How do Households Suppress the Price of Tail Risk?

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Abstract

We examine how households dampen the price of tail risk in equity markets through their demand for Short Put Products (SPPs) – a globally popular class of structured products that offers high headline rates in exchange for implicitly selling a deep out-of-the money put option. Using a comprehensive dataset covering all index-linked SPP issuances worldwide since market inception, we document that higher SPP volumes are associated with a lower variance risk premium, especially at low moneyness and during times of market stress. A natural experiment based on a ban on a specific index-linked SPP reinforces the case for a causal interpretation. To uncover the underlying mechanism and quantify the equilibrium effects, we develop a structural model in which households can invest in the equity index, a risk-free asset, and SPPs. Risk-averse financial intermediaries optimize the headline rate offered on SPPs, match exogenous demand for put options, and hedge imperfectly by trading the underlying equity index. In this framework, SPP volume enters the pricing kernel and thereby affects the price of all puts. When volatility rises, stronger demand for SPPs – driven by higher headline rates – exerts increasing downward pressure on option prices, particularly at low strikes. Our findings reveal how household appetite for innovative security designs, likely shaped by non-standard preferences or beliefs, can significantly affect the cost of tail risk insurance for other market participants and carry broader implications for financial stability.

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I. Introduction

The variance risk premium, particularly at low strike, is large and difficult to reconcile with standard risk models, pointing to a persistently high market price of insurance against tail risk.[1] While financial intermediaries are the primary absorbers of this tail risk in normal times, writing puts to institutional investors seeking downside protection (Lakonishok, Lee, Pearson and Poteshman, 2007) and thereby influencing the variance risk premium (Gârleanu, Pedersen and Poteshman, 2009; Barras and Malkhozov, 2016), this supply is fragile. Regulatory risk limits and capital requirements force these intermediaries to curtail their positions precisely when demand for protection is highest, causing their supply of put options to drop and even reverse during periods of market stress (Chen, Joslin and Ni, 2019; Almeida and Freire, 2026), thereby affecting risk sharing in the economy.[2] Together, these facts raise the question of which agents can offer an alternative and more stable absorber of tail risk, including during periods of stress. Can households play this role, and if so, through which channel can it happen? The answer might have implications both for asset pricing and financial stability: household participation in tail risk markets may shape the equilibrium price of insurance and the resilience of its supply when it is most needed.

At first sight, households may appear ill-suited to provide insurance against tail risk, as they are widely viewed as highly risk averse and even loss averse. Yet, in the past two decades, households have increasingly allocated their savings to innovative financial products whose designs embed tail risk exposure, such as retail structured products (Celerier and Vallee, 2017; Vokata, 2023), structured funds (Gao, Hu, Kelly, Peng and Zhu, 2024), and retirement savings products (Barbu and Sen, 2024).[3] Among retail structured products, Short Put

[1] See for instance Dew-Becker and Giglio (2025) for a list of works on this topic.

[2] While investors with high risk appetite may step in as marginal suppliers, they do so only at elevated premia (Jurek and Stafford, 2015).

[3] Although these products embed options, their connection to option markets differs substantially from retail option trading in both magnitude and nature. Structured products are long-term investments that often embed deep out-of-the-money options, whereas retail option trades are typically short-dated and frequently in-the-money. Structured products are held by a broad segment of households – up to 15% in Europe – while retail option trading remains concentrated among a small group of investors allocating relatively

Products (SPPs) are the most widely sold globally. Through SPPs, households implicitly sell large quantity of deep out-of-the-money put options in exchange for high coupons, effectively turning them into suppliers of tail risk insurance. The size of SPP markets relative to underlying option markets, which is especially large at low moneyness and for certain equity indices, suggests that households' appetite for tail risk may significantly affect volatility prices.[4] As shown in Figure 1, for the three main equity indices – the Eurostoxx 50, Nikkei 225, and S&P 500, – SPP outstanding volumes represent approximately 25%, 15% and 5%, respectively, of open interest in out-of-the-money put options with maturities exceeding three months.

INSERT FIGURE 1

In light of this phenomenon, we use the development of the SPP market to explore our initial research question and investigate the mechanism and implications: Does households' demand for SPPs affect volatility prices, and particularly so for deeply out-of-the-money options? Does the magnitude of these pricing effects increase in periods of market stress? If so, what is the underlying mechanism? And what are the implications in terms of financial stability?

