Asset pricing in an economy with sentiment swings and price feedback*

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Abstract

This paper develops a continuous-time equilibrium model in which the beliefs of a representative agent are influenced by a stochastic sentiment process. The agent's sentiment evolves endogenously, reacting to past asset price movements and occasionally exhibiting jumps, or large mood swings. Sentiment tends to self-correct: sentiment jumps become more likely as sentiment deviates more from economic fundamentals. Our model is highly tractable allowing us to solve for the equilibrium risk free rate, equity premium and stock price in closed-form. A two-way feedback loop emerges: asset prices influence sentiment, and sentiment, in turn, affects asset prices, despite sentiment having no direct impact on fundamentals. In equilibrium, this feedback generates endogenous price distortions that sometimes get corrected in the form of jumps, or "Minsky moments." Empirically, we validate the model's sentiment dynamics, its implied predictive regressions, and show that the model successfully matches key unconditional and conditional asset pricing moments.

Keywords: Asset Prices, Sentiment, Price Feedback
JEL Classification: G02, G12

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1. Introduction

Keynes (1936) noted that investor psychology, or "animal spirits," can drive financial markets away from fundamental values. Markets have indeed been shown to be sensitive to changes in sentiment, i.e., to shifts in investor views toward financial assets ranging from optimism (bullishness) to pessimism (bearishness). Sometimes, these shifts are tied to real changes in the economy, such as unexpected monetary or regulatory policies or geopolitical events. Often, however, sentiment moves for reasons with little to no connection to underlying fundamentals. As documented in Shiller (1981), much of the volatility in stock prices cannot be explained by changes in economic fundamentals. A vast literature, discussed below, has established both theoretically and empirically that investor sentiment can affect asset prices.

A remaining challenge is to develop a tractable sentiment-based equilibrium asset-pricing model that can jointly deliver (i) belief-driven price jumps without corresponding jumps in fundamentals, (ii) empirically realistic post-jump dynamics of excess returns and realized volatility following sentiment shocks, and (iii) a quantitative fit that extends beyond a small set of unconditional moments to the joint distribution and time-series behavior of the key equilibrium variables. Our framework is designed to address this gap: it matches moments for the risk-free rate, dividend yield, excess return, and realized volatility, both unconditionally and across sentiment regimes, and it passes additional time-series diagnostics by aligning model-implied series with their empirical counterparts using several statistical tests.

We embed sentiment into a standard endowment-economy asset-pricing environment with a representative investor, where sentiment is modeled as a stochastic belief distortion that tilts the investor's expectations about dividend growth, capturing optimistic versus pessimistic outlooks. Our sentiment specification builds on the extrapolative expectations framework of Barberis et al. (2015) and Jin and Sui (2022), but we enrich it in two key ways. First, we introduce a stochastic feedback loop between prices and sentiment: price movements affect sentiment, and sentiment in turn feeds back into prices. This mechanism allows the model to generate realistic boom–bust dynamics even when fundamentals change little. Second, the model generates sentiment-driven jumps in asset prices—discrete price moves that can occur without jumps in fundamentals—because sentiment itself can jump when it becomes sufficiently extreme. Such sentiment jumps can give rise to "Minsky moments," in which prices self-correct abruptly and seemingly without cause to an observer who does not account for the role of sentiment.

A key feature behind these events is that jump risk is state dependent. In our model, the jump intensity of sentiment rises when sentiment becomes more extreme, capturing abrupt mood swings: when optimism becomes elevated, downward reversals become more likely, and conversely when pessimism is pronounced, upward shifts become more likely. This flexibility not only helps generate Minsky-style corrections, but also improves the model's ability to reproduce the observed dynamics

of risk premia and realized volatility around large sentiment movements.

In addition, we equip the representative agent with recursive preferences, which disentangle risk aversion from the preference for early versus late resolution of uncertainty. These preferences are necessary for the model to deliver empirically consistent predictions for how the price–dividend ratio responds to sentiment. When sentiment becomes more optimistic, stock prices rise relative to dividends and the dividend yield compresses, and vice versa. The strength—and even the sign—of this valuation channel is governed by the elasticity of intertemporal substitution: only when investors are sufficiently willing to shift consumption over time does greater optimism map into higher price–dividend ratios, in line with the data. By contrast, under time-additive CRRA preferences, our general equilibrium framework would instead imply the counterfactual prediction that a more optimistic market environment depresses the price–dividend ratio.[1]

Finally, despite the endogeneity of both prices and sentiment, the model remains analytically tractable: we solve for equilibrium quantities—in particular, the equity risk premium, its volatility, the price–dividend ratio, and the risk-free rate—in closed form. The framework therefore yields testable predictions for how equilibrium quantities depend on sentiment, for which we find strong empirical support. In our model, the endogenous risk-free rate is driven by sentiment: when optimistic investors would like to tilt portfolios toward the risky asset, market clearing requires the short rate to rise to temper this reallocation, a mechanism for which we also find empirical evidence. While sentiment is the main driver of time-series variation in the risk-free rate and the price–dividend ratio, the flexible sentiment dynamics—in particular feedback effects and jumps—substantially improve the model's ability to match conditional moments of the equity risk premium and its volatility. In particular, the model predicts that both the equity risk premium and its volatility decline with sentiment, which we validate empirically.

Much of the model's tractability comes from assuming that investors are myopic about the future evolution of their own sentiment: they condition on the current sentiment state when forming expectations, but do not internalize future sentiment fluctuations. This assumption is empirically plausible and has concrete pricing implications: it yields additional sentiment-related terms in the closed-form expressions for the equity premium and return volatility that improve the model's ability to match key asset-pricing moments.

Beyond matching standard asset-pricing moments, the model also generates realistic joint time-series behavior for sentiment and asset prices. The interaction of price feedback and sentiment jumps produces empirically plausible dynamics of excess returns and realized volatility in the aftermath of

[1] More precisely, the CRRA specification predicts a positive relationship between the price–dividend ratio and sentiment only when the coefficient of risk aversion lies between zero and one, which is empirically less relevant. Moreover, even under recursive utility, a positive association is obtained only if the elasticity of intertemporal substitution exceeds one.

large sentiment moves. In equilibrium, sentiment-based models typically imply a negative relationship between the equity risk premium and sentiment: when the representative agent becomes overly optimistic, expected returns must fall to clear the market and prevent excessive risky investment. Our model reproduces this negative relationship, but it is also flexible enough to generate short-run positive co-movement between excess returns and sentiment jumps, a pattern that is difficult to obtain in more restrictive specifications, especially when the feedback channel is shut down. Moreover, the model can match the differential response of realized volatility following positive and negative sentiment jumps. This performance relies on the flexible jump specification, whereas a model with only diffusive sentiment risk lacks this mechanism.

A major challenge in studying sentiment is quantifying it. To make sure that our empirical results are not specific to the respective sentiment index we use, we work with two distinct proxies for investor sentiment: The first index we use is based on the sentiment survey by the American Association of Individual Investors (AAII), which collects investors' stock market expectations over the following six months. The second index we employ is the news sentiment index (SSW) developed by Shapiro et al. (2022), which measures economic sentiment based on lexical analysis of economics-related news articles (see the Online Appendix). We use these indices to discipline the sentiment process and to test the model's equilibrium implications. In particular, we validate each component of the sentiment dynamics—price feedback in the drift, state dependence in diffusive volatility, and state-dependent jump intensities—and find strong empirical support for these features. We then test, via standard predictive regressions, the model's implications for the joint behavior of the risk-free rate, dividend yield, equity premium, and realized volatility across sentiment states and horizons. Finally, we calibrate the model to match both unconditional moments and moments conditional on sentiment regimes. The results indicate that incorporating sentiment jumps, especially in an asymmetric form, significantly improves the model's ability to match these (un) - conditional moments, and for aligning model-implied time series with their empirical counterparts - in particular for reproducing observed volatility dynamics.

2. Related Literature

Our paper is most closely related to Barberis et al. (2015) and Jin and Sui (2022). The X-CAPM model of Barberis et al. (2015) proposes a heterogeneous-agent model with a fraction of investors extrapolate future price changes based on past price movements, while others maintain fully rational beliefs. Extrapolative agents' beliefs are driven by a sentiment measure that is a deterministic function of past prices. In equilibrium, prices reflect a weighted average of these heterogeneous beliefs; extrapolators amplify price momentum following positive shocks and exacerbate downturns after negative shocks. The X-CAPM matches survey evidence that investors expect future returns to continue recent trends, excess volatility of aggregate prices relative to fundamentals, and aligns

with return predictability implied by autocorrelations in price-dividend ratios (see Barberis (2018) for an overview).

Going back further in time, a first important seminal paper based on sentiment-based asset pricing is De Long et al. (1990), who assumes a class of noise traders who form beliefs about asset values that are not fully justified by fundamentals, while arbitrage by rational investors is risky and limited, so mispricings caused by sentiment may persist. The model's key result is that noise traders can not only survive but influence equilibrium prices and earn higher expected returns as compensation for the additional noise trader risk they create. Lee et al. (1991) argued that shifts in the demand of sentiment-driven (noise) trader can explain the persistent divergence between a closed-end fund's market price and its net asset value.

Beyond the noise trader framework, models have delved into the psychological origins of sentiment. Barberis et al. (1998) proposed a model where investors are subject to conservatism (slow updating of beliefs) and the representativeness heuristic. These biases cause patterns of underreaction to new information at short horizons and overreaction at long horizons - in effect, generating price momentum and reversals that align with an "investor sentiment" cycle. In their model, sentiment isn't just random noise but arises from systematic cognitive biases. Daniel et al. (1998) develop a theory based on overconfidence (investors overestimating the precision of their private information) and self-attribution bias (crediting successes to skill and failures to bad luck). Overconfident investors initially push prices away from fundamentals, causing excess volatility, and as reality eventually sets in, prices mean-revert. Hong and Stein (1999) assume two types of traders: newswatchers, who receive information gradually and trade based on fundamentals, and momentum traders, who base decisions on past price trends. Because news is incorporated slowly (via newswatchers), prices initially underreact to fundamental information. However, momentum traders amplify price trends, causing price momentum in the short to medium term.

Many models with agents holding heterogeneous beliefs that differ from the objective data-generating measure (a form of static sentiment) derive asset prices that show patterns of either temporary or permanent mispricing: see, e.g., Harrison and Kreps (1978), Scheinkman and Xiong (2003), Xiong and Yan (2010) among others. Dumas et al. (2009) present a two-agent heterogeneous belief equilibrium, wherein an overconfident agent employs a distinct filter to incorporate a public signal regarding dividend the growth rate. Sentiment manifests as a filtering gap regarding dividend growth rate. They compute explicit equilibrium consumption and the stochastic discount factor dependent on sentiment. Martin and Papadimitriou (2022) show how sentiment can amplify speculation.

By contrast, our framework departs from existing extrapolative-expectations models along two main dimensions: (i) how sentiment enters belief formation and (ii) how the feedback between prices and sentiment is modeled. We let sentiment distort beliefs about dividend growth, rather than about asset prices directly. In addition, sentiment is an autonomous stochastic state variable with

both diffusive and jump innovations. This stands in contrast to the X-CAPM, where sentiment is a deterministic function of current and past prices, implying a tight (indeed, perfect) link between innovations in sentiment and innovations in asset prices.

Compared to existing sentiment models that incorporate a feedback mechanism, our approach is also more flexible. Instead of imposing a mechanical mapping from prices into sentiment, sentiment mean-reverts toward a time-varying target that summarizes recent price momentum. This formulation preserves the core economic idea that price trends shape beliefs, while allowing for imperfect comovement between sentiment and prices. Moreover, sentiment includes an idiosyncratic noise component, which can be interpreted as capturing fluctuations in disagreement or noise in belief formation, and it breaks the deterministic price–sentiment connection. Finally, the jump component in sentiment allows for abrupt mood swings and provides a natural mechanism for sudden valuation corrections. We show empirically that these features are present in the data and quantitatively important for the models asset-pricing implications.

Including jumps in sentiment is important for two reasons. First, as we will show, there is clear evidence in the data that sentiment jumps. Second, extrapolative expectations can give rise to the ramp up phase of a bubble (where prices feed into sentiment leading to higher and higher valuation, see Barberis et al. (2018)), but they do not provide a mechanism for the bubble to suddenly reverse course and crash. Our model can generate both phases. Another difference is that the risk-free asset in the X-CAPM has a fixed return by assumption, whereas it is determined endogenously in our model, and as a result also depends on sentiment in ways that can be analyzed and confronted with the data. As for the implications of the model, we show that our extension of the X-CAPM results in a more realistic equity premium (which the X-CAPM has difficulty fitting), a time-varying and sentiment-dependent volatility for stock returns (which is constant in the X-CAPM), and jumps in asset prices resulting from sudden mood swings in sentiment.

Jin and Sui (2022) provide a variation on beliefs extrapolation along the following dimensions. They model the sentiment dynamic via a filtering approach in a two-state regime switching model. By contrast, we allow our sentiment process to exhibit price feedback through mean-reversion that tracks price momentum, and jumps. We also obtain a realistic equity premium with a low relative risk aversion, and small variation in beliefs. Another paper which contains a dynamic sentiment framework is Maenhout et al. (2025). There, dynamic beliefs arise endogenously due to agents' attitude toward alternative models, i.e. the worst-case model that the decision-maker guards against can be viewed as representing endogenously distorted pessimistic beliefs. Also related are models of diagnostic expectations, which refer to a specific form of extrapolative belief formation in which agents overweight future outcomes that are especially representative of recent news: see Bordalo et al. (2019), Nagel and Xu (2022) for a related model, and Bordalo et al. (2022) for an overview. Our feedback form of sentiment accounts for fading memory or representativeness heuristic by letting

the mean-reversion level of sentiment be driven by the recent momentum of asset prices.[2]

Empirical literature. Unlike fundamental valuation quantities such as earnings, cash flows, etc., sentiment is an intangible, often subjective concept. The literature has developed various proxies and indices for sentiment, which can be categorized into a few broad types, and established the following empirical results regarding the influence of sentiment on asset prices:[3]

I. Survey-Based Measures: Some of the earliest sentiment measures come from surveys of investors or consumers. For example, the American Association of Individual Investors (AAII) has conducted a weekly survey since 1987 asking individual investors if they are bullish, bearish, or neutral on the stock market. Similarly, Investors Intelligence reports the proportion of bullish vs. bearish newsletters. These direct sentiment readings capture investors' stated expectations. Another prominent survey is the University of Michigan Consumer Sentiment Index, which measures general consumer confidence. While aimed at consumers, it has been used as a broad sentiment proxy for the stock market. Lemmon and Portniaguina (2006) find that changes in consumer sentiment are correlated with stock returns, especially for stocks that are small and harder to arbitrage; see also Jiang et al. (2019). Our model is also consistent with the findings of Greenwood and Shleifer (2014) who analyzed how investors form expectations of future stock market returns, and how those expectations compare to actual realized returns. Drawing on six survey-based data sources spanning 1963-2011, they found that investors consistently exhibit extrapolative expectations, meaning they expect recent market trends to continue. These subjective expectations are highly correlated across survey types and with recent past returns; they also found that they substantially overestimate future returns, revealing a persistent sentiment bias in investors' beliefs. Da et al. (2021) provide further empirical evidence on how investors form short term beliefs about stock prices.

II. Analysts forecasts: Several papers, for instance, Kozak et al. (2018), Delao and Myers (2021), Buraschi et al. (2021), Maenhout et al. (2025), Delao and Myers (2024), and Bastianello et al. (2025) construct subjective expectations based on analysts forecasts and surveys.

III. Market-Based Measures: Zweig (1973) proposed using the closed-end fund discount as a sentiment indicator and found that wide discounts preceded higher subsequent stock returns, consistent with a contrarian interpretation where extreme pessimism is bullish. Baker and Wurgler (2006) (B&W) constructed a composite sentiment index that is a linear combination of several market-based sentiment proxies, including the average closed-end fund discount (inversely related to sentiment), market turnover (as high volume may indicate speculative enthusiasm), the number of IPOs (initial

[2]In macroeconomics, López-Salido et al. (2017) employ a forecasting regression of GDP log difference and document sentiment-induced shifts (via changes in credit spreads) in the supply of credit. Hirshleifer et al. (2015) embed an extrapolative expectations rule, where agents overweight recent output or dividend growth into a production-based, recursive preferences model. This bias generates a sentiment component. Maxted (2024) shows that the interaction of behavioral and financial frictions generates macro-financial cycles of investment dynamics, risk-premia spikes, and recovery patterns, highlighting sentiment as both an amplifier and trigger of financial crises.

[3]Other proxies are media-based and social media/internet-based.

public offerings) and average first-day IPO returns, the equity share in new issues and the dividend premium (in euphoric times investors may chase growth stocks with no dividends). They then showed that this composite sentiment index predicts the cross-section of future returns: when their sentiment index is high (indicating bullish sentiment waves), subsequent returns are relatively low for young, small, speculative stocks. Huang et al. (2015) use a partial least squares method to extract an optimized sentiment index from similar inputs and claim it outperforms the B&W-index in predictive power. Baker et al. (2012) constructed sentiment indices for six major stock markets and found that both a global sentiment component and a local, country-specific, sentiment influence return. Other market-based proxies include options and volatility indicators. The put-call ratio (volume of put options vs. calls) is often seen as a sentiment gauge (a high put volume indicates fear). The VIX index, often dubbed the "investor fear gauge," reflects implied volatility on S&P 500 options and tends to spike during market panic. While VIX primarily measures expected volatility, it correlates inversely with sentiment. Additionally, market liquidity can serve as an indirect sentiment measure: unusual liquidity (narrow bid-ask spreads, high turnover) might indicate an overoptimistic market as irrational traders provide liquidity by trading more. Birru and Young (2022) show the predictive ability in the cross-section of sentiment is substantially larger in times of higher uncertainty.[4]

Outline of the paper. Section 3 introduces the model. Section 4 establishes the dynamics of the equilibrium prices, the stock's excess return and the risk free rate. Section 5 describes the data. Section 6 validates empirically the sentiment process. Section 7 tests the predictions of the model regarding the relationships between the endogenous equilibrium prices and investor sentiment. Section 8 presents the calibration results. Section 9 concludes. An Online Appendix contains all proofs and the empirical investigation and calibration of a second sentiment index.

3. The Model

We adopt a standard endowment economy in the spirit of Lucas (1978) with a representative agent who optimally invests in a financial market and consumes the proceeds. We assume there is a risk-free asset in zero net supply and a risky asset in unit net supply representing a claim to a stream of future dividends. The representative agent's behavior is affected by changing beliefs about the dynamics of the dividends, summarized by a sentiment process fluctuating between different levels of optimism and pessimism and responds to any momentum in the asset price. Further, the sentiment can jump to self-correct (i.e., revert to less extreme values) with an intensity that increases as sentiment becomes more disconnected from reality. As a result, in equilibrium the risk-free rate and stock price depend on the current output of the economy as well as market beliefs. Jumps in the stock's price can reflect either jumps in the fundamentals (dividends) or jumps in beliefs (sentiment).

[4]They also employ the AAII investor sentiment index we also study in this paper along with the Michigan Consumer Sentiment index, B&W-index and its refinement developed by Huang et al. (2015) as well as the conference board consumer confidence index.

