All or Nothing: Indivisible Goods, Relative Wealth Concerns, and Attitudes to Risk

Bingcun Dai* Pavel Savor† Mungo Wilson‡

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Abstract

We study an endowment economy with heterogeneous agents and two complementary consumption goods, one of which is indivisible. Because it affects their ability to acquire indivisible goods, agents care about relative, not just absolute, wealth. Although agents have standard log-CES preferences, they avoid small risks while preferring large ones. Poorer agents exhibit a preference for lottery-like assets with negative expected returns and also invest a smaller share of wealth in assets with positive expected returns. In consequence, richer agents have a higher expected return on wealth and inequality is expected to increase.

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*Said Business School, Oxford University. Email: bingcun.dai@sbs.ox.ac.uk.
†Driehaus College of Business, DePaul University. Email: psavor@depaul.edu.
‡Said Business School, Oxford University, and Oxford-Man Institute. Email: mungo.wilson@sbs.ox.ac.uk.

I. Introduction

Why do economic agents care about inequality? In a standard endowment economy with homogeneous preferences, agents are concerned with the level of their own endowments and prices, but not with relative endowments. Since prices depend on aggregate endowments, it immediately follows that agents do not care about inequality in itself, as we show below. Yet recent research, notably the work of Piketty (2014) and Saez (2021), suggests that relative endowments represent an important issue for many agents, holding their own endowment levels and prices constant.[1] The failure of mainstream economic theory to account for inequality concerns is perhaps one of its more perplexing aspects to non-economists. Moreover, if agents exhibit strong concerns about relative wealth, some predictions of standard models will not be correct, such as the prediction that rising living standards lead people to consume more leisure (Keynes, 1930). And if, as we argue in this study, relative wealth concerns themselves induce increases in inequality, then understanding why agents care about inequality is crucial to reducing it and assessing which policy interventions are likely to be effective. For example, an implication of the theory of inequality concerns proposed in this paper is that relatively poorer agents effectively end up insuring richer agents against aggregate risk, making poorer households less able to bear aggregate risk. Consequently, policies that promote retirement investments in risky assets may actually increase inequality, as they are less likely to appeal to poorer agents.

Motivated by the lack of inequality concerns in standard models, many studies simply assume that agents care about relative wealth and proceed from there. This approach, however, does not by itself explain why inequality matters. Furthermore, the assumptions about the ways in which agents care will determine the consequences for behavior, often with very different ultimate implications.[2]

[1] Saez (2021) argues that the tremendous increase in the provision of public services by governments of developed economies in the past century represents strong evidence that voters dislike inequality.

[2] For example, Roussanov (2010) uses a utility function which is concave in an agent's own wealth and linear in the ratio of her own wealth to average wealth. Such agents will underdiversify their investments relative to a standard model, but not gamble. By contrast, Friedman and Savage (1948) famously assume that agents' preferences are concave in their own wealth but convex in status, giving rise to a desire to gamble. Other papers make assumptions about keeping up with the Joneses (e.g., DeMarzo et al., 2008), which result in herd-like investment behavior and therefore bubbles, but not gambling in the sense of Friedman and Savage. Pástor

In this paper, we seek to characterize a minimal departure from the standard endowment economy required to generate inequality (relative wealth) concerns. We then explore properties of equilibria of such a minimally different economy to see which predictions of standard models no longer hold and which novel predictions arise. One crucial, usually implicit, assumption of standard models is that all goods are divisible. Indivisibility, though, is a key feature of many important goods in the real world.[3] For instance, it appears that most prosperous households do not wish to share their primary accommodation, so housing is often treated as indivisible, even if it varies in quality (Piazzesi et al., 2007). Matching relationships such as marriages or schools also seem difficult or impossible to share without removing their utility (Cole et al., 1992; Garratt and Marshall, 1994), and medical treatment programs are usually of limited usefulness unless substantially completed. Furthermore, these important goods are complementary to everyday consumption goods such as food or energy. From our perspective, the key aspect of these goods is that the agent with the highest valuation secures the additional available unit of the good, and that agent is usually the richer one. Even when wealth is not necessarily what determines who wins the additional unit, as in the case of spousal or school matching, there exists a strong tournament-like element.

Allowing for the existence of at least one complementary indivisible good in a closed endowment economy with multiple goods is a sufficient condition for some agents with standard preferences to exhibit strong relative wealth concerns. These concerns are cardinal, not ordinal: the greater their relative wealth, the greater the agents' welfare, in contrast to agents who only care about their rank. Indivisible complements exert particularly strong effects in our model when they are scarce: as the available quantity of such goods increases, inequality concerns abate much faster than prices decline. We believe this is a plausible prediction, as people seem to care more about inequality of access to elite educational institutions or cutting-edge medical

and Veronesi (2021) assume an agent's utility is concave in an agent's own consumption and decreasing in the variance of cross-sectional consumption shares of all agents (a measure increasing in inequality), resulting in better-off groups of agents eventually preferring to reduce inequality at the expense of personal consumption.

[3]The assumption that a good is indivisible can be regarded as an approximation for a good whose marginal utility is much higher when it does not have to be shared with other agents. Viewed in this light, the class of goods which are (effectively) indivisible is much larger than just strictly indivisible goods.

treatments (both indivisible and scarce) than inequality of access to prestige cars or club memberships (arguably indivisible but not so scarce). Relative to previous related work, a key part of our approach is to relax the assumption of one-to-one matching, under which every agent can obtain one unit in equilibrium, albeit of varying quality. In our model, some agents can obtain no units while others obtain more than one.

Once inequality concerns are generated by indivisible goods, they give rise to various interesting phenomena such as a desire to gamble. This is because indivisible goods, among other mechanisms, introduce a convexity into indirect utility. In a two-agent economy, for example, agents have indirect utility of wealth of the Friedman-Savage form. These preferences are locally concave, except at equality of wealth, but convex over a wider range. As noted by Friedman and Savage (1948), Becker et al. (2005), and Chetty and Szeidl (2007), gambling increases expected utility for both agents provided the poorer agent can become the richer agent if she wins. In contrast to other papers, we study gambles in which agents have equal bargaining power. As a result, these gambles are not in general actuarially fair, with the initially richer agent receiving better terms. Consequently, we predict that in economies in which inequality concerns arise from the existence of scarce indivisible complements, inequality will increase over time and at an accelerating rate.

In the example which we study in the paper, richer agents pay poorer agents a price for the indivisible good which depends positively on the poorer agent's wealth. Thus, richer agents have relative wealth concerns; holding their own wealth constant, they prefer the poorer agent's wealth to be lower. The poorer agent's welfare is a leveraged function of her own wealth, while the richer agent's welfare is a develeraged function of the poorer agent's wealth. In the presence of aggregate risk, the richer agent's wealth is lower in bad times, making her worse off, but so is the poorer agent's wealth, making the richer agent better off than she would have been otherwise. For poorer agents there is no such offsetting effect. In consequence, richer agents are naturally hedged against common risks, which reduce the wealth of all agents, while the opposite is true for poorer agents. Richer agents are then more willing to bear aggregate risk,

which in equilibrium has a positive expected return, under precisely the same conditions and for the same reasons that 'background' labor income risk can increase an agent's risk aversion (Eeckhoudt and Kimball, 1992).

This greater participation in positive expected return risk further increases expected growth in inequality. These results suggest that an important application of this model may be in finance, since the same mechanism that generates inequality concerns also generates lower participation by the relatively poor agents. They are also consistent with household finance research such as Bach et al. (2020) and Fagereng et al. (2020), who find that richer households tend to earn higher returns on their investments, in part because they take greater risks. Our findings can explain Piketty's (2014) claim that high returns on capital are what causes inequality to increase.

Finally, our model can generate mass participation in lotteries by rational agents with standard preferences. In an extension with one rich agent and many identical poor agents, the poor agents participate in lotteries which are designed to produce one rich winner. This rich winner can then extract a higher price for the indivisible goods, paid to the mass of poor agents.

What measures could mitigate these inequality concerns and thus, paradoxically, reduce the expected increase in inequality? One answer is to produce more units of the indivisible good, but this is likely to be inefficient and often impossible. Another may be to produce different types of scarce indivisible goods, or goods of the same type but different qualities. One intervention that is unlikely to be effective on its own is tax-funded redistribution, since in our model much of the inequality is 'manufactured' in the sense of Rosen (1997) through gambling. While taking resources from the richer and transferring to the poorer will reduce inequality, this reduction will only be temporary, since poorer agents will want to use their increased wealth to enter into gambles which deliver for them a negative expected return and even, in the extreme, restore the pre-status quo prior to the redistribution.

Indivisible goods are not the only mechanism for generating endogenous relative wealth concerns. Such concerns may potentially also arise in an otherwise standard economy with

heterogeneous preferences or in a more complex production economy. Nonetheless, we view indivisible goods as a particularly straightforward and plausible source of these concerns in even very simple settings. Our approach is complementary to work in psychology and political economy that studies the origins of relative wealth concerns, including ingrained aversion to inequality and worries that inequality will undermine democracy and quality of governance.

Ng (1965) shows how complementary indivisible goods give rise to indirect utility of the Friedman-Savage type once wealth becomes large enough to buy an additional unit. Crossley et al. (2016) argue that Ng's insight can show that credit-constrained agents gamble in the presence of financial market frictions to achieve consumption goals, as in Herskowitz (2021) and as theorized in Hartley and Farrell (2002). These studies treat gambling as a second-best source of finance in the presence of financial market frictions such as liquidity constraints and do not consider the equilibrium. Our contribution is to show how, in equilibrium, competition for these complementary goods gives rise to relative wealth concerns, and how these relative wealth concerns alter behavior relative to a standard model with divisible goods, even in frictionless markets.

The next section explains how agents compete for goods in our model and how relative wealth concerns arise in the presence of an indivisible good. Section 3 describes gambling in a setting without aggregate risk, first in bilateral gambles between two agents and then in lotteries among many poor agents. Section 4 details how aggregate risk changes investment choices, and Section 5 concludes.

II. Indivisible Goods and Relative Wealth

In this section, we describe a trading equilibrium in an endowment economy with two goods. Our analysis extends to settings with an arbitrary number of agents, two complementary goods, of which one is divisible (the numeraire) and one indivisible, and arbitrary aggregate endowments of both goods. However, our results are restricted to endowments in which agents with higher endowments of the divisible good also have weakly higher endowments of the indivisible good.

We make this assumption so we can identify which agents are richer without knowing the price of the indivisible good. The more general case of arbitrary endowments we leave for future research.

The other restrictive assumption we make is that prices are established through an ascending price auction, with all agents non-strategic price takers. When there are few agents (our base case has three), this usually means that buyers obtain most or all of the surplus at low levels of inequality.[4] Our idea is that there are fewer rich than poor agents in most relevant economic settings, and a relatively simple way to capture that fact without having to explicitly model numbers of each type is to assume an auction mechanism in which richer agents (who are usually buyers) capture most of the surplus because poorer people have to compete against each other. Of course, there are situations where there are more rich would-be buyers than poorer sellers (e.g., a sale of a rare painting), but in our view they are less common.

Our aim is to show how each agent's indirect utility depends not only on her own initial endowment but also on the distribution of wealth across other agents in the economy. Having provided the general result, we illustrate it with more specific examples.