We address these questions both empirically and theoretically. First, we assemble a comprehensive dataset that includes all SPPs issued globally since the inception of the retail market for structured products, along with detailed data on option prices and open interest. We provide reduced-form evidence of a robust link between realized volatility, SPP issuance, and volatility prices. To gain identification, we exploit a natural experiment in South Korea – one of the most active market for SPPs– where regulators temporarily banned the issuance of a specific index-linked SPP. Second, we develop and estimate a structural model to flesh out

modest amounts. Therefore, despite high trading volumes, retail option trading, especially zero-days-to-expiry (0DTE) contracts, primarily affects *intraday realized* volatility (Adams, Fontaine and Ornthanalai, 2025; Dim, Eraker and Vilkov, 2024; Vasquez, Amaya, Pearson and Garcia-Ares, 2025), whereas structured products are more likely to influence the long-run equilibrium of *implied* volatility, i.e. volatility prices.

[4]While identifying empirically the exact drivers of households' demand is not the focus of this study, our model shows that it cannot be explained within a fully rational framework. This may affect the resilience of the supply, which we explore theoretically.

the economic mechanism and quantify the equilibrium effects of the SPP market development through a counterfactual analysis. We find that only non-standard preferences can explain household demand for SPPs, and that because financial intermediaries hedge imperfectly, SPP volumes affects the stochastic discount factor, thereby influencing put prices. Overall, we estimate that SPPs reduce implied volatility by 2 to 5 percentage points, or up to 15%, with stronger effects during periods of heightened realized volatility, when demand for SPPs is particularly high.

Our dataset covers the global issuance of SPPs over the 2005-2019 period and includes key product characteristics for each SPP, including the underlying asset, maturity, detailed payoff design, issuance volume, and issuance date.[5] The payoff formula of these products depends on the performance of an underlying asset – typically an equity index or a stock – and is determined by two parameters: the *headline rate*, a fixed coupon paid periodically until maturity, and the *barrier*, usually set between 50% and 70% of the underlying price at issuance. At maturity, the investor receives their capital back if and only if the underlying asset is above the barrier. If the final value of the underlying falls below the barrier, the investor's final payment is adjusted for the full negative performance of the underlying asset. Hence, SPPs offer a fixed coupon in most scenarios, while fully exposing retail investors to left-tail risk.[6] These products are also often labeled as Yield-Enhancement Products (YEPs), although this terminology does not accurately capture their economic substance in our view.

We focus our analysis on equity index-linked SPPs, which account for 188,000 products and $650 billion in cumulated issuances across North America, Europe, and Asia over the 2005-2019 period, and represent around half of total SPP volumes. The twelve most popular underlying indices, for which we collect detailed data on the volatility surface and open interest, account for 99% of the equity index-linked SPPs. These indices also represent

[5]Data source is similar to the one in Celerier and Vallee (2017), except that the dataset for this study extends beyond Europe and covers a longer period.

[6]Many of these products also embed an autocall feature, meaning that the product is redeemed early if the underlying asset is above the barrier at any coupon date. This features limits the upside for the investor, while keeping the downside largely unchanged.

over 70% of global market capitalization, illustrating their relevance for financial stability. Median maturity, headline rate, and barrier levels in our sample are three years, 7.5%, and 60%, respectively. We extract headline rates – a central outcome of interest– using a text analysis algorithm applied to the product payoff descriptions. Finally, outstanding volumes and the open interest for put option of corresponding strikes and maturity amount to 13 and 67 billion dollars on average in 2015, suggesting a possible significant impact of SPPs on the price of tail risk.

Our empirical analysis yields four key findings. First, on the option market front, we find that higher SPP issuance volume predicts lower *implied* volatility. We control for *realized* volatility in all specifications, and find a similar effect on the volatility premium, defined as the spread between implied and realized volatility. This suggests that *vega*-hedging at issuance, rather than dynamic *gamma*-hedging, by the intermediaries is the central mechanism at play, as gamma hedging would primarily affect implied volatility via *realized* volatility. The effect of SPP volumes on implied volatility is strongest around the 60% moneyness level, corresponding to the most common SPP barrier, but is evident across all moneyness levels below 100%. Second, the effect is larger when the size of the SPP market relative to the corresponding option market is larger, such as for the Kospy 200, or the Hang Seng China Enterprise (HSCEI) indices. Third, the suppressing effect of SPP issuance on volatility prices is particularly pronounced following significant drops in the underlying index, when early redemption of outstanding products becomes unlikely. Fourth, on the SPP market front, we show that issuance volumes rise with realized volatility. This is likely because higher realized volatility leads to higher headline rates offered by SPPs, as the premium of the embedded put options increases with the underlying asset's volatility.

