Menu of Contracts in Imperfectly Competitive Markets with Adverse Selection

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Abstract

I develop an empirical framework to study screening in adversely selected markets with imperfect competition. I use it to illustrate that, due to a contractual externality, increasing capital requirements, increasing the Federal Reserve rate, or decreasing competition can increase lending and welfare by changing lenders' incentives to screen. I provide an empirical application in the context of consumer credit and show that, maturities are distorted relative to the second best. Model parameters are recovered using a linear regression of prices on quantities, controlling for contract market shares.

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1 Conflict-of-interest disclosure statement

I have nothing to disclose

2 Introduction

In many markets with adverse selection, firms use menus of contracts to make their customers reveal their private information through their choices (i.e., screening). Examples include the insurance market (Handel, Hendel, and Whinston 2015; Einav, Finkelstein, and Tebaldi 2019); retail credit markets (e.g., Taburet, Polo, and Vo 2024 for mortgages, and Nelson 2025 for credit cards); and the housing rental market (Humphries, Nelson, Nguyen, Dijk, and Waldinger 2024, Calder-Wang and Kim 2024).

Yet, the vast majority of empirical work on selection markets typically relies on quantitative models with a single contract and exogenous characteristics à la Akerlof (see Einav, Finkelstein, and Mahoney 2021 for a survey, Handel, Hendel, and Whinston 2015, Veiga and Weyl 2016 or Mahoney and Weyl 2017 for recent examples). As a consequence, these frameworks are ill-suited to analyze policies or market forces that operate through contract design. The relative scarcity of empirical work incorporating screening largely reflects the limitations of the theoretical literature.[1] Existing screening models vastly focus on extreme market structures – such as monopoly (Stiglitz 1977) or perfect competition (Rothschild and Stiglitz 1976) – and often encounters issues of equilibrium existence and tractability, even in highly stylized environments.

This paper bridges theory and empirical research on adverse selection by developing an empirical framework to study screening in selection markets. This paper makes two contributions. First, I analyse theoretically how market power affects contract terms and welfare in selection markets. Due to a contractual externality, contracts are distorted relative to first- and second-best benchmarks. I show that decreasing competition or increasing contract origination costs (i.e., capital requirements in the banking market) can increase quantities traded (i.e., loan amounts) and lead to a Pareto improvement. The second contribution is to quantify those frictions by bringing the model to the data. I do so using a novel research design, building on Berry and Haile (2014). The approach uses the lender's optimal pricing equation as a moment condition to recover demand-semi elasticities. It leverages the idea that lenders have better information about demand elasticities than econometricians, that

1. For instance, in Einav, Finkelstein, and Mahoney (2021): "Indeed, it may well be that endogenous contracting, which is the focus of RS (Rothschild and Stiglitz), rather than mispricing alone (as in Akerlof), is the most important dimension to consider in the context of selection markets, but the empirical intractability of the original RS framework has so far limited its impact on empirical work."

this information is incorporated into prices, and is proportional to the relative market shares of contracts in screening models. I show through a simulation exercise that my estimation approach outperforms standard methods à la Berry, Levinsohn, and Pakes 1995 in settings where the econometrician has imperfect information about borrowers' characteristics and available choice sets. I provide a working example in the context of consumer credit, using data from Hertzberg, Liberman, and Paravisini (2018).

The model incorporates imperfect competition in an otherwise standard screening model à la Rothschild and Stiglitz (1976). Following the empirical industrial organization literature, I model competition using a discrete choice framework in the spirit of McFadden (1981). To highlight the connection to the empirical application, I present my model in the context of credit markets. However, the framework is general enough to capture labour markets, insurance markets, or other financial markets in which contracts are exclusive (see Attar, Mariotti, and Salanié 2011).

In the baseline model, a contract consists of a loan amount and an interest rate. There are two borrower types (high and low default probabilities), with quasilinear utility over the present value of the loan. Borrowers want the same loan size but have private information about their willingness to pay (WTP) for each dollar borrowed, and about their default probabilities. To study adverse selection, I allow borrowers' WTP to be positively correlated with their default probabilities. Lenders can either pool borrowers (i.e., offer a single contract) or screen them (i.e., offer two separate contracts). Because high default borrowers have a higher WTP, screening is achieved by making high-default borrowers self-select into higher-rate, higher loan-size contracts. Screening allows lenders to price each borrower differently, thereby potentially restoring perfect information pricing. However, it also requires that low-default borrowers get a smaller loan than high-default borrowers, which is not what would happen in the first-best. When the cost of screening is higher than the cost of pooling, lenders should pool borrowers. However, this is not always possible as a lender can take advantage of a competitor's pooling contract to steal the most profitable borrowers (i.e., cream-skimming).

To model borrowers' choice of lender, I assume that the borrowers' utility contains an additive borrower-bank-specific random demand shock with a continuous probability distribution function as in Rochet and Stole (2002). This shock can be interpreted as borrowers having search costs or being imperfectly informed about the contracts offered by each bank

(as in Varian 1980, or Rochet and Stole 2002) or by bank product characteristics valued heterogeneously by borrowers (as in Hotteling 1929). When the shocks follow an extreme value distribution, their variance parameterizes the product demand elasticity. In the limit case in which the variance of the shocks tends to infinity, each lender behaves like a monopolist. When the variance tends to zero, borrowers' demand elasticity becomes infinite, as in the perfect competition case.

Using this discrete choice approach has two benefits. First, it makes the model more realistic thanks to demand functions that nest various degrees of competition, from perfect competition to monopoly. Second, it yields convenient analytical properties relative to existing screening models (e.g., the existence of a pure strategy equilibrium and the continuity of demand) and allows the use of standard estimation techniques from industrial organisation for demand estimation (e.g., Berry, Levinsohn, and Pakes 1995, Berry and Haile 2014).

Allowing for imperfect competition through a discrete-choice framework has three main implications for the analysis. First, it resolves the non-existence of a pure-strategy Nash equilibrium highlighted by Rothschild and Stiglitz (1976). In my model, a pure-strategy equilibrium always exists when firms' profit functions are continuous and concave, and can be characterized in closed form in some special cases. This simple condition for existence ensures flexibility when adapting the model to different empirical setups. The fact that lenders play pure strategies is uncommon in screening models (see, for instance, Lester, Shourideh, Venkateswaran, and Zetlin-Jones 2019) and matters for empirical tractability, since mixed strategies are typically unobserved – and plausibly not played – in most applications.

Second, I show that, depending on the level of competition, the effects of increasing the capital requirement or raising the monetary policy rate can be expansionary rather than contractionary. The mechanism is as follows. When competition is intense, lenders cannot cross-subsidise and must screen and price each contract at marginal cost.[2] As a result, low-WTP borrowers are credit rationed so that high-WTP borrowers have incentives to self-select the high rate contract. Due to competitive pressure, increases in contract-specific origination costs are fully passed through to the contract interest rates. Thus, increasing the origination cost of high-loan-size contracts weakens borrowers' incentives to self-select, thereby forcing lenders to ration credit to low STP borrowers even more. By contrast, when competition

2. The concavity assumption renders deviations that rely on increasing cross-subsidisation – i.e., deviations from screening in Rothschild and Stiglitz (1976) – unprofitable.

is limited, the same increase in the contract origination costs can increase lending. The reason is the following. The higher origination cost reduces the potential profitability (i.e., the surplus) of lending to high-WTP borrowers. As lower competition restores a lender's ability to cross-subsidise, the reduction in profits can induce lenders to lower interest rates in that segment. Lowering the interest rate relaxes incentive compatibility constraints, allowing the bank to expand lending to the other, relatively more profitable market segment. When lenders screen using maturity, the same effect follows with quantitative easing policies, which affect the cost of originating contracts with different maturities.

Third, competition among lenders generates a contractual externality, for which I derive a sufficient statistic. The inefficiency arises because, when lenders design their menus of contracts, their choices affect the composition of competitors' borrower pools and, consequently, competitors' screening costs. This contractual externality leads to inefficient outcomes in the parameter regions where the equilibrium characterised by Rothschild and Stiglitz (1976) fails to exist. I show that market equilibria can be inefficient in a second-best sense, so that an informationally constrained social planner can design a menu of contracts that makes both firms and borrowers better off. The second-best allocation can be implemented by a government-sponsored entity that purchases loans and sets fees strategically to influence the cost of loan origination. This implementation mirrors the role played by Fannie Mae and Freddie Mac in the mortgage market, providing a rationale for their intervention as a mechanism to restore market efficiency. Similarly, reducing competition (e.g., by approving more mergers) can increase lending and, in some cases, generate Pareto improvements for both borrowers and lenders. The reason is that it reduces the extent to which one lender's menu of contracts affects the composition of its competitors' demand, thereby giving lenders greater flexibility to screen borrowers efficiently – albeit at the cost of increased market power.

To ensure the model can be applied to various empirical setups, I extend it to support an arbitrary number of borrower types and lenders, as well as both adverse and positive selection, and endogenous default probabilities. The number of contract terms can exceed two, but this comes with restrictions on preference heterogeneity, as multidimensional screening problems are generally intractable (see, for instance, Rochet 2024). The tractability of the model allows me, once the parameters are estimated, to assess these trade-offs quantitatively using the sufficient statistic or counterfactual simulations.

Due to the presence of asymmetric information, I propose an alternative approach to

that of Crawford, Pavanini, and Schivardi (2018) or Berry, Levinsohn, and Pakes (1995) for recovering the model parameters. The identification leverages the idea that the lenders and the borrowers have better information than the econometrician about the menus available, and that this information is reflected in the equilibrium contracts. As a result, my approach does not require the estimation of default elasticities, nor does it require data on menus of contracts offered by other lenders to recover demand elasticities as usually done in the industrial organisation literature (see, for instance, Berry, Levinsohn, and Pakes 1995). This method is particularly convenient because these elasticities are difficult to estimate under adverse selection, given that borrowers who choose different contracts are not comparable.

Formally, I propose to identify and estimate the model parameters based on the optimal pricing equation that I use as a moment condition. The optimal pricing is composed of the marginal cost plus a markup. The markup can be further decomposed into a perfect-information markup – which depends on the inverse demand semi-elasticity – and an information rent – which equals the inverse demand semi-elasticity times a variable related to the information rent offered in the monopoly case. The information rent depends on the relative market share of contracts within the same menu, which is observable in my setup. The moment condition enables me to recover the marginal cost, the information rent, and the demand semi-elasticity as a constant and the slopes of a linear regression of price on quantities and contract relative market shares.

To illustrate the approach, I provide an application using data from an online lending platform (Lending Club) as in Hertzberg, Liberman, and Paravisini (2018).[3] I use the estimates to decompose the equilibrium interest rate into a break-even price, a perfect information markup, and an information premium or discount. The information premium divided by the perfect information markup of the contract designed for high-default borrowers is a sufficient statistic for the existence of the contractual externality.

The identification of the key parameter of the sufficient statistic relies on variation in the relative market shares of contracts within the same menu. Interpreting the regression error term through the lens of the theoretical model, the exclusion restrictions state that the relative market shares between high- and low-maturity contracts are uncorrelated with the

3. Although contracts are not formally exclusive in this market, the fact that Hertzberg, Liberman, and Paravisini (2018) find that screening with menus happens in practice shows that using Rothschild and Stiglitz (1976) framework (exclusive contracts) instead of Attar, Mariotti, and Salanié (2011) (non-exclusive contracts) is a valid approach in this context.

(unobserved) demand elasticity with respect to the loan present value or unobserved product characteristics. As I observe all contract characteristics, the key concern is unobserved demand heterogeneity. To mitigate these endogeneity concerns, I use variation in relative market shares across borrower segments with different observable risk categories. Therefore, the identifying assumption is that, conditional on observables, borrowers' risk categories are uncorrelated with their demand elasticities with respect to the loan's present value.[4] If this assumption is not satisfied, the estimation recovers a weighted average of the demand elasticities.

As in Hertzberg, Liberman, and Paravisini (2018), I find that high-default borrowers self-select into higher-maturity loans. Thanks to the structural model, I can also characterize the distortions caused by adverse selection and imperfect competition. I show that prices are close to the first-best case, but maturities are far from the second-best. Borrowers who choose high-maturity contracts are less price elastic but get an asymmetric information discount that mitigates the competition channel. The information rent is equal to 15 percent of the loan's present value. Using sufficient statistics, I show that both borrowers and lenders would be better off if lenders could cross-subsidize customers in order to increase the maturity of their lower-maturity contracts while maintaining incentives to self-select. The threat of cream skimming prevents this situation from emerging. Lowering competition or increasing the cost of originating low maturity contracts leads to a Pareto improvement.

The rest of the paper is structured as follows. Section 3 provides a literature review. Section 4 describes the model setup. Section 5 analyzes the equilibrium properties. Section 6 discusses extensions as well as identification approaches. Section 7 illustrates how to adapt the model by examining a specific empirical application: Consumer credit.

3 Literature Review

My paper contributes to the growing empirical literature on adverse selection. A first strand of the literature developed tests for adverse selection using reduced-form approaches (Chiap-

4. The demand elasticity to the loan present value is different from the elasticity to prices as it incorporates heterogeneity in elasticity due to heterogeneous default probability. Thus, our proposed instrument still satisfies the exclusion restrictions when borrowers search for approval as in Agarwal, Grigsby, Hortaçsu, Matvos, Seru, and Yao (2024).

pori and Salanie 2000, Karlan and Zinman 2009, Hertzberg, Liberman, and Paravisini 2018). Following Einav, Finkelstein, and Cullen (2010), a second strand of the literature developed models to estimate the cost of adverse selection. Those papers are based on models with a single contract, such as Akerlof (1978), and generally use a sufficient statistic approach. Examples include Veiga and Weyl (2016) for a model with one contract and two characteristics (price and quality) offered by a monopolist, and Mahoney and Weyl (2017) for a framework using Bresnahan (1989) conduct approach to model competition. The conduct approach is problematic because it lacks a clear interpretation outside of the perfect competition and monopoly cases.

In credit markets, the literature relies less on the sufficient statistic approach and instead has primarily focused on structural models in which lenders can only choose interest rates. For instance, Crawford, Pavanini, and Schivardi (2018) develops a structural model with adverse selection based on the positive correlation test approach of Chiappori and Salanié (2002). Their focus is on the market for business loans and as a result do not focus on screening. They use a framework based on Akerlof (1978) with a linear pricing schedule for loan size and abstract from unobserved preference heterogeneity, making it impossible to use for screening. It is not obvious to extend as adverse selection breaks the contraction mapping used to solve empirical industrial organisation models with multiproduct firms, preventing us from performing counterfactual analysis. Nelson (2025) and Darmouni (2020) estimate the magnitude of informational friction in credit markets using, respectively, a dynamic model inspired by Rust (1987) and a discrete choice model with hidden types. Brugues (2025) focuses on moral hazard using a dynamic contracting structural model. Yannelis and Zhang 2021 uses a sufficient statistic approach to assess welfare in the banking market using an Akerlof model.

My paper differs from previous empirical works in that it uses a model with endogenous menus of contracts to characterize the screening distortions. Endogenizing menus is important since, as in my empirical application, prices may be close to their first-best allocation but maturities or other characteristics may be far from their first or second-best values. My paper can thus provide insight into the range of products offered and can be used for welfare analysis and to perform counterfactual simulations.

The closest paper is that by Taburet, Polo, and Vo (2024), which uses a structural model of screening for default probability under imperfect competition. The paper endogenises

menus using an approach similar to that for firm entry, as in Wollmann (2018). This approach is computationally demanding and not robust to initial conditions, which limits the paper's counterfactual analysis to the social planner problem. In contrast, my paper is more tractable and focuses on the theoretical properties of a screening model with imperfect competition. I also propose a different research design to identify the model parameters, which uses the model's first-order conditions to recover demand elasticities. This paper's approach can thus be applied in different set-ups where data on menus and borrowers' choices for a single lender are observable (as in Handel and Kolstad 2015), rather than when only contracts chosen by borrowers are observable (as in credit registries).[5]

This paper is related to the theoretical literature on adverse selection, particularly the strand studying screening with menus in selection markets. Most papers assume either perfect competition (Rothschild and Stiglitz 1976) or monopoly (à la Stiglitz 1977), and rely on new solution concepts to analyse the perfect competition case. These equilibrium concepts are used because pure strategy Nash equilibria may not exist, and mixed strategy equilibria are difficult to compute even in stylised frameworks (see Dasgupta and Maskin 1986 for a proof of existence in mixed strategies and Farinha Luz 2017 for the computation of the mixed strategy equilibrium in a Rothschild and Stiglitz 1976 model). Recent examples of perfect competition models are those of Azevedo and Gottlieb (2017) – which proposes an novel solution concept based on robustness to small perturbation of fundamentals – and Guerrieri, Shimer, and Wright (2010) – with a pure strategy characterization of the equilibrium using a model in which each principal can match with at most one agent. Azevedo and Gottlieb (2017) microfound their solution concept with a game with capacity constraints and one contract being offered, I extend that microfoundation to allow for imperfect competition and menus of contract, analyse its impact, and discuss the estimation and identification of the model parameters.

The closest paper is Lester, Shourideh, Venkateswaran, and Zetlin-Jones 2019, which shares the idea of incorporating imperfect competition into a screening model. They use a search model à la Burdett and Judd (1983) to model imperfect competition in an otherwise standard screening model with goods of different quality. Burdett and Judd (1983) is a setup in which a fraction of borrowers are perfectly inelastic, and another fraction is perfectly elas-

5. The advantage of proprietary data with respect to credit registers is that one can observe the menus, the disadvantage is that one cannot observe the contract offered by other banks.

tic. The central difference lies in the use of a different modelling approach to competition, motivated by my goal of bringing the model to the data. The first benefit is the connection to standard demand models used in the industrial organisation literature and allowing to derive intuitive optimal pricing equations that can be used as moments to recover the information rents, marginal costs and markups. The second benefit is analytical tractability. The modelling is convenient as it allows the use of standard tools developed for continuous and convex problems. As such, my modelling choice yields a pure-strategy equilibrium under general conditions. It also allows me to characterise the optimal contract using the model's first-order condition, making the economic forces transparent and connecting them, following Rochet and Stole (2002) change of variable, to those described in textbook monopoly models, such as Stiglitz (1977). I obtain results similar to those in Lester, Shourideh, Venkateswaran, and Zetlin-Jones 2019 on the impact of competition on contract terms and welfare, but my approach yields a sufficient statistic that allows me to quantify the friction. Given my focus on the lending market, I also examine a novel comparative static with respect to origination costs.

4 Model Set-up

The economy is populated with two groups of agents: borrowers and lenders. I also refer to the second group as banks. There is a discrete number of borrowers indexed by j. They have private information about their type, indexed by $i \in \Gamma$ and denoted by the vector $\Theta_i := (\theta_i, \sigma_i)$, where θ_i is a vector that parameterizes the borrower's willingness to pay for contract terms and σ_i their demand semi-elasticity. The number of type i borrowers is denoted n_i. The distinction between the borrower j and its type i arises because lenders can only learn about the borrower's type (i.e., the borrower's characteristics indexed by i).

There is a finite number of lenders $|B| \in \mathbb{N}$ indexed by $b \in B$. Lenders can offer a menu of contracts $(X_c, R_c)_c$ to make borrowers reveal their private information, where X_c is a vector of contract c characteristics (for instance, the loan size, and whether the loan is collateralised), and R_c is the price of the contract (for instance, the discounted sum of the loan monthly payments). Each contract is indexed by $c := ib$, where i denotes the borrower type for which the contract is designed and b denotes the bank that designs the contract.

The following section describes the behaviour of these two groups. To focus on screening,

I model the agent's decision at contract origination and employ a parsimonious reduced-form approach (as, for instance, Crawford, Pavanini, and Schivardi 2018 or Chetty and Finkelstein 2013) for events that occur after origination.

4.1 Borrowers

Borrowers may select at most one contract from a single lender.[6] They choose the contract and bank (c and b) that maximizes their utility. The utility consists of two elements: the utility derived from contract c – denoted $u_i(X_c, R_c; \theta_i)$ in which $\theta_i := (\theta_{id})_d$ is a vector that drives the marginal rate of transformation between contract terms – and a lender b specific "taste shocks", denoted $\sigma_i^{-1}\epsilon_{jb}$. $(\epsilon_{jb})_b$ are drawn from a continuous distribution function.

Formally, borrower j of type i chooses the bank $b \in B$ that offers them the best deal $c \in \mathcal{M}_b$, where $\mathcal{M}_b$ is the menu offered by bank b:

$$b_{ij} = \arg\max_{b \in B} \left\{ \max_{c \in \mathcal{M}_b} \left\{ u_i(X_c, R_c; \theta_i) + \sigma_i^{-1}\epsilon_{jb} \right\} \right\} \tag{1}$$

The utility function $u_i(X_c, R_c; \theta_i)$ is quasi-linear in R_c and has the form $u_i(X_c, R_c; \theta_i) := \alpha_i[\sum_{d=1}^{D} \theta_{id} v_d(X_{cd}) - R_c]$ with v_d being a one-to-one continuous function. I denote $X_c := (X_{cd})_d$ the vector of contract c characteristics. It has dimension D. The quasi-linear utilities assumption is necessary to ensure the screening model is well-behaved,[7] and is used in the vast majority of structural empirical papers.[8] The utility $u_i(\cdot)$ can thus be interpreted as a linear approximation of the true utility around the interest rate observed in the data.[9] Appendix B provides an example of a micro-foundation for the utility function where $\alpha_i R$ can be interpreted as the loan present value and θ_i as the vector of willingness to pay for each contract characteristic.

6. We focus on exclusive contracts, which implies that the setup is more adapted to markets such as the one for mortgages, car loans or small firms.

7. See, for instance, Rochet (2024) or Rothschild and Stiglitz 1976)

8. See, for instance, Crawford, Pavanini, and Schivardi (2018), Benetton, Gavazza, and Surico (2021) for credit market examples, Landais, Nekoei, Nilsson, Seim, and Spinnewijn (2021) for insurance, and Chetty (2012) for the labour market.

9. Although the linearity might seem restrictive from a theoretical point of view, most empirical setups only allow for the recovery of local estimates of the utility around the equilibrium observed in the data. The functional form affects the counterfactual simulations, but those are based on extrapolations as the data cannot disciplinate which functional form is correct away from the data equilibrium.

The set of contracts offered by bank b (i.e., $\mathcal{M}_b$) contains the option of not borrowing. The utility when not borrowing is denoted $u_i(0,0;\theta_i)$, where $(0,0)$ is a contract with a price of zero and characteristics X set to zero. This modelling flexibly captures the market exit margin from the choice of lender, allowing me to analyse the impact of these two channels in Section 5 separately. It also makes the demand similar to the nested-logit model, heavily used in the empirical literature.[10] Section C discusses several other modeling alternative to study borrower participation.[11]

The ϵ_{jb} term can be interpreted as borrower-bank specific taste shocks. The shocks allow, as shown in, for instance, Rochet and Stole (2002) or Hotteling (1929), for imperfect substitution between contracts from different lenders and thus model imperfect competition. It is a random variable with a continuous probability distribution function, and σ_i^{-1} parameterizes the variance of the shocks. Section 5, which analyses equilibrium contracts, illustrates how σ parametrises the demand semi-elasticity and can thus capture various degrees of competition. I discuss the interpretation of the taste shape more in depth in Appendix D. The fact that the shocks are borrower (i.e., j) and not type (i.e., i) specific is done for empirical tractability, as it implies that observed contract market shares are equal to the probability of a borrower i choosing a contract c.

