Monetary Shocks and Long-Run Neutrality in Asset Pricing*

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Abstract

This paper studies a novel condition, an asset-pricing version of monetary "Long-Run Neutrality." Building on a literature studying high-frequency monetary surprises, our non-parametric test of Long-Run Neutrality reveals violations, especially during the period of unconventional monetary policy. This rejects most monetary models but admits models where investors worry about central bankers' impact on uncertainty and growth. We develop in detail two particular implications of Long-Run Neutrality violations. First, such violations resolve puzzling patterns of disconnect between equity and bond markets in FOMC windows. Second, monetary shocks recovered from asset prices will necessarily be polluted by risk premium effects.

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A modern analysis of monetary policy must go beyond its control of short-term interest rates. Nowadays, central banks access a wide-ranging toolkit that includes policies like forward guidance, quantitative easing, state-contingent asset market interventions, and targeted communication about the economy. According to the existing evidence, these "unconventional policies" permeate the whole spectrum of financial markets, including both short-term and long-term interest rates as well as riskier assets like stocks and other corporate securities. Transmission to these markets is often thought to reflect the influence central bankers have on expectations about the future, on uncertainties, and even on the pricing of risks. With this variety of different interventions and mechanisms, we are living in a brave new world with multi-faceted and complex monetary policy.[1]

In this paper, we provide an organizing framework to synthesize these emerging perspectives. Our framework zooms in on a critical condition—*Long-Run Neutrality (LRN)*—that says, roughly speaking, the Fed exerts at most a transitory influence on asset pricing. More precisely, LRN says that the Fed does not impact marginal utility (a.k.a., the pricing kernel) in permanent ways. A violation of LRN, by contrast, says that the Fed can permanently impact marginal utility. Whether or not LRN holds turns out to be central to our understanding of how monetary transmission works and the approach researchers should take to identify monetary shocks.

If LRN fails, several implications follow. First, most of our theoretical models are wrong. Most models equate LRN to conventional notions of monetary neutrality, i.e., to the idea that monetary policy does not affect real economic variables in the long run, and so the majority of the literature implicitly assumes that LRN holds. Second, in response to monetary news, risky asset markets and Treasury bonds can move in distinct ways. This implication connects to an empirical literature that asks whether equity and bond markets are connected or disconnected during monetary announcements (Bernanke and Kuttner, 2005; Bauer et al., 2023; Nagel and Xu, 2024; Boehm and Kroner, 2025). Third, researchers cannot identify certain monetary shocks by looking at asset markets, whereas they could do so if LRN were to hold. According to classical asset-pricing theory, asset markets co-mingle investor beliefs with risk premia; LRN is the necessary and sufficient condition for separating beliefs and risk premia in the context of monetary announcements. Consequently, many existing approaches that extract monetary shocks from asset

[1]For instance, forward guidance attempts to directly manage beliefs about the future path of short rates. Asset purchases, by absorbing risk from the market, can potentially increase investors' risk-bearing capacity and thereby alter risk prices. Such asset purchases are not one-offs but rather part of a state-contingent plan, in which case their impacts can appear upon announcement and through a mix of both expectations and risk premia. Speeches by central bankers can both convey an outlook on the economy as well as illuminate the bank's own policy stance. Such communication can calm or aggravate financial markets, depending on whether investors agree or disagree with how the Fed positions itself.

markets implicitly assume LRN (Gürkaynak et al., 2005b; Swanson, 2021; Haddad et al., 2025). The paper fleshes out all these implications in detail.

In our paper, we develop a non-parametric test of LRN. Our test builds on the insight that the return of the growth-optimal portfolio in excess of a long-maturity bond identifies the permanent component of the pricing kernel (Alvarez and Jermann, 2005). LRN implies this investment strategy has exactly a zero return on Fed announcements. To repeat, the return should not only be zero on average but zero on each and every announcement. To implement this test, we proxy the growth-optimal portfolio and long-maturity bond with equities and Treasury bonds, respectively, and study their return dynamics in high-frequency windows around Fed announcements. Of course, it is paramount to obtain good proxies for the growth-optimal portfolio and long-maturity bond. We therefore devote significant attention to exploring various different proxies and find our results are extremely robust.

Our evidence suggests LRN is violated. Indeed, equity and long-term bonds behave very differently near Fed announcements. The largest disconnect between equity and long-term bonds comes during the 2009-2015 period, when various unconventional monetary policies were introduced. Through the lens of our framework, it looks like the Fed's foray into these unconventional policies first scared the market (increased marginal utility sharply) and then built up confidence over time (decreasing marginal utility over the course of several years). One interpretation is that the market was learning about the efficacy of the Fed's new toolkit. Our framework also suggests that conventional monetary policies (i.e., short-rate shocks) are not the source of LRN violations.

Let us now revisit and elaborate on the implications of our evidence. First, our results resolve some puzzles regarding equity-bond disconnect. Under LRN, we prove that equities and bonds should always move together in response to monetary shocks: the R-squared from regressing equity returns on the yield curve and its changes should be close to one. LRN violations thus allow us to potentially rationalize the empirically-documented equity-bond disconnect in FOMC windows (Boehm and Kroner, 2025). Conversely, our result that LRN seems to hold better for conventional monetary shocks can also explain the tight equity-bond connection in response specifically to short-rate shocks (Nagel and Xu, 2024). These empirical results are not in conflict and, in fact, support an interpretation whereby the Fed can move stock and bond risk premia either in the same or opposite directions (Cieslak and Pang, 2021). Given these results, using long-term bond yields as a reduced-form sufficient statistic for monetary policy shocks (van Binsbergen and Grotteria, 2024) may be valid only in a subset of cases.

Second, our results clarify when monetary shocks can and cannot be identified from

asset markets. Asset markets are a natural arena to explore because of the richness in financial claims covering many horizons (e.g., far into the future), at many levels of contingency (e.g., isolating specific aspects of the probability distribution), and available at a high frequency. We prove that, in a large class of environments, LRN is a necessary and sufficient condition for such shock-identification. LRN violations, specifically vis-à-vis unconventional policies, imply biases likely arise when attempting to extract forward guidance shocks (Gürkaynak et al., 2005b), quantitative easing shocks (Swanson, 2021), and asset-purchase promises (Haddad et al., 2025). These policies all hinge on beliefs, and so extracting their shocks from market-implied forecast revisions is polluted by risk premium effects, if LRN fails. By contrast, the fact that LRN seems to hold better for conventional monetary policy suggests that short-rate shocks can be well-identified from asset markets.[2]

The paper concludes by quantifying the effects of LRN violations. We study a simple calibrated two-factor affine asset pricing model. For our exercises, it is important to have more shocks than state variables, so that the Treasury yield curve does not simply "reveal" all the underlying shocks and mechanically connect stock and bond markets. Our model thus entertains three types of shocks: a short-rate shock, a risk price shock, and a flight-to-quality shock (which increases risk prices and simultaneously decreases the short rate). The shock structure aligns with the risk premium decomposition proposed by Cieslak and Pang (2021). The source of LRN violations is precisely the fact that risk prices are time-varying and influenced by monetary policy. In this model, we obtain a degree of equity-bond disconnect broadly in line with Nagel and Xu (2024) and Boehm and Kroner (2025): equities and bonds respond on average very similarly to short-rate shocks, whereas the R-squared from regressing equity returns on the yield curve and its changes is below 30%. We then show that attempts to extract monetary shocks lead to sizeable biases, especially in two situations: (i) when studying forecast revisions to objects that depend non-trivially on the term structure of risk premia; and (ii) when attempting to extract shocks in "bad times" with high marginal utility.

Literature review. Our starting point builds on a literature that decomposes the pricing kernel into permanent and stationary components (Alvarez and Jermann, 2005; Hansen and Scheinkman, 2009; Bakshi and Chabi-Yo, 2012; Qin and Linetsky, 2017; Corsetti et al., 2023). We then use this decomposition to develop a model-free test of the Long-Run Neutrality (LRN) condition. What is new is connecting monetary policy and the perma-

[2] A common approach to identifying short-rate shocks examines high-frequency changes to Fed Funds futures prices on the FOMC meeting day (Krueger and Kuttner, 1996; Rudebusch, 1998; Kuttner, 2001; Rudebusch, 2002; Bernanke and Kuttner, 2005; Piazzesi and Swanson, 2008).

nent component of the pricing kernel, rather than studying the permanent component in general or on average.

An existing literature on "FOMC announcement effects" has also studied returns near Fed announcements. Several authors have argued that equity risk premia are strongly influenced by monetary policy announcements, including both actions and communications (Pearce and Roley, 1985; Rosa, 2011; Savor and Wilson, 2013, 2014; Lucca and Moench, 2015; Ai and Bansal, 2018; Cieslak et al., 2019; Bianchi et al., 2022a,b; Bauer et al., 2023). A related literature examines FOMC announcement effects in government bonds (Ederington and Lee, 1993; Gürkaynak et al., 2005a; Beber and Brandt, 2006; Faust et al., 2007; Hanson and Stein, 2015; Hillenbrand, 2021; Hanson et al., 2021). A smaller and more recent literature directly compares stocks and bonds in FOMC windows (Cieslak and Pang, 2021; Nagel and Xu, 2024; Boehm and Kroner, 2025; Ai and Fu, 2025). Our contribution is primarily a tighter connection between our data analysis and our theory. For one, the bonds we investigate are longer-maturity than those commonly studied, an important contribution given our test asks for an arbitrarily long-maturity bond. Second, we study a variety of proxies for the growth-optimal portfolio, leveraging the data in Aleti and Bollerslev (2024) to go significantly beyond the S&P500 or the value-weighted stock market. Finally, after extracting proxies for the permanent component of the pricing kernel, we investigate how it loads on specific types of monetary news.

When thinking about monetary shock identification, we connect to a literature seeking to measure the impact of forward guidance, asset purchase plans, and central bank communication. More specifically, we connect to a literature that takes a "reduced-form" approach to identification, e.g., extracting a "path" factor from long-term bond yields (Gürkaynak et al., 2005b; Swanson, 2021; Altavilla et al., 2019; Lunsford, 2020), projecting outcomes on long rates directly (van Binsbergen and Grotteria, 2024), or projecting outcomes on communication directly (Brand et al., 2010; Hansen and McMahon, 2016; Leombroni et al., 2021; Gómez-Cram and Grotteria, 2022). As we show, such reduced-form approaches do not capture pure shocks to investor beliefs; they are polluted by risk premia effects, given LRN fails. Haddad et al. (2025) take existing approaches a step further by laying out explicit identification assumptions that restrict the dynamics of the pricing kernel, hoping to recover investor surprises purged of risk premium effects. However, their identification assumptions are subsumed by LRN, meaning their recovered shocks still contain biases from risk premia effects.

Our theoretical results on shock identification connect closely to a broader issue in asset pricing, so-called recovery theory (Ross, 2015; Borovička et al., 2016; Qin and Linetsky, 2016). According to this literature, investor beliefs are not revealed by asset prices,

because beliefs are co-mingled with other permanent components of marginal utility. In the context of monetary policy, there is a nuance: we do not seek beliefs themselves, but rather shocks to beliefs. And this is why the key assumption of recovery theory, *absence* of a permanent component in marginal utility, is replaced by the weaker assumption that such permanent component be *invariant* to monetary policy (i.e., LRN).

An important caveat: LRN is a necessary identification assumption because we take a reduced-form approach, in line with the literature above. With more model structure, monetary shock identification is possible without full-blown LRN. For example, Backus et al. (2022) identify monetary rules versus discretionary shocks under a condition that roughly says "LRN holds on average" rather than path-wise. In our model too, if one assumes rational expectations and exploits the underlying structural dynamics, identification is possible with less than LRN. We refrain from this approach for two reasons. First, if new types of monetary shocks arise over time (e.g., forward guidance, asset purchases), then past estimates of the structural dynamics are less informative about the future. Second, if investors hold distorted beliefs, then estimating the objective dynamics does not recover investors' forecast revisions. To elaborate on this issue of how model structure can relieve the identification burden, our appendix connects more explicitly with Backus et al. (2022), contrasting our approaches in a unified framework.

Finally, our results on LRN speak to structural models of various types. As we show, the most pervasive models implicitly assume LRN holds, because they equate LRN with conventional monetary neutrality. These include standard New Keynesian models with "traditional" utility functions (e.g., Woodford, 2003; Galí, 2015), versions with habit utility (e.g., Pflueger and Rinaldi, 2022), and heterogeneous-agent extensions (e.g., Gertler and Karadi, 2011; Kekre et al., 2024). By contrast, in models where economic growth and uncertainty are priced sources of risks, growth and uncertainty shocks comprise the permanent component of the pricing kernel (Bansal and Yaron, 2004; Beaudry and Portier, 2004; Dew-Becker, 2014; Bidder and Dew-Becker, 2016; Christiano et al., 2014; Fajgelbaum et al., 2017; Bianchi et al., 2018, 2023; Bretscher et al., 2023). Our documented LRN violations suggest, through the lens of such models, that the Fed influences growth and uncertainty, potentially at longer horizons than currently acknowledged.

1 Motivating Examples: What is Long-Run Neutrality?

We begin with a sequence of example models to develop an understanding of our asset pricing version of Long-Run Neutrality (LRN). All the environments have no arbitrage opportunities and hence possess a real stochastic discount factor (SDF) process S. Fur-

thermore, the models are Markovian in some stationary state vector X. In this type of environment, the SDF can be decomposed into permanent and transitory components (Hansen and Scheinkman, 2009)

$$\frac{S_{t+T}}{S_t} = \exp(\eta T)\frac{e(X_t)}{e(X_{t+T})}\frac{H_{t+T}}{H_t}, \tag{1}$$

where η is a constant, $e(\cdot)$ is a deterministic function, and H_t is a martingale. We assume existence of a unique SDF factorization (1).[3]

Let us elaborate on the permanent-transitory factorization to understand it both statistically and economically. Statistically, the SDF factorization is similar in spirit to conventional permanent-transitory decompositions (e.g., Beveridge and Nelson, 1981). A crucial difference is this factorization applies to multiplicative rather than additive processes (i.e., the cumulative SDF S_T/S_0 is the product of all the one-period SDFs S_{t+1}/S_t).

The transitory component is tightly connected to the yield curve. The T-period discount bond price is $\mathbb{E}_t[\frac{S_T}{S_t}]$. Consider an arbitrarily long-maturity discount bond, whose holding period return is the limit $R^{\infty}_{t,t+1} := \lim_{T\to\infty} \mathbb{E}_{t+1}[\frac{S_T}{S_t}]/\mathbb{E}_t[\frac{S_T}{S_t}]$. Using (1), this long bond return is equal to

$$R^{\infty}_{t,t+1} = \exp(-\eta)\frac{e(X_{t+1})}{e(X_t)}\lim_{T\to\infty}\frac{\mathbb{E}_{t+1}[\frac{1}{e(X_T)}\frac{H_T}{H_{t+1}}]}{\mathbb{E}_t[\frac{1}{e(X_T)}\frac{H_T}{H_t}]} = \exp(-\eta)\frac{e(X_{t+1})}{e(X_t)}, \tag{2}$$

where the last equality holds if X_t is stochastically stable under the probability measure generated by H, which we implicitly assume (see footnote 3). The link between the long bond and the transitory component is essential to our empirical tests.

Finally, a fruitful perspective on the martingale H is as a change-of-measure. Let $\hat{\mathbb{E}}$ be the expectation operator induced by H. Given we have identified the transitory component with $R^{\infty}_{t,t+1}$, we know that the time-t price of payoff Y_{t+1} can be computed as

$$\text{price}_t[Y_{t+1}] = \hat{\mathbb{E}}\left[(R^{\infty}_{t,t+1})^{-1}Y_{t+1}\right]$$

[3]Existence holds under general conditions (Hansen and Scheinkman, 2009). In many cases, uniqueness fails. If multiple SDF factorizations satisfy (1), we follow Proposition 1 of Borovička et al. (2016) in picking the unique one such that X is stationary and ergodic under the probability measure $\hat{\mathbb{P}}$ induced by H (i.e., defined by $\hat{\mathbb{P}}(A) := \mathbb{E}(1_A H_T)$ for all sets $A \in \mathscr{F}_T$). For the purposes of this paper, non-uniqueness will not be relevant to the monetary policy questions, which is why we sidestep these issues.

Compare this to the traditional risk-neutral measure which computes the same price as

$$\text{price}_t[Y_{t+1}] = \mathbb{E}^*\left[(R_t^f)^{-1}Y_{t+1}\right],$$

where R_t^f is the one-period riskless rate, and $\mathbb{E}^*$ is the risk-neutral expectation. Both formulas embed risk adjustments. The traditional risk-neutral expectation eliminates one-period risks, and so the appropriate discount rate is the one-period riskless rate. By contrast, $\hat{\mathbb{E}}$ eliminates long-term risks, so it uses the long bond return for discounting. Given this perspective, we sometimes refer to $\hat{\mathbb{E}}$ as the *long-run risk-neutral* expectation.

1.1 Long-Run Neutrality (LRN)

The literature has established that the SDF is primarily driven by permanent shocks (Alvarez and Jermann, 2005; Bakshi and Chabi-Yo, 2012). In other words, the permanent component H accounts for most of the volatility in S. This can be understood intuitively as follows. If permanent shocks are the main risks investors worry about, they consider long-term bonds to be attractive assets, which pushes down equilibrium term premia relative to other risk premia. The empirical fact that term premia are low relative to equity risk premia thus reveals the outsized importance of H. Despite the current consensus that H matters a lot unconditionally, it is not clear whether or not H plays a significant role in monetary policy transmission.

We define *Long-Run Neutrality* (LRN) as invariance of H to monetary policy. Under LRN, monetary policy cannot exert a permanent impact on marginal utility. That is not to say that monetary policy cannot affect risk pricing. On the contrary, monetary policy might affect the state vector X, hence the SDF through the transitory component $e(X)$.

Let us comment on real versus nominal SDFs. We study real SDFs because that is a case where LRN is less obvious and has strong implications for monetary transmission. In almost every monetary model, monetary policy can exert a permanent impact on the price level P (e.g., printing money eventually leads to a permanently higher price level). Consequently, monetary policy will always impact the nominal SDF S/P in permanent ways. Much less obvious is whether or not monetary policy impacts S permanently.

The objective of the examples below will be to determine H and articulate what LRN means for that environment. It will become clear that most existing monetary models align LRN with conventional macroeconomic notions of monetary neutrality, thereby implicitly assuming LRN holds. We illustrate this point and then provide an instructive example where LRN and conventional neutrality are different.

1.2 Standard New Keynesian models

In most versions of the representative-agent New Keynesian model, which is the dominant paradigm of monetary analysis, the SDF is

$$\frac{S_t}{S_0} = \exp(-\delta t)\left(\frac{C_t}{C_0}\right)^{-\gamma}, \tag{3}$$

where C is aggregate consumption, δ is the time-discount rate, and γ is the coefficient of relative risk aversion. Clearly, the permanent component of S emerges solely from the permanent component of C. Therefore, LRN is precisely the statement that monetary policy does not impact the long-run level of consumption. Our asset-pricing version of LRN coincides here with the conventional macro notion of monetary neutrality. A strong consensus among macroeconomists maintains conventional monetary neutrality, and hence LRN is presumed to hold in this model.

The same insight—the equivalence of LRN to conventional macro neutrality—holds in generalized versions of the model. For instance, Pflueger and Rinaldi (2022) considers a New Keynesian model with habit preferences, while Gertler and Karadi (2011) and Kekre and Lenel (2022) consider versions with heterogeneity. These extensions are interesting in that they allow more interesting dynamics for risk premia. However, as shown in Appendix C, the SDFs in such models take the form

$$\frac{S_t}{S_0} = \exp(-\delta t)\left(\frac{C_t}{C_0}\right)^{-\gamma}\frac{f(X_t)}{f(X_0)}, \tag{4}$$

where f is some function and X is a vector of stationary state variables (e.g., the habit stock or agent's wealth shares). Consequently, the permanent component of S still coincides with the permanent component of C. Put differently, these extensions of the basic New Keynesian model only add transitory variation to the SDF. So long as we believe in conventional macro neutrality, LRN is not violated in any of these models.

We pause to make an important distinction between LRN and FOMC announcement premia. They are not the same. For example, in models with heterogeneity (Appendix C.2), any wealth redistribution caused by monetary policy is a priced risk, so FOMC announcements carry a risk premium. Yet since the wealth distribution is stationary, this is a "transitory shock" to the SDF under decomposition (1), and so, absent other forces or considerations, LRN still holds. For this reason, LRN is a weaker condition than zero FOMC announcement premia. By contrast, LRN does typically coincide with

zero FOMC announcement premia in representative-agent models.[4]

1.3 Models with priced news shocks

Next, we present an example with priced news shocks. For conceptual clarity, we impose trend-stationary consumption in this model following Bansal et al. (2010), and so conventional macro neutrality automatically holds in this example. Yet LRN can still be violated, in contrast to the previous models. (Appendix D.2 studies the more typical version with non-stationary consumption which has quantitatively larger LRN deviations.)

Aggregate consumption evolves as follows with stochastic drift and volatility,

$$\log C_{t+1} - \log C_t = \mu_C + g_{t+1} - g_t,$$

where the dynamics of g are given by

$$\begin{aligned} g_{t+1} &= \mu_g + \exp(-\lambda_g) g_t + \sqrt{v_t}\sigma_g \Delta W^{(1)}_{t+1} \\ v_{t+1} &= \mu_v + \exp(-\lambda_v) v_t + \sqrt{v_t}\sigma_v \Delta W^{(2)}_{t+1} \end{aligned}$$

The shocks to (g, v) are standard normals, $\Delta W_{t+1} \sim \text{Normal}(0, I)$.[5] Think of the first shock as producing a transitory boom, while the second shock produces a transitory rise in volatility. There are no permanent shocks to consumption here, by construction.

Suppose the representative investor has recursive preferences as in Kreps and Porteus (1978) and Epstein and Zin (1989), with unitary elasticity of intertemporal substitution

[4] In the context of frictionless representative-agent models models, Ai and Bansal (2018) prove that a non-zero announcement premium arises if and only if the agent possesses preferences with "generalized risk sensitivity" (GRS). Indeed, in the CRRA and habit utility models, LRN holds, as we have argued in the text; meanwhile, these are examples without GRS preferences, so the theorem of Ai and Bansal (2018) implies announcement premia are zero. In Section 1.3, the representative agent does have GRS preferences, and so announcement premia can exist in principle. But if LRN holds, monetary policy conveys no news about future consumption, which then implies a zero announcement premium.

Therefore, if we restrict attention to prominent representative-agent models, a non-zero FOMC announcement premium implies LRN is violated. Still, given that risk premia are notoriously difficult to estimate, there is value to our focus on LRN. For example, the literature expresses some doubt as to whether or not average FOMC returns are true risk premia or simply reflect slow responses to recent macroeconomic announcements or small-sample biases (Alam, 2022; Johannes et al., 2023; Knox et al., 2024). To the extent that LRN violations can be detected more precisely and at higher-frequency, we are able to shed light on the nature of FOMC announcement premia without computing average returns.

[5] In this discrete-time model, it is technically possible that v becomes negative in simulations. For the continuous-time dynamics that eliminate this possibility, see Appendix D.1.

(EIS). The investor's continuation value satisfies the following recursion

$$U_t = (1 - \exp(-\delta)) \log C_t + \exp(-\delta) \frac{\log \mathbb{E}_t[\exp((1-\gamma)U_{t+1})]}{1-\gamma},$$

and the one-period SDF is given by

$$\frac{S_{t+1}}{S_t} = \exp(-\delta) \frac{C_t}{C_{t+1}} \frac{\exp[(1-\gamma)U_{t+1}]}{\mathbb{E}_t[\exp[(1-\gamma)U_{t+1}]]}. \tag{5}$$

The interesting innovation in this class of preferences is the final term $\frac{\exp[(1-\gamma)U_{t+1}]}{\mathbb{E}_t[\exp[(1-\gamma)U_{t+1}]]}$ which only emerges if $\gamma \neq 1$ and is a martingale. Variables which impact continuation utility, even if they do not impact long-term consumption, will potentially be part of the permanent component of the SDF. In this example, consumption is trend-stationary, and so the SDF's entire permanent component is given by this final term.

We can solve for indirect utility to compute this permanent component as

$$\frac{H_{t+1}}{H_t} = \exp\Big\{ -\frac{1}{2}(\gamma-1)^2 v_t b' \Sigma b - \sqrt{v_t}(\gamma-1) b' \Sigma \Delta W_{t+1} \Big\}.$$

where $\Sigma := \text{diag}(\sigma_g, \sigma_v)$ and $b := (b_g, b_v)'$ is given by $b_g = \frac{1-\exp(-\delta)}{1-\exp(-\delta-\lambda_g)}$ and $b_v < 0$ that solves a quadratic equation (see Appendix D.1 for these calculations). The important insight is that H is stochastic, with exposure to both the growth and volatility shocks, despite the fact that these only affect consumption in a transitory way. Consequently, LRN here requires that monetary policy does not affect (i) the consumption growth rate g or (ii) its uncertainty v.

This example presents an interesting possibility that monetary news causes violations of LRN, even though conventional macro neutrality holds. The example may also suggest that LRN should not be expected to hold empirically. Indeed, through the lens of models where news shocks are priced, which are by now a bedrock of modern asset pricing, monetary authorities who influence the probability distribution of future real outcomes, even in the short run, will often violate LRN.[6]

[6]New Keynesian models with similar properties to our example economies include Kung (2015) and Bianchi et al. (2022a). In Kung (2015), monetary policy can affect R&D investment, hence growth rates. In Bianchi et al. (2022a), monetary policy occasionally modifies its reaction function, which induces stochastic volatility. For these effects to be large quantitatively, however, it must be the case the monetary policy affects growth or uncertainty persistently, even if not permanently. To see this, notice that parameters $b := (b_g, b_v)'$ that govern the shock exposures of H are both decreasing in the reversion speeds λ_g and λ_v.

