The Distributional Effects of Asset Returns

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Abstract

We study how asset returns shape wealth inequality in a heterogeneous-agent model with endogenous portfolio choices estimated to match three empirical facts: the aggregate share of safe assets, the wealth distribution, and the positive correlation between wealth and returns. Our main finding is that lower expected returns increase inequality: a one percentage point decline in the discount rate permanently raises the top 10% consumption share by 6.6%. This runs counter to the intuition from Piketty's (2014) $r - g$ formula. To reconcile these findings, we derive a generalized formula that incorporates the consumption-to-wealth ratio. The mechanism operates through saving: lower returns compress households' active saving rates outside the Pareto tail relative to those within it. As a result, inequality **within** the Pareto tail is largely unaffected, but inequality **between** the Pareto tail and the rest of the distribution responds strongly to changes in asset returns.

Keywords: Asset returns, household heterogeneity, Pareto tail.

JEL classifications: D14, D31, E21, G11.

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1 Introduction

How do asset returns affect inequality? This question has garnered increasing attention since the seminal work of Piketty (2014). Recent contributions include Hubmer et al. (2020), Bach et al. (2020), Fagereng et al. (2020), Kuhn et al. (2020), Greenwald et al. (2021), Fagereng et al. (2022), Gomez (2022), and Gomez and Gouin-Bonenfant (2024), among others. Changes in asset returns affect the market value of household wealth, thereby directly influencing the wealth distribution. However, if household consumption remains unchanged, these valuation effects may not generate distributional welfare consequences (Cochrane, 2020). A central question, thus, is whether asset returns drive distributional welfare effects through consumption.

In this paper, we show that endogenous portfolio choices play a crucial role in shaping the welfare effects of asset returns. As documented in various studies, e.g., Fagereng et al. (2020) and Bach et al. (2020), wealthy households hold high-yield assets and earn higher returns than poorer households. This implies that the rich primarily accumulate wealth through returns, whereas the poor rely on "active savings"—savings out of labor income (Bach et al., 2017).[1] When expected returns decline, asset prices rise, increasing aggregate wealth. Consequently, poor households struggle to maintain their wealth shares, as their active savings shrink relative to aggregate wealth. This slows their wealth accumulation compared to the rich, ultimately increasing inequality in wealth, capital income, and consumption.

To quantify this effect, we analyze a heterogeneous-agent model with endogenous portfolio choices. The model is set in a Lucas (1978) economy featuring uninsurable idiosyncratic shocks to labor and capital income, along with aggregate disaster shocks as in Barro (2006). Agents have Epstein and Zin (1989) preferences with idiosyncratic risk-aversion shocks (Barro et al., 2022). Low-risk-aversion agents allocate more wealth to riskier assets and earn higher returns, while high-risk-aversion agents prefer safer, lower-return assets. We estimate the model using the simulated method of moments (SMM) to match key moments of the joint wealth-return distribution. A permanent one percentage point decline in the discount rate reduces expected returns and raises consumption by the top 10% by 6.6% permanently, demonstrating significant and lasting distributional welfare effects.

The distributional effects hinge critically on heterogeneous savings, which emerge endogenously. Agents with high risk aversion allocate savings to safer, low-return portfolios, so their wealth share declines, bringing them closer to their borrowing constraints. As a result, they increase precautionary saving and reduce consumption relative to wealthier, low-risk-aversion agents, who invest in risky, high-return portfolios and accumulate wealth more rapidly.

[1] Throughout, we define the active saving rate as savings out of labor income divided by wealth.

Because precautionary saving motives are stronger among poorer agents, lower returns impose a negative (positive) income effect on the poor (rich), redistributing consumption from poor to rich. This effect depends on the active saving rate—labor income less consumption as a fraction of wealth. Agents with higher active saving rates are worse off when discount rates are lower. Since active saving rates decline with wealth, lower returns to saving redistribute consumption from the poor to the rich.

To illustrate this mechanism, we compare our model with an alternative that features heterogeneous discount factors, as in Krusell and Smith (1998). In that model, the distributional effects are exactly opposite: lower returns make the rich worse off, because they have a stronger saving motive due to a lower discount rate. These contrasting results are associated with opposite correlations between active saving rates and wealth. In our heterogeneous-risk-aversion model, active saving rates decrease with wealth, consistent with the data, whereas in the Krusell and Smith (1998) model they increase with wealth, contrary to the data.

We use our model to analyze the Pareto tail of the wealth distribution. In a steady-state framework, Piketty and Zucman (2015) show that the Pareto exponent of the right tail decreases with $r - g$ (here, r is the aggregate return on wealth, while g is the growth rate). This implies that top wealth inequality rises with $r - g$, as suggested by Piketty (2014). In contrast, our findings indicate that lower returns increase inequality, contradicting the $r - g$ hypothesis. Revisiting Piketty and Zucman (2015), we derive an "extended Piketty formula," showing that top wealth inequality depends on $r - g - c$, where c is the consumption-to-wealth ratio at the Pareto tail. The $r - g$ formula holds when this ratio is fixed, as in Piketty and Zucman (2015). However, when c is endogenous (as in our model), the broader $r - g - c$ framework applies.

More importantly, we find that changes in key parameters affect $r - g$ and c by roughly the same magnitude, leaving $r - g - c$ largely unchanged. Consequently, inequality **within** the Pareto tail is relatively insensitive to asset returns. However, inequality **between** the Pareto tail and the rest of the distribution responds strongly to asset returns through differential effects on active savings relative to wealth.

We draw two main conclusions. First, focusing solely on inequality **within** the Pareto tail may overlook the broader impact of asset returns, as most effects occur at the lower end of the wealth distribution. Second, these effects operate primarily through the active saving rate rather than the $r - g - c$ formula.

We complement our analysis by examining transitional dynamics across steady states. Variations in wealth inequality along transition paths are primarily driven by unanticipated capital gains, which disproportionately benefit the rich, who hold larger equity shares. During transitions, inequality rises with realized returns, including unanticipated capital gains.

We add to the computational literature on heterogeneous-agent models by employing a novel approach that extends Campbell (1998), Reiter (2009), Winberry (2018), and Ahn et al. (2018). These studies combine global approximations along individual state variables with local approximations around the deterministic steady state. We follow a similar strategy, but our local approximation differs: instead of standard perturbation, we use the Taylor projection method developed by Levintal (2018) and implemented in Fernández-Villaverde and Levintal (2018) and Barro et al. (2022). The key advantage is applicability to portfolio-choice models, as the approximation point incorporates the full volatility of aggregate shocks. Perturbation, by contrast, is performed around a steady state with no aggregate shocks, where portfolio decisions are indeterminate.

Our paper contributes to the broad literature on income and wealth inequality (Piketty and Saez, 2003; Piketty, 2014; Saez and Zucman, 2016; Piketty et al., 2018). More specifically, we build on recent research examining the welfare implications of asset returns. Glover et al. (2020) analyze a recessionary shock mirroring the Great Recession and find that welfare losses increase with age. Fagereng et al. (2022) estimate the distributional effects of asset-price dynamics in Norway since 1994, finding significant welfare transfers from the young to the old due to the sustained rise in asset prices. The mechanisms in these studies operate through the life-cycle profile of financial transactions: the young are primary asset buyers, whereas the old are sellers; thus, lower returns shift welfare from buyers to sellers.

Our paper is most closely related to Greenwald et al. (2021), who examine the long-term distributional effects of a permanent decline in the discount rate in general equilibrium. In their model, the primary source of heterogeneity is variation in labor income across the life cycle: low returns harm younger individuals, whose stronger saving demand stems from accumulating wealth for retirement. We complement their analysis by exploring heterogeneity driven by endogenous portfolio choices rather than life-cycle considerations.[2]

Other related papers examine the role of asset returns in explaining U.S. wealth inequality, including model-based explanations (Gomez, 2022), model-free approaches (Kuhn et al., 2020), and partial-equilibrium analyses (Cioffi, 2021; Catherine et al., 2023).

In our model, heterogeneity in saving behavior arises endogenously. Agents with lower returns accumulate wealth more slowly and gradually approach their net-worth constraint, thereby increasing precautionary savings. The resulting increase in precautionary savings among poorer agents raises asset prices and lowers returns. In response, the rich increase consumption and reduce savings. This creates heterogeneous saving demand across the wealth distribution, with the poor exhibiting a stronger precautionary motive. Consequently, low

[2]In contrast to both papers, Glover et al. (2020) analyze a temporary shock, while Fagereng et al. (2022) conduct comparative-statics analysis at the individual level, treating asset prices as given. Catherine et al. (2023) study exposure to interest-rate risk in a life-cycle model.

returns disproportionately harm the poor, whose saving demand is greater.

We highlight the active saving rate as a key variable in determining the distributional effects of asset returns at the steady state. We show that lower returns impose a redistributive income effect: agents with high (low) active saving rates are worse (better) off. We complement this finding by showing analytically that under regularity conditions, the active saving rate serves as a sufficient statistic for the welfare effects of asset returns, *holding wealth and labor-income shares constant*. This complements the theoretical analysis of Fagereng et al. (2022); our contribution extends the analysis to a general-equilibrium steady state.

Previous studies have recognized heterogeneous returns as a key driver of wealth inequality (Benhabib et al., 2011, 2015, 2016; Atkeson and Irie, 2020; Hubmer et al., 2020; Xavier, 2021; Gaillard et al., 2023). These studies assume exogenous return heterogeneity, whereas our model generates it endogenously through portfolio choices. Related models with endogenous portfolios include Krusell and Smith (1997), Cagetti and De Nardi (2006), Favilukis (2013), Lei (2019), Kacperczyk et al. (2019), and Gomez (2022). Our primary contribution is quantitative: we match the joint distribution of wealth and returns. While Gomez (2022) also studies this joint distribution, his focus is on the top tail, whereas we examine the entire wealth distribution. Crucially, we find that asset returns influence inequality primarily through saving decisions at the lower end of the distribution.

Our portfolio-choice model builds on a long tradition of two-agent models with heterogeneous risk aversion, including Dumas (1989), Benninga and Mayshar (1993), Wang (1996), Guvenen (2009), Longstaff and Wang (2012), Gârleanu and Panageas (2015), and Barro et al. (2022), among others. These models often generate excessive wealth inequality, as high-risk-aversion agents' wealth and consumption shares tend to converge to zero. We mitigate this by imposing a non-negativity constraint on financial wealth, which we consider empirically plausible. This constraint acts as a reflecting barrier, preventing excessive expansion of the wealth distribution (Gabaix, 2009). However, it introduces significant computational challenges, which we overcome using our novel solution method.

Gomez and Gouin-Bonenfant (2024) document the rise in U.S. wealth inequality **within** the Pareto tail over recent decades and attribute it to declining asset returns. Their framework assumes limited capital-market access: agents with investment opportunities (entrepreneurs) borrow from others (rentiers) and earn an internal return exceeding the external return. A decline in the external return raises the internal return, increasing return heterogeneity and, consequently, wealth inequality within the Pareto tail. In contrast, we assume free capital-market access, so asset returns move in roughly tandem. Under these conditions, Pareto inequality is largely insensitive to asset returns in the steady state, though it may vary along a transition path.

The paper is structured as follows. Section 2 presents the model. Section 3 outlines our computational approach. Section 4 describes the estimation. Section 5 presents comparative statics. Section 6 compares our model to two alternative benchmarks of wealth inequality: the heterogeneous discount factor model and the entrepreneurial model. Section 7 analyzes the distributional effects of asset returns. Section 8 concludes. Two online appendices provide analytical derivations and computational details.

2 The model

We consider an economy with a continuum of agents indexed by i on the unit interval. Agents have Epstein and Zin (1989) preferences over consumption sequences $C_{i,t}$:

$$U_{i,t}^{1-\theta} = (1-\beta)\, C_{i,t}^{1-\theta} + \beta \left(\mathbb{E}_{i,t} U_{i,t+1}^{1-\gamma_{i,t}} \right)^{\frac{1-\theta}{1-\gamma_{i,t}}}, \tag{1}$$

where β is the discount factor, and $\mathbb{E}_{i,t}$ denotes the expectation over future aggregate and idiosyncratic shocks. For most of the analysis, we focus on unit intertemporal elasticity of substitution (IES), obtained as $\theta \to 1$. The time discount rate is $\rho \equiv \beta^{-1} - 1$.

Our key deviation from the standard Epstein-Zin setup is that risk aversion $\gamma_{i,t}$ is an uninsurable idiosyncratic shock following a 2-state Markov process with values $\gamma_1 < \gamma_2$. Variations in risk aversion can be interpreted as reflecting deaths and births within the same dynasty. Agents internalize the stochastic nature of risk aversion in their decision-making.

Output Y_t is exogenous (Lucas, 1978) and follows $\Delta \log Y_t = g + \epsilon_t^Y - b d_t$, where ϵ_t^Y is a business-cycle shock with bounded support, d_t is a rare disaster shock (0 or 1), and b is the disaster size. As in Barro (2006), this shock helps match the equity premium without extreme risk aversion. We focus on U.S. post-WWII samples without realized disasters.

A share $1-\alpha$ of output is labor income, while α is paid as dividends. Agents trade equity and short-term bonds, so their budget constraint is:

$$C_{i,t} + P_t E_{i,t} + Q_t B_{i,t} = \left(1 + \epsilon_{i,t}^L\right)(1-\alpha)\, Y_t + \left(1 + \epsilon_{i,t}^E\right)(\alpha Y_t + P_t)\, E_{i,t-1} + B_{i,t-1}. \tag{2}$$

Here, $E_{i,t}$ and $B_{i,t}$ are equity and bonds acquired in t at prices P_t and Q_t, respectively, while $\epsilon_{i,t}^L$ and $\epsilon_{i,t}^E$ are uninsurable bounded shocks to labor income and stock returns.[3]

The labor-income shock $\epsilon_{i,t}^L$ follows a zero-mean 2-state Markov process, calibrated to match labor-income shares of the top 10% and bottom 90%. The stock-return shock $\epsilon_{i,t}^E$ is a zero-mean i.i.d. shock, calibrated to match cross-sectional wealth return variation. A law of

[3] In our model, all stocks bear uninsurable risk. A natural extension would distinguish between private and public equity, following Angeletos (2007).

large numbers ensures that idiosyncratic shocks average out:

$$\int \epsilon_{i,t}^{L} di = \int \epsilon_{i,t}^{E} di = 0.$$

Equity and bond returns are:

$$R_{i,t}^{e} = \frac{\left(1+\epsilon_{i,t}^{E}\right)\left(\alpha Y_t + P_t\right)}{P_{t-1}}, \quad R_t^{f} = \frac{1}{Q_{t-1}}.$$

Thus, total wealth return is $R_{i,t}^{w} = R_t^{f} + \lambda_{i,t-1}\left(R_{i,t}^{e} - R_t^{f}\right)$, where $\lambda_{i,t}$ is the equity portfolio share:[4]

$$\lambda_{i,t} = \frac{P_t E_{i,t}}{P_t E_{i,t} + Q_t B_{i,t}}.$$

Trade in financial assets is subject to three constraints:

$$P_t E_{i,t} + Q_t B_{i,t} \geq 0 \tag{3}$$

$$E_{i,t} \geq 0 \tag{4}$$

$$P_t E_{i,t} \leq \bar{\lambda}\left(P_t E_{i,t} + Q_t B_{i,t}\right). \tag{5}$$

Constraint (3) prevents negative net worth, constraint (4) prohibits short-selling equity, and constraint (5) caps leverage. The "net worth" constraint (3) is crucial, as it sets a lower bound on financial wealth, ensuring a stationary wealth distribution. In the unconstrained model of Barro et al. (2022), agents' wealth growth (including human capital) follows an exogenous random process, leading to unbounded inequality (see Gabaix, 2009 on random growth models). The "net worth" constraint stabilizes wealth distribution by increasing growth rates near its bound. Constraint (5) limits debt issuance via an upper bound $\bar{\lambda} > 1$. While constraints (4) and (5) are empirically plausible, they are not essential for our results.

Agents maximize their preferences (1) subject to the constraints (2)-(5). For convenience, let $\xi_{i,t+1}$ denote the stochastic discount factor:

$$\xi_{i,t+1} = \beta \frac{C_{i,t}^{\theta}}{C_{i,t+1}^{\theta}} \frac{U_{i,t+1}^{\theta-\gamma_{i,t}}}{\left(\mathbb{E}_{i,t} U_{i,t+1}^{1-\gamma_{i,t}}\right)^{\frac{\theta-\gamma_{i,t}}{1-\gamma_{i,t}}}}. \tag{6}$$

[4] If net worth $P_t E_{i,t} + Q_t B_{i,t}$ approaches zero, $\lambda_{i,t}$ is its limit. Constraints (3)-(5) ensure the limit is either zero or one.

Then, the consumption-saving decision is determined by the first-order condition:

$$\left\{ \begin{array}{l|l} \mathbb{E}_{i,t}\xi_{i,t+1}R^w_{i,t+1} = 1 & P_tE_{i,t} + Q_tB_{i,t} > 0 \\ \mathbb{E}_{i,t}\xi_{i,t+1}R^w_{i,t+1} \leq 1 & P_tE_{i,t} + Q_tB_{i,t} = 0 \end{array} \right. , \tag{7}$$

and the portfolio decision is determined by:

$$\left\{ \begin{array}{l|c} \mathbb{E}_{i,t}\xi_{i,t+1}\left(R^e_{i,t+1} - R^f_{t+1}\right) \leq 0 & E_{i,t} = 0 \\ \mathbb{E}_{i,t}\xi_{i,t+1}\left(R^e_{i,t+1} - R^f_{t+1}\right) = 0 & 0 < P_tE_{i,t} < \bar{\lambda}\left(P_tE_{i,t} + Q_tB_{i,t}\right) \\ \mathbb{E}_{i,t}\xi_{i,t+1}\left(R^e_{i,t+1} - R^f_{t+1}\right) \geq 0 & P_tE_{i,t} = \bar{\lambda}\left(P_tE_{i,t} + Q_tB_{i,t}\right) \end{array} \right. . \tag{8}$$

Finally, we have market-clearing conditions in the goods market, the stock market, and the bond market:

$$\int C_{i,t}di = Y_t \tag{9}$$

$$\int E_{i,t}di = 1 \tag{10}$$

$$\int B_{i,t}di = 0. \tag{11}$$

The cross-sectional distribution. Let $W_{i,t}$ denote the financial wealth of agent i in period t (including dividend and interest income):

$$W_{i,t} = \left(1 + \epsilon^E_{i,t}\right)\left(\alpha Y_t + P_t\right)E_{i,t-1} + B_{i,t-1}. \tag{12}$$

Aggregate wealth is given by:

$$W_t = \alpha Y_t + P_t. \tag{13}$$

Then, we can define the normalized wealth of agent i as a fraction of aggregate wealth:

$$\mu_{i,t} = \frac{W_{i,t}}{W_t},$$

and $\boldsymbol{\mathcal{F}}_t\left(\mu, \gamma, \epsilon^L\right)$ as the joint cumulative distribution function of normalized wealth, risk aversion, and the labor-income shock:

$$\boldsymbol{\mathcal{F}}_t\left(\mu, \gamma, \epsilon^L\right) = \int \mathbb{1}\left(\mu_{i,t} \leq \mu, \gamma_{i,t} \leq \gamma, \epsilon^L_{i,t} \leq \epsilon^L\right)di, \tag{14}$$

where $\mathbb{1}(\cdot)$ is the indicator function.

Recursive equilibrium. Let $\mathbf{x}_{i,t} = \left(\mu_{i,t}, \gamma_{i,t}, \epsilon^L_{i,t}\right)$ denote the vector of individual state variables of agent i in period t. Analogously, let $\mathbf{X}_t = (Y_t, \boldsymbol{\mathcal{F}}_t)$ denote the aggregate state variables in period t, which include output and the cross-sectional distribution. When we solve the model, we detrend all the variables by output, and thus, we can exclude output from the state vector. However, for notational clarity, we define the solution of the non-detrended model here.