3.1 The economy

We assume there is a single consumption good. Risk is described by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathbb{P}$ represents the objective probability measure.[5] The dividend rate D_t at which an owner of a share of the stock receives the consumption good is assumed to follow the exogenous dynamics

$$\frac{dD_t}{D_{t-}} = \mu_D dt + \sigma_D dW_t^D + dJ_t^D \tag{3.1}$$

for a drift $\mu_D \in \mathbb{R}$, a volatility $\sigma_D \geq 0$, a Brownian motion W^D and a compensated compound Poisson process J^D of the form

$$J_t^D = \sum_{1 \leq i \leq N_t^D} \left(e^{Y_i^D} - 1 \right) - \lambda_D \phi_D t \tag{3.2}$$

where N^D denotes a Poisson process with constant intensity $\lambda_D \geq 0$ and i.i.d. random variables Y_i^D, $i \geq 1$, such that $\phi_D = \mathbb{E}[e^{Y_1^D} - 1]$ and $\mathbb{E}[e^{-\gamma Y_1^D}]$ are finite, where $\gamma > 1$ is the relative risk aversion of the representative agent; see Section 3.4 below. We assume W^D, N^D and Y_i^D, $i \geq 1$, to be independent.[6]

3.2 Investor's sentiment

We depart from a standard Lucas endowment economy in that we assume that the representative agent's has beliefs about future dividend payments that fluctuate around the objective measure $\mathbb{P}$ according to changing market sentiment. We model the investor's sentiment with a $[a, b]$-valued Jacobi jump-diffusion with dynamics

$$\mathrm{d}\eta_t \;=\; \kappa(m_t - \eta_t)\mathrm{d}t \;+\; \sigma_\eta(\eta_t)\,\mathrm{d}W_t^\eta - (\eta_{t-} - a)\mathrm{d}J_t^- \;+\; (b - \eta_{t-})\mathrm{d}J_t^+ \tag{3.3}$$

for constants $a < 0 < b$, a constant mean-reversion speed $\kappa \geq 0$, a stochastic mean-reversion level m_t and volatility function σ_η defined below, a Brownian motion $W_t^\eta = \rho\, W_t^D + \sqrt{1 - \rho^2}\, W_t$, where $\rho \in [-1, 1]$ is a constant correlation coefficient and W is a Brownian motion independent of W^D and J^D. The neutral sentiment level is $\eta_t = 0$, while $\eta_t > 0$ represents optimism and $\eta_t < 0$ pessimism.

The sentiment process η is meant to summarize different effects affecting the beliefs of market participants. The dynamics in (3.3) allow the sentiment process η_t to capture realistic behavioral patterns such as extrapolation, mean-reversion, feedback effects and sudden mood swings, captured by the jump processes $J^{\eta-}$ and $J^{\eta+}$. As we show later the domain $[a, b]$ of η is based on the extreme points of growth rate expectations. The three key components of the sentiment process in (3.3) are:

[5] All equalities and inequalities between random quantities will be understood in the $\mathbb{P}$-almost sure sense.

[6] Since W^D and J^D are independent martingales, the dividend process D_t defined in Equation (3.1) admits the following explicit solution $D_t = D_0 \exp\left(\left[\mu_D - \frac{1}{2}\sigma_D^2 - \lambda_D \phi_D \right] t + \sigma_D W_t^D + \sum_{i=1}^{N_t^D} Y_i^D \right)$.

1. **Feedback part**: Our model includes a feedback channel through which sentiment is influenced by momentum in past stock prices. Denote by $\bar{S}_t^l = \frac{1}{l}\int_{t-l}^t S_u du$, the stock price average on $[t-l, t]$. We describe stock price momentum with the variable

$$M_t = \log \frac{\bar{S}_t^l}{\bar{S}_t^L} \tag{3.4}$$

for two lag parameters $0 < l < L$. The short-term average $\bar{S}_t^l$ is a measure of recent stock price levels that is more volatile than the longer-term mean S_t^L. The values (l, L) control how the investor's sentiment respond to past prices, including how reactive sentiment is to recent price changes and how far back in the past the inference relies on, capturing memory effects. We then model the mean-reversion level m_t in (3.3) as follows

$$m_t = -a\frac{2}{\pi}\arctan(c_n M_t)\, 1_{\{M_t<0\}} + b\frac{2}{\pi}\arctan(c_p M_t)\, 1_{\{M_t\geq 0\}} \tag{3.5}$$

for sensitivity parameters $c_n, c_p > 0$. Note that this transformation is strictly increasing and maps 0 to 0. In particular, positive price momentum M_t makes η_t trend to a mean-reversion level $m_t > 0$, while a negative price momentum M_t leads to a mean-reversion level $m_t < 0$. In other words, the agent's sentiment follows (stochastically) the stock price trend, becoming more (resp. less) optimistic when asset prices are rising (resp. falling).
As we will see, this generates in our model in equilibrium a feedback loop between sentiment and price. For example, a price surge creates euphoria on social media (many posts about getting rich, fear of missing out), which further boosts demand and prices, in a self-reinforcing "greed" part of the cycle. Conversely, when prices start falling, sentiment on forums turns bearish or panicky, potentially accelerating the sell-off (the "fear" part of the cycle).

2. **Variance part**: We parametrize the diffusive volatility function $\sigma_\eta \colon [a, b] \to \mathbb{R}_+$, as follows

$$\sigma_\eta(\eta) = s_\eta\sqrt{(\eta - a)(b - \eta)} \quad \text{for a constant } s_\eta > 0,$$

which is a symmetric hump at the midpoint $\frac{a+b}{2}$ of the interval $[a, b]$. The volatility term σ_η is maximal for η_t at the midpoint and minimal for η_t at the boundaries of the interval $[a, b]$.

3. **Jump part**: The downward/upward jump $J^{\eta-}/J^{\eta+}$ are compensated jump processes given by

$$J_t^{\eta-} = \sum_{i=1}^{N_t^{\eta-}} \left(1 - e^{-Y_i^{\eta-}}\right) - \lambda_t^{\eta-}\phi_{\eta-}t \quad \text{and} \quad J_t^{\eta+} = \sum_{i=1}^{N_t^{\eta+}} \left(1 - e^{-Y_i^{\eta+}}\right) - \lambda_t^{\eta+}\phi_{\eta+}t \tag{3.6}$$

for inhomogeneous Poisson processes $N^{\eta-}$, $N^{\eta+}$ with stochastic intensities $\lambda_t^{\eta-}$, $\lambda_t^{\eta+}$, i.i.d. non-negative jump magnitudes $Y_i^{\eta-}, Y_i^{\eta+}$, $i \geq 1$, and expected jump sizes $\phi_{\eta-} = \mathbb{E}[1 - e^{-Y_1^{\eta-}}]$, $\phi_{\eta+} = \mathbb{E}[1 - e^{-Y_1^{\eta+}}]$. We assume $J^{\eta-}$ and $J^{\eta+}$ to be conditionally independent of W^D, J^D, W^η given the stochastic intensities $\lambda^{\eta-}$ and $\lambda^{\eta+}$. We assume the jump intensities $\lambda_t^{\eta-}$ and

$\lambda_t^{\eta+}$ depend on current sentiment and to be of the form $\lambda_t^{\eta-} = g_-(\eta_t)$ and $\lambda_t^{\eta+} = g_+(\eta_t)$, with $g_-, g_+ \colon [a,b] \to \mathbb{R}_+$. As we will later show, the data indicates that g_- is increasing and g_+ decreasing. This means that negative (positive) jumps become more likely if sentiment is excessively optimistic (pessimistic), which leads to tipping points and a self-correcting behavior of sentiment: the larger the gap between sentiment and reality, the more likely sentiment is to jump in the direction of closing that gap, see (8.3) for a parametrization of g_+, g_-.

Section 6 supports each component in equation (3.3) by empirical evidence in the sentiment data.

3.3 Investor's beliefs

The sentiment process aggregates market beliefs by making the representative agent more or less optimistic about the size of future dividends. We model this by distorting the objective measure $\mathbb{P}$, under which dividend rates grow with drift μ_D in (3.1), to a new probability measure $\mathbb{P}_t^\eta$ – the agent's time-varying subjective belief – under which dividend rates are expected to grow with a different subjective drift.[7] To do that, we let for every $t \geq 0$, $\mathbb{P}_t^\eta$ be a probability measure[8] under which[9]

$$W_{t+s}^{D,t} - W_t^{D,t} = W_{t+s}^D - W_t^D - \left(\frac{\eta_t}{\sigma_D}\right) s, \quad s \geq 0, \tag{3.7}$$

and $W_{s+t}^\eta - W_t^\eta$, $s \geq 0$, are Brownian motions with correlation ρ. All other random quantities keep their $\mathbb{P}$-distributions under $\mathbb{P}_t^\eta$. In terms of the $\mathbb{P}_t^\eta$-Brownian motion $W_s^{D,t}$, the dynamics of the dividend rate, given in (3.1), becomes

$$\frac{dD_s}{D_{s-}} = (\mu_D + \eta_t)\, ds + \sigma_D dW_s^{D,t} + dJ_s^D, \quad s \geq t. \tag{3.8}$$

At time t, the representative agent believes that during $[t, \infty)$, the dividend rate will grow with drift $\mu_D + \eta_t$. This makes the representative agent time-inconsistent since she is not taking into account that her beliefs might change in the future. Indeed, it would be strange to assume that an agent continues to hold distorted beliefs $\mathbb{P}_t^\eta \neq \mathbb{P}$ if she is aware that they do not match the reality and is furthermore anticipating how her distorted beliefs will change. The natural agent is unaware that her beliefs are distorted. In any event, individuals are often behaving in time-inconsistent ways, and our representative agent is meant to be an aggregate of different market participants whose beliefs are

[7] Specifically we allow that the drift of the data generating process can lie a priori in an interval around zero. Conceptually this allows for two different interpretations. First, it may capture (drift) uncertainty about GDP-growth, as in Maenhout (2004). Secondly, and considered in this paper, it results in bounds of the market's belief dispersion. Thus sentiment can be seen as the fraction of agents tending to the respective extreme views.

[8] The measure $\mathbb{P}_t^\eta$ can be constructed from $\mathbb{P}$ iteratively. Let $(\mathcal{F}_s^{D,\eta})_{s\geq 0}$ be the filtration generated by the dividend process $(D_s)_{s\geq 0}$ and the sentiment process $(\eta_s)_{s\geq 0}$. For every horizon $T > t$, define $\mathbb{P}_t^\eta$ on $\mathcal{F}_T^{D,\eta}$ through the Radon–Nykodim derivative $d\mathbb{P}_t^\eta / d\mathbb{P}_t = \exp\left(\theta\eta_t(W_T^D - W_t^D) - \theta^2\eta_t^2(T-t)/2\right)$.

[9] We scale sentiment η_t by the dividend volatility parameter σ_D so that our interpretation of η_t is by how much our representative agent over- or underestimates the dividend growth rate μ_D, see Equation (3.8). Without this scaling, the investor over- or underestimates the dividend growth rate by $\sigma_D\eta_t$.

influenced by news, momentum in past stock prices as well as each other.[10]To illustrate this notion of sentiment-based beliefs, note that the investor's forecast error $\epsilon_s = g^D_{t+s} - \mathbb{E}^{\mathbb{P}^\eta}_t[g^D_{t+s}]$ of the dividend's growth rate g^D_{t+s} at time $t+s$ creates a predictable component of the objective forecasting error $\mathbb{E}^{\mathbb{P}}_t[\epsilon_s] = -\eta_t$. The form of the forecasting error reveals the myopic behavior towards the perception of sentiment. Conceptually, the agent inside the model is unable to anticipate any change of her own sentiment and is in particular unaware of the dynamic model of η_t in (3.3). Iterated forecasts carry over any forecasting errors, thus leading to the aforementioned time-inconsistency. This also implies a constant predicted growth rate $\mathbb{E}^{\mathbb{P}^\eta}_t[g^D_{t+s}] = \mu_D + \eta_t$ for any $s \geq 0$. At the same time, this belief system is still forward-looking, in the sense of Bordalo et al. (2022), as new information enter the sentiment process by market prices and other shocks.

3.4 Investor preferences

At time $t \geq 0$, the representative agent believes that the future develops according to the measure $\mathbb{P}^\eta_t$ and does not take into account that her beliefs might change. Hence, in the investor's view, the only source of randomness are the stochastic dividend rates $(D_s)_{s\geq t}$. We assume that at time $t \geq 0$, the representative agent ranks future consumption streams $(C_s)_{s\geq t}$ according to the stochastic differential utility V^t_t given recursively by

$$V^t_s = \mathbb{E}^{\mathbb{P}^\eta_t}\left[\int_s^\infty f(C_u, V^t_u)du \mid \mathcal{F}^D_s\right],\ s \geq t, \tag{3.9}$$

where $(\mathcal{F}^D_s)_{s\geq 0}$ is the filtration generated by the dividend process $(D_s)_{s\geq 0}$ and $f\colon (0,\infty)\times(-\infty,0) \to (-\infty,0)$ is a function of the form

$$f(c,v) = \frac{\beta(1-\gamma)v}{1-\frac{1}{\psi}}\left(\left(\frac{c}{((1-\gamma)v)^{\frac{1}{1-\gamma}}}\right)^{1-\frac{1}{\psi}} - 1\right)$$

with discounting parameter $\beta > 0$, the coefficient of relative risk aversion (RRA) γ, and the elasticity of intertemporal substitution (EIS) $\psi > 1$; see Duffie and Epstein (1992). In general, the empirically relevant cases are when $\gamma > 1$ and $\psi \in (0.5, 2)$. Moreover, if the parameter restriction $\gamma > 1/\psi$ is satisfied, then the recursive utility encodes a preference for early resolution of uncertainty.

4. The Equilibrium

Compared to a standard equilibrium derivation, the innovation in the model is that sentiment affects prices which themselves feed back into the agent's sentiment. To derive the equilibrium risk-free rate r_t and ex-dividend price S_t of the stock, we start with the partial equilibrium optimal investment and consumption plan of the agent. At time $t \geq 0$, she holds a fraction π_t of her wealth X_t in the

[10]The agent inside the model is unable to anticipate changes in her own sentiment.This myopic behavior results in time inconsistency. The behavioral biases explored in this paper address the belief formation process, see Barberis (2018) for an overview. Here, the representative agent holds a myopic belief, a form of short-termism. In the presence of sentiment, sudden fluctuation of mood swings become a key driver in the belief formation process.

stock and invests $(1-\pi_t)X_t$ in the risk-less asset. Therefore, if the investor's consumes at rate C_t, her wealth evolves according to

$$\frac{dX_t}{X_{t-}} = \pi_t \frac{dS_t + D_t dt}{S_{t-}} + (1-\pi_t) r_t dt - \frac{C_t}{X_{t-}} dt. \tag{4.1}$$

Remark 1. It is shown in the Online Appendix that in this economy a well-defined equilibrium exists if and only if the primitive parameters satisfy the condition

$$-a < \mu_D - \frac{\gamma \sigma_D^2}{2} - \lambda_D \zeta_D(\gamma) < \frac{\psi}{\psi - 1}\beta - b, \tag{4.2}$$

where $\zeta_D(\gamma) = \mathbb{E}\left[e^{Y_1^D} - \frac{\gamma - e^{(1-\gamma)Y_1^D}}{\gamma - 1}\right]$. We assume (4.2) is satisfied throughout the paper.

4.1 Equilibrium stock price and risk-free rate

The equilibrium is defined by clearing the market for the consumption good, the risk-free asset and the stock. Since we consider an endowment economy populated by one investor with a risk-free asset in zero net supply and a stock in unit net supply, in equilibrium, the investor must hold all her wealth in the stock and consume all dividends. Therefore, the market clearing conditions are $\pi_t = 1$ and $C_t = D_t$. Moreover, in contrast to our representative investor, we take into account that both dividend rates $(D_t)_{t\geq 0}$ and sentiments $(\eta_t)_{t\geq 0}$ fluctuate randomly, as opposed to only the dividends, which is what the representative agent assumes.

Proposition 1 (Equilibrium stock price and the risk-free rate)**.** *In an endowment economy populated by an investor with subjective sentiment-dependent preferences given by* (3.9) *and dividend dynamics of the form* (3.1)*, the equilibrium stock price is given by*

$$S_t = \frac{D_t}{k_t}, \quad k_t = A - \left(1 - \frac{1}{\psi}\right)\eta_t, \tag{4.3}$$

where $A = \beta - \left(1 - \frac{1}{\psi}\right)\left[\mu_D - \frac{1}{2}\gamma\sigma_D^2 - \lambda_D \zeta_D(\gamma)\right]$*. The risk-free rate* r_t *is given by*

$$r_t = \beta + \frac{\mu_D}{\psi} - \frac{\gamma}{2}\left(1 + \frac{1}{\psi}\right)\sigma_D^2 - \lambda_D\left(\xi_D(\gamma) - \left(1 - \frac{1}{\psi}\right)\zeta_D(\gamma)\right) + \frac{1}{\psi}\eta_t. \tag{4.4}$$

where $\xi_D(\gamma) = \mathbb{E}\left[(e^{Y_1^D} - 1)(1 - e^{-\gamma Y_1^D})\right]$.[11]

First, the equilibrium stock price in equation (4.3) increases when the agent becomes more optimistic only if the elasticity of intertemporal substitution (EIS) is relatively high, that is, $\psi > 1$.[12] Intuitively, more optimistic sentiment increases investors' willingness to pay for a given dividend

[11] Note that, for $\gamma > 1$, $(e^y - 1)\left(1 - e^{-\gamma y}\right)$ and $e^y - \frac{\gamma - e^{(1-\gamma)y}}{\gamma - 1}$ are both zero for $y = 0$ and positive for $y \neq 0$. Therefore, $\xi_D(\gamma), \zeta_D(\gamma) \geq 0$ and $\xi_D(\gamma), \zeta_D(\gamma) > 0$ if $\mathbb{P}[Y_1^D \neq 0] > 0$. Similarly, it can be shown that $\xi_D(\gamma) > \zeta_D(\gamma)$ if and $\mathbb{P}[Y_1^D \neq 0] > 0$.

[12] Recursive utility allows to fit the data better for the following reasons. Consider a few special cases: First, if

stream, pushing up stock prices and therefore compressing the dividend yield. However, this intuition only holds true if investors are more willing to defer consumption, favoring growth over dividends. In this case, optimism (higher η_t) boosts expected future returns, leading investors to accept a lower dividend yield as they anticipate higher capital gains, which is in line with empirical evidence.

Second, regarding the equilibrium risk-free rate in Equation (4.4), we observe the following: The first term, which is the subjective discount factor β, implies that an impatient representative agent, i.e. a high β, requires a higher risk-free rate in equilibrium in order for the agent to be willing to forgo current for future consumption. Next, as is the case for the equilibrium stock price, our interest rate is significantly influenced by the EIS ψ. For instance, with a high EIS, the agent is more flexible about shifting consumption. Therefore, a given increase in expected growth μ_D does not dramatically change the relative value of tomorrow vs. today, because the agent is more willing to substitute inter-temporally, which implies that the short rate is less sensitive to changes in expected growth μ_D. Next, when dividend growth becomes more risky, i.e. either diffusive dividend risk (σ_D) or dividend jump risk λ_D increases, the equilibrium risk free rate decreases. This effect is muted whenever the EIS is large.[13] The last term in the risk-free rate equation is driven by sentiment, i.e. $\frac{\eta_t}{\psi}$. Higher sentiment makes the representative agent want to invest more aggressively into the stock, but because the risk-free bond is in zero net supply, this reallocation cannot occur in aggregate and market clearing instead requires the equilibrium short rate to rise. This effect is governed by the agent's EIS, i.e. his or her willingness to substitute consumption inter-temporally in response to a change in sentiment.