1.4 Affine asset-pricing models

Finally, we consider a very common asset-pricing model with conditionally log-normal dynamics. There is a latent state vector that follows a stationary VAR[7]

$$X_{t+1} = AX_t + B\Delta W_{t+1}, \tag{6}$$

where $\Delta W_{t+1} \sim \text{Normal}(0, I)$. Assume the SDF is given by

$$\frac{S_{t+1}}{S_t} = \exp\Big[-r_t - \frac{1}{2}\|\pi_t\|^2 - \pi_t'\Delta W_{t+1}\Big], \tag{7}$$

where $r_t = \rho_0 + \rho_1' X_t$ is the one-period risk-free rate, and $\pi_t = \pi_0 + \Pi X_t$ is the one-period risk price vector. The permanent component of the SDF is given by

$$\frac{H_{t+1}}{H_t} = \exp\Big[-\frac{1}{2}\|\pi_t - B'v\|^2 - (\pi_t - B'v)\cdot\Delta W_{t+1}\Big] \tag{8}$$

where $v = -[I - (A - B\Pi)']^{-1}\rho_1$ (see Appendix B.1). The shock-exposure of H is the short-run risk price π_t plus additional long-run risk compensation from $-B'v$.

In this class of models, LRN requires time-invariant risk pricing of monetary policy shocks. Indeed, if LRN is to hold, then every shock $\Delta W^{(i)}$ that can be impacted by monetary policy must have $\pi_t^{(i)} = [B'v]^{(i)}$. LRN thus requires that the short-run risk price $\pi_t^{(i)}$ be constant (and equal to a knife-edge value). Non-monetary risk prices can be time-varying, but monetary risk prices must be constant. We caution that constant monetary risk pricing is an overly restrictive characterization of LRN that need not hold generally. For example, in the models of Section 1.2, monetary policy could induce time-varying risk pricing through its transitory real effects while still retaining LRN.[8]

[7] These types of models are often specified as reduced-form models, but they also nest more structural versions such as the preferred-habitat model of Vayanos and Vila (2021). In the preferred-habitat interpretation, the state variables include the short rate as well as various "demand factors."

[8] For instance, in the standard New Keynesian model with CRRA utility, a monetary policy that temporarily modifies the conditional volatility of consumption induces time-variation in risk pricing (e.g., the risk price vector in a conditionally log-normal model with CRRA utility is $\gamma\sigma_{C,t}$, where $\sigma_{C,t}$ is the conditional shock exposure of consumption growth). In the example with habit utility (Appendix C.1), real effects on consumption growth today modify the habit stock tomorrow, which translates into risk price dynamics. Finally, in the example with heterogeneity (Appendix C.2), monetary policy that redistributes wealth also affects risk pricing. In all of these examples, if the real effects of monetary policy are transitory, then LRN holds despite time-varying risk prices.

2 Testing Long-Run Neutrality

In this section, we construct a simple non-parametric test of Long-Run Neutrality (LRN). This test builds on insights by Alvarez and Jermann (2005) and Bakshi and Chabi-Yo (2012) in proxying the permanent and transitory components of the SDF. We evaluate the test with some novel evidence, followed by a comparison to the existing literature.

2.1 A non-parametric test

Our test takes the difference between two portfolios: (i) an infinite-maturity bond with return $R^{\infty}_{t,t+\Delta}$; and (ii) the growth-optimal portfolio with return $R^{*}_{t,t+\Delta}$. Recall that the holding period return on the infinite-maturity bond is given by

$$R^{\infty}_{t,t+\Delta} = \exp(-\eta\Delta)\frac{e(X_{t+\Delta})}{e(X_t)}. \tag{9}$$

On the other hand, the growth-optimal portfolio return $R^{*}_{t,t+\Delta}$ is defined as the maximal log return: it is the time-$(t+\Delta)$ measurable return R that maximizes $\mathbb{E}_t[\log(R)]$ subject to $\mathbb{E}_t[\frac{S_{t+\Delta}}{S_t}R] = 1$, the solution of which is $R^{*}_{t,t+\Delta} = \frac{S_t}{S_{t+\Delta}}$ (Bansal and Lehmann, 1997). In an incomplete market, where there are multiple possible SDFs, the growth-optimal portfolio will proxy the entropy-minimizing SDF. Putting these results together, and using the SDF factorization (1), the excess return of the growth-optimal portfolio relative to the infinite-horizon bond is

$$\log(R^{*}_{t,t+\Delta}) - \log(R^{\infty}_{t,t+\Delta}) = \log\left(\frac{H_t}{H_{t+\Delta}}\right) \tag{10}$$

over any horizon Δ. The result in (10) holds in even more general environments than the one considered here—e.g., in non-Markovian environments (Qin and Linetsky, 2017).

Under LRN, the excess return $\log(R^{*}_{t,t+\Delta}) - \log(R^{\infty}_{t,t+\Delta})$ should be identically zero in a short window around monetary announcements. This is our test. Notice that the return should be *identically zero*, not zero on average. In particular, higher moments of $\log(R^{*}_{t,t+\Delta}) - \log(R^{\infty}_{t,t+\Delta})$ will be particularly informative about LRN, even if this long-short portfolio earns zero on average.

The most important question is how to proxy $R^{\infty}_{t,t+\Delta}$ and $R^{*}_{t,t+\Delta}$. In our baseline results, we suppose $R^{\infty}_{t,t+\Delta}$ is well-approximated by 30-year Treasuries, and $R^{*}_{t,t+\Delta}$ is well-approximated by the aggregate stock market. Under these proxies, LRN says that monetary policy impacts the stock market and 30-year Treasuries in an identical way.

After presenting these baseline results, we will perform extensive robustness. We will also explore robustness to the fact that we use nominal rather than real returns.

2.2 Baseline evidence on Long-Run Neutrality

Following much of the literature, we consider scheduled FOMC decision announcements. The FOMC meets eight times per year on a pre-announced schedule.[9] Since 1994, the FOMC has published statements of the policy decisions, and since 2011, the statements have been followed by press conferences by the Fed Chair, initially every other announcement and subsequently, starting in 2019, after each announcement. Until 2011, statements were released at 14:15 ET. The time changed in 2011, alternating between 12:30 and 14:15 ET, depending on whether the meeting was followed by a press conference. Currently, statements are published at 14:00 ET, and the press conference is held at 14:30 ET.

Our baseline analysis relies on the E-mini S&P 500 returns as a proxy for R^* and the 30-year Treasury bond (T-bond) futures returns as a proxy for R^∞. We obtain futures prices at a one-minute frequency from TickData.com. The 30-year T-bond futures is the longest maturity consistently available throughout our sample period. The actual delivery can take place in bonds with maturities of 15 years and above. Our high-frequency sample starts in September 1997 when the E-mini S&P 500 futures contract was introduced and runs through December 2023, covering 210 scheduled FOMC announcements and 71 press conferences. Section 2.3 analyzes the robustness of our results to alternative proxies for R^* and R^∞.

We consider event windows from 24 hours before, narrowly around, and up to 24 hours after the FOMC decision announcements and press conferences. Table 1 summarizes the distribution of log returns of E-mini futures in excess of 30-year T-bond futures in different windows. The results in Panel A show that equities have done particularly well in the 24 hours before the FOMC announcement, earning on average 26 bps higher returns than bonds during the 1997–2023 sample. This result is consistent with, albeit somewhat weaker than, the original Lucca and Moench (2015) finding based on the 1994–2011 sample. While on average equities performed similarly to bonds in the narrow window of $\pm$15 minutes around the announcements, they underperformed bonds by about 20 bps in the 24 hours after the announcement. Panel B summarizes returns around press conferences, showing a broadly similar pattern.

[9] In 2020, one scheduled meeting was canceled.

Panel A. Monetary policy decision (MPD) announcements

	N	Mean	SE(mean)	SD	Skew	Kurtosis
(−24h,−15m)	210	26.0	8.2	118.2	1.8	18.6
(−15m,+15m)	210	-1.3	4.1	59.5	-1.5	11.3
(−15m,+24h)	210	-20.1	12.3	178.2	-0.8	6.0

Panel B. Press conferences (PC)

	N	Mean	SE(mean)	SD	Skew	Kurtosis
(−24h,−15m)	71	18.5	9.1	76.5	0.1	3.6
(−15m,+15m)	71	9.2	4.6	38.9	1.2	5.6
(−15m,+60m)	71	2.1	8.1	67.9	0.3	4.4
(+60m,+24h)	71	-42.0	19.0	159.9	-1.0	5.7

Table 1. Summary statistics. The table reports summary statistics for log returns on S&P500 E-mini futures minus log returns on 30-year Treasury bond futures, in basis points, in various windows around scheduled monetary FOMC decision announcements and press conferences. A futures "return" in window $(t, t+\Delta)$ is defined as $F_{t+\Delta}/F_t$, where F denotes the futures price. The sample covers FOMC meetings from 1997:09 through 2023:12, with press conferences introduced in April 2011.

The narrow event windows are particularly informative given that the FOMC-driven news is plausibly the main source of variation in asset prices in those windows. The equity-bond portfolio returns show a volatility of nearly 60 bps in the $\pm$15 minutes window of the decision announcement and 40 bps in the $\pm$15 minutes around the start of the press conference, which cover the opening remarks by the Fed Chair. For comparison, the volatility is 27 bps in the $\pm$15-minutes window around 14:00 ET on all other days in the 1997:09–2023:12 sample, i.e., less than half of that around the FOMC decision announcements. The insignificant average $\pm$15-minutes announcement return in Table 1 thus masks a sizable time variation in returns around announcements.

To illustrate the time-series variation, Figure 1 plots the cumulative equity-bond portfolio returns obtained by summing the $\pm$15-minute window decision announcement returns across announcements. The cumulative returns show a persistent downward trend in the first half of the sample, reaching -1004 bps at the November 2009 meeting, and from then onward a persistent upward trend through the end of our sample. Thus, equities underperformed long-term bonds on FOMC announcements from the late 1990s through the first rounds of the quantitative easing implemented after the Global Financial Crisis but outperformed long-term bonds afterward through the most recent period.

Under LRN, assuming the proxies for R^* and R^∞ are valid, this time series should be a flat line at zero, which is dramatically rejected. Later, we will examine the timing



Figure 1. Cumulative returns in a narrow window around FOMC policy announcements. The figure plots cumulative log returns on a long-short portfolio of S&P 500 E-mini futures in excess of 30-year Treasury bond futures. The returns are calculated from -15 to $+15$ minutes around scheduled FOMC decision announcements (MPD). The sample period covers 210 meetings from 1997:09 through 2023:12.

of large moves in the long-short portfolio, to investigate which types of shocks drive LRN violations. But already, one can see that conventional monetary policy is less likely to generate large LRN violations: the dramatic upward drift between 2009-2015 is entirely in a period where the zero lower bound was binding and all monetary policies were "unconventional." Under the interpretation of H as the permanent component of marginal utility, the 2009-2015 period suggests the Fed was reducing marginal utility in permanent ways with its announcements.

We first convert our preliminary observations into a statistical test. If LRN holds, the FOMC-driven news should move equity and T-bond returns one for one, implying that a regression of T-bond returns on equity returns should have a slope coefficient (beta) and an R-squared both equal to one. Figure 2 shows that these predictions are rejected in the data. Bond-equity betas are statistically different from one across various windows around decision announcements and press conferences. While the $\pm$15-minute betas are positive, they are significantly below one, reaching 0.32 for decision announcements and 0.22 for press conferences, with R-squareds of 9.9% and 20.1%, respectively. Outside narrow windows, the evidence against LRN strengthens further, with even lower R-squared and betas becoming negative.

To assess how often the LRN condition could potentially hold in our sample, we compute high-frequency betas and R-sqaureds using realized covariances and variances of one-minute equity and T-bond returns in the $\pm$15 window around decision announce-

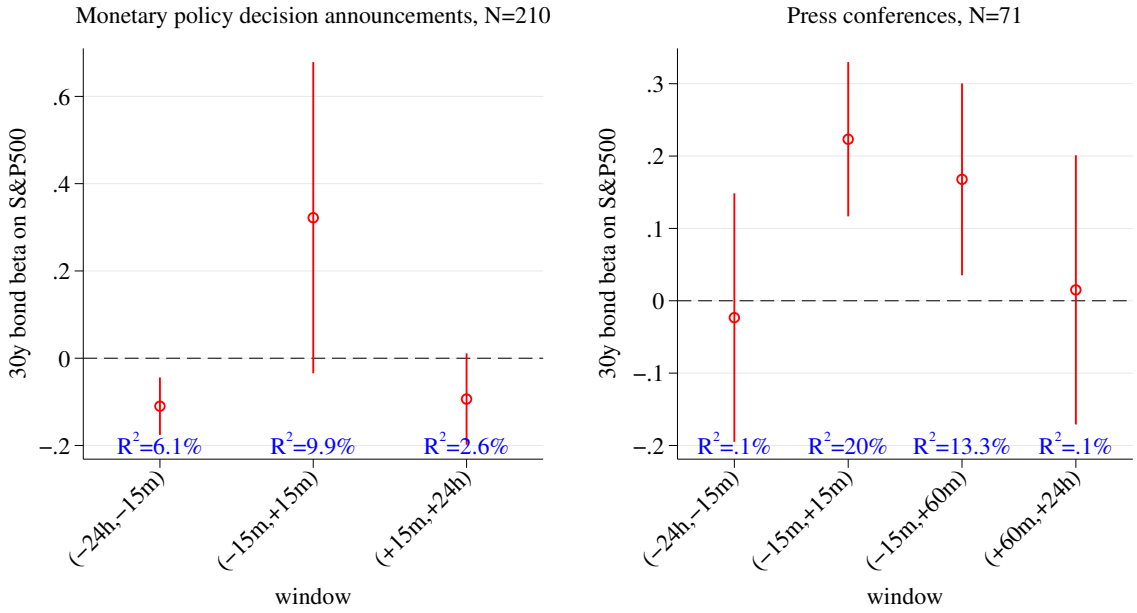


Figure 2. Bond betas. The figure presents betas of 30-year Treasury bond futures returns on S&P 500 E-mini futures returns in different windows around scheduled FOMC announcements and press conferences. Robust 95% confidence intervals are included.

ments. The histograms in Figure 3 indicate that 90% of announcements feature a beta below 0.5 and an R-squared below 0.42, again suggesting that LRN remains violated most of the time.

2.3 Robustness: Alternative R^* and R^∞ proxies

One concern with the evidence above relates to using E-mini S&P 500 futures and 30-year Treasury bond futures to proxy for R^* and R^∞, respectively. Here, we explore alternatives.

As an alternative measure of R^*, we use the high-frequency returns on the tangency constructed by Aleti and Bollerslev (2024), extending the adversarial machine-learning approach of Chen, Pelger and Zhu (2024). The Aleti-Bollerslev (AB) portfolio returns reflect both market timing and anomaly returns and thus can be markedly different from the S&P 500 returns. For instance, the AB portfolio offers an annualized Sharpe ratio of 1.1 compared to 0.31 on the S&P 500 E-mini. Regressions of AB portfolio returns on E-mini returns gives an R-squared of only 4% overall and only up to 10% in ±15-minute FOMC event windows. The high-frequency AB tangency portfolio returns are available through December 2020.

For the long bond, we consider the 10-year and 30-year "Ultra" Treasury bond fu-

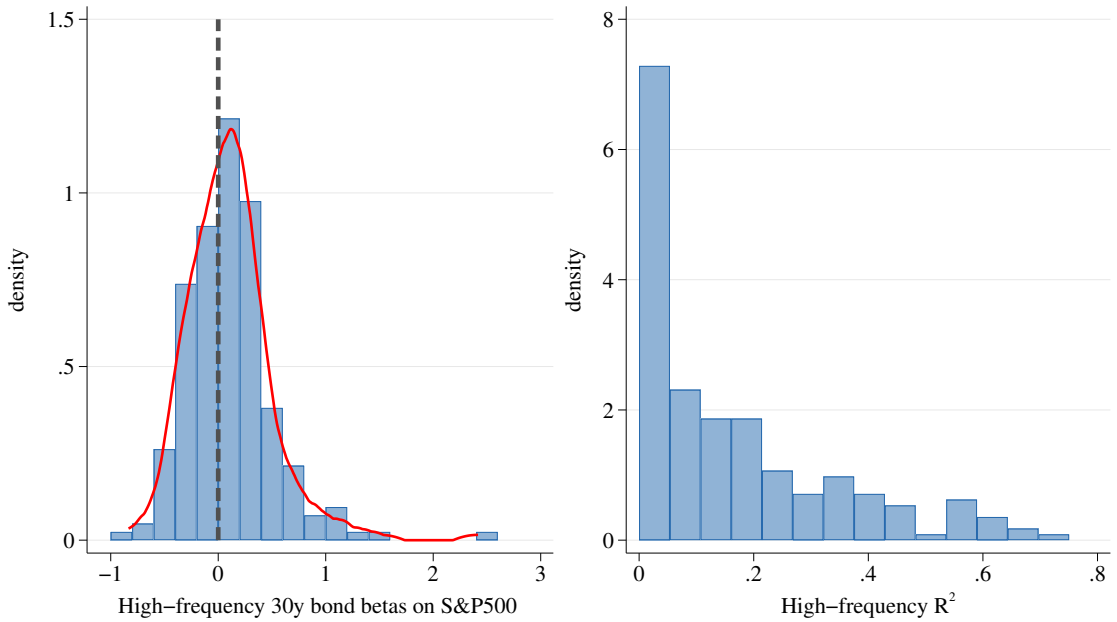


Figure 3. High-frequency bond betas. The figure presents the distribution of high-frequency realized betas of the 30-year Treasury bond futures returns on S&P 500 E-mini futures returns. For each ±15-minutes window around scheduled FOMC decision announcements, the beta and R-squared are calculated from realized covariances and variances of one-minute returns.

tures, in addition to the "classic" 30-year bond futures used above. Relative to this 30-year "classic" contract, the 30-year Ultra contract is tied to a longer-maturity bond but is only available for half the sample, starting with the March 2010 meeting.[10]

These alternative proxies for R^* and R^∞ support the main conclusion that Fed-induced news violates the LRN condition. As a starting point, cumulative log returns on equity-bond portfolios constructed using the AB tangency and the 30-year Ultra bond returns, in analogy to Figure 1, consistently demonstrate persistent LRN deviations and the upward trend post financial crisis (see Appendix Figures E.1 and E.3).

To provide a more granular view of the LRN deviations, Figure 4 displays scatter plots of bond returns at different maturities against the E-mini and AB returns in FOMC decision windows. If LRN holds, the data should lie approximately along the 45-degree line (dashed line in the graphs). A large share of monetary policy events clearly violate this prediction. Irrespective of the proxies we use, stocks and bonds move in opposite directions in more than one-third of meetings. Furthermore, modulo differences in sample periods, the scatter plots look similar for any pair of stock and bond proxies, which connects to some existing results in the literature. On the equity side, Savor and Wilson

[10]The CME introduced the 30-year Ultra T-bond futures in February 2010. The Ultra contract requires delivery of a bond with at least 25-year maturity. At that time, the range of eligible maturities for the "classic" 30-year T-bond futures was adjusted from a 15-to-30-year range to a 15-to-25-year range.

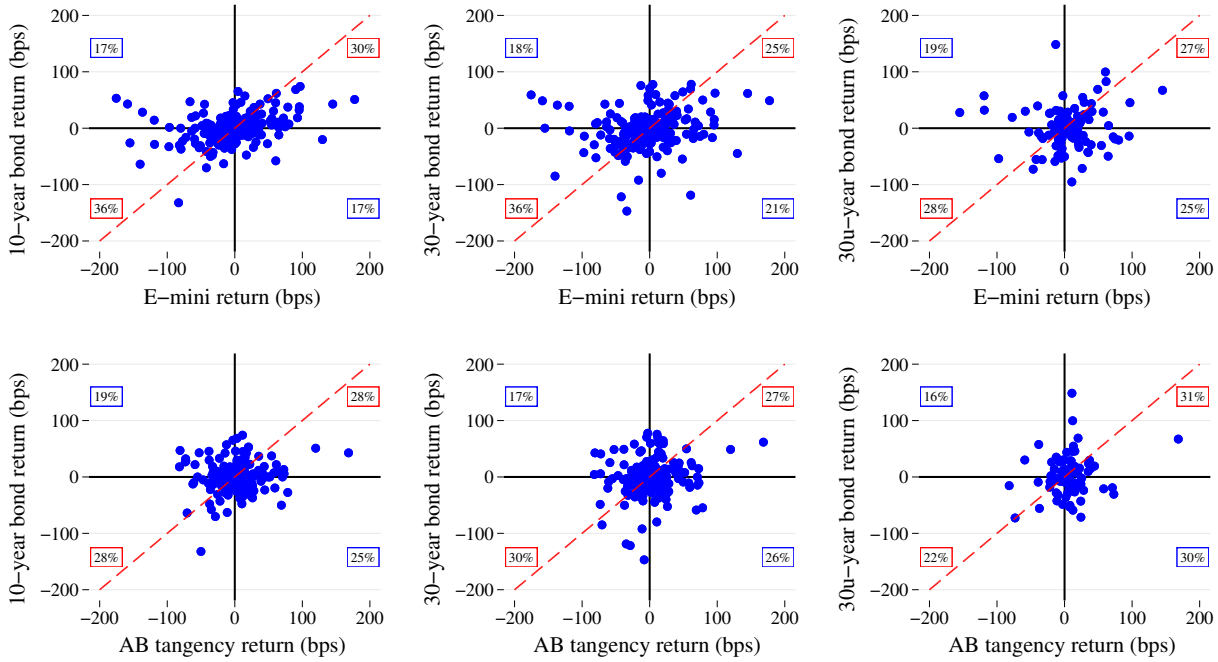


Figure 4. Relationship between different proxies for $\log R^*$ and $\log R^\infty$ around MPD announcements. The figure presents scatter plots of log returns on the 10- and 30-year, and 30-year Ultra (30u) bond futures against E-mini and Aleti-Bollerslev (AB) in the ±15-minute windows around MPD announcements. The dashed line is a 45-degree line. The percentages reported in each subplot are a fraction of meetings falling in the respective quadrant (numbers are rounded to the nearest integer). Extreme outliers are excluded for clarity of the graphs. The sample period is 1997:09–2023:12 for the E-mini, 10- and 30-year bond futures, 1997:02–2020:12 for the AB tangency portfolio. For the 30-year Ultra Treasury futures the samples start in 2010:03.

(2014) and Lucca and Moench (2015) show that stock returns are much better-explained by the CAPM on FOMC announcement days, relative to other days. On the bond side, Nagel and Xu (2024) show in their appendix that FOMC announcement day returns of long-term bonds (maturities 9, 15, 20, and 30 years) are statistically indistinguishable from each other on average and in response to short-rate shocks.[11]

Given that none of the proxies is perfect, we additionally construct a test, based on the sign of returns, which is robust to misspecification. In particular, suppose the unobserved growth-optimal return $\log(R^*_{t,t+\Delta})$ possesses the same sign in FOMC event windows as our empirical proxy $\log(R^m_{t,t+\Delta})$ (where R^m is either the E-mini or the AB tangency portfolio). Similarly, suppose the unobserved long bond return $\log(R^\infty_{t,t+\Delta})$ possesses the same sign in FOMC event windows as the finite-maturity bond futures

[11] In ±15 minutes' MPD windows, returns on E-mini futures and AB tangency portfolio have the same sign in 68% of announcements, while returns on the 10- and 30-year bond futures have the same sign in 85% of announcements. The signs of returns on the classic 30-year and 30-year Ultra bond futures agree in 90% of announcements over the available sample period.

$\log(R^T_{t,t+\Delta})$ (where R^T is either the 10-year, 30-year, or 30-year Ultra). Under these sign restrictions and the LRN condition, the signs of $\log(R^m_{t,t+\Delta})$ and $\log(R^T_{t,t+\Delta})$ must be identical in each FOMC event window.[12]

Table 2 displays the results of the sign test, by regressing an indicator for a positive bond return on an indicator for a positive stock return, in ±15-minute windows around monetary policy decision announcements. R-squared values are consistently below 0.1 and the test clearly rejects that the regression slopes are 1, implying robust violations of LRN. A nontrivial fraction of observations that violate the LRN comes from large returns moving equities and bonds in opposite directions, making it further unlikely that violations are driven by noise. For example, 30-year bond and E-mini (AB tangency) returns moved by more than 0.5 standard deviations in opposite directions in 6.2% (7.9%) of meetings. These fractions are even higher for the 30-year Ultra bond in the post 2010 sample, at 11.8% for E-mini and 9.3% for AB returns.

	(1)	(2)	(3)
Dependent Variable:	$\mathbb{1}\{r_t^{(10Y)} > 0\}$	$\mathbb{1}\{r_t^{(30Y)} > 0\}$	$\mathbb{1}\{r_t^{(30YU)} > 0\}$
Panel A. E-mini S&P 500 futures			
$\mathbb{1}\{r_t^{E\text{-}mini} > 0\}$	0.311***	0.210***	0.113
	(0.066)	(0.067)	(0.095)
Obs	210	210	110
R^2	0.097	0.045	0.013
Sample	1997:09-2023:12	1997:09-2023:12	2010:03-2023:12
Panel B. AB Tangency portfolio			
$\mathbb{1}\{r_t^{AB} > 0\}$	0.123*	0.147**	0.0852
	(0.072)	(0.071)	(0.111)
Obs	191	191	86
R^2	0.015	0.022	0.007
Sample	1997:02-2020:12	1997:02-2020:12	2010:03-2020:12

Table 2. Sign test regressions. The table reports regressions of $\mathbb{1}\{r^T_{t,t+\Delta} > 0\}$ on $\mathbb{1}\{r^m_{t,t+\Delta} > 0\}$, where $r^T_{t,t+\Delta}$ is the log return on the 10-, 30-, or 30-year Ultra Treasury futures, and $r^m_{t,t+\Delta}$ the log return on either E-mini futures or AB tangency portfolio in the ±15-minute window around monetary policy decision announcements. Robust standard errors are reported in parentheses (***p<0.01, **p<0.05, *p<0.1).