To strengthen causal identification, we exploit a natural experiment in South Korea, where regulators temporarily banned the issuance of HSCEI-linked SPPs in September 2015. We find that, while realized volatility declines after the ban – as markets stabilize after the market crash – implied volatility increases. This result holds in a difference-in-differences

specification controlling for index and month fixed effects, supporting a causal interpretation of the link between SPP issuances and volatility prices.

These empirical facts support the case that SPP issuances suppress volatility prices, and particularly so in times of stress when equity markets are down or volatility is high. Household appetite for SPPs would therefore make them play the somewhat unexpected role of volatility price stabilizer. These patterns are persistent at a relatively long horizon, and therefore do not correspond to the micro-structure concept of price impact. This persistence suggests that the asset pricing phenomenon we are exploring results from long run demand and supply equilibrium effects, which calls for a demand-based option pricing framework that endogenizes the design of SPPs and volatility prices, and delivers a stationary equilibrium.

We therefore develop such a theoretical framework to investigate the economic mechanism that can rationalize our empirical findings, and precisely characterize and quantify the equilibrium effects of the SPP market development. Financial intermediaries, who issue SPPs to households and trade options and the underlying asset to hedge SPP contracts, are a natural focus of our framework. When structuring short put products, intermediaries hedge the associated market risks. The typical hedge for a SPP requires the intermediaries to short a portfolio of puts on the underlying asset and short the dividend exposure through dividend swaps. If competitive intermediaries could hedge their risks perfectly, as would be the case in a Black-Scholes-Merton economy, options would be redundant assets and hedging would have no effect on option prices. In reality, however, even intermediaries cannot hedge options perfectly because trading cannot be implemented in continuous time due to transactions costs and because the volatility of the underlying evolves stochastically over time (Figlewski, 1989). In addition, intermediaries are sensitive to risk, for instance due to capital constraints and agency costs.

We accordingly develop a structural model with these features, which we estimate to rigorously describe and quantify the impact of household demand for SPPs on volatility prices. The economy is populated by a representative household, a representative dealer, and

outside investors with exogenous demand for options, in the style of Gârleanu et al. (2009), who all trade in discrete time. The investment set includes a riskless asset, an equity index, an SPP, and a set of plain-vanilla European puts on the equity index. Financial markets are driven by an exogenous state vector consisting of the prevailing interest rate and the stochastic volatility of the stock index. The dealer sets the headline rate and put prices, matches the demand for the SPP from households and puts from institutional investors, and partially hedges by trading the stock index. In line with the intermediary asset pricing literature, the dealer has CARA utility. The household can invest in the risk-free asset, the equity index, and the SPP, but cannot trade options. Each period, household demand for the SPP is affected by their time-invariant preferences, the endogenous SPP design (that is, the headline rate that it offers given a fixed maturity and barrier), and time-varying market conditions. This setting allows for substantial flexibility in modeling household preferences and beliefs.

We estimate the model by likelihood-based inference using ata on SPPs lined to the Eurostoxx 50, the underlying with the largest market share globally. For each level of the risk-free rate and the equity index volatility, the model provides the household demand for the SPP, the SPP headline rate set by the dealer, and the implied volatility of each put. We compare these values with monthly data on the swap rate, equity index return, SPP headline rate, and implied volatility of puts, which allows us to estimate the structural parameters of the model by maximum likelihood.

The model predicts that household demand for SPPs and headline rates increase when volatility increases and interest rates decrease, which is consistent with the data. However, the demand for the SPP is null when the representative household has Epstein-Zin Utility. In contrast, we do obtain significant household demand for SPPs when households underweight the left-tail risk, for instance when the household neglects to some extent the probability that the SPP may not pay the headline rate. This results carry implications regarding the resilience of the volatility stabilizing mechanism we describe, as households' demand might

drop if losses become salient.

To quantify the impact of the effect on volatility prices, we conduct a counterfactual analysis. We find that the implied volatility of a put contract on the equity index with a 60% strike is ca. 2 percentage points higher in the absence of the SPP compared to a market in which the SPP is traded. Furthermore, the equilibrium impact of SPPs increases with both the volume of issuance of SPPs, and the volatility of the equity index.

Our results suggest that the development of markets for innovative securities can affect the supply and demand equilibrium for volatility prices by incorporating non-standard household preferences into prices through intermediaries hedging strategies. Crucially, this impact is concentrated on the left-tail risk as households' appetite for taking this exposure, whether driven by rational or behavioral preferences, drives a security design that implicitly sells deep out-of-the-money put options. As option markets prices are key inputs for financial institutions risk management models, and as certain economic agents likely have elastic demand for downside protection, the effects we document have the potential to influence risk-taking, and in turn financial stability. At the same time, one may question the resilience of the supply of tail risk insurance originating from households, especially if the demand for SPP results from biased beliefs that may update, especially in case of losses. Our event study on the ban on HSCEI-linked SPPs in South Korea in 2015 illustrates how a sudden drop in housholds' demand leads to a surge in the price of tail risk.[7] More broadly, our results speak to the equilibrium effects of a change in the macro-structure of financial markets, in our case the introduction of a new asset class that allows some investors to express their preferences through their demand for these assets.