4.2 Equity risk premium and its volatility

To derive the equity risk premium and its associated volatility, we proceed under the objective measure $\mathbb{P}$. Using the results in Proposition 1, we can derive the following result.

Proposition 2 (Equity premium and volatility)**.** *The expected return rate from holding the stock is*

$$R_t = \frac{\mathbb{E}[dS_t \mid \mathcal{F}_t^{D,\eta}]}{S_t dt} + \frac{D_t}{S_t},$$

where $\mathcal{F}^{D,\eta}$ is the filtration generated by the dividend rate process D and the sentiment process η.

$\psi = 1$, then $A = \beta$, price-dividend ratio becomes constant $S_t/D_t = 1/\beta$ which is inconsistent with the data. The same holds if the agent has log-utility. Next, if our agent were to have time-separable CRRA utility, i.e. setting $\psi = 1/\gamma$ in our model, then for $\gamma > 1$, the model makes the counterfactual prediction that the price-dividend ratio is lower when investor sentiment is optimistic $\eta \to 1$, i.e. $\frac{\partial}{\partial \eta_t}\left(\frac{S_t}{D_t}\right) = (1-\gamma)/\Big(\beta - (1-\gamma)\big[\mu_D - \frac{1}{2}\gamma\sigma_D^2 - \lambda_D\zeta_D(\gamma) + \eta_t\big]\Big)^2 < 0$ for $\gamma > 1$. Finally, in the recursive utility case only if $\psi > 1$ then higher sentiment leads to a higher price-dividend ratio which is consistent with empirical facts, i.e. $\frac{\partial}{\partial \eta_t}\left(\frac{S_t}{D_t}\right) = \frac{\psi-1}{\psi}/k_2^2 > 0$ if $\psi > 1$. For these reasons we will subsequently focus mainly on the case $\psi > 1$.

[13] To see this, first, the diffusive term $\frac{\gamma}{2}(1 + \frac{1}{\psi})\sigma_D^2$ clearly decreases as the EIS increases. Second, for the jump risk term, i.e. $\lambda_D(\xi_D(\gamma) - (1 - \frac{1}{\psi})\zeta_D(\gamma))$, notice that $\xi_D(\gamma) - (1 - \frac{1}{\psi})\zeta_D(\gamma) \geq \xi_D(\gamma) - \zeta_D(\gamma) > 0$ since $1 - \frac{1}{\psi} \in (0, 1)$ and thus, consistent with the intuition of how the EIS affects the impact of diffusive risk, a higher EIS also reduces the impact of dividend jump risk on the risk free rate.

Then the equity premium is given by

$$R_t - r_t = \gamma\sigma_D^2 + \lambda_D\xi_D(\gamma) - \eta_t + \frac{1}{\bar{\eta}-\eta_t}\Bigg\{\kappa(m_t-\eta_t) + \frac{\sigma_\eta^2(\eta_t)}{\bar{\eta}-\eta_t} + \rho\sigma_D\sigma_\eta(\eta_t) + \lambda_t^{\eta-}\int \frac{(\eta_t - d_t^-(y))^2}{\bar{\eta}-d_t^-(y)}\,\nu^-(\mathrm{d}y) \;+\; \lambda_t^{\eta+}\int \frac{(\eta_t - d_t^+(y))^2}{\bar{\eta}-d_t^+(y)}\,\nu^+(\mathrm{d}y)\Bigg\}, \tag{4.5}$$

and the associated conditional variance, defined as $\sigma_t^2 = \frac{\mathbb{E}\left[\left(dS_t - \mathbb{E}[dS_t|\mathcal{F}_t^{D,\eta}]\right)^2 \,\middle|\, \mathcal{F}_t^{D,\eta}\right]}{S_t^2 dt}$*, is*

$$\sigma_t^2 = \sigma_D^2 + \lambda_D\mathbb{E}\left[(e^{Y_1^D}-1)^2\right] + \frac{1}{\bar{\eta}-\eta_t}\left\{\frac{\sigma_\eta^2(\eta_t)}{(\bar{\eta}-\eta_t)} + 2\rho\sigma_D\sigma_\eta(\eta_t)\right\} + \lambda_t^{\eta-}\int\left(\frac{\eta_t - d_t^-(y)}{\bar{\eta}-d_t^-(y)}\right)^2\nu^-(\mathrm{d}y) + \lambda_t^{\eta+}\int\left(\frac{\eta_t - d_t^+(y)}{\bar{\eta}-d_t^+(y)}\right)^2\nu^+(\mathrm{d}y) \tag{4.6}$$

with a constant $\bar{\eta} = \frac{A}{1-\frac{1}{\psi}} = \frac{\psi}{\psi-1}\beta - \mu_D + \frac{1}{2}\gamma\sigma_D^2 + \lambda_D\zeta_D(\gamma)$*, varying sentiment levels* $d_t^-(y) = e^{-y}\eta_t + (1-e^{-y})a$ *and* $d_t^+(y) = e^{-y}\eta_t + (1-e^{-y})b$*, and distributions* ν^-, ν^+ *of* $Y_1^{\eta-}, Y_1^{\eta+}$ *on* $\mathbb{R}_+$*.*[14]

The first two terms in equation (4.5) are standard risk premia arising in a model with a fully rational investor (η_t=0). These two terms compensate investors for the two sources of risk inherent in the dividend process, diffusive and jump.

The first new effect, $-\eta_t$, is a "misjudgment term" which reduces the risk premium when investors are overly optimistic and increase it when they are overly pessimistic. This is a general equilibrium effect, whereby prices have to adjust such that the market clearing conditions are satisfied. Given that in partial equilibrium the investor's demand for the stock is an increasing function of her sentiment, prices in equilibrium need to adjust to clear the market for the stock: that is, prices must be so that the investor does not wish to invest too much in the stock when she is too optimistic and conversely does not wish to under-invest when pessimistic. This effect is consistent with the vast majority of the empirical evidence in the literature, as discussed in the introduction: high sentiment often signals overvaluation and lower subsequent returns, while low sentiment often corresponds to undervaluation and higher subsequent returns.

The first term in the curly bracket[15] In equation (4.5), $\kappa(m_t-\eta_t)$, captures the feedback from past price movements into current sentiment. When the signal exceeds current sentiment (i.e. $m_t > \eta_t$), the investor tends to become more optimistic about future returns, which in turn increases the equity premium. This mechanism can endogenously generate return momentum: rising prices elevate m_t, which raises η_t, further boosting perceived returns and reinforcing upward price pressure.

[14] It follows from condition (4.2) that $\bar{\eta} > b$. Therefore, all fractions in the bracket of formula (4.5) are non-negative.

[15] The factor $\frac{1}{\bar{\eta}-\eta_t}$ in front of the curly bracket in equation (4.5) depends on the critical value $\bar{\eta}$ and amplifies all further sentiment-based premium terms. For instance, in our leading parametrization, η_t can take values in $[a,b] = [-2\%, 2\%]$ with $\bar{\eta} = 4{,}6\%$. However, η_t is normally contained in $[-1\%, 1\%]$, so that $\frac{1}{\bar{\eta}-\eta_t} \approx 25$.

The second term $\frac{1}{\bar{\eta}-\eta_t}\sigma_\eta^2(\eta_t)$ reflects the market price of diffusive belief-risk volatility, even though the investor does not fully internalize or hedge future sentiment evolution due to time-inconsistency. The term is positive, irrespectively of optimism or pessimism, and reveals a compensation for the "unhedged stochasticity of misperception", priced by the market despite the investor's myopia.

The third term $\rho\sigma_D\sigma_\eta$ captures the priced risk arising from contemporaneous shocks to dividends and investor beliefs. A positive ρ implies that optimistic belief shifts tend to occur in conjunction with good fundamental (i.e., dividend) news, which inflates the risk premium.

The two last two terms in the curly bracket in equation (4.5)[16] capture the effect of extreme positive or negative sentiment swings on the excess return, which are both positive and therefore increase the equilibrium excess return.[17] In Section 6.4, we introduce the following parameterization of the sentiment jump intensities: $\lambda_t^{\eta-} = \bar{\lambda}_- \left[-\log\left(b-\eta_t\right)\right]^{\zeta_{\eta-}}$ and $\lambda_t^{\eta+} = \bar{\lambda}_+ \left[-\log\left(\eta_t-a\right)\right]^{\zeta_{\eta+}}$ for two positive constants $\zeta_{\eta-}$ and $\zeta_{\eta+}$. This functional form implies that as investor sentiment becomes extreme, either very optimistic or pessimistic, sentiment can abruptly change. Moreover, as the agent becomes overly optimistic, the likelihood of downward jumps increases while simultaneously the likelihood of an upward jump in sentiment decreases towards zero and vice versa. To complete the specification of the model, we assume that the two sentiment jump size distributions $\nu^{\pm}(\mathrm{d}y)$ are Gamma distributed with location and scale parameters $\alpha_\pm$ and $\beta_\pm$. The risk compensation for the risk of jumps is always positive, like all risk compensations, no matter whether it is favorable or unfavorable to the investor, for all values of η.

Moreover, apart from the baseline terms $\gamma\sigma_D^2 + \lambda_D\xi_D(\lambda)$- all remaining components in equation (4.5) are scaled by the price-dividend ratio. To see this, note that $\frac{S_t}{D_t} = \frac{\psi}{\psi-1}\frac{1}{\bar{\eta}-\eta_t}$. Thus, when the EIS exceeds one, the price-dividend ratio is high and thereby raises the equity risk premium, provided that the term in the curly brackets is positive. Furthermore, if momentum switches sign and given the elevated valuations, as can happen in our framework when m_t falls below sentiment η_t, the equity risk premium falls and may even become negative, see Section 4.3 for a detailed discussion.

Next, equation (4.6) shows the conditional variance. In a standard rational agent asset pricing model, this conditional variance would be equal to the first two constant terms $\sigma_D^2 + \lambda_D\mathbb{E}[(e^{Y_1^D}-1)^2]$. All the remaining terms are driven by the investor's sentiment and, both diffusive and jump terms, increase conditional equilibrium variance. As a result of these additional terms, the volatility of the stock return is higher due to the presence of sentiment than in the standard rational model.

[16] Given the critical level $\bar{\eta} > b$, the integrands are ratios of distances of $\bar{\eta}$ and η_t with respect to the moving sentiment level $d_t(y)$, an exponentially weighted convex combination between sentiment and its potentially binding boundary value. For the downward (upward) jumps, $d_t(y)^-$ moves from η_t to a ($d_t(y)^+$ moves from b to η_t).

[17] The sentiment-dependent terms in equation (4.5) are non-standard risk premium terms: these terms appear despite the investor not accounting for the full stochastic evolution of sentiment in a forward-looking way. In a standard rational model, risk premia arise due to covariance with marginal utility. In our case, the investor perceives that the dividend growth is higher when η_t is high but fails to fully internalize how the sentiment itself will evolve. Since sentiment in reality evolves stochastically, this risk introduces an exposure to unhedgeable belief shocks, which is compensated.

Next, Figure 1 shows plots of the equity risk premium and conditional volatility as functions of the investor's sentiment for a range of economically plausible parameter values.

[Figure 1 about here.]

As can be inferred from the figure, the relationship between excess return as well as its associated volatility is negative. First, in the special case where the sentiment jump intensities are constant, i.e. $\alpha_{\eta-} = \alpha_{\eta+} = 0$ (see the legend entry "independent"), the excess return almost becomes linear in sentiment, suggesting that all the effect from changing the investor's sentiment arises due to the leading term $-\eta_t$, which is the misjudgment term of the agent in the equity risk premium formula (4.5) above.[18] This negative relationship is explained by the above-mentioned general equilibrium effect whereby prices need to adjust in such a way that an overly optimistic investor does not invest too much in the risky asset and vice versa. Generalizing the intensities to allow for sudden sentiment swings at the boundaries, i.e. $\alpha_{\eta-}, \alpha_{\eta+} > 0$, shows that the equilibrium excess return is not only more elevated, but also bends upward at both extreme pessimism and optimism, suggesting that a large part of the risk compensation is derived from abrupt swings in the investor's sentiment.

Lastly, a similar conclusion can be drawn when analyzing the effect of extreme investor sentiment swings on volatility, i.e. at the upper (lower) boundary, the risk of sudden downward (upward) jumps in sentiment increase equilibrium volatility substantially. The bottom panel in Figure 1 plots the model-implied equity premium as a function of the investor sentiment (η_t) and its mean-reversion target (m_t). Interestingly, when optimism is very high, the equity premium can turn negative, if m_t is sufficiently low. This behavior stands in contrast to standard consumption-based asset pricing models.[19]

4.3 Booms, crashes, and Minsky moments

In this section, we analyze the relationship between stock prices, excess returns and sentiment, and how our model can generate sizable boom and bust cycles even if there is hardly a change in the fundamental. To this end, we simulate stock price and sentiment dynamics where we compare a model with no feedback effect and no jumps in sentiment, to model with feedback effects but no jumps in sentiment to our full sentiment model that includes both feedback effects and jumps in sentiment. In Figure 2 we depict a typical simulated trajectories of the stock price.

[Figure 2 about here.]

As the figure illustrates, dividends (dashed red line) display steady growth with minimal variation - sharply contrasting with the much more volatile path of the full sentiment model (solid black line).

[18] The blue curve bends downwards at the extreme sentiment levels since $\sigma(\eta)$ peaks at $\eta_t = 0$ and vanishes at the boundaries.

[19] In the rational expectations model, the equity premium simplifies to a positive constant $\gamma\sigma_D^2 + \lambda_D\xi_D(\lambda)$. Introducing stochastic volatility or stochastic dividend jump intensity preserves the positivity of the equity premium.

Around year 2 of the simulation, the stock price enters a pronounced boom phase, triggered by an upward jump in sentiment (first vertical dashed blue line). This expansion persists until roughly year 6, when a sudden and severe drop in sentiment causes a sharp decline in the stock price. Following this swing, the simulated price remains subdued and continues to fall until about year 10, after which it begins a new boom cycle. During this phase, the stock price rises above what both the feedback and no-feedback models imply, only to plunge again shortly after year 20, despite hardly any change in dividend growth.[20] Lastly, the model with feedback effects (dash-dotted blue line) generates stock price paths that fall between those of the full model and the version without both feedback and sentiment jumps. Relative to the no-feedback specification, feedback endogenously amplifies price movements and strengthens momentum, making booms rise higher and busts fall further. Overall, the feedback mechanism reinforces both upward and downward market dynamics.

We now turn our attention to the equity risk premium to understand how the current level of sentiment and the feedback function m_t affect this forecast. Figure 3 below showcases the dependence of the excess return on sentiment and the feedback effect m_t.

[Figure 3 about here.]

The top panel in the figure shows the equity premium surface for different sentiment and feedback m_t. A noticeable observation is the large area where the equity risk premium is negative, which is commonly referred to as a bust cycle. In our framework, such a regime arises when two events occur at the same time: First, sentiment becomes excessively optimistic: the gap between actual and expected sentiment, $\eta_t - m_t > 0$, widens, and η_t approaches its upper bound $\bar{\eta}$. Economically, this implies that investors extrapolate recent good news and push optimism toward this upper limit, and thus, the price-dividend ratio rises sharply. Second, even though sentiment remains elevated, the likelihood of a downward sentiment jump, $\lambda_t^{\eta-}$, is still moderate rather than extreme.[21] When this occurs, overvalued stock prices become susceptible to crash risk, i.e. the equity premium $\mu_{S,t}$ may become negative.[22] This high-valuation configuration is fragile: once sentiment reverts or jumps downward, prices correct sharply - thus, our framework is able to capture a classic Minsky moment.

Lastly, we examine whether the model can reproduce the empirically relevant co-movement between large *sentiment jumps* and excess returns.[23] To this end, the bottom panel of Figure 3 plots

[20] Shiller (2000) documents how waves of "irrational exuberance", overconfidence and overreaction to past returns, inflate price-dividend ratio far beyond historical means, setting the stage for subsequent crashes. These dynamics are what our sentiment jumps aim to capture: both the inflation of a bubble and its sudden collapse when sentiment turns.

[21] The combination of overconfidence and a low risk of a downward jump in sentiment is reflected by a negative value of the term in the curly braces of equation (4.5).

[22] From a technical side, because the bond is in zero net supply while equity is in unit net supply, the representative agent cannot reallocate out of the stock because the market clearing forces the agent to hold the risky asset. Instead equilibrium prices and the endogenous risk-free rate adjust, so that the market clearing condition is met.

[23] From an empirical point of view, Figure 12 in Section 7 provides strong statistical evidence that excess returns tend to be positive following upward jumps in sentiment.

the model-implied excess return as a function of the current sentiment level, for several fixed values of the feedback function m_t. These curves indicate in which regions the model is able to generate positive co-movement between sentiment and excess returns, i.e. upward (downward) sentiment shock is followed by a positive (negative) excess return.

Start with the benchmark without price feedback into sentiment, $m_t = 0$ (black dashed line). Even in this case, the model can deliver positive excess returns after an increase in sentiment, but only over a limited range: excess returns are positive from extreme pessimism (-2%) up to roughly 1%, i.e., wherever the black dashed line lies above the zero line. Conversely, generating a negative excess return after a decrease in sentiment is difficult without feedback: it requires sentiment to start from a highly optimistic region, roughly between 1% and 2%, where the black dashed line falls below zero. Thus, without feedback the sign link between sentiment shocks and excess returns is present but weak and strongly state-dependent.

Allowing for feedback effects substantially improves the model's ability the capture this positive association between sentiment and excess returns. To see this, consider the case when sentiment is close to its lower bound ($a = -2\%$) and feedback is strong and positive (blue dash-dotted line).[24] In that case, the excess return remains strictly positive across all sentiment states. In this regime, an upward jump in sentiment is therefore always followed by a positive (though diminishing) excess return as sentiment rises. On the other hand, when m_t is very low, negative excess returns become pervasive —as at the onset of a correction when prices have been falling—in which case the model produces negative excess returns over a wide range of sentiment values, extending down to approximately -0.5%.

In summary, feedback effects allow the model to match realistic time-series behavior across a much broader set of sentiment states than a no-feedback specification. In particular, they generate the empirically observed pattern that large positive (negative) sentiment shocks can be followed by large positive (negative) excess returns - a feature that emerges only when the feedback channel is active and can switch sign.

5. The Data

Stock Prices, Dividends and Riskfree Rate. The S&P 500 (logarithmic) returns including dividends obtained from the CRSP database serve as a proxy for the risky-asset return.[25] The risk-free rate is the one-month (constant maturity) Treasury bill rate, which we construct using the daily effective fed funds rate from the Fred St. Louis Database (Ticker "DFF") and the one-year maturity

[24] As a real world example, consider a recovery episode following a crash. Prices begin to rebound, so the short-run average $\bar{S}_t^l$ rises above the long-run average $\bar{S}_t^L$, turning the feedback function positive and potentially large. At the same time, sentiment may remain depressed, i.e. sentiment remains low because investors are still cautious, while the feedback signal is positive because recent price changes are strong relative to the longer-run trend. This regime implies that $\eta_t < m_t$ so that positive excess returns after upward sentiment shocks can occur which is consistent with the empirical evidence.