A recursive equilibrium is defined by the function $\mathbf{G}$:

$$\begin{pmatrix} P_t \\ Q_t \end{pmatrix} = \mathbf{G}\left(\mathbf{X}_t\right),$$

the function $\mathbf{g}$:

$$\begin{pmatrix} C_{i,t} \\ E_{i,t} \\ B_{i,t} \end{pmatrix} = \mathbf{g}\left(\mathbf{x}_{i,t}, \mathbf{X}_t\right),$$

and the function $\mathbf{H}$:

$$\boldsymbol{\mathcal{F}}_{t+1} = \mathbf{H}\left(\mathbf{X}_t, \Delta \log Y_{t+1}\right),$$

such that in each period t the optimality conditions (7)-(8) and the budget constraint (2) hold for each agent i, the market-clearing conditions (9)-(11) hold, and the distribution function satisfies equation (14), given the exogenous processes of $\gamma_{i,t}$, $\epsilon^L_{i,t}$, $\epsilon^E_{i,t}$ and Y_t.

3 Solution method

Introducing heterogeneous risk aversion in an incomplete-markets heterogeneous-agent model poses novel computational challenges. Prior models typically assume complete markets (e.g., Gârleanu and Panageas, 2015) or allow simple aggregation (e.g., Barro et al., 2022).

Our method blends global and local approximations. The effects of individual state variables, which may be highly nonlinear, are handled using a global method. In contrast, the effects of aggregate state variables, which are nearly linear locally, are treated using a local method. Similar techniques appear in Campbell (1998), Reiter (2009), Winberry (2018), and Ahn et al. (2018).

We depart from earlier work by adopting a local solution method different from perturbation. Most prior studies rely on perturbation techniques, which are ill-suited to our model. First, approximating around a non-stochastic steady state eliminates the aggregate equity

premium and induces certainty-equivalence behavior. Second, perturbation methods often struggle to accurately approximate the equity premium, even at high orders (Fernández-Villaverde and Levintal, 2018), which is particularly problematic given the central role of disaster risk in our framework.

To address this issue, we use the Taylor projection method of Levintal (2018), which has proven effective for models with rare disasters and a large equity premium (e.g., Fernández-Villaverde and Levintal, 2018; Barro et al., 2022). The Taylor projection generalizes perturbation by leveraging derivatives of the equilibrium conditions in a similar manner. Like perturbation, it relies on Taylor's theorem to compute a locally accurate solution near the approximation point.

Taylor projection offers two key advantages over perturbation. First, it applies at any state-space point, not just the steady state, and does not require a steady state to exist. Second, it avoids certainty equivalence and fully accounts for model volatility. Unlike standard perturbation (unless taken to higher orders), it effectively captures asset price effects of risk and uncertainty, which are central to our model.[5]

The computational appendix details our method and reports accuracy measures; here, we outline key ideas. Following Krusell and Smith (1998), we approximate $\boldsymbol{\mathcal{F}}_t\left(\mu, \gamma, \epsilon^L\right)$ using M moments. The first moments, conditional on risk aversion and labor income, suffice to accurately capture local dynamics, though additional moments could be used if needed.

Second, we approximate a distribution function that satisfies the M moments and use it for agent integration, following Winberry (2018). To refine the approximation, we introduce K additional moments, ensuring the function satisfies $M + K$ moments. Since these extra moments have little quantitative impact on local dynamics, we omit them from the state vector and treat them as locally constant, significantly reducing computational costs.

Third, we approximate an auxiliary function $P_{t+1} = \mathbf{P}\left(\mathbf{X}_t, \Delta \log Y_{t+1}\right)$, which maps the current aggregate state and future shocks to the next stock price (Kubler and Schmedders, 2003; Cao et al., 2023). This step is crucial for deriving our recursive solution.

Fourth, for simulations, we discretize the cross-sectional distribution on a dense grid. Starting from an arbitrary initial distribution, we solve the model and simulate along a zero-shock path. We iteratively solve at several points (since Taylor projection applies anywhere) until convergence. This point is the stochastic steady state of the model.[6] For simplicity, we

[5] Barro et al. (2022) applies this method to a two-agent model related to Gârleanu and Panageas (2015), where it performed well. As a local method, it may fail when aggregate state variables exhibit extreme nonlinearities or kinks; however, in our model, these variables are moments of the wealth distribution that evolve smoothly. See the appendix for details.

[6] The deterministic steady state has constant aggregate variables and no aggregate shocks, though agents still face idiosyncratic risk. The stochastic steady state also has constant aggregate variables but assumes zero aggregate shock realizations, while agents anticipate future shocks. This concept preserves the effects of

refer to the stochastic steady state as the "steady state" in our results.

4 Estimation

This section outlines our parameter calibration and estimation. We calibrate parameters either directly from the data or using commonly used parameters in the literature. The remaining parameters are estimated via the SMM. We assess the estimation by evaluating the model fit for untargeted moments. Table 1 summarizes the parameter values.

Table 1: Parameter values: Benchmark

Parameter	Value	Comments
A. Calibrated parameters		
Inverse IES θ	1	Conventional
Capital income share α	1/3	Conventional
Maximum leverage $\bar{\lambda}$	2	Rarely binding
Output growth		
g (mean in normal periods)	0.02	Barro and Ursúa (2008)
σ^Y (volatility in normal periods)	0.023	Barro and Ursúa (2008)
p^{disaster} (disaster probability)	0.037	Barro and Ursúa (2012)
b (disaster size)	$-\log(1-0.21)$	Barro and Ursúa (2012)
Labor-income shock (ϵ_{it}^L)		
values		
ϵ_1^L (low income)	-1/6	Match the bottom 90% and top 10% of the U.S. labor-income distribution in 1960-1970 from Piketty et al. (2018)
ϵ_2^L (high income)	3/2	
population shares		
θ_1^L (low income)	0.9	
θ_2^L (high income)	0.1	
probability of remaining at the low-income state		
π_{11}^L	0.9972	Chetty et al. (2014)
B. Estimated parameters		
Markov process of risk aversion (γ_{it})		SMM estimates to match six moments: aggregate return on wealth, risk-free rate, wealth shares of top 10% and top 0.1%, excess return of top 10%, and standard deviation of returns on wealth across households.
values		
γ_1	3.7	
γ_2	10.0	
transition probabilities		
π_{11}^γ	0.992	
π_{22}^γ	0.998	
Time discount factor (β)	0.959	
Idiosyncratic stock-return volatility (σ^E)	0.052	

aggregate uncertainty while keeping aggregate variables fixed.

4.1 Calibrated parameters

We set the inverse IES to $\theta = 1$ (later, we report results for alternative values, including $\theta = 1/2$). The capital income share follows the standard calibration $\alpha = 1/3$. Maximum leverage, or the equity portfolio share, is set at $\bar{\lambda} = 2$, implying a maximum wealth return of 9.8% (given the targeted asset returns). This constraint binds infrequently in simulations, affecting only 8% of the population and 3% of total wealth.

The post-WWII mean per-capita growth rate of U.S. GDP is $g = 0.02$ with a standard deviation $\sigma^Y = 0.023$ (Barro and Ursúa, 2008). The disaster calibration follows Barro and Ursúa (2012), defining disasters as rare events with GDP contractions of at least 10%. Using data from 40 countries (1870-2009), Barro and Ursúa (2012) estimate a 3.7% annual disaster probability and an average GDP contraction of 21%.

The labor-income shock $\epsilon^L_{i,t}$ takes values ϵ^L_1 and ϵ^L_2, corresponding to the bottom 90% and top 10% of the labor-income distribution. We calibrate these based on the 1960-1970 U.S. distribution, treating it as a long-run steady state. During this period, the average labor income of individuals in the top 10% was roughly three times larger than that of the bottom 90% (Piketty et al., 2018), which implies the values $\epsilon^L_1 = -1/6$ and $\epsilon^L_2 = 3/2$ with the corresponding population shares $\theta^L_1 = 0.9$ and $\theta^L_2 = 0.1$, respectively. Given infinitely lived agents and persistent shocks, we use dynastic data for persistence. From Chetty et al. (2014), we estimate a 0.92 probability that a child of parents in the bottom 90% remains there at age 30, translating to an annual persistence probability of 0.9972.

4.2 Estimation moments

The remaining parameters include the 2-state Markov process for risk aversion, the discount factor β, and the volatility of the i.i.d. stock-return shock. We estimate these six parameters via SMM, targeting six moments: the risk-free rate, aggregate wealth return, wealth shares of the top 10% and top 0.1%, the excess return on wealth of the top 10%, and the cross-sectional variation in wealth returns.

The asset-pricing moments and the wealth distribution moments are standard in the literature. Asset returns are taken from Jordà et al. (2019a), and top wealth shares from Piketty et al. (2018). Following Hubmer et al. (2020), we focus on 1960–1970, a period of stable wealth shares and asset returns, which we interpret as a steady state.

The moments that require more discussion are those governing the portfolio distribution. First, the excess return on wealth of the top 10% is taken from Hubmer et al. (2020). To compute excess returns, Hubmer et al. (2020) consider four asset classes held by U.S. households—risk-free assets, public equity, private equity, and housing—and decompose re-

turns in each class into aggregate and idiosyncratic components. Aggregate returns are taken from U.S. studies.[7] The idiosyncratic component is borrowed from Bach et al. (2020) using Swedish data.[8] Second, the cross-sectional variation in returns on wealth is estimated by Fagereng et al. (2020) using administrative data from Norway.

This leads us to target the distribution of portfolio *returns* rather than portfolio *composition*. Focusing on returns follows Hubmer et al. (2020) and a growing literature emphasizing heterogeneous returns as a key driver of wealth inequality (Benhabib et al., 2011, 2015, 2016). We capture this through the excess return earned by the top 10%, which exceeds the population average. This moment is the primary observation disciplining the portfolio distribution in our estimation.

The positive correlation between wealth and returns is well documented. Earlier empirical work based on the Survey of Consumer Finances, summarized in Guiso and Sodini (2013) and Gomes et al. (2021), shows that wealthier households allocate larger portfolio shares to high-return assets. More recent administrative data reinforce this finding: Bach et al. (2020) show that wealthy Swedish households hold riskier portfolios and earn higher expected returns, while Fagereng et al. (2020) find a strong positive correlation between returns and wealth in Norway.

The key advantage of targeting portfolio returns is that the return process summarizes state-contingent payoffs in a single object. In contrast, portfolio composition is inherently high-dimensional. Many portfolios can generate the same return process, but the reverse is not true. Under complete markets, for instance, the return process is well-defined, whereas composition is indeterminate because identical payoffs can be replicated by multiple asset combinations.

From a practical perspective, matching portfolio composition is challenging because household portfolios are extremely heterogeneous and include asset classes absent from our stylized model, which features only equity and safe bonds. The most important omissions are housing and mortgage debt, shaped by considerations outside our framework: life-cycle patterns, indivisibility, regulation, and other institutional features.[9] In Section 4.5, we discuss how housing behaves in a disaster model and offer an interpretation consistent with our results.

[7] Returns on public and private equity are from Kartashova (2014), and housing returns are from Jordà et al. (2019a).

[8] The positive wealth-return correlation obtained by Hubmer et al. (2020) is driven primarily by the aggregate return components, as rich households tend to hold assets with higher expected returns.

[9] Our model also abstracts from government debt and foreign assets and liabilities, which are important components of U.S. household portfolios.

4.3 Estimation results

Table 2 compares simulated and empirical moments. The model closely matches the targeted moments (in bold). In particular, it reproduces jointly the asset returns and the wealth distribution observed in the data. Notably, the model replicates the fat tail of the wealth distribution, a key focus of the literature. Our contribution lies in explaining this fat tail through endogenous portfolio choices that generate heterogeneous wealth returns.

Table 2: Wealth and returns: Model vs. data

	Model	Data
Returns (%)		Jordà et al. (2019a) (1950-2017)
return on equity	**5.6**	5.6
return on bonds	**1.4**	1.4
Wealth shares (%)		Piketty et al. (2018) (1960-1970)
bottom 50%	5.6	1.1
50-90%	24.2	28.6
top 10%	**70.2**	70.3
top 1%	29.3	28.1
top 0.1%	**9.8**	9.9
Excess returns (%)		Hubmer et al. (2020) (1967)
bottom 50%	0.0	0.6
50-90%	2.5	2.2
top 10%	**5.1**	5.1
top 1%	4.9	7.0
top 0.1%	4.8	8.8
		Fagereng et al. (2020) (Norway 2005-2015)
Return volatility (%)	**7.8**	7.8
		Gorton et al. (2012) (1952-2011)
Safe to total assets (%)	29.2	32.0

Steady-state moments. Targeted moments are in bold, with data samples in parentheses. All data are from the U.S., except return volatility, which is based on Norwegian data. Returns on (unlevered) equity and bonds correspond to the total wealth return and the bill rate from Jordà et al. (2019a). Excess return is the portfolio return above the risk-free rate, estimated by Hubmer et al. (2020). Return volatility captures variation across households and over time.

The asset-pricing implications of the model are similar to those in Barro (2009). In particular, the price–dividend ratio is relatively stable, and the volatility of equity returns is therefore driven primarily by the volatility of dividend growth. In a stochastic simulation without realized disasters, the annual volatilities are 0.7% for the price–dividend ratio, 2.3% for aggregate dividend growth (equal to output growth), 2.5% for the aggregate return on

unlevered equity, and 0.1% for the risk-free rate.[10]

Figure 1 plots the optimal consumption-to-wealth ratio, which decreases in wealth for all agents and converges to $1 - \beta$ within the top 1% of the wealth distribution. In contrast, Figure 2 highlights stark differences in portfolio choices by risk aversion. Low-risk-aversion agents hold leveraged equity portfolios ($\lambda_{i,t} > 1$), with leverage declining as wealth rises, since they borrow against labor income, which dominates at low wealth levels.[11] High-risk-aversion agents avoid equity at low wealth and, even above the threshold, allocate less than 50% of their wealth to equity.

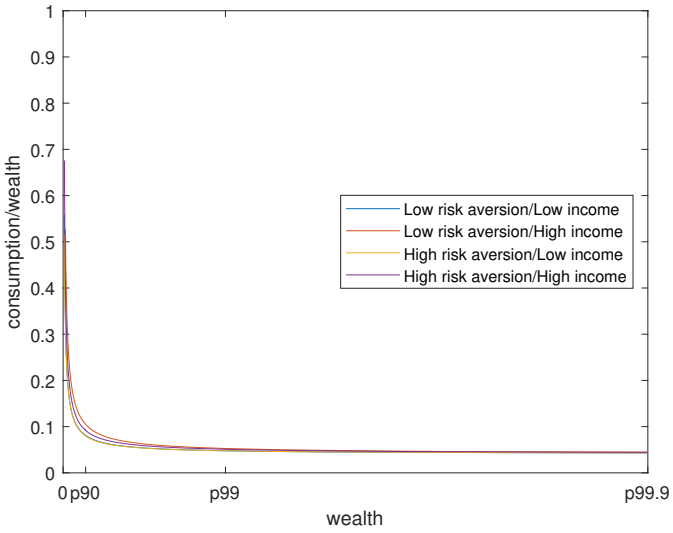


Figure 1: Consumption/wealth ratio

The optimal consumption-to-wealth ratio ($C_{i,t}/W_{i,t}$) across wealth levels and agent types in the steady state. The horizontal axis represents wealth, with markers at the 90th, 99th, and 99.9th percentiles.

Table 3 shows the distribution of types across wealth groups. As expected, low-risk-aversion agents dominate the upper tail of wealth due to higher returns and faster wealth accumulation. Among the top 0.1%, 87.4% are low-risk-aversion types (76.9% with low labor income, 10.5% with high labor income). Moving down the wealth distribution, high-risk-aversion agents become more prevalent, comprising 99.5% of agents in the bottom 50%.

[10] A natural extension that would increase asset-price volatility is to allow for heterogeneity in the IES, as in Gârleanu and Panageas (2015).

[11] Risk-tolerant agents borrow against their total assets, which include both equity and human capital. At low levels of wealth, equity represents a small share of total assets, so the debt-to-equity ratio is high. As wealth increases, the share of equity in total assets rises, and the debt-to-equity ratio falls.

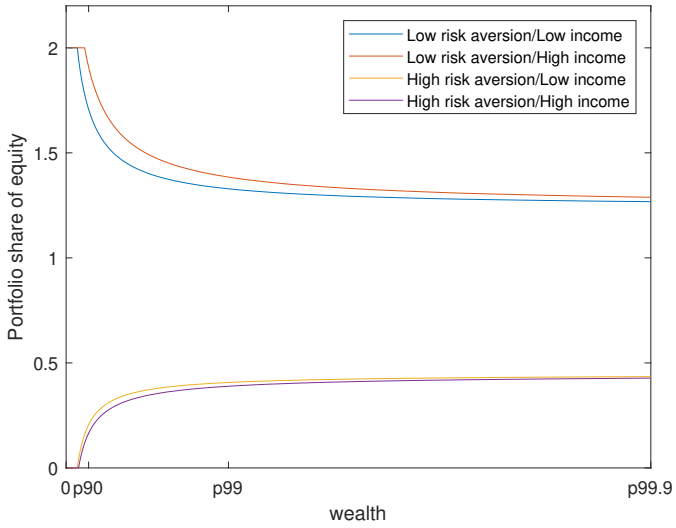


Figure 2: Portfolio share of equity

The optimal equity portfolio share ($\lambda_{i,t}$) across wealth levels and agent types in the steady state. The horizontal axis represents wealth, with markers at the 90th, 99th, and 99.9th percentiles.

Table 3: Distribution of types (model)

	Low risk aversion Low income	Low risk aversion High income	High risk aversion Low income	High risk aversion High income
bottom 50%	0.005	0.000	0.992	0.003
50-90%	0.245	0.010	0.565	0.180
top 10%	0.633	0.141	0.141	0.085
top 1%	0.722	0.148	0.111	0.019
top 0.1%	0.769	0.105	0.112	0.014

Notes: Steady-state distribution of agent types across wealth groups. Entries denote population shares within each wealth group and sum to 1 across each row.

Agents continuously move along the wealth distribution. Figure 3 illustrates the wealth dynamics of four agent types (high/low risk aversion and high/low labor income) from different starting wealth levels. A key pattern emerges: at low wealth (e.g., bottom 50%, upper-left panel), labor-income heterogeneity drives wealth accumulation: high-income agents accumulate faster, while portfolio choices matter less. At high wealth (e.g., top 0.1%, lower-right panel), portfolio decisions dominate, and low-risk-aversion agents accumulate wealth much faster, regardless of labor income.

The forces governing wealth dynamics can be understood through Figures 1–3. Consider

the extremely wealthy agents. At the very top of the distribution, consumption–wealth ratios are fairly similar across types (Figure 1), but portfolio choices differ markedly (Figure 2). Rich agents who transition from low to high risk aversion shift their portfolios from high-risk/high-return to low-risk/low-return allocations. Consequently, they begin to lose wealth shares over time, as shown in the lower-right panel of Figure 3.

Table 4: Source of wealth mobility

	Wealth growth	Low γ	High ϵ^L	High ϵ^E
Aggregate population shares		0.18	0.10	0.50
A. Moving upward		fraction of time in this state		
From P50 to P75	3.3%	0.29	0.17	0.51
From P75 to P90	2.3%	0.57	0.46	0.56
From P90 to P99	4.0%	0.98	0.74	0.65
From P99 to P99.9	2.6%	1.00	0.62	0.67
B. Moving downward		fraction of time in this state		
From P99.9 to P99	-2.6%	0.08	0.00	0.47
From P99 to P90	-4.0%	0.11	0.44	0.49
From P90 to P75	-2.3%	0.02	0.02	0.47
From P75 to P50	-3.3%	0.22	0.14	0.51

Table 4 sheds light on wealth mobility in our model. The table tracks agents who move across the 50th, 75th, 90th, 99th, and 99.9th wealth percentiles over a 50-year simulation. For agents who move up or down the distribution, the table reports the fraction of time they spend in states that accelerate wealth accumulation. These "good states" are: low risk aversion (γ), high labor income (ϵ^L), and high idiosyncratic stock-return shocks (ϵ^E). As a benchmark, the first row reports the unconditional probability of each good state; that is, the aggregate population shares. Hence, if an agent spends more time in a good state than the population share implies, this indicates that the agent was relatively fortunate in drawing shocks that promote wealth growth.

The results show that low risk aversion is essential for upward mobility, and its importance increases at the top of the distribution. For example, agents who move from the 99th to the 99.9th percentile spend nearly 100% of their time in the low-γ state. High labor income also plays a notable role even at high wealth levels. Agents moving from the 90th to the 99th percentile spend 74% of the time in the high-ϵ^L state, reflecting that high-labor-income agents have higher active saving rates (see Table 9, Panel A, discussed below).

