1, \triangle x_t^2, \triangle a_t^1) = 0$ for $(\triangle x_t^1, \triangle x_t^2, \triangle a_t^1) = (0, 0, 0)$.

Throughout the analysis, we make following assumptions.

[21] In the literature of development, a typical interpretation of primary and secondary goods is agricultural and manufactured products respectively, which would be useful in framing rather initial phase of economic developments. The setting could also capture more developed economies where manufactured products partially attain widespread use. Concretely, manufactured products such as automobile and personal computers could be framed as components of primary good in analyzing the situation of their widespread use while some other state-of-the-art products such as AI relevant products could be framed as secondary good.

[22] The hierarchical preference structure concerning primary and secondary assets intends to distinguish households' demand for land as primary (i.e., residential) houses from that as secondary houses. Here, note that the setting separately treats the supplies of primary and secondary lands (so that they are traded in different markets). One consistent interpretation of the setting would be such that there exists some regulation that protects certain section of land solely for the usage of residence (hence modeled as primary good).

[23] All the properties of f are satisfied by, for example, CES functions. Note that the properties of constant return to scale and constant elasticity of substitution are not necessarily required for function f.

Assumption 1 (Productivity improvement process)**.**
i) (Productivity improvements) $\alpha_{t-1}^j \leq \alpha_t^j$, for $j = 1, 2$ and every $t = 1, 2, ...$
ii) (Maximum productivity) There exists $\bar{t} \geq 0$ such that $\alpha_t^j = \bar{\alpha}^j$ for every $t \geq \bar{t}$.

Assumption 2 (Productivity and asset availability)**.**
i) (Universal satiation on primary goods) $\frac{\bar{x}^1}{\alpha_0^1} < 1$.
ii) (No universal satiation on secondary goods) $1 - \frac{\bar{x}^1}{\bar{\alpha}^1} < \frac{\bar{x}^2}{\bar{\alpha}^2}$.
iii) (Possibility of partial satiation of secondary goods) $\frac{\bar{x}^2}{\alpha_0^2} < 1$.
iv) (Universal satiation on primary assets) $A_0^1 = \bar{a}^1$ and
v) (Sufficient availability of secondary asset) $A_0^2 > \frac{1}{\kappa}\bar{\alpha}^j$.

Given the assumed productivities, observe that the efficient labor distribution $\{n_t^{1e}, n_t^{2e} = 1 - n_t^{1e}\}$ for period t becomes as follows:

$$n_t^{1e} = \frac{\bar{x}^1}{\alpha_t^1}, \quad n_t^{2e} = \frac{\alpha_t^1 - \bar{x}^1}{\alpha_t^1}. \tag{13}$$

Part i) of Assumption 2 sets the stage for examining sufficiently developed economy. Part ii) is to assume away uninteresting situations where all produced goods are no more scarce. Part iii) is a key assumption in demonstrating situations of partial satiation (i.e., situations where only some fraction of households consume secondary good to the satiation level), wherein asset bubble is possible to arise. Part iv) allows us to focus on trades of secondary asset while fixing the supply of primary assets at the efficient level. Part v) ensures that the aggregate volume of secondary asset is sufficient for producing secondary good. We also assume binding labor demand in the secondary sector, which ensures full employment in the analysis. For the statement, let $\bar{n}^{2e} := \frac{\bar{\alpha}^1 - \bar{x}^1}{\bar{\alpha}^1}$.

Assumption 3 (Binding labor demand in the secondary sector)**.**
$MRS_{ax} \leq \frac{1}{\kappa}$ on $(a, x) = (a^e, \bar{x}^e)$, where $a^e := \bar{a}^1 + A_0^2$ and $\bar{x}^e := \bar{x}^1 + \bar{\alpha}^2\bar{n}^{2e}$.

Timeline in each period

The inherent dynamics of each period in the full framework is analyzed with three phases. The first phase is for bank lending to firms. The second phase is for labor markets. The third phase is for good and asset markets.

The reminder of this section introduces a partial framework that is composed of good and asset markets (second phase; Section 3.1) together with labor market (third phase; Section 3.2) given bank lending to firms (thus, omitting the first phase). Then, under the partial framework, Section 4 shows preliminary analysis on optimal money supply and Section 5 summarizes the results on excess money supply and asset bubble. The full framework is presented in Section 6, introducing banks lending (first phase; Section 6.1). The results on monetary policy under full framework are presented in Section 7.

3.1 Good and asset markets

For the phase of good and asset markets, we let asset markets open firstly, then good markets open. Thus, firms can use current holding assets for the current production of goods.

3.1.1 Good trades

Now, we first see the functioning of goods markets, then proceed to see the asset markets. Let p_t^j denote price of good $j = 1, 2$ for period $t = 0, 1, 2, ..,$ where p_0^j is given exogenously. Concerning the prices of primary good, certain price lower and upper bounds are effectively imposed. First, concerning the price lower bound, we assume firms in the primary sector collectively dispose goods in the face of excessive production. Formally, let Y_t^1 denote the supply of the primary good while $X_t^1 = \alpha^1 n_t^1$ denote the produced amount. Then, the collective disposal of the primary good together with the price lower bound are expressed with parameter $0 < \mu \leq 1$ as follows:

$$Y_t^1 = \begin{cases} \bar{x}^1 & \text{if } X_t^1 \geq \bar{x}^1 \text{ and } \frac{\int_{i \in I} m_t^{i,1}}{\bar{x}^1} \leq \mu p_{t-1}^1 \\ X_t^1 & \text{otherwise,} \end{cases} \tag{14}$$

where $m_t^{i,j}$ denotes the expenditure for good j by household $i \in I$ in period t. We will see that the above formulation indicates that any excess production is disposed while the clearing price in period t is realized as μp_{t-1}^1, which hence serves as the price lower bound. The formulation is understood such that the producing firms in the primary sector form a coalition to engage in collective disposal so as to sustain the price of primary good.[24] Notably, the formulation indicates a rather weak coalition—compared with monopoly or oligopoly—in the sense that the coalition controls the aggregate supply only in the case of excess production. Also, in the perspective that a coalition reacts to excess production, μ indicates responsiveness in engaging in collective disposal.

Next, concerning the price upper bound of the primary good, we assume a government conditional intervention that effectively brings certain price cap. Specifically, the government cuts off an excessive purchasing power with respect to the purchases of primary goods. For given $m_t^{i,1}$, let $\hat{m}_t^{i,1}$ denote the purchasing power after the intervention, then the intervention is formally expressed as follows:

$$\hat{m}_t^{i,1} = \begin{cases} \min(m_t^{i,1}, w_t^{min}) & \text{if } X_t^1 < \bar{x}^1 \\ m_t^{i,1} & \text{otherwise,} \end{cases} \tag{15}$$

where $w_t^{min} := min(w_t^1, w_t^2)$. The setting indicates that in case of an insufficient production, the maximum purchasing power of an expenditure is constrained to the level of the

[24] A collective disposal against an excessively produced goods is widely observed, for example under an agricultural cooperative.

minimum wage across industries. This hinders households with larger purchasing power from squeezing the others, that is, in purchasing primary good, effectively any household cannot spend more than the minimum wage. In this perspective, the intention of the assumed government intervention is understood as to ensure the minimum standard of living.[25]

The clearing condition for primary good in period t is given as follows:

$$p_t^1 = \begin{cases} p_{t-1}^1 & \text{if } X_t^1 = \bar{x}^1 \\ \max\left(\mu p_{t-1}^1, \frac{\int_{i\in I} \hat{m}_t^{i,1}}{Y_t^1}\right) & \text{otherwise.} \end{cases} \tag{16}$$

Observe that the first line of the clearing condition demonstrates price stickiness of the primary good. In analytical perspective, it prevents potentially multiple clearing prices of primary good.[26]

For clearing of secondary good, let $X_t^2 = F_t^2(n_t^2, \kappa a_t^2)$ denote the aggregate production realized by the inputs (n_t^2, a_t^2), while Y_t^2 denote the aggregate supply of secondary good in period t. The lag structure in production indicates that $Y_t^2 = (1-\theta)X_t^2 + \theta X_{t-1}^2$. Then, the clearing condition for secondary good in period t is given as follows.

$$p_t^2 = \frac{\int_{i\in I} m_t^{i,2}}{Y_t^2}. \tag{17}$$

Stabilizing role of lower and upper bounds of primary good

As shown in Appendix A.1, the existence of price lower and upper bounds serve to stabilize our monetary economy, avoiding possible process of instability and destruction in which households flood to move across the two sectors and finally settle in the primary sector while the secondary sector being abandoned. In this perspective, the price bounds are not mere friction in the functioning of the markets, but placed as built-in mechanisms to ensure the sustainability of our monetary economy.

[25] The government intervention that sets certain maximum purchasing power effectively sets price caps. In reality, there are many cases in setting price caps on food or essentials against inflation.

[26] Note that the current formulation is understood as a convenient way to address price stickiness within our limited focus on the situations where households hold enough budget. In fact, the condition is not self-consistent when households hold sufficiently small budget such that p_{t-1}^1 cannot be realized. In covering such cases, we could formulate certain sequential clearing process instead of simultaneous clearing. More concretely, let $k = 1, 2, ...$ denote round in each period and we consider clearing of goods and assets in each round. The key setting in realizing price stickiness of primary good is to let households form adoptive expectation on the primary good while form rational expectation on the other prices. Here, the adoptive expectation can be formulated with $E_k[p_{t,k}^1] = p_{t,k-1}^1$ where $p_{t,k-1}^1 = p_{t-1}^1$. The clearing process can be shown to terminate given the price lower and upper bounds of the primary good.

3.1.2 Asset trades

Assets are not allowed to be sold in short.[27] Assets can be traded by households and firms, and we refer to each subject $i \in I \cup J$ as a trader in the phase of asset trades. We examine two modes of trades called "fundamental mode" and "speculative mode." In fundamental mode, each trader can simply buy or sell asset. Meanwhile, in speculative mode, each trader cannot simply buy or sell asset, instead a trader must buys (sells) back when the trader has initially sold (bought). Thus, for any trade made in speculative mode, there is only temporary transfer of asset and no net transfer in the end. Only fundamental mode is available for primary asset, while both fundamental and speculative modes are available for trades of secondary asset. For the sake of analysis, we examine whole clearing process of assets with two sequential stages; the first stage is for fundamental mode of trades for both primary and secondary assets, while the second stage is for speculative mode of trades for secondary asset as well as trades of fundamental mode for primary asset. Given this sequence, the second stage opens only when there is no finite value clearing price of secondary asset in the first stage. Note that alongside the sequential stage structure of asset clearing process, we will examine decision making of consumers and firms under each of the first and second stage.

Clearing of fundamental trades

In each period $t = 1, 2, ...$, market clearing for fundamental asset trades is attained for given expenditure of asset purchase $\{m_t^{i,a_l;f} > 0\}_{i \in I \cup J}$ and asset selling $\{\triangle a_t^{i,l;f} > 0\}_{i \in I \cup J}$ for each asset $l = 1, 2$. Accordingly, we say trader $i \in I \cup J$ as a buyer when $m_t^{i,a_l;f} > 0$ and as a seller when $\triangle a_t^{i,l;f} > 0$. Let $S_t^{l;f}$ and $B_t^{l;f}$ respectively denote the set of sellers and buyers in period t engaging in the first mode asset trades of asset $l = 1, 2$. Let $M_t^{a_l;f} := \int_{i \in B_t^{l;f}} m_t^{i,a_l;f}$ and $\triangle A_t^{l;f} := \int_{i \in S_t^{l;f}} \triangle a_t^{i,l;f}$. Then, market clearing is formally stated as follows: for asset $l = 1, 2$,

$$p_t^{a,l} = \frac{M_t^{a_l;f}}{\triangle A_t^{l;f}}, \text{ while} \tag{18}$$

$$p_t^{a,l} \triangle a_t^{i,l;f} = m_t^{i,a_l;f} \text{ for each trader } i \in I \cup J. \tag{19}$$

Note that Equation (19) specifies the obtained assets for each buyer, while specifies the obtained money for each seller.

Clearing of speculative trades

In making speculative trades, observe that traders are essentially interested in the difference of the relevant pair of prices. Let $\triangle p_t^{a,2s}$ denote the difference of prices for speculative asset trades, where the positive (negative) sign shows an increase (decrease) in prices for the corresponding two consecutive trades. We treat $\triangle p_t^{a,2s}$ as the clearing price for the corresponding speculative trades.[28] Observe that when speculative trades are made at non-zero price gap $\triangle p_t^{a,2;s} \neq 0$, there necessarily exists net loser (i.e., trader

[27] Technically, this eliminates sustainable asset deflation.

[28] When making speculative trades in the form of buy-then-sell, if finance for the initial buy would be

who faces capital loss). Concretely, for the case of $\triangle p_t^{a,2;s} > 0$, traders who make sell-then-buy trades are net losers. It is immediate that net winners (i.e., traders who face capital gain) have infinite demand in making the same direction of trades. Facing with the potential infinite demand, we let traders submit holding money or secondary assets as margin with which monetary expenditure in making initial purchase is financed. Let v_t^i denote the monetary value of the margin submitted by each trader $i \in \{I \cup J\}$ and let $V_t := \int_{i \in I} v_t^i$ denotes the aggregate value of submitted margin. Then, market clearing is given such that the total gain is distributed alongside the relative size of each value of the margin. Meanwhile, for the case of $\triangle p_t^{a,2;s} < 0$, profitable trades are now in the form of sell-then-buy. Here, the aggregate gain is distributed alongside the volume of assets which are initially provided.

Formally, market clearing for speculative trades is attained for given submitted margins $\{v^i > 0\}_{i \in I \cup J}$ and initial asset selling $\{\triangle a_t^{i,2;s} < 0\}_{i \in I \cup J}$. Let S_t^s and B_t^s respectively denote the set of initial sellers and initial buyers in period t engaging in the second mode asset trades. Then, let $\triangle A_t^{2;s} := \int_{i \in S_t^s} \triangle a_t^{i,l;f}$. Capital gain or loss for trader $i \in I \cup J$ becomes as follows:[29]

$$m_t^{i,a_2;s} = \begin{cases} -\triangle p_t^{a_2;s} \triangle a_t^{i,2;s} & \text{for } i \in S_t^s \text{ (initial seller)} \\ \triangle p_t^{a_2;s} \triangle A_t^{2;s} \left(\frac{v_t^i}{V_t}\right) & \text{for } i \in B_t^s \text{ (initial buyer)} \end{cases}, \tag{20}$$

Here, note that $m_t^{i,a_2;s} < 0$ indicates capital gain (negative expenditure) while the reverse sign indicates capital loss.

Given $\triangle p_t^{a,2;s}$, we let clearing price of secondary asset be formed as follows:

$$p_t^{a_2} := p_{t-1}^{a_2} + \triangle p_t^{a_2;s}. \tag{21}$$

Note that this formulation addresses price stickiness in a simple manner. It is interpreted that the price formed in the previous period (i.e., $p_{t-1}^{a_2}$) works as the "current price" in engaging in any speculative trade. Also, observe that if there occurs no speculative trade, the current price forms the price in the period (i.e., $p_t^{a_2}$), though there has not been found

provided based on margin finance so that holding of certain own money is necessary for making such speculative trades, the information of the absolute price level for the initial buy would matter for decision making. Regarding this point, we consider certain form of margin finance but allow arbitrary finance so that the information of absolute price levels does not matter in making relevant decisions.

[29] We assume that banks can provide arbitrary short-term finance for speculative trades. Actually, this is not an innocuous assumption, as we explicitly examine interbank settlements and the role of reserve there. In fact, when a bank makes short-term finance, if it is expected that the provided fund is actually used for purchase and generates interbank obligation for the financing bank, the amount of holding reserve could bind the provision of short-term finance. In this context, we amount to presume that the pair of trades for each speculative trade is made with sufficiently short lag (e.g., within a day) against settlement deadline; thus it does not cause any associated actual monetary payment and holding reserve is never binding.

any clearing price of secondary asset with fundamental mode of trades.[30]

In the analysis, we allow potentially limited access for speculative trades, such that certain ratio of households in the primary sector cannot engage in speculative trades.[31] Formally, we introduce an exogenous parameter $0 \leq \lambda \leq 1$ and let $\lambda = |I_t^{1;1}|/|I_t^1|$ for every $t = 1, 2...$, where $I_t^{1;z}$ with $z = 0, 1$ respectively denotes the households who are not entitled to engage in speculative trades and those who are. Let $I_t^{1;z}$ for $z = 0, 1$ be realized at the end of the stage for labor shift in each period.

3.1.3 Firm decision: fundamental trades

For fundamental mode of asset trades, firms in secondary sector engage in trades of secondary asset when they face shortage or redundancy for their production. For clarity of the statement, we say firm $j \in J^2$ faces shortage of asset for production when $\frac{1}{\kappa} n_t^{2,j} > a_{t-1}^{2,j}$, while faces redundancy when $\frac{1}{\kappa} n_t^{2,j} < a_{t-1}^{2,j}$, where $n_t^{2,j}$ and $a_t^{2,j}$ respectively denote the realized inputs of labor and secondary asset of firm j in period t.[32] In examining asset markets, we let the amount of shortage forms demand while the amount of redundancy forms supply. Furthermore, we let each of the formed supply and demand be inelastic to price. Formally, let $\triangle a_t^{j,2;f}$ denote the demand (the negative sign shows supply) of secondary asset by firm $j \in J^2$. Then, we have $\triangle a_t^{j,2;f} = \frac{1}{\kappa} n_t^{2,j} - a_{t-1}^{2,j}$.

Note that here we examine the formation of associated demand and supply separately from the firm's profit maximization. This is for our analytical purpose in excluding any efficiency loss associated with shortage or redundancy of asset for production, while focusing on potential efficiency/welfare loss associated with asset bubble. Concerning the effect on money/credit supply, we let banks provide arbitrary finance through credit creation for purchases of assets to fill shortage. Meanwhile, for the sale of redundant asset, we let the corresponding amount of credit is effectively demolished.[33]

[30] As shown in the analysis, no existence of clearing price of secondary asset in the first stage typically occurs under severe inequality in terms of both income and wealth (i.e., holding of secondary asset). There, excess demand for secondary asset is not resolved under any finite price. Technically, one way of consistent treatment of clearing price would be to adopt infinite value clearing price (thus any excess demand must be zero). Instead, this article posits stickiness of price given no possibility of trade.

[31] The setting of limited access to speculative trades aim to capture the existence of diffusion process of information such that households in the primary sector are only gradually aware of gain through speculative trades during a bubble. Note that we assume limited access only for households in the primary sector. The reason of allowing every household in the secondary sector to access speculative trades is that any limited access on those households has no effect in welfare perspective given the existence of a bubble, while the analysis on the existence of a bubble becomes somewhat cumbersome.

[32] In aggregate term, there occurs no redundancy of asset for production since we focus on sequences of productivity improvements; still, we allow individual firms to face such redundancy.

[33] In positing a consistent specification that explains the formation of demand, we let any purchase of secondary asset that fills shortage of asset is always financed by banks in the form of equity finance. Assuming that equity finance does not incur any cost for the firm (i.e., no dividend payment), the purchase does not affect the firm's profit. Concerning the supply, we let any revenue from selling of secondary asset that reduces redundant asset is always used for equity share cancellation (through buyback). Thus,

3.1.4 Firm decision: speculative trades

For speculative mode of asset trades, firms in the secondary sector which hold secondary asset can participate. Here, firms' engagement in speculative trades is framed as bubble riding behavior. For analytical purpose in demonstrating potential negative welfare effects of bubble riding behavior, we assume limited managerial resource such that engaging in speculative trades decrease the firm's productivity by $0 < \delta < 1$ (i.e., the firm's productivity becomes $(1-\delta)\alpha_t^2$).[34] We consider profit maximizing firms such that each firm $j \in J$ maximizes own profit given $\triangle p_t^{a,2;s}$ and the relevant good prices to choose margin level $v_t^j > 0$ when making buy-then-sell trade while choose initial asset sale $\triangle a_t^{j,2;s} < 0$ when making sell-then-buy trade. For the sake of notational simplicity, let $F_t^j := F^2(n_t^{2,j}, a_t^{2,j})$ and $p_{t,t+1}^2 := p_t^2(1-\theta) + p_{t+1}^2\theta$. Now, profit maximizing behavior of firm $j \in J$ becomes as follows: For given $p_{t,t+1}^2$, $\triangle p_t^{a,2;s}$, $\triangle A_t^{2;s}$ and $V_t \geq p_{t,1}^{a,2} a_{t-1}^{j,2}$,

$$(v_t^j, \triangle a_t^{j,2;s}) = \begin{cases} (p_{t-1}^{a,2} a_{t-1}^{j,2}, 0) & \text{if } \triangle p_t^{a,2;s} > 0 \text{ and } -\triangle p_t^{a_2;s} \triangle A_t^{2;s} \left(\frac{v_t^i}{V_t}\right) > p_{t,t+1}^2 \delta F_t^j. \\ (0, -a_t^{j,2}) & \text{if } \triangle p_t^{a,2;s} < 0 \text{ and } \triangle p_t^{a_2;s} \triangle a_t^{i,2;s} > p_{t,t+1}^2 \delta F_t^j. \\ (0,0) & \text{otherwise.} \end{cases} \tag{22}$$

Note that in the first case firm i makes buy-then-sell speculative trades while makes sell-then-buy speculative trades in the second case.

3.1.5 Consumer decision in the stage of asset fundamental trades

Households are assumed to be price takers in simultaneously making decisions on good and asset trades. We first examine household decisions under the first stage where all assets are traded in fundamental mode. Let $P_t := (p_t^1, p_t^2, p_t^{a,1}, p_t^{a,2}, \triangle p_t^{a_2;s})$ show relevant prices which households take given in making decision. Then, let $D_t^{i;f} := (\{m_t^{i,j}\}_{j=1,2}, \{m_t^{i,a_l;f}, \triangle a_t^{i,l;f}\}_{l=1,2})$ denote the decision on good trades and fundamental asset trades by household $i \in I$. Thus, household $i \in I$ maximizes utility specified in Equation (12) given relevant prices P_t to choose $D_t^{i;f}$ while satisfying the budget constraint shown below.

$$\Sigma_{j=1}^2 m_{t,k}^{i,j} + \Sigma_{l=1}^2 m_{t,k}^{i,a_l;f} + s_t^i = w_t^i + s_{t-1}^i, \tag{23}$$

where we abuse notation in letting $w_t^i \in \{w_t^1, w_t^2\}$ and $s_t^i \geq 0$ respectively denote the wage and saving of household $i \in I$ in period t.

---

the associated revenue does not affect the firm's profit (while the effect is understood as balance sheet reduction).

[34] It is discussed there emerge firms engaging in short-term asset trades in bubble era. The behavior is referred to as Zai-teku in Japanese. (Literal meaning is "Financing technology")

3.1.6 Consumer decision in the stage of speculative trades

For the second stage with speculative asset trades for secondary asset, let $D_t^{i;s} := (\{m_t^{i,j}\}_{j=1,2}, \{m_t^{i,a_1;f}, \triangle a_t^{i,1;f}\}, \{v_t^i, \triangle a_t^{i,2;s}\}$ denote the household decision. Thus, household $i \in I$ maximizes utility specified in Equation (12) given prices P_t as well as $\triangle A_t^{2;s}$ and V_t to choose $D_t^{i;s}$ while satisfying budget constraint shown below.

$$\Sigma_{j=1}^2 m_t^{i,j} + m_t^{i,a_1;f} + m_t^{i,a_2;s} + s_t^i = w_t^i + s_{t-1}^i, \tag{24}$$

where

$$m_t^{i,a_2;s} = -\triangle p_t^{a_2;s} \triangle a_t^{i,2;s} + \triangle p_t^{a_2;s} \triangle A_t^{2;s} (\frac{v_t^i}{V_t}). \tag{25}$$

Also, in submitting margin to engage in buy-then-sell speculative trade, each household $i \in I$ faces following constraint:

$$0 \leq v_t^i \leq p_{t-1}^{a_2} a_{t-1}^{i,2} + w_t^i + s_{t-1}^i. \tag{26}$$

Here, note that each household cannot use primary asset to finance margin.

Note that in making speculative trades in the form of buy-then-sell, we allow households to obtain money required for the initial purchase as long as the net payments satisfies the constraint shown in Equation (24). In addition, household $i \in I$ faces inability of short-selling, which is formally expressed as follows.

$$-a_{t-1}^{i,2} \leq \triangle a_t^{i,2;s} \leq 0. \tag{27}$$

Household $i \in I_t^{1;0}$ cannot engage in speculative trades for secondary assets as follows.

$$\triangle a_t^{i,2;s} = 0, \text{ and } v_t^i = 0. \tag{28}$$

3.1.7 Realization of good and asset trades

We spare the term "equilibrium" for the analysis of the whole dynamics throughout periods. Here, instead we use the term "realization" for the corresponding analysis within each period. In examining realization of good and asset trades in period t, we take given initial situation Ω_t^{GA} that is composed of productivities $(\alpha_t^j)_{t=1,2}$, labor allocation (I_t^1, I_t^2), participation to speculative trade $(I_t^{1;0}, I_t^{1;1})$, initial asset holdings $(\{a_{t-1}^{i,l}\}_{l=1,2;i\in I}, \{a_{t-1}^{j,2}\}_{j\in J^2})$ savings $\{s_{t-1}^i\}_{i\in I}$ and current wages (w_t^1, w_t^2). Also, we take given X_{t-1}, $P_{t-1} = (p_{t-1}^1, p_{t-1}^2, p_{t-1}^{a,1}, p_{t-1}^{a,2})$ and p_{t+1}^2. Realization of good and asset trades are examined in the following two steps.

Step 1. Realization in the first stage

Derive prices P_t, the associated household trades $\{D^{i;f}\}_{i\in I}$ and firms trades $\{\triangle a_t^{j,2;f}\}_{j\in J^2}$ that satisfy good clearing condition (stated in Section 3.1.1), asset clearing condition for the first stage (stated in Section 3.1.2 concerning clearing of fundamental trades both for asset 1 and 2), firm asset trade behavior for the first stage (stated in Section 3.1.3) and

consumer decision for the first stage (stated in Section 3.1.5). If there is no consistent clearing prices, then fix the holdings of primary asset of households as $a_t^{i,1} = a_{t-1}^{i,1}$ for every $i \in I$ and accordingly let $m_t^{i,a_1;f} = 0$ for every $i \in I$ while setting $p_t^{a,1} = p_{t-1}^{a,1}$, and derive the other prices and relevant trades in the same manner as stated above.

Step 2. Realization in the second stage

When Step 1 still never finds consistent clearing prices, then proceed to examine realization in the second stage for goods and assets clearing as follows. Now, derive prices P_t, the associated household trades $\{D^{i;s}\}_{i\in I}$ and firms trades $\{(v_t^j, \triangle a_t^{j,2;s})\}_{j\in J^2}$ that satisfy good clearing condition (stated in Section 3.1.1), asset clearing condition for the second stage (stated in Section 3.1.2 concerning clearing of fundamental trades for asset 1 while clearing of speculative trades for asset 2), firm asset trade behavior for the second stage (stated in Section 3.1.4) and consumer decision for the second stage (stated in Section 3.1.6).