[25] Center for Research in Security Prices, US Stock Database, obtained from Wharton Research Data Services.

zero coupon yield obtained from the Federal Reserve.[26] We aggregate them to the weekly/monthly frequency depending on the respective sentiment data.

Sentiment data. Investor sentiment can be driven by a combination of factors such as economic indicators, news, market trends, and psychological factors. To try to capture market sentiment, we use two different proxies. The common goal of these two indexes is to measure investor sentiment, but they vary significantly, not only in terms of their construction and methodology but also in terms of the input data they employ the Investor Sentiment Survey Index by the American Association of Individual Investors (AAII). To produce this index, individual investors are surveyed weekly about their views on where the stock market is heading over the next six months. Their vote is expressed in terms of being bullish, neutral or bearish. We construct two sentiment proxies from this index. First, we compute the weekly spread between the bearish and bullish percentages and, second, we divide this difference by the percentage of neutral votes.[27]

Constructing sentiment proxy. The sentiment process η_t is restricted to the interval $[a, b]$, where $a < 0 < b$. These limits represent how much the agent overestimates, which corresponds to b, or underestimates, which corresponds to a, the dividend growth rate. As a starting point, we focus on the symmetric case, i.e., $a = -\theta$ and $b = \theta$, with $\theta = 2\%$. Given that the long run dividend growth rate is about 2% in the data, this implies that the agent believes real dividend growth can be as low as 0% or as high as 4% in the extreme cases of pure pessimism and optimism. Next, to map the original sentiment process to this interval, we use the arc-tangent function, i.e.,[28]

$$\eta_t = f(x; \theta, \varsigma) = \theta \frac{2}{\pi} \arctan(\varsigma x) \in [-\theta, \theta], \tag{5.1}$$

where $\varsigma > 0$ is a scaling parameter. For each of the two sentiment indexes, we select an appropriate scaling parameter ς with the aim of minimally changing the statistical properties of the original sentiment data beyond mapping into the desired interval.[29] In Figure 4 we plot the transformed

[26] We construct the one-month maturity yield by linear interpolation between the effective fed funds rate and the one year yield. We also construct the three-month interest rate and compare it to the three-month (constant maturity) obtained from the Fred St. Louis Database (Ticker "DGS3MO"). We smooth the resulting two interest rate data sets with a Hampel-filer as the data exhibit occasional spikes. Their correlation is 0.991.

[27] More information about the index construction method can be found at this link: AAII. This index is available at a weekly frequency, starts on July 24th, 1987. In the Online Appendix in Section S.3 we use another sentiment index, the news sentiment index of Shapiro et al. (2022) (referred to as SSW), to verify that our empirical predictions are not specific to the sentiment index we have used here in the main text. This index measures economic sentiment on a daily frequency based on lexical analysis of economics-related news articles, our results remain largely unchanged.

[28] In general, to map the sentiment process to the closed interval $[a, b]$, the following transformation can be used $f(x; \varsigma, a, b) = a + \frac{b-a}{2}\left(1 + \frac{2}{\pi}\arctan(\varsigma x)\right) \in [a, b]$. Other transformation functions such as sigmoid, the hyperbolic function or any cumulative distribution function could serve as possible transformation. The arc-tangent preserves extreme values in investor sentiment. Other transformations alter the original sentiment data more as they compress the extreme values stronger and the transition to the boundaries is faster compared to the arc-tangent transformation.

[29] We set the transform parameter $\varsigma = 2$. The transformation leaves the statistical properties of the resulting sentiment indexes invariant. We compute the correlations between the original standardized sentiment data, i.e. mean zero and variance one, and our newly transformed sentiment index, both in levels and in the first differences. The

sentiment proxy.

[Figure 4 about here.]

As Figure 4 shows, the sentiment index exhibit broadly speaking positive co-movement with the business cycle, i.e. it tends to bottom around recession dates (indicated by the shaded areas) and reach its highest levels at times when the economy is booming.[30]

6. Validating the Sentiment Dynamics

As described in Section 3.2, our model relies on specific assumptions regarding the evolution of the sentiment process. We now empirically validate each assumption separately, regarding the drift, volatility and jump components of the sentiment dynamics in equation (3.3). To this end, we start by presenting a recursive jump detection rule which will be central to our analysis to distinguish the diffusive part from the jump part of our sentiment model. Then, we first examine whether the drift of the sentiment indeed incorporates feedback from stock prices through the function m_t, by testing this model for predicting changes in sentiment η_t against a model where sentiment mean-reverts to a fixed value, say, the neutral sentiment level of 0. Second, we analyze whether the diffusive volatility is indeed hump-shaped in the sentiment level η_t. Third, we examine whether the upward $\lambda_t^{\eta,+}$ and downward jump $\lambda_t^{\eta,-}$ intensities depend on the investor's sentiment level, and whether positive (resp. negative) jumps are more likely at highly pessimistic (resp. optimistic) sentiment levels. Lastly, we analyze the resulting distribution of positive and negative jumps in sentiment.

6.1 Identification of jumps

An empirical challenge that arises in our context is the fact that the naive volatility estimator based on the changes in sentiment $\Delta\eta_t$ contains both, the volatility contribution of the diffusive term $\sigma_\eta(\eta)$, plus the contribution of the jump terms $J_t^{\eta+}$ and $J_t^{\eta-}$ to the overall variability of the η_t.[31] In order to obtain an estimator of the diffusive volatility component only, we remove the jumps detected based on a recursive procedure.[32] More specifically, in order to disentangle sentiment jumps from

corresponding correlations for the AAII index are 0.991 in levels and 0.981 in first differences.

[30]The raw series has mean 0.066, volatility 0.18, skewness -0.06, kurtosis 2.93, an empirical range $[-0.54, 0.62]$, and AR(1)$= 0.706$. After applying the arc transformation in (5.1), the mean moves to $= 0.4\%$ and volatility drops to 1.1%; the distribution is modestly left-skewed -0.296 with lower kurtosis 2.28. The range tightens to $[-1.88\%, 1.78\%]$, while persistence is basically unchanged (AR(1)$= 0.71$).

[31]Because $\sigma_\eta(\eta_t)$ in (3.3) is state-dependent (hump-shaped Jacobi volatility) and the feedback effect function m_t ties sentiment to price momentum, the scale of $\Delta\eta_t$ changes across regimes even when no jump occurs. On top of that, the jump terms $J_t^{\eta,\pm}$ with state-dependent intensities $\lambda_t^{\eta,\pm}$ inject sporadic spikes that inflate the naive variance estimate. Therefore, a fixed cutoff based on an unconditional (or nave rolling) sample volatility estimate is misleading, as it tends to wrongly classify large movements in sentiment in high-volatility regimes and misses genuine discontinuities in low-volatility regimes, and thereby confounds stochastic volatility with jumps.

[32]We estimate the diffusive component of the sentiment process $\sigma_\eta(\eta_t)$ as the square root of the sample variance

sentiment volatility, we employ the following jump detection rule

$$\mathcal{I}_t := \mathbb{I}_{\{|\Delta\eta_t| > k_J \sigma_\eta(\eta_t)/\sqrt{m}\}}, \qquad Z_t := \mathcal{I}_t \Delta\eta_t \tag{6.1}$$

where k_J is a positive constant, $\sigma_\eta(\eta_t)$ denotes the diffusive component of sentiment volatility, m the sampling frequency and Z_t represents the extracted jump size.[33] In Figure 5 we plot the diffusive volatility obtained using the recursive method outlined above.

[Figure 5 about here.]

As can be seen from Figure 5, sentiment changes cluster over time. Moreover, there is heteroskedasticity in the diffusive volatility of sentiment changes, which leads to a constantly varying jump detection threshold and at times hovers far away from its unconditional level. This implies that the time-varying jump detection threshold detects jumps in periods of relative tranquility, which a naive jump detection threshold based on the fixed, unconditional volatility, is not able to do, as it only classifies the largest movements in sentiment as jumps. Conversely, in periods with elevated sentiment volatility, large sentiment changes are not necessarily classified as jumps as the diffusive volatility estimator increases above its unconditional level. Therefore, our recursive jump detection rule yields a jump-robust scale and makes the classification approximately scale-invariant across regimes, reducing the risk of mistaking volatility swings for genuine jumps.

6.2 Sentiment drift and price feedback

Out-of-sample forecasts of sentiment with stock price feedback. Our first empirical test is to show that adding a feedback mechanism improves the predictability of changes in sentiment of our model compared to a model where the long-run mean of sentiment is fixed at 0. To this end, we compare R^2 values from forecast regressions. As for the specification of the feedback function m_t, we focus on the symmetric price feedback effects, i.e. we specialize m_t in (3.5) as follows

$$m_t = \theta \frac{\pi}{2} \arctan(qM_t), \; q > 0, \tag{6.2}$$

estimator, where we iteratively remove the detected jumps. To be more precise, $\sigma_\eta(\eta_t)$ is estimated over a time period of d days by calculating the number of jumps over this time period using (6.1), remove all the detected jumps in $\Delta\eta_t$, then recalculate the volatility $\sigma_\eta(\eta_t)$ once more, followed by checking again whether there are still jumps detected in the time series of $\Delta\eta_t$ until no further jumps are detected in the sample.

[33] We use the *median absolute deviation* (MAD) to make the initial diffusive volatility estimator robust to jumps. By definition,

$$\text{MAD} = \text{median}_i\big(\,|x_i - \text{median}_j(x_j)|\,\big), \qquad \hat{\sigma}_{\text{MAD}} = \frac{\text{MAD}}{\Phi^{-1}(0.75)} \approx 1.4826\,\text{MAD},$$

where Φ^{-1} is the standard Normal quantile function. This reduces the first-round upward bias in the recursive de-jumping step, yields an initial threshold $k_J\hat{\sigma}$ that is sufficiently *stringent* to reveal jumps that an over-inflated scale would miss, and - after removing flagged increments ensures that subsequent re-estimation on the filtered sample - provides a jump-robust local estimate of the diffusive scale. Section S.2 of our online appendix provides more details.

where $\theta = -a = b$ and $q = c_n = c_p$ is the (symmetric) feedback strength parameter. The price momentum in log units, $M_t = \log \frac{\bar{S}_t^l}{\bar{S}_t^L}$ in (3.4), drives the long run mean of sentiment. So the model predicts that if the stock market has been going up (resp. down) over the past days/weeks, investor sentiment tracks (or mean-reverts to) a more optimistic (resp. pessimistic) level. To address this mechanism, we run a standard forecasting regression of the form

$$\Delta\eta_{t+h} = \eta_{t+h} - \eta_t = \beta_0 + \beta_{FB}\,(m_t - \eta_t) + \epsilon_{t+h}, \tag{6.3}$$

where ϵ_{t+h} denotes the forecast error at horizon h (measured in days). Lastly, in order to find reasonable parameter estimates for the feedback strength parameter q, the lag parameters l and L, we maximize the R^2 obtained from the regression in Equation (6.3) above. In Figure 6 below, we plot the sentiment process η_t together with the optimally parametrized feedback effect function m_t.

[Figure 6 about here.]

Figure 6 shows that the estimated price feedback function m_t positively co-moves with the investor sentiment.[34] When the S&P 500 index level has been increasing relative to its recent average level, investor sentiment becomes more optimistic, and vice versa. In addition, while the estimated price feedback function m_t fluctuates around the neutral sentiment level of zero, there are periods of prolonged and persistent deviations of m_t from that neutral sentiment level. Finally, as the bottom panel shows, the predictive R^2 is improved using the model with feedback as it outperforms its counterpart at all forecast horizons h. The out-performance differs across the forecast horizon, i.e. for relatively short h, the out-performance of the feedback model is significantly larger compared to the case when h is larger.

Granger causality tests. In this section, we test for feedback effects from prices to sentiment (as assumed by the dynamics of sentiment) using Granger causality tests. To this end, we perform a set of Granger causality tests, based on bivariate VAR models, to investigate the causal relationship between the market excess return and investor sentiment. Formally, we test two null hypotheses for forecast horizons $h \in \{1, 2, 4, 12\}$ measured in weeks

(i) $\mathcal{H}_0^{R \not\to S}$: lagged returns do *not* Granger-cause sentiment.

(ii) $\mathcal{H}_0^{S \not\to R}$: lagged sentiment does *not* Granger-cause returns.

Thus, a rejection of the null hypothesis would indicate that there is an effect, i.e. lagged values of sentiment (market returns) can predict or Granger-cause current market returns (sentiment). In Figure 7 we report the p-values associated with the Granger causality tests.

[Figure 7 about here.]

[34] Moreover, the sample correlation between the estimated feedback effect function m_t and sentiment η_t is 0.43.

The Granger causality results show that the null hypothesis can be rejected with strong statistical evidence for both tests, i.e. whether the sentiment Granger-causes excess returns or returns Granger-causes sentiment. Interestingly, the feedback effect from past returns into sentiment is highly statistically significant (indicated by the "$< 1\%$" labels), which implies that past returns have significant forecasting power for future sentiment levels. Therefore, these Granger causality tests further strengthen our modeling assumption that past returns are due to current impact sentiment levels, captured by the function m_t in equation (3.5).

6.3 Sentiment volatility

In our model for the sentiment process, the volatility function $\sigma_\eta(\eta) = s_\eta\sqrt{(\eta - a)(b - \eta)}$, is hump-shaped in the level of sentiment and peaks at $\eta_t = 0$ (in the symmetric case where $\theta = -a = b$). We now examine whether the empirical evidence is consistent with this specification. To be able to measure the volatility of the sentiment data, given the potential presence of jumps in sentiment, we first employ a recursive jump detection rule, which is described in Section 6.1 above. This allows for a separation of the total quadratic variation of the sentiment process into diffusive and jump components. Figure 8 shows a scatterplot of the estimated diffusive volatility estimated on a rolling basis (over twenty-five non-overlapping weeks), plotted against the average sentiment level over the same period.

[Figure 8 about here.]

As can be seen from Figure 8, the fact that the lowess fit peaks at the sentiment neutral $\eta_t = 0$ and overall hump shape are consistent with the modeling assumptions on the diffusive volatility component of the sentiment dynamics.

6.4 Sentiment jumps

We analyze the distribution of positive and negative jumps in sentiment. More precisely, our model implies that the positive and negative jumps in sentiment are given by $z_i^{\eta,+} = (b - \eta_{t-})(1 - e^{-y_i^{\eta,+}})$ and $z_i^{\eta,-} = -(\eta_{t-} - a)(1 - e^{-y_i^{\eta,-}})$, see equation (3.3). Next, solving these equations for $y_i^{\eta,+}$ and $y_i^{\eta,-}$ gives

$$y_i^{\eta,+} = -\log\Big(1 - \frac{z_i^{\eta,+}}{b - \eta_{t-}}\Big), \qquad y_i^{\eta,-} = -\log\Big(1 + \frac{z_i^{\eta,-}}{\eta_{t-} - a}\Big) \tag{6.4}$$

whose distribution should follow a Gamma distribution with shape and scale parameters $\alpha_\pm$ and $\beta_\pm$. In Figure 9 below, we plot the histogram of both $y_i^{\eta,+}$ and $y_i^{\eta,-}$.

[Figure 9 about here.]

With a jump detection threshold $k_J = 2.5$ from equation (6.1) the algorithm detects 24 (30) upward

(downward) sentiment jumps of average size 0.44% (0.36%) at a weekly frequency.[35]

[Table 1 about here.]

Table 1 also shows a slightly lower dispersion (variance 1.47% vs. 1.54%).[36] Negative jumps are more right-skewed and heavy-tailed (skewness 0.69, kurtosis 3.72) than positive jumps (0.57, 3.46), implying relatively higher probability of large negative jump sizes.

Jump intensity at different sentiment levels. Our last model validation step asks whether upward (resp. downward) jumps in sentiment become more likely when the investor becomes excessively pessimistic (resp. optimistic). To empirically validate this assumption, we compute the percentiles of the sentiment process and then count, conditional on the level of sentiment η_t, how many upward and downward jumps were detected at different sentiment levels. Table 2 summarizes the results.

[Table 2 about here.]

The specification of our model suggests that when the investor becomes very optimistic (resp. pessimistic), downward (resp. upward) jumps in sentiment become more likely. This assumption holds in the data, as the number of upward (resp. downward) jumps in sentiment decreases (resp. increases) with the level of sentiment. For instance, the top two rows in Table 2 with jump detection threshold $k_J = 2.5$, shows there are 20 upward jumps compared to 5 downward jumps in sentiment when sentiment is below its median value, i.e., the 50%-quantile, but there are 3 upward jumps and 24 downward jumps when sentiment is above the median. A more granular decomposition of sentiment levels into quantiles lead to the same conclusion: the higher the sentiment level, the more (resp. less) likely downward (resp. upward) jumps are. Overall, we conclude that the model proposed in Section 3.2 for the sentiment dynamics is supported by the empirical evidence.

7. Observable Implications of the Model

In this Section, we test the implications of the model concerning the dependence of the endogenous equilibrium variables on sentiment. We further analyze how jumps in the investor's sentiment affect the excess return and its associated realized volatility.

Implied predictive regressions. As stated in Proposition 1 and 2, the model implies that, in equilibrium, the equity risk premium, its volatility, the risk-free rate and the stock dividend yield all depend on investor sentiment in closed form. To this end, we regress these variables onto the two proxies for investor sentiment at different forecast horizons h. Let $r_{e,t} := R_t - r_t$ define the log excess

[35] With this jump detection method, on average every 18 months (15 months) a positive (negative) jump occurs.

[36] Recall $\mathbb{E}[Y] = \alpha_\pm\beta_\pm$, $\text{Var}[Y] = \alpha_\pm\beta_\pm^2$, skewness $\mathbb{S} = 2/\sqrt{\alpha_\pm}$ and kurtosis $\mathbb{K} = 3 + \frac{6}{\alpha_\pm}$ for a Gamma distribution.

(total, cum dividends) return of the S&P 500 index over the risk free rate and let $RV_t(r_{e,t})$ be its associated realized volatility[37]. Then, we run the following predictive regressions,

$$Y_{t+h} = \beta_0 + \beta_\eta \eta_t + \epsilon_{t+h},$$

where $Y_t \in \{r_t,\ k_t,\ r_{e,t},\ RV_t(r_{e,t})\}$ and $h = 1, \ldots, H$ represents the forecast horizon. In Figures 10 and 11 we summarize our results for different forecast horizons.

[Figure 10 about here.]

[Figure 11 about here.]

As shown in Figure 10, regardless of the forecast horizon h the estimated regression coefficients are in line with the model's predictions. Statistical significance, however, varies depending on the forecast horizon h for all variables except for the dividend yield where the regression coefficients are statistically significant for all forecast horizons. For the excess return and risk-free rate regressions, the results become significant for a forecasting horizon of a few weeks. This is not surprising since both variables are well known to be quite noisy for short h. As can be inferred from Figure 11 this observation is also in line with the increase in explanatory power of sentiment on excess returns and the risk-free rate as we increase the forecast horizon. By contrast, the negative relation between investor sentiment and realized volatility is strongest at the shortest forecast horizon.