An important finding is that idiosyncratic return shocks play only a modest role in wealth mobility. Among agents moving up the distribution, the incidence of high stock-return shocks

lies between 0.51 and 0.67, only slightly above the unconditional probability of 0.50. Similarly, agents who move down the distribution spend between 0.47 and 0.51 of their time in the high-stock-return state. Since these shocks are i.i.d., their effects dissipate quickly. By contrast, shocks to risk aversion and labor income are highly persistent, and this persistence primarily drives wealth mobility in the model.

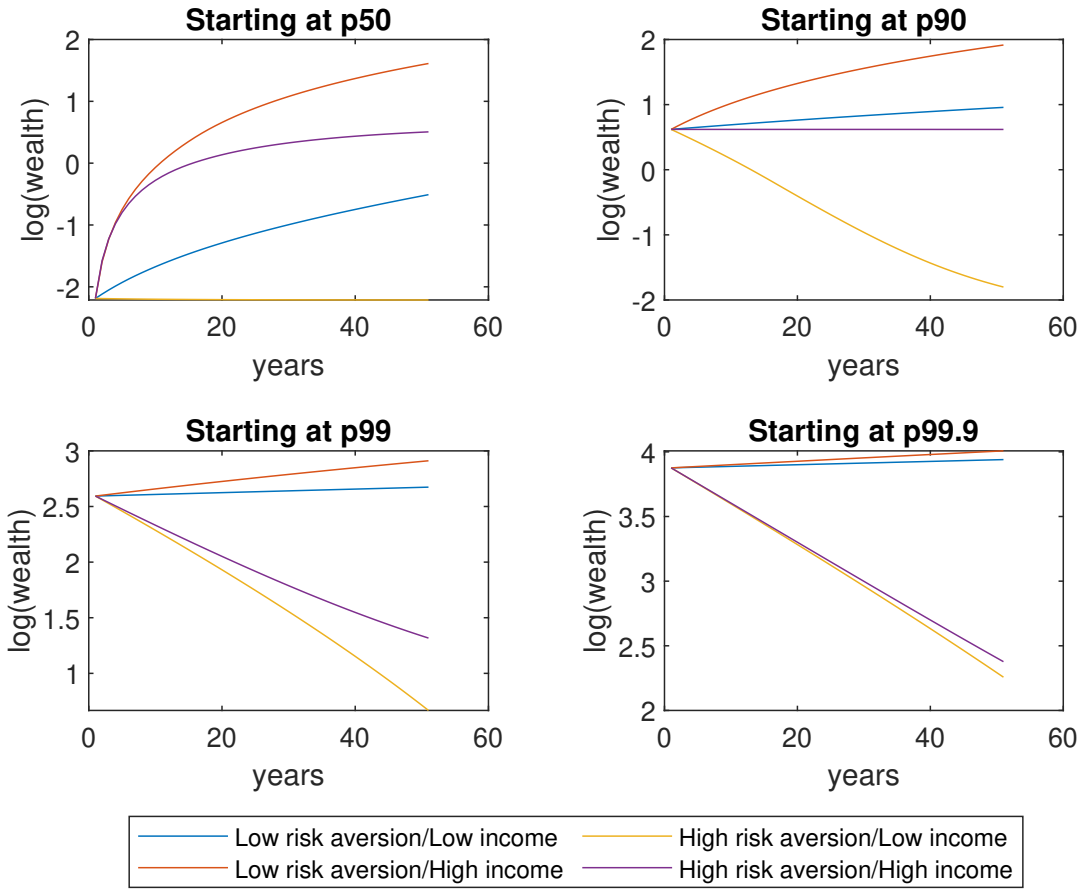


Figure 3: Wealth mobility across the distribution

Dynamics of log wealth for the four agent types at the stochastic steady state, starting from different wealth levels. Idiosyncratic stock-return shocks are set to zero for clarity. Panels show trajectories from the median (upper-left), 90th percentile (upper-right), 99th percentile (lower-left), and 99.9th percentile (lower-right).

Next, we assess the model's performance by examining untargeted moments (in normal font in Table 2). Starting with the wealth shares of the bottom 50% and the 50th–90th percentiles, the model fits well but slightly overestimates wealth for the bottom 50%. At these wealth levels, inequality is mainly driven by labor-income heterogeneity. For computa-

tional reasons, labor income is modeled as taking only two values, which limits labor-income inequality and, consequently, underestimates wealth inequality at the lower end.

Excess return is the return on wealth above the risk-free rate. Hubmer et al. (2020) estimate excess returns for different wealth percentiles in the U.S. economy. Our SMM targets the estimated excess return for the top 10% and matches it exactly. Excess returns within the bottom 90% are not targeted but are still replicated fairly accurately. In particular, the bottom 50% in the model save mainly in risk-free assets, so their excess return is zero, compared to 0.6% in the data.

However, the model fails to generate a steep rise in excess returns within the top 10%. For instance, the excess return of the top 0.1% is 8.8% in the data but only 4.8% in the model. This gap may reflect tax and regulatory considerations absent from our framework. For example, access to private equity funds is severely limited in the U.S.[12] Since much of the excess returns to the top 0.1% come from private equity, this represents an important source of heterogeneity that we do not capture. We leave this issue for future research and focus on the top 10%, which the model matches fairly well.[13]

Table 5 presents untargeted moments related to income, consumption, and savings. The model closely replicates the income share of the top 10%. However, it slightly underestimates income inequality within the bottom 90%, which could be improved with a finer labor-income grid at additional computational cost. Inequality within the top 10% is also slightly underestimated, linked to the underestimation of returns at the top. For the top 10%, our main focus, the simulated income share is 36.1%, closely matching the 35.4% observed in the data.

Krueger et al. (2016) use data from the Consumer Expenditure Survey (CEX) and the Panel Study of Income Dynamics (PSID) to estimate the U.S. consumption distribution in 2006. Table 5 compares their estimates to our simulations.[14] The model captures the fat right tail but overstates overall inequality. It implies a top 10% consumption share of 37.8%, compared to 26.4% (CEX) and 29.8% (PSID) in the data. Beyond the well-known difficulties in measuring consumption inequality (Attanasio and Pistaferri, 2016), part of the gap reflects the omission of progressive taxes. Without a redistributive tax system, matching top wealth shares requires higher top consumption shares to slow wealth accumulation at the top.

Next, we examine the "active saving rate" defined by Bach et al. (2017) as the ratio of

[12]The SEC defines "accredited investors"—those who can invest in some private equity—as individuals who earn over $200,000 per year or have a net worth exceeding $1 million (excluding the value of their home). "Qualified purchasers," who can invest in a larger range of private equity, must hold a wealth of $5 million or more, not including their primary residence or property used in the normal conduct of business.

[13]Another obstacle to matching excess returns within the top 10% is that the portfolio equity share of low-risk-aversion agents falls with wealth, as shown in Figure 2.

[14]The model is calibrated to 1960–1970, a period with likely lower consumption inequality than 2006 (Attanasio and Pistaferri, 2014). Early-period consumption data are scarce.

Table 5: Income, consumption, and savings: Model vs. data

	model	data
Income shares[1] **(%)**		Piketty et al. (2018) (1960-1970)
bottom 50%	28.5	18.5
50-90%	35.4	46.2
top 10%	36.1	35.4
top 1%	11.7	12.2
top 0.1%	3.7	4.2
Consumption shares[2] **(%)**		Krueger et al. (2016) (PSID/CEX 2006)
bottom 60%	33.1	31.9-34.3
60-90%	29.1	38.2-39.3
top 10%	37.8	29.8-26.4
top 1%	13.0	8.2-5.1
Active saving rate[3] **(%)**	%	Bach et al. (2017) (Sweden)
bottom 50%	0.8	14.3
50-90%	-1.7	0.0
top 10%	-4.2	-3.7
top 1%	-4.2	-5.2
top 0.1%	-4.1	-6.5

[1] Sorted by income.
[2] Sorted by consumption.
[3] Sorted by wealth. Active saving rate is saving out of labor income as a ratio of wealth.

savings out of labor income to wealth. As shown below, the active saving rate plays an important role in the model. We use the subscript k to denote a wealth fractile (e.g., the top 10%). The active saving rate of fractile k is

$$s_k = \frac{\int_{i\in k} \left[\left(1+\epsilon^L_{i,t}\right)(1-\alpha)\, Y_t - C_{i,t}\right] di}{\int_{i\in k} W_{i,t} di}.$$

Table 5 reports estimates of the active saving rate in Sweden from Bach et al. (2017) and compares them to our simulated rates, since no equivalent U.S. study exists. A key pattern in both the data and the model is that the active saving rate declines with wealth. The simulated rates closely match the data; for instance, the top 10% active saving rate is -4.2% in the model versus -3.7% in Sweden. Within the top 10%, saving rates fall significantly in the data but rise slightly in the model. This discrepancy stems from the model's underestimation of return heterogeneity at the top, as discussed earlier. For this reason, our analysis below focuses on the top 10%.[15]

[15] Fagereng et al. (2025) study the saving-to-income ratio (not saving-to-wealth) using Norwegian data,

4.4 Endogenous portfolios

A key contribution of our model is that the positive wealth–return correlation is generated through endogenous portfolio choices. By comparison, Hubmer et al. (2020) propose a model in which wealth returns depend exogenously on wealth. They attribute all return heterogeneity to special investment skills that rise exogenously with wealth. As a result, their framework abstracts from the substantial heterogeneity in household portfolios observed in the data and from the strong correlation between portfolio choices and realized returns.

Conversely, in our approach, return heterogeneity arises from heterogeneous portfolio decisions. This feature makes the framework suitable for studying how inequality responds to shocks that affect agents' portfolios. For instance, an aggregate shock that moves stock prices differently from bond prices affects inequality in our model but not in Hubmer et al. (2020), where returns are fixed exogenously. Section 7 examines the distributional effects of aggregate shocks that change asset returns differentially. Addressing these issues requires a model in which portfolio choices are endogenous.

The previous literature on endogenous portfolio choices under heterogeneous risk aversion has focused primarily on two-agent complete-markets models, e.g., Dumas (1989) and Gârleanu and Panageas (2015). Of these, Barro et al. (2022) is closest to ours. It is instructive to compare our results to Barro et al. (2022) along three dimensions: (1) the share of safe assets in the economy; (2) the distribution of wealth; and (3) the distribution of household portfolios. Barro et al. (2022) study a model with heterogeneous risk aversion similar to ours but abstract from constraints on trade in financial securities (henceforth the "unconstrained model"). Their framework matches the aggregate share of safe assets, estimated at 32% of total assets by Gorton et al. (2012). Likewise, our model generates a safe-asset share of 29.2% (untargeted), close to this empirical benchmark.

The key difference concerns wealth and portfolio distributions. The unconstrained model of Barro et al. (2022) does not generate a well-defined cross-sectional distribution of wealth or portfolios. Consumption and portfolio choices scale proportionally with total wealth (including human capital), implying that wealth grows at a random rate independent of current wealth. As Gabaix (2009) shows, such "random-growth" models do not admit a stationary wealth distribution, as wealth inequality diverges over time. Consequently, Barro et al. (2022) cannot reproduce the U.S. wealth distribution or household portfolio distribution.

finding it increases strongly with wealth (including capital gains). Our model shows the same pattern: steady-state saving-to-income ratios are 4%, 12%, and 25% for the bottom 50%, middle 40%, and top 10%. Fagereng et al. (2025) also find that a measure excluding capital gains is fairly flat up to the 90th percentile and declines slightly at the top. In our model, this ratio excluding capital gains is 4% and 6% for the bottom 50% and middle 40%, consistent with Fagereng et al. (2025). We differ at the top 10%, where it drops to −9%. However, when Fagereng et al. (2025) exclude retained earnings of publicly listed firms, the top 10% pattern aligns with ours, suggesting households treat retained earnings like capital gains.

We extend Barro et al. (2022) by imposing non-tradability of human capital and a non-negativity constraint on financial wealth. These constraints are essential for pinning down a stationary wealth distribution. With them in place, our model jointly accounts for three salient empirical features: (1) the aggregate quantity of safe assets, (2) the highly skewed wealth distribution, and (3) the positive correlation between wealth and portfolio returns. All three arise from a unified mechanism based solely on heterogeneous risk aversion, indicating that this form of heterogeneity is quantitatively important for understanding the joint distribution of wealth and portfolios.

4.5 Housing in disaster models

The portfolio choice in our model involves two assets: risky equity and safe bonds. Households at the top of the distribution hold a larger fraction of their portfolios in risky assets, which is consistent with SCF data summarized by Guiso and Sodini (2013) and Gomes et al. (2021) and presented in Table 6. We abstract from the substantial heterogeneity in assets observed in the data, particularly owner-occupied housing and mortgage debt, which are central components of household portfolios. For households in the bottom 90% of the U.S. wealth distribution, housing constitutes 50–60% of gross assets (Table 6). A full treatment of housing is beyond the scope of this paper. Nevertheless, we show in this section that interpreting housing through a rare-disaster lens does not materially alter our main conclusions.

The prevalence of housing and mortgage debt among households in the bottom 90% implies that leverage declines strongly with wealth. For example, Table 6 shows that direct bond holdings net of debt amount to −241% of wealth for the bottom 50%, −20% for the middle 40%, and 5% for the top 10%.[16] When we incorporate indirect net fixed-income positions through ownership of firms and pension funds and exclude non-mortgage debt, the numbers become more moderate (−119% for the bottom 50% and 11% for the top 10%), but the qualitative pattern remains: poorer households are substantially more leveraged than richer ones.[17] This empirical pattern is opposite to the prediction of our model, which abstracts from housing demand.

These observations naturally raise the question: how would results change if housing were added to the model? Would housing behave more like a risky asset or a safe one? To answer

[16] Table 6 defines "Net Bonds" as bond-like fixed-income assets minus total debt liabilities. Negative values imply net debt positions. The model's definition of debt is narrower, as it is assumed to be risk-free.

[17] Indirect positions are constructed by attributing fixed-income assets held by pension funds (from the U.S. Financial Accounts) to households based on SCF pension wealth, and corporate bonds/debt (from the Financial Accounts) to households based on SCF stock holdings. Non-mortgage debt consists primarily of consumer credit and student loans, which are largely unrelated to wealth accumulation and are therefore outside our model.

Table 6: Asset and debt composition across the wealth distribution

	0-50%	50-90%	Top 10%	top 1%	Top 0.1%
Assets			percent of assets		
Safe assets	6.1	7.6	9.8	10.4	6.9
Stocks	0.7	3.5	16.7	20.0	20.3
Pension	9.7	21.6	19.5	11.1	6.7
Business	1.0	4.5	26.1	37.8	51.9
House (owner occupied)	63.1	49.6	15.3	8.8	3.8
Other real estate	2.4	7.0	10.4	9.5	7.8
Vehicles	15.7	5.0	0.9	0.5	0.4
Other assets	1.3	1.2	1.4	1.9	2.1
			percent of assets		
Debt	72.5	22.9	4.9	2.3	1.3
Mortgages	42.0	18.0	3.4	1.4	0.4
Student loans	16.1	0.9	0.0	0.0	0.0
Vehicle loans	6.6	1.4	0.1	0.0	0.0
Credit cards	3.0	0.6	0.0	0.0	0.0
Other debt	4.7	1.9	1.3	0.9	0.8
Net bonds			percent of net-worth		
Direct holdings	-241	-20	5	8	6
Direct and indirect	-231	-12	9	10	6
excl. non-mortgage debt	-119	-5	11	11	7
houses equivalent to bonds	111	59	27	20	11

Source: Survey of Consumer Finances, 2019. Safe assets include bank deposits, money market funds, CDs, bonds, and mutual funds that invest in bonds (including half the value of combination mutual funds). Stocks include directly held stocks and mutual funds that invest in stocks. Pensions include IRAs and similar pension funds, annuities, trusts, and life insurance accounts. Other real estate includes residential property not occupied by the owner and net equity in non-residential real estate.

this, we restrict attention to owner-occupied housing—the dominant form held by households. Although housing has many special features (indivisibility, illiquidity, durability, regulatory constraints), as emphasized by Piazzesi and Schneider (2016), here we focus narrowly on its role as a store of value within a rare-disaster framework.

In such an environment, the riskiness of an asset is determined primarily by its performance during disasters. Assets that experience large losses in disasters are risky and command premia in normal times; such assets are held disproportionately by low-risk-aversion households. Thus, the key question is: how do housing returns behave in disasters?

Jordà et al. (2019a) address this question using annual data for 16 countries from 1870 to 2015. They document a stark contrast between stocks and housing during economic disasters. Stock returns fall sharply (as shown in Barro, 2006, and Barro and Ursúa, 2008), whereas housing returns remain comparatively stable. Moreover, housing tends to outperform short-term bonds during disaster episodes, likely because it provides partial inflation protection

that is often elevated during such episodes. Excluding WWII (which Jordà et al., 2019a, regard as an outlier due to heavy government intervention in capital markets) real returns on equity during disasters average −19.7%, compared to 0.7% on housing and −5.3% on short-term bonds. Thus, housing is an excellent hedge against disaster shocks.

These findings indicate that, in a disaster environment, housing behaves much more like a safe asset than risky equity. We apply this insight in Table 6, where the last row classifies owner-occupied housing as safe. Under this interpretation, the portfolio share of safe assets (now including housing) declines with wealth—precisely the pattern generated by our model. Households in the bottom 90% hold substantial safe assets in the form of housing, which provides disaster insurance. In our model, this insurance role is played by bonds; in the data, it is largely provided by housing.

Other mechanisms reinforce the interpretation of housing as relatively safe. For example, Sinai and Souleles (2005) argue that owner-occupied housing hedges rent risk, implying that more risk-averse households may hold housing as insurance against rental price fluctuations. A full treatment of such mechanisms is beyond the scope of this paper, but they are consistent with housing acting as a relatively safe asset in a disaster setting.[18]

5 Comparative statics

We next examine the key drivers of wealth inequality in the model. Table 7 analyzes how parameter changes affect key moments. We adjust one or two parameters relative to the benchmark and compute the new stationary equilibrium. For brevity, we focus on two cases: varying idiosyncratic shocks and the capital income share.

The role of the idiosyncratic shocks. We begin by shutting down all idiosyncratic shocks except labor-income shocks (column (2) vs. (1)). Wealth inequality declines sharply, especially at the top. The wealth share of the top 0.1% drops from 9.8% to 1.0%. Thus, our model aligns with the literature: labor-income shocks alone cannot generate a fat-tailed wealth distribution (Benhabib and Bisin, 2018) unless extreme skewness is assumed (Castañeda et al., 2003).

Column (2) corresponds to the economy studied by Krueger and Lustig (2010), where relative risk aversion is constant, labor income is subject to idiosyncratic shocks independent of aggregate shocks, and stocks are subject to aggregate shocks only. As Krueger and Lustig

[18]This interpretation suggests relatively low returns on housing, contrasting with the high average returns documented by Jordà et al. (2019a). In a related paper, Jordà et al. (2019b) refer to this discrepancy as a "housing return puzzle" and argue that standard asset-pricing frameworks, including rare-disaster models, cannot explain it. Resolving this puzzle is beyond our scope. We conjecture it may reflect a combination of factors, such as high mortgage and financing costs, indivisibility, and liquidity constraints.