Lemma 1 (Realization of good and asset trades)**.**

We take given initial situation Ω_t^{GA} together with relevant production $X_{t-1} \geq 0$ and prices $\{P_{t-1}, p_{t+1}^2\}$. Under $a_{t-1}^{i,1} = \bar{a}^1$ for every $i \in I$ (equal holding of primary asset), together with $w_t^j > p_{t-1}^1 \bar{x}^1$ for $j = 1, 2$ (sufficient wages), we have following results I)-III):

I) (Existence) There always exist finite prices P_t that realize good and asset trades.

II) (Price and consumption of primary good) we have:

i) If $Y_t^1 = \bar{x}^1$, then $p_t^1 = p_{t-1}^1$ and $x_t^{i,1} = \bar{x}^1$ for every $i \in I$.

ii) If $Y_t^1 > \bar{x}^1$, then $p_t^1 = \mu p_{t-1}^1$ and $x_t^{i,1} = \bar{x}^1$ for every $i \in I$.

iii) If $Y_t^1 < \bar{x}^1$, then $p_t^1 = w_t^{min}$ and $x_t^{i,1} = Y_t^1 < \bar{x}^1$ for every $i \in I$.

III) (Necessary consumption inequality for the occurrence of speculative trade) If there occurs any speculative trade, then any net loser household $i \in I$ through speculative trades must consume $x_t^{i,2} = \bar{x}^2$.

IV) (Multiple realizations of speculative trades) If there exists realization with $P := \triangle p_t^{a,2} \neq 0$, then there exists realization with any $0 \leq \triangle p_t^{a,2} < P$ for $P > 0$ while with any $P < \triangle p_t^{a,2} \leq 0$ for $P < 0$.

Proof. See Appendix A.2. □

3.2 Labor shift phase

We consider a sequential labor shift under a system of discrete and infinite rounds $k = 1, 2, ...$ We assume that households move in the unit of a group rather than individually. Specifically, in each round, a labor shift from one sector to the other can be made by at most one group of households in the size of $\epsilon_n > 0$, which is assumed sufficiently small. This setting of group-based labor shift is to allow equilibrium analysis by letting a household internalize the effect of own labor shift on good and asset prices.[35] In each

[35] In fact, if labor shift is made on each household basis, technically there would be no equilibrium but we observe a disequilibrium oscillation around certain point.

round, a group of households is randomly picked up when there exist multiple households who want to move out from each status quo sector. When there exists no household who wants to move out, the labor shift process terminates and we say the corresponding labor distribution (n_t^1, n_t^2) is realized.

In each round, each household maximizes utility by choosing own working sector. In their decisions, we let households rationally expect clearing prices and the associated consequences of own consumption and asset holding. Note that rationality requires consistency such that the expected situation must be realized when a household becomes one of the last decision makers in the process. There remains some subtle issue on off-path expectation (i.e, labor shift arises also in the next round). This matters in case of multiple clearing prices, and note that such multiplicity only concerns the price for speculative trades (i.e., $\triangle p_{t,2}^{a,2}$) and relevant only when there potentially exists a speculative trade. We let arbitrary set of clearing prices (i.e., P_t) given the choice of own labor shift be implied by rational expectation when it turns out to be off-path expectation.

Another issue in forming expectation is whether a household in the primary sector will be entitled to engage in speculative trades or not (i.e., whether a household will be included in $I_t^{1;0}$ or $I_t^{1;1}$). As we let $\lambda = \frac{|I_t^{1;1}|}{|I_t^1|}$ be satisfied for any period t, we let each household endows probability λ in engaging in speculative trades if the household decides to work in the primary sector.[36]

Let M_t^j denote the available fund for employment in each sector $j = 1, 2$ in period t. Then, the competitive wage schedule for sector $j = 1, 2$ becomes as follows:

$$w_t^j = \frac{M_t^j}{n_t^j}. \tag{29}$$

4 Social optimality and preliminary results on optimal money supply

We say allocation $((x_t^i, a_t^i)_{i\in I}, \int_{j\in J} a_t^{j,2})$ is social optimal in period t if it is Pareto efficient and also attains equality across households. The relevant resource constraints

[36] If households in the primary sector are heterogeneous in holding secondary assets, differences in $(I_t^{1:0}, I_t^{1;1})$ could lead to different clearing prices. The analysis in this article avoids such difficulty by limiting our discussion on the cases of homogeneous holding of secondary asset in the primary sector.

are shown as follows:

$$
\begin{aligned}
n_t^1 + n_t^2 &\leq 1. && (30)\\
\int_{i\in I} a_t^{i,1} &\leq A^1 = \bar{a}^1. && (31)\\
\int_{i\in I} a_t^{i,2} &\leq A^2. && (32)\\
X_t^1 &\leq F_t^1(n_t^1). && (33)\\
X_t^2 &\leq F_t^2(n_t^2, a_t^{J,2}). && (34)\\
\int_{i\in I} x_t^{i,1} &\leq X_t^1. && (35)\\
\int_{i\in I} x_t^{i,2} &\leq (1-\theta)X_t^2 + \theta X_{t-1}^2. && (36)
\end{aligned}
$$

Note that in discussing social optimality in period t, previous period production X_{t-1}^2 is taken as given. Social optimal allocation is characterized in the following Lemma.

Definition 1 (Social optimal)**.**

Given X_{t-1}^2 and resource constraints (31)-(36), allocation in period t is social optimal when following i)-vi) are satisfied:
i) (Labor allocation) $n_t^j = n_t^{je}$ for $j = 1, 2$.
ii) (Production) $X_t^1 = F_t^1(n_t^1)$ and $X_t^2 = F_t^2(n_t^2, a_t^{J,2})$ with $a_t^{J,2} := \int_{j\in J^2} a_t^{j,2} = \frac{1}{\kappa} n_t^2$.
iii) (Consumption) $x_t^{i,1} = \bar{x}^1$ and $x_t^{i,2} = Y_t$ with $Y_t = (1-\theta)X_t^2 + \theta X_{t-1}^2$ for every $i \in I$.
vi) (Asset allocation) $a_t^{i,1} = \bar{a}^1$ and $a_t^{i,2} = A^2 - \frac{1}{\kappa} n_2^e$ for every $i \in I$.

Let Ω_t^L denote the initial situation in entering the labor shift process in period t. Concretely, let Ω_t^L be composed of monetary allocation (M_t^1, M_t^2), initial labor distribution (I_{t-1}^1, I_{t-1}^2), initial asset holdings $(\{a_{t-1}^{i,l}\}_{l=1,2;i\in I}, \{a_{t-1}^{j,2}\}_{j\in J^2})$, savings $\{s_{t-1}^i\}_{i\in I}$ and productivities $(\alpha_t^j)_{t=1,2}$.

Lemma 2 (Efficiency of labor distribution and money supply)**.**

We take given initial situation Ω_t^L as well as X_{t-1}, P_{t-1} and p_{t+1}^2. Also, let $s_{t-1}^i = 0$ for every $i \in I$ (no unnecessary saving), $a_{t-1}^{J,2} = \frac{1}{\kappa} n_{t-1}^2$, (previous efficient production) $M_t^1 = \bar{x}^1 p_{t-1}^1$ (just sufficient money allocation to primary sector) and $n_{t-1}^2 \leq n_t^{2e}$ (possible productivity improvement in primary sector). Furthermore, suppose that asset intensity parameter κ in production of secondary good is sufficiently large (triviality of the effect of asset input for production), and also we restrict our focus on the realizations of good and asset trades where no speculative trade occurs, and accordingly let households expect no occurrence of speculative trade even in off-path expectation during the labor shift process (ignorance of speculative trade).

Then, efficiency in labor distribution $(n_t^1, n_t^2) = (n_t^{1e}, n_t^{2e})$ is realized under monetary allocation (M_t^1, M_t^2) if:

$$
M_t^2 \geq \frac{n_t^{2e}}{n_t^{1e}} M_t^1. \qquad (37)
$$

Proof. See Appendix A.3. □

Remark for Lemma 2: 1. Wage inequality

There are two remarks for the statements of Lemma 2. Firstly, when Relationship (37) is satisfied with inequality, it indicates realization of efficient labor distribution (i.e., $(n_t^1, n_t^2) = (n_t^{1e}, n_t^{2e})$) even though the wage in secondary sector is higher than that in primary sector (i.e., $w_t^2 = M_t^2/n_t^2 > M_t^1/n_t^1 = w_t^1$). Observe that this apparently contradicts with Law of one price as every household holds equal labor. A critical setting that causes this phenomenon is sequential labor process in which each mover internalizes the effect of own move. In fact, if each mover never internalizes the effect of own move, that is, all relevant prices of good and asset are assumed fixed in the labor shift decision, then each mover is better to move whenever the household expects to gain higher wage. Another critical setting given the internalization setting is the existence of price lower and upper bounds. As illustrated in Observation 2, if there existed sufficiently large wage discrepancy without effective price bounds, then there would arise cascade of labor shift toward sector 2 in the current period. This is because households in secondary sector could squeeze those in primary sector in purchasing primary good. Observation 2 further indicates that the cascade of labor shift in the current period would then cause reverse cascade of labor shift in the successive period, and consequently end up in destruction of the economic structure (i.e., no labor in secondary sector). Given this observation, the price lower and upper bounds serve to stabilize economy against situations of excess money supply (i.e., $M_t^2/n_t^{2e} > M_t^1/n_t^{1e}$), while bringing sectoral wage inequality.

Remark for Lemma 2: 2. Attainability of labor efficiency

Turning to another issue, observe that Lemma 2 does not include "only if" part. Thus, the lemma is silent about whether or not the efficiency in labor distribution can be attained when Relationship (37) is not satisfied. On this point, given monetary allocation that does not satisfy Relationship (37), suppose that labor shift process attains the equal wage at some round. Then, in the next round, moving to sector 2 has two competing effects on the utility of the mover. One effect is through a decrease in monetary wage, while the competing effect is through an increase in the aggregate production of good 2. If the holding asset of the moving household is sufficiently highly valued, the household could sell asset to buy up secondary goods and by doing so the household possibly could attain higher utility level even under the situation where the wage would be smaller by moving to sector 2. Whether the former effect dominates or not would depend particularly on properties of household preferences. Though we do not have a general result on this point (and the lack of a general result itself does not matter for the purpose of our analysis), next observation shows that the former effect always dominates for the case of Cobb-Douglas utility function given the initial equal asset allocation.

Observation 1 (Case of Cobb-Douglas)**.**

We take as given all the conditions for Lemma 2 to be satisfied. In addition, consider initial equal allocation also for secondary asset as $a_{t-1}^{i,2} = a_{t-1}^{i',2}$ *for every* $i, i' \in I$*. Here,*

we restrict our focus on Cobb-Douglas (ordinal) utility function of the representative form $f(a,x) = a^{\nu}x^{1-\nu}$ *with* $0 < \nu < 1$. *Then, efficiency in labor distribution* $(n_t^1, n_t^2) = (n_t^{1e}, n_t^{2e})$ *is realized under monetary allocation* (M_t^1, M_t^2) *if and only if Relationship (37) is satisfied.*

Proof. See Appendix A.4. □

Given Lemma 2, we have following result on economic equality.

Lemma 3 (Optimal money supply and equality)**.**

We take as given all the conditions for Lemma 2 to be satisfied. Furthermore, suppose that monetary allocation (M_t^1, M_t^2) *is made so that Relationship (37) is satisfied with equality as follows:*

$$M_t^2 \quad = \quad \frac{n_t^{2e}}{n_t^{1e}} M_t^1. \tag{38}$$

Then, we have following results. Firstly, for arbitrary allocation of secondary asset $\{a_{t-1}^{i,2}\}_{i\in I}$*, we have following result i):*

i) (income equality) $w_t^1 = w_t^2$.

In addition, if $a_{t-1}^{i,l} = a_{t-1}^{i',l}$ *for* $l = 1, 2$ *and for every* $i \in I$ *(i.e., equal asset allocation), we have following results ii) and iii).*

ii) (consumption equality) $x_t^{i,j} = x_t^{i',j}$ *for* $j = 1, 2$*, for every* $i, i' \in I$.

iii) (wealth equality) $a_t^{i,l} = a_t^{i',l}$ *for* $l = 1, 2$ *and for every* $i, i' \in I$.

Proof. All the results i)-iii) are immediate given the proof for Lemma 2. □

The next proposition explicitly shows the required additional money to realize efficiency of labor distribution and equal wages against a productivity improvement in the primary sector.

Lemma 4 (Optimal monetary expansion)**.**

We take as given all the conditions for Lemma 3-i) to be satisfied. Furthermore, let Relationship (38) be satisfied also for period $t-1$. *Also, let* $\alpha_t^1 = \rho\alpha_{t-1}^1$ *with* $\rho > 1$. *Then, efficient labor allocation together with equal wage is realized in period* t *by letting* $M_t^2 = \triangle M + M_{t-1}^2$ *with* $M_t^1 = M_{t-1}^1$*, where* $\triangle M$ *is specified as follows:*

$$\triangle M \quad = \quad w_{t-1}(n_t^{2e} - n_{t-1}^{2e}) + n_t^{2e}\triangle w, \tag{39}$$

where $w_{t-1} := w_{t-1}^1 = w_{t-1}^2$, $n_t^{1e} := \frac{\bar{x}^1}{\alpha_t^1} = \frac{1}{\rho}n_{t-1}^{1e}$ *and* $n_t^{2e} = 1 - n_t^{1e} = \frac{\rho-1}{\rho}n_{t-1}^{1e}$, $\triangle w := w_t^2 - w_{t-1} = \frac{n_{t-1}^{1e}w_{t-1}}{n_t^{1e}} - w_{t-1} = \frac{n_{t-1}^{1e} - n_t^{1e}}{n_t^{1e}}w_{t-1} = (\rho-1)w_{t-1}$.

Furthermore, under equal initial allocation also for secondary asset as $a_{t-1}^{i,2} = a_{t-1}^{i',2}$ *for every* $i, i' \in I$*, monetary expansion specified in (39) realizes social optimal allocation in period* t.

Proof. See Appendix A.5. □

Lemma 4 shows a decomposition of the required money in maintaining efficient labor distribution together with equal wage against an increase in demand of labor in sector 2. The first term indicates the required additional money that facilitates the increased labor demand under the assumption of the previous wage level. Observe that a labor shift toward sector 2 has an upward impact on the wage of sector 1, and thus, the wage of sector 2 must be increased accordingly. The second term shows the required additional money against the wage impact where $\triangle w$ expresses the resultant wage impact. Figure 1 illustrates the decomposition.

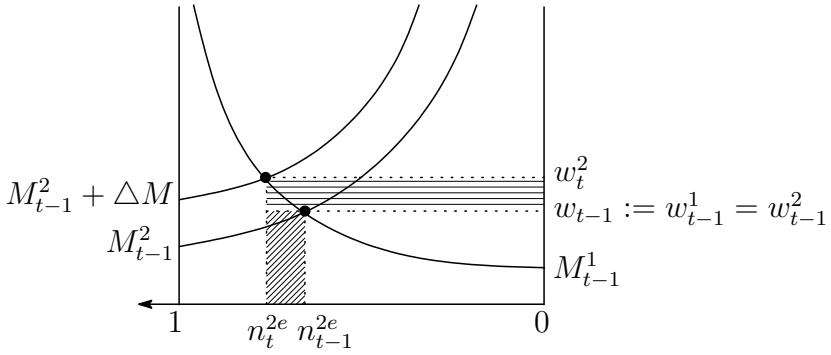


Figure 1: Note: the figure graphically shows the decomposition of $\triangle M$ shown in Proposition 4. The horizontal axis shows labor inputs, where the starting point to see the corresponding labor inputs to secondary sector is set at the right side (show with 0). The decomposition is illustrated such that the first term $w_{t-1}(n^{2e}_t - n^{2e}_{t-1})$ is shown with the shaded area with diagonal lines while the second term $n^{2e}_t \triangle w$ is shown with that with horizontal lines.

We prepare to see the implications on nominal side of the economy when monetary expansion is optimally made.

Lemma 5 (Nominal implications of optimal monetary expansion)**.**

We take as given all the conditions for Lemma 4 for the results of optimal monetary expansion to be satisfied Furthermore, let $X^2_{t-2} = X^2_{t-1}$. *Accordingly, denote* $\triangle Y := Y^2_t - Y^2_{t-1} = (\alpha^2_t - \alpha^2_{t-1})(1-\theta)n^{2e}_{t-1} + \alpha^2_t(1-\theta)(n^{2e}_t - n^{2e}_{t-1})$.

Then, under the optimal monetary expansion $\triangle M$ *that satisfies Equation (39), we have following consequences:*

i) (Wages) $w_t = \triangle w + w_{t-1}$, *where* $\triangle w$ *is defined in Lemma 4.*

ii) (Goods prices) $p^2_t > \tilde{p}^2_t := \left(p^2_{t-1} + \frac{\triangle M}{Y^2_{t-1}}\right)\frac{Y^2_{t-1}}{Y^2_{t-1}+\triangle Y}$ *and* $\lim_{\kappa\to\infty} p^2_t = \tilde{p}^2_t$.

iii) (Relative prices of good and asset) $\lim_{\kappa\to\infty} \frac{p^{a,2}_t}{p^2_t} > \frac{p^{a,2}_{t-1}}{p^2_{t-1}}$ *and* $\frac{p^{a,2}_t}{p^2_t} > \lim_{\kappa\to\infty} \frac{p^{a,2}_t}{p^2_t}$.

Proof. See Appendix A.6. □

Lemma 5 says that the optimal development is always accompanied with wage inflation, while implications on good and asset prices are more intricate. Concerning the

price of secondary good, there exists two competing effects: the upward force is brought by an increase in money supply (i.e., $\triangle M$) while the downward force is brought through an increase in production (i.e., $\triangle Y$). Furthermore, by the definition of $\triangle Y$, an increase in production is brought by two routes: one route is through productivity improvement of secondary sector (i.e., $(\alpha_t^2 - \alpha_{t-1}^2)(1-\theta)n_{t-1}^{2e}$), while the other route is through labor shift brought by productivity improvement of primary sector (i.e., $\alpha_t^2(1-\theta)(n_t^{2e} - n_{t-1}^{2e})$). Observe that the good price becomes deflationary under sufficiently large $\triangle Y$ where the downward force on the good price becomes dominant. Given the price formation of secondary good, the price formation of secondary asset could be understood with three factors as follows. Firstly, homotheticity of preferences forces to maintain the constant relative price to the secondary good price given no change in good supply (i.e., $\triangle Y = 0$) and no change in asset supply to households (i.e., $\kappa \to \infty$). Given this, even without no change in asset supply (i.e., κ goes to infinity), since there must occurs an increase in good supply, the relative asset price increase as $\lim_{\kappa\to\infty} \frac{p_t^{a,2}}{p_t^2} > \frac{p_{t-1}^{a,2}}{p_{t-1}^2}$. On this basis, when there occurs a decrease in asset supply (i.e., finite κ), the relative asset price further increases as $\frac{p_t^{a,2}}{p_t^2} > \lim_{\kappa\to\infty} \frac{p_t^{a,2}}{p_t^2}$.

5 Results on excess money supply and asset bubble

We say asset bubble occurs in period t when $\triangle p_t^{a_2;s} > 0$ is realized. Remember that Lemma 1-III) indicates that certain extreme consumption inequality is a necessary condition for the occurrence of asset bubble. In addition, we will see that extreme wealth inequality is another necessary condition. Given this, in arguing the occurrence of asset bubble in the perspective of dynamics throughout periods, we need to demonstrate how such extreme unequal situation could be attained in some period. On this issue, fully dynamic analysis is provided in later section, while here we proceed to demonstrate it within our current preliminary framework of one period.

As will be done in the full analysis, we aim to focus on the dynamics which originally starts with an social optimal situation, where particularly secondary asset is equally held by all households. Furthermore, we restrict our focus on the situation of stable production throughout periods such that there arises no productivity improvement. Given this, Proposition 1 implies that excess monetary expansion kicks in asset accumulation process wherein secondary asset is accumulated to sector 2 households while satisfying equal asset allocation within each sector. Also, under sufficiently large excess money supply in each period, it would surely attain an extreme unequal asset holding where households in sector 1 have no secondary asset.

Proposition 1 (Excess money supply and asset accumulation dynamics)**.**

We take given all the conditions for Lemma 2 to be satisfied. Furthermore, let Relationship (39) be satisfied with inequality. In addition, suppose that $\alpha_t^2 = \alpha_{t-1}^2$ *and* $n_{t-1}^2 = n_t^{2e}$ *(stable production),* $a_{t-1}^{i,l} = a_{t-1}^{i',l}$ *for* $l = 1, 2$ *and for every* $i, i' \in I^j$ *for* $j = 1, 2$ *(equal asset allocation within each sector),* $a_{t-1}^{i,2} \leq a_{t-1}^{i',2}$ *for* $i \in I_t^1$ *and* $i' \in I_t^2$*. (sectoral*

unequal asset allocation), and $M_t^2 > \frac{n_t^{2e}}{n_t^{e1}} M_t^1$ *(excess money supply). Let* i_t^1 *and* i_t^2 *denote the representative household of* I_t^1 *and* I_t^2*, respectively. Also, denote the maximum possible asset holding of secondary asset by* i_t^2 *with* $\bar{a}_t^2 := A^2/n_t^{2e}$*.*

Then, there exits threshold $\bar{M} = \frac{\alpha_t^2}{\alpha_t^2 - \bar{x}^2} \frac{\alpha_t^1 - \bar{x}^1}{\alpha_t^1} M_t^1$ *and we have function* $\tilde{a}(M_t^2)$ *which is weakly increasing and also* $\tilde{a} = \bar{a}_t^2$ *for any* $M_t^2 \geq \bar{M}$*, with which we have:*

i) (Widening wealth inequality and convergence) For arbitrary $M_t^2 > \frac{n_t^{2e}}{n_t^{1e}} M_t^1$*, if* $a_{t-1}^{i,2} < \tilde{a}(M_t^2)$ *for* $i = i_t^2$*, then* $a_{t-1}^{i,2} < a_t^{i,2} \leq \tilde{a}$ *for* $i = i_t^2$ *and* $\frac{A^2 - n_t^2 \tilde{a}}{n_t^1} \leq a_t^{i,2} < a_{t-1}^{i,2}$ *for* $i = i_t^1$*.*

ii) (Attainability of maximum wealth inequality) If $M_t^2 > \bar{M}$*, then there exists* $a_{t-1}^{i,2} < \tilde{a} = \bar{a}_t^2$ *such that* $a_t^{i,2} = \bar{a}_t^2$ *for* $i = i_t^2$ *and* $a_t^{i'2} = 0$ *for* $i' = i_t^1$*.*

Proof. See Appendix A.7. □

In Proposition 1, $\bar{M}$ serves as a threshold for money supply which distinguishes whether it is mild excess money supply ($M \leq \bar{M}$) or severe excess money supply ($M > \bar{M}$) in terms of the induced consumption level of secondary goods particularly for I_t^2. Concretely, let $\bar{M} := n_t^{2e} \bar{w}$, where $\bar{w}$ is the wage level of sector 2 such that it is just sufficient to purchase secondary good to the satiation level given no asset trade. Formally, $\bar{w}$ is determined by the following equation.

$$\frac{\bar{w} - p_t^1 \bar{x}^1}{(\bar{w} - p_t^1 \bar{x}^1) n_t^{2e} + (w_t^1 - p_t^1 \bar{x}^1) n_t^{1e}} \alpha_t^2 n_t^{2e} = \bar{x}^2 \tag{40}$$

The proof of the lemma in Appendix A.7 shows that $\bar{M}$ is represented in the form shown in Proposition 1.

In forming some intuition of the results, see Figure 2 that shows a situation for Proposition 1-i). It shows a diagram constructed in a form of Edgeworth Box with respect to aggregate households I_t^1 and I_t^2. Given M_t^2 and also $a_{t-1}^{2,1} n_t^2$, which denotes the initial aggregate asset holding of I_t^2 , we let $x_t^{2;max}$ denote the amount of secondary good purchased by I_t^2 when there had not occurred any asset trade. Observe that the aggregate expenditure for secondary good is not affected by any occurrence of asset trade as long as every household does not make positive saving. Given this, we can interpret $(a_{t-1}^{2,1} n_t^2, x_t^{2;max})$ as the initial endowment for I_t^2, and in this perspective, the corresponding initial allocation is indicated with point z_0 in the diagram. Given M_t^2, we let $\tilde{a}$ indicate the (hypothetical) initial asset holding of I_t^2 with which there would not occur any asset trade, and let point $\tilde{z}$ denote the associated allocation in the diagram. Given the (actual) initial allocation z_0, good and asset clearing realizes allocation z_1 in the diagram. For the illustrated case, it is apparent that for given $a_{t-1}^{2,1} n_t^2 < \tilde{a}$, we have $a_{t-1}^{2,1} n_t^2 < a_t^{2,1} n_t^2 < \tilde{a}$. Proposition 1-i) has an implication on full dynamics such that under stable production, when some fixed level of relatively mild excess money (in the sense of $M_t^2 \leq \bar{M}$) is provided for every period, it demonstrates an asset accumulation process that converges to wealth equality with the level of $\tilde{a} \leq \bar{a}_t^2$. Note that there exist cases where wealth inequality with the level of $\tilde{a}$ would not be attained in finite periods (as in the case of Figure 2), while there also

exist cases where it is attained in some finite period (as discussed in the proof shown in Appendix A.7).

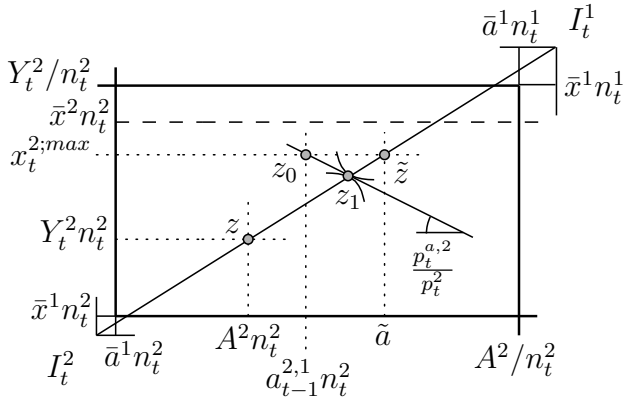


Figure 2: Note: The diagram shows the situation with two aggregate households I_t^1 and I_t^2, and the left-bottom point is the origin for I_t^2 while the right-top point is that for I_t^1. The inner box with bold line particularly shows the allocation concerning secondary good and asset given the holding of primary good and asset to the satiation levels for every household. The straight line between the origins for I_t^1 and I_t^1 shows the offer curve, which is derived from points of tangency of indifference curves concerning function $f(x, a)$. In the diagram, equal allocation of good and asset is shown with z, wherein $(A_t^2, n_t^2 Y_t^2 n_t^2)$ is allocated to I_t^2. The initial asset holding of $i \in I_t^2$ is denoted with $a_{t-1}^{2,1} := \int_{i \in I_t^2} a_{t-1}^{i,1}$.

