Bypassing the Shareholders: Activist Settlements

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Abstract

I provide a theoretical framework of negotiations and settlements between activist investors and boards. The activist can demand that his proposal be implemented right away or demand board seats. If rejected, the activist can launch a proxy fight. Contrary to common concerns, value-destroying projects are typically not implemented following settlements, but rather after the activist *wins* proxy fights with shareholders' endogenous support. Yet, shareholders would be *better off* by committing to supporting the activist more. Nevertheless, banning settlements may still benefit shareholders even if the activist implements only value-creating projects. The model yields many predictions consistent with existing empirical evidence.

KEYWORDS: Allocation of Control, Authority, Bargaining, Boards of Directors, Corporate Governance, Settlement, Shareholder Activism, Shareholder Voting, Voice.
JEL CLASSIFICATION: C70, C78, D72, D74, D82, G23, G34

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1 Introduction

Shareholder activism has become widespread over the recent decades. Under many jurisdictions, one path utilized by activist investors to exert control on firms is to challenge boards with a "proxy fight" (i.e., a contested election for board seats). However, there is another path that an activist can pursue to exert influence: The activist can negotiate directly with the incumbent board, and if the incumbent agrees to the activist's demands, they reach a settlement, thereby effectively sidestepping the shareholders. As documented by Gow et al. (2023), Bebchuk et al. (2020), Schoenfeld (2020), and Klein and Zur (2009), such settlements are common and their number has surpassed the number of proxy fights.[1]

Importantly, there have been a lot debates and concerns about activist settlements, the lack of shareholders' say in them, and whether these settlements should be allowed in the first place. In a summary of its survey of large shareholders, referring to the settlements where the activist obtains board seats, Ernst & Young stated that "some (institutional investors) feel that board seats should not be a bargaining chip, as this practice distorts the board's election process."[2] BlackRock, the world's largest asset manager, has expressed that "there is a real concern among investors that standard negotiated settlements—such as giving board seats to a dissident or announcing a stock buyback—may favor short-term gains at the expense of long-term performance," voicing its worry about not only settlements that give board seats to activists but also those that implement activists' proposal outright, such as a share buyback.[3]

However, in spite of the prevalence of activist settlements and the concerns surrounding them, they have not received much attention in the theoretical literature. The objective of this paper is to provide a theoretical framework to study implications of settlements for shareholders, as well as to compare different kind of settlements.

To that end, I develop a model of negotiations and settlements between activists and incumbent boards, where the activist can run a proxy fight if the negotiations fail. I find that

[1] Klein and Zur (2009) document that in 81-86% of the cases where activists have obtained board representation, they have obtained it through settlements rather than proxy fights. Gow et al. (2023) and Bebchuk et al. (2020) report similar statistics, where the latter also emphasize the upward trend in settlements. For more details on these papers, see Section 3.2.2.

[2] See Reuters, "Big Funds Push Back Against Activist Investor Settlements," 7/18/2016.

[3] See HLS Forum on Corporate Governance and Financial Regulation, "Getting Along with BlackRock", 11/06/2017. Another large asset manager, State Street, has stated that "settlement agreements entered into rapidly between boards and activists and without the voice of long-term shareholders concern us, as we see evidence of short-term priorities compromising longer-term interests." (see State Street Global Advisors, "Protecting Long-Term Shareholder Interests In Activist Engagements," April 2020). Besides institutional investors, the SEC and policymakers have expressed concerns about activist short-termism as well, and they have suggested rule changes to combat it. For details, see the footnote in Section 5.4.

even though shareholders are left out of settlement negotiations, contrary to common concerns, value-destroying projects are typically not implemented following settlements, but rather after the activist (endogenously) *wins* a proxy fight with shareholders' support. However, almost paradoxically, the level of support that shareholders give to the activist in a proxy fight is *lower* than the level of support that maximizes ex-ante shareholder value. Yet, allowing for settlements can still *harm* shareholders, and this is true even if the activist's interests are aligned with those of shareholders.

As alluded to in BlackRock's quote above, there are two types of settlements that can be reached between activists and incumbent boards. In a "board settlement," the activist obtains board seats and hence acquires some control in the boardroom to execute his agenda.[4] On the other hand, in what I call an "action settlement", the incumbent agrees to implement the activist's proposal right away.[5] I show that comparing action and board settlements not only yields many predictions that are consistent with the existing empirical evidence, but also generates further implications for shareholder value: Even though the *ex-post* shareholder value upon *reaching* a board settlement is lower, *demanding* a board settlement can yield a higher *ex-ante* shareholder value.

The model is as follows. There is an activist and a firm that is run by its incumbent board. The activist can either settle with the incumbent board or run a proxy fight to replace it. There is a project that the firm can implement. The incumbent board is privately informed about the value of the project but it also enjoys private benefits from keeping the status quo. The activist is uninformed about the project's value but he is more willing to implement the project than the incumbent. That said, the activist can have a bias of his own regarding the project. The novel feature of my model is that the activist can demand a settlement. Specifically, he can demand an action settlement or a board settlement. The latter provides the activist not only with some decision authority over the project, but also (through his position on the board) with better information regarding the value of the project. To the best of my knowledge, not only this is the first paper in the blockholder theory literature that models board settlements, but also more

[4]For example, in 2016, the activist hedge fund Starboard Value approached Yahoo with the aim of selling its core business, reached a settlement that gave it four seats on the board, and then succeeded in the sale. Yahoo's core business was sold to Verizon later that year for $4.8 bn. See CNBC, "Yahoo adds 4 directors in settlement with activist investor Starboard", 4/27/2016; The New York Times, "Yahoo's Sale to Verizon Ends an Era for a Web Pioneer", 7/24/2016.

[5]For example, in 2020, after being pressured by the hedge fund Third Point, Walt Disney agreed to prioritize its streaming service Disney+ and substantially increase the investment in it by undergoing a major reorganization, although Third Point did not have any presence on the board of Walt Disney. See Wall Street Journal, "Disney Elevates Streaming Business in Major Reorganization", 10/12/2020; CNN, "Disney+ is up to nearly 74 million subscribers", 11/12/2020.

generally, it is the first theoretical paper in the economics literature on bargaining that studies negotiations over access to information and decision making authority. Importantly, board settlements are very common: Bebchuk et al. (2020) document that 84% of the settlement contracts with activist hedge funds between 2000-2013 resulted in the appointment of new directors to the board.

I start the analysis in Section 3 by considering the subgame where the activist has demanded a *board settlement* (with the goal of potentially implementing the project after taking board seat(s)). The incumbent accepts the board settlement demand both when the project's NPV is sufficiently high and when the NPV is very low, as illustrated in Figure 1. The reason the incumbent is willing to accept giving board seats to the activist in the latter case is that an activist who joins the board and learns that the NPV is very low will choose not to implement the project. Similarly, the incumbent also accepts the settlement when the NPV is high (even though in this case, the activist implements the project whenever he achieves decision authority after joining the board), because compared to facing the risk of a proxy fight, the incumbent suffers less from the implementation of the project (due to the shares it owns in the firm).[6] On the other hand, if the project's NPV is moderate, then the incumbent's private benefits from keeping the status quo dominate, and as a result, the incumbent rejects the board settlement and the activist sometimes runs a proxy fight.

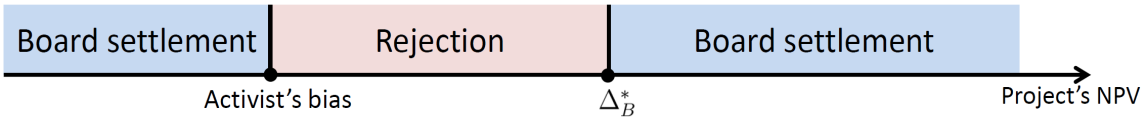


Figure 1 - The incumbent's equilibrium behavior if the activist demands a board settlement.

The dynamics described above have several key implications for shareholder value. The main insight behind these implications (as well as behind the rest of the results in the paper) is that the information conveyed by the incumbent's response to the activist's demand is a critical factor that determines both the activist's decision to run a proxy fight and shareholders' support for the activist.

First, despite common concerns mentioned earlier, value-destroying projects are never implemented following board settlements, but they are implemented with positive probability after the activist *wins* a proxy fight, even though winning the proxy fight requires getting sufficient support from the shareholders. This result follows due to the shareholders' inference

[6] As I show in the analysis, all results in the paper continue to hold even if the incumbent owns very few shares of the firm.

about the project from the incumbent's decision to reject the settlement. Intuitively, for a negative NPV project to be implemented following a settlement, it has to be that the incumbent's rejection threshold (Δ_B^* in Figure 1) is negative. However, the shareholders would then never support the activist in a proxy fight, which would invalidate the activist's proxy fight threat and in turn eliminate any incentive of the incumbent to accept the settlement in the first place. Therefore, in equilibrium, for any negative NPV project that the activist is willing to implement, the incumbent rejects the settlement (i.e., $\Delta_B^* > 0$), and the activist runs a proxy fight with positive probability.

Moreover, the activist sometimes wins this proxy fight, resulting in implementation of value-destroying projects. The reason the activist can win a proxy fight is because the incumbent rejects the settlement also for some positive NPV projects (again, because $\Delta_B^* > 0$), and hence shareholders support the activist with positive probability in a proxy fight. This is why negative NPV projects are not implemented upon reaching a settlement, but rather after the activist wins a proxy fight with the endogenous support of the shareholders. Therefore, the common argument that compared to proxy fights, the lack of shareholders' say in settlements makes it more likely for activists to implement value-destroying projects after a settlement is significantly flawed.

In fact, I also find that shareholders' ability to vote in the proxy contest may *harm* ex-ante shareholder value, even though shareholders behave rationally and do not have any bias in my model: In particular, in any activist campaign that takes place in equilibrium, shareholders would be better off by committing to always supporting the activist in a proxy fight. This is because such a commitment would increase the likelihood of a settlement (i.e., result in a lower Δ_B^*), and thereby result in a larger increase in the probability that the project is implemented when it has a high NPV.

Nevertheless, I also show that even if the activist's interests are aligned with shareholders' interests (i.e., even if he prefers to implement the project if and only if its NPV is positive), his ability to reach settlements can harm *ex-ante* shareholder value, because it may reduce the probability that the project is implemented. For example, this occurs if a board settlement does not give the activist a high level of control on the board or the activist's cost of the proxy fight is high, both of which are consistent with the empirical evidence documented in the literature (Gantchev (2013), Bebchuk et al. (2020)). Therefore, it is true that the existence of settlements may destroy shareholder value, although it does not occur because of the common concerns described earlier.

Next, to derive empirical predictions and further policy implications, I compare action and

board settlements in Section 4.[7] I show that the model yields many predictions consistent with the existing empirical evidence: First, the likelihood of reaching a settlement is higher when the activist demands a board settlement than when he demands an action settlement, as documented by Gantchev (2013). Second, as information asymmetry increases, the likelihood that an activist campaign results in a board (action) settlement increases (decreases). Bebchuk et al. (2020) explicitly test this prediction and point out that their finding is consistent with my model. Third, the average ex-post shareholder value conditional on reaching an action settlement is larger compared to conditional on reaching a board settlement. This finding is not only consistent with the evidence in Bebchuk et al. (2020), as again pointed out in their paper, but also consistent with Becht et al. (2009). I derive further predictions in Section 4.

Then, I focus on the additional implications for shareholder value, and I show that abnormal stock returns around settlements may not be an accurate measure to evaluate the implications of settlements for shareholder value. In particular, note that the last empirical prediction described above might give the impression that the type of settlement that the activist can demand should be restricted to an action settlement (e.g., by prohibiting board settlements), consistently with institutional investors' concern that "board seats should not be a bargaining chip." However, despite the *lower* ex-post shareholder return after *reaching* a board settlement, *demanding* a board settlement can result in *higher* ex-ante shareholder value than demanding an action settlement. This is because demanding a board settlement not only leads to a higher likelihood of reaching a settlement, but it can also increase the likelihood of the activist launching a proxy fight upon the incumbent's rejection of the settlement. As a result, making it harder for activists to acquire board representation via settlements can hurt shareholders.

Finally, in Section 5, I consider several extensions, which include: (*i*) micro-founding the activist's bias via short-termism, (*ii*) the activist not always learning the value of the project upon joining the board, (*iii*) allowing the incumbent to make a counter-offer to the activist's settlement demand, (*iv*) adding a stage at the beginning of the game where the incumbent can voluntarily choose to disclose the project's NPV to the activist (e.g., via an information sharing agreement), (*v*) allowing the incumbent to have a bias in favor of implementing the project, and (*vi*) allowing for shareholder proposals and short sales. These extensions show robustness of the results and generate further implications.

[7] I also show that all of the implications for shareholder value that were derived above (where it was assumed that the activist demands a board settlement) continue to be valid in the case where the activist demands an action settlement, and the intuition behind these implications are the same.

1.1 Related literature

My paper is related to the literature on corporate governance and shareholder activism. In general, there are two kinds of governance mechanisms: voice and exit.[8,9] My paper belongs to the strand of literature that focuses on voice. Typically, this strand does not distinguish between different types of intervention methods and builds on the notion that the activist can force his intervention on the firm without persuading the incumbent or shareholders (e.g., Admati et al. (1994), Burkart et al. (1997), Maug (1998), Noe (2002), Edmans and Manso (2011), Back et al. (2018), Edmans et al. (2019), and Burkart and Lee (2022)).[10] In contrast, my paper includes settlements as a form of voice mechanism, alongside a proxy fight. Moreover, in my model, the success of the activist's intervention attempts depends on the belief of the incumbent regarding the activist's threat of running a proxy fight and on the belief of the shareholders regarding the value the activist will create in the event of a proxy fight. The role of proxy fights in exerting control is extensively studied in literature (e.g., Shleifer and Vishny (1986), Harris and Raviv (1988), Jarrow and Leach (1989, 1991), Bhattacharya (1997), Maug (1999), Yılmaz (1999), Bebchuk and Hart (2001), Gilson and Schwartz (2001), Levit and Malenko (2011), and Corum and Levit (2019)). Differently from this literature, my paper focuses on settlements and their interaction with the activist's decision to run a proxy fight.

The closest papers to mine are other papers that model activist interventions as a sequence of escalating steps (e.g., Song (2017), Levit (2019), Pi (2021)), which are surprisingly few relative to the large size of the overall voice literature. These models and my paper share the idea that voice (i.e., a proxy fight) is an outcome of a failure to resolve the conflict by other means. Song (2017) studies the effects of different degrees of reputation concerns on the activist's threat of intervention and exit; Levit (2019) combines voice and exit but focuses on persuasion (i.e., communication of private information) by the activist; and Pi (2021) analyzes the activist's choice of coalition size. In contrast, my model focuses on settlements as a form of bargaining: I include a board settlement in the set of the activist's possible demands along with

[8] For surveys on voice and exit, see, e.g., Edmans (2014) and Edmans and Holderness (2017). For a survey that is focused on financial intermediaries as blockholders, see Dasgupta, Fos, and Sautner (2021).

[9] Broadly speaking, the exit literature studies how a blockholder can influence the decisions of the manager of a firm, not through voice, but because the exit of the blockholder can reveal negative information about the firm, thereby punishing the manager. For the literature on exit, see, e.g., Admati and Pfleiderer (2009), Edmans (2009), Dasgupta and Piacentino (2015), and Cvijanovic, Dasgupta, and Zachariadis (2022). For other models related to exit, where again a blockholder's trading impacts the manager's decisions, see, e.g., Khanna and Mathews (2012) and Goldman and Strobl (2013).

[10] For further papers, see the surveys in footnote 8 above.

an action settlement, and I show that the information content of the incumbent's response to the activist's settlement demand is a key factor behind the activist's choice of his demand, his incentives to run a proxy fight, and shareholders' voting behavior in a proxy fight. In addition, contrary to these papers, an important focus of my paper is the implications of settlements for shareholders.

In the structural estimation literature, Gantchev (2013) and Johnson and Swem (2021) also model interventions by activists as escalating steps. In particular, to estimate the costs of various stages of activism, Gantchev (2013) builds a sequential decision model where the activist decides at each stage whether to exit or escalate his intervention tactic. Johnson and Swem (2021) quantify the dynamic effects of the activist's reputation over time. In contrast to these papers, a novel feature of my framework is that I explicitly model the critical differences between a proxy fight and settlements, as well as within settlements.

Cohn and Rajan (2013) also study the effect of an activist investor on the board's decision-making. However, in their model the role of the activist is to produce information, and the board acts as an unbiased arbitrator between the management and the activist with the aim of maximizing shareholder value. In contrast, my model treats the board and management as a monolithic entity whose interests are not fully aligned with those of shareholders, and I study the relation among different kind of intervention methods (i.e., settlements and proxy fight) the activist can utilize.

My paper is also related to the literature on bargaining under asymmetric information (see Kennan and Wilson (1993) for an early survey). A novel aspect of my paper relative to this literature is that I allow the parties to negotiate on two different dimensions, as opposed to one dimension: actions and board composition. The latter is effectively bargaining over rights on access to information and decision making authority, which is a novel aspect of my paper relative to the bargaining literature. In this sense, my paper is related to Eraslan and Yılmaz (2007) who consider bargaining with securities that allow eventual payoffs to depend on privately held information at the time of negotiations. More generally, the theoretical framework in my paper is relevant for any setting that shares a similar bargaining protocol (e.g., applications in political science).

2 Setup

Consider a model with (i) an activist investor, (ii) a publicly traded firm which is targeted by the activist and is initially run by its incumbent board of directors, and (iii) shareholders of the target firm. The activist (incumbent) owns n^a (n^i) fraction of the shares of the firm, and shareholders own the rest of the shares. There is no constraint on dispersion of these shareholders, i.e., they can be a continuum of atomistic shareholders, a single shareholder, or anything in between. The total number of shares is normalized to one. While I assume $n^a > 0$ and $n^i > 0$, Section 5.5 allows activist short-selling by considering $n^a < 0$.

There is a project (e.g., a spin-off) that the firm can implement. I use "project" and "action" interchangeably. Denote by $x = 1$ if the project is implemented, and $x = 0$ otherwise (i.e., status quo). Relative to the status quo, the project creates a value of Δ per share for the shareholders (I assume that all shareholders are aligned). In contrast, the incumbent's and activist's payoff from implementation is $n^i\Delta - B^i$ and $n^a\Delta - B^a$, respectively.

Here, $B^i > 0$ represents the private benefits that the incumbent board loses if the project is implemented, which is an important friction in the model.[11] For example, if a division of the firm is spun off, the incumbent board may lose some of its private benefits since it will manage a smaller firm. More generally, these private benefits can represent perks, perquisites, salary, etc. the incumbent obtains from keeping the status quo. While I assume $B^i > 0$ in the main analysis, I show in Section 5.4 that the model also applies to the cases where $B^i < 0$ with a slight change in the interpretation of the setup.

Similarly, B^a represents the private benefits that the activist loses in the target if the project is implemented. While I do not make an assumption about the sign of B^a, as I show in the analysis, the more interesting effects of this bias arises when $B^a < 0$ (i.e., the activist has some bias *in favor of* implementing the project, as opposed to the incumbent).[12] Moreover, I show in Section 5.4 that negative B^a arises endogenously under activist short-termism (which is a common concern as mentioned in the introduction and further elaborated in Section 5.4).

When referring to Δ, I use "NPV", "value", and "return" of the project interchangeably. Δ is distributed according to a continuous probability density function f (and cumulative dis-

[11] If $n^i\Delta < B^i < \Delta$, then implementing the project is the efficient outcome even when the incumbent's private benefits are considered.

[12] See Anabtawi and Stout (2008) for a detailed discussion regarding common conflicts of interest between an activist and other shareholders (short-termism, self-dealing, etc.).

tribution function F) with full support on $(l, B^i/n^i)$.[13] Here, the distribution f also represents the activist's and shareholders' prior information about Δ. On the other hand, the incumbent privately knows Δ (e.g., since it has access to confidential information) and the activist cannot learn Δ unless he joins the board, which is another key friction in the model. Note that Δ represents the shareholder value created by the project *relative* to the status quo, and therefore the source of the incumbent's private information may stem from the shareholder value created (i) by implementing the project, as well as (ii) by keeping the status quo, where the latter captures any plan that the incumbent may be formulating to pursue instead of the project. Importantly, any of these two may involve nonpublic information. I introduce an extension in Section 5.2 where the incumbent can voluntarily disclose Δ to the activist at the beginning of the game (e.g., via information sharing agreements).

The timeline of the game consists of three phases, as summarized in Figure 2. In the first phase (i.e., "negotiations" phase), the activist and the incumbent negotiate. This phase consists of two stages: "demand" and "response" stages. (i) First, at the demand stage, the activist decides whether to demand an action settlement or a board settlement. Demanding an action settlement means that the activist demands that the incumbent implement the project. In contrast, demanding a board settlement means that the activist demands board seat(s) which give him an "authority" of $\alpha_B \in (0, 1]$ on the board. I define the activist's authority α_B as the probability that he has decision authority in the board decision-making phase, as explained below. (ii) Second, at the response stage, the incumbent decides whether to accept or reject the activist's settlement demand. If the incumbent accepts a settlement, the activist cannot run a proxy fight due to the standstill provisions of the settlement.[14] If the incumbent accepts an action settlement, then the project is implemented, payoffs are realized, and the game ends. While I assume in the main model that the incumbent cannot make a counter

[13] Note that the distribution of Δ implies that $\Delta < B^i/n^i$. Indeed, if $n^i\Delta \geq B^i$, then it is straightforward to see that the incumbent would implement the project without any pressure, even before the firm is targeted by an activist. Therefore, I do not consider $n^i\Delta \geq B^i$ in the model.

[14] Indeed, 92-97% of settlements between activists and firms in 2020-2022 have prohibited activists from launching proxy fights due to standstill provisions. See Section "Standstill Provisions" in HLS Forum on Corporate Governance, "2022 U.S. Shareholder Activism and Activist Settlement Agreements," 1/5/2023.

settlement offer to the activist, I relax this assumption in Section 5.3.

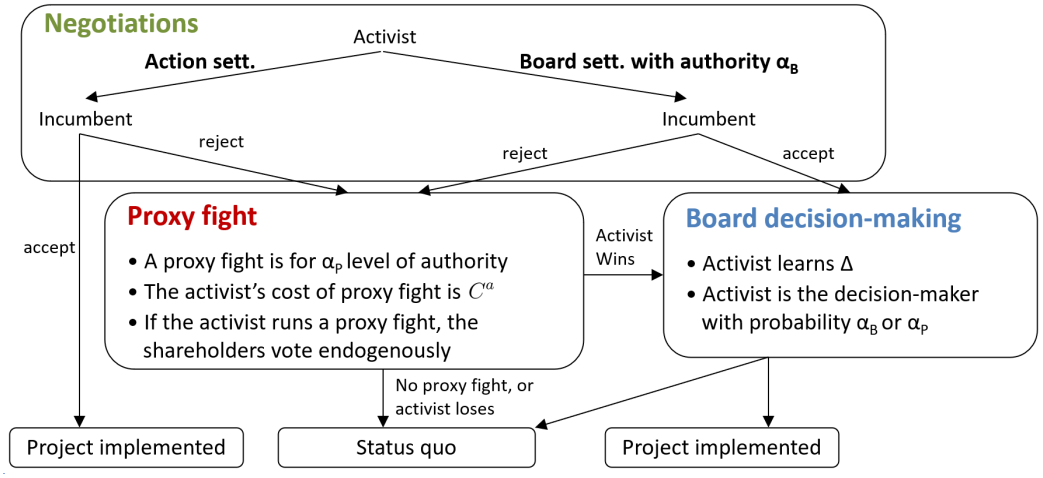


Figure 2 - The timeline

The second phase (i.e., "proxy fight" phase) occurs if no settlement is reached in the first phase, and it consists of two stages as well: "proxy fight" and "voting" stages. (i) First, at the proxy fight stage, the activist decides whether to launch a proxy fight for a board authority of $\alpha_P \in (0, 1]$. If the activist runs a proxy fight, then he incurs a cost of $C^a > 0$,[15] and the incumbent incurs a cost of $C^i > 0$. (ii) Second, at the "voting stage", shareholders vote if the activist has launched a proxy fight, and the activist's proxy fight succeeds if and only if the activist receives at least τ fraction of all shareholder votes, where $\tau \in (0, 1)$. In other words, the voting of shareholders is endogenous. Note that an implicit assumption is that the activist (incumbent) does not hold enough shares to get elected if everyone else votes for the incumbent (activist). I focus on equilibria where the shareholders do not play weakly dominated strategies, as standard in literature. If the activist wins a proxy fight, then he obtains an authority of α_P on the board. If the activist has not launched a proxy fight or loses it, the payoffs are realized, and the game ends.

The third and final phase (i.e., "board decision-making" phase) takes place if the activist has achieved some board authority of $\alpha > 0$, either through board settlement or proxy fight.

[15] The parameter C^a can also be interpreted as the opportunity cost the activist incurs by not liquidating the stake in the target. This opportunity cost can be due to forgoing other new investment opportunities, or alternatively, it can also be due to having to liquidate other existing investments to meet investor outflows (i.e., liquidity shock). In either case, if this cost is too high, then the activist will choose not to run a proxy fight and instead will choose to liquidate his stake in the firm.

This phase also consists of two stages: "learning" and "implementation" stages. (i) First, at the "learning stage", the activist learns Δ, because he obtains access to confidential information that the incumbent has (in Section 5.1, I relax this assumption and instead assume that the activist does not always learn Δ upon obtaining board seats). I assume that with positive probability, the NPV Δ is low, i.e.,

$$P[\Delta < \min\left\{B^a/n^a, \left(B^i - C^i\right)/n^i\right\}] > 0. \tag{1}$$

(ii) Second, at the "implementation stage", the activist obtains decision authority with probability α, and the incumbent obtains decision authority otherwise.[16] Whoever has the decision authority decides whether to implement the project. Payoffs are realized, and the game ends.

Payoffs. Denote by $e = 1$ if a proxy fight is launched, and $e = 0$ otherwise. Recall that $x = 1$ if the project is implemented, and $x = 0$ otherwise. Therefore, the shareholders' payoff is $\Pi^s(\Delta, e) = x \cdot \Delta$ per share they own in the firm. Given that the incumbent and the activist own a stake of n^i and n^a, respectively, their payoffs are $\Pi^i(\Delta, e) = x \cdot (n^i\Delta - B^i) - e \cdot C^i$ and $\Pi^a(\Delta, e) = x \cdot (n^a\Delta - B^a) - e \cdot C^a$.

Illustration of frictions: Jana Partners vs. Qualcomm. As described above, two of the important frictions of the model are that (I) the incumbent board is biased against the project, but (II) the incumbent has some private information about Δ, which the activist may learn after joining the board. One of the examples to illustrate both of these frictions is the activist campaign by the hedge fund Jana Partners against Qualcomm in 2015. Specifically, in this campaign, Jana Partners approached the company with the proposal of considering a split of the firm, even though Qualcomm stated that "it looks at its corporate structure from time to time but that earlier reviews have concluded shareholders are better off with the current configuration."[17] However, Jana stated that "What we think they ought to do is just a transparent review of the client businesses," also adding that "so we are not definitely saying that they should split it up."[18] These statements by Jana demonstrate that the activist was both (I) trying to pressure the incumbent to take whichever is the right decision for the

[16] Note that in practice, α is a function of the number of the board seats that the activist obtains. To simplify the exposition, I do not directly model the number of seats the activist acquires. Instead, via α, I capture the decision making authority that these seats give to the activist.

[17] See Wall Street Journal, "Activist Puts Pressure on Qualcomm", 4/13/2015; and Wall Street Journal, "Qualcomm to Conduct Strategic Review, May Consider Breakup", 7/20/2015.

[18] See CNBC, "Qualcomm should consider breakup: Jana Partners founder", 4/13/2015.

shareholders, and (II) simultaneously admitting that he did not know which one was the right decision himself but the incumbent should know, which are consistent with the first and second frictions above, respectively. Moreover, also consistently with the second friction, even though two Jana-backed directors joined Qualcomm's board in a settlement afterwards, upon a careful review these directors also concluded that it was best not to split the firm, suggesting that they obtained new information after joining the board.[19]

2.1 Preliminary analysis and parameter restrictions

Before analyzing each settlement seperately, in this section I provide some preliminary results, followed by parameter specifications. Throughout the paper, I solve for the Perfect Bayesian Equilibria of the game, where I allow for mixed strategies. The proofs of the propositions in Section 3 are in Appendix A. The rest of the proofs (including proofs of all lemmas and corollaries) are in the online appendix.

Lemma 1 *At the implementation stage:*

(i) *If the activist has obtained the decision authority (i.e., by acquiring board seats via a proxy fight or a board settlement), then he implements the project if* $\Delta > b^a$ *and does not implement it if* $\Delta < b^a$*, where* $b^a \equiv B^a/n^a$*.*

(ii) *If the incumbent board has kept its decision authority and an action settlement has not been reached, then the incumbent does not implement the project.*

The activist's payoff from implementing the project is $n^a\Delta - B^a$, or equivalently, $\Delta - b^a$ per share he owns in the firm, where $b^a \equiv B^a/n^a$. Therefore, at the implementation stage, the activist strictly prefers implementing the project if and only if $\Delta < b^a$, i.e., the change Δ in the share value from implementing the project is larger than his bias b^a per share he owns. Similarly, the incumbent's payoff if the project is implemented is $n^i\Delta - B^i$, or equivalently, $\Delta - b^i$ per share it owns, where $b^i \equiv B^i/n^i$. Since in addition Δ is always strictly smaller than its private benefits b^i per share from keeping the status quo, the incumbent strictly prefers not to implement the project.[20]

[19] See Wall Street Journal, "Qualcomm Decides Against Breakup", 12/15/2015. In particular, the article reports that "the six-member committee that evaluated prospects for a breakup included the two Jana-backed board members ... but the committee and board of directors *unanimously* concluded that the current structure worked best" (emphasis added).

[20] For the motivation behind the assumption $\Delta < b^i$, see footnote 13 in Section 2.

Next, I specify the parameters of the model. In particular, throughout the paper I assume

$$C^a < \bar{C}^a \equiv \alpha_P n^a E\left[\max\left\{0, \Delta - b^a\right\}\right], \tag{2}$$

$$0 < E\left[\Delta | \Delta \geq b^a\right], \tag{3}$$

because whenever any of these two conditions is violated, the activist has no impact on the firm whatsoever if he demands an action settlement or if settlements are not allowed: In particular, in equilibrium, no settlement is reached, the activist never runs a proxy fight, and the project is never implemented. In Lemma 3 in Online Appendix D, I show that this is the *unique* equilibrium if the activist has demanded an action settlement or if no settlement is allowed, for the reasons I discuss below. While there might be other equilibria if the activist has demanded a board settlement, this is still the reasonable equilibrium, as I also explain below and in more detail in Online Appendix D.[21]

Intuitively, if (2) or (3) is violated, the incumbent has no incentive to accept any settlement. This is because even if the incumbent rejects the activist's settlement demand for all Δ, then the activist does not run a proxy fight: If (3) is violated, then the shareholders do not elect the activist in a proxy fight, because the activist destroys shareholder value on average if he wins a proxy fight. Similarly, if (2) is violated, then the activist's cost of proxy fight C^a is too high or his stake n^a is too small for him to make a profit from running a proxy fight, even if the shareholders always support him. Therefore, (2) and (3) are *necessary* conditions to ensure that the activist has an impact on the firm with positive probability. Note that (2) also implies that $b^a < b^i$, i.e., the activist is more willing to implement to project than the incumbent is.[22] Otherwise, the project is never implemented, and the activist again has no effect on the firm.

Throughout the paper, I denote the probability that the activist runs a proxy fight if no settlement is reached by ρ. I also refer to ρ as the activist's "proxy fight threat". Conditional on running a proxy fight, I denote the probability that the activist wins by σ.

[21] To begin with, when (2) or (3) is violated, the only equilibrium that always exists is the one where the project is never implemented (because the activist never runs a proxy fight), as shown in parts (i) and (ii) of Proposition 11 in Online Appendix D. Moreover, it is the unique equilibrium if C^a is sufficiently high. Similarly, as shown in parts (i) and (iii) of the same proposition, if $0 > E\left[\Delta | \Delta \geq b^a\right]$, then it is the unique equilibrium that satisfies the Grossman and Perry (1986) criterion.

[22] This is because if $b^a \geq b^i$, then $\alpha_P n^a E\left[\max\left\{0, \Delta - b^a\right\}\right] = 0$ since $\Delta < b^i$ for all b^i. However, this would yield a contradiction with (2) since $C^a > 0$.

3 Analysis: Board settlement

In this section, I analyze the subgame where the activist has demanded a board settlement. In Section 4, I analyze action settlements and compare with board settlements.

3.1 Equilibrium Characterization

The proposition below characterizes the equilibrium. Recall that if the incumbent accepts the board settlement, then the activist learns Δ, and with probability α_B the activist decides whether to implement the project.

Proposition 1 *Suppose that the activist has demanded a board settlement. Then, an equilibrium of this subgame where a proxy fight is on the equilibrium path exists, it is unique, and in this equilibrium:*

(*i*) *The incumbent accepts the settlement if and only if* $\Delta \in (l, b^a) \cup (\Delta_B^*, b^i)$*, where* $\Delta_B^* \in (b^a, b^i)$ *is given by*

$$\Delta_B^* \equiv b^i - \frac{C^i/n^i}{\alpha_B/\rho_B^* - \sigma_B^*\alpha_P}, \tag{4}$$

(*ii*) *Upon rejection, the activist runs a proxy fight with probability* $\rho^* = \rho_B^* > 0$*, where*

$$\rho_B^* \equiv \min\left\{1, \alpha_B/[\sigma_B^*\alpha_P + C^i/[n^i(b^i - \hat{\Delta}_B(\sigma_B^*))]]\right\}, \tag{5}$$

and $\hat{\Delta}_B(\sigma)$ *is given by the unique solution of* [23]

$$C^a = \sigma\alpha_P n^a E[\Delta - b^a | b^a \leq \Delta \leq \hat{\Delta}_B]. \tag{6}$$

(*iii*) *If the activist runs a proxy fight, he wins with some probability* $\sigma_B^* \in (0, 1]$*, where* σ_B^* *is unique.*

Besides the equilibrium described in Proposition 1 above, the only other equilibrium that may exist is the one where the incumbent accepts the board settlement for all Δ. However, as I show in Online Appendix E, this equilibrium does not always exist. In contrast, the equilibrium

[23] As I show in the last step of the proof of Proposition 1, in equilibrium σ_B^* always satisfies $C^a < \sigma_B^*\alpha_P n^a E[\Delta - b^a | b^a \leq \Delta \leq b^i]$. Therefore, $\hat{\Delta}_B(\sigma_B^*) \in (b^a, b^i)$.

in Proposition 1, where a proxy fight is on the equilibrium path, always exists. Indeed, empirically, Gantchev (2013) documents that activists' demand for board seats is rejected in about 61% of the time and activists often run a proxy fight afterwards.[24] Nevertheless, in Online Appendix E, I also show that even if the equilibrium where the board settlement is accepted for all Δ exists, it is not robust to the perfect sequential equilibrium criterion of Grossman and Perry (1986), while the equilibrium in Proposition 1 is robust. Therefore, throughout the analysis, if the activist demands a board settlement, then I focus on the equilibrium described in Proposition 1 above.

Let us start by analyzing the equilibrium for a given σ, which is the probability that the activist wins a proxy fight if he launches one. As long as the activist runs a proxy fight with positive probability, the incumbent's strategy follows the structure described in Proposition 1, which is illustrated in Figure 3. To understand this strategy, there are two cases to consider. First, suppose that $\Delta < b^a$. Then, the incumbent does not have anything to fear from having the activist on the board, since the activist will learn that $\Delta < b^a$ and hence will not push to implement the project. On the other hand, the incumbent faces the risk of a proxy fight if it rejects. Therefore, the incumbent strictly prefers to accept the board settlement for all $\Delta < b^a$. For a detailed example to illustrate these dynamics for $\Delta < b^a$, see Jana Partners vs. Qualcomm in Section 2.

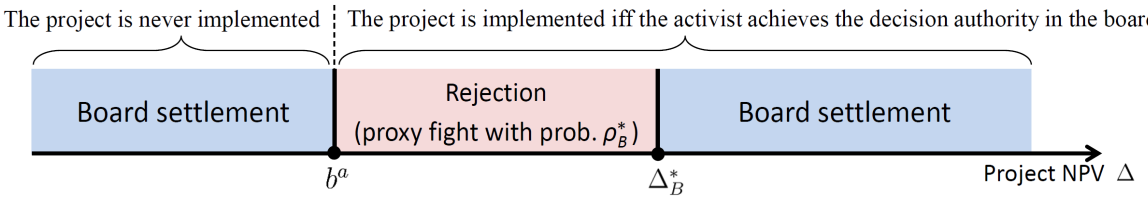


Figure 3 - Illustration of the equilibrium if the activist demands a board settlement.

Next, consider the other case, i.e., $\Delta \geq b^a$. Then, as described in Proposition 1, the incumbent follows a threshold strategy, as again depicted in Figure 3. To see why, note that the incumbent knows that if the activist gets the decision authority on the board then he will implement the project. For given project value Δ and probability ρ that the activist runs a proxy fight upon rejection, accepting the board settlement gives the incumbent a payoff of $\alpha_B n^i(\Delta - b^i)$, while rejecting gives it an expected payoff of $\rho[-C^i + \sigma\alpha_P n^i(\Delta - b^i)]$. The latter follows because the incumbent incurs a cost of C^i due to the proxy fight itself, and whenever

[24] See panel B of table 3 in Gantchev (2013).

$\Delta \geq b^a$, the activist implements the project with probability α_P upon winning a proxy fight. Also recall that $\Delta - b^i < 0$ for all Δ.

Let us take a closer look at the trade-off that the incumbent faces when it decides to accept the board settlement for a given $\Delta \geq b^a$: On the one hand, rejecting it bears the risk of facing a proxy fight and hence the associated expected cost of ρC^i, while accepting the settlement eliminates the risk that the activist will launch a proxy fight (due to the settlement's standstill provisions, which empirically exist in almost every settlement).[25] On the other hand, the probability that the project will be implemented following a rejection, $\rho\sigma\alpha_P$, is smaller than the probability it will be implemented following a settlement, α_B (indeed, if it were $\rho\sigma\alpha_P \geq \alpha_B$, then the incumbent would accept the board settlement for all Δ, which cannot take place in the equilibrium where proxy fight is on the equilibrium path).[26] As a result, the incumbent's incentive to accept the settlement is strictly increasing for all $\Delta \geq b^a$. Therefore, for Δ values that are sufficiently large, the incumbent is better off by accepting the board settlement instead facing the risk of proxy fight. In contrast, for smaller Δ, the incumbent is willing to incur the cost C^i of a proxy fight in exchange for a lower probability that the project is eventually implemented. To be precise, for any $\Delta \geq b^a$, the incumbent accepts the board settlement if and only if $\alpha_B n^i(\Delta - b^i) > \rho[-C^i + \sigma\alpha_P n^i(\Delta - b^i)]$, or equivalently if and only if $\Delta > \Delta_B(\rho;\sigma)$, where

$$\Delta_B(\rho;\sigma) \equiv b^i - \frac{C^i/n^i}{\alpha_B/\rho - \sigma\alpha_P}, \tag{7}$$

which is higher than b^a as long as the activist's proxy fight threat ρ is not too high. Although Δ_B could become lower than b^a if ρ were to be very high, this would imply that the incumbent accepts for all Δ, which is again not possible in an equilibrium with a proxy fight is on the equilibrium path.[27] Indeed, note that Δ_B is decreasing in the probability ρ of a proxy fight upon rejection.

To see how the equilibrium level of proxy fight threat ρ^* is pinned down for a given σ, note that upon rejection of board settlement, the activist infers that the project's NPV Δ is in the interval $[b^a, \Delta_B]$. The activist is willing to run a proxy fight upon rejection if and only if there is sufficient potential to increase the value of the firm. This is because the activist's payoff

[25] For statistics on standstill provisions, see footnote 14 in Section 2.

[26] Formally, $\rho^*\sigma^*\alpha_P < \alpha_B$ follows from the next footnote.

[27] To put it formally, in this equilibrium, $\Delta_B^* > b^a$ because $\rho^* < \frac{\alpha_B}{\sigma^*\alpha_P + C^i/[n^i(b^i - b^a)]}$ as I show in steps 4 and 5 of the proof of Proposition 1. Note that this condition on ρ^* also ensures that $\rho^* < \frac{\alpha_B}{\sigma^*\alpha_P}$, implying that the denominator in (7) is always positive in equilibrium.

is given by $\sigma\alpha_P n^a \max\{0, \Delta - b^a\} - C^a$ upon running a proxy fight. In particular, if Δ_B is higher, the expected value of Δ upon the incumbent's rejection is larger. Therefore, if Δ_B is higher, the expected profit of the activist from running a proxy fight is larger, increasing the activist's incentives to run a proxy fight upon rejection. On the other hand, as explained above, the incumbent's rejection threshold Δ_B itself is decreasing in the likelihood ρ of a proxy fight upon rejection. For these reasons, Δ_B and ρ together constitute a fixed point equation for any given σ.

Indeed, as I show in the proof of Proposition 1, as long as σ is not below a certain threshold $\underline{\sigma}_B \in (0, 1)$, the solution to the fixed point equation mentioned above is uniquely given by $\rho^* = \rho_B(\sigma) > 0$, which is identical to (5) with the exception that σ_B^* is replaced with σ. In turn, $\rho_B(\sigma)$ pins down $\Delta_B(\sigma)$, resulting in (4) (again, with the exception that σ_B^* is replaced with σ).

Finally, to see how σ is determined, note that as the probability σ that the activist wins a proxy fight decreases, the incumbent becomes less willing to accept the settlement, resulting in an increase in $\Delta_B(\sigma)$.[28] As a result, as σ decreases, the expected shareholder value if the activist wins a proxy fight increases (which is given by $\alpha_P E[\Delta|\Delta \in [b^a, \Delta_B]]$), in turn increasing the shareholders' incentives to vote for the activist. Therefore, Δ_B and σ together constitute their own fixed point equation, yielding a unique σ^* in equilibrium, denoted by σ_B^*. More details on how σ^* is determined in equilibrium are provided in Section 3.2.1 below. Moreover, as shown in the proof of Proposition 1, σ_B^* is always larger than the threshold $\underline{\sigma}_B$ mentioned above, resulting in $\sigma_B^* > 0$.[29]

Remark regarding voting. Note that while σ represents the probability that the activist *wins* conditional on running a proxy fight, it can also be interpreted as the probability that shareholders vote for the activist as a block. To be more precise, for any equilibrium throughout the paper where the activist wins a proxy fight with probability $\sigma^* \in [0, 1]$, there exists an identical equilibrium where with probability σ^* all shareholders vote for the activist, and with

[28] More formally, this is because for $\Delta_B(\sigma)$ to not increase, ρ would have to increase to compensate for the decrease in σ. However, if $\Delta_B(\sigma)$ does not increase as σ decreases, then the activist's expected payoff $\sigma\alpha_P n^a E[\Delta - b^a|\Delta \in [b^a, \Delta_B]] - C^a$ from running a proxy fight would decrease as well, which would yield a contradiction with an increase in ρ.

[29] This is because if $\sigma_B^* \leq \underline{\sigma}_B$ (i.e., the shareholder support for the activist is too low), then the activist would never run a proxy fight in equilibrium. However, then the incumbent would reject the settlement for all $\Delta \geq b^a$, and hence the shareholders would strictly prefer to support the activist in a proxy fight, because their expected payoff from electing the activist would be weakly larger than $\alpha_P E[1_{\{\Delta \geq b^a\}} \cdot \Delta] > 0$, which is positive due to (3). Importantly, since this payoff is positive, it would yield a contradiction with $\sigma_B^* < 1$.

probability $1-\sigma^*$ all shareholders vote for the incumbent (if $\sigma^* \in (0,1)$ then shareholders are indifferent between these two options). However, I do not assume anywhere in the paper that shareholders vote as a block, because such an assumption is not required for any of the results. While this implies that there might be multiple voting strategy profiles that lead to the same σ when shareholders are indifferent between electing the activist and the incumbent,[30] this multiplicity has no material effect on the equilibrium. Instead, the relevant aspect of shareholders' voting for the equilibrium characterization is that they do not play weakly dominated strategies, as mentioned in the setup. In other words, they vote as if they are pivotal: Whenever the expected shareholder value is *strictly* larger if the activist (incumbent) wins, all of them vote for the activist (incumbent).