4.2 Lenders

This section describes how each lender designs their menu of contracts.

Lenders do not observe individual borrower characteristics $(\Theta_i, \epsilon_{jb})$, but they know the population distribution and form rational expectations about the menus their competitors offer. Lenders can use a menu of contracts to elicit borrowers' private information (Θ_i) through their choices. They simultaneously make their menu offers, treating other lenders' offers as given (a Nash equilibrium). Then consumers look at the menus and make their choices. After the contract is chosen, borrower of type i defaults on their loan with probability d_i. d_i is unobserved by the lender and is correlated with Θ_i to capture adverse or positive

10. See for instance, McFadden (1978) and Grigolon and Verboven (2014)

11. As shown in Section 5, the inclusion of $u_i(0,0)$ in the choice set $\mathcal{M}_b$ makes the model similar to benchmark models: monopoly as in Stiglitz (1977) when $\sigma_i \to 0$, or perfect competition as in Rothschild and Stiglitz (1976) when $\sigma_i \to \infty$.

selection. The reason the default probability d_i does not appear directly in the borrower's utility function is to allow lenders and borrowers to have different beliefs.

4.3 Derivation of the Lender's Maximization Problem

Before presenting the lender problem, I show that it can be expressed as an incentive compatibility problem and that lenders cannot use a menu to learn about the borrower bank-specific shocks (ε_{jb}):

Lemma 1: *The allocation of the game in which lenders can choose the number of contracts and the contracts' terms is the same as the one in which lender b designs a contract* (X_{ib}, R_{ib}) *for each borrower type i subject to the following incentive compatibility constraint:*

$$(IC_{ib}): \ u_i(X_{ib}, R_{ib}; \theta_i) \geqslant u_i(X_{\tilde{i}b}, R_{\tilde{i}b}; \theta_i) \ \forall i, \tilde{i} \in \Gamma \tag{2}$$

Proof: See Appendix G.

The constraint states that borrower i must prefer the contract ib designed for them rather than any other contract $\tilde{i}b$ designed by bank b for borrower of type $\tilde{i}$. The assumption that the random shocks are additive allows us to eliminate them from equation (2). It allows the parameter σ to affect bank selection but not contract selection, making it a convenient way to analyse competition and screening separately.

4.4 Lender's Maximization Problem

Each lender b maximises the sum of their expected profits in each market segment i, subject to the incentive compatibility constraints (IC_{ib}). The profit in market segment i is composed of the expected number of type i borrowers that accepts the contract ib (denoted $D_{ib}(u_i(X_{ib}, R_{ib}; \theta_i); u_{-ib})$), times the net present value of lending to that borrower via the contract ib (denoted $NPV_{ib}(X_{ib}, R_{ib}; d_i)$). $u_{-ib} := (u_i(X_{i\tilde{b}}, R_{i\tilde{b}}; \theta_i))_{\tilde{b} \in B \setminus \{b\}}$ is the vector of utilities the borrower gets when borrowing from other banks than bank b. Formally, we have:

$$\max_{\{(X_{ib},R_{ib})\in C_b\}} \sum_{i\in\Gamma} \overbrace{D_{ib}(u_i(X_{ib},R_{ib};\theta_i);u_{-ib})}^{Demand} \cdot \overbrace{NPV_{ib}(X_{ib},R_{ib};d_i)}^{Net\ Present\ Value\ of\ lending} \tag{3}$$

$$s.t.\ (IC_{ib}) \text{ defined in equation (2)}$$

The demand function $D_{ib}(u_i(X_{ib},R_{ib};\theta_i);u_{-ib}) := E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\mathcal{I}_b]$ is derived from the borrower maximization problem defined by equation (1) by integrating over the taste shocks (ϵ_{jb}). $\mathcal{I}_b$ is lender b information set; it contains the distributions of $(\epsilon_{jb})_{ij}$ for each borrower j conditional on their choice of contract and type i, and the lender b expectations about the menus of contracts other lenders offers.

C_b in a compact set that represent the set of feasible menus of contracts. The fact that it is bounded is important for equilibrium existence. It is not restrictive, as it can be made arbitrarily large and, in practice, interest rates and contract terms are naturally bounded by regulation (e.g., interest rate caps).

I parametrize the net present value of lending NPV_{ib} the as follows:

$$NPV_{ib} := E[\alpha_i^B(d_i)|\theta_i]R_{ib} - mc_b(X_{ib}) \tag{4}$$

The function mc captures the marginal cost of lending via contract ib. mc is convex and increasing in X. I set $mc_b(0)$ to zero so that the contract $(0,0)$ can be interpreted as rejecting the borrower's application.

The quantity $E[\alpha_i^B(d_i)|\theta_i]R_{ib}$ can be interpreted as the expected present value of the loan from the lender's perspective. α_i^B is a function of borrower i default probability d_i, which captures how the bank discounts the face value of the debt R. The default probability d_i is unobserved[12] by the bank and can be correlated with the borrower's willingness to pay θ_i. If the correlation is positive, the market is adversely selected, and screening using a menus provide useful information about about d_i. For simplicity, we assume that ϵ_{jb} is uncorrelated with default conditional on knowing the borrower type i. Appendix B presents a potential microfoundation for the borrower-lender problem.

12. d_i can be partially observed by the lender; what is important is that knowing Θ_i can provide addtional information about d_i to the lender.

4.5 Utility Representation

In this section, I use a change of variables to express the lender problem in terms of promised utilities, as in, for instance, Rochet and Stole (2002). This formulation is analytically convenient and useful for intuition, as it makes the analysis similar to the textbook model of screening under monopoly. It is used to derive many results in section 5.

Before stating the utility representation, let us first introduce some notation. I express the demand as $D_{ib}(u_i)$ instead of $D_{ib}(u_i(X_{ib}, R_{ib}; \theta_i); u_{-i})$, where $u_i := (u_{ib})_b$ is a vector composed of the utility derived by borrower i from each contract designed for them $b \in B$. I also denote $S_{ib}(X_c) := E[\alpha_i^B(d_i)|\theta_i]\theta_i' v(X_c) - mc_b(X_c)$ the surplus function, with $\theta_i' v(X_c) := \sum_{d=1}^{D} \theta_{id} v_d(X_{cd})$. $S_{ib}(X_c)$ is the amount of profit generated by lending if lenders were to price each loan so that it extracts all the borrower's utility (i.e., $\theta_i' v(X_c)$). Using a change of variable to express the lender profit function for market segment i in terms of borrower utility, I denote $\Pi_{ib}(u_{ib}, X_{ib}; u_{-ib}) := D_{ib}(u_{ib}, u_{-ib})[S(X_{ib}) - u_{ib}]$. The lender maximization problem thus has the following utility representation:

Lemma 2: Utility representation. *The lender maximization problem (3) can be written:*

$$\max_{(u_{ib}, X_{ib})_{ib} \in C_b} \sum_{i \in \Gamma} D_{ib}(u_i)[S_{ib}(X_{ib}) - u_{ib}] \tag{5}$$
$$s.t.\ (IC_{ij}):\ u_{ib} \geqslant u_{jb} + (\theta_i - \theta_j) v(X_{jb}), \forall i, j \in \Gamma$$

This representation highlights that, absent incentive compatibility constraints, the problem is a standard nonlinear pricing model. In that case, lender b would thus chooses contract charcteristics (X_{ib}) to maximise total surplus, $S_{ib}(X_{ib})$, and uses the interest rate – equivalently, promised utility u_{ib} – to allocate that surplus between the borrower and the lender.

With assymetric information, the lender has to maintain borrowers' incentives to self-select. When the incentive-compatibility constraint (IC_{ij}) binds, higher willingness-to-pay borrowers (e.g., $\theta_i > \theta_j$) earn an information rent, $(\theta_i - \theta_j) v(X_{jb}) \geqslant 0$, relative to the borrower j with a lower willingness to pay. The lender can reduce these rents either by compressing profits in lower segments (i.e., lowering X_{jb}, at the cost of lower profits) or by using interest

rates to cross-subsidise market segments (i.e., increasing u_{ib} while decreasing u_{jb}).

5 Analysis of the Equilibrium Contracts

This section derives the main comparative statics of the paper. I first present, in Section 5.1, general technical results on the model properties such as equilibrium existence. Section 5.2 characterises the equilibrium contracts for a tractable parametrisation of the model and discusses economic intuition. Sections 5.3 and 5.4 provide a positive and normative analysis of the equilibrium contracts. To illustrate the key theoretical mechanisms, I focus on a simple parametrisation of the model and, in the spirit of Einav, Finkelstein, and Cullen (2010), I discuss the model using a graphical representation of the lenders' problem.

5.1 Existence

This section establishes the existence of a pure-strategy Nash equilibrium, a central concern in this class of models. Those conditions ensure that, assuming lenders play pure strategies and use the model's first-order conditions to recover model parameters, the model is model-consistent and usable in my framework. Those approaches are comonly used in empirical work as in, for instance, Berry, Levinsohn, and Pakes 1995. I provide intuition for why the pure strategy equilibrium may fail to exist in many models in the literature and how the modelling assumptions restore its existence in Section 5.3.

I also provide, as a corollary, specific functional forms commonly used in empirical work that satisfy the conditions for the existence of a pure strategy equilibrium. One of these functional forms is used in the next subsection to analyse the equilibrium properties in closed form in a symmetric equilibrium.

Theorem 1: Existence and convergence to separating contracts.
Existence – *A pure strategy Nash equilibrium exists when the profit functions* $\Pi_{ib}(u_{ib}, X_{ib}; u_{-ib})$ *are continuous and concave in* u_{ib} *and* X_{ib} *for any* u_{-ib}*, and continuous in* u_{-ib}*. The set of feasible contracts* (C_b) *for each bank* b *must be convex and bounded.*

Corollary: Example of functional form – *The condition for existence is satisfied*

when the tastes shocks (ϵ_{ib}) are extreme value distributed, the lender face a capacity constraint as in Azevedo and Gottlieb (2017) (i.e., $D_{ib}(u_{ib}; u_{-ib}) = n_i f(u_{ib})\mathbf{1}_{f(u_{ib})\leqslant k} + n_i k \mathbf{1}_{f(u_{ib})>k}$ with $f(u_{ib}) := \frac{exp(\sigma u_{ib})}{(exp(\sigma u_{ib})+\sum_{m\neq b} exp(\sigma u_{im})}$) and the surplus functions S_{ib} are concave with $-S''_{ib} > 0$ in contract terms X. The capacity constraint is such that: $k \leqslant \bar{k} := \min\{\min_X[\sigma \frac{S'(X)^2}{2(-S''(X))\bar{D}} + 1]^{-1}, \frac{1}{2}\}$ with $\bar{D}$ being the demand faced by bank b when all banks offer the lower rate and best contract terms X and bank b offers the highest rate and worst term in C_b.
In the special case where the surplus functions are linear, and there are two borrower types. The capacity constraint is $k < \frac{1}{2}$ and σ must be high enough (see Appendix G.3.2 for a bound on σ).

Proof: See Appendix G.3 for the proof. Section G.3.1 for the existence proof, and Appendix G.3.2 for the functional form result.

The existence result follows from a standard fixed-point argument for games with convex strategy sets and continuous payoffs (see, for instance, Glicksberg 1952). These tools apply naturally in my setting because the discrete-choice structure delivers continuous demand functions, unlike much of the existing literature that assumes perfect competition and therefore uses demand functions that generate discontinuities (e.g., Rothschild and Stiglitz 1976).

The assumption about continuous demand is also useful because it guarantees the existence of a *pure-strategy* equilibrium. This property is novel with respect to existing models, such as Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), that must be solved using mixed strategies. It matters for empirical tractability, since mixed strategies are typically unobserved – and plausibly not played – in most applications. Unlike Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), the proof is not restricted to linear functional form or to homogeneous banks. The simple concavity condition for existence ensures flexibility when adapting the model to different empirical setups. It ensures that the first order conditions can be used to recover model parameters.

The capacity constraint introduced here is a convenient way to ensure concavity of the lender's profit function. This assumption is very common, as under the standard logit demand system, concavity already requires a capacity constraint $k < \frac{1}{2}$. Allowing the con-

straint to depend on σ follows Azevedo and Gottlieb (2017), who adopt a related device[13] to microfound their solution concept.

Economically, the constraint can be interpreted as a tractable reduced-form way to introduce convex lending costs. In practice, such capacity limits may reflect scarce loan officers, limited analytics capacity, or constraints in transaction-proccapacity-constraint assumption is that itconstraint assumption is that it does not bind in equilibrium. As a result, it does not distort the equilibrium first-order and ensures that locally unprofitable deviations also remain unprofitable when scaled up. In this sense, the capacity constraint (k) is a technical restriction that rules out profitable "large" deviations without affecting equilibrium allocations (as long as $k < \bar{k}$). Figure 2 in section 5.3 discusses the economic content of this assumption and clarifies how it relates to potential equilibrium non-existence.

5.2 Equilibrium Contracts

I start by describing the parametrisation used for the equilibrium analysis. I then analyze the equilibrium contracts under perfect information before solving for the imperfect information case.

5.2.1 Parametrization for the Theoretical Analysis

To highlight the model's main economic forces, this section considers a stripped-down version of the framework, closely related to Rothschild and Stiglitz (1976). In this simplified environment, the equilibrium admits a closed-form characterization when lenders are homogeneous. Avoiding numerical methods sharpens the intuition, clarifies the mechanisms at work, and makes transparent how functional-form assumptions–such as demand elasticities and participation constraints–shape the model's predictions.

I consider an economy with two borrower types, indexed by B and G. Type B has a higher willingness to pay (i.e., $\theta_B > \theta_G$) and a higher default probability, denoted d_B. For simplicity, I assume that borrowers and lenders use the same discount rates $(1 - d_i)$. As a result, as in Rothschild and Stiglitz (1976), the market exhibits adverse selection: high-willingness-to-pay borrowers are more likely to default. Finally, I assume that lending

13. Their micro-foundation focuses on a perfectly competitive environment (i.e., $\sigma \to \infty$) without menus.

generates positive net present value for both types, implying that, under the first-best, both borrower types are served.

Borrower utilities are assumed to be linear in both X and R. On the cost side, marginal lending costs are constant and equal to mc for $X < \bar{X}$, and infinite for $X \geqslant \bar{X}$. These linearity assumptions are adopted for expositional convenience and do not affect the underlying economic intuition.

A more general analysis is provided in Appendix J, where I relax the assumptions about the number of borrowers and the functional forms. Issues with multidimensional screening (i.e., when the number of borrowers and contract characteristics is higher than two) are discussed in Section 6.2. As shown, for instance, in Rochet 2024 or Carroll 2017, characterisation of the equilibrium contracts in the multidimensional case requires, even in stylised cases, additional strong assumptions about the joint distribution of preferences.

5.2.2 Perfect Information Case

To gain intuition about the economic forces driving lenders' decisions, I first start by solving the perfect information case. Formally, I assume that lenders observe all borrower characteristics except for the demand shocks (i.e., the parameters (Θ_i, d_i) are observable but ε_{jb} are not). Because lenders do not observe ε_{jb}, the demand functions are continuous, which simplifies the analysis.

Using the first-order conditions of model 5 without the incentive compatibility constraints, we have that lenders set contract characteristics (X^{PI}) to maximise social surplus. We thus have X equal to $\bar{X}$ (equation (6)). Lenders use the interest rates to split the surplus between borrowers and lenders (equation (7)). How the surplus is split depends on the demand semi-elasticity (denoted $\varepsilon_i^{-1} := \frac{D_{ib}(u_i)}{\partial_{u_{ib}} D_{ib}(u_i)}$): when lenders offer products that are close substitutes (e.g., σ high), they have low market power and thus equate the loan present value to the contract marginal cost ($mc(X_i)$).

The optimal contract terms (X^{PI}, R^{PI}) are characterized by:

$$\text{Contract Terms}: \quad \overbrace{X_i^{PI}}^{\text{Equilibrium Loan Size}} = \bar{X} \tag{6}$$

$$\text{Pricing}: \quad \underbrace{(1-d_i)R_i^{PI}}_{\text{Loan Present value}} = \underbrace{mc(X_i^{PI})}_{\text{Break even price}} + \underbrace{\varepsilon_i^{-1}}_{\text{Markup}} \tag{7}$$

Expressing the debt face value as in equation (7) is useful for two reasons. First, when the demand shocks (ε_{jb}) are independent and extreme value distributed, the (inverse) semi-elasticity is constant in a symmetric equilibrium and takes the closed-form value $\frac{1}{\sigma_i\left(1-\frac{n_i}{|B|}\right)}$, so the right-hand side depends only on deep parameters.[14] Second, written in this form, the first-order condition maps directly into the regression specification used in the empirical analysis.

5.2.3 Imperfect Information Case

I now turn to the imperfect-information environment, in which lenders do not observe borrowers' characteristics, including $(\Theta_i, \varepsilon_{jb})$.

How screening is implemented: Under perfect information, lenders choose the same loan size X for both types but charge different interest rates. If these perfect-information contracts were offered as a menu, all borrowers would strictly prefer the lower-rate contract, so the allocation would violate incentive compatibility. To induce self-selection, lenders can exploit the fact that high-default borrowers also have a higher willingness to pay, $\theta_B > \theta_G$. They can screen by offering a menu with two contracts – one contract with a lower X and one with a higher X. Sorting is achieved by setting the price of the high-loan contract so high that it is too expensive for the low-willingness-to-pay borrower only (i.e, $X_B \geqslant X_G$).

Monopoly case: When demand $D_{ib}(u_{ib}; u_{-ib})$ is a constant when $u_{ib} \geqslant u_i(0,0) = \bar{u}$ and equal to zero otherwise, the model becomes similar to the textbook principal agent model of

14. I define the inverse semi-elasticity as $\varepsilon^{-1} := \frac{D_{ib}(u_{ib})}{\partial_{u_{ib}} D_{ib}(u_{ib})}$. With logit demand, $\varepsilon^{-1} = \frac{1}{\sigma_i\left(1-\frac{n_i}{|B|}\right)}$ in a symmetric Nash equilibrium, where n_i denotes the number of borrowers in segment i and $|B|$ the number of lenders.

screening under a monopolistic lender (see, for instance, Tirole 1988). This demand structure arises in my model in the limit $\sigma \to 0$ when the demand-shock terms (ε_{jb}) are extreme-value distributed and independent.

We now analyze screening incentives under monopoly. As in Tirole (1988), it is optimal to offer borrower B the first-best loan size and to set the interest rate R_G so that borrower G receives utility equal to his outside option $u_G(0, 0; \theta_G)$. The incentive-compatibility constraint of borrower B is binding. The monopolist can satisfy this constraint either by lowering the loan size offered to borrower G or by reducing the interest rate charged to borrower B (i.e., providing them an information rent).

The lender therefore faces the following trade-off. Reducing credit rationing and increasing profits in the G segment requires granting an information rent to B borrowers, which in turn lowers profits in the B segment. Rewriting the first-order conditions of problem (5), one can show that pooling is optimal whenever:

$$IR_B := \frac{D_G(u_G)\, S'_G(X_{Gb})}{D_B(u_B)\, (\theta_B - \theta_G)\, v'(X_{Gb})} > 0 \tag{8}$$

The term IR_B has a natural interpretation. The numerator captures the benefit of granting an information rent: it allows the lender to increase lending to G borrowers, generating an additional marginal surplus $S'_G(X_G)$ for each of the D_G borrowers in that segment. The denominator captures the cost of this rent: the informational rent $(\theta_G - \theta_B)v'(X_B)$ must be paid to all B borrowers, yielding a total cost of $D_B(u_B)(\theta_G - \theta_B)v'(X_B)$.

The following paragraphs generalise this analysis to all degrees of competition. It shows that perfect competition pushes lenders toward marginal-cost pricing in each market segment. Competitive pressure thus limits lenders' ability to use interest rates to manage incentives to self-select, forcing them to rely on credit rationing (X_G) to sort borrowers. This does not necessarily hold in Rothschild and Stiglitz (1976), because of profitable deviations that rely of cross-subsidizing market segments (i.e., pooling). Concave profit functions eliminate those deviations (Theorem 1), I provide intuition for why this restriction works. I also show that, as competition weakens, lenders recover pricing power, allowing them to use cross-subsidies to screen. The amount of cross-subsidy is proportional to IR_B.

Imperfect information contracts: Let us now analyse the contract terms formally. Rearranging the first-order conditions of problem 5 and using the fact that, as in Rothschild and Stiglitz (1976), the incentive compatibility constraint of the B borrower is binding (denoted IC_B, see Appendix G.2) for a proof), the contracts for an interior solution (i.e., for $u_{ib} \geqslant u_i(0,0)$ and $X_G < \bar{X}$) are:

$$\textbf{No distortion at the top:} \quad X_B = \bar{X} \tag{9}$$

$$\textbf{Credit rationing at the bottom } (IC_B)\textbf{:} \; X_G = \bar{X} - \frac{R_B - R_G}{\theta_B} \leqslant \bar{X} \tag{10}$$

$$\textbf{Pricing:} \quad \underbrace{\alpha_i R_i}_{\substack{Loan\ Present\ Value}} = \underbrace{mc(X_i)}_{\substack{break\ even\ price}} + \underbrace{\varepsilon_i^{-1}}_{\substack{Perfect\ information\ markup}} + \underbrace{\varepsilon_i^{-1} IR_i}_{\substack{Information\ discount/premium}} \tag{11}$$

With $IR_i = I \cdot [\mathbf{1}_{i=G} - \mathbf{1}_{i=B} \frac{D_{Gb}}{D_{Bb}}]$, $I := \frac{S'_G}{(\theta_B - \theta_G) v'_G}$, and $S'_i := (1-d_i)\theta_i - mc_i,\ i \in \{G, B\}$. As in the previous section, in a symmetric Nash equilibrium, the semi-elasticities are $\varepsilon_i^{-1} = \frac{1}{\sigma_i(1 - \frac{n_i}{|B|})}$ and the demand is $D_{ib} = \frac{n_i}{|B|}$ for each bank.

Equations (9) and (10) mirror the classic results of Rothschild and Stiglitz (1976). Equation (9) restates the "no distortion at the top" result. Moving X_B away from $\bar{X}$ would reduce the surplus generated by the loan and does not help with self-selection. Equation (10) corresponds to the incentive-compatibility constraint for type B. It states that sustaining self-selection requires a distortion in loan size that is tied to the interest-rate wedge across contracts: the extent of credit rationing, $\bar{X} - X_B$, increases with the spread $\frac{R_B - R_G}{\theta_B}$. The lender thus faces a similar tradeoff as the one discussed in the monopoly case, but now they also have to take competition into account.