We proceed to see the case with $M_t^2 > \bar{M}$. Observe that $M_t^2 > \bar{M}$ indicates $x_t^{2;max} > \bar{x}^2 n_t^2$. Figure 3 shows such a situation. It illustrates a situation where I_t^2 attains the satiation level consumption of secondary good (i.e., $x_t^{2,2} n_t^2 = \bar{x}^2 n_t^2$) but asset is only partially accumulated. Note that the indifference curve of I_t^2 on z_1 consists of a vertical line above z_1 (when ignoring cost of excess consumption) and a downward sloping curve below z_1 which is given by function $f(x, a)$. Given this, when we take relative price $p_t^{a,2}/p_t^2$ with which I_t^1 maximizes utility on shown z_1, then I_t^2 maximizes utility under the same relative price.

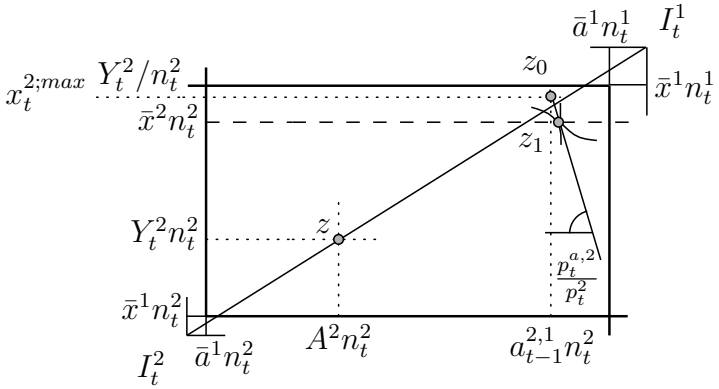


Figure 3:

For the statement of Proposition 1-ii), see Figure 4. The diagram shows that given

initial allocation z_0, where I_t^2 holds sufficient secondary asset, then good and asset clearing realizes z_1 that attains maximum wealth inequality. Observe that the shape of the indifference curve of I_t^2 that goes through z_1 has the same property with that in Figure 3. Also, for the part of I_t^1, given that there will never occur trade of primary asset, the indifference curve that goes through z_1 is relevant for the leftside of z_1 but irrelevant for the rightside of z_1. Given these indifference curves and the associated marginal rate of substitutions on z_1, it is immediate that there exists relative price $p_t^{a,2}/p_t^2$ that realize shown z_1 if $a_{t-1}^{2,1} n_t^2$ is sufficiently large. Proposition 1-ii) has an implication on full dynamics such that under stable production, when some fixed level of relatively severe excess money (in the sense of $M_t^2 > \bar{M}$) is provided for every period, it demonstrates an asset accumulation process that surely attains maximum wealth inequality as $\tilde{a} = \bar{a}_t^2$ in some finite period.

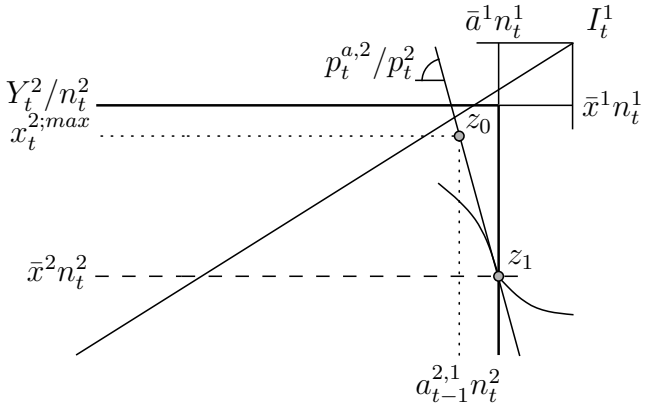


Figure 4:

In examining the occurrence of asset bubble, following lemma clarifies necessary conditions concerning money supply and initial wealth inequality.

Proposition 2 (Necessary conditions for speculative trade)**.**

Given all the conditions for Proposition 1 to be satisfied, if there occurs any speculative trade in period t, then we have following i) and ii):

i) (Severe excess money supply) $M_t^2 > \bar{M}$.

ii) (Extreme wealth inequality) $a_{t-1}^{i,2} = 0$ for every $i \in I_t^1$ and $a_{t-1}^{i,2} = \bar{a}^2$ for every $i \in I_t^2$.

Proof. See Appendix A.8. □

Given the situations stated in Proposition 2, we have following results on the occurrence of asset bubble.

Proposition 3 (Occurrence of asset bubble)**.** [37]

[37] Observe that statement I of the current lemma also indicates the condition for the occurrence of negative asset bubble. Furthermore, statements of II) and III) are also true for the occurrence of negative asset bubble.

We take given all the conditions for Proposition 1 to be satisfied. Furthermore, we focus on situations with $a_{t-1}^{i,2} = 0$ *for every* $i \in I_t^1$ *and* $a_{t-1}^{i,2} = \bar{a}_t^2$ *for every* $i \in I_t^2$ *(extreme wealth inequality),* $n_t^j = n_t^{j'}, a_t^{j,2} = a_t^{j',2}$ *for every* $j, j' \in J^2$ *(symmetry of firms) and* $M_t^2 > \bar{M}$ *(severe excess money supply).*

Then, we have following results I)-III):

I) (Speculative trade under asset bubble) Given $\lambda = |I_t^{1;1}|/|I_t^1| > 0$ *and the occurrence of any speculative trade (i.e.,* $\triangle p_t^{a_2;s} \neq 0$*), it presents as asset bubble (i.e.,* $\triangle p_t^{a_2;s} > 0$*) if and only if there exists household* $i \in I_t^{1,1}$ *who engages in some speculative trade.*

II) (Existence without bubble riding) There always exists asset bubble wherein $m_t^{j,a_2;s} = 0$ *for every* $j \in J^2$*.*

III) (Existence with bubble riding) There exists asset bubble wherein $m_t^{i,a_2;s} = m^{i',a_2;s}$ *for every* $i, i' \in I_t^2$ *(symmetry participation) and* $m_t^{j,a_2;s} < 0$ *for every* $j \in J^2$ *(bubble riding) only if* $M_t^2 > \bar{M}^b$*, where* $\bar{M}^b := n_t^{2e}\bar{w}^b$ *and* $\bar{w}^b$ *is defined by the following equation.*

$$\frac{\bar{w}^b - p_t^1\bar{x}^1}{(\bar{w}^b - p_t^1\bar{x}^1)n_t^{2e} + (w_t^1 - p_t^1\bar{x}^1)n_t^{1e}}\Big(1 - (1-\theta)\delta\Big)\alpha_t^2 n_t^{2e} = \bar{x}^2. \tag{41}$$

Proof. See Appendix A.9. □

Part I) of Proposition 3 highlights the indispensable role of $I_t^{1;1}$ in the formation of asset bubble, and thus serves as the basis in arguing the consequence on inequality. Part II) establishes the occurrence of asset bubble, while part III) provides a necessary condition for bubble riding to occur. There, Equation (41) can be easily understood through rewriting Equation (40) for the case with production loss that is caused by bubble riding by firms. When all producing firms engage in bubble riding, observe that $X_t = (1-\delta)\alpha_t^2 n_t^{2e}$ while supply of secondary good is given by $Y_t^2 = (1-\theta)X_t + \theta X_{t-1}$. Since it is assumed $X_{t-1} = \alpha_t^2 n_t^{2e}$, loss in good supply brought by bubble riding becomes $((1-\theta)\delta)\alpha_t^2 n_t^{2e}$.

In understanding nominal implications of the occurrence of asset bubble, we pose the concept of "bubble money" which refers to the funds poured into speculative asset trades. We examine how bubble money could affect money flow toward good markets and consequently affects good prices. Proposition 4 provides the relevant results in this perspective.

Proposition 4 (Nominal implications of asset bubble)**.**

We take as given all the conditions for Proposition 3 to be satisfied. Let $P_t = (p_t^1, p_t^2, p_t^{a,1}, p_t^{a,2}, \triangle p_t^{a_2;s})$ *with* $\triangle p_t^{a_2;s} = 0$ *denote clearing prices without any speculative trade. Furthermore, suppose that there exists clearing prices* $\tilde{P}_t = (\tilde{p}_t^1, \tilde{p}_t^2, \tilde{p}_t^{a,1}, \tilde{p}_t^{a,2}, \triangle \tilde{p}_t^{a_2;s})$ *with asset bubble as* $\triangle \tilde{p}_t^{a_2;s} > 0$*, with which* $s_t^i = 0$ *for every* $i \in I_t^{1;1}$ *(no excess money transfer). Accordingly, let* $\tilde{Y}_t^2$ *and* Y_t^2 *denote supply of secondary good for the case with bubble and without bubble, respectively.*

Then, by letting $\triangle M_t^{1;1,g2}$ *and* $\triangle M_t^{2,g2}$ *denote the additional expenditure of respective households* $I_t^{1;1}$ *and* I_t^2 *in purchasing secondary good under* $\tilde{P}_t$ *compared to that under* P_t*,*

we have following results for given $M_t^{a_2;s} := \int_{i\in I_t^2} m_t^{i,a_2;s}$ *(bubble money):*

I) (Direct effect on good expenditure) $\triangle M_t^{1;1,g2} = \frac{\int_{i\in I_t^{1;1}} v_t^i}{V_t} M_t^{a_2;s}$.

II) (Indirect effect on good expenditure) $\triangle M_t^{2,g2} = M^F + \beta \triangle M_t^{1;1,g2}$, *where:*

$$\begin{aligned} M^F &:= \frac{Y_t^2 - \tilde{Y}_t^2}{\tilde{Y}_t/n_2^e - \bar{x}^2} p_t^2 \bar{x}^2. \\ \beta &:= \frac{\bar{x}^2}{\tilde{Y}_t^2/n_2^e - \bar{x}^2}. \end{aligned}$$

III) (Total effect on good price) $\tilde{p}_t^2 = p_t^{2;r} + \frac{1}{\tilde{Y}_t}(\triangle M_t^{1;1,g2} + \triangle M_t^{2,g2})$, *where* $p_t^{2;r} := \frac{Y_t^2}{\tilde{Y}_t^2} p_t^2$.

Proof. See Appendix A.10. □

Part I and II) of Proposition 4 in combination reveal how bubble money (i.e., the aggregate net inflow into speculative trade $M_t^{a_2;s}$) will generate money flow to good markets. The generated money flow consists of two streams. One stream ($\triangle M_t^{1;1,g2}$) is directly caused by bubble money such that it is initially poured by I_t^2 to $I_t^{1;1}$ through speculative trades, and then $I_t^{1;1}$ forwards the funds to good market aiming to buy up secondary good. Since this added stream has effect in shrinking consumption of I_t^2, it necessarily arises the other stream ($\triangle M_t^{2,g2}$) that is initiated by I_t^2 to good market in maintaining the satiation level of secondary good consumption. Thus, in the perspective of the effects on expenditure on secondary good, we can grasp the implication on money flow such that bubble money has direct effect (brought by the former stream) and also the indirect effect (brought by the latter stream). Part III) summarizes the effect on the good price in this perspective.

Concerning the components of the indirect effect shown in Part II), the fixed effect (i.e., M^F) indicates the additionally required funds even without any additional money flow in the direct route (i.e., $\triangle M_t^{1;1,g2}$), and thus, apparently it arises only when $\tilde{Y}_t^2 < Y_t^2$, while the coefficient of the marginal effect (i.e., β) indicates the additionally required funds against a unit change in the money flow of the direct route (i.e., $\triangle M_t^{1;1,g2}$). Given this, observe that part II) has implication on the effect of bubble riding such that when bubble riding brings larger decrease in production (i.e., smaller $\tilde{Y}_t^2$), then both the marginal and fixed effect become larger, and thus larger funds are poured into good markets in total.

Concerning the components of the formation of the good price shown in Part III), the first term ($p_t^{2;r}$) is interpreted as the reference price in evaluating the price impact, while the second term shows the impact in this perspective. Given this, observe that when bubble riding brings larger decrease in production, the total effect is decomposed into following three effects. Firstly, decrease in production has a price raising effect through increasing the associated money flow (in terms of $\triangle M_t^{1;1,g2} + \triangle M_t^{2,g2}$) as indicted in Part II). Secondly, decrease in production has a leverage effect (i.e., $\frac{1}{\tilde{Y}_t^2}$) in amplifying the impact through the first effect. Thirdly, decrease in production has an effect in raising the reference price $p_t^{2;r}$.

Note that the impact on good price indicated by Proposition 4 is presented as comparative statics, and does not reveal the impact in the dynamic perspective. In fact, suppose that asset bubble continues to arise throughout periods under stable production and sustained with the same level of bubble money, then we will not observe any price change of secondary good except for the initial period of the bubble occurrence, while observing increasing asset prices throughout periods. Thus, continuing asset bubble consistently demonstrates asset inflation in the dynamic perspective, as formally stated in a later part.

We proceed to see the implications of asset bubble on production efficiency. Following Proposition 5 summarizes the results.

Proposition 5 (Asset bubble and efficiency)**.**

We take as given all the conditions for Proposition 4 to be satisfied. Then, we have $\tilde{Y}_t^2 < Y_t^2$ *if and only if* $M_t^{a_2;s} > \bar{M}^{a_2;s} := \frac{V_t}{v_t^J} p_{t,t+1}^2 \delta Y_t^2$*, where* $v_t^J := p_{t-1}^{a,2}(1/\kappa) n_t^{2e}$ *and* $V_t = v_t^J + w_t^1 \lambda n_t^{1e}$*.*

Proof. See Appendix A.11. □

Proposition 5 points out that given relevant good prices $p_{t,t+1}^2$, the base wage level w_{t-1}^1, potential production Y_t^2 and efficient labor distribution (n_t^{1e}, n_t^{2e}), production inefficiency during bubble (i.e, $\tilde{Y}_t^2 < Y_t^2$) is more likely to arise when: i) larger bubble money is poured (larger $M_t^{a_2;s}$), ii) bubble riding causes less impairment effect on production (smaller δ), iii) asset intensity in production is higher (larger κ), iv) household participation to speculative trades is smaller (smaller λ) and v) the initial asset price is higher (higher $p_{t-1}^{a,2}$).

The lemma also has implication on the dynamics, particularly concerning factor v) above. Observe that when the initial asset price $(p_{t-1}^{a,2})$ in each period is higher, it gives larger share of margin (v^J/V_t), and thus more attracts bubble riding in the corresponding period. In a dynamic context in which asset bubble is sustained in a form of asset inflation, observe that even when asset bubble itself is not swelling throughout periods (in terms of bubble money), the cumulative increase in asset prices has effect in attracting bubble riding through inflating the value of their submitting margin.

We proceed to see the implications of asset bubble on inequality. Proposition 6 summarizes the results.

Proposition 6 (Asset bubble and inequality)**.**

We take as given all the conditions for Proposition 4 to be satisfied. Also, use notations of $(x_t^{i,2}, a_t^{i,2})$ *for the case without speculative trade, while* $(\tilde{x}_t^{i,2}, \tilde{a}_t^{i,2})$ *for the case with asset bubble. Then, given* $0 < \lambda = |I_t^{1;1}|/|I_t^1| < 1$*, we have following results.*
I) (Stable wealth inequality) $\tilde{a}_t^{i,2} = a_t^{i,2} = a_{t-1}^{i,2}$ *for every* $i \in I$*.*
II) (Regressive transfer of consumption)
i) (Baseline inequality without bubble) $x_t^{i,2} = \bar{x}^2$ *for every* $i \in I_t^2$*, while* $x_t^{i,2} = \frac{Y_t^2 - n_t^{2e}\bar{x}}{n_t^{1e}} < \bar{x}^2$ *for every* $i \in I_t^1$*.*

ii) (No effect on riches) $\tilde{x}_t^{i,2} = \bar{x}^2$ *for every* $i \in I_t^2$.

iii) (Gain for asset trade participants) $\tilde{x}_t^{i,2} > x_t^{i,2}$, *for every* $i \in I_t^{1;1}$.

iv) (Loss for asset trade non-participants) $\tilde{x}_t^{i,2} < x_t^{i,2}$, *for every* $i \in I_t^{1;0}$.

v) (Implication of the size of bubble) Within each domain of $\triangle M^{a_2;s} < \triangle \bar{M}^{a_2;s}$ *(no bubble riding) and* $\triangle M^{a_2;s} \geq \triangle \bar{M}^{a_2;s}$ *(bubble riding), larger* $\triangle M^{a_2;s}$ *leads to strictly larger* $\tilde{x}_t^{i,2}$ *for* $i \in I_t^{1;1}$ *and strictly smaller* $\tilde{x}_t^{i,2}$ *for* $i \in I_t^{1;0}$

Proof. See Appendix A.12. □

Given that asset holding is never affected by the occurrence of bubble as stated in Part I), we can focus on the consequence on consumption in examining the implication of bubble on inequality. Part II-i) provides the benchmark inequality in terms of consumption of secondary good. We could say that I_t^2 forms a group of riches since they always attain the satiation level of consumption, while I_t^1 forms a group of poors. Given this, Part II-ii)-iv) indicates that the occurrence of bubble effectively realizes transfer among the group of poors, and such transfer is grasped as regressive transfer. Figure 5 illustrates the nature of the indicated regressive transfer, while also illustrating the implication of the size effect stated in II-v). The figure presents cumulative consumption of secondary good in a way to form a Lorenz curve, thus the straight line on the whole domain serves as the line of perfect equality. The kinked line with the kink on n_t^{1e} shows the situation of maximum unequal consumption without bubble, which is realized under the situation for Proposition 6. Given this, the size of the shaded area indicates further inequality (in the perspective of Gini coefficient) that could be brought by the occurrence of asset bubble.

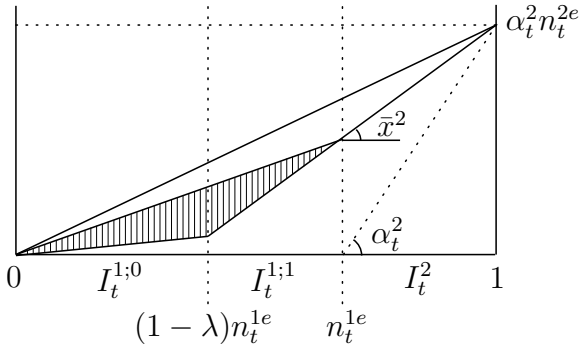


Figure 5: Note: the figure shows the cumulative consumption of secondary good for the case of no bubble riding (i.e., $\tilde{Y}_t^2 = \alpha_t^2 n_t^{2e}$) in the manner of forming Lorenz curve, where the cumulation is made in the order of $I_t^{1;0}$, $I_t^{1;1}$ and I_t^2.

6 Full framework

6.1 Bank lending phase

Under the assumed price inflexibility on the primary goods and the resultant inflexible nature of labor transfer across sectors, the essential macroeconomic role of the whole

banking system is grasped as allowing credit creation to foster the secondary sector. For the concrete setting, we consider a set of banks $B = [0, B]$ that is unchanged throughout periods. We let the banks lend only to firms in the secondary sector. To avoid inessential cumbersome in the analysis, we let each bank lend to at most one firm and each firm borrows from at most one bank. Accordingly, we abuse notation such that when bank $b \in [0, B]$ makes lending to a firm, we refer to the firm as firm b.

We assume that the central bank implements policy rates so that they are incorporated in the marginal cost of making lending by firms. While specific ways of implementation are discussed in Section 8, here we let the profit of bank $b \in B$ which makes lending L_t^b be expressed as follows.

$$\Pi_t^b = i_t^L L_t^b - \xi[(1-\theta)X_t^b p_t^2 i_t^R + \theta X_t^b p_{t+1}^2 i_{t+1}^R], \tag{42}$$

where i_t^L is the market bank lending rate for the loans made in period t, while i_t^R and i_{t+1}^R indicate policy rates in period t and $t+1$, respectively. Note that we have $X_t^b = \alpha_t^2 n_t^{b,2}$ when the firm does not engage in bubble riding, while $X_t^b = (1-\delta)\alpha_t^2 n_t^{b,2}$ for the case of bubble riding. Given these, $(1-\theta)X_t^b p_t^2$ shows the current sale of the borrowing firm while $\theta X_t^b p_{t+1}^2$ shows the sale of the firm in the following period. It is assumed that the effective reserve demand in each period is proportional to the sales in the corresponding period, and the ratio is denoted with ξ. Section 8 elaborates consistent settings for this assumption. On this basis, the policy rate is incurred as unit cost of the effective reserve demand. For the sake of analysis, we examine competitive situation by imposing zero profit condition $\Pi_t^b = 0$ for every $b \in B$ and every $t = 1, 2, ...$, which gives

$$i_t^L L_t^b = \xi[(1-\theta)X_t^b p_t^2 i_t^R + \theta X_t^b p_{t+1}^2 i_{t+1}^R]. \tag{43}$$

Concerning clearing condition of firm loan market, observe that zero profit of firms indicates that the repayment for lending in period t equals the aggregate earning of the borrowing firm. Also, the aggregate earning of firms consists of sales and the gain from bubble riding. Let $M_t^a := \int_{j \in J^2} m_t^{j,a_2;s}$ denotes the gain from bubble riding. Then, the clearing condition of firm loan market is expressed as follows.

$$(1 + i_t^L)L_t = M_t^a + \Theta_{t,t} p_t^2 Y_t^2 + \Theta_{t,t+1} p_{t+1}^2 Y_{t+1}^2, \tag{44}$$

where $\Theta_{t,t} := \frac{(1-\theta)X_t^2}{Y_t^2}$ and $\Theta_{t,t+1} := \frac{\theta X_t^2}{Y_{t+1}^2}$ show the market shares of the current firms (i.e., firms who borrow in period t) in period t and $t+1$, respectively. Notable feature of Equation (44) is that current market interest rate i_t^L depends also on future sale. This feature allows future credit supply to affect current market interest rate, as elaborated later.

6.2 Equilibrium and steady state

Equilibrium is defined throughout periods. Following equations show relevant inter-period relationships: for $t = 1, 2, ...$,

$$M_{t+1}^1 = p_t^1 \hat{X}_t^1. \tag{45}$$

$$a_{t+1}^{i,l} = a_t^{i,l} + \triangle a_t^{i,l;f} \text{.for } l = 1, 2 \text{ and for every } i \in I \cup J. \tag{46}$$

In defining equilibrium, we let $\Omega_0 = (\alpha_0, I_0, w_0, p_0, M_0, a_0, s_0, i_0^R)$ denotes the initial economy, where $\alpha_0 := (\alpha_0^1, \alpha_0^2)$, $I_0 = (I_0^{1;0}, I_0^{1;1}, I_0^2)$, $w_0 := (w_0^1, w_0^2)$, $p_0 := (p_0^1, p_0^2, p_0^{a,1}, p_0^{a,2})$, $M_0 := (M_0^1, M_0^2)$ with $M_0^2 = 0$, $a_0 := \{a_0^{i,l}\}_{l=1,2,i\in I\cup J}$ and $s_0 := \{s_0^i\}_{i\in I}$.

Also in defining equilibrium, we let the price of secondary good in the following period is always consistent with the corresponding current equilibrium conditions: i) firms' bubble riding condition (shown in Section 3.1.4), ii) bank incentive condition (Equation (43)), and iii) loan market clearing condition (Equation (44)). We refer to this requirement as Inter-period price consistency in the following definition of equilibrium.

Definition 2 (Equilibrium)**.**

We take given exogenous values of $(\lambda, \mu, \kappa, \theta)$, initial economy Ω_0, sequence of productivity improvements $(\alpha_t^1, \alpha_t^2)_{t=2,3,...}$, and monetary policy $(i_t^L)_{t=2,3...}$. Then, the economy throughout periods $t = 0, 1, 2, ...$ is attained as a (strong) equilibrium if it is consistent with following i)-iv):

i) Realization of good and asset trades (explained in Section 3.1).

ii) Labor shift incentive and competitive wage schedule (explained in Section 3.2).

iii) Bank zero profit condition and loan market clearing condition (Equations (43) and (44)).

iv) Inter-period relationships (Equations (45) and (46)).

Note that the bank zero profit condition in the above condition iii) is required to be satisfied also for period 0 for an economy to be attained as a strong equilibrium. In this regard, we add to say that the economy is attained as a weak equilibrium if all the equilibrium conditions i)-iv) except for the bank zero profit condition for period 0 are satisfied. For the following analyses, we let "equilibrium" without mentioning strong or weak always refer to strong equilibrium, since most of the analyses examines strong equilibrium.

6.3 Fundamental macroeconomic equations

For our purpose, we are interested in equilibrium where money supply is sufficiently made such that $M_t^2 \geq \frac{n_t^{2e}}{n_t^{1e}} M_t^1$ is satisfied for every $t = 0, 1, 2, ...$ This indicates $p_t^1 = p_t^0$ for every $t = 1, 2, ...$ in equilibrium. Let $M_t^P := \int_{i\in I} m_t^{i,2}$, then, on the basis of inter-period price consistency, p_t^2 is expressed as follows.

$$p_t^2 = \frac{M_t^P}{Y_t^2} = \frac{L_t + M_t^e - M_t^a - S_{t-1,t}}{Y_t^2}, \tag{47}$$

where $M_t^e := \frac{1}{\kappa} n_t^2 - \int_{j\in J^2} a_{t-1}^{j,2}$ indicates equity finance and $S_{t-1,t} := \int_{i\in I} (s_t^i - s_{t-1}^i)$ denotes the aggregate net increase in household saving.

Using Equation (47), bank incentive condition (Equation (43)) in aggregate term and loan clearing condition (Equation(44)) derive the following fundamental macroeconomic

equations given positive credit supply for every period.

$$i_t^L = \xi[\frac{M_t^P}{L_t}\Theta_{t,t}i_t^R + \frac{M_{t+1}^P}{L_t}\Theta_{t,t+1}i_{t+1}^R]. \tag{48}$$

$$1 + i_t^L = \frac{M_t^a}{L_t} + \frac{M_t^P}{L_t}\Theta_{t,t} + \frac{M_{t+1}^P}{L_t}\Theta_{t,t+1}. \tag{49}$$

Here, focus on equilibrium where $M_t^a = 0$ (no bubble) and $S_{t-1,t} = 0$ (no saving) for every $t = 0, 1, 2, ...$ Then, equilibrium inter-period relationship across monetary policy, money supply and production is succinctly given by combining equations (48) and (49).

$$\frac{L_t + M_t^e}{L_t}\Theta_{t,t}\big(1 - \xi i_t^R\big) + \frac{L_{t+1} + M_{t+1}^e}{L_t}\Theta_{t,t+1}\big(1 - \xi i_{t+1}^R\big) = 1. \tag{50}$$

7 Analysis on monetary policy

Throughout the analysis, we let the lower bound of policy rate be zero (i.e., $i_t^R \geq 0$) for every period $t = 0, 1, 2, ...$ We prepare to show robust existence of weak equilibrium with zero credit supply in the following lemma.