By contrast, in an otherwise identical economy in which both goods are divisible, agents' indirect utilities depend only on their own and aggregate endowments of the two goods, not on how the endowments are distributed across agents. Before setting out the results for an economy with indivisible goods, we thus first explain why inequality does not matter in an economy with only divisible goods.

A. Why inequality is irrelevant with divisible goods

Trivially, in an endowment economy with a single good, where more is better, no agent's welfare can be altered by changing any other agent's endowment, holding the first agent's endowment constant. All agents simply consume their own endowments, indivisible or not. In a two-good economy in which agents have identical preferences, agents care about the levels of each good

[4] Assuming the opposite, that prices are established by descending auction, changes some but not all of our main results, usually by reversing the positions of the relatively poor and rich.

in their own endowment and about the relative price of the two goods.[5] This relative price depends on the ratio of the aggregate endowment of each good. An agent whose own relative endowment is the same as the aggregate relative endowment cannot be made better or worse off by changing the distribution of the aggregate endowment across agents, holding her own endowments constant.

To make this concrete, consider an economy with three agents $i \in \{1, 2, 3\}$ and two divisible goods C and H. Each agent is endowed with $(C_i^{\mathrm{e}}, H_i^{\mathrm{e}})$, with aggregate endowments $\Omega_C = \sum_i C_i^{\mathrm{e}}$ and $\Omega_H = \sum_i H_i^{\mathrm{e}}$ fixed. Preferences take the log-CES form

$$u(C_i, H_i) = \log\left(C_i^{\rho} + H_i^{\rho}\right)^{1/\rho}, \quad \rho < 0,$$

so that C and H are complements.

When both goods are divisible, the competitive equilibrium depends only on the aggregate endowments (Ω_C, Ω_H). The relative price of H in terms of C is determined by the aggregate scarcity ratio

$$q = \left(\frac{\Omega_C}{\Omega_H}\right)^{1-\rho}.$$

Given q, each agent chooses her optimal bundle. Her welfare then depends only on her own endowment $(C_i^{\mathrm{e}}, H_i^{\mathrm{e}})$ together with the aggregate endowments, but not on how the remainder of Ω_C and Ω_H is distributed across other agents.

For example, suppose we fix agent 1's endowment and vary the distribution of C between agents 2 and 3, keeping Ω_C constant. Inequality between agents may rise or fall, but aggregate endowments remain the same. As a result, the equilibrium relative price q is unchanged, and so is agent 1's welfare. In this sense, with divisible goods inequality in endowments has no effect on individual welfare. As we will see in the next subsection, this is no longer true if one of the goods is indivisible: in that case the distribution of endowments directly affects welfare.

[5] Agents with heterogeneous preferences can exhibit relative wealth concerns, and such concerns could also arise in a production economy, even when all goods are divisible.

B. Model setup

Let there be N agents indexed by $i = 1, 2, \ldots, N$. The economy contains two goods: one divisible good C and an indivisible good H. Each agent must consume at least one unit of H: this base unit is not tradable and reflects a structural requirement. It is not counted in agent i's endowment and can be interpreted as a normalization of her utility function. Agents can acquire additional units of H through market exchange.

Each agent i is endowed with $C_i^{\mathrm{e}} \in \mathbb{R}_+$ units of the divisible good and $H_i^{\mathrm{e}} \in \mathbb{R}_+$ of claims on the tradable portion of the indivisible good, denominated in units of H at the prevailing market price, which enable them to purchase an equivalent number of units of the physical good H. In the case of fractional ownership, for example two agents owning a claim to a half of a unit each, an auctioneer sells the underlying unit. Each agent then receives a share of the proceeds from the sale, allocated in proportion to their ownership stake equal to one-half in the present case. As we are ruling out strategic behavior, agents will agree to sell their claim in exchange for proceeds proportional to their stake. The total supply of tradable indivisible goods is given by $H^{\mathrm{supply}} \in \mathbb{Z}_+$.[6] We make the following restrictive assumption about the distribution of the endowments:

Assumption 1 (**Wealth–Endowment Monotonicity**)**.**
For all i and j, if $C_i^{\mathrm{e}} > C_j^{\mathrm{e}}$ then $H_i^{\mathrm{e}} \geq H_j^{\mathrm{e}}$.

Assumption 1 requires that if agent i has more divisible wealth than agent j, then she has at least as large an endowment of the indivisible good as well. While certainly not general, the assumption enables us to say with assurance which agent is relatively richer. A counterexample would be relatively poor, but still wealthy, owners of large endowments of housing selling off their assets to newly rich entrepreneurs, the topic of many early twentieth century novels!

We treat good H as indivisible in the sense that $u(C, H) = u(C, \mathrm{floor}(H))$, where $\mathrm{floor}(H)$ is the largest integer less than H. The idea is that having to consume only part of a unit or share a unit with another essentially renders the unit valueless to the consumer.

[6] $\mathbb{R}_+ = [0, \infty)$ is the set of nonnegative real numbers and $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$ is the set of nonnegative integers.

Let H_i denote the number of additional (tradable) units of H that agent i consumes, so that her total consumption of good H is $\tilde{H}_i = 1 + H_i$. Let the price of one unit of tradable H be denoted $q > 0$. Since the good is homogeneous, all units trade at the same price. Agent i's final consumption of the divisible good is denoted $C_i \in \mathbb{R}_+$, and is determined residually from her budget constraint:

$$C_i + qH_i \leq C_i^{\mathrm{e}} + qH_i^{\mathrm{e}}. \tag{1}$$

Each agent i has a utility function of CES form, with complementarity between C and H:

$$u_i\left(C_i, \tilde{H}_i\right) = \log\left(C_i^{\rho} + \tilde{H}_i^{\rho}\right)^{1/\rho},\ \rho < 0. \tag{2}$$

While a standard utility function over bundles of goods, CES utility with $\rho < 0$ can have difficulties accommodating decision problems over indivisible goods, as there are some endowments $\{C_i^{\mathrm{e}}, H_i^{\mathrm{e}}\}$ for which willingness to pay for an additional unit of H is infinite (for example, $\{2, 0\}$ when $\rho = -1$.) Note that the problem does not arise when the second good is divisible, as the agent can just pay a very high price for a smaller amount than one whole unit of the second good. We assume that actual endowments lie strictly within these extreme cases and leave it to future research to develop more general utility functions.

Given price q, agent i chooses $H_i \in \mathbb{Z}_+$ to solve:

$$\max_{H_i \in \mathbb{Z}_+, qH_i \leq C_i^{\mathrm{e}} + qH_i^{\mathrm{e}}} u_i\left(C_i, \tilde{H}_i\right) = \log\left(\left((C_i^{\mathrm{e}} + qH_i^{\mathrm{e}} - qH_i)^{\rho} + (1 + H_i)^{\rho}\right)^{1/\rho}\right). \tag{3}$$

This characterizes agent i's demand for additional units of H as a function of price. Aggregate demand for additional units of H at price q is then:

$$H^{\mathrm{demand}}(q) = \sum_{i=1}^{n} H_i^*(q), \tag{4}$$

where $H_i^*(q)$ is the solution to agent i's problem. A competitive equilibrium price q^* is a price

such that:

$$H^{\text{demand}}(q^*) = H^{\text{supply}}. \tag{5}$$

C. Pricing Indivisible Goods

So far, our setup is standard, with the exception of having the second good be indivisible. To derive equilibrium prices q^*, we must somewhat deviate from the standard framework due to the stepwise nature of the demand curves. As usual in this setting, we assume non-strategic behavior, in particular price-taking by all agents.

Definition 1 (**Reservation Price**)**.**
For $s \in \mathbb{Z}_+$, the *reservation price* $q_i^{(s)}$ is the price at which agent i is indifferent between consuming s and $s+1$ additional units of H. Formally,

$$u_i\Big(C_i^{\text{e}} + q_i^{(s)} H_i^{\text{e}} - q_i^{(s)} s,\, 1+s\Big) = u_i\Big(C_i^{\text{e}} + q_i^{(s)} H_i^{\text{e}} - q_i^{(s)}(s+1),\, 1+s+1\Big), \tag{6}$$

where utility takes the form $u_i(C, \tilde{H}) = \log\left(C^\rho + \tilde{H}^\rho\right)^{1/\rho}$ with $\rho < 0$.

Each agent's sequence of reservation prices $\{q_i^{(s)}\}$ defines the thresholds at which her demand decreases from $s+1$ to s units.

Let $q = q_i^{(s)}$, $f(x) = x^\rho$, $C_s = C_i^{\text{e}} + q(H_i^{\text{e}} - s)$, and $H_s = 1 + s$. Then (6) is equivalent to

$$L(q,s) = R(s), \quad L(q,s) := f(C_s) - f(C_s - q), \quad R(s) := f(H_s + 1) - f(H_s).$$

Lemma 1 (**Existence and Uniqueness**)**.**
i. **Net-buyer case** ($s > H_i^{\text{e}} - 1$). There exists a unique solution $q_i^{(s)} \in (0, \bar{q}_s)$, where $\bar{q}_s = \frac{C_i^{\text{e}}}{s+1-H_i^{\text{e}}}$.
ii. **Net-seller case** ($s \le H_i^{\text{e}} - 1$). There exists a unique solution $q_i^{(s)} \in (0, \hat{q}_s]$ if and only if $L_{\min}(s) \le R(s)$, where $\hat{q}_s = \frac{C_i^{\text{e}}}{s+1-H_i^{\text{e}}+\phi}$ and $\phi = \left[\left(\frac{H_i^{\text{e}}-s}{H_i^{\text{e}}-s-1}\right)^{\frac{1}{1-\rho}} - 1\right]^{-1}$.

Proof. Net-buyer case. For fixed s, $L(\cdot, s)$ is continuous and strictly decreasing on $(0, \bar{q}_s)$. More-

over, $\lim_{q\to 0^+} L(q,s) = 0$ and $\lim_{q\to \bar{q}_s^-} L(q,s) = -\infty$. Since $R(s) \in (-\infty, 0)$, the Intermediate Value Theorem guarantees a solution, and strict monotonicity ensures uniqueness.

Net-seller case. Here, $L(\cdot, s)$ is continuous on $(0,\infty)$, strictly decreasing on $(0, \hat{q}_s)$, strictly increasing on $(\hat{q}_s, \infty)$, and attains a unique minimum at $\hat{q}_s$, where $L_{\min}(s) := L(\hat{q}_s, s) < 0$. Furthermore, $\lim_{q\to 0^+} L(q,s) = \lim_{q\to\infty} L(q,s) = 0$.

- If $R(s) = L_{\min}(s)$, there is exactly one solution $q_i^{(s)} = \hat{q}_s$.
- If $L_{\min}(s) < R(s)$, there are two solutions $q_i^{(s,-)} \in (0, \hat{q}_s)$ and $q_i^{(s,+)} \in (\hat{q}_s, \infty)$. Since we assume an ascending-price auction, the relevant (unique) solution lies on the decreasing branch: $q_i^{(s)} \in (0, \hat{q}_s)$. ∎

Remark 1.

In the net-buyer case a reservation price always exists, since the agent's willingness to pay for the next unit is well defined. In the net-seller case, however, existence requires that the agent's consumption endowment not be "too high." Intuitively, when C_i^{e} is large, the marginal utility of additional consumption becomes negligible, while the loss of housing is costly due to complementarity ($\rho < 0$). In that case, no finite price equates the trade-off, and the reservation price fails to exist.