Table 7: Comparative statics

	benchmark	$\gamma_1 = \gamma_2$, $\epsilon^E = 0$	$\gamma_1 = \gamma_2$	$\epsilon^E = 0$	$\epsilon^L = 0$	$\alpha = 0.5$	$\theta = 1/2$	$g + 1\%$	$\theta = 1/2$, $g + 1\%$	$\rho - 1\%$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Wealth share (%)										
bottom 50%	5.6	0.1	12.3	11.3	11.2	4.2	7.5	5.6	7.6	7.3
50-90%	24.2	49.3	42.0	27.3	17.3	18.0	29.3	24.1	28.1	22.2
top 10%	70.2	50.6	45.7	61.4	71.6	77.8	63.2	70.3	64.3	70.5
top 1%	29.3	7.9	7.9	21.5	31.9	42.6	20.4	29.3	20.9	29.4
top 0.1%	9.8	1.0	1.1	5.5	12.3	20.7	5.1	9.9	5.3	10.0
Consumption share (%)										
bottom 50%	27.5	27.9	33.7	27.5	33.2	20.8	27.6	27.5	27.3	26.3
50-90%	34.7	44.9	41.4	35.7	31.3	28.8	37.2	34.7	36.6	33.5
top 10%	37.8	27.1	24.9	36.8	35.5	50.4	35.1	37.8	36.1	40.3
top 1%	13.0	3.9	3.8	10.9	12.9	24.7	9.6	13.0	9.9	14.0
top 0.1%	4.1	0.5	0.5	2.6	4.7	11.7	2.3	4.1	2.4	4.4
Portfolio share of equity (%)										
bottom 50%	1	100	141	0	2	1	0	1	0	1
50-90%	60	100	100	54	88	64	68	60	68	60
top 10%	122	100	89	139	118	114	127	122	125	123
top 1%	118	100	87	138	111	111	121	118	120	117
top 0.1%	115	100	86	137	109	110	118	115	117	115
Active saving rate (%)										
bottom 50%	0.8	16.0	-5.5	0.3	3.4	0.4	0.4	0.8	0.8	1.7
50-90%	-1.7	-3.6	-4.0	-1.5	-4.8	-2.2	-2.3	-1.7	-1.9	-0.7
top 10%	-4.2	-3.6	-3.6	-4.1	-4.4	-4.2	-4.7	-4.2	-4.2	-3.3
top 1%	-4.2	-3.6	-3.8	-4.1	-4.2	-4.1	-4.6	-4.2	-4.2	-3.2
top 0.1%	-4.1	-3.6	-4.0	-4.1	-4.1	-4.1	-4.6	-4.1	-4.1	-3.2
Between term (%)										
bottom 50%	-0.2	-16.9	-0.4	-0.1	-3.2	-0.2	-0.2	-0.2	-0.2	-0.1
50-90%	-0.1	-0.0	-0.0	0.0	1.7	-0.1	-0.2	-0.1	-0.2	-0.1
top 10%	0.0	-0.0	0.1	-0.0	0.0	0.0	0.1	0.0	0.1	0.0
top 1%	0.1	-0.0	0.4	-0.0	0.1	0.0	0.2	0.1	0.2	0.1
top 0.1%	0.1	-0.0	0.6	-0.0	0.1	0.0	0.3	0.2	0.3	0.2
Aggregate variables (%)										
r^e	5.6	5.8	6.2	5.1	5.9	5.9	5.9	6.6	6.4	4.5
r^f	1.4	1.5	1.1	1.7	1.9	1.8	1.8	2.4	2.4	0.4
r^d	3.5	3.7	4.1	3.0	3.8	3.8	3.8	3.5	3.3	2.5
g	2.0	2.0	2.0	2.0	2.0	2.0	2.0	3.0	3.0	2.0
c	4.2	5.2	5.3	4.2	4.2	4.1	4.7	4.2	4.2	3.2
$r^e - g$	3.6	3.8	4.2	3.1	3.9	3.9	3.9	3.6	3.4	2.5
$r^e - g - c$	-0.6	-1.4	-1.0	-1.1	-0.3	-0.2	-0.9	-0.6	-0.8	-0.7
Pareto	1.9452	23.2637	8.0307	2.6253	1.7283	1.4541	2.6473	1.9426	2.6104	1.9425

Steady-state outcomes for different parameter values. Each column follows the benchmark estimation except for the parameters listed in the header. Agents are sorted by wealth. In columns (2) and (3), risk aversion matches the level required to reproduce the equity premium in a representative-agent model. r^e, r^f, and r^d denote equity return, bond return, and dividend yield. c is the consumption-to-wealth ratio of the top 0.1%. Pareto is the Pareto exponent for the top 0.1%.

(2010) show, all agents optimally choose the same equity-only portfolio. This is exactly what we obtain numerically in column (2).

Against this benchmark, column (3) introduces the idiosyncratic stock-return shock (ϵ^E). The volatility of this shock (σ^E in Table 1) largely determines cross-sectional variation in returns on wealth, estimated at 7.8% by Fagereng et al. (2020) and taken as one of our targeted moments.

The idiosyncratic stock-return shock has conflicting effects on wealth inequality. Two channels increase inequality: first, idiosyncratic stock-market risk increases return heterogeneity, thereby generating greater wealth inequality. Second, precautionary savings of the rich rise more than those of the poor because rich households' income is more exposed to stock-market risk, whereas poor households earn mainly labor income; this differential response further increases inequality. However, a third channel works in the opposite direction. Poor households increase their equity share because idiosyncratic stock-return risk affects only a small fraction of their total income, making them less sensitive to it. This portfolio adjustment raises the wealth returns of the poor relative to those of the rich, thereby reducing inequality.

The introduction of the idiosyncratic stock-return shock (ϵ^E) in column (3) reduces wealth inequality relative to column (2). Here, the dominant channel is endogenous portfolio adjustment: poor households substantially increase their equity shares, accelerating their wealth accumulation relative to the rich and reducing inequality.

These conclusions change once the model includes heterogeneous risk aversion. Column (4) reports the case with heterogeneous risk aversion but with $\epsilon^E = 0$. Turning on ϵ^E in column (1) increases wealth inequality. Portfolio choices of poor households respond very little—see the equity share of the bottom 50% in columns (4) and (1), which stays close to zero. This is because portfolios of the bottom 50% are primarily constrained by the no-short-sale condition (4), leaving their allocations largely unaffected by idiosyncratic stock-market risk. Hence, the dominant channels are (i) increased heterogeneity in realized returns and (ii) stronger precautionary savings by wealthy households. Both forces raise inequality when idiosyncratic stock-market risk increases.

We conclude that the effect of idiosyncratic stock-market risk on wealth inequality is model-specific. This highlights the importance of endogenous portfolio decisions. If all households are constrained to save exclusively in risky capital (e.g., Hubmer et al., 2020), idiosyncratic return risk unambiguously increases wealth inequality. Once portfolio choices become endogenous, however, this conclusion may reverse, depending on model structure.

The role of the capital income share α. Column (6) in Table 7 increases the capital income share from $\alpha = 1/3$ to $\alpha = 1/2$, significantly affecting wealth inequality. The wealth share of the top 10% rises from 70.2% to 77.8%. The impact on consumption inequality is even larger, with the top 10% consumption share increasing from 37.8% to 50.4%. The key mechanism is income redistribution: a higher capital share reduces earnings for poorer agents, who rely primarily on labor income, lowering their savings and slowing wealth accumulation.

6 Alternative models of wealth inequality

This section compares our heterogeneous-risk-aversion model to two alternative mechanisms widely used to explain wealth inequality. The first, following Krusell and Smith (1998), relies on heterogeneity in discount factors: households with higher β save more and accumulate wealth faster. The second, developed by Quadrini (2000) and Cagetti and De Nardi (2006), emphasizes heterogeneity in investment opportunities: some households (entrepreneurs) persistently earn higher returns on capital than others. This entrepreneurial view is closely aligned in spirit with our framework if one interprets low-risk-aversion households as "entrepreneurs." The crucial distinction is that, in our model, such households earn higher returns because they optimally take on more financial risk, not because they possess superior investment technology.

To compare these mechanisms on equal footing, we extend our benchmark model to nest all three approaches. First, to introduce heterogeneous discount factors, we replace the constant β with a household-specific process $\beta_{i,t}$ following a two-state idiosyncratic Markov chain with values β^{low} and β^{high}. The stochastic discount factor (6) becomes

$$\xi_{i,t+1} = \beta_{i,t} \frac{1-\beta_{i,t+1}}{1-\beta_{i,t}} \frac{C_{i,t}^{\theta}}{C_{i,t+1}^{\theta}} \frac{U_{i,t+1}^{\theta-\gamma_{i,t}}}{\left(\mathbb{E}_t U_{i,t+1}^{1-\gamma_{i,t}}\right)^{\frac{\theta-\gamma_{i,t}}{1-\gamma_{i,t}}}}.$$

Second, to incorporate heterogeneity into investment opportunities, we allow for persistent differences in households' expected stock returns. We postulate the following process for the idiosyncratic stock-return component $\epsilon_{i,t}^{E}$:

$$\epsilon_{i,t+1}^{E} = \alpha_{i,t}^{E} + \zeta_{i,t+1}^{E},$$

where $\zeta_{i,t+1}^{E}$ is a zero-mean i.i.d. shock and $\alpha_{i,t}^{E}$ is a persistent component that depends on entrepreneurial status, which takes one of two values (entrepreneur or non-entrepreneur) with fixed transition probabilities. To ensure that aggregate stock returns satisfy market clearing,

we define

$$\alpha_{i,t}^{E} = \begin{cases} \alpha^{E}\left(\frac{1}{E_t^{\text{entrepreneurs}}} - 1\right) & \text{Entrepreneur} \\ -\alpha^{E} & \text{Non-Entrepreneur} \end{cases}$$

where $\alpha^E \geq 0$ and $E_t^{\text{entrepreneurs}}$ denotes the aggregate equity share held by entrepreneurs. This guarantees that $\int E_{i,t}\alpha_{i,t}^{E}\, di = 0$.

When $\alpha^E = 0$, all agents have identical investment opportunities; when $\alpha^E > 0$, entrepreneurs earn higher expected equity returns. Because entrepreneurial status follows a persistent two-state Markov chain, the return premium is likewise persistent.

We therefore analyze three models: (1) the heterogeneous-risk-aversion model (our benchmark), (2) the heterogeneous-discount-factor model (heterogeneity in β only), and (3) the entrepreneurial model (heterogeneity in expected stock returns only).[19] The following subsections discuss the differences between these models.

6.1 The joint distribution of wealth and returns

We start by exploring the implications of the three models for the joint distribution of wealth and returns. Each model is estimated by SMM using the same moments as in the benchmark. Since the heterogeneous-discount-factor and entrepreneurial models cannot match the high excess returns of the top 10%, we exclude that moment from their SMM targets and require the remaining moments to match exactly. The resulting implications are summarized in Table 8.

All three models reproduce the empirical wealth distribution. They differ sharply, however, in the distribution of returns on wealth. The heterogeneous-risk-aversion model generates a positive relationship between wealth and returns, consistent with the data: richer households earn higher returns by allocating larger portfolio shares to equity. In contrast, both the heterogeneous-discount-factor and entrepreneurial models generate a *negative* relationship. In these models, poorer households hold riskier portfolios because labor income dominates their total income, thereby making them less sensitive to fluctuations in capital income. This force persists even in the entrepreneurial model, despite entrepreneurs earning higher expected equity returns (see Table 9 for returns on wealth by agent type).

The models also differ substantially in their ability to match the aggregate share of safe assets. The heterogeneous-risk-aversion model generates a safe-asset share close to the data (29.2% vs. 32.0%). By contrast, the heterogeneous-discount-factor and entrepreneurial models (calibrated to match the large equity premium) produce safe-asset shares of only 10.2% and 8.2%, respectively. These findings indicate that heterogeneity in risk aversion plays a cen-

[19] A natural extension would combine all three sources of heterogeneity.

Table 8: Alternative models of wealth inequality

	Heterogeneous risk aversion	Heterogeneous discount factors	Entrepreneurial model	data
Returns (%)				
equity	**5.6**	**5.6**	**5.6**	5.6
bonds	**1.4**	**1.4**	**1.4**	1.4
Wealth shares (%)				
bottom 50%	5.6	3.0	0.3	1.1
50-90%	24.2	26.7	29.4	28.6
top 10%	**70.2**	**70.3**	**70.3**	70.3
top 1%	29.3	30.0	28.7	28.1
top 0.1%	**9.8**	**9.9**	**9.9**	9.9
Excess returns (%)				
bottom 50%	0.0	8.4	6.9	0.6
50-90%	2.5	5.3	4.3	2.2
top 10%	**5.1**	3.6	4.1	5.1
top 1%	4.9	3.3	4.1	7.0
top 0.1%	4.8	3.2	4.0	8.8
Active saving rate (%)				
bottom 50%	0.8	-6.1	-3.5	14.3
50-90%	-1.7	-4.2	-3.2	0.0
top 10%	-4.2	-2.9	-3.4	-3.7
top 1%	-4.2	-2.7	-3.4	-5.2
top 0.1%	-4.1	-2.6	-3.4	-6.5
Return volatility (%)	**7.8**	**7.8**	**7.8**	7.8
Safe to total assets (%)	29.2	10.2	8.2	32.0

Targeted moments in bold.

tral quantitative role in simultaneously matching the safe-asset share, the wealth distribution, and the cross-sectional distribution of portfolio returns.

6.2 Drivers of wealth inequality

We now examine the mechanisms that generate wealth inequality across the three models. Table 9 reports sources of wealth accumulation by agent type in each model, decomposing wealth growth into two components: the active saving rate (non-financial income less consumption as a fraction of wealth) and the return on wealth. For each agent type, these components are reported separately at fixed wealth levels corresponding to the 50th, 90th,

99th, and 99.9th percentiles. Agents are classified as high- or low-labor-income. Within each group, agents differ in risk aversion (Panel A), discount rate (Panel B), or investment skill (Panel C). We discuss how these types of heterogeneity affect the wealth distribution.

Table 9: Drivers of wealth growth in alternative models of wealth inequality

	(1)	(2)	(3)	(4)
	A. Heterogeneous Risk Aversion			
	Low labor income		**High labor income**	
	Low γ	**High γ**	**Low γ**	**High γ**
Wealth percentile	*Active saving rate (%)*			
P50	-0.3	0.3	50.9	62.5
P90	-5.2	-5.1	-1.4	-0.1
P99	-4.5	-4.4	-4.1	-3.9
P99.9	-4.3	-4.3	-4.2	-4.2
	Return on wealth (%)			
P50	9.4	1.4	9.4	1.4
P90	8.2	2.2	9.0	2.1
P99	6.8	3.1	7.0	3.0
P99.9	6.5	3.2	6.6	3.2
	B. Heterogeneous Discount Factors			
	Low labor income		**High labor income**	
	High β	**Low β**	**High β**	**Low β**
Wealth percentile	*Active saving rate (%)*			
P50	10.9	-8.1	59.9	45.8
P90	-2.0	-6.0	2.0	-2.7
P99	-2.3	-4.9	-1.7	-4.5
P99.9	-2.2	-4.6	-2.1	-4.5
	Return on wealth (%)			
P50	9.3	9.3	9.3	9.3
P90	5.4	5.4	5.7	5.6
P99	4.7	4.7	4.8	4.8
P99.9	4.6	4.6	4.6	4.6
	C. Entrepreneurial Model			
	Low labor income		**High labor income**	
	Entr.	**Non-entr.**	**Entr.**	**Non-entr.**
Wealth percentile	*Active saving rate (%)*			
P50	18.8	-8.5	94.6	104.7
P90	-4.1	-4.6	-0.8	-1.4
P99	-3.7	-3.8	-3.2	-3.3
P99.9	-3.5	-3.5	-3.4	-3.4
	Return on wealth (%)			
P50	10.2	8.0	10.2	8.0
P90	6.9	4.1	7.4	4.2
P99	5.9	3.7	6.1	3.8
P99.9	5.7	3.6	5.8	3.7

Notes: Active saving rate and return on wealth of the four agent types in each model at fixed wealth levels (50th, 90th, 99th, and 99.9th percentiles).

Heterogeneous saving behavior. The table highlights a fundamental difference in saving behavior between the heterogeneous-risk-aversion and heterogeneous-discount-factor models. In the heterogeneous-discount-factor model (Panel B), households with the same wealth and labor income earn roughly the same return on wealth, independent of their discount factor. What distinguishes them is their consumption-to-wealth ratio, reflected in substantial differences in active saving rates. High-β households consume less and save more, as seen by comparing columns (1) vs. (2) or (3) vs. (4) in Panel B.

By contrast, in the heterogeneous-risk-aversion model (Panel A), differences in consumption-to-wealth ratios across types are modest. Active saving rates are therefore similar across agents with the same wealth and labor income (columns (1) vs. (2), or (3) vs. (4)). The primary source of heterogeneity in wealth growth instead comes from differences in *returns on wealth*: low-γ types earn higher returns than high-γ types because they choose riskier portfolios.

This comparison shows that the two models generate heterogeneity in saving behavior for fundamentally different reasons. Risk-aversion heterogeneity produces heterogeneous saving through *differences in financial income*: agents with different γ earn different returns on wealth. By contrast, heterogeneity in discount factors creates heterogeneous saving through differences in consumption-wealth ratios (and thus in active saving rates), not through differences in financial returns.

These differences will play an important role in Section 7, where we examine the distributional effects of asset returns. We show there that the welfare and distributional consequences of changes in asset returns differ sharply depending on the source of heterogeneity. In particular, a decline in asset returns increases inequality under heterogeneous risk aversion but reduces inequality under heterogeneous discount factors. Hence, the mechanism generating wealth inequality is critical for policy analysis.

Heterogeneous investment skills. Panel C in Table 9 shows wealth accumulation in the entrepreneurial model, where some agents ("entrepreneurs") have access to higher expected equity returns. This model is closely related to the heterogeneous-risk-aversion model (Panel A) in that some agents earn higher returns than others. Table 9 shows that in both models (Panels A and C), the main driver of heterogeneous wealth growth is heterogeneity in *returns*, reflected in sizable differences in returns on wealth compared with much smaller differences in active saving rates. In this sense, the two models share a similar mechanism for generating wealth inequality.

Despite this similarity, the two models generate sharply different steady-state portfolio distributions, as discussed in Subsection 6.1. The heterogeneous-risk-aversion model matches

the *positive* wealth–return correlation observed in the data, whereas the entrepreneurial model produces a *negative* correlation. Thus, the entrepreneurial model is inconsistent with a central empirical fact about the cross-section of returns.

The entrepreneurial model requires stronger assumptions about financial frictions, imposing restricted access to capital investments on non-entrepreneurs. Moreover, entrepreneurs cannot form partnerships with non-entrepreneurs to share their higher returns. Our benchmark model does not rely on such restrictions: all agents face the same asset menu, and differences in returns arise endogenously from differences in risk appetite.

The empirical evidence on special investment skills is mixed. Bach et al. (2020) find that return heterogeneity among wealthy Swedish households is fully accounted for by differences in risk exposure, consistent with our heterogeneous-risk-aversion mechanism. In contrast, Fagereng et al. (2020) provide evidence suggestive of persistent investment-skill heterogeneity. Our results indicate that variation in risk aversion is essential: without it, the wealth–return correlation becomes negative. Nonetheless, investment skills may provide a complementary source of wealth inequality, potentially correlated with risk aversion. Exploring such interactions is a natural next step.

The sources of heterogeneous returns (risk aversion vs. investment skills) may matter for tax policy. Guvenen et al. (2023) show that a wealth tax shifts the tax burden toward low-return households. If low returns reflect poor investment skills, a wealth tax reallocates resources toward more productive agents (high-return entrepreneurs), potentially increasing aggregate output. This rationale does not apply when return heterogeneity reflects differences in risk aversion, since higher returns do not signal greater productivity. In that case, a wealth tax need not reallocate resources in a way that increases output.

A full evaluation would require extending the model from an endowment economy to a production economy, where taxes can influence capital accumulation and output through precautionary savings and portfolio choice.

7 The distributional effects of asset returns

We turn now to the distributional effects of asset returns. We analyze steady-state implications in Subsections 7.1–7.2 and transitional dynamics in Subsections 7.3–7.4. We present our results through numerical simulations, but complement them where possible with analytical results to clarify the economic intuition.

The wealth distribution is defined in equation (14) over agents' normalized wealth $\mu_{i,t} \equiv W_{i,t}/W_t$, the ratio of individual to aggregate (per capita) wealth. The evolution of the wealth distribution is determined by the law of motion of $\mu_{i,t}$. The growth rate of $\mu_{i,t}$, denoted $g^{\mu}_{i,t+1}$,

can be decomposed into two terms:

$$g^{\mu}_{i,t+1} \equiv \frac{\Delta \mu_{i,t+1}}{\mu_{i,t}} = (s_{i,t} - s_t) + \left(r^{w}_{i,t+1} - r^{e}_{t+1}\right), \tag{15}$$

where $s_{i,t}$ and s_t denote the active saving rates of agent i and the aggregate economy, and $r^{w}_{i,t+1}$ and r^{e}_{t+1} denote log returns on individual and aggregate wealth.[20] The active saving rate of agent i is labor income net of consumption as a fraction of wealth:

$$s_{i,t} = \frac{I_{i,t} - C_{i,t}}{W_{i,t}}, \tag{16}$$

where $I_{i,t} \equiv (1 + \epsilon^{L}_{i,t})(1 - \alpha)Y_t$ denotes agent i's labor income.