Lemma 6 (Robust existence of equilibrium with zero credit supply)**.**

We take as given exogenous parameter values $(\lambda, \mu, \kappa, \theta)$*, arbitrary initial economy and productivity improvement sequence* $(\alpha_t^1, \alpha_t^2)_{t=1,2,...}$*. Then, there always exists a weak equilibrium with* $L_t = 0$ *for every* $t = 1, 2, ...$ *against any monetary policy* $(i_t^R)_{t=2,3,...}$*.*

While Lemma 6 poses the potential issue of instability of equilibrium with positive credit supply, we focus on characterization of equilibrium while omitting stability analysis.[38]

We firstly introduce the concept of steady state and relevant result.

Definition 3 (Steady state)**.**

Given exogenous values of $(\lambda, \mu, \kappa, \theta)$, we say initial economy Ω_0 is realized as a steady state when the economy attains an equilibrium under the environment of following i)-ii) and also the equilibrium satisfies following iii)- vi): for every $t = 1, 2, ...,$

i) (stable productivities) $\alpha_t^j = \alpha_0^j$ for $j = 1, 2$.

ii) (stable policy rates) $i_t^R = i_0^R$.

iii) (stable production) $X_t^j = X_0^j$ for $j = 1, 2$.

[38] Lemma 6 could indicate instability of equilibrium with positive credit supply such that any expected loss in bank profit at some period (caused through tight monetary policy in a general sense) could cause a cascade of credit shrink toward backward periods and finally settles at an equilibrium with zero credit supply. It could indicate subtlety of conducting monetary policy in practice. Note that formal analysis on the stability of equilibrium in this sense would require explicit examination of clearing process of lending market, which remains implicit under the current framework that resorts to zero profit condition of banks and rational expectation.

iv) (stable labor distribution) $I_t^j = I_0^j$ for $j = 1, 2$.
v) (stable credit supply) $L_t = L_0$.
vi) (stable wealth distribution) $\{a_t^{i,l}\}_{i \in I}$ for $l = 1, 2$.

Definition 4 (Optimal steady state)**.**

We say that a steady state is optimal when the associated allocation attains social optimality (see Definition 1).

For the statement of the following lemma, let $\alpha := \{\alpha^1, \alpha^2\}$ shows a pair of stable productivity sequences.

Lemma 7 (Existence of optimal steady state)**.**

For given parameter values of $(\lambda, \mu, \kappa, \theta)$ together with α, we have:
i) there always exists an initial economy that is realized as an optimal steady state, and
ii) such a steady state is realized only under zero policy rates.

Proof. See Appendix A.13. □

We proceed to discuss possibility of optimal monetary policy. Given initial optimal steady state, we say monetary policy $(i_t^R)_{t=1,2,...}$ is optimal when it is consistent with an equilibrium with $n_t^j = n_t^{je}$ for $j = 1, 2$ and every $t = 1, 2, ...$ In showing our impossibility result on optimal monetary policy, we make some preparation. For given productivity improvement sequence $(\alpha_t^1, \alpha_t^2)_{t=1,2,...}$, we let (ϵ_α^1, T) with $\epsilon_\alpha^1 > 0$ satisfying $\alpha_T^j < \alpha_{T+1}^j$ denote a perturbation in sector 1 that generates another productivity improvement sequence $(\tilde{\alpha}_t^1, \alpha_t^2)_{t=1,2,...}$ such that $\tilde{\alpha}_T^1 = \alpha_T^1(1 + \epsilon_\alpha^1) \leq \alpha_{T+1}^1$, while $\tilde{\alpha}_t^1 = \alpha_t^1$ for every $1 \leq t < T$ and every $t > T$. Meanwhile, we let (ϵ_α^2, T) with $\epsilon_\alpha^2 > 0$ denote a perturbation in sector 2 that generates another productivity improvement sequence $(\alpha_t^1, \tilde{\alpha}_t^2)_{t=1,2,...}$ such that $\tilde{\alpha}_T^2 = \alpha_T^2(1 + \epsilon_\alpha^2)$ for every $t \geq T$, while $\tilde{\alpha}_t^2 = \alpha_t^2$ for every $t \geq 1$.

Definition 5 (Generical impossibility)**.**

We take as given exogenous parameter values $(\lambda, \mu, \kappa, \theta)$, initial optimal steady state Ω_0 and productivity improvement sequence $(\alpha_t^1, \alpha_t^2)_{t=1,2,...}$. We say optimal monetary policy is generically impossible against perturbation in sector $j \in \{1, 2\}$ if there exists no optimal monetary policy, or even when there exists some optimal monetary policy, there no more exists optimal monetary policy against any perturbation (ϵ_α^j, T) that satisfies $\epsilon_\alpha^j > 0$ and $T > 0$.

Lemma 8 (Generical impossibility of optimal monetary policy)**.**

We take as given exogenous parameter values $(\lambda, \mu, \kappa, \theta)$ with sufficiently large κ, initial optimal steady state Ω_0 and productivity improvement sequence $(\alpha_t^1, \alpha_t^2)_{t=1,2,...}$. Then, if future temporal cease of productivity improvements for consecutive 3 periods is expected to arise at any period (i.e., for $j = 1, 2$, $\alpha_T^j = \alpha_{T+1}^j = \alpha_{T+2}^j$ for some $T \geq t$ and for any $t > 0$), optimal monetary policy is generically impossible against perturbation in sector 1 and also that in sector 2.

Proof. See Appendix A.14. □

Given the impossibility result, we analyze monetary policy which primarily pursues efficiency in labor allocation (efficiency-first) while pursuing economic equality is of secondary importance (equality-second). Specifically, we examine following type monetary policy.

Definition 6 (Modestly efficiency-first monetary policy)**.**

We say monetary policy $(i_t^R)_{t=1,2,...}$ is modestly efficiency-first when $(L_t)_{t=1,2,...}$ is realized in equilibrium such that: for every $t = 1, 2, ...$, by denoting $L_t^* := \frac{n_t^{2e}}{n_t^{1e}} M_0^1$,
i) (Efficiency-first) $L_t \geq L_t^*$
ii) (Equality-second) $L_t > L_t^*$ only if $i_t^R = 0$. Equivalently, we can write as $(L_t - L_t^*) i_t^R = 0$.

The equality-second property ii) is well-defined under our assumption of rational expectation of banks particularly concerning future credit supply. In fact, observe that if $(L_t \geq L_t^*, i_t^R = 0)$ is consistent with Equation (50) for period $t-1$, then when we take larger $i_t^R > 0$, we can always take certain larger L_t to satisfy the equation. This indicates that when we adopt efficiency-first monetary policy without requiring equality-second property, arbitrary larger $L_t > 0$ could be realized in equilibrium by imposing i_t^R be accordingly larger. On this point, although simultaneous rise in i_t^R and L_t is required under our analysis with rational expectation, we could interpret it in a dynamic context such that preceding increase in credit supply (L_t) that have been brought by some accommodative stance of monetary policy (efficiency-first but with looser care of inequality) must be regulated by explicit rise in policy rate (i_t^R) in ex-post. In this perspective, equality-second property serves to restrain unnecessary excess credit supply in pursuing efficiently-first property, and it consequently serves to restrain unnecessary inequality among households.

The next lemma provides an explicit procedure to derive equilibrium credit supply and policy rate for each period under modestly efficiency-first monetary policy.

Lemma 9 (Unique realization of credit supply and policy rate)**.**

We take as given exogenous parameter values $(\lambda, \mu, \alpha, \kappa, \theta)$ with $\kappa = \infty$, optimal steady state Ω_0 and productivity improvement sequence $(\alpha_t^1, \alpha_t^2)_{t=1,2,...}$. Then, under modestly efficiency-first monetary policy, in any equilibrium where there occurs no speculative trade and no saving, $(L_t, i_t^R)_{t=1,2,...}$ is uniquely realized as follows:
i) $L_{t+1} = \max\{L_{t+1}^*, L_{t+1}^0\}$, where

$$L_{t+1}^0 := \frac{1 - \Theta_{t,t}(1 - \xi i_t^R)}{\Theta_{t,t+1}} L_t.$$

ii) $i_{t+1}^R = \max\{i_{t+1}^{R,0}, 0\}$, where $i_{t+1}^{R,0}$ is given by the following equation.

$$1 - \xi i_{t+1}^{R,0} := \frac{1 - \Theta_{t,t}(1 - \xi i_t^R)}{\Theta_{t,t+1}} \frac{L_t}{L_{t+1}^*}.$$

Proof. See Appendix A.15. □

In examining the implication of modestly efficient-first monetary policy on excess credit, next lemma prepares to show that once excess credit has somehow arisen, then it must be sustained under stable economic environment.

Lemma 10 (Persistent excess credit under stable productivity sequence)**.**

We take as given all conditions for Lemma 9 to be satisfied. Then, under modestly efficiency-first monetary policy, in any equilibrium where there occurs no speculative trade and no saving, we have: for every $t = 1, 2, ...,$
if $L_{t-1} > L^*_{t-1}$ *(already excess credit),* $\alpha^j_t = \alpha^j_{t-1}$ *for* $j = 1, 2$ *(stable productivity) and* $i^R_{t-1} = 0$*, then* $L_t = L_{t-1} > L^*_t$ *and* $i^R_t = 0$*.*

Proof. See Appendix A.16. □

Lemma 10 provides a clue to discuss the occurrence of assert bubble since Proposition 3 shows that persistent excess credit under stable economic environment is a critical situation of asset bubble. Still, we have several issues in showing the occurrence of asset bubble. Firstly, Lemma 10 is silent on how excess credit could arise. Also, note that Lemma 10 does not ensure persistent nature of excess credit even when productivity sequence is eventually stable as stated in Lemma 10. In fact, somehow arisen excess credit could vanish before stable environment is attained. Facing with these issues, we proceed to examine the conditions for the occurrence of excess credit and also whether and when the arisen excess credit could be persistent.

7.1 Emergence of excess credit

We proceed to investigate two base situations of productivity improvements. Concretely, we examine once-and-for-all productivity improvements for each of primary and secondary sector which are formally defined as follows.

Definition 7 (One-time growth/development)**.**

Given (α^1_0, α^2_0), let $(\triangle\alpha^j, T)$ with $j \in \{1, 2\}$ and $T \geq 1$ refer to the sequence of productivity improvements $(\alpha^1_t, \alpha^2_t)_{t=1,2,...}$ that satisfy following i) and ii):
i) (improvement sector) $\alpha^j_T = \alpha^j_{T-1} + \triangle\alpha^j$ for $t = T$ while $\alpha^j_t = \alpha^j_{t-1}$ for every period $t \neq T$ and
ii) (non-improvement sector) for $j' \neq j$, $\alpha^{j'}_t = \alpha^{j'}_{t-1}$ for every $t = 1, 2, ...$

To simplify terminology, we refer to $(\triangle\alpha^j, T)$ with $j = 1$ as one-time development in period T, while that with $j = 2$ as one-time growth in period T.

Proposition 7 (One-time growth: no persistent effect on excess credit)**.**

We take as given exogenous parameter values $(\lambda, \mu, \alpha, \kappa, \theta)$ *with* $\kappa = \infty$*, optimal steady state* Ω_0 *and one-time growth* $(\triangle\alpha^2, T)$*. Then, under modestly efficiency-first monetary policy, excess credit never arises for period* $t \geq T + 1$*. More specifically, any equilibrium must satisfy:*

i) (Zero policy rate) $i^R_t = 0$ *for every period* $t = 1, 2, ...$

ii) (Temporary excess credit) $L_T > L^*_T := \frac{n^{2e}_t}{n^{1e}_t} M^1_0$ *and* $L_t = L^*_t$ *for every* $t \neq T$*.*

iii) (Size of excess credit)

$$\frac{L_T}{L^*_T} = 1 + (1 - \theta)\frac{\triangle\alpha^2}{\alpha^2_0}. \tag{51}$$

Proof. See Appendix A.17. □

Intuitively, Equation (50) indicates that since the expected growth in the following period (period t) has negative effect on the sales of the current borrowing firms (which borrow at period $t-1$) through lower share in the following period (i.e., $\Theta_{t-1,t}$), this negative effect needs be augmented by the corresponding rise of credit supply in the following period (period t) so that lending banks to the current firms will not make any loss. This essentially explains the occurrence of excess credit at the period of growth. Given this, the growth in the current period (period t) has positive effect on the sales of the current borrowing firms through higher share in the current period (i.e, $\Theta_{t,t}$). This positive effect on the sales need be countered by the corresponding decrease in credit supply in the following period (period $t+1$) so that lending banks never earn excess profit. Here, since policy rate at the growth period becomes zero (i.e., $i_t^R = 0$) under modestly efficient-first monetary policy, the required decrease in credit supply (i.e., $L_{t+1} - L_t$) is always equal to the previous increase in credit supply (i.e., $L_t - L_{t-1}$). Thus, there arises no persistent excess credit for the case of one-time growth. The next proposition shows that it is not the case for one-time development.

Proposition 8 (One-time development: persistent effect on excess credit)**.**

We take as given exogenous parameter values $(\lambda, \mu, \alpha, \kappa, \theta)$ *with* $\kappa = \infty$*, optimal steady state* Ω_0 *and one-time development* $(\triangle\alpha^1, T)$*. Then, under modestly efficiency-first monetary policy, excess credit arises and persistent for every period* $t \geq T+1$ *in any equilibrium without speculative trade and no saving. More specifically, there exists an equilibrium without speculative trade and no saving only if:*
i) (Temporary policy rate increase) $i_T^R > 0$ *and* $i_t^R = 0$ *for every* $t \neq T$*, where* i_T^R *is given by the following equation.*

$$1 - \xi i_T^R \;=\; \theta\Big(1 + \frac{\triangle\alpha^1/\alpha_0^1}{1 - \bar{x}^1/\alpha_0^1}\Big)^{-1} + (1-\theta)\Big(1 + \triangle\alpha^1/\alpha_0^1\Big)^{-1}. \tag{52}$$

ii) (Lagged impact and persistent excess credit) $L_t = L_{T+1} > L_T^*$ *for every* $t \geq T+2$ *while* $L_t = L_t^*$ *for* $t = 1, 2, ..., T$*.*
iii) (Size of excess credit)

$$\frac{L_{T+1}}{L_T^*} \;=\; \frac{1}{\theta} - \Big(\frac{1-\theta}{\theta}\Big)\Big(\frac{1}{1 + \triangle\alpha^1/\alpha_0^1}\Big). \tag{53}$$

Proof. See Appendix A.18. □

To see the intuition for the occurrence of persistent excess credit, observe that one-time development indicates an increase in optimal credit supply at the time of development. In reference to Equation (50), although the expected development (in period t) has negative effect on the sales of currently borrowing firms (which borrow at period $t-1$) through the expected lower share (i.e., $\Theta_{t-1,t}$), the positive effect on the expected sales through necessary increase in credit supply (i.e., L_t^*/L_{t-1}^*) is shown to dominate the negative effect.

Thus, the expected net excess profit needs be eliminated by the associated rise in policy rate (i.e., $i_t^R > 0$). Then, the rise in policy rate at the period of development (period t) in turn has negative effect on the profits of the current borrowing firms (period t). Though the firms face positive effect on their profits through higher current share (i.e., $\Theta_{t,t}$), this positive effect is dominated by the negative effect brought through the rise in policy rate. Thus, the net negative effect needs be augmented by the associated increase in the expected credit supply (period $t+1$). Here, according to Lemma 10, once excess credit supply arises while there no more exists development (nor growth), it becomes persistent excess credit.

Concerning the determination of the policy rate at the time of development, Part i) of Proposition 8 provides comparative statics such that the policy rate becomes higher for the economy that faces: i) larger $\frac{\triangle\alpha^1}{\alpha_0^1}$ (larger scale of development), ii) larger $\frac{\bar{x}^1}{\alpha_0^1}$ (lower development level) and iii) larger θ (larger lag in production), while iv) $\frac{\bar{x}^2}{\alpha_0^2}$ (growth level) has no effect. To see the intuition for these results, observe that Condition (52) is derived by

$$1-\xi i_T^R \;=\; \Big(\frac{L^*_{T-1}}{L^*_T}\Big)\Big(\frac{\Theta_{T-2,T-1}}{\Theta_{T-1,T}}\Big). \tag{54}$$

Here, let $L^*_{\frac{T}{T-1}} := \frac{L^*_T}{L^*_{T-1}}$ (optimal credit ratio) and $\Theta_{\frac{T-1,T}{T-2,T-1}} := \frac{\Theta_{T-1,T}}{\Theta_{T-2,T-1}}$ (expected share ratio) In this perspective, to see property i), observe that

$$\frac{\partial L^*_{\frac{T}{T-1}}}{\partial(\triangle\alpha^1/\alpha_0^1)} > 0 \text{ and } \frac{\partial \Theta_{\frac{T-1,T}{T-2,T-1}}}{\partial(\triangle\alpha^1/\alpha_0^1)} > 0. \tag{55}$$

For property ii), we have

$$\frac{\partial L^*_{\frac{T}{T-1}}}{\partial(\bar{x}^1/\alpha_0^1)} > 0 \text{ while } \frac{\partial \Theta_{\frac{T-1,T}{T-2,T-1}}}{\partial(\bar{x}^1/\alpha_0^1)} < 0. \tag{56}$$

Here, the former effect (on the optimal credit ratio) dominates the latter effect (on the expected share ratio). For property iii), we have

$$\frac{\partial L^*_{\frac{T}{T-1}}}{\partial\theta} = 0 \text{ and } \frac{\partial \Theta_{\frac{T-1,T}{T-2,T-1}}}{\partial\theta} > 0. \tag{57}$$

Lastly, for property iv), we have

$$\frac{\partial L^*_{\frac{T}{T-1}}}{\partial(\bar{x}^2/\alpha_0^2)} = 0 \text{ and } \frac{\partial \Theta_{\frac{T-1,T}{T-2,T-1}}}{\partial(\bar{x}^2/\alpha_0^2)} = 0. \tag{58}$$

Also, Part iii) of Proposition 8 indicates that excess credit ratio ($\frac{L_{T+1}}{L^*_T}$) becomes larger for the economy that faces: i) larger $\frac{\triangle\alpha^1}{\alpha_0^1}$ (larger scale of development) and ii) smaller θ (smaller lag in production), while iii) $\frac{\bar{x}^1}{\alpha_0^1}$ (development level) and $\frac{\bar{x}^2}{\alpha_0^2}$ (growth level) have no effect. To see the intuition, observe that Equation (53) is derived by

$$\frac{L_{T+1}}{L^*_T} \;=\; \frac{1-\Theta_{T,T}(1-\xi i_T^R)}{\Theta_{T,T+1}} \tag{59}$$

with $\Theta_{T,T+1} = \theta$. To see property i), observe

$$\frac{\partial \Theta_{T,T}}{\partial(\triangle\alpha^1/\alpha_0^1)} > 0 \text{ and } \frac{\partial i_T^R}{\partial(\triangle\alpha^1/\alpha_0^1)} > 0. \tag{60}$$

while the combined effect of these is given as

$$\frac{\partial \Theta_{T,T}(1-\xi i_T^R)}{\partial(\triangle\alpha^1/\alpha_0^1)} < 0. \tag{61}$$

To see property ii), observe

$$\frac{\partial \Theta_{T,T}}{\partial \theta} < 0 \text{ and } \frac{\partial i_T^R}{\partial \theta} > 0 \tag{62}$$

while the combined effect of these is given as

$$\frac{\partial \Theta_{T,T}(1-\xi i_T^R)}{\partial \theta} < 0. \tag{63}$$

However, the indicated increasing effect through $1-\Theta_{T,T}(1-\xi i_T^R)$ against an increase in θ is dominated by the lowering effect through $1/\Theta_{T,T+1}$.[39] To see property iii) with respect to the effect of $(\bar{x}^1/\alpha_0^1)$, observe

$$\frac{\partial \Theta_{T,T}}{\partial(\bar{x}^1/\alpha_0^1)} > 0 \text{ and } \frac{\partial i_T^R}{\partial(\bar{x}^1/\alpha_0^1)} > 0 \tag{64}$$

while the combined effect of these is canceled out as

$$\frac{\partial \Theta_{T,T}(1-\xi i_T^R)}{\partial(\bar{x}^1/\alpha_0^1)} = 0. \tag{65}$$

Lastly, concerning the effect of a change in $(\bar{x}^2/\alpha_0^2)$, observe that it has no effect on both $\Theta_{T,T}$ and i_T^R.

On the basis of the occurrence of persistent excess credit shown in Proposition 8, next proposition demonstrates the occurrence of asset bubble.

Proposition 9 (Development and asset inflation)**.**

We take as given exogenous parameter values $(\lambda, \mu, \alpha, \kappa, \theta)$ with $\kappa = \infty$ and sufficiently large $\lambda < 1$, optimal steady state Ω_0 and one-time development $(\triangle\alpha^1, T)$. Then, under modestly efficient-first monetary policy, we have following results I) and II).
I) (Occurrence of asset bubble) There exists an equilibrium with asset bubble if and only if

$$\frac{\triangle\alpha^1}{\alpha_0^1} > \theta\Big(\frac{\bar{x}^1/\alpha_0^1}{1-\bar{x}^2/\alpha_0^2} - 1\Big). \tag{66}$$

[39] In words, an increase in i_T^R serves to decrease the profit of banks in period T, which needs to be augmented by an increase in excess credit in period $T+1$. Also, a decrease in the sales share of those banks in period T (decrease in Θ) itself also serves to decrease the profit. However, an increase in the sales share of the banks in period $T+1$ serve to amplify the profit increasing effect of an increase in excess credit in period $T+1$, and this effect dominates the former effects.

II) (Realization as persistent asset inflation) Furthermore, if Equation (66) is satisfied, there exists an equilibrium with persistent asset inflation from period T on such that

i) (Persistent asset bubble) For every $t \geq T$, $\triangle p_t^{a_2;s} = \triangle p_T^{a_2;s} > 0$ while $\triangle p_t^{a_2;s} = 0$ for every $t < T$.

ii) (Stable good price during bubble) $p_t^j = p_{T+1}^j$ for every $t \geq T+1$.

Proof. See Appendix A.19. □

Part I) of Proposition 9 indicates that asset bubble is more likely to arise under the economy with: i) larger $\frac{\triangle \alpha^1}{\alpha_0^1}$ (larger scale of development), ii) lower $\frac{\bar{x}^2}{\alpha_0^2}$ (higher growth level), iii) lower $\frac{\bar{x}^1}{\alpha_0^1}$ (higher development level) and iv) lower θ (smaller production lag). To see the intuition, observe that Condition (66) is derived by $\frac{L_{T+1}}{L_T^*} > \frac{\bar{M}}{L_T^*}$, where threshold value $\bar{M}$ for the occurrence of asset bubble is derived in Proposition 1. Concretely, we have threshold value ratio (to the optimal credit supply) as

$$\frac{\bar{M}}{L_T^*} = \Big(\frac{1}{1-\bar{x}^2/\alpha_0^2}\Big)\Big(\frac{\bar{x}^1/\alpha_0^1}{1+\triangle\alpha^1/\alpha_0^1}\Big). \tag{67}$$

while equilibrium excess credit ratio ($\frac{L_{T+1}}{L_T^*}$) is shown in Proposition 8. In this perspective, to see property i), observe

$$\frac{\partial(L_{T+1}/L_T^*)}{\partial(\triangle\alpha^1/\alpha_0^1)} > 0 \text{ and } \frac{\partial(\bar{M}/L_T^*)}{\partial(\triangle\alpha^1/\alpha_0^1)} > 0, \tag{68}$$

while the effect on (L_{T+1}/L_T^*) dominates as

$$\frac{\partial(L_{T+1}/L_T^*)}{\partial(\triangle\alpha^1/\alpha_0^1)} > \frac{\partial(\bar{M}/L_T^*)}{\partial(\triangle\alpha^1/\alpha_0^1)}. \tag{69}$$

To see property ii), observe

$$\frac{\partial(L_{T+1}/L_T^*)}{\partial(\bar{x}^2/\alpha_0^2)} = 0 \text{ and } \frac{\partial(\bar{M}/L_T^*)}{\partial(\bar{x}^2/\alpha_0^2)} > 0. \tag{70}$$

To see property iii), observe

$$\frac{\partial(L_{T+1}/L_T^*)}{\partial(\bar{x}^1/\alpha_0^1)} = 0 \text{ and } \frac{\partial(\bar{M}/L_T^*)}{\partial(\bar{x}^1/\alpha_0^1)} > 0. \tag{71}$$

To see property iv), observe

$$\frac{\partial(L_{T+1}/L_T^*)}{\partial\theta} < 0 \text{ and } \frac{\partial(\bar{M}/L_T^*)}{\partial\theta} = 0. \tag{72}$$

Part II) of Proposition 9 addresses limitation of monetary policy in restraining asset bubble. Observe that to restrain asset bubble either ex ante or ex post, persistent excess credit must be somehow decreased. However, given zero profit condition of banks, there is no way for monetary policy to do this. In fact, under our model framework, committing to

raise future policy rate (i.e., i^R_{t+1}) will always be reacted by the associated increase in future credit supply (i.e., L_{t+1}) so that zero profit of banks is to be maintained (concerning firms which borrow at period t), and thus, it merely ends up in exacerbating excess credit.[40]

7.2 Attenuation of excess credit

We proceed to discuss how the emerged persistent excess credit could be affected through subsequent productivity changes. Following proposition shows no effect is brought by any growth sequence.

Proposition 10 (Subsequent sequential growth: No effect on persistent excess credit)**.**

We take as given exogenous parameter values $(\lambda, \mu, \alpha, \kappa, \theta)$ *with* $\kappa = \infty$ *and sufficiently large* $\lambda < 1$*, and also given optimal initial steady state* Ω_0*. Given* $\triangle\alpha^1_1 > 0$ *and* $\triangle\alpha^1_2 = 0$ *together with* $\triangle\alpha^2_1 = \triangle\alpha^2_2 = 0$ *(occurrence of excess credit), let* $\triangle\alpha^1_t = 0$ *for every* $t \geq 3$ *(no subsequent development), while* $\triangle\alpha^2_t \geq 0$ *for* $t = 3, 4...., T$ *and* $\triangle\alpha^2_t = 0$ *for every* $t \geq T+1$ *(subsequent sequential growth). Then, under modestly efficiency-first monetary policy, we have* $L_t = L_2$ *for every* $t \geq T+1$ *in any equilibrium without speculative trade nor saving.*

Proof. The proof is given in the same manner with the proof for Proposition 7, and thus omitted. □

Next proposition shows possible attenuation of persistent excess credit through subsequent sequential development. In the statement of the result, we focus on the situations of "natural attenuation," that indicates attenuation of persistent excess credit without any rise in policy rates.