The pricing equation (11) is a key novelty of the paper. It provides a transparent characterization of pricing incentives under asymmetric information and delivers a moment condition that can be used to recover model parameters in the empirical application. This closed-form expression relies on the continuity of demand implied by the discrete-choice structure.

To sharpen intuition, I decompose equation (11) into three components.[15] The first two terms coincide with the perfect-information benchmark (equation (7)): the marginal cost of

15. As shown in Appendix J, the same decomposition extends to the general environment.

lending, $mc(X_i)$, and the perfect-information markup, ε^{-1}. The third term, $\varepsilon^{-1}IR_i$, captures an information discount (or premium).

Let us now analyse the expression $\varepsilon^{-1}IR_i$ to gain insight into screening incentives. First, $\varepsilon^{-1}IR_i \to 0$ when $\sigma \to \infty$. The intuition is that lenders must price each market segment at marginal cost to avoid losing customers to competitors. In that case, the lender must thus maintain incentives to self-select using quantities rather than prices. This result is not specific to the functional form used and relies on the profit function being convex (see Appendix G.3.3 for the convergence result in the general model). The next section provides intuition for why this result holds.

Second, IR_i is negative for the high-default type B and positive for the low-default type G. It implies that lenders optimally compress markups in the B segment while raising markups in the G segment relative to the perfect-information pricing rule. Intuitively, shifting markups in this way helps sustain self-selection by making the G contract less attractive to type-B borrowers (as shown in equation (11): $\bar{X} - X_B = \frac{R_B - R_G}{\theta_B}$). This distortion in the G market segment does not appear in monopoly models, as the G borrower is at the participation constraint.

Third, the term IR_i depends on $I := \frac{S'(X_G)}{(\theta_B - \theta_G)v'_G(X_G)}$, which captures the key trade-off discussed in the monopoly section. Recall that lenders seek to maximize surplus while preserving incentives for self-selection (see Problem 5). Holding the number of borrowers fixed, increasing surplus in market G – that is, raising $S'(X_G) \cdot n_G$ – requires relaxing incentive constraints by granting a larger information rent, proportional to $(\theta_B - \theta_G)v'(X) = (\theta_B - \theta_i)$. The key difference with the textbook monopoly model is that this information rent can be delivered either by lowering the interest rate in the G segment (in which case it is paid to n_G borrowers) or by lowering the interest rate in the B segment (in which case it is paid to n_B borrowers). The later explains why the ratio $\frac{n_G}{n_B}$ multiplies I in $IR_B = -I \cdot \frac{n_G}{n_B}$. IR_i thus captures the relative cost of these two ways of "financing" the information rent.

Level of credit rationing: Let us now rewrite equation (10) using equation (11) to get

the level of credit rationing:

$$X_G = \bar{X} - \frac{R_B - R_G}{\theta_B} \tag{12}$$

$$= \overbrace{\frac{1}{S'_{BG}}[S'_B\bar{X}}^{perfect\ competition\ distortion\ level} + \underbrace{\frac{\varepsilon_G^{-1}}{\alpha_G} - \frac{\varepsilon_B^{-1}}{\alpha_B}}_{competition\ channel\ (:=CC)} + \overbrace{\frac{\varepsilon_G^{-1} IR_G}{\alpha_G} + \frac{\varepsilon_B^{-1}|IR_B|}{\alpha_B})}^{cross-subsidy\ channel\ (:=CM)} \tag{13}$$

With $S'_{BG} := (1 - d_G)\theta_B - mc_G$, where mc_G is the marginal cost of lending X_G.

The same three elements as in the pricing equations (11) are present in equation (13). $CC := \frac{1}{S'_{BG}} S'_B \bar{X}$ captures the level of credit rationing if lenders price each borrower at their marginal costs. $CM := \frac{\varepsilon_G^{-1}}{\alpha_G} - \frac{\varepsilon_B^{-1}}{\alpha_B}$ is the relative perfect information markups and $\frac{\varepsilon_G^{-1} IR_G}{\alpha_G} + \frac{\varepsilon_B^{-1}|IR_B|}{\alpha_B}$ captures the impact of information discount and premiums on credit rationing.

5.3 Positive Analysis of the Equilibrium

In this section, I analyze the properties of the equilibrium contracts. I derive two comparatives statics: one with respect to the demand semi-elasticity (parametrized by σ), the other with respect to marginal costs (mc_i, refers to the marginal costs of the i contract). The results are summarized in Figure 3. For simplicity of the exposition, I consider the case in which $\sigma_G = \sigma_B$ so that I do not have to carry different σ_i in the notation.

Comparative statics: Competition.

(i) The equilibrium is separating when $\sigma \to \infty$, each lender break even on each borrower type.

(ii) As long as $u_i \geqslant u_i(0,0)$, a decrease in competition (i.e., σ decreases) decreases the amount of credit rationing when the cost of screening is low (i.e., when $\partial_{\sigma^{-1}}(CM + CC) > 0$, CC and CM defined in equation (13)).

The condition $u_i \geqslant u_i(0,0)$ holds when demand elasticities are large enough, that is when σ_i^{-1} is lower than $\bar{\sigma} := \frac{S(X_i) - u_i(0,0)}{1 + IR_B}$.

Proof: See Appendix G.4

The first result holds in the more general model when the profit functions satisfy the conditions of Theorem 1, the market is adversely selected and preferences are well ordered (see Appendix G.3.3 for the convergence result in the general model).

The following discussion and figures 1 and 2 provide intuition for the result. I plot the borrower indifference curves (blue for type B, green for type G) and the lenders' break-even conditions for each segment (light grey lines). To simplify the graphical exposition, I assume a fixed origination cost, i.e., $mc(X) = f \in \mathbb{R}_+$, so that the break-even loci are horizontal.

As in Rothschild and Stiglitz (1976), let us proceed in two steps. First, let us notice that a competitive equilibrium in which borrowers choose the same contract cannot exist. Indeed, consider lenders break even on their pooling contract: they make profits on G borrowers and losses on B borrowers. Such a contract cannot be offered in equilibrium as a competitor can make profits by attracting only the profitable G borrowers ("cream-skim"). Figure 1 illustrates this point. Second, let us now consider separating contracts. In Rothschild and Stiglitz (1976), the separating contracts cannot be an equilibrium when the number of low default borrowers is high. The reason is that, in that case, there exists a pooling contract that is profitable and makes all borrowers better off (the CP pooling contract in figure 2).

In my model, the concavity assumption prevents the above mentioned deviation from screening from being profitable. To gain intuition, consider the market is close to perfect competition and that screening contracts are offered. Assume that the number of low-default borrowers is small, so a lender can try to exploit a deviation from screening. Let us first consider local deviations. Lender can implement a small increase in the loan size for a low-default borrower (i.e., moving toward pooling contracts), and extract some additional profit from it. However, to maintain incentives, the lender must also provide an informational rent (i.e., a lower rate) to high-default borrowers, thereby reducing their profits on this market segment. Due to the demand shocks, small deviations from screening do not attract the full market. Furthermore, as this type of deviation is more valuable to high-default borrowers(in figure 2, CP is on the G borrower indifference curve but on a higher indifference curve than CB for the B borrower),[16] they will be disproportionately attracted first, making it

16. This is because high default borrowers have a higher willingness to pay for loan size. Thus, the

unprofitable. As the size of the deviation increases (and approaches the Rothschild and Stiglitz (1976) pooling contracts), the fraction of good borrowers attracted rises faster than the proportion of bad borrowers. Yet due to the concavity assumption, the lender's profit declines as the cost of serving borrowers rises.

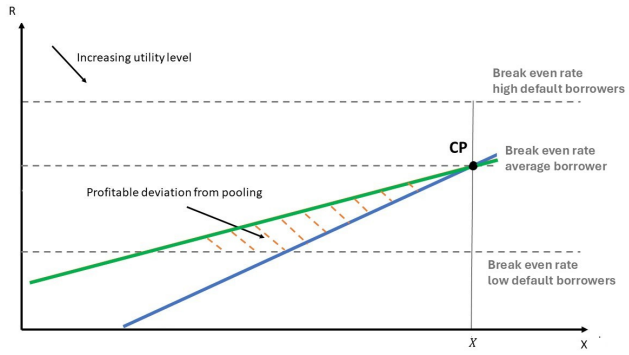


Figure 1: Perfect competition case: Deviation from pooling always exists

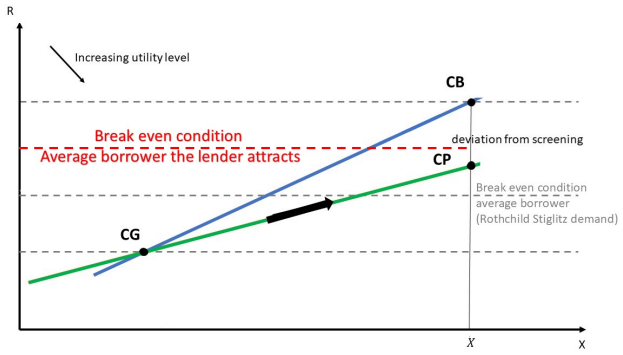


Figure 2: Perfect competition case: Deviation from screening

The second result on changes in σ can be derived from equation (13). The intuition is the following. When borrowers become less price elastic, lenders have incentives to increase interest rates R for both contracts to make more profits. However, as shown in equations (10) and (13), if lenders want to increase the B contract interest rate relatively more than the G contract one, they will have to credit ration G borrowers to maintain incentives to self-select. Credit rationing decreases lenders' profits on the G market segment due to both an intensive and an extensive margin. Lenders thus face a trade-off: if they want to extract more from B borrowers, they have to forgo profits on the G market segment.

Changes in σ thus affect credit rationing by changing the relative surplus that can be extracted from market segments (captured by the relative demand semi-elasticities CC), and providing lenders with the possibility to cross-subsidise (captured by CM). When $CC > |CM|$, relatively more surplus can be generated in the G market segment, and thus, credit rationing decreases as σ goes down. This condition is more likely to happen when the surplus generated by one additional dollar lent in the G market segment is high (S_G

information rent increase they receive to maintain self-selection is higher than the utility the low-default borrower gains by having a higher loan size.

high) and when the cost of screening is low (i.e., the information rent required to maintain incentive $\Delta\theta$ is low) so that CM is low. This decomposition of the effect into CC and CM highlights that seemingly innocuous parametrization of the demand impacts the strength of those two channels. This is particularly relevant for counterfactual simulations where the functional form assumptions are used to extrapolate the behavior of the economy away from the equilibrium observed in the data.

Let us now discuss the third result. When the outside option of borrowing is the same for the two borrower types ($u_i(0,0)$), the constraint for the low-default borrower (i.e., G borrowers) is binding first. This happens when σ is low enough (i.e., lower than $\bar{\sigma}$). Thus, if $\sigma \leqslant \bar{\sigma}$, lenders cannot increase the rate on the G contract to extract more surplus as σ decreases. In that case, a decrease in σ leads to more credit rationing as lenders are incentivised to distort the G market segment to make more profits on the B market segment.

Comparative statics: Marginal cost of lending.

Increasing the marginal cost of lending of low loan contracts decreases the credit rationing level and interest rate spread under high competition (i.e., $\sigma \geqslant \bar{\sigma}_{mc} := \frac{\alpha_G \partial_{mc_G} CC + CM + CC}{S'_B \bar{X}}$). The effect goes in an opposite direction under low competition.

Increasing the marginal cost of lending of high loan contracts increases the amount of credit rationing.

Proof: See Appendix G.4

To analyze the result on the marginal costs, I decompose the effect into a perfect competition and an imperfect competition channel as in equation (13).

Let us consider a situation in which the marginal cost of the G contract increases. Under perfect competition, the cost has to be passed through to interest rates. As a result, the incentive compatibility constraint is relaxed and lenders can increase the loan size of the G contract.[17]

Under imperfect competition, there is an additional effect. The increase in the marginal

17. Increasing the marginal cost of B contracts has the opposite effect, as it tightens the incentive compatibility constraints. When both marginal costs increase, the second effect dominates as interest rates increase relatively more in the B contract because they are scaled up by default probabilities.

cost decreases the surplus of lending via the G contract. As the potential surplus for G contracts decreases, lenders have less incentive to provide an information rent to B borrowers in order to lower the credit constraint on G contracts.

Implication for banking: Banking policies often affect the marginal cost of lending but are not analyzed through the lens of screening models. Different policies can be affected depending on the contract characteristics used to screen. For instance, in markets where lenders screen with maturity (see Hertzberg, Liberman, and Paravisini 2018 for empirical evidence), our model implies that the effect of quantitative easing (QE) likely depends on the degree of competition. This is because maturity plays the same role as interest rate in the model (it affects the loan present value), and that QE affect marginal costs of originating contracts with different maturity. In markets where lenders screen with Loan-to-Values (see Taburet, Polo, and Vo 2024 for empirical evidence), then capital requirements are the relevant policy. LTV plays the same role as loan size in the model, and capital requirements are often LTV-specific.

Summary: I summarise the result of the two comparative statics in figure 3. When the marginal cost increases relatively more for the G contract, the equilibrium region moves from the dotted lines to the solid lines.

5.4 Normative Analysis of the Equilibrium

In this subsection, I show that a contractual externality may cause excessive screening. This implies that there is room for policy interventions. I show how the implementation of Government Sponsored Enterprises like Fannie Mae and Freddie Mac, or lowering competition, can mitigate the externality.

Contractual externality: To get a lower bound on the contractual externality, I show that there exists a menu of contracts that makes all borrowers and lenders better off. I do so by solving for the optimal contract of an informationally constrained social planner. The social planner maximizes lenders' profits subject to the same incentive compatibility

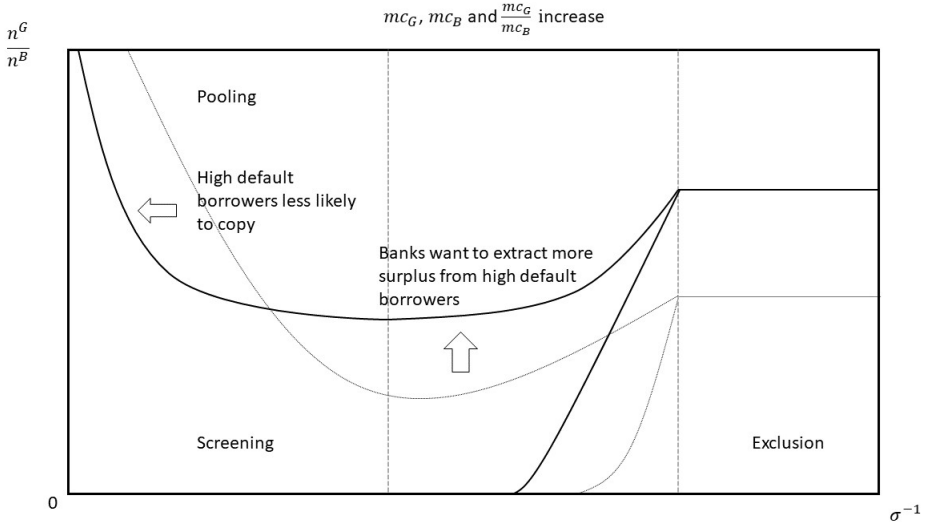


Figure 3: Equilibrium regions when marginal costs increases

constraints as each lender. However, the social planner faces an additional constraint: they must provide at least as much utility to borrowers as the market contracts for a given competition level (denoted (u_B^{eq}, u_G^{eq})). Formally, the problem of the social planner is the same as (3) but with constant market shares (n_i) and the outside option being u_i^{eq} instead of $u(0,0)$.

The key advantage of the social planner with respect to the market is that they internalise that cream-skimming can be ex-post inefficient. They are thus able to offer pooling contracts.

Using the first-order conditions, I derive the following proposition:

Sufficient Statistic for the Contractual Externality.

Banks should pool borrowers together at $\bar{X}$ if and only if:

$$\frac{\alpha_G \theta_G - mc_G}{(\theta_B - \theta_G)} \frac{n_G}{\alpha_B n_B} > 1 \tag{14}$$

However, except under monopoly, the competitive equilibrium switching point is different than the social planner one (14).

The second best can be implemented by having a Government Sponsored Entreprise buy

the loan at its present value and set G-fees (i.e., choose the marginal costs mc_B for loan with characteristic $\bar{X}$ and mc_G for loan with characteristics below $\bar{X}$). Denoting (L_G^{eq}, R_G^{eq}) the decentralised equilibrium contracts for the G borrower. The marginal costs (mc_G, mc_B) must be set such that:

$$mc_G : mc_G(\bar{X}) + \varepsilon_G(1 + IR_G) = \theta_G[\bar{X} - L_G^{eq}] + R_G^{eq} \tag{15}$$

$$mc_B : X_G = \bar{X} \text{ with } X_G \text{ defined in equation 13} \tag{16}$$

Proof: See Appendix H.1.

The above results show the conditions under which the equilibrium is inefficient in the second-best sense. The friction emerges for the following reasons. Maintaining customers' incentives to self-select is costly as it may require distorting contracts relative to the first best. When the distortions are too high, it is more efficient to pool borrowers instead. Yet, if pooling contracts are offered and competition is high, a competitor can take advantage of a pooling contract by introducing a product that steals the most profitable customers only (cream-skimming). In short, the friction arises because lenders do not internalize how their screening strategies (e.g., cream-skimming) change the types of borrowers selecting competitors' products – and thus the cost of lending via those products.

This friction exists only when pooling is a Pareto improvement over screening so that cream-skimming deviations are ex-post-inefficient. Equation (14) captures the condition for the latter to be true. The numerator is the benefit of increasing X_G while the denominator represents the costs. Increasing the loan size allows the social planner to generate more profits $(\alpha_G\theta_G - mc_G)$ per G borrowers. However, to maintain incentives to self-select, lenders have to decrease the interest rate on the B market segment, leading to a loss of $\alpha_B(\theta_B - \theta_G)$ per B borrower.

Equation (14) can be used to test for the existence of the contractual externality. For instance, one can use a revealed preference approach to recover borrowers' willingness to pay and relative contracts' shares to get the relative numbers of borrowers. The marginal cost can be recovered either using accounting data or by estimating it using the interest rate first order conditions as standard in the industrial organisation literature.

Effect of Competition and marginal costs on welfare

Decreasing competition or increasing the marginal cost of lending via low loan size contracts can lead to a Pareto improvement when the number of high default borrowers is low.

Proof: Appendix H.2

The intuition behind the above result is that the changes in marginal costs or competition can lower the incentives to implement cream-skimming deviations. For instance, decreasing competition can make G borrowers better off by lowering their credit constraint at the cost of a higher markup; it makes B borrowers get a higher information rent but also a higher markup. Finally, a decrease in competition allows lenders to extract more surplus from their customers. When the number of high default borrowers is low, the positive effects dominate for each agent. Figure 4 summarises the set of parameters for which decreasing competition is a Pareto improvement: the number of high borrowers must be relatively large, and competition high.

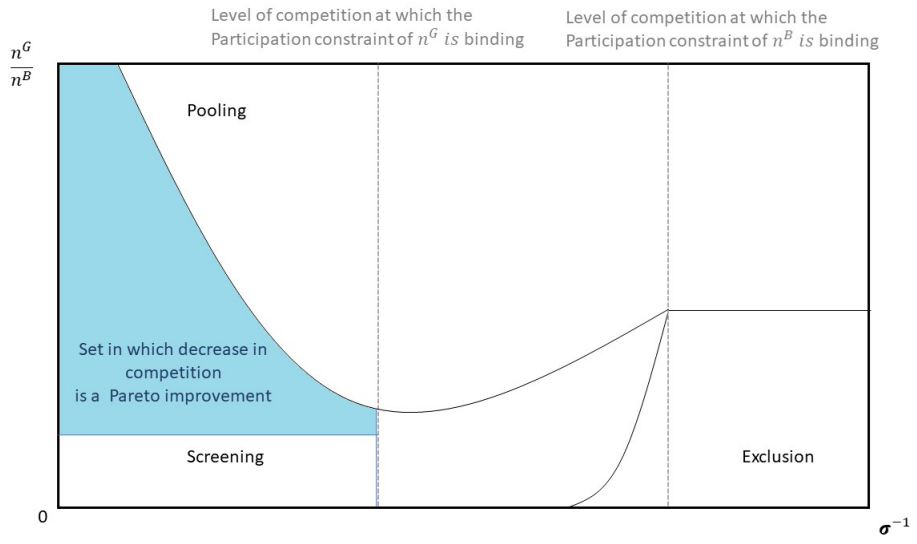


Figure 4: Pareto set

6 Extensions and Bringing the Model to the Data

6.1 Arbitrary Number of Borrowers and General Utility Function

Appendix J analyzes the general case with utility functions satisfying Theorem 1, an arbitrary number of borrower type and contracts featuring two terms (i.e., $dim(\theta) > 1$).

The intuition remains the same as in the two type case: lower willingness to pay borrower get credit constrained and pay an asymmetric information premium due to high willingness to pay borrower.

6.2 Multidimensional Screening: $dim(\theta) > 1$ and $dim(\Gamma) > 2$

Characterizing the equilibrium is challenging when the number of contract terms and the number of agents' types are greater than two (i.e., $dim(\theta) > 1$ and $dim(\Gamma) > 2$).[18] In that case, the set of incentive compatibility constraints that are binding is endogenous to contract terms, which leads to tractability issues (see, for instance, Rochet 2024 or Carroll 2017).

As a result, the vast majority of the empirical and theoretical literature focuses on the situation in which $dim(\theta_i) = 1$ and endogenizes prices and at most one contract characteristic.[19] This assumption does not restrict other parameters (i.e., $(\Theta_i \backslash \{\theta_i\}, d_i,)$ to be heterogeneous conditional on θ_i.

The focus on one dimension is not as restrictive as it seems. Indeed, the recent theoretical literature on robust contract theory (see, for instance, Carroll 2017) showed that when the principal does not know the joint distribution of preferences and evaluates mechanisms according to the worst-case scenario, the optimum for the principal is to screen each component separately.

Alternatively, when utilities are linear, one can show that only one characteristic is used to screen between two borrowers. The characteristic is the one that is less costly to distort (i.e., the one that has the lowest shadow cost: $\lambda_{ij} := \frac{\partial_X S_j(X)}{\theta_i - \theta_j}$ where $\theta_i > \theta_j$ and $S_j(X)$ is the social surplus). When the characteristic that has the lowest shadow cost is the same for each pair of borrowers, then the problem boils down to a unidimensional screening situation

18. With each element of θ_i being random variables that are not perfectly correlated

19. See, for instance, Veiga and Weyl (2016) for an empirical model without menus, and Azevedo and Gottlieb (2017) for empirical applications, and Einav, Finkelstein, and Mahoney (2021) for a review.

as well.