3.2 Implications for shareholder value

In this section, I derive the implications of board settlements for shareholder value. As I show and elaborate in Online Appendix B, these implications also hold for action settlements due to the same intuition discussed below.

3.2.1 Activist's bias and shareholder voting

A common concern expressed by institutional investors is that settlements may harm shareholders. In particular, as mentioned in the introduction, this concern usually relies on the argument that activists effectively bypass other shareholders in settlements and hence may use these settlements to force firms to implement value-destroying projects due to their (e.g., short-termist) motives. However, in this section, I show that negative NPV projects are typically not implemented following board settlements, but rather after the activist *wins* proxy fights with shareholders' endogenous support. Despite this result, I also find that shareholders would be better off by supporting the activist *more* in a proxy fight compared to their equilibrium level of support, even though shareholders vote rationally in the model and do not have any bias. Importantly, in all of these results, the inference that *shareholders* make from the incumbent's rejection of the settlement plays a critical role.

I start with the first result in the following proposition. Recall from Proposition 1 that σ_B^* denotes the likelihood that the activist wins a proxy fight conditional on launching a proxy fight after the rejection of a board settlement.

[30] For example, if the shareholder base is completely dispersed, indexing each shareholder by $j \in J$ and denoting the probability that shareholder j votes for the activist by σ_j, for any given σ there is a continuum of voting strategy profiles $\{\sigma_j\}_{j\in J}$ that yield the same σ.

Proposition 2 *Suppose that the activist has demanded a board settlement. Then, a negative NPV project is implemented only if $b_a < 0$. Specifically, if $b_a < 0$, then:*

(*i*) *The activist runs and wins a proxy fight with positive probability.*

(*ii*) *The probability that a negative NPV project is implemented is zero after a settlement is reached, but it is positive after the activist wins a proxy fight.*

Moreover, there is $\bar{b}_a < 0$ such that $\sigma_B^* < 1$ if and only if $b_a < \bar{b}_a$.

The proposition above states that, interestingly, a negative NPV project is never implemented after a board settlement is reached, but almost paradoxically, if $b^a < 0$ then it is implemented by the activist with positive probability after winning a proxy fight with the support of the shareholders (in Section 5.4, I show that $b^a < 0$ arises endogenously under activist short-termism). To understand this result, recall that the incumbent follows the rejection strategy illustrated in Figure 4 below. Importantly, the threshold Δ_B^* is positive in equilibrium. This is because otherwise (i.e., if $\Delta_B^* \leq 0$), the shareholders would infer in a proxy fight that the NPV is negative, and therefore they would never support the activist. However, this would completely eliminate the credibility of the activist to run a proxy fight, and in turn, eliminate any incentive of the incumbent to accept the settlement for any $\Delta \geq b^a$ in the first place.[31] Therefore, the endogenous voting behavior of the shareholders ensures that $\Delta_B^* > 0$. Moreover, the incumbent's other threshold is at b^a, as also shown in Figure 4. This is because the reason why this second threshold (which is lower than Δ_B^*) exists is that the incumbent knows that if the NPV is very low (i.e., smaller than b^a), then the activist will not implement the project upon joining the board.

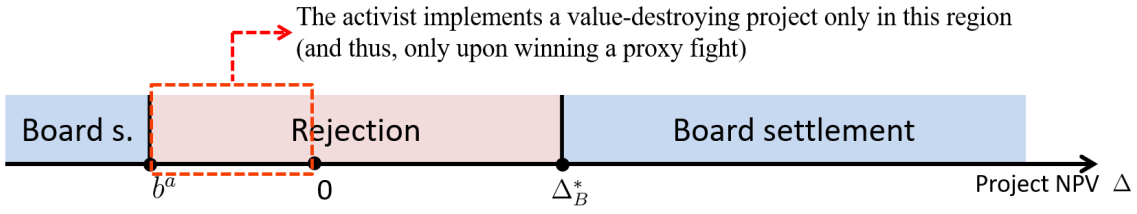


Figure 4 - Illustration of the equilibrium if the activist has a bias $b^a < 0$.

Therefore, for any $\Delta \in [b^a, 0)$, the incumbent never accepts the settlement demand of the activist. Moreover, upon rejection, the activist runs and wins a proxy fight with positive

[31] In other words, this would yield a contradiction with $\Delta_B^* \leq 0$.

probability. This result follows immediately from Proposition 1, which states that $\rho^* > 0$ and $\sigma^* > 0$. Nevertheless, let us take a moment to understand this result better, as well as why $\sigma^* < 1$ if and only if b^a is low, which will also be helpful for understanding other results further below. Simplifying the notation of the incumbent's rejection threshold $\Delta_B(\rho_B(\sigma);\sigma)$ for a given σ to $\Delta_B(\sigma)$, note that $\Delta_B(\sigma)$ is decreasing in σ, because the incumbent becomes more incentivized the accept a settlement if the activist is more likely to win a proxy fight. Therefore, in a proxy fight, the shareholders' expected payoff $\pi_B^s(\sigma)$ from electing the activist is strictly lower if σ is higher, where

$$\pi_B^s(\sigma) = \alpha_P E[\Delta | b^a \leq \Delta \leq \Delta_B(\sigma)]. \tag{8}$$

Indeed, if σ is too low, then the activist never runs a proxy fight, and hence $\Delta_B(\sigma) = b^i$, resulting in $\pi_B^s > 0$ (due to (3)), making it strictly profitable for the shareholders to elect the activist.[32] Therefore, σ has to be sufficiently high such that the activist runs a proxy fight with positive probability. As σ increases further, $\Delta_B(\sigma)$ strictly decreases, reducing $\pi_B^s(\sigma)$ as a result and yielding a unique $\sigma_B^* > 0$ in equilibrium. In particular, as also described in Proposition 2 above, if the bias b^a of the activist is not too negative (i.e., $b^a \geq \bar{b}^a$), then $\pi_B^s(\sigma) \geq 0$ even if $\sigma = 1$, resulting in $\sigma_B^* = 1$ in equilibrium.[33] However, if the activist is highly biased in favor of implementing the project (i.e., $b^a < \bar{b}^a$), then $\pi_B^s(\sigma) < 0$ if $\sigma = 1$, and hence the equilibrium σ_B^* is uniquely given by $\pi_B^s(\sigma_B^*) = 0$ and satisfies $\sigma_B^* \in (0,1)$. To put it differently, the shareholders always support the activist in a proxy fight if their payoff from doing so is positive (which occurs when b^a is not too low), and they must be indifferent between electing him and re-electing the incumbent if they do not fully support the activist (which occurs when $b^a < \bar{b}^a$).

To summarize, going to back to Figure 4 above, for any negative NPV for which the activist is willing to implement the project (i.e., for any $\Delta \in [b^a, 0)$), the incumbent always rejects the board settlement, and the activist runs and wins the proxy fight with positive probability. This

[32] Note that if the activist never runs a proxy fight, then the incumbent may reject the board settlement for some or all $\Delta < b^a$. However, the incumbent's rejection strategy for any $\Delta < b^a$ is irrelevant for shareholders' voting decision at a proxy fight, because if $\Delta < b^a$ then the project is never implemented anyway (regardless of whether the activist acquires board control).

[33] Note that if $b^a \geq \bar{b}^a$, then $\pi_B^s(\sigma) \geq 0$ even if b^a is negative and $\sigma = 1$, because $\Delta_B(\sigma)$ is sufficiently large compared to b^a. Indeed, for all σ, $\Delta_B(\sigma)$ satisfies $\Delta_B(\sigma) \geq b^a + C^a/n^a$, because otherwise the activist would never run a proxy fight upon the incumbent's rejection (since $\Delta \leq \Delta_B(\sigma)$ would imply that Δ is too low for him to make a profit, or more formally, $\alpha_P n^a E[\Delta - b^a | b^a \leq \Delta \leq \Delta_B] < C^a$), in turn eliminating any incentive of the incumbent to accept the settlement for any $\Delta \geq b^a$ in the first place.

implies that a negative NPV project is implemented with positive likelihood in equilibrium, but only following a successful proxy fight. Therefore, contrary to common arguments, the lack of shareholders' say in settlements does not actually hurt them.

Moreover, due to the interaction between settlements and proxy fights, I find that the shareholders' ability to vote in a proxy fight can actually harm them, even though the shareholders act rationally as described above. In particular, as I elaborate below, I show that the ex-ante shareholder value would be *larger* if shareholders were to commit to electing the activist with higher probability than the equilibrium probability, even though this would result in a *negative* average ex-post shareholder return upon the activist's successful proxy fight. In other words, in equilibrium, the shareholders *undersupport* the activist in a proxy fight. This result, which is in a strong contrast with the result above that the activist implements value-destroying projects *only after* winning a proxy fight (as explained following Proposition 2), is formalized in the following proposition.

Proposition 3 *Consider any campaign where the activist has demanded a board settlement. If the shareholders commit to any $\sigma > \sigma_B^*$, then compared to any $\sigma \leq \sigma_B^*$:*

(*i*) *The ex-ante shareholder value is strictly larger.*

(*ii*) *The expected ex-post shareholder value conditional on the activist winning a proxy fight is strictly smaller. More specifically, it is negative for any $\sigma > \sigma_B^*$, while it is nonnegative for any $\sigma \leq \sigma_B^*$.*

To understand this result, note that if the shareholders do not always support the activist in a proxy fight upon the incumbent's rejection (i.e., $\sigma_B^* < 1$), then it must be $\sigma_B^* \in (0, 1)$, as explained following (8) above. In other words, the shareholders are indifferent between supporting the activist and supporting the incumbent. Therefore, the expected ex-post shareholder value if the incumbent wins the proxy fight (which is zero) and if the activist wins the proxy fight should be equal. Since the incumbent rejects the settlement if and only if $\Delta \in [b^a, \Delta_B(\sigma_B^*)]$, this implies that the expected value of Δ within this interval should be equal to zero, i.e., $E\left[\Delta | b^a \leq \Delta \leq \Delta_B(\sigma_B^*)\right] = 0$ as illustrated in Figure 5. Moreover, as also shown in Figure 5 with blue solid line, in this interval of Δ, the probability that the project is implemented is $\rho_B(\sigma_B^*)\sigma_B^*\alpha_P$, i.e., the probability that the activist runs and wins a proxy fight and he achieves decision authority on the board (recall that for a given σ, $\rho_B(\sigma)$ represents the probability that

the activist runs a proxy fight upon rejection of a board settlement).

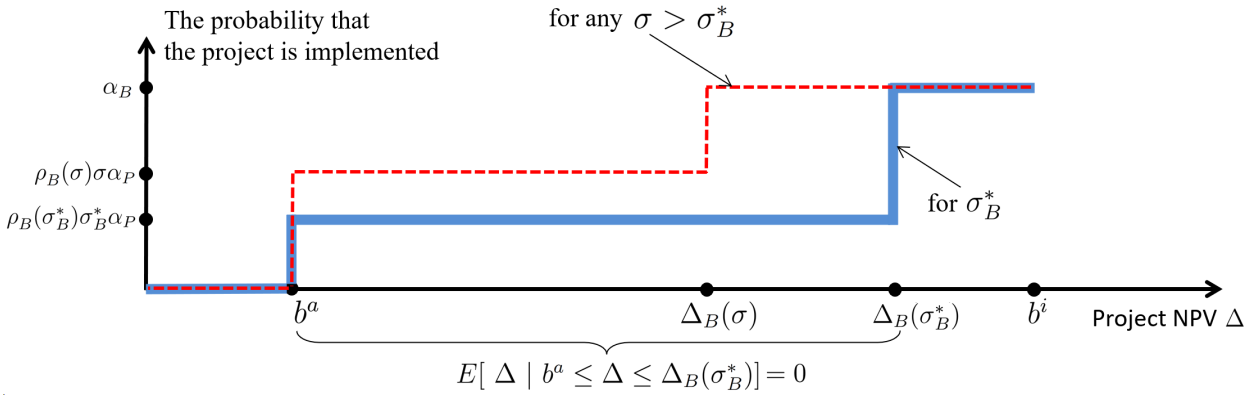


Figure 5 - The probability that the projected is implemented if the activist demands a board settlement: for equilibrium $\sigma_B^* < 1$ (blue solid line), and for any higher $\sigma > \sigma_B^*$ (red dashed line).

Now, instead suppose that the shareholders committed to supporting the activist more, i.e., the activist wins the proxy fight with probability $\sigma > \sigma_B^*$ if he runs a proxy fight. Then, this would incentivize the incumbent to accept the settlement more in the first place, leading to a lower $\Delta_B(\sigma)$, and if the incumbent rejects, it would also result in a higher probability of project implementation $\rho_B(\sigma)\sigma\alpha_P$. The resulting probability of project implementation as a function of Δ is illustrated with red dashed line in Figure 5. Importantly, compared to σ_B^*, σ leads to a *strictly larger* increase in the likelihood that the project is implemented for *higher* NPV region $\Delta \in (\Delta_B(\sigma), \Delta_B(\sigma_B^*)]$ than for the *lower* NPV region $\Delta \in [b^a, \Delta_B(\sigma)]$.[34] Combining this with the fact that the expectation of Δ in the combination of these two regions is zero (i.e., $E[\Delta | b^a \leq \Delta \leq \Delta_B(\sigma_B^*)] = 0$ as mentioned above), the *ex-ante* expected shareholder value is always strictly larger for σ than for σ_B^*. However, this also implies that for σ, the *ex-post* expected shareholder value conditional on the activist winning a proxy fight is negative (i.e., $\pi_B^s(\sigma) = \alpha_P E[\Delta | b^a \leq \Delta \leq \Delta_B(\sigma)] < 0$), while it is zero for σ_B^*.

The intuition behind these results is as follows. For $\Delta \geq b^a$, in equilibrium the probability with which the project is implemented is larger if a settlement is reached *than* if a settlement is not reached, because the only the reason why the incumbent rejects a settlement and faces the

[34] Note that $\rho_B(\sigma)\sigma\alpha_P < \alpha_B$ always holds, because otherwise the incumbent would accept the board settlement for all Δ. This is because if $\rho_B(\sigma)\sigma\alpha_P \geq \alpha_B$, then for any $\Delta \geq b^a$, rejecting the board settlement would result in a higher probability that the project is implemented, compared to accepting the settlement (more specifically, this probability is $\rho_B(\sigma)\sigma\alpha_P$ in the former case, and it is α_B in the latter). Since the incumbent also saves the cost of a proxy fight by accepting the settlement, it would always accept.

risk of a proxy fight is to reduce the probability that the project is implemented. Moreover, (again for $\Delta \geq b^a$) the settlement is reached for higher Δ and a proxy fight is realized only for smaller Δ. Since in addition Δ is equal to zero on average in the combination of these two regions, compared to σ_B^*, commitment to any $\sigma > \sigma_B^*$ results in a strictly larger ex-ante expected shareholder value by expanding the region of Δ where a settlement is reached.[35] However, in the absence of commitment, the shareholders do not elect the activist with a higher likelihood than the equilibrium likelihood. This is because if they did, then the incumbent's rejection threshold would decrease, and hence the shareholders would anticipate that the activist will destroy value on average upon winning the proxy fight.

The takeaway in this section is that in any activist campaign realized in equilibrium, if the shareholders could commit to not making any inference from the incumbent's rejection and instead always support the activist in a proxy fight, this would benefit the shareholders due to its effect on the incentives of the incumbent to accept the settlement. In other words, shareholders' rational behavior in a proxy contest may end up hurting themselves. Moreover, Proposition 2 further above implies that this problem occurs *only if* b^a is sufficiently lower than zero (i.e., the activist prefers to implement the project even if it destroys a lot of shareholder value), because only then the shareholders do not always support the activist in a proxy fight.

Robustness to parameter specifications. A question of interest might be whether Proposition 3 above continues to hold if $E[\Delta|\Delta \geq b^a] < 0$ (i.e., the activist is so much biased that b^a violates the parameter restriction (3)). Then, if the shareholders commit to supporting such an activist in a proxy fight, the expected shareholder value created after the activist runs and wins a proxy fight might be so low that it might potentially result in a negative ex-ante shareholder value,[36] even though it is still true that no value-destroying project would be implemented following a settlement. In contrast, in the absence of shareholder commitment, such an activist never runs a proxy fight in equilibrium,[37] because he would always lose (i.e., $\sigma_B^* = 0$). However, this eliminates any credibility the activist would have against the incumbent, and hence in equilibrium, the activist would *not* incur any cost to negotiate a settlement with the

[35] Note that a board settlement is also reached if $\Delta < b^a$, where the project is never implemented. However, this is true under both σ_B^* and any $\sigma > \sigma_B^*$. Therefore, to compare the differential effect of σ to σ_B^* on the ex-ante shareholder value, it is sufficient to focus on $\Delta \geq b^a$.

[36] Nevertheless, the opposite might also occur: That is, committing to support the activist can still result in a higher ex-ante shareholder value even if (3) is violated, due to the same reason explained following Proposition 3 (i.e., due to the shareholder value created following a settlement).

[37] This result is formalized by parts (i) and (iii) of Proposition 11 in Online Appendix D.

incumbent or to launch an activist campaign in the first place.[38] This is the reason behind the qualifier "in any activist campaign realized in equilibrium" in the first statement of the previous paragraph and "Consider any campaign" at the beginning of Proposition 3. Importantly, it is also why Proposition 3 continues to be valid regardless of whether (3) holds. Therefore, it is also still true that shareholders (weakly) undersupport the activist in a proxy fight, regardless of (3).[39]

3.2.2 Does the existence of settlements benefit shareholders?

Proposition 2 in the previous section and the discussion following it describe how board settlements do not destroy value ex-post. Proposition 8 in Online Appendix B shows that the same result holds for action settlements as well. This may give the impression that the existence of settlements should benefit shareholders. However, I show that even if the activist's interests aligned with those of shareholders (i.e., $b^a = 0$), allowing for settlements may not be good for shareholders, because it may reduce the probability that the project is implemented. The proposition below shows that if the activist has demanded a board settlement, then this occurs when α_B is low or C^a is high, both of which are consistent with the existing empirical evidence as discussed below (recall that α_B represents the level of decision authority that the activist obtains if the incumbent agrees to a board settlement, and C^a denotes the activist's cost of a proxy fight). Proposition 10 in Online Appendix B shows that the same result holds for action settlements if C^a is high (note that α_B is not applicable in the case of an action settlement).

Proposition 4 *Denote the ex-ante shareholder value (the ex-ante payoff of the activist) by Π_s^B (Π_a^B) if the activist has demanded a board settlement, and by $\Pi_s^\varnothing$ ($\Pi_a^\varnothing$) if settlements are not allowed.*

(*i*) *There exists $\hat{\alpha}_B > 0$ such that $\Pi_s^B < \Pi_s^\varnothing$ if $\alpha_B < \hat{\alpha}_B$. In addition, for any α_B, conditional on the activist demanding a board settlement, as α_B decreases:*

(*i.a*) *The likelihood of reaching a settlement increases.*

[38] Gantchev (2013) reports that the demand negotiations stage is highly expensive, with average costs of $2.94 million for the activist. Becht et al. (2009) also suggest that activists' unobservable costs play an important role in negotiations with the incumbent (see their discussion on p. 3096).

[39] This is because if (3) is violated, then as explained above the shareholders never support the activist in equilibrium anyway (i.e., $\sigma_B^* = 0$), and if (3) is satisfied, then Proposition 3 implies that the shareholders are strictly better off by committing to supporting the activist more than their equilibrium support and are never better off by supporting him less.

(*i.b*) *The likelihood of a proxy fight decreases, and it goes to zero as* $\alpha_B \to 0$.

(*ii*) *There exists* $\hat{C}_a$ *such that* $\Pi_s^B < \Pi_s^\varnothing$ *if and only if* $C_a > \hat{C}_a$.[40]

(*iii*) *There exists* $\tilde{C}_a$ *such that* $\Pi_a^B > \Pi_a^\varnothing$ *if* $C_a > \tilde{C}_a$.[41]

Let us begin with part (i). Before discussing the intuition behind the results, let us take a look at how the existing empirical evidence maps to part (i). Indeed, the evidence in the literature suggests it is possible that the reality falls in the region where $\Pi_B^s < \Pi_\varnothing^s$: To begin with, Bebchuk et al. (2020) document that while the mean board size is about 8-9 directors, on average activist hedge funds obtain only about 2 seats in board settlements, indicating low α_B.[42] Moreover, the empirical evidence documented in the literature is also consistent with the implications of low α_B described in parts (i.a) and (i.b) of Proposition 4: Klein and Zur (2009) analyze 2003-2005 and report that conditional on obtaining board representation, activist hedge funds as well as other kinds of activists have obtained it without even filing for a proxy fight in an overwhelming majority of their campaigns (81% and 86%, respectively).[43] Gow et al. (2023) report that between 2004 and 2016, activist hedge funds obtained 1,245 board seats though settlements, in contrast with only 378 seats through director elections.[44] In addition, Bebchuk et al. (2020) document that, over the second half their data sample (i.e., between 2007-2013), the number of activist hedge fund campaigns ending with a settlement is about 3.5 times the number of campaigns ending with a voted proxy fight (recall that the vast majority of the settlements in their sample are board settlements).[45] Moreover, they also document that this ratio is about 2 in the first half of their sample (2000-2006), leading the authors to emphasize the upward trend in settlements.

Even though Proposition 4 applies for any bias b^a of the activist (as also shown in the proof), to understand the intuition as clearly as possible, let us focus on the case where the activist's interests are aligned with those of shareholders (i.e., $b^a = 0$) since b^a does not play a major role for the results in this section. If hypothetically settlements were not an option, then

[40] Moreover, I show in the proof of the proposition that there exists $\tilde{\alpha}_B > 0$ such that if $\alpha_B < \tilde{\alpha}_B$ then $\hat{C}^a < \bar{C}^a$, where $\bar{C}^a$ is given by (2).

[41] Moreover, $\tilde{C}^a < \bar{C}^a$ as I show in the proof.

[42] See panel C of table 5 and Section 6.2 in Bebchuk et al. (2020).

[43] See table VI in Klein and Zur (2009).

[44] See table 3 in Gow et al. (2023).

[45] For the number of proxy fights and settlements in Bebchuk et al. (2020), see table 1.

the activist would always run a proxy fight,[46] because the expected value of the project would be high and there would be no additional information that the activist would learn before deciding to launch a proxy fight. Thus, whenever $\Delta > 0$, the project would be implemented with probability α_P.

Compared to this hypothetical scenario, demanding a board settlement results in a lower ex-ante shareholder value for two reasons if α_B is low: (a) Conditional on reaching a board settlement, if $\Delta > \Delta_B^*$, it reduces the likelihood that the activist can implement the project from α_P to α_B, and also (b) for $\Delta \in [0, \Delta_B^*]$, it reduces the likelihood ρ^* of a proxy fight upon the incumbent's rejection, in turn again decreasing the probability that the project is implemented from α_P to $\rho^* \alpha_P$. In particular, the shareholders prefer that the activist always runs a proxy fight, because they do not internalize the cost of a proxy fight but the activist does. However, as α_B decreases, ρ^* decreases as well, because the incumbent becomes more willing to accept the board settlement, signalling lower Δ to the activist upon rejection. This also explains parts (i.a) and (i.b) of Proposition 4.

The intuition behind part (ii) is the same: Upon the incumbent's rejection of settlement, the activist will often choose not to launch a proxy fight even though it is better to do so for the shareholders, because the activist infers that Δ is not high and avoids incurring the cost of a proxy fight by not launching one. This is why in part (ii), I find that if the activist's cost of proxy fight is high, the ex-ante shareholder value is lower when the activist demands a board settlement compared to the case where settlements are not allowed. Indeed, Gantchev (2013) finds that proxy fights are highly costly for activists: He reports that a campaign ending in a proxy fight has average costs of \$10.7 million for the activist and that these costs are equal to the two-thirds of the mean abnormal activist return.

Figure 6 below provides a numerical example: Even though $b^a = 0$, the ex-ante shareholder value when the activist demands a board settlement is smaller than in the case where settlements are not allowed, as depicted in panel (a). However, even though the shareholders are *worse off*, panel (b) shows that if the activist's cost of proxy fight is high, then he is strictly *better off* by demanding a board settlement (compared to not demanding any settlement), since it reduces the need for a proxy fight. This is also shown formally by part (iii) of Proposition 4.

[46] Formally, for any b^a, this is because the shareholders would always support the activist in a proxy fight due to (3), and hence the activist's expected payoff from running a proxy fight ($\alpha_P n^a E[\max\{0, \Delta - b^a\}] - C^a$) would be positive due to (2).

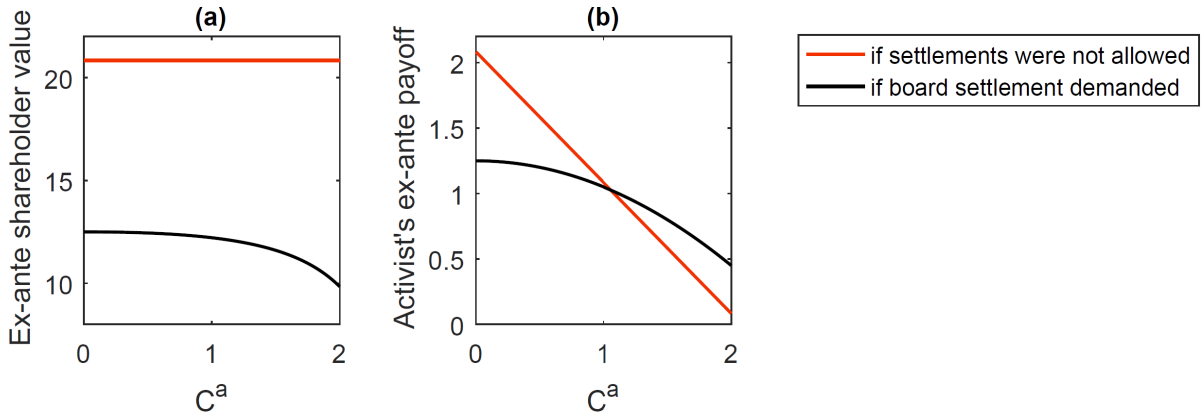


Figure 6 - As a function of C^a: (a) ex-ante shareholder value, and (b) activist's ex-ante payoff. Parameters: $b^a = 0, b^i = 100, l = -20, n^a = 10\%, C^i/n^i = 5, \alpha_P = 0.5, \alpha_B = 0.3$, and $\Delta \sim U(-20, 100)$.

Remark. Note that while activists obtain two seats on average in board settlements (Bebchuk et al. (2020)) and this implies that α_B is low as mentioned above, it does not imply that $\alpha_B = 0$. This is because even one or two seats might change the tide in the boardroom. For example, suppose that the true NPV satisfies $\Delta > \max\{0, b^a\}$, and also suppose that the board consists of 9 directors, where 5 directors are against implementing the project because they will lose private benefits, and 4 directors are in favor of implementing it because they have no or little private benefits from keeping the status quo. Then, in the absence of the activist, the majority of the board votes against implementing the project. However, if the activist acquires two seats in the board, then the majority of the board votes in favor of implementing the project. Also note that while this may give the impression that either $\alpha_B = 1$ or $\alpha_B = 0$, this is not true: If there is any uncertainty about how individual directors will vote (for example, their preferences are not fully clear or might change over time), this implies $\alpha_B \in (0, 1)$.

4 Analysis: Action settlement and comparison with board settlement

In this section, I analyze action settlements and then compare them with board settlements.

4.1 Action settlement

The proposition below characterizes the equilibrium for the case where the activist has demanded an action settlement.

Proposition 5 *Suppose that the activist has demanded an action settlement. Then, an equilibrium of this subgame exists, it is unique, and in equilibrium:*

(*i*) *The incumbent accepts the action settlement if and only if* $\Delta > \Delta_A^*$, *where*

$$\Delta_A^* \equiv b^i - \frac{C^i/n^i}{1/\rho_A^* - \sigma_A^* \alpha_P} \in \left(b^a, b^i\right), \tag{9}$$

(*ii*) *Upon rejection, the activist runs a proxy fight with probability* $\rho^* = \rho_A^* > 0$, *where*

$$\rho_A^* \equiv \min\left\{1, 1/[\sigma_A^* \alpha_P + C^i/[n^i(b^i - \hat{\Delta}_A(\sigma_A^*))]]\right\}, \tag{10}$$

and $\hat{\Delta}_A(\sigma)$ *is given by the unique solution of* [47]

$$C^a = \sigma \alpha_P n^a E[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}_A]. \tag{11}$$

(*iii*) *If the activist runs a proxy fight, he wins with some probability* $\sigma_A^* \in (0, 1]$, *where* σ_A^* *is unique.*

Proposition 5 states that the incumbent follows a threshold strategy, as illustrated in Figure 7 below. To see why, note that the incumbent faces the following trade-off: Rejecting the action settlement reduces the probability that the project is implemented, but it also entails an additional expected cost of ρC^i because the activist launches a proxy fight with probability ρ. Therefore, the incumbent's incentive to accept the action settlement is monotonic with respect to the project's value. As a result, the incumbent accepts the action settlement if and only if the project's NPV Δ is higher than the threshold Δ_A^*, because only then the incumbent's payoff from implementing the project is higher (due to the shares it owns in the firm) than facing the risk of a proxy fight.

In turn, how ρ_A^* and σ_A^* are determined in equilibrium is similar to the case where the activist demands a board settlement. For a more detailed discussion of Proposition 5, including how ρ_A^* and σ_A^* are determined, see Online Appendix B.

[47] As I show in the last step of the proof of Proposition 5, in equilibrium σ_A^* always satisfies $C^a < \sigma_A^* \alpha_P n^a E[\max\{0, \Delta - b^a\} | \Delta \leq b^i]$. Therefore, $\hat{\Delta}_A(\sigma_A^*) \in (b^a, b^i)$.

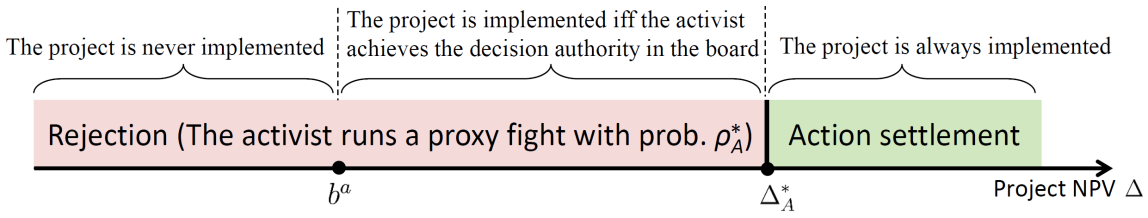


Figure 7 - Illustration of the equilibrium if the activist demands an action settlement.

Robustness of the implications for shareholder value. Section 3.2 formalized and discussed many implications of board settlements for shareholder value. As I formally show and elaborate in Online Appendix B, all of these implications continue to hold for the case where the activist demands an action settlement as well, due to the same intuition discussed in Section 3.2.

4.2 Comparison of board and action settlements

In this section, I compare board and action settlements to derive empirical predictions and additional implications for shareholder value. To that end, I first discuss the trade-offs between board and action settlements by comparing the subgames in Section 3.1 and Section 4.1. To begin with the advantages of board settlements, comparing these two subgames yields a key result of this section: The incumbent's rejection of board settlement reveals its private information about the project's value more precisely than the rejection of action settlement reveals. The following corollary formalizes this result, which follows from Propositions 1 and 5. Note that while I assume in the main model that the incumbent cannot make a counter-offer to the activist, I relax this assumption in Section 5.3.

Corollary 1 *The incumbent board rejects an action settlement if and only if* $\Delta \leq \Delta_A^*$*, and it rejects a board settlement if and only if* $\Delta \in [b_a, \Delta_B^*]$*, where* $\Delta_B^* \leq \Delta_A^*$.

Comparing action and board settlements, the difference in the activist's inference between these two cases is important, because the activist's threat of a proxy fight depends on the information revealed by the incumbent. Upon rejection of an action settlement, the activist infers that the project's NPV might be lower than his bias b^a, as illustrated in Figure 8 below. For this reason, the activist does not have much incentive to run a proxy fight upon rejection. Since the incumbent is aware of this dynamic, it is not very likely to accept the action settlement

in the first place (i.e., Δ_A^* is high).

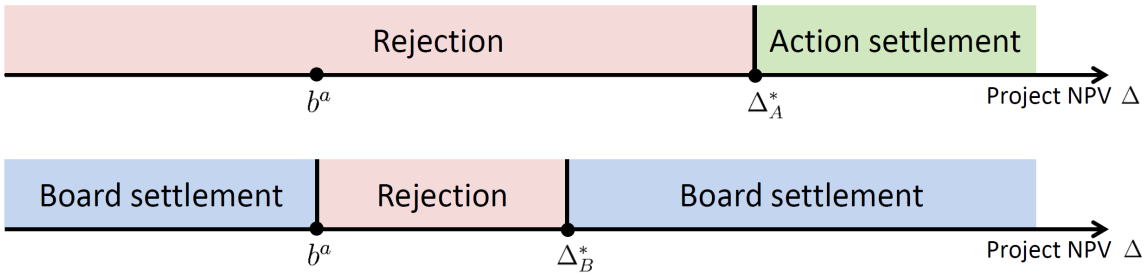


Figure 8 - Illustration of the incumbent's equilibrium behavior under different settlement demands.

In contrast, if a board settlement is rejected, then the activist infers that the project's NPV must be higher than his bias b^a. Therefore, demanding a board settlement can create a more credible threat of running a proxy fight upon rejection, in turn leading to a higher likelihood of reaching a settlement in the first place (i.e., low Δ_B^*). Importantly, the gap $\Delta_A^* - \Delta_B^*$ is larger when demanding a board settlement plays a stronger role in increasing the credibility of the activist's proxy fight threat, as we will see in Proposition 6 below.

Overall, the advantages of board settlements described above show why activists may prefer to demand board settlements over action settlements. Indeed, Bebchuk et al. (2020) report that 84% of the settlement contracts in their sample resulted in the appointment of new directors to the board. However, the discussion above does not imply that it is always optimal for the activist to demand a board settlement instead of an action settlement. This is because an important drawback of board settlement is that it may not give the activist full decision authority on the board (which corresponds to the cases where $\alpha_B < 1$). There are several reasons why this might be the case: (a) First, the activist may not be able to or may not want to get too many seats on the board. Indeed, empirically, activists do not get the full control of the board in board settlements, and they obtain 2 seats on average as mentioned earlier (Bebchuk et al. (2020)). This might be because, for example, valuable information might be lost in the boardroom if too many incumbent directors leave, hurting the firm value as a result. Or, alternatively, the activist may be under too much legal liability if he acquires too many seats. (b) Second, the activist may not always be able to exercise his decision authority after a board settlement even if he obtains full control. In particular, this is a result that arises endogenously in the extension of Section 5.1.

Importantly, whenever $\alpha_B < 1$, the project is less likely to be implemented after reaching a board settlement than after reaching an action settlement, even if $\Delta \geq b^a$. As a result, despite the other advantages of board settlement mentioned above, the activist may nevertheless choose to demand an action settlement instead, as we will see next.

4.2.1 The type of settlement in equilibrium

This section endogenizes the kind of settlement that the activist demands in equilibrium. As discussed above, compared to action settlement, a disadvantage of board settlement is that conditional on reaching a settlement, it may lead to a lower probability of implementing the project (in particular, when $\alpha_B < 1$). However, as also highlighted above, a key advantage of demanding board settlement (over action settlement) is that it can create a higher credibility of a proxy fight threat, in turn incentivizing the incumbent to accept the settlement more often. Importantly, this advantage of board settlement plays a stronger role as the *information asymmetry* between the activist and the incumbent increases, because then it becomes more difficult for the activist to convince the incumbent that he has sufficient incentives to run a proxy fight upon rejection of an action settlement. Therefore, as I show in the proposition below, the activist demands a board settlement if and only if information asymmetry is high. As I elaborate when discussing empirical predictions later, the implications of this proposition are consistent with the empirical evidence provided in the literature, including Bebchuk et al. (2020), who test multiple predictions of my model.

Proposition 6 *Suppose that Δ is uniformly distributed over (l, b^i),*[48] *and let $\underline{l} \equiv b^i - \alpha_P \frac{(b^i-b^a)^2}{2C^a/n^a}$.*[49] *Then,*

(*i*) *As l decreases, ρ_B^* does not change, ρ_A^* decreases, and $\lim_{l \downarrow \underline{l}} \rho_A^* = 0$.*

(*ii*) *As l decreases, Δ_B^* does not change, Δ_A^* increases, and $\lim_{l \downarrow \underline{l}} \Delta_A^* = b^i$.*

(*iii*) *There exists a threshold $\hat{l}_a > \underline{l}$ such that the activist demands a board settlement if $l < \hat{l}_a$, and demands an action settlement otherwise.*

where Δ_B^*, ρ_B^*, Δ_A^*, and ρ_A^* are given by (4), (5), (9), and (10), respectively.

[48] While I impose a uniform distribution on Δ, this is because it significantly simplifies the exposition. In an unreported analysis, I confirm that Proposition 6 as well as other results continue to hold for any distribution of Δ.

[49] As I show at the beginning of the proof of Proposition 6, (2) implies that $\underline{l} < b^a$.

Note that a lower l captures higher information asymmetry between the activist and the incumbent. I impose the restriction that the upperbound of Δ stays fixed at $b^i = B^i/n^i$ as information asymmetry increases, because B^i represents the private benefits of the incumbent from keeping the status quo and n^i represents its stake in the firm, and hence the agency problem of the incumbent is not relevant if $n^i\Delta \geq B^i$, which is equivalent to $\Delta \geq b^i$. In other words, the actual range of Δ might expand beyond b^i, but then the incumbent will implement the project himself anyway, before the activist even shows up. Therefore, when the activist arrives, the distribution of Δ will be bounded from above by b^i.

Also note that I show at the end of the proof of Proposition 6 that if α_B is not too high, then $\hat{l}_a$ specified in the proposition is not too large. In other words, I show that there is $\underline{\alpha}_B \in (0,1)$ such that if $\alpha_B < \underline{\alpha}_B$, then there always exists some values of l for which the activist demands an action settlement.[50]

As alluded to above, the intuition behind Proposition 6 is that as information asymmetry increases, the relative credibility of a proxy fight threat created by demanding a board settlement (instead of an action settlement), ρ_B^*/ρ_A^*, gets higher. In turn, this increases the relative likelihood of reaching a settlement if the activist demands a board settlement (i.e., the gap $\Delta_A^* - \Delta_B^*$ widens), as illustrated in Figure 9 below.

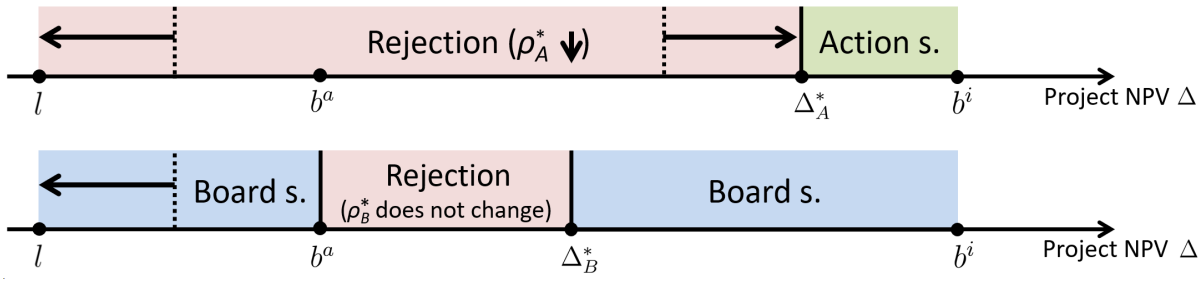


Figure 9 - Changes in settlement regions and proxy fight likelihoods as information asymmetry increases.

To understand this intuition better, first suppose that the activist has demanded an action settlement. Then, as information asymmetry increases, the likelihood that the project has a low NPV conditional on the incumbent's rejection increases, and therefore the likelihood ρ_A^* that the activist runs a proxy fight upon rejection decreases (all the way down to zero as l decreases to $\underline{l}$). As a result, as again shown in Figure 9, the incumbent becomes less likely to accept the action settlement, that is, Δ_A^* increases (all the way up to b^i as l decreases to $\underline{l}$).

[50] For instance, the numerical example in Figure 10 in Section 4.2.3 satisfies this property.

Therefore, as information asymmetry increases, the activist's profit from demanding an action settlement decreases (all the way down to zero as l decreases to $\underline{l}$).

Next, suppose that the activist has demanded a board settlement instead. Then, since the board settlement is accepted when the NPV is low, the information revealed by the incumbent's rejection about the NPV is not affected by higher information asymmetry. This implies that the activist's likelihood ρ_B^* of running a proxy fight upon rejection does not change as information asymmetry increases. In turn, the incumbent's incentives to reject the settlement does not change either (i.e., Δ_B^* does not change). As a result, demanding a board settlement *dampens* the negative effect of higher information asymmetry on the activist's profit. To see this more clearly, note that the activist's payoff from demanding a board settlement is zero if $\Delta < b^a$, where a board settlement is always reached and the project is never implemented. Moreover, as information asymmetry increases, *conditional on* $\Delta \geq b^a$, the activist's expected payoff from demanding a board settlement does not change (because Δ_B^* and ρ_B^* do not change). Thus, as information asymmetry increases, the activist's expected payoff from demanding a board settlement decreases proportionally with $P(\Delta \geq b^a)$,[51] but it does not converge to zero (unlike his expected payoff from demanding an action settlement, which decreases faster with information asymmetry and hence converges to zero as l decreases to $\underline{l}$, as explained above). Therefore, as information asymmetry increases, the activist switches from demanding an action settlement to demanding a board settlement.

As I discuss next, the results above yield many predictions consistent with the empirical evidence documented in literature.

4.2.2 Empirical Predictions

In this section, I present and discuss the empirical predictions of the model. I start with some of the predictions implied by Proposition 6.

Empirical prediction 1.

(*i*) *As information asymmetry increases, the probability that the activist runs a proxy fight conditional on rejection of an action (board) settlement decreases (does not change).*

[51] Note that this payoff is always positive, because the activist obtains a positive expected payoff from reaching a board settlement and a nonnegative expected payoff from running a proxy fight (if the latter were negative, he would never run a proxy fight).

(*ii*) *Conditional on the activist demanding a board (action) settlement, the likelihood that the incumbent accepts strictly increases (decreases) with information asymmetry.*

(*iii*) *The activist demands a board (action) settlement if information asymmetry is high (low).*

Parts (i), (ii), and (iii) of this prediction are implied by parts (i), (ii), (iii) of Proposition 6, respectively. Parts (i) and (iii) follow immediately. To better understand part (ii) of the prediction, note that part (ii) of Proposition 6 states that as information asymmetry increases, Δ_A^* increases and Δ_B^* does not change. Therefore, as information asymmetry increases, the set of NPV Δ where an action settlement is reached shrinks, while the set of Δ where a board settlement is reached expands, as also illustrated by Figure 9 in the previous section. This observation results in part (ii) of the prediction above.

Note that the prediction above utilizes the type of settlement *demanded* by the activist. However, by combining parts (ii) and (iii) of this prediction, it is possible to make a separate prediction about the unconditional likelihood that a board or action settlement is *reached* without relying on which one the activist has demanded:

Empirical prediction 2. *As information asymmetry increases, the likelihood that an activist campaign results in a board (action) settlement increases (decreases).*

Importantly, this prediction is consistent with Bebchuk et al. (2020), who explicitly test my prediction: Indeed, across several specifications, they find a negative relation between the extent of information asymmetry and the probability that a settlement directly specifies actions (e.g., the sale or merger of the firm, CEO departure), leading the authors to point out the consistency of their finding with my model.[52]

All of the predictions so far have been with respect to different levels of information asymmetry, and they are direct implications of Proposition 6. The next set of empirical predictions go one step further and utilize a different result. In particular, as discussed at the beginning of Section 4.2, another important advantage of demanding a board settlement (over action settlement) is that it increases the incumbent's incentives to accept the settlement even when $\Delta \geq b^a$. This relative advantage of board settlement was formalized by $\Delta_B^* \leq \Delta_A^*$ in Corollary 1, which continues to hold under endogeneity of the settlement type.[53] The next two empirical

[52] See Sections 5.1.3 and 5.2.3 in Bebchuk et al. (2020) for details of this empirical finding and a discussion of how their finding fits my model.