Proposition 11 (Subsequent sequential development: natural attenuation of persistent excess credit through)**.**

We take as given exogenous parameter values $(\lambda, \mu, \alpha, \kappa, \theta)$ *with* $\kappa = \infty$ *and sufficiently large* $\lambda < 1$*, and also given optimal initial steady state* Ω_0*. Given* $\triangle\alpha^1_1 > 0$ *and* $\triangle\alpha^1_2 = 0$ *together with* $\triangle\alpha^2_1 = \triangle\alpha^2_2 = 0$ *(occurrence of excess credit), let* $\triangle\alpha^1_t > 0$ *for every* $t = 3, 4, ..., T$ *and* $\triangle\alpha^1_t = 0$ *for every* $t \geq T+1$ *(subsequent sequential development), while* $\triangle\alpha^2_t = 0$ *for every* $t \geq 3$ *(no subsequent growth). Then, under modestly efficiency-first monetary policy, we have all of following i)-iii) in any equilibrium without speculative trade nor saving if and only if* $L^*_T = L_2$ *is attained, that is,* $\alpha^1_T = \bar{x}^1 \frac{M^1_0 + L_2}{M^1_0}$*:*

i) $i^R_t = 0$ *for every* $t \geq 3$ *("naturality" requirement),*

ii) $L_t > L^*_t$ *for every* $t = 3, 4..., T$ *(sustained temporal excess credit) and*

iii) $L_t = L^*_t$ *for every* $t \geq T+1$ *(no persistent excess credit).*

Proof. See Appendix A.20. □

[40] In applying to realistic context where the zero profit property is not always valid, Proposition 9 could imply necessity of negative bank profit when persistent excess credit is to be decreased and consequently asset bubble is restrained. Formal discussion in this direction needs reshaping the whole framework, which is beyond the scope of this article.

Proposition 11 indicates that once excess credit has been emerged, it would not be "naturally" attenuated (while maintaining zero policy rates) if subsequent economy continues to be "stagnant" in realizing development (in the sense that the optimal credit in each subsequent period is short of the initially emerged excess credit).[41] In this perspective, the result indicates that a typical dynamics that enhances asset inflation is the combination of steep development and subsequent stagnation in development, while steady development would not necessarily enhance asset inflation since the emerged excess credit could be sufficiently attenuated by subsequent development sequence.

7.3 Cross effect on excess credit

We denote one-time simultaneous development and growth with $(\triangle\alpha^1, \triangle\alpha^2, T)$, which indicates i) $\alpha^j_T = \alpha^j_{T-1} + \triangle\alpha^j$ with $\alpha^j > 0$ for $j = 1, 2$ and ii) $\alpha^j_t = 0$ for $j = 1, 2$ and every $t \neq T$.

Proposition 12 (One-time simultaneous development and growth: non-linear effect on excess credit)**.**

We take as given exogenous parameter values $(\lambda, \mu, \alpha, \kappa, \theta)$ *with* $\kappa = \infty$*, optimal steady state* Ω_0 *and one-time simultaneous development and growth* $(\triangle\alpha^1, \triangle\alpha^2, T)$*. Then, under modestly efficiency-first monetary policy, in any equilibrium without speculative trade and no saving, excess credit for given* $\triangle\alpha^1$ *is characterized with threshold values* $\triangle_1$ *and* $\triangle_2$ *with respect to* $\triangle\alpha^2$ *such that:*

I) (attenuating region) For $\triangle\alpha^2 \leq \triangle_1$*, we have:*

i) (persistency periods) $L_{t+1} = L_t > L^*_T$ *for every* $t \geq T+1$ *with* $L_T = L^*_T$ *and*

ii) (decreasing excess credit) $\frac{\partial L_{T+1}}{\partial \triangle\alpha^2} < 0$.

II) (amplifying region) For $\triangle_1 \leq \triangle\alpha^2 \leq \triangle_2$*, we have:*

i) (persistency periods) $L_{t+1} = L_t > L^*_T$ *for every* $t \geq T+2$ *with* $L_t = L^*_T$ *for* $t = T, T+1$ *and*

ii) (increasing excess credit) $\frac{\partial L_{T+1}}{\partial \triangle\alpha^2} > 0$.

III) (maximum amplification region) For $\triangle_2 \leq \triangle\alpha^2$*, we have:*

i) (persistency periods) $L_{t+1} = L_t > L^*_T$ *for every* $t \geq T+2$ *with* $L_t = L^*_T$ *for* $t = T, T+1$ *and*

ii) (constant excess credit) $\frac{\partial L_{T+1}}{\partial \triangle\alpha^2} = 0$ *and* $L_{T+2} > L^*_T(\triangle\alpha^2 = 0)$.

Proof. See Appendix A.21. □

8 Implementation of bank incentive condition

In this section, we illustrate two typical ways of implementing bank incentive condition that is presented as Equation (43). One implementation is called interbank settlement

[41] Note that the proof indicates that attenuation is also possible while experiencing further development that eventually leads the optimal credit in some period to go beyond the initially emerged excess credit. But such development must be associated with positive policy rate in some period, and thus, it would be grasped as attenuation against a renewed situation of development with positive policy rate.

regime, while the other is called required reserve regime.

8.1 Additional settings

To allow detailed discussion, we explicitly let banks hold reserve in their central bank accounts. Reserve plays two roles: one is to settle interbank settlements and the other is to satisfy the required reserve ratio that is set by the central bank. In implementing the bank incentive condition, the central bank provides both deposit and lending facility, and can separately control the rates of deposit and lending facilities. Also, the central bank can control R_t^b for every $b \in B$ and $t = 1, 2, ..,$ where R_t^b shows the reserve holding of bank $b \in B$ at the beginning of period t. We assume away any interbank loan market to simplify our discussion.

Concerning the repayment structure of bank lending, here we let each firm who borrows in period t makes maximal repayment at the end of period t and makes the residual repayment at the end of period $t+1$. Given zero profit of firms, the repayment of a loan that has been provided in period t is made such that the whole sale of the indebted firm in each period is used for the repayment.

Setting of interbank settlements

We assume interbank obligations are formed through household payments to firms in purchasing secondary goods. By assuming symmetry formation of interbank obligations, we let each bank symmetrically face both temporal needs of reserve and temporal increase in reserve within each period. Concretely, for bank $b \in B$ which makes lending in period t, we let $R_{t,t}^{b,S}$ and $R_{t,t+1}^{b,S}$ denote the temporal needs of reserve to process the relevant interbank settlements in period t and $t+1$, respectively. At the same time, we let $R_{t,t}^{b,S}$ and $R_{t,t+1}^{b,S}$ also refer to temporal increase in reserve in the corresponding periods. We assume that $R_{t,t}^{b,S}$ and $R_{t,t+1}^{b,S}$ linearly depends on the sales in the corresponding periods with the ratio $\xi^S \in (0,1)$.[42]

Observe that the whole sales of firm $b \in B$ (which has borrowed from bank $b \in B$) in period t in the amount of L_t^b consists of the current sales $(1-\theta)X_t^b p_t^2$ and the sales in the following period $\theta X_t^b p_{t+1}^2$. Accordingly, we have $R_{t,t}^{b,S} = \xi^S(1-\theta)X_t^b p_t^2$ and $R_{t,t+1}^{b,S} =$

[42]We could elaborate the formation of ξ^S with some more detailed specification. To see the point, consider a situation where arbitrary firm j and the employees have bank accounts in the same bank and call the bank "home bank" for those households and the firm. In this setting, payments of wages are always processed within each bank, and thus, there never arises interbank settlements. Then, we let households purchase secondary goods from firms who have borrowed from some other banks as well as their home banks. Specifically, let ξ^{S1} ratio of each payment in purchasing secondary good flows to banks other than home bank. Thus, in aggregate term, ξ^{S1} ratio of payments forms the corresponding amount of interbank obligations. Now, suppose that ξ^{S2} ratio of interbank obligation needs be settled through reserve transfer across banks. Given formed interbank obligations, let ξ^{S3} denote the ratio of which each bank faces temporal need of reserve in processing own obligations. Under symmetry setting, we would have $\xi^{S3} = \frac{1}{2}$. These settings demonstrates a decomposition of ξ^S such that $\xi^S = \xi^{S1}\xi^{S2}\xi^{S3}$. ξ^{S1} depends on the distribution of firms in terms of their home banks. Meanwhile, ξ^{S2} would depend on the functioning of the corresponding interbank settlement system.

$\xi^S(1-\theta)X_t^b p_{t+1}^2$. Using $Z_{t,t} := \frac{\alpha_t^2 p_t^2}{w_t^2}$ and $Z_{t,t+1} := \frac{\alpha_t^2 p_{t+1}^2}{w_t^2}$, $R_{t,t}^{b,S}$ and $R_{t,t+1}^{b,S}$ are represented as follows:

$$R_{t,t}^{b,S} = \xi^S(1-\theta)Z_{t,t}L_t^b. \tag{73}$$
$$R_{t,t+1}^{b,S} = \xi^S\theta Z_{t,t+1}L_t^b. \tag{74}$$

Setting of reserve requirements

Let ξ^R denotes the required ratio of reserve. For bank $b \in B$ who makes lending in period t, let $R_{t,t}^{b,R}$ and $R_{t,t+1}^{b,R}$ denote the needs of reserve in period t and $t+1$, respectively. To simplify the discussion, we let each reserve requirement is calculated based on the current period average deposit size, for which average is taken after the whole selling of consumption good is made and before any repayment of firm loans is made.[43] Thus, the required reserve deposit base equals the sales of the borrower firm in each period. Formally, we have:

$$R_{t,t}^{b,R} = \xi^R(1-\theta)Z_{t,t}L_t^b. \tag{75}$$
$$R_{t,t+1}^{b,R} = \xi^R\theta Z_{t,t+1}L_t^b. \tag{76}$$

8.2 Two regimes of implementation

In discussing implementation of bank incentive condition, we prepare to clarify the available policy choices. Let $i_t^{R,L}$ denote the interest rate of lending facility. Concerning deposit facility, we prepare two relevant interest rates $i_t^{R,D;l}$ an $i_t^{R,D;e}$, where $i_t^{R,D;l}$ is the interest rate for reserve holding below and equal to the required reserve and $i_t^{R,D;e}$ is the interest rate for the excess reserve (i.e., reserve holding that exceeds the required reserve).

The central bank determines the reserve levels $\{R_t^b\}_{t=1,2,...}$ while setting the ratio of reserve requirement ξ^R, and choosing which of $i_t^{R,L}$, $i_t^{R,D;e}$ and $i_t^{R,D;l}$ are used as the policy rate i_t^R for each period $t = 1, 2, ...$

Interbank settlement regime

One approach of implementation is to utilize interbank settlements rather than resorting to reserve requirements. Concretely, let the central bank set $\xi^R = 0$ and $R_t^b = 0$ for every $b \in B$ and every period $t = 1, 2, ...$ Concerning the policy rate, let the central bank set $i_t^{R,L} = i_t^R$ and $i_t^{R,D;l} = i_t^{R,D;e} = 0$ for every period $t = 1, 2, ...$ Under these choices, bank incentive condition is successfully implemented with $\xi = \xi^S$.[44] Note that if the central bank had made a change as $i_t^{R,D;e} = i_t^R$ for every $t = 1, 2, ...,$ then the bank

[43] Note that any fluctuation in bank deposits associated with interbank payments has no effect on the required reserve deposit base under the current setting.

[44] To see the point more clearly, we could elaborate the setting so that sales of a firm in each period is formed through 4 sub-periods and the supplying firms in the period are grouped into two groups. Let sales of a firm in each sub-period has certain fluctuation such that the firms in one group face relatively smaller sales in periods 1 and 4 (e.g., $(9, 11, 11, 9)$) while the firms in the other group does so in periods

incentive condition would be no more valid since temporal reserve increase associated with interbank settlements would now earn the interest rate of $i^{R,D;e} = i^R_t$ and it would cancel out the cost of temporal reserve needs brought through interbank settlements.[45]

Reserve requirement regime

The other approach of implementation fits with the situation where banks face large excess reserve. To demonstrate the situation, let R^b_t for every bank $b \in B$ and $t = 1, 2, ...$ be set sufficiently high so that each temporal reserve need would be fully covered. On this basis, let $\xi^R > 0$ be set sufficiently small in reference to $\{R^b_t\}_{b\in B, t=1,2,...}$ and target credit supply such that the required reserve under the target credit would not surpass any of $\{R^b_t\}_{b\in B}$ in each period $t = 1, 2, ...$ Concerning the policy rate, let the central bank now set $i^{R,D;e}_t = i^{R,L}_t = i^R_t$ and $i^{R,D;l}_t = 0$ for every period $t = 1, 2, ...$ Under these choices, observe that an increase in lending of a bank brings the associated increase in reserve requirement, which means an decrease in excess reserve. Thus, it brings the associated lost interest earning from the excess reserve. Thus, bank incentive condition is successfully implemented with $\xi = \xi^R$.

9 Extension: Direct effect of policy rates on credit supply

In the baseline setting, the effects of monetary policy are examined through following two equations:

(Firm loan market clearing condition)

$$1 + i^L_t = \frac{L_t + M^e_t}{L_t}\Theta_{t,t} + \frac{L_{t+1} + M^e_{t+1}}{L_t}\Theta_{t,t+1}. \tag{77}$$

(Bank incentive condition)

$$i^L_t = \xi\Big(\frac{L_t + M^e_t}{L_t}\Theta_{t,t}i^R_t + \frac{L_{t+1} + M^e_{t+1}}{L_t}\Theta_{t,t+1}i^R_{t+1}\Big). \tag{78}$$

2 and 3 (e.g., $(11, 9, 9, 11)$). Given homogeneous lending of each bank and accordingly homogeneous aggregate sales of each firm in each period, banks who face net outflow of reserve in sub-period 1 (in the amount of $2 = 11 - 9$ in the example) necessarily faces net inflow of reserve in sub-period 3, while the other type of banks oppositely face net outflow in sub-period 3 while face net inflow in sub-period 1. Here, required reserve for interbank settlements correspond to the amount of net outflow. By imposing zero deposit rate for excess reserve, temporary net inflow will never earn deposit rate of reserve but only temporary net outflow brings loss in the opportunity of earning deposit rate.

[45] For successful implementation under the interbank settlement regime, note that we also assume that temporal reserve increase associated with interbank settlements cannot be used for interbank lending. Considering the nature of interbank settlement process wherein the whole payment network structure and the order of settlements essentially depend on the customers' payment behavior, both the timing of the occurrence and the duration of temporal reserve increase would not necessarily be predictable, and thus, it would not form stable source in providing interbank loan.

Particularly, a change in policy rates could have effects on credit supply only indirectly through the bank incentive condition in making lending to firms that is shown as Equation (78), while it has no effect through the loan market clearing condition.

In this section, we provide a generalization of the baseline setting to demonstrate additional direct effect of policy rates on overall credit supply, which is grasped through the generalized loan market clearing condition that replaces Equation (77), which becomes as follows:

(Generalized firm loan market clearing condition)

$$\begin{aligned} 1+i_t^L &= \frac{(L_t^F+M_t^e)+(L_t^H-\xi^H i_t^R L_{t-1}^H)}{L_t^F}\Theta_{t,t} \\ &\quad +\frac{(L_{t+1}+M_{t+1}^e)+(L_{t+1}^H-\xi^H i_{t+1}^R L_t^H)}{L_t^F}\Theta_{t,t+1}, \end{aligned} \tag{79}$$

In demonstrating Equation (79), we add to examine loans to households and accordingly add to examine a new group of banks we call housing-loan banks which specialize in providing housing loans, and let L_t^F and L_t^H denote the aggregate lending to firms and households in period t, respectively, while letting ξ^H denote the reserve relevant parameter for housing-loan banks. Observe that the direct effect of policy rates is demonstrated with explicit appearance of i_t^R and i_{t+1}^R in Equation (79). The associated generalized bank incentive condition in providing loan to firms becomes as follows:

(Generalized bank incentive condition in providing firm loans)

$$\begin{aligned} i_t^L &= \xi\Big(\frac{(L_t^F+M_t^e)+(L_t^H-\xi^H i_t^R L_{t-1}^H)}{L_t^F}\Theta_{t,t} i_t^R \\ &\quad +\frac{(L_{t+1}+M_{t+1}^e)+(L_{t+1}^H-\xi^H i_{t+1}^R L_t^H)}{L_t^F}\Theta_{t,t+1} i_{t+1}^R\Big). \end{aligned} \tag{80}$$

Setting with home equity loan

As the form of housing-loan, we specifically examine home equity type loan. Concretely, each household can borrow money from banks using holding primary asset as collateral. We let home equity of household $i \in I$ at the beginning of period t be evaluated as $p_{t-1}^1 a_{t-1}^{i,1}$. Since we assume households are short-sighted, they intend to borrow as much home equity loan as they can in each period. Given this observation, so as to allow households to make repayments by forming a new loan, we let repayments of home equity loans contracted at the beginning of period t terminates at the beginning of period $t+1$. Households who have contracted home equity loan in period t will face the associated repayments in period $t+1$, but they can make repayments by revolving home equity loan. When a household revolves home equity loan, we let the household need to make the interest payment but not need to repays the principle.[46] Thus, the available money for the household is the funds obtained through current loan netted by the interest payment for

[46] We effectively examine open-ended type home equity loan.

the previously contracted loan. Formally, by letting i_t^H denote the interest rate of home equity loan provided in period t, the budget of household $i \in I$ in period t becomes:[47]

$$w_t^i + p_t^1 a_t^{i,1} - i_t^H p_{t-1}^1 a_{t-1}^{i,1}, \tag{81}$$

where $i_t^H p_{t-1}^1 a_{t-1}^{i,1}$ shows the interest payment.

Profit of housing-loan bank $b \in B$ in providing loan in period t in the size of $L_t^{b,H}$ is represented as follows:

$$\Pi_t^b = i_t^H L_t^{b,H} - i_t^R \xi^H L^{b,H}. \tag{82}$$

We assume competitive supply of home equity loan such that housing-loan banks earn zero profit in equilibrium, and thus, we have:

$$i_t^H = i_t^R \xi^H. \tag{83}$$

Here, the formation of unit cost of housing loan $i_t^R \xi^H$ can be examined in a similar way to the case of firm loans that are discussed in Section 8.[48] Given these observations, the aggregate credit supply in period t becomes:

$$(L_t^F + M_t^e) + (L_t^H - \xi^H i_t^R L_{t-1}^H). \tag{84}$$

This gives the generalized conditions shown in Equations (79) and (80). Particularly, in equilibrium with stable prices of primary goods and stable holding of primary assets such that $p_t^1 = p_0^1$ and $a_t^{i,1} = a_0^{i,1}$ for every $i \in I$ and for every $t = 1, 2, ...$, the aggregate housing loan also becomes stable as $L_t^H = L_1^H = p_0^1 a_0^1$ for every $t = 1, 2, ...$

10 Concluding remarks

This paper established a new dynamic monetary (non-stochastic) general equilibrium framework, which is dubbed as "credit-driven" economy. This article demonstrated that the framework provides a consistent and consolidated view on the issues of segregated boom, asset bubble and monetary policy, while it formalizes the concept of equilibrium excess credit and alongside fundamentally reframes the analysis of asset bubble and monetary policy. Since this article largely focused on normative analysis, there is limitation particularly in addressing realistic functioning of monetary policy in affecting real economy. In enhancing the proposed framework for the purpose of positive analysis, there would exist various directions such as reframing with non-rational expectation of banks, addressing heterogeneity across households, allowing unemployment and enabling to analyze demographic dynamics.

[47] We effectively

[48] Note that ξ^H can be different from ξ depending on the assumed situation. In fact, since we assume housing-loan banks specialize in providing housing-loan under the current setting, the whole payments of secondary goods are transferred to the accounts of non housing loan banks (firm loan banks), and thus, ξ^H would tend to be much higher than ξ.

A Appendix

A.1 Stabilizing role of price lower and upper bounds

The purpose of this section is to formally discuss the roles of price lower and upper bounds in stabilizing monetary economy. In order to see the essential unstable feature of monetary economy, the current section adopts a simpler model which we call cash economy where banks or credits are omitted. Thus, we let (M_0^1, M_0^2) show the initial cash distribution for sector 1 and 2. Also, we let $M_{t+1}^j = \int_{i\in I} m_t^{i,j}$. Let $M = M_0^1 + M_0^2$ shows the aggregate cash.

Furthermore, we omit (both primary and secondary) assets from our analysis, and exclude lag structure in production (i.e., $\theta = 1$). We examine constant productivities as $\alpha_t^j = \alpha^j$ for every $t = 0, 1, 2, ...$ for $j = 1, 2$. Accordingly, production technology of secondary sector becomes $F_t^2(n_t^2) = \alpha^2 n_t^2$. Also, utility function is simplified as follows.

$$u((x_t^j)_{j=1,2}) = f(x) - c(\triangle x_t^1, \triangle x_t^2), \tag{85}$$

where x denotes consolidated consumption as below.

$$x : \quad = \quad \begin{cases} \min\{x_t^1, \bar{x}^1\} & \text{if } x_t^1 < \bar{x}^1 \\ \bar{x}^1 + x_t^2 & \text{if } x_t^1 \geq \bar{x}^1 \text{ and } x_t^2 < \bar{x}^2 \\ \bar{x}^1 + \bar{x}^2 & \text{otherwise.} \end{cases}$$

To examine the roles of price lower and upper bound, we examine an economy with the minimum price lower-bound together with no price upper-bound. Specifically, let $\epsilon_p > 0$ be sufficiently small positive value. Then, we replace formulation (14) in Subsection 3.1 with the following formulation.

$$\hat{X}_{t,k}^1 \quad = \quad \begin{cases} \bar{x}^1 & \text{if } X_t^1 \geq \bar{x}^1 \text{ and } \frac{\int_{i\in I} m_t^{i,1}}{\bar{x}^1} \leq \epsilon_p \\ X_t^1 & \text{otherwise.} \end{cases} \tag{86}$$

In excluding price upper-bound, we let $\hat{m}_t^{i,1} = m_t^{i,1}$ instead of conditional price cap introduced with formulation (15) in Subsection 3.1.

Following observation reveals the role of price lower and upper bounds in stabilizing monetary economy.

Observation 2 (Unstablity and destruction of cash economy)**.**

We let the initial economy be given by arbitrary equal and efficient steady state. Consider a once-and-for-all productivity shock in primary sector that arises in period 1. Formally, for $t \geq 1$, let $\alpha_t^1 = \alpha^{1'} > \alpha^1$, where Assumptions i)-iii) are maintained with $\alpha^{1'}$. Then, the economy must experience following dynamics:
i) (Instability)

i-1) (Upstream cascade) For period $t = 2$, sufficiently large portion of labor shifts to the secondary sector. Formally, $\frac{n_2^2}{n_2^1} = \frac{M - \bar{x}^1 \epsilon_p}{\bar{x}^1 \epsilon_p}$.

i-2) (Reversal cascade) For period $t = 3$, all labor shifts to the primary sector, that is, $(I_t^0, I_t^1) = (I, \emptyset)$.

ii) (Destruction) The economy attains a steady state for period $t \geq 3$ with $(I_t^0, I_t^1) = (I, \emptyset)$ *and* $(M_t^1, M_t^2) = (M, 0)$.

Proof.

For period 1, suppose that there occurs no labor shift. Given this, good market clearing process attains $p_1^1 = \epsilon_p$. Given these clearing prices, households have no incentive to move to other sector. Thus, we have $(n_1^1, n_1^2) = (n_0^1, n_0^2)$. Since $m_1^{i,1} = \epsilon_p \bar{x}^1$ for every $i \in I_0^1$, we have $(M_2^1, M_2^2) = (\epsilon_p \bar{x}^1, M - \epsilon_p \bar{x}^1)$.

For the labor shift process in period 2, households always move from primary to secondary sector in the initial round since the resultant production of primary good still exceeds demand (i.e., $X_2^1 > \bar{x}^1$). In this logic, households keep moving toward secondary sector as long as the resultant production is equal to or larger than the demand (i.e., $X_2^1 \geq \bar{x}^1$). Now, consider a round k where the resultant production has become equal to the demand. When a household moves to secondary sector in round $k+1$, it causes insufficient supply (i.e., $X_2^1 < \bar{x}^1$). In that case, the clearing process of good prices indicates a price increase of primary good. Here, since there is no price cap, households in secondary sector can squeeze those in primary sector. As there is sufficiently small cash in primary sector, each household in secondary sector can attain the satiation level of consumption of primary good while each household in primary sector consequently can consume only below the satiation level of primary good. Thus, there always occur labor shift toward secondary sector in round $k+1$. Such labor shift continues until the wages become equal. Once the equal wages would attain, there is no incentive to move across sectors. Thus, (n_2^1, n_2^2) is derived by $\frac{M - \bar{x}^1 \epsilon_p}{n_2^1} = \frac{\bar{x}^1 \epsilon_p}{n_2^2}$. This proves part i-1).

Given the derived labor distribution (n_2^1, n_2^2), observe that production of primary good is short of demand (i.e., $X_2^1 < \bar{x}^1$). Since wages are equal across sectors, every household spends all the holding cash to buy primary goods. Thus, we have $(M_3^1, M_3^2) = (M, 0)$. Given the cash distribution, labor shift process in period 3 apparently attains complete labor shift towards the primary sector, that is, $(I_3^1, I_3^2) = (I, 0)$. This proves part i-2).

Then, given the labor distribution in period 3, clearing process of good markets trivially indicates no spending to secondary good and all the cash is again used to buy primary goods. Thus, we have $(M_4^1, M_4^2) = (M, 0)$. Given the labor distribution realized in period 3, there is no more incentive to move across sectors in period 4. This indicates a steady state with $(I_t^1, I_t^2) = (I, 0)$ and $(M_t^1, M_t^2) = (M, 0)$ for $t \geq 3$. This proves part ii) and completes the whole proof. □

A.2 Proof of Lemma 1

Proof of I) and II)

Firstly we see the case i) $X_t^1 = \bar{x}^1$. The clearing condition of primary good indicates $p_t^1 = p_{t-1}^1$. Since $w_t^i > p_{t-1}^1 \bar{x}^1$, we have $m_t^{i,1} = p_{t-1}^1 \bar{x}^1$ and $x_t^{i,1} = \bar{x}^1$ for every $i \in I$. Given this, for secondary good and secondary asset, Step 2 of the realization process ensures the existence of the prices $p_t^2, p_t^{a,2}$ and the associated trades. For primary asset, there occurs no trade in any case given the setting of $a_t^{i,1} = \bar{a}^1$ for every $i \in I$. Concerning the price $p_t^{a,1}$, it is indeterminate (any price level is consistent) in any case. Note that even in the case where there exists household $i \in I$ with $a_t^{i,2} = 0$, the household i cannot increase own utility through selling primary asset (and purchasing secondary good), thus price of primary asset is irrelevant for the household's decision.

We proceed to see the case ii) $X_t^1 > \bar{x}^1$. The clearing condition of primary good indicates $p_t^1 = \mu p_{t-1}^1$. Since $w_t^i > p_{t-1}^1 \bar{x}^1$, we have $m_t^{i,1} = \mu p_{t-1}^1 \bar{x}^1$ and $x_t^{i,1} = \bar{x}^1$ for every $i \in I$. The rest of the proof is the same as the case i).

For the case iii) $X_t^1 < \bar{x}^1$, the clearing condition of primary good now indicates $p_t^1 = w_t^{min}$. Given $w_t^j \geq w_t^{min}$ by the definition, we have $m_t^{i,1} = w_t^{min}$ and $x_t^{i,1} = X_t^1 < \bar{x}^1$ for every $i \in I$. The insufficient consumption of primary good indicates indifference in engaging in trades of secondary good and secondary asset. In any case, Step 2 of the realization process ensures the existence of the prices and the associated trades.