6.3 Bringing the Model to the Data

There are two key options to recover the model parameters. The first one follows Berry, Levinsohn, and Pakes (1995). It consists of recovering the demand elasticity using a revealed preference approach and inverting the pricing equation to recover the marginal cost. The second approach is similar to Berry and Haile (2014): the model's first-order conditions are directly used as a moment condition to recover both the demand and the supply parameters.

Appendix K discusses in detail the identification and estimation of the model parameters for types of datasets commonly used in banking or insurance.

Section 7 is a working example of how to bring the model to the data.

7 Empirical Application

To illustrate how to bring the model to the data, I provide an empirical application using data from a key paper on screening in credit markets: Hertzberg, Liberman, and Paravisini (2018). I discuss the empirical setting in Section 7.1, how I adapt my framework to the setting in Section 7.2. Finally, I discuss identification and estimation as well as the results in Sections 7.3 and 7.4.

7.1 Data and Setting

I use data from LendingClub (hereafter LC), an online lending platform, in 2013. LC offers unsecured consumer loans with amounts ranging from $1,000 to $35,000. During this period, LC was the largest online lending platform in the United States, originating $4.4 billion in consumer loans across 45 states. Its closest competitor, Prosper Marketplace, originated $1.6 billion in loans in the same year.

The LendingClub dataset provides an ideal setting to study screening with contract menus, as it contains the full set of information observed by LC at loan origination. In particular, the data include the contract menu offered to each borrower, the borrower's contract choice, LC's internal credit risk assessment, and subsequent loan performance.

The borrowing process unfolds as follows. Before selecting a loan amount or maturity, LC assigns each applicant to one of 25 risk categories using a proprietary credit-risk assessment algorithm. The algorithm relies on hard information from the borrower's credit report – such as FICO score, outstanding debt, and repayment history – as well as reported income.

Within each assigned risk category, all borrowers are offered the same menu of contracts. Borrowers choose the loan amount and select a maturity of either 36 or 60 months, although for smaller loan amounts, only the shorter maturity is available.

7.2 Model Set-up

The model parametrisation is similar to that in Section 5: borrowers have linear utility over loan size and the present value of monthly payments; the lender's marginal costs of originating a loan are also linear in loan size.

For the model to be able to match the data, I consider the following extensions: I endogenise monthly payments (or equivalently, interest rates), loan sizes, and maturities; I allow default probabilities to depend on the monthly payment-to-income ratio; I also allow for an arbitrary number of borrowers and heterogeneous lenders.

The following paragraph discusses how I include those extensions while keeping the model tractable. First, I consider that each borrower i has a maximum monthly payment capacity mp_i.[20] I assume that conditional on the variables observed by the lender (i.e., the borrower risk category) and contract terms, the payment capacity mp_i is not correlated with default probabilities nor the demand elasticity parameter (σ). This assumption ensures that, as observed in the data, the contract interest rate does not depend on the amount borrowed – and thus mp_i. Second, I consider that default probabilities depend on contract characteristics through monthly payments only,[21] and increasing maturity is costly for the lender.

Given the linearity in preferences and the maximum monthly capacity assumptions, the optimal contract max out the borrower's monthly payment capacity (mp_i), as it is more efficient to minimize the maturity of the contract as much as possible. This modelling makes the model very tractable. The monthly payment mp_i being maxed out can be interpreted as a monthly payment targeting assumption. Payment targeting is consistent with empirical

20. This can be micro-founded by a wage process equal to zero with some probability and W otherwise. Borrowers default with probability one if the monthly payment exceeds their monthly payment capacity.

21. Any correlation between maturity and default is thus due to selection rather than moral hazard.

evidence (see Argyle, Nadauld, and Palmer 2020 for an example using the car loan market, Graham and Harvey 2001 or their CFO Survey).

With those assumptions, the model is similar to the one analyzed in Section 5: one element of the present value (i.e., the monthly payment) is maxed out, leaving the lender to use maturity and loan size to screen. Appendix M shows that formally.

I am agnostic about the number of types (θ_i, σ_i). Appendix (N) shows that the equilibrium conditions of the model with an arbitrary number of borrower types – but with LC choosing to offer two optimally – and the optimality conditions with two borrower types are identical when there is a discrete number of types.

7.3 Identification and Estimation

I recover the model parameters by exploiting the model's optimal pricing equations as moment conditions. The estimation proceeds in two steps. In Step 1, I construct an estimate of each loan's present value, which requires estimating expected default probabilities. Following standard practice in the literature, I proxy expected default with realized default outcomes (see, for example, Benetton 2018 and Crawford, Pavanini, and Schivardi 2018).

In Step 2, I regress the estimated present value of the loan on contract terms and interpret the resulting coefficients through the lens of the model to recover the structural parameters.

Appendix K discusses alternative approaches that can be used to recover the model parameters.

7.3.1 Default Probability Estimates (Step 1):

I start by calculating the average default probability on long and short-maturity contracts conditional on the borrower's investment grade and the monthly payment-to-income ratio (denoted PI_i). Formally, I run the following regression:

$$1 - Default_i = \delta_i^{gt} + \beta \frac{Montly\ Payment_i}{Income_i} + \beta^M \mathbf{1}_{\{T_i=60\ Month\}} + e_i \tag{17}$$

$Default_i$ is a dummy equal to 1 if the loan is reported as charged off after the loan should have been completely repaid. δ_i^{gt} are investment grade-time fixed effects. The superscript g

indexes the grade and t the time. $\frac{Montly\ Payment_i}{Income_i}$ is the monthly payment to yearly income ratio. I normalise the standard deviation to one. β^M is the coefficient of interest. This coefficient captures whether high default borrowers self-select into different loans.

Denoting λ_i the hazard rate, I can recover the model parameter ($\alpha_i \approx \lambda_i$) as the hazard rate:

$$\lambda_i := -\frac{log(E[1 - Default_i | g_i, T_i, \frac{Montly\ Payment_i}{Income_i}])}{T_i} \tag{18}$$

7.3.2 Marginal costs, Demand Elasticities, Asymmetric Information Premium and Discounts (Step 2):

To recover the model parameters, I adopt an approach closely related to the conduct literature (see, for example, Berry and Haile 2014). Specifically, I regress the loan's present value on contract terms and interpret the resulting coefficients through the first-order conditions of the structural model. Unlike Berry and Haile (2014), which uses this strategy to identify a conduct parameter, I exploit the same regression to recover the demand semi-elasticity, information rents, and the marginal cost of lending.

In practice, I exploit the model's first-order condition with respect to maturity in Model 7.2. This condition yields two equations—one for each maturity observed in the data. These equations involve the same objects as in the theoretical pricing condition analyzed in equation (11) of Section 5. The loan present value must be equal to the cost of originating the loan – which depends on the loan size – and a markup. The markup equals the inverse demand semi-elasticity times 1 plus the information discount or premium (IR_i). The information premium or discount depends on the contract's relative market share, which is directly observable in the data. As a result, the optimal pricing condition can be expressed as a linear regression of prices on quantities, controlling for relative market share. Variation in loan size allows for the recovery of the marginal costs (mc_i), while variation in relative market share identifies the equilibrium demand semi-elasticities times some part of the information rent (I).

Let me first present the regression model before discussing identification. For the high-maturity contract, I regress the present value of the loan on loan size, a constant, and the relative contract share ($\frac{n_{36}}{n_{60}}$), where 36 denotes the low-maturity contract. For the high-maturity contract, I regress the present value of the loan on the loan size and a constant. I parametrize the present value of the loan R as a linear approximation of the continuous

time annuity formula ($\rho R := mp[1 - e^{\rho\lambda T}] \approx \rho\lambda mpT$ where mp is the monthly payment, λ is the hazard rate and T is the maturity so that $\alpha mp\rho T$ is the approximate loan present value). All coefficients on the right-hand side should thus be interpreted as divided by ρ. This implies that the equation identifies the relative value of each component of the pricing equation.

Formally, I estimate the following two regressions for each month using ordinary least squares, where $(mc_{T_i}, \sigma_{T_i}, \beta, \alpha)$ are the parameters to estimate:

$$\overbrace{E[R_i|g_i, T_i = 60]}^{Present\ Value} = \overbrace{mc_{T_i}Ł_i}^{Break\ even} + \overbrace{\sigma_{T_i}}^{perfect information markup} + \overbrace{\beta\frac{n_{36g}}{n_{60g}}}^{Information\ rent:\ -\sigma_{T_i}\frac{n_{36g}}{n_{60g}}I_g} + e_i \quad (19)$$

$$\underbrace{E[R_i|g_i, T_i = 36]R_i}_{Present\ value} = \underbrace{mc_{T_i}Ł_i}_{Break\ even} + \underbrace{\tilde{\alpha}_{T_i}}_{Total\ markup:\ \sigma_{T_i}(1+IR_g)} + \tilde{e}_i \quad (20)$$

The right-hand side of equations 19 and 20 directly maps into the three elements discussed in Section 5: the marginal cost (mc), the perfect information markup (ε^{-1}), and the information rent ($\varepsilon^{-1}IR$). Abusing the notation, I denote σ the inverse demand semi-elasticity with respect to the loan present value in regression 19 instead of ε^{-1} in equation 11.

Once the parameters of equations 19 and 20 are estimated, I can recover the information premium and discount, and the demand semi-elasticity, the following way. The coefficient β is interpreted as the information premium: $\sigma_{60}I$. I can thus recover $\hat{I}$ as $\frac{\hat{\beta}}{\hat{\sigma}_{60}}$. The coefficient $\tilde{\alpha}$ captures the perfect information markup and the asymmetric information premium $\sigma_{36}[1 + IR]$. I thus recover $\hat{\sigma}_{36}$ as $\frac{\tilde{\alpha}}{1+\hat{I}R}$.

Identification: Because equations (19) and (20) are derived from the model's first-order conditions, they represent equilibrium relationships among endogenous objects. Accordingly, these equations should not be interpreted as identifying the causal impact of contract terms on the loan's present value.

However, using the first-order condition as a moment condition does not eliminate the need for exclusion restrictions. In practice, the econometrician may be unable to observe or include all the terms appearing in the pricing equation, or may lack sufficient variation to

flexibly accommodate the required coefficient heterogeneity. Those gaps between the theory and the data justify the presence of an error term in the regression. The following paragraphs discuss the identifying restrictions implied by these limitations.

To map equation (11) exactly into regression (19), the coefficient β would need to vary by maturity and risk category. However, the data do not provide sufficient variation to allow for such a flexible parameterization. As a result, the error term e_i should be interpreted as capturing unobserved heterogeneity in demand elasticities and information rents.[22] By contrast, it is unlikely that e_i captures heterogeneity in costs or unobserved product characteristics, as the marginal cost is allowed to vary flexibly at the contract–month level.

The above argument implies the following exclusion restriction for the coefficient β: changes in relative market shares, $\frac{n_{36g}}{n_{60g}}$, must be uncorrelated with the average demand semi-elasticity with respect to the loan present value. To address potential endogeneity concerns, I exploit variation in relative market shares across investment grades to estimate β. The identifying assumption is therefore that the average demand sensitivity σ_B of borrowers selecting 60-month maturity contracts is the same across risk categories g.

As the investment grade goal is to predict default (i.e., the borrower cost), not demand sensitivities, this assumption is plausible. Notice that, according to the model, I recover (σ) the (inverse) demand semi-elasticity with respect to the loan present value, not the elasticity relative to the interest rates. The reason is that the present value already incorporates subjective discount factors reflecting default probabilities. The concern that the exclusion restrictions may be violated because high-default borrowers are less sensitive to interest rate changes – owing to their lower expected repayment frequency– does not apply in this context.

A similar argument holds for the variation in loan size. Conditional on the market segment, the loan size variation should be uncorrelated with deviation from the average demand elasticities σ_{T_i}. To mitigate endogeneity concerns, I use a granular-instrument approach. That is, I instrument the loan size using the residual from a loan size regression, controlling for contract characteristics and observable borrowers' characteristics. The underlying assumption is that the search cost heterogeneity is uncorrelated with the residual demand. The results are very similar with and without the instrument. This is likely because the

22. Formally, e_i includes heterogeneity in demand sensitivities, $\sigma_i - \mathbb{E}[\sigma_i \mid g_i, T_i]$, as well as heterogeneity in information rents, for example $IR_i\sigma_i - \mathbb{E}[IR_i\sigma_i \mid g_i, T_i]$, where $IR_g := \frac{\frac{\theta_{36g}}{\alpha_{36g}} - mc_T}{\theta_{60g} - \theta_{36g}}$ in equation 20.

variation in loan size is uncorrelated with lender choices. This is expected if, for instance, all lenders would offer the same borrower similar loan size.

Discussion about the method and simulation: My estimation approach leverages the fact that the lender has superior information relative to the econometrician, and that this information is reflected in the contract terms. By putting more structure on the data, I can thus recover the model parameters without data on contracts offered by other lenders. This allows me to overcome the data limitation preventing me from directly estimating demand elasticities using a discrete choice model as in, for instance, Crawford, Pavanini, and Schivardi (2018).

Even if contracts chosen by borrowers going to other lenders were available to the econometrician (e.g., credit registry), my approach would still be useful as it relies on different exclusion restrictions than methods based on demand estimation. My approach is particularly useful in banking as direct estimation of demand elasticities is problematic in that setting. Indeed, the standard approach (see, for instance, Berry, Levinsohn, and Pakes 1995) requires data on borrowers' choice sets. As data on choice sets are not available in banking, one has to predict it using comparable borrowers (see Crawford, Pavanini, and Schivardi 2018). This step induces both measurement error and selection on unobservables. Appendix O formalises the identification concerns.

Compared to direct demand estimation approaches (Crawford, Pavanini, and Schivardi 2018), my method is more sensitive to misspecification of the demand function (i.e., demand semi-elasticity not being approximately linear around the equilibrium contract), but not to the definition of the choice set.[23]

I show, in appendix P, that in simulations in which I compare the performance of Crawford, Pavanini, and Schivardi (2018) and my approach when the econometrician does not know the borrower choice set, and show that my approach performs better. My approach thus trades off the bias coming from selection on unobservables and choice set mispecification for a demand functional form assumption.

My key specification assumption – that LC considers that the demand semi-elasticity is approximately constant around contract terms – is plausible. Indeed, standard functional

23. See, for instance, Benetton 2018 for a discussion on how including different products in the choice set leads to different results.

forms used in the literature satisfy this property: logit model when market share are small (or in a symmetric equilibrium), and regression of log market shares on prices.

Although not exploited in this empirical application, the linearity of the model allows the use of the full set of tools developed in the reduced-form literature, such as difference in difference. This is one main advantage of this model compared to a theory-oriented framework such as Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019). The simplicity of the model also makes the exclusion restrictions transparent, as the error term has a structural interpretation. A caveat of the approach is that results depend on the interpretation of the coefficient of the linear regression. Given that markup and adverse selection are not directly observable, putting some structure is, however, necessary.

7.4 Results

Table 1 reports the regression results of the default regressions. The average default probability is 15 percent. Borrowers that chose high maturity contracts are α is 4% more likely to see their loan charged off. The implied average hazard rate is $4 \cdot 10^{-3}$ and the one of borrowers with high maturity contract is $2.7 \cdot 10^{-3}$. The hazard rate allows taking into account the fact that with constant default probabilities, longer loans are more likely to be charged off.

The results of the present value regressions are reported in Tables 2 and 3. All coefficients are significant and have the sign predicted by the theory. Figure (5) provides a visual decomposition of the present value of the loan. All else equal, high-maturity contracts are more expensive to originate (the marginal cost is 1.5 per dollar lent instead of instead of 1.2). Borrowers that self-select into low-maturity contracts are less likely to default (from step 1). They are also very elastic, so they do not pay any markups. This is consistent with the fact that searching costs are very low in the online lending market (as in Agarwal, Grigsby, Hortaçsu, Matvos, Seru, and Yao 2024). Borrowers who self-select into high-maturity contracts are more likely to default (from step 1). They are less price elastic. Their perfect information markup would be 0.2 dollars per dollar borrowed. However, because of adverse selection, they get an information report that cancels this markup.

Overall, due to screening, the prices are close to the perfect competition and perfect information case. However, low-default borrowers get a lower maturity compared to what they

should get under perfect information. To understand whether there is too much screening, I use the social planner framework described in Section 5.4.



Figure 5: Decomposition of the present value of the average loan (normalized by loan size)

Sufficient Statistic. Using Proposition 1, I find that: $\frac{n_{36g}}{n_{60g}} I = 1.32 > E[\theta_i | g_i, T_i = 60] = 0.99$. This result implies that there is excessive screening. Due to the contractual externality, low default borrowers get a lower maturity than what they should get under perfect information but also lower maturity than what they should get in the second best scenario (informationally constrained social planner). Decreasing competition or increasing the marginal cost of lending via low-maturity contracts can thus lead to a Pareto improvement. The exact cost of the distortion requires extrapolating preferences away from their local estimates. Using the linear model assumption, low-default borrowers would prefer to go all the way to 60-month contracts.

Distribution of willingness to pay: To test if the inequality constraint assumption delivers reasonable results, I implement step 2 of Appendix K. I use equation (87) to recover the distribution of borrowers' willingness to pay. For each borrower, I calculate the implied loan size that would give them the same monthly payment if they were to change maturity. I consider only borrowers for whom the alternative combination of loan size and maturity is feasible, given the menus offered by LC.

For a given investment grade, the inequality allows me to recover one lower bound for the willingness to pay off high-maturity borrowers. I use the interest rate variation across borrower investment grades to recover the shape of the distribution of willingness to pay. The identifying assumption is that the distribution of marginal utility of loan size ($\alpha \sim G$) does not change with investment grades.

This result also implies that increasing the marginal cost of originating low-maturity contracts can lead to higher maturities being offered and lower levels of credit rationing. It is a welfare-improving policy.

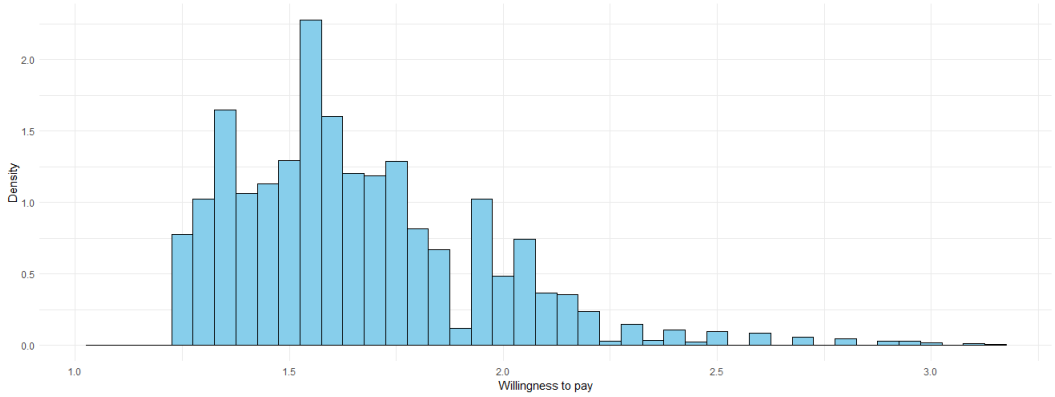


Figure 6: Borrowers' willingness to pay (all market segment pooled)

Figure (6) report the distribution of willingness to pay. The average willingness to pay is 1.6. Interpreting the value in terms of a two-period model, the results imply that for each dollar borrower in period 0, a borrower is willing to pay up to 1.5 dollars in period 2.

8 Conclusion

This paper develops and analyzes the properties of a new screening framework that can be brought to the data to study the contract term distortions that stem from screening.

The paper carefully analyzes the existence of equilibrium, one of the central theoretical issues of screening models. I show that the equilibrium always exists and characterize it in closed form.

The paper then brings the model to the data. The model's tractability allows for discussion of exclusion restriction and provides a structural interpretation of a linear regression of prices on quantity. I propose an approach that is robust to adverse selection and does not rely on estimating borrowers' choice set and default elasticities, which is problematic in the context of adverse selection.

The tractability of the model also opens the way to study a wide set of counterfactuals with endogenous menus of contracts. Existing models with endogenous menus, such as Wollmann (2018), use numerical methods that are computationally demanding and are often not robust to the starting point.

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A Tables

Table 1: Survival Probabilities

	Loan is fully paid at maturity
Term: 60 months	-4***
	(0.0118)
Monthly payment to income ratio	-2***
	(0.01445)
Subgrade fixed effect	Yes
Average number of defaults at maturity	15

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

Table 2: Present value regression (60-month maturity)

	Present Value
Intercept	$4e^{3***}$
	$(1.5e^{2})$
Loan Size	1.5***
	$(5.5e^{-3})$
Market share ratio	-9.3 e^{3***}
	$(1.3e^{2})$
Average present value	34,438
R^2	0.98

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

Table 3: Present value regression (36-month maturity)

	Present Value
Intercept	82**
	(25.4)
Loan Size	1.2***
	($1.5e^{-3}$)
Average present value	16,940
R^2	0.98

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p $<$ 0.01, **p $<$ 0.05, *p $<$ 0.1

ONLINE APPENDIX

B Example of Micro-foundation

This section introduces a micro-foundation for the borrower's utility function.

The following example, provides insight into interpreting the quantity $E[\alpha_i^B(d_i)|\theta_i]R_{ib}$ and the correlation between default (d_i) and borrowers' willingness to pay (θ_i). Let us consider that the borrower's utility is increasing in loan size (X) quasi-linear in the loan present value. With an exogenous default probability of d_i, a discount rate of β_i and a bullet payment of R, the loan present value is: $\beta_i(1-d_i)R$. The borrower utility is thus $\gamma_i v(X) - \beta_i(1-d_i)R$, where $\gamma_i v(X)$ is the utility derived from the loan. Lenders would like to know $(1-d_i)R$ but cannot observe $(1-d_i)$. However, they can improve their prediction of the borrower default probability parameter d_i using a menu. Indeed, the lender can leverage the idea that if a borrower chooses a contract with a loan size of \$1,000 while they had access to a contract a loan size of \$1,100 for an interest rate increase of, say, 100 bps, it must be that their willingness to pay for loan size is lower than 100 bps. Lender can then use the fact that borrowers' willingness to pay (θ_i) is informative of default probabilities, and learn about their correlation using realized default data. For instance, using our example, we have $\theta_i = \frac{\gamma_i}{\beta_i(1-d_i)}$, so the lender can infer d_i from θ by inverting the equation (conditional on $\frac{\gamma_i}{\beta_i}$).