Both $s_{i,t}$ and $r^{w}_{i,t+1}$ are endogenous and depend on the agent's normalized wealth $\mu_{i,t}$. Thus, equation (15) defines a stochastic law of motion for $\mu_{i,t}$ that depends on $s_{i,t}(\mu_{i,t}) - s_t$ and $r^{w}_{i,t+1}(\mu_{i,t}) - r^{e}_{t+1}$, which we label the "saving term" and "portfolio term," respectively.

The law of motion (15) induces a stochastic steady state in which realized aggregate shocks are zero and the cross-sectional distribution of $\mu_{i,t}$ is time-invariant. This steady state is the focus of our analysis below.

Any change in the steady state must involve a change in the saving-term function $s_{i,t}(\mu_{i,t}) - s_t$ or the portfolio-term function $r^{w}_{i,t+1}(\mu_{i,t}) - r^{e}_{t+1}$. As we show below, the portfolio term is relatively insensitive to parameters affecting asset returns. Hence, steady-state changes are driven primarily by the saving term. If the saving-term function stays constant, the steady state does not change. Thus, our analysis studies how model parameters affect the saving term $s_{i,t} - s_t$, *holding normalized wealth* $\mu_{i,t}$ *constant*.

We define the dynamics of the wealth distribution through wealth-share growth of fractiles (e.g., bottom 50%, top 10%). The wealth-share growth of fractile k is (time subscript omitted):

$$g^{\mu}_{k} = (s_k - s) + (r^{w}_{k} - r^{e}) + \tau_k. \tag{17}$$

This is analogous to equation (15) with one additional component, τ_k, which captures net wealth transfers into fractile k from other parts of the distribution. Following Gomez (2023), we refer to τ_k as the "between term." Quantitatively, it plays no meaningful role in our analysis.

[20] To derive equation (15), note that individual wealth evolves according to $W_{i,t+1}/W_{i,t} = (1 + s_{i,t})R^{w}_{i,t+1}$, while aggregate wealth satisfies $W_{t+1}/W_t = (1 + s_t)R^{e}_{t+1}$. Taking logs and subtracting yields the growth rate of $\mu_{i,t}$, with saving rates log-linearized.

7.1 Steady state

We start by analyzing the stochastic steady state, where the wealth distribution is constant, i.e., $g_k^\mu = 0$ for all fractiles k.[21] Substituting in equation (17) yields:

$$(\bar{s}_k - \bar{s}) + (\bar{r}_k^w - \bar{r}^e) + \bar{\tau}_k = 0, \tag{18}$$

where bars denote steady-state values. A key feature of our model is that the return on wealth increases with wealth while the active saving rate declines, as shown in Tables 2 and 5.[22] Consequently, the saving term $\bar{s}_k - \bar{s}$ decreases with wealth, while the portfolio term $\bar{r}_k^w - \bar{r}^e$ increases.

Consider a permanent decline in the time discount rate ρ. Table 7 shows the steady-state effects. The benchmark estimate sets $\rho \equiv \beta^{-1} - 1 = 4.3\%$ (column 1). In column (10), we reduce it to $\rho = 3.3\%$. The dividend yield r^d (the inverse of the price-dividend ratio) declines from 3.5% to 2.5%, a 40% increase in the price-dividend ratio. As a result, the top 10% wealth share rises slightly from 70.2% to 70.5%, while their consumption share increases more substantially, from 37.8% to 40.3%, implying a 6.6% permanent rise in their consumption. In a representative-agent model, changes in ρ affect asset prices but not consumption—a result that does not hold with heterogeneous agents.

To interpret these numbers, we explore how a permanent decline in ρ affects the three terms in equation (18). We start with the last two (portfolio term and between term), which are fairly insensitive to ρ. We then turn to the saving term and show that it drives the key results.

The portfolio term $\bar{r}_k^w - \bar{r}^e$ can be written as $\bar{r}_k^w - \bar{r}^e = \left(\bar{\lambda}_k - 1\right)\left(\bar{r}^e - \bar{r}^f\right)$, where $\bar{\lambda}_k$ denotes the equity share in fractile k's portfolio. The portfolio term comprises the equity premium $\bar{r}^e - \bar{r}^f$ and the portfolio equity share $\bar{\lambda}_k$. How does lower ρ affect these two variables?

The simulation results in Table 7 show that the equity premium and portfolio decisions are largely insensitive to permanent changes in the discount rate; compare columns (1) and (10). Surprisingly, these findings resemble frictionless models. Under complete markets, the equity premium is independent of ρ. Likewise, in the classical CRRA portfolio model of Merton (1969), optimal portfolios depend on risk aversion and the equity premium, but not on ρ. Despite market incompleteness and asset trade constraints, our model retains these insights from unconstrained asset pricing and portfolio theory. We conclude that the steady-state

[21] This definition allows for reallocation of agents across different parts of the wealth distribution. Changes in the wealth share of a given fractile reflect both within-fractile accumulation (the first two terms in equation (18)) and a between-fractile component capturing agents moving across fractiles due to idiosyncratic shocks (the "between term" $\bar{\tau}_k$).

[22] The model replicates these empirical patterns across the bottom 50%, middle 50–90%, and top 10% of the wealth distribution.

portfolio term $\bar{r}_k^w - \bar{r}^e$ is nearly unaffected by permanent changes in ρ.

The third term in equation (18) is the between term $\bar{\tau}_k$. Gomez (2023) derives this term analytically for a class of models similar to, though not identical to, ours, showing that $\bar{\tau}_k$ depends on the volatility of wealth growth. In our model, this volatility is primarily driven by exogenous idiosyncratic shocks to equity and labor income. Thus, changes in ρ should have little effect on $\bar{\tau}_k$. Indeed, columns (1) and (10) in Table 7 confirm that $\bar{\tau}_k$ remains nearly unchanged when ρ changes.

Since both the portfolio term and the between term are largely insensitive to the discount rate, the key distributional effects operate through the saving term. Therefore, the saving term will be the focus of our analysis. At the individual level, the saving-term function $s_{i,t}(\mu_{i,t}) - s_t$ depends on normalized wealth $\mu_{i,t}$. Hence, we study how the saving term adjusts, holding normalized wealth $\mu_{i,t}$ constant.

The individual active saving rate, defined in equation (16), can be decomposed as

$$s_{i,t} = \frac{Y_t}{W_t} \cdot \frac{I_{i,t}/Y_t - C_{i,t}/C_t}{\mu_{i,t}}, \tag{19}$$

where C_t, Y_t, and W_t denote aggregate consumption, output, and wealth, and market clearing implies $C_t = Y_t$. At the aggregate level, normalized consumption and wealth equal one, and aggregate labor income equals $(1-\alpha)Y_t$. Thus, $s_t = -\alpha Y_t/W_t$, so the saving term is

$$s_{i,t} - s_t = \frac{Y_t}{W_t} \cdot \left(\frac{I_{i,t}/Y_t - C_{i,t}/C_t}{\mu_{i,t}} + \alpha \right). \tag{20}$$

We analyze the effect of the discount rate on the saving term in two steps. First, we hold consumption fixed and examine the mechanical effect. Then, we allow consumption to adjust endogenously and discuss the total effect.

The first step is purely mechanical. Holding normalized wealth $\mu_{i,t}$ and consumption fixed, a decline in ρ reduces the dividend yield and consequently lowers the output–wealth ratio Y_t/W_t. Because the saving term $s_{i,t} - s_t$ decreases in wealth, a lower output–wealth ratio reduces the saving term of poorer agents and increases that of wealthier agents. Holding consumption fixed, this mechanical effect would slow wealth accumulation at the bottom relative to the top. In equilibrium, however, poorer agents reduce consumption, partially offsetting this channel. Table 7 confirms that wealth shares change modestly, while the main distributional impact appears in consumption.

To see this more formally, consider the intertemporal budget constraint of agent i:[23]

$$0 = W_{i,t} + \mathbb{E}_t \sum_{j=0}^{\infty} Q_{i,t,t+j} \left(I_{i,t+j} - C_{i,t+j} \right),$$

where $1/Q_{i,t,t+j} = R^w_{i,t+1} \cdots R^w_{i,t+j}$ is the cumulative return on wealth of agent i from period t to $t+j$. Agent i's wealth can be written as:

$$W_{i,t} = \frac{W_{i,t}}{W_t} \mathbb{E}_t \sum_{j=0}^{\infty} M_{t,t+j} \alpha Y_{t+j},$$

where W_t is aggregate wealth, αY_t is aggregate dividend income, and $M_{t,t+j}$ is the discount factor determining the equilibrium stock price. This implies that $W_{i,t}$ is a claim on a fixed share of future dividend income (equal to $W_{i,t}/W_t$).

Note that $\alpha Y_t = Y_t - I_t$, where I_t denotes aggregate labor income. Using market clearing $C_t = Y_t$, we can express the intertemporal budget constraint of agent i as:

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} Q_{i,t,t+j} \left(I_{i,t+j} - C_{i,t+j} \right) - \mathbb{E}_t \sum_{j=0}^{\infty} M_{t,t+j} \frac{W_{i,t}}{W_t} \left(I_{t+j} - C_{t+j} \right).$$

Holding $\frac{W_{i,t}}{W_t}$ constant, the net effect of lower discount rates (i.e., higher discount factors) on the intertemporal budget constraint depends on the timing of agent i's active saving. Heuristically, an agent who saves more than the aggregate at shorter horizons and dissaves more at longer horizons is worse off when discount rates fall, because higher discount factors place more weight on future negative saving.

For instance, if $Q_{i,t,t+j} \approx M_{t,t+j}$ for each $j \geq 0$, then:

$$0 \approx \mathbb{E}_t \sum_{j=0}^{\infty} M_{t,t+j} \left[\left(I_{i,t+j} - C_{i,t+j} \right) - \frac{W_{i,t}}{W_t} \left(I_{t+j} - C_{t+j} \right) \right].$$

A positive (negative) bracket implies that agent i saves more (less) than the aggregate relative to his wealth share. For $j = 0$, this is equivalent to a positive (negative) saving term $s_{i,t} - s_t$, as defined in equation (20). For $j \geq 1$, the bracketed term is a slightly different measure but conveys the same idea. Agents with positive saving terms today and negative saving terms in the future incur a negative income effect when discount rates fall, because the compounding effect on the present value of long-term negative saving terms exceeds that on short-term

[23] Iterate forward the budget constraint $W_{i,t+1}/R^w_{i,t+1} = W_{i,t} + I_{i,t} - C_{i,t}$, take expectations, and apply the transversality condition $\lim_{T \to \infty} \mathbb{E}_t W_{i,T+1} / \left(R^w_{i,t+1} \cdots R^w_{i,T+1} \right) = 0$.

positive saving terms.

Given the negative income effect on the poor and positive effect on the rich, consumption inequality adjusts: poor agents reduce consumption while rich agents increase it. The endogenous response of consumption can be illustrated using analytical results derived in the Appendix. The following proposition summarizes the effect on consumption:

Proposition 1 *Assume $\theta = 1$ (unit IES) and consider a decline of identical magnitude in the discount rate and all asset returns. Holding labor income and normalized wealth $\mu_{i,t}$ fixed, poor agents reduce consumption and rich agents increase consumption.*

The mechanism underlying this result is heterogeneity in precautionary savings. Poor agents have a stronger precautionary motive, so they are worse off when returns are lower. We can show further that, holding normalized wealth constant, agents with a stronger precautionary motive have a higher saving term (equation (20)) and are affected more negatively by lower discount rates and returns (see Appendix for details):

Proposition 2 *Assume $\theta = 1$ (unit IES) and consider a decline of identical magnitude in the discount rate and all asset returns. Holding labor income and normalized wealth $\mu_{i,t}$ fixed, the saving term is a sufficient statistic for the effect on consumption: a higher saving term implies a stronger decline in consumption.*

We now turn to the effect of a lower discount rate on the steady-state wealth distribution. As explained, the steady-state distribution changes only if the saving-term function $s_{i,t}(\mu_{i,t}) - s_t$ changes (since the portfolio term is fairly stable). The saving-term function changes for two reasons: (i) the mechanical effect operating through the aggregate output–wealth ratio Y_t/W_t; (ii) the endogenous response of consumption $C_{i,t}$ summarized in Proposition 1. If these effects cancel exactly, the saving-term function remains unchanged, and so does the steady-state wealth distribution. In this case, a lower discount rate, ρ, increases consumption inequality but leaves wealth inequality unchanged.

The simulation results in Table 7, columns (1) and (10), show that a lower discount rate changes the wealth distribution only modestly. Top wealth shares hardly change, whereas top consumption shares increase substantially. These findings suggest that the net effect on the saving-term function is relatively small at the top of the wealth distribution.

Analytical results in the Appendix confirm this interpretation:

Proposition 3 *Under the same assumptions as in Proposition 1, the change in the saving term is close to zero for very wealthy agents and ambiguous for other agents.*

In summary, the long-run distributional effects of a lower discount rate have two components. First, *holding the steady-state wealth distribution fixed*, consumption inequality increases. Second, allowing the wealth distribution to adjust to the new steady state yields an additional, albeit modest, effect. The overall impact on consumption inequality is thus largely driven by the first component.

We conclude by examining the effect of a higher growth rate g. This experiment is insightful because it keeps preferences constant, making the interpretation of changes in consumption as welfare effects straightforward. With unit elasticity of intertemporal substitution ($\theta = 1$), an increase in g leads to a proportional rise in asset returns, while the price-dividend ratio and output–wealth ratio remain unchanged. Consequently, the saving term in equation (18) is unaffected, and the steady state remains intact. Table 7 confirms this numerically: column (8) increases g from 2% to 3%, raising asset returns by one percentage point yet leaving the dividend yield and the distribution of wealth and consumption unchanged (compare columns (1) and (8)).

Next, consider $\theta = 1/2$, implying an IES of 2. Column (7) in Table 7 shows the steady state for $\theta = 1/2$ with $g = 2\%$, while column (9) reports the same θ with $g = 3\%$. A permanent change Δg shifts asset returns by approximately $\theta \Delta g$, while the effect on the dividend yield is $\Delta r^d \approx (\theta - 1)\,\Delta g$.[24] Thus, with $\theta = 1/2$, a rise in g lowers the dividend yield and the output–wealth ratio. As a result, the saving rate of the poor declines more than that of the rich, generating inequality dynamics similar to those under a lower discount rate.

Table 7 shows the effect of increasing g from 2% to 3% under $\theta = 1/2$ (columns (7) and (9)). The consumption share of the top 10% rises from 35.1% to 36.1%, while their wealth share increases from 63.2% to 64.3%. Portfolios remain nearly unchanged, the equity premium shows little variation, and the between terms remain stable, consistent with our previous analysis.

These results demonstrate that changes in asset returns have significant permanent distributional welfare effects. A one percentage point decline in the dividend yield (due to a lower discount rate) raises long-run consumption of the top 10% by 6.6%. Similarly, a 0.5 percentage point decline (from higher growth under IES = 2) increases their consumption by 2.9%. Combining these results, the semi-elasticity of the top 10% consumption with respect to the dividend yield is approximately -6.

Net borrowing versus saving motives. Are the rich better off because they are net borrowers who benefit from lower borrowing costs? No.

[24] In steady state, equity returns satisfy (to first order) $r^e = r^d + g$. Since $\Delta r^e \approx \theta \Delta g$, it follows that $\Delta r^d \approx (\theta - 1)\,\Delta g$.

To see this, consider a complete-markets environment with i.i.d. labor-income shocks and heterogeneous risk aversion, as in Barro et al. (2022). In such models, low-risk-aversion agents are net borrowers, as in our economy. Nevertheless, we show in the Appendix that under unit IES ($\theta = 1$), agents' consumption is independent of the discount rate (holding wealth and labor-income shares constant). Moreover, the saving term is identically zero for all agents, and portfolio choices are invariant to the discount rate (Merton, 1969). Hence, despite the presence of net borrowers and creditors, the discount rate does not affect consumption. This is consistent with our interpretation: the saving term is zero for all agents, so heterogeneity in saving motives, the key force in our model, is absent.

By contrast, in incomplete markets, heterogeneity in precautionary saving motives emerges endogenously. Agents with low equity exposure accumulate wealth more slowly and are pushed closer to their borrowing constraints, strengthening their precautionary motive relative to wealthier agents. As a result, declines in returns impose a negative income effect on poorer households, reducing their consumption relative to the rich. It is this heterogeneity in saving motives (rather than net borrowing positions per se) that drives the steady-state distributional effects in our model.

An additional illustration is provided in Table 7, which compares columns (1) and (8). The only difference is the growth rate, which is higher in column (8). Hence, the interest rate is higher in column (8), yet the consumption distribution is unchanged. This shows that differences in net bond positions alone cannot account for the response of consumption to changes in returns.

Heterogeneity in saving motives may also arise from life-cycle effects, absent from our model. Young agents typically hold little wealth but earn high labor income and therefore save for retirement, implying a positive active saving rate. By contrast, retired agents have little or no labor income and consume out of accumulated wealth, implying a negative active saving rate. Glover et al. (2020) and Greenwald et al. (2021) study life-cycle models and show that declines in asset returns redistribute consumption from young to old. This mechanism is consistent with our analysis, which emphasizes heterogeneity in active saving rates as the key driver of distributional effects.

Finally, our analysis in this section is conducted in an endowment economy, where output is exogenous and unaffected by household behavior. How would results change in a production economy? First, lower asset returns would increase the steady-state capital–output ratio. In our endowment framework, this corresponds to a higher price-dividend ratio (i.e., a lower dividend yield). Hence, the distributional effects described in this section would remain qualitatively unchanged.

A production economy may also feature an endogenous capital-income share, depending

on the production function. In our model, the capital-income share α is fixed. However, Table 5 (columns 6 vs. 1) shows that a higher α amplifies wealth and consumption inequality. This mechanism could become operative in a production environment in which asset returns influence the steady-state capital-income share.

7.2 $r - g - c$ and the Pareto tail

Piketty (2014) argues that wealth inequality increases with $r - g$, where r represents the return on aggregate wealth (equivalent to r^e in our model). To formalize this idea, Piketty and Zucman (2015) show that the right tail of the stationary wealth distribution approximates a Pareto distribution, with the Pareto exponent decreasing in $r - g$ (implying greater top wealth inequality as $r - g$ rises).[25]

Table 7 reports the Pareto exponent for the top 0.1% alongside $r-g$. The results contradict Piketty and Zucman (2015). When $r-g$ declines from 3.6% to 2.5% due to lower ρ, the Pareto exponent remains nearly unchanged (compare columns (1) and (10)). Similarly, when $r - g$ decreases from 3.9% to 3.4% due to higher growth (under IES = 2), the Pareto exponent shows little variation (compare columns (7) and (9)).

To explain these results, consider the law of motion of the wealth of agent i:

$$W_{i,t+1} = (W_{i,t} + I_{i,t} - C_{i,t})\, R^w_{i,t+1},$$

where $I_{i,t}$ denotes labor income. At the steady state, aggregate wealth grows at a rate $e^g - 1$. Hence, normalized wealth $\mu_{i,t} = W_{i,t}/W_t$ follows:

$$\mu_{i,t+1} = \frac{Y}{W} \cdot \frac{I_{i,t}}{Y_t} \cdot \exp\left(r^w_{i,t+1} - g\right) + \mu_{i,t} \exp\left(r^w_{i,t+1} - g - c_{i,t}\right),$$

where Y/W is fixed at the steady state, $r^w_{i,t+1} \equiv \log R^w_{i,t+1}$, and $c_{i,t} \equiv -\log\left(1 - C_{i,t}/W_{i,t}\right)$.

The classical result of Kesten (1973) states that if $I_{i,t}/Y_t$, $c_{i,t}$, and $r^w_{i,t+1}$ are i.i.d., then $\mu_{i,t}$ converges to a stationary distribution with a right Pareto tail. The Pareto exponent decreases with $r - g - c$, the mean of $r^w_{i,t+1} - g - c_{i,t}$.[26] For zero labor income ($I_{i,t} = 0$), Gomez (2022) derives the Pareto exponent analytically in continuous time.