Impact of jumps in sentiment on stock returns. Our model predicts that a jump in sentiment immediately affects both the excess return and its associated volatility. To test this prediction empirically, we employ our recursive jump detection rule outlined in Section 6.1. In Figure 12 we show the impact of jumps in sentiment on asset price moments based on our two sentiment proxies.

[Figure 12 about here.]

First, the top left panel shows that of the 84 jumps detected in the S&P500 index, i.e., 4.1% of the observations 13 jumps can be traced back to sentiment and 71 are idiosyncratic jumps. Next, in the top right panel, we show the contemporaneous (same week) change in the S&P 500 index given a jump in sentiment was detected. The panel reveals a positive correlation of 0.53 between jumps in sentiment and index changes, which indicates that a positive jump in investor sentiment

[37] The empirical proxy for our model implied variance of the returns is the realized variance estimator, which we construct as follows: Let S_t denote the asset price at time t, and define the *log-return* over interval $[t_{i-1}, t_i]$ as $r_i = \log S_{t_i} - \log S_{t_{i-1}}$. Suppose we observe mh returns over a fixed interval. The *realized variance* (RV) for that period is then defined as $RV_{t+h} = \sum_{i=1}^{mh} r_i^2$. This is a non-parametric estimator of the quadratic variation of the log-price process over the interval, assuming arbitrarily fine sampling. Since we are working with daily data but use weekly frequency for our regression analysis, we set $m = 5$ (=days per week).

is associated with an increase in the S&P500 within the same week and vice versa for a negative jump in sentiment. Next, in the bottom two panels of Figure 12 we show the cumulative excess return and the average realized volatility over the next four weeks after a sentiment jump has been detected. As can be inferred from the bottom left panel, a sentiment jump elicits a large reaction in asset markets, a positive jump in sentiment is associated with an average excess return of 38% one week after and, analogously, excess returns are very negative (-52%) after a negative jump in sentiment has occurred.[38] As discussed in Section 4.3, reconciling this empirical pattern with theoretical asset pricing models is challenging. According to Proposition 2 in Section 4.2, the leading sentiment term is "$-\eta_t$," which we refer to as the misjudgment term. Consequently, standard models with no feedback effect and jumps in sentiment imply a negative relationship between sentiment and excess returns (see also bottom panel in Figure 3 with $m_t = 0$). However, our framework incorporates a feedback mechanism m_t into the sentiment process. Under specific conditions, this feature allows the model to generate short-run positive momentum in excess returns following sentiment jumps, aligning with the empirical evidence. Finally, the observed response of realized volatility to sentiment jumps is consistent with economic intuition: positive sentiment jumps are associated with declines in volatility of the excess return, while negative sentiment jumps increase it. This empirical finding is also captured by our model. As shown in the bottom panel of Figure 1, the model generally implies a negative relationship between sentiment and realized volatility.

8. Moment Matching: Model vs. Data

This section evaluates the model's quantitative performance by comparing a set of targeted empirical moments to their model-implied counterparts for four endogenous objects: the risk-free rate, the dividend yield, the equity premium, and return volatility. We also study two restricted variants of the baseline specification (i) a symmetric-jump model and (ii) a no-jump model to isolate which features are essential for matching the data.

8.1 Calibration procedure

We calibrate the parameter vector Θ by minimizing the distance between empirical moments and model-implied moments, augmented with a regularization (penalty) term that disciplines the sentiment jump-size parameters. Let μ^{data} denote the stacked vector of empirical moments and $\mu^{\text{model}}(\Theta)$ the corresponding model-implied moments. Our baseline loss is

$$\mathcal{L}_{\text{mom}}(\Theta) = \Big(\mu^{\text{model}}(\Theta) - \mu^{\text{data}}\Big)' W \Big(\mu^{\text{model}}(\Theta) - \mu^{\text{data}}\Big), \tag{8.1}$$

where $W = \text{diag}(w_1, \ldots, w_K)$ is a diagonal weighting matrix.

We simultaneously match both unconditional and sentiment-conditional moments. Specifically, we include: (i) unconditional moments (no conditioning on sentiment); (ii) conditional moments

[38] We test whether the average excess return after a positive or negative jump (one week) in sentiment has occurred and found strong evidence that the null hypothesis can be rejected. The t-stat is t=2.69 with a p-value of 0.989%.

under a two-way split at the median sentiment level (low: $\eta_t \leq q_{50}$; high: $\eta_t > q_{50}$); and (iii) conditional moments under a three-way split using terciles (low: $\eta_t \leq q_{33}$; medium: $q_{33} < \eta_t \leq q_{66}$; high: $\eta_t > q_{66}$). This yields $i \in \{1, \ldots, 6\}$ sentiment regimes and $j \in \{1, \ldots, 4\}$ moment types (risk-free rate, dividend yield, excess return, volatility), so that

$$\mathcal{L}_{\text{mom}}(\Theta) = \sum_{i=1}^{6} \sum_{j=1}^{4} w_{ij} \Big(\mu_{ij}^{\text{model}}(\Theta) - \mu_{ij}^{\text{data}} \Big)^2. \tag{8.2}$$

Dividend growth parameters are disciplined using the annual average real dividend growth rate and volatility, with fitted values $\mu_D = 1.9\%$ and $\sigma_D = 3.2\%$. We set $\theta = 2\%$, implying sentiment bounds $\eta_t \in [-2\%, 2\%]$; under the representative agent's subjective expectation, dividend growth rates then lie in $[-0.1\%, 3.9\%]$. To construct the feedback function m_t we use $l = 8$, $L = 80$, and feedback strength $q = 8$.

Sentiment jumps $Y^{\eta,\pm}$ follow a Gamma distribution, $Y^{\eta,\pm} \sim \text{Gamma}(\alpha_{\eta,\pm}, \beta_{\eta,\pm})$ (shape–scale parameterization), which implies $\mathbb{E}[e^{-Y}] = (1+\beta)^{-\alpha}$. Next, we parameterize the time-varying jump intensities as follows

$$\lambda_t^{\eta+} = \bar{\lambda}_+ \left[-\log\left(\eta_t - a\right) \right]^{\zeta_{\eta+}}, \quad \lambda_t^{\eta-} = \bar{\lambda}_- \left[-\log\left(b - \eta_t\right) \right]^{\zeta_{\eta-}}, \qquad \zeta_{\eta\pm} \geq 0, \tag{8.3}$$

so that the likelihood of downward jumps in sentiment rises in optimistic states and vise versa. The curvature parameters $\zeta_{\eta\pm}$ control the speed at which the probability of downward and upward jumps increase as investor sentiment becomes more and more extreme.

In addition to matching the moments in (8.2), we augment the objective with a penalty term that disciplines the Gamma jump-size parameters $(\alpha_{\eta,\pm}, \beta_{\eta,\pm})$ so that the model is not only consistent with the asset price moments, but also with the jump size distribution we extracted from the sentiment time series. Let $(\widehat{\alpha}_{\eta,\pm}, \widehat{\beta}_{\eta,\pm})$ denote benchmark values estimated from the sentiment time series via maximum-likelihood estimation. We define the penalty as follows

$$\mathcal{P}(\Theta) = \sum_{s \in \{+,-\}} \left[\left(\frac{\alpha_{\eta,s} - \widehat{\alpha}_{\eta,s}}{\widehat{\alpha}_{\eta,s}} \right)^2 + \left(\frac{\beta_{\eta,s} - \widehat{\beta}_{\eta,s}}{\widehat{\beta}_{\eta,s}} \right)^2 \right], \tag{8.4}$$

and estimate parameters by minimizing the augmented loss

$$\min_{\Theta} \quad \mathcal{L}(\Theta) = \mathcal{L}_{\text{mom}}(\Theta) + \omega_J \, \mathcal{P}(\Theta), \tag{8.5}$$

where $\omega_J \geq 0$ governs how tightly we regularize jump-size parameters relative to the asset moments.[39]

[39] We incorporate this additional penalty term for the following reasons: First, without regularization the optimizer can push (α, β) to extreme values that artificially fit pricing moments by exploiting the nonlinear mapping from jump sizes into risk premia and volatility, yielding economically implausible jump distributions. Second, the penalty improves numerical stability because jump integrals and exponential terms become ill-behaved precisely in those extreme regions, which can otherwise generate spurious local minima or non-convergence.

Our *asymmetric* (full) model uses the 14-parameter vector

$$\Theta_{\text{asym}} = (\beta, \gamma, \psi, \kappa, s_\eta, \rho, \bar{\lambda}_+, \bar{\lambda}_-, \zeta_{\eta+}, \zeta_{\eta-}, \alpha_{\eta,+}, \alpha_{\eta,-}, \beta_{\eta,+}, \beta_{\eta,-}),$$

and the penalty $\mathcal{P}(\Theta)$ in (8.5) applies to both $(\alpha_{\eta,+}, \beta_{\eta,+})$ and $(\alpha_{\eta,-}, \beta_{\eta,-})$. In the *symmetric-jump* model with 10 parameters, we impose $\bar{\lambda}_+ = \bar{\lambda}_- = \bar{\lambda}$, $\zeta_{\eta+} = \zeta_{\eta-} = \zeta_\eta$, $\alpha_{\eta,+} = \alpha_{\eta,-} = \alpha_\eta$, $\beta_{\eta,+} = \beta_{\eta,-} = \beta_\eta$. The penalty is then applied to the common jump-size parameters

$$\mathcal{P}_{\text{sym}}(\Theta) = \left(\frac{\alpha_\eta - \widehat{\alpha}_\eta}{\widehat{\alpha}_\eta}\right)^2 + \left(\frac{\beta_\eta - \widehat{\beta}_\eta}{\widehat{\beta}_\eta}\right)^2, \tag{8.6}$$

where $(\widehat{\alpha}_\eta, \widehat{\beta}_\eta)$ are the corresponding symmetric benchmarks (e.g., pooled across jump signs or constructed as an average of $(\widehat{\alpha}_{\eta,\pm}, \widehat{\beta}_{\eta,\pm})$). Finally, in the *no-jump* model with 6 parameters we set $\lambda_t^{\eta\pm} \equiv 0$, so the model does not have any jumps in sentiment. Accordingly, we remove the jump-size penalty from the optimization by setting $\omega_J = 0$ (equivalently, dropping $\mathcal{P}$) and we omit $(\alpha_{\eta,\pm}, \beta_{\eta,\pm})$ from the calibrated parameter vector.

8.2 Moment matching: results

Table 3 reports the empirical moments (Emp) and their model-implied counterparts (MI) for the four targeted variables across unconditional and sentiment-sorted regimes.

[Table 3 about here.]

Overall, the asymmetric jump specification delivers a tight fit both unconditionally and across sentiment bins, while imposing symmetry on jump parameters substantially worsens the fit - most visibly for the equity premium and return volatility. Shutting down sentiment jumps altogether preserves the unconditional fit reasonably well, but it largely eliminates the model's ability to reproduce the conditional moments of excess returns and volatility.

Turning to the asymmetric specification, the model matches the unconditional targets closely: for example, the risk-free rate is 2.93 (MI) versus 2.98 (Emp), the equity premium is 6.82 versus 6.86, and volatility is essentially exact at 16.53. The only noticeable unconditional discrepancy is the dividend yield, which is modestly overstated (2.22 versus 2.15). Conditioning on sentiment, the model continues to track the data well: it reproduces the higher equity premium and volatility in low-sentiment states and the lower premium in high-sentiment states, and it also captures the non-monotonic volatility pattern across terciles - volatility is lowest at moderate sentiment (Emp: 15.77) and higher in both low and high sentiment bins (Emp: 17.90 and 15.94), a shape the model matches closely. In addition, the calibration procedure produces jump size parameters $\alpha_\pm$ and $\beta_\pm$ which are very close to the estimated jump size parameters fitted to the extracted sentiment jumps,[40] suggesting that the model is able to reproduce asset price moments and simultaneously capture the

[40] Compare MLE estimates in Table 1 to calibrated parameters in "Asymmetric" regime in Table 3 above.

sentiment jumps we extracted from our indices using our recursive jump detection rule. Next, imposing symmetry on jump parameters materially degrades the fit. While the risk-free rate remains only moderately too high (e.g., 3.15 vs. 2.98 unconditionally), the model severely overshoots both the equity premium and volatility (unconditionally 12.98 vs. 6.86 for excess returns and 37.95 vs. 16.53 for volatility), and this distortion persists across sentiment regimes, reflected in the higher criterion value ($\mathcal{L}^{\dagger} = 0.965$ vs. 0.048). Finally, shutting down sentiment jumps preserves a decent unconditional fit, but it largely removes the model's ability to generate the observed state dependence: return volatility becomes comparatively flat across sentiment bins (MI values clustered around 16–17) and fails to reproduce the low-at-mid, high-at-extremes pattern. Consistent with this, the error criterion increases substantially to $\mathcal{L}^{\ddagger} = 1.793$.[41]

Our final empirical validation assesses whether the model can replicate not only (conditional) *average* moments, but also the *time-series variation* in the risk-free rate, dividend yield, excess return, and realized volatility. To this end, for each variable, we run the following regression

$$Y_t^{\text{emp}} = \alpha + \beta\, Y_t^{\text{model}} + \varepsilon_t, \tag{8.7}$$

where Y_t^{emp} denotes the empirical series and Y_t^{model} its model-implied counterpart. We evaluate time-series performance along three dimensions. First, β should be positive and statistically different from zero, indicating that the model and data co-move in the right direction. Second, explanatory power should be high, as captured by the adjusted R^2. Third, we test whether the slope is close to one; the final column reports the p-value for $H_0\colon \beta = 1$, which diagnoses whether the model matches the scale of empirical fluctuations rather than merely their sign. Table 4 summarizes the results.

[Table 4 about here.]

Overall, the time-series regressions show the following: the model's success hinges on allowing for sentiment jumps. First, staring with the risk-free rate: across all specifications the slope estimates are reasonably close to one although imprecisely estimated. Moreover, the adjusted R^2 is modest, indicating that the model's risk-free-rate dynamics do capture some of the empirical behavior of the empirical risk-free rate dynamics, although the statistical significance is weak. Next, the results for the dividend yields show that the our model-implied dividend yield is able to capture a larger part of the variation of the empirical dividend yield. First, the slope estimates are all larger than zero and highly statistically significant - apart from the symmetric model where the statistical significance falls slightly - and reasonably close to one, which suggests that the model-implied dividend yield does capture some of the time series variation of the empirical dividend yield. Moreover, compared to the risk-free rate regressions, the adj. R^2 is now approximately 5%, which corresponds to an almost

[41] Recall that $\mathcal{L}^{\ddagger}$ excludes the jump-size penalty by construction so that this loss criteria is, conditional on the same moment fit, lower than the loss criteria ($\mathcal{L}^{\dagger}$) we impose on the asymmetric and symmetric sentiment jump model.

10 fold increase in explanatory power.[42] However, the economically relevant differences emerge for equity-market variables - in particular the jumps in sentiment matter. For excess returns, all models generate a strong positive co-movement with the data (Adj. R^2 between 0.36 and 0.43), but the *joint* behavior of returns and volatility is where jumps matter. The full asymmetric jump model produces a positive volatility slope ($\beta = 0.893$) and which can not be distinguished from a unit slope ($p = 0.851$), i.e., it gets the sign and broadly the scale of volatility fluctuations right. In contrast, the symmetric-jump restriction collapses the volatility linkage: the slope is close to zero ($\beta = 0.117$) and the unit-slope hypothesis is decisively rejected ($p = 0.0001$). The no-jump model produces counterfactual results by generating a *negative* and strongly significant volatility slope ($\beta = -3.378$). Thus, these results show that sentiment jumps are crucial for the model to reproduce the empirical volatility dynamics without sign reversals and without significantly distorted scaling. Without jumps (or under symmetry), the model may still track parts of return variation, but it fails precisely where the data are most unforgiving: the time-series behavior of market volatility.

9. Conclusions

We develop a continuous-time asset pricing equilibrium model in which the agent is subject to a stochastic sentiment that tracks any momentum in asset prices, and sometimes experiences sudden and abrupt mood swings. Our framework is highly tractable, leading to closed form expressions for the values and dynamics of the endogenous variables, namely the risk-free rate, dividend yield, stock price, equity premium and its volatility. Our model allows for feedback effects from prices into sentiment and vice versa leading to realistic boom-and-bust cycles in asset prices driven by shifts in sentiment. Like models based on extrapolative expectations, our model can generate the ramp up phase of asset pricing bubbles, but unlike them it can also generate their rapidly deflating phase.

We then empirically validate each component of our sentiment model and find strong statistical support for our sentiment process specification. Next, our model leads to a set of theoretical predictions which we test empirically based on two entirely differently constructed sentiment indices. Interestingly, despite their differences, the empirical dependence of our model-implied variables on sentiment is not only in line with the model's predictions but also statistically significant at different forecast horizons. Finally, calibrating our model to the data we find that our model is able to replicate not only a set of targeted conditional and unconditional moments. The statistical test results suggest that the dynamics of the model-implied variables are closely aligned with their empirical counterparts. Within its class, the model is parsimonious, in the sense that restricted versions of the model are unable to reproduce these dynamics, unlike the full model.

Among other asset classes, cryptocurrencies provide a particularly compelling case for the sentiment-driven pricing implied by the model, including feedback loops between prices and sentiment, and jumps in both. Cryptocurrencies have no clear intrinsic value or cash flow, and their prices are

[42] The adjusted R^2 for the risk-free rate and dividend yield regressions are identical across specifications, as the sentiment index enters linearly and without a jump component in both expressions. See Proposition 1.

driven largely by speculative demand and narratives fertile ground for sentiment effects. And cryptocurrency markets, being global and trading continuously 24/7, exhibit this feedback cycle at high speed – booms and busts that take years in stocks can unfold in months or weeks in cryptocurrencies.