This paper makes contributions to multiple strands of the literature. Our work expands the burgeoning literature of market macrostructure covered in Haddad and Muir (2025) in a new dimension: asset prices are impacted by how financial institutions connect different asset classes or capital market segments to respond to a specific demand from a class of

[7]There also was a more recent ban on Kospi2-linked SPPs in February 2024.

investor previously excluded. This role is different from traditional financial intermediation, and is economically closer to what banks do when structuring securitization (Duffie and Rahi, 1995). It also ties to work identifying spillovers between distinct but related capital markets, such as derivative markets and their underlying asset markets (Calvet, Gonzalez-Eiras and Sodini, 2004; Henderson, Pearson and Wang, 2015).

Relatedly, our findings connect to models that emphasize supply and demand factors, such as the funding constraints financial intermediaries face in certain states of the world, or market segmentation, as key drivers of asset prices. Vayanos and Vila (2009) and Greenwood and Vayanos (2014) explain the term structure of riskless returns in a segmented supply and demand framework., where risk-averse intermediaries trade with end clients with strong preferences for specific-maturity bonds, hence driving price and return variations across different maturities. Our work relates closely to the literature studying options markets in this framework. Gârleanu et al. (2009) develop a demand-based option pricing model and show that demand pressures from the put-call imbalance explain cross-sectional variations in volatility skewness across U.S. equity options. Chen et al. (2019) document that financial intermediaries switch from supplier to consumer of left-tail insurance in the option market during times of market stress. Almeida and Freire (2026) establish that demand for options from institutional investors is crucial to explain the pricing kernel. Our contribution is to uncover a novel, micro-founded source of supply for left-tail risk insurance, and quantify its equilibrium implications for asset pricing and financial stability.

The paper also adds to the literature on the variance risk premium (Bakshi, Carr and Wu, 2008; Bollerslev, Tauchen and Zhou, 2009; Todorov, 2010; Han and Zhou, 2012; Cao and Han, 2013) and its recent compression (Dew-Becker and Giglio, 2025). Our paper identifies a new type of economic agents receiving the risk premium, and provide a complementary explanation to Dew-Becker and Giglio (2025) for rationalizing its recent evolution.

Finally, this study speaks to the general equilibrium effects of the class of financial products studied in Henderson et al. (2015), Celerier and Vallee (2017), Vokata (2021), Calvet,

Célérier, Sodini and Vallee (2023), and Vokata (2023), retail structured products, which have been shown to be effective at catering to, or mitigating, household behavioral biases. Barbu and Sen (2024) study SPPs in the insurance annuity context.

The paper is organized as follows. Section II describes the market for SPPs and the data. Section III describes our empirical analysis and its results. Section IV presents our structural model. Section V develops its estimation. Section VI concludes.

II. Background and Data

A. SPPs: Definition and Market Development

Short Put Products (SPPs), also commonly referred to as *Yied Enhancement Products* (YEPs), are the most popular category of retail structured products. Retail structured products include any fixed-maturity investment products marketed to retail investors that offer a payoff varying automatically and non-linearly with the performance of an underlying asset.[8] Typically designed with embedded options, these products leave no room for discretionary investment decisions before maturity. While the underlying assets are primarily equity indices and individual stocks, they may also include commodities, fixed-income assets, or alternative indices.

A defining feature of SPPs is that their payoff at maturity embeds the sale of a put option, exposing retail investors to tail risk. More precisely, the capital remains protected on the downside as long as the underlying asset stays *above* a predefined *barrier*. If the underlying falls below this barrier, the investor participates in its negative performance. This downside exposure allows intermediaries to enhance the coupon payments offered at regular intervals until maturity – referred to as the *headline rate*. As a result, the payoff formula of a standard SPP is determined by two key parameters, the *headline rate* – the

[8]Exchange traded funds, which have payoffs that are a linear function of a given underlying financial index, are not retail structured products.

fixed coupon paid at regular interval until maturity, and the *barrier* – typically set between 50% and 80% of the underlying asset's price at issuance. Figure 2 illustrates the payoff diagram of a typical SPP at maturity, while Panel A of Figure 3 presents a histogram of barriers observed in our dataset.