Proof of III)

Suppose that there occurs some speculative trade but there exists no household $i \in I$ with $x_t^{i,2} = \bar{x}^2$. Then, there must exist household $i \in I$ who engages in speculative trade and becomes net loser since any firm never decides to become net loser. Given P_t, household $i \in I$ can increase utility by buying up secondary good by using money lost through engaging in speculative trades. To maximize utility, household $i \in I$ decides to buy up secondary good until $x_t^{i,2} = \bar{x}^2$ or use up all money and cease to engage in any speculative trade. Thus, any household $i \in I$ who is net loser must consume $x_t^{i,2} = \bar{x}^2$. This completes the proof of III).

Proof of IV)

If there exists realization with $P := \triangle p_t^{a,2} \neq 0$, then Part III) of the current lemma states that every net loser attains maximum possible consumption. Thus, the net losers are indifferent in providing money through speculative trades. Let M denotes the aggregate supply of money (i.e., lost money) by the net losers. Then, arbitrary $0 \leq M' \leq M$ is consistent with utility maximization of the net losers. Also, concerning the households engaging in sell-buy trades, if they are net losers (that is, $P > 0$), then they are indifferent concerning the amount of selling assets; meanwhile, if they are net winners ($P < 0$), then they have incentive in selling all holding asset. Thus, we can safely fix the amount of initial selling asset A in examining multiplicity of clearing prices for speculative trades. Given this, there always exists realization with $0 \leq M' \leq M$ that attains $P' = \frac{M'}{A}$ for given $P > 0$ while attains $P' = -\frac{M'}{A}$ for given $P < 0$.

A.3 Proof of Lemma 2

It is sufficient to examine two cases: 1) $n^1_{t-1} = n^{e1}_t$ and 2) $n^1_{t-1} > n^{e1}_t$. For the case 1), suppose that there has not occurred any labor shift. Since $X^1_t = \bar{x}^1$, according to Part II of Lemma 1, we have $p^1_t = p^1_{t-1}$ and $x^{i,1}_t = \bar{x}^1$ for every $i \in I$. Given this, we examine the incentive of household $i \in I^1_{t-1}$ in moving to sector 2 in initial round $k = 1$. The labor shift indicates $n^1_t = n^1_t - \epsilon_n < n^{1e}_t$, and thus, $X^1_t < \bar{x}^1$. According to Part II of Lemma 1, $x^{i,1}_t < \bar{x}^1$. This indicate a decrease in utility, thus there is no incentive to move. Next, we examine the incentive of household $i \in I^2_{t-1}$ in moving to sector 1 in initial round $k = 1$. By the labor shift in the unit of n_ϵ, there is no change in the supply of primary good while the supply of secondary good would surely decrease. Also, the moving household would necessarily face lower wage. Thus, the utility after moving is expected to decrease since the relative purchasing power (i.e., relative size of the budget) would decrease while good supply would not increase. Thus, there is no incentive for any household to move to sector 1.

We proceed to the case 2) $n^1_{t-1} > n^{1e}_t$. For household $i \in I^1_{t-1}$, moving to sector 2 in initial round $k = 1$ would realize larger purchasing power while supply of secondary good surely increases. Note that the labor shift toward sector 2 is accompanied with decrease in available secondary asset for households since certain fraction is used for production. Still, since we assume asset intensity parameter κ is sufficiently large, the effect on the decrease in asset supply is safely ignored. Thus, there exist households who move to sector 2 in the successive rounds until the efficient labor distribution $n^1_t = n^{1e}_t$ is attained. Once the efficient labor distribution is attained, there occurs no more labor shift as is straight from the proof for the case i).

A.4 Proof of Observation 1

Suppose that $M^2_t < \frac{n^e_2}{n^e_1} M^1_t$. Then, if the efficient labor distribution is attained in period t, there must exist round $k \geq 1$ in the labor shift process for which a potential mover $i \in I^1_{t-1}$ faces smaller expected wage. Let $w^j_{t,k}$ denote the expected wage of sector $j = 1, 2$ that would be realized when a unit of household moves to sector 2. It is sufficient to show that any household $i \in I^1_{t-1,k-1}$ has no incentive to move to sector 2 at round k under the situation of $w^1_{t,k-1} = w^2_{t,k-1}$.

Let $B^2_{t,k}$ refer to the expected budget of household $i \in I^1_{t-1,k-1}$ in terms of secondary good after moving to sector 2, that is, $B^2_{t,k} = \frac{w^2_{t,k} - p^1_{t-1}\bar{x}^1}{E_{t,k}[p^2_t]}$, where $E_{t,k}[p^2_t]$ denote rationally expected price of secondary good when labor shift terminates at the end of round k. Also, let $Y_{t,k}$ denote the expected supply of secondary good if there exists labor shift at round k in period t. Observe $Y_{t,k} = \theta\alpha^2 n^2_{t-1} + (1-\theta)\alpha^2 n^2_{t,k} = Y_{t,k-1} + (1-\theta)\alpha^2 n_\epsilon$, where we simplify notation with $\alpha^2 := \alpha^2_t$.

We are going to show that $\lim_{\kappa\to\infty} B^2_{t,k-1} > \lim_{\kappa\to\infty} B^2_{t,k}$ for arbitrary $n_\epsilon > 0$ and also $\frac{\partial \lim_{\kappa\to\infty} B^2_{t,k}}{\partial n_\epsilon}|_{n_\epsilon=0} < 0$. For this purpose, observe $\frac{M^1_t}{M^2_t} = \frac{n^1_{t,k-1}}{n^2_{t,k-1}}$, $w^2_{t,k} = M^2_t/(n^2_{t,k-1} + n_\epsilon)$ and

$p^1_{t-1}\bar{x}^1 = M^1_t$. Also, we have $\frac{1}{E_{t,k}[p^2_t]} = \frac{Y_{t,k-1}+(1-\theta)\alpha^2 n_\epsilon}{M^2_t + M^{2,a_2}_{t,k}}$, where $M^{2,a_2}_{t,k}$ denotes the money supplied to purchase secondary asset (in the amount of $\frac{1}{\kappa}n_\epsilon$) for production. Observe that $\lim_{\kappa\to\infty} M^{2,a_2}_{t,k} = 0$. Then, we have:

$$\begin{aligned}
\lim_{\kappa\to\infty} B^2_{t,k} &= \Big(\frac{1}{n^2_{t,k-1}+n_\epsilon} - \frac{M^1_t}{M^2_t}\Big)\big(Y_{t,k-1}+(1-\theta)\alpha^2 n_\epsilon\big) \\
&= \Big(\frac{1}{n^2_{t,k-1}+n_\epsilon} - \frac{n^1_{t,k-1}}{n^2_{t,k-1}}\Big)\big(Y_{t,k-1}+(1-\theta)\alpha^2 n_\epsilon\big) \\
&= \Big(\frac{n^2_{t,k-1} - n^1_{t,k-1}(n^2_{t,k-1}+n_\epsilon)}{n^2_{t,k-1}(n^2_{t,k-1}+n_\epsilon)}\Big)\big(\theta\alpha^2 n^2_{t-1} + (1-\theta)\alpha^2(n^2_{t,k-1}+n_\epsilon)\big) \\
&= \Big(\frac{n^2_{t,k-1} - n^1_{t,k-1}(n^2_{t,k-1}+n_\epsilon)}{(n^2_{t,k-1})^2}\Big)\Big(\frac{n^2_{t,k-1}}{n^2_{t,k-1}+n_\epsilon}\theta\alpha^2 n^2_{t-1} + (1-\theta)\alpha^2 n^2_{t,k-1}\Big) \\
&= \Big(\frac{(1-n^1_{t,k-1})n^2_{t,k-1} - n^1_{t,k-1}n_\epsilon}{(n^2_{t,k-1})^2}\Big)\Big(\frac{n^2_{t,k-1}}{n^2_{t,k-1}+n_\epsilon}\theta\alpha^2 n^2_{t-1} + (1-\theta)\alpha^2 n^2_{t,k-1}\Big) \\
&= \Big(1 - \frac{n^1_{t,k-1}}{(n^2_{t,k-1})^2}n_\epsilon\Big)\Big(\frac{n^2_{t,k-1}}{n^2_{t,k-1}+n_\epsilon}\theta\alpha^2 n^2_{t-1} + (1-\theta)\alpha^2 n^2_{t,k-1}\Big)
\end{aligned}$$

Since $w^1_{t,k-1} = w^2_{t,k-1}$, observe $\lim_{\kappa\to\infty} B^2_{t,k-1} = \theta\alpha^2 n^2_{t-1} + (1-\theta)\alpha^2 n^2_{t,k-1}$. Thus, we have $\lim_{\kappa\to\infty} B^2_{t,k-1} > \lim_{\kappa\to\infty} B^2_{t,k}$ for $n_\epsilon > 0$. Note that this result is sufficient for the current observation to be satisfied for the simplified model without asset that is introduced in Appendix A.1. To examine effects through potential asset trades, we need more detailed result. Let $X := 1 - \frac{n^1_{t,k-1}}{(n^2_{t,k-1})^2}n_\epsilon$ and $Z := \frac{n^2_{t,k-1}}{n^2_{t,k-1}+n_\epsilon}\theta\alpha^2 n^2_{t-1} + (1-\theta)\alpha^2 n^2_{t,k-1}$. Then, $\frac{\partial \lim_{\kappa\to\infty} B^2_{t,k}}{\partial n_\epsilon} = \frac{\partial X}{\partial n_\epsilon}Z + \frac{\partial Z}{\partial n_\epsilon}X$. Here, we have $\frac{\partial X}{\partial n_\epsilon} = -\frac{n^1_{t,k-1}}{(n^2_{t,k-1})^2} < 0$, $Z > 0$, $\lim_{n_\epsilon\to 0} X = 1$ and $\frac{\partial Z}{\partial n_\epsilon} = -\frac{n^2_{t,k-1}}{(n^2_{t,k-1}+n_\epsilon)^2}\theta\alpha^2 n^2_{t-1}$. Thus, $\frac{\partial B^2_{t,k}}{\partial n_\epsilon}|_{n_\epsilon=0} < 0$.

Now, we proceed to show that budget $\lim_{\kappa\to\infty} B^2_{t,k}$ is insufficient to gain higher utility even when household $i \in I^1_{t,k-1}$ can sell assets after moving to sector 2 at round k. Figure 6 illustrates the procedure. For preparation, observe that $\lim_{\kappa\to\infty} E_{t,k}[p^2_t] = \lim_{\kappa\to\infty} E_{t,k-1}[p^2_t] = M^2_t/Y^2_t$. Thus, let $p^2_t = M^2_t/Y^2_t$. Given this, let $(a_0, x_0) := (\bar{a}^1 + a^i_{t-1}, \bar{x}^1 + w^2_{t,k-1}/p^2_t)$ denote the pair of expected asset holding and consumption when the household will not move in round k. Here, note that there will occur no asset trade given realized equality. Then, let z denote the required budget in terms of secondary good such that the household would attain utility level $u(a_0, x_0)$ through asset trades after moving to sector 2. In addition, let (a', x') denote the pair of asset holding and consumption that is attained under utility maximization given budget (in terms of secondary good) z and the initial holding of asset a_0. In Figure 6, from the ratio of the sides of the corresponding triangle, we have $\frac{x'}{a''} = MRS_{ax}|_{(a,x)=(a',x')}$. Here, observe $MRS_{ax}|_{(a,x)=(a',x')} = \frac{\nu}{1-\nu}\frac{x'}{a'}$ from the definition. From these equations, we have:

$$a'' = \frac{1-\nu}{\nu}a'. \tag{87}$$

Also, from similarity of the corresponding triangles, $x' : z = a'' : a' + a'' - a_0$. Thus, we have:

$$z = \frac{x'}{a''}(a' + a'' - a_0). \tag{88}$$

Using equation (87), equation (88) becomes:

$$z = \big(\frac{\nu}{1-\nu}\big)\big(\frac{x'}{a'}\big)\big(\frac{1}{\nu}a' - a_0\big). \tag{89}$$

Let $\bar{u} = u(a_0, x_0) = u(a', x')$. Then, we have:

$$a' = \big(\frac{x'}{a'}\big)^{-1+\nu}\bar{u}. \tag{90}$$

Using equation (90), equation (89) becomes:

$$z = \big(\frac{1}{1-\nu}\big)\big(\frac{x'}{a'}\big)^{\nu}\bar{u} - \big(\frac{\nu}{1-\nu}\big)\big(\frac{x'}{a'}\big)a_0. \tag{91}$$

From homotheticity of preference over good consumption and asset holding, together with the effects on the production (increase by $(1-\theta)\alpha^2 n_\epsilon$) and the aggregate asset holding for household (no effect as $\lim_{\kappa\to\infty}(1/\kappa)n_\epsilon = 0$) , we have:

$$\frac{x'}{a'} = \frac{x_0 + (1-\theta)\alpha^2 n_\epsilon}{a_0}. \tag{92}$$

Using equation (92) together with $\bar{u} = a_0^{\nu}x_0^{1-\nu}$, equation (91) becomes:

$$\begin{aligned} z = &\big(\frac{1}{1-\nu}\big)x_0^{1-\nu}\big(x_0 + (1-\theta)\alpha^2 n_\epsilon\big)^{\nu} \\ &-\big(\frac{\nu}{1-\nu}\big)\big(x_0 + (1-\theta)\alpha^2 n_\epsilon\big). \end{aligned} \tag{93}$$

Let $X := x_0 + (1-\theta)\alpha^2 n_\epsilon$. Then, $\frac{\partial z}{\partial n_\epsilon} = \frac{\partial z}{\partial X}\frac{\partial X}{\partial n_\epsilon}$. Thus, we have:

$$\frac{\partial z}{\partial n_\epsilon} = \big(\frac{\nu}{1-\nu}X^{\nu-1}x_0^{1-\nu} - \frac{\nu}{1-\nu}\big)(1-\theta)\alpha^2. \tag{94}$$

$$\tag{95}$$

Since $\lim_{n_\epsilon\to 0} X = x_0$, we have $\frac{\partial z}{\partial n_\epsilon}|_{n_\epsilon=0} = 0$. Combining with $\frac{\partial B^2_{t,k}}{\partial n_\epsilon}|_{n_\epsilon=0} < 0$, we have $z > \frac{\lim_{\kappa\to\infty} B^2_{t,k}}{p_t^2}$ for sufficiently small $n_\epsilon > 0$, and this completes the proof.

A.5 Proof of Lemma 4

From Lemma 2, we have $M^2_{t'} = \frac{n^{2e}_{t'}}{n^{1e}_{t'}}M^1_{t'}$ for $t' = t$ and $t-1$. Thus, we have $\triangle M = (\frac{n^{2e}_t}{n^{1e}_t} - \frac{n^{2e}_{t-1}}{n^{1e}_{t-1}})M^1_t$. Since $M^1_t = n^{1e}_{t-1}w_{t-1}$, we have:

$$\begin{aligned} \triangle M &= n^{2e}_t\frac{n^{1e}_{t-1}}{n^{1e}_t}w_{t-1} - n^{2e}_{t-1}w_{t-1} \\ &= n^{2e}_t(\frac{n^{1e}_{t-1}}{n^{1e}_t}w_{t-1} - w_{t-1}) + (n^{2e}_t w_{t-1} - n^{2e}_{t-1}w_{t-1}) \\ &= n^{2e}_t\triangle w + w_{t-1}(n^{2e}_t - n^{2e}_{t-1}). \end{aligned}$$

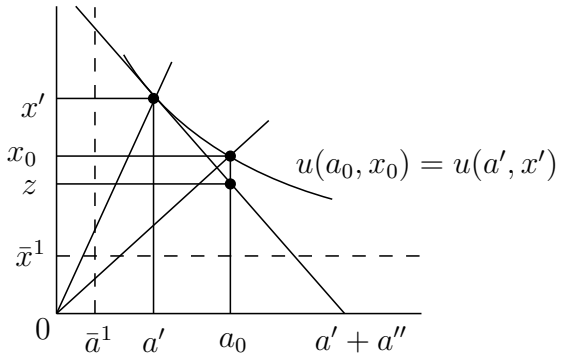


Figure 6:

Combining with the results on economic equality brought by Lemma 3, it is immediate that the above $\triangle M$ realizes the social optimal allocation in period t.

A.6 Proof of Lemma 5

Part i) is immediate. For Part ii), let E_t denote the money supplied for the purchase of asset by firms. Then, we have:

$$\begin{aligned} p_t^2 &= (M_t^2 + E_t)/Y_t^2 \\ &= (M_{t-1}^2 + \triangle M + E_t)/(Y_{t-1}^2 + \triangle Y) \\ &= \frac{M_{t-1}^2 + \triangle M + E_t}{Y_{t-1}^2} \frac{Y_{t-1}^2}{Y_{t-1}^2 + \triangle Y} \\ &= \Big(p_{t-1}^2 + \frac{\triangle M + E_t}{Y_{t-1}^2}\Big) \frac{Y_{t-1}^2}{Y_{t-1}^2 + \triangle Y}. \end{aligned}$$

Observe that $E_t > 0$ for finite κ while $\lim_{\kappa \to \infty} E_t = 0$.

For Part iii), observe that efficient labor shift $\triangle n$ is accompanied by the increase of asset input with $\frac{1}{\kappa}\triangle n$, which means decrease of supply of secondary asset for households. Thus, under homotheticity of preferences, we have $\frac{p_t^{a,2}}{p_t^2} > \lim_{\kappa \to \infty} \frac{p_t^{a,2}}{p_t^2}$. Also, given infinite κ, good supply increases while asset supply is fixed. Thus, also under homotheticity of preferences, we have $\lim_{\kappa \to \infty} \frac{p_t^{a,2}}{p_t^2} > \frac{p_{t-1}^{a,2}}{p_{t-1}^2}$.

A.7 Proof of Proposition 1

Given $M_t^2 \geq \frac{n_t^{2e}}{n_t^{1e}}$, Lemma 2 indicates there occurs no labor shift and $(n_t^1, n_t^2) = (n_t^{1e}, n_t^{2e})$ is maintained. In deriving the stated expression for $\bar{M}$, observe that $\bar{w}$ is deter-

mined by the following equation.

$$\begin{aligned} & \frac{\bar{w}-p_t^1\bar{x}^1}{(\bar{w}-p_t^1\bar{x}^1)n_t^{2e}+(w_t^1-p_t^1\bar{x}^1)n_t^{1e}}\alpha_t^2 n_t^{2e}=\bar{x}^2 \\ \Leftrightarrow \quad & \frac{\alpha_t^2\bar{w}n_t^{2e}-p_t^1\bar{x}^1\alpha_t^2 n_t^{2e}}{\bar{w}n_t^{2e}-p_t^1\bar{x}^1 n_t^{2e}+w_t^1 n_t^1-p_t^1\bar{x}^1 n_t^{1e}}=\bar{x}^2 \\ \Leftrightarrow \quad & \frac{\alpha_t^2\bar{w}n_t^{2e}-p_t^1\bar{x}^1\alpha_t^2 n_t^{2e}}{\bar{w}n_t^{2e}+(w_t^1 n_t^{1e}-p_t^1\bar{x}^1)}=\bar{x}^2. \end{aligned}$$

By substituting $p_t^1\bar{x}^1=M_t^1$ and $w_t^1=M_t^1/n_t^{1e}$ together with the implied relationship of $p_t^1\bar{x}^1=w_t^1 n_t^{1e}$, we have:

$$\begin{aligned} & \frac{\alpha_t^2\bar{M}-M_t^1\alpha_t^2 n_t^{2e}}{\bar{M}}=\bar{x}^2 \\ \Leftrightarrow \quad & (\alpha_t^2-\bar{x}^2)\bar{M}=M_t^1\alpha_t^2 n_t^{2e} \\ \Leftrightarrow \quad & \bar{M}=\frac{\alpha_t^2}{\alpha_t^2-\bar{x}^2}n_t^{2e}M_t^1. \end{aligned}$$

By substituting $n_t^{2e}=\frac{\alpha_t^1-\bar{x}^1}{\alpha_t^1}$, we have

$$\bar{M}=\frac{\alpha_t^2}{\alpha_t^2-\bar{x}^2}\frac{\alpha_t^1-\bar{x}^1}{\alpha_t^1}M_t^1. \tag{96}$$

In the following proof, we utilize diagram that is constructed in a form of Edgeworth box as shown Figure 2. In referring to the diagram, we divide into two cases for our proof. Take the intersection between the offer curve and the line that indicates holding of secondary asset in the amounts of $(\bar{a}^1 n_t^2+\frac{A^2}{n_t^2},0)$ for I_t^2 while $(\bar{a}_t^1 n_t^1,0)$ for I_t^1. For situations where the location of the intersection indicates that the consumption of secondary good by I_t^2 is larger than the satiation level (i.e., $\bar{x}^2 n_t^2$), we say Case i), and let Case ii) refer to the other cases. Observe that Figure 2 shows a situation of Case i).

Proof of Case i)

As illustrated in Figure 2, given $\frac{n_t^2}{n_t^1}M_t^1\leq M_t^2\leq\bar{M}$, we always have $Y_t^2 n_t^2\leq\bar{x}^2 n_t^2\leq x_t^{2;max}$. For our purpose, it is sufficient to examine situations where z_0 is located leftside of or on the offer curve while located rightside of or on the vertical line that indicates equal asset allocation (i.e., $a_{t-1}^{i,2}n_t^2=A^2 n_t^2$). Then, given such z_0, we can always take (hypothetical) initial allocation $\tilde{z}$ within the inner box (i.e.,$\tilde{z}$ indicates an allocation that gives positive consumption of secondary good as well as positive holding of secondary asset for both I_t^1 and I_t^2). We set $\tilde{a}$ as the holding of secondary asset of I_t^2 on allocation $\tilde{z}$. Note that from the construction, $\tilde{a}(M_t^2)$ is strictly increasing. Also, given quasi-convexity and differentiability of function $f(a,x)$, we always have unique relative price $p_t^{a,2}/p_t^2$ that realizes allocation (shown with z_1 in the diagram) that satisfies $a_{t-1}^{i,2}<a_t^{i,2}<\tilde{a}$ for $i=i_t^2$ and $\frac{A^2-n_t^2\tilde{a}}{n_t^1}<a_t^{i,2}<a_{t-1}^{i,2}$ for $i=i_t^1$.

For cases of $M_t^2>\bar{M}$, we have $\tilde{a}=\bar{a}_t^2$. For given z_0, take allocation z_1 so that it locates on the horizontal line that indicates $x_t^{i,2}n_t^2=\bar{x}^2 n_t^2$ for I_t^2 and maximize utility of I_t^2 given

the associated relative price $\frac{p_t^{a,2}}{p_t^2}$. Figure 3 illustrate such a situation. Suppose that we can take such z_1 within the inner box (i.e., z_1 indicates $a_t^{i,2}n_t^2 < \bar{a}_t^2 n_t^2$). Then, observe that the indifference curve of I_t^2 that goes through z_1 consists of vertical line (when ignoring cost from excess consumption) above z_1 and downward sloping curve below z_1 that is formed by function $f(x, a)$. Thus, z_1 is always consistent with utility maximization of I_t^2 given the associated relative price. If there exists no such z_1, take z_1 so that it gives $(\bar{x}^2 n_t^2, \bar{a}_t^2 n_t^2)$ to I_t^2. Concerning the shape of the indifference curve of I_t^1 that goes through z_1, given that there will never occur trade of primary asset, we can safely ignore the lower-right domain of z_1. For the corresponding indifference curve of I_t^2, it has the same property as discussed above. Thus, given the shapes of indifference curves of I_t^1 and I_t^2, there always exist relative price $\frac{p_t^{a,2}}{p_t^2}$ with which z^1 is consistent with utility maximization of I_t^1 and I_t^2 given no trade of primary asset. This proves part i) of the current lemma.

Next, for the proof of part ii), see Figure 4. Take allocation z_1 so that it gives $(\bar{x}^2 n_t^2, \bar{a}^2 n_t^2)$ to I_t^2. Remember the discussed shapes of the associated indifference curves of I_t^1 and I_t^2 on z_1 hold. Thus, given $M_t^2 > \bar{M}$ and the associated $x_t^{2;max} > \bar{x}^2 n_t^2$, we can always take z_0 with sufficiently large $a_{t-1}^{2,1} n_t^2$ so that there exists relative price $p_t^{a,2}/p_t^2$ with which each of I_t^1 and I_t^2 maximizes utility on z_1. This completes the proof for Case i).

Proof of Case ii)

For arbitrary $\frac{n_t^2}{n_t^1} M_t^1 \leq M_t^2$, it is sufficient to examine situations where z_0 is located left-side of or on the offer curve while located right-side of or on the vertical line that indicates equal asset allocation (i.e., $a_{t-1}^{i,2} n_t^2 = A^2 n_t^2$), and also located within the inner box or on the right-side line of the box. Given z_0, take (hypothetical) initial allocation $\tilde{z}$ wherein there would arise no asset trade while assuming primary asset can also be traded. Then, there are two cases for the location of $\tilde{z}$. For cases where $\tilde{z}$ is located inside the inner box where trades of primary good and asset are irrelevant, set $\tilde{a}$ on the offer curve as in Case i). For this case, the existence of relative price that uniquely realizes z_1 inside the inner box in the diagram is immediate from the proof of Case i).

For cases where $\tilde{z}$ is outside the inner box, we set $\tilde{a} = \bar{a}^2 n_t^2$. Take allocation z_1 so that it is on the offer curve and actually the associated relative price $p_t^{a,2}/p_t^2$ realizes utility maximization of I_t^2 within the inner box (i.e., $a_t^{2,1} n_t^2 < \bar{a}_t^2$). When there exists such allocation z_1, then it is consistent with part i) of the current lemma. If there exits no such allocation, take allocation z_1 so that it maximizes utility of I_t^2 while realizing maximum wealth inequality (i.e., on the right-side line of the inner box) Figure 7 illustrates the situation. Suppose that we can take such z_1 while indicating $x_t^{i,2} n_t^2 \leq \bar{x}^2 n_t^2$ for $i = i_t^2$. Then, concerning the shape of the indifference curve of I_t^1 on z_1, since there would occur no trade of primary asset in any case, we safely ignore the left side domain of z_1, and the indifference curve for the left side of z_1 becomes a downward sloping curve that is given by $f(x, a)$. Since z_1 is located above the offer curve, it is always consistent also with utility maximization of I_t^1. When there exists no such z_1, then take z_1 so that it gives $\bar{x}^2 n_t^2$ secondary good and maximum secondary asset $\bar{a}^2 n_t^2$ to I_t^2. Given the discussed shapes of the indifference curves of I_t^1 and I_t^2 on z_1, since z_1 is located above the offer curve, there

always exists relative price $\frac{p_t^{a,2}}{p_t^2}$ with which z^1 is consistent with utility maximization of I_t^1 and I_t^2 given no trade of primary asset. Observing $\tilde{a}(M_t^2)$ is weakly increasing by construction, we complete the proof for part i).