[53] In particular, note that Corollary 1 established the inequality $\Delta_B^*(l) \leq \Delta_A^*(l)$ for a given l (by comparing

predictions utilize this result.

Empirical prediction 3. *The likelihood of reaching a settlement is strictly higher when the activist demands a board settlement than when he demands an action settlement.*

This prediction is consistent with Gantchev (2013), who documents that activists succeed in only about 7% of the cases where they aim to reach a settlement over the action they are demanding, in a stark contrast to 39% of the cases where they aim to reach a board settlement.[54]

Moreover, Gantchev (2013) also finds that between the two types of settlements, the number of cases where the activist *demands* a board settlement is smaller (i.e., 203 vs. 342). However, Bebchuk et al. (2020) find that an overwhelming majority (84%) of settlements *reached* between activists and boards specify director additions. The strong contrast between these two findings might be seemingly puzzling, especially considering that both papers focus on the campaigns by hedge fund activists and significantly overlap in their sample periods.[55] However, the prediction above helps reconcile the empirical evidence in these two papers.

The second prediction implied by $\Delta_B^* \leq \Delta_A^*$ relates to shareholder value and is as follows.

Empirical prediction 4. *The average ex-post shareholder value is strictly larger upon reaching an action settlement than upon reaching a board settlement or upon a proxy fight by the activist.*

This empirical prediction compares the average ex-post shareholder value conditional on various outcomes: board settlement, action settlement, and proxy fight. An important difference between action and board settlements is that while an action settlement takes place if and only if $\Delta > \Delta_A^*$, a board settlement takes place if and only if $\Delta < b^a$ or $\Delta > \Delta_B^*$. Moreover, $\Delta_B^* \leq \Delta_A^*$. In other words, reaching an action settlement bears the positive news that the project NPV is high, while reaching a board settlement bears the negative news that the project might actually have a lower NPV, or even worse, might be worthless.

For these reasons, the shareholder value upon reaching an action settlement is strictly larger

two subgames). Because $\Delta_B^*(l)$ does not change with respect to l (by part (ii) of Proposition 6), this implies that $\Delta_B^*(l_1) = \Delta_B^*(l_2) \leq \Delta_A^*(l_2)$ for any two values l_1 and l_2 that l can take. Therefore, under the endogeneity of the settlement type, the incumbent's rejection thresholds realized on the equilibrium path always satisfy $\Delta_B^* \leq \Delta_A^*$ as well.

[54] See panel B of table 3 in Gantchev (2013).

[55] Specifically, the sample period in Gantchev (2013) is 2000-2007, and it is 2000-2013 in Bebchuk et al. (2020).

than both upon reaching a board settlement and upon a successful proxy fight. This result is also consistent with the existing empirical evidence. Bebchuk et al. (2020) find that the average announcement return for *all* settlements in their sample is about 1.2% (as noted earlier, vast majority of these settlements specify director additions). In contrast, they also find that the average announcement return for settlements that contract *actions* (e.g., CEO departure, strategic transactions) is 2.6-8.9% higher compared to settlements that do not contract such actions, again leading the authors to point out the consistency with my model for this finding as well.[56] Becht et al. (2009) document that in their study of activist campaigns, announcement of the implementation of an action demanded by the activist (e.g., restructuring, CEO turnover, payout) results in statistically significant positive abnormal returns, while announcement of director turnover results in negative and insignificant returns.[57]

Finally, the next empirical prediction is again regarding the average ex-post shareholder value, but it is with respect to information asymmetry and follows from Proposition 6.

Empirical prediction 5. *As information asymmetry increases, the average ex-post shareholder value conditional on reaching a board (action) settlement decreases (increases).*

The intuition behind this prediction can be understood once again from Figure 9 in the previous section. Indeed, as information asymmetry increases, the project's NPV needs to be higher for the incumbent to accept an action settlement (i.e., Δ_A^* increases), and therefore reaching an action settlement bears even better news. In contrast, the incumbent accepts a board settlement for the same set of positive NPVs (i.e., Δ_B^* does not change), and in addition it also accepts for the whole set of NPVs $\Delta < b^a$, where the project is never implemented. Since the latter set becomes larger as information asymmetry increases, reaching a board settlement bears even worse news.

[56] See table 10 and the accompanying discussion in Section 7 in Bebchuk et al. (2020). Specifically, they find that the average announcement return is 8.1–8.9% higher if the settlement contracts merging or selling the target firm or selling part of its asset, and 2.6–2.7% higher if the settlement contracts departure of the CEO.

[57] See table 8 in Becht et al. (2009). Note that although the UK fund that they study does not typically launch a proxy contest to acquire board seats, the risk of losing director seats still exists for the incumbent board due to the difference of governance rules in the UK compared to the US: As Becht et al. (2009) puts, "At annual general meetings, the statutory rule in the United Kingdom is cumulative majority voting, meaning that each and every director must receive a majority of the 'yes' votes cast to be elected (excluding abstentions)."

4.2.3 Further implications for shareholder value

As discussed in the previous section, one of the model's predictions (specifically, prediction 4) is that the average ex-post shareholder value conditional on reaching a board settlement is lower compared to conditional on reaching an action settlement. This prediction, which is consistent with various empirical evidences listed following prediction 4, can also be seen in panel (b) of Figure 10 below. Therefore, combined with institutional investors' concern that "board seats should not be a bargaining chip" in settlements,[58] a relevant question is whether shareholders would be better off if activists were incentivized to demand an action settlement instead of a board settlement (e.g., by making it harder for activists to obtain board seats via settlements). Note that this question is different than the one that was studied back in Section 3.2.2, which compared the case where the activist demands a board settlement with the case where neither board settlements nor action settlements are allowed.

In contrast with the criticisms of board settlements, I find that if information asymmetry is high, then (i) compared to action settlement, demanding a board settlement results in *higher ex-ante* shareholder value, even though (ii) the *ex-post* shareholder value conditional on reaching a board settlement is at its *lowest* level, both in absolute terms as well as relative to an action settlement. I begin with the former result, which is formalized by the proposition below for any bias b^a of the activist (also recall that lower l represents higher information asymmetry).

Proposition 7 *Suppose that Δ is uniformly distributed over (l, b^i).*[59] *There exists a threshold $\hat{l}_s > \underline{l}$ such that $\Pi_B^s > \Pi_A^s$ if $l < \hat{l}_s$, and $\Pi_B^s \leq \Pi_A^s$ otherwise.*[60]

This result is also illustrated in panel (a) of Figure 10 below. Intuitively, even though the shareholders' threshold $\hat{l}_s$ typically does not coincide with the activist's threshold $\hat{l}_a$,[61] the advantages of demanding a board settlement (over action settlement) increase with information asymmetry for the shareholders as well, due to the same reason explained in Section 4.2.1: That is, as information asymmetry increases, the activist can keep his proxy fight threat stable by demanding a board settlement, but this threat gets weaker if he demands an action settlement. This matters for the shareholders through its impact on the probability that the

[58] See Reuters, "Big Funds Push Back Against Activist Investor Settlements," 7/18/2016.

[59] Again, I impose a uniform distribution on Δ in order to simplify the exposition, and I verify in an unreported analysis that Proposition 7 continues to hold under any distribution of Δ.

[60] Recall that $\underline{l}$ was defined in Proposition 6.

[61] Recall that $\hat{l}_a$ was defined in Proposition 6 as well.

project is implemented. In particular, as information asymmetry increases, Proposition 6 implies that *conditional on* the project's NPV being positive: (i) if the activist has demanded an action settlement, then the probability that the project is implemented decreases down to zero,[62] however, (ii) if the activist has demanded a board settlement, this probability does not change.[63] For these reasons, the shareholders prefer that the activist demand a board settlement if and only if information asymmetry is higher than a certain threshold ($l < \hat{l}_s$).

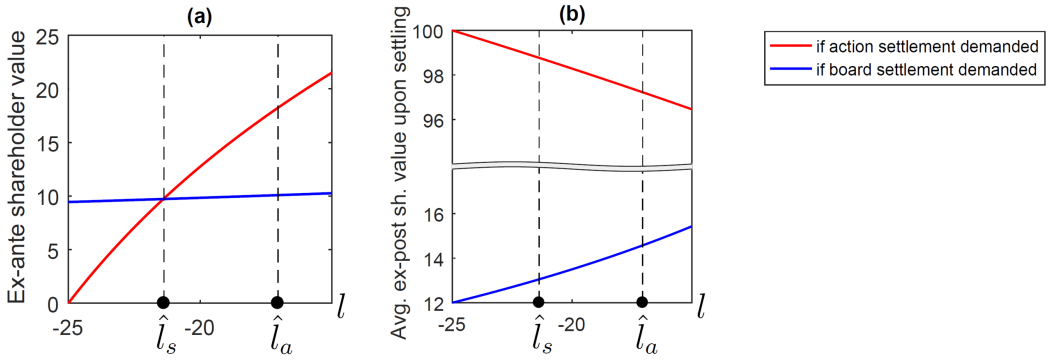


Figure 10 - As a function of info. asymmetry: (a) ex-ante shareholder value, and (b) average ex-post shareholder value conditional on reaching a settlement. Parameters: $b^a = 0, b^i = 100, C^a/n^a = 20, C^i/n^i = 5, \alpha_P = 0.5, \alpha_B = 0.3$, and $\Delta \sim U(l, b^i)$.

However, if information asymmetry is high (i.e., if $l < \hat{l}_s$, which implies that board settlements do benefit shareholders more than action settlements), then the average *ex-post* shareholder value conditional on reaching a board (action) settlement is at its lowest (highest) level, as again can be observed in panel (b) of Figure 10 above. This result is not specific to this example, and it holds generally, as formalized by the corollary below (which directly follows from Corollary 1 and Proposition 6; for further intuition see the discussions after empirical predictions 4 and 5 in Section 4.2.2). Therefore, for evaluating when board settlements are better for shareholders than action settlements, abnormal stock returns around settlements may not be an accurate measure, and they might actually point to the opposite conclusion.

[62] Formally, this is because the settlement region converges to an empty set (i.e., Δ_A^* converges to b) and the likelihood of a proxy fight ρ_A^* converges to zero.

[63] While the activist may implement a negative NPV project (specifically, after winning a proxy fight), note that the ex-ante shareholder value cannot be negative, because if it were than the activist would never win a proxy fight.

Corollary 2 *Denote the average ex-post shareholder value upon reaching an action (board) settlement by π_A^s (π_B^s). Then, for any two values l_1 and l_2 that l can take, if $l_2 < \hat{l}_s < l_1$, then $\pi_B^s(l_2) < \pi_B^s(l_1) < \pi_A^s(l_1) \leq \pi_A^s(l_2)$.*

5 Extensions

I study several extensions of the main model in order to show robustness of the results and to generate further implications. In this section, I summarize these extensions. For a complete description of the extensions, more detailed discussions, and formalization of the results, refer to Online Appendix F.

5.1 Imperfect learning upon joining the board

In Online Appendix F.1, I relax the assumption that the activist always learns the value of the project if he joins the board, and I instead assume that he learns it with probability $q \in (0, 1)$. I find that the results from the main model continue to hold, becase the equilibrium structure is preserved both for the case where the activist demands a board settlement and for the case where he demands an action settlement.

To start with the former case, the equilibrium described in Proposition 1 continues to be valid, with the exception that α_B is replaced with $\alpha_B q$. Intuitively, upon reaching a board settlement, the activist cannot distinguish whether the NPV Δ is higher or lower than his bias b^a, and thus he implements the project only if he learns that $\Delta \geq b^a$. An additional implication of this result is that the activist is not able to exercise his decision authority after reaching a board settlement even if he obtains full control, that is, even if $\alpha_B = 1$.

Regarding the latter case, the equilibrium in Proposition 5 continues to be valid as well, where the only change is that α_P is replaced with $\alpha_P q$. This is because upon winning a proxy fight upon rejection of an action settlement, the activist again implements the project only if he learns that $\Delta \geq b^a$.[64]

Moreover, as I also discuss in the online appendix, the empirical predictions of the main model get even stronger under this extension, because the dynamics described above amplify the forces that were already present in Section 4.2.

[64] Note that if the activist wins a proxy fight upon rejection of a *board* settlement, then α_P remains unchanged, because the activist infers from the rejection of board settlement that the project's NPV is higher than his bias b^a, even when he is unable to directly observe the NPV.

5.2 Incumbent's disclosure and information sharing agreements

In Online Appendix F.2, I study an extension where I add a disclosure stage at the very beginning of the game: I assume that, with some probability q, the incumbent has the option to voluntarily disclose the NPV to the activist (e.g., via information sharing agreements), where this disclosure is also verifiable. I find that under reasonable parameter specifications, there exists a threshold such that the incumbent discloses the NPV if and only if the NPV is lower than this threshold, and moreover, the activist ends his campaign following disclosure (i.e., he does not demand any settlement and does not run a proxy fight). Intuitively, disclosure eliminates the activist's incentives to run a proxy fight since it shows that the NPV is low. Following nondisclosure, however, the equilibrium is exactly as it was for the main model (i.e., as described in Propositions 1 and 5). As a result, as long as it is not always possible for the incumbent to disclose a given NPV (i.e., $q < 1$, for example due to Regulation Fair Disclosure, as elaborated in Online Appendix F.2.1), the results and empirical predictions from the main model continue to hold in this extension as well.

5.3 Incumbent's counter-offer

An assumption made in the main model is that when demanding a settlement, the activist makes a take-it-or-leave-it offer to the incumbent board. However, in practice, sometimes boards make counter-offers to the activist. Therefore, in Online Appendix F.3, I assume that after the activist makes his demand, the incumbent can make a counter-offer to the activist.

First of all, I find that if the activist demands a board settlement, there is always an equilibrium that is identical to the one in Proposition 1 (and therefore all of the other results in Section 3 continue to hold without any change whatsoever). The reason is that the incumbent has no incentive to counter with an action settlement offer, because the probability that the project is implemented is lower after a board settlement reached, and thus the incumbent always prefers a board settlement over an action settlement.[65]

Next, consider the case where the activist demands an action settlement. The discussion above may give the impression that whenever $\Delta < b^a$, the incumbent will always counter with a board settlement (because the activist never implements the project upon joining the board when $\Delta < b^a$). However, this is not true: In fact, I find that if α_B is low (which is consistent with the existing empirical evidence, e.g., Bebchuk et al. (2020)), then there always exists an

[65] Moreover, for the same reason, if $\alpha_B < 1$ then there exists no equilibrium where the incumbent counters with an action settlement. For reasons why it might be $\alpha_B < 1$, refer to the discussion in Section 4.2.

equilibrium that is identical to the equilibrium characterized by Proposition 5 (and thus all of the other results and predictions in Section 4 continue to hold as well). This is because even though the incumbent and the activist are aligned when the NPV is *low*, this does not imply that they will reach a board settlement (when the activist's initial demand is an action settlement), because such a settlement would affect the incentives of the incumbent to counter with a board settlement when the NPV is *high*, and in turn the incentives of the activist to accept the counter-offer.

To see this more clearly, suppose for a moment that if the activist demands an action settlement, the incumbent counters with a board settlement when $\Delta < b^a$, and the activist accepts. However, as I also formally show in the online appendix, then the incumbent would never accept the action settlement, and instead it would counter with a board settlement when Δ is high as well, because the incumbent prefers it to an action settlement.[66] Importantly, accepting the board settlement prevents the activist from running a proxy fight later on, due to the standstill provision of the settlement as mentioned earlier. Therefore, if α_B is low, the activist prefers rejecting a board settlement counter-offer, and instead runs a proxy fight. As a result, the incumbent's decision boils down to either accepting the activist's initial settlement demand or rejecting it, as in the main model.

In turn, the activist continues to make his settlement demand as specified in Proposition 6. Note that regardless of how small α_B is, the activist continues to demand a board settlement if information asymmetry is high, due to reasons explained following Proposition 6.

5.4 Incumbent with $b^i < 0$ and micro-foundation of activist's $b^a < 0$

Another set of assumptions of the main model was about the incumbent's and activist's biases, B^i and B^a. First, relative to the status quo, the incumbent was assumed to be biased *against* the project on the table (i.e., $B^i > 0$, or equivalently, $b^i = B^i/n^i > 0$), however it could be that the incumbent is biased *in favor of* the project (i.e., $B^i < 0$). Indeed, as I explain in Online Appendix F.4, if $B^i < 0$ then the main model and its analysis continue to hold if the labels of "status quo" and "project" are simply switched.

Second, it was also assumed in the main model that the activist has an exogenous bias B^a, and it was shown in Section 3.2.1 as well as Online Appendix B.2 that this bias has interesting implications if $B^a < 0$ (or equivalently, if $b^a = B^a/n^a < 0$). In Online Appendix F.5, motivated

[66] Note that if Δ is high, then the incumbent prefers reaching board settlement over the risk of facing a proxy fight as well, due to the costs of a proxy fight as explained after Proposition 1.

by the concerns expressed by institutional investors, policymakers, and the SEC about activist short-termism,[67] I micro-found the activist's bias under short-termism. In addition, in the same online appendix, I discuss a second extension where there is uncertainty about the activist's skills. Importantly, in both of these extensions, the activist's bias B^a arises endogenously and indeed satisfies $B^a < 0$.

5.5 Shareholder proposals, short sales, and greenmail

In this section, I consider other actions that might be available to activists in their campaigns. To begin with, shareholder proposals were not considered in the main model, because they are typically nonbinding. Nevertheless, I study an extension in Online Appendix F.6 where a proxy fight is replaced with a binding shareholder proposal, and I show the robustness of the results as well as empirical predictions to this modification. Similarly, another potential concern could be whether short selling would affect the results in the paper. However, I do not find this to be the case in the extension I analyze in Online Appendix F.7, where I allow short sales.

Remark regarding greenmail. Another concern could be whether activists utilize greenmailing. In the context of activism, greenmail refers to the practice where the target firm buys back the activist's stake at a premium in order to convince the activist to give up his campaign. However, empirical literature documents that greenmail is very rare in activism.[68] Therefore, I do not incorporate greenmailing in the model.

[67] For such concerns raised by institutional investors, see the second paragraph in the introduction. For concerns expressed by the SEC and policymakers about activist short-termism, as well as rule changes they have suggested to combat it, see sec.gov, "Activism, Short-Termism, and the SEC: Remarks at the 21st Annual Stanford Directors," 6/23/2015; and baldwin.state.gov, "U.S. Senator Tammy Baldwin Introduces Bipartisan Legislation to Strengthen Oversight of Predatory Hedge Funds," 8/31/2017. Indeed, many of these rule changes were formally proposed by the SEC in 2022 (see, e.g., WSJ, "SEC Proposes Giving Activist Investors Less Time to Report Positions,"2/10/2022; and HLS Forum on Corporate Governance and Financial Regulation, "SEC Proposes Updates to Schedule 13D/G Reporting," 2/15/2022).

[68] See, e.g., Section 8.5.2 in Bebchuk et al. (2020). In particular, they find that only 2% of the settlements during their sample period were followed by a privately negotiated buyback that enabled the activist to sell shares to the issuer. Furthermore, they report that in a large majority of these cases, the buyback was executed without a premium to the market price, concluding that they find no evidence supporting the greenmail concern.

6 Conclusion

In this paper, I provide a theoretical framework of settlements between activist investors and boards. The activist can demand that his proposal be implemented right away ("action settlement") or demand board seats ("board settlement"), which also gives the activist access to better information. The key insight in my findings is that the information conveyed by the incumbent's response is a critical factor that determines both the activist's decision to run a proxy fight and shareholders' support for the activist.

The model delivers many interesting implications of settlements for shareholders. To begin with, contrary to common concerns, value-destroying projects are typically not implemented following settlements, but rather after the activist *wins* a proxy fight with the *endogenous* support of shareholders. However, in any activist campaign that takes place in equilibrium, shareholders would be better off by always supporting the activist in a proxy fight, even though it would also result in the average ex-post shareholder return of proxy fights being *negative*. I also show that even if the activist is unbiased (i.e., he prefers to implement the project if and only if doing so increases shareholder value), allowing for settlements can *harm* shareholders.

Moreover, comparing action and board settlements yields many empirical predictions consistent with the existing empirical evidence (Becht et al. (2009), Gantchev (2013), Bebchuk et al. (2020)) and further implications for shareholder value: Among other results, I find that compared to action settlement, the average *ex-post* shareholder value conditional on *reaching* a board settlement is smaller, but *demanding* a board settlement can result in higher *ex-ante* shareholder value.

While the application chosen in my paper is shareholder activism, the theoretical framework is relevant for any setting that has a similar bargaining protocol. For instance, after a general election in many countries, a coalition government needs to be formed if the leading party has not received sufficient votes. When the political parties negotiate how to form such a coalition, they can negotiate over allocation of control on certain policies (akin to board settlement in my model) or negotiate directly over implementing these policies (akin to action settlement). Moreover, if this negotiation fails and a coalition is not formed, then the parties go to a re-election (akin to a proxy fight), demonstrating the relevance of the model presented in this paper for settings outside of shareholder activism. Therefore, one of the potential routes for future research is to tailor and study the model in this paper for such applications.

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A Appendix: Proofs for the propositions in Section 3

This appendix provides the proofs of the propositions in Section 3. The results that are mentioned but not formalized in these sections are formalized and proven in the online appendix. Proofs for lemmas and corollaries are provided in Online Appendix C.1.

Proof of Proposition 1. I first solve for the equilibrium for any given $\sigma \in [0,1]$, which is a major part of the proof. Then, I endogenize σ.

Note that the incumbent's payoff from accepting the board settlement is $\alpha_B n_i(\Delta - b_i)$ if $\Delta \geq b_a$, and zero if $\Delta < b_a$. This is because upon reaching a board settlement, the activist implements the project with probability α_B if $\Delta \geq b_a$, and he never implements it if $\Delta < b_a$. The likelihood that the project is implemented is similar if the activist wins a proxy fight, with the exception that α_B is replaced by α_P. Therefore, for any given ρ, the incumbent's payoff from rejecting the settlement is $\rho\left[-C_i + \sigma\alpha_P n_i(\Delta - b_i)\right]$ if $\Delta \geq b_a$, and $-\rho C_i$ otherwise. The proof consists of the following steps.

Step 1: First, note that if $\rho = 0$, then due to the arguments above, the incumbent always rejects the settlement if $\Delta \geq b_a$.

Step 2: Second, I characterize the incumbent's strategy for any given $\sigma \in [0,1]$ and $\rho > 0$. Note that then, again by the arguments made above, the incumbent accepts the board settlement for all $\Delta < b_a$. Therefore, it remains to derive the incumbent's response for $\Delta \geq b_a$. Note that if $\Delta \geq b_a$, the incumbent accepts the board settlement if and only if

$$\alpha_B n_i(\Delta - b_i) > \rho\left[-C_i + \sigma\alpha_P n_i(\Delta - b_i)\right]. \tag{12}$$

If $\rho < \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, then (12) is equivalent to $\Delta > \Delta_B(\rho;\sigma)$, where $\Delta_B(\rho;\sigma)$ is given by (7) and satisfies $\Delta_B(\rho;\sigma) \in (b_a, b_i]$ (due to $\rho < \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$). Indeed, it cannot be $\rho^* \geq \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$ in any equilibrium where proxy fight is on the equilibrium path (which is the type of equilibrium that is considered in this proposition). This is because if $\rho^* \geq \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, then either (a) the incumbent rejects board settlement with positive probability if and only if $\Delta = b_a$, which yields a contradiction with $\rho^* > 0$ because the activist would never run a proxy fight upon rejection, or (b) the incumbent accepts board settlement for all Δ, which again yields a contradiction because it would imply that there is no proxy fight on the equilibrium path.

Step 3: Third, I show that for any given $\sigma \leq \underline{\sigma}_B$, where

$$\underline{\sigma}_B \equiv C_a / \{\alpha_P n_a E\left[\Delta - b_a | \Delta \geq b_a\right]\}, \tag{13}$$

there is no equilibrium where a proxy fight is on the equilibrium path. Suppose there is. Then, $\rho^* > 0$, and hence by the second step above, the incumbent accepts the settlement for all $\Delta < b_a$, and if $\Delta \geq b_a$, then the incumbent rejects the settlement if and only if $\Delta \in [b_a, \Delta_B(\rho^*;\sigma)]$, where $\Delta_B(\rho^*;\sigma) < b_i$ since $\rho^* > 0$. Therefore, the activist's expected

payoff from running a proxy fight upon rejection is given by π_a, where

$$\pi_a(\sigma) = \sigma\alpha_P n_a E[\Delta - b_a | \Delta \in [b_a, \Delta_B(\rho^*;\sigma)]] - C_a, \tag{14}$$

which is negative because $\sigma \leq \underline{\sigma}_B$ and $\Delta_B(\rho^*;\sigma) < b_i$. However, this yields a contradiction with $\rho^* > 0$.

Step 4: Fourth, I show that for any given $\sigma > \underline{\sigma}_B$ (note that $\underline{\sigma}_B < 1$ due to (2)), there always exists an equilibrium where a proxy fight is on the equilibrium path, it is unique, and this equilibrium is given by parts (i) and (ii) of the proposition, where σ_B^* is simply replaced with σ. Note that to show this result, it is sufficient to prove that for any given $\sigma > \underline{\sigma}_B$, there indeed exists an equilibrium where a proxy fight is on the equilibrium path, and in this equilibrium, ρ^* is uniquely given by

$$\rho_B(\sigma) \equiv \min\left\{1, \alpha_B/(\sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_B(\sigma))])\right\} \tag{15}$$

and it satisfies $\rho^* \in (0, \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]})$ (because then, due to the second step above, the incumbent's equilibrium threshold is simply equal to $\Delta_B(\rho^*;\sigma)$, and $\Delta_B(\rho^*;\sigma) \in (b_a, b_i)$). Note that ρ_B, which is given by (15), is equivalent to

$$\rho_B(\sigma) = \min\left\{1, \alpha_B/\hat{\alpha}_L(\sigma)\right\}, \tag{16}$$

where

$$\hat{\alpha}_L(\sigma) \equiv \sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_B(\sigma))], \tag{17}$$

where, in turn, $\hat{\Delta}_B$ is given by (6) and satisfies $\hat{\Delta}_B \in (b_a, b_i)$ due to $\sigma > \underline{\sigma}_B$ and $C_a > 0$. Also recall that upon rejection of the board settlement, the activist infers that $\Delta \in [b_a, \Delta_B(\rho^*;\sigma)]$, and hence the activist's expected payoff from running a proxy fight upon rejection is given by (14) above. Importantly, this payoff $\pi_a(\sigma)$ is strictly positive if $\Delta_B(\rho^*;\sigma) > \hat{\Delta}_B$, it is equal to zero if $\Delta_B(\rho^*;\sigma) = \hat{\Delta}_B$, and it is negative if $\Delta_B(\rho^*;\sigma) < \hat{\Delta}_B$. With this observation in mind, there are two cases to consider in order to show that there exists an equilibrium where a proxy fight is on the equilibrium path, it is unique, and in this equilibrium ρ^* is equal to (15) and satisfies $\rho^* \in (0, \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]})$.

First, suppose that $\alpha_B \geq \hat{\alpha}_L$. Then, plugging (17) in for $\hat{\alpha}_L$, this implies that $\alpha_B > \sigma\alpha_P$ and $\hat{\Delta}_B \leq b_i - \frac{C_i/n_i}{\alpha_B - \sigma\alpha_P}$. Therefore, $\hat{\Delta}_B \leq \Delta_B(\rho;\sigma)$ if $\rho = 1$, and $\hat{\Delta}_B < \Delta_B(\rho;\sigma)$ for all $\rho \in [0, 1)$. Importantly, then it has to be $\rho^* = 1$. Indeed, if $\rho^* = 1$, then $\hat{\Delta}_B \leq \Delta_B(\rho^*;\sigma)$, and therefore $\pi_a \geq 0$ (i.e., the activist's expected payoff from running a proxy fight upon rejection is nonnegative). On the other hand, if any $\rho^* \in (0, 1)$ were to be an equilibrium, then it would imply that $\hat{\Delta}_B < \Delta_B(\rho^*;\sigma)$. However, this would yield a contradiction, because $\pi_a > 0$. Similarly, it cannot be $\rho^* = 0$, because then a proxy fight would not be on the equilibrium path. Therefore, $\rho^* = 1$ is the unique equilibrium where proxy fight is on the equilibrium path. Note that $\rho^* = 1 < \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, where the inequality follows from

$\hat{\Delta}_B > b_a$ and $\alpha_B \geq \hat{\alpha}_L$ (since $\alpha_B \geq \hat{\alpha}_L$ implies that $1 \leq \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_B)]}$). Moreover, also note that $\Delta_B(\rho^*;\sigma) = b_i - \frac{C_i/n_i}{\alpha_B - \sigma\alpha_P} \in (b_a, b_i)$, because $\alpha_B > \sigma\alpha_P$ and $b_a < \hat{\Delta}_B \leq b_i - \frac{C_i/n_i}{\alpha_B - \sigma\alpha_P}$.

Second, suppose that $\alpha_B < \hat{\alpha}_L$. Note that $\frac{\alpha_B}{\alpha_B/\hat{\alpha}_L} - \sigma\alpha_P > 0$ and $\Delta_B(\alpha_B/\hat{\alpha}_L;\sigma) = \hat{\Delta}_B \in (b_a, b_i)$. Importantly, then it must be $\rho^* = \alpha_B/\hat{\alpha}_L$. Indeed, if $\rho^* = \alpha_B/\hat{\alpha}_L$, then $\Delta_B(\rho^*;\sigma) = \hat{\Delta}_B$, and therefore $\pi_a = 0$, and hence $\rho = \alpha_B/\hat{\alpha}_L$ is a best response of the activist. On the other hand, if $\rho^* < \alpha_B/\hat{\alpha}_L$, then it would imply that $\hat{\Delta}_B < \Delta_B(\rho^*;\sigma)$. However, then $\pi_a > 0$, yielding a contradiction with $\rho^* < 1$. Similarly, $\rho^* > \alpha_B/\hat{\alpha}_L$ is not possible either. This is because if $\rho^* > \alpha_B/\hat{\alpha}_L$, then either (a) $\rho^* \geq \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, which is not possible in any equilibrium where proxy fight is on the equilibrium path (as explained in the second step above), or, (b) $\alpha_B/\hat{\alpha}_L < \rho^* < \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$ and hence $\Delta_B(\rho^*;\sigma) \in (b_a, \hat{\Delta}_B)$, implying $\pi_a < 0$ and therefore yielding a contradiction with $\rho^* > 0$. Therefore, $\rho^* = \alpha_B/\hat{\alpha}_L$ is the unique equilibrium where proxy fight is on the equilibrium path. Finally, also note that $\rho^* = \alpha_B/\hat{\alpha}_L < \frac{\alpha_B}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, where the inequality follows from $\hat{\alpha}_L > \sigma\alpha_P + C_i/[n_i(b_i - b_a)]$. This completes the characterization of the equilibrium for any given $\sigma > \underline{\sigma}_B$.

Step 5: Finally, I endogenize σ, and I show that there exists a unique σ_B^* that yields an equilibrium where proxy fight is on the equilibrium path. Note that by the third step above, if $\sigma \leq \underline{\sigma}_B$ then such an equilibrium does not exist. Therefore, it is sufficient to focus attention on $\sigma > \underline{\sigma}_B$ (note that $\underline{\sigma}_B < 1$ due to (2)). Then, due to the fourth step above, for any given $\sigma > \underline{\sigma}_B$, if the activist wins a proxy fight upon incumbent's rejection then the expected shareholder value is $\alpha_P E[\Delta | \Delta \in [b_a, \Delta_B(\rho_B(\sigma);\sigma)]]$, which is strictly decreasing in σ (this is because $\Delta_B(\rho_B(\sigma);\sigma) \in (b_a, b_i)$ by the fourth step above, and $\Delta_B(\rho_B(\sigma);\sigma)$ is strictly decreasing in σ due to the arguments in Section 3.1). Since the shareholders' expected payoff is zero if the activist loses a proxy fight upon rejection, to show that there exists a unique equilibrium σ_B^*, it is sufficient to prove $\lim_{\sigma \downarrow \underline{\sigma}_B} \alpha_P E[\Delta | \Delta \in [b_a, \Delta_B(\rho_B(\sigma);\sigma)]] > 0$. Indeed, this always holds, because $\lim_{\sigma \downarrow \underline{\sigma}_B} \hat{\Delta}_B(\sigma) = b_i$ by the definitions of $\underline{\sigma}_B$ and $\hat{\Delta}_B$, which in turn imply that $\lim_{\sigma \downarrow \underline{\sigma}_B} \rho_B(\sigma) = 0$ (where ρ_B is given by (15)). Therefore, $\lim_{\sigma \downarrow \underline{\sigma}_B} \Delta_B(\rho_B(\sigma);\sigma) = b_i$, which implies $\lim_{\sigma \downarrow \underline{\sigma}_B} \alpha_P E[\Delta | \Delta \in [b_a, \Delta_B(\rho_B(\sigma);\sigma)]] > 0$ as desired, due to (3). Also note that the equilibrium Δ_B^* has to satisfy $\Delta_B^* > 0$, because otherwise $\alpha_P E[\Delta | \Delta \in [b_a, \Delta_B^*]] < 0$ (since $\Delta_B^* > b_a$) and hence the shareholders would deviate to always supporting the incumbent in a proxy fight (i.e., deviate to $\sigma = 0$). ■

Proof of Proposition 2. Note that except for the last statement, all of the statements in the proposition follows directly from combining Lemma 1 with Proposition 1. Note that $\Delta_B^* > 0$, because otherwise Proposition 1 implies that it must be that $b_a < \Delta_B^* \leq 0$, which would imply a contradiction with $\sigma_B^* > 0$ (since the shareholders would be strictly worse off by electing the activist in a proxy fight, compared to re-electing the incumbent). Therefore, it remains to show that there exists $\bar{b}_a < 0$ such that $\sigma_B^* < 1$ if and only if $b_a < \bar{b}_a$. To show this result, I actually prove a stronger result: I show that that σ_B^* weakly decreases as b_a decreases.

Suppose that there exists $b_a'' < b_a'$ such that $\sigma_B^*(b_a'') > \sigma_B^*(b_a')$. Note that the expected shareholder value conditional on the activist winning a proxy fight after incumbent's rejection

of a board settlement is given by $\pi_s^B = \alpha_P E\left[\Delta | \Delta \in [b_a, \Delta_B^*]\right]$, where $\Delta_B^* > b_a$ by part (i) of Proposition 1. Since the shareholder value is zero if the incumbent is re-elected, and since $\sigma_B^* > 0$ again by Proposition 1, $0 < \sigma_B^*(b_a') < \sigma_B^*(b_a'') \leq 1$ implies $\pi_s^B(b_a') = 0 \leq \pi_s^B(b_a'')$, which in turn implies $b_a < \Delta_B^*(b_a') \leq \Delta_B^*(b_a'')$ (since $\Delta_B^* > b_a$ as mentioned above). Note that the activist's expected payoff from running a proxy fight upon the incumbent's rejection of a board settlement is given by $\pi_a^B = \sigma_B^* \alpha_P n_a E\left[\Delta - b_a | \Delta \in [b_a, \Delta_B^*]\right] - C_a$. Then, $b_a < \Delta_B^*(b_a') \leq \Delta_B^*(b_a'')$ and $\sigma_B^*(b_a') < \sigma_B^*(b_a'')$ imply that $\pi_a^B(b_a'') > \pi_a^B(b_a')$, resulting in $\rho_B^*(b_a'') \geq \rho_B^*(b_a')$. However, combining $\rho_B^*(b_a'') \geq \rho_B^*(b_a')$ with $\sigma_B^*(b_a'') > \sigma_B^*(b_a')$, part (i) of Proposition 1 implies $\Delta_B^*(b_a'') < \Delta_B^*(b_a')$ (because $\Delta_B^* \in (b_a, b_i)$), yielding a contradiction. ■

Proof of Proposition 3. Below, steps 6-7 prove part (i), and steps 8-9 prove part (ii). The rest of the steps are preliminary steps.

Step 1: First, given that the activist has demanded a board settlement, for any given $\sigma \in [0,1]$, I derive the expected shareholder value $\Pi_s^B(\sigma)$. By the fourth step in the proof of Proposition 1, for any given $\sigma > \underline{\sigma}_B$ (where $\underline{\sigma}_B$ is given by (13) and $\underline{\sigma}_B < 1$ due to (2)), there always exists an equilibrium where a proxy fight is on the equilibrium path, it is unique, and this equilibrium is given by parts (i) and (ii) of Proposition 1, where σ_B^* is simply replaced with σ. Therefore, for any $\sigma > \underline{\sigma}_B$,

$$\Pi_s^B(\sigma) = \rho_B(\sigma)\sigma \int_{b_a}^{\Delta_B(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B(\sigma)}^{b_i} \alpha_B \Delta dF(\Delta), \tag{18}$$

where $\Delta_B(\sigma)$ and $\rho_B(\sigma)$ are given by (4) and (5), respectively, and where σ_B^* is again replaced with σ. Note that $\Delta_B(\sigma) \in (b_a, b_i)$ by the fourth step of the proof of Proposition 1. Also note that in equilibrium, $\sigma_B^* > \underline{\sigma}_B$ by the fifth step in the proof of Proposition 1.

In contrast, if $\sigma \leq \underline{\sigma}_B$, then by the third step in the proof of Proposition 1, there is no equilibrium where a proxy fight is on the equilibrium path. However, given $\sigma \leq \underline{\sigma}_B$, there always exists an equilibrium where the incumbent always rejects the settlement for all $\Delta \geq b_a$, and the activist never runs a proxy fight. Indeed, this is an equilibrium because then the activist's expected payoff from running a proxy fight upon rejection is weakly smaller than $\sigma \alpha_P n_a E[\Delta - b_a | \Delta \geq b_a] - C_a$, which is nonpositive due to $\sigma \leq \underline{\sigma}_B$ and (13). Therefore, it is a best response of the activist to never run a proxy fight (i.e., $\rho(\sigma) = 0$). In turn, it is a best response for the incumbent to reject the settlement for all $\Delta \geq b_a$.[69] Therefore, in this equilibrium, $\Pi_s^B(\sigma) = 0$.

Note that given $\sigma \leq \underline{\sigma}_B$, even though there might also exist an equilibrium where the incumbent accepts the board settlement for all Δ, I choose the equilibrium described above (i.e., where the incumbent rejects the settlement for all $\Delta \geq b_a$) for three reasons: (a) First

[69] Note that for all $\Delta < b_a$, the incumbent is indifferent between accepting and rejecting the settlement, because the activist does not implement the project upon reaching a board settlement. Therefore, there is actually a continuum of equilibria, where the incumbent accepts the settlement with any probability for any $\Delta < b_a$, while it rejects with certainty for all $\Delta \geq b_a$. However, the differences in these equilibria has no material impact, because for any $\Delta < b_a$, regardless of whether the incumbent accepts the settlement or not, the project is never implemented.

of all, recall that in the proof of Proposition 1, for any $\sigma > \underline{\sigma}_B$, the equilibrium described by parts (i) and (ii) of Proposition 1 is chosen (for the reasons for this selection, please see the discussion right after Proposition 1), where σ_B^* is again replaced with σ. Importantly, in this equilibrium, plugging (5) in (4) implies that $\Delta_B^*(\sigma) \geq \hat{\Delta}_B(\sigma)$, where $\hat{\Delta}_B(\sigma) \geq b_a + C_a/n_a$ due to (6). Therefore, for any $\sigma > \underline{\sigma}_B$, the incumbent rejects the settlement at least for $\Delta \in [b_a, b_a + C_a/n_a]$, which is a set with nonzero measure. For this reason, for the region where shareholder support for the activist is lower (i.e., $\sigma \leq \underline{\sigma}_B$), it would not be reasonable to choose the equilibrium where the incumbent accepts the settlement for all Δ. (b) Second, the equilibrium where the incumbent always accepts the settlement does not always exist: In particular, it does not exist if $C_i/n_i < (\alpha_B - \underline{\sigma}_B \alpha_P)(b_i - b_a)$. This is because by the arguments made at the beginning of the proof of Proposition 1, for any $\Delta > b_a$, the incumbent strictly prefers to reject the board settlement if $\alpha_B n_i(\Delta - b_i) < \rho\left[-C_i + \sigma\alpha_P n_i(\Delta - b_i)\right]$, or equivalently, $\rho C_i < (\alpha_B - \rho\sigma\alpha_P)\, n_i(b_i - \Delta)$. In contrast, the equilibrium where the incumbent rejects the settlement for all $\Delta \geq b_a$ always exists (as shown above). (c) Third, if the equilibrium where the incumbent rejects the settlement for all $\Delta \geq b_a$ is chosen for the region where $\sigma \leq \underline{\sigma}_B$, then, $\rho(\sigma)$ and $\Pi_s^B(\sigma)$ in this region is continuous with $\rho_B(\sigma)$ and $\Pi_s^B(\sigma)$ in the region where $\sigma > \underline{\sigma}_B$ (again derived from the fourth step in the proof of Proposition 1), because (6) and (13) imply that $\lim_{\sigma \downarrow \underline{\sigma}_B} \hat{\Delta}_B(\sigma) = b_i$, in turn (4) and (5) yield $\lim_{\sigma \downarrow \underline{\sigma}_B} \rho_B(\sigma) = 0$ and $\lim_{\sigma \downarrow \underline{\sigma}_B} \Delta_B(\sigma) = b_i$, and thus, (18) implies $\lim_{\sigma \downarrow \underline{\sigma}_B} \Pi_s^B(\sigma) = 0$. However, this continuity at $\sigma = \underline{\sigma}_B$ cannot be achieved with the equilibrium where the incumbent always accepts the settlement, because the latter equilibrium yields $\Pi_s^B(\sigma) = \alpha_B E[1_{\{\Delta \geq b_a\}}\Delta] > 0$ (where the inequality follows from (3)).

Step 2: I show that $\Delta_B(\sigma)$ is strictly decreasing in σ if $\sigma > \underline{\sigma}_B$. Recall from step 1 that for any $\sigma > \underline{\sigma}_B$, $\Delta_B(\sigma)$ and $\rho_B(\sigma)$ are given by (4) and (5), respectively, where σ_B^* is simply replaced with σ. In turn, (5) implies that another way to express $\Delta_B(\sigma)$ is

$$\Delta_B(\sigma) = \begin{cases} b_i - \frac{C_i/n_i}{\alpha_B - \sigma\alpha_P}, & \text{if } \alpha_B > \sigma\alpha_P + \frac{C_i/n_i}{b_i - \hat{\Delta}_B(\sigma)}, \\ \hat{\Delta}_B(\sigma), & \text{otherwise.} \end{cases} \tag{19}$$

Therefore, there are three cases to consider: First, suppose that $\alpha_B < \sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_B(\sigma))]$. Then, $\Delta_B(\sigma) = \hat{\Delta}_B(\sigma)$, where $\hat{\Delta}_B(\sigma)$ is strictly decreasing in σ for all $\sigma > \underline{\sigma}_B$, due to (1), (2), and (13). Second, suppose that $\alpha_B > \sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_B(\sigma))]$. Then, $\Delta_B(\sigma) = b_i - \frac{C_i/n_i}{\alpha_B - \sigma\alpha_P}$ is again strictly decreasing in σ. Third, suppose that $\alpha_B = \sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_B(\sigma))]$. Then, $\Delta_B(\sigma)$ is again strictly decreasing in σ, because both $\hat{\Delta}_B(\sigma)$ and $b_i - \frac{C_i/n_i}{\alpha_B - \sigma\alpha_P}$ are strictly decreasing in σ.