2.4 Robustness: Real versus nominal returns

Note that we have been expositing LRN as a condition for the *real* SDF, whereas our baseline results use nominal returns. We thus explore robustness to this discrepancy. In particular, let $\tilde{S}$, $\tilde{R}^*$, and $\tilde{R}^\infty$ be the nominal SDF, growth-optimal return, and long bond.

[12]We thank Jonathan Wright for suggesting a test of this type.

The nominal SDF is $\tilde{S}_t = S_t/P_t$, and the nominal growth-optimal return is $\tilde{R}^*_{t,t+\Delta} = \tilde{S}_t/\tilde{S}_{t+\Delta}$. Using these identities, we may express our real quantity of interest as follows:

$$\log R^*_{t,t+\Delta} - \log R^\infty_{t,t+\Delta} = \underbrace{\log \tilde{R}^*_{t,t+\Delta} - \log \tilde{R}^\infty_{t,t+\Delta}}_{\text{nominal excess return}} + \underbrace{\log \tilde{R}^\infty_{t,t+\Delta} - \log R^\infty_{t,t+\Delta}}_{\text{nominal minus TIPS return}} - \underbrace{\log P_{t+\Delta}/P_t}_{\text{inflation}\approx 0}$$

In any sticky price model, the inflation term will be zero at high frequency around monetary announcements. Therefore, the next set of results focuses on the nominal minus TIPS return, which captures the change in breakeven inflation rate.

While Fed announcements can naturally lead to revisions in inflation expectations and inflation risk premia—both of which affect the second term in the above decomposition—they are unlikely to generate the LRN violations that we document. To demonstrate this, we rely on daily zero-coupon yields constructed following Gürkaynak et al. (2007, 2010), as higher-than-daily-frequency TIPS data are not readily available.

Assuming that Fed news is orthogonal to other information released on FOMC days, we test whether nominal and TIPS bonds respond differently to Fed announcements. Specifically, we project daily changes in the nominal and TIPS zero-coupon 20-year yields, as well as the corresponding breakeven inflation rate, onto high-frequency 30-year bond futures returns around FOMC events. We focus on the 20-year yield because it is the longest maturity available for TIPS.

We reject the hypothesis that nominal and TIPS yield changes respond differently to Fed news. The loading of breakeven rate changes on high-frequency bond returns is statistically and economically indistinguishable from zero for both the announcement and press conference windows, with R-squared values around 1%. By extension, high-frequency bond returns explain similarly sizable fractions of the variation in nominal and TIPS yield changes, with R-squared values exceeding 25%.[13] Regression details are reported in Appendix Table E.1. Overall, these results suggest that LRN violations primarily pertain to the permanent component of the *real* SDF.

2.5 Relationship to the existing literature on announcement effects

Our empirical results relate to a large literature on FOMC announcement effects on stocks and bonds. We discuss this literature and then explain how our results contribute to its findings for the purpose of testing LRN.

[13]These findings are consistent with the stability of survey-based long-term inflation expectations and the small magnitude of inflation risk premia over our sample period (see, e.g., Cieslak and Pflueger, 2023). Relatedly, Hanson and Stein (2015) show that on FOMC days, long-maturity breakeven inflation rates are largely insensitive to short-rate shocks.

First, there is evidence that stock returns are higher than long-term bond returns around FOMC meeting days, *on average*. In particular, Lucca and Moench (2015) show (for 1994–2011) that the S&P500 return was on average 33 bps higher in the 24 hours before the FOMC announcements, relative to other days. By contrast, Hillenbrand (2021) shows (for 1989–2021) that 30-year Treasury returns were approximately 13.8 bps to 18.6 bps higher per day in a 3-day window around the FOMC announcements, relative to other days.[14] Using longer samples and additionally studying non-FOMC macro announcement effects, Savor and Wilson (2013, 2014) document broadly similar results. This evidence suggests that the average returns on stocks and long-term bonds differ both in magnitudes and the timing around the FOMC announcements.

A parallel strand of research examines responses to monetary policy *surprises* rather than average announcement returns. This literature relies on surprises identified from high-frequency short-term interest rate changes around FOMC announcements. In recent work, Nagel and Xu (2024) revisit the seminal analysis of Bernanke and Kuttner (2005) and argue that stock returns on FOMC announcement days, in response to short-rate shocks, are entirely attributable to a change in the yield curve. More specifically, they maturity-match equity dividend strips to Treasury bonds and show that the returns of these two portfolios, on average in response to a short-rate shock, are equal, suggesting that the yield curve can explain the stock market reaction to conventional monetary policy. We can understand their result through the lens of LRN. Indeed, suppose we approximate their procedure by regressing $\log R^* - \log R^\infty$ onto the short-rate shock z in FOMC windows $(t, t+\Delta)$.[15] The slope coefficient of this regression,

$$\beta^{NX} = -\frac{\text{Cov}[\log(H_{t+\Delta}/H_t),\, z_{t,t+\Delta}]}{\text{Var}[z_{t,t+\Delta}]}, \tag{11}$$

equals zero if and only if z and $\log H$ are uncorrelated, which is the case if LRN holds for short-rate shocks. Thus, we interpret Nagel and Xu (2024) as implying that conventional monetary policy has the LRN property. We present a more formal calculation in our numerical example in Section 4.

It is worth explaining how our analysis and empirical results differ from Nagel and

[14] We impute this range for the 30-year bond average return using evidence in Hillenbrand (2021) that 30-year Treasury yields decline between 0.46 bps to 0.62 bps more per day in the 3-day window surrounding FOMC meetings. We use the duration-approximation $\log(R^T_{t,t+\Delta}) \approx -T(y_{t+\Delta} - y_t)$, with $T = 30$.

[15] This approximation is justified by the following. First, almost all the stock market's value comes from long-horizon dividends (e.g., in a Gordon growth model with 2% dividend growth rate and 8% discount rate, only 33% of value is due to years 1-7). Second, Nagel and Xu (2024) show that all long-term bonds return the same in response to z (they show that forward rates past 8 years do not move in response to z), so R^∞ is a suitable proxy for the collection of long-term bonds for this exercise.

Xu (2024). First, the theoretically appropriate object for us is not a Treasury whose duration matches that of the stock market but rather one whose duration is as long as possible. To the extent that all long-term bonds behave similarly around monetary announcements, this difference is however not essential to our interpretation. Second, and more importantly, with tools such as quantitative easing and forward guidance, and the Fed's extensive reliance on communication in general, monetary policy does much more than change the short rate. By looking at returns during the FOMC announcements, we are effectively pooling responses to all monetary shocks, whereas Nagel and Xu (2024) look only at short-rate shocks. This is a major difference between our studies, and it reveals itself through the low R-squared that Nagel and Xu (2024) find when they regress stock returns on short-rate shocks (see also Kroencke et al., 2021, for a similar result on low FOMC-window correlation of equities and short rates). Third, the effects *on average* mask tremendous time-variation: in our data, focusing on 30-minute windows around FOMC announcements, equities and long-term bonds do earn similar *average* returns (Table 1, row 2), but their returns are only weakly correlated (Figures 2 and 3).

The literature analyzing high-frequency returns in FOMC windows goes beyond short-rate shocks. Pioneering this line of research, Gürkaynak et al. (2005b) study a "path factor" that disproportionately affects long-term Treasuries. FOMC windows may include shocks beyond those affecting bond yields. For example, Boehm and Kroner (2025) show that the high-frequency yield curve behavior on FOMC announcements fails to fully explain stock and foreign exchange returns, terming the common component of the residuals a "Fed non-yield shock." By documenting a significant variation in the equity-bond portfolio around FOMC events, our evidence aligns with the presence of such a factor, for which we provide an economic interpretation in terms of SDF properties. In our subsequent theoretical analysis in Section 3.2, we draw deeper connections between LRN and the "Fed non-yield shock" in Boehm and Kroner (2025).

We conclude this section by noting that the evidence from the extant literature, while suggestive, does not speak directly to the LRN of the Fed-driven news. The samples vary substantially in length, choice of event windows, and are often inconsistent between equities and bonds, with little direct evidence on the behavior of long-short equity-bond portfolios. Importantly, most studies document average announcement effects, ignoring higher moments, whereas our violations are critically only visible in the volatility of the long-short portfolio. Furthermore, given our LRN condition implies an exact theoretical portfolio, we go beyond the existing literature in exploring alternatives to the oft-used S&P500 and 10-year Treasury returns.

3 Implications of Long-Run Neutrality Violations

Long-Run Neutrality offers a unified framework for interpreting empirical findings regarding the impact of monetary policy on asset prices. Specifically, our framework sheds light on two key issues: (i) the observed disconnect between equity and bond markets documented in contemporary studies (e.g., Nagel and Xu, 2024; Boehm and Kroner, 2025); (ii) the ability to identify monetary shocks using asset price data, explored in some recent research (e.g., Gürkaynak et al., 2005b; Swanson, 2021; Haddad et al., 2025).

3.1 General theoretical setting

The setting we use to develop our implications is a Markovian, continuous-time environment, based on Hansen and Scheinkman (2009) and Borovička et al. (2016). We work in continuous time here, for two reasons. First, this permits us to naturally delineate between "typical shocks" that occur all the time and "monetary shocks" that occur only at specific dates. Second, we can obtain our results even allowing for some types of nonlinearities which are less easily accommodated in discrete time. (Appendix B provides analogous theoretical results and implications in a simpler discrete-time VAR setting.)

Beliefs. Let the probability measure $\mathbb{P}$ represent investor beliefs. Rational expectations is not assumed: $\mathbb{P}$ may or may not coincide with the objective probability. We work exclusively in the realm of investors' subjective beliefs, because our second implication is about identifying changes relative to the market beliefs. Whether or not $\mathbb{P}$ is rational will not matter for the first implication regarding equity-bond disconnect.

States and shocks. There is a stationary N-dimensional economic state X. The evolution of X is perturbed by two types of shocks. There are *non-monetary shocks* that are defined only by the fact that they occur continuously (e.g., day-to-day news about the economy). By contrast, *monetary shocks* occur only at specific dates. Separating these two types of shocks allows us to easily "zoom in" on monetary announcements, during which the predominant source of variation is monetary news. Note that our monetary shocks will be relatively unrestricted (e.g., X can move in very flexible ways on monetary dates), and so the reader should think of a broad notion of monetary shocks that includes short-rate news but also other Fed-driven news about the macroeconomy.

The details of the shock structure are as follows. Non-monetary shocks are modeled by the increments to W, which is a d-dimensional Brownian motion under $\mathbb{P}$. Monetary shocks are modeled by the following Poisson jump process. The (state-dependent) ar-

rival rate of monetary announcements is $\lambda(X_{t-})$. We let M_t be the counter for announcements, so $dM_\tau = 1$ if and only if τ is an announcement date. At an announcement date τ, monetary shocks are given by the d-dimensional vector ξ_τ. Denote the probability distribution of ξ_τ by $\nu(d\xi \mid X_{\tau-})$, which is allowed to depend on the state $X_{\tau-}$ just prior. For simplicity, assume the mean of ξ_τ is equal to zero, so that "expected jumps" are implicitly reflected in the drifts of the state vector.

Modeling monetary shocks as Poisson jumps is a tractable simplification. Whereas monetary announcement dates are deterministic and known in advance, one can think of randomness in these dates as capturing announcements during which some surprises actually occur. Furthermore, during some times of crisis, emergency actions and statements by the central bank can take place. The technical advantage of modeling monetary shocks in this way, as opposed to deterministic dates, is that we preserve a stationary and Markovian structure of our economy, allowing us to directly apply the permanent-transitory decomposition (1).[16]

We have assumed that, at any time, there are d potential shocks, whereas there are N state variables (i.e., $\dim(W) = \dim(\xi) = d$ and $\dim(X) = N$). Importantly, we allow for more shocks than state variables ($d > N$). This assumption implies that while asset prices reveal state variables X, they may not reveal the individual driving shocks.

In case there are more shocks than states ($d > N$), we introduce a vector Y of additional observables (with dimension $d - N$) that allows us to infer all the shocks. Let us refer to the composite vector (X, Y) as the observables. Like the state evolution dX_t, we assume the evolution of auxiliary variables dY_t depends only on X_{t-}, so that the observables (X, Y) form a Markov process with a "triangular" structure whereby only X_{t-} matters for the probability distribution of dX_t and dY_t. (An example with $d > N$ is the long-run risks model of Section 1.3, which has 3 shocks and 2 state variables.)

In summary, the state vector and auxiliary observables are the Markov jump-diffusion

$$dX_t = \mu(X_{t-})dt + \sigma(X_{t-})dW_t + \Omega_X \xi_t dM_t \tag{12}$$

$$dY_t = \alpha(X_{t-})dt + \psi(X_{t-})dW_t + \Omega_Y \xi_t dM_t, \tag{13}$$

where Ω_X and Ω_Y are $N \times d$ and $(d - N) \times d$ matrices, respectively, such that the square matrix $(\Omega_X', \Omega_Y')'$ is non-singular. The sequence of information sets $(\mathscr{F}_t)_{t\geq 0}$ available to investors is generated by histories of W, M, ξ, and X_0, or equivalently by (X, Y).

SDF and Long-Run Neutrality. We assume there exists a real stochastic discount factor

[16] In Piazzesi (2005), monetary shocks can arrive at both Poisson and deterministic dates.

(SDF) process S whose increment is given by

$$\frac{dS_t}{S_{t-}} = -r(X_{t-})dt - \pi(X_{t-}) \cdot dW_t + \big(\exp[\kappa(X_{t-}) \cdot \xi_t] - 1\big)dM_t - \chi_S(X_{t-})dt, \tag{14}$$

where $S_0 = 1$ and $\chi_S(x)dt$ is the jump compensator.[17] The variable $r(x)$ denotes the short-term real interest rate, while $\pi(x)$ and $\kappa(x)$ denote risk prices associated to the non-monetary and monetary shocks.

In this environment, Hansen and Scheinkman (2009) show how to obtain a factorization of the SDF as

$$\frac{S_{t+T}}{S_t} = \exp(\eta T)\frac{e(X_t)}{e(X_{t+T})}\frac{H_{t+T}}{H_t}. \tag{15}$$

In (15), $\exp(\eta T)$ is a positive eigenvalue of the horizon-T pricing operator; $e(\cdot)$ is the associated positive eigenfunction; and H_t is a martingale under $\mathbb{P}$. Note that because the dynamics are Markovian in X, the stationary piece does not depend on Y but only on X. We assume existence and uniqueness of an SDF factorization (15).

Definition 1. *We say that monetary policy possesses* Long-Run Neutrality (LRN) *if*

(i) The evolution of dH_t is independent of the monetary shock dM_t;

(ii) The evolution of dH_t only depends on monetary-insensitive economic states $X_t^{\perp}$,

where monetary-insensitivity is defined by $\mathbb{E}[f(X^{\perp}_{\tau+T}) \mid X_\tau] - \mathbb{E}[f(X^{\perp}_{\tau+T}) \mid X_{\tau-}] = 0$ for all bounded functions $f(\cdot)$, all $T > 0$, and all monetary announcement dates τ.

Definition 1 specifies LRN in terms of asset markets, via the martingale H. Condition (i) rules out direct effects of monetary policy. Condition (ii) rules out indirect policy effects, whereby monetary policy affects H_{t+T} by moving X_t. For H to satisfy Definition 1, it must take the form

$$H_t = \exp\Big[-\frac{1}{2}\int_0^t \|\beta(X_s^{\perp})\|^2 ds - \int_0^t \beta(X_s^{\perp}) \cdot dW_s\Big] \tag{16}$$

for some function $\beta : \mathbb{R}^{\dim(X^{\perp})} \mapsto \mathbb{R}^N$, where $X^{\perp}$ denotes the sub-vector of monetary insensitive states. One can interpret β as the exogenous long-run risk price associated with non-monetary shocks dW. By condition (i), there is a zero long-run risk price for monetary shocks dM.

[17] In general, the compensator is the term that captures the conditional mean of the jump component. In this case, the compensator is given by $\chi_S(x) = \lambda(x)\int\big(\exp[\kappa(x)\cdot\xi] - 1\big)\nu(d\xi \mid x)$.

3.2 Implication 1: Equity-bond disconnect

Our first implication of LRN violations pertains to the degree of disconnect between equities and Treasury bonds in response to monetary shocks (Boehm and Kroner, 2025). These findings can be explained if LRN fails for some types of monetary policies.

The idea will be to relate the growth-optimal return R^* to the yield curve. To implement this, we make an assumption about the structure of equilibrium yields, which nests a large class of models that fits into the more general setting above. Let $y_t^{(T)}$ denote the equilibrium log yield of a risk-free discount bond with time-to-maturity $T > 0$.

Assumption 1 (Non-degenerate yields)**.** *The yield curve satisfies $y_t^{(T)} = g(X_t, T)$ for some function g. In addition, assume that g is non-degenerate in the sense that the state X_t can be "inverted" from the yield curve $(y_t^{(T)})_{T>0}$.*

Assumption 1 holds in a variety of special cases. For example, one important special case is when equilibrium yields are affine in the state vector, i.e., $y_t^{(T)} = \mathcal{B}_0^{(T)} + \mathcal{B}^{(T)} X_t$ for some family of constants $\{\mathcal{B}_0^{(T)}, \mathcal{B}^{(T)} : T > 0\}$. To obtain this affine result, all one needs is that the risk-neutral dynamics of the short rate $r(X_t)$ are given by an affine jump-diffusion (Duffie and Kan, 1996; Duffie et al., 2000). This can hold if the dynamics of X and S have a particular affine structure—see Lemma A.2 for such a result which slightly extends Duffie et al. (2000). But more generally, all we require here is that the yield curve reveals the state vector. Such invertability is likely to hold very broadly, since the yield curve is an infinite-dimensional object, while the state vector X is finite-dimensional. Under invertability, the changes in the yield curve on monetary announcement dates τ (when $dM_\tau = 1$) reveal $dX_\tau = \Omega_X \xi_\tau$.

Next, recall that the growth-optimal return is the inverse of the SDF, so its instantaneous return is

$$d \log R_t^* = \Big[r(X_{t-}) + \frac{1}{2} \pi(X_{t-}) + \chi_S(X_{t-}) \Big] dt + \pi(X_{t-}) \cdot dW_t - \kappa(X_{t-}) \cdot \xi_t dM_t.$$

On monetary announcement τ, the growth-optimal return is given by $-\kappa(X_{\tau-})' \xi_\tau$.

Given Assumption 1, regressing the high-frequency monetary announcement return $d \log R_\tau^*$ on arbitrary functions of the current and past yield curve is equivalent to regressing $d \log R_\tau^*$ onto $\varphi(X_\tau, X_{\tau-})$ for some function φ. We define *equity-bond disconnect* as any situation where the fit of such a regression is imperfect.

Definition 2 (Equity-bond disconnect)**.** *Consider solving*

$$\min_{\varphi} \mathbb{E} \big\| d \log R_\tau^* - \varphi(X_\tau, X_{\tau-}) \big\|^2, \quad \tau \in \big\{ t : dM_t = 1 \big\}. \tag{17}$$

We say equity-bond disconnect *arises if the optimal objective in (17) is non-zero.*

In this nonlinear regression, a non-zero residual requires LRN violations. To see this, notice that the SDF factorization (15) implies that, on monetary announcements,

$$d \log R^*_\tau = \log e(X_\tau) - \log e(X_{\tau-}) - d \log H_\tau.$$

Consequently, if LRN holds so that $d \log H_\tau = 0$, the solution to (17) will be $\varphi(\tilde{x}, x) = \log e(\tilde{x}) - \log e(x)$ and the fit will be perfect (R-squared of 1). We have thus proved

Proposition 1. *If Assumption 1 and LRN hold, then equity-bond disconnect does not arise.*

What happens if LRN is violated? In that case, equity-bond disconnect can arise, but, crucially, it requires that the state vector does not fully span shocks to the SDF. This is possible in our setting because the number of monetary shocks ($\dim(\xi) = d$) potentially exceeds the number of state variables ($\dim(X) = N \leq d$). To understand the spanning requirement is quite simple: suppose to the contrary that on monetary announcements, the SDF is spanned by shocks to the state, i.e.,

$$d \log R^*_\tau = -\kappa(X_{\tau-}) \cdot (X_\tau - X_{\tau-}),$$

which holds if the number of monetary shocks coincides with the number of state variables (e.g., $d = N$). Under this spanning condition, the solution to (17) will be $\varphi(\tilde{x}, x) = -\kappa(x) \cdot (\tilde{x} - x)$ and the fit will again be perfect, proving

Proposition 2. *If Assumption 1 holds and the monetary shocks to S_τ are spanned by $X_\tau - X_{\tau-}$, then equity-bond disconnect does not arise.*

Summarizing, equity-bond disconnect requires two conditions: a violation of LRN and monetary-induced marginal utility shocks that are unspanned by the yield curve. Through the lens of this result, the findings of Boehm and Kroner (2025) form additional indirect evidence that LRN fails. The advantage of our approach is that it identifies LRN directly: we do not require any regression, nor is our test of LRN confounded by a spanning condition (the second requirement for equity-bond disconnect).

3.3 Implication 2: Identification of monetary shocks

We now consider the question of whether and how to recover "policy shocks" from asset prices. The assumption here is that the econometrician does not know the investor beliefs $\mathbb{P}$. Recall that we do not assume rational expectations here—it could be that $\mathbb{P}$

is distorted relative to the objective probability. More specifically, the econometrician wants to learn shocks using *only data on asset prices*, an inherently "reduced-form" approach. Appendix B.2 provides an in-depth comparison between reduced-form shock identification and structural approaches as in Backus et al. (2022). Appendix B.3 provides a simple discrete-time VAR example with "forward guidance" that motivates why a researcher might want to recover investor surprises about interest rates.

In this environment, we define a *policy plan* by a function $\phi(X_t)$ which maps the states into some action or outcome. Many examples fit into this general setting. One example would be where $\phi(x) = r(x)$ is simply the short rate. Another example, studied in Haddad et al. (2025), models state-contingent asset purchase plans.

Critically, our setting can accommodate pure announcements of future interventions, in addition to contemporaneous actions. For instance, suppose $\phi(x) = \bar{\phi}(x^{(1)})$ depends only on the first state variable, while the second state $x^{(2)}$ governs the long-run mean of $x^{(1)}$. In that case, a monetary shock to $x^{(2)}$ acts as a pure announcement shock with no intervention today.

Our *identification* question is, roughly speaking, whether we can infer shocks to policy plans from financial markets. In the two-state example above, inferring the shock to the policy plan is tantamount to inferring a change in the probability distribution of $(X^{(1)}_{\tau+t})_{t>0}$. More generally, we seek to identify, at a monetary announcement τ,

$$z^T_\tau = \Delta\mathbb{E}_\tau[\phi(X_{\tau+T})], \quad T > 0. \tag{18}$$

where $\Delta\mathbb{E}_\tau[\phi(X_{\tau+T})] := \mathbb{E}_\tau[\phi(X_{\tau+T})] - \mathbb{E}_{\tau-}[\phi(X_{\tau+T})]$ refers to the high-frequency monetary shock. We define our notion of identification more precisely shortly, after we explain what it means to "use financial markets."

We propose to use financial markets in the following way. The basic idea, building on Borovička et al. (2016), is that the martingale H in the factorization (15) may be used as a change-of-measure from $\mathbb{P}$ to the long-run risk-neutral measure $\hat{\mathbb{P}}$, defined by

$$\hat{\mathbb{P}}(F) := \mathbb{E}[H_t \mathbf{1}_F], \quad \forall F \in \mathscr{F}_t. \tag{19}$$

Similar to how financial markets reveal the traditional risk-neutral measure, they also reveal this long-run risk-neutral measure (see Lemma A.3 in the appendix). Namely, expectations like $\hat{\mathbb{E}}[f(X_t) \mid X_0]$, for any f, coincide with the price of some asset.

Except under the very special degenerate situation $H \equiv 1$, investor beliefs $\mathbb{P}$ will not coincide with the recovered $\hat{\mathbb{P}}$, as explained by Borovička et al. (2016). But our goal is less ambitious. We do not seek $\mathbb{P}$ directly but rather investor surprises or *belief shocks*.

As long as the gap, in some sense, between $\mathbb{P}$ and $\hat{\mathbb{P}}$ remains constant before and after monetary policy announcements, we may hope that

$$\Delta\hat{\mathbb{E}}_\tau[\phi(X_{\tau+T})] = \Delta\mathbb{E}_\tau[\phi(X_{\tau+T})]. \tag{20}$$

If equality (20) were to hold, then identification would be straightforward: the left-hand-side is observable from financial markets as a change in some asset price, so the belief shock on the right-hand-side is too. Intuitively, it is easier to recover belief shocks, because they difference out any unobservable level effect in beliefs. The key question is which conditions enable (20). As we will see, the critical condition is LRN.

Furthermore, if we use asset prices in the way prescribed by Lemma A.3, the only way to infer policy shocks $(z_\tau^T)_{T>0}$ is if condition (20) holds. A variant of this has been shown in Borovička et al. (2016).[18] For this reason, we define identification as follows.

Definition 3 (Identification)**.** *Suppose the function $\phi(\cdot)$ is known. We say that* policy shocks are identified from asset markets *if (20) holds for all $T > 0$.*

Remark 1 (Risk-neutral measure)**.** *An alternative is to attempt identification using the usual risk-neutral measure $\mathbb{P}^*$. And then, instead of (20), the identification condition would be*

$$\Delta\mathbb{E}_\tau^*[\phi(X_{\tau+T})] = \Delta\mathbb{E}_\tau[\phi(X_{\tau+T})].$$

Intuitively, so long as the "gap" between $\mathbb{P}$ and $\mathbb{P}^*$ remains constant before and after a monetary announcement, belief shocks could be inferred from shocks under the risk-neutral measure. However, such a assumption is overly strong, as it requires monetary policy to have no impact on risk pricing. *By contrast, our approach to using asset markets will impose a weaker condition (i.e., LRN), whereby monetary policy can exert an arbitrary transitory impact on marginal utility, just not a permanent one.*

To obtain formal results, we will make an assumption such that the entire economy is quasi-linear. Essentially, the drifts of the state vector are linear, and their diffusions are exogenous to policy. We view this as a mild assumption that can always be satisfied as an approximation, by sufficiently enriching the state vector.