Benhabib et al. (2011) extend Kesten (1973) to Markov processes under regularity conditions, building on de Saporta (2005). While our model does not strictly satisfy their condi-

[25] We define the Pareto tail as the top 0.1% and the "Pareto exponent" as the parameter ζ in the Pareto counter CDF $\Pr(\tilde{x} \geq x) = kx^{-\zeta}$.

[26] Formally, the Pareto exponent ζ satisfies $\mathbb{E}e^{\phi\zeta} = e^{-\zeta(r-g-c)}$, where $\phi \equiv \left(r^w_{i,t+1} - g - c_{i,t}\right) - (r - g - c)$ is a zero-mean i.i.d. variable; see Gabaix (2009). Thus, ζ decreases in $r - g - c$, holding ϕ's distribution constant.

tions, their result provides a useful approximation for the right tail of the wealth distribution. Along the right tail, $r^w_{i,t+1} - g - c_{i,t}$ approximates a Markov process.[27] By Benhabib et al. (2011) and de Saporta (2005), the stationary right-tail distribution is approximately Pareto with an exponent decreasing in $r - g - c$, where c is the consumption-to-wealth ratio at the Pareto tail. In other words, top wealth inequality increases with $r - g - c$. If the top consumption-to-wealth ratio is fixed, we recover the $r-g$ result of Piketty and Zucman (2015). However, in our model, this ratio is endogenous, making the $r - g - c$ formula applicable.

Consider a one percentage point decline in ρ (columns (1) vs. (10) in Table 7). While $r - g$ falls by roughly one percentage point, $r - g - c$ remains nearly unchanged, and the Pareto exponent shows little variation, consistent with our $r - g - c$ formula. Similarly, a one percentage point increase in g (under IES > 1) reduces $r - g$ by 0.5 percentage points (columns (7) vs. (9)), yet $r - g - c$ and the Pareto exponent remain stable. Thus, wealth inequality **within** the Pareto tail appears largely insensitive to asset returns.

However, these results do not imply that **overall** inequality is unresponsive to asset returns. On the contrary, inequality **between** the Pareto tail and the rest of the distribution shifts significantly, especially in terms of consumption inequality. In this range, asset returns influence inequality mainly through differential effects on the active saving rate. Notably, these effects run counter to Piketty's (2014) $r - g$ formula: lower expected returns actually increase inequality.

We conclude that changes in asset returns do not necessarily alter wealth inequality **within** the Pareto tail, as adjustments in the consumption-to-wealth ratio can keep $r - g - c$ relatively stable. Furthermore, inequality **within** the Pareto tail is often an inadequate measure of **overall** inequality. Asset returns can significantly impact **overall** inequality without affecting the Pareto tail, primarily through the active saving rate rather than the $r - g - c$ formula.

7.3 Transitional dynamics

Our next step is to analyze how asset returns affect the cross-sectional distribution along a transition path between steady states. We show that wealth and consumption inequality increase with the price–dividend ratio during the transition. The key driver is unanticipated capital gains, which disproportionately benefit the rich, who hold larger equity shares.

Figure 4 shows the impulse response to an unanticipated one percentage point decline in the discount rate. The drop in ρ leads to a sudden increase in the price-dividend ratio,

[27] At the right tail, the consumption-to-wealth ratio $c_{i,t}$ converges to a constant under $\theta = 1$ or follows a Markov process under $\theta \neq 1$, depending on risk aversion. The log return $r^w_{i,t+1}$ is determined by $\gamma_{i,t}$, which follows a Markov process.

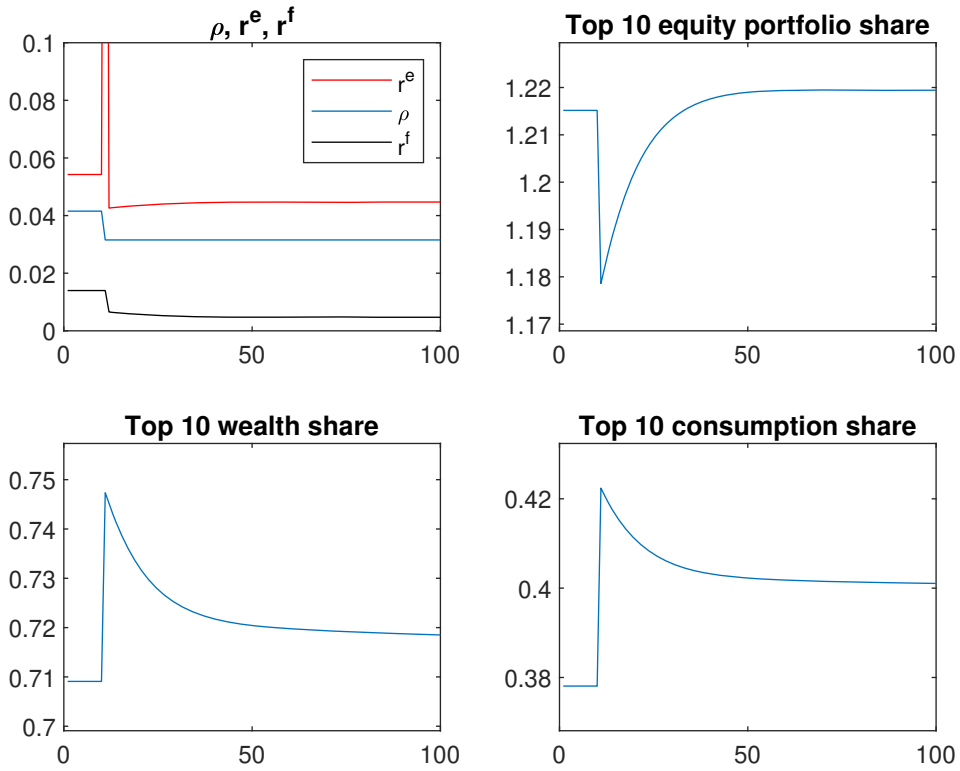


Figure 4: Impulse response of the top 10%: Permanent change in ρ (IES = 1)

Note: The figure presents the impulse response of the top 10% to a permanent decline in ρ.

generating capital gains for stockholders, who are disproportionately wealthy. As a result, wealth and consumption shares of the top 10% rise. This effect on equity prices stems from their longer maturity compared to bonds.

Figure 4 also illustrates how a temporarily higher r^e raises top wealth shares along the transition path, aligning with Piketty's (2014) explanation for rising wealth inequality in the late 20th century. However, this holds only during the transition, when capital gains exceed initial expectations. Once the economy reaches the new steady state, r^e falls below its initial level, yet wealth and consumption inequality remain elevated. Thus, the correlation between asset returns and top wealth shares is positive along a transition path but negative at the steady state.

The left panel of Figure 5 shows the transitional dynamics of the capital-income share of the top 10%, where capital income includes dividend income (but excludes capital gains) and interest income. The figure shows a substantial rise in the capital-income share of the top 10% that persists in the long run, suggesting that variations in asset returns are not merely

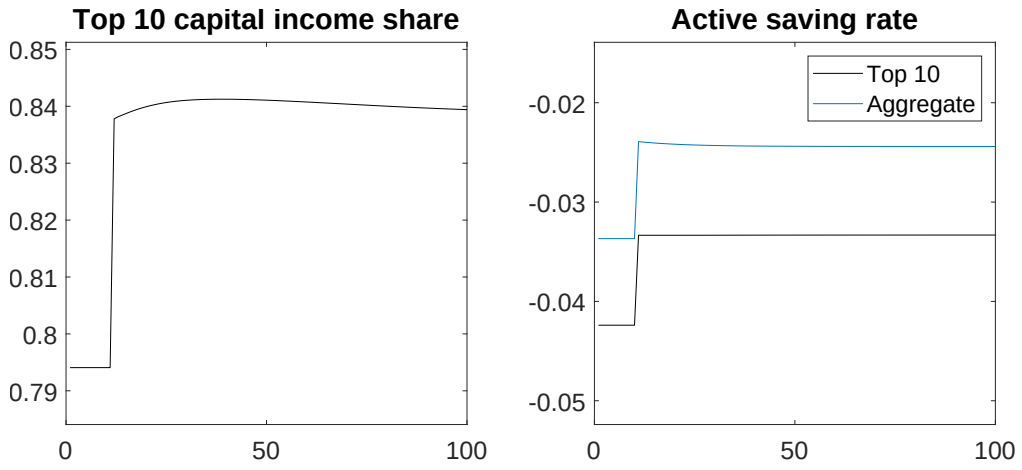


Figure 5: Impulse response of the top 10%: Permanent change in ρ (IES = 1)

Note: Impulse response of the top 10% to a permanent decline in ρ. Capital income share excludes capital gains. Active saving rates shown for the top 10% and the aggregate economy.

passive effects of asset-price revaluations but generate real changes in capital income with tangible welfare effects. These findings align with Irie (2023), who shows that the rise in top wealth shares over recent decades has been accompanied by a proportional increase in their capital-income share.

The right panel of Figure 5 presents the active saving rates of the top 10% and of the aggregate economy along the transition path. The two series track each other closely, indicating that the saving term of the top 10% (defined as the difference between the two) remains largely unchanged. Thus, the mechanical force that would increase the saving term of the top 10% through a lower output–wealth ratio, as implied by equation (20), is nearly fully offset by higher consumption and wealth shares.

The transitional dynamics may differ in a production economy without capital adjustment costs. In such an environment, Tobin's q is fixed and capital gains are zero, so a change in the discount factor would generate a much smoother transition in asset prices. However, this contradicts the data: changes in expected returns are typically accompanied by sizable movements in price-dividend ratios. Introducing capital-adjustment costs resolves this tension. With properly calibrated adjustment costs, a fall in discount rates raises Tobin's q and generates capital gains, thereby reproducing—at least qualitatively—the transitional dynamics described in this section.

This analysis highlights two distinct channels through which asset returns influence wealth distribution. At the steady state, the saving term in equation (17) dominates, whereas along

a transition path, the portfolio term prevails.[28]

These results rely on two key properties of the model. First, the return on wealth increases with wealth. Second, the active saving rate declines with wealth. Both properties are supported by the data and arise endogenously from portfolio choices rather than being imposed ex ante.

In the next subsection, we conduct a similar experiment in a model where the key mechanism is heterogeneous discount factors, as in Krusell and Smith (1998). We show that the effects of asset returns on inequality are reversed, because the model generates opposite correlations between wealth and returns and between wealth and active saving rates.

7.4 Heterogeneous discount factors

Much of the literature has explored heterogeneous discount factors as a driver of wealth inequality (Krusell and Smith, 1998). In Section 6, we showed that heterogeneity in discount factors can reproduce observed wealth inequality but violates the data on two dimensions: it generates a negative correlation between wealth and returns, and consequently a positive correlation between wealth and active saving rates, whereas the data show opposite signs (Table 8). In this section, we show that the distributional effects of asset returns in the heterogeneous-discount-factor model are also reversed, due to misestimated correlations between returns and active saving rates and wealth. We conclude that the source of wealth inequality matters for welfare analysis.

The details of the heterogeneous-discount-factor model are described in Section 6. Figure 6 presents the impulse response to a permanent decline in discount rates, with $\Delta \log \beta^{\text{low}} = \Delta \log \beta^{\text{high}} = .01$.[29] The distributional effects contrast sharply with the heterogeneous-risk-aversion model in Figure 4. On impact, top wealth and consumption shares decline in Figure 6 but rise in Figure 4. This stems from differing portfolio allocations: in the heterogeneous-discount-factor model (Table 8), the poor bear more stock-market risk than the rich, thereby earning higher returns as equity prices increase.

In the long run, the heterogeneous-discount-factor model converges to lower consumption inequality but higher wealth inequality, whereas in the heterogeneous-risk-aversion model, both increase.[30] Crucially, the models predict opposite distributional effects of asset returns. In the heterogeneous-discount-factor model (Figure 6), the rich consume less when returns fall, while in the heterogeneous-risk-aversion model (Figure 4), they consume more. This difference

[28] In the working-paper version (Fernández-Villaverde and Levintal, 2024), we show that this channel accounts for a significant portion of U.S. wealth inequality dynamics.

[29] Under complete markets, this would affect only asset prices, leaving consumption plans unchanged.

[30] Here, the decline in rich agents' consumption is so strong that they accumulate a larger wealth share in the new steady state than in the initial one.

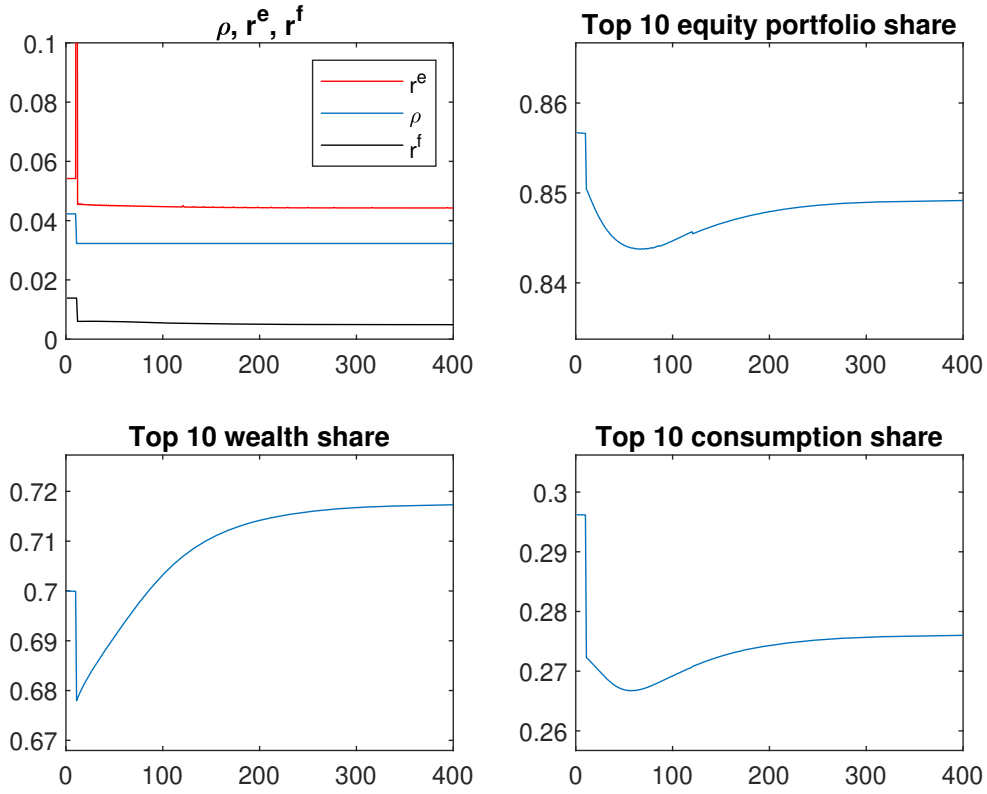


Figure 6: β-shock model: Impulse response to a permanent change in ρ (IES = 1)

Notes: The upper-left plot presents the population average of ρ.

arises because, in the heterogeneous-discount-factor model, the rich have a higher active saving rate than the poor (contrary to the data), causing lower returns to shift consumption from the rich to the poor.

In summary, the correlation between wealth and returns is crucial for determining the distributional effects of asset returns. When this correlation is positive (as in the data), the active saving rate declines with wealth, making lower returns (higher asset prices) beneficial for the rich in both the short and the long run. Conversely, when the wealth-return correlation is negative, these effects reverse.

8 Conclusions

We have examined the distributional effects of asset returns and found that lower expected returns increase long-run inequality. To reconcile this finding with Piketty's (2014) $r - g$ formula, we derived an extended version, $r - g - c$, which applies when the consumption-

to-wealth ratio c is endogenous. Crucially, asset returns influence steady-state inequality primarily through their differential impact on the active saving rate rather than through $r - g - c$.

Our findings highlight portfolio decisions as a key determinant of the distributional effects of asset returns. Models that fail to capture the positive correlation between wealth and returns produce incorrect distributional effects in both the short and the long run. Incorporating endogenous portfolio choices in heterogeneous-agent models poses significant computational challenges, but given their importance, addressing them is worthwhile. We hope the methodological contributions presented here will facilitate future research on heterogeneous portfolios and inequality.

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Online Appendix A: Analytical Results

This appendix derives the analytical results used in Section 7.1. Consider an optimization problem identical to that of agent i, except the agent can trade human capital—but only with identical agents. Let $L_{i,t}$ denote human capital units purchased by agent i in period t (including his endowment), and let $P^L_{i,t}$ be the human capital price in this market (index i indicates that only identical agents participate). The symmetric equilibrium is $L_{i,t} = 1$. Thus, this new optimization problem is equivalent to the original one (henceforth, the "equivalent problem").

To solve the equivalent problem, let $I_{i,t} = \left(1 + \epsilon^L_{i,t}\right)(1-\alpha)\, Y_t$ denote labor income per unit of human capital in period t. Total "cash at hand" of agent i, including human capital, is:

$$A_{i,t} = W_{i,t} + L_{i,t-1}\left(P^L_{i,t} + I_{i,t}\right), \tag{A-1}$$

where financial assets $W_{i,t}$ are defined in (12). The new budget constraint is:

$$C_{i,t} + P_t E_{i,t} + Q_t B_{i,t} + P^L_{i,t} L_{i,t} = A_{i,t}. \tag{A-2}$$

The agent maximizes (1) subject to (A-2) and (3)–(5). The additional control variable is $L_{i,t}$, with optimality condition:

$$\mathbb{E}_{i,t}\xi_{i,t+1}\frac{P^L_{i,t+1} + I_{i,t+1}}{P^L_{i,t}} = 1. \tag{A-3}$$

Let $r^L_{i,t} = I_{i,t}/P^L_{i,t}$ denote the "dividend yield" of human capital.

Since utility (1) is homothetic and constraints (A-2) and (3)–(5) are homogeneous, all decisions of agent i in period t are proportional to $A_{i,t}$. Following Coen-Pirani (2005), we normalize all variables by $A_{i,t}$ and differentiate the normalized version of (1) with respect to normalized consumption $C_{i,t}/A_{i,t}$. Optimal consumption under $\theta = 1$ is:

$$C_{i,t} = (1-\beta)\, A_{i,t}. \tag{A-4}$$

Substituting (A-1) with equilibrium $L_{i,t-1} = 1$, dividing by Y_t, and rearranging:

$$\frac{C_{i,t}}{C_t} = (1-\beta)\left(\frac{W_{i,t}}{W_t}\frac{W_t}{Y_t} + (1-\alpha)\left(\frac{P^L_{i,t}}{I_{i,t}} + 1\right)\frac{I_{i,t}}{I_t}\right),$$

where C_t denotes aggregate consumption (equal to aggregate output) and $I_t = (1-\alpha)\, Y_t$ denotes aggregate labor income. Substituting aggregate financial wealth from equation (13)

and using the dividend yields of equity ($r_t^d = \alpha Y_t/P_t$) and human capital ($r_{i,t}^L = I_{i,t}/P_{i,t}^L$):

$$\frac{C_{i,t}}{C_t} = \alpha \frac{\rho}{1+\rho} \frac{1+r_t^d}{r_t^d} \frac{W_{i,t}}{W_t} + (1-\alpha) \frac{\rho}{1+\rho} \frac{1+r_{i,t}^L}{r_{i,t}^L} \frac{I_{i,t}}{I_t}, \tag{A-5}$$

The active saving rate of agent i is:

$$s_{i,t} = \frac{I_{i,t} - C_{i,t}}{W_{i,t}} = \frac{Y_t}{W_t} \frac{(1-\alpha)\frac{I_{i,t}}{I_t} - C_{i,t}/C_t}{W_{i,t}/W_t} = \frac{1}{\alpha} \frac{r_t^d}{1+r_t^d} \frac{(1-\alpha)\frac{I_{i,t}}{I_t} - C_{i,t}/C_t}{W_{i,t}/W_t},$$

where the third equality substitutes (13) and the dividend yield. The aggregate active saving rate is:

$$s_t = -\frac{r_t^d}{1+r_t^d}.$$

Thus, the saving term of agent i is:

$$s_{i,t} - s_t = \frac{r_t^d}{1+r_t^d} \left(\frac{1}{\alpha} \frac{(1-\alpha)\frac{I_{i,t}}{I_t} - C_{i,t}/C_t}{W_{i,t}/W_t} + 1 \right).$$

Substituting equation (A-5) and rearranging:

$$s_{i,t} - s_t = \left(\frac{r_t^d}{1+r_t^d} - \frac{\rho}{1+\rho} \right) + \frac{1-\alpha}{\alpha} \frac{r_t^d}{1+r_t^d} \left(1 - \frac{\rho}{1+\rho} \frac{1+r_{i,t}^L}{r_{i,t}^L} \right) \frac{I_{i,t}/I_t}{W_{i,t}/W_t}. \tag{A-6}$$

First, we show that under complete markets $r_t^d = \rho$. Complete markets imply that $\xi_{i,t+1}$ is identical across agents, so we can drop the i subscript and aggregate (A-3):

$$\mathbb{E}_t \xi_{t+1} \frac{P_{t+1}^L + (1-\alpha) Y_t}{P_t^L} = 1,$$

where $P_t^L = \int P_{i,t}^L di$. The dividend yield of aggregate human capital equals that of aggregate equity. Aggregating (A-1) yields $\int A_{i,t} di = Y_t + Y_t/r_t^d$. Aggregating (A-4) and substituting $\int C_{i,t} di = Y_t$ yields $r_t^d = \rho$ under complete markets.