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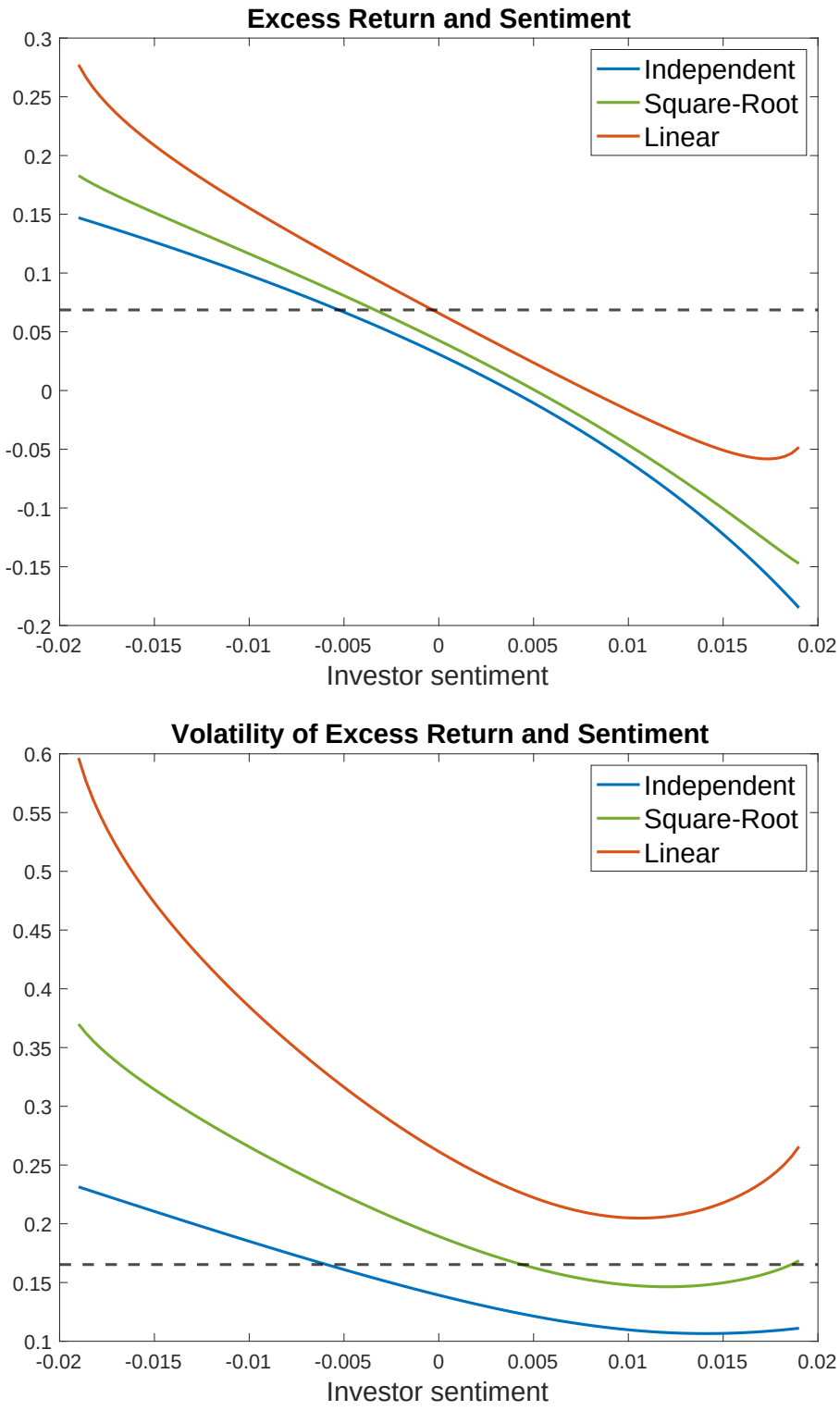


Figure 1: Sentiment and price feedback effects

Notes: The figure shows the model equity risk premium (see Equation 2 and its corresponding volatility (see Equation 4.6. The positive sentiment jump intensity $\lambda_t^{\eta+} = \bar{\lambda}_+ [-\log(\eta_t - a)]^{\zeta_{\eta+}}$ and the negative sentiment jump intensity $\lambda_t^{\eta-} = \bar{\lambda}_- [-\log(b - \eta_t)]^{\zeta_{\eta-}}$, $\zeta_{\eta\pm} > 0$ and we consider a symmetric interval for the sentiment process η_t, i.e. $a = -\theta$, and $b = -a$. The associated parameter values are: $\beta = 0.02$, $\gamma = 7$, and $\psi = 1.5$, $\mu_D = 0.023$, $\sigma_D = 0.0485$, $\lambda_D = 0$, $\theta = 0.02$, $m_t = 0$ (no feedback effect), $\kappa = 0.3$, $\sigma_\eta = 0.025$, $\rho = 0.2$ and the sentiment jump parameters are symmetric, i.e. $\bar{\lambda}_{\eta\pm} = \bar{\lambda}_\eta = 0.01$, $\alpha_{\eta\pm} = \alpha_\eta = 10$ and $\beta_{\eta\pm} = \beta_\eta = 0.25$. The black-dashed lines correspond to the empirical unconditional averages of the equity risk premium (6.86%) and its corresponding volatility (16.53%). The legend labels "independent", "square-root" and "linear" refer to $\zeta_\eta = 0, 0.5, 1$.

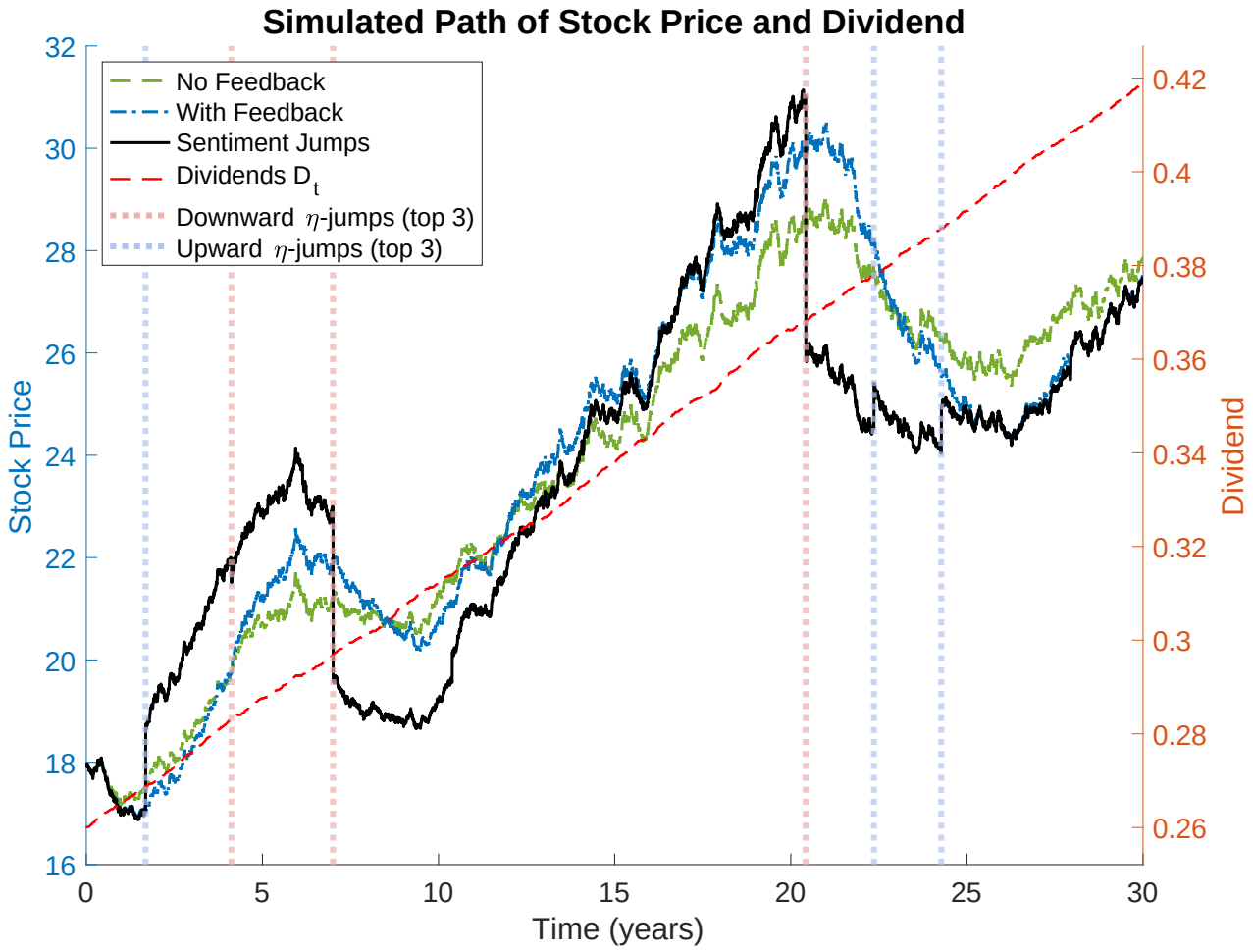


Figure 2: Simulated stock price dynamics using different versions of the model

Notes: The solid black line (Sentiment jumps, left y-axis) corresponds to the full model, i.e. model with feedback effects and jumps in sentiment. The dashed blue line corresponds to a model where there are no jumps in sentiment $\lambda_t^{\eta\pm} = 0$, but we allow for feedback effects (with feedback, left y-axis). Lastly, the dashed green line (no feedback) corresponds to a model in which both sentiment jumps and the feedback effect is turned off, i.e. $\lambda_t^{\eta\pm} = m_t = 0$ (no feedback, left y-axis). The red dashed line corresponds to the dividend process D_t (right, y-axis). We simulate the model on a daily frequency, i.e. $\Delta = 1/m$ where $m = 252$ corresponds to the number of trading days per year T. The feedback parameters are set to $l_p = 20$ and $l_P = 200$ and the feedback effect strength parameter is set to $q = 15$. Regarding the dividend parameters, we left the mean growth rate at $\mu_D = 0.023$ but scaled down the volatility parameter σ_D by a factor 20. All the other parameters are the same as shown in Figure 1 above. The red (blue) vertical lines correspond to the largest three downward (upward) jumps detected in sentiment.

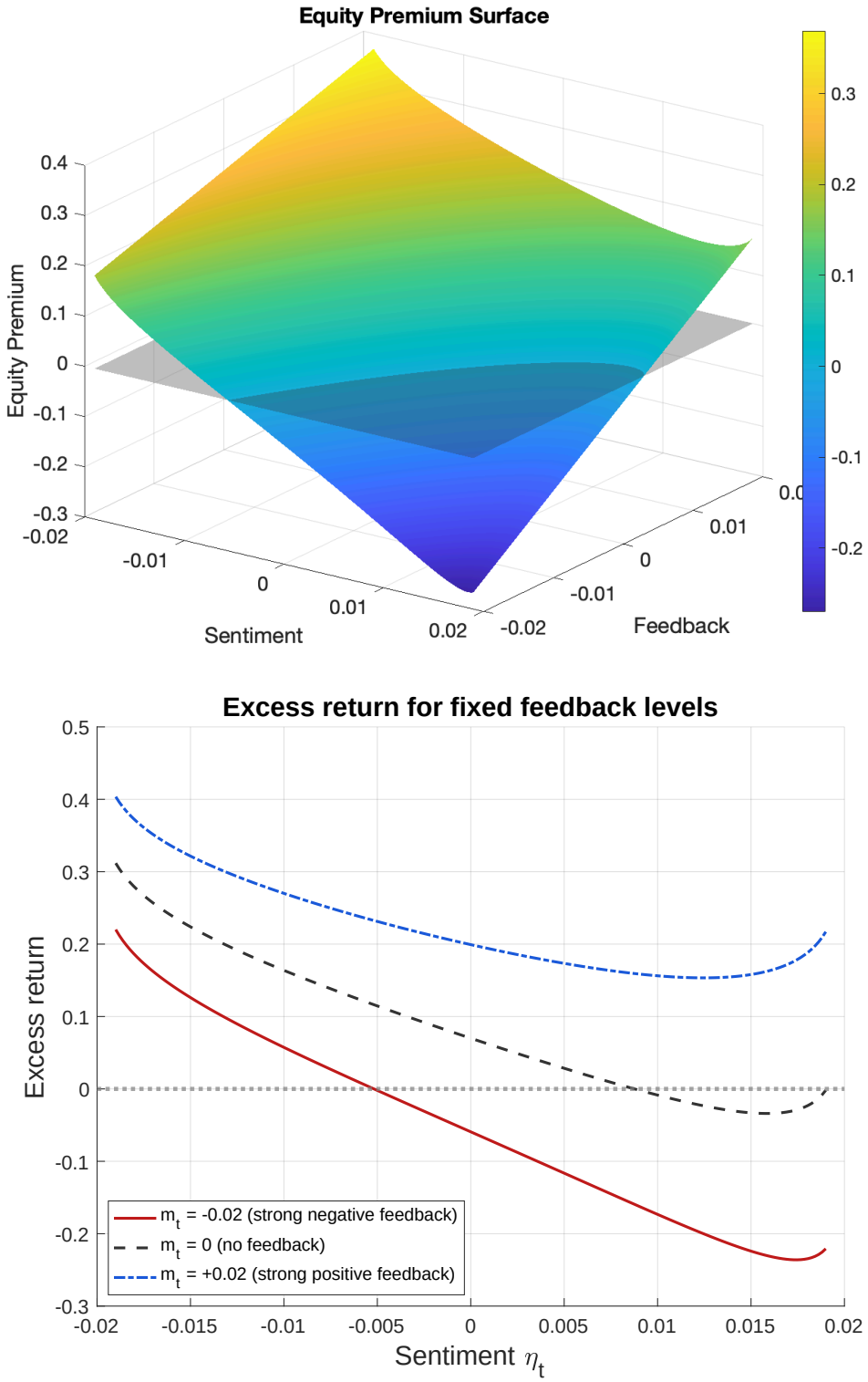


Figure 3: Excess return, sentiment and price feedback effects

Notes: Except for the feedback effect function m_t, all parameters are the same as shown in Figure 1 above.

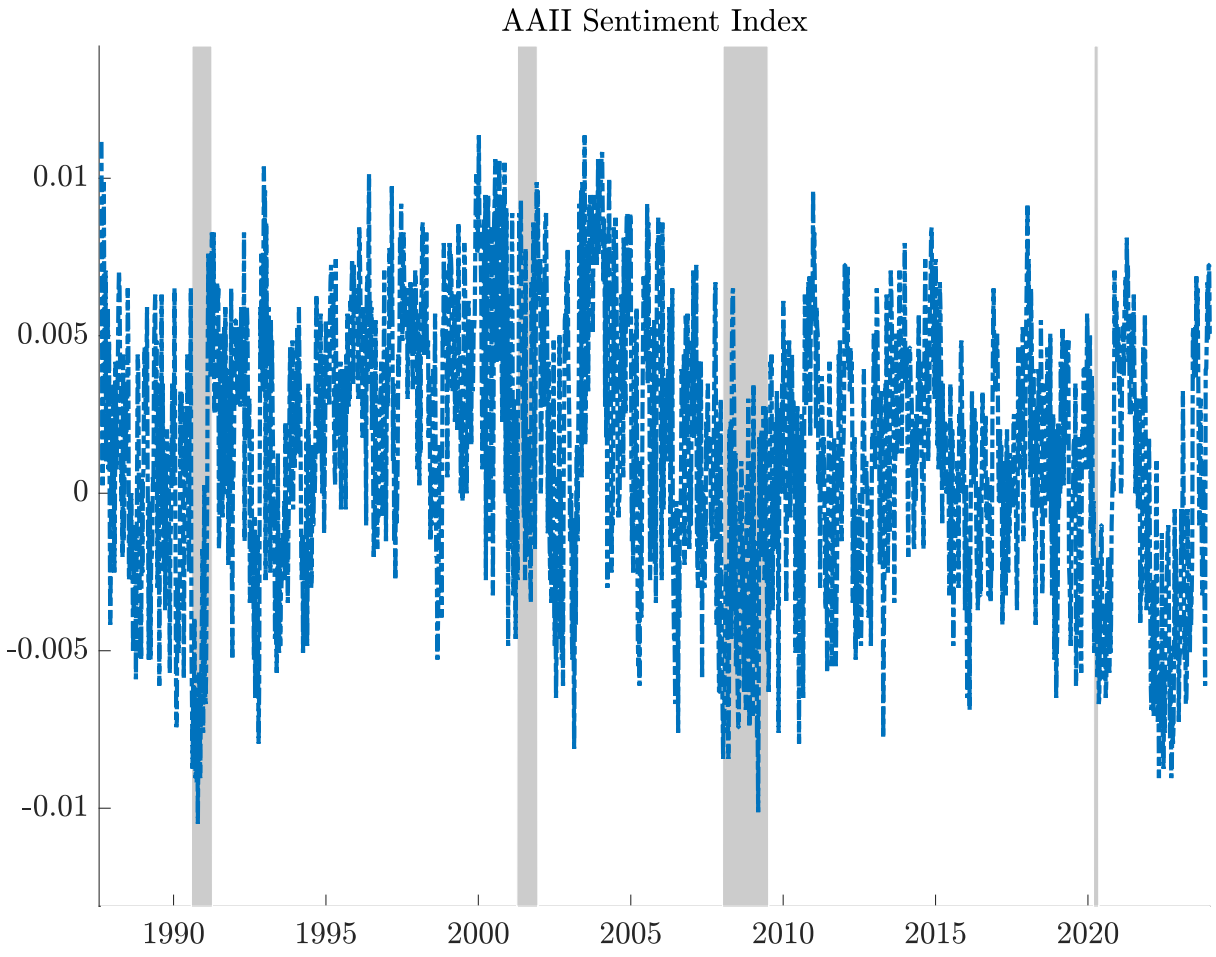


Figure 4: Transformed sentiment proxy

Notes: The figure shows the transformed (as shown in Equation (5.1)) sentiment indexes of the AAII sentiment at weekly frequency. Higher (lower) levels of the sentiment proxies imply higher levels of optimism (pessimism). Shaded areas correspond to NBER recession periods.

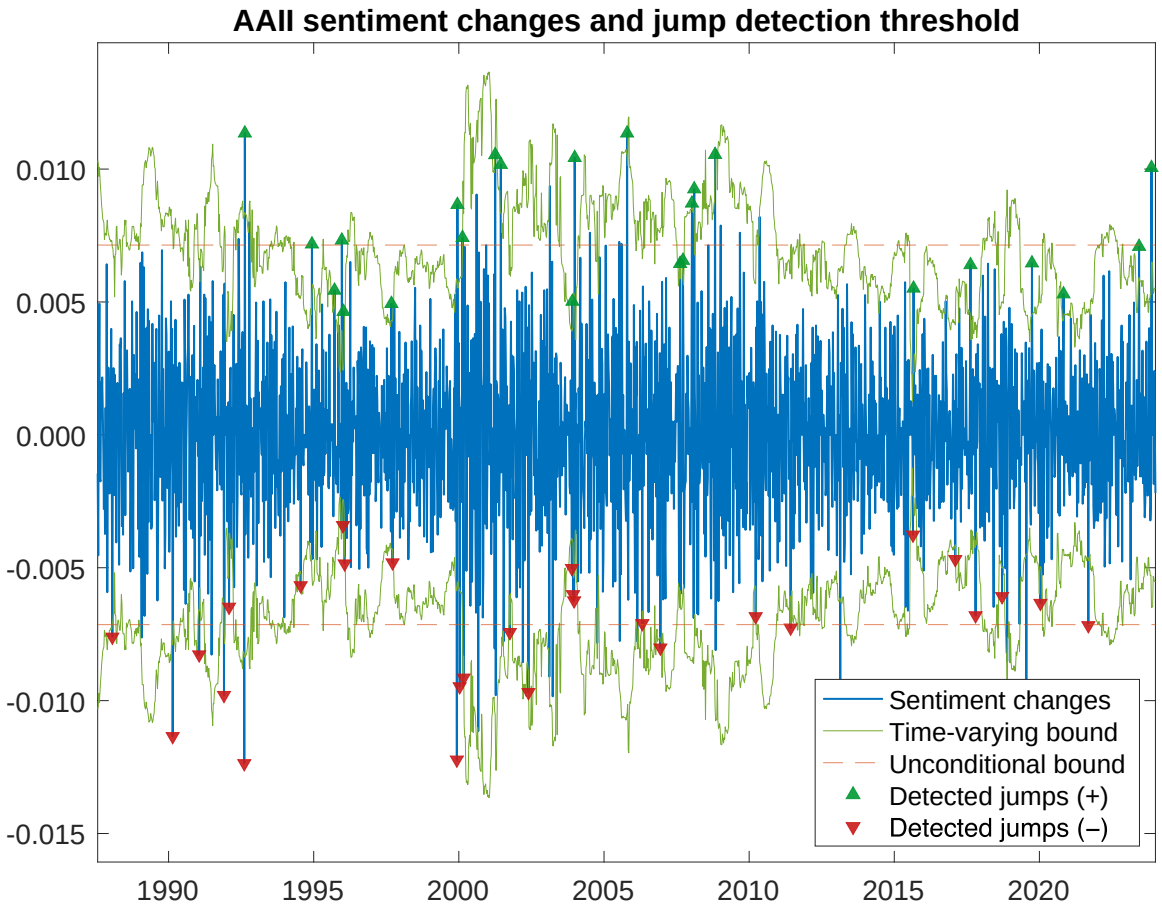


Figure 5: Sentiment changes and jump detection

Notes: The figure shows the changes in investor sentiment and the corresponding unconditional $\pm k_J \bar{\sigma}_d$ (dashed orange line) and conditional jump detection threshold $\pm k_J \sigma_d(\eta_t)$ (green solid line). The identification of jumps is based on the detection rule in Equation (6.1). $\bar{\sigma}_d$ and $\sigma_d(\eta_t)$ represent the unconditional and conditional diffusive volatility of sentiment changes. Green (red) upward (downward) pointing arrows indicate positive (negative) detected jumps. The estimator of the diffusive volatility is based on 25 weekly observations using a rolling overlapping window. The jump detection constant is set to $k_J = 2.5$.