INSERT FIGURE 2

INSERT FIGURE 3

The first SPPs were introduced in Europe in the early 2000s, since the inception of the retail market for structured products. Since then, their adoption has grown steadily worldwide, with volumes surging across Asia, Europe, and the U.S. alongside the broader expansion of retail structured products. As of 2023, global outstanding volumes of retail structured products exceed $2 trillion, with Asia accounting for 50% and the U.S and Europe each representing 25%.[9] While synthetic Capital Guarantee Products (CGPs) initially dominated the market, SPPs have since emerged as the leading design in most regions, representing nearly 40% of total issuances in 2017 and around $400 billion in outstanding volumes by 2024.[10] A large share of SPPs also embed an autocall feature, meaning that the product is redeemed early if the underlying asset trades above a pre-defined level at a coupon date. This features limits the upside for the investor as the headline rate is likely to be paid only during a short period, and therefore allows to raise the headline rate the product offer. This feature keeps the downside for the investor largely unchanged.

B. Hedging SPP Issuances and the Market for Out of the Money Puts

When issuing structured products, banks typically mitigate the risk of large payouts to investors by buying or selling options that offset these liabilities. In the case of SPPs, the issuing bank sells put options that mirror the exposure of the end user, or roll similar

[9]See SRP Global Market Review: H1 2024.

[10]See Calvet et al. (2023) for more details on CGPs.

positions with a shorter maturity. Banks often rely on a combination of initial hedging with options and dynamic vega-hedging using the underlying asset. This latter approach is necessary because the autocall feature creates a Vanna exposure, i.e. a sensitivity of the product vega to changes in the underlying price (and volatility) .

A key characteristic of SPPs is the large relative size of their market compared to the put option markets where banks hedge. To hedge these products, banks typically sell deep out-of-the money puts with relatively long maturity. However, the open-interest on these markets is generally limited, often not exceeding a few hundred billion dollars. The largest option market is in the U.S., where institutional investors such as pension funds and hedge funds actively participate and buy puts to protect their portfolios against pronounced drops. Europe follows as the second-largest market, while Asia – despite having the largest share of SPP outstanding volumes as of 2015 – has a relatively small options market.

C. Data

C.1. SPPs Volumes, Underlying Assets, and Payoff Formula

We obtain detailed information on all retailed structured products issued globally since market inception, including SPPs, from a specialized private data provider. For each product, we observe key characteristics such as the underlying asset, maturity, exact payoff structure, issuance volume, issuance and redemption date.

While existing literature on retail structured products has primarily focused on specific geographic areas, such as Europe and the U.S., this study leverages the dataset's global coverage, exploiting data from products issued in North America, Europe and Asia. This comprehensive coverage provides a rich cross-section for analysis and is crucial for our study, as a large share of these products are structured on foreign underlying assets. Consequently, observing both domestic and foreign issuances is essential to fully capture the demand pressure on put option markets.

Our dataset includes all SPPs issued globally since market inception that matured before 2020. In total, it covers 362,000 products issued across 52 countries, representing $1.3 trillion of cumulated issuance (in current dollars). To enhance the accuracy of each product's headline rate and barrier, we apply a textual analysis algorithm to the payoff description, following the approach in Celerier and Vallee (2017). This algorithm also enables us to identify key payoff features that influence headline rates.[11]

We focus our analysis on index-linked SPPs, which represent 45% of SPP total volumes, 188 000 products, and $650 billion in cumulated issuances over the 2005-2019 period. Our focus on index-linked SPPs is motivated by two key factors. First, we have access to high-quality volatility surfaces for the top 12 indices, which collectively cover more than 99% of index-linked SPPs. Second, these 12 indices represent over 70% of global market capitalization, making their impact on financial stability particularly relevant. The 12 indices, ranked by decreasing SPP market share size, are Eurostoxx 50, Nikkei, S&P 500, Hang Seng China Enterprise Index, Kospi 200, FTSE 100, Russell, Hang Seng Index, CAC 40, Swiss Market Index, DAX, and MIB Index.

C.2. Implied Volatility

For the three main SPP underlying indices – Eurostoxx 50, Nikkey, and S&P500 – we obtain a monthly panel dataset of implied volatility surfaces from a trading desk of a global investment bank. This dataset spans 2002 to 2019 and provides implied volatility at the underlying asset, moneyness, and maturity levels. For each index at time t, we observe implied volatility for moneyness levels ranging from 50% to 150% in 10% increments, with maturities of 3, 12, and 24 months. This data is of higher quality than the one from commercial vendor because the trading desk continuously model the volatility surface, even in the absence of listed trades.