[18] Indeed, suppose z_τ^T is identified by financial markets, according to the procedure of Lemma A.3. Then, the same procedure also identifies $\hat{z}_\tau^T := \Delta\hat{\mathbb{E}}_\tau[\phi(X_{\tau+T})]$. Why? Proposition 2 of Borovička et al. (2016) says that the observable asset prices can be obtained by either $\mathbb{E}_0[S_t f(X_t)]$ or $\hat{\mathbb{E}}_0[\hat{S}_t f(X_t)]$, i.e., using either (i) probability measure $\mathbb{P}$ and SDF S, or (ii) probability measure $\hat{\mathbb{P}}$ and SDF $\hat{S}$, where $\hat{S}_t := S_t H_0 / H_t$. Therefore, the same asset price data that identify z_τ^T also identify $\hat{z}_\tau^T$. In that case, $z_\tau^T = \hat{z}_\tau^T$ must hold.

Assumption 2 (Quasi-linear dynamics). *The state dynamics and policy plan take the form*

$$\mu(x) = A_0 + Ax \tag{21}$$
$$\sigma(x) = B(x^\perp) \tag{22}$$
$$\phi(x) = \phi_0 + \phi_1 \cdot x, \tag{23}$$

for some $N \times 1$ vector A_0, an $N \times N$ matrix A, a function $B : \mathbb{R}^{dim(X^\perp)} \mapsto \mathbb{R}^{N \times d}$ that only depends on monetary-insensitive states, a constant ϕ_0, and an $N \times 1$ vector ϕ_1.

We first show that LRN is sufficient for identification. Under Assumption 2 and LRN, the dynamics of X_t under $\hat{\mathbb{P}}$ are the same as those under $\mathbb{P}$, less the change-of-drift $B(x^\perp)\beta(X_t^\perp)$ that arises due to the change-of-measure (16):

$$\hat{\mu}(x) = \mu(x) - B(x^\perp)\beta(x^\perp).$$

To obtain forecast revisions in linear environments, what matters is the change in the drift before and after an announcement, i.e., $\Delta\mu(x)$. Because $B(x^\perp)\beta(x^\perp)$ are monetary-invariant, we have that the change in drift under $\hat{\mathbb{P}}$ and $\mathbb{P}$ are the same: $\Delta\hat{\mu}(x) = \Delta\mu(x)$. For this reason, (20) holds under LRN.

Proposition 3. *If Assumption 2 and LRN hold, then the policy shocks $(z_\tau^T)_{T \geq 0}$ are identified from asset markets alone.*

On the other hand, LRN is generically necessary for identification. To gain intuition in the simplest case, suppose monetary policy does not directly affect H but it indirectly affects H through X (i.e., condition (i) of Definition 1 holds, but condition (ii) is violated). In that case, rather than the form (16), H_t takes the form

$$H_t = \exp\Big[-\frac{1}{2}\int_0^t \|\beta(X_s)\|^2 ds - \int_0^t \beta(X_s) \cdot dW_s\Big] \tag{24}$$

for some function $\beta(\cdot) : \mathbb{R}^N \mapsto \mathbb{R}^d$ that depends non-trivially on the monetary-sensitive states. The drift of X_t under the long-run measure $\hat{\mathbb{P}}$ is given by

$$\hat{\mu}(x) = \mu(x) - B(x^\perp)\beta(x).$$

Since $\beta(x)$ is affected by monetary policy, we have that $\Delta\hat{\mu}(x) = \Delta\mu(x) - B(x^\perp)\Delta\beta(x)$. While $\hat{\mu}(x)$, hence $\Delta\hat{\mu}(x)$, is observable (since the long-run measure $\hat{\mathbb{P}}$ is observable), the change to risk prices $\Delta\beta(x)$ is unobservable, meaning that the monetary shock to the investor-perceived drift, $\Delta\mu(x)$, is also unobservable.

This non-identification argument can be formalized and generalized via

Proposition 4. *If Assumption 2 holds but LRN fails, then, generically, the policy shocks $(z_\tau^T)_{T>0}$ cannot be identified from asset markets alone.*

Whereas Proposition 3 demonstrated the sufficiency of LRN for identifying monetary shocks, Proposition 4 demonstrates the corresponding necessity result. (The qualification "generically" is to rule out the knife-edge case with IID monetary shocks and constant monetary risk prices.)

Remark 2 (SDF proxies and identification)**.** *If we assume rational expectations and have a good proxy for the growth-optimal portfolio $R^*_{t+1} = S_t/S_{t+1}$, then we can easily identify shocks to beliefs by essentially "purging" the market-implied beliefs of their marginal utility effects. However, this approach is fraught by two issues: (i) if agents hold subjective beliefs, this approach would identify shocks under the objective measure rather than investors' subjective one; (ii) such an approach requires having a good proxy for R^*. While our empirical tests rejecting LRN do entertain several proxies for R^*, it was never critical that we had the exact proxy, as our "robust sign test" in Section 2.3 makes particularly clear.*

4 Quantitative example

So far, we have seen that LRN fails empirically and that, theoretically, this could have important implications. Namely, LRN violations could induce (i) equity-bond disconnect in FOMC windows and (ii) biases in attempting to extract monetary shocks from asset prices. But while LRN fails in a literal sense, this may be of minuscule quantitative importance. In this section, we seek to *quantify* the size of the effects of LRN violations. For this purpose, we use the flexible affine asset-pricing model described in Section 1.4. More details on this model are contained in Appendix B.1.

4.1 Model setup and calibration

Recall from Section 1.4 that the state variables follow $X_{t+1} = AX_t + B\Delta W_{t+1}$, a linear VAR, while the SDF takes the log-linear form $\frac{S_{t+1}}{S_t} = \exp[-r_t - \frac{1}{2}\|\pi_t\|^2 - \pi_t'\Delta W_{t+1}]$, where both the short rate r_t and market price of risk π_t are affine functions of X_t. We use a parsimonious specification with two state variables ($\dim(X) = N = 2$) and three shocks ($\dim(\Delta W) = d = 3$). A large literature on the term structure of interest rates shows that two state variables can capture the lion's share of yield curve dynamics, so our specification, while simple, is nevertheless rich enough to be empirically relevant.

At the same time, specifying more shocks than states is guided by Proposition 2, which says that this is a requirement to achieve equity-bond disconnect.

The two states correspond to a short rate factor and a factor that governs market prices of risk. Mathematically, this corresponds to the following. First, the short rate is given by $r_t = \rho_0 + \rho_1' X_t$, where we impose $\rho_1 = (1, 0)'$. This means that $X^{(1)}$ is in units of the one-period interest rate. Second, the market price of risk is given by $\pi_t = \pi_0 + \Pi X_t$, where we impose $\Pi = \begin{bmatrix} 0 & \pi_1 \end{bmatrix}$, where π_1 is a 3×1 vector. We will further normalize $\|\pi_1\| = 1$, so that the $X^{(2)}$ is in units of a Sharpe ratio. The assumption that a single factor $X^{(2)}$ controls bond risk premia also aligns with existing literature (Cochrane and Piazzesi, 2005; Cieslak and Povala, 2015).

The three shocks will be (i) a pure short-rate shock; (ii) a pure risk-price shock; and (iii) a flight-to-quality shock that depresses the short-rate as it raises risk prices. Mathematically, the shock-exposure matrix in the state dynamics takes the form

$$B = \begin{bmatrix} b_{11} & 0 & b_{13} \\ 0 & b_{22} & b_{23} \end{bmatrix},$$

where $b_{11} > 0$, $b_{22} > 0$, and $b_{23} > 0 > b_{13}$. The idea in this section is that all three shocks are potentially influenced by monetary policy. This is a simplification. One could easily add more shocks that are non-monetary in nature (as in the continuous-time model of Section 3), but this would complicate the notation and distract from the main points. Importantly, while a single factor $X^{(2)}$ controls risk prices, this factor is exposed to two shocks, making it difficult to extract the shocks from bond returns alone.

In this setting, several important asset-pricing objects are available in closed form (see Appendix B.1). Discount bond yields are affine in the state vector, with coefficients that can be computed analytically. As a consequence, the model satisfies the invertability Assumption 1, whereby the yield curve reveals the state vector.

The permanent-transitory factorization is also available analytically, with H given in (8). As this formula for H makes clear, the fact that risk prices π_t are time-varying is enough to generate violations of LRN. For this reason, the risk-price and flight-to-quality shocks, which both shift π_t, will be key drivers of LRN violations.

Table 3 describes our calibration of the model and some of its unconditional moments. The calibration successfully produces a low and smooth short rate, a slightly upward-sloping yield curve, long-term bonds with a realistic amount of return volatility, and a growth-optimal log risk premium that is similar to the equity premium. Recall we are modeling *real* bonds here, and so we ask the model to produce a yield curve slope and long-term bond volatilities slightly below their nominal counterparts. (Also, since

long-maturity bonds all behave very similarly in our calibration, it is unimportant here which maturity is used as a proxy for the long bond.) The volatility of the growth-optimal log return is equal to the volatility of the log SDF, and so its high value (0.410) is consistent with high Sharpe ratios observed empirically (Hansen and Jagannathan, 1991). The final two rows present shares of SDF volatility (measured by entropy) that are due to its permanent and transitory components, with the overwhelming majority coming from the permanent component, as in the data (Alvarez and Jermann, 2005).

Moment	Description	Value
$\mathbb{E}[r]$	Short rate mean	0.020
$\text{Std}[r]$	Short rate volatility	0.017
$\mathbb{E}[y^{(10)} - r]$	Average term spread	0.001
$\mathbb{E}[y^{(\infty)} - r]$	Average term spread (long bond)	0.002
$\text{Std}[\log R^{(10)}]$	10-year bond return volatility	0.041
$\text{Std}[\log R^{\infty}]$	Long bond return volatility	0.043
$\mathbb{E}[\log R^* - r]$	Growth-optimal risk premium	0.057
$\text{Std}[\log R^* - r]$	Growth-optimal volatility	0.410
$L(\Delta H)/L(\Delta S)$	Share of permanent risks in SDF	0.968
$L(\Delta G)/L(\Delta S)$	Share of transitory risks in SDF	0.012

Table 3. Unconditional asset-pricing moments. This table presents model-implied moments of bonds and the SDF. In the final two rows, the operator $L(V) := \log \mathbb{E}[V] - \mathbb{E}[\log V]$ refers to the entropy operator on a positive random variable V, ΔV refers to $\frac{V_{t+1}}{V_t}$, and G represents the transitory component of the SDF (i.e., $\Delta S = \Delta G \Delta H$). Calibration: $\rho_0 = 0.02$, $\pi_0 = (-0.15, -0.10, -0.15)'$, $\pi_1 = -3^{-1/2}(1,1,1)'$, $b_{11} = 0.01$, $b_{12} = 0$, $b_{13} = -0.007$, $b_{21} = 0$, $b_{22} = 0.20$, $b_{23} = 0.10$, $a_{11} = 0.7$, $a_{12} = 0$, $a_{21} = 0$, $a_{22} = 0.7$.

Figure 5 displays the average shock exposures of the growth-optimal return $\log R^*$ and the long bond $\log R^{\infty}$ in this calibration. All three shocks are "bad" shocks and generate sizable declines in the growth-optimal return. But the shocks affect long-term bonds differently. The flight-to-quality shock is particularly interesting, as it sends the growth-optimal portfolio and the long bond in opposite directions. The dichotomy between the risk-price and flight-to-quality shocks relates to the findings of Cieslak and Pang (2021), which separates monetary risk premium shocks into two types: a "common risk premium" that affects stocks and bonds with the same sign (like our risk-price shock), versus a "hedging risk premium" that oppositely affects stocks and bonds (like our flight-to-quality shock). Additionally, our setting clarifies why ex-post stock returns are needed to separate these two types of risk premium shocks, despite a one-factor structure to bond risk premia: like us, Cieslak and Pang (2021) feature more shocks than state variables, and so the yield curve by itself does not span all the shocks.

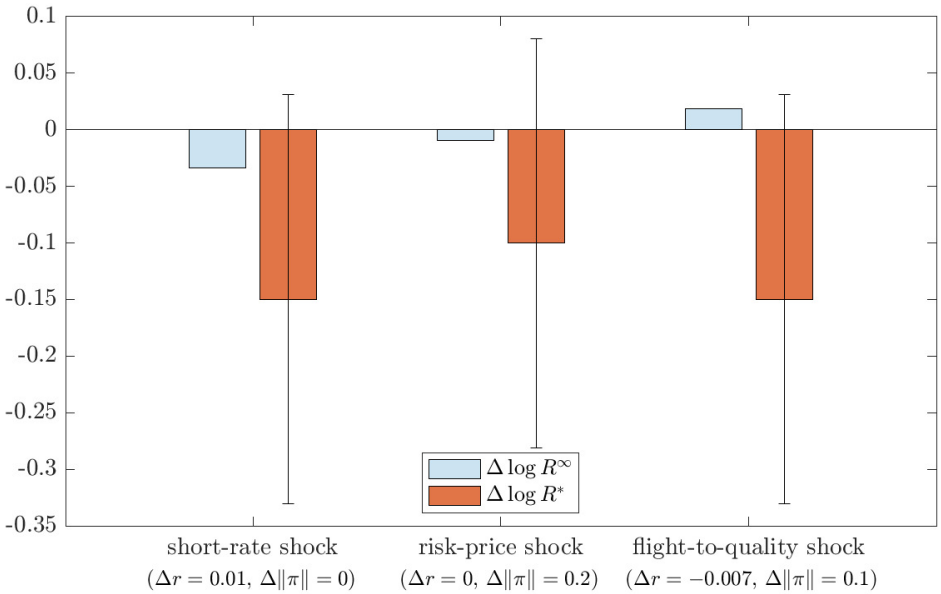


Figure 5. Shock exposures of key returns. The figure presents the responses of the growth-optimal portfolio return $\log R_t^*$ and the long bond return $\log R_t^{\infty}$ to a one standard deviation shock in each of the three shocks. The bars represent the average responses, while the thin error bands represent $[-1, +1]$ standard deviations of the response.

4.2 Equity-bond disconnect

We now turn to two key quantitative exercises regarding equity-bond disconnect, with results reported in Table 4. In these results, we proxy "equity" by the growth-optimal portfolio R^*. Panel A presents the results from a regression of $\log R^* - \log R^{\text{bond}}$ onto monetary policy surprises, where R^{bond} is "duration-matched" to R^*, in the spirit of Nagel and Xu (2024). Panel B presents the results from a regression of $\log R^*$ onto the yield curve and its changes, as in Boehm and Kroner (2025). We now describe these exercises in more detail.

For Panel A of Table 4, we first construct a bond portfolio with a similar cash-flow "duration" as R^*. To do this, we assume the growth-optimal portfolio features cash flows F_t that are co-integrated with aggregate dividends, implying an average growth rate of roughly $g := \mathbb{E}[\log F_{t+1}/F_t] = 0.02$. Then, we apply the approximation that, according to the Gordon growth formula, the prices of dividend strips decay at rate $\delta := \mathbb{E}[\log R^*] - g$, i.e., $\mathbb{E}_t[S_{t+T}F_{t+T}] \approx S_t F_t \exp(-\delta T)$. A duration-matched bond portfolio should thus put weight $(1 - \exp(-\delta))\exp(-\delta n)$ on the n-period discount bond return. In our calibration, we have $\delta = 0.057$, so the bond portfolio features a duration of about 20 years. This calibration is reasonably close to duration estimates for the US stock market.[19]

[19] Weber (2018) estimates that the average US stock has a cash flow duration of 19, while Van Binsbergen (2020) considers US equity market calibrations for δ between 0.02 and 0.06.

Nagel and Xu (2024) show that equities and a duration-matched bond portfolio feature similar high-frequency loadings on conventional monetary policy surprises, with modest R-squareds below 7%. Panel A shows that our model can match this result. Our regression of $\log R^*_{t+1} - \log R^{\text{bond}}_{t+1}$ onto the short-rate shock $z_{t+1} := \rho_1 \cdot B\Delta W_{t+1}$ produces a slope coefficient of $\beta_z \approx -0.48$ with an R-squared of 3.6%. The slope coefficient implies a 25bp short-rate shock leads to an excess return $\log R^* - \log R^{\text{bond}}$ of -12bp on average, which is a trivial magnitude economically.

A. Nagel-Xu exercise		B. Boehm-Kroner exercise	
α	0.077	α	0.099
SE(α)	(0.001)		
β_z	-0.480	$\beta_{\Delta X^{(1)}}$	-6.935
SE(β_z)	(0.104)	$\beta_{\Delta X^{(2)}}$	-0.809
R-squared	0.036	R-squared	0.270

Table 4. Nagel-Xu and Boehm-Kroner regressions. Panel A displays results from a regression of $\log R^*_{t+1} - \log R^{\text{bond}}_{t+1}$ onto a constant and the short-rate shocks $z_{t+1} := \rho_1 \cdot B\Delta W_{t+1}$, where $\log R^{\text{bond}}_{t+1} := \sum_{n=1}^{\infty}(1-\exp(-\delta))\exp(-\delta n)\log R^{(n)}_{t+1}$ is a weighted-average of log bond returns, with weights governed by the decay rate $\delta := \mathbb{E}[\log R^*_{t+1}] - 0.02 = 0.057$. Panel B displays results from a regression of $\log R^*_{t+1}$ onto a constant, the lagged state vector X_t, and the changes to the state vector $\Delta X_{t+1} = X_{t+1} - X_t$. These regressions are done on 100,000 years of simulated data.

For Panel B of Table 4, we are primarily interested in the goodness-of-fit. Given that the yield curve reveals a space spanning X, we obtain identical regression residuals from either (i) regressing $\log R^*_{t+1}$ onto the lagged yield curve and its contemporaneous changes, or (ii) regressing $\log R^*_{t+1}$ onto the lagged state variables X_t and their changes $X_{t+1} - X_t$. We report the results from approach (ii), specifically an R-squared of 27%. Our result is in the same ballpark as Boehm and Kroner (2025), which obtains R-squareds between 25% and 40% from a similar regression in FOMC windows.

Putting these results together, Table 4 shows that our model can quantitatively match the degree of equity-bond disconnect observed around FOMC meetings. In particular, in response to short-rate shocks, equities and bonds move to a similar degree on average, once equity duration is accounted for (Panel A). On the other hand, equities and bonds move very differently in general, as the yield curve cannot even come close to spanning equity market dynamics (Panel B). As discussed earlier, these results suggest that conventional monetary policy satisfies the LRN property, while unconventional policies (forward guidance, asset-purchase plans, communication in general) violate LRN.

4.3 Biases in shock identification

Finally, to what extent can monetary policy shocks can be extracted from financial markets? As we argued theoretically, if LRN is violated, then any attempt to identify shocks from asset prices will inevitably lead to biases. How large are these biases?

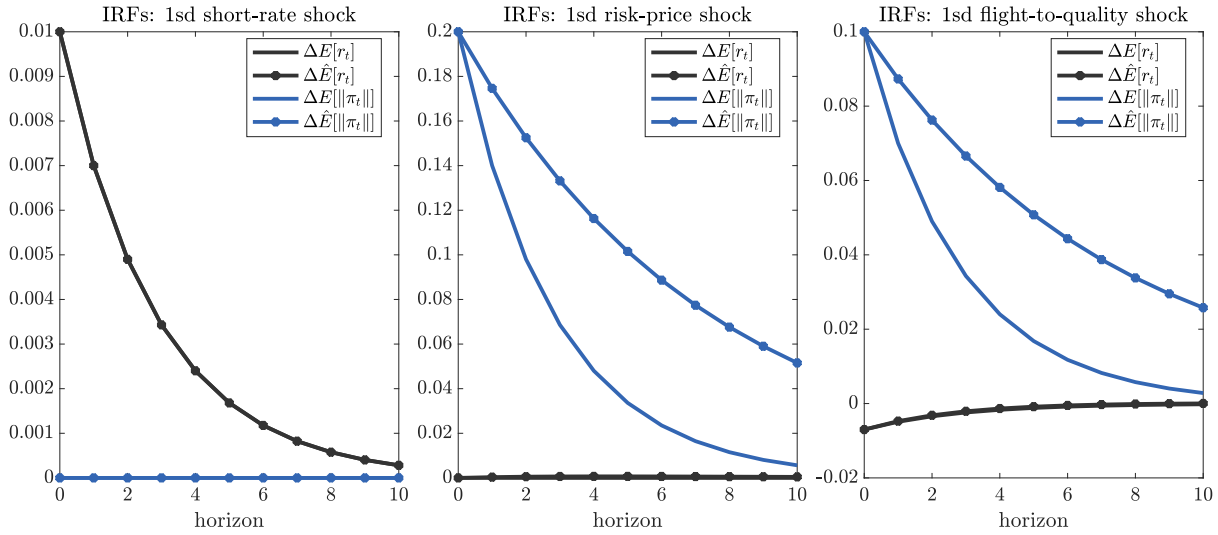


Figure 6. Biases in market-implied forecast revisions. The figure presents forecast revisions regarding the two factors (the short rate and price of risk factor). The three panels present the results for the three shocks, respectively. Forecasts are computed under the investor belief (expectation operator $\mathbb{E}$) and the long-run risk-neutral measure (expectation operator $\hat{\mathbb{E}}$).

Figure 6 reports the magnitude of biases from wrongly assuming LRN in the calibrated model. We compute forecast revisions for each of the two state variables under the investor's belief $\mathbb{P}$ and under the long-run risk-neutral measure $\hat{\mathbb{P}}$. There are never any deviations in the short-rate forecast revisions, because the persistence of the short-rate is identical under $\mathbb{P}$ and $\hat{\mathbb{P}}$, which results from risk price dynamics being independent of the short rate. But there are large deviations in risk-price forecast revisions, particularly at longer horizons. Therefore, if the Fed policies impact risk pricing (i.e., shocks 2 and 3), market-implied forecast revisions feature quantitatively significant biases.[20]

Figure 7 presents another way to visualize the biases in shock extraction that may be more relevant to practice. Imagine a researcher consults the financial market, inappropriately assumes LRN, and identifies a one standard deviation "easing shock," i.e.,

[20] Appendix D.2 performs an identical exercise in the context of a nonlinear long-run risks model with stochastic growth and uncertainty. We obtain some related conclusions: there are no biases when recovering belief updates to growth shocks, whereas significant biases arise when attempting to recover belief updates to volatility shocks. Volatility shocks in the long-run risks model, being the drivers of time-varying risk prices, are the key challenge to recovering forecast revisions. Therefore, if the Fed affects uncertainty (e.g. Johannes et al., 2023), and uncertainty shocks are priced, monetary shocks will be recovered with biases.

they obtain $\Delta\hat{W}_{t+1}^{(i)} = -1$ for shock i. How wrong is this inference? In other words, what was the actual monetary shock $\Delta W_{t+1}^{(i)}$? Figure 7 shows that the actual monetary easing can be substantially smaller, especially in "bad times" when risk prices are large in magnitude. This helps us understand the findings of Haddad et al. (2025), who argue that asset-purchase promises are introduced in bad times when financial markets are in distress. If that is the case, financial markets are likely to imply a substantial intervention, even if the actual intervention is modest. Intuitively, when risk prices are large, even a modest intervention can induce a large change to asset prices. By wrongly assuming LRN, we essentially ignore this state-contingent impact of policies and attribute everything to the size of the policy shock instead.

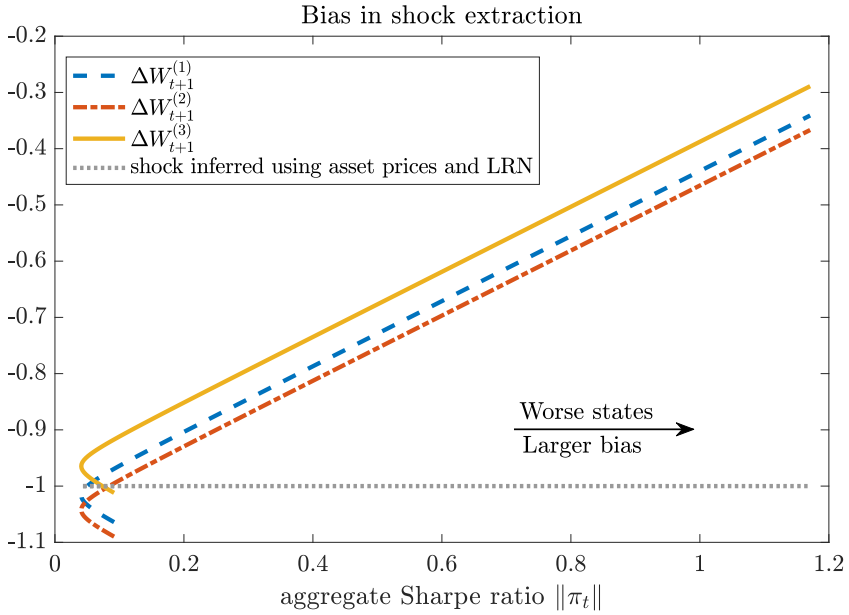


Figure 7. Biases in market-implied shocks. The figure displays the actual shocks ΔW_{t+1}, given the market-implied counterpart is $\Delta\hat{W}_{t+1} = -(1,1,1)'$, as a function of the Sharpe ratio $\|\pi_t\|$. To calculate the actual shocks, we use the relation $\Delta W_{t+1} = \Delta\hat{W}_{t+1} - \pi_t + B'v$.

Interestingly, the result that shock inference is biased holds similarly for all three of the shocks, including the pure short-rate shock, despite the earlier result that short-rate forecast revisions feature no biases. The reason for this discrepancy is that, while short-rate shocks do not themselves move risk prices, the risk price on short-rate shocks is still time-varying. A different way to understand the difference between Figures 6-7 is the following: Figure 6 plots the dynamic forecast revisions $\Delta\mathbb{E}X_t$ and $\Delta\hat{\mathbb{E}}X_t$ assuming the initial perturbation ΔX_0 is known, while Figure 7 is about the bias to the initial shock, $X_0 - \mathbb{E}_{-1}[X_0]$ versus $X_0 - \hat{\mathbb{E}}_{-1}[X_0]$.