To fix ideas, let us interpret the contract as a mortgage. In period 0, the borrower chooses a contract (L, m, M) that maximizes their utility. L is the loan size, m is the monthly payment value, and M is the maturity. The borrower gets utility $\alpha_i H(L)$ from housing each period until time T as long as they do not default. After the loan is originated, the borrower also derives utility from consumption ($v(c_t)$). They receive income Y_t each period $t > 0$, and must make scheduled payments of m until maturity M. The borrower may be subject to a permanent income shock that drives their income to zero permanently. The shock can happen each period with probability $d_i \in [0, 1)$.[24] In case of default, borrowers loose the house and gets utility $\alpha_i H(K)$ from housing and $v(0)$ for consumption until date T. For simplicity of the notation, I normalise $(\alpha_i H(K) + v(0))$ to zero.[25]

24. This modelling approach aligns with Hertzberg, Liberman, and Paravisini (2018), who, in the context of consumer credit, demonstrate that borrowers self-select based on private information about their income processes, rather than on differences in risk aversion. It is also supported by evidence from the Bank of England's NGM survey, which identifies the primary causes of default as increases in other bills or unexpected expenses (21 percent), and job loss, unemployment, or wage reductions (67 percent).

25. To isolate the screening mechanism, the model abstracts from modelling strategic default in the repayment period by considering that lenders can verify whether a borrower has experienced a genuine shock in

Denoting the survival probability $\delta_i := (1 - d_i)$, the expected utility derived from the contract is:

$$u_i(L, K, m, M) = \alpha_i H(L) + \sum_{t=1}^{T} \delta_i^{t-1} \{(1 - d_i)[\alpha_i H(L) + v(c_t)] + d_i[\overbrace{v(0) + \alpha_i H(K)}^{=0}]\} \quad (21)$$

$$with\ c_t = Y_t - m_t$$

Considering v is linear (i.e., $v(Y - R) = \beta_i(Y - R)$), we have:

$$u_i(L, K, m, M) = \alpha_i H(L) + \frac{1 - \delta_i^T}{1 - \delta_i}[\alpha_i H(L) - (1 - d_i)\beta_i Y] - \frac{1 - \delta_i^M}{1 - \delta_i}(1 - d_i)\beta_i R] \quad (22)$$

Linearising u_i around default probabilities equal to zero, we have:

$$u_i(L, K, m, M) \approx \alpha_i H(L) + T[1 - (T - 1)d_i]\alpha_i[H(L) + (1 - d_i)\beta_i Y] - M[1 - (M - 1)d_i](1 - d_i)\beta_i R \quad (23)$$

Denoting $R = Mm$ and $X = (L, M)$, the utility (23) fits into the general functional form defined in Section 4.1.

Linearizing the present value expression allows borrower type and loan maturity to enter utility in a multiplicatively separable form. This functional form is desirable, as it ensures that the model remains well-behaved when loan maturity is endogenized. The linearization is not required when one does not want to endogenize maturity.

The approximation error introduced by this linearization is small in settings where default probabilities are low, as is typically the case in most empirical applications. For example taking standard values for the mortgage market (a thirty-year maturity, constant monthly payments, and a yearly default probability of 1 percent) the linear approximation results in an error of approximately 2 percent relative to the true present value.[26].

Appendix F discuss the modeling choice on default and discuss how to endoenize it.

the second period. In practice, strategic default may be limited due to the use of recourse loans or credit score penalties following default.

26. The approximation error does not depend on the collateral and monthly payment amounts

C Borrower Participation in the Borrowing Market

The model is flexible enough to allow for borrowers' entry and exit decisions. There are several modelling options. I discuss here the pros and cons of each.

First, one can include the option of not borrowing directly in the set of banks the lender can borrow from (i.e., $\mathbf{B}$, defined in equation 1). Although the bulk of the analysis is maintained with this hypothesis (existence of the equilibrium, characterization of the contracts), this modeling does not allow for a closed form characterization of the equilibrium and does not flexibly separate the borrower entry margin from the choice of bank margin.

As a result, I use the following approach in the main theoretical analysis of the equilibrium contracts. Formally, I model exit inside the set of contracts offered by banks (i.e., all $\mathcal{M}_b$ sets contain the contract $(0,0)$). The advantage of this is that it makes the participation independent of ϵ_j, which makes the model more tractable and separates the intensive and extensive margins.[27]

The economic justification for this modeling is based on the following interpretation of the ϵ variable as transport costs. Before getting a loan, the borrower j of type i first gets their transport cost drawn. The draw implies a certain ordering of contracts as in equation (24). The borrower then chooses to go to the bank (denoted b_0 as in equation 24) to get their preferred contract (denoted c_0) if it yields a higher utility than the option of not borrowing (i.e., $u_{ic_0b_0} \geqslant u_i(0,0)$). As the transport costs are independent of borrowers' preferences, there is no value for the lender in designing a new contract once the lender comes to his bank (see Lemma 1 below for a formal proof). Each bank thus faces the additional constraint $u_{icb} \geqslant u_i(0,0)$.[28] Section 5 shows that, when competition is high, the constraint is not binding. In that case, the model is equivalent to the ones used by the vast majority

27. From an empirical perspective, separating the entry margin from the competition among banks is common practice. In fact, Lester, Shourideh, Venkateswaran, and Zetlin-Jones 2019 and a vast majority of empirical industrial organisation empirical papers in banking (See, for instance, Benetton 2018, Crawford, Pavanini, and Schivardi 2018) do not model borrower entry decision. In the mortgage market, it is justified by the empirical evidence of Andersen, Badarinza, Liu, Marx, and Ramadorai (2021) and Benetton, Gavazza, and Surico (2021) showing that borrowers' entry decision in the mortgage market is very inelastic to loan prices and characteristics.

28. Another approach is to use a nested logit approach. Formally, borrowers choose in period 0, before knowing their ϵ draws, whether to enter or not the market. They enter if their expected utility $E_\epsilon[b_{ij}|\Theta_i]$ is greater than their outside option denoted U_i. Conditional on participation, they do not exit the market. With logit demand, this approach also allows for a closed-form characterization of a symmetric equilibrium.

of the empirical literature which abstracts from the entry margin.

D Interpretation of the taste shock

Taste shocks $\sigma_i^{-1}\epsilon_{jb}$ are the main departure from the classic principal-agent model. The shocks allow, as shown in, for instance, Rochet and Stole (2002) or Hotteling (1929), for imperfect substitution between contracts from different lenders. Section 5, in the spirit of Rochet and Stole (2002), illustrate that changes in σ_i are relevant for thinking about changes in competition. While σ_i does not affect borrowers willingness to pay – and thus their choice of contract within a bank – , they affect the degree of substitution across lenders' contracts. For instance, as shown in Appendix E, when the taste shocks are extreme value distributed – a common assumption in empirical models – the demand function faced by lenders has a logit form. In that case, σ_i parametrizes the demand semi-elasticity to changes in utility $(u_i(X,R))$. Intuitively, $\sigma_i \to \infty$ implies that the taste shocks are irrelevant, so borrowers choose the lender that offers them the best contract. As taste shocks differ for different lenders, $\sigma_i \to 0$ implies that borrowers will be stuck in a particular bank.

The taste shocks can be interpreted in several ways: as reflecting imperfect information (e.g., Varian 1980), or as capturing bank-specific characteristics–such as brand reputation, proximity to branches, or customer service quality–that are fixed at the bank level but valued heterogeneously across borrowers, or as search or transportation costs (e.g., Rochet and Stole 2002 or Appendix E).

The theoretical interpretation of these shocks has implications for welfare. For instance, a decrease in σ_i may reflect either an increase in bank product variety, which can enhance welfare, or a rise in the frequency of borrower choice errors, which may reduce welfare. In practice, however, these interpretations are often observationally equivalent: a revealed preference approach cannot distinguish between consistent mistakes and deliberate choices.

No matter the interpretation, these shocks create some degree of market power which affects pricing incentives (R), contract terms (X) and borrower utility $u_i(X,R)$. I analyse those distortions in Section 5. The section illustrates that changes is σ is a convenient way to model competition and study how it affects $u_i(X,R)$.[29]

29. Also, once the model parameters are recovered, one can impose a particular interpretation when conducting counterfactual analyzes. σ is thus a convenient tool for studying competition.

E Example of demand function

The ϵ_{jb} draw implies a certain ordering of contracts (indexed by $x \in [\![0, n]\!]$) posted by lenders. For concision, I denote the utilities u_{ic_x} instead of $u_i(X_{c_x}, R_{c_x})$. Without loss of generality, I normalize the transport cost ϵ_{i0} of the preferred lender to 0 so that other ϵ can be interpreted as the cost of visiting one more lender. For borrower j of type i, we have the following ordering of contracts:

$$\overbrace{u_{ic_0}}^{V(u_{ic_0},0)} \geqslant \overbrace{u_{ic_1}}^{Contract\ utility} - \overbrace{\sigma_{ib_0}^{-1}\epsilon_{jb_1}}^{\text{“}Transportation\ cost\text{”}} \geqslant ... \geqslant u_{ic_n} - \sigma_{ib_n}^{-1}\epsilon_{jb_n} \tag{24}$$

Equation (24) implies a demand function faced by lenders. To fix ideas, let us consider that lenders do not observe the transportation cost $(\epsilon_{jb})_{jb}$ and that the $(\epsilon_{jb})_{jb}$ are independent conditional on borrower preferences $(\Theta_i := (\theta_i, \sigma_i, \alpha_i))$. In that case, lenders cannot sort borrowers on their transportation cost (see Lemma 1 for a formal proof). When $(-\epsilon_{jb})_{jb}$ are extreme value distributed, equation (24) implies the following demand faced by bank b:

$$D_{ib}(u_i) := E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i] \tag{25}$$

$$= \overbrace{n_i}^{market\ size} \overbrace{\frac{exp(\sigma_i u_{ib})}{\sum_{x\in\mathbf{B}} exp(\sigma_i u_{ix})}}^{Probability\ i\ chooses\ b} \tag{26}$$

$$\implies \frac{\partial D_{ib}(u_i)}{\partial u_{ib}} \frac{1}{D_{ib}(u_i)} = \sigma_i(1 - D_{ib}(u_i)) \tag{27}$$

$u_{ib} := \arg\max_{c\in\mathcal{M}_b}\{u_i(X_c, R_c)\}$ is the borrower i utility when choosing the best contract offered by bank b. $u_i := (u_{ib})_{b\in\mathbf{B}}$ denotes the vector of utilities.

In that example, σ drives the demand elasticity and is thus a convenient way to model competition. When σ tends to infinity, holding utilities constant, the demand becomes close to perfectly elastic. I refer to this situation as the perfect competition case. When σ tends to zero, the demand is inelastic. I refer to this as the monopoly case.

F Endogeneous Default

Endogenous default: The microfoundation abstracts from the effect of contract terms on default outcomes. This simplification is made primarily for tractability but provides a reasonable approximation in settings where the sensitivity of default to contract terms–particularly interest rates–is low in the neighborhood of the equilibrium (see, for example, Nelson 2025). Such environments include cases where default is primarily driven by large, exogenous income shocks–such as job loss or redundancy–which are cited as major causes of default in retail lending markets, according to the Bank of England's NGM survey.

One can allow for endogenous default as long as the resulting utility derived from contracts is additively separable. There are different ways to achieve this. For instance, one can linearize the utility function $u(X, R)$ defined in equation (21) around the equilibrium contract terms, taking into account that default is also a function of contract terms. Alternatively, one can assume that default is piece-wise linear with one kink, defined as functions of monthly payments. Finally, if the borrower's default probability does not directly impact the utility derived from the loan (as in the case of consumer credit so that $T = 0$ in equation (23)), one can use the following functional form for default: $d(\theta_i, m) = 1 - f(\theta_i) - \beta_i \cdot m^{-1}$ with $\beta_i \in \mathbf{R}$ and f being a function of θ_i.

G Proofs

G.1 Proof of Lemma 1

We start by showing that lenders cannot create menus to separate borrowers with the same type (Θ_i) but different draws ϵ_j once they come to the bank. Indeed, borrower i to choose contract c over $\tilde{c}$ if and only if: $u_i(X_c, R_c; \theta_i) + \sigma_i^{-1}\epsilon_{jb} \geqslant u_i(X_{\tilde{c}}, R_{\tilde{c}}; \theta_i) + \sigma_i^{-1}\epsilon_{jb}$ which in independent of $\sigma_i^{-1}\epsilon_{jb}$. A similar argument can be made when ϵ_{jb} multiplies u_i. Conditional on choosing a contract from their bank, the lender can separate borrowers only based on their preferences, θ_i.

We now show that the lenders' profits and borrowers' utilities in the game in which the lender choose the number of contracts they offer and their characteristics is equivalent to problem (3).

Let us consider that each lender design a menu of contracts knowning: That borrower behave according to equation (1), the menus offered by other lenders, and the distribution of borrowers characteristics (Θ, ϵ). Denoting N_b the number of contract offered and $(X_{cb}, R_{cb})_{c\in[\![1,N_b]\!]}$ the menu, the lender b problem is:

$$\max_{\{(X_{cb},R_{cb})_{c\in[\![1,N]\!],N}\}} E[\sum_{i\in\Gamma} \overbrace{E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i]}^{Demand} \quad \overbrace{NPV_i}^{Net\ Present\ Value\ of\ lending} \quad |\mathcal{I}_b] \tag{28}$$

With b_{ij} defined in equation (1)

The contracts allow to screen borrowers. Given the assumption of Lemma 1 a borrower i with epsilon draws j coming in bank b will choose the contract c over any other contract $\tilde{c}$ offered by the lender if:

$$u_i(X_c, R_c; \theta_i) + \sigma_i^{-1}\epsilon_{jb} > u_i(X_{\tilde{c}}, R_{\tilde{c}}; \theta_i) + \sigma_i^{-1}\epsilon_{jb} \tag{29}$$

$$\iff u_i(X_c, R_c) > u_i(X_{\tilde{c}}, R_{\tilde{c}}) \tag{30}$$

Given that the number of contracts N in the menu does not affect borrowers' utility, the only reason to offer additional contracts is to separate borrowers by their type. Conditional on full separation, allowing for more contracts than the type of borrowers does not increase borrowers' profits or lender utilities. Similarly to Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), we can thus without loss of generality restrict our attention to the following lender problem:

$$\max_{\{(X_{ib},R_{ib})\}} E[\sum_{i\in\Gamma} \overbrace{E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i]}^{Demand} \quad \overbrace{NPV_i}^{Net\ Present\ Value\ of\ lending} \quad |\mathcal{I}_b] \tag{31}$$

$$u_i(X_{ib}, R_{ib}) \geqslant u_i(X_{\tilde{i}b}, R_{\tilde{i}b}) \ \forall(i, \tilde{i}) \tag{32}$$

With b_{ij} defined in equation (1)

Notice that this formulation allows lender b to condition his menus on the menus offered

by other lenders.

G.2 Proof Lemma 2

The incentive compatibility of the B and G borrowers can be written as: $(\theta^B - \theta^G)X^B \geqslant u_B - u_G \geqslant (\theta^B - \theta^G)X_G$ which imply $X_B \geqslant X_G$. Using the promised utility version of the model, we get that X^B is not distorted and thus equal to $\bar{X}$ and that the incentive compatibility constraint of the B borrower is binding. The incentive compatibility of G can also be written as $\bar{X} - X_G = \frac{R_B - R_G}{\theta^B}$, which implies that $R_B \geqslant R_G$.

G.3 Proof of Theorem 1

To simplify the algebra, I start by rewriting the lender problem using a change of variable.

I use the following change of variable:

$$u_i := v(X_i; \theta_i) - R_i \iff R_i = v(X_i; \theta_i) - u_i. \tag{33}$$

The bank's objective function for a particular borrower type is thus:

$$\Pi_i(X_i, u_i) := D(u_i)\pi(X_i, u_i) \tag{34}$$

$$with\ \pi(X_i, u_i) := (1 - d_i)R_i - mc(X_i, u_i). \tag{35}$$

$$and\ D : \text{is a continuous demand function} \tag{36}$$

The borrower incentive compatibility constraints can be rewritten:

$$g(u_i, u_j, X_j) := u_i - [u_j + \Delta v(X_j; \theta_{ij})] \geqslant 0 \tag{37}$$

The subscript i denotes the borrower type and contract designed for that type. $\Delta v(X_j; \theta_{ij}) := v(X_j; \theta_i) - v(X_j; \theta_j)$.

Using the change of variable, I rewrite the bank maximization the following way:

$$\max_{\{(X_{ib},u_{ib})\in\mathcal{F}\}} \sum_{i\in\Gamma} \overbrace{D_{ib}(u_{ib})}^{Demand}\{ \overbrace{S_{ib}(X_i)}^{Social\ Surplus} -\alpha_i u_i\} \tag{38}$$

$$s.t.\ (IC_{ib}):\ g(u_i,u_j,X_j)\geqslant 0,\ \forall i,j\in\Gamma$$

I denote $\Pi_b(X,u):=\sum_{i\in\Gamma} D_{ib}(u_{ib})\{S_{ib}(X_i)-\alpha_i u_i\}$ the lender objective function. $(X,u):=(X_{ib},u_{ib})_{ib}$ is the vector of contracts and utilities.

G.3.1 Existence

The conditions for the existence of a pure strategy equilibrium are taken from Glicksberg (1952). The assumptions are: the objective function Π_{ib} is continuous in $(X_{ib},u_{ib})_b$, the feasible set is convex, nonempty and compact, and Π_b is quasi-concave in the variables chosen by lender b (i.e., $(X_{ib},u_{ib})_i$). For tractability, I assume that Π_b is concave instead of quasi-concave.

(i) Convex set: Dropping the i index I denote x_c each element of the vector X. I assume that $v(X)=\sum_c v_c(x_c)$, where v_c are a one-to-one mapping. I denote $Z_c:=v_c(x_c)$. The constraint 37 thus define a convex set in (u_i,u_j,Z_j) as g is affine in (u_i,u_j,Z_j).

(ii) Concavity: I now derive conditions for the objective function Π_i to be concave by looking at whether the Hessian matrix is negative semi-definite. I look first at the conditions for the case in which X is of dimension 1. I generalise those conditions below using the fact that the Hessian is a bordered block diagonal matrix. We now need to show that the first leading principal minor is negative (i.e., $D''\pi-2D'\leqslant 0$) and that the second-order leading principal minor is positive (i.e., $(D''\pi-2D')S''\geqslant (S'D')^2$) so that the Hessian matrix is negative semi-definite. Denoting $D':=\partial_{\alpha u}D(u)$ and D'' the second derivative, and dropping the i index, we have the following hessian:

$$\nabla^2\Pi(u,X) = \begin{pmatrix} D''(u)\pi(u,X) - 2D'(u) & D'(u)\,S'(X) \\ D'(u)\,S'(X) & D(u)\,S''(X) \end{pmatrix}. \tag{39}$$

The matrix is negative definite if the leading principal minor has alternate signs. In the case in which X is a scalar, we have:

$$D''(u)\pi(u,X) - 2D'(u) \leqslant 0 \tag{40}$$

$$[D''(u)\pi(u,X) - 2D'(u)]D(u)\,S''(X) \geqslant (D'(u)\,S'(X))^2 \tag{41}$$

When demand D_i is concave in $\alpha_i u_i$, the first equation holds provided the profits in each market segment are not too negative. This does not necessarily hold because of cross subsidization across type. When the surplus function is concave, the second equation holds if the surplus curvature is sufficiently large. Intuitively, those condition prevents the pooling deviation from being profitable when demand elasticities are high (See, for instance, figure 1 and 2 and section 5.3 for a discussion).

Section G.3.2 shows that those two conditions are satisfied for certain functional forms, such as with logit demand with a capacity constraint. The capacity constraint allows the objective function to remain concave even when the demand semi-elasticity is large.

For the general case in which X is a vector, we have that:

$$\nabla^2 S(X) \prec 0 \quad \text{and} \tag{42}$$

$$D''(u)\pi(u,X) - 2D'(u) \;-\; \frac{\big(D'(u)\big)^2}{D(u)}\,\nabla S(X)^\top \big(\nabla^2 S(X)\big)^{-1} \nabla S(X) \leqslant 0. \tag{43}$$

$\nabla^2 S(X) \prec 0$ is satisfied when $S(X)$ is strictly concave.

G.3.2 Examples of demand functions

Examples of functional forms satisfying Theorem 1: The following examples satisfy the conditions of existence. Those functional forms are convenient because they are

widely used in empirical works and macroeconomics and feature closed-form solutions for homogeneous lenders.

(i) Logit with capacity constraint: I show that the Hessian is negative semi-definite for the logit case with a capacity constraint. The proof proceeds in two steps: (A) first, I show that the first principal minor is negative, then (B) I show that the second principal minor is positive.

(A) The proof proceeds in two steps: (a) we show that there exists a point such that $\Pi'(u) = 0$, then (b) we use this condition to bound profits and derive the conditions under which the first principal minor is negative.

(a) $lim_{u\to\infty} = -\infty$ and $lim_{u\to S(X)} = 0$. Since Π_{bi} is continuous, there exists a point such that $\Pi'_{bi}(u) = 0$.

(b) Using $\Pi'(u) = 0$, we get that $\pi = \frac{1}{\sigma(1-D)}$. Plugging it into equation 40, we get:

$$\sigma^2 D(1-D)[1-2D]\frac{1}{\sigma(1-D)} - 2\sigma D(1-D) \leqslant 0 \tag{44}$$

$$\iff \sigma D \geqslant 0 \tag{45}$$

Equation 44 implies that the is no local minimum holding X. Thus, we have: $D\pi \leqslant \frac{D^*}{\sigma(1-D^*)} \leqslant \frac{1}{\sigma(1-D^*)}$ where D^* solves the first order condition $\Pi'(u) = 0$. We also have $\pi \leqslant \frac{a}{\sigma}$ and $a < 2$. Plugging it into equation 40, we have:

$$\sigma^2 D(1-D)[1-2D]\frac{1}{\sigma(1-D^*)} - 2\sigma D(1-D) \leqslant 0 \tag{46}$$

$$\implies \frac{1-2D}{(1-D^*)} < \frac{1}{(1-D^*)} < 2 \; when, \; D^* < \frac{1}{2} \tag{47}$$

The equation 47 implies that when the demand faces a capacity constraint $k \leqslant \frac{1}{2}$, the first principal minor is always negative.

(B) We derived the conditions under which 40 is always satisfied for all X. Given those conditions, we now show that equation 41 is also always satisfied with a capacity constraint.

Plugging equation 46 into 41 and simplifying the σ, we have:

$$D(1-D)\{2-\sigma[1-2D]\pi\}D(-S'') \geqslant \sigma S'^2[D(1-D)]^2 \tag{48}$$

If $D(1-D)=0$, the equation is satisfied. Otherwise, we can divide by $D(1-D)$ to get:

$$\{2-\sigma[1-2D]\pi\}D(-S'') \geqslant \sigma S'^2 D(1-D) \tag{49}$$

Considering that u belongs to a bounded set, we can get a minimum value for D equal to $\bar{D}$. Using the bound on π, and the capacity constraint $D < \frac{k}{\sigma}$ to get $\sigma D(1-D) < \sigma D < k$, we have:

$$[(2-a) - 2a\frac{k}{\sigma}]D \geqslant k\frac{S'(X)^2}{(-S'')} \tag{50}$$

$$\implies k \leqslant \min_X \frac{\bar{D}(2-a)}{[\frac{S'(X)^2}{(-S''(X))} + 2a\frac{\bar{D}}{\sigma}]} \tag{51}$$

The equation is thus satisfied if the capacity constraint k is small enough, $S(X)$ flat enough or curved enough so that equation 50 is satisfied and profits are concave. We can derive a similar condition using equation 43 when X is a vector.