For all 12 indices, we obtain volatility surfaces, prices, realized volatility, and open interest

[11]The code is available in the Internet Appendix.

from the OptionMetrics IvyDB Global Indices dataset. For each index at time t, we observe implied volatility across different strike prices. We merge these volatility surfaces with index price data to express strikes as a percentage of the underlying asset price. To ensure data reliability, we validate OptionMetrics data for strikes above 80% by comparing it against the investment bank's volatility surface.

Finally, we complement our dataset with swap rates for distributing countries, sourced from Bloomberg. Specifically, we include German swap rates for all European countries, along with swap rates for Canada, China, Hong Kong, Japan, South Korea, Singapore, Sweden, Switzerland, Taiwan, the UK, and the U.S.

C.3. Monthly Panel

To perform our empirical analysis, we construct a monthly panel of underlying indices spanning the 2005-2019 period. For each month-underlying observation, the panel includes issuance and outstanding volumes of SPPs, volatility prices at three maturity across different strikes, as well as average headline rates, realized volatility, and swap rates.

For each index, issuance volumes represent the total volume of SPPs that use the index as an underlying asset, divided by the total number of underlying assets in each product. This adjustment accounts for the fact that 88% of SPPs have more than one underlying asset, with an average of 2.5 underlying assets per product. To measure outstanding volumes, we compute the rolling sum of issuances, subtracting the rolling sum of matured products while accounting for early redemptions.

The final panel consists of 2,160 observations spanning 180 months across twelve indices.

D. Summary Statistics

Figure 4 illustrates the growth of the SPP market across the U.S., Europe, and Asia since 2005, highlighting its significant size relative to corresponding put option markets. The figure demonstrates the rapid adoption of SPPs in Europe and Asia, as well as their

more recent expansion in the U.S.. On average over the period, global outstanding volumes of index-linked SPPs accounted for 16% of the open interest in index-linked put options worldwide.

INSERT FIGURE 4

Tables I provides summary statistics on all variables included in our empirical analysis. The strike of the embedded put option ranges from 50% of the initial value of the underlying (10^{th} percentile) to 70% (90^{th} percentile). The median maturity is 3 years. On average, SPPs offer a headline rate and excess ex-post return of 6.5% (median 6%) and 4.9% (median 5.1%), respectively. In addition, most SPPs earned positive excess returns, as confirmed in Panel B of Figure 3. Panel B of Figure 3 reports the distribution of ex post excess returns across SPPs.

INSERT TABLE I

The second panel of Tables I provides summary statistics on our monthly panel of SPP volumes, volatility prices, and put option open interest. While the median outstanding volumes of SPPs amount to 3.4 billion $, the median open interest of put options of strikes at 80% or below amounts and maturity higher than three months amounts to 12.5 billion $, suggesting a possible large impact of SPPs on the price of tail risk.

III. Empirical Results

We establish two sets of reduced-form findings. On the SPP market side, we provide evidence that net issuance volumes increase when realized volatility is higher, as the headline rates SPPs offer are higher under these market conditions. On the option market side, we find that the price of volatility decreases when SPP volumes increase. This relationship is more pronounced when the market has experienced a significant drop recently, as the SPP autocall feature is then unlikely to be triggered. A natural experiment in South Korea leveraging

a targeted SPP ban strengthens a causal interpretation of the relationship between SPP volumes and volatility prices.

A. Realized Volatility and SPP Issuances

We first document that the headline rate of SPPs significantly varies with market conditions, namely realized volatility and the level of interest rates. In Figure 5, we plot the average headline rates of SPP linked to the EuroStoxx 50 and the 90-day realized volatility for that index between 2005 and 2020. We observe a clear positive correlation between these two quantities. This relationship is intuitive as the premium of a put increases when the volatility of the underlying asset is high, and SPPs embed a short position in puts. The relationship between headline rates and interest rates has been first uncovered in Celerier and Vallee (2017), although on a shorter and narrower sample.

INSERT FIGURE 5

To more precisely measure the relationship between headline rates and market conditions, we run the following regressions at the SPP issuance level:

$$\log(HeadlineRate_i) = \alpha + \beta_v HistVol_{a,t} + \beta_r IR_t \gamma_i + \epsilon_i, \tag{1}$$

where $HeadlineRate_i$ is the headline rate of issuance i, $HistVol_t$ is the 90-day realized volatility in month t of the underlying asset a, and IR_t is the 3-year CMS swap rate in the country of issuance in month t, and γ_i are a set of fixed effects, namely month, underlying assets, product design and country fixed effects. Regression coefficients are presented in Table II. We observe that even in this saturated specification the level of realized volatility is highly predictive of the headline rate offered by a SPP, with an elasticity around 0.4. Interest rates also predict headline rate, which is consistent with banks aiming to preserve the headline rate when interests come down, as documented in Celerier and Vallee (2017). The economic magnitude of this relationship is less pronounced than the one with volatility,

which is to be expected as given that volatility has a more direct effect on the designs that banks can structure.