5 Conclusion

We propose a new way of looking at monetary policy, with an asset-pricing version of monetary *Long-Run Neutrality (LRN)* as our center-piece. LRN posits that monetary policy does not affect the permanent component of marginal utility. Whether or not LRN holds has profound consequences.

If LRN holds, then we can rest easy. Many "standard" monetary models are admissible. Central banks impact risk premia primarily through term premia. And all monetary shocks can be recovered from a sufficiently rich set of asset prices.

But if LRN is violated, we must engage with some novel issues. Almost all "standard" monetary models are wrong; instead, we may need to entertain that some monetary policies primarily affect uncertainty and long-term growth. Central banks impact risk premia in nuanced ways; for example, sometimes they move equity and bond risk premia in tandem and other times in opposition. And many monetary shocks cannot be recovered purely from asset prices, without additional theoretical restrictions.

Our empirical evidence suggests LRN is violated, and so we are in the latter world with a litany of open questions. Moreover, our results suggest a dichotomy between conventional and unconventional monetary policies, with LRN holding for short-rate shocks but not for forward guidance, asset purchase plans, and the like.

Broadly speaking, a world without LRN is a world with more potent central banks. By permanently affecting marginal utility, monetary policy has the potential to change risk-taking behavior and thus actions of agents whose decisions involve risk-return tradeoffs. This power can be beneficial, allowing policy to better achieve its objectives. But given our nascent understanding of this mechanism, there can also be unintended consequences: central bankers can "make mistakes" and create uncertainty. Put differently, the Fed may now have an instrument that is harder to control than the conventional short-term interest rate, and so its "grip on the steering wheel is not as tight as it otherwise might be" (Stein, 2013). More research should investigate how the Fed can manage its ability to manipulate long-term risk premia.

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Appendix:

Risk Premia, Subjective Beliefs, and Forward Guidance

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April 8, 2026

A Proofs

A.1 Change-of-measure with jumps

We describe how to compute measure changes in our jump-diffusion environment. Under investor beliefs $\mathbb{P}$, the state dynamics are given by

$$dX_t = \mu(X_{t-})dt + \sigma(X_{t-})dW_t + \Omega_X \xi_t dM_t. \tag{A.1}$$

Note that jump arrivals, which occur with intensity $\lambda(x)$, and jump sizes, which have conditional distribution $\nu(\xi \mid x)$, are conditionally independent, conditional on X_{t-}. For this reason, we have that $\mathbb{E}_{t-}[\xi_t dM_t] = \lambda(X_{t-})dt \times \int \xi \nu(d\xi \mid X_{t-})$. (We normalized the jump distribution to be mean-zero in the main text, but the calculations below apply more generally.) And so the drift of X_t under $\mathbb{P}$ is $\mu(x) + \lambda(x)\Omega_X \int \xi \nu(d\xi \mid x)$.

Introduce a positive martingale G that follows

$$\frac{dG_t}{G_{t-}} = \omega(X_{t-}) \cdot dW_t + \Big(\exp[\zeta(X_{t-}) \cdot \xi_t] - 1\Big)\Big(dM_t - \lambda(X_{t-})dt\Big), \quad H_0 = 1. \tag{A.2}$$

Using G as a Radon-Nikodym derivative, define the probability measure $\mathbb{P}^G$ by $\mathbb{P}^G(B) := \mathbb{E}[G_t \mathbf{1}_B]$ for each $B \in \mathscr{F}_t$. Applications in the paper of this generic G include the risk-neutral density process (i.e., Q in Lemma A.2 below) and the permanent component of the SDF (i.e., H throughout). We can then compute dynamics under $\mathbb{P}^G$, using some standard results.

Lemma A.1. *If $f(t,x)$ is any C^2 function, the drift $\mu_f^G(t,x) := \frac{1}{dt}\mathbb{E}^G[df(t,X_t) \mid X_{t-} = x]$ is given by*

$$\begin{aligned}\mu_f^G(t,x) = \frac{\partial f(t,x)}{\partial t} + \Big[\mu(x) + \sigma(x)\omega(x)\Big] \cdot \frac{\partial f(t,x)}{\partial x} + \frac{1}{2} trace\Big[\sigma(x)\sigma(x)' \frac{\partial^2 f(t,x)}{\partial x \partial x'}\Big] \\ + \lambda(x) \int \Big[f(x + \Omega_X \xi) - f(x)\Big] \exp[\zeta(x) \cdot \xi] \nu(d\xi \mid x),\end{aligned} \tag{A.3}$$

provided the integral is finite.

Proof of Lemma A.1. Note that since G is a martingale and change-of-measure,

$$\mathbb{E}_t^G[f(T, X_T) - f(t, X_t)] = \frac{1}{G_t}\mathbb{E}_t[\int_t^T d(G_s f(s, X_s))],$$

so the drift of $f(t, X_t)$ under $\mathbb{P}^G$ is given by the drift of $G_t f(t, X_t)$ under $\mathbb{P}$. Thus, to obtain equation (A.3), we apply Itô's formula (with jumps) to $G_t f(t, X_t)$ and scale the

resulting drift by $1/G_{t-}$. See also Section 5 of Hansen and Scheinkman (2009). □

As an example of Lemma A.1, we can compute the drift of X_t under $\mathbb{P}^G$ by choosing the identity function $f(t,x) = x$, giving

$$\frac{1}{dt}\mathbb{E}^G[dX_t \mid X_{t-} = x] = \mu(x) + \underbrace{\sigma(x)\omega(x) + \lambda(x)\Omega_X \int \xi \exp[\zeta(x)\cdot\xi]\nu(d\xi \mid x)}_{\text{drift distortion under } \mathbb{P}^G}$$

The drift "distortion" of X_t under $\mathbb{P}^G$ is given by the latter two terms.

A.2 Results from Section 3.2

Lemma A.2. *Consider the general model of Section 3.1. We make the following "affine jump-diffusion hypotheses." Assume $r(x) = \rho_0 + \rho_1' x$ (affine short rate), $\mu(x) = A_0 + A_1 x$ (affine drift), $(\sigma(x)\sigma(x)')_{ij} = (\Sigma_0)_{ij} + (\Sigma_1)_{ij}\cdot x$ (affine covariances), $\sigma(x)\pi(x) = \varpi_0 + \varpi_1 x$ (affine Brownian risk premia), $\lambda(x) = \lambda_0 + \lambda_1' x$ (affine arrival rate of jumps), $\kappa(x) = \kappa_0$ (constant jump risk premia), and $\nu(\xi \mid x) = \nu_0(\xi)$ independent of x (state-invariant jump distribution). Furthermore, assume these functions are "well-behaved" in that they satisfy the integrability requirements of Duffie et al. (2000). Then, the price of a T-horizon discount bond takes the exponential-affine form*

$$p_t^{(T)} = \exp\left[\alpha(t) + \beta(t)\cdot X_t\right]$$

for some $\alpha(t)$ and $\beta(t)$ that are deterministic functions of time.

Proof of Lemma A.2. The approach is to guess-and-verify that $p_t^{(T)} := \mathbb{E}_t[\frac{S_T}{S_t}]$ takes the stated exponential-affine form, which we denote by the function $f(t,x) := \exp[\alpha(t) + \beta(t)\cdot x]$. As a preliminary, we let $Q_t/Q_0 = \exp[\int_0^t r(X_s)ds]S_t/S_0$ be the risk-neutral density process. We are tasked with verifying the pricing equation, which can be written $\exp(-\int_0^t r(X_s)ds)f(t,X_t) = \mathbb{E}_t^Q[\exp(-\int_0^T r(X_s)ds)f(T,X_T)]$. In other words, following Proposition 5 of Duffie et al. (2000), it suffices to show that $\Psi_t := \exp(-\int_0^t r(X_s)ds)f(t,X_t)$ is a $\mathbb{P}^Q$-martingale for some $\alpha(t)$ and $\beta(t)$.

Note that Q is a positive local martingale in the form of G from (A.2), with coefficients

$$\omega(x) = -\pi(x)$$
$$\zeta(x) = \kappa(x)$$

Applying the result of Lemma A.1 with the function $f(t,x) = \exp[\alpha(t) + \beta(t)\cdot x]$, and using the coefficients of Q from above, we can obtain the drift of $f(t,X_t)$ under $\mathbb{P}^Q$, and then apply Itô's product rule to obtain the drift of Ψ_t under $\mathbb{P}^Q$ as

$$\mu^Q_\Psi(t,x) = \Psi_t\Big\{ -r(x) + \dot\alpha(t) + \dot\beta(t)'x + \beta(t)'\big[\mu(x) - \sigma(x)\pi(x)\big] + \frac{1}{2}\big[\beta(t)'\sigma(x)\sigma(x)'\beta(t)\big] \\ + \lambda(x)\int \big(\exp[\beta(t)'\Omega_X\xi] - 1\big)\exp[\kappa(x)\cdot\xi]\nu(d\xi \mid x)\Big\}$$

Using the "affine jump-diffusion hypotheses" in the lemma, this drift becomes

$$\mu^Q_\Psi(t,x) = \Psi_t\Big\{ -\rho_0 - \rho_1'x + \dot\alpha(t) + \dot\beta(t)'x + \beta(t)'\big[A_0 + A_1x - \varpi_0 - \varpi_1x\big] + \frac{1}{2}\beta(t)'\big[\Sigma_0 + \Sigma_1\cdot x\big]\beta(t) \\ + (\lambda_0 + \lambda_1'x)\int \big(\exp[\beta(t)'\Omega_X\xi] - 1\big)\exp[\kappa_0\cdot\xi]\nu_0(d\xi)\Big\}$$

For Ψ_t to be a $\mathbb{P}^Q$-martingale, this drift must be zero, leading to the pair of ODEs:

$$\dot\alpha(t) = \rho_0 - (A_0 - \varpi_0)'\beta(t) - \frac{1}{2}\beta(t)'\Sigma_0\beta(t) - \lambda_0\int \big(\exp[\beta(t)'\Omega_X\xi] - 1\big)\exp[\kappa_0\cdot\xi]\nu_0(d\xi)$$
$$\dot\beta(t) = \rho_1 - (A_1 - \varpi_1\mathbb{I})'\beta(t) - \frac{1}{2}\beta(t)'\Sigma_1\beta(t) - \lambda_1\int \big(\exp[\beta(t)'\Omega_X\xi] - 1\big)\exp[\kappa_0\cdot\xi]\nu_0(d\xi)$$

For $f(T,x) = 1$, these ODEs must be solved subject to the terminal conditions $\alpha(T) = 0$ and $\beta(T) = 0$. Assuming these ODEs have a solution, and making the "well-behaved" assumptions of Duffie et al. (2000), we have that Ψ_t is indeed a proper $\mathbb{P}^Q$-martingale, completing the proof. □

Proof of Propositions 1-2. Given in the text before the statements of the propositions. □

A.3 Results from Section 3.3

Lemma A.3. *Asset prices reveal* $\hat{\mathbb{P}}\{X_{\tau+T} \in \mathcal{B} \mid X_\tau\}$ *for every Borel set* $\mathcal{B}$ *and all* τ, T.

Proof of Lemma A.3. Using the definition of $\hat{\mathbb{P}}$ in (19), the SDF factorization (15), and result (2) on the long bond, compute

$$\hat{\mathbb{P}}\{X_{\tau+T} \in \mathcal{B} \mid X_\tau\} = \mathbb{E}\Big[\frac{H_{\tau+T}}{H_\tau}\mathbf{1}_{\{X_{\tau+T}\in\mathcal{B}\}} \mid X_\tau\Big] \\ = \mathbb{E}\Big[\frac{S_{\tau+T}}{S_\tau}R^\infty_{\tau,\tau+T}\mathbf{1}_{\{X_{\tau+T}\in\mathcal{B}\}} \mid X_\tau\Big],$$

where $R^{\infty}_{\tau,\tau+T} := \int_{\tau}^{\tau+T} dR^{\infty}_t$ is the cumulative return on the long bond. The final object is the price of the claim $R^{\infty}_{\tau,\tau+T}\mathbf{1}_{\{X_{\tau+T}\in\mathcal{B}\}}$. This price is observable in the financial market. □

Proof of Proposition 3. Let $m^x_t := \mathbb{E}^x[X_t]$ denote the (investor-perceived) conditional mean of X_t, starting from point x. By applying Itô's formula to X_t, we have that m^x_t solves the differential equation (since the compensated monetary shock has zero mean)

$$\frac{d}{dt}m^x_t = \mathbb{E}^x[\mu(X_t)]$$

subject to the initial condition $m^x_0 = x$. Specializing to the linear drift from (21), the ODE becomes

$$\frac{d}{dt}m^x_t = \mathbb{E}^x[A_0 + AX_t] = A_0 + Am^x_t$$

This ODE is affine, and the solution takes the well-known form

$$m^x_t = \exp(At)\Big[x + \int_0^t \exp(-As)A_0 ds\Big].$$

We may then compute

$$m^{X_\tau}_t - m^{X_{\tau-}}_t = \exp(At)(X_\tau - X_{\tau-}).$$

(The interpretation of $\exp(At)$ is as the Taylor series $\sum_{k=0}^{\infty}\frac{t^k}{k!}A^k$.)

On the other hand, using assumption (22) and LRN, we have that the drift of X_t under $\hat{\mathbb{P}}$ is $A_0 + AX_t - B(X^{\perp}_t)\beta(X^{\perp}_t)$, where $X^{\perp}$ are the monetary-insensitive states. Compute similarly an equation for $\hat{m}^x_t := \hat{\mathbb{E}}^x[X_t]$:

$$\begin{aligned}
&\hat{m}^{X_\tau}_t - \hat{m}^{X_{\tau-}}_t \\
&= A\int_0^t \Big(\hat{\mathbb{E}}^{X_\tau}[X_{\tau+s}] - \hat{\mathbb{E}}^{X_{\tau-}}[X_{\tau+s}]\Big)ds - \int_0^t \Big(\hat{\mathbb{E}}^{X_\tau}[B(X^{\perp}_{\tau+s})\beta(X^{\perp}_{\tau+s})] - \hat{\mathbb{E}}^{X_{\tau-}}[B(X^{\perp}_{\tau+s})\beta(X^{\perp}_{\tau+s})]\Big)ds \\
&= A\int_0^t \Big(\hat{m}^{X_\tau}_s - \hat{m}^{X_{\tau-}}_s\Big)ds,
\end{aligned}$$

where the last line uses the monetary-insensitivity of $X^{\perp}$ and the fact that $\mathbb{P}$ and $\hat{\mathbb{P}}$ are mutually absolutely continuous. The solution to this integral equation is

$$\hat{m}^{X_\tau}_t - \hat{m}^{X_{\tau-}}_t = \exp(At)(X_\tau - X_{\tau-}).$$

Since $\hat{m}_t^{X_\tau} - \hat{m}_t^{X_{\tau-}}$ is observable from asset prices (by Lemma A.3), we have that $m_t^{X_\tau} - m_t^{X_{\tau-}}$ is observable. By assumption (23), we obtain

$$\mathbb{E}^{X_\tau}[\phi(X_{\tau+T})] - \mathbb{E}^{X_{\tau-}}[\phi(X_{\tau+T})] = \phi_1 \cdot (m_T^{X_\tau} - m_T^{X_{\tau-}})$$

so long as we know ϕ_1. □

Proof of Proposition 4. Suppose, leading to contradiction, that z_τ^T is identified by asset price data, i.e., (20) holds:

$$\begin{aligned}\mathbb{E}\big[\frac{H_{\tau+T}}{H_\tau}\phi(X_{\tau+T}) \mid X_\tau\big] - \mathbb{E}\big[\frac{H_{\tau+T}}{H_{\tau-}}\phi(X_{\tau+T}) \mid X_{\tau-}\big] \\ = \mathbb{E}\big[\phi(X_{\tau+T}) \mid X_\tau\big] - \mathbb{E}\big[\phi(X_{\tau+T}) \mid X_{\tau-}\big].\end{aligned} \tag{A.4}$$

Using the fact that (A.4) holds for all X_τ and $X_{\tau-}$, we must have

$$\mathbb{E}\big[H_T\phi(X_T) \mid X_0 = x\big] = \mathbb{E}\big[\phi(X_T) \mid X_0 = x\big] + \alpha(T), \tag{A.5}$$

where $\alpha(T)$ may depend on T but is independent of x. Recall $\phi(x) = \phi_0 + \phi_1 \cdot x$. Without loss of generality, let us assume that ϕ_1 has non-zeros in all of its entries. It must therefore generically be the case that

$$\mathbb{E}\big[H_T X_T \mid X_0 = x\big] = \mathbb{E}\big[X_T \mid X_0 = x\big] + \epsilon(T), \tag{A.6}$$

where $\epsilon(T)$ is an N-dimensional vector independent of x.

Now, the dynamics of H in general take the form

$$\frac{dH_t}{H_{t-}} = -\beta(X_{t-}) \cdot dW_t - \big(\exp[\zeta(X_{t-}) \cdot \xi_t] - 1\big)dM_t \tag{A.7}$$

for some functions $\beta(\cdot)$ and $\zeta(\cdot)$, and where recall that the intensity of dM_t is $\lambda(X_{t-})$ and the distribution of ξ_t is $\nu(\xi \mid X_{t-})$. If LRN fails, it must generically be the case that dH_t depends on X_{t-}, including at least some states that are not monetary-insensitive. (We must include the term "generically," because of the following knife-edge case. LRN can be violated and yet $dH_t \perp X_{t-}$ holds if $\zeta(x) \equiv \zeta_0$ constant, the announcement arrival rate $\lambda(x) \equiv \lambda_0$ is constant, and the jump probability distribution $\nu(\xi \mid x) = \nu_0(\xi)$ is independent of x.) We may summarize both cases by using Lemma A.1 to write the drift

of X_t under $\hat{\mathbb{P}}$ as

$$\hat{\mu}(x) = \underbrace{A_0 + Ax}_{\mu(x)} - \Big(\underbrace{\sigma(x)\beta(x) - \lambda(x)\Omega_X \int \xi \exp[\zeta(x)\cdot\xi]\nu(d\xi \mid x)}_{:=\Gamma(x)} \Big). \tag{A.8}$$

In (A.8), the function $\Gamma(x)$ is a non-constant function of x that depends non-trivially on at least some monetary-sensitive states, generically. Using (A.8), compute

$$\begin{aligned} \mathbb{E}^x\left[H_T X_T\right] - \mathbb{E}^x\left[X_T\right] &= \hat{\mathbb{E}}^x\left[X_T\right] - \mathbb{E}^x\left[X_T\right] \\ &= A\int_0^T \left(\hat{\mathbb{E}}^x\left[X_t\right] - \mathbb{E}^x\left[X_t\right]\right)dt - \int_0^T \hat{\mathbb{E}}^x\left[\Gamma(X_t)\right]dt \end{aligned} \tag{A.9}$$

Guess that (A.6) holds, and replace in (A.9) to get

$$\epsilon(T) = A\int_0^T \epsilon(t)dt - \int_0^T \hat{\mathbb{E}}^x\left[\Gamma(X_t)\right]dt. \tag{A.10}$$

Clearly, the last term in (A.10) generically depends on x, contradicting the guess that $\epsilon(T)$ is independent of x. Thus, z_τ^T is not identified. □

B Discrete-time VAR setting

B.1 The general exponential-affine setting

Consider the following asset-pricing model. There is a potentially-latent state vector that follows a stationary VAR

$$X_{t+1} = AX_t + B\Delta W_{t+1}, \tag{B.1}$$

We assume A is stable and B is full-rank but not necessarily square (meaning there can be more shocks than state variables). We denote $N = \dim(X)$ and $d = \dim(W)$ to be the state and shock dimensionality. The SDF follows

$$\frac{S_{t+1}}{S_t} = \exp\Big[-r_t - \frac{1}{2}\pi_t'\pi_t - \pi_t'\Delta W_{t+1}\Big], \tag{B.2}$$

where

$$r_t = \rho_0 + \rho_1' X_t \tag{B.3}$$

is the one-period risk-free rate, and

$$\pi_t = \pi_0 + \Pi X_t \tag{B.4}$$

is the one-period risk price vector. In that case, the risk-neutral dynamics of the state are given by

$$\begin{aligned} X_{t+1} &= A_0^* + A^* X_t + B\Delta W_{t+1}^*, \\ \text{where} \quad A_0^* &:= -B\pi_0 \\ \text{and} \quad A^* &:= A - B\Pi, \end{aligned} \tag{B.5}$$

where ΔW_{t+1}^* is a Normal shock under the risk-neutral distribution.

In this conditionally-lognormal setting, the equilibrium yield-to-maturity for an n-period risk-free zero-coupon bond can be computed as an affine function of the state vector

$$y_t^{(n)} = \frac{1}{n}\Big[\mathcal{B}_0^{(n)} + \mathcal{B}^{(n)} X_t\Big] \tag{B.6}$$

where the coefficients $\mathcal{B}_0^{(n)}$ and $\mathcal{B}^{(n)}$ can be computed recursively via

$$\mathcal{B}^{(n)} := \rho_1'(I - A^*)^{-1}(I - (A^*)^n),$$
$$\mathcal{B}_0^{(n)} := n\rho_0 + \Big(\sum_{i=1}^{n-1} \mathcal{B}^{(i)}\Big) A_0^* - \frac{1}{2}\sum_{i=1}^{n-1} \mathcal{B}^{(i)} BB' (\mathcal{B}^{(i)})'.$$

This exponential-affine solution is standard in the literature. Given (B.6), we will assume going forward an "invertability" condition such that observing the entire yield curve is enough to infer (a rotation of) the state vector X_t.

We characterize the permanent and transitory components in the SDF from (1) as

$$\frac{S_{t+1}}{S_t} = \exp\Big[\eta - v'(X_{t+1} - X_t)\Big]\frac{H_{t+1}}{H_t}, \tag{B.7}$$
$$\text{where} \quad \frac{H_{t+1}}{H_t} = \exp\Big[-\frac{1}{2}\|\pi_t - B'v\|^2 - (\pi_t - B'v)\cdot \Delta W_{t+1}\Big]$$
$$v = -[I - (A - B\Pi)']^{-1}\rho_1$$
$$\eta = -\rho_0 - v'B\pi_0 + \frac{1}{2}v'BB'v$$

The shock-exposure of H is the short-run risk price π_t plus $-B'v$, which is additional long-run risk compensation.[21]

In this setting, let us decompose the shocks into monetary and non-monetary shocks. The monetary shocks are the first k shocks, i.e., $\Delta M := (\Delta W^{(1)}, \ldots, \Delta W^{(k)})$. Accordingly, let $B_M := B^{(:,1-k)}$ denote the first k columns of the exposure matrix B, let $\Pi_M := \Pi^{(1-k,:)}$ denote the first k rows of Π, let $\pi_{M,0}$ denote the first k elements of π_0, and hence let $\pi_{M,t} := \pi_{M,0} + \Pi_M X_t$ denote the first k risk prices. The shock-exposure of H to the monetary shocks ΔM is the $k \times 1$ vector $\pi_{M,t} - B_M' v$. Consequently, Long-Run Neutrality

[21] To obtain this, guess-and-verify that the function characterizing the transitory piece takes the form $e(x) = \exp(v'x)$. Then, given the form of the SDF in (B.2), we must have

$$\frac{H_{t+1}}{H_t} = \exp\Big[-\rho_0 + \eta - (\rho_1' + v'(I - A))X_t - \frac{1}{2}\pi_t'\pi_t - (\pi_t' - v'B)\Delta W_{t+1}\Big]$$

For H to be a martingale, we must have

$$\frac{1}{2}\pi_t'\pi_t + \rho_0 + \eta + (\rho_1' + v'(I - A))X_t = \frac{1}{2}(\pi_t' - v'B)(\pi_t' - v'B)'$$
$$\Rightarrow \rho_0 + \eta + (\rho_1' + v'(I - A))X_t = \frac{1}{2}v'BB'v - v'B\pi_0 - v'B\Pi X_t$$

By imposing this equation holds for every X_t, we obtain the result.

(LRN) says that

$$\text{LRN} \quad \iff \quad \pi_{M,t} - B_M' v = 0. \tag{B.8}$$

Using $\pi_{M,t} = \pi_{M,0} + \Pi_M X_t$, we can write LRN as

$$\pi_{M,0} - B_M' v = 0 \tag{B.9}$$

$$\Pi_M = 0 \tag{B.10}$$

Condition (B.9) says that "LRN holds on average" (since the unconditional mean of X_t is zero), while condition (B.10) says that monetary risk prices are constants. Useful for Section B.2 is to count the number of restrictions here: there are k restrictions in (B.9) and $d \times k$ restrictions in (B.10), for a total of $k(d+1)$, where recall k is the dimension of monetary shocks and d is the overall shock dimensionality.

B.2 Comparison of reduced-form and structural approaches

This section compares our approach to identification, which follows the "reduced-form" tradition, to a more structural approach. We do so by comparing directly to Backus, Chernov, Zin and Zviadadze (2022), hereafter BCZZ. This comparison is apt, because BCZZ use an exponential-affine framework identical to Appendix B.1 and employ an assumption about the long-run impact of monetary policy on asset markets. Whereas our paper requires a strong assumption for identification (LRN), BCZZ require something much weaker (roughly speaking, "LRN on average"). This discrepancy arises precisely because BCZZ fully exploit the structure of their model to aid in shock identification. By contrast, we lean heavily on a high-frequency approach to extract surprises, and then exploit LRN to identify these surprises with monetary shocks, all the while refraining from leaning on the full structure of the model.

B.2.1 Identification problems and their solutions

We start by discussing the identification problems studied by our paper (hereafter, CK) and BCZZ. CK are interested in computing investors' forecast revisions for an arbitrary variable (e.g., belief updates about the future interest rate, which maps into forward guidance). By contrast, BCZZ are interested in identifying the monetary policy disturbance contemporaneously. We nest both papers in assuming that they are trying to learn about some variable of the form $z_t = \zeta \cdot X_t$.