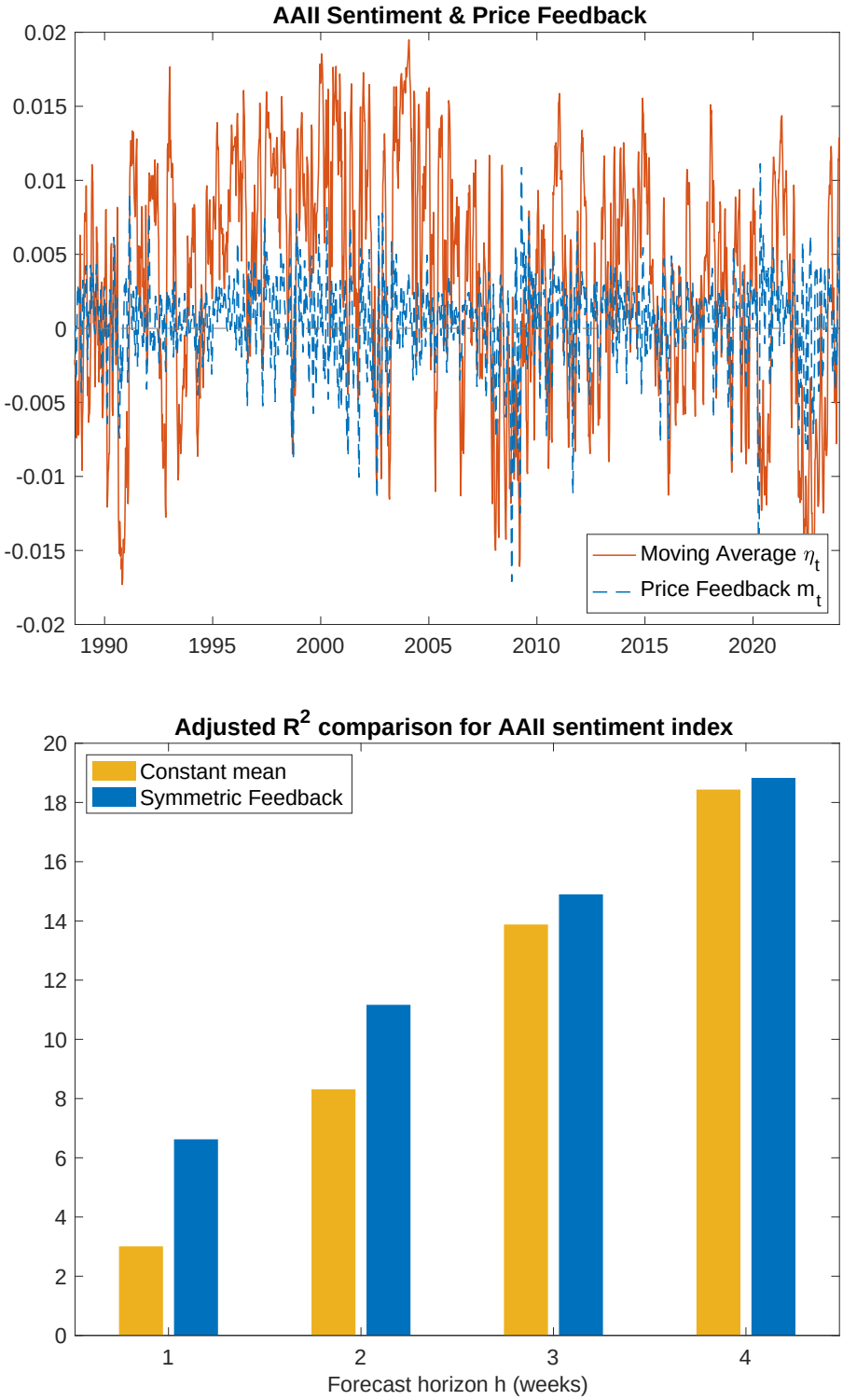


Figure 6: Sentiment and price feedback effects

Notes: The top panel of the figure shows the time series of investor sentiment and the optimally parametrized price feedback function m_t, which is obtained by maximizing the predictive R^2's from the regression in Equation (6.3). The calibrated parameters are, $l_p = 4$, $L_p = 52$, both measured in weeks, and $q = 8$ is the feedback strength parameter. The bottom panel of the figure shows the adjusted R^2's obtained from the regression in Equation (6.3) for both the model with feedback and the model in which m_t is set to zero.

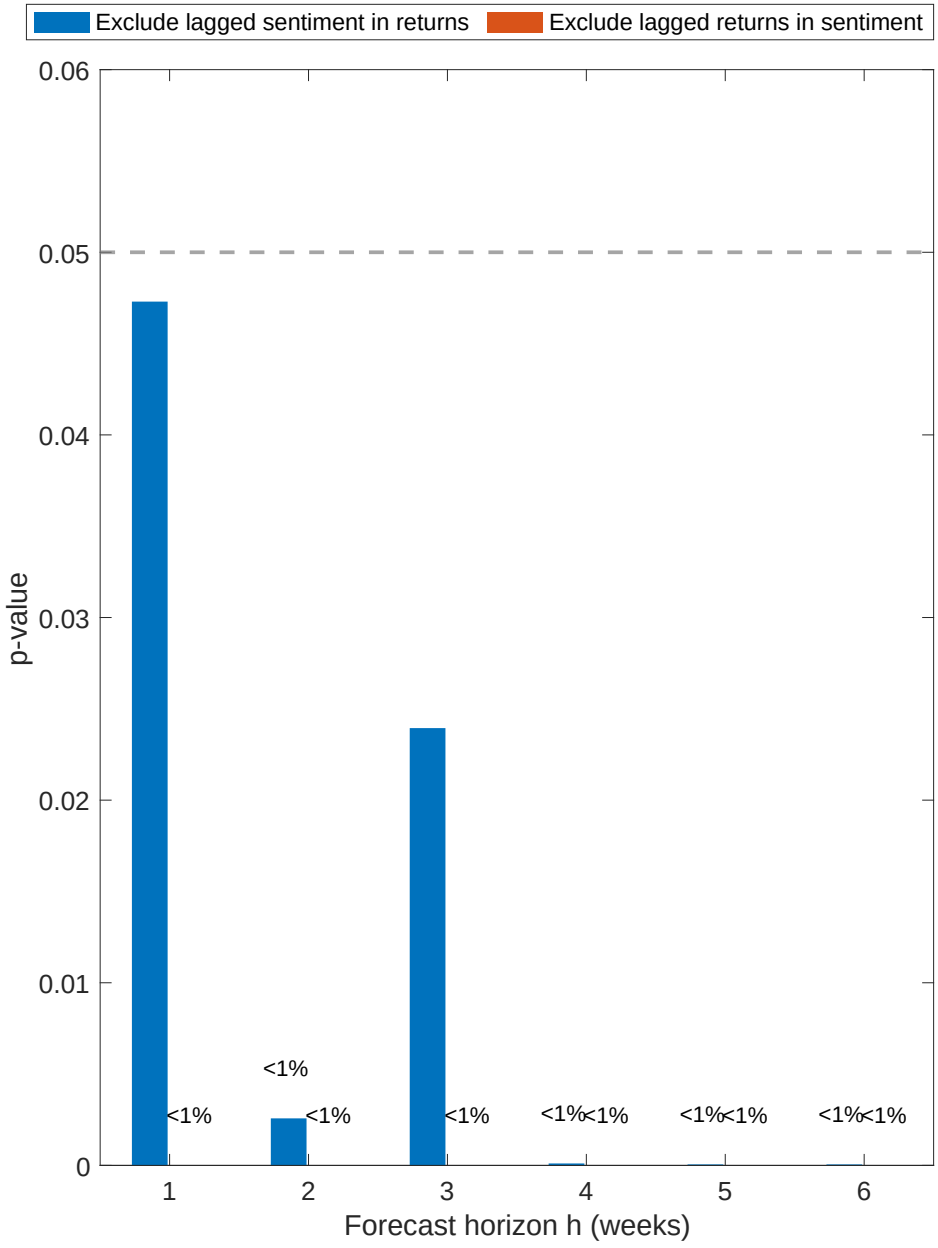


Figure 7: Granger Causality Test Results

Notes: The table reports the p-values of the Granger causality test based on a bi-variate (excess return at forecast horizon h and sentiment) VAR model where the lag length p was set to 3. The labels "<1%" indicate that the associated p-value is less than 1%.

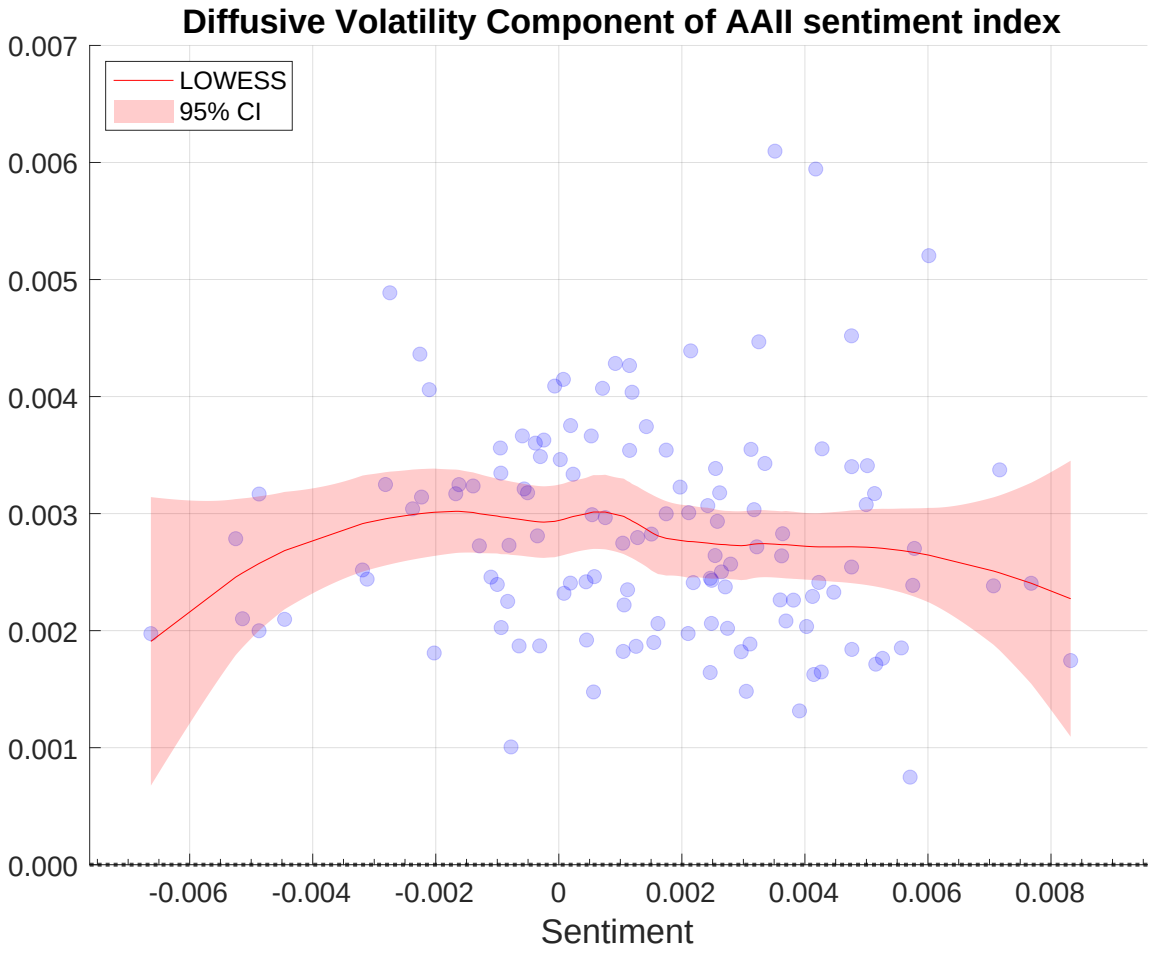
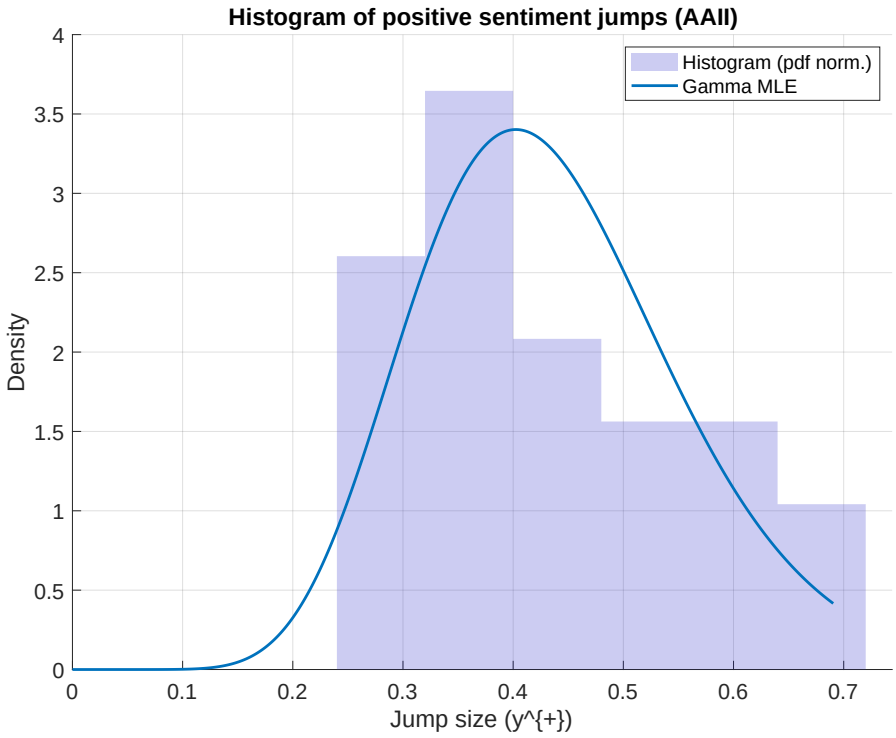


Figure 8: Sentiment changes and jump detection

Notes: The figure shows the estimated diffusive volatility component, estimated using non-overlapping observations, plotted against the level of investor sentiment. The solid red curve indicates the locally weighted smoothing linear regression fit (lowess) with smoothing parameter set to 2/3 together with the associated 95% bootstrapped confidence intervals. The diffusive volatility component is obtained using the recursive jump detection rule as outlined in Section 6.1. The jump detection constant k_J is set to 2.5. To estimate the diffusive volatility component, we use 25 observations.



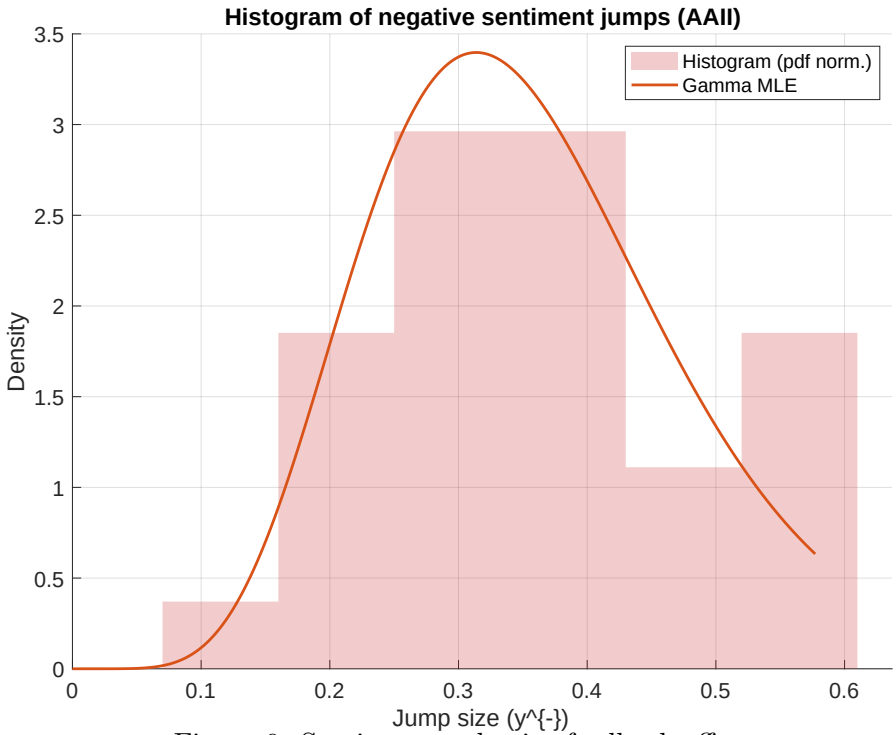


Figure 9: Sentiment and price feedback effects

Notes: The figure shows the distribution of positive and negative jumps, that is, $y_i^{\eta,+}$ and $y_i^{\eta,-}$, respectively. The jump detection procedure is detailed in Section 6.1. The solid blue (red) curve correspond to the estimated Gamma density with shape and scale parameters $\alpha_\pm$ and $\beta_\pm$. We multiplied the negative jumps by minus one to report only positive numbers. Each bar height is normalized as follows $\text{height}_i = \frac{\text{count}_i}{N \times \text{BinWidth}_i}$, where N is the total number of observations so that area (height $\times$ width), $\text{area}_i = h_i \cdot \text{binWidth}_i$, with $h_i = \frac{c_i}{N \, \text{binWidth}_i}$ where c_i counts the number of observation in each bin i, sums to 1.