Lemma 2 (**Monotonicity of Reservation Prices**)**.**

Let $q_i^{(s)}$ be well-defined as in Lemma 1 for each integer s. Then the sequence $\{q_i^{(s)}\}$ is strictly decreasing in s.

Proof. Fix s and evaluate the residual at level $s+1$ using $q_i^{(s)}$:

$$\begin{aligned} D_{s+1}(q_i^{(s)}) &:= L(q_i^{(s)}, s+1) - R(s+1) \\ &= L(q_i^{(s)}, s+1) - L(q_i^{(s+1)}, s+1) \\ &= \big[L(q_i^{(s)}, s+1) - L(q_i^{(s)}, s)\big] - \big[R(s+1) - R(s)\big]. \end{aligned}$$

Since $f(x) = x^\rho$ with $\rho < 0$, f is strictly convex. Hence the forward difference $x \mapsto f(x+1) - f(x)$

is strictly increasing, so $R(s+1) > R(s)$. Thus, the term in the second set of brackets is positive. Similarly, $x \mapsto f(x) - f(x - q_i^{(s)})$ is strictly increasing in x, so the term in the first set of brackets is negative. Therefore,

$$D_{s+1}(q_i^{(s)}) = L(q_i^{(s)}, s+1) - L(q_i^{(s+1)}, s+1) < 0. \tag{7}$$

Now, for level $s+1$, the map $q \mapsto L(q, s+1)$ is strictly decreasing on its feasible domain—$(0, \bar{q}_{s+1})$ in the net-buyer case or $(0, \hat{q}_{s+1}]$ in the net-seller case.

- If $q_i^{(s)}$ lies in the feasible domain, then $q_i^{(s)}$ is on the decreasing branch. By (7), the residual is negative, so $q_i^{(s+1)} < q_i^{(s)}$.
- If $q_i^{(s)}$ lies outside the feasible domain, then $q_i^{(s)} \geq \bar{q}_{s+1}$ (or $> \hat{q}_{s+1}$), which implies trivially that $q_i^{(s+1)} < q_i^{(s)}$.

In both cases, $q_i^{(s+1)} < q_i^{(s)}$. ∎

Lemma 3 (**Reservation prices and endowments**).
Fix an integer $s \in \mathbb{Z}_+$ and suppose agent i's reservation price $q_i^{(s)}$ is well defined as in Lemma 1. Then $q_i^{(s)}$ is strictly increasing in both endowments C_i^{e} and H_i^{e}.

Proof. The reservation price $q_i^{(s)}$ solves $L(q, s) = R(s)$, with

$$L(q,s) = f(C_s) - f(C_s - q), \quad C_s = C_i^{\mathrm{e}} + q(H_i^{\mathrm{e}} - s), \quad f(x) = x^{\rho},\ \rho < 0,$$

and where $R(s)$ depends only on s. Define

$$F(q; C_i^{\mathrm{e}}, H_i^{\mathrm{e}}) := L(q,s) - R(s).$$

By Lemma 1, along the relevant (decreasing) branch we have $F_q < 0$. By the Implicit Function Theorem,

$$\frac{\partial q_i^{(s)}}{\partial C_i^{\mathrm{e}}} = -\frac{F_{C_i^{\mathrm{e}}}}{F_q}, \qquad \frac{\partial q_i^{(s)}}{\partial H_i^{\mathrm{e}}} = -\frac{F_{H_i^{\mathrm{e}}}}{F_q}.$$

Since f is strictly convex,

$$F_{C_i^{\mathrm{e}}} = f'(C_s) - f'(C_s - q) > 0, \qquad F_{H_i^{\mathrm{e}}} = q\big[f'(C_s) - f'(C_s - q)\big] > 0.$$

With $F_q < 0$, it follows that both derivatives are strictly positive. Therefore, $q_i^{(s)}$ is strictly increasing in both C_i^{e} and H_i^{e}. ∎

Remark 2.

The reservation price $q_i^{(s)}$ reflects agent i's marginal willingness to pay for an additional unit of the indivisible good. While $q_i^{(s)}$ depends on both endowments $(C_i^{\mathrm{e}}, H_i^{\mathrm{e}})$, it is not straightforward to compare reservation prices across agents unless one dominates the other in both dimensions. In particular, under assumption 1, if agent i dominates agent j in both endowments, then she has greater purchasing power at every price level, and one can show that $q_i^{(s)} > q_j^{(s)}$.

Individual demand is piecewise constant:

$$H_i^*(q) = \max\left\{s \in \mathbb{Z}_+ : q < q_i^{(s)}\right\}, \tag{8}$$

with steps at the set $\mathcal{Q}_i := \left\{q_i^{(0)}, q_i^{(1)}, ...\right\}$. Aggregate demand $H^{\mathrm{demand}}(q) = \sum_{i=1}^n H_i^*(q)$ is constant on intervals between points in the union $\mathcal{Q} := \bigcup_{i=1}^n \mathcal{Q}_i$ and is therefore a step function.

Proposition 1 (**Equilibrium Price Characterization**)**.**

If an equilibrium price exists, it must belong to $\mathcal{Q}$. Sorting the reservation prices $q^{(1)} < q^{(2)} < ... < q^{(M)}$, let $D^{(m)} := \lim_{q \uparrow q^{(m)}} H^{\mathrm{demand}}(q)$. The equilibrium price q^* is the lowest $q^{(m)}$ satisfying $D^{(m)} \geq H^{\mathrm{supply}}$ and $H^{\mathrm{demand}}\left(q^{(m)}\right) \leq H^{\mathrm{supply}}$. In such a case, the pivotal agent is the one whose demand drops by one unit at q^*, and her reservation price determines the equilibrium.

Definition 2 (**Indirect Utility**)**.**

Given q^*, agent i's indirect utility is

$$V_i(q^*; C_i^{\mathrm{e}}, H_i^{\mathrm{e}}) = \log\left(((C_i^{\mathrm{e}} + q^* H_i^{\mathrm{e}} - q^* H_i^*(q^*))^\rho + (1 + H_i^*(q^*))^\rho)^{1/\rho}\right). \tag{9}$$

Unlike in a standard endowment economy, q^* depends on the distribution of wealth, both divisible and indivisible. Therefore, indirect utility of (almost) every agent depends on the distribution of wealth and in general agents exhibit relative wealth concerns. This result is in contrast to a standard economy with only divisible goods, in which indirect utility of every agent only depends on her own absolute endowments and on relative aggregate endowments. This is the first key result in our paper.

The rest of this section explores the simplest examples to further build intuition.

D. Example: Three agents, one tradable indivisible unit

We apply the previous results to a simple case with three agents ($N = 3$) and one unit of the indivisible good ($H^{\text{supply}} = 1$). For notational clarity, we relabel the agents as $\iota \in \{\text{R}, \text{P}, \text{VP}\}$ denoting richer (R), poor (P) and very poor (VP). Endowments of the divisible good satisfy $C^{\text{e}}_{\text{R}} > C^{\text{e}}_{\text{P}} > C^{\text{e}}_{\text{VP}}$, so that under Assumption 1 we also have $H^{\text{e}}_{\text{R}} \geq H^{\text{e}}_{\text{P}} \geq H^{\text{e}}_{\text{VP}}$. Throughout all figures, we set the log-CES parameter to $\rho = -1$.

Because the two goods are complementary, the richer agent has a higher valuation for a unit of the indivisible good. If the indivisible endowment happens to be $\{H^{\text{e}}_{\text{R}}, H^{\text{e}}_{\text{P}}, H^{\text{e}}_{\text{VP}}\} = \{1, 0, 0\}$, then R keeps and consumes the unit and no trade occurs. For all other endowments permitted under Assumption 1, at least one other agent owns a claim to a fraction of the indivisible good, so trade takes place. We assume an ascending price auction, with all agents behaving as price takers. For simplicity, in what follows we take $\{H^{\text{e}}_{\text{R}}, H^{\text{e}}_{\text{P}}, H^{\text{e}}_{\text{VP}}\} = \{1/2, 1/2, 0\}$.

The exercise of interest is to vary the distribution of divisible endowments between R and P while keeping the aggregate endowment constant. Let i denote the agent whose divisible endowment is fixed and j the agent whose endowment is scaled:

$$C^{\text{e}}_j = kC^{\text{e}}_i, \qquad C^{\text{e}}_{\text{VP}} = \Omega - C^{\text{e}}_{\text{R}} - C^{\text{e}}_{\text{P}},$$

where Ω denotes the constant aggregate divisible endowment. When $k < 1$, agent i is richer than j; when $k > 1$, she is poorer. The labels R and P refer to the absolute wealth ranking,

whereas i and j denote relative roles in this comparative exercise. Accordingly, the mapping between (i, j) and (R,P) depends on k. We consider an example in which k is not too high.[7]

The auction mechanism determines the equilibrium price q^* through the reservation value at which aggregate demand equals the single unit of supply. Each agent exits when the price reaches the point where she is indifferent between keeping and selling her share of the indivisible good. By Lemma 3, an agent's reservation value increases with her divisible endowment. At VP's reservation price, both R and P still demand at least one unit each, so aggregate demand exceeds supply and the market does not clear. When the price reaches P's reservation value, her demand falls from one unit to zero, so aggregate demand drops exactly to one and the market clears. The equilibrium price is therefore always given by Agent P. When $k < 1$, agent j is poorer (so $j = \text{P}$ and $i = \text{R}$), and her reservation value $q_j^{(0)}$ determines the price, which rises with k. When $k > 1$, agent i is poorer (so $i = \text{P}$ and $j = \text{R}$), and her reservation value $q_i^{(0)}$ pins down the price. Because i's endowment is fixed, $q_i^{(0)}$ does not vary with k, so the equilibrium price remains flat as inequality grows.

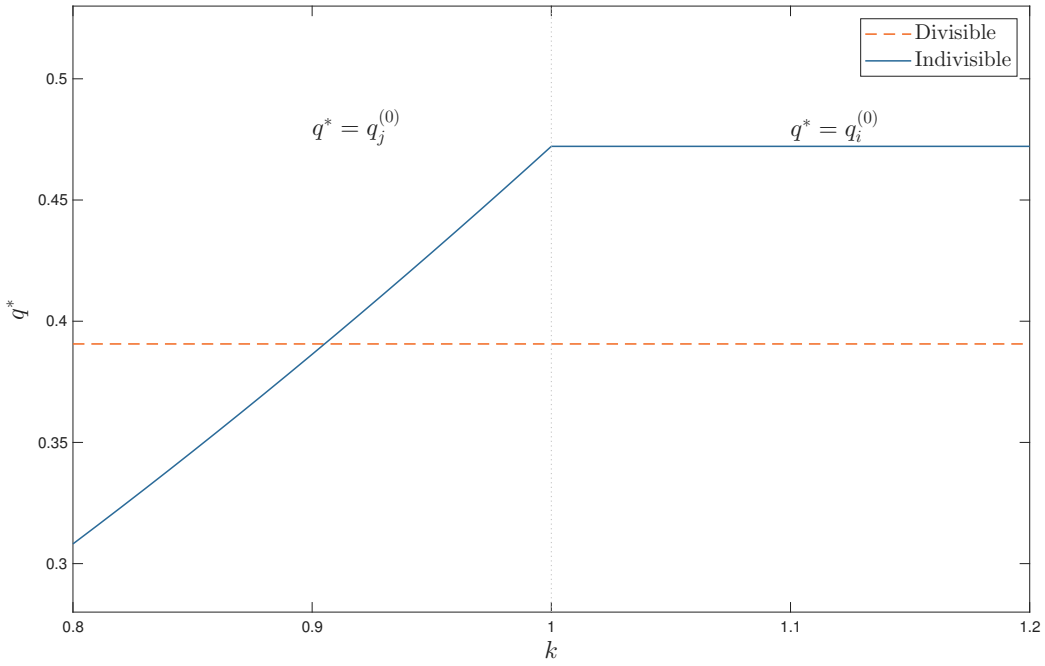
Figure 1a illustrates this mechanism. With indivisibilities, the price is determined by the poorer agent's reservation value and is therefore sensitive to inequality when the poorer agent's endowment is the one that varies. The dashed line shows the benchmark with a divisible second good, where the price depends only on aggregate wealth and is unaffected by inequality.