Step 3: For any two values σ_2, σ_1 of σ such that $\sigma_2 > \sigma_1 > \underline{\sigma}_B$, I show that it must be $\rho_B(\sigma_2) \cdot \sigma_2 > \rho_B(\sigma_1) \cdot \sigma_1$. Suppose that there exist $\sigma_2 > \sigma_1 > \underline{\sigma}_B$ such that $\rho_B(\sigma_2) \cdot \sigma_2 \leq \rho_B(\sigma_1) \cdot \sigma_1$. Then, it must be that $\rho_B(\sigma_2) < \rho_B(\sigma_1)$. In turn, since the fourth step of the proof of Proposition 1 implies that $\Delta_B(\sigma) = b_i - \frac{\rho_B C_i/n_i}{\alpha_B - \rho_B\sigma\alpha_P} \in (b_a, b_i)$, it follows that $\rho_B(\sigma_2) \cdot \sigma_2 \leq \rho_B(\sigma_1) \cdot \sigma_1$ and $\rho_B(\sigma_2) < \rho_B(\sigma_1)$ together imply that $\Delta_B(\sigma_2) > \Delta_B(\sigma_1)$, yielding

a contradiction with step 2 above.

Step 4: Note that for $\sigma = \sigma_B^*$, $\Pi_s^B(\sigma_B^*) > 0$ and the first term in (18) is nonnegative. To see this, note that if the first term in (18) were to be negative, then the shareholders would deviate to always supporting the incumbent in a proxy fight, yielding a contradiction with $\sigma_B^* > 0$ (which was shown in Proposition 1). In turn, since the first term in (18) is nonnegative, this implies that $\Pi_s^B(\sigma_B^*) > 0$, because $\Delta_B(\sigma_B^*) < b_i$ by Proposition 1.

Step 5: I show that for $\sigma > \underline{\sigma}_B$, it must be $\rho_B(\sigma)\sigma\alpha_P < \alpha_B$. Suppose that there exists $\sigma > \underline{\sigma}_B$ such that $\rho_B(\sigma)\sigma\alpha_P \geq \alpha_B$. However, then, (4) yields a contradiction with $\Delta_B(\sigma) \in (b_a, b_i)$ (recall by step 1 above that $\Delta_B(\sigma)$ is given by (4) where σ_B^* is simply replaced with σ, and also that $\Delta_B(\sigma) \in (b_a, b_i)$).

Step 6: I show that for any $\sigma > \sigma_B^*$, it must be $\Pi_s^B(\sigma) > \Pi_s^B(\sigma_B^*)$. Recall that $\sigma_B^* > \underline{\sigma}_B$ (as noted in step 1). Therefore, for any $\sigma > \sigma_B^*$,

$$\begin{aligned}
\Pi_s^B(\sigma) &= \rho_B(\sigma)\sigma \textstyle\int_{b_a}^{\Delta_B(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B(\sigma)}^{\Delta_B(\sigma_B^*)} \alpha_B \Delta dF(\Delta) + \int_{\Delta_B(\sigma_B^*)}^{b_i} \alpha_B \Delta dF(\Delta) \\
&> \rho_B(\sigma)\sigma \textstyle\int_{b_a}^{\Delta_B(\sigma)} \alpha_P \Delta dF(\Delta) + \rho_B(\sigma)\sigma \int_{\Delta_B(\sigma)}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B(\sigma_B^*)}^{b_i} \alpha_B \Delta dF(\Delta) \\
&\geq \rho_B(\sigma_B^*)\sigma_B^* \textstyle\int_{b_a}^{\Delta_B(\sigma)} \alpha_P \Delta dF(\Delta) + \rho_B(\sigma_B^*)\sigma_B^* \int_{\Delta_B(\sigma)}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B(\sigma_B^*)}^{b_i} \alpha_B \Delta dF(\Delta) \\
&= \Pi_s^B(\sigma_B^*)
\end{aligned}$$

where the first and last equalities follow from (18); the first inequality follows from combining $\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) \geq 0$ (by step 4), $\rho_B(\sigma)\sigma\alpha_P < \alpha_B$ (by step 5), $\Delta_B(\sigma) < \Delta_B(\sigma_B^*)$ (by step 2), and $b_a < \Delta_B(\sigma)$ (by step 1); and the second inequality follows from combining $\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) \geq 0$ and $\rho_B(\sigma)\sigma > \rho_B(\sigma_B^*)\sigma_B^*$ (by step 3).

Step 7: I show that for any $\sigma < \sigma_B^*$, it must be $\Pi_s^B(\sigma_B^*) > \Pi_s^B(\sigma)$. Again, recall that $\sigma_B^* > \underline{\sigma}_B$. There are two cases to consider. First, suppose that $\sigma \leq \underline{\sigma}_B$. Then, $\Pi_s^B(\sigma_B^*) > 0 = \Pi_s^B(\sigma)$, where the inequality follows from step 4, and $\Pi_s^B(\sigma) = 0$ follows from step 1. Second, suppose that $\sigma > \underline{\sigma}_B$. Then,

$$\begin{aligned}
\Pi_s^B(\sigma_B^*) &= \rho_B(\sigma_B^*)\sigma_B^* \textstyle\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B(\sigma_B^*)}^{\Delta_B(\sigma)} \alpha_B \Delta dF(\Delta) + \int_{\Delta_B(\sigma)}^{b_i} \alpha_B \Delta dF(\Delta) \\
&> \rho_B(\sigma_B^*)\sigma_B^* \textstyle\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) + \rho_B(\sigma_B^*)\sigma_B^* \int_{\Delta_B(\sigma_B^*)}^{\Delta_B(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B(\sigma)}^{b_i} \alpha_B \Delta dF(\Delta) \\
&> \rho_B(\sigma)\sigma \textstyle\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) + \rho_B(\sigma)\sigma \int_{\Delta_B(\sigma_B^*)}^{\Delta_B(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B(\sigma)}^{b_i} \alpha_B \Delta dF(\Delta) \\
&= \Pi_s^B(\sigma)
\end{aligned}$$

where the first and last equalities follow from (18); the first inequality follows from combining $\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) \geq 0$ (by step 4), $\rho_B(\sigma_B^*)\sigma_B^*\alpha_P < \alpha_B$ (by step 5), and $\Delta_B(\sigma_B^*) < \Delta_B(\sigma)$ (by step 2); and the second inequality follows from combining $\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) \geq 0$, $\Delta_B(\sigma_B^*) < \Delta_B(\sigma)$, and $\rho_B(\sigma_B^*)\sigma_B^* > \rho_B(\sigma)\sigma$ (by step 3).

Step 8: Denoting by π_s^B the expected ex-post shareholder value conditional on the activist winning a proxy fight upon rejection of a board settlement, I show that $\pi_s^B(\sigma) \geq 0$ for all $\sigma \leq \sigma_B^*$. Since $\sigma_B^* > \underline{\sigma}_B$ by step 1, there are two cases to consider: First, suppose that

$\sigma \leq \underline{\sigma}_B$. Then, the incumbent always rejects the settlement by step 1, and hence $\pi_s^B(\sigma) = \alpha_P E[1_{\{\Delta \geq b_a\}} \cdot \Delta]$, which is positive due to (3). Second, suppose that $\underline{\sigma}_B < \sigma \leq \sigma_B^*$. Then, by step 1, $\pi_s^B(\sigma) = \alpha_P E[\Delta | \Delta \in [b_a, \Delta_B(\sigma)]]$ and $\Delta_B(\sigma) > b_a$. Importantly, step 4 implies $\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) \geq 0$, because $\sigma_B^* > 0$, $\rho_B(\sigma_B^*) > 0$, and $\Delta_B(\sigma_B^*) > b_a$ due to Proposition 1. Combining $\int_{b_a}^{\Delta_B(\sigma_B^*)} \alpha_P \Delta dF(\Delta) \geq 0$ and $\Delta_B(\sigma_B^*) > b_a$ with step 2 implies $\pi_s^B(\sigma) \geq 0$ for all $\underline{\sigma}_B < \sigma \leq \sigma_B^*$.

Step 9: I show that $\pi_s^B(\sigma) < 0$ for all $\sigma > \sigma_B^*$, where π_s^B is defined in the previous step. Note that for there to be any $\sigma > \sigma_B^*$, it must be that $\sigma_B^* < 1$. Combining this with part (iii) of Proposition 1 implies $\sigma_B^* \in (0, 1)$. In other words, in a proxy fight upon rejection of a board settlement, the shareholders are indifferent between electing the activist and re-electing the incumbent. Since the shareholder value is zero if the incumbent is re-elected, it must be that $\pi_s^B(\sigma_B^*) = 0$. Combining this with $\pi_s^B(\sigma) = \alpha_P E[\Delta | \Delta \in [b_a, \Delta_B(\sigma)]]$ (which again follows from step 1 due to $\sigma > \sigma_B^* > \underline{\sigma}_B$, where $\Delta_B(\sigma) > b_a$) and step 2 implies $\pi_s^B(\sigma) < 0$ for all $\sigma > \sigma_B^*$. ■

Proof of Proposition 4. The proof consists of several steps. Among these steps, steps 8-13 prove part (ii), steps 14-17 prove part (i), and step 18 proves part (iii). The rest of the steps are preliminary steps. The steps are as follows.

Step 1: First, note that if the activist demands a board settlement, the equilibrium is given by Proposition 1, and hence Π_s^B is given by

$$\Pi_s^B(C_a) = \sigma_B^*(C_a)\rho_B^*(C_a) \int_{b_a}^{\Delta_B^*(C_a)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B^*(C_a)}^{b_i} \alpha_B \Delta dF(\Delta). \tag{20}$$

Step 2: Second, I derive $\Pi_s^{\varnothing}$. To that end, suppose that the activist cannot make any settlement demand. Note that if the activist runs and wins a proxy fight, then the expected shareholder value is given by $\int_{b_a}^{b_i} \alpha_P \Delta dF(\Delta)$, which is positive due to (3). Therefore, the shareholders always support the activist in a proxy fight. As a result, the activist's expected payoff from running a proxy fight is given by $\Pi_a^{\varnothing} = n_a \int_{b_a}^{b_i} \alpha_P (\Delta - b_a) dF(\Delta) - C_a$, which is also positive due to (2). Therefore, the activist always runs a proxy fight. As a result,

$$\Pi_s^{\varnothing} = \int_{b_a}^{b_i} \alpha_P \Delta dF(\Delta) > 0. \tag{21}$$

Step 3: Note that if $\sigma_B^* < 1$, this implies that when the activist runs a proxy fight upon rejection of a board settlement, shareholders are indifferent between supporting the activist and supporting the incumbent, because also $\sigma_B^* > 0$ by Proposition 1. Therefore, the expected shareholder value if the activist runs and wins a proxy fight upon rejection of a board settlement must be zero. As a result, it also follows that Δ_B^* satisfies the equation $E[\Delta | \Delta \in [b_a, \Delta_B^*]] = 0$. Note that since $\Delta_B^* > b_a$ by part (i) of Proposition 1, (3) implies that there exists a unique $\Delta_0 \in (b_a, b_i)$ such that $\Delta_B^* = \Delta_0$. Also note that $b_a < 0$ due to $\sigma_B^* < 1$, because if $b_a \geq 0$ then in any equilibrium upon the rejection of a board settlement, the shareholders strictly prefer that the activist is elected in a proxy fight, resulting in $\sigma_B^* = 1$. Finally, also note that $\Delta_0 > 0$.

Step 4: I show that if $\sigma_B^* < 1$ for any $C_a = C_a'$, then $\sigma_B^* < 1$ for any $C_a = C_a'' < C_a'$. Note that for any C_a, the activist's expected payoff from running a proxy fight upon rejection of a board settlement is given by $\pi_a^*(C_a) = n_a \sigma_B^* \alpha_P E\left[\Delta - b_a | b_a \leq \Delta \leq \Delta_B^*\right] - C_a$. Moreover, for any C_a, $\rho_B^* > 0$ due to Proposition 1, and hence it must be $\pi_a^*(C_a) \geq 0$. Suppose that there exists $C_a'' < C_a'$ such that $\sigma_B^*(C_a') < 1$ and $\sigma_B^*(C_a'') = 1$. Then, at $C_a = C_a''$, the expected shareholder value if the activist runs and wins a proxy fight upon rejection of a board settlement must be nonnegative, i.e., $E[\Delta | \Delta \in [b_a, \Delta_B^*(C_a'')]] \geq 0$. When combined with $\sigma_B^*(C_a') < 1$ and step 3 above, this implies that $\Delta_B^*(C_a'') \geq \Delta_B^*(C_a')$. Then, due to $\pi_a^*(C_a') \geq 0$ and $\sigma_B^*(C_a') < \sigma_B^*(C_a'') = 1$, this implies that $\pi_a^*(C_a'') > 0$ and hence $\rho_B^*(C_a'') = 1 \geq \rho_B^*(C_a')$. However, since $\sigma_B^*(C_a') < \sigma_B^*(C_a'')$, due to Proposition 1, this implies that $\Delta_B^*(C_a'') < \Delta_B^*(C_a')$, yielding a contradiction.

Step 5: Note that if $\sigma_B^* < 1$, then $\Pi_s^B = \int_{\Delta_0}^{b_i} \alpha_B \Delta dF(\Delta)$, where Δ_0 is as defined in step 3. Note that this expression for Π_s^B follows from the definition of Π_s^B in step 1, and the result in step 3 that Δ_B^* satisfies the equation $E[\Delta | \Delta \in [b_a, \Delta_B^*]] = 0$.

Step 6: Note that if $\sigma_B^* = 1$, then it must be that $\Delta_B^* \geq \Delta_0$, because otherwise, the definition of Δ_0 in step 3 implies that in a proxy fight, the shareholders always support the incumbent, yielding a contradiction with $\sigma_B^* = 1$.

Step 7: Note that if $\sigma_B^* = 1$ for some $C_a = C_a'$, then for all $C_a > C_a'$: (a) Step 4 implies that $\sigma_B^* = 1$, and hence (b) (20) implies that $\Pi_s^B(C_a)$ is continuous in C_a, because given that $\sigma_B^* = 1$, $\Delta_B^*(C_a)$ and $\rho_B^*(C_a)$ given by (4) and (5) are continuous in C_a.

Step 8: In this step, I start the proof of part (ii). In particular, in this step, I show that if $\sigma_B^*(C_a') < 1$ for some $C_a = C_a'$, then $\Pi_s^B(C_a) \leq \Pi_s^B(C_a')$ for all $C_a > C_a'$. Since $\Pi_s^B(C_a') = \int_{\Delta_0}^{b_i} \alpha_B \Delta dF(\Delta)$ due to step 5, and in addition $\Pi_s^B(C_a) = \int_{\Delta_0}^{b_i} \alpha_B \Delta dF(\Delta)$ if $\sigma_B^*(C_a) < 1$ (again due to step 5), it is sufficient to show this result for C_a that satisfies $\sigma_B^*(C_a) = 1$. Indeed, for any such C_a, (20) implies

$$\begin{aligned} \Pi_s^B(C_a) &= \rho_B^*(C_a) \textstyle\int_{b_a}^{\Delta_B^*(C_a)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B^*(C_a)}^{b_i} \alpha_B \Delta dF(\Delta) \\ &= \rho_B^*(C_a) \textstyle\int_{\Delta_0}^{\Delta_B^*(C_a)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B^*(C_a)}^{b_i} \alpha_B \Delta dF(\Delta) \qquad (22) \\ &\leq \textstyle\int_{\Delta_0}^{b_i} \alpha_B \Delta dF(\Delta) = \Pi_s^B(C_a'), \end{aligned}$$

where the second equality follows from the definition of Δ_0 in step 3 and $\Delta_B^*(C_a) \geq \Delta_0$ by step 6; the inequality follows from $\Delta_0 > 0$ in step 3 and $\rho_B^*(C_a)\alpha_P \leq \alpha_B$ due to (5) and $\sigma_B^*(C_a) = 1$; and the last equality follows from step 5.

Step 9: I show that if $\sigma_B^*(C_a') = 1$ for some $C_a = C_a'$, then $\Pi_s^B(C_a'') \leq \Pi_s^B(C_a')$ for all $C_a'' > C_a'$. This result follows because $\sigma_B^*(C_a') = \sigma_B^*(C_a'') = 1$ (by step 4), and hence

$$\begin{aligned} \Pi_s^B(C_a'') &= \rho_B^*(C_a'') \textstyle\int_{\Delta_0}^{\Delta_B^*(C_a'')} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B^*(C_a'')}^{b_i} \alpha_B \Delta dF(\Delta) \\ &\leq \rho_B^*(C_a') \textstyle\int_{\Delta_0}^{\Delta_B^*(C_a')} \alpha_P \Delta dF(\Delta) + \int_{\Delta_B^*(C_a')}^{b_i} \alpha_B \Delta dF(\Delta) = \Pi_s^B(C_a'), \end{aligned}$$

where the first and last equalities follow from (22) in step 8, and the inequality follows because whenever $\sigma_B^*(C_a) = 1$: (a) $\rho_B^*(C_a)$ is weakly decreasing in C_a due to (5), (b) $\Delta_B^*(C_a)$ is weakly increasing in C_a due to (4), (c) $\rho_B^*(C_a)\alpha_P \leq \alpha_B$ due to (5), and (d) $\Delta_0 > 0$ by step 3.

Step 10: I show that $\Pi_s^B(C_a)$ is continuous for all C_a. Note that if $\sigma_B^*(C_a) < 1$ for all C_a, this holds due to step 5, and if $\sigma_B^*(C_a) = 1$ for all C_a, then it holds due to step 7b. Therefore, it remains to prove this result for the case where $\sigma_B^*(C_a) < 1$ for some C_a and $\sigma_B^*(C_a) = 1$ for some C_a. Let $\underline{C}_a \equiv \inf\{C_a : \sigma_B^*(C_a) = 1\}$, which exists due to step 4. Moreover, steps 4, 5, and 7b together directly imply that $\Pi_s^B(C_a)$ is continuous for all $C_a \neq \underline{C}_a$. Therefore, it remains to show that $\Pi_s^B(C_a)$ is continuous at $C_a = \underline{C}_a$ as well. Since (4) and (5) imply that (20) is continuous w.r.t. σ_B^* (holding C_a constant) and C_a (holding σ_B^* constant), it is sufficient to show that $\sigma_B^*(C_a)$ is continuous at $C_a = \underline{C}_a$. I show this result in two substeps: (a) First, I prove that $\sigma_B^*(C_a)$ is continuous for all $C_a < \underline{C}_a$. To see why, note that step 4 implies that $\sigma_B^* < 1$ for all $C_a < \underline{C}_a$, and thus step 3 implies that $\Delta_B^*(C_a) = \Delta_0$ for all $C_a < \underline{C}_a$. In turn, for $\Delta_B^*(C_a)$ to remain constant for all $C_a < \underline{C}_a$, it follows from (4) and (5) that $\sigma_B^*(C_a)$ has to be continuous for all $C_a < \underline{C}_a$, because not only (4) is continuous w.r.t. C_a (holding σ_B^* constant) but it is also strictly decreasing w.r.t. σ_B^* (holding C_a constant) due to the arguments in Section 3.1. (b) Second, I show that $\lim_{C_a \uparrow \underline{C}_a} \sigma_B^*(C_a) = 1$. Suppose that $\lim_{C_a \uparrow \underline{C}_a} \sigma_B^*(C_a) \neq 1$, which implies that $\lim_{C_a \uparrow \underline{C}_a} \sigma_B^*(C_a) < 1$ (note that the limit exists due to the previous substep). Recall that $\Delta_B^*(C_a) = \Delta_0$ for all $C_a < \underline{C}_a$ due to step 3. Since $\sigma_B^*(C_a)$ is continuous for all $C_a < \underline{C}_a$ (by the last substep) and thus (4) is continuous in C_a for all $C_a < \underline{C}_a$ as well, this implies that (4) evaluated at the pair of $C_a = \underline{C}_a$ and $\sigma_B^* = \lim_{C_a \uparrow \underline{C}_a} \sigma_B^*(C_a) < 1$ is equal to Δ_0 as well. Since (4) is strictly decreasing w.r.t. σ_B^* (holding C_a constant) as noted in the previous substep and (4) is continuous w.r.t. C_a (holding σ_B^* constant), this in turn implies that there exists $\delta > 0$ such that $\Delta_B^*(C_a) < \Delta_0$ for all $C_a \in (\underline{C}_a, \underline{C}_a + \delta)$ (because $\sigma_B^*(C_a) = 1$ for all such C_a). However, then for all $C_a \in (\underline{C}_a, \underline{C}_a + \delta)$, the shareholders are strictly worse off by electing the activist in a proxy fight upon the rejection of a board settlement (because $E[\Delta | \Delta \in [b_a, \Delta_B^*(C_a)]] < 0$ due to combining the definition of Δ_0 in step 3 with $\Delta_B^* > b_a$ in Proposition 1), yielding a contradiction with $\sigma_B^*(C_a) = 1$.

Step 11: $\Pi_s^B(C_a)$ is weakly decreasing in C_a, which immediately follows from steps 7a and 8-10.

Step 12: Steps 10 and 11 imply there exists $\hat{C}_a$ such that $\Pi_s^B < \Pi_s^\varnothing$ if and only if $C_a > \hat{C}_a$.

Step 13: While step 12 above completes the proof of part (ii), in this step I also show that there exists $\tilde{\alpha}_B > 0$ such that if $\alpha_B < \tilde{\alpha}_B$, then $\hat{C}_a < \bar{C}_a$, where $\bar{C}_a$ is given by (2). Note that due to (21) and step 12, it is sufficient to show that for any given C_a, $\lim_{\alpha_B \downarrow 0} \Pi_s^B(C_a) = 0$. Indeed, this result follows from (20) because for any $\delta > 0$, (5) implies that there exists $\alpha_B' > 0$ such that if $\alpha_B < \alpha_B'$, then $\rho_B^*(\alpha_B) < \delta$ (note that $-\infty < b_a < \hat{\Delta}_B(\sigma_B^*(\alpha_B)) < b_i < \infty$ is always satisfied in equilibrium due to part (ii) of Proposition 1).

Step 14: In this step, I start proving part (i). To begin with, note that (21) and the result $\lim_{\alpha_B \downarrow 0} \Pi_s^B = 0$ in step 13 imply that there exists $\hat{\alpha}_B > 0$ such that $\Pi_s^B < \Pi_s^\varnothing$ if $\alpha_B < \hat{\alpha}_B$.

Step 15: I prove part (i.a) by showing that conditional on the activist demanding a board settlement, as α_B decreases, Δ_B^* weakly decreases. To prove by contradiction, suppose that

there exist α'_B and α''_B such that $\alpha''_B < \alpha'_B$ and $\Delta^*_B(\alpha''_B) > \Delta^*_B(\alpha'_B)$. Then, $\sigma^*_B(\alpha''_B) = 1 \geq \sigma^*_B(\alpha'_B)$ due to the same arguments made in steps 3 and 6. In turn, the activist's expected payoff π^*_a (given in step 4) from running a proxy fight upon rejection of a board settlement satisfies $\pi^*_a(\alpha''_B) > \pi^*_a(\alpha'_B)$. Therefore, $\rho^*_B(\alpha''_B) \geq \rho^*_B(\alpha'_B)$. Combining this with $\sigma^*_B(\alpha''_B) \geq \sigma^*_B(\alpha'_B)$ and $\alpha''_B < \alpha'_B$, part (i) of Proposition 1 yields a contradiction with $\Delta^*_B(\alpha''_B) > \Delta^*_B(\alpha'_B)$.

Step 16: Next, I show that conditional on the activist demanding a board settlement, ρ^*_B weakly decreases as α_B decreases. To prove by contradiction, suppose that there exist α'_B and α''_B such that $\alpha''_B < \alpha'_B$ and $\rho^*_B(\alpha''_B) > \rho^*_B(\alpha'_B)$. Note that then, $\rho^*_B(\alpha'_B) \in (0,1)$ by part (ii) of Proposition 1. Also note that the activist's expected payoff π^*_a from running a proxy fight upon rejection of a board settlement was given in step 4, and $\rho^*_B(\alpha'_B) \in (0,1)$ implies $\pi^*_a(\alpha'_B) = 0$. Since $\Delta^*_B(\alpha''_B) \leq \Delta^*_B(\alpha'_B)$ (by the previous step), there are two subcases to consider:

Step 16a: Suppose that $\Delta^*_B(\alpha''_B) = \Delta^*_B(\alpha'_B)$. Then, $\rho^*_B(\alpha''_B) > \rho^*_B(\alpha'_B)$ implies $\sigma^*_B(\alpha''_B) \geq \sigma^*_B(\alpha'_B)$ (because if $\sigma^*_B(\alpha''_B) < \sigma^*_B(\alpha'_B)$ then $\pi^*_a(\alpha''_B) < 0$ due to $\pi^*_a(\alpha'_B) = 0$, yielding a contradiction with $\rho^*_B(\alpha''_B) > 0$). However, combining $\sigma^*_B(\alpha''_B) \geq \sigma^*_B(\alpha'_B)$, $\rho^*_B(\alpha''_B) > \rho^*_B(\alpha'_B)$, and $\alpha''_B < \alpha'_B$, part (i) of Proposition 1 yields a contradiction with $\Delta^*_B(\alpha''_B) = \Delta^*_B(\alpha'_B)$.

Step 16b: Suppose that $\Delta^*_B(\alpha''_B) < \Delta^*_B(\alpha'_B)$. Then, $\rho^*_B(\alpha''_B) > \rho^*_B(\alpha'_B)$ implies $\sigma^*_B(\alpha''_B) > \sigma^*_B(\alpha'_B)$ (because if $\sigma^*_B(\alpha''_B) \leq \sigma^*_B(\alpha'_B)$ then $\pi^*_a(\alpha''_B) < 0$ due to $\pi^*_a(\alpha'_B) = 0$, yielding a contradiction with $\rho^*_B(\alpha''_B) > 0$). Then $\sigma^*_B(\alpha'_B) < 1$, and hence step 3 implies that $\Delta^*_B(\alpha'_B) = \Delta_0$. However, then $\Delta^*_B(\alpha''_B) < \Delta^*_B(\alpha'_B)$ implies $\Delta^*_B(\alpha''_B) < \Delta_0$, yielding a contradiction with steps 3 and 6 (since these steps imply that it must be $\Delta^*_B(\alpha''_B) \geq \Delta_0$).

Step 17: Next, I complete the proof of part (i.b). To that end, since the incumbent accepts the board settlement if and only if $\Delta \in (l, b_a) \cup (\Delta^*_B, b_i)$ (due to part (i) of Proposition 1), combining with steps 15 and 16 implies that conditional on the activist demanding a board settlement, the likelihood of a proxy fight decreases as α_B decreases. Therefore, to complete the proof of part (i.b), it is sufficient to show that $\lim_{\alpha_B \downarrow 0} \rho^*_B(\alpha_B) = 0$. Indeed, this result holds because as it was shown in step 13 above, for any $\delta > 0$, there exists $\alpha'_B > 0$ such that if $\alpha_B < \alpha'_B$, then $\rho^*_B(\alpha_B) < \delta$.

Step 18: In this step, I prove part (iii), which consists of the two substeps below. Note that I also show in step 18b below that $\tilde{C}_a < \bar{C}_a$.

Step 18a: Note that if the activist does not make any settlement demand, then his expected payoff is $\Pi^{\varnothing}_a = -C_a + n_a \alpha_P E\left[\max\left\{0, \Delta - b_a\right\}\right]$, where the shareholders always support the activist in a proxy fight due to step 2 above. Importantly, $\lim_{C_a \uparrow \bar{C}_a} \Pi^{\varnothing}_a(C_a) = 0$ due to (2).

Step 18b: To complete the proof of part (iii), there are two cases to consider: (I) First, suppose that $\sigma^*_B(C_a) < 1$ for all $C_a < \bar{C}_a$. Then, step 3 above implies that $\Delta^*_B(C_a) = \Delta_0$ for all $C_a < \bar{C}_a$, where $\Delta_0 < b_i$. Since the activist's payoff when the incumbent rejects a board settlement is nonnegative (because not running a proxy fight is always an option for the activist), this implies that $\Pi^B_a(C_a) \geq n_a \int_{\Delta_0}^{b_i} \alpha_B(\Delta - b_a) dF(\Delta)$ for all $C_a < \bar{C}_a$. In turn, since $\Delta_0 \in (b_a, b_i)$ again by step 3, this implies that $\Pi^B_a(C_a) > 0$ for all $C_a < \bar{C}_a$, and also $\lim_{C_a \uparrow \bar{C}_a} \Pi^B_a(C_a) > 0$. Combining with step 18a, this implies that there exists $\tilde{C}_a < \bar{C}_a$ such that $\Pi^B_a > \Pi^{\varnothing}_a$ if $C_a > \tilde{C}_a$, as desired. (II) Second, suppose that there exists $C'_a < \bar{C}_a$ such that

$\sigma_B^*(C_a') = 1$. Then, step 7a above implies that $\sigma_B^*(C_a) = 1$ for all $C_a \in (C_a', \bar{C}_a)$. Recall from step 2 of the proof of Proposition 3 that another way to express Δ_B^* is (19), where σ is replaced with σ_B^* in equilibrium. This implies that for all $C_a \in (C_a', \bar{C}_a)$, $\Delta_B^*(C_a) = b_i - \frac{C_i/n_i}{\alpha_B - \alpha_P}$ if $\alpha_B > \alpha_P + \frac{C_i/n_i}{b_i - \hat{\Delta}_B(1|C_a)}$ and $\Delta_B^*(C_a) = \hat{\Delta}_B(1|C_a)$ otherwise. In turn, since (1), (2), and $C_a > 0$ together imply that $\hat{\Delta}_B(1|C_a) \in (b_a, b_i)$ for all $C_a \in (C_a', \bar{C}_a)$ and also $\lim_{C_a \uparrow \bar{C}_a} \hat{\Delta}_B(1|C_a) \in (b_a, b_i)$ (where $\hat{\Delta}_B(1|C_a)$ is given by part (ii) of Proposition 1), this implies that $\Delta_B^*(C_a) \in (b_a, b_i)$ for all $C_a \in (C_a', \bar{C}_a)$, and also $\lim_{C_a \uparrow \bar{C}_a} \Delta_B^*(C_a) \in (b_a, b_i)$. Since the activist's payoff when the incumbent rejects a board settlement is nonnegative (because not running a proxy fight is always an option for the activist), this implies that $\Pi_a^B(C_a) \geq n_a \int_{\Delta_B^*(C_a)}^{b_i} \alpha_B(\Delta - b_a) dF(\Delta) > 0$ for all $C_a \in (C_a', \bar{C}_a)$, and also $\lim_{C_a \uparrow \bar{C}_a} \Pi_a^B(C_a) > 0$. Combining with step 18a, this implies that there exists $\tilde{C}_a < \bar{C}_a$ such that $\Pi_a^B > \Pi_a^\varnothing$ if $C_a > \tilde{C}_a$, as desired. ■

INTERNET APPENDIX

B Online appendix for action settlement (Section 4.1)

Recall that in Section 4.1, the subgame where the activist has demanded an action settlement was analyzed. This appendix provides additional discussion for this section, as well as formalization of the results that were not formalized in the main text. Below, I start with a more detailed discussion of Proposition 5, and then show and discuss the robustness of the implications for shareholder value.

B.1 Discussion of Proposition 5

If the activist demands an action settlement, then the incumbent follows a threshold strategy, as described by Proposition 5 and depicted in Figure 7. To see why, let us start by analyzing the equilibrium for a given σ. Note if the incumbent accepts the action settlement, then the incumbent's payoff is $n^i(\Delta - b^i)$, because the project is implemented with certainty. Also note that if the incumbent accepts the action settlement, the activist does not launch a proxy fight.[70] In contrast, if the activist rejects the action settlement, then the incumbent's expected payoff is $\rho[-C^i + \sigma\alpha_P n^i(\Delta - b^i)]$ whenever $\Delta \geq b^a$, because the activist runs a proxy fight with probability ρ and implements the project with probability $\rho\sigma\alpha_P$ (which is the probability that the activist runs and wins a proxy fight and acquires the decision authority on the board), where $\rho\sigma\alpha_P < 1$ always holds in equilibrium.[71] Moreover, if $\Delta < b^a$, the activist does not implement the project upon winning a proxy fight, and therefore the incumbent's incentive to reject the settlement is larger. Therefore, the incumbent's incentive to accept the settlement is strictly increasing for all Δ. As a result, for any ρ, there is a threshold $\Delta_A(\rho;\sigma)$ such that the incumbent accepts the action settlement if and only if $\Delta > \Delta_A(\rho;\sigma)$, where

$$\Delta_A(\rho;\sigma) \equiv b^i - \frac{C^i/n^i}{1/\rho - \sigma\alpha_P}, \tag{23}$$

which is higher than b^a as long as the activist's proxy fight threat ρ is not too high. Although Δ_A could become equal to or lower than b^a if ρ were to be very high, this never takes place in equilibrium (because otherwise, the activist would have no incentive to launch a proxy fight upon rejection). Indeed, note that Δ_A is decreasing in the probability ρ of a proxy fight upon rejection.

[70] An important reason is the standstill provisions, which are very common in settlements (see footnote 15 in Section 2 for some statistics). Another reason is that once the incumbent agrees to implement the activist's proposal, there is no reason left for the activist to run a proxy fight.

[71] Formally, this is because (10) implies that in equilibrium, if $\sigma_A^*\alpha_P = 1$, then $\rho_A^* < 1$. Intuitively, it is because if it were $\rho^*\sigma^*\alpha_P = 1$ in equilibrium, then the incumbent would accept the action settlement for all $\Delta \geq b^a$, however this would yield a contradiction as described in the discussion following (23).

In turn, again for a given σ, ρ^* is determined in a similar fashion to the case where the activist demands a board settlement. In particular, as Δ_A increases, the activist's incentives to run a proxy fight upon rejection increases. At the same time, Δ_A is decreasing in the activist's proxy fight threat ρ. Therefore, Δ_A and ρ together form a fixed point equation.

As a result, for any given σ, if σ is not too low (which always holds in equilibrium),[72] then this fixed point equation uniquely pins down ρ^* as $\rho^* = \rho_A(\sigma) > 0$, which is given by (10), and in turn, $\Delta_A(\sigma)$ is given by (9), where the only exception is that σ_A^* everywhere is replaced with σ. Moreover, due to the same intuition described in the Section 3.1, a decrease in σ results in an increase in $\Delta_A(\sigma)$ and hence increases $\alpha_P E\left[1_{\{\Delta \geq b^a\}} \cdot \Delta | \Delta \leq \Delta_A\right]$ (which is the expected payoff of the shareholders from electing the activist in a proxy fight). This yields a unique σ^* in equilibrium, which is denoted by σ_A^* and is positive.

B.2 Robustness of the implications for shareholder value

Section 3.2 formalized and discussed many implications of board settlements for shareholder value. As I formally show and elaborate below, all of these implications continue to be hold for the case where the activist demands an action settlement as well, due to the same intuition discussed in Section 3.2.

To begin with, a negative NPV project is never implemented after an action settlement is reached. This is because $\Delta_A^* > 0$ always holds in equilibrium, since shareholders would never support the activist in a proxy fight if it were $\Delta_A^* \leq 0$, which would eliminate the activist's incentives to run a proxy fight and thus the incumbent's incentives to accept the settlement. Therefore, for any negative NPV project that the activist is willing to implement (i.e., $\Delta \in [b^a, 0)$), in equilibrium the incumbent rejects the action settlement, and hence the activist implements the project with positive probability after running and winning a proxy fight.

As a result, a negative NPV project is implemented with positive probability after the activist runs and wins a proxy fight with the (endogenous) support of the shareholders. Indeed, if the activist were to never run or win a proxy fight,[73] then the incumbent would always reject the action settlement, making it profitable for shareholders to elect the activist and in turn making it profitable for the activist to run a proxy fight.[74]

[72] Formally, this condition corresponds to σ being larger a certain threshold $\underline{\sigma}_A \in (0,1)$. Indeed, as I show in the proof of Proposition 5, $\sigma_A^* > \underline{\sigma}_A$ always holds in equilibrium. This is because, similar to the intuition described in footnote 30 in Section 3.1, if $\sigma_A^* \leq \underline{\sigma}_A$ then the incumbent never accepts the settlement, and as a result, it becomes strictly profitable for the shareholders to elect the activist in a proxy fight, yielding a contradiction with $\sigma_A^* < 1$.

[73] Note that if the activist were to never win a proxy fight, he would have no incentive to launch a proxy fight.

[74] This is because if the incumbent were to always reject the settlement, then (3) implies that shareholders would be strictly better off by electing the activist. In turn, (2) implies that the activist would strictly prefer to run a proxy fight.

The results above are formalized in the proposition below.

Proposition 8 *Suppose that the activist has demanded an action settlement. Then, a negative NPV project is implemented only if $b_a < 0$. Specifically, if $b_a < 0$, then:*

(*i*) *The activist runs and wins a proxy fight with positive probability.*

(*ii*) *The probability that a negative NPV project is implemented is zero after a settlement is reached, but it is positive after the activist wins a proxy fight.*

Moreover, there is $\bar{b}_a^A < 0$ such that $\sigma_A^* < 1$ if and only if $b_a < \bar{b}_a^A$.

Next, in any activist campaign realized in equilibrium, shareholders would be better off by committing to supporting the activist in a proxy fight more, as shown in the proposition below. This is because such a commitment would increase the incumbent's incentives to accept the action settlement (i.e., reduce the rejection threshold from Δ_A^* to some $\Delta_A' < \Delta_A^*$), and as a result, it would yield a larger increase in the probability that the project is implemented when it has a high NPV (i.e., $\Delta \in (\Delta_A', \Delta_A^*]$) compared to lower NPV (i.e., $\Delta \in [b^a, \Delta_A')$).

However, whenever such a commitment changes the voting outcome (i.e., whenever $\sigma_A^* < 1$), it also results in π_A^s (the average ex-post shareholder return conditional on the activist's victory in the proxy fight) being negative, as also formalized by the proposition below. The reason is that $\sigma_A^* < 1$ implies π_A^s is already nonpositive without such a commitment, and this in turn implies that π_A^s becomes negative under such a commitment since Δ_A^* decreases.

Proposition 9 *Consider any campaign where the activist has demanded an action settlement. If the shareholders commit to any $\sigma_A > \sigma_A^*$, then compared to any $\sigma_A \leq \sigma_A^*$:*

(*i*) *The ex-ante shareholder value is strictly larger.*

(*ii*) *The expected ex-post shareholder value conditional on the activist winning a proxy fight is strictly smaller. More specifically, it is negative for any $\sigma_A > \sigma_A^*$, while it is nonnegative for any $\sigma_A \leq \sigma_A^*$.*

Finally, despite the results above, I show in part (i) of the proposition below that even if the activist is unbiased (i.e., $b^a = 0$) and he demands an action settlement, allowing for settlements can destroy ex-ante shareholder value, because it can reduce the overall probability that the project is implemented (note that this result is not limited to $b^a = 0$, and it applies for any bias of the activist). Specifically, this happens if the activist's cost of a proxy fight C_a is high, because then the activist often does not run a proxy fight upon the rejection of the action settlement (even though shareholders prefer that he always runs a proxy fight), as the incumbent's rejection signals to the activist that Δ is not high. In contrast, he always runs a proxy fight in the hypothetical scenario where settlements were not allowed.[75]

Moreover, as shown in parts (ii) and (iii) below, if settlements are allowed, and C_a is high or $b_a = 0$, then the activist strictly prefers demanding an action settlement over making no settlement demand, due to saving the cost of a proxy fight.

[75] See footnote 47 in Section 3.2.2.

Proposition 10 *Denote the ex-ante shareholder value (the ex-ante payoff of the activist) by* Π_s^A *(*Π_a^A*) if the activist has demanded a board settlement, and by* $\Pi_s^\varnothing$ *(*$\Pi_a^\varnothing$*) if settlements are not allowed. Recall that* $\bar{C}_a$ *is given by (2).*

(i) There exists $\hat{C}_a^A$ *such that* $\Pi_s^A < \Pi_s^\varnothing$ *if and only if* $C_a > \hat{C}_a^A$*, where* $\hat{C}_a^A < \bar{C}_a$*.*

(ii) There exists $\tilde{C}_a^A$ *such that* $\Pi_a^A > \Pi_a^\varnothing$ *if* $C_a > \tilde{C}_a^A$*, where* $\tilde{C}_a^A < \bar{C}_a$*.*

(iii) If $b_a = 0$*, then* $\Pi_a^A > \Pi_a^\varnothing$ *for any* $C_a < \bar{C}_a$*.*

B.3 Proofs for action settlement (Section 4.1)

Proof of Proposition 5. I first solve for the equilibrium for any given $\sigma \in [0,1]$, which is a major part of the proof. Then, I endogenize σ.

Step 1: First, for any given $\sigma \in [0,1]$, I characterize the incumbent's strategy at the response stage. Note that the incumbent's payoff from accepting the action settlement is $n_i(\Delta - b_i)$ for all Δ. On the other hand, if the incumbent rejects, then for any given ρ, its payoff is $\rho\left[-C_i + \sigma\alpha_P n_i(\Delta - b_i)\right]$ if $\Delta \geq b_a$, and $-\rho C_i$ otherwise. This is because if the incumbent rejects the settlement, the activist runs a proxy fight with probability ρ (which the activist wins with probability σ), and upon winning a proxy fight, the activist implements the project only if $\Delta \geq b_a$ (in which case he implements it with probability α_P).

As a result, the incumbent accepts the settlement if and only if the NPV Δ is larger than a certain threshold. In particular, if $\rho < \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, then the incumbent accepts the action settlement if and only if $\Delta \geq b_a$ and $n_i(\Delta - b_i) > \rho\left[-C_i + \sigma\alpha_P n_i(\Delta - b_i)\right]$. Note that the latter inequality is equivalent to $\Delta > \Delta_A(\rho;\sigma)$, where $\Delta_A(\rho;\sigma)$ is given by (23) and $\Delta_A(\rho;\sigma) > b_a$ is satisfied if $\rho < \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$. Even though the incumbent's threshold becomes lower and satisfies $\Delta_A(\rho;\sigma) \leq b_a$ if ρ were so high that $\rho \geq \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, this never happens in equilibrium, because if it did then the activist would never run a proxy fight upon rejection (since he would infer with certainty that $\Delta \leq b_a$ upon rejection), contradicting with $\rho \geq \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$. Indeed, it is verified below that $\Delta_A(\rho^*;\sigma) > b_a$ always holds in equilibrium.

Related, also note that the incumbent rejects the action settlement for some $\Delta \in (l, b_i)$. To see this, recall from above that when $\Delta \in (l, b_a)$, the incumbent's payoff is $n_i(\Delta - b_i)$ if it accepts the action settlement, and it is $-\rho C_i$ if it rejects. Therefore, a necessary condition for the incumbent to accept the action settlement for all Δ is $l - b_i \geq -C_i/n_i$, which contradicts with (1).

Step 2: Second, note that in any equilibrium for any given $\sigma \in [0,1]$, due to arguments made above, upon rejection of the action settlement the activist infers that the project return Δ is smaller than the incumbent's threshold $\Delta_A(\rho^*;\sigma)$, where $\rho^*(\sigma)$ denotes the probability that the activist runs a proxy fight upon rejection of action settlement. Therefore, the activist's expected payoff from running a proxy fight upon rejection is given by π_a, where

$$\pi_a = \sigma\alpha_P n_a E\left[\max\{0, \Delta - b_a\} \,|\Delta \leq \Delta_A(\rho^*;\sigma)\right] - C_a.$$

Step 3: Third, I show that if $\sigma \leq \underline{\sigma}_A$ where

$$\underline{\sigma}_A \equiv C_a/\{\alpha_P n_a E\left[\max\left\{0, \Delta - b_a\right\}\right]\}, \tag{24}$$

then the equilibrium exists, it is unique, and in equilibrium, the incumbent always rejects the settlement and the activist never runs a proxy fight. Note that to show this, it is sufficient to prove that if $\sigma \leq \underline{\sigma}_A$, then $\rho^*(\sigma) = 0$ (i.e., the activist never runs a proxy fight upon rejection). This is because then, due to the arguments made in the first step above, the incumbent always rejects the settlement. The proof of this step consists of two substeps: First, note that $\rho^*(\sigma) = 0$ indeed constitutes an equilibrium given σ, because $\Delta_A(0;\sigma) = b_i$ and hence $\pi_a \leq 0$. Second, it cannot be that $\rho^*(\sigma) > 0$. This is because if $\rho^* > 0$, then $\Delta_A(\rho^*;\sigma) < b_i$, and hence $\pi_a < 0$, which yields a contradiction with $\rho^* > 0$.