INSERT TABLE II

We also investigate whether headline rates, and relatedly realized volatility, are predictive of SPP net issuance volumes.[12] We regress the net issuance volume of issuance for SPP linked to a given index on quartiles of realized volatility, calculated at the index level. Consistent with household preferences for high headline rates, we observe that SPP issuance volume is significantly higher when headline rates offered are high, and relatedly when realized volatility is high. Together, these findings highlight the endogenous nature of SPP design and volume.

INSERT TABLE III

B. SPP Volumes and Volatility Prices

We now turn to studying whether SPP issuances predicts the level of implied volatility. In Panel A of Figure 1, we plot the average yearly issuance volume of SPP linked to a given index, scaled by the index open interest. In Panel B, we plot the volatility risk premium for the three indices, i.e. the difference between the 1 year implied volatility and the 90-day realized volatility, at the 60 to 100% moneyness, with 10% increments. We observe that the volatility risk premium is negatively correlated with the amount of SPP issuance, and that this relationship is more pronounced for moneyness around 60%.

This relationship is confirmed when looking at the broader sample of indices. We plot the average volatility premium over SPP outstanding volumes scaled by open interest in figure 6 over the 12 indices we previously study. We observe a strong negative relationship in this cross-section.

[12]Net issuance is the volume of SPP issuance in a month minus the volume that is redeemed in that month, early or not. Banks typically issue a new SPP when a given issuance is redeemed, to encourage investors to roll-over their capital. Similar to mutual funds, net flows better capture the demand for the asset class, and the change of notional that banks need to hedge.

INSERT FIGURE 6

To gain identification and identify the existence of this relationship in the time-series, we now run the following specification:

$$IV^{u}_{K=k,t} = \alpha + \beta SPPVolumes + \beta_v HistVol_{a,t} + \beta_r IR_t + \gamma_t + \gamma^u + \varepsilon^u_t, \tag{2}$$

where $IV^{u}_{K=k,t}$ is the 1-year implied volatility of underlying asset u and moneyness k, in month t, $SPPVolume^{u}_{t}$ is the SPP volume with underlying asset u in month t. γ_t and γ^u are month and underlying fixed effects. Standard errors are clustered at the month level.

Table IV provides the coefficient estimates when we use the proprietary volatility surface for the three main indices, S&P 500, Eurostoxx 50, and Nikkei 225. We use absolute SPP issuance volumes in Column 1, and scale SPP issuance volumes by indices market capitalization in Columns 2 to 8. The main advantage of this proprietary dataset of volatility surface is that we can span moneyness from 50 to 100% (Columns 3 to 8).

Overall, we observe that the amount of SPP issuance is predictive of lower implied volatility. This result is robust when running regressions in levels or in logs, and when scaling volumes by indices market capitalization. When looking at the cross-section of moneyness, we observe that the relationship is more pronounced at 60%, and decreases as moneyness rises, consistent with the cross-sectional figure

INSERT TABLE IV

To better visualize these relationships, we plot in Figure 7 the coefficient estimates for dummies for quintiles of issuance volumes in Panel A, and strikes, in Panel B in a regression where the dependent variable is the 1 year implied volatility (in level). These figures suggest a concave relationship between SPP volumes and implied volatility, and confirm the monotonic relationship relative to moneyness.

INSERT FIGURE 7

Finally, Figure 2 confirm that we obtain similar result when using the full sample of index-linked SPP and volatility surface data from option metrics.

INSERT TABLE V

C. Market Downturns

Table VI shows the dampening effects of SPP outstanding volumes in periods of market downturns. We create dummies for index yearly returns below 15%, 25% and 35%. As expected given the well known negative correlation between volatility and returns, we observe that implied volatility increases during market downturns. However, this increase is significantly lower when outstanding SPP volumes are high. The economic magnitude is large, as changing quartiles of SPP outstanding volume can affect implied volatility under market stress by several percentage points. The magnitude is larger when the market drop is more pronounced. This result is consistent with SPPs playing a particularly pronounced dampening role on the price of tail risk in times of stress.