Figure 1b reports indirect utility for the fixed-endowment agent i. In the divisible economy, utility depends only on own and aggregate wealth and is flat in k. With an indivisible good, however, welfare depends on relative wealth. When i is richer ($k < 1$), her utility declines as j's endowment rises, because she must pay a higher price to buy the indivisible unit. When i is poorer ($k > 1$), her welfare is unaffected by changes in k, since her reservation value pins down the price and does not vary. Therefore, indivisibilities create a channel through which relative wealth affects welfare.

Finally, note that allocations are less efficient when goods are indivisible. Agents who wish

[7] At very high levels of inequality, which we do not consider in this subsection, other results hold consistent with Lemma 3.

(a) Price of good H



(b) Indirect utility of Agent i

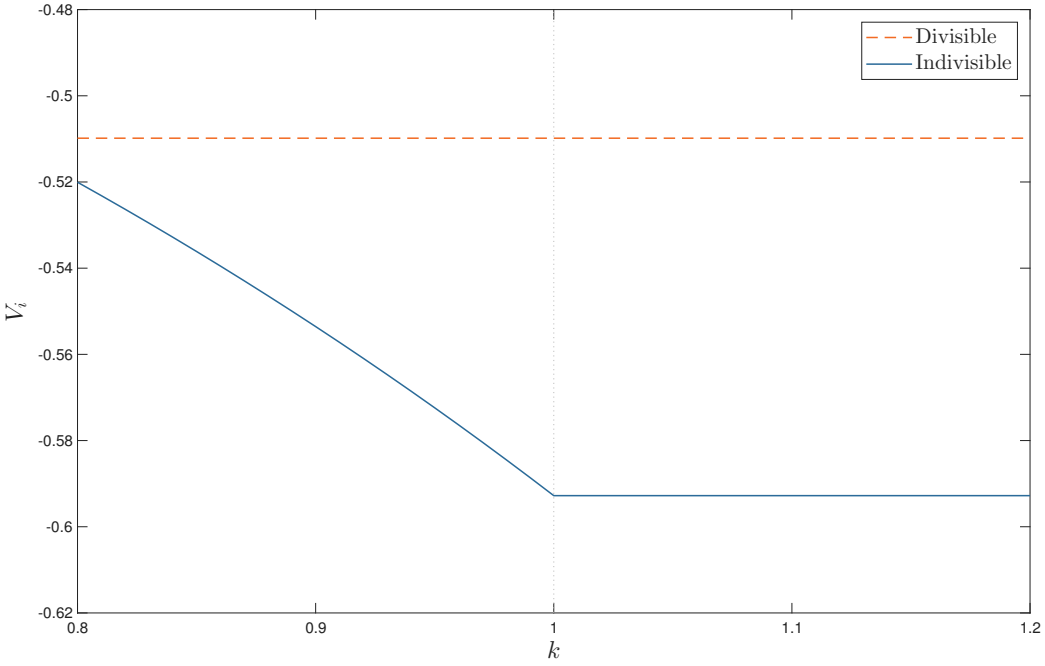


Figure 1. Equilibrium with three agents and one unit. Calibration: $\rho = -1$, $\Omega/C_i^{\mathrm{e}} = 2.5$, $C_i^{\mathrm{e}} = 1$, $k \in [0.8, 1.2]$, where Ω denotes aggregate endowment.

to diversify consumption cannot do so fully, and poorer agents in particular are worse off than in a divisible economy. With more units of the indivisible good, the analysis becomes more

complex, but the same principles apply: relative wealth concerns persist, though they weaken as the indivisible good becomes less scarce. Methods based on one-to-one matching, such as those used in some models of indivisible good allocation (e.g. Becker et al., 2005), are therefore not suitable in this setting.

In summary, in an endowment economy with indivisible goods, welfare depends not only on own and aggregate wealth but also on the distribution across agents. Indivisibilities introduce relative wealth concerns that can affect both prices and welfare, providing a channel through which inequality shapes attitudes to risk. It is to this effect on attitudes to risk that we turn next.

III. Gambling without Aggregate Risk

We now turn to a two-agent economy with one tradable unit of the indivisible good. In the previous section, our objective was to isolate the effect of relative wealth while keeping one agent's own wealth unchanged, which required a third agent, enabling us to change relative wealth while keeping aggregate wealth constant. Here the goal is different: we study gambles, in which wealth shifts between agents while aggregate wealth stays fixed. This simpler framework is therefore the natural one for analyzing how indivisibilities shape attitudes to risk. In the bilateral gamble setting, we show that indivisibilities generate Friedman-Savage-type preferences: agents dislike small risks but are willing to accept larger gambles. We then extend the analysis to a multi–agent setting and find that lotteries among the poor can exist, allowing them to pool resources to create a winner who can bid more aggressively against the rich.

A. Bilateral gambles

Figure 2 shows how indirect utility depends on relative endowments of divisible wealth, holding aggregate, but not agent's own, wealth constant. The x-axis increases an agent's own endowment while holding the aggregate endowment constant (i.e., decreasing the other agent's endowment at the same time). The y-axis shows indirect utility V. The main point is that the richer agent has utility of the form $u(C_{\mathrm{R}}^{\mathrm{e}} + q_{\mathrm{P}}^{(0)} H_{\mathrm{R}}^{\mathrm{e}} - q_{\mathrm{P}}^{(0)}, 2)$, increasing in her own endowment $C_{\mathrm{R}}^{\mathrm{e}}$ but

decreasing in the poorer agent's endowment $C_{\mathrm{P}}^{\mathrm{e}}$, since $q_{\mathrm{P}}^{(0)}$ increases in $C_{\mathrm{P}}^{\mathrm{e}}$. Thus, the richer agent exhibits relative wealth concerns, which arise entirely endogenously in equilibrium. The poorer agent has welfare of the form $u(C_{\mathrm{P}}^{\mathrm{e}} + q_{\mathrm{P}}^{(0)} H_{\mathrm{P}}^{\mathrm{e}}, 1)$. While the poorer agent has no relative wealth concerns (in this two-agent setting), her welfare is a leveraged function of her own wealth.

Except at exact equality, where there is an inflection point, agents are locally risk-averse at all levels of wealth. However, over larger gambles with outcomes above and below equality, agents are risk-loving, as in Friedman and Savage. As noted by Chetty and Szeidl (2007) in a different context, this form of welfare function addresses the Rabin (2000) critique of expected utility theory. Rabin shows that investors' attitudes to small and large risks are generally inconsistent with standard expected utility, because given how much agents appear to dislike small risks, they should avoid almost all large risks. But in our setting, agents can dislike small risks and still like larger risks.

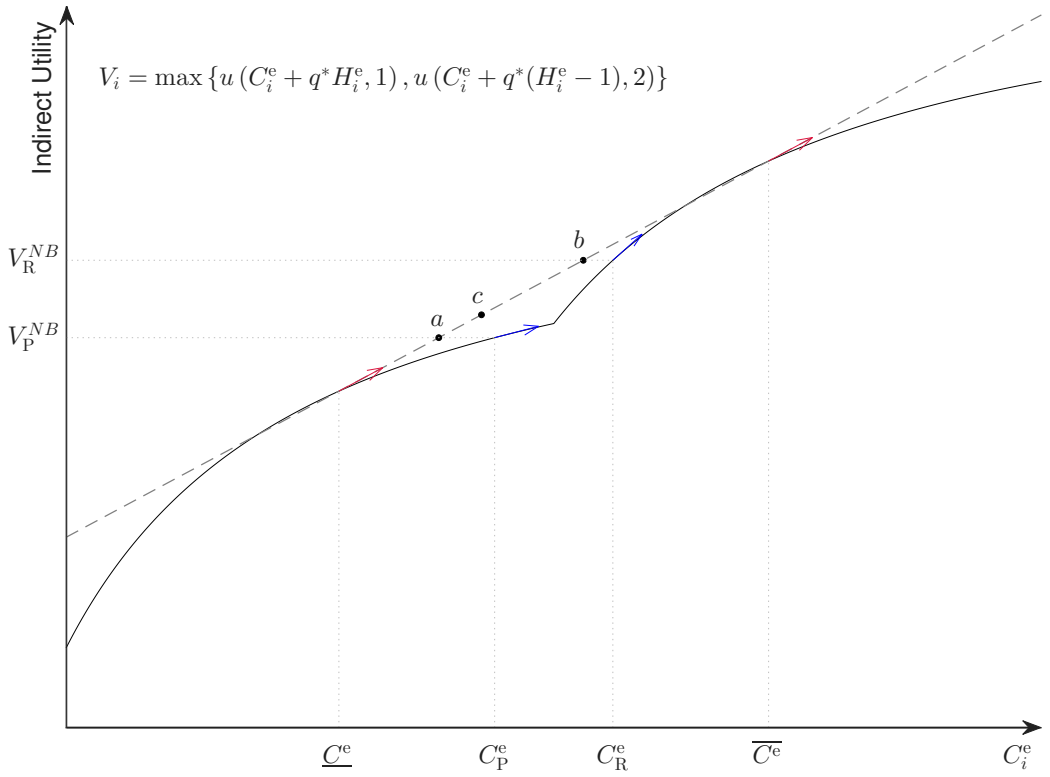


Figure 2. Indirect utility of Agent i with constant aggregate wealth

Between divisible wealth levels $\underline{C^{\mathrm{e}}}$ and $\overline{C^{\mathrm{e}}}$, both agents prefer to accept zero-sum side bets

(gambles) involving transfers of the divisible good from the loser to the winner. The dashed line tangent to V at both $\underline{C^{\mathrm{e}}}$ and $\overline{C^{\mathrm{e}}}$ represents the expected welfare obtainable by optimal side bets between the two agents. (All convex combinations of any other pairs of divisible wealth levels would result in dashed lines lying strictly below this tangent line.) As all the points on the dotted line between these levels of divisible wealth lie above the points on the solid line beneath, agents prefer to gamble, with the richer agent obtaining higher probabilities of winning (the exact magnitude depends on bargaining power or, equivalently, the equilibrium criterion). Importantly, in this case the marginal utility of divisible wealth at both tangency points is the same. Thus, given an aggregate endowment of $\Omega = \underline{C^{\mathrm{e}}} + \overline{C^{\mathrm{e}}}$, the lower bound $\underline{C^{\mathrm{e}}}$ is identified by:

$$u_C(\underline{C^{\mathrm{e}}} + q_{\mathrm{P}}^{(0)} H_{\mathrm{P}}^{\mathrm{e}}, 1) = u_C(\Omega - \underline{C^{\mathrm{e}}} + q_{\mathrm{P}}^{(0)} H_{\mathrm{R}}^{\mathrm{e}} - q_{\mathrm{P}}^{(0)}, 2), \tag{10}$$

where $q_{\mathrm{P}}^{(0)}$ is also evaluated at $\underline{C^{\mathrm{e}}}$. In the region between $\underline{C^{\mathrm{e}}}$ and $\overline{C^{\mathrm{e}}}$, the price function $q^* = q_{\mathrm{P}}^{(0)}$. Also, the marginal utility u_C of the richer agent is higher than the poorer agent's at all points strictly between the two tangency points, with the difference largest at near-equality.