Step 4: Fourth, I show that for any given $\sigma > \underline{\sigma}_A$ (note that $\underline{\sigma}_A < 1$ due to (2)), the equilibrium exists, it is unique, and it is given by parts (i) and (ii) of the proposition, where σ_A^* is simply replaced with σ. Note that to show this result, it is sufficient to prove that for any given $\sigma > \underline{\sigma}_A$, an equilibrium exists, and in equilibrium, ρ^* is uniquely given by

$$\rho_A(\sigma) \equiv \min\{1, 1/(\sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_A(\sigma))])\} \tag{25}$$

and it satisfies $\rho^* \in (0, \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]})$ (because then, due to the arguments made in the first step above, the incumbent's equilibrium rejection threshold is simply equal to $\Delta_A(\rho^*;\sigma)$, and $\Delta_A(\rho^*;\sigma) \in (b_a, b_i)$). Note that ρ_A, which is given by (25), is equivalent to

$$\rho_A(\sigma) = \begin{cases} 1, & \text{if } \hat{\Delta}_A \leq b_i - \frac{C_i/n_i}{1-\sigma\alpha_P} \text{ and } \sigma\alpha_P < 1, \\ \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_A)]} \in (0,1), & \text{otherwise,} \end{cases} \tag{26}$$

where $\hat{\Delta}_A(\sigma)$ is given by (11) and satisfies $\hat{\Delta}_A(\sigma) \in (b_a, b_i)$ due to $\sigma > \underline{\sigma}_A$ and $C_a > 0$. Recall by the second step above that the activist's expected payoff from running a proxy fight upon rejection is given by π_a. Importantly, this payoff π_a is strictly positive if $\Delta_A(\rho^*;\sigma) > \hat{\Delta}_A$, it is equal to zero if $\Delta_A(\rho^*;\sigma) = \hat{\Delta}_A$, and it is negative if $\Delta_A(\rho^*;\sigma) < \hat{\Delta}_A$. With this observation in mind, there are three cases to consider in order to show that ρ^* is equal to (25) and satisfies $\rho^* \in (0, \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]})$.

First, suppose that $\hat{\Delta}_A \leq b_i - \frac{C_i/n_i}{1-\sigma\alpha_P}$ and $\sigma\alpha_P < 1$. Then, $\hat{\Delta}_A \leq \Delta_A(\rho;\sigma)$ if $\rho = 1$, and $\hat{\Delta}_A < \Delta_A(\rho;\sigma)$ for all $\rho \in [0,1)$. Importantly, then it has to be $\rho^* = 1$. Indeed, if $\rho^* = 1$, then $\hat{\Delta}_A \leq \Delta_A(\rho^*;\sigma)$, and therefore $\pi_a \geq 0$ (i.e., the activist's expected payoff from running a proxy fight upon rejection is nonnegative). On the other hand, if any $\rho^* < 1$ were to be an equilibrium, then it would imply that $\hat{\Delta}_A < \Delta_A(\rho^*;\sigma)$. However, then $\pi_a > 0$, which would yield a contradiction with $\rho^* < 1$. These arguments confirm that $\rho^* = 1$. Note that $\rho^* = 1$ satisfies $\rho^* < \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$, because $\hat{\Delta}_A \leq b_i - \frac{C_i/n_i}{1-\sigma\alpha_P}$ implies that $1 \leq \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_A)]}$, where $\hat{\Delta}_A$ satisfies $\hat{\Delta}_A > b_a$. Also note that $\Delta_A(\rho^*;\sigma) > b_a$, because $\Delta_A(\rho^*;\sigma) = b_i - \frac{C_i/n_i}{1-\sigma\alpha_P} \geq \hat{\Delta}_A > b_a$.

Second, suppose that $\sigma\alpha_P = 1$. Then, it cannot be $\rho^* = 1$ in equilibrium. This is because if $\rho^* = 1$, then the incumbent would accept action settlement for all $\Delta > b_a$ (since $n_i(\Delta - b_i) > [-\rho^* C_i + \rho^* \sigma \alpha_P n_i(\Delta - b_i)]$ for all $\Delta > b_a$), and hence the activist would never run a proxy fight upon rejection, yielding a contradiction (recall from the first step that the incumbent rejects the action settlement for some $\Delta \in (l, b_i)$). Therefore, it has to be that $\rho^* < 1$. Moreover, it must be $\Delta_A(\rho^*; \sigma) = \hat{\Delta}_A$. This is because if $\Delta_A(\rho^*; \sigma) > \hat{\Delta}_A$, then $\pi_a > 0$, which implies that $\rho^* = 1$, yielding a contradiction. On the other hand, if $\Delta_A(\rho^*; \sigma) < \hat{\Delta}_A$, then $\pi_a < 0$, and hence $\rho^* = 0$. However, this would imply that $\Delta_A(\rho^*; \sigma) = b_i$, yielding a contradiction with $\Delta_A(\rho^*; \sigma) < \hat{\Delta}_A < b_i$. Therefore, it must be that $\Delta_A(\rho^*; \sigma) = \hat{\Delta}_A$ (which makes the activist indifferent between running and not running a proxy fight upon rejection). In turn, due to (23), this equality uniquely pins down ρ^*, which is given by $\rho^*(\sigma) = \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_A)]}$. Note that this implies $\rho^* < \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$ and $\rho^* \in (0, 1)$ since $\sigma\alpha_P = 1$ and $\hat{\Delta}_A \in (b_a, b_i)$.

Third, suppose that $b_i - \frac{C_i/n_i}{1-\sigma\alpha_P} < \hat{\Delta}_A$ and $\sigma\alpha_P < 1$. Then, the proof is almost identical to the previous case. In particular, it must be that $\Delta_A(\rho^*; \sigma) = \hat{\Delta}_A$. This is because if $\Delta_A(\rho^*; \sigma) > \hat{\Delta}_A$, then $\pi_a > 0$, which implies that $\rho^* = 1$. However, this would imply that $\Delta_A(\rho^*; \sigma) = b_i - \frac{C_i/n_i}{1-\sigma\alpha_P}$ and thus $\Delta_A(\rho^*; \sigma) < \hat{\Delta}_A$, yielding a contradiction. On the other hand, if $\Delta_A(\rho^*; \sigma) < \hat{\Delta}_A$, then $\pi_a < 0$, and hence $\rho^* = 0$. However, this would imply that $\Delta_A(\rho^*; \sigma) = b_i$, yielding a contradiction with $\Delta_A(\rho^*; \sigma) < \hat{\Delta}_A$. Therefore, it must be that $\Delta_A(\rho^*; \sigma) = \hat{\Delta}_A$. In turn, this equality uniquely pins down ρ^*, which is again given by $\rho^*(\sigma) = \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - \hat{\Delta}_A)]}$. Note that this implies $\rho^* < \frac{1}{\sigma\alpha_P + C_i/[n_i(b_i - b_a)]}$ (since $\hat{\Delta}_A \in (b_a, b_i)$) and also implies $\rho^* \in (0, 1)$ (since $b_i - \frac{C_i/n_i}{1-\sigma\alpha_P} < \hat{\Delta}_A$ and $\hat{\Delta}_A < b_i$). This completes the characterization of the equilibrium for any given $\sigma > \underline{\sigma}_A$.

Step 5: Fifth, note that for any given $\sigma > \underline{\sigma}_A$, in equilibrium the incumbent's rejection threshold can be also expressed as

$$\Delta_A(\rho_A(\sigma); \sigma) = \max\left\{\hat{\Delta}_A(\sigma), b_i - \frac{C_i/n_i}{1 - \sigma\alpha_P}\right\}. \tag{27}$$

Indeed, this follows from plugging (26) in (23).

Step 6: Finally, I endogenize σ, and I show that there exists a unique σ_A^* that yields an equilibrium. Note that by the third step above, if $\sigma \leq \underline{\sigma}_A$ then the incumbent always rejects the settlement. In turn, if the activist runs a proxy fight upon rejection, then the shareholders deviate to always supporting the activist, because the expected shareholder value is $\alpha_P E\left[1_{\{\Delta \geq b_a\}} \cdot \Delta\right]$ if the activist wins a proxy fight, which is positive due to (3). However, this yields a contradiction with $\sigma < 1$, which is implied by $\sigma \leq \underline{\sigma}_A$ due to (2).

Therefore, it is sufficient to focus attention on $\sigma > \underline{\sigma}_A$. Then, due to the fourth step above, for any given $\sigma > \underline{\sigma}_A$, if the activist wins a proxy fight upon incumbent's rejection then the expected shareholder value is $\alpha_P E\left[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A(\rho_A(\sigma); \sigma)\right]$, which is strictly decreasing in σ (as it can be seen from the fifth step above). Since the shareholders' expected payoff is zero

if the activist loses a proxy fight upon rejection, to show that there exists a unique equilibrium σ_A^*, it is sufficient to prove $\lim_{\sigma \downarrow \underline{\sigma}_A} \alpha_P E\left[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A(\rho_A(\sigma);\sigma)\right] > 0$. Indeed, this always holds, because $\lim_{\sigma \downarrow \underline{\sigma}_A} \hat{\Delta}_A(\sigma) = b_i$ by the definitions of $\underline{\sigma}_A$ and $\hat{\Delta}_A$, which in turn implies that $\lim_{\sigma \downarrow \underline{\sigma}_A} \rho_A(\sigma) = 0$ (where ρ_A is given by (25)). Therefore, $\lim_{\sigma \downarrow \underline{\sigma}_A} \Delta_A(\rho_A(\sigma);\sigma) = b_i$, which implies $\lim_{\sigma \downarrow \underline{\sigma}_A} \alpha_P E\left[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A(\rho_A(\sigma);\sigma)\right] > 0$ as desired, due to (3). Also note that the equilibrium Δ_A^* has to satisfy $\Delta_A^* > 0$, because otherwise $\alpha_P E\left[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A^*\right] < 0$ (since $\Delta_A^* > b_a$) and hence the shareholders would deviate to always supporting the incumbent in a proxy fight (i.e., deviate to $\sigma = 0$). ■

Proof of Proposition 8. Note that except for the last statement, all of the statements in the proposition follows directly from combining Lemma 1 with Proposition 5. Note that $\Delta_A^* > 0$, because otherwise Proposition 5 implies that it must be that $b_a < \Delta_A^* \leq 0$, which would imply a contradiction with $\sigma_A^* > 0$ (since the shareholders would be strictly worse off by electing the activist in a proxy fight, compared to re-electing the incumbent). Therefore, it remains to show that there exists $\bar{b}_a^A < 0$ such that $\sigma_A^* < 1$ if and only if $b_a < \bar{b}_a^A$. To show this result, I actually prove a stronger result: I show that that σ_A^* weakly decreases as b_a decreases.

Suppose that there exists $b_a'' < b_a'$ such that $\sigma_A^*(b_a'') > \sigma_A^*(b_a')$. Note that the expected shareholder value conditional on the activist winning a proxy fight after the incumbent's rejection of an action settlement is given by $\pi_s^A = \alpha_P E\left[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A^*\right]$, where $\Delta_A^* > b_a$ by (9). Since the shareholder value is zero if the incumbent is re-elected, and since $\sigma_A^* > 0$ by Proposition 5, $0 < \sigma_A^*(b_a') < \sigma_A^*(b_a'') \leq 1$ implies $\pi_s^A(b_a') = 0 \leq \pi_s^A(b_a'')$, which in turn implies $b_a < \Delta_A^*(b_a') \leq \Delta_A^*(b_a'')$ (since $\Delta_A^* > b_a$ as mentioned above). Note that the activist's expected payoff from running a proxy fight upon the incumbent's rejection of an action settlement is given by $\pi_a^A = \sigma_A^* \alpha_P n_a E\left[\max\left\{0, \Delta - b_a\right\} | \Delta \leq \Delta_A^*\right] - C_a$. Then, $b_a < \Delta_A^*(b_a') \leq \Delta_A^*(b_a'')$ and $\sigma_A^*(b_a') < \sigma_A^*(b_a'')$ imply that $\pi_a^A(b_a'') > \pi_a^A(b_a')$, resulting in $\rho_A^*(b_a'') \geq \rho_A^*(b_a')$. However, combining $\rho_A^*(b_a'') \geq \rho_A^*(b_a')$ with $\sigma_A^*(b_a'') > \sigma_A^*(b_a')$, (9) implies $\Delta_A^*(b_a'') < \Delta_A^*(b_a')$, yielding a contradiction. ■

Proof of Proposition 9. Below, steps 6-7 prove part (i), and steps 8-9 prove part (ii). The rest of the steps are preliminary steps.

Step 1: First, given that the activist has demanded an action settlement, for any given $\sigma \in [0,1]$, I derive the expected shareholder value $\Pi_s^A(\sigma)$. By the fourth step in the proof of Proposition 5, for any $\sigma > \underline{\sigma}_A$ (where $\underline{\sigma}_A$ is given by (24) and $\underline{\sigma}_A < 1$ due to (2)), the equilibrium exists, it is unique, and it is given by parts (i) and (ii) of Proposition 5, where σ_A^* is simply replaced with σ. Therefore, for any $\sigma > \underline{\sigma}_A$,

$$\Pi_s^A(\sigma) = \rho_A(\sigma)\sigma \int_{b_a}^{\Delta_A(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A(\sigma)}^{b_i} \Delta dF(\Delta), \tag{28}$$

where $\Delta_A(\sigma)$ and $\rho_A(\sigma)$ are given by (9) and (10), respectively, and where σ_A^* is again replaced with σ. Note that $\Delta_A(\sigma) \in (b_a, b_i)$ by the fourth step of the proof of Proposition 5. Also note that in equilibrium, $\sigma_A^* > \underline{\sigma}_A$ by the sixth step in the proof of Proposition 5.

In contrast, if $\sigma \leq \underline{\sigma}_A$, then by the third step in the proof of Proposition 5, the equilibrium

exists, it is unique, and in equilibrium, the incumbent always rejects the settlement and the activist never runs a proxy fight. Therefore, for any $\sigma \leq \underline{\sigma}_A$, $\Pi_s^A(\sigma) = 0$.

Step 2: I show that $\Delta_A(\sigma)$ is strictly decreasing in σ if $\sigma > \underline{\sigma}_A$. Indeed, this follows from (27), which is the alternative expression for $\Delta_A(\sigma)$ that was derived in the fifth step of the proof of Proposition 5 (note that 11 implies that $\hat{\Delta}_A(\sigma)$ is strictly decreasing in σ for all $\sigma > \underline{\sigma}_A$, due to (1), (2), and (24)).

Step 3: For any two values σ_2, σ_1 of σ such that $\sigma_2 > \sigma_1 > \underline{\sigma}_A$, I show that it must be $\rho_A(\sigma_2) \cdot \sigma_2 > \rho_A(\sigma_1) \cdot \sigma_1$. Suppose that there exist $\sigma_2 > \sigma_1 > \underline{\sigma}_A$ such that $\rho_A(\sigma_2) \cdot \sigma_2 \leq \rho_A(\sigma_1) \cdot \sigma_1$. Then, it must be that $\rho_A(\sigma_2) < \rho_A(\sigma_1)$. In turn, since the fourth step of the proof of Proposition 5 implies that $\Delta_A(\sigma) = b_i - \frac{\rho_A C_i/n_i}{1-\rho_A \sigma \alpha_P} \in (b_a, b_i)$, it follows that $\rho_A(\sigma_2) \cdot \sigma_2 \leq \rho_A(\sigma_1) \cdot \sigma_1$ and $\rho_A(\sigma_2) < \rho_A(\sigma_1)$ together imply that $\Delta_A(\sigma_2) > \Delta_A(\sigma_1)$, yielding a contradiction with step 2 above.

Step 4: Note that for $\sigma = \sigma_A^*$, $\Pi_s^A(\sigma_A^*) > 0$ and the first term in (28) is nonnegative. To see this, note that if the first term in (28) were to be negative, then the shareholders would deviate to always supporting the incumbent in a proxy fight, yielding a contradiction with $\sigma_A^* > 0$ (which was shown in Proposition 5). In turn, since the first term in (28) is nonnegative, this implies that $\Pi_s^A(\sigma_A^*) > 0$, because $\Delta_A(\sigma_A^*) < b_i$ by Proposition 5.

Step 5: I show that for any $\sigma > \underline{\sigma}_A$, it must be $\rho_A(\sigma)\sigma\alpha_P < 1$. Suppose that there exists $\sigma > \underline{\sigma}_A$ such that $\rho_A(\sigma)\sigma\alpha_P = 1$. However, then $\rho_A(\sigma) = \sigma = \alpha_P = 1$, which yields a contradiction with (26) in the proof of Proposition 5.

Step 6: I show that for any $\sigma > \sigma_A^*$, it must be $\Pi_s^A(\sigma) > \Pi_s^A(\sigma_A^*)$. Recall that $\sigma_A^* > \underline{\sigma}_A$ (as noted in step 1). Therefore, for any $\sigma > \sigma_A^*$,

$$\begin{aligned}
\Pi_s^A(\sigma) &= \rho_A(\sigma)\sigma \textstyle\int_{b_a}^{\Delta_A(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A(\sigma)}^{\Delta_A(\sigma_A^*)} \Delta dF(\Delta) + \int_{\Delta_A(\sigma_A^*)}^{b_i} \Delta dF(\Delta) \\
&> \rho_A(\sigma)\sigma \textstyle\int_{b_a}^{\Delta_A(\sigma)} \alpha_P \Delta dF(\Delta) + \rho_A(\sigma)\sigma \int_{\Delta_A(\sigma)}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A(\sigma_A^*)}^{b_i} \Delta dF(\Delta) \\
&\geq \rho_A(\sigma_A^*)\sigma_A^* \textstyle\int_{b_a}^{\Delta_A(\sigma)} \alpha_P \Delta dF(\Delta) + \rho_A(\sigma_A^*)\sigma_A^* \int_{\Delta_A(\sigma)}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A(\sigma_A^*)}^{b_i} \Delta dF(\Delta) \\
&= \Pi_s^A(\sigma_A^*)
\end{aligned}$$

where the first and last equalities follow from (28); the first inequality follows from combining $\int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) \geq 0$ (by step 4), $\rho_A(\sigma)\sigma\alpha_P < 1$ (by step 5), $\Delta_A(\sigma) < \Delta_A(\sigma_A^*)$ (by step 2), and $b_a < \Delta_A(\sigma)$ (by step 1); and the second inequality follows from combining $\int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) \geq 0$ and $\rho_A(\sigma)\sigma > \rho_A(\sigma_A^*)\sigma_A^*$ (by step 3).

Step 7: I show that for any $\sigma < \sigma_A^*$, it must be $\Pi_s^A(\sigma_A^*) > \Pi_s^A(\sigma)$. Again, recall that $\sigma_A^* > \underline{\sigma}_A$. There are two cases to consider. First, suppose that $\sigma \leq \underline{\sigma}_A$. Then, $\Pi_s^A(\sigma_A^*) > 0 = \Pi_s^A(\sigma)$, where the inequality follows from step 4, and $\Pi_s^A(\sigma) = 0$ follows from step 1. Second, suppose

that $\sigma > \underline{\sigma}_A$. Then,

$$\begin{aligned}\Pi_s^A(\sigma_A^*) &= \rho_A(\sigma_A^*)\sigma_A^* \int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A(\sigma_A^*)}^{\Delta_A(\sigma)} \Delta dF(\Delta) + \int_{\Delta_A(\sigma)}^{b_i} \Delta dF(\Delta) \\ &> \rho_A(\sigma_A^*)\sigma_A^* \int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) + \rho_A(\sigma_A^*)\sigma_A^* \int_{\Delta_A(\sigma_A^*)}^{\Delta_A(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A(\sigma)}^{b_i} \Delta dF(\Delta) \\ &> \rho_A(\sigma)\sigma \int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) + \rho_A(\sigma)\sigma \int_{\Delta_A(\sigma_A^*)}^{\Delta_A(\sigma)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A(\sigma)}^{b_i} \Delta dF(\Delta) \\ &= \Pi_s^A(\sigma)\end{aligned}$$

where the first and last equalities follow from (28); the first inequality follows from combining $\int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) \geq 0$ (by step 4), $\rho_A(\sigma_A^*)\sigma_A^*\alpha_P < 1$ (by step 5), and $\Delta_A(\sigma_A^*) < \Delta_A(\sigma)$ (by step 2); and the second inequality follows from combining $\int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) \geq 0$, $\Delta_A(\sigma_A^*) < \Delta_A(\sigma)$, and $\rho_A(\sigma_A^*)\sigma_A^* > \rho_A(\sigma)\sigma$ (by step 3).

Step 8: Denoting by π_s^A the expected ex-post shareholder value conditional on the activist winning a proxy fight upon rejection of an action settlement, I show that $\pi_s^A(\sigma) \geq 0$ for all $\sigma \leq \sigma_A^*$. Since $\sigma_A^* > \underline{\sigma}_A$ by step 1, there are two cases to consider: First, suppose that $\sigma \leq \underline{\sigma}_A$. Then, the incumbent always rejects the settlement by step 1, and hence $\pi_s^A(\sigma) = \alpha_P E[1_{\{\Delta \geq b_a\}} \cdot \Delta]$, which is positive due to (3). Second, suppose that $\underline{\sigma}_A < \sigma \leq \sigma_A^*$. Then, by step 1, $\pi_s^A(\sigma) = \alpha_P E[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A(\sigma)]$ and $\Delta_A(\sigma) > b_a$. Importantly, step 4 implies $\int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) \geq 0$, because $\sigma_A^* > 0$, $\rho_A(\sigma_A^*) > 0$, and $\Delta_A(\sigma_A^*) > b_a$ due to Proposition 5. Combining $\int_{b_a}^{\Delta_A(\sigma_A^*)} \alpha_P \Delta dF(\Delta) \geq 0$ and $\Delta_A(\sigma_A^*) > b_a$ with step 2 implies $\pi_s^A(\sigma) \geq 0$ for all $\underline{\sigma}_A < \sigma \leq \sigma_A^*$.

Step 9: I show that $\pi_s^A(\sigma) < 0$ for all $\sigma > \sigma_A^*$, where π_s^A is defined in the previous step. Note that for there to be any $\sigma > \sigma_A^*$, it must be that $\sigma_A^* < 1$. Combining this with part (iii) of Proposition 5 implies $\sigma_A^* \in (0, 1)$. In other words, in a proxy fight upon rejection of an action settlement, the shareholders are indifferent between electing the activist and re-electing the incumbent. Since the shareholder value is zero if the incumbent is re-elected, it must be that $\pi_s^A(\sigma_A^*) = 0$. Combining this with $\pi_s^A(\sigma) = \alpha_P E[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A(\sigma)]$ (which again follows from step 1 due to $\sigma > \sigma_A^* > \underline{\sigma}_A$, where $\Delta_A(\sigma) > b_a$) and step 2 implies $\pi_s^A(\sigma) < 0$ for all $\sigma > \sigma_A^*$. ■

Proof of Proposition 10. The proof consists of several steps. Among these steps, steps 8-13 prove part (i), step 14 proves part (ii), and step 15 proves part (iii). The rest of the steps are preliminary steps. The steps are as follows.

Step 1: First, note that if the activist demands an action settlement, the equilibrium is given by Proposition 5, and hence Π_s^A is given by

$$\Pi_s^A(C_a) = \sigma_A^*(C_a)\rho_A^*(C_a) \int_{b_a}^{\Delta_A^*(C_a)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*(C_a)}^{b_i} \Delta dF(\Delta), \tag{29}$$

Step 2: Second, note that by the second step in the proof of Proposition 4, $\Pi_s^{\varnothing}$ is given by (21).

Step 3: Note that if $\sigma_A^* < 1$, this implies that when the activist runs a proxy fight upon

rejection of an action settlement, shareholders are indifferent between supporting the activist and supporting the incumbent, because also $\sigma_A^* > 0$ by Proposition 5. Therefore, the expected shareholder value if the activist runs and wins a proxy fight upon rejection of an action settlement must be zero. As a result, it also follows that Δ_A^* satisfies the equation $E[\Delta|\Delta \in [b_a, \Delta_A^*]] = 0$. Note that since $\Delta_A^* > b_a$ by part (i) of Proposition 5, (3) implies that there exists a unique $\Delta_0 \in (b_a, b_i)$ such that $\Delta_A^* = \Delta_0$. Also note that $b_a < 0$ due to $\sigma_A^* < 1$, because if $b_a \geq 0$ then in any equilibrium upon the rejection of an action settlement, the shareholders strictly prefer that the activist is elected in a proxy fight, resulting in $\sigma_A^* = 1$. Finally, also note that $\Delta_0 > 0$.

Step 4: I show that if $\sigma_A^* < 1$ for any $C_a = C_a'$, then $\sigma_A^* < 1$ for any $C_a = C_a'' < C_a'$. Note that for any C_a, the activist's expected payoff from running a proxy fight upon rejection of an action settlement is given by $\pi_a^*(C_a) = n_a \sigma_A^* \alpha_P E\left[\max\{0, \Delta - b_a\} | \Delta \leq \Delta_A^*\right] - C_a$. Moreover, for any C_a, $\rho_A^* > 0$ due to Proposition 5, and hence it must be $\pi_a^*(C_a) \geq 0$. Suppose that there exists $C_a'' < C_a'$ such that $\sigma_A^*(C_a') < 1$ and $\sigma_A^*(C_a'') = 1$. Then, at $C_a = C_a''$, the expected shareholder value if the activist runs and wins a proxy fight upon rejection of an action settlement must be nonnegative, i.e., $E[\Delta|\Delta \in [b_a, \Delta_A^*(C_a'')]] \geq 0$. When combined with $\sigma_A^*(C_a') < 1$ and step 3 above, this implies that $\Delta_A^*(C_a'') \geq \Delta_A^*(C_a')$. Then, due to $\pi_a^*(C_a') \geq 0$ and $\sigma_A^*(C_a') < \sigma_A^*(C_a'') = 1$, this implies that $\pi_a^*(C_a'') > 0$ and hence $\rho_A^*(C_a'') = 1 \geq \rho_A^*(C_a')$. However, since $\sigma_A^*(C_a') < \sigma_A^*(C_a'')$, due to Proposition 5, this implies that $\Delta_A^*(C_a'') < \Delta_A^*(C_a')$, yielding a contradiction.

Step 5: Note that if $\sigma_A^* < 1$, then $\Pi_s^A = \int_{\Delta_0}^{b_i} \Delta dF(\Delta)$, where Δ_0 is as defined in step 3. Note that this expression for Π_s^A follows from the definition of Π_s^A in step 1, and the result in step 3 that Δ_A^* satisfies the equation $E[\Delta|\Delta \in [b_a, \Delta_A^*]] = 0$.

Step 6: Note that if $\sigma_A^* = 1$, then it must be that $\Delta_A^* \geq \Delta_0$, because otherwise, the definition of Δ_0 in step 3 implies that in a proxy fight, the shareholders always support the incumbent, yielding a contradiction with $\sigma_A^* = 1$.

Step 7: Note that if $\sigma_A^* = 1$ for some $C_a = C_a'$, then for all $C_a > C_a'$: (a) Step 4 implies that $\sigma_A^* = 1$, and hence (b) (29) implies that $\Pi_s^A(C_a)$ is continuous in C_a, because given that $\sigma_A^* = 1$, $\Delta_A^*(C_a)$ and $\rho_A^*(C_a)$ given by (9) and (10) are continuous in C_a.

Step 8: In this step, I start the proof of part (i). In particular, in this step, I show that if $\sigma_A^*(C_a') < 1$ for some $C_a = C_a'$, then $\Pi_s^A(C_a) \leq \Pi_s^A(C_a')$ for all $C_a > C_a'$. Since $\Pi_s^A(C_a') = \int_{\Delta_0}^{b_i} \Delta dF(\Delta)$ due to step 5, and in addition $\Pi_s^A(C_a) = \int_{\Delta_0}^{b_i} \Delta dF(\Delta)$ if $\sigma_A^*(C_a) < 1$ (again due to step 5), it is sufficient to show this result for C_a that satisfies $\sigma_A^*(C_a) = 1$. Indeed, for any such C_a, (29) implies

$$\begin{aligned} \Pi_s^A(C_a) &= \rho_A^*(C_a) \int_{b_a}^{\Delta_A^*(C_a)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*(C_a)}^{b_i} \Delta dF(\Delta) \\ &= \rho_A^*(C_a) \int_{\Delta_0}^{\Delta_A^*(C_a)} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*(C_a)}^{b_i} \Delta dF(\Delta) \qquad (30) \\ &\leq \int_{\Delta_0}^{b_i} \Delta dF(\Delta) = \Pi_s^A(C_a'), \end{aligned}$$

where the second equality follows from the definition of Δ_0 in step 3 and $\Delta_A^*(C_a) \geq \Delta_0$ by

step 6; the inequality follows from $\Delta_0 > 0$ in step 3 and $\rho_A^*(C_a)\alpha_P \leq 1$; and the last equality follows from step 5.

Step 9: I show that if $\sigma_A^*(C_a') = 1$ for some $C_a = C_a'$, then $\Pi_s^A(C_a'') \leq \Pi_s^A(C_a')$ for all $C_a'' > C_a'$. This result follows because $\sigma_A^*(C_a') = \sigma_A^*(C_a'') = 1$ (by step 4), and hence

$$\begin{aligned}\Pi_s^A(C_a'') &= \rho_A^*(C_a'') \int_{\Delta_0}^{\Delta_A^*(C_a'')} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*(C_a'')}^{b_i} \Delta dF(\Delta) \\ &\leq \rho_A^*(C_a') \int_{\Delta_0}^{\Delta_A^*(C_a')} \alpha_P \Delta dF(\Delta) + \int_{\Delta_A^*(C_a')}^{b_i} \Delta dF(\Delta) = \Pi_s^A(C_a'),\end{aligned}$$

where the first and last equalities follow from (30) in step 8, and the inequality follows because whenever $\sigma_A^*(C_a) = 1$: (a) $\rho_A^*(C_a)$ is weakly decreasing in C_a due to (10), (b) $\Delta_A^*(C_a)$ is weakly increasing in C_a due to (9), (c) $\rho_A^*(C_a)\alpha_P \leq 1$, and (d) $\Delta_0 > 0$ by step 3.

Step 10: I show that $\Pi_s^A(C_a)$ is continuous for all C_a. Note that if $\sigma_A^*(C_a) < 1$ for all C_a, this holds due to step 5, and if $\sigma_A^*(C_a) = 1$ for all C_a, then it holds due to step 7b. Therefore, it remains to prove this result for the case where $\sigma_A^*(C_a) < 1$ for some C_a and $\sigma_A^*(C_a) = 1$ for some C_a. Let $\underline{C}_a \equiv \inf\{C_a : \sigma_A^*(C_a) = 1\}$, which exists due to step 4. Moreover, steps 4, 5, and 7b together directly imply that $\Pi_s^A(C_a)$ is continuous for all $C_a \neq \underline{C}_a$. Therefore, it remains to show that $\Pi_s^A(C_a)$ is continuous at $C_a = \underline{C}_a$ as well. Since (9) and (10) imply that (29) is continuous w.r.t. σ_A^* (holding C_a constant) and C_a (holding σ_A^* constant), it is sufficient to show that $\sigma_A^*(C_a)$ is continuous at $C_a = \underline{C}_a$. I show this result in two substeps: (a) First, I prove that $\sigma_A^*(C_a)$ is continuous for all $C_a < \underline{C}_a$. To see why, note that step 4 implies that $\sigma_A^* < 1$ for all $C_a < \underline{C}_a$, and thus step 3 implies that $\Delta_A^*(C_a) = \Delta_0$ for all $C_a < \underline{C}_a$. In turn, for $\Delta_A^*(C_a)$ to remain constant for all $C_a < \underline{C}_a$, it follows from (9) and (10) that $\sigma_A^*(C_a)$ has to be continuous for all $C_a < \underline{C}_a$, because not only (9) is continuous w.r.t. C_a (holding σ_A^* constant) but it is also strictly decreasing w.r.t. σ_A^* (holding C_a constant) since (9) is given by plugging σ_A^* in (27) in step 5 of the proof of Proposition 5 (where $\sigma_A^* > \underline{\sigma}_A$ by step 6 of the same proof). (b) Second, I show that $\lim_{C_a \uparrow \underline{C}_a} \sigma_A^*(C_a) = 1$. Suppose that $\lim_{C_a \uparrow \underline{C}_a} \sigma_A^*(C_a) \neq 1$, which implies that $\lim_{C_a \uparrow \underline{C}_a} \sigma_A^*(C_a) < 1$ (note that the limit exists due to the previous substep). Recall that $\Delta_A^*(C_a) = \Delta_0$ for all $C_a < \underline{C}_a$ due to step 3. Since $\sigma_A^*(C_a)$ is continuous for all $C_a < \underline{C}_a$ (by the last substep) and thus (9) is continuous in C_a for all $C_a < \underline{C}_a$ as well, this implies that (9) evaluated at the pair of $C_a = \underline{C}_a$ and $\sigma_A^* = \lim_{C_a \uparrow \underline{C}_a} \sigma_A^*(C_a) < 1$ is equal to Δ_0 as well. Since (9) is strictly decreasing w.r.t. σ_A^* (holding C_a constant) as noted in the previous substep and (9) is continuous w.r.t. C_a (holding σ_A^* constant), this in turn implies that there exists $\delta > 0$ such that $\Delta_A^*(C_a) < \Delta_0$ for all $C_a \in (\underline{C}_a, \underline{C}_a + \delta)$ (because $\sigma_A^*(C_a) = 1$ for all such C_a). However, then for all $C_a \in (\underline{C}_a, \underline{C}_a + \delta)$, the shareholders are strictly worse off by electing the activist in a proxy fight upon the rejection of an action settlement (because $E[\Delta|\Delta \in [b_a, \Delta_A^*(C_a)]] < 0$ due to combining the definition of Δ_0 in step 3 with $\Delta_A^* > b_a$ in Proposition 5), yielding a contradiction with $\sigma_A^*(C_a) = 1$.

Step 11: $\Pi_s^A(C_a)$ is weakly decreasing in C_a, which immediately follows from steps 7a and 8-10.

Step 12: Steps 10 and 11 imply there exists $\hat{C}_a^A$ such that $\Pi_s^A < \Pi_s^\varnothing$ if and only if $C_a > \hat{C}_a^A$.

Step 13: Next, I complete the proof of part (i) by showing that $\hat{C}_a^A < \bar{C}_a$, where $\bar{C}_a$ is given by (2). Note that due to steps 10 and 11 above and (21), it is sufficient to show that $\lim_{C_a \uparrow \bar{C}_a} \Pi_s^A(C_a) = 0$. In turn, the proof of this limit consists of two substeps:

Step 13a: First, I show that there exists $\tilde{C}_a < \bar{C}_a$ such that $\sigma_A^*(C_a) = 1$ for all $C_a > \tilde{C}_a$. Note that if $b_a \geq 0$, then $\tilde{C}_a = 0$ by the arguments made in step 3. Instead, if $b_a < 0$, let $\tilde{C}_a$ be the value of C_a that satisfies $\hat{\Delta}_A(1|C_a) = \Delta_0$, where $\hat{\Delta}_A(\sigma|C_a)$ is given by part (ii) of Proposition 5 and Δ_0 was defined in step 3 above. Note that such $\tilde{C}_a$ exists and is unique since $\Delta_0 \in (b_a, b_i)$. Moreover, $\tilde{C}_a < \bar{C}_a$ due to (2). Importantly, then for all $C_a > \tilde{C}_a$, (27) in the proof of Proposition 5 implies that $\Delta_A^*(C_a) > \Delta_0$. Hence, due to the same arguments made in step 3 above, for all $C_a > \tilde{C}_a$ the shareholders strictly prefer the activist to be elected in a proxy fight, yielding $\sigma_A^*(C_a) = 1$ as desired.

Step 13b: Since $\sigma_A^*(C_a) = 1$ for all $C_a \in (\tilde{C}_a, \bar{C}_a)$ (by step 13a), parts (i) and (ii) of Proposition 5 imply that for all $C_a \in (\tilde{C}_a, \bar{C}_a)$, $\Delta_A^*(C_a)$ and $\rho_A^*(C_a)$ are continuous in C_a. Moreover, $\lim_{C_a \uparrow \bar{C}_a} \hat{\Delta}_A(1|C_a) = b_i$ due to (2) and the definition of $\hat{\Delta}_A(\sigma|C_a)$ in part (ii) of Proposition 5. Therefore, (10) implies that $\lim_{C_a \uparrow \bar{C}_a} \rho_A^*(C_a) = 0$, and hence in turn (9) implies that $\lim_{C_a \uparrow \bar{C}_a} \Delta_A^*(C_a) = b_i$. As a result, (29) implies that $\lim_{C_a \uparrow \bar{C}_a} \Pi_s^A(C_a) = 0$, as desired.

Step 14: In this step, I prove part (ii). Proof of this result has three substeps:

Step 14a: First, due to step 2 in the proof of Proposition 4,

$$\Pi_a^{\varnothing} = -C_a + n_a \int_{b_a}^{b_i} \alpha_P(\Delta - b_a) dF(\Delta) > 0 \tag{31}$$

and the shareholders always support the activist in a proxy fight.

Step 14b: Second, note that due to step 13a above, there exists $C_a' < \bar{C}_a$ such that $\sigma_A^*(C_a) = 1$ (i.e., when the activist demands an action settlement) for all $C_a > C_a'$.

Step 14c: Third, I prove part (ii) by showing that $\Pi_a^A(C_a) > \Pi_a^{\varnothing}(C_a)$ for all $C_a > C_a'$. Combining step 14b with Proposition 5 implies that for all $C_a > C_a'$,

$$\Pi_a^A = \rho_A^* \int_{b_a}^{\Delta_A^*} \left(n_a \alpha_P(\Delta - b_a) - C_a\right) dF(\Delta) + n_a \int_{\Delta_A^*}^{b_i} (\Delta - b_a) dF(\Delta). \tag{32}$$

Importantly, (32) is strictly larger than (31), because $b_a < \Delta_A^* < b_i$ (by (9) in Proposition 5) and $\rho_A^* = 1$ if $\int_{b_a}^{\Delta_A^*} \left(n_a \alpha_P(\Delta - b_a) - C_a\right) dF(\Delta) > 0$ (because then the activist strictly prefers to run a proxy fight upon rejection of an action settlement), concluding the proof of part (ii).

Step 15: In this step, I prove part (iii). Proof of this result is similar to the proof of part (ii), and it also has three substeps:

Step 15a: First, $\Pi_a^{\varnothing}$ is again given by (31), where $b_a = 0$.

Step 15b: Second, note that due to $b_a = 0$, the arguments made in step 3 imply that $\sigma_A^* = 1$ (i.e., when the activist demands an action settlement) for any $C_a < \bar{C}_a$.

Step 15c: Third, I show that $\Pi_a^A(C_a) > \Pi_a^{\varnothing}(C_a)$ for all $C_a < \bar{C}_a$. This holds because combining step 15b with Proposition 5 implies that for all $C_a < \bar{C}_a$, $\Pi_a^A(C_a)$ is given by (32) where $b_a = 0$. In particular, (32) is strictly larger than (31) due to the same reasons described in step 14c, concluding the proof of part (iii). ■

C Online appendix: Proofs for the propositions in Section 4.2

Proof of Proposition 6. Note that l satisfies the parameter restrictions (1) and (2) if and only if $l \in (\underline{l}, \bar{l})$, where $\underline{l}$ is defined in the statement of Proposition 6 and

$$\bar{l} \equiv \min\{b^a, b - c_P\}. \tag{33}$$

To see why, note that under uniform distribution of Δ, the parameter restrictions (2) and $l < b^a$ (which is implied by (1)) are equivalent to $b - \alpha_P \frac{(b-b^a)^2}{2\kappa} < l < b^a$.[76] Combining with (1), this yields $l \in (\underline{l}, \bar{l})$. Also note that again due to uniform distribution, (3) is satisfied if and only if $-b < b^a$ (because $l < b^a$ as mentioned above).

I start by proving parts (i) and (ii). First, I show that ρ_B^* and Δ_B^* do not change with l. To see this, note that by part (i) of Lemma 2 in Online Appendix C.2, σ_B^* does not change in l. In addition, $\hat{\Delta}_B(\sigma_B^*(l)|l)$ (which is given by (6)) does not change with l, because (1) implies $l < b^a$. Combining these, (5) implies that ρ_B^* does not change with l, and in turn, (4) implies that Δ_B^* does not change with l.

Second, I show that $\lim_{l \downarrow \underline{l}} \rho_A^* = 0$ and $\lim_{l \downarrow \underline{l}} \Delta_A^* = b$. Part (ii) of Lemma 2 shows that there exists $\tilde{l}_\sigma > \underline{l}$ such that $\sigma_A^* = 1$ if $l \leq \tilde{l}_\sigma$. Moreover, given $\sigma_A^*(l) = 1$, $\hat{\Delta}_A$ (which is given by (11)) strictly decreases with l (again, due to $l < b^a$). In addition, given $\sigma_A^*(l) = 1$, $\lim_{l \downarrow \underline{l}} \hat{\Delta}_A = b$, because $\lim_{l \downarrow \underline{l}} \alpha_P E^l [\max\{0, \Delta - b^a\}] = \kappa$, where $E^l[\cdot]$ denotes the expectation for a given l. Combining these, (10) implies that $\lim_{l \downarrow \underline{l}} \rho_A^* = 0$ and that ρ_A^* weakly decreases as l decreases if $l \leq \tilde{l}_\sigma$, and in turn (9) implies that $\lim_{l \downarrow \underline{l}} \Delta_A^* = b$ and that Δ_A^* weakly increases as l decreases if $l \leq \tilde{l}_\sigma$.

Third, I show that as l decreases, ρ_A^* weakly decreases and Δ_A^* weakly increases. Note that due to arguments made at the end of the previous step, it is sufficient to show this result for $l > \tilde{l}_\sigma$. Note that again by part (ii) of Lemma 2, σ_A^* is continuous w.r.t. l, $\sigma_A^* < 1$ if $l > \tilde{l}_\sigma$, and σ_A^* weakly increases as l decreases. Since $\sigma_A^* > 0$ by Proposition 5, this implies that $\sigma_A^* \in (0, 1)$. In other words, when the activist runs a proxy fight upon rejection of an action settlement, shareholders are indifferent between supporting the activist and supporting the incumbent. Therefore, it must be that the shareholders' expected payoff if the activist wins a proxy fight is equal to zero. Since Proposition 5 implies that this payoff is given by $\alpha_P E^l[1_{\{\Delta \geq b^a\}} \cdot \Delta | \Delta \leq \Delta_A^*]$ where $\Delta_A^* > b^a$, it must be that as l changes, Δ_A^* stays constant so that this payoff remains equal to zero. Combining this with the earlier result that σ_A^* weakly increases as l decreases, (9) implies that ρ_A^* weakly decreases as l decreases.

Next, I prove part (iii), which consists of several steps. To that end, as the first step, I make a few notes. Denote by $\Pi_A^a(l)$ ($\Pi_B^a(l)$) the activist's expected payoff from demanding action settlement (board settlement) for a given l. If the activist demands an action settlement (board

[76] This is because if Δ has a uniform distribution, then $\kappa < \alpha_P E[\max\{0, \Delta - b^a\}] \Leftrightarrow \kappa < \alpha_P Prob(\Delta \geq b^a) E[\Delta - b^a | \Delta \geq b^a] \Leftrightarrow \kappa < \alpha_P \frac{b-b^a}{b-l} \left(\frac{b+b^a}{2} - b^a\right) \Leftrightarrow b - \alpha_P \frac{(b-b^a)^2}{2\kappa} < l$.

settlement), the equilibrium is given by Proposition 5 (Proposition 1), where σ_A^* and σ_B^* are continuous w.r.t. l due to Lemma 2. Therefore, for all $l \in (\underline{l}, \bar{l})$, $\Pi_A^a(l)$ ($\Pi_B^a(l)$) is continuous w.r.t l. This is because the probability density function $f(\Delta) = \frac{1}{b-l}$ is continuous in l for all $\Delta \in (l, b)$, and as a result, all of $\hat{\Delta}_A(l)$, $\rho_A^*(l)$ and $\Delta_A^*(l)$ are continuous in l, while $\hat{\Delta}_B(l)$, $\rho_B^*(l)$, and $\Delta_B^*(l)$ do not change with l (recall that σ_B^* does not change with l). Also note that if $\rho_A^*(l) < \gamma$, then the activist's expected payoff upon the rejection of the incumbent is zero, because if $k = \kappa$ then he is indifferent between running a proxy fight and not, where the latter yields a payoff of zero. In the remainder of the proof, consider any two values l_1, l_2 of l such that $l_2 < l_1$. Recall that $l_1 < b^a$ due to (33).