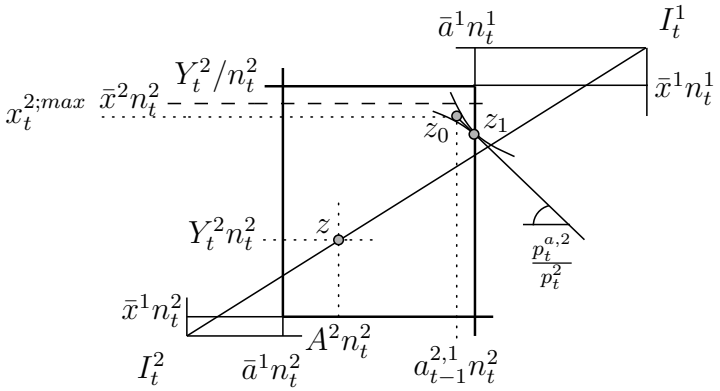


Figure 7:

For the proof of part ii), see Figure 8. In the same manner of the proof for Case i), take allocation z_1 so that it gives $\bar{x}^2 n_t^2$ secondary good and maximum secondary asset $\bar{a}^2 n_t^2$ to I_t^2. Then, given $M_t^2 > \bar{M}$ and the associated $x_t^{2;max} > \bar{x}^2 n_t^2$, we can always take z_0 with sufficiently large $a_{t-1}^{2,1} n_t^2$ so that there exists relative price $p_t^{a,2}/p_t^2$ with which each of I_t^1 and I_t^2 maximizes utility on z_1. This completes the whole proof.

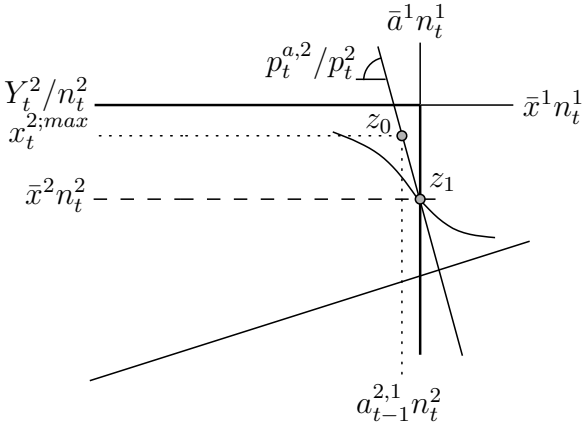


Figure 8:

A.8 Proof of Proposition 2

Proof of i)

When $M \leq \bar{M}$, suppose that there occurs any speculative trade. Lemma 2-III) indicates that any net loser $i \in I$ in speculative trades must attain $x_t^{i,2} = \bar{x}^2$. However, given no labor shift and no fundamental trade of secondary asset, $\frac{n_t^{2e}}{n_t^{1e}} M_t^1 < M \leq \bar{M}$ indicates that $x_t^{i,2} < \bar{x}^2$ for the net loser i in either case of $i \in I_t^1$ or $i \in I_t^2$, and this completes the proof.

Proof of ii)

Given no labor shift, we have $I_t^j = I_{t-1}^j$ for $j = 1, 2$. Suppose that $a_{t-1}^{i,2} < \bar{a}^2$ for $i = i_t^2$, which also indicates $a_{t-1}^{i,2} > 0$ for $i = i_t^1$. For our purpose, it is sufficient to show that there always occur any fundamental trade of secondary asset. In proceeding our proof, we divide the situations into two cases Case i) and ii) in the same manner as in the proof for Proposition 1.

Case i)

Proof of Proposition 1 proves for the situations where z_0 is located left-side of or on the offer curve while located right-side of or on the vertical line that indicates equal asset allocation (i.e., $a_{t-1}^{i,2} n_t^2 = A^2 n_t^2$). Thus, the remaining situations for our proof are those where z_0 is located between the offer curve and the right-side line of the inner box. When z_0 is located sufficiently left-side, there apparently exists unique interior solution on the offer curve. For z_0 that is located sufficiently right-side and there exists no solution on the offer curve, take candidate z_1 such that it allocates maximum consumption $\bar{x}_t^2 n_t^2$ for I_t^2 and also I_t^1 maximizes utility given the associated relative prices $\frac{p_t^{a,2}}{p_t^2}$. The discussion on the properties of the relevant indifference curves on such z_1 that is made in the proof of Proposition 1 indicates that such z_1 is always consistent with utility maximization for the part of I_t^2. This completes the proof for Case i).

Case ii)

Proof of Proposition 1 proves for the situations where z_0 is located left-side of or on the offer curve while located right-side of or on the vertical line that indicates equal asset allocation (i.e., $a_{t-1}^{i,2} n_t^2 = A^2 n_t^2$), and also located within the inner box or on the right-side line of the box. Thus, the remaining situations for our proof are those where z_0 is located between the offer curve and the right-side line of the inner box. Given such z_0, observe that we always find unique interior solution that realizes allocation located on the offer curve and also within the inner box. This completes the whole proof for part ii) of the current lemma.

A.9 Proof of Proposition 3

Proof of I)

Given $\alpha_t^2 = \alpha_{t-1}^2$ and $n_{t-1}^2 = n_t^{2e}$ (stable production) and $M_t^2 > \bar{M}$ (excess money supply), Lemma 2 indicates no occurrence of labor shift in period t. Suppose that there occurs speculative trade with $\triangle p_t^{a_2;s} \neq 0$. Then, given labor distribution (I_t^1, I_t^2), the assumption of $a_{t-1}^{i,2} = 0$ for every $i \in I_t^1$ and $a_{t-1}^{i,2} = \bar{a}_t^2$ for every $i \in I_t^2$ indicates $a_t^{i,2} = 0$ for every $i \in I_t^1$ and $a_t^{i,2} = \bar{a}_t^2$ since there occurs no fundamental trade of secondary asset. Since we do not allow short-selling of assets, the initial selling can be made by household $i \in I_t^2$ or firms in secondary sector, but not by any household $i \in I_t^1$. Suppose that it turns out as negative bubble as $\triangle p_t^{a_2;s} < 0$. Then, any trader who engages in initial selling becomes a net winner. Here, Lemma 1-III) states that any net loser in speculative trade must satisfy $x_t^{i,2} = \bar{x}^2$. Since $M_t^2 > \bar{M}$, only households of I_t^2 can be net losers.

Thus, any speculative trade is only possible between firms and I_t^2. This proves "if" part of statement I).

Concerning "only if" part of statement I), suppose that there occurs asset bubble as $\triangle p_t^{a_2;s} > 0$. Then, for any household $i \in I_t^{1;1}$ has incentive in engaging in profitable buy-then-sell speculative trade since otherwise every household $\triangle p_t^{a_2;s} \neq 0$ cannot attain the satiation level of secondary good consumption. Since each household $i \in I_t^{1;1}$ can submit nontrivial margin as $v_i = w^1 > 0$, every household $i \in I_t^{1;1}$ always engages in buy-then-sell speculative trade and consequentially obtains nontrivial profit.

Proof of II)

Given the assumed extreme wealth inequality together with severe excess money supply, there exists no clearing prices that realizes fundamental trade of secondary asset. Then, each household $i \in I_t^2$ cannot use up money through purchasing satiation levels of primary and secondary goods. Such redundant money becomes the source of asset bubble. Let sufficiently small portion of such redundant money be lost through engaging in sell-then-buy speculative trade. Observe that since the associated price impact is sufficiently small, firms will not engage in any buy-then-sell speculative trade facing with relatively large profit loss from selling goods. Given this, the lost money of I_t^2 must flow to $I_t^{1;1}$. Since any household $i \in I_t^{1;1}$ cannot attain the satiation level of secondary good consumption, all of $I_t^{1;1}$ will participate in buy-then-sell speculative trade through submitting the whole available margin $v_i = w_t^1$. Observe that the whole gained monetary profits through speculative trades by $I_t^{1;1}$ flow to buy up secondary good. Since each $i \in I_t^2$ still holds sufficient redundant money, they can always pay the necessary amount to buy up secondary good to the satiation level. This completes the proof of part II).

Proof of III)

Observe that if asset bubble occurs, then any net loser must be $i \in I_t^2$. Also, if $i \in I_t^2$ turns out as a net loser in speculative trade, then the household must attain the satiation level of secondary good consumption. Here, suppose that firms ride bubble. Then, since all producing firms ride bubble given the assumption of symmetry size of producing firms, supply of secondary good in period t becomes

$$\begin{aligned} Y_t &= (1-\theta)X_t + \theta X_{t-1} \\ &= (1-\theta)(1-\delta)\alpha_t^2 n_t^{2e} + \theta\alpha_t^2 n_t^{2e} \\ &= \Big(1-(1-\theta)\delta\Big)\alpha^2 n_t^{2e}. \end{aligned}$$

When we let $\bar{w}^b$ be the wage level of secondary sector that is just sufficient for each $i \in I_t^2$ to purchase secondary good to the satiation level, $\bar{w}^b$ is given by Equation (41). Thus, if asset bubble occurs, we have $w_t^2 > \bar{w}^b$. Therefore, we have $M_t^2 > \bar{M}^b := n_t^{2e}\bar{w}^b$. This completes the whole proof.

A.10 Proof of Proposition 4

Since we confine us to the situation where any saving will not occur (i.e., $s_t^i = 0$) for every $i \in I$, households $I_t^{1,1}$ use all the money gained from speculative trades to buy up secondary good. Then, from the definition of $\triangle M_t^{1;1,g2}$ and $M_t^{a_2;s}$, part I) is immediate.

Concerning trades of secondary good, given the increased expenditure $\triangle M_t^{1;1,g2}$ as specified in part I), I_t^2 will also increase their expenditure in order to attain the satiation level of consumption. Thus, the additional increase in expenditure denoted with $\triangle M_t^{2,g2}$ must satisfy

$$(\frac{\triangle M_t^{2,g2}}{n_2^e} + p_t^2 \bar{x}^2) \frac{1}{p_t^2 Y_t^2 + \triangle M_t^{1;1,g2} + \triangle M_t^{2,g2}} \tilde{Y}_t^2 = \bar{x}^2. \tag{97}$$

Rearranging the equation, we have
$(\frac{\tilde{Y}_t^2}{n_2^e} - \bar{x}^2) \triangle M_t^{2,g_2} = (Y_t^2 - \tilde{Y}_t^2) p_t^2 \bar{x}^2 + \bar{x}^2 \triangle M_t^{1;1,g2}$. Thus, we have $\triangle M_t^{2,g2} = M^F + \beta \triangle M_t^{1;1,g2}$ as stated in the current lemma.

Now, given $\triangle M_t^{1;1,g2}$ and $\triangle M_t^{2,g2}$, clearing of secondary good market indicates $\tilde{p}_t^2 \tilde{Y}_t^2 = p_t^2 Y_t^2 + \triangle M_t^{1;1,g2} + \triangle M_t^{2,g2}$. Rearranging this, we have $\tilde{p}_t^2 = \frac{1}{\tilde{Y}_t^2}(p_t^2 Y_t^2 + \triangle M_t^{1;1,g2} + \triangle M_t^{2,g2})$ and further rearrangements immediately derives the form shown in the lemma.

A.11 Proof of Proposition 5

Proof. When $p_{t-1}^{a_2} < p_t^{a_2} = p_{t-1}^{a_2} + \triangle p_t^{a_2;s}$, each firm $j \in J$ participate in speculative trade (i.e., puts margin $v^j = p_{t,1}^{a,2} a_{t-1}^{j,2}$) if and only if $-m_t^{j,a_2;s} > \delta p_{t,t+1}^2 F^2(n_t^{2,j}, \frac{1}{\kappa} n_t^{2,j})$, where $p_{t,t+1}^2 = p_t^2(1-\theta) + p_{t+1}^2 \theta$. Given symmetry of firms, the inequality is equivalent to $-M_t^{J,a_2;s} > \delta p_{t,t+1}^2 F^2(n_2^e, \frac{1}{\kappa} n_2^e)$, where $M_t^{J,a_2;s} := \int_{j \in J_t^2} m_t^{j,a_2;s}$. We examine firms' incentive of deviation from the situation of full participation to speculative trades. We have $-M_t^{J,a_2;s} = \frac{v_t^J}{V_t} M_t^{a_2;s}$, where $v_t^J = p_{t-1}^{a,2}(1/k) n_t^{2e}$ and $V_t = v_t^J + w_t^1 \lambda n_t^{1e}$. Given these v_t^J and V_t, the condition becomes $\frac{v_t^J}{V_t} M_t^{a_2;s} > \delta p_{t,t+1}^2 F^2(n_2^e, \frac{1}{\kappa} n_2^e)$. □

A.12 Proof of Proposition 6

Proof.

Given $a_{t-1}^{i,2} = 0$ for $i \in I_t^1$ and $M_t^2 > \bar{M}$, any finite $p_t^{a,2}$ is never consistent with utility maximization of household $i \in I_t^2$. Thus, realization in the first stage fails and proceed to the second stage where no trade of secondary asset is given. This prove Part I).

Given Part I), $M_t^2 > \bar{M}$ indicates Part II-i). Part II-ii) holds since household $i \in I_t^2$ will not engage in any speculative trade that leads to $x_t^{i,2} < \bar{x}^2$. According to Proposition 3-I), the occurrence of asset bubble indicates that every $i \in I_t^{1;1}$ engages in some speculative trade. Those households $I_t^{1;1}$ always engage in profitable speculative trades, and use the profits to buy up secondary good. This is only possible by decreasing consumption of $I_t^{1;0}$. Thus, combining with the assumption of $0 < \lambda < 1$, we have Part II-iii) and iv). Part II-v) is immediate by observing that larger $\triangle M_t^{a_2;s}$ gives larger purchasing power for $I_t^{1;1}$ against $I_t^{1;0}$ given the same supply of secondary good. □

A.13 Proof of Lemma 7

Given (n_0^{1e}, n_0^{2e}), take $w_0^1 = w_0^2 = M_0^1/n_0^{1e}$, $p_0^1 = M_0^1/\alpha_0^1 n_0^{1e}$ and $L_0 = L_0^* := \frac{n_0^{2e}}{n_0^{1e}} M_0^1$. Then, we always have some p_0^2 and $p_0^{a,2}$ that satisfies the conditions for the realization of good and asset trades. Furthermore, take $i_0^R = 0$. Then, it is immediate that the obtained initial economy constitutes an optimal steady state. This proves statement i) of the current lemma.

In proving statement ii), let $i_0^R > 0$ while maintaining the other elements of the derived initial economy. Then, Equation (50) is no more satisfied, and this completes the whole proof.

A.14 Proof of Lemma 8

Given sufficiently large κ, we can safely ignore any potential impacts on M_t^e for every $t = 1, 2, ..$ that could be brought by perturbation in either section $j = 1, 2$. Now, suppose that optimal monetary policy is realized in equilibrium.

I) Case: perturbation of sector 2

Take arbitrary perturbation (ϵ^2, T). Firstly, suppose that the optimal monetary policy is now given with the same i_t^R for every $t = 1, 2, ..., T-1$. Then, since $\Theta_{T-1,T}$ becomes smaller while there is no change in all of $\Theta_{T-1,T-1}$, L_{T-1}^* and L_T^*, in order to satisfy Equation (50) for $t = T-1$, we have smaller i_T^R for the case with $i_T^R > 0$ while we have larger L_T for the case of $i_T^R = 0$. Observe that larger L_T indicates $L_T > L_T^*$, thus optimal monetary policy is no more available. Concerning the case of smaller i_T^R with $i_T^R \geq 0$, in order to satisfy Equation (50) for $t = T$, we have lager i_{T+1}^R since the perturbation never affects any of L_T^*, L_{T+1}^*, $\Theta_{T,T}$, $\Theta_{T,T+1}$. Given this, in order to satisfy Equation (50) for $t = T+1$, we have smaller i_{T+2}^R for the case with $i_{T+2}^R > 0$ while we have larger L_{T+2} for the case of $i_{T+2}^R = 0$. This discussion is applicable to the remaining periods in the same manner. Thus, if there exists consecutive periods T' and $T'+1$ such that $i_{T'}^R = i_{T'+1}^R = 0$ under optimal monetary policy, perturbation in period $T \geq T'$ will be no more consistent with optimal monetary policy.

Here, since we assume there always exists period $T'' \geq T$ such that $\alpha_t^j = \alpha_{T''}^j$ for $j = 1, 2$ for $t = T''+1, T''+2, T''+3$, we have $L_t^* = L_T^*$ for $t = T'', T''+1, T''+2, T''+3$ and $\Theta_{t,t} + \Theta_{t,t+1} = 1$ for $t = T''+1, T''+2$. Thus, in order to satisfy Equation (50) for $t = T''+1$, we need to have $i_{T''+1}^R = i_{T''+2}^R = 0$ (since $\Theta_{T''+1,T''+1} + \Theta_{T''+1,T''+2} = 1$). This shows the contradiction given no change in i_t^R for $t = 2, 3, ...$

Now, suppose that Equation (50) for $t = T-1$ is maintained by fixing i_T^R. This leads to decrease in i_{T-1}^R if optimal monetary policy is realized. By applying the previous logic in the backward direction, contradiction is similarly achieved since we have $i_0^R = 0$ under the original optimal monetary policy.

II) Case: perturbation in sector 1

Next, take arbitrary perturbation (ϵ^1, T). Let L_t^* and $L_t^{*,\epsilon}$ respectively denote the optimal credit supplies before introducing perturbation and that after the introduction. In the same manner, we define notations of $\Theta_{t,t}^{\epsilon}$, $\Theta_{t,t+1}^{\epsilon}$, $n_t^{1e,\epsilon}$, $n_t^{2e,\epsilon}$ and $X_t^{2,\epsilon}$. Observe that $n_T^{1e,\epsilon} = \frac{\bar{x}^1}{(1+\epsilon_\alpha^1)\alpha_T^1} = \Big(\frac{1}{1+\epsilon_\alpha^1}\Big) n_T^{1e}$. Using this, we have

$$\begin{aligned}
X_T^{2,\epsilon} - X_T^2 &= \alpha_T^2 n_T^{2e,\epsilon} - X_T^2 \\
&= \alpha_T^2 (1 - n_T^{1e,\epsilon}) - X_T^2 \\
&= \alpha_T^2 \Big(1 - \frac{n_T^{1e}}{1+\epsilon_\alpha^1}\Big) - X_T^2 \\
&= \alpha_T^2 \frac{n_T^{2e} + \epsilon_\alpha^1}{1+\epsilon_\alpha^1} - X_T^2 \\
&= (X_T^2 + \alpha_T^2 \epsilon_\alpha^1)\Big(\frac{1}{1+\epsilon_\alpha^1}\Big) - X_T^2 \\
&= \Big(\frac{1}{1+\epsilon_\alpha^1}\Big) X_T^2 + \Big(\frac{\epsilon_\alpha^1}{1+\epsilon_\alpha^1}\Big)\alpha_T^2 - X_T^2 \\
&= \Big(\frac{-\epsilon_\alpha^1}{1+\epsilon_\alpha^1}\Big) X_T^2 + \Big(\frac{\epsilon_\alpha^1}{1+\epsilon_\alpha^1}\Big)\alpha_T^2 \\
&= \Big(\frac{-\epsilon_\alpha^1}{1+\epsilon_\alpha^1}\Big) n_T^{2e} \alpha_T^2 + \Big(\frac{\epsilon_\alpha^1}{1+\epsilon_\alpha^1}\Big)\alpha_T^2 \\
&= \Big(\frac{\epsilon_\alpha^1}{1+\epsilon_\alpha^1}\Big) n_T^{1e} \alpha_T^2
\end{aligned}$$

Then, we have

$$\begin{aligned}
\frac{L_T^{*,\epsilon}}{L_{T-1}^{*,\epsilon}} &= \Big(\frac{n_T^{2e,\epsilon}}{n_T^{1e,\epsilon}} M_0^1\Big) \frac{1}{L_{T-1}^*} \\
&= \frac{L_T^*}{L_{T-1}^*}\Big(\frac{n_T^{2e,\epsilon}}{n_T^{1e,\epsilon}} \frac{n_T^{1e}}{n_T^{2e}}\Big) \\
&= \frac{L_T^*}{L_{T-1}^*}\Big(\frac{n_T^{1e}}{1-n_T^{1e}}\Big)\Big(\frac{1}{n_T^{1e,\epsilon}} - 1\Big) \\
&= \frac{L_T^*}{L_{T-1}^*}\Big(\frac{n_T^{1e}}{1-n_T^{1e}}\Big)\Big(\frac{1+\epsilon_\alpha^1}{n_T^{1e}} - 1\Big) \\
&= \frac{L_T^*}{L_{T-1}^*}\Big(\frac{1-n_T^{1e}+\epsilon_\alpha^1}{1-n_T^{1e}}\Big) \\
&= \frac{L_T^*}{L_{T-1}^*}\Big(1 + \frac{\epsilon_\alpha^1}{n_T^{2e}}\Big)
\end{aligned}$$

Also, we have

$$\begin{aligned}
\Theta^{\epsilon}_{T-1,T} &= \frac{\theta X^2_{T-1}}{\theta X^2_{T-1} + (1-\theta) X^{2,\epsilon}_T} \\
&= \Theta_{T-1,T}\Big(\frac{\theta X^2_{T-1} + (1-\theta) X^2_T}{\theta X^2_{T-1} + (1-\theta) X^{2,\epsilon}_T}\Big) \\
&= \Theta_{T-1,T}\Big[1 + (1-\theta)\Big(\frac{X^{2,\epsilon}_T - X^2_T}{\theta X^2_{T-1} + (1-\theta) X^2_T}\Big)\Big]^{-1} \\
&= \Theta_{T-1,T}\Big[1 + (1-\theta)\Big(\frac{1}{\theta X^2_{T-1} + (1-\theta) X^2_T}\Big)\Big(\frac{\epsilon^1_\alpha}{1+\epsilon^1_\alpha}\Big) n^{1e}_T \alpha^2_T\Big]^{-1}
\end{aligned}$$

Here, when we define $F(\theta)^{-1} := (1-\theta)\Big(\frac{1}{\theta X^2_{T-1} + (1-\theta) X^2_T}\Big)$, we have $F(\theta) = \frac{\theta}{1-\theta} X^2_{T-1} + X^2_T$. Thus, since $\frac{\partial F}{\partial \theta} > 0$, we have $\frac{\partial F^{-1}}{\partial \theta} < 0$. Using $F^{-1}(\theta) < F^{-1}(0)$, we have

$$\begin{aligned}
\Theta^{\epsilon}_{T-1,T} &> \Theta_{T-1,T}\Big[1 + \Big(\frac{1}{X^2_T}\Big)\Big(\frac{\epsilon^1_\alpha}{1+\epsilon^1_\alpha}\Big) n^{1e}_T \alpha^2_T\Big]^{-1} \\
&= \Theta_{T-1,T}\Big[1 + \Big(\frac{1}{\alpha^2_T n^{2e}_T}\Big)\Big(\frac{\epsilon^1_\alpha}{1+\epsilon^1_\alpha}\Big) n^{1e}_T \alpha^2_T\Big]^{-1} \\
&= \Theta_{T-1,T}\Big[1 + \Big(\frac{n^{1e}_T}{n^{2e}_T}\Big)\Big(\frac{\epsilon^1_\alpha}{1+\epsilon^1_\alpha}\Big)\Big]^{-1}
\end{aligned}$$

Thus, we have

$$\frac{L^{*,\epsilon}_T}{L^{*,\epsilon}_{T-1}}\Theta^{\epsilon}_{T-1,T} > \Big(\frac{L^*_T}{L^*_{T-1}}\Theta_{T-1,T}\Big)\Big(1 + \frac{\epsilon^1_\alpha}{n^{2e}_T}\Big)\Big[1 + \Big(\frac{n^{1e}_T}{n^{2e}_T}\Big)\Big(\frac{\epsilon^1_\alpha}{1+\epsilon^1_\alpha}\Big)\Big]^{-1} \tag{98}$$

Relationship (98) indicates $\frac{L^{*,\epsilon}_T}{L^{*,\epsilon}_{T-1}}\Theta^{\epsilon}_{T-1,T} > \frac{L^*_T}{L^*_{T-1}}\Theta_{T-1,T}$ when

$$\begin{aligned}
& \frac{\epsilon^1_\alpha}{n^{2e}_T} > \Big(\frac{n^{1e}_T}{n^{2e}_T}\Big)\Big(\frac{\epsilon^1_\alpha}{1+\epsilon^1_\alpha}\Big) \\
\Leftrightarrow\ & \frac{1}{n^{1e}_T} > \frac{1}{1+\epsilon^1_\alpha} \\
\Leftrightarrow\ & \epsilon^1_\alpha > n^{1e}_T - 1.
\end{aligned}$$

Since $n^{1e}_T < 1$, we always have $\frac{L^{*,\epsilon}_T}{L^{*,\epsilon}_{T-1}}\Theta^{\epsilon}_{T-1,T} > \frac{L^*_T}{L^*_{T-1}}\Theta_{T-1,T}$ for arbitrary $\epsilon^1_\alpha > 0$. When we apply this observation to Equation (50) for $t = T-1$, perturbation (ϵ^1_α, T) must cause larger i^R_T to sustain $L_{T-1} = L^{*,\epsilon}_{T-1}$ and $L_T = L^{*,\epsilon}_T$.

Next, we examine Equation (50) for $t = T$. Our aim is to show that $i^{R,\epsilon}_{T+1} < i^R_{T+1}$. Since $\Theta^{\epsilon}_{T,T+1} < \Theta_{T,T+1}$, it suffices to show that $\Theta^{\epsilon}_{T,T}(1 - \xi i^{R,\epsilon}_t) < \Theta_{T,T}(1 - \xi i^R_t)$. From Equation (50) for $t = T-1$, we have

$$1 - \xi i^{R,\epsilon}_T = \Big(1 - \Theta_{T-1,T-1}(1 - \xi i^R_t)\Big)\Big(\frac{L^{*,\epsilon}_T}{L^*_{T-1}}\Theta^{\epsilon}_{T-1,T}\Big)^{-1}$$

Thus, we have

$$\begin{aligned}\Theta^{\epsilon}_{T,T}(1-\xi i^{R,\epsilon}_T) &= \Theta^{\epsilon}_{T,T}\Big(1-\Theta_{T-1,T-1}(1-\xi i^R_t)\Big)\Big(\frac{L^{*,\epsilon}_T}{L^*_{T-1}}\Theta^{\epsilon}_{T-1,T}\Big)^{-1}\\ &= \Big(1-\Theta_{T-1,T-1}(1-\xi i^R_t)\Big)\Big(\frac{L^{*,\epsilon}_T}{L^*_{T-1}}\frac{\Theta^{\epsilon}_{T-1,T}}{\Theta^{\epsilon}_{T,T}}\Big)^{-1}\\ &= \Big(1-\Theta_{T-1,T-1}(1-\xi i^R_t)\Big)\Big(\frac{L^{*,\epsilon}_T}{L^*_{T-1}}\frac{\theta}{1-\theta}\frac{X^2_{T-1}}{X^{2,\epsilon}_T}\Big)^{-1}. \qquad (99)\end{aligned}$$

Here, we have

$$\begin{aligned}\frac{L^{*,\epsilon}_T}{X^{2,\epsilon}_T} &= \Big(\frac{n^{2e,\epsilon}_T}{n^{1e,\epsilon}_T}M^1_0\Big)\Big(\alpha^2_T n^{2e,\epsilon}_T\Big)^{-1}\\ &= \Big(\frac{n^{2e}_T}{n^{1e,\epsilon}_T}M^1_0\Big)\Big(\alpha^2_T n^{2e}_T\Big)^{-1}\\ &> \Big(\frac{n^{2e}_T}{n^{1e}_T}M^1_0\Big)\Big(\alpha^2_T n^{2e}_T\Big)^{-1} = \frac{L^*_T}{X^2_T}.\end{aligned}$$

Substituting into Equation (99), we have

$$\begin{aligned}\Theta^{\epsilon}_{T,T}(1-\xi i^{R,\epsilon}_T) &< \Big(1-\Theta_{T-1,T-1}(1-\xi i^R_t)\Big)\Big(\frac{L^*_T}{L^*_{T-1}}\frac{\theta}{1-\theta}\frac{X^2_{T-1}}{X^2_T}\Big)^{-1}\\ &= \Theta_{T,T}(1-\xi i^R_T).\end{aligned}$$

Thus, we confirm $i^{R,\epsilon}_{T+1} < i^R_{T+1}$. For the case where $L^{*,\epsilon}_{T+1}$ is realized, we turn to examine Equation (50) for $t \geq T+1$. Now, since the perturbation never affects all of $\Theta_{t,t}$, $\Theta_{t,t+1}$, L^*_t and L^*_{t+1}, we have oscillation such that $i^{R,\epsilon}_{T+2} > i^R_{T+2}$, $i^{R,\epsilon}_{T+3} < i^R_{T+3}$ and so on. This leads to contradiction at some period in the same logic for Case I), and completes the whole proof.