In the neighborhood of the inflection point, a small increase in the poorer agent's divisible endowment has a small effect on welfare, provided it doesn't thereby make her the richer agent, as she already has a lot of the divisible good.[8]

This observation can also rationalize status anxiety in this region: a small decrease in the richer agent's wealth, holding the poorer agent's wealth constant, can have a large negative affect on the richer agent's welfare. In this respect, the welfare function depicted in Figure 2

[8]For example, when $H_{\mathrm{R}}^{\mathrm{e}} = H_{\mathrm{P}}^{\mathrm{e}} = 1/2$, the inflection point is at equality of divisible endowment, $C^{\mathrm{e}} = \Omega/2$. In the neigbhourhood of this point, the richer agent's marginal utility of divisible wealth exceeds the poorer agent's by

$$\begin{aligned} &u_C\left(\tfrac{\Omega}{2} + \varepsilon - \tfrac{1}{2}q_{\mathrm{P}}^{(0)}, 2\right) - u_C\left(\tfrac{\Omega}{2} - \varepsilon + \tfrac{1}{2}q_{\mathrm{P}}^{(0)}, 1\right) \\ &\quad = \underbrace{\left(u_C\left(\tfrac{\Omega}{2} - \tfrac{1}{2}q_{\mathrm{P}}^{(0)}, 2\right) - u_C\left(\tfrac{\Omega}{2} + \tfrac{1}{2}q_{\mathrm{P}}^{(0)}, 1\right)\right)}_{\triangle_0} + \underbrace{\left(u_{CC}\left(\tfrac{\Omega}{2} - \tfrac{1}{2}q_{\mathrm{P}}^{(0)}, 2\right) + u_{CC}\left(\tfrac{\Omega}{2} + \tfrac{1}{2}q_{\mathrm{P}}^{(0)}, 1\right)\right)}_{\triangle_1} \varepsilon + O(\varepsilon^2). \end{aligned}$$

For small ε , the leading term $\triangle_0$ dominates. Moreover, $\triangle_0 > u_C\left(\frac{\Omega}{2} + \frac{1}{2}q_{\mathrm{P}}^{(0)}, 2\right) - u_C\left(\frac{\Omega}{2} + \frac{1}{2}q_{\mathrm{P}}^{(0)}, 1\right) > 0$,because $u_{CC}(\cdot) < 0$ and $u_{CH}(\cdot) > 0$.

resembles habit close to the subsistence level (Campbell and Cochrane, 1999) or keeping-up-with-the-Joneses' preferences (Chan and Kogan, 2002). Unlike those utility functions, however, this status anxiety arises endogenously in equilibrium from competition between agents, and implies that agents wish to gamble.

An agent's outside option is not to enter into a side bet, in which case the richer agent (agent R, whose initial endowment of the divisible good $C_{\mathrm{R}}^{\mathrm{e}} = kC_{\mathrm{P}}^{\mathrm{e}}$, $k \geq 1$) acquires the additional unit for q^*. For a side bet to occur, both agents must be weakly better off in expected utility terms compared to their outside option. Otherwise at least one agent refuses the side bet. As a solution criterion we use the Nash bargaining solution with equal bargaining power, which in the case of two agents just maximizes the sum of their expected utilities and then splits the surplus from gambling. Our results are qualitatively robust to using alternative solution criteria, such as giving one agent all of the bargaining power.

If agent R has all the bargaining power, agent P is driven to her outside option, so that agent P's expected utility from the side bet $V_{\mathrm{P}}^{\mathcal{B}}$ equals her utility from not betting $V_{\mathrm{P}}^{\mathcal{NB}}$. This point is indicated, for an initial wealth level $C_{\mathrm{P}}^{\mathrm{e}}$, in Figure 2 as point (a). Not surprisingly, agent P's probability of winning a side bet is low at this point. Likewise if agent P has all the bargaining power, agent R is driven to her outside option, indicated by point (b) for an initial wealth level $C_{\mathrm{R}}^{\mathrm{e}}$, with a high probability of the poorer agent winning. With equal bargaining power they split the surplus and the gamble is shown at point (c). For two agents, this also corresponds to each agent receiving their Shapley value. All points on the dashed line represent possible side bets which maximize the surplus expected utility relative to no side bets.

An alternative solution criterion (used by Becker et al., 2005, Garratt and Marshall, 1994, and Freeman, 1996, among others) is to require all bets to be actuarially fair in the sense that each agent's expected total wealth (including the value of implicit holdings of indivisible goods) is the same under the bet as under no bet. We use Nash bargaining instead for three reasons. First, we are interested in exploring circumstances in which standard risk-averse agents will accept bets with negative expected returns. Second, the Nash Bargaining/Shapley criterion, although

not universal, is more standard in solving games of this type. Third, we will shortly introduce aggregate risk, in which agents' expected wealth varies across states under any criterion.

We now show how these Nash bargaining side bets depend on relative wealth, and give sufficient conditions under which the poorer agent expects a negative payoff, decreasing as she becomes relatively poorer.

Definition 3 (**Nash Bargaining Side Bets**)**.**

A *Nash bargaining side bet* is given by a triple

$$\mathcal{B} \equiv \{p, \mathcal{G}_{\mathrm{R}}, \mathcal{G}_{\mathrm{P}}\},$$

where $p \in [0,1]$ is the probability that the poorer agent P wins, $\mathcal{G}_{\mathrm{R}}$ is the transfer of divisible wealth from the richer agent R to P if P wins, and $\mathcal{G}_{\mathrm{P}}$ is the transfer from P to R if R wins.

If no side bet is made, R acquires the indivisible good at price q^*, yielding utilities $V_{\mathrm{R}}^{\mathcal{NB}}$ and $V_{\mathrm{P}}^{\mathcal{NB}}$, which constitute each agent's outside option. Under equal bargaining power, the side bet is determined by

$$\max_{p,\mathcal{G}_{\mathrm{R}},\mathcal{G}_{\mathrm{P}}} \ \mathbb{E}[V_{\mathrm{R}}] + \mathbb{E}[V_{\mathrm{P}}], \tag{11}$$

subject to feasibility, with the surplus split equally relative to outside options.

Let $\overline{C^{\mathrm{e}}}$ and $\underline{C^{\mathrm{e}}}$ denote the wealth allocations induced by the bet, where $\underline{C^{\mathrm{e}}}$ is defined by (10) and $\overline{C^{e}} = \Omega - \underline{C^{\mathrm{e}}}$. Then

$$\mathcal{G}_{\mathrm{R}} = C_{\mathrm{R}}^{\mathrm{e}} - \underline{C^{\mathrm{e}}}, \qquad \mathcal{G}_{\mathrm{P}} = C_{\mathrm{P}}^{\mathrm{e}} - \underline{C^{\mathrm{e}}}.$$

The Nash bargaining probability assigned to P satisfies

$$p = \tfrac{1}{2} - \tfrac{1}{2}\frac{V_{\mathrm{R}}^{\mathcal{NB}} - V_{\mathrm{P}}^{\mathcal{NB}}}{V^{\mathcal{W}} - V^{\mathcal{L}}}, \tag{12}$$

where $V^{\mathcal{W}}$ and $V^{\mathcal{L}}$ denote the utilities corresponding to $\overline{C^{\mathrm{e}}}$ and $\underline{C^{\mathrm{e}}}$, respectively.

Definition 4 (**Actuarially Fair Side Bets**)**.**

An *actuarially fair side bet* is a triple

$$\mathcal{B}_{\text{fair}} \equiv \{p_{\text{fair}}, \mathcal{G}_{\text{R}}, \mathcal{G}_{\text{P}}\},$$

with the same transfers $\mathcal{G}_{\text{R}}$ and $\mathcal{G}_{\text{P}}$ as in Definition 3, and where each agent has zero expected return. Equivalently, actuarial fairness is characterized by

$$0 \;=\; p_{\text{fair}}\,\mathcal{G}_{\text{R}} \;-\; (1 - p_{\text{fair}})\,\mathcal{G}_{\text{P}},$$

which yields

$$p_{\text{fair}} = \frac{\mathcal{G}_{\text{P}}}{\mathcal{G}_{\text{R}} + \mathcal{G}_{\text{P}}} = \tfrac{1}{2} - \tfrac{1}{2}\,\frac{\Omega/2 - C_{\text{P}}^{\text{e}}}{\Omega/2 - \underline{C}^{\text{e}}}.$$

Lemma 4 (**Monotonicity in** k)**.**
Suppose $C_{\text{R}}^{\text{e}} = kC_{\text{P}}^{\text{e}}$ with $k \in [1, \overline{k}]$ and Ω fixed. Then both p and p_{fair} are strictly decreasing in k, with boundary values $p(1) = p_{\text{fair}}(1) = 0.5$ and $p(\overline{k}) = p_{\text{fair}}(\overline{k}) = 0$.

Proof. The outside-option utility difference $V_{\text{R}}^{\mathcal{NB}} - V_{\text{P}}^{\mathcal{NB}}$ is strictly increasing in k, while the spread $V^{\mathcal{W}} - V^{\mathcal{L}}$ is constant. Substituting into the expression for p yields the claim; the same reasoning applies to p_{fair}. ∎

Lemma 5 ($p \leq p_{\text{fair}}$)**.**
For all $k \in [1, \overline{k}]$,

$$p(k) \;\leq\; p_{\text{fair}}(k),$$

with equality if and only if $k \in \{1, \overline{k}\}$, provided that

$$r\,\mathcal{A}(C_{\text{R}}, 2) \;\geq\; \lambda\,\mathcal{A}(C_{\text{P}}, 1) + (1 - \lambda)\mathcal{A}(C_{\text{P}} - q^*, 2),$$

where $\mathcal{A}(C, H) = -u_{CC}(C, H)/u_C(C, H)$ is absolute risk aversion w.r.t. C (holding H fixed),

$C_{\mathrm{R}} = C_{\mathrm{R}}^{\mathrm{e}} + q^*(H_{\mathrm{R}}^{\mathrm{e}} - 1)$, $C_{\mathrm{P}} = C_{\mathrm{P}}^{\mathrm{e}} + q^* H_{\mathrm{P}}^{\mathrm{e}}$, $r = u_C(C_{\mathrm{R}}, 2)/u_C(C_{\mathrm{P}}, 1)$, and

$$\lambda = \frac{(1 - H_{\mathrm{P}}^{\mathrm{e}})u_C(C_{\mathrm{P}} - q^*, 2) - H_{\mathrm{P}}^{\mathrm{e}} u_C(C_{\mathrm{R}}, 2)}{(1 - H_{\mathrm{P}}^{\mathrm{e}})u_C(C_{\mathrm{P}} - q^*, 2) + H_{\mathrm{P}}^{\mathrm{e}} u_C(C_{\mathrm{P}}, 1)} \ \in \ (0, 1).$$

Proof. By Lemma 4, both p and p_{fair} are strictly decreasing in k and coincide at $k = 1$ and $k = \overline{k}$. Let $t := C_{\mathrm{P}}^{\mathrm{e}} = \Omega/(1 + k)$; then t ranges over a compact interval and p_{fair} is affine in t by Definition 4. A direct calculation shows

$$\frac{d^2 p}{dt^2} \ \geq \ 0 \quad \Longleftarrow \quad r\,\mathcal{A}(C_{\mathrm{R}}, 2) \ \geq \ \lambda\,\mathcal{A}(C_{\mathrm{P}}, 1) + (1 - \lambda)\,\mathcal{A}(C_{\mathrm{P}} - q^*, 2).$$

Hence $p(\cdot)$ is convex in t while $p_{\mathrm{fair}}(\cdot)$ is linear in t, and they meet at both endpoints. Therefore $p \leq p_{\mathrm{fair}}$ on the whole interval, with strict inequality in the interior when the right-hand side inequality holds strictly. ∎

Remark 3.