Second, note that combining parts (i) and (ii) with Proposition 1 and the result that σ_B^* does not change with l implies that $\Pi_B^a(l_2) = \frac{b-l_1}{b-l_2}\Pi_B^a(l_1)$.This is because $\rho_B^*(l_1) = \rho_B^*(l_2)$, $\Delta_B^*(l_1) = \Delta_B^*(l_2)$, $\sigma_B^*(l_1) = \sigma_B^*(l_2)$, and

$$\Pi_B^a(l) = \rho_B^*(l) \int_{b^a}^{\Delta_B^*(l)} \left(\sigma_B^*(l)\alpha_P(\Delta - b^a) - \kappa\right) \frac{1}{b-l} d\Delta + \int_{\Delta_B^*(l)}^{b} \alpha_B(\Delta - b^a)\frac{1}{b-l} d\Delta, \tag{34}$$

for any l, where the term $\frac{1}{b-l}$ in the integrals stems from the uniform distribution of Δ (that is, for any given l, $f(\Delta) = \frac{1}{b-l}$ for all $\Delta \in (l, b)$).

Third, I show that $\Pi_A^a(l_2) \leq \frac{b-l_1}{b-l_2}\Pi_A^a(l_1)$. To that end, Proposition 5 implies

$$\Pi_A^a(l) = \rho_A^*(l) \int_l^{b^a} (-\kappa) \frac{1}{b-l} d\Delta + \rho_A^*(l) \int_{b^a}^{\Delta_A^*(l)} \left(\sigma_A^*(l)\, \alpha_P(\Delta - b^a) - \kappa\right) \frac{1}{b-l} d\Delta + \int_{\Delta_A^*(l)}^{b} (\Delta - b^a)\frac{1}{b-l} d\Delta \tag{35}$$

for any l (where the term $\frac{1}{b-l}$ in the integrals again stems from the uniform distribution of Δ). Recall that by Lemma 2, σ_A^* is weakly decreasing in l, and there exists $\tilde{l}_\sigma > \underline{l}$ such that $\sigma_A^* = 1$ if $l \leq \tilde{l}_\sigma$ and $\sigma_A^* < 1$ if $l > \tilde{l}_\sigma$. Note that to prove $\Pi_A^a(l_2) \leq \frac{b-l_1}{b-l_2}\Pi_A^a(l_1)$, it is sufficient to show $(b - l_2)\Pi_A^a(l_2) \leq (b - l_1)\Pi_A^a(l_1)$ for any l_2 and l_1 that satisfes $\tilde{l}_\sigma \leq l_2 < l_1$ and for any l_2 and l_1 that satisfies $l_2 < l_1 \leq \tilde{l}_\sigma$.

I start with $\tilde{l}_\sigma \leq l_2 < l_1$. Recall that for any $l > \tilde{l}_\sigma$, $\sigma_A^* < 1$ and $\sigma_A^* > 0$ (where the latter follows from Proposition 5). Then, as shown in the third step of the proof of parts (i) and (ii) above, it must be that $\Delta_A^*(l)$ stays constant w.r.t. l if $l \geq \tilde{l}_\sigma$ (the reason this statement holds for $l = \tilde{l}_\sigma$ is because $\Delta_A^*(l)$ is continuous w.r.t. l). There are two subcases to consider: (a) First, suppose that $\sigma_A^*(l_2) = \sigma_A^*(l_1)$. Since $\Delta_A^*(l_2) = \Delta_A^*(l_1)$, then (9) implies that $\rho_A^*(l_2) = \rho_A^*(l_1) > 0$ (where the inequality follows from $\Delta_A^* < b$ by Proposition 5). Hence, $l_2 < l_1 < b^a$ implies that $(b - l_2)\Pi_A^a(l_2) < (b - l_1)\Pi_A^a(l_1)$. (b) Second, suppose that $\sigma_A^*(l_2) > \sigma_A^*(l_1)$. Then, since $\Delta_A^*(l_2) = \Delta_A^*(l_1)$, (9) implies that $\rho_A^*(l_2) < \rho_A^*(l_2)$. This implies that $\rho_A^*(l_2) < \gamma$, and hence (35) reduces to $\Pi_A^a(l_2) = \int_{\Delta_A^*(l_2)}^{b} (\Delta - b^a)\frac{1}{b-l_2} d\Delta$, because the activist's expected payoff upon the rejection of the incumbent is zero, as noted in the first step above. Note that the latter payoff (which is represented by the sum of the first two terms in (35)) is always nonnegative for any l, because otherwise the activist would deviate to never

running a proxy fight (i.e., $\rho_A^*(l) = 0$), which would yield a contradiction with this payoff being negative. Hence, comparing the above expression for $\Pi_A^a(l_2)$ with $\Pi_A^a(l_1)$ which is given by (35) implies $(b - l_2)\,\Pi_A^a(l_2) \leq (b - l_1)\,\Pi_A^a(l_1)$.

Next, suppose that $l_2 < l_1 \leq \tilde{l}_\sigma$. Note that for any $l \leq \tilde{l}_\sigma$, $\sigma_A^*(l) = 1$ as mentioned above. Therefore, $\rho_A^*(l)$ given by (10) in Proposition 5 is weakly increasing in l, and strictly increasing in l if $\rho_A^*(l) < \gamma$ (because $\hat{\Delta}_A$ is strictly decreasing in l). There are three cases to consider: (a) First, suppose that $\rho_A^*(l_2) = \gamma$. Then, $\rho_A^*(l_1) = \gamma$ as well, and hence $\Delta_A^*(l_2) = \Delta_A^*(l_1)$ by (9). Therefore, (35) and $l_2 < l_1 < b^a$ imply that $(b - l_2)\,\Pi_A^a(l_2) < (b - l_1)\,\Pi_A^a(l_1)$. (b) Next, suppose that $\rho_A^*(l_2) < \gamma$ and $\rho_A^*(l_1) < \gamma$. Then, (35) reduces to $\Pi_A^a(l_i) = \int_{\Delta_A^*(l_i)}^{b} (\Delta - b^a)\frac{1}{b - l_i} d\Delta$ for any $l_i \in \{l_1, l_2\}$, because the activist's expected payoff upon the rejection of the incumbent is zero, as noted in the first step above. Moreover, (9) implies that $b^a < \Delta_A^*(l_1) < \Delta_A^*(l_2) < b$, because $\rho_A^*(l)$ is strictly increasing in l when $\rho_A^*(l) < \gamma$. Therefore, again $(b - l_2)\,\Pi_A^a(l_2) < (b - l_1)\,\Pi_A^a(l_1)$. (c) Finally, suppose that $\rho_A^*(l_2) < \gamma$ and $\rho_A^*(l_1) = \gamma$. Then, since $\rho_A^*(l)$ is increasing in l as mentioned above, there exists $\tilde{l} \in (l_2, l_1]$ such that $\rho_A^*(l) < \gamma$ for all $l \in [l_2, \tilde{l})$ and $\rho_A^*(l) = \gamma$ for all $l \in [\tilde{l}, l_1]$. Hence,

$$(b - l_2)\,\Pi_A^a(l_2) < \lim_{l \uparrow \tilde{l}} (b - l)\,\Pi_A^a(l) = \lim_{l \downarrow \tilde{l}} (b - l)\,\Pi_A^a(l) \leq (b - l_1)\,\Pi_A^a(l_1), \tag{36}$$

where the first inequality follows from the substep (b) above, the equality follows from the continuity of $\Pi_A^a(l)$ w.r.t. l for all $l \in (\underline{l}, \bar{l})$, and the last inequality follows from substep (a), in turn yielding $(b - l_2)\,\Pi_A^a(l_2) < (b - l_1)\,\Pi_A^a(l_1)$ again (note that the last inequality in (36) is weak because it could be that $\tilde{l} = l_1$). This completes the third step of the proof of part (iii).

Fourth, I show that $\Pi_A^a(l) > 0$ and $\Pi_B^a(l) > 0$ for any $l \in (\underline{l}, \bar{l})$. As mentioned at the beginning of the proof, any $l \in (\underline{l}, \bar{l})$ satisfies the parameter restrictions of the model. Therefore, for all $l \in (\underline{l}, \bar{l})$, Propositions 5 and 1 imply that $\Delta_A^*(l) \in (b^a, b)$ and $\Delta_B^*(l) \in (b^a, b)$, respectively. Since the activist's expected payoff conditional on the incumbent's rejection cannot be negative (because not running a proxy fight is always an option, which gives the activist a payoff of zero), it must be that $\Pi_A^a(l) > 0$ and $\Pi_B^a(l) > 0$.

Fifth, I show that $\lim_{l \downarrow \underline{l}} \Pi_B^a(l) > 0$. As mentioned in the previous step, $\Delta_B^*(l) \in (b^a, b)$ for all $l \in (\underline{l}, \bar{l})$. Moreover, also note that due to part (ii), $\Delta_B^*(l') = \Delta_B^*(l'')$ for any l', l'' that satisfy $\underline{l} < l'' < l' < \bar{l}$. Therefore, it must be $\lim_{l \downarrow \underline{l}} \Delta_B^*(l) < b$. Since $|\underline{l}| < \infty$ and the activist's expected payoff upon the rejection of the incumbent can never be negative (as also mentioned in the previous step), this implies that $\lim_{l \downarrow \underline{l}} \Pi_B^a(l) > 0$.

Sixth, note that $\lim_{l \downarrow \underline{l}} \Pi_A^a(l) = 0$, which directly follows from $\lim_{l \downarrow \underline{l}} \rho_A^*(l) = 0$ (part (i)) and $\lim_{l \downarrow \underline{l}} \Delta_A^*(l) = b$ (part (ii)).

Finally, I show there exists a threshold $\hat{l}_a \in (\underline{l}, \bar{l}]$ such that the activist demands a board settlement if and only if $l < \hat{l}_a$. Indeed, combining steps 2-6 above implies that either (a) $\Pi_B^a(l) > \Pi_A^a(l)$ for all $l \in (\underline{l}, \bar{l})$ (i.e., $\hat{l}_a = \bar{l}$, that is, the threshold $\hat{l}_a$ is higher than all values that l can take), or (b) there exists a unique $\hat{l}_a \in (\underline{l}, \bar{l})$ such that $\Pi_B^a(l) > \Pi_A^a(l)$ for all $l \in (\underline{l}, \hat{l}_a)$, and $\Pi_B^a(l) \leq \Pi_A^a(l)$ for all $l \in [\hat{l}_a, \bar{l})$.

While the final argument above completes the proof of part (iii), I also show that if α_B is

sufficiently small, then case (a) described in the previous step never holds. More formally, there exists $\underline{\alpha}_B > 0$ such that if $\alpha_B < \underline{\alpha}_B$, then $\hat{l}_a \in (\underline{l}, \bar{l})$. To see this result, recall that by the fourth step above, $\Pi^a_A(l) > 0$ and $\Pi^a_B(l;\alpha_B) > 0$ for any given $l \in (\underline{l}, \bar{l})$ and $\alpha_B > 0$ (the argument α_B was not made explicit in the fourth step, because it was not relevant until now). Note that it is sufficient to show that for any given $l \in (\underline{l}, \bar{l})$, there exists $\alpha'_B > 0$ such that if $\alpha_B < \alpha'_B$, then $\Pi^a_B(l;\alpha_B) < \Pi^a_A(l)$. Indeed, this result holds due to (34), because for any $l \in (\underline{l}, \bar{l})$ and $\delta > 0$, (5) implies that there exists $\alpha'_B > 0$ such that if $\alpha_B < \alpha'_B$, then $\rho^*_B(l;\alpha_B) < \delta$ (note that $\hat{\Delta}_B(\sigma^*_B(l;\alpha_B)|l,\alpha_B) < b < \infty$ is always satisfied in equilibrium due to the footnote in part (ii) of Proposition 1, and $b^a < \hat{\Delta}_B(\sigma^*_B(l;\alpha_B)|l,\alpha_B)$ due to (6)). ■

Proof of Proposition 7. As the first step, recall from the proof of Proposition 6 that l satisfies the parameter restrictions (1) and (2) if and only if $l \in (\underline{l}, \bar{l})$, where $\bar{l}$ is given by (33) and $\underline{l}$ is defined in the statement of Proposition 6. Recall that (1) implies $l < b^a$. Also note that again uniform distribution implies that (3) is satisfied if and only if $-b < b^a$ (because $l < b^a$ as mentioned above).

Second, denoting by $\Pi^s_A(l)$ ($\Pi^s_B(l)$) the ex-ante shareholder value from demanding an action settlement (board settlement) for a given l, Proposition 5 and Proposition 1 imply

$$\Pi^s_A(l) = \rho^*_A(l)\sigma^*_A(l)\int_{b^a}^{\Delta^*_A(l)} \alpha_P \Delta \tfrac{1}{b-l} d\Delta + \int_{\Delta^*_A(l)}^{b} \Delta \tfrac{1}{b-l} d\Delta, \tag{37}$$

$$\Pi^s_B(l) = \rho^*_B(l)\sigma^*_B(l)\int_{b^a}^{\Delta^*_B(l)} \alpha_P \Delta \tfrac{1}{b-l} d\Delta + \int_{\Delta^*_B(l)}^{b} \alpha_B \Delta \tfrac{1}{b-l} d\Delta, \tag{38}$$

respectively. Note that the term $\frac{1}{b-l}$ in the integrals stems from the uniform distribution of Δ, that is, $f(\Delta) = \frac{1}{b-l}$ for all $\Delta \in (l, b)$. Moreover, both $\Pi^s_A(l)$ and $\Pi^s_B(l)$ are continuous in l for all $l \in (\underline{l}, \bar{l})$ (because $\sigma^*_A(l)$ and $\sigma^*_B(l)$ are continuous w.r.t. l by Lemma 2 in Online Appendix C.2, and hence $\rho^*_A(l)$, $\Delta^*_A(l)$, $\rho^*_B(l)$, and $\Delta^*_B(l)$ are continuous w.r.t. l by Propositions 5 and 1).

Third, I show that for any two values l_1 and l_2 that l can take such that $l_2 < l_1$, it must be

$$\Pi^s_A(l_2) \leq \tfrac{b-l_1}{b-l_2}\Pi^s_A(l_1). \tag{39}$$

Recall that by Lemma 2, σ^*_A is weakly decreasing in l, and there exists $\tilde{l}_\sigma > \underline{l}$ such that $\sigma^*_A = 1$ if $l \leq \tilde{l}_\sigma$ and $\sigma^*_A < 1$ if $l > \tilde{l}_\sigma$. Note that to prove (39), it is sufficient to show $(b-l_2)\Pi^s_A(l_2) \leq (b-l_1)\Pi^s_A(l_1)$ for any l_2 and l_1 that satisfes $\tilde{l}_\sigma \leq l_2 < l_1$ and for any l_2 and l_1 that satisfies $l_2 < l_1 \leq \tilde{l}_\sigma$.

I start with $\tilde{l}_\sigma \leq l_2 < l_1$. Recall that for any $l > \tilde{l}_\sigma$, $\sigma^*_A < 1$ and $\sigma^*_A > 0$ (where the latter follows from Proposition 5). This implies that the expected shareholder value if the activist wins a proxy fight upon the incumbent's rejection of an action settlement demand is equal to zero. Hence, $\int_{b^a}^{\Delta^*_A(l)} \alpha_P \Delta \frac{1}{b-l} d\Delta = 0$. Moreover, it must be that $\Delta^*_A(l)$ stays constant w.r.t. l if $l \geq \tilde{l}_\sigma$ (the reason this statement holds for $l = \tilde{l}_\sigma$ is because $\Delta^*_A(l)$ is continuous w.r.t. l). Therefore, for all $l \geq \tilde{l}_\sigma$, (37) implies that $\Pi^s_A(l) = \int_{\Delta^*_A(l)}^{b} \Delta \frac{1}{b-l} d\Delta$. Since $\Delta^*_A(l)$ is constant

w.r.t. l if $l \geq \tilde{l}_\sigma$, in turn it must be that $(b - l_2)\,\Pi_A^s(l_2) = (b - l_1)\,\Pi_A^s(l_1)$.

Next, suppose that $l_2 < l_1 \leq \tilde{l}_\sigma$. Note that for any $l \leq \tilde{l}_\sigma$, $\sigma_A^*(l) = 1$ as mentioned above. Moreover, $\rho_A^*(l_2) \leq \rho_A^*(l_1)$ and $\Delta_A^*(l_2) \geq \Delta_A^*(l_1)$ by parts (i) and (ii) of Proposition 6, respectively, and the first term in (37) is nonnegative for any $l_i \in \{l_1, l_2\}$ (because this term represents the expected shareholder value upon the incumbent's rejection, and this payoff has to be weakly greater than zero since the shareholders receive a payoff of zero if they re-elect the incumbent in a proxy fight or if the activist does not run a proxy fight). Note that this also implies $\Delta_A^*(l_1) \geq 0$. Combining these with (37) implies $(b - l_2)\,\Pi_A^s(l_2) \leq (b - l_1)\,\Pi_A^s(l_1)$.

Fourth, note that for any two values l_1 and l_2 that l can take such that $l_2 < l_1$, it must be

$$\Pi_B^s(l_2) = \tfrac{b - l_1}{b - l_2}\Pi_B^s(l_1). \tag{40}$$

Indeed, this follows from combining (38) with the results that $\rho_B^*(l)$ and $\Delta_B^*(l)$ do not change with l (by parts (i) and (ii) of Proposition 6) and $\sigma_B^*(l)$ does not change with l (by part (i) of Lemma 2).

Fifth, I show that $\Pi_A^s(l) > 0$ and $\Pi_B^s(l) > 0$ for any $l \in (\underline{l}, \bar{l})$. As mentioned at the beginning of the proof, any $l \in (\underline{l}, \bar{l})$ satisfies the parameter restrictions of the model. Therefore, for all $l \in (\underline{l}, \bar{l})$, Propositions 5 and 1 imply that $\Delta_A^*(l) \in (b^a, b)$ and $\Delta_B^*(l) \in (b^a, b)$, respectively. Importantly, the first term in (37) and (38) cannot be negative, because they represent the shareholders' payoff upon the incumbent's rejection of a settlement (if these terms were negative, then the shareholders would always support the incumbent in a proxy fight, which would give them a payoff of zero instead). Since in addition $-b < b^a$ (as noted at the beginning of the proof), it follows that $\Pi_A^s(l) > 0$ and $\Pi_B^s(l) > 0$.

Sixth, I show that $\lim_{l \downarrow \underline{l}} \Pi_B^s(l) > 0$. As mentioned in the previous step, $\Delta_B^*(l) \in (b^a, b)$ for all $l \in (\underline{l}, \bar{l})$. Moreover, also note that by part (ii) of Proposition 6, $\Delta_B^*(l') = \Delta_B^*(l'')$ for any l', l'' that satisfy $\underline{l} < l'' < l' < \bar{l}$. Therefore, it must be $\lim_{l \downarrow \underline{l}} \Delta_B^*(l) \in (b^a, b)$. Since $|\underline{l}| < \infty$, $-b < b^a$, and the shareholders' expected payoff upon the rejection of the incumbent can never be negative (as also mentioned in the previous step), this implies that $\lim_{l \downarrow \underline{l}} \Pi_B^s(l) > 0$.

Seventh, note that $\lim_{l \downarrow \underline{l}} \Pi_A^s(l) = 0$, which directly follows from combining (37) with $\lim_{l \downarrow \underline{l}} \rho_A^*(l) = 0$ and $\lim_{l \downarrow \underline{l}} \Delta_A^*(l) = b$. Note that these limits follow from part (i) and part (ii) of Proposition 6, respectively.

Finally, eighth, I show there exists a threshold $\hat{l}_s \in (\underline{l}, \bar{l}]$ such that $\Pi_B^s(l) > \Pi_A^s(l)$ if $l < \hat{l}_s$, and $\Pi_B^s(l) \leq \Pi_A^s(l)$ if $l \geq \hat{l}_s$. Indeed, combining steps 3-7 above implies that either (a) $\Pi_B^s(l) > \Pi_A^s(l)$ for all $l \in (\underline{l}, \bar{l})$ (i.e., $\hat{l}_s = \bar{l}$, that is, the threshold $\hat{l}_s$ is higher than all values that l can take), or (b) there exists a unique $\hat{l}_s \in (\underline{l}, \bar{l})$ such that $\Pi_B^s(l) > \Pi_A^s(l)$ for all $l \in (\underline{l}, \hat{l}_s)$, and $\Pi_B^s(l) \leq \Pi_A^s(l)$ for all $l \in [\hat{l}_s, \bar{l})$.

While the final argument above completes the proof of the proposition, before concluding, I also show that if α_B is sufficiently small, then case (a) described in the previous step never holds. More formally, there exists $\underline{\alpha}_B > 0$ such that if $\alpha_B < \underline{\alpha}_B$, then $\hat{l}_s \in (\underline{l}, \bar{l})$. To see this result, recall that by the fifth step above, $\Pi_A^s(l) > 0$ and $\Pi_B^s(l; \alpha_B) > 0$ for any given $l \in (\underline{l}, \bar{l})$ and $\alpha_B > 0$ (the argument α_B was not made explicit in the fifth step, because it was not

relevant until now). Note that it is sufficient to show that for any given $l \in (\underline{l}, \bar{l})$, there exists $\alpha_B'' > 0$ such that if $\alpha_B < \alpha_B''$, then $\Pi_B^s(l;\alpha_B) < \Pi_A^s(l)$. Indeed, this result holds due to (38), because for any $l \in (\underline{l}, \bar{l})$ and $\delta > 0$, (5) implies that there exists $\alpha_B'' > 0$ such that if $\alpha_B < \alpha_B''$, then $\rho_B^*(l;\alpha_B) < \delta$ (note that $\hat{\Delta}_B(\sigma_B^*(l;\alpha_B)|l,\alpha_B) < b < \infty$ is always satisfied in equilibrium due to the footnote in part (ii) of Proposition 1, and $b^a < \hat{\Delta}_B(\sigma_B^*(l;\alpha_B)|l,\alpha_B)$ due to (6)). ■

C.1 Proofs for the corollaries in Section 4.2

Proof of Corollary 1.

Note that it directly follows from Proposition 5 and Proposition 1 that the incumbent board rejects an action settlement if and only if $\Delta \leq \Delta_A^*$, and it rejects a board settlement if and only if $\Delta \in [b_a, \Delta_B^*]$.

I prove $\Delta_B^* \leq \Delta_A^*$ by contradiction. Suppose that $\Delta_B^* > \Delta_A^*$. Note that by part (iii) of Proposition 5, $\sigma_A^* > 0$. This implies that the expected shareholder value if the activist wins a proxy fight upon the incumbent's rejection of an action settlement is nonnegative, which in turn implies that $\alpha_P E\left[1_{\{\Delta \geq b_a\}} \cdot \Delta | \Delta \leq \Delta_A^*\right] \geq 0$. Since $l < b_a$ (due to (1)), $b_a < \Delta_A^*$ (due to Proposition 5), and $\Delta_A^* < \Delta_B^*$, then it must be that $\alpha_P E\left[\Delta | \Delta \in [b_a, \Delta_B^*]\right] > 0$, which is the expected shareholder value if the activist wins a proxy fight upon the incumbent's rejection of a *board* settlement. Therefore, if the activist runs a proxy fight upon the incumbent's rejection of a board settlement, the shareholders always support the activist, i.e., $\sigma_B^* = 1$.

Next, note (5) implies that another way to express (4) is (19), where $\sigma = \sigma_B^*$. Also note that in equilibrium, by steps 5-6 in the proof of Proposition 5, Δ_A^* can also be expressed as (27), where $\sigma = \sigma_A^*$. Since in addition $\sigma_B^* = 1 \geq \sigma_A^*$, (11) and (6) imply that $\Delta_B(\sigma_B^*)$ given by (19) is weakly smaller than $\Delta_A(\sigma_A^*)$ given by (27), because $\hat{\Delta}_B(\sigma_B^*) \leq \hat{\Delta}_A(\sigma_A^*)$. However, this yields a contradiction with $\Delta_B^* > \Delta_A^*$. ■

Proof of Corollary 2. Note that due to Propositions 5 and 1, $\pi_A^s = E[\Delta | \Delta > \Delta_A^*]$ and $\pi_B^s = \alpha_B E[1_{\{\Delta \geq b^a\}} \cdot \Delta | \Delta < b^a \text{ or } \Delta > \Delta_B^*]$. Suppose that Δ is uniformly distributed over (l, b). Then, for any l, π_A^s and π_B^s becomes

$$\pi_A^s(l) = \tfrac{\Delta_A^* + b}{2}, \tag{41}$$

$$\pi_B^s(l) = \alpha_B \tfrac{b - \Delta_B^*}{b - \Delta_B^* + b^a - l} \tfrac{\Delta_B^* + b}{2}, \tag{42}$$

where the term $\frac{b-\Delta_B^*}{b-\Delta_B^*+b^a-l}$ in π_B^s represents the conditional probability $P(\Delta > \Delta_B^* \mid \Delta < b^a \text{ or } \Delta > \Delta_B^*)$. Also recall that $l < b^a$ due to (1).

To prove the corollary, first note that $\pi_B^s(l_2) < \pi_B^s(l_1)$ if $l_2 < \hat{l}_s < l_1$. This directly follows from (42), because part (ii) of Proposition 6 implies that Δ_B^* does not change with l, and Proposition 2 implies that $\Delta_B^* \geq 0$.

Second, note that $\pi_B^s(l_1) < \pi_A^s(l_1)$. This again directly follows from (41) and (42), because $\Delta_B^*(l_1) \leq \Delta_A^*(l_1)$ due to Corollary 1, and $\Delta_B^* \geq 0$ and $l < b^a$ as noted above.

Finally, $\pi_A^s(l_1) \leq \pi_A^s(l_2)$ due to part (ii) of Proposition 6. ■

C.2 Supplemental results for proofs

This appendix provides the supplemental results that are utilized by some of the proofs in Online Appendix C.

Lemma 2 *Suppose that Δ is uniformly distributed over (l, b), and let $\underline{l} \equiv b - \frac{\alpha_P}{2\kappa}(b - b^a)^2$.*[77] *Then:*

(*i*) *As l decreases, σ_B^* does not change for any given $\alpha_B \in (0, 1]$.*

(*ii*) *There exists $\tilde{l}_\sigma > \underline{l}$ such that $\sigma_A^* = 1$ if $l \leq \tilde{l}_\sigma$ and $\sigma_A^* < 1$ if $l > \tilde{l}_\sigma$. Moreover, σ_A^* is continuous and weakly decreasing in l.*

Proof of Lemma 2. First, note that l satisfies the parameter restrictions (1) and (2) if and only if $l \in (\underline{l}, \bar{l})$, where $\underline{l}$ is defined in the statement of Lemma 2 and

$$\bar{l} \equiv \min\{b^a, b - c_P\}. \tag{43}$$

To see why, note that under uniform distribution of Δ, the parameter restrictions (2) and $l < b^a$ (which is implied by (1)) are equivalent to $b - \frac{\alpha_P}{2\kappa}(b - b^a)^2 < l < b^a$.[78] Combining with (1), this yields $l \in (\underline{l}, \bar{l})$. Also note that again due to uniform distribution, (3) is satisfied if and only if $-b < b^a$ (because $l < b^a$ as mentioned above).

Second, I make a few notes. Denote by $\Pi_A^a(l)$ ($\Pi_B^a(l)$) the activist's expected payoff from demanding an action settlement (board settlement) for a given l. If the activist demands an action settlement (board settlement), the equilibrium is given by Proposition 5 (Proposition 1). In the remainder of the proof, consider any two values l_1, l_2 of l such that $l_2 < l_1$.

Third, I prove part (i) by contradiction. Suppose that $\sigma_B^*(l_2) \neq \sigma_B^*(l_1)$. Then, Proposition 1 implies that the pair $(\Delta_B^*(l_2), \rho_B^*(l_2))$ is not equal to the pair $(\Delta_B^*(l_1), \rho_B^*(l_1))$. Since Proposition 1 also implies that the equilibrium where proxy fight is on the equilibrium path is unique, it is sufficient to show that $(\Delta_B^*(l_2), \rho_B^*(l_2), \sigma_B^*(l_2))$ constitutes an equilibrium for $l = l_1$ as well.

Indeed, since $l_2 < l_1 < b^a$, the fourth and fifth steps in the proof of Proposition 1 imply that given $l = l_1$, $\Delta_B^*(l_2)$ and $\rho_B^*(l_2)$ are best responses to $\sigma = \sigma_B^*(l_2)$ as well as to each other (because they are also best reponses at $l = l_1$). In turn, given $l = l_1$, $\sigma = \sigma_B^*(l_2)$ is also a best response to $(\Delta_B^*(l_2), \rho_B^*(l_2))$ (because given $(\Delta_B^*(l_2), \rho_B^*(l_2))$, the shareholder's expected payoff if the activist wins a proxy fight is the same regardless of $l = l_1$ or $l = l_2$, and in particular, it is given by $\alpha_P \frac{b^a + \Delta_B^*(l_2)}{2}$ due to uniform distribution of Δ). Therefore, the strategy profile $(\Delta_B^*(l_2), \rho_B^*(l_2), \sigma_B^*(l_2))$ constitutes an equilibrium for $l = l_1$ as well, yielding the desired contradiction and concluding the proof of part (i).

[77] As I show at the beginning of the proof of Lemma 2, (2) implies that $\underline{l} < b^a$.

[78] This is because if Δ has a uniform distribution, then $\kappa < \alpha_P E[\max\{0, \Delta - b^a\}] \Leftrightarrow \kappa < \alpha_P Prob(\Delta \geq b^a) E[\Delta - b^a | \Delta \geq b^a] \Leftrightarrow \kappa < \alpha_P \frac{b - b^a}{b - l}\left(\frac{b + b^a}{2} - b^a\right) \Leftrightarrow b - \alpha_P \frac{(b - b^a)^2}{2\kappa} < l$.

Fourth, note that due to Proposition 5, for a given l, the shareholders' expected payoff if the activist wins the proxy fight upon rejection of an action settlement is given by

$$\pi_A^s(l) = \alpha_P E^l[1_{\{\Delta \geq b^a\}} \cdot \Delta | \Delta \leq \Delta_A^*] = \alpha_P \frac{\Delta_A^* - b^a}{b-l} \frac{\Delta_A^* + b^a}{2}, \tag{44}$$

where $E^l[\cdot]$ denotes the expectation for a given l, and the last equality in (44) follows from the uniform distribution of Δ. Also note that $b^a < \Delta_A^*$, again by Proposition 5.

Fifth, I show that σ_A^* weakly decreases in l. I prove by contradiction. Suppose that there exists two values l_1, l_2 of l such that $l_2 < l_1$ and $\sigma_A^*(l_1) > \sigma_A^*(l_2)$. Note that this implies also $\sigma_A^*(l_2) < 1$. Then, by part (iii) of Proposition 5, it must be $\sigma_A^*(l_2) \in (0,1)$. Hence, this implies that $\pi_A^s(l_2) = 0$, where π_A^s is given by (44). Moreover, $\sigma_A^*(l_1) > \sigma_A^*(l_2)$ implies that $\Delta_A^*(l_1) < \Delta_A^*(l_2)$ due to (27), which is given in the fifth step of proof of Proposition 5 (note that by the last step of the same proof, in equilibrium σ_A^* always satisfies $\sigma_A^* > \underline{\sigma}_A$). However, $\Delta_A^*(l_1) < \Delta_A^*(l_2)$ in turn implies that $\pi_A^s(l_1) < \pi_A^s(l_2)$, which yields a contradiction with $\pi_A^s(l_2) = 0$ and $\sigma_A^*(l_1) > 0$.

Sixth, I show that σ_A^* is continuous in l. Suppose that it is not. Then, there exist some $\epsilon > 0$ and a value of l, denoted by l', such that there is no $\delta > 0$ where $l \in (l' - \delta, l' + \delta)$ implies that $\sigma_A^*(l) \in (\sigma_A^*(l') - \epsilon, \sigma_A^*(l') + \epsilon)$. Since σ_A^* weakly decreases in l (which was shown in the previous step), this implies that letting

$$\begin{aligned} &\hat{\Delta}'_A \equiv \hat{\Delta}_A(\sigma_A^*(l') | l = l') \\ &\delta' \equiv \epsilon \tfrac{\alpha_P}{2\kappa} [\hat{\Delta}'_A - b^a]^2, \end{aligned}$$

(where $\hat{\Delta}_A(\sigma|l)$ is given by (11) in Proposition 5), there are two possible cases: (a) There exists $l'' \in (l' - \delta', l')$ such that $\sigma_A^*(l'') \geq \sigma_A^*(l') + \epsilon$, or (b) there exists $l'' \in (l', l' + \delta')$ such that $\sigma_A^*(l'') \leq \sigma_A^*(l') - \epsilon$.

Consider case (a). I start by showing that $\hat{\Delta}_A(\sigma_A^*(l'')|l = l'') < \hat{\Delta}'_A$ is satisfied. To see why this inequality holds, note that (11) implies that for any l and σ,

$$\kappa = \sigma \alpha_P E^l[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}_A(\sigma|l)] = \sigma \alpha_P \frac{\hat{\Delta}_A - b^a}{\hat{\Delta}_A - l} \left(\frac{\hat{\Delta}_A + b^a}{2} - b^a \right) \tag{45}$$

$$\Leftrightarrow l = \hat{\Delta}_A(\sigma|l) - \sigma \tfrac{\alpha_P}{2\kappa} [\hat{\Delta}_A(\sigma|l) - b^a]^2. \tag{46}$$

Since this has to hold for any l, plugging $l = l'$ yields

$$l' = \hat{\Delta}'_A - \sigma_A^*(l') \frac{\alpha_P}{2\kappa} [\hat{\Delta}'_A - b^a]^2 \tag{47}$$

Combining this with $l'' > l' - \delta'$ yields

$$l'' > l' - \delta' = \hat{\Delta}'_A - \sigma_A^*(l') \tfrac{\alpha_P}{2\kappa} \left[\hat{\Delta}'_A - b^a \right]^2 - \delta' = \hat{\Delta}'_A - [\sigma_A^*(l') + \epsilon] \tfrac{\alpha_P}{2\kappa} [\hat{\Delta}'_A - b^a]^2,$$

and this inequality implies that (due to the equivalence in (45)-(46))

$$\kappa < [\sigma_A^*(l') + \epsilon]\, \alpha_P E^{l''}[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}'_A] \leq \sigma_A^*(l'')\alpha_P E^{l''}[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}'_A],$$

where last inequality follows because $\sigma_A^*(l'') \geq \sigma_A^*(l') + \epsilon$. Since $\hat{\Delta}_A(\sigma; l)$ is unique for any given l and σ, $\kappa < \sigma_A^*(l'')\alpha_P E^{l''}[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}'_A]$ implies that $\hat{\Delta}_A(\sigma_A^*(l'')|l = l'') < \hat{\Delta}'_A$.

Due to definition of $\hat{\Delta}'_A$, this implies that $\hat{\Delta}_A(\sigma_A^*(l'')|l = l'') < \hat{\Delta}_A(\sigma_A^*(l')|l = l')$, which in turn implies $\Delta_A^*(l'') < \Delta_A^*(l')$ due to (27) and $\sigma_A^*(l'') > \sigma_A^*(l')$. Then, due to (44), it must be that $\pi_A^s(l'') < \pi_A^s(l')$. However, $\sigma_A^*(l') < \sigma_A^*(l'') \leq 1$ implies that $\pi_A^s(l') \leq 0$, and hence this implies $\pi_A^s(l'') < 0$, yielding a contradiction with $\sigma_A^*(l'') > 0$ in part (iii) of Proposition 5.

Next, consider case (b). The proof for this case is very similar to the previous case. In particular, I start by showing that $\hat{\Delta}_A(\sigma_A^*(l'')|l = l'') > \hat{\Delta}'_A$ is satisfied. To see why this inequality holds, note that (47) again holds, and hence combining it with $l'' < l' + \delta'$ yields

$$l'' < l' + \delta' = \hat{\Delta}'_A - \sigma_A^*(l')\tfrac{\alpha_P}{2\kappa}[\hat{\Delta}'_A - b^a]^2 + \delta' = \hat{\Delta}'_A - [\sigma_A^*(l') - \epsilon]\, \tfrac{\alpha_P}{2\kappa}[\hat{\Delta}'_A - b^a]^2,$$

and this inequality implies that (due to the equivalence in (45)-(46))

$$\kappa > [\sigma_A^*(l') - \epsilon]\, \alpha_P E^{l''}[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}'_A] \geq \sigma_A^*(l'')\alpha_P E^{l''}[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}'_A],$$

where last inequality follows because $\sigma_A^*(l') - \epsilon \geq \sigma_A^*(l'')$. Since $\hat{\Delta}_A(\sigma; l)$ is unique for any given l and σ, $\kappa > \sigma_A^*(l'')\alpha_P E^{l''}[\max\{0, \Delta - b^a\} | \Delta \leq \hat{\Delta}'_A]$ implies $\hat{\Delta}_A(\sigma_A^*(l'')|l = l'') > \hat{\Delta}'_A$.

Due to definition of $\hat{\Delta}'_A$, this implies that $\hat{\Delta}_A(\sigma_A^*(l'')|l = l'') > \hat{\Delta}_A(\sigma_A^*(l')|l = l')$, which in turn implies $\Delta_A^*(l'') > \Delta_A^*(l')$ due to (27) and $\sigma_A^*(l'') < \sigma_A^*(l')$. Then, due to (44), it must be that $\pi_A^s(l'') > \pi_A^s(l')$. However, $\sigma_A^*(l'') < \sigma_A^*(l') \leq 1$ implies that $\pi_A^s(l'') \leq 0$, and hence this implies $\pi_A^s(l') < 0$, yielding a contradiction with $\sigma_A^*(l') > 0$ in part (iii) of Proposition 5. This completes the proof of the sixth step.

Finally, I prove that there exists $\tilde{l}_\sigma > \underline{l}$ such that $\sigma_A^* = 1$ if $l \leq \tilde{l}_\sigma$ and $\sigma_A^* < 1$ if $l > \tilde{l}_\sigma$. Note that if $b^a \geq 0$, then this result is straightforward, because $\sigma_A^* = 1$ for all l since in equilibrium it has to be $\pi_A^s > 0$ due to (44) and $\Delta_A^* > b^a$ (where the latter follows from Proposition 5). Therefore, it remains to show the result for $b^a < 0$. In turn, since it is already shown that σ_A^* is continuous and weakly decreasing in l, it is sufficient to show that $\tilde{l}' > \underline{l}$ and $\sigma_A^*(l) = 1$ for all $l \in (\underline{l}, \tilde{l}')$, where

$$\tilde{l}' \equiv -b^a - 4\tfrac{\alpha_P}{2\kappa}\left(b^a\right)^2.$$

First, I show that $\tilde{l}' > \underline{l}$. Indeed,

$$\begin{aligned} &\underline{l} = b - \tfrac{\alpha_P}{2\kappa}(b - b^a)^2 < -b^a - 4\tfrac{\alpha_P}{2\kappa}\left(b^a\right)^2 \\ &\Leftrightarrow 0 > b + b^a - \tfrac{\alpha_P}{2\kappa}(b - 3b^a)(b + b^a) \Leftrightarrow 0 > 1 - \tfrac{\alpha_P}{2\kappa}(b - 3b^a) \Leftrightarrow \alpha_P \tfrac{(b-3b^a)}{2} > \kappa \end{aligned}$$

where the second equivalence follows from the first equivalence because due to (1) and uniform distribution of Δ, (3) is equivalent to $b + b^a > 0$; and the last equivalence holds because due

to $b^a < 0$, (1), and uniform distribution of Δ, (2) implies $\alpha_P \frac{b-b^a}{2} > \kappa$. Therefore, $\tilde{l}' > \underline{l}$ holds.

Second, I show that $\sigma_A^*(l) = 1$ for all $l \in (\underline{l}, \tilde{l}')$. Suppose that there exists $l' \in (\underline{l}, \tilde{l}')$ such that $\sigma_A^*(l') \neq 1$. Then, it must be $\sigma_A^*(l') < 1$. Importantly, $\hat{\Delta}_A(\sigma_A^*(l')|l = l') > -b^a$ (where again $\hat{\Delta}_A$ is given by (27)), because

$$\kappa > \alpha_P E^{l'}[\max\{0, \Delta - b^a\} | \Delta \leq -b^a] \Leftrightarrow \kappa > \alpha_P \tfrac{-b^a - b^a}{-b^a - l'} \left(\tfrac{-b^a + b^a}{2} - b^a\right) \Leftrightarrow \kappa > 2\alpha_P \tfrac{(b^a)^2}{-b^a - l'} \Leftrightarrow l' < \tilde{l}',$$

which holds. In turn, $\hat{\Delta}_A(\sigma_A^*(l')|l = l') > -b^a$ implies $\Delta_A^*(l') > -b^a$ due to (27), which is given in the fifth step of the proof of Proposition 5 (note that by the last step of the same proof, in equilibrium σ_A^* always satisfies $\sigma_A^* > \underline{\sigma}_A$). However, then, (44) implies that $\pi_A^s(l') > 0$, yielding a contradiction with $\sigma_A^*(l') < 1$. ■

D Online appendix for parameter restrictions (Section 2.1)

In this appendix, I provide further details as well as formal results regarding the parameter restrictions (2) and (3) made in Section 2.1. I begin with the case where (a) the activist has demanded an action settlement or (b) settlements are not allowed.

Lemma 3 *Suppose that (2) or (3) is violated, and either (a) the activist has demanded an action settlement, or (b) settlements are not allowed. Then, the equilibrium is unique, and in equilibrium, no settlement is reached, the activist never runs a proxy fight, and the project is never implemented.*[79]

Proof of Lemma 3. First, note that regardless of whether (2) and (3) hold, Lemma 1 continues to hold.

Second, note that if the activist never runs a proxy fight upon incumbent's rejection, then the incumbent never accepts the settlement (and hence the project is never implemented). This is because the incumbent's payoff from accepting the action settlement is negative (since $\Delta < b$), while its payoff is zero if it rejects and the activist does not run a proxy fight.

Third, I show that the strategy profile described in the lemma indeed constitutes an equilibrium. Given that the activist never runs a proxy fight upon rejection, it is a best response for the incumbent to reject the settlement for all Δ (and hence the project is never implemented). This is because then, its payoff from rejecting is zero, while its payoff from accepting is always nonpositive. In turn, if (2) is violated, then given that the incumbent always rejects the settlement, it is a best response for the activist to never run a proxy fight (because the activist's expected payoff from running a proxy fight is then $\sigma\alpha_P E[\max\{0, \Delta - b^a\}] - k \leq$

[79] If settlements are not allowed, then other equilibria exist in the knife-edge cases where $0 = E[\Delta|\Delta \geq b^a]$ or $C^a = \bar{C}^a$. However, none of these other equilibria exist if $0 > E[\Delta|\Delta \geq b^a]$ or $C^a > \bar{C}^a$. Therefore, as the distribution of Δ converges from $0 > E[\Delta|\Delta \geq b^a]$ to $0 = E[\Delta|\Delta \geq b^a]$ (or from $C^a > \bar{C}^a$ to $C^a = \bar{C}^a$), the equilibrium described in Lemma 3 is the unique *continuous* equilibrium.

$\sigma\alpha_P E\left[\max\left\{0, \Delta - b^a\right\}\right] - \kappa \leq 0$). Similarly, if (3) is violated, then again given that the incumbent always rejects the settlement, it is a best response for the shareholders to never elect the activist in a proxy fight (because their payoff is zero if the incumbent wins, and it is $\alpha_P E[1_{\{\Delta \geq b^a\}} \cdot \Delta] \leq 0$ if the activist wins), and hence in turn it is a best response for the activist to never run a proxy fight.