A.15 Proof of Lemma 9

We focus on equilibrium without any speculative trade. Since Equation (37) is always satisfied under modestly efficient-first monetary policy, Lemma 2 indicates $n^j_t = n^{je}_t$ for $j = 1, 2$ and every $t = 1, 2, ...$ in equilibrium. Given this, observe that $(X^2_T)_{t=1,2,...}$ is also uniquely realized. We thus have unique $(\Theta_{t,t}, \Theta_{t,t+1})_{t=1,2,...}$. Given this, when we apply Equation (50) to period 1, we have unique (i^R_1, L_1). Specifically, to be consistent with modestly efficient-first monetary policy, if $i^{R,0}_1 < 0$, then $i^R_1 = 0$ and $L_1 = L^0_1 > L^*_1$. Meanwhile, if $i^{R,0}_1 > 0$, then $i^R_1 = i^{R,0}_1$ and $L_1 = L^*_1$. When we apply Equation (50) for any period $t > 1$ given (i^R_{t-1}, L_{t-1}), we have unique (i^R_t, L_t) in the same manner. This completes our proof.

A.16 Proof of Lemma 10

Observe that if $L_{t-1} > L^*_{t-1}$, $\alpha^j_t = \alpha^j_{t-1}$ for $j = 1, 2$ and $i^R_{t-1} = 0$, we have $\Theta_{t-1,t-1} = 1-\theta$, $\Theta_{t-1,t} = \theta$. Thus, to satisfy Equation (50) for period $t-1$, we have $L_t \geq 0$. Observe

that $L_t > L_{t-1} > L_t^*$ is consistent only with $i_t^R > 0$, while such $(i_t^R > 0, L_t > L_t^*)$ is not allowed under modestly efficient-first monetary policy. Thus, we have $L_t = L_{t-1}$ and $i_t^R = 0$.

A.17 Proof of Proposition 7

When applying Equation (50) for period $t = 1, 2, ..., T-2$, observe $\Theta_{t,t} = 1-\theta$ and $\Theta_{t,t+1} = \theta$. Given this, $L_t = L_t^*$ for $t = 1, 2, .., T-2$ is brought by $i_{t+1}^R = 0$ for $t = 1, 2, .., T-2$. Next, when examining Equation (50) for period $t = T-1$, observe that $\Theta_{T-1,T-1} = 1-\theta$ and $\Theta_{T-1,T} = \frac{\theta X_{T-1}^2}{\theta X_{T-1}^2 + (1-\theta)X_T^2} = \theta \frac{\alpha_0^2}{\alpha_0^2 + (1-\theta)\triangle\alpha^2} < \theta$ under one-time growth in period T. Thus, we have $L_T > L_T^*$ while $i_T^R = 0$. Specifically, we have

$$\begin{aligned}
L_T &= (1 - \Theta_{T-1,T-1}(1 - \xi i_{T-1}^R))\frac{L_{T-1}}{\Theta_{T-1,T}(1 - \xi i_T^R)} \\
&= \frac{1 - \Theta_{T-1,T-1}}{\Theta_{T-1,T}} L_{T-1} \\
&= \theta \frac{\alpha_0^2 + (1-\theta)\triangle\alpha^2}{\theta\alpha_0^2} L_{T-1} \\
&= \frac{\alpha_0^2 + (1-\theta)\triangle\alpha^2}{\alpha_0^2} L_{T-1} \\
&= \big(1 + (1-\theta)\frac{\triangle\alpha^2}{\alpha_0^2}\big) L_{T-1}.
\end{aligned}$$

We proceed to apply Equation (50) for period T, then we have $\Theta_{T,T} = 1 - \Theta_{T-1,T} = (1-\theta)\frac{\alpha_0^2 + \triangle\alpha^2}{\alpha_0^2 + (1-\theta)\triangle\alpha^2} > 1-\theta$ and $\Theta_{T,T+1} = \theta$. Here, supposing $i_{T+1}^R = 0$, we have

$$\begin{aligned}
L_{T+1} &= (1 - \Theta_{T,T}(1 - \xi i_T^R))\frac{L_T}{\Theta_{T,T+1}(1 - \xi i_{T+1}^R)} \\
&= \frac{1 - \Theta_{T,T}}{\Theta_{T,T+1}} L_T \\
&= \frac{1}{\theta} \frac{\theta\alpha_0^2}{\alpha_0^2 + (1-\theta)\triangle\alpha^2} L_T \\
&= \frac{\alpha_0^2}{\alpha_0^2 + (1-\theta)\triangle\alpha^2} L_T \\
&= L_{T-1} = L_{T+1}^*.
\end{aligned}$$

Thus, we confirm $i_{T+1}^R = 0$ and $L_{T+1} = L_{T+1}^*$. Lastly, when applying Equation (50) for period $t = T+1, T+2, ...$, we always have $\Theta_{t,t} = 1-\theta$ and $\Theta_{t,t+1} = \theta$. Thus, we have $L_{t+1} = L_t = L_{t+1}^*$ and $i_{t+1}^R = 0$ for every $t = T+1, T+2, ...$ This completes our proof.

A.18 Proof of Proposition 8

When applying Equation (50) for period $t = 1, 2, ..., T-2$, observe $\Theta_{t,t} = 1-\theta$ and $\Theta_{t,t+1} = \theta$. Given this, $L_t = L_t^*$ for $t = 1, 2, .., T-2$ is brought by $i_{t+1}^R = 0$ for

$t = 1, 2, .., T-2$. Next, when examining Equation (50) for period $t = T-1$, observe that $\Theta_{T-1,T-1} = 1-\theta$ while $\Theta_{T-1,T} = \frac{\theta\alpha_0^2 n_{T-1}^{2e}}{\theta\alpha_0^2 n_{T-1}^{2e} + (1-\theta)\alpha_0^2 n_T^{2e}} = \theta \frac{\alpha_0^2}{\alpha_0^2 + (1-\theta)(n_T^{2e} - n_{T-1}^{2e})/n_{T-1}^{2e}} < \theta$ under one-time development in period T. Note that different from the case of one-time growth, $L_T^* > L_{T-1}^*$. Thus, to see whether $i_T^R > 0$ or not, we examine whether $\frac{L_T^*}{L_{T-1}^*}\Theta_{T-1,T} > \theta$ or not. On this point, the proof of Lemma 8 for the case of perturbation in sector 1 ensures that $\frac{L_T^*}{L_{T-1}^*}\Theta_{T-1,T} > \frac{L_{T-1}^*}{L_{T-2}^*}\Theta_{T-2,T-1} = \theta$. Thus, we have $L_T = L_T^*$ while $i_T^R > 0$.

Given $L_{T-1} = L_{T-1}^*$ and $L_T = L_T^*$, Equation (50) for $t = T-1$ indicates

$$\begin{aligned} & \Theta_{T-1,T-1} + \frac{L_T}{L_{T-1}}\Theta_{T-1,T}(1 - \xi i_T^R) = 1 \\ \Leftrightarrow \quad & 1 - \xi i_T^R = \Big(\frac{L_{T-1}^*}{L_T^*}\Big)\Big(\frac{1-\Theta_{T-1,T-1}}{\Theta_{T-1,T}}\Big) \end{aligned}$$

Here, observe $\frac{L_{T-1}^*}{L_T^*} = \frac{n_{T-1}^{2e} n_T^{1e}}{n_{T-1}^{1e} n_T^{2e}}$ and $\frac{1}{\Theta_{T-1,T}} = \frac{\theta\alpha_0^2 n_{T-1}^{2e} + (1-\theta)\alpha_0^2 n_T^{2e}}{\theta\alpha_0^2 n_{T-1}^{2e}} = 1 + \frac{1-\theta}{\theta}\frac{n_T^{2e}}{n_{T-1}^{2e}}$. Also, since $n_t^{1e} = \frac{\bar{x}^1}{\alpha_t^1}$, observe $\frac{n_{T-1}^{1e}}{n_T^{1e}} = \frac{\alpha_0^1 + \triangle\alpha^1}{\alpha_0^1}$ and $\frac{n_T^{2e}}{n_{T-1}^{2e}} = \frac{\triangle\alpha^1 + \alpha_0^1 - \bar{x}^1}{\alpha_0^1 - \bar{x}^1}\frac{\alpha_0^1}{\alpha_0^1 + \triangle\alpha^1}$. Using these, we have

$$\begin{aligned} 1 - \xi i_T^R &= \Big(\frac{n_{T-1}^{2e} n_T^{1e}}{n_{T-1}^{1e} n_T^{2e}}\Big)\Big(\theta(1 + \frac{1-\theta}{\theta}\frac{n_T^{2e}}{n_{T-1}^{2e}})\Big) \\ &= \theta\Big(\frac{n_{T-1}^{2e} n_T^{1e}}{n_{T-1}^{1e} n_T^{2e}}\Big) + (1-\theta)\Big(\frac{n_T^{1e}}{n_{T-1}^{1e}}\Big) \\ &= \theta\Big(1 + \frac{\triangle\alpha^1}{\alpha_0^1 - \bar{x}^1}\Big)^{-1} + (1-\theta)\Big(\frac{\alpha_0^1 + \triangle\alpha^1}{\alpha_0^1}\Big)^{-1} \\ &= \theta\Big(1 + \frac{\triangle\alpha^1/\alpha_0^1}{1 - \bar{x}^1/\alpha_0^1}\Big)^{-1} + (1-\theta)\Big(1 + \triangle\alpha^1/\alpha_0^1\Big)^{-1} \end{aligned}$$

Concerning the size of the excess credit, using Equation (50) for $t = T$ with $i_{T+1}^R = 0$, we

have

$$\begin{aligned}
\frac{L_{T+1}}{L_T^*} &= \frac{1-\Theta_{T,T}(1-\xi i_T^R)}{\Theta_{T,T+1}} \\
&= \frac{1}{\theta}\Big(1-\Theta_{T,T}(1-\xi i_T^R)\Big) \\
&= \frac{1}{\theta}\Big(1-\Theta_{T,T}\Big(\frac{L_{T-1}^*}{L_T^*}\Big)\Big(\frac{1-\Theta_{T-1,T-1}}{\Theta_{T-1,T}}\Big)\Big) \\
&= \frac{1}{\theta}\Big(1-\Big(\frac{L_{T-1}^*}{L_T^*}\Big)\Big(\frac{\Theta_{T,T}}{\Theta_{T-1,T}}\theta\Big)\Big) \\
&= \frac{1}{\theta}-\Big(\frac{L_{T-1}^*}{L_T^*}\Big)\Big(\frac{1-\Theta_{T-1,T}}{\Theta_{T-1,T}}\Big) \\
&= \frac{1}{\theta}-\Big(\frac{L_{T-1}^*}{L_T^*}\Big)\Big(\frac{1}{\Theta_{T-1,T}}-1\Big) \\
&= \frac{1}{\theta}-\Big(\frac{n_{T-1}^{2e}n_T^{1e}}{n_{T-1}^{1e}n_T^{2e}}\Big)\Big(1+\frac{1-\theta}{\theta}\frac{n_T^{2e}}{n_{T-1}^{2e}}-1\Big) \\
&= \frac{1}{\theta}-\Big(\frac{1-\theta}{\theta}\Big)\Big(\frac{n_T^{1e}}{n_{T-1}^{1e}}\Big) \\
&= \frac{1}{\theta}-\Big(\frac{1-\theta}{\theta}\Big)\Big(\frac{\alpha_0^1}{\alpha_0^1+\triangle\alpha^1}\Big) \\
&= \frac{1}{\theta}-\Big(\frac{1-\theta}{\theta}\Big)\Big(\frac{1}{1+\triangle\alpha^1/\alpha_0^1}\Big) > 1.
\end{aligned}$$

Thus, we confirm $i_{T+1}^R = 0$ and $L_{T+1} > L_{T+1}^* = L_T^*$.

Lastly, when applying Equation (50) for period $t = T+1, T+2, ...$, we always have $\Theta_{t,t} = 1-\theta$ and $\Theta_{t,t+1} = \theta$. Thus, we have $L_{t+1} = L_t$ and $i_{t+1}^R = 0$ for every $t = T+1, T+2, ...$ This completes our proof.

A.19 Proof of Proposition 9

Proposition 2 indicates that asset bubble is possible to occur in some period only if $L_{T+1} > \bar{M} = (\frac{\alpha_0^2}{\alpha_0^2-\bar{x}^2})(\frac{\alpha_0^1+\triangle\alpha^1-\bar{x}^1}{\alpha_0^1+\triangle\alpha^1})M_0^1$. Here, remember $L_T^* = \frac{n_T^{2e}}{n_T^{1e}}M_0^1 = (\frac{1}{n_T^{1e}}-1)M_0^1 = (\frac{\triangle\alpha^1+\alpha_0^1-\bar{x}^1}{\bar{x}^1})M_0^1$. Thus, we have

$$\frac{\bar{M}}{L_T^*} = \Big(\frac{1}{1-\bar{x}^2/\alpha_0^2}\Big)\Big(\frac{\bar{x}^1/\alpha_0^1}{1+\triangle\alpha^1/\alpha_0^1}\Big)$$

From Proposition 8, we have

$$\begin{aligned}
& \frac{L_{T+1}}{L_T^*} > \frac{\bar{M}}{L_T^*} \\
\Leftrightarrow \quad & \frac{1}{\theta}\Big[1-(1-\theta)\Big(\frac{1}{1+\triangle\alpha^1/\alpha_0^1}\Big)\Big] > \Big(\frac{1}{1-\bar{x}^2/\alpha_0^2}\Big)\Big(\frac{\bar{x}^1/\alpha_0^1}{1+\triangle\alpha^1/\alpha_0^1}\Big) \\
\Leftrightarrow \quad & \theta\Big(\frac{1}{1-\bar{x}^2/\alpha_0^2}\Big)\Big(\frac{\bar{x}^1/\alpha_0^1}{1+\triangle\alpha^1/\alpha_0^1}\Big) + (1-\theta)\Big(\frac{1}{1+\triangle\alpha^1/\alpha_0^1}\Big) < 1 \\
\Leftrightarrow \quad & \frac{1}{1+\triangle\alpha^1/\alpha_0^1}\Big[\theta\Big(\frac{\bar{x}^1/\alpha_0^1}{1-\bar{x}^2/\alpha_0^2}\Big) + (1-\theta)\Big] < 1 \\
\Leftrightarrow \quad & \theta\Big(\frac{\bar{x}^1/\alpha_0^1 + \bar{x}^2/\alpha_0^2 - 1}{1-\bar{x}^2/\alpha_0^2}\Big) + 1 < 1 + \triangle\alpha^1/\alpha_0^1 \\
\Leftrightarrow \quad & \theta\Big(\frac{\bar{x}^1}{\alpha_0^1} + \frac{\bar{x}^2}{\alpha_0^2} - 1\Big) < \frac{\triangle\alpha^1}{\alpha_0^1}\Big(1-\frac{\bar{x}^2}{\alpha_0^2}\Big) \\
\Leftrightarrow \quad & \theta\Big(\frac{\bar{x}^1}{\alpha_0^1} - (1-\frac{\bar{x}^2}{\alpha_0^2})\Big)\Big(1-\frac{\bar{x}^2}{\alpha_0^2}\Big)^{-1} < \frac{\triangle\alpha^1}{\alpha_0^1} \\
\Leftrightarrow \quad & \theta\Big(\frac{\bar{x}^1/\alpha_0^1}{1-\bar{x}^2/\alpha_0^2} - 1\Big) < \frac{\triangle\alpha^1}{\alpha_0^1}.
\end{aligned}$$

We proceed to examine the reverse direction of the statement in the current Lemma. Firstly, observe that if Equation (66) is satisfied, then Proposition 1 indicates that in any equilibrium, there exists some period $T-1$ where extreme wealth inequality is attained such that $a_{T-1}^{i,2} = 0$ for every $i \in I_{T-1}^1$ and $a_{T-1}^{i,2} = \bar{a}^2$ for every $i \in I_{T-1}^2$. Given this, Proposition 3 ensures the existence of asset bubble in period T. On this basis, we examine the existence of equilibrium with persistent asset bubble such that $\triangle p_t^{a_2;s} = \triangle p_T^{a_2;s}$ and $M_t^{a_2;s} = M_T^{a_2;s}$ for every $t \geq T$. Existence of such equilibrium is ensured under sufficiently large $\lambda < 1$, since maximum possible size of $M_T^{a_2;s}$ ensures $s_T^i = 0$ for every $i \in I_T^2$ while it will never cause $s_T^i > 0$ for any $i \in I_T^{1;1}$, and maximum possible $M_t^{a_2;s} = M_T^{a_2;s}$ for every $t \geq T$. This proves part I). Part II) is immediate under an equilibrium with persistent asset bubble.

A.20 Proof of Proposition 11

We first show the "if" part. Observe that

$$\begin{aligned}
L_T^* = L_2 \quad & \Leftrightarrow \quad \frac{n_T^{2e}}{n_T^{1e}} M_0^1 = L_2 \\
& \Leftrightarrow \quad \frac{1-\bar{x}^1/\alpha_T^1}{\bar{x}^1/\alpha_T^1} = \frac{L_2}{M_0^1} \\
& \Leftrightarrow \quad \frac{\alpha_T^1}{\bar{x}^1} = \frac{M_0^1 + L_2}{M_0^1} \\
& \Leftrightarrow \quad \alpha_T^1 = \bar{x}^1 \frac{M_0^1 + L_2}{M_0^1}.
\end{aligned}$$

Now, suppose $L_t = L_t^*$. Also, suppose that $i_t^R = 0$ for every $t = 3, 4, ..., \bar{t}$ with arbitrary $\bar{t} \geq 4$. Then, given $i_2^R = 0$, we have

$$\begin{aligned} L_{\bar{t}+1}^0 &= \frac{\Theta_{\bar{t}-1,\bar{t}}}{\Theta_{\bar{t},\bar{t}+1}} L_{\bar{t}}^0 \\ &= \Big(\frac{\Theta_{\bar{t}-1,\bar{t}}}{\Theta_{\bar{t},\bar{t}+1}}\Big)\Big(\frac{\Theta_{\bar{t}-2,\bar{t}-1}}{\Theta_{\bar{t}-1,\bar{t}}}\Big) L_{\bar{t}-1}^0 \\ &= \Pi_{t=2}^{\bar{t}}\Big(\frac{\Theta_{t-1,t}}{\Theta_{t,t+1}}\Big) L_2 \\ &= \frac{\Theta_{1,2}}{\Theta_{\bar{t},\bar{t}+1}} L_2 \\ &= \theta\Big(\frac{\theta n_{\bar{t}}^{2e}}{\theta n_{\bar{t}}^{2e} + (1-\theta) n_{\bar{t}+1}^{2e}}\Big)^{-1} L_2 \\ &= \theta\Big(1 + \frac{1-\theta}{\theta}\frac{n_{\bar{t}+1}^{2e}}{n_{\bar{t}}^{2e}}\Big) L_2 \geq L_2. \end{aligned}$$

Since $L_{\bar{t}+1}^* < L_T^* = L_2$, we have $L_{\bar{t}+1} = \max\{L_{\bar{t}+1}^*, L_{\bar{t}+1}^0\} \geq L_2$. Since $\frac{n_{t+1}^{2e}}{n_t^{2e}} > 1$ for every $t = 3, 4, ..., T-1$, we have $L_t > L_t^*$ and $i_t^R = 0$ for every $t = 2, 3, ..., T$. Also, for $t \geq T$, since $n_{t+1}^{2e} = n_t^{2e}$, we have $L_{t+1}^0 = L_2$. Thus, we have $L_t = L_2 = L_t^*$ for every $t \geq T+1$.

For the "only if" part, it suffices to show that if $L_T^* \neq L_2$, then at least one of statements i)-iii) is not attained. First, suppose that $L_T^* < L_2$. Then, observe that $L_{T+1}^* = L_T^* < L_2 = L_{T+1}^0$ and $i_t^R = 0$ for every $t \geq 3$. Thus, we have $L_{T+1} = \max\{L_{T+1}^*, L_{T+1}^0\} = L_{T+1}^0 = L_2 > L_{T+1}^*$, and we confirm that statement iii) is not satisfied. Next, suppose that $L_T^* > L_2$. Then, there always exists period $t' \geq 3$ such that for every $3 \leq t \leq t'$, $i_t^R = 0$ and $L_{t'+1}^0 = L_2 < L_{t'+1}^*$. Thus, we have $i_{t'+1}^R > 0$, which confirm that statement i) is not attained. This completes our whole proof.

A.21 Proof of Proposition 12

We first show that there exist some threshold values $\triangle^1$ and $\triangle^2$ for $\triangle\alpha^2$ such that
I) for $0 \leq \triangle\alpha^2 \leq \triangle^1$, we have
  i) $i_T^R > 0$ and $i_{T+1}^R = 0$, while
  ii) i_T^R is decreasing to $\triangle\alpha^2$.
II) for $\triangle^1 \leq \triangle\alpha^2 \leq \triangle^2$, we have
  i) $i_T^R > 0$ and $i_{T+1}^R > 0$, while
  ii) i_T^R is decreasing to $\triangle\alpha^2$ and i_{T+1}^R is increasing to $\triangle\alpha^2$.
III) for $\triangle^2 \leq \triangle\alpha^2$, we have
  i) $i_T^R = 0$ and $i_{T+1}^R > 0$, while
  ii)i_{T+1}^R is constant.

Firstly, when $\triangle\alpha^2 = 0$, we have $i_T^R > 0$ and $i_{T+1}^R = 0$. Observe that $i_T^{R,0}$ is decreasing to $\triangle\alpha^2$, while $i_{T+1}^{R,0}$ is increasing to $\triangle\alpha^2$. Particularly, when $i_T^{R,0} = 0$, we can show that

$i^R_{T+1} > 0$. To see this, given $i^R_T = 0$ and $L_T = L^*_T$, we have

$$\begin{aligned} 1 - \xi i^{R,0}_{T+1} &= \frac{\Theta_{T-1,T}}{\Theta_{T,T+1}} \frac{L^*_T}{L^*_{T+1}} \\ &= \frac{\Theta_{T-1,T}}{\Theta_{T,T+1}}. \end{aligned}$$

Since $\Theta_{T,T+1} = \theta$ and $\Theta_{T-1,T} < \theta$, we have confirmed $i^R_{T+1} > 0$. This shows the existence of $\triangle^1$ that separate the cases I) and II). Particularly, $\triangle_1$ is determined through $i^{R,0}_{T+1} = 0$ given $i^R_{T-1} = 0$ and $i^R_T = i^{R,0}_T > 0$. Next, observe that $i^{R,0}_T = 0$ always exists for some $\triangle\alpha^2 = \triangle^2$. for $\triangle\alpha^2 \geq \triangle^2$, we have

$$\begin{aligned} 1 - \xi i^{R,0}_{T+1} &= \frac{\Theta_{T-1,T}}{\Theta_{T,T+1}} \frac{L^0_T}{L^*_{T+1}} \\ &= \frac{\Theta_{T-1,T}}{\Theta_{T,T+1}} \frac{\Theta_{T-2,T-1}}{\Theta_{T-1,T}} \frac{L^*_0}{L^*_T} \\ &= \frac{\Theta_{T-2,T-1}}{\Theta_{T,T+1}} \frac{L_{T-1}}{L^*_T} \\ &= \frac{L^*_0}{L^*_T}. \end{aligned}$$

Thus, case III) is confirmed.

We let cases I)-III) above correspond to cases I)-III) in the proposition. To see the statements for case I) in the proposition, we have $i^R_T > 0$ and $i^R_t = 0$ for every $t \geq T+1$. Furthermore, the property of $\frac{\partial i^R_T}{\partial \triangle\alpha^2} < 0$ indicates $\frac{\partial L_{T+1}}{\partial \triangle\alpha^2} < 0$ while $(i^R_T > 0, i^{R,0}_{T+1} = 0)$ indicates $L_{T+1} > L^*_T$. These facts prove statements i) and ii). For case II), we have $i^R_{T+1} > 0$ and $i^R_t = 0$ for every $t \geq T+2$, while the property of $\frac{\partial i^R_{T+1}}{\partial \triangle\alpha^2} > 0$ indicates $\frac{\partial L_{T+2}}{\partial \triangle\alpha^2} > 0$. For case III), we have $i^R_{T+1} > 0$ and $i^R_t = 0$ for every $t \geq T+2$, while the property of $\frac{\partial i^R_{T+1}}{\partial \triangle\alpha^2} = 0$ indicates $\frac{\partial L_{T+2}}{\partial \triangle\alpha^2} = 0$. Also, given $1 - \xi i^R_{T+1} = \frac{L^*_0}{L^*_T}$, we have

$$\begin{aligned} L_{T+2} &= L^0_{T+2} \\ &= \frac{1 - (1 - \Theta_{T,T+1})(1 - \xi i^R_{T+1})}{1 - \Theta_{T+1,T+2}} L_{T+1} \\ &= \frac{1 - (1 - \theta)(L^*_0/L^*_T)}{1 - \theta} L^*_T \\ &= \Big[\frac{1}{1 - \theta} - \frac{L^*_0}{L^*_T}\Big] L^*_T. \end{aligned}$$

Since $\frac{1}{1-\theta} > 1$ and $\frac{L^*_0}{L^*_T} < 1$, we have $\frac{L_{T+2}}{L^*_T} > 1$. This competes the whole proof.

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Declaration of Generative AI and AI-assisted technologies in the writing process

There is nothing to declare about the use of any Generative AI or AI-assisted technologies.

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