The condition of Lemma 5 compares (normalized) curvature of marginal utilities across agents. The left-hand side is the rich agent's absolute risk aversion, scaled by the ratio of her marginal utility to the poor agent's. The right-hand side is a convex combination of the poor agent's absolute risk aversions, evaluated at her indifference bundles with one or two indivisible units, with weights determined by indivisible-good endowments and relative marginal utilities. When the condition holds, the utility frontier is more curved on the rich agent's side, which makes the poorer agent's winning probability p convex in relative wealth: starting from equality, p falls faster than linearly as she becomes relatively poorer, so the odds become increasingly tilted toward the rich. In all of the examples we consider, the condition holds everywhere in the relevant region, most strongly near equality of C_i^e.

The actuarially fair probability p_{fair} provides a benchmark in which expected transfers balance so that neither agent gains on average, while the Nash bargaining probability p reflects outside-option asymmetries and divides the surplus accordingly. As shown in Figure 3, both probabilities

decline monotonically with the wealth ratio k, starting at 0.5 under equality and converging to zero at the upper bound $\overline{k}$. At equality, outside options are identical and the surplus is shared symmetrically, but as agent R's wealth rises and agent P's falls, R's outside option improves while P's deteriorates. To preserve equal shares of the surplus, the gamble must tilt increasingly in R's favor, and thus Nash bargaining assigns the poorer agent strictly less favorable odds than actuarial fairness ($p < p_{\text{fair}}$), with equality only at the boundaries. The wedge between the two probabilities captures how differences in outside options and risk aversion disadvantage the poorer agent, making p_{fair} a natural upper bound on the probability she can obtain.

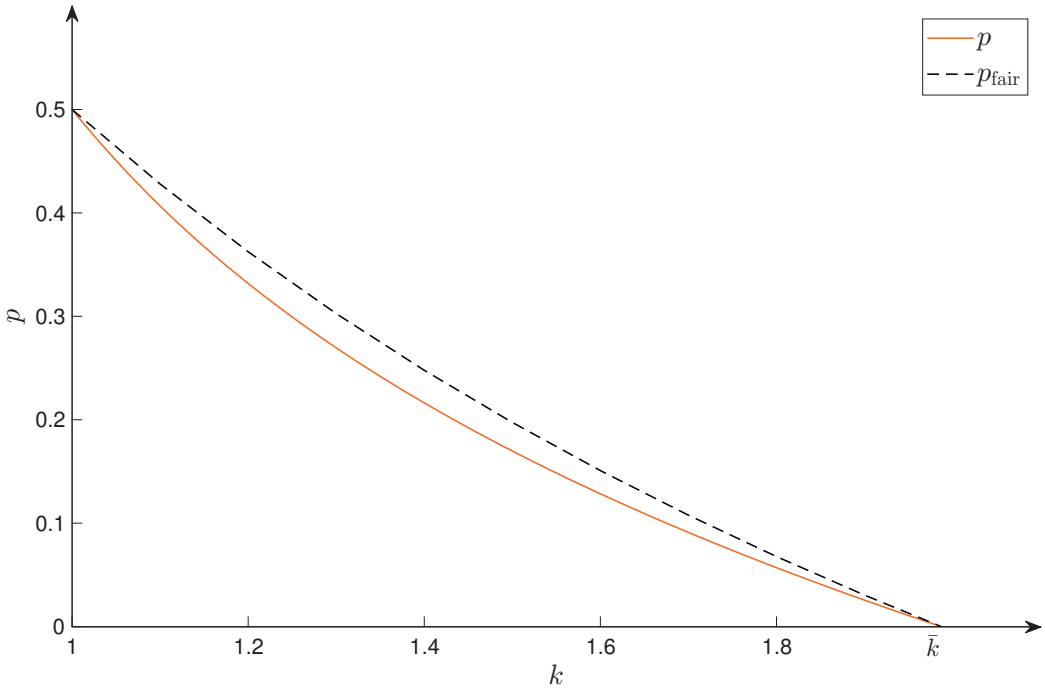


Figure 3. The Nash bargaining probability p (solid) and the actuarially fair probability p_{fair} (dashed) as functions of the wealth ratio $k = C_{\text{R}}^{\text{e}}/C_{\text{P}}^{\text{e}}$, with $\Omega = 4$, and $\rho = -1$.

Proposition 2 (Expected Returns).

Fix Ω and $k \in [1, \overline{k}]$. Under the condition of Lemma 5, the poorer agent's expected return from the Nash-bargaining side bet satisfies

$$\mathbb{E}[\Delta C_{\text{P}}] \;\leq\; 0,$$

with equality if and only if $k \in \{1, \overline{k}\}$. Consequently, the richer agent's expected return satisfies

$$\mathbb{E}[\Delta C_{\mathrm{R}}] \;\geq\; 0,$$

with strict inequalities for all interior $k \in (1, \overline{k})$.

Proof. By Definition 4, actuarial fairness is characterized by $0 = p_{\text{fair}}\,\mathcal{G}_{\mathrm{R}} - (1 - p_{\text{fair}})\,\mathcal{G}_{\mathrm{P}}$. Hence, for any p,

$$\mathbb{E}[\Delta C_{\mathrm{P}}] = p\,\mathcal{G}_{\mathrm{R}} - (1-p)\,\mathcal{G}_{\mathrm{P}} = \big(\mathcal{G}_{\mathrm{R}} + \mathcal{G}_{\mathrm{P}}\big)\big(p - p_{\text{fair}}\big).$$

Lemma 5 gives $p \leq p_{\text{fair}}$ (strictly $<$ on $k \in (1, \overline{k})$), which yields the stated inequalities for $\mathbb{E}[\Delta C_{\mathrm{P}}]$. For the richer agent,

$$\mathbb{E}[\Delta C_{\mathrm{R}}] = (1-p)\,\mathcal{G}_{\mathrm{P}} - p\,\mathcal{G}_{\mathrm{R}} = -\mathbb{E}[\Delta C_{\mathrm{P}}],$$

so $\mathbb{E}[\Delta C_{\mathrm{R}}] \geq 0$ (and > 0 in the interior). ∎

Therefore, in situations with scarce indivisible goods, or other real-world features which convexify equilibrium welfare functions such as status concerns, we should expect to see relatively poor agents with equal bargaining power accept negative expected-return bets when the gains from winning are greater than the losses from losing (i.e., when the payoffs from the gambles are positively skewed). As the poorer agents become relatively poorer (but with $k < \overline{k}$) these bets become, from the point of view of the poorer agent, both more positively skewed and with a more negative expected payoff: the two features of such a side bet are inseparable. Our model predicts that agents will dislike small risks but, provided they are relatively poor, will seek out positively skewed side bets with negative expected returns, with the skew increasing and the expected returns decreasing as they become relatively poorer. Consistent with this prediction, empirical evidence on skewness-loving behavior has been documented in equity markets (Mitton and Vorkink, 2007, Boyer et al., 2010, Bali et al., 2011, Amaya et al., 2015, Lin and Liu, 2018).

There still exist zero-expected-return side bets which both agents would strictly prefer to no betting. However, given equal or superior bargaining power, agent R strictly prefers gambles

with a positive expected return that agent P is also willing to accept over no betting.

B. Lotteries among the poor

In the two-agent case of side bets, the gamble always produces the same pair of wealth allocations, with initial inequality affecting only the probability that the poorer agent obtains the better outcome. Moving from this two-agent setting to a many-agent environment introduces a different mechanism. When several poorer agents pool resources in a lottery, the winner does not leap to the top allocation but instead gains enough wealth to raise her willingness to pay, thereby increasing the price faced by the rich agent. Lotteries thus offer a parallel channel to side bets: both alter the terms of trade, but through distinct dynamics—side bets by fixing outcomes with probabilities tied to inequality, and lotteries by concentrating wealth among poorer agents to strengthen one bidder.

Definition 5 (**Poor-Agents Lottery**)**.**
Let there be a group of $n \geq 2$ identical, equally poor agents whose endowments are $\{C_n^{\mathrm{e}}, H_n^{\mathrm{e}}\}$ such that $nC_n^{\mathrm{e}} + C_{\mathrm{R}}^{\mathrm{e}} = \Omega$ and $nH_n^{\mathrm{e}} + H_{\mathrm{R}}^{\mathrm{e}} = 1$. A symmetric lottery among the poor agents is the pair

$$\mathcal{T} \equiv \{\, n,\ \tau_n \,\}, \qquad \tau_n > 0,$$

with n identical participants, each paying a ticket price τ_n into a common pot. One participant is selected uniformly at random as the sole winner. If each participant's pre-lottery divisible wealth is C_n^{e}, the post-lottery wealth of agent i is

$$C_i^{\mathcal{W}} = C_n^{\mathrm{e}} + (n-1)\tau_n \quad (i \text{ wins}), \qquad C_i^{\mathcal{L}} = C_n^{\mathrm{e}} - \tau_n \quad (i \text{ loses}),$$

so that $\frac{n-1}{n} C_i^{\mathcal{L}} + \frac{1}{n} C_i^{\mathcal{W}} = C_n^{\mathrm{e}}$. If nobody participates in the lottery, agent i sells her share of the indivisible good to the rich agent at her reservation price and obtains

$$V_i^{\mathcal{NT}} = u(C_n^{\mathrm{e}} + q(C_n^{\mathrm{e}})H_n^{\mathrm{e}}, 1),$$

where $q(\cdot)$ solves

$$u(x + q(x)H_n^{\mathrm{e}}, 1) = u(x + q(x)(H_n^{\mathrm{e}} - 1), 2).$$

Assuming $C_i^{\mathcal{W}} \leq C_{\mathrm{R}}^{\mathrm{e}}$ (the winner remains poorer than the rich agent), expected utility from playing the lottery is

$$V_i^{\mathcal{T}} = \tfrac{n-1}{n} u(C_i^{\mathcal{L}} + q(C_i^{\mathcal{W}})H_n^{\mathrm{e}}, 1) + \tfrac{1}{n} u(C_i^{\mathcal{W}} + q(C_i^{\mathcal{W}})H_n^{\mathrm{e}}, 1),$$

and participation occurs iff $V_i^{\mathcal{T}} > V_i^{\mathcal{NT}}$.