Fourth, finally, I show that there exists no other equilibrium than the one described in the lemma. Note that due to the second step above, it is sufficient to show that there is no equilibrium where $\rho^* > 0$ (i.e., there is no equilibrium where if the incumbent rejects, the activist runs a proxy fight with positive probability). To prove by contradiction, suppose that there exists such an equilibrium. Then due to the same arguments made in the first step of the proof of Proposition 5 (note that the arguments made in this step do not rely on parameter assumptions (2) and (3)), the incumbent rejects if and only if $\Delta < \Delta_A$ for some $\Delta_A \in (b^a, b)$. Moreover, since (1) implies $l < b^a$, it has to be that proxy fight is on the equilibrium path. There are two subcases to consider: First, suppose that (2) is violated. However, then this implies that upon the incumbent's rejection, the activist's expected payoff from running a proxy fight (which is given by $\sigma\alpha_P E\left[\max\left\{0, \Delta - b^a\right\} | \Delta \leq \Delta_A\right] - k \leq \sigma\alpha_P E\left[\max\left\{0, \Delta - b^a\right\} | \Delta \leq \Delta_A\right] - \kappa$) is negative because $l < b^a < \Delta_A < b$. Hence, the activist never runs a proxy fight upon the incumbent's rejection, yielding a contradiction with $\rho^* > 0$. Second, suppose that (3) is violated. However, then this implies that upon the incumbent's rejection, the shareholders' payoff from electing the activist (which is given by $\alpha_P E[1_{\{\Delta \geq b^a\}} \cdot \Delta | \Delta \leq \Delta_A]$) is negative, because again $l < b^a < \Delta_A < b$. In contrast, their payoff is zero if the incumbent wins. Hence, the shareholders never support the activist, which yields a contradiction with $\rho^* > 0$, because the activist never incurs the cost of a proxy fight that he will always lose. ■

For the intuition behind this lemma, see the discussion after (2) and (3) in Section 2.1.

Next, consider the case where the activist has demanded a board settlement. Suppose that the incumbent always rejects the activist's board settlement demand. Then, under certain parameter conditions, there is a unique equilibrium: In this equilibrium, the activist never runs a proxy fight upon the incumbent's rejection, and as a result, the project is never implemented. Specifically, this is the unique equilibrium if any of the following conditions hold: (a) $C^a > \alpha_P n^a E\left[\max\left\{0, \Delta - b^a\right\}\right]$, (b) $b^a \geq b^i$, or (c) $E\left[\Delta | \Delta \geq b^a\right] < 0$. Intuitively, Lemma 1 implies that the activist's expected payoff if he runs and wins a proxy fight is $\alpha_P n^a E\left[\max\left\{0, \Delta - b^a\right\}\right] - C^a$, which is negative if (a) holds. If instead condition (b) or (c) holds, then the activist's interests are too misaligned with those of shareholders: Specifically, if b^a is so high that $b^a \geq b$, then the activist's bias against the project is even stronger than the incumbent's, and hence the activist never implements the project even if he obtains decision authority. If b^a is so low that $E\left[\Delta | \Delta \geq b^a\right] < 0$, then the activist destroys shareholder value on average after winning a proxy fight (because he is too much in favor of the project), and as a result the shareholders never elect him in a proxy fight. As a result, the activist never runs a proxy fight if any of the conditions (a)-(c) above holds.

In turn, since the only reason why the incumbent may accept the activist's settlement demand is to avoid a proxy fight, this implies that if any of the conditions (a)-(c) holds, the reasonable equilibrium is the one described above: The incumbent always rejects the settlement demand, and as a result, due to the arguments above, the activist never runs a proxy fight upon the incumbent's rejection, and the project is never implemented. Indeed, part (i) of the lemma below shows that this equilibrium exists if any of (a)-(c) holds, or more generally, if (2) or (3) is violated.

Moreover, parts (i) and (ii) of the same lemma shows that if C^a is sufficiently high, then the *unique* equilibrium is the one where the activist has no impact on the firm whatsoever (i.e., he does not run any campaign): The project is never implemented because the activist never runs a proxy fight upon the incumbent's rejection of the settlement. Similarly, parts (i) and (iii) of the lemma shows that if $0 > E\left[\Delta|\Delta \geq b^a\right]$, then this is the *unique* equilibrium that satisfies the Grossman and Perry (1986) criterion.

Proposition 11 *If (2) or (3) is violated, and the activist has demanded a board settlement:*

(*i*) *There always exists an equilibrium where the incumbent always rejects the activist's board settlement demand, the activist never runs a proxy fight upon rejection, and the project is never implemented. Moreover, this equilibrium always satisfies the Grossman and Perry (1986) criterion.*

(*ii*) *If $C^a \geq \alpha_P n^a (b^i - b^a)$, then in any equilibrium, the activist never runs a proxy fight and the project is never implemented.*

(*iii*) *If $E\left[\Delta|\Delta \geq b^a\right] < 0$, then the activist never runs a proxy fight in any equilibrium, and any equilibrium where the project is implemented with positive probability does not satisfy the Grossman and Perry (1986) criterion.*

Proof of Proposition 11.

Note that regardless of whether (2) and (3) hold, Lemma 1 continues to hold.

First, I prove part (i). The proof of the existence of this equilibrium is identical to the third step of the proof Lemma 3, and hence is not repeated here. Also note that this equilibrium satisfies the Grossman and Perry (1986) criterion, since the activist's proxy fight decision does not rely on off-equilibrium beliefs about Δ. This is because the incumbent's rejection of the settlement is on the equilibrium path.

Next, I prove part (ii). If $\kappa \geq \alpha_P (b - b^a)$, then for any given Δ, the activist's payoff from running a proxy fight (i.e., $\sigma \alpha_P \max\left\{0, \Delta - b^a\right\} - k$) is always negative (recall that $\Delta < b$). Therefore, the activist never runs a proxy fight. In turn, the incumbent never accepts the board settlement if the activist implements the project with positive probability upon joining the board. Hence, it has to be that in any equilibrium, the project is never implemented.

Finally, I prove part (iii). Suppose that $E\left[\Delta|\Delta \geq b^a\right] < 0$. I prove part (iii) in three steps. (a) I begin by showing that the activist never runs a proxy fight in any equilibrium. To prove

by contradiction, suppose that there exists an equilibrium where the activist runs a proxy fight with positive probability. Then, due to the same arguments made in the second step of the proof of Proposition 1 (note that the arguments made in this step do not rely on any parameter assumption), the incumbent rejects if and only if $\Delta \in [b^a, \Delta_B]$ for some $\Delta_B < b$. Specifically, for there to be a proxy fight with positive probability on the equilibrium path, it also has to be that $\Delta_B > b^a$. Then, if the activist runs a proxy fight and wins, the expected shareholder value is $\alpha_P E[\Delta|\Delta \in [b^a, \Delta_B]] < \alpha_P E[\Delta|\Delta \geq b^a] < 0$, where the first inequality holds because $\Delta_B < b$. In contrast, shareholder value is zero if the incumbent wins. However, then the shareholders never support the activist, and the activist never incurs the cost of a proxy fight, yielding a contradiction.

(b) Next, I show that in any equilibrium where the project is implemented with positive probability, it has to be that the incumbent accepts the board settlement for all Δ. To prove by contradiction, suppose that there exists an equilibrium where the project is implemented with positive probability, and the incumbent rejects the board settlement for some Δ. Then, by substep (a) above, it has to be that the activist never runs a proxy fight upon incumbent's rejection. Then, it has to be that in equilibrium, for some Δ the project is implemented with positive probability after the incumbent accepts the board settlement. However, for any such Δ, the incumbent strictly prefers to deviating to rejecting the board settlement, because then the project is never implemented (since the activist never runs a proxy fight), yielding a contradiction.

(c) Also note that in any equilibrium where the incumbent accepts the board settlement for all Δ, it has to be that $\rho^* > 0$ (i.e., the activist runs a proxy fight with positive probability if the incumbent rejects). This is because if $\rho^* = 0$, then the incumbent strictly prefers to reject the board settlement for all $\Delta > b^a$.

(d) Finally, I complete the proof of part (iii). Note that due to substeps (a)-(c) above, it is sufficient to show that the equilibrium where $\rho^* > 0$ and the incumbent accepts board settlement for all Δ does not satisfy the Grossman and Perry (1986) criterion. To that end, denote by μ the activist's and shareholders' off-equilibrium belief about Δ upon the incumbent's rejection. Then, according to this criterion, this belief μ must be "credible". To put it more formally, if there exists any off-equilibrium belief ν such that under this belief, the incumbent weakly prefers to reject the settlement for some Δ (due to the resulting $\rho(\nu)$ and $\sigma(\nu)$), then μ must satisfy this property (i.e., μ must be equal to one of such ν). Therefore, the proof of this final step consists of two further substeps.

(d.1) First, I show there exists a credible off-equilibrium belief which does not sustain the equilibrium where $\rho^* > 0$ and the incumbent accepts board settlement for all Δ. In particular, the activist's and shareholders' off-equilibrium belief $\nu = (l, b)$ (that is, the belief that the incumbent rejects for all Δ) is credible according to the Grossman and Perry (1986) criterion. This is because under this belief, the resulting $\rho(\nu)$ and $\sigma(\nu)$ indeed makes the incumbent weakly prefer to reject the settlement for all Δ, and strictly reject for all $\Delta > b^a$. More formally, under this belief, the equilibrium of the subgame starting with the incumbent's response is given by part (i) of the proposition; specifically, the activist never runs a proxy

fight (because this is always an equilibrium by part (i) of the proposition, and there cannot be any other subgame equilibrium by substep (a) above). Therefore, the incumbent indeed weakly prefers to reject the settlement for all Δ, and strictly reject for all $\Delta > b^a$. This also implies that under this credible belief, it is never the case that the incumbent accepts the settlement for all Δ.

(d.2) Since the credible belief described in the substep (d.1) above leads to the incumbent's rejection of the settlement for some Δ, the second substep is to show that if the activist's and shareholders' off-equilibrium belief μ about Δ leads to the incumbent's acceptance for all Δ, then μ cannot satisfy the credibility criterion of Grossman and Perry (1986). To prove by contradiction, suppose that μ is credible. Then, under the belief μ, it has to be that the incumbent *weakly* prefers to reject the settlement for *some* Δ (for the belief μ to be credible), and it also has to be that the incumbent *weakly* prefers to accept the settlement *for all* Δ (so that the equilibrium is sustained). Note that under this belief μ, the likehood of a proxy fight $\rho(\mu)$ upon the incumbent's rejection cannot be $\rho(\mu) = 0$, because then the incumbent would not accept the settlement for any $\Delta > b^a$. Therefore, the only remaining possibility is $\rho(\mu) > 0$. However, then the incumbent accepts the settlement for all $\Delta < b^a$. Moreover, by the second step in the proof of Proposition 1, it has to be $\rho(\mu) < \frac{\alpha_B}{\sigma(\mu)\alpha_P + c_P/(b-b^a)}$, because otherwise either (I) the incumbent weakly prefers to reject the settlement if and only if $\Delta = b^a$, which would imply that $\mu = \{b^a\}$ and hence $\rho(\mu) = 0$, yielding a contradiction, or (II) the incumbent *strictly* prefers to accept the board settlement for all Δ, which also yields a contradiction. Therefore, for the belief μ to be credible, it must be $\rho(\mu) < \frac{\alpha_B}{\sigma(\mu)\alpha_P + c_P/(b-b^a)}$. However, then, again by the second step in the proof of Proposition 1, the incumbent would *strictly* prefer to reject the settlement for some Δ. This would imply that the belief μ does not sustain the equilibrium, since it is not possible that the incumbent accepts the settlement for all Δ under this belief. This yields another contradiction, concluding the proof. ■

E Online appendix for Section 3.1: Equilibrium selection

This appendix provides the formalization and proofs for the equilibrium selection in Section 3.1 that was mentioned following Proposition 1. To that end, I analyze a generalized version of the model where the activist's cost of proxy fight has a distribution, rather than being a constant.

Recall in the main model (Section 2), the activist's cost of proxy fight was C^a, which was a constant. The extension in this appendix makes the activist's cost of proxy fight C^a continuous. To distinguish the notation from the main model, in this extension denote C^a instead by C_c^a, and suppose that C_c^a is distributed according to a continuous probability density function g (and cumulative distribution function G) with full support on $[\underline{C}^a, \tilde{C}^a]$ for some $0 < \underline{C}^a < \tilde{C}^a$. I also assume that

$$\underline{C}^a < \alpha_P n^a E\left[\max\left\{0, \Delta - b^a\right\}\right], \tag{48}$$

which is similar to the parameter restriction (2) imposed in the main model (indeed, as the distribution of C_c^a converges to C^a, condition (48) reduces to (2)). As in the main model, I impose this restriction to ensure that the activist has some impact on the firm's decisions with positive probability. To simplify the analysis, I let $b^a \geq 0$.

The results of this appendix are as follows: (a) In Proposition 12, I show that if the activist demands a board settlement, then the equilibrium where proxy fight is on the equilibrium path always exists, it survives the Grossman and Perry (1986) criterion, and it converges to the equilibrium described in Proposition 1 as $\underline{C}^a$ and $\tilde{C}^a$ converge to C^a. (b) On the other hand, in Proposition 13, I show that if the activist demands a board settlement, then the only other equilibrium (besides the one described in Proposition 12) is the equilibrium where the incumbent accepts the board settlement for all Δ, and this equilibrium exists only if $\alpha_B \leq \alpha_P + C^i/(n^i(b^i - b^a))$. However, in the same proposition, I also show that this equilibrium does not survive the Grossman and Perry (1986) criterion.

I start the formal results with Proposition 12 below.

Proposition 12 *Suppose that the activist has demanded a board settlement, and $b^a \geq 0$. Then, an equilibrium of this subgame where a proxy fight is on the equilibrium path exists, it is unique, and in this equilibrium there exists a unique $\rho_B^c \in (0,1]$ such that:*

(*i*) *The incumbent accepts the settlement if and only if $\Delta \in (l, b^a) \cup (\Delta_B^c, b^i)$, where $\Delta_B^c \in (b^a, b^i)$ is given by*

$$\Delta_B^c \equiv b^i - \frac{C^i/n^i}{\alpha_B/\rho_B^c - \sigma_B^c \alpha_P}, \tag{49}$$

(*ii*) *Upon rejection, the activist runs a proxy fight with probability $\rho^* = \rho_B^c$.*

(*iii*) *If the activist runs a proxy fight, he wins with probability $\sigma_B^c = 1$.*

Moreover, this equilibrium always satisfies the Grossman and Perry (1986) criterion, and as $\underline{C}^a$ and $\tilde{C}^a$ converge to C^a (i.e., as the distribution of C_c^a converges to C^a): ρ_B^c converges to ρ_B^*, σ_B^c converges to σ_B^*, and Δ_B^c converges to Δ_B^*, where ρ_B^*, σ_B^*, and Δ_B^* are given by Proposition 1.

Next, in Proposition 13 below, I show that if the activist demands a board settlement, then the only other equilibrium (besides the one described in Proposition 12) is the equilibrium where the incumbent accepts the board settlement for all Δ, and this equilibrium exists only if $\alpha_B \leq \alpha_P + C^i/(n^i(b^i - b^a))$. However, in this proposition I also show that this equilibrium does not survive the Grossman and Perry (1986) criterion.

Proposition 13 *Suppose that the activist has demanded a board settlement, and $b^a \geq 0$. Then, an equilibrium of this subgame without any proxy fight on the equilibrium path exists only if $\alpha_B \leq \alpha_P + C^i/(n^i(b^i - b^a))$. In this equilibrium, the incumbent accepts the board settlement for all Δ. However, this equilibrium never satisfies the Grossman and Perry (1986) criterion.*

E.1 Proofs for Online Appendix E

This section provides all the proofs for the results in this extension, that is, all the results listed in Online Appendix E above.

Proof of Proposition 12. First, note that $b^a \geq 0$ implies that in any equilibrium where a proxy fight is on the equilibrium path, the shareholders always support the activist when he runs a proxy fight, i.e., $\sigma_B^c = 1$. This is because if the activist runs a proxy fight, his expected payoff from doing so must be nonnegative, which in turn implies that the shareholder value is positive if the activist wins the proxy fight (because $b^a \geq 0$ and the activist needs to incur the cost of a proxy fight but shareholders do not), while the shareholders' payoff is zero if the incumbent is re-elected.

Second, the previous step and Lemma 1 imply that in any equilibrium where proxy fight is on the equilibrium path, for any given ρ, the incumbent's payoff from rejecting the settlement is $\rho[-c_P + \alpha_P(\Delta - b)]$ if $\Delta \geq b^a$, and $-\rho c_P$ otherwise. On the other hand, the incumbent's payoff from accepting the board settlement is $\alpha_B(\Delta - b)$ if $\Delta \geq b^a$, and zero if $\Delta < b^a$.

Third, I start characterizing the incumbent's strategy at the response stage. To that end, in this step, I prove that the incumbent accepts the board settlement for all $\Delta < b^a$. Suppose that this is not true, and the incumbent rejects for some $\Delta < b^a$. Then, due to the incumbent's payoff structure described in the previous step, this implies that $\rho^* = 0$ in equilibrium. In turn, this implies that the incumbent rejects the board settlement for all $\Delta > b^a$. Therefore, if $k = \kappa^c$, then the activist's expected payoff from running a proxy fight upon rejection is weakly larger than $\alpha_P E[\max\{0, \Delta - b^a\}] - \kappa^c$. However, (48) implies that this payoff is positive at least for some κ^c. In particular, it implies that $\rho^* = \gamma G(\alpha_P E[\max\{0, \Delta - b^a\}]) > 0$, yielding a contradiction.

Fourth, I characterize the incumbent's strategy for $\Delta \geq b^a$. Note that if $\Delta \geq b^a$, the incumbent accepts the board settlement if and only if $\alpha_B(\Delta - b) > \rho[-c_P + \alpha_P(\Delta - b)]$. Note that this inequality is equivalent to $\Delta > \Delta_B(\rho)$, where $\Delta_B(\rho)$ is given by (7) and satisfies $\Delta_B(\rho) \in (b^a, b]$ if $\rho < \frac{\alpha_B}{\alpha_P + c_P/(b-b^a)}$. Indeed, it cannot be $\rho^* \geq \frac{\alpha_B}{\alpha_P + c_P/(b-b^a)}$ in any equilibrium where proxy fight is on the equilibrium path (which is the type of equilibrium that is considered in this proposition). This is because if $\rho^* \geq \frac{\alpha_B}{\alpha_P + c_P/(b-b^a)}$, then either (a) the incumbent rejects board settlement if and only if $\Delta = b^a$, which yields a contradiction because then the activist would never run a proxy fight upon rejection (since $\underline{\kappa} > 0$), or (b) the incumbent accepts board settlement for all Δ, which again yields a contradiction because it would imply that there is no proxy fight on the equilibrium path.

Fifth, I show that there exists a unique $\rho_B^c \in (0, \frac{\alpha_B}{\alpha_P + c_P/(b-b^a)})$ such that $\rho^* = \rho_B^c$ yields an equilibrium (then, due to the arguments above, the equilibrium where proxy fight is on the equilibrium path exists, it is unique, in this equilibrium the incumbent's rejection threshold Δ_B is simply equal to $\Delta_B(\rho_B^c)$, and $\Delta_B(\rho_B^c) \in (b^a, b)$). Note that upon rejection of the board settlement, the activist infers that $\Delta \in [b^a, \Delta_B]$. Therefore, if $k = \kappa^c$, then the activist's expected payoff from running a proxy fight upon rejection is given by $\pi^a = \alpha_P E[\Delta - b^a | \Delta \in [b^a, \Delta_B]] - \kappa^c$. Therefore, whenever $k = \kappa^c$, the activist runs a proxy

fight if $\alpha_P E\left[\Delta - b^a | \Delta \in [b^a, \Delta_B]\right] > \kappa^c$ and does not run if $\alpha_P E\left[\Delta - b^a | \Delta \in [b^a, \Delta_B]\right] < \kappa^c$. As a result, the likelihood of a proxy fight upon rejection of a board settlement is given by

$$\rho(\Delta_B) = \gamma \text{Prob}(\alpha_P E\left[\Delta - b^a | \Delta \in [b^a, \Delta_B]\right] > \kappa^c) = \gamma G(\alpha_P E\left[\Delta - b^a | \Delta \in [b^a, \Delta_B]\right]). \quad (50)$$

Importantly, $\rho(\Delta_B)$ is continuous and weakly increasing in Δ_B. Moreover, (48) implies that $\rho(b) > 0$, and $\underline{\kappa} > 0$ implies that $\rho(b^a) = 0$. On the other hand, $\Delta_B(\rho)$ is continuous and strictly decreasing in ρ at any $\Delta_B \in (b^a, b)$. Moreover, $\Delta_B(0) = b$ and $\Delta_B(\frac{\alpha_B}{\alpha_P + c_P/(b-b^a)}) = b^a$. Therefore, there exists a unique $\rho_B^c \in (0, \frac{\alpha_B}{\alpha_P + c_P/(b-b^a)})$ that solves the fixed point equation $\rho = \gamma G(\alpha_P E\left[\Delta - b^a | \Delta \in [b^a, \Delta_B(\rho)]\right])$. Moreover, this solution also satisfies $\rho_B^c \leq \gamma$. Hence, the equilibrium where proxy fight is on the equilibrium path exists, it is unique, and in this equilibrium $\rho^* = \rho_B^c$.

Sixth, note that this equilibrium satisfies the Grossman and Perry (1986) criterion, since the activist's proxy fight decision does not rely on off-equilibrium beliefs about Δ. This is because the incumbent's rejection of the settlement is on the equilibrium path.

Seventh, note that due to the arguments made in the first step of this proof, $b^a \geq 0$ implies not only $\sigma_B^c = 1$ for this proposition, but it also implies $\sigma_B^* = 1$ for Proposition 1. Therefore, as $\underline{\kappa}$ and $\tilde{\kappa}$ converge to κ, σ_B^c converges to σ_B^*.

Eighth, finally, I show that as $\underline{\kappa}$ and $\tilde{\kappa}$ converges κ, ρ_B^c converges to ρ_B^* given by (5) (which implies that Δ_B^c converges to Δ_B^* due to the fifth and seventh steps above). Noting that κ satisfies (2), the proof of this step consists of two substeps.

First, I show that if κ satisfies $\alpha_B \geq \gamma \hat{\alpha}_L(1)$, then ρ_B^c converges to γ, where $\hat{\alpha}_L(\cdot)$ is given by (17) in the proof of Proposition 1. Note that due to that proof (and also because $\sigma_B^* = 1$ by the seventh step above), $\alpha_B \geq \gamma \hat{\alpha}_L(1)$ implies that $\rho_B^* = \gamma$. Suppose that ρ_B^c converges to some other value $\rho_B^f \neq \gamma$. Since $\rho_B^c \leq \gamma$, then it must be $\rho_B^f < \gamma$. However, then $\sigma_B^c = 1$ and (49) implies that Δ_B^c converges to

$$\Delta_B^f \equiv b - \frac{c_P}{\alpha_B/\rho_B^f - \alpha_P}. \quad (51)$$

Moreover, note that $\alpha_B \geq \gamma \hat{\alpha}_L(1)$ implies that $\hat{\Delta}_B(1) \leq b - \frac{c_P}{\alpha_B/\gamma - \alpha_P}$, and therefore, $\rho_B^f < \gamma$ in turn implies that $\Delta_B^f > \hat{\Delta}_B(1)$. Let $\varepsilon \equiv \Delta_B^f - \hat{\Delta}_B(1)$. Then, there exists some $\delta > 0$ such that if $|\tilde{\kappa} - \underline{\kappa}| < \delta$, then $\Delta_B^c > \hat{\Delta}_B(1) + \varepsilon/2$ and $\tilde{\kappa} < \alpha_P E[\Delta - b^a | \Delta \in [b^a, \hat{\Delta}_B(1) + \varepsilon/2]]$. Note that the last inequality follows from (6), $\sigma_B^* = 1$, and the fact that $\tilde{\kappa}$ converges to κ. In turn, $\kappa^c \leq \tilde{\kappa} < \alpha_P E[\Delta - b^a | \Delta \in [b^a, \Delta_B^c]]$ combined with (50) imply that $\rho(\Delta_B^c) = \gamma$ for all $|\tilde{\kappa} - \underline{\kappa}| < \delta$, yielding a contradiction with $\rho_B^f \neq \gamma$. Therefore, ρ_B^c converges to γ.

Second, I show that if κ satisfies $\alpha_B < \gamma \hat{\alpha}_L(1)$, then ρ_B^c converges to (5), or equivalently, to $\alpha_B/\hat{\alpha}_L(1)$ (since $\sigma_B^* = 1$ by the seventh step above). Note that $\alpha_B < \gamma \hat{\alpha}_L(1)$ implies that $\rho_B^* = \alpha_B/\hat{\alpha}_L(1) < \gamma$. Suppose that ρ_B^c converges to some other value $\rho_B^f \neq \alpha_B/\hat{\alpha}_L(1)$. Then, noting (49) implies that Δ_B^c converges to (51) (because $\sigma_B^c = 1$ also by the seventh step), there are further two subcases to consider:

(a) Suppose that $\rho_B^f < \alpha_B/\hat{\alpha}_L(1)$. This implies that $\Delta_B^f > \hat{\Delta}_B(1)$. Let $\varepsilon \equiv \Delta_B^f - \hat{\Delta}_B(1)$. Then, there exists some $\delta > 0$ such that if $|\tilde{\kappa} - \underline{\kappa}| < \delta$, then $\Delta_B^c > \hat{\Delta}_B(1) + \varepsilon/2$ and $\tilde{\kappa} < \alpha_P E[\Delta - b^a | \Delta \in [b^a, \hat{\Delta}_B(1) + \varepsilon/2]]$. Note that the last inequality follows from (6), $\sigma_B^* = 1$, and the fact that $\tilde{\kappa}$ converges to κ. In turn, $\kappa^c \leq \tilde{\kappa} < \alpha_P E[\Delta - b^a | \Delta \in [b^a, \Delta_B^c]]$ combined with (50) imply that $\rho(\Delta_B^c) = \gamma$ for all $|\tilde{\kappa} - \underline{\kappa}| < \delta$, yielding a contradiction with $\rho_B^f < \alpha_B/\hat{\alpha}_L(1) < \gamma$.

(b) Next, suppose that $\rho_B^f > \alpha_B/\hat{\alpha}_L(1)$. This implies that $\Delta_B^f < \hat{\Delta}_B(1)$. Let $\varepsilon \equiv \hat{\Delta}_B(1) - \Delta_B^f$. Then, there exists some $\delta > 0$ such that if $|\tilde{\kappa} - \underline{\kappa}| < \delta$, then $\Delta_B^c < \hat{\Delta}_B(1) - \varepsilon/2$ and $\underline{\kappa} > \alpha_P E[\Delta - b^a | \Delta \in [b^a, \hat{\Delta}_B(1) - \varepsilon/2]]$. Note that the last inequality follows from (6), $\sigma_B^* = 1$, and the fact that $\underline{\kappa}$ converges to κ. In turn, $\kappa^c \geq \underline{\kappa} > \alpha_P E[\Delta - b^a | \Delta \in [b^a, \Delta_B^c]]$ combined with (50) imply that $\rho(\Delta_B^c) = 0$ for all $|\tilde{\kappa} - \underline{\kappa}| < \delta$, yielding a contradiction with $\rho_B^f > \alpha_B/\hat{\alpha}_L(1) > 0$. Therefore, (a) and (b) together imply that ρ_B^c converges to $\alpha_B/\hat{\alpha}_L(1)$, concluding the proof. ■

Proof of Proposition 13. First, note that in any equilibrium where the activist runs a proxy fight upon the incumbent's rejection with positive probability (regardless of whether the incumbent's rejection is on- or off-the-equilibrium path), the shareholders always support the activist, i.e., $\sigma_B = 1$. This is because if the activist runs a proxy fight, his expected payoff from doing so must be nonnegative, which in turn implies that the shareholder value is positive if the activist wins the proxy fight (because $b^a \geq 0$ and the activist needs to incur the cost of a proxy fight but shareholders do not), while the shareholders' payoff is zero if the incumbent is re-elected. Note that this result holds even if the incumbent's rejection is off-the-equilibrium path. This is because the activist does not have any private information relative to the shareholders before joining the board, and hence the shareholders' and the activist's beliefs about Δ must coincide upon the incumbent's rejection (when the activist is deciding whether to run a proxy fight).

Second, I show that in any equilibrium, the activist runs a proxy fight with positive probability upon rejection (i.e., $\rho^* > 0$). Suppose that $\rho^* = 0$. Note that the incumbent's payoff from accepting the board settlement is $\alpha_B(\Delta - b)$ if $\Delta \geq b^a$, and zero if $\Delta < b^a$. On the other hand, for any given ρ, the incumbent's payoff from rejecting the settlement is $\rho[-c_P + \alpha_P(\Delta - b)]$ if $\Delta \geq b^a$, and $-\rho c_P$ otherwise (due to the previous step). Therefore, if $\rho^* = 0$, then the incumbent rejects the board settlement for all $\Delta > b^a$. Then, the activist's expected payoff from running a proxy fight upon rejection is weakly larger than $\alpha_P E[\max\{0, \Delta - b^a\}] - \kappa^c$. However, (48) implies that this payoff is positive at least for some κ^c. In particular, it implies that $\rho^* \geq \gamma G(\alpha_P E[\max\{0, \Delta - b^a\}]) > 0$, yielding a contradiction.

Third, note that since $\rho^* > 0$ is satisfied in any equilibrium (due to the previous step), it immediately follows that in any equilibrium, the incumbent accepts the board settlement for all $\Delta < b^a$. Moreover, it also follows that in any equilibrium where proxy fight is not on the equilibrium path, it must be that the incumbent accepts the settlement for all Δ.

Therefore, as the fourth step, I show that an equilibrium where the incumbent accepts the board settlement for all $\Delta \geq b^a$ exists only if $\alpha_B \leq \gamma G(\alpha_P(b - b^a))(\alpha_P + c_P/(b - b^a))$. Recall that if $\Delta \geq b^a$, the incumbent weakly prefers to accept the settlement if and only if $\alpha_B(\Delta - b) \geq \rho[-c_P + \alpha_P(\Delta - b)]$. Therefore, the incumbent weakly prefers to accept for

all $\Delta \geq b^a$ if and only if $\alpha_B \leq \rho(c_P/(b-b^a)+\alpha_P)$. This inequality is not satisfied for any $\rho \in [0, \gamma G(\alpha_P(b-b^a))]$ if $\alpha_B > \gamma G(\alpha_P(b-b^a))(\alpha_P+c_P/(b-b^a))$, however it is satisfied for all $\rho \in [\frac{\alpha_B}{\alpha_P+c_P/(b-b^a)}, \gamma G(\alpha_P(b-b^a))]$ if $\alpha_B \leq \gamma G(\alpha_P(b-b^a))(c_P/(b-b^a)+\alpha_P)$. Also note that it has to be $\rho \leq \gamma G(\alpha_P(b-b^a))$ in equilibrium (even though the proxy fight is off-the-equilibrium path), because the activist's payoff after winning a proxy fight is strictly smaller than $\alpha_P(b-b^a)-\kappa^c$ (since $\Delta < b$ for all Δ). Therefore, there exists an equilibrium where the incumbent weakly prefers to accept the settlement for all Δ only if $\alpha_B \leq \gamma G(\alpha_P(b-b^a))(\alpha_P + c_P/(b-b^a))$.

It remains to show that the equilibrium where the incumbent accepts the settlement for all Δ never satisfies the Grossman and Perry (1986) criterion. To that end, denote by μ the activist's and shareholders' off-equilibrium belief about Δ upon the incumbent's rejection. Then, according to this criterion, this belief μ must be "credible". To put it more formally, if there exists any off-equilibrium belief ν such that under this belief, the incumbent weakly prefers to reject the settlement for some Δ (due to the resulting $\rho(\nu)$ and $\sigma(\nu)$), then μ must satisfy this property (i.e., μ must be equal to one of such ν). Therefore, the proof of this final step consists of two substeps.

First, I show that there exists a credible off-equilibrium belief which does not sustain the equilibrium described in this proposition. In particular, the activist's and shareholders' off-equilibrium belief $\nu = [b^a, \Delta_B^c]$ (that is, the belief that the incumbent rejects if and only if $\Delta \in [b^a, \Delta_B^c]$) is credible according to the Grossman and Perry (1986) criterion, where Δ_B^c is given by (49). This is because under this belief, the resulting $\rho(\nu)$ and $\sigma(\nu)$ indeed makes the incumbent weakly prefer to reject the settlement for some Δ (in particular, if and only if $\Delta \in [b^a, \Delta_B^c]$). More formally, under this belief, the equilibrium of the subgame starting with the incumbent's response is as given by Proposition 12: Specifically, $\rho^* = \rho_B^c$ and $\sigma^* = \sigma_B^c$, and therefore the incumbent rejects if and only if $\Delta \in [b^a, \Delta_B^c]$. This also implies that under this credible belief, it is never the case that the incumbent accepts the settlement for all Δ.

Since the credible belief described in the first substep above leads to the incumbent's rejection of the settlement for some Δ, the second substep is to show that if the activist's and shareholders' off-equilibrium belief μ about Δ leads to the incumbent's acceptance for all $\Delta \geq b^a$, then μ cannot satisfy the credibility criterion of Grossman and Perry (1986). To prove by contradiction, suppose that μ is credible. Then, under the belief μ, it has to be that the incumbent *weakly* prefers to reject the settlement for *some* Δ (for the belief μ to be credible), and it also has to be that the incumbent *weakly* prefers to accept the settlement *for all* $\Delta \geq b^a$ (so that the equilibrium is sustained). Note that under this belief μ, the likehood of a proxy fight $\rho(\mu)$ upon the incumbent's rejection would be given by $\rho(\mu) = \gamma \text{Prob}(\sigma(\mu)\alpha_P E[\max\{0, \Delta - b^a\} | \Delta \in \mu] > \kappa^c) = \gamma G(\sigma(\mu)\alpha_P E[\max\{0, \Delta - b^a\} | \Delta \in \mu])$. It cannot be $\rho(\mu) = 0$, because then the incumbent would not accept the settlement for any $\Delta \geq b^a$. Therefore, the only remaining possibility is $\rho(\mu) > 0$, which also implies that $\sigma(\mu) = 1$ by the first step. However, then, by the third step, the incumbent accepts the settlement for all $\Delta < b^a$. Moreover, by the fourth step in the proof of Proposition 12, it has to be $\rho(\mu) < \frac{\alpha_B}{\alpha_P+c_P/(b-b^a)}$, because otherwise either (a) the incumbent weakly prefers to reject the settlement if and only if $\Delta = b^a$, which would imply that $\mu = \{b^a\}$ and hence $\rho(\mu) = 0$,

yielding a contradiction, or (b) the incumbent *strictly* prefers to accept the board settlement for all Δ, which also yields a contradiction. Therefore, for the belief μ to be credible, it must be $\rho(\mu) < \frac{\alpha_B}{\alpha_P + c_P/(b-b^a)}$. However, then, again by the fourth step in the proof of Proposition 12, the incumbent would *strictly* prefer to reject the settlement for some Δ. This would imply that the belief μ does not sustain the equilibrium, since it is not possible that the incumbent accepts the settlement for all Δ under this belief. This yields another contradiction, concluding the proof. ■

F Online appendix for Extensions (Section 5)

In this appendix, I study several extensions of the main model. For a summary of these extensions, see Section 5.

F.1 Imperfect learning upon joining the board (Section 5.1)

In this extension, I relax the assumption that the activist always learns the value of the project if he joins the board. I show that not only all the results from the main model are robust to this extension, but also the empirical predictions get even stronger. Moreover, this section also implies an endogenous upperbound on α_B for the main model by showing that the activist may not always be able to exercise his decision authority after reaching a board settlement even if he obtains full control.

In terms of the specific assumptions, I assume that if the activist joins the board via winning a proxy fight or reaching a board settlement, then he receives a signal $s = \Delta$ with probability $q \in (0, 1)$, and he does not receive any signal with probability $1 - q$. Note that if $q = 1$, then the model reduces to the main model described in Section 2 (where the activist always learns Δ after joining the board). I also assume that if the activist's beliefs about the NPV at the implementation stage is equal to the prior distribution, he does not implement the project. To simplify the analysis below, I achieve this by assuming $E[\Delta - b^a] < 0$.[80]

I also assume that the activist's cost of proxy fight C^a satisfies

$$C^a < \check{C}^a \equiv q\alpha_P n^a E[\max\{0, \Delta - b^a\}], \tag{52}$$

[80] However, even if $E[\Delta - b^a] \geq 0$, I verify in an unreported analysis that similar results are obtained in a broader setup where the activist suffers some reputation costs if the project he implements ends up destroying a lot of firm value. To be specific, in this broader setup, I assume that if the activist implements the project, $n^a(\Delta - b^a)$ in the activist's payoff $\Pi^a(\Delta, e)$ in Section 2 is replaced by $n^a(\Delta - b^a) + \mu(\Delta)$, where $\mu(\Delta)$ is the additional payoff of the activist from the reputation he gains or loses from implementing the project. Here, I assume that $\mu(\Delta)$ is weakly increasing in Δ, $\mu(0) \geq 0$, and $E[n^a(\Delta - b^a) + \mu(\Delta)] < 0$. The last inequality implies that the activist does not implement the project if his information set is not more precise than the prior distribution at the implementation stage. Under this broader setup, all results of the extension continue to hold, where the only additional change is that the activist's bias b^a per share he owns is replaced with $b^a - \mu(\Delta')/n^a$, where Δ' denotes the unique solution of $n^a(\Delta - b^a) + \mu(\Delta) = 0$.

which is the generalization of the parameter restriction made in (2). Indeed, if C^a is so large that (52) is violated, then there always exists an equilibrium where the activist never runs a proxy fight upon rejection, and hence the incumbent has no incentive to accept any settlement and the project is never implemented.

Under this modification of the model, the equilibria described in Propositions 1 and 5 continue to be valid, with the exception that α_B in the former is replaced with $\alpha_B q$, and α_P in the latter is replaced with $\alpha_P q$. To understand this, first consider the former case, where the activist has demanded a board settlement. Then, upon rejection of a board settlement, the activist infers that $\Delta \geq b^a$, and therefore implements the project if he wins a proxy fight and becomes the decision-maker on the board, regardless of whether he receives any signal. However, upon *acceptance* of board settlement, the activist cannot immediately infer whether $\Delta \geq b^a$ or $\Delta < b^a$: In particular, compared to the prior distribution, the project's NPV Δ is more likely to be lower than b^a and the expected NPV is lower. Therefore, even if the activist achieves decision authority after the incumbent accepts a board settlement, the activist implements the project if and only if he receives a signal about the project that its NPV is higher than b^a. Since the activist learns the NPV with probability q, this means that even though the activist is the decision-maker on the board with probability α_B after reaching a board settlement, he implements the project only with probability $\alpha_B q$ when $\Delta \geq b^a$. Therefore, the equilibrium given by Proposition 1 continues to be valid, with the exception that α_B is replaced with $\alpha_B q$ (hence, it implies an endogenous upperbound on α_B for the main model). Compared to the main model, the activist's reduced ability to implement the project upon reaching a board settlement increases the incumbent's incentives to accept the settlement, resulting in a lower Δ_B^*, as illustrated by the top figure in Figure 11.

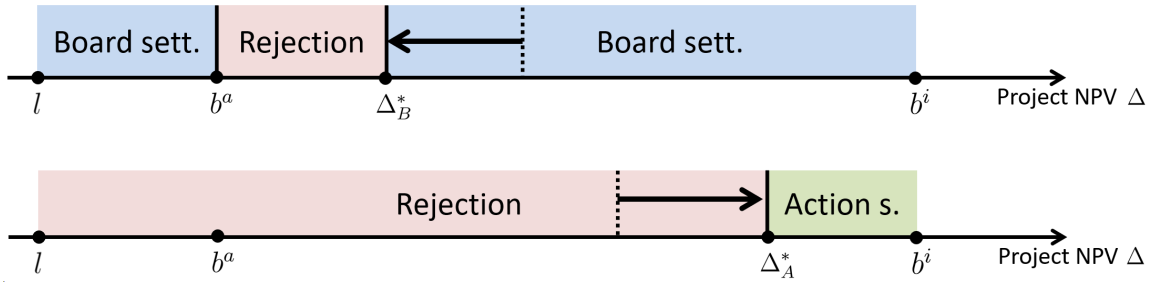


Figure 11 - The changes in the incumbent's rejection thresholds w.r.t. to the main model

Next, suppose that the activist has demanded an action settlement instead. Note that upon rejection of an action settlement (that is, conditional on $\Delta \leq \Delta_A^*$), the project's NPV is more likely to be lower than b^a and the expected NPV is lower, compared to the prior distribution. Therefore, even if the activist runs and wins a proxy fight upon rejection and obtains decision authority over implementation of the project, he implements the project if and only if he receives a signal about the project that its NPV is higher than b^a. For this reason, after a successful proxy fight, even though the activist is the decision-maker with probability α_P, when $\Delta \geq b^a$ he implements the project with probability $\alpha_P q$. Therefore, the

equilibrium given by Proposition 5 still constitutes an equilibrium, with the exception that α_P is replaced with $\alpha_P q$. Compared to the main model where $q = 1$, the activist's reduced ability to implement the project decreases his incentives to run a proxy fight upon rejection and increases the incumbent board's incentives to reject the settlement, resulting in a higher Δ_A^*, as illustrated by the bottom figure in Figure 11.

Due to preservation of the equilibrium structure as described above, the results from the main model continue to hold, as elaborated below. Moreover, as also detailed below, the empirical predictions of the main model get even stronger under this extension. This is because the changes explained above makes the inequality $\Delta_B^* \leq \Delta_A^*$ in Corollary 1 stronger, since they strengthen the insight behind these predictions that was introduced in Section 4.2: That is, compared to the main model, the incumbent's rejection of board settlement reveals its private information about the project's value *even more* precisely than the rejection of action settlement reveals, because Δ_B^* decreases while Δ_A^* increases.

The details on the robustness of the results and empirical predictions of the model are as follows:

(a) As explained above, the equilibrium described in Proposition 1 in Section 3.1 continue to be valid, with the exception that α_B is replaced with $\alpha_B q$. As a result, Propositions 2 and 3 in Section 3.2.1 continues to hold without any change, because these results do not depend on the magnitude of α_B. Proposition 4 in Section 3.2.2 continues to hold as well, where the only changes are that $\bar{C}^a$ is replaced with $\check{C}^a$ due to the generalization of parameter specification to (52), and α_B is again replaced with $\alpha_B q$.

(b) Also also explained above, the equilibrium described in Proposition 5 in Section 4.1 continue to be valid, with the exception that α_P is replaced with $\alpha_P q$. Therefore, Propositions 8 and 9 in Online Appendix B continues to hold without any change, because they do not depend on the magnitude of α_P. Proposition 10 in the same appendix continues to hold as well, where the only change is that $\bar{C}^a$ is replaced with $\check{C}^a$ due to the generalization of parameter specification to (52).

(c) Corollary 1 in Section 4.2 continues to hold, because under this extension Δ_A^* gets weakly larger and Δ_B^* gets weakly smaller, as explained above.

(d) Proposition 6 in Section 4.2.1 continues to hold as well, subject to the only change that α_P in $\underline{l}$ is replaced with $\alpha_P q$.[81] This is because even though α_P is replaced with $\alpha_P q$ if the activist demands an action settlement and α_B is replaced with $\alpha_B q$ if the activist demands a board settlement, the rest of the proof of Proposition 6 continues to hold.

(e) The robustness of the results above from the main model implies that all of the empirical predictions of the main model (as described in Section 4.2.2) continue to hold. Moreover, due to (c) above, predictions 3 and 4 get even stronger.