The identification problem in CK is the following. They wish to use asset markets to identify investors' forecast revision about future values of z at any horizon:

$$\mathbb{E}_t[z_{t+T}] - \mathbb{E}_{t-1}[z_{t+T}], \quad T > 0 \tag{B.11}$$

Assuming z_t is observable, the states X_t are observable, and a long enough time series is available, we can identify the coefficients ζ by simple projection. Motivated by this, CK simply *assume* that these coefficients are observable.

At that point, the key question is how much to exploit the model structure. By using the state dynamics, (B.11) can be rewritten as:

$$\zeta'(A^T X_t - A^{T+1} X_{t-1}), \quad T > 0. \tag{B.12}$$

Consequently, assuming rational expectations and assuming the econometrician observes the state X_t for a long enough time series, he can estimate the persistence matrix A and then compute (B.12). This is a "structural" approach.

CK does not take this structural route. Instead, CK asks whether (B.11) can be learned from asset markets alone, refraining from using direct observation of X_t and direct estimation of A. This abstinence from fully exploiting the structure can be valuable in the case that new types of monetary shocks arise (e.g., forward guidance, asset purchases)—a type of non-stationarity. Non-rational expectations constitutes another important justification for a less structural approach: running regressions can only obtain the objective state dynamics, which may differ from the subjective persistence matrix A. The downside of a reduced-form approach, however, is that the identification problem becomes very challenging and requires the strong LRN assumption, which recall from (B.9)-(B.10) embeds $k(d+1)$ total restrictions (where k is the number of monetary shocks, and $d-k$ is the number of non-monetary shocks).

By contrast, the identification problem in BCZZ is the following. Given the affine model, suppose economic growth can be written

$$g_t = \gamma_0 + \gamma_1 \cdot X_t. \tag{B.13}$$

Next, assume a Taylor-type rule

$$r_t = \tau_0 + \tau_g g_t + z_t, \tag{B.14}$$

so that τ_0 is the "target rate," τ_g captures the rule's response to growth, and z_t is the

monetary policy disturbance. (BCZZ's Taylor rule also includes inflation, but that is not needed for the points here.) BCZZ assume that z_t is not observable, and it cannot be inferred directly either because the Taylor rule parameters (τ_0, τ_g) are also as yet unobserved. As a first step to identification, we may use equations (B.3), (B.13) and (B.14) to obtain the following $N+1$ restrictions:

$$\tau_0 = \rho_0 - \tau_g \gamma_0 \tag{B.15}$$

$$\zeta = \rho_1 - \tau_g \gamma_1 \tag{B.16}$$

Since growth and the interest rate are observable, we may assume that (ρ_0, ρ_1) and (γ_0, γ_1) are observable. However, ignorance about τ_g precludes direct inference about τ_0 and ζ. In other words, we have $N+2$ unknown parameters (τ_0, τ_g, ζ) but only $N+1$ restrictions in (B.15)-(B.16). BCZZ need only add a single scalar restriction for identification—this permits something much weaker than LRN.

For their scalar condition, BCZZ study the following second-moment condition:

$$\mathbb{E}\left[z_{t+1} \log\left(\frac{H_{t+1}}{H_t} \right) \mid X_t = 0 \right] = 0. \tag{B.17}$$

This says, roughly, that monetary shocks z do not affect H, at least on average. Substituting the affine formulation for z and the permanent component H into (B.17), BCZZ equivalently assume:

$$\begin{aligned} 0 &= \mathbb{E}\Big[(\zeta \cdot X_{t+1}) \Big(-\frac{1}{2} \|\pi_t - B'v\|^2 - (\pi_t - B'v) \cdot \Delta W_{t+1} \Big) \mid X_t = 0 \Big] \\ &= \mathbb{E}[(\zeta' B \Delta W_{t+1})(\pi_0' \Delta W_{t+1} - v' B \Delta W_{t+1})] \\ &= \zeta' B (\pi_0 - B'v) \end{aligned} \tag{B.18}$$

Using the fact that $\zeta' B = \zeta' \begin{bmatrix} B_M & 0 \end{bmatrix}$, which says that non-monetary shocks by definition do not impact the monetary disturbance z, we have that (B.18) is equivalent to

$$\zeta' B_M (\pi_{M,0} - B_M' v) = 0 \tag{B.19}$$

Going back to the "LRN-on-average" condition (B.9), we see that (B.19) is imposing LRN-on-average but for a particular linear combination of monetary shocks.

In summary, the problems of CK and BCZZ are distinct but complementary. CK assume that ζ is known and try to obtain forecast revisions about $z_{t+T} = \zeta \cdot X_{t+T}$ without relying fully on the VAR structure. BCZZ instead assume that ζ is unknown and try to

estimate it, using the restriction that z is the monetary policy disturbance to a particular Taylor rule. The required identification assumptions in CK are strong because the problem is hard: we want to identify, non-parametrically, forecast revisions about future economic states. The required identification assumptions in BCZZ are much weaker, because their problem is lower-dimensional: they only need to identify a small number of unknown Taylor rule parameters.

B.2.2 More complex Taylor rules

To further understand the difference between CK and BCZZ, we consider more elaborate versions of the Taylor rule. This fundamentally alters the required assumptions in BCZZ, because they define the policy disturbance as the interest rate movements after netting out the rule component. We consider an extension in which: (i) the monetary policy rule contains feedback from a larger set of economic variables; and (ii) the monetary policy rule itself features nonlinear responses to these variables. The key result is that for sufficiently complex Taylor rules, the required identification conditions approach LRN, even for the structural approach in BCZZ. This highlights the importance of an accurate assumption on the Taylor rule.

First, suppose the set of variables is now the vector G_t, which may include growth g_t but also includes a set of other observables (e.g., financial conditions, asset prices, inflation, employment). Suppose

$$G_t = \Gamma_0 + \Gamma_1 X_t \tag{B.20}$$

where $N_G := \dim(G)$ is the dimensionality of G, with $1 \leq N_G \leq N$. Second, the Taylor rule is

$$r_t = \tau_0 + \tau_G(X_t) \cdot G_t + z_t, \tag{B.21}$$

where $\tau_G(X_t) = \bar{\tau}_{G,0} + \bar{\tau}_{G,1} X_t$ is an affine function into $\mathbb{R}^{N_G}$ (i.e., $\bar{\tau}_{G,0}$ is an $N_G \times 1$ vector and $\bar{\tau}_{G,1}$ is an $N_G \times N$ matrix). Thus, the Taylor rule conditions on a larger set of variables and its responsiveness is a function of market conditions.

How many identification assumptions are needed? To accommodate this nonlinear setup, we must allow the discretionary shock to also be nonlinear, so we conjecture $z_t = \zeta(X_t) \cdot X_t$ for some function $\zeta(\cdot)$. Use equations (B.3), (B.4) (B.20), and (B.21) to

obtain the following restrictions:

$$\tau_0 = \rho_0 - \bar{\tau}'_{G,0}\Gamma_0 \tag{B.22}$$

$$\zeta(X_t) = \rho_1 - \bar{\tau}'_{G,1}\Gamma_0 - \Gamma'_1\tau_G(X_t) \tag{B.23}$$

This implies that the form of $\zeta(x)$ is affine, which we can write as

$$\begin{aligned} \zeta(x) &= \bar{\zeta}_0 + \bar{\zeta}_1 x, \\ \text{where} \quad \bar{\zeta}_0 &:= \rho_1 - \bar{\tau}'_{G,1}\Gamma_0 - \Gamma'_1\bar{\tau}_{G,0} \\ \bar{\zeta}_1 &:= -\Gamma'_1\bar{\tau}_{G,1} \end{aligned} \tag{B.24}$$

There are $N^2 + N \times N_G + N + N_G + 1$ parameters to be estimated, namely the collection $(\tau_0, \bar{\tau}_{G,0}, \bar{\tau}_{G,1}, \bar{\zeta}_0, \bar{\zeta}_1)$. Meanwhile, equations (B.22)-(B.23) constitute $N^2 + N + 1$ restrictions. This leaves $N_G \times (N+1)$ degrees of freedom. It is easy to see that N_G of these degrees of freedom come from the endogenous variables in the Taylor rule, while $N \times N_G$ of these degrees of freedom come from the state-dependence of the reaction function.

Suppose we wish to add restrictions via only the long-run component of the SDF, i.e., via H.[22] Focusing on the impact monetary shocks have on H, we may restrict both $\pi_{M,0}$ and Π_M, which have $k(d+1)$ parameters in total (recall $k = \dim(M)$ is the dimension of the monetary shocks and $d = \dim(W)$ is the dimension of all shocks). Identification requires $N_G(N+1)$ restrictions, and we have $k(d+1)$ parameters available to restrict. As an example, suppose that there are just as many shocks as states ($d = N$) and that there are just as many monetary shocks as inputs into the Taylor rule ($k = N_G$). In that case, both the level and time-variation in monetary risk prices must be completely restricted. LRN in (B.8) is one example of such a condition that embeds precisely $k(d+1)$ restrictions.

In summary, the complexity of endogeneity in the Taylor rule substantially raises the hurdle for identification. In some cases, which are hard to rule out a priori (i.e., time-varying reaction function and multiple components to the Taylor rule), shock identification essentially requires a condition of the dimensionality of LRN, which is much stronger than the LRN-on-average condition adopted by BCZZ.

[22] For every real variable in G, a reasonable thing to do following BCZZ, is to assume that variable is neutral to monetary policy in the long-run. But for every nominal or financial variable in G, we require an asset-pricing style neutrality condition. For these asset-pricing neutrality conditions, we follow BCZZ in focusing on the long-run component of the SDF.

B.3 In-depth example with forward guidance and subjective beliefs

B.3.1 Setup

Consider a three-state model

$$X_t = \begin{bmatrix} g_t \\ r_t \\ f_t \end{bmatrix} = \begin{bmatrix} \text{(demeaned) growth rate} \\ \text{(demeaned) interest rate} \\ \text{forward guidance} \end{bmatrix}.$$

We assume that g_t and r_t are observable, whereas f_t is a latent factor. Suppose the state evolves dynamically according to

$$X_{t+1} = A^o X_t + B\Delta W^o_{t+1}, \tag{B.25}$$

where $\Delta W^o_{t+1} \sim \text{Normal}(0, I)$ is a 3-dimensional vector of Normal shocks. For the purpose of this example, think of the shocks ΔW^o_{t+1} as containing a "real shock" and two "monetary shocks" that are completely governed by central bank actions—if B were a diagonal matrix, we could assign these labels to the individual shocks, but that is not necessary for our purposes. Equation (B.25) governs the *objective* state dynamics (hence the "o" superscript), which may differ from agents' *perceived* dynamics that we detail below.

Let us specify the persistence matrix A^o. Assume

$$A^o = \begin{bmatrix} a^o_{gg} & a^o_{gr} & a^o_{gf} \\ a^o_{rg} & a^o_{rr} & a^o_{rf} \\ 0 & 0 & a^o_{ff} \end{bmatrix}. \tag{B.26}$$

and that A^o is stable. Note that a^o_{gr} and a^o_{gf} capture the effects of short rates and forward guidance on growth. By contrast, a^o_{rg} captures the "feedback effect" of growth into the interest rate rule. And a^o_{rf} captures the transmission from forward guidance into the short-rate. In fact, the entire purpose of including f in this system is to add an additional factor governing future short rates. Finally, forward guidance f_t evolves as a univariate AR(1) independently of (g_t, r_t), a setup that is not necessary to any result but is transparent. We make no assumptions about B.

We allow agents' beliefs to potentially be distorted. While we take no stand here, belief distortions could come from multiple sources: pure cognitive biases, imperfect information, finite samples with imperfect priors, etc. To keep things simple, we consider

a belief distortion that modifies the persistence of X_t. Under agents' beliefs

$$X_{t+1} = AX_t + B\Delta W_{t+1}, \quad \Delta W_{t+1} \sim \text{Normal}(0, I). \tag{B.27}$$

We will assume A, like A^o, is a stable matrix. The notation $\mathbb{E}$ will stand for the subjective expectation operator for agents in our model, which may or may not coincide with the objective expectation $\mathbb{E}^o$.

Thus, agents perceive

$$\Delta W_{t+1} = \Delta W^o_{t+1} - L_t, \quad \text{where} \quad L_t := B^{-1}\big(A - A^o\big)X_t, \tag{B.28}$$

to be a standard Normal shock. One interpretation is that the vector L_t represents investors' time-varying degree of optimism.

B.3.2 The causal effects of monetary policy

Consider the question "what are the causal effects of monetary policy on future growth?" Here, there are two components of policy: short rates and forward guidance.

The *short-rate shock* at time τ defined as

$$z^r_\tau := r_\tau - \mathbb{E}[r_\tau \mid X_{\tau-1}]. \tag{B.29}$$

In this paper, we take as given the ability to non-parametrically identify the short rate shock from data. In particular, assume the existence of a financial market (e.g., Fed Funds futures) whose price corresponds to $\mathbb{E}[r_\tau \mid X_{\tau-1}]$ at time $\tau - 1$. It is technically appropriate to use the investor expectation $\mathbb{E}$ here, because financial markets reflect investor beliefs. A *forward-guidance shock* at time τ is defined as

$$z^f_\tau := f_\tau - \mathbb{E}[f_\tau \mid X_{\tau-1}]. \tag{B.30}$$

Unlike the short-rate shock, we do not assume the forward-guidance shock is non-parameterically identified. This will be a key challenge. For completeness, also define the *growth shock* $z^g_\tau := g_\tau - \mathbb{E}[g_\tau \mid X_{\tau-1}]$. Stack the shocks into

$$z_\tau := \left(z^g_\tau, z^r_\tau, z^f_\tau\right)' = B\Delta W_\tau.$$

These shocks are reduced-form in nature, but that is not the key issue. The complication,

instead, comes from the distorted beliefs: the reduced-form shocks

$$z_\tau = B\Delta W^o_\tau - (A - A^o)X_{\tau-1} \tag{B.31}$$

are related to the true structural shocks ΔW^o, with a bias that is a function of lagged X.

What are the causal effects of monetary policy on $g_{\tau+t}$? Because we do not have a full structural model of monetary action, we *define* all causal effects by the following IRFs

$$D_h := \frac{\partial}{\partial w'}\mathbb{E}^o\left[g_{\tau+h} \mid B\Delta W^o_\tau = w\right] = \begin{bmatrix}1\\0\\0\end{bmatrix} \cdot (A^o)^h \tag{B.32}$$

The second and third entries of this IRF tell us the effects of the short-rate and forward-guidance shocks. Suppose we seek D_h.

A first convenient result is that, to measure D_h empirically, we may run a regression of future growth $g_{\tau+h}$ onto z_τ, the lagged state $X_{\tau-1}$, and a constant. In other words, we can employ the *subjective shocks* and need not obtain proxies for the objective shocks; this simplification is beneficial because some shocks like z^r will be identified from financial market data and therefore reflect investors' beliefs. In principle, this "local projection" approach, highlighted by Jordà (2005), flexibly estimates the effect of shocks on future g at various horizons. The Jorda approach is considered to be more robust to model misspecification than the alternative that estimates the VAR(1) and computes $(A^o)^h$. For emphasis, we give this procedure a name:

Procedure 1. *The estimate of b_h from the regression*

$$g_{\tau+h} = a_h + b_h' z_\tau + c_h' X_{\tau-1} + \epsilon_{\tau+h} \tag{B.33}$$

is a consistent estimator of D_h.

Why does Procedure 1 work? An important, and underappreciated, feature is the lagged state control $X_{\tau-1}$: in a distorted-belief world, this control is necessary as it captures the discrepancy between the objective and subjective shocks—see equation (B.31). Approaches in the literature, instead, have mostly hoped to find objective shocks to monetary policy. If a researcher succeeds in measuring an objective shock, there is no need to control for drivers of belief distortions. But if instead one merely measures a subjective surprise, omitting the lagged states $X_{\tau-1}$ effectively, and perhaps wrongly, assumes rational expectations holds.

A simple numerical example highlights the importance of controlling for $X_{\tau-1}$. We calibrate A^o, A, and B to roughly match the evidence in Cieslak (2018), who compares the

objective dynamics of employment growth and short rates to the subjective dynamics of these same variables from surveys. Details of this calibration are provided in Appendix B.4.1. The results for the calibrated A^o and A imply agents perceive a smaller feedback of growth to future short rates, as well as higher persistences of r and especially of f.

After calibrating, we ask: what is the impact of wrongly assuming that z^r_τ and z^f_τ are objective shocks, instead of perceived investor surprises? Assuming rational expectations corresponds to implementing Procedure 1 without controlling for the lagged state $X_{\tau-1}$. On the other hand, controlling for $X_{\tau-1}$ corrects for belief distortions and therefore yields the correct effect of forward guidance. Figure B.1 plots the outcomes of these two regressions. The lines labelled "True IRF" come from correctly implementing Procedure 1, while the lines labelled "Wrong IRF" come from forgetting to control for $X_{\tau-1}$ (and thereby implicitly assuming rational expectation). One notices a large discrepancy between these measures, so much that even the signs of the effect can be opposite in the two approaches.

As an aside, we note that the sign-flip from our numerical example may provide an alternative mechanism to generate what looks like a "Fed information effect." For example, some studies argue that monetary tightening can be associated with higher output, investment, or stock prices if the central bank reveals positive information through its policy decision (Romer and Romer, 2000; Melosi, 2017; Nakamura and Steinsson, 2018; Miranda-Agrippino and Ricco, 2021). Through the lens of our framework, the positive response to monetary tightening is evidence of misspecification. The true structural effect of monetary tightening re-emerges with the proper controls for investors' belief distortion.

To summarize, the obstacles to implementing Procedure 1 are twofold. First, as mentioned, we need to control for $X_{\tau-1}$ in our estimation. But since X includes the unobservable forward-guidance variable f, we will need a method to recover f. What is this method? Second, we require the shocks z_τ. We will continue to assume that the short-rate shock z^r_τ is observed from financial markets. But how can we recover z^f_τ? Given the forward-looking nature of financial markets, let us broach the possibility that asset prices can inform us about both of f and z^f.[23]

[23]We do not discuss the growth shock z^g further because its inclusion is already emphasized by the existing literature on the "information effect" of monetary policy. In particular, one should include z^g in Procedure 1, because it may be correlated to monetary surprises z^r and z^f (and so omission of z^g results in an omitted variable bias). For example, if the central bank responds to high-frequency growth information, one needs to control for this growth information to get an appropriate monetary policy instrument (Miranda-Agrippino and Ricco, 2021; Bauer and Swanson, 2023a,b; Andrade and Ferroni, 2021).

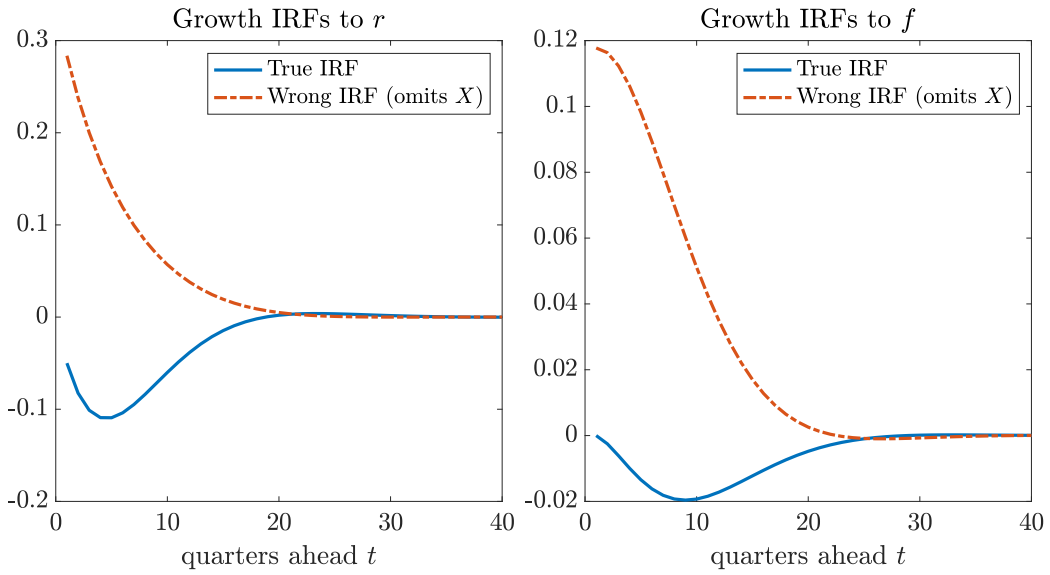


Figure B.1. Growth IRFs to forward guidance. Regressions of $g_{\tau+t}$ on the forward-guidance shock z^f_τ with and without the lagged state $X_{\tau-1}$ as a control. The solid curve controls for $X_{\tau-1}$ and coincides with the "True IRF." The dashed curve omits $X_{\tau-1}$ which yields the "Wrong IRF" and corresponds to assuming rational expectations. The calibrations of A^o and A are given in Appendix B.4.1, equation (B.42). The calibration of B is a diagonal matrix with $\sigma_g = 0.0025/4$, $\sigma_r = 0.010/4$, and $\sigma_f = 0.050/4$.

B.3.3 Identifying forward guidance from asset prices

We need to recover a time series for f, as well as its perceived shock z^f. Since f is primarily about future short-term interest rates, observation of the market's expectation of future short rates should suffice in place of f. A natural place to look for these market beliefs are Treasury bond markets.

The problem with using asset markets is that they do not directly reveal the market's expectation, but rather a risk-adjusted expectation. To address this discrepancy, we will first write down a standard affine term structure model (Duffie and Kan, 1996; Dai and Singleton, 2002; Duffee, 2002; Ang and Piazzesi, 2003) and then impose sufficient structure on the model. In doing so, we will be explicit about which conditions allow us to invert risk adjustments and recover the market expectation.

Pricing model. We add a lognormal asset pricing model, as in Section B.1. We have the objective stochastic discount factor (SDF):

$$\frac{S^o_{t+1}}{S^o_t} = \exp\Big[-r_t - \frac{1}{2}\pi_t'\pi_t - \pi_t'\Delta W^o_{t+1}\Big], \tag{B.34}$$

We can also re-write the SDF in terms of the investor measure:

$$\frac{S_{t+1}}{S_t} = \exp\Big[-(\bar{r}+r_t) - \frac{1}{2}(\pi_t+L_t)'(\pi_t+L_t) - (\pi_t+L_t)'\Delta W_{t+1}\Big]. \tag{B.35}$$

The variable $\exp[L_t'\Delta W^o_{t+1} - \frac{1}{2}L_t'L_t] = \frac{S^o_{t+1}/S_{t+1}}{S^o_t/S_t}$ changes the probability measure from the objective one to investors' subjective one, while S represents investor marginal utility. Notice that investors' perceived risk prices are $\pi_t + L_t$.

In this setting, the risk-neutral dynamics of the state are given by

$$\begin{aligned} X_{t+1} &= A^*_0 + A^* X_t + B\Delta W^*_{t+1}, \\ \text{where} \quad A^*_0 &:= -B\pi_0 \\ \text{and} \quad A^* &:= A^o - B\Pi, \end{aligned} \tag{B.36}$$

where ΔW^*_{t+1} is a Normal shock under the risk-neutral distribution. At this point, given r_t and π_t are affine in the state vector, we can solve for bond yields of all maturities exactly as in Section B.1. The solution is $y^{(n)}_t = \frac{1}{n}[\mathcal{B}^{(n)}_0 + \mathcal{B}^{(n)}X_t]$.

The challenge: extracting forward guidance. Can we use the model solution to identify f? Since bond yields are affine in the factors, we should be able to invert for the factor time series, given data on any three maturities. The setting here is even simpler because the growth rate and one-period yield (short rate) are observable. So if we have a single n-period bond ($n > 1$), we can use its yield to obtain the state vector as

$$X_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mathcal{B}^{(n)} & & \end{bmatrix}^{-1} \begin{bmatrix} g_t \\ r_t \\ ny^{(n)}_t - \mathcal{B}^{(n)}_0 \end{bmatrix}.$$

This expression suggests that f can be obtained via data on bond yields and an estimation of the asset-pricing model.

Unfortunately, this identification logic is incorrect in general, as Appendix B.4.2 explains in more detail. Bond yields themselves do not reveal *particular latent factors*. This is a well-known identification issue in affine term-structure models with latent state variables like our forward guidance variable (Hamilton and Wu, 2012). Without specific knowledge of A, B, or Π, we cannot decide whether the underlying state vector is X_t or some rotation $\hat{X}_t$ of the state, as X and $\hat{X}$ lead to the same pricing implications. This issue goes beyond our particular example. Indeed, the example above represents the "best-case scenario" where we know the true model is a three-factor model with two of

the factors observable. Even then, the sole latent factor cannot be identified. The problem intensifies if the environment is more complex, e.g., if there are additional latent factors driving bond yields or bond risk premia. Therefore, bringing in bond yields generically does not help us identify the causal effects of forward guidance. (As Appendix B.4.2 discusses, only in a knife-edge case—where forward-guidance is the unique latent variable and where its correlations with all observable states are known—can f be identified from bond yields.)

Surprises to future short rates: a resolution? Given the impossibility of identifying f, hence z^f, let us now ask a humbler question: can our asset-pricing model help us identify belief updates about future short rates? That is, can we obtain

$$z_\tau^{r,t} := \mathbb{E}[r_{\tau+t} \mid X_\tau] - \mathbb{E}[r_{\tau+t} \mid X_{\tau-1}] = (0,1,0)A^t B \Delta W_\tau \tag{B.37}$$

from asset-price data?