Incomplete markets increase aggregate precautionary savings, pushing asset prices up and r_t^d down. Hence, under incomplete markets $r_t^d < \rho$.

To prove Proposition 1, consider a decline in the discount rate with returns falling by a similar magnitude: $\Delta \frac{\rho}{1+\rho} = \Delta \frac{r_t^d}{1+r_t^d} = \Delta \frac{r_{i,t}^L}{1+r_{i,t}^L} < 0$. Hold the agent's type and normalized wealth fixed and consider the effect on consumption, defined by equation (A-5). First, under incomplete markets $r_t^d < \rho$, so the first term on the right-hand side of equation (A-5) rises, with the effect increasing in wealth. Second, poor agents have a stronger precautionary saving

motive, lacking sufficient wealth to self-insure. As a result, the dividend yield on human capital ($r^L_{i,t}$) is higher for poor agents, reflecting a larger risk premium. The effect on the second term is therefore negative for the very poor (where $r^L_{i,t} > \rho$) and increasing in wealth. The total effect on equation (A-5) is thus negative for the very poor (with $W_{i,t}/W_t = 0$ and $r^L_{i,t} > \rho$) and increasing in wealth.

To prove Proposition 2, consider an agent anticipating low future labor income. This agent has a stronger motive to save for consumption smoothing, so human capital value is low and $r^L_{i,t}$ is high. Similarly, an agent facing higher idiosyncratic labor-income risk has a stronger precautionary motive, lower human capital value, and higher $r^L_{i,t}$. Thus, stronger saving demand (whether from precautionary or consumption-smoothing motives) is associated with higher $r^L_{i,t}$. Holding the agent's type and normalized wealth fixed, equation (A-6) implies that the saving term rises with $r^L_{i,t}$.

Moreover, the saving term serves as a sufficient statistic for the welfare impact of asset returns, holding type and normalized wealth constant. A higher saving term implies a higher $r^L_{i,t}$ through equation (A-6). Then equation (A-5) implies that if $\Delta\frac{\rho}{1+\rho} = \Delta\frac{r^d_t}{1+r^d_t} = \Delta\frac{r^L_{i,t}}{1+r^L_{i,t}} < 0$, an agent with higher $r^L_{i,t}$ is worse off than one with lower $r^L_{i,t}$, other things equal.

To prove Proposition 3, note that the first term on the right-hand side of equation (A-6) is unaffected by the change in ρ and r^d_t, since $\frac{r^d_t}{1+r^d_t}$ and $\frac{\rho}{1+\rho}$ fall by equal magnitudes. For the very rich, the second term is close to zero, so the saving term hardly changes. For other agents, the effect is ambiguous.

Online Appendix B: Computational Procedure

This appendix details our computational procedure. Following Winberry (2018), we globally approximate individual variables over a grid, capture the cross-sectional distribution with a finite set of moments, and locally approximate aggregate variables. We then apply the Taylor projection method of Levintal (2018) to approximate local effects of aggregate variables. Since the Taylor projection can be applied at any state-space point, we begin with an arbitrary initial distribution and simulate forward, iteratively solving until the distribution converges.

The appendix is structured as follows. Section A detrends the model and simplifies notation. Section B approximates individual decisions over a grid of wealth nodes. Section C captures the cross-sectional distribution. Section D derives market-clearing conditions. Section E introduces an auxiliary variable to forecast the future price-dividend ratio, used in Section F to express wealth distribution dynamics recursively. Section G summarizes model conditions in recursive form. Section H details the solution and simulation.

A The detrended model

Define the variables $y_{t+1} = \frac{Y_{t+1}}{Y_t}$, $p_t = \frac{P_t}{\alpha Y_t}$, $q_t = Q_t$, $u_{i,t} = \frac{U_{i,t}}{Y_t}$, $c_{i,t} = \frac{C_{i,t}}{Y_t}$, $e_{i,t} = E_{i,t}$, and $b_{i,t} = \frac{B_{i,t}}{Y_t}$. Then, we can rewrite the Epstein-Zin utility function (1) as follows:

$$u_{i,t}^{1-\theta} = (1-\beta_{i,t})\, c_{i,t}^{1-\theta} + \beta_{i,t} z_{i,t}^{1-\theta}, \tag{B-1}$$

where $z_{i,t}$ is the (detrended) Epstein-Zin certainty-equivalence variable:

$$1 = \mathbb{E}_t \left(\frac{u_{i,t+1} y_{t+1}}{z_{i,t}} \right)^{1-\gamma_{i,t}}, \tag{B-2}$$

and $\beta_{i,t}$ and $\gamma_{i,t}$ are assumed to be time-varying.

The stochastic discount factor for this general case is:

$$\xi_{i,t+1} = \beta_{i,t} \frac{1-\beta_{i,t+1}}{1-\beta_{i,t}} \frac{c_{i,t}^{\theta}}{c_{i,t+1}^{\theta}} \frac{u_{i,t+1}^{\theta-\gamma_{i,t}}}{z_{i,t}^{\theta-\gamma_{i,t}}} y_{t+1}^{-\gamma_{i,t}}. \tag{B-3}$$

Define the (detrended) wealth (including labor income):

$$w_{i,t} = \left(1+\epsilon_{i,t}^{L}\right)(1-\alpha) + \alpha\,(1+p_t)\left(1+\epsilon_{i,t}^{E}\right) e_{i,t-1} + \frac{b_{i,t-1}}{y_t}. \tag{B-4}$$

Note that $w_{i,t}$ is equal to $\frac{\left(1+\epsilon_{i,t}^{L}\right)(1-\alpha)Y_t+W_{i,t}}{Y_t}$ in the main text.

The portfolio share of equity is given by:

$$\lambda_{i,t} = \frac{\alpha p_t e_{i,t}}{w_{i,t} - c_{i,t}}. \tag{B-5}$$

Thus, the budget constraint (2) can be rewritten as:

$$c_{i,t} + \alpha p_t e_{i,t} + q_t b_{i,t} = w_{i,t}, \tag{B-6}$$

and the future returns on equity and bonds are:

$$R_{i,t+1}^{e} = \left(1 + \epsilon_{i,t+1}^{E}\right) \frac{1 + p_{t+1}}{p_t} y_{t+1} \tag{B-7}$$

$$R_{t+1}^{f} = 1/q_t. \tag{B-8}$$

In addition, define the total portfolio return at the maximum portfolio share $\bar{\lambda}$ by:

$$R_{i,t+1}^{B} = R_{t+1}^{f} + \bar{\lambda}\left(R_{i,t+1}^{e} - R_{t+1}^{f}\right). \tag{B-9}$$

The optimality conditions are given by equations (7) and (8) in the main text. Equation (7) becomes:

$$\begin{cases} 0 = \mathbb{E}_t\left(\xi_{i,t+1} R_{t+1}^{f} - 1\right) & \left| \begin{array}{c} w_{i,t} - c_{i,t} > 0\,,\ \lambda_{i,t} < \bar{\lambda} \end{array} \right. \\ 0 = \mathbb{E}_t\left(\xi_{i,t+1} R_{i,t+1}^{B} - 1\right) & \left| \begin{array}{c} w_{i,t} - c_{i,t} > 0\,,\ \lambda_{i,t} = \bar{\lambda} \end{array} \right. \\ c_{i,t} = w_{i,t} & \left| \begin{array}{c} \text{else} \end{array} \right. \end{cases}, \tag{B-10}$$

while equation (8) becomes:

$$\begin{cases} 0 = \mathbb{E}_t\left(\xi_{i,t+1} R_{i,t+1}^{e} - 1\right) & \left| \begin{array}{c} w_{i,t} - c_{i,t} > 0\,,\ 0 < \lambda_{i,t} < \bar{\lambda} \end{array} \right. \\ e_{i,t} = 0 & \left| \begin{array}{c} w_{i,t} - c_{i,t} > 0\,,\ 1 > \mathbb{E}_t \xi_{i,t+1} R_{i,t+1}^{e} \end{array} \right. \\ \alpha p_t e_{i,t} = \bar{\lambda}\left(w_{i,t} - c_{i,t}\right) & \left| \begin{array}{c} w_{i,t} - c_{i,t} > 0\,,\ 1 > \mathbb{E}_t \xi_{i,t+1} R_{t+1}^{f} \end{array} \right. \\ e_{i,t} = 0 & \left| \begin{array}{c} \text{else} \end{array} \right. \end{cases}. \tag{B-11}$$

Finally, the market-clearing conditions (9)-(11) are now:

$$\int c_{i,t} di = 1 \tag{B-12}$$

$$\int e_{i,t} di = 1, \tag{B-13}$$

where the bond market clears by Walras' law.

Individual wealth $w_{i,t}$ follows the law of motion in equation (B-4). The idiosyncratic shocks $\gamma_{i,t}$, $\beta_{i,t}$, and $\epsilon^L_{i,t}$ follow Markov processes. The idiosyncratic shock $\epsilon^E_{i,t}$ is an i.i.d. discrete shock, and y_t is an aggregate i.i.d. discrete shock.

B Approximating individual decisions over a grid

Next, we approximate individual decisions over a grid and derive optimality conditions at the grid nodes. Following Winberry (2018), with some differences in implementation, agents are classified into types based on idiosyncratic Markov shocks γ, β, and ϵ^L, with total types denoted I. Henceforth, subscript $i = 1, \ldots, I$ refers to type in period t, while $j = 1, \ldots, I$ refers to type in period $t+1$.[31] The transition probability from i to j is π_{ij}. We drop time subscripts and use $'$ for next-period values.

Optimality conditions are subject to two constraints. The "portfolio constraint" requires $0 \leq \lambda \leq \bar{\lambda}$, where λ is the equity portfolio share defined in (B-5). The "net-worth constraint" ensures $w - c \geq 0$.

The portfolio constraint is computationally challenging since the portfolio decision may or may not hit its bounds. We approximate the portfolio choice under three "portfolio states": (i) Unconstrained; (ii) Constrained at the lower bound; (iii) Constrained at the upper bound. The portfolio state is indexed by superscript S, taking values U (Unconstrained), E (Equity constraint), and B (Bond constraint). The actual state is endogenous. If the portfolio choice under U respects the constraint bounds, the endogenous state is U. If it violates the lower bound, the state is E; otherwise, B.

B.1 The grid

Fix a grid of N nodes over $[0, \bar{x}]$, where $\bar{x} > 0$ is a fixed parameter. Denote the vector of nodes by:

$$\mathbf{x} = (x_1, \ldots, x_N), \tag{B-14}$$

where $x_1 = 0$ and $x_N = \bar{x}$. Nodes are indexed by $n = 1, \ldots, N$. For instance, c_{in} denotes consumption of agent type i at node n.

For each agent type $i = 1, \ldots, I$ and portfolio state $S \in \{U, E, B\}$, define vectors of

[31] Note the change in notation: in the main text and Section A, subscript i indexes individuals over the unit interval. Here and in subsequent sections, it indexes agent types, determined by risk aversion, discount rate, and labor income.

individual variables over the grid:

$$\mathbf{C}_i^S = \left(C_{i1}^S, \ldots, C_{iN}^S\right) \tag{B-15}$$

$$\mathbf{E}_i^S = \left(E_{i1}^S, \ldots, E_{iN}^S\right) \tag{B-16}$$

$$\mathbf{Z}_i^S = \left(Z_{i1}^S, \ldots, Z_{iN}^S\right), \tag{B-17}$$

where C_{in}^S, E_{in}^S, and Z_{in}^S are endogenous variables determined in period t by agent type i at node n in portfolio state S.[32] Since $S \in \{U, E, B\}$ and i takes I values, the total number of variables in equations (B-15)–(B-17) is $3 \times 3 \times I \times N$.

B.2 The wealth cutoff of the net-worth constraint

We now derive the wealth cutoff at which the net-worth constraint binds (for each agent type i and portfolio state S). The cutoff is endogenous and used below to determine whether the constraint binds.

Let c_{i1}^S and w_{i1}^S denote consumption and wealth at the first node ($n = 1$) for agent type i in portfolio state S. Define consumption at the first node by $c_{i1}^S = \left(w_{i1}^S + h\right) \exp\left(C_{i1}^S\right)$, where $h = (1 - \alpha)\, p > 0$ measures per capita human capital. This ensures numerically that $c_{i1}^S > 0$ (provided h is sufficiently larger than w_{i1}^S).

Define the first node ($n = 1$) as the wealth cutoff: the net-worth constraint is slack for wealth above w_{i1}^S and binds otherwise. Since $w_{i1}^S = c_{i1}^S$:

$$w_{i1}^S = \frac{h \exp\left(C_{i1}^S\right)}{1 - \exp\left(C_{i1}^S\right)}. \tag{B-18}$$

B.3 The interpolation algorithm

We use the following algorithm to interpolate (or extrapolate) individual decisions for a given wealth level:

Algorithm 1 *For wealth w of agent type i, we compute consumption $c_i(w)$, Epstein-Zin variable $z_i(w)$, and equity decision $e_i(w)$. We first compute these for a given portfolio state $S \in \{U, E, B\}$, then derive the portfolio state endogenously.*

Given w, compute the distance from the wealth cutoff w_{i1}^S defined in (B-18):

$$x_i^S(w) = \log(w + h) - \log\left(w_{i1}^S + h\right).$$

[32] In practice, we eliminate vectors $\mathbf{E}_i^S$ for $S = E, B$, since they follow analytically from the corner solution. For notational clarity, we treat them here as unknown endogenous variables.

Use vectors (B-14)–(B-17) *to interpolate:*

$$C_i^S(w) = interpolate\left(x_i^S(w), \boldsymbol{x}, \boldsymbol{C}_i^S\right)$$
$$E_i^S(w) = interpolate\left(x_i^S(w), \boldsymbol{x}, \boldsymbol{E}_i^S\right)$$
$$Z_i^S(w) = interpolate\left(x_i^S(w), \boldsymbol{x}, \boldsymbol{Z}_i^S\right).$$

The notation $y = interpolate(x, \boldsymbol{x}, \boldsymbol{y})$ *denotes the value of* y *given* x*, interpolated through node and value vectors* $\boldsymbol{x}$ *and* $\boldsymbol{y}$*. We use piecewise linear interpolation.*

For agent type i *in portfolio state* S*, compute:*

$$c_i^S(w) = \begin{cases} (w+h)\exp\left(C_i^S(w)\right) & x_i^S(w) > 0 \\ w & else \end{cases} \tag{B-19}$$

$$e_i^S(w) = \frac{1}{\alpha p} \times \begin{cases} -h + (w+h)\exp\left(E_i^S(w)\right) & x_i^S(w) > 0 \\ -h + \left(w_{i1}^S + h\right)\exp\left(E_{i1}^S\right) & else \end{cases} \tag{B-20}$$

$$z_i^S(w) = \begin{cases} (w+h)\exp\left(Z_i^S(w)\right) & x_i^S(w) > 0 \\ \left(w_{i1}^S + h\right)\exp\left(Z_{i1}^S\right) & else \end{cases}. \tag{B-21}$$

When $x_i^S(w) \leq 0$*, the net-worth constraint binds, so consumption equals wealth as in equation* (B-19)*. Additionally, when this constraint binds, the equity decision and Epstein-Zin variable become independent of* w*, taking values at the first grid node as in equations* (B-20)–(B-21)*.*

Finally, derive the portfolio state and the corresponding endogenous variables, depending on the portfolio decision in the unconstrained state $(S = U)$*:*

$$c_i(w), e_i(w), z_i(w) = \begin{cases} c_i^U(w), e_i^U(w), z_i^U(w) & 0 \leq \alpha p e_i^U(w) \leq \bar{\lambda}\left(w - c_i^U(w)\right) \\ c_i^E(w), e_i^E(w), z_i^E(w) & \alpha p e_i^U(w) < 0 \\ c_i^B(w), e_i^B(w), z_i^B(w) & else \end{cases}. \tag{B-22}$$

B.4 The optimality conditions

For each agent type i in portfolio state S, define wealth at node n by:

$$w_{in}^S = -h + \left(w_{i1}^S + h\right)\exp(x_n),$$

where w_{i1}^S is the wealth cutoff defined in equation (B-18).

Use Algorithm 1 to compute $c_i^S\left(w_{in}^S\right)$, $e_i^S\left(w_{in}^S\right)$, and $z_i^S\left(w_{in}^S\right)$, through equations (B-19)-(B-21). To simplify notation, denote these variables c_{in}^S, e_{in}^S, and z_{in}^S, respectively. The bond

decision comes from the budget constraint (B-6):

$$b_{in}^{S} = \frac{w_{in}^{S} - c_{in}^{S} - \alpha p e_{in}^{S}}{q}.$$

Next period wealth follows from equation (B-4):

$$w_{ijn}^{\prime S} = \left(1 + \epsilon_{j}^{\prime L}\right)(1-\alpha) + \alpha\left(1+p^{\prime}\right)\left(1+\epsilon^{\prime E}\right)e_{in}^{S} + \frac{b_{in}^{S}}{y^{\prime}}. \tag{B-23}$$

Here, $w_{ijn}^{\prime S}$ denotes the future wealth of an agent at node n that is currently in portfolio state S and switches from type i to type j, conditional on the future realizations of the stock-return idiosyncratic shock $\epsilon^{\prime E}$, the aggregate shock $y^{\prime}$, and the price/dividend ratio $p^{\prime}$.

Use Algorithm 1 to compute $c_j\left(w_{ijn}^{\prime S}\right)$, $e_j\left(w_{ijn}^{\prime S}\right)$, and $z_j\left(w_{ijn}^{\prime S}\right)$ through equation (B-22). To simplify notation, denote these variables $c_{ijn}^{\prime S}$, $e_{ijn}^{\prime S}$ and $z_{ijn}^{\prime S}$, respectively. Evaluate equations (B-1) and (B-3) as follows:

$$\begin{aligned} u_{ijn}^{\prime S} &= \left[(1-\beta_j)\left(c_{ijn}^{\prime S}\right)^{1-\theta} + \beta_j\left(z_{ijn}^{\prime S}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \\ \log \xi_{ijn}^{\prime S} &= \log\beta_i + \log\left(1-\beta_j\right) - \log\left(1-\beta_i\right) + \theta\log c_{in}^{S} - \theta\log c_{ijn}^{\prime S} \\ &\quad + \left(\theta - \gamma_i\right)\left[\log u_{ijn}^{\prime S} - \log z_{in}^{S}\right] - \gamma_i \log y^{\prime}. \end{aligned}$$

Evaluate equation (B-2) for each agent type $i = 1, \ldots, I$, node $n = 1, \ldots, N$, and portfolio states $S = \{U, E, B\}$:

$$0 = \mathbb{E}\sum_{\epsilon^E}\pi^{E}\sum_{j}\pi_{ij}\left[\left(\frac{u_{ijn}^{\prime S} y^{\prime}}{z_{in}^{S}}\right)^{1-\gamma_i} - 1\right]. \tag{B-24}$$

The net-worth constraint is (weakly) slack over the grid. Hence, optimality conditions (B-10)-(B-11) hold in the following form:

$$\begin{cases} 0 = \mathbb{E}\sum_{\epsilon^E}\pi^{E}\sum_{j}\pi_{ij}\left[\xi_{ijn}^{\prime S} R^{\prime f} - 1\right] & S = \{U, E\} \\ 0 = \mathbb{E}\sum_{\epsilon^E}\pi^{E}\sum_{j}\pi_{ij}\left[\xi_{ijn}^{\prime S} R^{\prime B} - 1\right] & S = B \end{cases} \tag{B-25}$$

$$\begin{cases} 0 = \mathbb{E}\sum_{\epsilon^E}\pi^{E}\sum_{j}\pi_{ij}\left[\xi_{ijn}^{\prime S} R^{\prime e} - 1\right] & S = U \\ e_{in}^{S} = 0 & S = E \\ \alpha p e_{in}^{S} = \bar{\lambda}\left(w_{in}^{S} - c_{in}^{S}\right) & S = B. \end{cases} \tag{B-26}$$

The returns $R^{\prime e}$, $R^{\prime f}$, and $R^{\prime B}$ are defined in equations (B-7)-(B-9). The expectation

operator $\mathbb{E}$ is taken over the distribution of the aggregate shock y'. The first summation runs over the values of the idiosyncratic stock-return shock ϵ^E with probabilities π^E, while the second summation runs over future types j with transition probabilities π_{ij}.