(ii) Logit with linear surplus, 2 borrower type:

The Hessian is not negative semi-definite when S is linear. Because the case where S is linear is useful for tractability, I derive the conditions for the profit function to be concave for the two borrower cases, indexed by H and L. H refers to the high-default borrower.

I use the fact that, in that case, only the IC constraint of the H borrower is binding (as in Rothschild and Stiglitz (1976)). I then use it to replace $X_L = \frac{u^H - u^L}{\Delta}$. The case of multiple borrowers with unidimensional screening must be solved numerically to verify that equation 41 is satisfied.

The objective function is:

$$f(u_H, u_L) = N_H D_H(u_H)\,(X - u_H) + N_L D_L(u_L)\Big(S\Big(\frac{u_H - u_L}{\Delta}\Big) - u_L\Big), \qquad \Delta := \theta_B - \theta_G$$

N_i is the mass of type i borrower.

The hessian is thus:

$$\nabla^2 f(u_H, u_L) = \begin{pmatrix} \frac{\partial^2 f}{\partial u_H^2} & \frac{\partial^2 f}{\partial u_H \partial u_L} \\ \frac{\partial^2 f}{\partial u_L \partial u_H} & \frac{\partial^2 f}{\partial u_L^2} \end{pmatrix} = \begin{pmatrix} H_{HH} & H_{HL} \\ H_{LH} & H_{LL} \end{pmatrix}, \qquad H_{HL} = H_{LH},$$

where

$$H_{HH} = N_H D''_H(u_H)\,(X - u_H) - 2D'_H(u_H) + N_L D_L(u_L)\,\frac{S''(z)}{\Delta^2},$$

$$H_{HL} = H_{LH} = N_L D'_L(u_L)\,\frac{S'(z)}{\Delta} - N_L D_L(u_L)\,\frac{S''(z)}{\Delta^2},$$

$$H_{LL} = N_L[D''_L(u_L)\,\big(S(z) - u_L\big) - 2D'_L(u_L)\Big(1 + \frac{S'(z)}{\Delta}\Big) + D_L(u_L)\Big(\frac{S''(z)}{\Delta^2} + 2\Big)]$$

I denote $z := \frac{u_H - u_L}{\Delta}$, We have $S'' = 0$ $S' = s \in R^+$.

Using the same argument as before, we have $H_{HH} < 0$ when D has a logit form and and $D < 0.5$. So the first principal minor is negative.

The second principal minor is the determinant of the hessian. It must be positive for the objective function to be concave. We must have:

$$H_{HH} \cdot H_{LL} > H_{HL}^2 \tag{52}$$

H_{HH} can be written $-\sigma N_H D_H(1 - D_H)[2 - \sigma(1 - 2D_H)\pi_H]$ and H_{LL} can be written $-N_L[\sigma D_L(1-D_L)[2(1+\sigma I) - \sigma(1-2D_L)\pi_L] - 2D_L]$. I denote $I := \frac{s}{\Delta}$. H_{HL}^2 can be written $\sigma^2 N_L^2 D_L^2 (1 - D_L)^2 I^2$.

Using the bound $\pi_i < \frac{k}{\sigma}$, with $k \in (0, \frac{1}{2})$ as in the previous proof, and the fact that $0 < D < \frac{1}{2}$, a sufficient condition for $\det(H) \geqslant 0$ is:

$$N_L N_H\big[\sigma D_H(1 - D_H)(2 - k)\big]\,\big[\sigma D_L(1 - D_L)\big(2(1 + \sigma I) - k\big) - 2D_L\big] \;\geqslant\; \sigma^2 N_L^2 D_L^2 (1 - D_L)^2 I^2 \tag{53}$$

Dividing both sides by $\sigma^2 D_L(1 - D_L) > 0$ yields:

$$\frac{N_H}{N_L} D_H(1 - D_H)(2 - k)\left[\big(2(1 + \sigma I) - k\big) - \frac{2}{\sigma(1 - D_L)}\right] \;\geqslant\; \frac{I^2}{4} \geqslant\; D_L(1 - D_L)\, I^2. \tag{54}$$

The equation is satisfied for σ high enough and utilities belonging to a bounded set.

Indeed, using the fact that the demand is concave, we have that $D_H(u) > D_H(\underline{u}) + \sigma \bar{D}_H(1 - \bar{D}_H)[u - \underline{u}] > 0$. In that case, the left-hand side goes to infinity faster than the demand goes to 0 when u belongs to a bounded set bounded below by $\underline{u}$ and above by $\bar{u}$. Where $\bar{D}_H$ is the demand at $u = \bar{u}$. This condition can be written as a constraint on the price belonging to a bounded set (as one can fix X to its first best value and rewrite the constraint on u as a constraint on R).

The above equation can thus be written as a constraint on the parameter k:

$$k \leqslant [2(1 + \sigma I) - \frac{2}{\sigma}] - \frac{I^2}{4} \cdot \frac{N_H}{N_L} A \tag{55}$$

With $A := \bar{D}_H[1 - 0.25]^{-1}$ with $\bar{D}_H$ being the minimum demand when other lenders offer the best feasible contract and lender b offers the worst for borrowers. The above condition is satisfied when σ is large (as the right-hand side is increasing in σ).

G.3.3 Proof Convergence

This section shows that: When the demand-semi elasticities tend to infinity (i.e. $\frac{\partial_{u_{ib}} D_{ib}(u_{ib}, u_{-ib})}{D_i(u_{ib}, u_{-ib})} \to \infty \ \forall (i,b), (u_{ib}, u_{-ib}), \forall (i,b,\tilde{b}))$, the market is adversely selected (i.e., $\alpha_i^B(\theta_i)$ decreasing in $\theta_i \in \mathbb{R}$) and $\frac{S'(X)}{v'(X)}$ bounded, the equilibrium features no cross-subsidy and full separation.

Rewriting the (IC_{ij}), using the change of variable $Z_{ic} := v_{ic}(X_{ic})$, denoting $\Delta_{ic} := \theta_i - \theta_j$ when $\theta_i > \theta_j$ we get:

$$\Delta_{ij} Z_i \geqslant u_i - u_j \geqslant \Delta_{ij} Z_j \tag{56}$$

Denoting the lagrange multiplier by λ_{ij} writing the first order conditions of problem 3, we get that:

$$(1 - d_i) R_i = mc(X) + \frac{D_i + \sum_{j:\theta_i > \theta_j} \lambda_{ij} - \sum_{j:\theta_i < \theta_j} \lambda_{ji}}{D_i'} \tag{57}$$

$$D_i S'(Z_i) = \sum_{j:\theta_i > \theta_j} \Delta_{ij} \lambda_{ij} - \sum_{j:\theta_i < \theta_j} \Delta_{ji} \lambda_{ji}, \ with \ \lambda_{ji} = \lambda_{ji} \geqslant 0 \tag{58}$$

In the case where preferences are ordered (that is, we can re-index borrower such that $i > j \implies \theta_{ic} > \theta_{jc}$ for all element c of the vector θ, this condition is trivially satisfied for $\theta \in \mathbb{R}$), the constraints 56 imply that the incentive compatibility constraints of the borrower i would first bind for borrowers $i+1$ or $i-1$. Indeed, as we must have $Z_i \geqslant Z_j \forall i > j$ and $\Delta_{ij} > \Delta_{i,j+x} \forall x > 0$, then if $u_i - u_{i-1} \geqslant \Delta_{i-1} Z_{i-1}$ satisfied, and $u_{i-1} - u_{i-2} \geqslant \Delta_{i-2} Z_{i-2}$ satisfied, we must have $u_i - u_{i-2} \geqslant u_i - u_{i-1} - u_{i-1} - u_{i-2} \geqslant \Delta_{i,i-1} Z_{i-1} + \Delta_{i-1,i-2} Z_{i-2} \geqslant \Delta_{i,i-1} Z_{i-2} + \Delta_{i-1,i-2} Z_{i-2} \geqslant \Delta_{i-1,i-2} Z_{i-2}$.

Starting from the borrower with the lowest willingness to pay l, we get that $0 \leqslant \lambda_{ll+1} = \frac{D_l S'(Z_l)}{\Delta_{ll+1}} \leqslant \frac{B}{\Delta_{ll+1}}$, where B is an upper bound on $S'(Z_l) := \frac{S'(X_l)}{v'(X_l)}$. Using these bounds on equation 57, we get that the pricing goes to the marginal costs when $\frac{D}{D'} \to \infty$. Doing it recursively for borrower $l+1$, we have that lenders price each borrower at marginal cost.

G.4 Proof of Comparative Statics

I use the equilibrium credit rationing equation (13) to derive the comparative statics. The equation, for an interior solution is:

$$X_G = \overbrace{\frac{1}{S'_{BG}}[S'_B \bar{X}}^{perfect\ competition\ distortion\ level} + \underbrace{\frac{\varepsilon_G^{-1}}{\alpha_G} - \frac{\varepsilon_B^{-1}}{\alpha_B}}_{competition\ channel} + \overbrace{\frac{\varepsilon_G^{-1} IR_G}{\alpha_G} + \frac{\varepsilon_B^{-1} |IR_B|}{\alpha_B})}^{cross-subsidy\ channel} \tag{59}$$

With $IR_i = I \cdot [\mathbf{1}_{i=G} - \mathbf{1}_{i=B} \frac{n_G}{n_B}]$, $I := \frac{S'_G}{\theta_B - \theta_G}$, and $S'_i := \theta_i - \frac{mc_i}{\alpha_i},\ i \in \{G, B\}$. $S'_{BG} := \theta_B - \frac{mc_G}{\alpha_G}$, mc_G is the marginal cost of lending X_G. As in the previous section, in a symmetric equilibrium, the semi-elasticities are $\varepsilon_i^{-1} = \frac{1}{\sigma_i (1 - \frac{n_i}{\#B})}$ and the demand are $D_i = \frac{n_i}{\#B}$ for each bank.

Competition: When $\sigma \to \infty$ $\varepsilon_i^{-1} \to 0$. In that case, we have an interior solution since $X_G < \bar{X}$ because $\frac{S'_B}{S'_{BG}} < 1$. The condition $u_{ib} \geqslant u_i(0,0)$ is satisfied because of the assumption of positive NPV.

I now take the derivative of X_G with respect to σ^{-1}, I denote $CM := \partial_{\sigma^{-1}} [\frac{\varepsilon_G^{-1}}{\alpha_G} - \frac{\varepsilon_B^{-1}}{\alpha_B}] = (\frac{1}{\alpha_G (1 - \frac{n_G}{B})} - \frac{1}{\alpha_B (1 - \frac{n_G}{B})}) < 0)$ and $CC := \partial_{\sigma^{-1}} [\frac{\varepsilon_G^{-1} IR_G}{\alpha_G} + \frac{\varepsilon_B^{-1} |IR_B|}{\alpha_B})] = \frac{IR_G}{\alpha_G (1 - \frac{n_G}{B})} - \frac{IR_B}{\alpha_B (1 - \frac{n_G}{B})} > 0$,

I get:

$$\partial_{\sigma^{-1}} X_G = CM + CC \tag{60}$$

$\partial_{\sigma^{-1}} X_G$ is positive when $CM + CC > 0$.

Marginal cost: Taking the derivative of X_G with respect to the marginal cost of G contract, we get that the condition for $\partial_{mc_G} X_G$ to be positive:

$$\partial_{mc_G} X_G > 0 \iff \frac{1}{\alpha_G} X_G - \sigma^{-1} \frac{\tilde{IR}'}{S'_{BG}} > 0 \tag{61}$$

With $\tilde{IR}' := \frac{\varepsilon_G'^{-1} \frac{1}{\theta_B - \theta_G}}{\alpha_G} + \frac{\varepsilon_B'^{-1} \frac{n_G}{n_B(\theta_B - \theta_G)}}{\alpha_B}$, and $\varepsilon_i'^{-1} := \frac{1}{1 - \frac{n_i}{B}}$.
The condition (61) is satisfied for σ larger than σ_M. With $\sigma_M := \frac{\alpha_G \tilde{IR}' + CM + CC}{S'_B \bar{X}}$.

Taking the derivative of X_G with respect to the marginal cost of G contract, we get that:

$$\partial_{mc_B} X_G < 0 \iff -\frac{1}{S'_{BG} \alpha_G} \bar{X} < 0 \tag{62}$$

Condition (62) always holds.

H Welfare Analysis

H.1 Proof Proposition 1

Using the utility representation, and noticing that as in the monopoly model, the participation constraints of the G borrower is binding (so $u_G = u_G^{eq}$) the problem is:

$$\max_{\{u_B, L_B\}} n_B \alpha_B [(\theta_B - mc_B) L_B - u_B] + n_G \alpha_G [(\theta_G - mc_G) L_G - u_G] \tag{63}$$

$$s.t.\ L_G = \frac{u_B - u_G^{eq}}{\theta_B - \theta_G} \tag{64}$$

Using the first order condition with respect to u_B, the social planner pool if:

$$n_B \alpha_B < n_G \alpha_G \frac{\theta_G - mc_G}{\theta_B - \theta_G} \tag{65}$$

Rearranging equation (65), we get equation 14.

Comparing this with Theorem 1 and the Comparative statistics on competition, we see that lenders do not pool when the social planner would pool (for instance, when competition σ is high).

For the implementation of the second best, we must have:

$$u_G = u_G^{eq} \Longleftrightarrow R_G = \theta_G L_G^{eq} - R_G^{eq} \tag{66}$$

$$X_G = \bar{X} - e \text{ with e arbitrarily small} \tag{67}$$

Using the pricing equation 11 to replace R_G and the loan size equation 13 to replace X_G, we get:

$$mc_G : mc_G(\bar{X}) + \varepsilon_G(1 + IR_G) = \theta_G[\bar{X} - L_G^{eq}] + R_G^{eq} \tag{68}$$

$$mc_B : X_G = \bar{X} - e \text{ with } X_G \text{ defined in equation 13 and e arbitrarily small} \tag{69}$$

H.2 Policies

To analyze whether policy interventions have a positive impact on welfare, I use the following lemma:

***Lemma 5: Equilibrium utility and profit levels (Interior Solution)**:*

$$u_B = (\alpha_B \theta_B - mc_B)\bar{X} - \pi_B \tag{70}$$

$$u_G = (\alpha_G \theta_G - mc_G)X_G - \pi_G \tag{71}$$

$$\Pi_B := n_B \pi_B = n_B \frac{1}{\varepsilon_B \alpha_B}[1 - I\frac{n_G}{n_B}] \tag{72}$$

$$\Pi_G := n_G \pi_G = n_G \frac{1}{\varepsilon_G \alpha_G}[1 + I] \tag{73}$$

With X_G defined in (59), $I := \frac{S'_G}{\theta_B - \theta_G}$, and $S'_i := \theta_i - \frac{mc_i}{\alpha_i},\ i \in \{G, B\}$.

Equations (70) and (71) state that borrowers' utility is the surplus generated by the loan minus the lender's profits per borrower. Equations (72) and (72) state that the lender's profits are the markup plus or minus the asymmetric information premium or discount.

Changes in competition: Idonote the semi-elasticities $\varepsilon = \tilde{\varepsilon}_i \sigma^{-1}$, where $\tilde{\varepsilon}_i$ is a free parameter. I analyze how changes in competition (σ^{-1}) affect the welfare of different market participants. For the welfare results to be independent of the welfare function used, I then examine whether there exists a situation in which a decrease in competition leads to Pareto improvements.

$$\partial_{\sigma^{-1}} u_B = -\frac{1}{\tilde{\varepsilon}_B \alpha_B}[1 - I\frac{n_G}{n_B}] \tag{74}$$

$$\partial_{\sigma^{-1}} u_G = (\alpha_G \theta_G - mc_G)\partial_{\sigma^{-1}} X_G - \frac{1}{\tilde{\varepsilon}_G \alpha_G}[1 - I] \tag{75}$$

$$\partial_{\sigma^{-1}}[\Pi_B + \Pi_G] = \frac{n_B}{\tilde{\varepsilon}_B \alpha_B}[1 + I\frac{n_G}{n_B}] + \frac{n_G}{\tilde{\varepsilon}_G \alpha_G}[1 + I] \tag{76}$$

The change in competition increase u_i and $\Pi_B + \Pi_G$ when:

$$I\frac{n_G}{n_B} > 1 \tag{77}$$

$$(\alpha_G\theta_G - mc_G)[CM + CC] > \frac{1}{\tilde{\varepsilon}_G\alpha_G}[1 + I] \tag{78}$$

$$\frac{n_B}{\tilde{\varepsilon}_B\alpha_B} + \frac{n_G}{\tilde{\varepsilon}_G\alpha_G} > n_G[\frac{1}{\tilde{\varepsilon}_B\alpha_B} - \frac{1}{\tilde{\varepsilon}_G\alpha_G}]I \tag{79}$$

The above equations are satisfied for $\frac{n_G}{n_B}$ large enough and the difference between default probabilities is small enough (for equation 79 to hold).

Decreases in competition decrease the interest rate of the B contract and increase the B borrower's utility when the asymmetric information discount is higher than the pure markup effect (i.e., $I\frac{n_G}{n_B} > 1$).[30] This happens when the surplus in the B market segment is large (i.e., I large) and when the cost of screening is low (i.e., $\theta_G - \theta_B$ low) and the relative number of G borrowers is high.

A decrease in competition increases G borrowers' rate but also relaxes the credit rationing. It increases welfare when the increase in price is compensated by the increase in loan size (see equation 78). This happens when the relative number of G borrowers is high or if the G market segment is relatively more elastic.

A decrease in competition has an ambiguous effect on lenders' profits. It increases lenders' profits if incentives to provide an information rent are low enough. When the interest rate elasticity of the G market segment is high and the benefit of screening is large, a decrease in competition can decrease profits. This is because a decrease in competition creates incentives to distort the B market segment more to extract surplus on the G market segment. However, since the G market segment is relatively more elastic, lenders begin to compete more on this market. Overall, the latter effect dominates.

As a result, when the number of high-default-borrowers is high enough and the G market's surplus is high enough, a decrease in competition is a Pareto improvement.

Changes in the marginal cost of lending: I analyze how changes in the marginal cost of lending (mc) affect the welfare of different market participants. For welfare results to

30. Because of the linearity assumption of the utility, the condition implies that lenders make negative profits on the B market segment. This is not necessarily true otherwise.

be independent of the welfare function used, I then examine whether there exists a situation in which a decrease in competition leads to Pareto improvements.

Let us denote mc_i the marginal cost of lending via i contract.

An increase in mc_G renders interest rates in the B market higher and the surplus in the G market lower, which renders the surplus in the B market relatively higher. This gives incentives to provide less information rent to B borrowers. A decrease in mc_B is passed through lower interest rates. This increases the welfare of B borrowers.

The effect of an increase in mc_G on G borrower is ambiguous. An increase in mc_G is partially passed through interest rates. It also lowers the relative surplus in the G market segment, which triggers incentives to increase the asymmetric information discount (i.e., the interest rate) so that more profits can be made in the B market segment. The rate increase also decreases cream-skimming incentives, so lenders can increase the cross-subsidy and the amount lent in the G market segment without the threat of losing their customers. Overall, welfare can increase. mc_B changes have no impact.

Increases in mc_G positively impact profits if the interest rate elasticity is higher in the G market segment. This is because an increase in mc_G triggers incentives to provide lower information rents. mc_B changes have no impact on lenders' profits in my model, because the cost is fully passed through borrowers.

As a result, an increase in mc_G coupled with a decrease in mc_B can be a Pareto improvement policy. The extreme-case scenario is equivalent to banning contracts with lower than $\bar{X}$ loan size, which prevents cream-skimming. This is a Pareto-efficient policy when cream skimming is ex-post inefficient.

I Utility Representation

Given that the incentive compatibility and individual rationality constraints are stated in terms of promised utilities and quantities, the literature usually transforms the lender's program from the price-characteristic space to the utility-characteristic space (see, for instance, Rochet 2024). This formulation helps to separate the effect of competition – through utilities – from the effect of imperfect information on product characteristics (X). It also makes the program similar to the workhorse monopoly model. I thus use the change of variable

$\tilde{u}_{ib} := v(X;\theta_i) - R_i$ and rewrite the bank maximization the following way:

$$\max_{\{(X_{ib},\tilde{u}_{ib})\in\mathcal{F}\}} E[\sum_{i\in\Gamma} \overbrace{D_{ib}(\alpha_i\tilde{u}_{ib})}^{Demand}\{ \overbrace{S_{ib}(X)}^{Social\ Surplus} -\alpha_i\tilde{u}_i\}|\mathcal{I}_b] \tag{80}$$

$$s.t.\ (IC_{ib}):\ \tilde{u}_{ib} \geqslant \tilde{u}_{\tilde{i}b} + \underbrace{\Delta\theta_{i\tilde{i}}v(X_{\tilde{i}})}_{Information\ rent}\ ,\ \forall i,\tilde{i}\in\Gamma$$

The demand function $D_{ib}(u_i) := E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i]$, is equal to the probability that borrower i chooses bank b conditional on the menus offered in the market. $u_i := (u_{ib})_b$ is the vector of promised utilities offered by the lenders to borrower i. Abusing the notation, I use D_{ib} and $D_{ib}(u_{ib})$ when unambiguous instead of $D_{ib}(u_i)$.

$S_{ib}(X) := \alpha_i v(X;\theta_i) - mc_b(X)$ is the social surplus generated by the loan.

$\mathcal{F}$ is the set of feasible contracts. When the modeling of exclusion is done through a contract $(0,0)$ in $\mathcal{M}_b$ or a participation constraint as described in Section C, I include those constraint on those set. The constraints are convex in promised utilities so the proofs of Lemma 2 remain unchanged.

The incentive compatibility constraint (IC_{ib}) can be rewritten as in the monopoly case. It highlights that if the lender wants to provide utility $\tilde{u}_{\tilde{i}b}$ to borrower $\tilde{i}$ with a contract with characteristic $X_{\tilde{i}}$, it has to provide at least $\tilde{u}_{\tilde{i}b} + \Delta\theta_{i\tilde{i}}v(X_{\tilde{i}})$ to borrower i. The term $\Delta\theta_{i\tilde{i}}v(X_{\tilde{i}}) := (\theta_i - \theta_{\tilde{i}})v(X_{\tilde{i}})$ is called an information rent in the literature.

This formulation of problem 3 highlights that lenders use contract characteristics (X) to maximize the social surplus $(S(X))$ and use interest rates (R or equivalently $\tilde{u}$) to split the surplus. Due to asymmetric information, lenders have to take into account that contract characteristics affect the information rent $(\Delta\theta_{i\tilde{i}}v(X_{\tilde{i}}))$ and thus the interest rate that they have to provide to borrowers to maintain incentives to self-select.