[81] In other words, $\underline{l}$ becomes $\underline{l} = b^i - \alpha_P q \frac{(b^i - b^a)^2}{2C^a/n^a}$. This is because if Δ has a uniform distribution, then (52) is equivalent to $C^a < \alpha_P q n^a E\left[\max\{0, \Delta - b^a\}\right] \Leftrightarrow C^a < \alpha_P q n^a Prob(\Delta \geq b^a) E\left[\Delta - b^a | \Delta \geq b^a\right] \Leftrightarrow C^a/n^a < \alpha_P q \frac{b^i - b^a}{b^i - l}\left(\frac{b^i + b^a}{2} - b^a\right) \Leftrightarrow b^i - \alpha_P q \frac{(b^i - b^a)^2}{2C^a/n^a} < l$.

(*f*) Proposition 7 in Section 7 continue to hold as well, where the only change is that α_P in $\underline{l}$ is replaced with $\alpha_P q$ for the same reason stated in (d) above. Finally, Corollary 2 continues to hold without any change.

F.2 Incumbent's disclosure and information sharing agreements (Section 5.2)

Recall that in the main model it is implicitly assumed that the incumbent board cannot disclose Δ to the activist unless the activist joins the board. In this extension, I relax this assumption and show that all the results and empirical predictions from the main model continue to hold.

In particular, in this extension I add a disclosure stage at the very beginning of the game: I assume that at this stage, with probability q the incumbent can voluntarily choose whether to disclose Δ to the activist (e.g., via information sharing agreements), and with probability $1-q$, no disclosure can take place. I also assume that disclosure is verifiable. The parameter restrictions imposed in this extension are as follows: First, $q < 1$, which means that the incumbent is not always able to disclose Δ to the activist, and second, if $\alpha_P < 1$ then

$$\frac{C^i/n^i}{1-\alpha_P} > b^i \qquad \text{or} \qquad \frac{C^a/n^a}{\alpha_P} + \frac{C^i/n^i}{1-\alpha_P} > b^i - b^a, \tag{53}$$

which means that a proxy fight is highly costly to the activist and/or to the incumbent. This condition is consistent with the empirical evidence in Gantchev (2013) and Fos and Tsoutsoura (2014).[82] For the discussion of the assumption $q < 1$, see Section F.2.1 below. Before that discussion, however, I first present and discuss the results.

Proposition 14 *Let* $d \equiv \max\{0, \frac{C^a/n^a}{\alpha_P} + b^a\}$. *Then,* $d \in (b^a, b^i)$, *and there exists an equilibrium where:*[83]

(*i*) *If the incumbent is able to disclose* Δ *to the activist, it discloses if and only if* $\Delta \leq d$.

(*ii*) *If the incumbent discloses* $\Delta \leq d$, *then no material outcome occurs (i.e., no settlement is reached, no proxy fight is launched, and the project is never implemented).*

[82] Consistently with high C^a, Gantchev (2013) reports that the activist's cost of a campaign ending with a proxy fight accounts for two thirds of the average abnormal return of the activist, implying that these costs are significant. Consistently with high C^i, Fos and Tsoutsoura (2014) find that facing a direct threat of removal is associated with \$1.3-2.9 million in foregone income until retirement for the median incumbent director. They also find that after a proxy fight, not only incumbent directors that were up for re-election during the proxy fight lose on average 0.71 on other boards, but also the other incumbent directors (who were not up for re-election) lose on average 0.45 seats on other boards.

[83] While the equilibrium is not necessarily unique, the potential multiplicity of equilibria stems from the indifference conditions of the incumbent, and it can be shown that the described equilibrium becomes the unique equilibrium if some small costs are incorporated to break these indifference conditions. An example of such costs would be disclosure costs, as well as costs that the incumbent suffers from conceding board seats to the activist.

(*iii*) *If the incumbent does not disclose* Δ*, then the equilibrium of this subgame is as described in Propositions 5 and 1.*

The proof of the proposition above is provided in Section F.2.2 below. The proposition states that there exists a threshold $d = \max\{0, \frac{C^a/n^a}{\alpha_P} + b^a\} \in (b^a, b^i)$ such that the incumbent discloses Δ if and only if $\Delta \leq d$. Intuitively, if no settlement is reached, then the activist never runs a proxy fight when $\Delta \leq d$ (because then his payoff from running a proxy fight is $\sigma^* \alpha_P n^a \max\{0, \Delta - b^a\} - C^a \leq 0$, where $\sigma^* = 0$ if $\Delta \leq 0$), and he always runs a proxy fight if $\Delta > d$ (because then he always wins a proxy fight, implying that his payoff from running a proxy fight is $\alpha_P n^a(\Delta - b^a) - C^a > 0$). In particular, when $\Delta \leq d$, the incumbent fends off the activist by disclosing that the project is worthless or does not have enough value to make a proxy fight profitable for the activist. Therefore, if $\Delta \leq d$ and the incumbent is able to disclose, the game practically ends after the disclosure, resulting in no action or board settlement (as well as no proxy fight). In contrast, if $\Delta > d$ were to be disclosed, then the activist would always run and win a proxy fight if no settlement is reached, and since the activist knows that this is costly for the incumbent, he would demand an action settlement after the disclosure (which the incumbent would end up accepting). Therefore, this incentivizes the incumbent not to disclose when $\Delta > d$. Indeed, since the incumbent is not always able to disclose a given Δ (that is, $q < 1$, as also elaborated in Section F.2.1 below), by not disclosing when $\Delta > d$, the incumbent can pool with the cases where the incumbent was not able to disclose $\Delta \leq d$.

Therefore, the results and empirical predictions of main model are robust to this extension as well. This is because as Proposition 14 above states, if $\Delta \leq d$ then with probability q no material outcome occurs; and in all other cases (i.e., with probability 1 if $\Delta > d$, and with probability $1 - q$ if $\Delta \leq d$), the equilibrium is as described in Propositions 5 and 1.

F.2.1 Motivating imperfect disclosure

Here, I motivate the assumption $q < 1$ made above, that is, with some positive probability, the incumbent will not be able to disclose Δ to the activist. Indeed, this assumption can be motivated by several arguments, which are detailed as follows:

Disclosing the value of the project: Note that for the incumbent to be able to disclose Δ to the activist, the incumbent may need to disclose information that was not disclosed publicly before. However, due to Regulation Fair Disclosure (Reg FD), the incumbent may then have to make this informaton public. This is because due to Reg FD, the incumbent has to make public disclosure of any material information disclosed outside the board to a shareholder that is likely to trade, such as an activist. However, public disclosure of such information may harm firm value (e.g., by giving an informational advantage to the competitors of the firm), especially considering that the incumbent had originally found it optimal not to share such information with the public. Moreover, if such a disclosure hurts firm value, it may also result in the breach of fiduciary duty of the incumbent. Therefore, such costs may make it prohibitively costly for the incumbent to disclose Δ to the activist.

Disclosing the value of the incumbent's preferred plans (i.e., "status quo"): Moreover, note that Δ represents the shareholder value created by the project *relative* to the "status quo", where status quo represents any plan the incumbent board prefers to pursue over the project. Importantly, since the incumbent may be formulating new plans to pursue, the source of the incumbent's private information about Δ may stem from the shareholder value created by pursuing these plans. Indeed, the details of such plans may not be available to the public. Therefore, if the incumbent were to share these plans with the activist (so that the activist can evaluate Δ), the incumbent may have to make public disclosure of some of this information, again due to Reg FD. However, due to the same arguments above, such a disclosure can have very costly consequences for the firm as well as for the incumbent board.

Information sharing agreeements: An imperfect solution for this problem is sometimes the incumbent can make such disclosures to the activist via exclusive information sharing agreements that can be made between the activist and the firm. However, such agreements are not without their problems: First, it is not fully clear whether such agreements legally satisfy Reg FD, which may make activists more hesitant with such agreements. Indeed, Schoenfeld (2020) tests this hypothesis and reports that shareholder contracts with information sharing covenants has dropped significantly with the introduction of Reg FD. Second, the information the activist can acquire via such agreements is likely to be more limited and less precise compared to the information he can acquire by having board seats. For these reasons, information sharing agreements are unlikely to be a perfect solution to eliminate the information asymmetry between the incumbent board and the activist. Indeed, Schoenfeld (2020) reports that higher information asymmetry is a key predictor of activist shareholder agreements and documents that the most common covenant type in the agreements is directorship covenants, surpassing the number of information sharing covenants.[84]

F.2.2 Proofs for incumbent's disclosure and information sharing agreements

Proof of Proposition 14. Note that $d < b$ since $\frac{\kappa}{\alpha_P} + b^a < b$ due to (2) and $\Delta < b$, and $d > b^a$ holds because $d = \frac{\kappa}{\alpha_P} + b^a$ and $\kappa > 0$.

If the incumbent discloses Δ, then the activist's payoff from running a proxy fight is nonpositive if $\alpha_P \max\{0, \Delta - b^a\} - \kappa \leq 0$, or equivalently, if $\Delta \leq \frac{\kappa}{\alpha_P} + b^a$. Similarly, if the incumbent discloses that $\Delta \leq 0$, then it is a best response for the shareholders to never elect the activist in a proxy fight. In turn, if $\Delta \leq \max\{0, \frac{\kappa}{\alpha_P} + b^a\}$, then it is a best response for the incumbent to disclose Δ and reject any settlement, and no proxy fight is launched as a result. As a result, the project is never implemented following such a disclosure due to Lemma 1. This completes part (ii).

The proof of parts (i) and (iii) consists of several steps. First, note that if the incumbent

[84] Moreover, since many board settlements also contain information sharing covenants, the number of information sharing agreements that do not grant the activist a board seat is even lower. E.g., see: http://clsbluesky.law.columbia.edu/2016/12/15/sullivan-cromwell-reviews-and-analyzes-2016-u-s-shareholder-activism/

discloses that $\Delta > d$ and no settlement is reached, then the activist always runs a proxy fight if $k = \kappa$ (because $\alpha_P \max\{0, \Delta - b^a\} - \kappa > 0$) and wins it (because $\Delta > 0$).

Second, if the incumbent has disclosed that $\Delta > d$ and the activist demands an action settlement, then the incumbent always accepts. This is because the incumbent's payoff from accepting the action settlement is strictly larger than rejecting it, that is, $\Delta - b > -c_P + \alpha_P(\Delta - b)$ for all $\Delta > d$, where the inequality holds because if $\alpha_P < 1$ then (53) holds.

Third, if the incumbent has disclosed that $\Delta > d$, the activist weakly prefers demanding an action settlement over demanding a board settlement. This is because his payoff if an action settlement is reached is $\Delta - b^a$, which is weakly larger than his payoff if a board settlement is reached ($\alpha_B(\Delta - b^a)$) as well as than his payoff if no settlement is reached ($\alpha_P(\Delta - b^a) - \kappa$), because $d > b^a$ as noted at the beginning of the proof.

Fourth, if $\Delta > d$, then the incumbent weakly prefers not to disclose Δ. To show this result, due to the second and third steps above, it is sufficient to show that the incumbent weakly prefers nondisclosure over disclosing and accepting an action settlement. Indeed, if it does not disclose, then a lowerbound on its payoff is $\Delta - b$, since at worst the activist will demand an action settlement (the incumbent's payoff from reaching a board settlement is weakly higher, since $\alpha_B(\Delta - b) \geq \Delta - b$). However, the incumbent's payoff from disclosing and accepting an action settlement is $\Delta - b$. Combining with the result in part (ii), this completes the proof of parts (i) and (iii).

Although the proof is complete, I also show that the results do not change even if the activist is prohibited from making any settlement demand. Note that then, the proof of part (ii) above does not change. Therefore, it remains to show the robustness of parts (i) and (iii). Indeed, as noted above, if the incumbent discloses $\Delta > d$ and no settlement is reached, then the activist always runs a proxy fight if $k = \kappa$ (because $\alpha_P \max\{0, \Delta - b^a\} - \kappa > 0$) and wins it (because $\Delta > 0$). Therefore, the incumbent's expected payoff from disclosing $\Delta > d$ is $\alpha_P(\Delta - b) - c_P$. Note that if the incumbent does not disclose Δ, then his expected payoff is weakly larger, i.e., it is $\rho(\sigma\alpha_P(\Delta - b) - c_P) \geq \alpha_P(\Delta - b) - c_P$. Therefore, for all $\Delta > d$, the incumbent weakly prefers nondisclosure over disclosing. ■

F.3 Incumbent's counter-offer (Section 5.3)

An assumption made in the main model is that when demanding a settlement, the activist makes a take-it-or-leave-it offer to the incumbent board. However, in practice, sometimes boards make counter-offers to the activist. Therefore, in this extension I assume that after the activist makes his demand, the incumbent can make a counter-offer to the activist. If no agreement is reached, then as in the main model, the activist decides whether to run a proxy fight, and in the case of a proxy fight, the shareholders decide whom to support.

The propositions below formalize the results described in Section 5.3. Intuition for these results are provided in Section 5.3 as well.

Proposition 15 (*i*) *Suppose that the activist has demanded a board settlement. Then, there always exists an equilibrium that is identical to the equilibrium in Proposition 1. In this*

equilibrium, the incumbent never makes a counter-offer. Moreover, if $\alpha_B < 1$, then there exists no equilibrium where the incumbent counters with an action settlement.

(*ii*) *Suppose that the activist has demanded an action settlement. There exists $\bar{\alpha}_B > 0$ such that if $\alpha_B < \bar{\alpha}_B$, then there always exists an equilibrium that is identical to the equilibrium in Proposition 5. In this equilibrium, the incumbent never makes a counter-offer.*

(*iii*) *Suppose that the activist has demanded an action settlement and $\alpha_B < 1$.*[85] *Suppose that there exists an equilibrium where the incumbent counters with a board settlement if $\Delta < b^a$, and the activist accepts this counter-offer. Then, in this equilibrium, the incumbent never accepts the action settlement, and there exists $\tilde{\Delta}_B < b$ such that the incumbent counters with a board settlement if $\Delta > \tilde{\Delta}_B$.*

Proof of Proposition 15. In the proof below, first I show that given the activist's settlement demand, each player's action described in the proposition constitutes a best response to each other. To that end, steps 1-3 prove part (i), and step 4 proves part (ii). Finally, in step 5, I show that the activist's settlement demand described at the beginning of the proposition constitutes a best response as well.

Step 1: First, suppose that the incumbent cannot make any counter-offer. Then, the incumbent's decision boils down to accepting or rejecting the activist's settlement demand, just as in the main model. Moreover, note that the equilibrium characterization implies that upon no settlement, ρ^* (the probability that the activist runs a proxy fight) as well as σ^* (the probability that he wins) are as characterized in Proposition 5 if the activist has demanded an action settlement and as in Proposition 1 if the activist has demanded a board settlement. Therefore, the incumbent's strategy described in Propositions 5 and 1 indeed constitutes a best-response for the incumbent. As a result, ρ^* (σ^*) characterized in Propositions 5 and 1 constitute a best response for the activist (shareholders) as well.

Step 2: Next, suppose that the activist has demanded a board settlement and the incumbent can counter. Note that the incumbent weakly prefers accepting the board settlement over deviating to countering with an action settlement if the activist's off-equilibrium beliefs about Δ upon such a deviation satisfies $\Delta > b^a$. This is because under such a deviation, the activist always accepts the action settlement, yielding a weakly lower payoff to the incumbent than accepting the activist's board settlement demand outright.

Also suppose that the activist and shareholders' off-equilibrium beliefs conditional on the incumbent rejecting and countering with a board settlement are $\Delta \in [b^a, \Delta_B^*]$, where Δ_B^* is given in Proposition 1 (i.e., these off-equilibrium beliefs are the same with on-the-equilibrium-path beliefs conditional on the incumbent rejecting and not making any counter-offer). Then, if no settlement has been reached (regardless of whether the incumbent counters with a board settlement or makes no counter-offer after rejecting the activist's settlement demand), the same ρ^* and σ^* as those characterized in Proposition 1 indeed constitute a best response for

[85] Recall that reasons for $\alpha_B < 1$ were discussed after Corollary 1 in Section 4.2.

the activist and shareholders, respectively. As a result, it is a best response for the incumbent to make no counter-offer, and instead behave as described in Proposition 1 as well. This is because Proposition 1 implies that given ρ^* and σ^*, the incumbent weakly prefers reaching a board settlement over not reaching such a settlement if $\Delta \in (l, b^a) \cup (\Delta_B^*, b)$, and it weakly prefers not reaching a board settlement over reaching such a settlement if $\Delta \in [b^a, \Delta_B^*]$.

Step 3: Finally, to complete the proof of part (i), suppose that the activist has demanded an action settlement, the incumbent can counter, and $\alpha_B < \gamma\alpha_P - \gamma\frac{\kappa}{b-b^a}$. Also suppose that the activist and shareholders' off-equilibrium beliefs are such that if the incumbent rejects the activist's action settlement demand and counters with a board settlement, then Δ is sufficiently high (specifically, it satisfies $\Delta > 0$ and $\alpha_B(\Delta - b^a) < \gamma\alpha_P(\Delta - b^a) - \gamma\kappa$).[86] Then, it is a best response for the activist to reject this counter-offer (and run a proxy fight whenever $k = \kappa$) and for the shareholders to always elect the activist. Therefore, in turn, it is a best response for the incumbent to never counter with a board settlement, because instead it can simply reject the activist's action settlement demand and make no counter-offer, which would yield weakly greater payoff to the incumbent since $\rho \leq \gamma$ and $\sigma \leq 1$.

In addition, in a similar spirit with the last step, suppose that the activist and shareholders' off-equilibrium beliefs conditional on the incumbent rejecting and countering with an action settlement are $\Delta \leq \Delta_A^*$, where Δ_A^* is given in Proposition 5 (i.e., these off-equilibrium beliefs are the same with on-the-equilibrium-path beliefs conditional on the incumbent rejecting and not making any counter-offer). Then, if no settlement is reached (regardless of whether the incumbent counters with an action settlement or makes no counter-offer after rejecting the activist's settlement demand), the same ρ^* and σ^* as those characterized in Proposition 5 indeed constitute a best response for the activist and shareholders, respectively. As a result, it is a best response for the incumbent to make no counter-offer, and instead behave as described in Proposition 5 as well. This is because Proposition 5 implies that given ρ^* and σ^*, the incumbent weakly prefers reaching an action settlement over not reaching such a settlement if $\Delta > \Delta_A^*$, and it weakly prefers not reaching an action settlement over reaching such a settlement if $\Delta \leq \Delta_A^*$.

Step 4: Next, I prove part (ii). To that end, suppose that the activist has demanded an action settlement, the incumbent can counter, and $\alpha_B \geq \gamma\alpha_P - \gamma\frac{\kappa}{b-b^a}$. Then, it is a best response for the activist to never run a proxy fight when the incumbent counters with a board settlement, because for any Δ, ρ', and σ', the activist's payoff from accepting the board settlement is weakly higher than his payoff from rejecting and running a proxy fight, i.e., $\alpha_B \max\{0, \Delta - b^a\} \geq \rho'\sigma'\alpha_P \max\{0, \Delta - b^a\} - \rho'\kappa$.[87] Moreover, note that the equilibrium

[86] Note that there exists $\Delta' \in (b^a, b)$ such that the inequality $\alpha_B(\Delta - b^a) < \gamma\alpha_P(\Delta - b^a) - \gamma\kappa$ is satisfied if and only if $\Delta > \Delta'$. That is because: (a) both sides of the inequality are linear in Δ, (b) the inequality is not satisfied for $\Delta = b^a$, (c) the inequality is satisfied for $\Delta = b$ due to $\alpha_B < \gamma\alpha_P - \gamma\frac{\kappa}{b-b^a}$, and (d) $b^a < b$ due to (2).

[87] This inequality is satisfied for all $\Delta < b$ because: (a) both sides of the inequality are linear in Δ, (b) the inequality is satisfied for $\Delta \leq b^a$, (c) the inequality is satisfied for $\Delta = b$ due to $\alpha_B \geq \gamma\alpha_P - \gamma\frac{\kappa}{b-b^a}$, and (d) $b^a < b$ due to (2).

characterization implies that upon no settlement, ρ^* as well as σ^* are as characterized in Proposition 1. Therefore, Proposition 1 implies that the incumbent weakly prefers reaching a board settlement over reaching no settlement if $\Delta \in (l, b^a) \cup (\Delta_B^*, b)$, and it weakly prefers reaching no settlement over reaching such a settlement if $\Delta \in [b^a, \Delta_B^*]$. Since in addition the incumbent weakly prefers reaching a board settlement over reaching an action settlement, it is a best response for the incumbent to always reject the action settlement, and counter with a board settlement if $\Delta \in (l, b^a) \cup (\Delta_B^*, b)$ and make no counter-offer if $\Delta \in [b^a, \Delta_B^*]$. In turn, the same ρ^* and σ^* as those characterized in Proposition 1 indeed constitute a best response for the activist and shareholders, respectively.

Step 5: Finally, I complete the proof of the proposition by showing that given the strategy profile described in Proposition 15, the activist's choice of settlement demand characterized at the beginning of the proposition is best response as well. Note that part (i) implies that in the case where $\alpha_B < \gamma\alpha_P - \gamma\frac{\kappa}{b-b^a}$, the subgame equilibria after the activist makes his settlement demand are identical with those in the main model (i.e., those in Propositions 5 and 1). Therefore, when deciding what settlement demand to make, the strategy described in Proposition 6 continues to maximize the activist's expected payoff.

Moreover, if $\alpha_B \geq \gamma\alpha_P - \gamma\frac{\kappa}{b-b^a}$, then parts (i) and (ii) imply that the activist's settlement demand is irrelevant in the cases where the incumbent is able to make a counter-offer: Because then, regardless of the activist's demand, the activist's expected payoff is equal to his expected payoff in Proposition 1. Therefore, when choosing what settlement to demand, the activist only takes into account the case where the incumbent will not be able to make a counter-offer. In turn, since part (i) again implies that in this case (i.e., the incumbent is unable to counter), the subgame equilibria after the activist makes his settlement demand are identical with those in the main model (i.e., those in Propositions 5 and 1), the strategy described in Proposition 6 continues to maximize the activist's expected payoff. ■

F.4 Incumbent with $b^i < 0$ (Section 5.4)

Recall that an assumption of the main model was that the incumbent is biased against the project on the table. Specifically, relative to the status quo, if the project is implemented then the incumbent's payoff was assumed to be $\Delta - b^i$ per share because it loses private benefits $b^i > 0$, as motivated in the setup (Section 2). However, it could also be that compared to keeping the status quo, the incumbent is biased in favor of the project (i.e., $b^i < 0$).[88] Suppose that this is the case. Then, letting $\Delta_s \equiv -\Delta$ and $b_s^i \equiv -b^i$, the incumbent's payoff per share from keeping the status quo (relative to implementing the project) can be expressed as $\Delta_s - b_s^i$, where $b_s^i > 0$.[89] Therefore, the main model and its analysis continue to hold if the labels of

[88] Also note that then the distribution of Δ would satisfy $\Delta > b^i$. This is because if $\Delta \leq b^i$, then the incumbent would prefer not to implement this project, and therefore it would not have or announce any intention to implement this project. Hence, it would not prompt any activist campaign in the first place (because the activist and the shareholders would infer that $\Delta < 0$ from $\Delta \leq b^i$).

[89] Moreover, the previous footnote implies that $\Delta_s < b_s^i$.

"status quo" and "project" are simply switched.

For example, consider the case where in the original model, the firm can undertake a spin-off but the incumbent is opposed to it (i.e., $b^i > 0$). Then, the project represents "the spin-off" while the status quo represents "not doing the spin-off," where the incumbent is in favor of the status quo. Now, consider the opposite case, where the incumbent is in favor of a spin-off (i.e., $b^i < 0$). Then, the original model and its results continue to hold, where the only change is that the project now represents "not doing the spin-off" and the status quo represents "the spin-off" (note that again, the status quo represents the action that the incumbent is in favor of). Therefore, the model applies just as well in the instances of $b^i < 0$.

F.5 Micro-foundation of the activist's bias $b^a < 0$ (Section 5.4)

In the main model, it was assumed that the activist has an exogenous bias b^a for the project, and it was shown in Section 3.2.1 and Online Appendix B.2 that this bias has interesting implications if $b^a < 0$ (i.e., the activist's bias is in favor of implementing the project). In the extensions below, such a bias b^a arises endogenously and indeed satisfies $b^a < 0$.

F.5.1 Short-termism

Motivated by the concerns expressed by institutional investors, policymakers, and the SEC about activist short-termism,[90] in this section I micro-found the activist's bias under short-termism. More specifically, in this extension, I assume that after the project is implemented, with probability $\phi \in (0, 1)$ the activist can sell his stake in the firm before the project's NPV Δ becomes public information. The market is perfectly liquid and hence the market maker does not infer any information about the activist's trading.

Consistent with the criticism against short-termism, the activist's ability to sell his stake before the NPV becomes public information creates incentives for the activist to focus on the short-term stock price at the expense of the long-term value. As a result, there exists a unique b^{a*} such that the activist implements the project if and only if $\Delta \geq b^{a*}$; and moreover $b^{a*} < 0$. Note that even if Δ is negative, the activist can still get a positive payoff from implementing the project and selling his stake, because with probability ϕ, Δ does not become public information by then and hence the stock price incorporates the possibility that Δ is positive (in contrast, if the activist does not implement the project, then the market observes this and the activist receives a payoff of zero). The following proposition formalizes the results, and its proof is provided in Section F.5.3 further below.

Proposition 16 *Suppose that the activist has achieved the decision-making authority in the board (through a board settlement or a successful proxy fight), and denote the cumulative probability distribution of Δ conditional on this event by $G(\cdot)$, where $G(0) < 1$ (i.e, the probability*

[90] For such concerns raised by institutional investors, see the second paragraph in the introduction. For concerns expressed by the SEC and policymakers about activist short-termism, as well as rule changes they have suggested to combat it, see the footnote in Section 5.4.

that $\Delta > 0$ is positive). Then, for any such $G(\cdot)$, there exists a unique b^{a*} such that the activist implements the project if and only if $\Delta \geq b^{a*}$. Moreover, $b^{a*} < 0$.

F.5.2 The activist's reputation about his skills

The activist's bias $b^a < 0$ emerges endogenously also when there is uncertainty about the activist's ability to implement the project. To that end, consider an extension where there are two types of activists: skilled and unskilled. In particular, suppose that when the activist achieves decision authority on the board, he can implement the project if and only if he is skilled. To keep the modification simple, also suppose that the activist learns his skill when he attempts to implement the project (which avoids further signalling complications). Note that if it is revealed that the activist is unskilled, then he would lose his credibility to run a proxy fight, and hence he would lose his ability to affect firm value. However, if the activist implements the project, then the market infers that the activist is skilled, which would benefit the activist in his future campaigns by giving him more credibility. Therefore, when there is uncertainty about the activist's skill, implementing the project gives the activist a payoff of $\Delta - b^a$ per share he owns, where $-b^a > 0$ due to the benefit he receives from revealing that he is skilled.

F.5.3 Proofs for micro-foundation of the activist's bias $b^a < 0$

Proof of Proposition 16. As described in the statement of the proposition, suppose that the activist has achieved the decision-making authority in the board. First, I show that the activist follows a threshold strategy. In particular, the activist's payoff from keeping the status quo (i.e., not implementing the project) is zero, since the market observes that the project is not implemented and the stock price reflects this information. In contrast, per share the activist owns in the firm, the activist's payoff implementing the project is Δ if he keeps his stake until the NPV is revealed (i.e., until it becomes public information). Therefore, for any $\Delta > 0$, the activist strictly prefers to implement the project, since his payoff by doing so and keeping his stake is Δ per share, which is positive and hence strictly greater than his payoff from not implementing the project, which is zero. Also note that per share the activist owns in the firm, his payoff implementing the project is V^* if he sells his stake before the NPV is revealed, where V^* is the market's expectation of the NPV conditional on the activist's implementation of the project. Therefore, if $\Delta < 0$, then the activist implements the project if and only if $\phi V^* + (1-\phi)\Delta \geq 0$, which is his expected payoff from implementing the project and then selling his stake with probability ϕ before the NPV is revealed. As a result, there exists some b^{a*} such that the activist implements the project if and only if $\Delta \geq b^{a*}$.

Second, note that $b^{a*} \leq 0$. This is because for any $\Delta > 0$, the activist strictly prefers to implement the project as noted in the previous step.

Third, note that the steps above imply that $V^* = \frac{1}{1-G(b^{a*})} \int_{b^{a*}}^{\infty} \Delta dG(\Delta)$, where $G(b^{a*}) < 1$ due to the previous step.

Fourth, note that b^{a*} is unique. This is because due to the third step, $\phi V^* + (1-\phi)\Delta$ is strictly increasing in Δ and b^{a*}.

Fifth, finally I show that $b^{a*} < 0$. To prove by contradiction, suppose that $b^{a*} \geq 0$. Then, $V^* > 0$. However, then there exists some $\Delta < 0$ such that $\phi V^* + (1-\phi)\Delta > 0$, and hence the activist strictly prefers implementing a project with such an NPV over not implementing it, yielding a contradiction with $b^{a*} \geq 0$. ■

F.6 Shareholder proposals (Section 5.5)

This section provides the shareholder proposal extension mentioned in Section 5.5 (for the short sales extension mentioned in the same section, see Online Appendix F.7). Shareholder proposals were not considered in the main model, because they are nonbinding in many jurisdictions. That said, in cases where they are binding, if the incumbent rejects settling then the activist might consider making a shareholder proposal for the project to be implemented instead of running a proxy fight for board seats.

However, I show below that even if the proxy fight is replaced with such a shareholder proposal, the main results as well as predictions of the model continue to hold. To see why, note that if a board settlement is rejected, then making a binding shareholder proposal would be akin to running a proxy fight for a board control of $\alpha_P = 1$, because in both cases the project is implemented with probability one if the activist wins.[91] Therefore, letting $\alpha_P = 1$ in Proposition 1, all of the results derived in Section 3 continue to hold without any change whatsoever.

Moreover, if the activist demands an action settlement then the incumbent follows a threshold strategy under this extension similar to the strategy it follows in the main model.[92] Therefore, the only difference compared to the main model is that upon rejection of an action settlement, if the shareholders vote for the activist's proposal then the project is implemented even if $\Delta < b^a$. Since this is an outcome that neither the activist nor the shareholders prefer, it reduces the credibility of the activist's threat (more specifically, the threat to make a shareholder proposal in this extension) upon rejection of an action settlement, in turn pushing up the incumbent's rejection threshold. Therefore, as elaborated below, these changes amplify the differences that are already present between an action and a board settlement in the main model, leading to the robustness of all of the results and predictions derived in Section 4 as well. In fact, for this reason some of the predictions get even stronger, as also explained below.

F.6.1 Modification of the setup

In this extension, I replace the proxy fight with a binding shareholder proposal for the project to be implemented. Therefore, the difference in this setup compared to the setup of the main

[91] Recall that Proposition 1 implies that in equilibrium, the incumbent rejects a board settlement only if $\Delta \geq b^a$.

[92] More specifically, under this extension, there again exists Δ_A^* such that the incumbent accepts the action settlement if and only if $\Delta > \Delta_A^*$.

model is that if the shareholders vote for the activist's shareholder proposal, then the project is implemented with probability one. To keep the notation parallel with the setup of the main model, if the activist decides to make a shareholder proposal, I denote the cost that the incumbent incurs by $C^i > 0$, and the cost that the activist incurs by $C^a > 0$. While C^i and C^a are likely to be smaller under this extension than in the main model, note that none of the parameter specifications made in the main model ((1), (2), and (3)) prevent lower C^i and C^a.[93] Therefore, none of the parameterizations made in the main model is violated in this extension. If the activist makes a shareholder proposal, the shareholders choose between voting for it and voting against it.

Note that if $E[\Delta] \leq 0$ or $n^a E[\Delta] - B^a \leq C^a$, then there always exits an equilibrium where the incumbent always rejects any settlement, the activist never makes a shareholder proposal, and therefore the project is never implemented. This is because in this equilibrium, if $E[\Delta] \leq 0$, then never supporting the activist's proposal is always a best response for the shareholders, and if $n^a E[\Delta] - B^a \leq C^a$, then never making a shareholder proposal is a best response for the activist. Therefore, to focus the attention on the parameter region where the activist has some impact on the firm's decisions with positive probability in any equilibrium, I assume that $E[\Delta] > 0$ and $n^a E[\Delta] - B^a > C^a$.

F.6.2 Equilibrium characterization

To begin with the analysis, note that if a board settlement is rejected, then making a binding shareholder proposal would be akin to running a proxy fight for a board control of $\alpha_P = 1$. More specifically, if the activist demands a board settlement, the same equilibrium described in Proposition 1 continues to exist, where α_P is set to $\alpha_P = 1$. This is because if the activist wins a proxy fight in the equilibrium characterized by Proposition 1 (where $\alpha_P = 1$), then the project is always implemented, which is also true if the activist's shareholder proposal wins.

Next, consider the case where the activist has demanded an action settlement. Then, the equilibrium described in Proposition 5 continues to exist, where α_P is set to $\alpha_P = 1$ and (11) is replaced with

$$C^a/n^a = \sigma E[\Delta - b^a | \Delta \leq \hat{\Delta}_A], \tag{54}$$

because if the shareholders vote for the activist's proposal then the project is implemented even if $\Delta < b^a$ (recall that $b^a \equiv B^a/n^a$). The derivation behind the equilibrium is very similar to the derivation in the main model as well: For any given ρ and σ, the incumbent's payoff from accepting the action settlement is $n^i\Delta - B^i$, while its payoff from rejecting and facing the risk of a shareholder proposal is $\rho(\sigma(n^i\Delta - B^i) - C^i)$. Therefore, the incumbent accepts the action settlement if and only if $\Delta > \Delta_A(\rho;\sigma)$, where $\Delta_A(\rho;\sigma)$ is again given by (23) (where α_P is again set to $\alpha_P = 1$), and $\Delta_A(\rho;\sigma)$ is strictly decreasing in ρ.[94] In turn, for a given Δ_A

[93] Also recall that the parameter C^a can also be interpreted as the cost the activist incurs by not liquidating the stake in the target (e.g., due to the opportunity cost of forgoing other new investments) or by not meeting investor outflows. In either case, if this cost is too high, then the activist will choose not to make a shareholder proposal and instead will choose to liquidate his stake.

[94] Note that the incumbent's threshold Δ_A again satisfies $\Delta_A > b^a$. This is because for Δ_A to become

and σ, the activist's expected payoff from making a shareholder proposal upon rejection of an action settlement is given by $\sigma n^a E[\Delta - b^a | \Delta \leq \Delta_A] - C^a$, which is strictly increasing in Δ_A. Therefore, Δ_A and ρ together constitute a fixed point equation for any given σ, and unless σ is too low,[95] the solution to this fixed point equation is given by (9) and (10) as a function of σ, where again the only difference is that α_P is set to $\alpha_P = 1$ and (11) is replaced with (54). Since $\Delta_A(\sigma)$ is also decreasing in σ, this in turn implies that the expected shareholder value conditional on the activist's shareholder proposal becoming successful (where this payoff is given by $E[\Delta | \Delta \leq \Delta_A(\sigma)]$) is decreasing in σ, and hence $\Delta_A(\sigma)$ and σ together constitute their own fixed point equation, yielding a unique σ_A^* in equilibrium.

F.6.3 Robustness of the results of the main model

In this extension, the results and empirical predictions from the main model continue to hold, and in fact the empirical predictions of the main model get even stronger. To see why, first note that as explained in Section F.6.2 above, if the activist demands a board settlement, the same equilibrium described in Proposition 1 continues to exist, where α_P is set to $\alpha_P = 1$. Therefore, all of the results derived in Section 3 (Propositions 1-4) continue to hold without any change, except that "proxy fight" everywhere is replaced with "shareholder proposal".

Second, as also explained in Section F.6.2 above, the equilibrium described in Proposition 5 continues to exist, where α_P is set to $\alpha_P = 1$ and (11) is replaced with (54). Due to this preservation of the equilibrium structure, Propositions 2-10 in Online Appendix B continue to hold without any change as well (except that again, "proxy fight" everywhere is replaced with "shareholder proposal"), due to the same intuition discussed for these propositions.

Finally, as also described in Section F.6.2 above, the only difference compared to the main model is that upon rejection of an action settlement, if the shareholders vote for the activist's proposal then the project is implemented even if $\Delta < b^a$. Since this is an outcome that neither the activist nor the shareholders prefer, it reduces the credibility of the activist's threat to make a shareholder proposal upon rejection of an action settlement, in turn (weakly) pushing up the incumbent's rejection threshold Δ_A^*. Therefore, these changes amplify the differences that are already present between an action and a board settlement in the main model, which not only leads to to the robustness of the results and predictions derived in the Section 4.2, but also makes some of these predictions even stronger:

(a) Corollary 1 continues to hold, because under this extension Δ_A^* gets weakly larger while Δ_B^* does not change (as explained in Section F.6.2 above).

equal to or lower than b^a, ρ would have to be high, but this never takes place in an equilibrium where a shareholder proposal is on the equilibrium path (because otherwise, the activist would have no incentive to make a shareholder proposal upon the incumbent's rejection of the action settlement).

[95] Indeed, in equilibrium, σ^* satisfies this constaint and is not too low. This is because if σ^* were too low, then the activist would never make a shareholer proposal in equilibrium. However, then the incumbent would always reject the settlement, and hence the shareholders would strictly prefer to support the activist's proposal, because their expected payoff from doing so would be $E[\Delta] > 0$. Importantly, since this payoff is positive, it would yield a contradiction with $\sigma^* < 1$.

(*b*) Proposition 6 continues to hold as well, where the only changes are that α_P is set to $\alpha_P = 1$ and $\underline{l}$ is replaced with $\underline{l} = 2(C^a/n^a + b^a) - b^i$.[96]

(*c*) The robustness of the results above from the main model implies that all of the empirical predictions of the main model (as described in Section 4.2.2) continue to hold under this extension as well, subject to the change that "proxy fight" everywhere is replaced with "shareholder proposal". Moreover, predictions 3 and 4 get even stronger since Δ_A^* gets weakly larger and Δ_B^* does not change.

(*d*) Proposition 7 continues to hold as well, where the only change is that α_P is set to $\alpha_P = 1$ and $\underline{l}$ is replaced with $\underline{l} = 2(C^a/n^a + b^a) - b^i$ (due to the same reason in the footnote in (b) above). Finally, Corollary 2 continues to hold without any change.

F.7 Short sales (Section 5.5)

In this section, I provide the short sales extension mentioned in Section 5.5 (for the shareholder proposal extension mentioned in the same section, see Online Appendix F.6 above). A potential concern could be whether short selling would affect the results in the paper. However, I do not find this to be the case. To see why, note that in most jurisdictions, an activist that has shorted the firm's stock would need to disclose this information when he solicits votes in a proxy fight,[97] and therefore either he would not be eligible to run a proxy fight or he would not be elected by the shareholders of the firm. Therefore, he would not run a proxy fight in the first place. Similarly, he would not be able to sit on the board as a result of a board settlement either, since his short position in the firm would conflict with his fiduciary duties as a board director.[98] Therefore, he would not demand any board settlement.

For these reasons, as I formally show below, an activist with a short interest in the firm would not have any effect on the firm. In particular, in the proposition below, I show that an activist with a short stake in the firm does not have any incentive to start a campaign and use a proxy fight threat or settlement (including action settlement, as I elaborate after the proposition) to pressure the incumbent for a particular decision regarding the project (i.e., pressuring the incumbent to implement the project, or alternatively, not to implement it). Note that this result does not preclude campaigns where an activist with a short stake in a firm *speculates* that the firm is overvalued (Appel and Fos (2023)). However, such campaigns do not involve proxy fights or settlements with the activist, and therefore are excluded from the analysis in this paper.

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[96] This is because if Δ has a uniform distribution, then $C^a < n^a E\left[\Delta - b^a\right] \Leftrightarrow C^a/n^a < \frac{b^i+l}{2} - b^a \Leftrightarrow 2(C^a/n^a + b^a) - b^i < l$.

[97] According to Securities and Exchange Commission (SEC) Rule 14a-9, activists are required to disclose their net economic exposure to the firm as part of the proxy solicitation process.

[98] Note that it could be that as a result of the board settlement, the activist himself does not join the board, but his nominees do. However, then these nominees would need to have no short interest in the firm, and since they would be the ones who are part of the decision-making process in the boardroom, it would make the activist's short position irrelevant.

Also note that while the following proposition exogenously assumes that the activist with a short stake either has no other bias (i.e., $b_S^a = 0$) or has an additional bias against implementing the project (i.e., $b_S^a > 0$), this is because as I argue in the remark made at the end of this appendix, any sensible setting that endogenizes the activist's stake would yield $b_S^a \geq 0$.

Proposition 17 *Suppose that with some probability $\varphi \in [0,1]$, the activist has obtained a short position in the target instead of a long position. In addition, denoting the bias of the activist when he has a short position by b_S^a, suppose that $b_S^a \geq 0$. Then, the activist that has a short stake has no impact on the firm outcome, and his expected payoff from initiating a campaign is nonpositive.*

The proof of the proposition above is as follows. Suppose that there exists an equilibrium where conditional on the activist starting a campaign, he has a short stake in the firm with some probability, and with the remaining probability he has a long position (as in the main model). Note that by the arguments made above Proposition 17, such an activist does not demand a board settlement or cannot run a proxy fight. Moreover, note that if the activist demands from the incumbent *not* to implement the project, then the incumbent indeed never implements the project, since its preferences are not to implement the project to begin with (since $\Delta < b$ for all Δ). However, the incumbent would not implement the project even if the activist was not present in the first place. Therefore, if the activist makes such a demand, he has no impact on the firm outcome.

Therefore, the only remaining action that is available for the activist with a short stake is to demand an action settlement (i.e., demand that the incumbent implement the project). There are two cases to consider. The first case is that it is perfectly revealed to the incumbent that the activist has a short stake. Then, the incumbent again never agrees to the activist's demand, and the activist has no recourse (since he cannot run a proxy fight). Therefore, the project is again never implemented.

Second, suppose that conditional on the activist making this demand, it cannot be perfectly inferred whether the activist has a short or long stake in the firm, i.e., the activist has a long position with probability ψ for some $\psi \in (0,1)$. Then, with probability $1-\psi$, the activist will not do anything if the incumbent rejects the action settlement. However, with probability ψ, the activist has a long position as in the main model, and hence the activist decides whether to launch a proxy fight in the same way he did in the main model. Moreover, if the activist ends up launching a proxy fight, it is perfectly revealed to the shareholders that the activist has a long position in the firm. Therefore, due to the same reasons discussed for Proposition 5 in the main model, the incumbent again follows a threshold strategy similar to that in Proposition 5, i.e., the incumbent accepts the action settlement if and only if $\Delta > \Delta_A^*$ for some $\Delta_A^* < b^i$. Importantly, $\Delta_A^* > 0$ due to the same reasons discussed for Proposition 8. However, then $E\left[\Delta | \Delta > \Delta_A^*\right] > 0$, which implies that the activist that has a short stake gets a negative expected payoff from demanding an action settlement whenever $b_S^a \geq 0$ (because the activist's expected payoff then is $E[-\Delta - b_S^a | \Delta > \Delta_A^*] < 0$ per share he has shorted). Therefore, he never makes such a demand either, and again, he has no impact on the firm outcome.

Remark regarding b_S^a. Note that the discussion above implies that even if $b_S^a < 0$ (i.e., the activist with a short stake has a bias in favor of implementing the project), the activist with a short stake has an effect on the firm outcome only if he demands an action settlement (i.e., when he demands that the firm implements the project). However, whenever this is the case, he always makes a loss on his short position itself since $E[-\Delta|\Delta > \Delta_A^*] < 0$. Therefore, under any sensible setting that endogenizes the activist's stake, this loss would yield a contradiction with the activist's short stake. In other words, any activist with a bias $b_S^a < 0$ will find it optimal to take a *long* position in the firm, rather than a short position. For this reason, Proposition 17 above focuses on the reasonable case of $b_S^a \geq 0$ for the activist with a short stake. Nevertheless, note that even if $b_S^a < 0$, the model implies that the activist that has a short stake never results in a negative NPV project being implemented (because as again discussed above, the only case where he can affect firm value is by demanding an action settlement, where the incumbent agrees if and only if $\Delta > \Delta_A^*$, where $\Delta_A^* > 0$).