C The cross-sectional distribution

Given wealth w, define a normalized wealth measure μ:

$$\mu = \frac{w + h}{1 + \alpha p + h}. \tag{B-27}$$

Here, wealth is augmented by per capita human capital h to ensure μ remains strictly positive. In the main text, μ is defined solely in terms of financial wealth (excluding labor income and human capital) for analytical clarity. However, this definition is computationally less convenient due to potential negative or zero values.

Let $\mathcal{F}_i(\mu)$ denote the CDF of μ across type i agents, where the mean is:

$$\boldsymbol{\mu}_i = \int \mu d\mathcal{F}_i(\mu). \tag{B-28}$$

Let θ_i denote the type i population share. Then, at the aggregate:

$$\sum_i \theta_i \boldsymbol{\mu}_i = 1. \tag{B-29}$$

Define a measure of wealth relative to type i mean $\zeta_i = \log \mu - \log \boldsymbol{\mu}_i$ and the cumulative wealth-share function relative to type i wealth $\Phi_i(\zeta) = \frac{1}{\boldsymbol{\mu}_i} \int_0^{\boldsymbol{\mu}_i \exp(\zeta)} \mu d\mathcal{F}_i(\mu)$.

We approximate the distribution by discretizing the cumulative wealth-share function $\Phi_i(\zeta)$. Specifically, discretize Φ by K fixed points over the interval $[0, 1-\tau]$ (for some $\tau > 0$), denoted $\boldsymbol{\Phi} = (\Phi_1, \ldots, \Phi_K)$. Find K values:

$$\boldsymbol{\zeta}_i = (\zeta_{i1}, \ldots, \zeta_{iK}), \tag{B-30}$$

that satisfy $\Phi_i(\zeta_{ik}) = \Phi_k \ \forall k = 1, \ldots, K$. The vectors $\boldsymbol{\zeta}_i$ and $\boldsymbol{\Phi}$ provide a discrete approximation of $\Phi_i(\zeta)$, where $\boldsymbol{\zeta}_i$ changes over time and $\boldsymbol{\Phi}$ is held fixed.

The complete distribution is represented by $\boldsymbol{\mu}_i$ and $\boldsymbol{\zeta}_i$ for all types $i = 1, \ldots, I$. We find that the information in $\boldsymbol{\mu}_i$ is sufficient to approximate the local dynamics of asset prices successfully. Hence, we include $\boldsymbol{\mu}_i$ in the vector of state variables. Since all $\boldsymbol{\mu}_i$'s aggregate to

1 through condition (B-29), we drop $\boldsymbol{\mu}_1$ and define the aggregate state variables as follows:

$$\mathbf{X} = [\boldsymbol{\mu}_2, \ldots, \boldsymbol{\mu}_I]. \tag{B-31}$$

The information in $\boldsymbol{\zeta}_i$ captures higher moments, which are crucial for accurate aggregation but less relevant for local dynamics. Therefore, we treat $\boldsymbol{\zeta}_i$ as locally fixed, significantly simplifying computations.

D Market-clearing conditions

Consider a certain cumulative wealth share $\Phi_q \in [0, 1]$. Compute the associated wealth measure $\zeta_{iq} = interpolate\left(\Phi_q, \boldsymbol{\Phi}, \boldsymbol{\zeta}_i\right)$ and use ζ_{iq} and equation (B-27) to compute the corresponding wealth $w_{iq} = \boldsymbol{\mu}_i\left(1 + \alpha p + h\right)\exp\left(\zeta_{iq}\right) - h$. Then, use Algorithm 1 to compute $c_i\left(w_{iq}\right)$ and $e_i\left(w_{iq}\right)$ through equation (B-22).

The market-clearing conditions (B-12)-(B-13) are obtained by integrating with respect to wealth shares:

$$1 = \sum_i \left\{ \left(1 + \alpha p + h\right)\theta_i \boldsymbol{\mu}_i \left(\int_0^1 \frac{c_i\left(w_{iq}\right)}{w_{iq} + h} d\Phi_q \right) \right\}$$
$$1 = \sum_i \left\{ \left(1 + \alpha p + h\right)\theta_i \boldsymbol{\mu}_i \left(\int_0^1 \frac{e_i\left(w_{iq}\right)}{w_{iq} + h} d\Phi_q \right) \right\}.$$

At high wealth levels, individual decisions scale proportionally with wealth. We use this property to approximate the integral over the top tail. For each agent type i, the top tail comprises the top τ wealth share of type i. Let $\bar{w}_i$ be an arbitrary wealth level within tail i, and apply Algorithm 1 to compute $c_i\left(\bar{w}\right)$ and $e_i\left(\bar{w}\right)$. These determine the consumption-to-wealth and equity-to-wealth ratios at the top tail. This method performs best when the equity market-clearing condition integrates over shares in augmented aggregate wealth $\left(\alpha p e_i\left(w_{iq}\right) + h\right) / \left(\alpha p + h\right)$ rather than financial wealth alone $e_i\left(w_{iq}\right)$. We adjust the equity market-clearing condition as follows:

$$1 = \sum_i \left\{ \left(1 + \alpha p + h\right)\theta_i \boldsymbol{\mu}_i \left(\int_0^1 \frac{\left(\alpha p e_i\left(w_{iq}\right) + h\right) / \left(\alpha p + h\right)}{w_{iq} + h} d\Phi_q \right) \right\}.$$

A quadrature approximates the integral over the region below the top tail. Define a set of Q quadrature nodes $\{\Phi_1, \ldots, \Phi_Q\} \in [0, 1 - \tau]$. The corresponding weights are $\omega_1, \ldots, \omega_Q$.

Approximate the market-clearing conditions as follows:

$$1 = \sum_i \left\{ (1 + \alpha p + h)\, \theta_i \boldsymbol{\mu}_i \left(\sum_{q=1}^{Q} \frac{c_i(w_{iq})}{w_{iq} + h} \omega_q + \frac{c_i(\bar{w}_i)}{\bar{w}_i + h} \tau \right) \right\} \tag{B-32}$$

$$1 = \sum_i \left\{ \frac{1 + \alpha p + h}{\alpha p + h} \theta_i \boldsymbol{\mu}_i \left(\sum_{q=1}^{Q} \frac{\alpha p e_i(w_{iq}) + h}{w_{iq} + h} \omega_q + \frac{\alpha p e_i(\bar{w}_i) + h}{\bar{w}_i + h} \tau \right) \right\}. \tag{B-33}$$

E Future price/dividend ratio

Let $\mathbf{H}$ denote the law of motion of the state vector $\mathbf{X}$:

$$\mathbf{X}' = \mathbf{H}(\mathbf{X}, y'), \tag{B-34}$$

where y' is the only aggregate shock in the model. Let the function $\mathbf{p}$ denote the solution of the price/dividend ratio $p = \mathbf{p}(\mathbf{X})$.

Let $p'^f(y')$ be the future price/dividend ratio forecasted in period t conditional on the future realization of y'. Namely, $p'^f(y')$ is defined by:

$$p'^f(y') = \mathbf{p}(\mathbf{X}') = \mathbf{p}(\mathbf{H}(\mathbf{X}, y')). \tag{B-35}$$

Note that $p'^f(y')$ is determined in period $t+1$, but it is known in period t, because it is conditional on the future realization of y', which is the only unknown future variable. This will enable us below to define the evolution of wealth as a function of current state variables and future aggregate shocks.

F Law of motion of the moments

This section computes the law of motion of $\boldsymbol{\mu}_i$, defined in equation (B-28). The normalized wealth of agent type i with wealth w_{iq} is $\mu_{iq} = \frac{w_{iq}+h}{1+\alpha p+h}$. Compute the bond decision through the budget constraint (B-6): $b_{iq} = \frac{w_{iq} - c_{iq} - \alpha p e_{iq}}{q}$. Here, c_{iq} and e_{iq} denote $c_i(w_{iq})$ and $e_i(w_{iq})$ from Section D, respectively. Future wealth is derived from equation (B-4):

$$w'_{ijq} = \left(1 + \epsilon'^L_j\right)(1 - \alpha) + \alpha \left(1 + p'^f(y')\right)\left(1 + \epsilon'^E\right) e_{iq} + \frac{b_{iq}}{y'}. \tag{B-36}$$

Here, w'_{ijq} represents the future wealth of agent type i with current wealth w_{iq}, transitioning to type j in period $t+1$, conditional on future realizations of y' and ϵ'^E. We use the future price/dividend ratio $p'^f(y')$ from equation (B-35), which depends on the current state $\mathbf{X}$

and future realization y'. Thus, equation (B-36) expresses future wealth in terms of known period-t variables and future shocks, enabling a recursive definition of wealth distribution dynamics.[33]

Define $h' = (1-\alpha)\, p'^f\,(y')$ and use equation (B-27) to define future normalized wealth:

$$\mu'_{ijq} = \frac{w'_{ijq} + h'}{1 + \alpha p'^f\,(y') + h'},$$

where h' ensures that μ'_{ijq} depends on variables known in period t and future shocks only.

It follows that the mean future normalized wealth of agent type j conditional on future realization y' is:

$$\boldsymbol{\mu}'_j\,(y') = \frac{1}{\theta_j} \sum_{\epsilon^E} \pi^E \sum_i \theta_i \boldsymbol{\mu}_i \pi_{ij} \int_0^1 \frac{\mu'_{ijq}}{\mu_{iq}} d\Phi_q.$$

Here, $\theta_i \boldsymbol{\mu}_i$ is the current share of type i agents in aggregate wealth, and $\theta_i \boldsymbol{\mu}_i \pi_{ij} d\Phi_q$ is the current wealth share associated with type i agents that switch to type j, whose wealth-share growth is $\frac{\mu'_{ijq}}{\mu_{iq}}$.

It follows that $\theta_i \boldsymbol{\mu}_i \pi_{ij} \int_0^1 \frac{\mu'_{ijq}}{\mu_{iq}} d\Phi_q$ is the future wealth share of agents that switch from i to j. Integrating across all types $i = 1, \ldots, I$ and future realizations of the idiosyncratic stock-return shock ϵ^E yields the future wealth share of type j agents (conditional on y'), namely $\theta_j \boldsymbol{\mu}'_j$. Multiplying by $\frac{1}{\theta_j}$ yields $\boldsymbol{\mu}'_j$.

To approximate the integral accurately, we follow the procedure used in equations (B-32)-(B-33):

$$\boldsymbol{\mu}'_j\,(y') = \frac{1}{\theta_j} \sum_{\epsilon^E} \pi^E \sum_i \theta_i \boldsymbol{\mu}_i \pi_{ij} \left\{ \sum_{q-1}^{Q} \frac{\mu'_{ijq}}{\mu_{iq}} \omega_q + \frac{\mu'_{ij}\,(\bar{w}_i)}{\mu_i\,(\bar{w}_i)} \tau \right\}. \tag{B-37}$$

G Summary of the model conditions

The state vector is defined in condition (B-31). The control variables include the variables over the grid, defined in equations (B-15)-(B-17), the price/dividend ratio p, and the price of a risk-free bond q.

We define additional control variables as follows. Suppose that the distribution of y' is discretized over Z states, where $y'_1, \ldots, y'_Z$ denote the possible realizations of y'. Define a new variable denoted p'^f_z, which is the future price/dividend ratio, defined in equation (B-35),

[33] In contrast, equation (B-23) defines future wealth in terms of p', an endogenous variable determined in $t+1$, preventing a recursive formulation.

conditional on realization y'_z:

$$p'^f_z = p'^f\left(y'_z\right). \tag{B-38}$$

Similarly, let $\boldsymbol{\mu}'_{iz}$ denote future mean normalized wealth conditional on realization y'_z determined by equation (B-37):

$$\boldsymbol{\mu}'_{iz} = \boldsymbol{\mu}'_i\left(y'_z\right). \tag{B-39}$$

Importantly, p'^f_z and $\boldsymbol{\mu}'_{iz}$ depend only on variables known in period t. Hence, we include them in the vector of control variables together with the other variables that are determined in period t: $\mathbf{Y} = \left[\left\{\mathbf{C}^S_i, \mathbf{E}^S_i, \mathbf{Z}^S_i, p'^f_z, \boldsymbol{\mu}'_{iz} | i = 1, \ldots, I, S = \{U, E, B\}, z = 1, \ldots, Z\right\}, p, q\right]$. The solution of $\mathbf{Y}$ is denoted:

$$\mathbf{Y} = \mathbf{G}\left(\mathbf{X}\right). \tag{B-40}$$

We can define the future mean normalized wealth as $\boldsymbol{\mu}'_i = \sum_z \mathbb{1}\left(y' = y'_z\right)\boldsymbol{\mu}'_{iz}$, where $\boldsymbol{\mu}'_{iz}$ is defined by equation (B-39) and known in period t. It follows that the future vector of aggregate state variables (B-31) can be written as a function of variables known in period t and the future aggregate shock, denoted:

$$\mathbf{X}' = \Theta\left(\mathbf{X}, \mathbf{Y}, y'\right), \tag{B-41}$$

where Θ is a known function. Substituting equation (B-40) yields the function $\mathbf{H}$ in equation (B-34):

$$\mathbf{H}\left(\mathbf{X}, y'\right) = \Theta\left(\mathbf{X}, \mathbf{G}\left(\mathbf{X}\right), y'\right).$$

The solution is identified by the optimality conditions (B-24)-(B-26), the market-clearing conditions (B-32)-(B-33), and conditions (B-38)-(B-39). Together, these conditions can be written as:

$$0 = \mathbb{E}\mathbf{F}\left(\mathbf{Y}', \mathbf{Y}, \mathbf{X}', \mathbf{X}, y'\right), \tag{B-42}$$

where $\mathbb{E}$ denotes expectations with respect to the distribution of the aggregate shock y'.[34] Equations (B-40), (B-41), and (B-42) complete the definition of the model.

In extended versions of the model, we allow the parameters ρ and g to change over time according to the following processes:

$$\rho' = \rho + \epsilon'_\rho$$
$$g' = g + \epsilon'_g,$$

[34] To define conditions (B-38)-(B-39) in the form of equation (B-42), we can use indicator functions that select that relevant realization y'_m.

where ϵ'_ρ and ϵ'_g are the shocks. To facilitate that, we include ρ and g in state vector $\mathbf{X}$. The corresponding processes are included in function $\mathbf{H}$ with the additional shocks ϵ'_ρ and ϵ'_g.

H Solving and simulating the model

The model solution (B-40) provides the effects of the cross-sectional distribution, approximated by the moments in $\mathbf{X}$, on the control variables $\mathbf{Y}$. We approximate this solution by the Taylor projection method developed by Levintal (2018). First, postulate a linear solution:[35]

$$\mathbf{Y} = \mathbf{X}\mathbf{b},$$

and substitute in equations (B-41) and (B-42) to get:

$$0 = \mathbb{E}\mathbf{F}\left(\Theta\left(\mathbf{X}, \mathbf{X}\mathbf{b}, y'\right)\mathbf{b}, \mathbf{X}\mathbf{b}, \Theta\left(\mathbf{X}, \mathbf{X}\mathbf{b}, y'\right), \mathbf{X}, y'\right). \tag{B-43}$$

Then, differentiate (B-43) with respect to $\mathbf{X}$:

$$0 = \mathbb{E}\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\left(\Theta\left(\mathbf{X}, \mathbf{X}\mathbf{b}, y'\right)\mathbf{b}, \mathbf{X}\mathbf{b}, \Theta\left(\mathbf{X}, \mathbf{X}\mathbf{b}, y'\right), \mathbf{X}, y'\right). \tag{B-44}$$

Notice that this notation is a shorthand for the full Jacobian of the composite equilibrium system with respect to $\mathbf{X}$.

We evaluate equations (B-43)–(B-44) at a given state $\mathbf{X}_0$ and find $\mathbf{b}$ satisfying these conditions. As shown in Levintal (2018), the solution is accurate locally around $\mathbf{X}_0$.

To simulate the model, we start with an arbitrary distribution of normalized wealth μ, represented by CDF $\mathcal{F}_i(\mu)$ for all i, discretized over a dense grid with 10^4 nodes. Let $\boldsymbol{\mathcal{F}}_t$ denote the simulated distribution in period t. Starting at $t = 0$, we compute first moments $\boldsymbol{\mu}_i$ $(\forall i)$ from equation (B-28) and store them in aggregate state $\mathbf{X}_t$ defined in equation (B-31). Additionally, we compute higher moments $\boldsymbol{\zeta}_i$ $(\forall i)$ from equation (B-30) and treat them as fixed parameters in equation (B-42). As discussed in the main text and Section C, local dynamics of higher moments are not essential for forecasting asset prices; forecasts based only on first moments (in $\mathbf{X}_t$) are sufficiently accurate. However, higher moments remain crucial for accurate aggregation, so they are included as fixed parameters. This significantly reduces computational costs.

We solve the model at $\mathbf{X}_t$ to obtain $\mathbf{Y}_t = \mathbf{G}(\mathbf{X}_t)$. Using this solution, we compute asset prices and individual decisions across the wealth distribution, deriving the next-period

[35] The method can be implemented at higher orders, but this is not necessary for our analysis. Also, $\mathbf{X}$ might include a constant if needed.

Table H.1: Approximation Errors (log10)

	Steady State	Simulation		Impulse response	
		max	mean	max	mean
Consumption Euler conditions					
bottom 50%	-4.1	-3.9	-4.1	-3.8	-4.1
50-90%	-3.5	-3.4	-3.5	-3.4	-3.5
top 10%	-3.4	-3.1	-3.4	-3.1	-3.3
top 1%	-3.6	-3.4	-3.6	-3.3	-3.6
top 0.1%	-3.5	-3.5	-3.6	-3.4	-3.6
Portfolio Euler conditions					
bottom 50%	-16.0	-5.0	-6.7	-5.0	-5.7
50-90%	-5.7	-4.0	-5.1	-3.9	-4.7
top 10%	-5.2	-3.3	-4.4	-3.1	-4.0
top 1%	-5.1	-3.2	-4.3	-3.0	-3.9
top 0.1%	-5.0	-3.2	-4.3	-3.0	-3.9
Market-clearing conditions					
Equity market	-4.3	-2.0	-2.6	-2.0	-2.5
Bond market	-4.7	-2.1	-2.7	-2.1	-2.6
Goods market	-3.3	-2.3	-3.0	-2.2	-2.8

Unit-free errors (log10 units) for consumption and portfolio Euler conditions and market-clearing conditions. Errors are reported at the stochastic steady state, during a simulation with business-cycle shocks but no realized disasters, and along an impulse response to a disaster shock.

distribution $\boldsymbol{\mathcal{F}}_{t+1}$. Repeating this process, we compute $\mathbf{X}_{t+1}$ and evaluate conditions (B-42) under the previous solution. If residuals exceed tolerance, we resolve at the new state. The simulation continues until the distribution converges.

Table H.1 presents approximation errors for Euler and market-clearing conditions across the simulated distribution $\boldsymbol{\mathcal{F}}_t$, reported for three regions of the state space: (1) the stochastic steady state, (2) a simulation with business-cycle shocks but no realized disasters, and (3) an impulse response to a disaster shock.