J Optimal Contracts

In this section, I use model 3 first-order condition to analyze how lender designs their menus for an arbitrary number of borrowers.

J.1 Characterization of the Contracts: Unobservable Type

I now consider that lenders cannot observe any of the borrowers' characteristics $(\Theta_i, d_i, \epsilon_{jb})$ and thus derive the first order condition of problem 3 with the incentive compatibility constraints. To get intuition, I consider the unidimensional screening case (i.e., $dim(\theta_i) = 1$).

J.1.1 Screening when $dim(\theta_i) = 1$

Motivated by example (23) and the empirical application, I focus here on a situation where lenders can choose interest rates, maturity and loan size so that θ_i is of dimension one. I interpret X as the loan size and R as the monthly payments times the maturity.

The multidimensional screening case is discussed in Section 6.

When $dim(\theta_i) = 1$, screening is achieved by making borrowers with a high willingness-to-pay (high θ) self-select into high loan high interest rate contracts. The set of incentive compatibility constraints can be simplified: Only the incentive compatibility constraints of the borrowers with close θ are relevant.

I thus order borrowers according to their θ value such that borrower i has a higher θ than borrower $i-1$. I index by I the borrower with the highest θ and 0 the borrower with the lowest. The relevant incentive compatibility constraints for borrower i are only the one of borrower $i+1$ ($i+1$ potentially wants to pretend to be of the i type) and the one of borrower $i-1$ (i potentially wants to pretend to be of the $i-1$ type).

For exposition purposes, I focus here on an interior solution and consider that $(\alpha_i, d_i, \sigma_i)$ are the average values of $(\alpha_j, d_j, \sigma_j)$ conditional on θ_i, where i index the different values of (θ_i) and j the borrower. The latter only affects equation (**??**) below and can be generalized by multiplying both sides by $\partial_{ui} D_i$ and taking the expectation conditional on the lender information set.

The optimal contracts is characterized by:

$$Contract\ Terms:\ X_i = \begin{cases} \overbrace{X_i^{II} := v^{-1}(v(X_{i+1}) - \frac{R_{i+1}-R_i}{\theta_{i+1}})}^{Binding\ IC\ constraint\ case} \text{ if } X_i^{II} \leqslant X_i^{PI} \\ X_i^{PI} \text{ if i=I or if } X_i^{II} > X_i^{PI} \end{cases} \tag{81}$$

$$Pricing: \quad \overbrace{\alpha_i R_i}^{Loan\ Present\ Value} = \overbrace{mc(X_i)}^{break\ even\ price} + \overbrace{\frac{D_i}{\partial_{u_i} D_i}}^{Perfect\ information\ markup} + \overbrace{\frac{D_i}{\partial_{u_i} D_i} IR_i(X_i, X_{i-1}, \frac{D_{i-1}}{D_i})}^{Information\ discount/premium}] \tag{82}$$

$IR_i(X_i, X_{i-1}, \frac{D_{i-1}}{D_i}) := \alpha_i^{-1}[\frac{\overbrace{\partial_{X_i} S_i(X_i)}^{Marginal\ social\ surplus}}{\underbrace{\Delta\theta_{i+1}\partial_{X_i} v(X_i)}_{Marginal\ information\ rent}} \mathbf{1}_{i\neq I} - \frac{D_{i-1}}{D_i} \frac{\partial_{X_{i-1}} S_{i-1}(X_{i-1})}{\Delta\theta_{i,i-1}\partial_{X_{i-1}} v(X_{i-1})} \mathbf{1}_{i\neq 0}]$ is an information distortion.

X^{PI} is the contract characteristic that solves equation (**??**), which is, the one that maximizes the social surplus $S(X)$.

Equation (81) highlights the tools that lenders can use to maintain borrowers' incentives to self-select: credit rationing (X_i^{II}) and cross-subsidies ($R_{i+1} - R_i$). It states that the contract characteristics are equal to their first best value (X^{PI}) when the pricing of contracts – characterized by equation (82) – makes them incentive-compatible. When the contract prices (R) are insufficient to maintain borrowers' incentives to self-select, the lender sets the contract characteristics such that the incentive compatibility constraint is binding (i.e., $X_i = X_i^{II}$).

As in Rothschild and Stiglitz (1976), there is no distortion at the top: The borrower with the highest willingness to pay (θ) gets the first best loan size. Other borrowers get a lower loan size, and potentially credit rationed. How credit rationed the borrower is (X_i^{II}) depends on the interest rate spread ($\frac{R_{i+1}-R_i}{\theta_{i+1}}$) and the loan size of the borrower's willingness to pay just above (X_{i+1}).

Equation (82) is the optimal pricing equation. It is a key novelty of the paper as the literature on selection markets does not use continuous demand systems and thus cannot rely on first-order conditions.

I rewrite the optimal pricing equation as the sum of three terms: a break-even price,

a perfect information markup, and an information distortion (i.e., the cross-subsidy term). This decomposition has two benefits. First, it provides intuition about the economic forces at play. Second, as discussed in Section K.1, the equation can directly be used as a moment condition to recover the model parameters.

The first two terms of equation (82) can be interpreted as the best response of bank b to its competitors' contracts if lender b had perfect information about its customer. Those components have already been analyzed in Section 5.2 as they appear in equation (7). The last term (IR_i) captures the impact of adverse selection. It provides intuition about the lender incentives to use credit rationing or cross-subsidies to maintain borrowers' incentives to self-select.

The first element of IR_i in equation (5.2.2) (i.e., $\frac{\partial_{X_i} S_i(X_i)}{\Delta\theta_{i+1}\partial_{X_i} v(X_i)}$) is positive and affect the rate of low default borrowers when $Corr(\theta_i, d_i) > 0$. I call that term the information premium. It prevents the higher default borrowers ($i+1$) from choosing the contract designed for the lower default borrower (i). The information premium formula captures the intensive margin costs and benefits of increasing interest rates above their perfect information values. The numerator represents the benefits of increasing the loan face value R (holding borrower utility constant): It relaxes the incentive compatibility constraint and thus increases the surplus generated by the loan by $\partial_X S$ for each dollar additional dollar lent. However, for a given increase in the loan face value R, there is a limit to how large the loan of the low-default borrower can increase without attracting the high-default borrower. This amount is exactly equal to one over the information rent.

The second term of IR_i (i.e., $\frac{D_{i-1}}{D_i}\frac{\partial_{X_{i-1}} S_{i-1}(X_{i-1})}{\Delta\theta_{i,i-1}\partial_{X_{i-1}} v(X_{i-1})}$) is negative and affects the rate of high default borrower when $Corr(\theta_i, d_i) > 0$. I call that term the information discount. This discount ensures high default borrowers have lower incentives to select the contract designed for the low default borrowers $i-1$. The intuition is the following. By providing a discount (i.e., lowering the interest rate of the i contract by $\Delta\theta_{i,i-1}\partial_{X_{i-1}} v(X_{i-1})$), the lender lower their profit margin on each i borrower they attracted (i.e., D_i). However, the information rent allows lenders to lower the amount of credit rationing and thus their profit margin on each $i-1$ borrower by $\partial_{X_i} S_i(X_i)$. This effect is multiplied by the number of customers they have (i.e., D_{i-1}).

The pricing distortion IR_i is multiplied by the demand semi-elasticity, which captures the extensive margin cost of playing with interest rates R (i.e., changing the promised utility

u_i level). When the demand semi-elasticity is high, increasing rates too much compared to the perfect information pricing is costly due to the effect on the number of customers. Thus, the lender has incentives to keep the demand unchanged and use more credit rationing.

J.1.2 Visual Representation of the lenders problem

Recall the equation for the credit rationig level is:

$$X_G = \overbrace{\frac{1}{S'_{BG}}[S'_B\bar{X}}^{perfect\ competition\ distortion\ level} + \underbrace{\frac{\varepsilon_G^{-1}}{\alpha_G} - \frac{\varepsilon_B^{-1}}{\alpha_B}}_{competition\ channel} + \overbrace{\frac{\varepsilon_G^{-1}IR_G}{\alpha_G} + \frac{\varepsilon_B^{-1}|IR_B|}{\alpha_B})}^{cross-subsidy\ channel} \tag{83}$$

With $S'_{BG} := \theta_B - \frac{mc_G}{\alpha_G}$, where mc_G is the marginal cost of lending X_G. As in the previous section, in a symmetric equilibrium, the semi-elasticities are $\varepsilon_i^{-1} = \frac{1}{\sigma_i(1-\frac{n_i}{|B|})}$ and the demand are $D_i = \frac{n_i}{|B|}$ for each bank.

The three elements of the pricing equations also appear in the loan size equation. The above intuition can be shown formally by analyzing the three elements of equation (13). The first term ($\frac{S'_B\bar{X}}{S'_{BG}}$) is the credit rationing level when lenders price each contract at their break-even rate. This pricing happens when competition is high. This is because high competition (i.e., ε high) forces lenders to break even on each market segment to prevent a competitor from stealing its most profitable borrowers (i.e., to cream-skim, see Figure **??** for graphical intuition).

The second term ($\frac{\varepsilon_G^{-1}}{\alpha_G} - \frac{\varepsilon_B^{-1}}{\alpha_B}$) captures the level of credit rationing if lenders were to price the contracts at their perfect information level. When lenders can extract a relatively more surplus from the B market segment because those borrowers are less elastic, they have incentives to maintain incentives to self-select by distorting – relative to the perfect information case – the G contract instead of the B contract.

The last term ($\frac{\varepsilon_G^{-1}IR_G}{\alpha_G} + \frac{\varepsilon_B^{-1}|IR_B|}{\alpha_B}$) is the spread between information discount and premium. This term captures that when competition is low, lenders can distort rates – relative to the perfect information case – in order to reduce the level of credit rationing. The terms $(IR_i)_i$ show that lenders have incentives to distort the contract designed for the market segment that generates a lower surplus (captured by $\frac{n_G}{n_B}$ in IR_G), and tend to pool borrowers when the

surplus generated by one dollar in the G market is lower than the cost of screening (captured by I in IR_i). $I := \frac{S'_G}{\theta_B - \theta_G}$ captures the cost and benefits of screening. S'_i is the social surplus generated by one dollar lent to borrower i. $\theta_B - \theta_G$ is the interest rate discount (i.e., the information rent in Rothschild and Stiglitz 1976). The information rent must be given to a B borrower to maintain the incentive to self-select when one additional dollar is lent to a G borrower (see Appendix I for more details).

To gain intuition about the forces at play, I plot borrower indifference curves (blue for B borrowers and green for G borrowers) and lenders' break-even conditions for each market segment (light grey horizontal lines) in Figures (7) and (8). To render the graph more legible, I consider the situation in which lenders must pay a fixed cost of originating contracts (i.e., $mc(X) = f \in \mathbf{R}^+$) so the grey lines are horizontal. I denote $(C_1, \tilde{C}_1)$ the perfect information contracts of bank b. C_2 and C_3 are the imperfect information contracts.

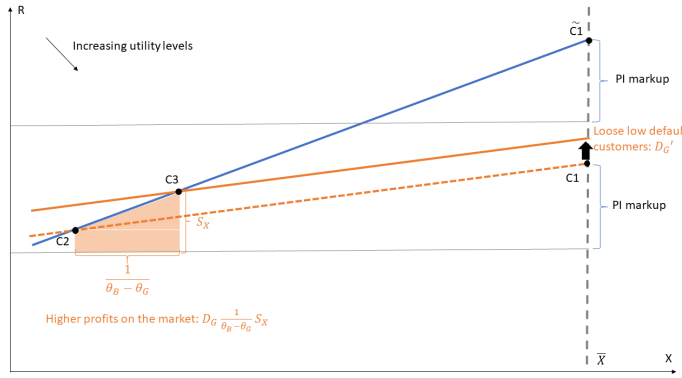


Figure 7: Effect of information premium: More profits per G customer (C_2 to C_3), loose G customers with respect to perfect information utility levels (C_1).

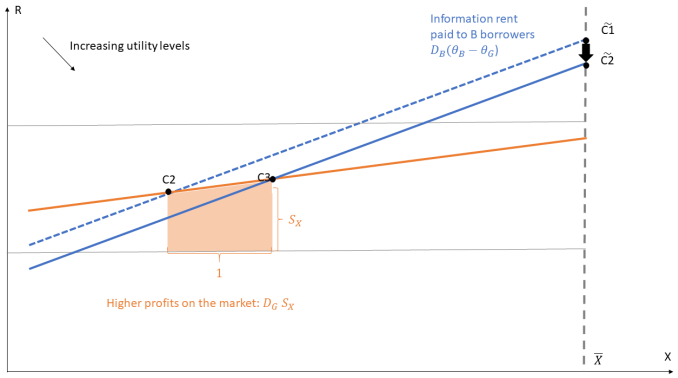


Figure 8: Effect of information discount: More profits per G customer (C_2 to C_3), loose B customers with respect to perfect information ($\tilde{C}_1$ to $\tilde{C}_2$).

Information rent: Figure 7 shows the effect of providing an information premium ($\varepsilon_G^{-1} IR_G = \frac{D_G}{\partial_R D_G} \cdot \frac{\partial_{X_i} S_i(X_i)}{\Delta\theta_{i+1} \partial_{X_i} v(X_i)}$ in equation (11)). It relaxes the incentive compatibility constraints and allows the firm to offer contract C_3 instead of C_2 to low default borrowers. This increases lender's profits by $\frac{\partial_{X_i} S_i(X_i)}{\Delta\theta_{i+1} \partial_{X_i} v(X_i)}$ where $\partial_{X_i} S_i(X_i)$ is the marginal surplus of lending one dollar and $\Delta\theta_{i+1} \partial_{X_i} v(X_i)$ is the information rent (i.e., the cost of painting screening incentives). However, this comes at the cost of losing customers ($\frac{D_G}{\partial_R D_G}$).

Figure 8 shows the effect of providing an information premium ($\varepsilon_B^{-1} IR_B = \frac{D_B}{\partial_R D_B} \cdot \frac{D_G \partial_{X_i} S_i(X_i)}{D_B \Delta\theta_{i+1} \partial_{X_i} v(X_i)}$ in equation (11)). It relaxes the incentive compatibility constraints and allows the firm to offer contract C_3 instead of C_2 to low default borrowers. This increases the lender's profits by $\partial_{X_i} S_i(X_i)$ to each low default borrower. However, this comes at the cost of having to provide an information rent ($\Delta\theta_{i+1} \partial_{X_i} v(X_i)$) to bad borrowers. The rent has both an intensive and an extensive margin effect: it is paid to existing borrowers and will attract new customers as well. The extensive margin is captured by ($\frac{D_B}{\partial_R D_B}$).

K Bringing the model to the data

This section develops a guide on how to bring the model to the data.

K.1 First Case: Proprietary Data on a Single Firm

The estimation of the model parameters is based on a three-step procedure. Step 1 recovers the default probabilities (d_i), which allows us to get α_i. Step 2 identifies the willingness to pay (θ), and step 3 the marginal costs (mc) as well as the demand elasticities parameter (σ). I use the general utility specification described by equation (23).

Step 1: Default Probabilities (d).

I assume that borrower i defaults if their disposable income W_{it} at time t is lower than the loan monthly payment m_{it}. I consider that disposable income is observable by the lender at origination and that the probability of it changing over time is exogenous and is the borrower's (partially) private information. In practice, lenders have a good sense of borrowers' disposable income through credit checks.

In addition to its tractability, this modelling assumption has a convenient implication: It allows recovering the key model parameters without estimating default elasticities to monthly payments (see Steps 2 and 3). Identifying default elasticities relies on comparing borrowers who are observationally equivalent but chose different contracts. Because of the selection on unobservables, finding the right control group is problematic when lenders screen.[31]

31. Alternatively, the default probability can be parametrized as $\theta = \alpha + \frac{\beta}{R}$. This approach also yields

Formally, denoting T_i the loan maturity in months, the borrower survival probability at maturity is:

$$S_i := E[\mathbf{1}_{\{\exists! t_i : m_{it} > W_{it}\}} | X_i^B, i \text{ chose contract } c] \tag{84}$$

X_i^B are borrowers' characteristics, contracts and menus characteristics or any other variables that are observable by the lender when the contract is originated, that predict the income or the contract selection process, and that are legal to use to make acceptance and rejection or pricing decisions.

For each observationally equivalent borrower choosing contract c (indexed by j $\in [\![1, n_c]\!]$), one can estimate the average survival probability ($\hat{S}_i$) using realised default at maturity:

$$\hat{S}_i := \sum_{j=1}^{n_c} \mathbf{1}_{\{j \text{ did not default}\}} \tag{85}$$

Using a constant hazards rate model, this yields the following constant monthly survival probability:

$$\hat{\alpha}_i := e^{-\hat{\lambda}_i}, \text{ with } \lambda_i : \ \hat{S}_i = e^{-\hat{\lambda}_i T_i} \tag{86}$$

The present value of a loan is then $[1 - e^{-\lambda T}]m \approx \lambda T m$. The constant hazards rate model is convenient when screening with maturity (see empirical application). In that case, λ plays the same role as α and Tm as R in the theoretical model. Constant hazard rates are consistent with assuming that the income process follows the same Bernoulli distribution in each period.

Step 2: Borrower preferences (θ_i).

I use a revealed preference approach to recover borrowers' preferences. The identification leverages the idea that if a borrower chooses a contract with a loan size of \$1,000 while

closed-form solutions.

they had access to a contract a loan size of \$1,100 for an interest rate increase of, say, 100 bps, it must be that their willingness to pay for loan size is lower than 100 bps. Figure (9) provides a visual representation of this argument. I plot the internet rate of the contract as a function of loan size (i.e., the pricing schedule) and the borrower indifference curve. When a continuum of contracts is offered, the optimality condition implies that the slope of the indifference curve (i.e., the willingness to pay) must be tangent to the pricing schedule at the contract chosen.

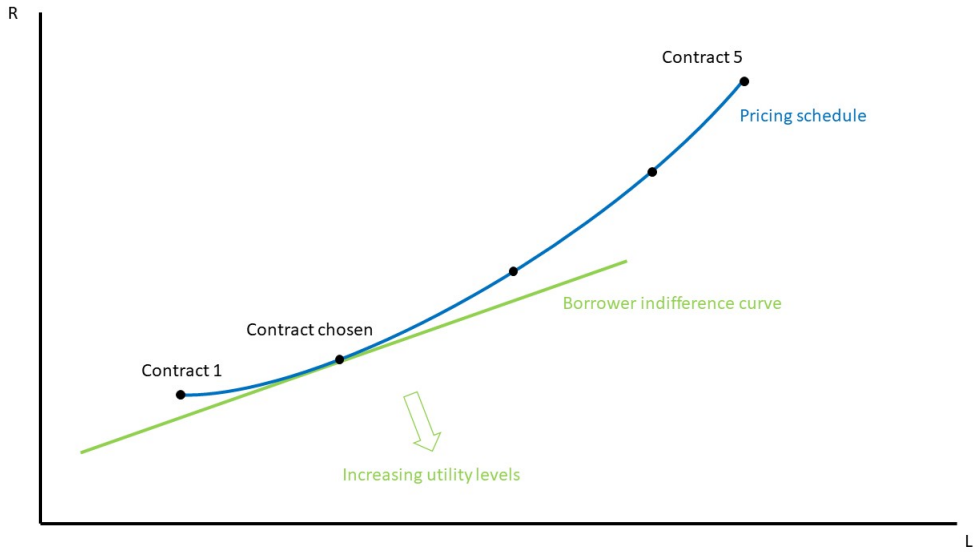


Figure 9: The slope of the pricing schedule at the contract chosen allows us to recover the borrower's willingness to pay θ_i

One challenge with this approach is that willingness to pay θ_i depends on their default probabilities (as highlighted in equation (23)), which are endogenous objects. This implies that the econometrician potentially needs to estimate the default probability if the borrower were to choose another contract. Because of adverse selection, this is problematic.

There is no standard way to address this point as the industrial organisation literature typically focuses on the market without moral hazard and thus estimates willingness to pay as a fundamental parameter of the borrowers' utilities.

As a solution to this problem, I propose to use contracts that yield the same monthly payments (leveraging the variety of maturity and loan size options offered in practice) so that default probability is plausibly constant for a given individual. Depending on the setting

and the data available, it may be preferable to depart from this assumption. The interested reader willing to do so can refer to, for instance, Taburet, Polo, and Vo (2024).

Using contracts with similar monthly payments to recover preferences is plausibly the relevant set of contracts in practice. Indeed, monthly payment targeting is consistent with empirical evidence (see Argyle, Nadauld, and Palmer 2020 for an example using the car loan market, Graham and Harvey 2001 or their CFO Survey). Furthermore, this consideration set also arises naturally in a model with linear utility and inefficient default (see the following footnote[32]). In that case, borrowers max out their maximum monthly payment capacity, so using contracts with the same maturity yields is the relevant set of moments to use.

Formally, I construct the moments using the fact that borrower i chooses the contract designed for them (contract i) if its indirect utility is higher than the one if they were to choose similar contracts (i-1 and i+1). Considering contracts with the same monthly payment and denoting G the probability distribution function of θ conditional on default probabilities, I get the following bounds:

$$u_i(R_i, L_i) \geqslant u_i(R_j, L_j) \ \forall j \implies a := \frac{R_i - R_{i+1}}{L_i - L_{i+1}} \leqslant \theta_i \leqslant b := \frac{R_i - R_{i-1}}{L_i - L_{i-1}} \tag{87}$$

L is the loan size, and R is the maturity times the monthly payments.

Relative contract market shares ($\frac{n_i}{\sum_c n_c}$) and the variables (a, b) are typically observable by the econometrician. An estimate of G is thus:

$$\hat{G}(a \leqslant \theta_i \leqslant b) = \frac{n_i}{\sum_c n_c} \tag{88}$$

Exogenous variation in a or b – i.e. variation uncorrelated with the distribution of θ (i.e., the G function) – allows the recovery of the full distribution non-parametrically. One can use variation in contract terms coming from either cross-sectional or time series variation. For instance, assuming that the distribution of borrower type stays constant over time, exogenous variation in the spread between contracts' interest rates allows the recovery of the distribution of borrowers' willingness to pay.

Parametric assumptions on G can help with power when the number of observations is

32. For instance with the following model: $\max_L \alpha L - p(rL > W)rL - (1-p)c$, $c > r\bar{W}$ and W follows a Bernoulli with its maximum value being $\bar{W}$.

small. Moral hazard, as long as its effects are homogeneous across types, can be dealt with by looking at variations of contracts other than contract i. For instance, Taburet, Polo, and Vo (2024) uses variation in the contract-specific marginal cost of lending to recover the preference heterogeneity in a model with moral hazard and adverse selection.

Step 3: Demand elasticities (σ_i).