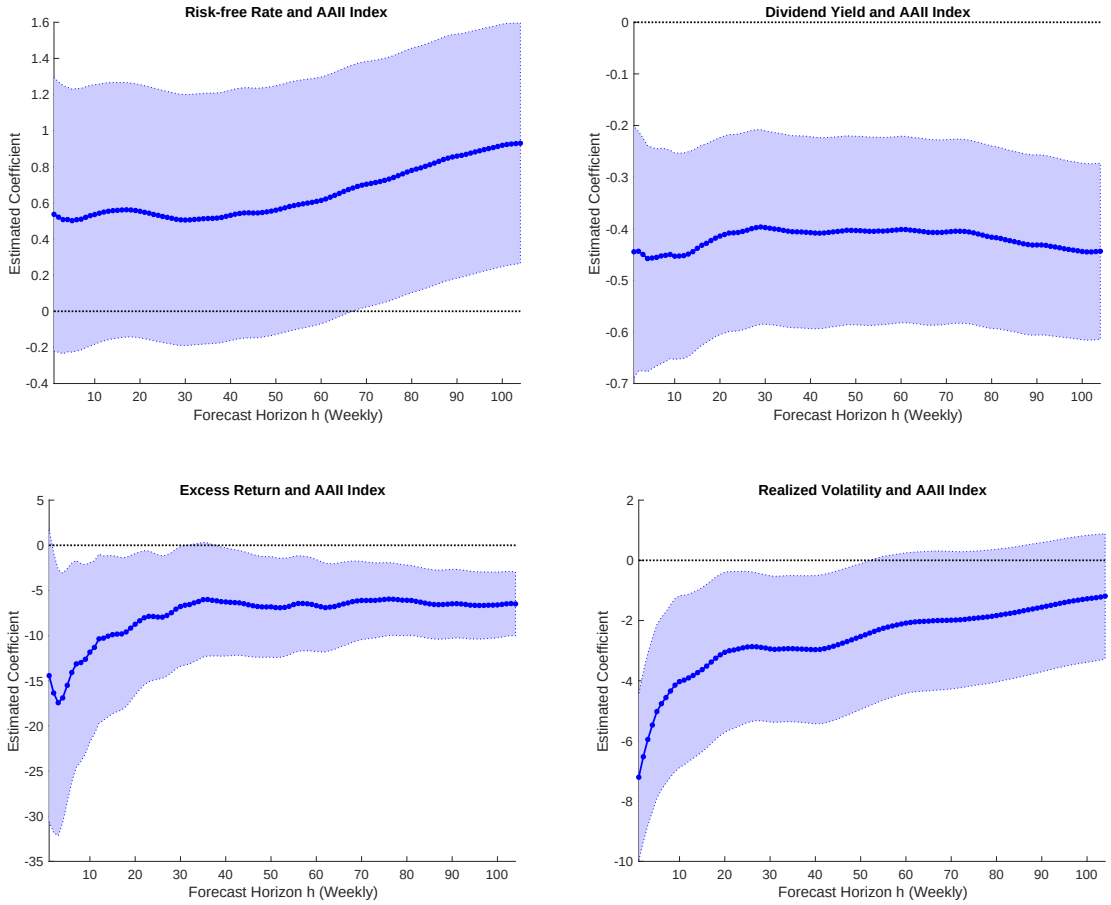


Figure 10: Forecast regression results based on AAII sentiment

Notes: The figure shows the estimated regression coefficients (solid blue line) of a regression equity risk premium, its realized volatility, the risk-free rate and the dividend yield onto the AAII sentiment index. For each plot, we compute a rolling linear moving-average of weekly sentiment using four weeks ($l = 4$). The light blue shaded areas represent HAC-adjusted 95% confidence intervals.

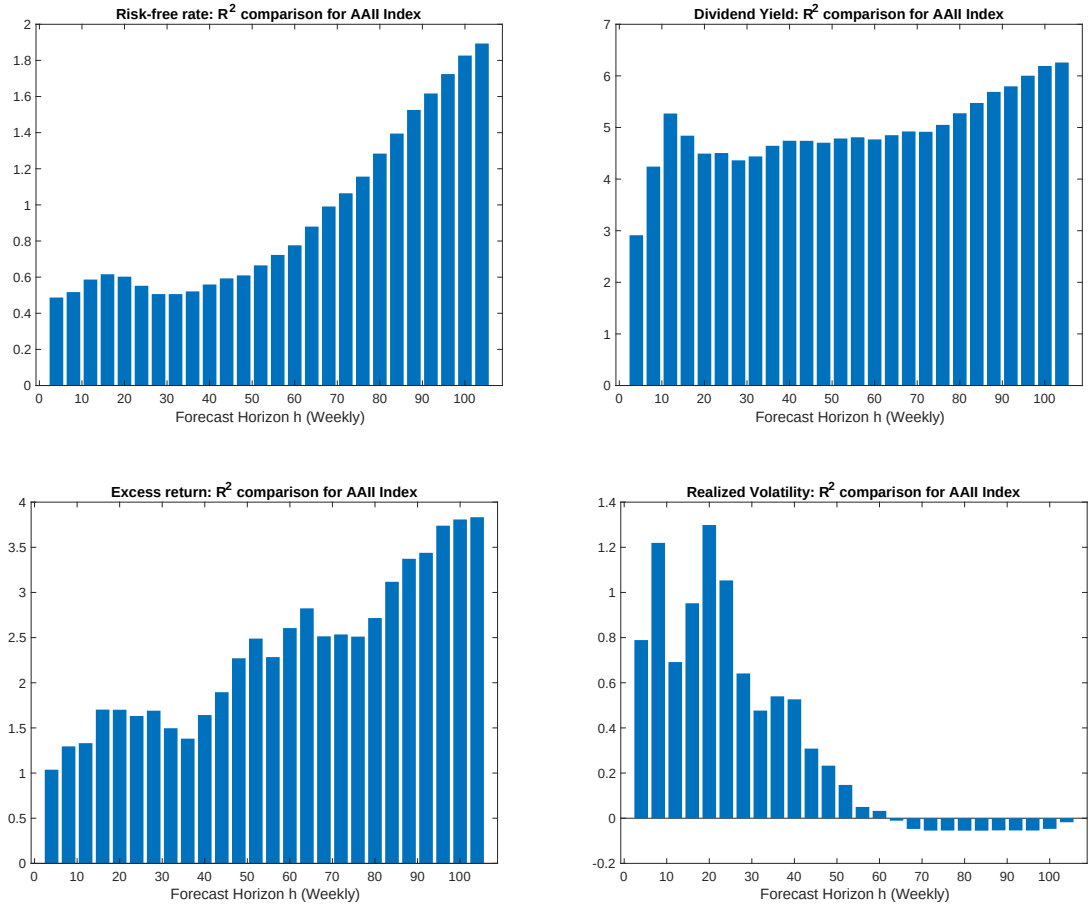


Figure 11: Forecast regression results based on AAII sentiment

Notes: The figure shows the adjusted R^2's based on the forecasting regressions in Equation (7). The legends refer to the number of observations l to compute the linear moving average of sentiment.

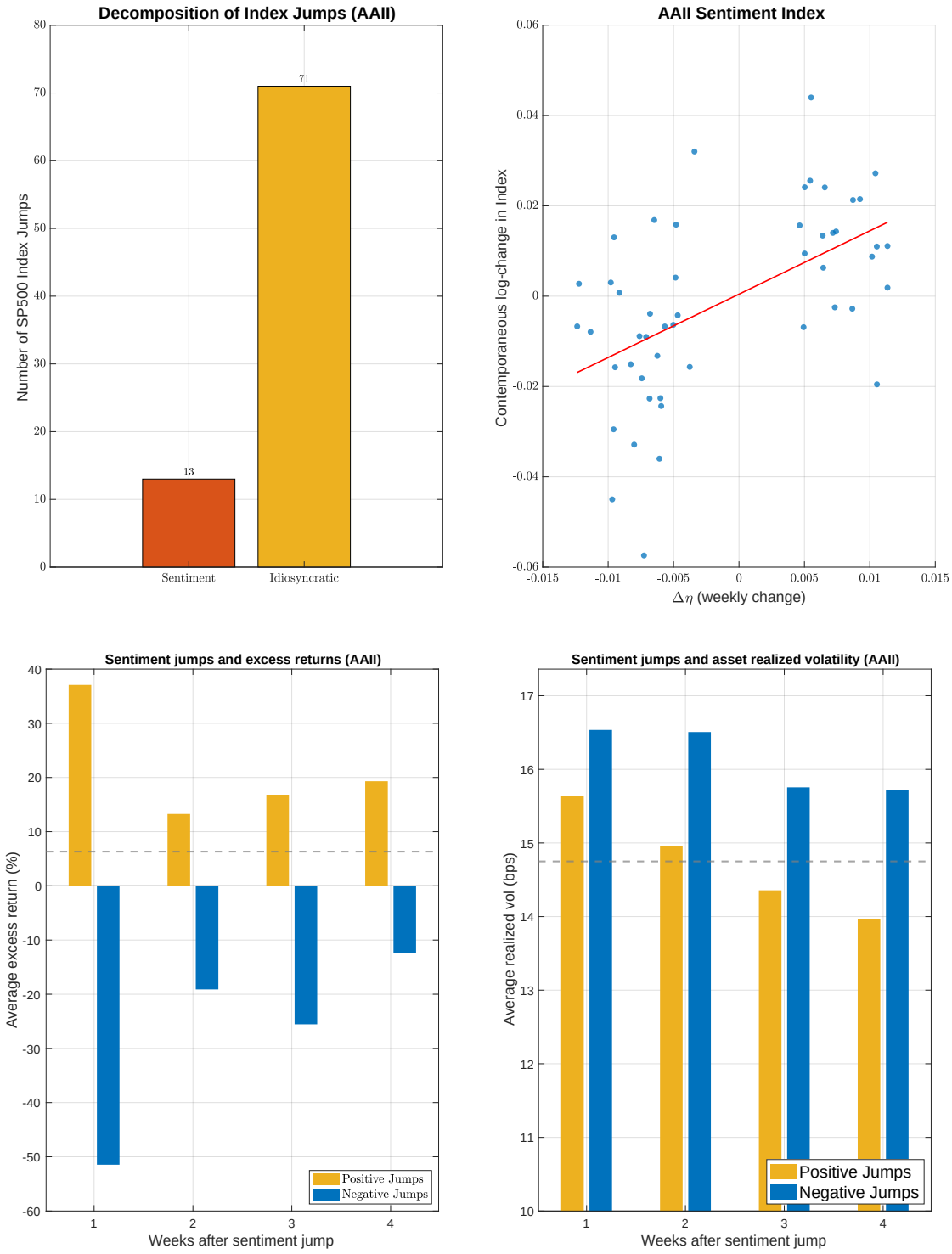


Figure 12: Impact of Sentiment Jumps

Notes: This figure analyzes the impact of sentiment jumps on the excess returns and realized volatility. For each figure, we use the recursive jump detection method as presented in Section 6.1. The jump threshold constant is set to $k_J = 3$. The total number of jumps in the S&P 500 index detected with this recursive jump detection rule is 84. The red bar in the top left panel counts the number of jumps that can be traced back to a jump in sentiment using a look back window of four weeks. The yellow bar indicates the number of jumps detected in the S&P 500 time series which could not be traced back to jumps in sentiment and where therefore labeled as "idiosyncratic" jumps. The top right panel shows the log return of the S&P 500 during the week a jump in sentiment got detected. The red line indicates the OLS regression estimate with estimated slope coefficient $\hat{\beta} = 1.38$ and associated HAC-robust $t-$stat of 4.542. The bottom right and left panels shows the average (up to four weeks after) cumulative excess return and associated average realized volatility given a jump in sentiment has occurred. The horizontal lines indicate the empirical average excess return and realized volatility in our data.

	Positive jumps (y^+)	**Negative jumps (y^-)**
Shape α	12.9442	8.3044
Scale β	0.0337	0.0430
Mean $\alpha\beta$	0.4362	0.3571
Variance $\alpha\beta^2$	0.0147	0.0154
Std. deviation $\sqrt{\alpha}\,\beta$	0.1212	0.1239
Coeff. of variation $1/\sqrt{\alpha}$	0.2779	0.3470
Skewness $2/\sqrt{\alpha}$	0.5559	0.6940
Kurtosis $3 + 6/\alpha$	3.4635	3.7225
Mode $(\alpha - 1)\beta$	0.4025	0.3141

Table 1: Gamma MLE fits for sentiment jump magnitudes.

		Up	Down	Up	Down	Up	Down
Jump Threshold	k_J	1.5	1.5	2	2	2.5	2.5
$\eta_t < q_{50}$	low	206	73	64	19	20	5
$\eta_t \geq q_{50}$	high	54	176	10	72	3	24
$\eta_t < q_{33}$	low	153	37	50	7	17	1
$q_{33} \leq \eta_t < q_{66}$	medium	90	80	24	30	6	8
$\eta_t \geq q_{66}$	high	24	137	3	56	1	21
$\eta_t < q_{25}$	very low	124	16	46	3	16	0
$q_{25} \leq \eta_t < q_{50}$	low	82	57	18	16	4	5
$q_{50} \leq \eta_t < q_{75}$	high	50	77	12	28	4	6
$\eta_t \geq q_{75}$	very high	11	104	1	46	0	19
$\eta_t < q_{20}$	very low	108	9	39	1	14	0
$q_{20} \leq \eta_t < q_{40}$	low	69	43	18	8	4	1
$q_{40} \leq \eta_t < q_{60}$	medium	54	51	13	21	4	5
$q_{60} \leq \eta_t < q_{80}$	high	27	66	6	27	2	10
$\eta_t \geq q_{80}$	very high	9	85	1	36	0	14

Table 2: Number of Detected Jumps by Sentiment Level

Notes: The table reports the number of upward and downward jumps conditional on sentiment when the sentiment level belongs to a given interval of its distribution, indexed by the p empirical percentile q_p. Jumps in sentiment are detected using the recursive jump detection rule discussed in Section 6.1.

Sentiment Regime	Risk-Free Rate		Dividend Yield		Excess Return		Volatility	
	Emp	MI	Emp	MI	Emp	MI	Emp	MI
Asymmetric jumps								
Unconditional	2.98	2.93	2.15	2.22	6.86	6.82	16.53	16.53
Low ($\leq q_{50}$)	2.73	2.69	2.26	2.29	8.68	8.75	17.22	17.24
High ($> q_{50}$)	3.24	3.16	2.04	2.16	5.02	4.89	15.83	15.82
Low ($\leq q_{33}$)	2.68	2.60	2.27	2.31	9.51	9.48	17.90	17.95
Mid (q_{33}–q_{66})	3.04	2.94	2.20	2.22	6.87	7.01	15.77	15.71
High ($> q_{66}$)	3.23	3.24	1.97	2.14	4.26	4.04	15.94	15.95
Symmetric jumps								
Unconditional	2.98	3.15	2.15	2.10	6.86	12.98	16.53	37.95
Low ($\leq q_{50}$)	2.73	2.99	2.26	2.24	8.68	17.53	17.22	44.87
High ($> q_{50}$)	3.24	3.31	2.04	1.96	5.02	8.42	15.83	31.00
Low ($\leq q_{33}$)	2.68	2.92	2.27	2.29	9.51	19.26	17.90	47.86
Mid (q_{33}–q_{66})	3.04	3.16	2.20	2.09	6.87	13.21	15.77	37.26
High ($> q_{66}$)	3.23	3.36	1.97	1.91	4.26	6.63	15.94	28.96
No sentiment jumps								
Unconditional	2.98	2.71	2.15	2.28	6.86	6.77	16.53	16.53
Low ($\leq q_{50}$)	2.73	2.48	2.26	2.35	8.68	8.61	17.22	16.31
High ($> q_{50}$)	3.24	2.94	2.04	2.21	5.02	4.92	15.83	16.76
Low ($\leq q_{33}$)	2.68	2.39	2.27	2.38	9.51	9.11	17.90	16.12
Mid (q_{33}–q_{66})	3.04	2.72	2.20	2.28	6.87	7.34	15.77	16.73
High ($> q_{66}$)	3.23	3.01	1.97	2.19	4.26	3.93	15.94	16.74

Model	Estimated Parameters
Asymmetric	$\beta = 0.026$, $\gamma = 7.332$, $\psi = 1.266$, $k_\eta = 0.866$, $s_\eta = 1.55\times10^{-4}$, $\rho = 0$, $\bar{\lambda}_+ = 0.005$, $\bar{\lambda}_- = 0.104$, $\zeta_+ = 4.793$, $\zeta_- = 2.379$, $(\alpha_+, \beta_+) = (12.624, 0.033)$, $(\alpha_-, \beta_-) = (8.276, 0.043)$; $\mathcal{L}^\dagger = 0.048$
Symmetric	$\beta = 0.029$, $\gamma = 5.882$, $\psi = 1.872$, $k_\eta = 0.513$, $s_\eta = 1.22\times10^{-4}$, $\rho = 1.03\times10^{-4}$, $\bar{\lambda} = 0.049$, $\zeta = 1.326$, $(\alpha, \beta) = (8.305, 0.069)$; $\mathcal{L}^\dagger = 0.965$
No Jumps	$\beta = 0.026$, $\gamma = 8.491$, $\psi = 1.307$, $k_\eta = 1.000$, $s_\eta = 0.577$, $\rho = 1.000$; $\mathcal{L}^\ddagger = 1.793$

Table 3: Calibration of model-implied moments to empirical moments for AAII

Notes: All numbers in the main panel are expressed in percent (%). Moments are computed at a one-year forecast horizon ($h = 52$ weeks). "Emp" denotes empirical moments and "MI" denotes model-implied moments. Conditional rows use sentiment quantile bins (median split: q_{50}; terciles: q_{33}, q_{66}).$\mathcal{L}^\dagger$ reports the *augmented* objective (moment loss plus the jump-size penalty on (α, β)) used for the Asymmetric and Symmetric jump models. $\mathcal{L}^\ddagger$ reports the *pure* moment loss used in the No-Jumps model (penalty weight set to zero and jump-size parameters not estimated), so criterion values are not strictly comparable across models.

Variable	Model	Beta (SE)	Adj. R^2	p-val (H_0: $\beta = 1$)
AAII Sentiment Index				
Risk-free rate	Full (asym)	0.7168 (0.6671)	0.0065	0.6713
	Symmetric	1.0599 (0.8112)	0.0065	0.9411
	No Jumps	0.7398 (0.6777)	0.0065	0.7010
Dividend yield	Full (asym)	1.8541^{***} (0.6570)	0.0512	0.1937
	Symmetric	0.8368^{*} (0.4413)	0.0512	0.7115
	No Jumps	1.6605^{***} (0.6217)	0.0512	0.2882
Excess return	Full (asym)	1.6543^{***} (0.3580)	0.4280	0.0678
	Symmetric	1.0492^{***} (0.2707)	0.3588	0.8558
	No Jumps	1.5461^{***} (0.3529)	0.4332	0.1218
Realized volatility	Full (asym)	0.8928 (0.5701)	0.0308	0.8509
	Symmetric	0.1171 (0.2301)	0.0183	0.0001
	No Jumps	-3.3776^{***} (1.1114)	0.0308	0.0001

Table 4: Regression of empirical time series on model-implied series

Notes: The table reports OLS regressions of empirical series on model-implied series for the AAII sentiment index. Standard errors are HAC-robust with lag length 7. Stars indicate significance for $H_0 : \beta = 0$: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The final column reports p-values for $H_0 : \beta = 1$.