Obtaining surprises about future rates is actually good enough for many purposes, for the following reason. Conditional on $X_{\tau-1}$, the correlation between $z_\tau^{r,0}$ and $z_\tau^{r,t}$ is imperfect:

$$\operatorname{corr}\left[z_\tau^{r,t}, z_\tau^{r,0} \mid X_{\tau-1}\right] = \frac{[A^t BB']_{2,2}}{\sqrt{[A^t BB' A^t]_{2,2}[BB']_{2,2}}}$$

Unless A is a diagonal matrix, which is the uninteresting case where monetary policy has no effects, this correlation will be below one for $t > 0$. Therefore, replacing z_τ^f in Procedure 1 with $z_\tau^{r,t}$ (for some $t > 0$) allows us to recover all desired objects: the coefficients on z_τ^r and $z_\tau^{r,t}$ will be the short-rate and forward-guidance effects, respectively. Intuitively, there is a two-factor structure to the short-rate path, and so z_τ^r and $z_\tau^{r,t}$ pick up different factors. In a richer model with additional latent factors driving short rates, we may want to include an entire collection $(z_\tau^{r,t})_{t=1}^T$ of surprises. The fact that these reduced-form surprises to future rates are "good enough" focuses our attention in the remainder of the paper on trying to extract the $z_\tau^{r,t}$ surprises, rather than forward guidance per se. For emphasis, we collect this discussion in the following procedure:

Procedure 2. *The estimate of* $(b_h^g, b_h^r, b_h^{r,t})$ *from the regression*

$$g_{\tau+h} = a_h + b_h^g z_\tau^g + b_h^r z_\tau^r + b_h^{r,t} z_\tau^{r,t} + c_h' X_{\tau-1} + \epsilon_{\tau+h} \tag{B.38}$$

is a consistent estimator of D_h.

To implement Procedure 2, we need a proxy for $z_\tau^{r,t}$. If investors are *risk-neutral*, we can recover these surprises. Under risk-neutrality, investor marginal utility features zero

risk-pricing ($\pi_t + L_t = 0$), so the recovered risk-neutral dynamics actually correspond to investors' perceived dynamics. In that case, we can recover the surprises in (B.37) via $z_\tau^{r,t} = \mathbb{E}^*[r_{\tau+t} \mid X_\tau] - \mathbb{E}^*[r_{\tau+t} \mid X_{\tau-1}]$, given estimates of A^* and B. Intuitively, risk-neutrality allows recovery because it implies an "Expectations Hypothesis" but under the investor subjective belief.

A relaxed version of risk-neutrality also permits shock recovery. Assume

$$\Pi = -B^{-1}(A - A^o). \tag{B.39}$$

Condition (B.39) says that investor perceived risk prices $\pi_t + L_t$ are time-invariant (the coefficient on X_t is zero). While time-invariance may seem quite restrictive, it turns out that it is both necessary and sufficient for identification of $B\Delta W$ in this model.

To understand sufficiency is fairly easy. Substitute (B.39) into the risk-neutral dynamics to obtain

$$A^* = A^o - B\Pi = A^o + BB^{-1}(A - A^o) = A.$$

If we know A^*, then we know A, which allows us to obtain perceived shocks as $X_{t+1} - AX_t = B\Delta W_{t+1}$. Intuitively, if investor perceived risk prices are constant, then a version of the Expectations Hypothesis holds: long-term bond yields capture investor beliefs about future short-term yields, with a constant shifter. The constant shifter is differenced out when studying belief surprises, rather than belief levels.

Necessity is harder to see. The important fact is that condition (B.39) is required to make investor-perceived long-run risk prices constant. To see this, compute the permanent component of the investor SDF S in (B.35) as

$$\frac{H_{t+1}}{H_t} = \exp\Big[-\frac{1}{2}\|\pi_t + L_t - B'v\|^2 - (\pi_t + L_t - B'v)\cdot\Delta W_{t+1}\Big],$$

where $v := -[I - (A - B\Pi)']^{-1}(0,1,0)'$. In our general environment of Section 3.3, we explain this permanent component in more detail, and we show that perceived shock identification requires H_{t+1}/H_t to be independent of X_t (under investor beliefs). Taking that result as given, identification thus requires $\pi_t + L_t = \pi_0 + [\Pi + B^{-1}(A - A^o)]X_t$ to be independent of X_t, which translates to condition (B.39).

As it turns out, this example sneakily permits looser identification conditions than the more general case covered in Section 3.3. In particular, we have assumed a very strong structure where *monetary shocks are iid*: they happen every period with the same time-invariant distribution. As our more general model in Section 3.3 shows, these type of iid monetary shocks are a knife-edge case without a reasonable intuition. In reality,

we expect monetary policy actions to depend on the state of the economy. Outside of this knife-edge case of random monetary interventions, shock identification will require the even stronger condition that H_{t+1}/H_t be *invariant to policy shocks*.

Remark 3 (Risk-neutral shocks)**.** *One limitation of our simple three-factor VAR is that, taking the setup literally, one could get away with using* risk-neutral surprises *rather than investor's perceived surprises. In particular, suppose we obtain* $z_\tau^{r,t,*} := \mathbb{E}^*[r_{\tau+t} \mid X_\tau] - \mathbb{E}^*[r_{\tau+t} \mid X_{\tau-1}]$ *from bond markets and run a modified version of Procedure 2 with* $z_\tau^{r,t,*}$ *in place of* $z_\tau^{r,t}$*. If the true model has* (g, r, f) *as the factors, then the modified procedure works—i.e., we recover both the short-rate and forward-guidance effects—with two possible caveats.*

First, while seemingly mundane, it is absolutely critical to control for $X_{\tau-1}$ *when running the variant of Procedure 2 with risk-neutral surprises. Controlling for* $X_{\tau-1}$ *soaks up the wedge between risk-neutral, subjective, and objective beliefs, so one can recover the correct structural effects. The literature often overlooks controls for drivers of belief distortions and risk adjustments; for example, Gürkaynak et al. (2005b) and Swanson (2021), by decomposing the effects of monetary policy on various asset markets, effectively employ risk-neutral surprises, but they do not control for* $X_{\tau-1}$*. Second, the term structure of investor forecast revisions* $(z_\tau^{r,t})_{t>0}$ *are closer to structural objects of interest than their risk-neutral counterparts* $(z_\tau^{r,t,*})_{t>0}$*. If so, measuring* $(z_\tau^{r,t})_{t>0}$ *is preferable. Within this second caveat, we see two specific issues: risk premia effects and multi-dimensional policy.*

Suppose forward guidance has risk premium effects in addition to its traditional impact on future short rates (Bauer et al., 2023). For instance, signals of higher future rates may lower growth but also reduce risks (like inflation risks in a richer model), with an ambiguous combined impact on long-term bond yields. The risk-neutral surprises $z_\tau^{r,t,*}$ *always contain this joint effect; a regression of economic outcomes onto* $z_\tau^{r,t,*}$ *cannot isolate the pure impact of foreward guidance. Depending on the size of risk premia effects,* $z_\tau^{r,t,*}$ *may even take an opposite sign to* $z_\tau^{r,t}$*.*[24]

In a world of multi-dimensional policy, risk-neutral surprises struggle to correctly attribute the relative impacts of various interventions. To see this, consider that in a general K-factor VAR, the risk-neutral and investor forecast revisions differ by

$$z_\tau^{r,t,*} - z_\tau^{r,t} = (0, 1, 0, \dots, 0)\Big\{\big[(A^*)^t - A^t\big]B\Delta W_\tau + (A^*)^t B\big[\pi_0 + \big[\Pi + B^{-1}(A - A^o)\big]X_{\tau-1}\big]\Big\}.$$

[24] To take an extreme but enlightening example where risk premia complicate inference, suppose A^* is a diagonal matrix. Bond yields $y_t^{(n)}$ will not reflect forward guidance in this world: the loading $\mathcal{B}^{(n)}$ will have zeros everywhere except for the short-rate entry. In fact, one can verify that $\text{corr}[z_\tau^{r,t,*}, z_\tau^{r,0,*} \mid X_{\tau-1}] = 1$ when A^* is diagonal, so that $z_\tau^{r,t,*}$ provides no additional information relative to the short-rate shock. This offsetting issue is related to the discussion surrounding "unspanned factors" in the term structure literature (e.g., yields may not contain all information about the SDF as in Collin-Dufresne and Goldstein, 2002; Cochrane and Piazzesi, 2005; Andersen and Benzoni, 2010; Duffee, 2011; Joslin et al., 2014).

The second term involving $X_{\tau-1}$ can be managed, assuming one controls for the lagged state as in Procedure 2. But the first term involves the perceived shocks ΔW_τ and cannot be controlled. To the extent that A^* differs from A^o, the risk-neutral and investor surprises can load very differently on the underlying shocks. In this world, the risk-neutral surprises cannot help answer questions like: at which horizon is forward guidance most effective? how does uncertainty management compare to traditional forward guidance? how does quantitative easing compare to forward guidance?

B.4 Additional details

B.4.1 Calibration of numerical example from Figure B.1

To get a sense for the consequences of misspecification, we provide a numerical example that is roughly calibrated to the evidence in Cieslak (2018). After calibrating, we illustrate how misspecification wrongly assuming rational expectations impacts the estimated IRF of growth to a forward-guidance shock. The result is Figure B.1 in the main text.

To calibrate, we mimic Cieslak (2018) and run the following two regressions in the model:

$$r_{t+j} = \beta_0^{(j)} + \beta_g^{(j)} g_t + \beta_r^{(j)} r_t + \text{residual} \tag{B.40}$$

$$\mathbb{E}_t[r_{t+j}] = \beta_0^{(j)} + \beta_g^{(j)} g_t + \beta_r^{(j)} r_t + \text{residual} \tag{B.41}$$

where $\mathbb{E}_t[r_{t+j}]$ denotes subjective expectations of interest rates, obtained from the Blue Chip Financial Forecasts. The results of these regressions, in the data, are presented in the bottom row of Table B.1.

	A. Dependent variable r_{t+j}				B. Dependent variable $\mathbb{E}_t[r_{t+j}]$			
	$j=1$ quarter		$j=4$ quarters		$j=1$ quarter		$j=4$ quarters	
	$\beta_g^{(1)}$	$\beta_r^{(1)}$	$\beta_g^{(4)}$	$\beta_r^{(4)}$	$\beta_g^{(1)}$	$\beta_r^{(1)}$	$\beta_g^{(4)}$	$\beta_r^{(4)}$
Model:	0.195	0.902	0.547	0.493	0.145	0.952	0.215	0.746
Data:	0.180	0.880	0.650	0.500	0.100	0.930	0.120	0.840

Table B.1. Short-rate forecasts in the model and data. Regressions of future short rates (Panel A) and survey-based expectations of future short rates (Panel B) on current growth and short rates (g_t, r_t). The calibration of B is a diagonal matrix with $\sigma_g = 0.0025/4$, $\sigma_r = 0.010/4$, and $\sigma_f = 0.050/4$. For the "Data" row, Cieslak (2018) proxies r_t by the Federal Funds Rate (with survey expectations obtained from the Blue Chip Financial Forecasts) and g_t by employment growth.

We set A^o and A in the model to roughly match these empirical results. We calibrate

$$A^o = \begin{bmatrix} 0.90 & -0.05 & 0 \\ 0.50 & 0.75 & 0.05 \\ 0 & 0 & 0.80 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0.90 & -0.05 & 0 \\ 0.45 & 0.80 & 0.05 \\ 0 & 0 & 0.99 \end{bmatrix}. \tag{B.42}$$

The comparison between our model and the data is presented in Table B.1. The fit, while not perfect, is good enough to illustrate our main points here. Relative to the econometric transition matrix, agents perceive a smaller feedback of growth to future short rates, as well as higher persistences of r and especially of f. The true and perceived dynamics of short rates in the model, plotted in Figure B.2, illustrate the discrepancy between A^o and A, especially the greater perceived persistence of the forward-guidance shock.

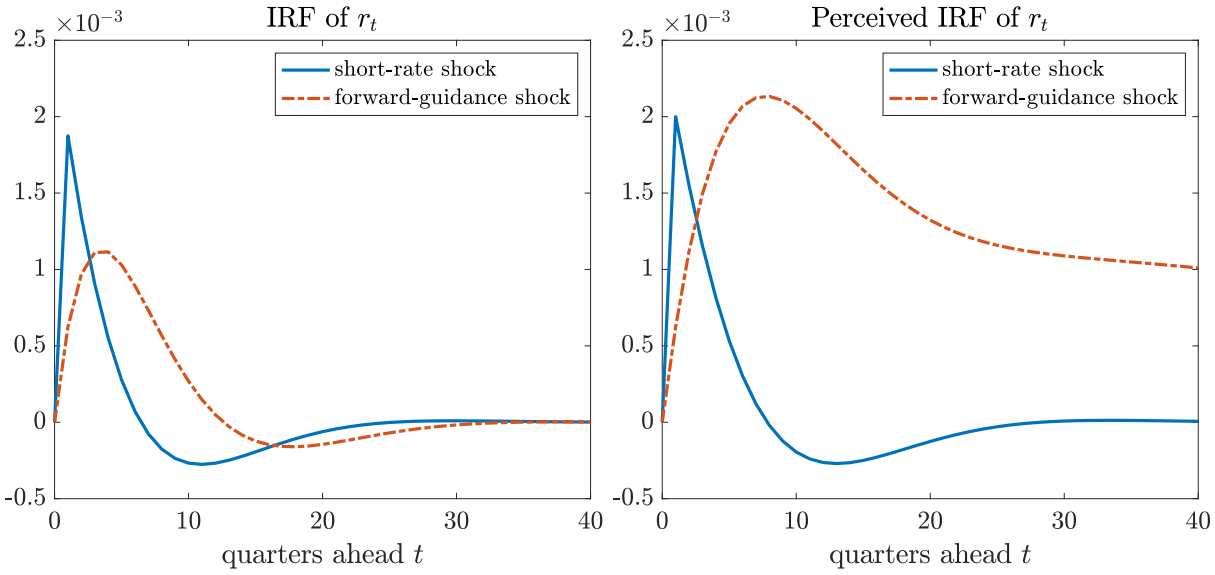


Figure B.2. IRFs of short rates to the two monetary shocks. The left panel plots the true IRFs. The right panel plots the perceived IRFs. The calibrations of A^o and A are given in (B.42). The calibration of B is a diagonal matrix with $\sigma_g = 0.0025/4$, $\sigma_r = 0.010/4$, and $\sigma_f = 0.050/4$.

B.4.2 Identifying latent factors from term structure models

We consider the three-factor VAR (B.25), paired with the SDF (B.34). In this model, the forward guidance variable f is latent. One would hope that asset prices, paired with the pricing model, allow us to "invert bond yields" for the latent factor, but this is not the case as we explain here.

For example, consider using the transformed state variable $\hat{X}_t = UX_t$, where

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ u_g & u_r & 1 \end{bmatrix}.$$

While the subsequent argument works for any U, this particular example acknowledges that (g, r) are observable and lets the latent factor be a linear combination of forward guidance with the observables. At the same time, suppose the risk price vector is $\hat{\pi}_t = \hat{\pi}_0 + \hat{\Pi}\hat{X}_t$, where $\hat{\pi}_0 = \pi_0$ and $\hat{\Pi} = \Pi U^{-1}$. Then, the SDF is identical to the original specification (i.e., $\hat{\pi}_t = \pi_t$) and the risk-neutral dynamics of $X_t = U^{-1}\hat{X}_t$ are identical to the original specification.[25]

To address the generic non-identification of latent factors, we must effectively make some assumption about U. A canonical approach starting with Hamilton and Wu (2012) assumes the state vector $\hat{X}$ is chosen such that its risk-neutral persistence is a diagonal matrix. This can be done by implicitly picking U^{-1} as the matrix of eigenvectors of $A^* = A^o - B\Pi$, since UA^*U^{-1} is the risk-neutral persistence of $\hat{X}$. But after making this choice, only $\hat{X}_t = UX_t$ is recovered from bond yields, not X_t itself. Without additional assumptions, bond yields do not reveal *particular latent factors*.

What can be done in light of the challenges to observing f? On the one hand, we are still able to extract $\hat{X}$ from bond yields, a state vector which spans the same space as X. This is good enough for the purpose of controlling for the past state: controlling for $\hat{X}_{\tau-1} = UX_{\tau-1}$ in Procedure 1 instead of $X_{\tau-1}$ will lead to identical inference for b_h. Thus, we may still recover the desired IRF in principle. On the other hand, failure to specifically recover f prevents us from constructing its shock z^f, which prevents us from actually implementing Procedure 1 as stated. We must necessarily exclude the forward-guidance shock z^f_τ and so cannot measure any forward-guidance effect, at least not in

[25] Indeed, the physical dynamics of $\hat{X} := UX$ are $\hat{X}_{t+1} = (UAU^{-1})\hat{X}_t + (UB)\Delta W^o_{t+1}$, so the risk-neutral dynamics for $\hat{X}$ are

$$\hat{X}_{t+1} = -UB\hat{\pi}_0 + (UAU^{-1} - UB\hat{\Pi})\hat{X}_t + (UB)\Delta\hat{W}^*_{t+1}$$

where $\Delta\hat{W}^*_{t+1} := \Delta W^o_{t+1} + \hat{\pi}_t$ is the risk-neutral shock. If $\hat{\pi}_0 = \pi_0$ and $\hat{\Pi} = \Pi U^{-1}$, then risk prices are

$$\hat{\pi}_t = \hat{\pi}_0 + \hat{\Pi}\hat{X}_t = \pi_0 + \Pi U^{-1}UX_t = \pi_t.$$

As a result, the risk-neutral shocks coincide: $\Delta\hat{W}^*_{t+1} = \Delta W^*_{t+1}$. Thus, the risk-neutral dynamics of $\hat{X}$ are

$$\hat{X}_{t+1} = UA^*_0 + UA^*U^{-1}\hat{X}_t + (UB)\Delta W^*_{t+1}$$

These dynamics imply the same risk-neutral dynamics for X_t displayed in (B.36). One can also easily show that, as a consequence, the solution for equilibrium bond yields is invariant to the choice of u_g and u_r.

this way.[26]

Although it is not our focus, excluding z_τ^f may also bias estimates of the causal impact of short-rate shocks. Indeed, note that $\text{Cov}[z_\tau^r, z_\tau^f \mid X_{\tau-1}] = (0,1,0)BB'(0,0,1)'$. So if we run Procedure 1 without z_τ^f, any correlation between short-rate and forward-guidance shocks will be impounded into the coefficient on z_τ^r.

What solves some issues in the simple baseline model is an assumption that forward-guidance shocks are orthogonal to short-rate and growth shocks. In other words, if we assume that the third row of B is proportional to $(0,0,1)$, then we can recover f from bond yields. This is because the unique matrix U that preserves this orthogonality is $U = I$, and so $\hat{X} = X$ is uniquely pinned down by yields. However, even armed with this orthogonality assumption, we re-encounter difficulties as soon as there are additional latent factors present. Examples of such problematic environments would be if forward guidance was described by a multi-factor structure, or if monetary policy managed either uncertainty or risk premia in addition to future short rates. In such case, f is once again unidentified without additional assumptions.

[26] A similar critique applies to the VAR approach in place of the Jorda projection. If we fully observed X, then we would not need to run Jorda projections; we could estimate the VAR directly via time series regressions, obtain A^o, and then construct the desired IRF $(A^o)^h$. But if we obtain only $\hat{X} = UX$, we would not recover A and would therefore obtain an incorrect IRF.

C LRN in extensions of the basic New Keynesian model

C.1 Habit-based models

The nature of LRN from the standard New Keynesian model is robust to many types of modifications that induce more interesting time-variation in risk premia. For instance, many modern versions of the New Keynesian model posit utility functions with external habits as a way of generating time-varying risk aversion (see Pflueger and Rinaldi, 2022). In many external habit settings, the CRRA utility flow $\frac{C_t^{1-\gamma}}{1-\gamma}$ is replaced by $\frac{(C_t - Z_t)^{1-\gamma}}{1-\gamma}$, where Z_t is some weighted-average of past aggregate consumption (Campbell and Cochrane, 1999; Santos and Veronesi, 2010). The SDF is then given by

$$\frac{S_t}{S_0} = \exp(-\delta t)\left(\frac{C_t}{C_0}\right)^{-\gamma}\left(\frac{1 - Z_t/C_t}{1 - Z_0/C_0}\right)^{-\gamma} \tag{C.1}$$

The key insight here is that Z/C is a stationary process. And so the permanent component of S is once again solely due to the permanent component in C.[27] Thus, LRN is once again the conventional assertion that monetary policy does not affect consumption in the long run. Even if monetary policy affects the real economy in the short-run, which then induces changes in future risk pricing through the impact on the habit process, these effects are transitory and do not constitute a violation of LRN.

C.2 Heterogeneous-agent models

A burgeoning literature studies heterogeneous-agent versions of the New Keynesian model. Such models, while generating many interesting dynamics, also do not inherently feature violations of LRN. Obviously, there are many ways to input heterogeneity into macro-finance models. We proceed with two examples that characterize a large class of settings.

[27] To verify this claim, let H_t be the permanent component of $C_t^{-\gamma}$, so that

$$\left(\frac{C_t}{C_0}\right)^{-\gamma} = \exp(\eta t)\frac{f(X_0^c)}{f(X_t^c)}\frac{H_t}{H_0}$$

for some η and function $f(\cdot)$. The state vector X^c above are any variables driving aggregate consumption dynamics. Now, add the stationary variable $X^z := Z/C$ to the state vector, $X := (X^c, X^z)'$. Then, H is also the permanent component of S, since

$$\frac{S_t}{S_0} = \exp(\eta t)\frac{f(X_0^c)}{f(X_t^c)}\frac{g(X_0^z)}{g(X_t^z)}\frac{H_t}{H_0}, \quad \text{where} \quad g(x) := (1-x)^\gamma.$$

Let us first consider an example with "participants" who can trade risky assets without constraints, alongside "non-participants" who cannot or do so with constraints (Basak and Cuoco, 1998; He and Krishnamurthy, 2013; Kekre et al., 2024). A recent literature starting from Gertler and Karadi (2011) embeds this friction into monetary models. The point of these models is that the participants do all the risk pricing, and their balance sheets thus matter for the transmission of monetary policy.

Let X_t denote the consumption share of the participants, and suppose they have CRRA utility with risk aversion γ. Then, $X_t C_t$ is the participants' consumption, and so the SDF is given by

$$\frac{S_t}{S_0} = \exp(-\delta t)\left(\frac{X_t}{X_0}\right)^{-\gamma}\left(\frac{C_t}{C_0}\right)^{-\gamma} \tag{C.2}$$

In all quantitative implementations of such models, assumptions are made to ensure a stationary wealth distribution (e.g., taxation, or a birth-and-death process as in OLG setups). Hence, X is a stationary variable. As a result, the permanent component of S once again stems solely from the permanent component of aggregate consumption, meaning that LRN holds if and only if long-run aggregate consumption is invariant to monetary policy.

As a second example, consider a setting in which agents face no constraints in trading aggregate securities but they differ in their risk preferences (Gârleanu and Panageas, 2015). Kekre and Lenel (2022) recently embeds this type of heterogeneity into a New Keynesian framework. These models also rely on redistribution to generate their asset-pricing implications: if monetary policy can redistribute wealth toward more risk-tolerant agents, they will reduce "aggregate risk aversion" which buoys asset prices.

The notion of LRN remains the same in the heterogeneous-preference model. So long as one agent type in the model has CRRA utility, the SDF continues to take the form in (C.2), where X is the consumption share of that agent type. For instance, Gârleanu and Panageas (2015) assumes that agents possess general recursive utilities, but their quantitative calibration features one agent type whose utility is approximately CRRA (namely, their risk aversion is approximately the inverse of their elasticity of intertemporal substitution).

An interesting novelty in these heterogeneous-agent models is the potential presence of an FOMC announcement risk premium. If monetary policy redistributes wealth among agent types, this constitutes a risk factor and will be priced. LRN does not, therefore, imply that FOMC announcement premia are zero. In this sense, one should think of LRN as a weaker condition than zero FOMC announcement premia.

D Long-run risks model

D.1 Calculations for the trend-stationary model in Section 1.3

We guess the value function takes the form $U_t = u_0 + \log C_t + u' X_t$, where $X_t = (g_t, v_t)'$. To maintain compact notation, write the dynamics of X_t as[28]

$$X_{t+1} = A_0 + AX_t + \sqrt{X_t^{(2)}}\Sigma \Delta W_{t+1},$$

$$\text{where} \quad A_0 = \begin{bmatrix} \mu_g \\ \mu_v \end{bmatrix}$$

$$A = \begin{bmatrix} \exp(-\lambda_g) & 0 \\ 0 & \exp(-\lambda_v) \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_g & 0 \\ 0 & \sigma_v \end{bmatrix}.$$

Plugging the utility guess into the Bellman equation, and writing $b := u + \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$, we obtain

$$u_0 = \frac{\exp(-\delta)}{1 - \exp(-\delta)} \left[\mu_C + b' A_0 \right]$$

$$\left(b - \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)\right)' = \exp(-\delta) \left[\left(b - \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)\right)' - b'(I - A) + \frac{1}{2}(1-\gamma) \left(\begin{smallmatrix} 0 \\ b'\Sigma\Sigma' b \end{smallmatrix}\right) \right]$$

Solving this latter equation delivers b, where we take the larger root in the quadratic equation for b_v (this ensures a non-explosive value function as discussed in the generalized model below in Appendix D.2). From here, we can use the solution for U_t to compute

$$\frac{H_{t+1}}{H_t} = \frac{\exp[(1-\gamma)U_{t+1}]}{\mathbb{E}_t[\exp[(1-\gamma)U_{t+1}]]},$$

which gives the expression in the text.

[28] Note that in this discrete-time model, it is technically possible that v becomes negative in simulations. The continuous-time versions of these processes are

$$dg_t = -\lambda_g(g_t - \mu_g)dt + \sigma_g \sqrt{v_t} dW_t^{(1)}$$
$$dv_t = -\lambda_v(v_t - \mu_v)dt + \sigma_v \sqrt{v_t} dW_t^{(2)},$$

where $W := (W^{(1)}, W^{(2)})$ is a Brownian motion. This is purely a technical detail, since the continuous-time limit of this model features the same solution form (e.g., for the value function and SDF).

D.2 General version with non-stationary consumption

We solve a standard long-run risks model as a laboratory to evaluate various procedures for extracting monetary policy's impacts on future interest rates. This model, with non-stationary consumption, is a generalized version of the example in Section 1.3.