I recover the demand elasticity using the pricing equation (82) as a moment condition. My approach contrasts with the standard industrial organisation approach, which estimates demand elasticities using a discrete choice model (e.g., logit in Berry, Levinsohn, and Pakes 1995) and uses the pricing equation (82) to recover marginal costs. There are two reasons to depart from the approach used in industrial organisation. First, data on choice sets at the individual level is not available; and using comparable borrowers to construct choice set is problematic due to bank specialization and selection on unobservables. Second, data on marginal lending costs at the individual level are less challenging to recover in banking: it is common practice to consider that the deposit rate plus the capital requirements constitute the marginal cost of lending. Central banks record those two elements.

The estimation procedure is as follows: Given an estimate of borrower's parameter (α_i, θ_i), and conditional on observing the cost of lending (mc) one can construct the asymmetric information premium and discount using the formula for IR from equation (82). In the case of linear preferences, we have:

$$\alpha_i \hat{IR}_i := [\frac{\alpha_i}{n_i} \frac{\theta_i - \frac{mc}{\alpha_i}}{\Delta\theta_i} 1_{i>1}] - [\frac{\alpha_{i+1}}{n_{i-1}} \frac{\theta_{i+1} - \frac{mc}{\alpha_{i+1}}}{\Delta\theta_{i+1}}] 1_{i<x} \tag{89}$$

Abusing the notation, I use σ_i for the equilibrium demand semi-elasticity instead of $\frac{1}{\sigma(1-\frac{n_i}{\mathbf{B}})}$. The demand elasticity of each group is recovered using the pricing equation (82):

$$\alpha_i R_i = \hat{mc}\text{Ł}_i + \varepsilon_i^{-1}[1 + IR_i] \implies E[\hat{\varepsilon}_i^{-1}|c] := R_i - \frac{\hat{mc}\text{Ł}_i}{E[\hat{\theta}_i|c]}[1 + \hat{IR}_i]^{-1} \tag{90}$$

Equation (90) is derived using the first-order condition of the lender problem with respect to maturity (T), holding the monthly payment constant (m). That way, thanks to the

assumption that only monthly payments affect defaults, I can recover the demand parameters without estimating default semi-elasticities, a problematic parameter to estimate in selection markets (see step 1).

By assuming that semi-elasticities are borrower-specific parameters (ε_i^{-1}), I avoid estimating demand elasticities using choice data as in for instance Crawford, Pavanini, and Schivardi (2018).[33] The constant semi-elasticity assumption can be justified in two ways. First, it arises naturally in the symmetric equilibrium model (the semi-elasticity is $\frac{1}{\sigma(1-\frac{n_i}{\mathbf{B}})}$ which are parameters in my model). Second, it can be interpreted as lenders considering the local borrowers' semi-elasticity as an approximation when setting contract terms.

Alternative to Steps 2 and 3.
Instead of estimating the demand parameters and the demand elasticity separately, one can run a linear regression based on the pricing equation (82). The variation in the loan size (L) allows the recovery of the marginal cost (mc), and the variation in information rents (IR) identifies the demand semi-elasticity (ε^{-1}). I illustrate this approach and discuss the exclusion restrictions in the empirical application (Section 7).

Formally, given that ($\alpha_i R_i$,Ł$_i$,IR_i) are observable or can be estimated, one can run the following regression:

$$\alpha_i R_i = \bar{m}c\text{Ł}_i + \bar{\varepsilon}_i^{-1}[1 + IR_i] + e_i \tag{91}$$

Where $\bar{\varepsilon}_i^{-1}$ is the 0^{th} order approximation of the demand semi-elasticity. The assumption that lenders consider demand semi-elasticities to be approximately a flat function is plausible as commonly used models imply constant approximately constant semi-elasticities. For instance, linear regression models of the log market share on product characteristics yield constant semi-elasticities. In logit models, the expression holds in a symmetric equilibrium. In an asymmetric equilibrium, the curvature of the semi-elasticity is equal to the market share D with logit demands, so the approximation error is small when the lender market share is small. In our empirical application, online lending firms compete with traditional

33. Similarly to the default elasticities, demand elasticities are problematic to estimate when there is a selection of unobservables, even with credit register type of data. This is because the econometrician only observes the contract chosen and thus has to estimate the contract offered by competitors using observational equivalent borrowers.

banks for consumer loans and have a relatively small market share compared to them.

K.2 Second Case: Credit Register Data

When data on menus are unavailable, one has to predict the menus using observational equivalent borrowers (see, for instance, Crawford, Pavanini, and Schivardi 2018).

In the context of my model, I need a parameterization flexible enough to feature random coefficients and separate choices of lender (i.e., competition) and contracts (i.e., screening). Using a nested logit with random coefficient is not empirically tractable. As a result, I propose to estimate the pricing schedule for each borrower and characteristics at the lender level and use it to recover the willingness to pay (θ) as in step 2. Then use a logit model with linear utilities to recover the demand elasticity using choice data. A standard parameterization of the utility is: $u^*_{i,b} + \mu_b + \sigma\varepsilon_{ib}$, with ε_{ib}, with $u^*_{i,b}$ estimated from step 2, μ_b being a bank fixed effect and ε_{ib} being extreme value distributed. This procedure is similar to the nested logit approach with random coefficients but is more flexible and tractable.

Alternatively, once can use the approach developed in the section called "Alternative to Steps 2 and 3" and implemented in the empirical section of the paper. This approach has the advantage compared to the logit model that one does not have to take a stand on the choice set available to the borrower.

L Borrower preference approximation

Using data on menus, I approximate the pricing schedule $(L_c, r_c, T_c)_c$, where r is the interest rate and T is the maturity using a spline function. This interpolation allows to use first-order conditions to derive the moment equation. I discuss alternative options in the paragraph in Section K.1 labelled "Heterogeneity within contracts".

Formally, I first calculate the present value of the loan $R_{i,c} = \sum_{t=1}^{T} \theta_i^t T_i$ for each contract c holding the monthly payment (T_i) constant. Then, I find the coefficients $(\gamma_1, \gamma_2, \gamma_3)$ such that $R_i(L) := \hat{\gamma}_{1,i} + \hat{\gamma}_{2,i} L + \frac{\hat{\gamma}_{3,i}}{2} L^2$ best match the data points $(R_{i,c})$.

I recover borrowers' preferences using a revealed-preference approach. For a given monthly payment and an implied pricing schedule, I have

$$u_i^* := \max_L u(L) := \alpha_i L - \theta_i R_i(L) \implies L_i = \frac{\alpha_i}{\hat{\gamma}_{3,i}\theta_i} - \frac{\hat{\gamma}_{2,i}}{\theta_i} \tag{92}$$

For each contract c, I can thus recover borrowers' average preference by inverting equation (92) and using the average default estimates:

$$E[\hat{\alpha}_i|c] = L_i\hat{\gamma}_{2,i}(1 + E[\hat{\theta}_i|c]) - \hat{\gamma_{2,i}} \tag{93}$$

Equation (93) states that the slope of the pricing schedule for the contract chosen allows us to recover the borrower's willingness to pay $\frac{\alpha_i}{\theta_i}$. Figure 5 provides a visual representation of the identification strategy. An identification threat would be the slope of the pricing schedules, which varies due to unobserved contract characteristics such as covenant. In that case, the econometrician is not identifying the willingness to pay for loan size but rather the willingness to pay for both covenant and loan size. The "Heterogeneity within contracts" paragraph discusses how to deal with this issue.

M Empirical model

I first show that it is optimal to max out monthly payments in that set-up. For simplicity, I first show this under perfect information.

Denoting u_{ib} the promised utility provided by contract i, the lender problem, replacing the present value of the loan by $L_i - u_i$, is (when the contract monthly payment is lower than mp_i):

$$\max_{u_i, L_i, T_i} D(u_i)[L_i - u_i - mc(L_i, T_i)] \tag{94}$$

Since the marginal cost of lending is increasing in maturity T_i, the optimal contract maxes out monthly payment by decreasing the maturity as much as possible.

This result also holds under imperfect information. Approximating the present value around default being close to zero as in equation 23, we get that the loan present value is:

$T(1-d)m$ where m is the monthly payment. This implies that the willingness to pay for maturity and monthly payment is the same, so the cost of screening with one versus the other is similar.

I now show that the model is similar to the one analyzed in Section 5.

Borrower i chooses contract i if it provides a higher utility than any other contract. Formally:

$$L_i(T_i, r_i; mp_i) - PV_i(T_i, r_i; mp_i) \geqslant L_i(T_j, r_j; mp_i) - PV_i(T_j, r_j; mp_i) \tag{95}$$

$L_i(T_j, r_j; mp_i)$ is the loan size that the borrower i would choose if they chose a contract with another maturity T_j at rate r_j. $PV(mp_i, T_i)$ is the present value of the loan as a function of maturity T_i.

Using the annuity formula, one can show that the heterogeneity in mp cancels out in the incentive compatibility constraints. For instance, in continuous time, the principal is $L = mp \cdot [1 - \frac{e^{-r_i T_i}}{r_i}]$ and the present value is $mp \cdot e^{-\lambda_i T_j}$. λ_i is the hazard rate, so that $e^{-\lambda_i}$ is the monthly survival probability. Formally, I linearize $e^{-\lambda_i T_j}$ and use the change of variable $\tilde{L}_i := [1 - \frac{e^{-r_i T_i}}{r_i}]$. Thus, if the distribution of maximum monthly payment mp_i is independent of the distribution of willingness to pay and θ_i and demand elasticities σ_i conditional on risk category, the optimal menus are independent on mp.

The model is similar to the one analyzed in Section 5. Indeed, the utility in continuous time, with a hazard rate λ, and linearizing the present value, is: $mp_i[1 - \frac{e^{r_i T_i}}{r_i}] - mp_i \lambda_i T_i$. Using the change of variable $L := [1 - \frac{e^{r_i T_i}}{r_i}]$ and $R := T$, we are back to the theoretical model.

N Empirical Application Model

I consider that there are N types of borrowers (i.e., N combinations of $(mp_i, \alpha_i, \theta_i, \sigma_i)$), where mp_i is the maximum monthly payment the borrower can repay. For each borrower type, I

construct an incentive compatibility constraint:

$$L_i - \theta_i \cdot \overbrace{mp_i T_i}^{R_i} \geqslant L_j(mp_i, T_j) - \theta_i \cdot mp_i T_j \tag{96}$$

where $L_j(mp_i, T_j)$ is the loan size that would be chosen by the borrower if they were to choose a contract with the other maturity.

The lender problem, when offering two contracts (indexed by l and h) is :

$$\max_{R.L} \sum_{i=1}^{l} \overbrace{n_i \phi(u_i)}^{Demand} \alpha_i [R_l - \frac{mc}{\alpha_i} L_h] + \sum_{i=h}^{n} n_i \phi(u_i) \alpha_i [R_h - \frac{mc}{\alpha_i} L_h] \tag{97}$$

$$with\ u_i := \theta_i L_i - R_i \tag{98}$$

$$\{h := l+1, l\} : u_h(L_h, R_h) := u_l(L_l, R_l) + (\theta_h - \theta_l) L_i \tag{99}$$

The model can be solved in two steps. First, the lender chose the marginal borrower (borrower h), which is indifferent between the two contracts. Second, the lender sets the optimal menu to separate the borrowers willing to pay above and below borrower h.

Denoting $\bar{u}$ and $\underline{u}$ the utility provided to the average borrower selecting a contract (the contracts are indexed by h and l), and denoting $\Delta_i := (\bar{\alpha} - \alpha_i)$ the difference of preferences between the average borrowers ($\bar{\alpha}$) and borrower i (α_i), the problem, for a given marginal borrower h, can be written:

$$\sum_{i=1}^{l} n_i \phi(\bar{u} + \Delta_i L_h) \alpha_i [(\bar{\theta} - \frac{mc}{\alpha_i}) L_h - \bar{u}] + \sum_{i=h}^{n} n_i \phi(\underline{u} + \Delta_i L_l) \alpha_i [(\underline{\theta} - \frac{mc}{\alpha_i}) L_h - \underline{u}] \tag{100}$$

$$\bar{u} + (\theta_h - \bar{\theta}) L_h = \underline{u} + (\theta_l - \underline{\theta}) L_l + (\theta_h - \theta_l) L_l \tag{101}$$

$\bar{u} + (\theta_h - \bar{\theta}) L_h = \underline{u} + (\theta_l - \underline{\theta}) L_l + (\theta_h - \theta_l) L_l$ is the incentive compatibility constraint of the borrower the lender choose as the marginal borrower (borrower h).

Replacing the constraint and taking the first-order conditions with respect to ($\bar{u}$ and $\underline{u}$), we obtain the same first-order conditions as in the two-borrower models.

O Bias with Crawford, Pavanini, and Schivardi (2018)

This section highlights the bias when using a revealed preference approach to recover borrowers' preferences. For exposition purpose, I consider that the borrower chose between two contracts: A fixed or a flexible interest rate contract. Because only chosen contract are observed, one has to simulate the outside option as in Crawford, Pavanini, and Schivardi (2018). Formally, I assume that borrowers choose the fixed rate (i.e., $Fixed = 1$) when:

$$Fixed = \begin{cases} 1 & \text{if } \beta X_i \geqslant \epsilon_i \\ 0 & \text{otherwise} \end{cases} \tag{102}$$

$\beta X_i \geqslant \epsilon_i$ can be interpreted as the difference in borrower utilities when choosing fixed or flexible contracts, X_i are the differences between the two contract characteristics.

For simplicity of the exposition, I use a linear probability model. The probability that the interest rate is flexible is βX_i. $Y_i = 1$ when the interest rate choice is fixed, zero otherwise, we have:

$$Y_i = \beta X_i + e_i = \beta_1 r_i^{fix} - \beta_2 r_i^{flex} + \delta_{flex} + e_i \tag{103}$$

$\beta X_i + e_i$ captures the utility difference between choosing a fixed or a flexible interest rate.

We typically do not observe the outside option (let us say r_i^{flex}). Thus, we estimate it $\hat{r}_i^{flex} = r_i^{flex} + \varepsilon_i + b_i$ where ε_i is an error term with mean zero and b_i is a bias term with mean different than zero.

Abstracting from the fact that we do not observe the flexible rate only for borrowers who chose fixed rate, An OLS regression would give (assuming the other parameters than β_2 are known):

$$-\beta_2 = \frac{-\beta_2 - cov(Y_i, \varepsilon_i + b_i)}{Var(r_i^{flex}) + Var(\varepsilon_i + b_i)} \tag{104}$$

There are two issues. First, there is a measurement error which biases the estimates

towards zero. Second, by construction, the estimation bias b_i is correlated with Y_i as the bias exists when $Y_i = 1$.

Those two issues are likely to be present when predictive interest rate regression there is low R^2, or when borrowers self-select based on their default probability and demand semi-elasticity (unobserved by the econometrician).

P Simulation

In this section, we simulate the performance of the estimation procedure proposed by Crawford, Pavanini, and Schivardi 2018 compared to the one proposed in this paper.

We compare the performance in two situations. One in which the borrower's choice set is unobservable. Another situation in which the econometrician observes the borrower's characteristics only partially.

For consistency, the simulation is based on the following model.

Borrowers: Borrowers have quasi-linear utilities in the loan repayment R:

$$u := F(L) - R + \sigma^{-1}\varepsilon \tag{105}$$

$F(L)$ is the utility coming from loan size.

Lenders: Lenders offer a take-it-or-leave-it contract to maximize static profit:

$$\max D(u)[\theta(R)R - mc(L)] \tag{106}$$

$\theta(R)$ is the survival probability.
$D(u)$ is the probability that the borrower accepts the contract.
$mc(L)$ is the marginal cost of originating the contract.
The first-order conditions yield:

$$\partial_R\{\theta(R)R\}F'(L) = mc \tag{107}$$

$$\theta(R)R = mc(L) + \frac{D(u)}{D'(u)}[\partial_R\{\theta(R)R\}] \tag{108}$$

For simplicity, let $\theta(R) = \bar{\theta} + \beta\frac{1}{R}$. In that case, the first-order conditions yield:

$$\bar{\theta}F'(L) = mc \implies L = F'^{-1}(\frac{mc}{\bar{\theta}}) \tag{109}$$

$$\bar{\theta}R + \beta = mc(L) + \frac{D(u)}{D'(u)}[\bar{\theta}] \tag{110}$$

P.1 Unobserved choice set

Setting: There are two markets. Each market has 2 lenders. There are N types of borrowers and they appear in both markets. Each individual borrower only borrows in their own market.

Data: As econometricians, we observe the bank and the contract chosen by each borrower. We do not observe each individual's true choice set. We assume that each borrower can access all 4 banks in both markets.

Conclusion: The old approach is subject to misspecification of choice set, while the new approach is robust to choice set. The old approach is the estimation procedure based on Crawford, Pavanini, and Schivardi (2018). The new approach is the one described in this paper.

Example: With $mc_1 = [0.1, 0.3], mc_2 = [0.15, 0.25]$

	Old Approach		New Approach	
	mc_1	mc_2	mc_1	mc_2
True	[0.1, 0.3]	[0.15, 0.25]	[0.1, 0.3]	[0.15, 0.25]
$N = 1,000$	[0.1749, 0.3638]	[0.2237, 0.3153]	[0.1105, 0.3277]	[0.1672, 0.2728]
$N = 5,000$	[0.1777, 0.3597]	[0.2251, 0.3129]	[0.1224, 0.3180]	[0.1662, 0.2658]
$N = 10,000$	[0.1771, 0.3636]	[0.2254, 0.3155]	[0.1510, 0.2792]	[0.1225, 0.2310]
$N = 50,000$	[0.1769, 0.3551]	[0.2233, 0.3091]	[0.0918, 0.2920]	[0.1432, 0.2431]

P.2 Average contract

Setting: There is one market and 2 lenders in the market. There are many types of borrowers. Lenders observe all characteristics of borrowers, while econometricians observe only a subset of characteristics. To econometricians, there are N types of observationally equivalent borrowers.

I assume that there are N_{sub} types of borrowers within each type of observationally equivalent borrowers, so there are $N \times N_{sub}$ types of borrowers in total, and therefore, $N \times N_{sub}$ pairs of contracts on the market.

Data: I first simulate $\sigma, Z, \beta, \bar{\theta}$ for N types of observational equivalent borrowers (call them main types), then for each main type, I simulate N_{sub} shocks separately for $\sigma, Z, \beta, \bar{\theta}$ to have N_{sub} types of borrowers (call them subtypes). For each subtype of borrowers, I solve for the optimal contract provided by each lender.

Estimation: As econometricians, we observe N_{sub} pairs of contracts for each main type of borrowers. We average over the N_{sub} contracts by the same lender to get the average contract, then treat the data as if all observationally equivalent borrowers have the same average contract.

Examples: With $mc = [0.1, 0.3]$, let N^{sim} be the number of simulated borrowers of each subtype.

Note: when using the old method, I can have N estimates of mc's and $sigma$'s, I report the median; when using the new method, there are two lenders so I can estimate two σ's. By default I report the median; if the two estimates deviate too much, I report both estimates.

In Table 5.2(A), I let

$$\bar{\theta}_{is} = \bar{\theta}_i + 0.03\epsilon_{is}$$
$$\beta_{is} = \beta_i + 0.03\epsilon_{is}$$
$$Z_{is} = Z_i + 0.05\epsilon_{is}$$

where $\epsilon_{is} \sim N(0, 1)$

N	N_{sub}	N^{sim}	Seed	$\hat{mc}^{old}$	$\hat{mc}^{new}$	σ^{sim}	$\hat{\sigma}^{old}$	$\hat{\sigma}^{new}$
4	1	10,000	12	[0.1037, 0.3056]	[0.1352, 0.3008]	2.3464	2.4243	3.0191
4	2	10,000	15	[0.1033, 0.3058]	[0.1692, 0.2676]	1.3270	1.3518	2.2898
4	5	10,000	15	[0.1046, 0.3065]	[0.1435, 0.2984]	1.3270	1.3469	1.7497
4	10	10,000	15	[0.1058, 0.3075]	[0.0958, 0.3018]	1.3270	1.3902	1.2943
4	20	10,000	15	[0.1058, 0.3096]	[0.1170, 0.2950]	1.3270	1.3775	1.4061
4	50	10,000	15	[0.1072, 0.3105]	[0.1114, 0.2915]	1.3270	1.4020	1.3387
6	1	5,000	14	[0.0763, 0.2816]	[0.1344, 0.3281]	2.1297	1.9750	3.3413
6	2	5,000	14	[0.0916, 0.2957]	[0.1118, 0.3033]	2.1297	2.1370	2.2704
6	5	5,000	14	[0.0993, 0.3016]	[0.1266, 0.3033]	2.1297	2.3024	2.4830
6	10	5,000	14	[0.0620, 0.2712]	[0.0943, 0.2992]	2.1297	1.9723	2.0404
6	20	5,000	14	[0.0946, 0.2964]	[0.2107, 0.3224]	2.1297	2.0064	[-29.0713, 3.0459]
6	50	5,000	14	[0.0879, 0.2944]	[0.1976, 0.3315]	2.1297	1.8750	[40.6687, 3.6556]

In the Table, I increase the level of variation within each main type, let

$$\bar{\theta}_{is} = \bar{\theta}_i + 0.05\epsilon_{is}$$
$$\beta_{is} = \beta_i + 0.05\epsilon_{is}$$
$$Z_{is} = Z_i + 0.075\epsilon_{is}$$

where $\epsilon_{is} \sim N(0, 1)$

Table 4: *
Table 5.2(A)

N	N_{sub}	N^{sim}	Seed	$\hat{mc}^{old}$	$\hat{mc}^{new}$	σ^{sim}	$\hat{\sigma}^{old}$	$\hat{\sigma}^{new}$
4	1	10,000	12	[0.1056, 0.3108]	[0.1572, 0.3108]	2.3464	2.4850	4.2893
4	2	10,000	15	[0.0998, 0.2966]	[0.1661, 0.3148]	1.3270	1.3960	2.4973
4	5	10,000	15	[0.0991, 0.3019]	[0.1629, 0.2883]	1.3270	1.3150	2.1587
4	10	10,000	15	[0.1134, 0.3218]	[0.1131, 0.2952]	1.3270	1.4639	1.3396
4	20	10,000	15	[0.1145, 0.3246]	[0.1497, 0.2854]	1.3270	1.4707	1.7762
4	50	10,000	15	[0.1134, 0.3233]	[0.1482, 0.2837]	1.3270	1.4532	1.7308
6	1	5,000	14	[0.0752, 0.2619]	[0.1206, 0.3204]	2.1297	3.0000	2.9915
6	2	5,000	14	[0.0938, 0.2981]	[0.1183, 0.3073]	2.1297	2.0714	2.4212
6	5	5,000	14	[0.0920, 0.2968]	[0.2161, 0.3844]	2.1297	1.9440	-14.1287
6	10	5,000	50	[0.0793, 0.2848]	[0.1540, 0.2739]	1.9090	1.5673	2.6733
6	20	5,000	50	[0.0653, 0.2766]	[0.1771, 0.2900]	1.9090	1.4215	4.5741
6	50	5,000	80	[0.0685, 0.2798]	[0.2255, 0.4574]	2.0264	1.4798	-5.5622