Proposition 3 (**Existence of a Beneficial Lottery**)**.**
There exists $\epsilon > 0$ such that for all $0 < \tau_n < \epsilon$ the lottery $\mathcal{T} = \{\, n,\ \tau_n \,\}$ is strictly preferred to nonparticipation:

$$V_i^{\mathcal{T}} > V_i^{\mathcal{NT}}.$$

Proof. Let $C_i^{\mathcal{L}} = C_n^{\mathrm{e}} - \tau_n$ and $C_i^{\mathcal{W}} = C_n^{\mathrm{e}} + (n-1)\tau_n$. Define

$$y(\tau_n) := V_i^{\mathcal{T}} - V_i^{\mathcal{NT}} = \tfrac{n-1}{n} u(C_i^{\mathcal{L}} + q(C_i^{\mathcal{W}})H_n^{\mathrm{e}}, 1) + \tfrac{1}{n} u(C_i^{\mathcal{W}} + q(C_i^{\mathcal{W}})H_n^{\mathrm{e}}, 1) - u\big(C_n^{\mathrm{e}} + q(C_n^{\mathrm{e}})H_n^{\mathrm{e}}, 1\big).$$

Clearly $y(0) = 0$, and y is differentiable at 0. Taylor's theorem with remainder yields

$$y(\tau_n) = \underset{:=\Phi}{y'(0)}\, \tau_n + o(\tau_n) \qquad (\tau_n \to 0^+).$$

A direct application of the chain rule gives

$$\Phi = (n-1)\, H_n^{\mathrm{e}}\, q'(C_n^{\mathrm{e}})\, u_C\big(C_n^{\mathrm{e}} + q(C_n^{\mathrm{e}})H_n^{\mathrm{e}}, 1\big).$$

In the CES-log specification with $\rho < 0$, implicit differentiation of $u(x + q(x)H_n^{\mathrm{e}}, 1) = u(x + q(x)(H_n^{\mathrm{e}} - 1), 2)$ implies $q'(C_n^{\mathrm{e}}) > 0$; moreover $u_C > 0$. Hence $\Phi > 0$.

By the definition of $o(\tau_n)$, for any $\eta > 0$ there exists $\epsilon > 0$ such that $0 < \tau_n < \epsilon \Rightarrow |o(\tau_n)| \leq$

$\eta\,\tau_n$. Choosing $\eta = \Phi/2$ gives, for all $0 < \tau_n < \epsilon$,

$$y(\tau_n) \;\geq\; \Phi\,\tau_n - |o(\tau_n)| \;\geq\; \tfrac{\Phi}{2}\,\tau_n \;>\; 0.$$

Therefore, for all sufficiently small $\tau_n > 0$, the lottery $\mathcal{T} = \{\, n, \tau_n \,\}$ is strictly preferred to nonparticipation: $V_i^{\mathcal{T}} > V_i^{\mathcal{NT}}$. ∎

Figure 4 shows how the expected utility of poor agents depends on the lottery fee relative to wealth. The solid line gives expected utility from participation, while the dashed line shows baseline utility from non-participation. Participation is strictly preferred across a wide range of fees, with the optimal lottery defined by the fee that makes the winner as wealthy as the rich agent. This stylized example illustrates why lotteries can remain attractive even when participation costs are relatively high.

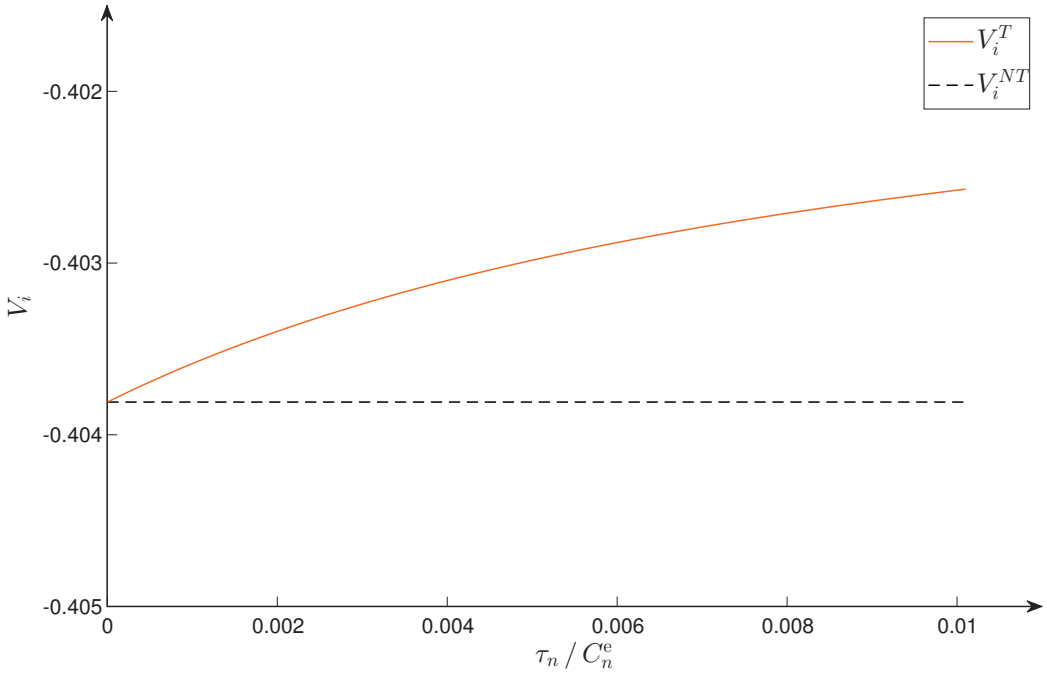


Figure 4. Expected utility of poor agents from playing (solid) versus not playing (dashed) the lottery, as a function of τ_n/C_n^{e}. Calibration: $n = 100$, $C_n^{\mathrm{e}} = 2$, $C_{\mathrm{R}}^{\mathrm{e}} = 4$, and $\rho = -1$.

Finally we note that lotteries like the above are vulnerable to free-riding (poor agents who don't pay τ_n still benefit from the uplift in price). They are also vulnerable to spoiling action

by the richer agent, but we are ignoring strategic behavior possibilities in this first study. The main take-away from this section is that normal, rational agents who care only about their own consumption dislike small risks, like large risks, and, provided they are sufficiently poor, will buy lottery tickets. In the next section, we will show that relatively poor agents also underinvest, compared to richer agents, in positive expected return assets.

IV. Equilibrium Risk-Taking under Aggregate Risk

We now consider a scenario with aggregate risk in the two–agent economy. In particular, we allow the aggregate endowment to vary across two equally likely states $\xi \in \{G, B\}$ according to

$$\Omega(G) = (\theta + \varepsilon)\Omega_0 \quad \Omega(B) = (\theta - \varepsilon)\Omega_0,$$

where $\theta \geq 1$ denotes the expected growth of the aggregate endowment, and $\varepsilon \in (0, \theta)$ introduces aggregate risk as a mean preserving spread of the zero-aggregate-risk case.

In the absence of gambles, inequality parameter k cannot vary across states., i.e. $C_{\mathrm{R}}^{\mathrm{e}}(\xi) = kC_{\mathrm{P}}^{\mathrm{e}}(\xi)$ for all ξ. With $k \geq 1$, agent R always acquires the additional unit and has higher welfare (except at equality). We have also analyzed all of the results in this section allowing agents to enter into side bets, and our results, both for side bets and for aggregate-risk bearing, are almost completely identical to our results for side bets in the absence of aggregate risk and for aggregate-risk bearing in the absence of gambles. To simplify exposition, we therefore treat them entirely separately in the paper.

A. Aggregate risk and welfare

The fact that agent R pays agent P her reservation price for the indivisible unit, which depends only on agent P's wealth, makes the role of aggregate risk very different from an economy with all divisible goods. Agent P's welfare is a leveraged function of her own wealth, while agent R's welfare is a deleveraged function of agent P's wealth. In effect, agent P provides insurance to agent R against the bad state by requiring a lower price for the indivisible good.

As a result, agent P's marginal utility of divisible consumption is much more sensitive to the aggregate endowment than agent R's, making agent P more reluctant to bear aggregate risk. This asymmetry implies that much of the benefit of an increase in aggregate wealth therefore accrues to agent P through a higher price of the indivisible unit, whereas agent R's divisible consumption remains comparatively state-insensitive. The price that must be paid to agent P forms a natural hedge against aggregate risk for agent R, and this hedge is provided by agent P, for whom the opposite is true.

To the best of our knowledge, this mechanism is novel. It has the potential to explain lower participation in assets exposed to aggregate risk, such as equities, in a straightforward fashion without requiring any agents to exhibit non-rationality or face investment frictions. It is also surely an important contributing factor to why inequality matters for welfare.

Proposition 4 (**Consumption Volatility**)**.**
Let $C_i(\xi)$, for $i \in \{\mathrm{R}, \mathrm{P}\}$, denote agent i's consumption of the divisible good at state ξ. Agent P's consumption is more volatile than agent R's, i.e.

$$\mathrm{Var}(C_{\mathrm{R}}) < \mathrm{Var}(C_{\mathrm{P}}),$$

provided that $k < \hat{k}$. Here $\hat{k} \in (1, \frac{1+H_{\mathrm{P}}^{\mathrm{e}}}{1-H_{\mathrm{P}}^{\mathrm{e}}})$ is the unique solution to

$$\hat{k} - 1 = 2H_{\mathrm{P}}^{\mathrm{e}} q'\left(\frac{(\theta-\varepsilon)\Omega_0}{1+\hat{k}}\right),$$

where $q(\cdot)$ is defined implicitly by $u(x + H_{\mathrm{P}}^{\mathrm{e}} q(x), 1) = u(x + H_{\mathrm{P}}^{\mathrm{e}} q(x) - q(x), 2)$.