Model setup. Aggregate consumption follows

$$\log C_{t+1} = \log C_t + g_t + \sqrt{v_t}\Delta W^{(1)}_{t+1},$$

where the expected growth rate g_t and growth variance v_t follow

$$\begin{aligned} g_{t+1} &= (1-a_g)\mu_g + a_g g_t + \sigma_g\sqrt{v_t}\Delta W^{(2)}_{t+1} \\ v_{t+1} &= (1-a_v)\mu_v + a_v v_t + \sigma_v\sqrt{v_t}\Delta W^{(3)}_{t+1}. \end{aligned}$$

The three shocks $\Delta W := (\Delta W^{(1)}, \Delta W^{(2)}, \Delta W^{(3)})' \sim N(0, I)$. In this discrete-time model, it is technically possible that v becomes negative in simulations. However, because the model solution is identical between discrete and continuous time, we will address this problem by simulating using a continuous-time version of the above dynamics.[29] We may stack the stationary state variables as $X := (g, v)$ and write their dynamics in vector form as

$$X_{t+1} = (I - A)\mu + AX_t + \sqrt{X^{(2)}_t}\Sigma\Delta W_{t+1}$$

$$\text{where} \quad \mu := \begin{bmatrix} \mu_g \\ \mu_v \end{bmatrix}$$

$$A := \begin{bmatrix} a_g & 0 \\ 0 & a_v \end{bmatrix}$$

$$\Sigma := \begin{bmatrix} 0 & \sigma_g & 0 \\ 0 & 0 & \sigma_v \end{bmatrix}$$

[29] The continuous-time versions of these processes are

$$\begin{aligned} d\log C_t &= g_t dt + \sqrt{v_t} dW^{(1)}_t \\ dg_t &= -\lambda_g(g_t - \mu_g)dt + \sigma_g\sqrt{v_t}dW^{(2)}_t \\ dv_t &= -\lambda_v(v_t - \mu_v)dt + \sigma_v\sqrt{v_t}dW^{(3)}_t, \end{aligned}$$

where $W := (W^{(1)}, W^{(2)}, W^{(3)})$ is a Brownian motion. We will have to calibrate λ_g and λ_v here, in order to match the same rate of mean-reversion from the discrete-time model. This is done by noting that $\exp(-\lambda_g) = a_g$ and $\exp(-\lambda_v) = a_v$ are the annual persistences.

The fact that all conditional volatilities are proportional to $\sqrt{v_t}$ will make the model analytically tractable.

Solution. The value function recursion (Bellman equation) for the representative agent with unitary EIS is given by

$$U_t = (1-\beta)\log C_t + \beta \frac{\log \mathbb{E}_t[\exp((1-\gamma)U_{t+1})]}{1-\gamma}$$

Guess that the value function is given by $U_t = \log C_t + u_0 + u \cdot X_t$ for some scalar u_0 and vector $u := (u_g, u_v)$. Substituting this guess into the Bellman equation gives the following three equations

$$\begin{aligned} u_0 &= \frac{\beta u'(I-A)\mu}{1-\beta} \\ u_g &= \frac{\beta}{1-\beta a_g} \\ 0 &= (\gamma-1)\sigma_v^2 u_v^2 + 2[\beta^{-1} - a_v]u_v + (\gamma-1)[1+\sigma_g^2 u_g^2] \end{aligned}$$

As explained in Hansen and Scheinkman (2009), the appropriate solution for u_v is the larger root of the above quadratic equation. This solution will be negative for $\gamma > 1$.

In this model, the SDF is

$$\frac{S_{t+1}}{S_t} = \beta \frac{C_t}{C_{t+1}} \frac{\exp[(1-\gamma)U_{t+1}]}{\mathbb{E}_t[\exp[(1-\gamma)U_{t+1}]]}.$$

Substituting the consumption dynamics and value function, we ultimately obtain

$$\frac{S_{t+1}}{S_t} = \beta \exp\Big\{ -(g_t + \frac{1}{2}v_t) + \gamma v_t - \pi_t \cdot \Delta W_{t+1} - \frac{1}{2}\|\pi_t\|^2 \Big\},$$

where the price of risk π_t is given by

$$\pi_t = \sqrt{v_t}\bar{\pi}, \quad \text{where} \quad \bar{\pi} := \begin{pmatrix} \gamma \\ (\gamma-1)\sigma_g u_g \\ (\gamma-1)\sigma_v u_v \end{pmatrix} := \begin{pmatrix} \bar{\pi}_C \\ \bar{\pi}_g \\ \bar{\pi}_v \end{pmatrix}.$$

Note that the log riskless rate $r_t := -\log(\mathbb{E}_t[S_{t+1}/S_t])$ is given by

$$r_t = -\log(\beta) + g_t + \frac{1}{2}v_t - \gamma v_t.$$

As we are interested in shocks that impact current and future interest rates, this model offers up $\Delta W^{(2)}$ and $\Delta W^{(3)}$ as candidate shocks.

Rotated shocks: current vs future rates. For rhetorical purposes, we rotate our growth and variance shocks to be about current versus future interest rates. We can write

$$r_{t+1} = r_t - (1 - a_g)(g_t - \mu_g) + (\gamma - \frac{1}{2})(1 - a_v)(v_t - \mu_v) + \sqrt{v_t}\sigma_r \Delta W^r_{t+1},$$

where

$$\Delta W^r_{t+1} := \frac{\sigma_g}{\sigma_r}\Delta W^{(2)}_{t+1} - (\gamma - \frac{1}{2})\frac{\sigma_v}{\sigma_r}\Delta W^{(3)}_{t+1}$$
$$\sigma_r := \sqrt{\sigma_g^2 + (\gamma - \frac{1}{2})^2\sigma_v^2}$$

Thus, ΔW^r captures *short-rate shocks*, whereas the shock

$$\Delta W^f_{t+1} := (\gamma - \frac{1}{2})\frac{\sigma_v}{\sigma_r}\Delta W^{(2)}_{t+1} + \frac{\sigma_g}{\sigma_r}\Delta W^{(3)}_{t+1}$$

is an orthogonal shock (i.e., $\Delta W^r \perp \Delta W^f$) that captures variation in future interest rates, holding fixed current rates. This latter shock is the one that allows us to conceptualize shocks to beliefs about future rates, as in the case of forward guidance—therefore, we refer to W^f as a *forward-guidance shock*.

Expectations Hypothesis. Recall that the "Expectations Hypothesis" measures beliefs under the risk-neutral measure $\mathbb{P}^*$, which is induced by the change-of-measure

$$\frac{H^*_{t+1}}{H^*_t} := \exp\Big[- \pi_t \cdot \Delta W_{t+1} - \frac{1}{2}\|\pi_t\|^2\Big].$$

In this conditionally log-normal environment, the dynamics of X under $\mathbb{P}^*$ are given by

$$X_{t+1} = (I - A^*)\mu^* + A^* X_t + \sqrt{X^{(2)}_t}\Sigma \Delta W^*_{t+1}$$

where $\Delta W^*_{t+1} \sim N(0, I)$ under $\mathbb{P}^*$, and the new persistent matrix A^* and long-run mean μ^* are given by

$$A^* := \begin{bmatrix} a_g & -\sigma_g \bar{\pi}_g \\ 0 & a_v - \sigma_v \bar{\pi}_v \end{bmatrix}$$
$$\mu^* := (I - A^*)^{-1}(I - A)\mu$$

The volatility matrix Σ is not impacted by the change-of-measure.

Long-Run Neutrality. Next, consider the permanent-transitory factorization of the SDF. Guess that the transitory piece is given by $e(x) = \exp(\kappa \cdot x)$ for some κ. We have

$$\begin{aligned}\frac{H_{t+1}}{H_t} &= \exp(-\eta)\frac{e(X_{t+1})}{e(X_t)}\frac{S_{t+1}}{S_t} \\ &= \exp\Big\{ -\eta - \log(\beta) - (g_t + \frac{1}{2}v_t) + \gamma v_t - \sqrt{v_t}(\bar{\pi} - \Sigma'\kappa)\cdot \Delta W_{t+1} - \frac{1}{2}v_t\|\bar{\pi}\|^2 + \kappa'(I-A)(\mu - X_t)\Big\}\end{aligned}$$

For this object to be a martingale, we require $\kappa := (\kappa_g, \kappa_v)'$ and η to satisfy

$$\begin{aligned}\eta &= -\log(\beta) + \kappa'(I-A)\mu \\ -1 &= (1-a_g)\kappa_g \\ 0 &= \frac{1}{2}\sigma_v^2\kappa_v^2 - [1 - a_v + \sigma_v\bar{\pi}_v]\kappa_v + \frac{1}{2}\sigma_g^2\kappa_g^2 - \sigma_g\bar{\pi}_g\kappa_g + \gamma - \frac{1}{2}\end{aligned}$$

where we have defined $\bar{\pi}_g := (\gamma - 1)\sigma_g u_g$ and $\bar{\pi}_v := (\gamma-1)\sigma_v u_v$. For the latter quadratic equation in κ_v, we pick the root that creates stochastic stability of the state vector X_t under the probability measure $\hat{\mathbb{P}}$ induced by the resulting H. The resulting H is given by

$$\frac{H_{t+1}}{H_t} = \exp\Big\{ -\frac{1}{2}v_t\|\bar{\pi} - \Sigma'\kappa\|^2 - \sqrt{v_t}(\bar{\pi} - \Sigma'\kappa)\cdot\Delta W_{t+1}\Big\}.$$

To determine which of the two roots to select for κ_v, note that, in this conditionally log-normal environment, the dynamics of X under $\hat{\mathbb{P}}$ are given by

$$X_{t+1} = (I - \hat{A})\hat{\mu} + \hat{A}X_t + \sqrt{X_t^{(2)}}\Sigma\Delta\hat{W}_{t+1}$$

where $\Delta\hat{W}_{t+1} \sim N(0, I)$ under $\hat{\mathbb{P}}$, and the new persistent matrix $\hat{A}$ and long-run mean $\hat{\mu}$ are given by

$$\begin{aligned}\hat{A} &:= \begin{bmatrix} a_g & \sigma_g^2\kappa_g - \sigma_g\bar{\pi}_g \\ 0 & a_v + \sigma_v^2\kappa_v - \sigma_v\bar{\pi}_v \end{bmatrix} \\ \hat{\mu} &:= (I - \hat{A})^{-1}(I - A)\mu\end{aligned}$$

To check whether X is stable under $\hat{\mathbb{P}}$ amounts to checking the stability of the matrix $\hat{A}$ under each of the two choices for κ_v.

Results: Biases in Belief Recovery. Now we are ready to define the two different ap-

proaches to measuring belief surprises. First, we may assume risk neutrality and extract beliefs via the Expectations Hypothesis, which assumes that long-term bond yields are averages of future short rates. Second, we may assume Long-Run Neutrality and extract beliefs via the "recovery" procedure described in Lemma A.3. For the purpose of calculating these belief surprises in the model, note that these two approaches equivalently extract beliefs under $\mathbb{P}^*$ and $\hat{\mathbb{P}}$, respectively, the two measures which are explained above. We will compare these two approaches to the correct belief surprises from the calibrated model.

The main result from this analysis is presented in Figure D.1. Suppose you asked the investor her beliefs about the expected path of interest rates after a monetary shock. Her answer, which is the object we seek to identify and is plotted in solid blue, depends on the type of monetary shock (short-rate versus forward-guidance) and the forecasting horizon. By contrast, we also plot what a researcher would extract by assuming the Expectation Hypothesis (dotted red) or Long-Run Neutrality (dashed yellow).

Interestingly, if the shock is a short-rate shock (top left panel), there is very little bias in extracting the shock. By contrast, there are very large biases in extracting forward-guidance shocks (top right panel). In particular, given a forward-guidance shock, investors typically update much more strongly about the future path of interest rates than the financial markets do. In the figure, which displays a tightening forward-guidance shock, investors fear a much stronger future tightening than markets.

Why is the bias so much stronger for forward-guidance shocks than short-rate shocks? The key is that, relative to current interest rates, future interest rates are disproportionately driven by uncertainty shocks. This matters because financial market-implied beliefs convey no biases about interest rates if the effect of monetary policy is on growth (bottom left panel)—assuming either the Expectation Hypothesis or Long-Run Neutrality would produce zero bias about the interest rate path if the shock is a growth shock. The reason is related to the IID nature of shocks in this model, which is the knife-edge case that Proposition 4 omits. In that knife-edge case, what matters for identification is something weaker than Long-Run Neutrality: that a particular shock induces zero time-variation in "long-run risk prices".[30] But the bias is substantial if monetary policy affects uncertainty (bottom right panel), and this bias increases with the horizon.

[30]Here, "long-run risk prices" refers to the loading of $\log H_{t+1}/H_t$ on the shock ΔW_{t+1}. In the IID shock case here, since "long-run risk prices" are independent of the current growth state, there is no bias in simply assuming Long-Run Neutrality. This independence turns out to hold given the linear structure of the growth state dynamics and the unitary EIS assumption on preferences. For a similar reason, namely that "short-run risk prices" are independent of the growth state, the Expectation Hypothesis assumption also produces zero bias.

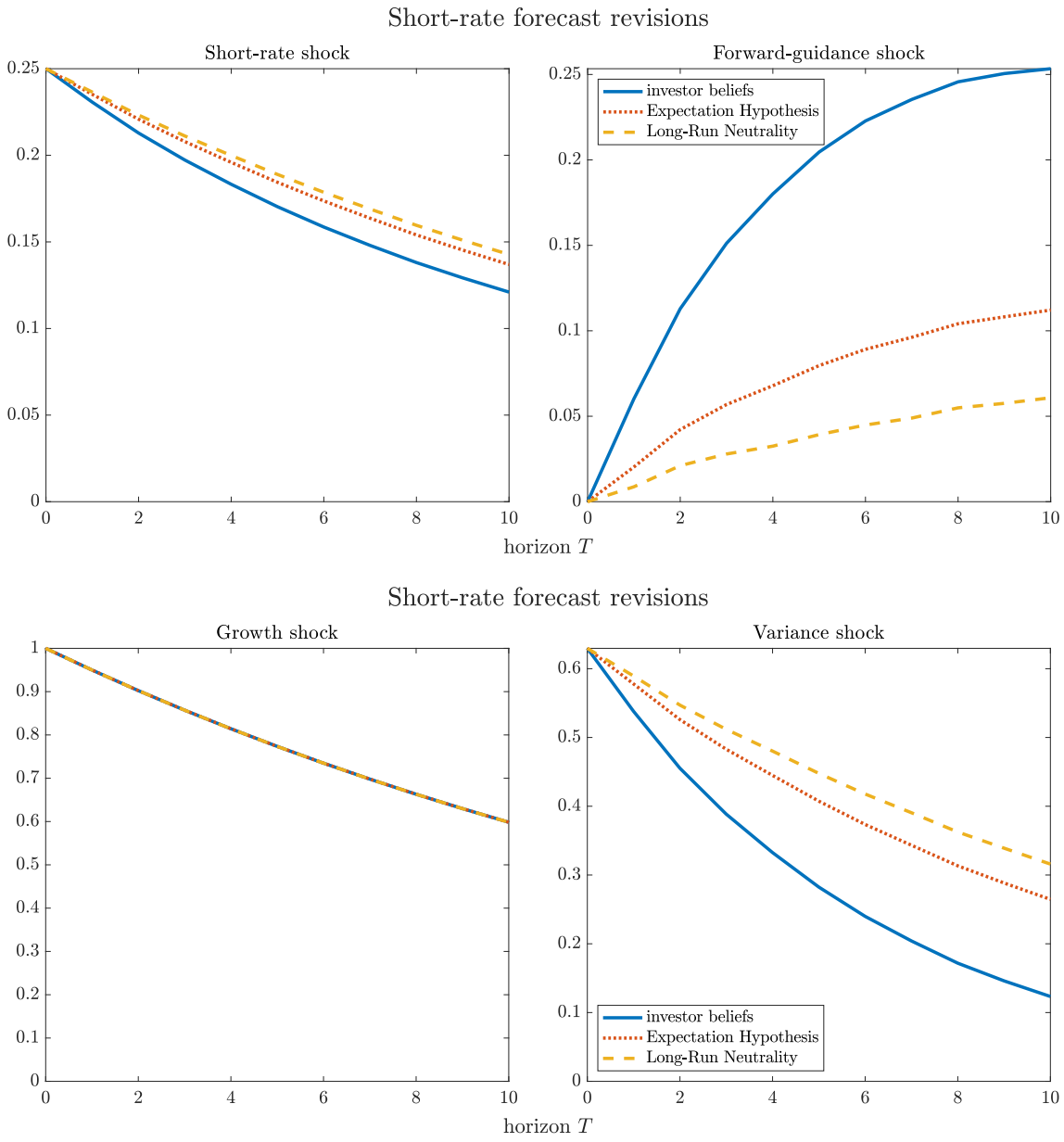


Figure D.1. Short-rate forecast revisions. The figure presents forecast revisions regarding short-term interest rates, at various forecast horizons, in the calibrated long-run risk model. Forecasts are computed under the investor belief, under the risk-neutral measure (i.e., labeled "Expectation Hypothesis"), and under the long-run risk-neutral measure (i.e., labeled "Long-Run Neutrality"). Calibration: $\beta = 0.99$, $\gamma = 10$, $\mu_g = 0.02$, $\mu_v = 0.04^2$, $a_g = 0.95$, $a_v = 0.85$, $\sigma_g = 0.005\sqrt{(1-a_g^2)/\mu_g}$, $\sigma_v = \frac{\mu_v}{2}\sqrt{(1-a_v^2)/\mu_v}$.

One additional takeaway is that Long-Run Neutrality can actually be worse as an identification assumption than the Expectation Hypothesis. Notice that in all the figures, the dashed yellow lines (LRN) are furthest from the solid blue ones (investor beliefs). The reason is simple: the model considered here features a prominent permanent component H that is actually even more volatile than the SDF S itself.

E Additional Tables and Figures

This appendix contains additional tables and figures, mainly from robustness exercises discussed in the text.

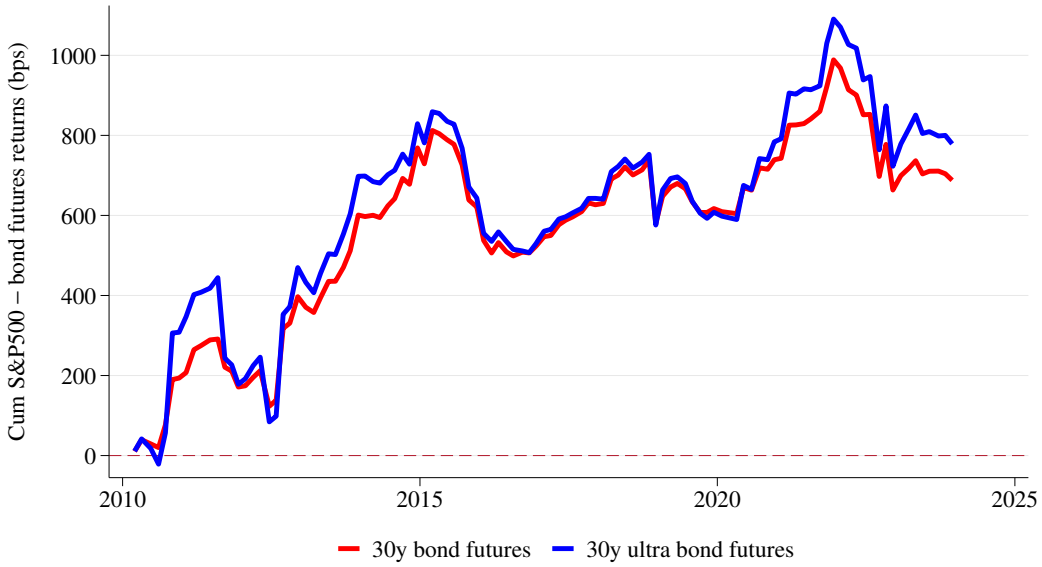


Figure E.1. Cumulative returns in a narrow window around FOMC policy announcements. The figure plots cumulative log returns on a long-short portfolio of S&P 500 E-mini futures in excess of 30-year Treasury bond futures. The two lines compare returns of the "classic" and the "ultra" 30-year bond futures contract, with the latter requiring a delivery bond basket with longer maturities. The returns are calculated from -15 to $+15$ minutes around scheduled FOMC announcements. The sample period covers 110 meetings from 2010:03, when the ultra contract becomes available, through 2023:12.

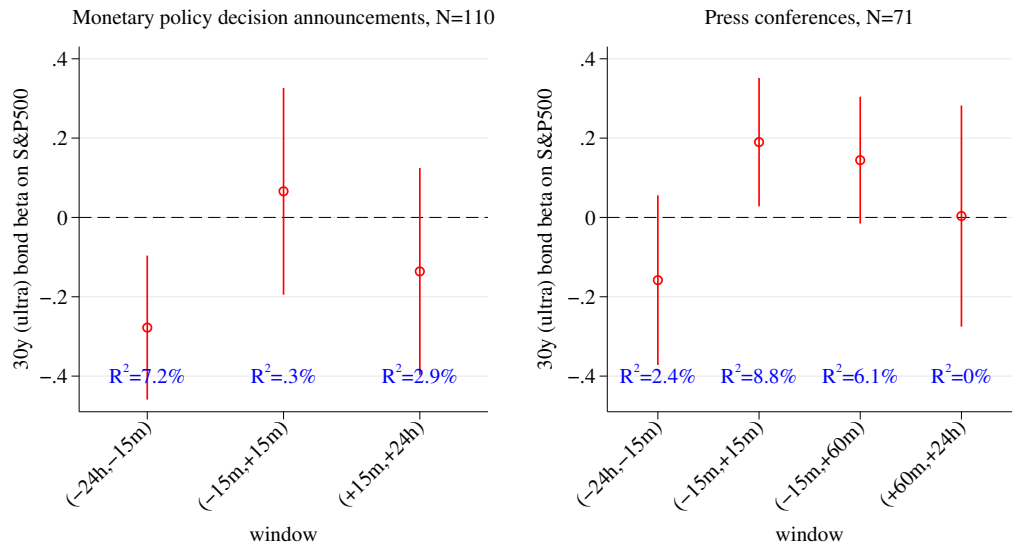


Figure E.2. Bond betas. The figure presents betas of 30-year Treasury ultra bond futures returns on S&P 500 E-mini futures returns in different windows around scheduled FOMC announcements and press conferences. Robust 95% confidence intervals are included.

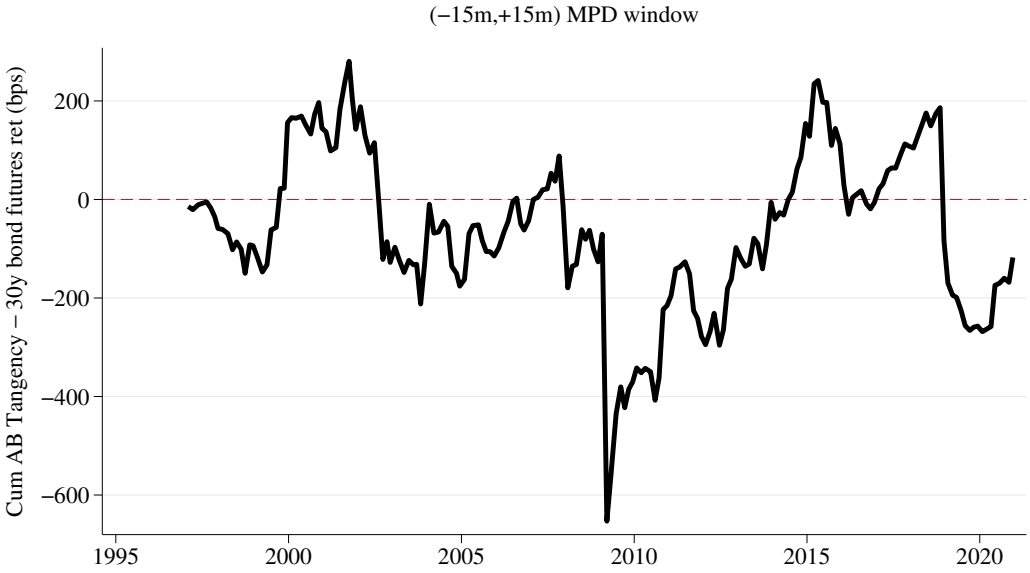


Figure E.3. Cumulative returns in a narrow window around FOMC policy announcements. The figure plots cumulative log returns on a long-short portfolio of Aleti-Bollerslev (AB) tangency portfolio in excess of 30-year Treasury bond futures. The returns are calculated from −15 to +15 minutes around scheduled FOMC announcements. The sample period covers 191 meetings from 1997:02 through 2020:12.

Panel A. Monetary policy decision (MPD) announcements

	(1) $\Delta_d y^{(n,20)}$	(2) $\Delta_d y^{(r,20)}$	(3) $\Delta_d(y^{(n,20)} - y^{(n,20)})$
$r^{(30)}_{(-15,+15)}$	-0.054***	-0.060***	0.0063
	(-8.18)	(-7.31)	(0.54)
Constant	-0.42	-0.95***	0.53**
	(-1.18)	(-2.73)	(2.09)
R^2	0.25	0.30	0.0087
N	199	199	199

Panel B. Press conferences (PC)

	(1) $\Delta_d y^{(n,20)}$	(2) $\Delta_d y^{(r,20)}$	(3) $\Delta_d(y^{(n,20)} - y^{(n,20)})$
$r^{(30)}_{(-15,+60)}$	-0.087***	-0.097***	0.010
	(-6.94)	(-4.89)	(0.65)
Constant	-0.42	-0.81	0.39
	(-0.98)	(-1.50)	(0.95)
R^2	0.39	0.37	0.011
N	71	71	71

Table E.1. Nominal and real return components. The table report regressions of daily yield changes on high-frequency bond returns in FOMC windows. The dependent variable are one-day change in the 20-year nominal zero-coupon yield in column (1), one-day change in the 20-year TIPS yield in column (2), and one-day change in the 20-year breakeven inflation rate computed as the difference in the nominal and TIPS yield changes. The explanatory variable is the high-frequency log return on the 30-year Treasury bond futures in the $\pm$15-minute window around FOMC decision announcements in Panel A and in $(-15,+60)$-minute press conference window in Panel B. The sample starts in 1999:02 in Panel A and in 2011:04 in Panel B, both ending in 2023:12.