Proof. Let $a = \frac{k}{1+k}$ and $b = \frac{1}{1+k}$. With two equally likely states G, B where $\Omega(G) = (\theta+\varepsilon)\Omega_0$ and $\Omega(B) = (\theta-\varepsilon)\Omega_0$, pre-transfer endowment is $C_{\mathrm{R}}^{\mathrm{e}}(\xi) = a\Omega(\xi)$, $C_{\mathrm{P}}^{\mathrm{e}}(\xi) = b\Omega(\xi)$. After transfers, with $h := H_{\mathrm{P}}^{\mathrm{e}}$,

$$C_{\mathrm{R}}(\xi) = a\Omega(\xi) - hq(C_{\mathrm{P}}^{\mathrm{e}}(\xi)), \qquad C_{\mathrm{P}}(\xi) = b\Omega(\xi) + hq(C_{\mathrm{P}}^{\mathrm{e}}(\xi)).$$

Define $\Delta\Omega = \Omega(G) - \Omega(B) = 2\varepsilon\Omega_0$ and $\Delta q = q(C_{\mathrm{P}}^{\mathrm{e}}(G)) - q(C_{\mathrm{P}}^{\mathrm{e}}(B))$. Then $\Delta C_{\mathrm{R}} = a\Delta\Omega - h\Delta q$ $\Delta C_{\mathrm{P}} = b\Delta\Omega + h\Delta q$. With two equally likely states,

$$\mathrm{Var}(C_{\mathrm{R}}) = \frac{1}{4}(\Delta C_{\mathrm{R}})^2, \qquad \mathrm{Var}(C_{\mathrm{P}}) = \frac{1}{4}(\Delta C_{\mathrm{P}})^2.$$

By the mean value theorem, there exists $\zeta \in (b\Omega(B), b\Omega(G))$ such that $\Delta q = q'(\zeta)b\Delta\Omega$. Cancelling $\Delta\Omega > 0$ gives

$$\mathrm{Var}(C_{\mathrm{R}}) < \mathrm{Var}(C_{\mathrm{P}}) \Longleftrightarrow k - 1 < 2hq'(\zeta). \tag{13}$$

Next, set $A(x) = x + hq(x)$, $B(x) = x - (1-h)q(x)$. The indifference condition $u(A(x), 1) = u(B(x), 2)$ yields

$$q'(x) = \frac{B^{\rho-1} - A^{\rho-1}}{hA^{\rho-1} + (1-h)B^{\rho-1}}, \qquad \rho < 0, \tag{14}$$

so $q'(x) \in (0, \frac{1}{1-h})$ and $q''(x) > 0$. Since $\zeta \in (b\Omega(B), b\Omega(G))$, we have $q'(\zeta) \geq q'(b\Omega(B))$. Hence from (13), a sufficient condition is

$$k - 1 < 2hq'(b\Omega(B)). \tag{15}$$

Define

$$S(k) = k - 1 - 2hq'\left(\frac{(\theta-\varepsilon)\Omega_0}{1+\hat{k}}\right).$$

At $k = 1$, $S(1) = -2hq'((\theta-\varepsilon)\Omega_0/2) < 0$. As $k \to \infty$, $q'(\cdot)$ is bounded, so $S(k) \to \infty$. Moreover,

$$S'(k) = 1 + 2hq''\left(\frac{(\theta-\varepsilon)\Omega_0}{1+k}\right)\frac{(\theta-\varepsilon)\Omega_0}{(1+k)^2} > 0,$$

so $S(k)$ is strictly increasing. By the intermediate value theorem there exists a unique $\hat{k} > 1$ with $S(\hat{k}) = 0$, i.e., solving the equation in the proposition. For all $k < \hat{k}$, we have $S(k) < 0$, hence condition (15) holds and therefore $\mathrm{Var}(C_{\mathrm{R}}) < \mathrm{Var}(C_{\mathrm{P}})$. Finally, since $q'(x) < \frac{1}{1-h}$, we obtain the outer bound $\hat{k} < \frac{1+h}{1-h}$. ∎

This represents a relatively high bound on k. Essentially, unless agent R is much richer than

agent P at date zero, agent P's consumption is more affected by the state at date 1. It follows that agent R's marginal utility of consumption is not overly sensitive to the state over a wide range of k, whereas agent P's marginal utility is much higher in the bad state than in the good state. Thus, in this range, agent P has a stronger desire to rebalance resources across states, hence reducing the aggregate risk she bears.

It is worth recalling why this is not the case when the second good is divisible. In equilibrium, agents' marginal utilities will be very similar to each other whatever the state, even though they differ across states, because each agent can equate their marginal utilities of each form of consumption and each will have the same marginal rate of substitution. As a result, each agent has the same demand for rebalancing resources across states.

B. The choice of how much risk to bear

In this last section, we examine what happens when agents are allowed to choose the level of aggregate risk they bear. There are two assets: a riskless asset with price normalized to one, which pays R_f at date 1, and a risky asset with price Q, which pays $R(\xi) = X(\xi)/Q$. The supplies of the two assets are S_f and S_X, implying aggregate divisible endowment

$$\Omega_0 = S_f + QS_X, \qquad \Omega(\xi) = X_f S_f + X(\xi) S_X.^{9}$$

At date 0, each agent allocates a fraction α_i of initial wealth to the risky asset, investing the remainder in the safe asset. Date-1 wealth is then

$$C_i^{\mathrm{e}}(\xi; \alpha_i) = [(1 - \alpha_i)R_f + \alpha_i R(\xi)]\, C_{i,0}^{\mathrm{e}},$$

[9]The ratio S_f/S_X is arbitrary. To close the model, we impose a normalization on the division of aggregate wealth between the two assets. Specifically, we fix the split between the risky and the risk-free asset in aggregate wealth in the good state. This anchors the system, while in all other states the division and returns adjust endogenously with aggregate risk.

and α_i is chosen to maximize expected utility,

$$\max_{\alpha_i} \mathbb{E}\left[V_i(C_i^{\mathrm{e}}(\xi;\alpha_i), H_i^{\mathrm{e}})\right].$$

In equilibrium, the sum of agents' demands must equal the supplies of the risky and risk-free assets.

In the benchmark case with a divisible second good, both agents allocate the same proportion of wealth to the risky asset, since they have identical risk preferences. By contrast, with indivisibilities, the poorer agent consistently invests a smaller fraction of wealth in the risky asset than the richer agent (see Figure 5). Both agents reduce risk-taking as aggregate risk increases, but the richer agent always holds relatively more risk. These results are almost identical when the agents are able to gamble as well as choose how much risk to bear.

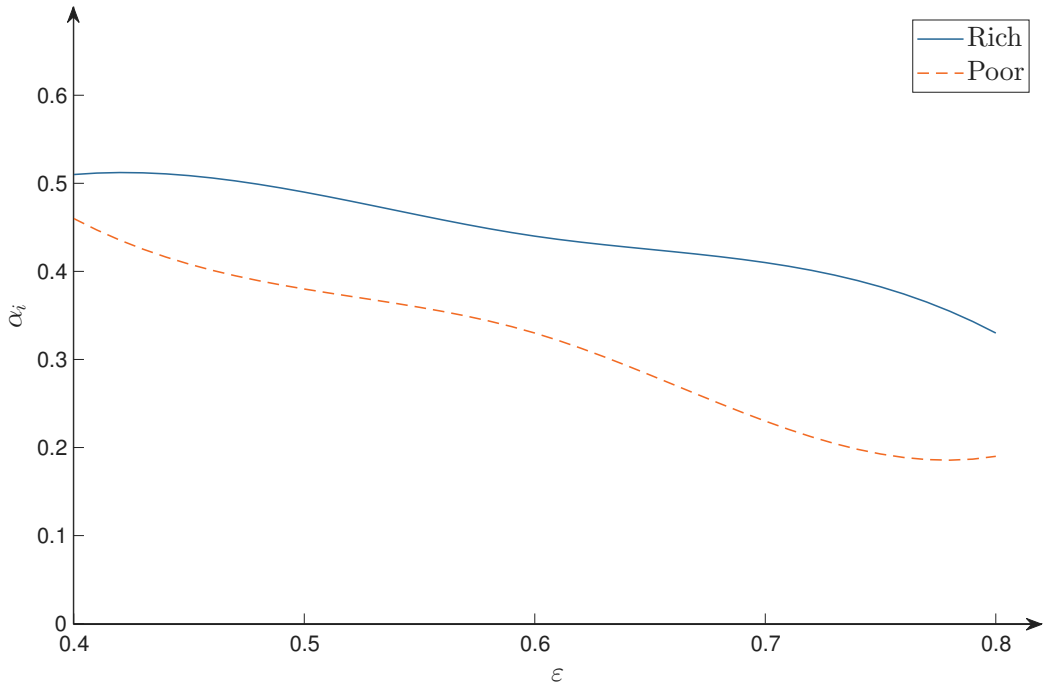


Figure 5. Equilibrium portfolio shares of the risky asset under different levels of aggregate risk. Calibration: $\Omega_0 = 2$, $\theta = 2$, $k = 1.6$, $\rho = -1$.

Since the risky asset is a leveraged claim on the aggregate endowment, an agent with a higher share of wealth invested in the risky asset will have a higher expected growth rate of wealth than

the growth rate of the aggregate endowment, which in turn will grow faster (in expectation) than the wealth of the agent with the lower share. Our results imply that the richer agent should on average increase their wealth faster than the economy can grow, resulting over time in a higher share of aggregate wealth. Thus, our equilibrium is consistent with Piketty's observation that returns on capital appear to be higher than returns for the aggregate economy.

Now that agents hold different shares of wealth in risky assets, given the previous observation, the degree of inequality of wealth k is expected to increase from date 0 to date 1. In principle, this alters the terms of the gamble entered into by the two agents. However, when we examine the optimal gamble in the presence of a riskfree asset, we find very little difference from the gambles discussed in the previous subsection, with all of our qualitative results still holding.

Indivisible goods thus provide a non-behavioral explanation for why relatively poor agents, under fairly realistic conditions, may allocate a smaller share of their wealth to assets with positive expected returns, such as equities, compared to richer agents. In combination with the previous section, which shows the same agents should also engage in negative expected return gambles under the same conditions, or even buy lottery tickets if poor enough, we can rationalize almost all the supposedly 'irrational' behavior observed of relatively poor agents without making any behavioral assumptions (Dimmock et al., 2021).

V. Conclusion

We develop a general framework for an endowment economy with multiple agents possessing heterogeneous wealth and two complementary goods, one of which is indivisible. Our model can explain attitudes to inequality that are difficult to rationalize in a standard divisible-goods setting with homogenous preferences. The indivisibility generates relative wealth concerns endogenously, even though agents care only about their own absolute consumption. In the two-agent case, we show that agents voluntarily engage in gambles in which the poorer agent accepts a negative expected return. The probability that the poorer agent wins such a gamble provides a measure of social mobility and is strictly decreasing in inequality. In this sense, inequality is 'manufactured'

as in Rosen (1997) and Becker et al. (2005), so tax-funded redistribution of initial wealth may not change ultimate inequality.

Introducing aggregate risk to our model further sharpens these dynamics, with the poorer agent optimally bearing less aggregate risk. When the aggregate endowment has positive expected growth, this means that the poorer agent invests less in 'good' risks. Consequently, the richer agent's wealth is expected to grow faster than the aggregate endowment, as argued by Piketty (2014).

In an extension with multiple identical poor agents and one rich agent, we show that agents may prefer to participate in lotteries in order to create a winner, who will drive up the price paid by the rich agent for the indivisible good. When the gambling and aggregate risk channels are combined, very little changes and relatively poor agents can simultaneously buy lottery tickets and underinvest in stocks.

In future work, we intend to study further implications of our framework, such as how the distribution of wealth affects welfare. In a dynamic setting, our approach should allow us to analyze how the distribution of wealth affects both the expected evolution of inequality and social mobility. Another key step is to extend the basic model to a production economy. Finally, the framework has different implications for finance than standard models: for example, when agents' wealth levels differ, so will their optimal portfolio allocations.

Indivisible goods generate a convexity in indirect utility, thus providing one possible microfoundation for Friedman and Savage's original study. In this respect, our paper is related to Chetty and Szeidl (2007) and also work in corporate finance such as Basak et al. (2020). However, we think that the implications of indivisible goods are wider and more important than just offering a foundation for Friedman-Savage preferences. We hope this paper will be a successful call to incorporate the demand for indivisible goods into the general framework.

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