INSERT TABLE VI

D. Natural Experiment

In September 2015, following a warning from the South Korean Financial Services Commission (FSC), the four largest SPP providers in South Korea decided to halt the issuance of HSCEI-linked SPPs. The warning came after a SPP provider announced that HSCEI-linked SPPs issued in April 2015 were expected to hit the barrier and trigger capital losses due to the sharp decline in the HSCEI index. The FSC cautioned SPP providers, stating that it might consider banning SPPs linked to certain indices. In response, all major SPP providers in South Korea ceased issuing HSCEI-linked SPPs. Panel A of Figure 8 illustrates the drop in the global issuance of HSCEI-linked SPPs after the warning. The effect is significant, as

South Korea accounts for more than 80% of the global HSCEI-index linked SPP market as of 2015.

INSERT FIGURE 8

Panel B of Figure 8 displays the associated impact on the volatility surface: following the regulator's warning, volatility prices increased, with the effect being particularly pronounced at lower strike prices. The figure plots the average HSCEI volatility surface in the year before and the year after the ban, highlighting the shift. The sharpness of the shock allows us to gain causal identification.

The endogeneity of the regulator's decision is a natural concern, given that it was triggered by changes in market conditions. However, if anything, this endogeneity would bias our estimates *downward*. The 35% drop in the HSCEI index is accompanied by several months of elevated *realized* volatility, as shown in Panel A of Figure 8. As market stabilize following the crash, one would expect *realized* volatility to decline, and subsequently *implied* volatility to fall as well. While we do observe a decrease in realized volatility, as shown in Figure A, the implied volatility of the HSCEI index increases.

INSERT TABLE VII

Results displayed in Table VII suggest an increase by over 3 percentage points in the HSCEI index implied volatility following the informal ban of linked SPP products. The Table shows the coefficient estimate of a dummy variable equal to one for HSCEI-index linked SPPs after September 2015 in a difference-in-differences model with index and month fixed effects. We restrict the sample period to one year before and one year after the informal ban. Columns 1 and 5 show the average effect across all strikes, with and without controls for realized volatility and the 3-year swap rate, respectively. Columns 2 to 5 and 6 to 8 show the effects at the 80, 90 and 100% strikes. Most of the effect is on the lowest strike, consistent with the embedded puts being deep out-of the money.

IV. Structural Model

In this section, we develop a quantitative equilibrium model, motivated by our empirical analysis, that jointly determines the design and volume of SPPs as well as the implied volatilities of put options. The model endogenizes the SPP headline rate – a central design parameter – while treating the barrier and maturity as fixed. Market conditions, represented by a bond yield and the level of equity volatility, enter as state variables, allowing us to condition endogenous outcomes on these factors. We use this theoretical framework to quantify equilibrium effects through counterfactual exercises and to analyze how household beliefs and preferences shape volatility prices.

A. Assets

We consider a discrete-time financial market defined at dates $t = 0, 1, \ldots, \infty$. Four securities are traded at each date t: (i) a riskless zero-coupon bond, (ii) an equity index fund, (iii) an SPP written on the equity index, and (iv) a set of plain-vanilla European puts on the equity index. The zero-coupon bond, the SPP, and the European puts all reach maturity at date $t + T$. In the empirical implementation of the model, the time index t corresponds to a month and the maturity parameter is set to $T = 36$ months, which corresponds to the median maturity of SPPs linked to the Eurostoxx 50.

Zero-coupon bond and equity index. The zero-coupon bond and the equity index have infinitely elastic supplies, so their returns are taken as exogenous to the model. Time-varying conditions in the bond and equity markets are specified by the exogenous state vector:

$$z_t = (z_{1,t}, z_{2,t})', \tag{3}$$

where $z_{1,t}$ is the annualized yield on the zero-coupon bond and $z_{2,t}$ drives the volatility of the equity index return, as we now explain.

We assume that the annualized yield, $z_{1,t}$, follows a first-order autoregression:

$$z_{1,t} - \mu_1 = \phi_1(z_{1,t-1} - \mu_1) + \sigma_1 \varepsilon_{1,t} \tag{4}$$

for every t, where μ_1, ϕ_1, and σ_1 are fixed parameters, and $\varepsilon_{1,t}$ is a stochastic shock. The net arithmetic return on the bond between t and $t+T$ is given by $R_{f,t:t+T} = (1+z_{1,t})^{T/12} - 1$.

Turning to the equity index, we denote by $R_{\text{M},t:t+1}$ its net arithmetic return between t and $t+1$, and by $R_{\text{M},\,t:t+T} = (1+R_{\text{M},t:t+1})\ldots(1+R_{\text{M},T-1:T}) - 1$ its net return between t and $t+T$. We specify that log excess return on equity between t and $t+1$ by a process with time-varying mean and volatility:

$$\ln(1+R_{\text{M},t:t+1}) - \ln(1+R^{(1)}_{f,t:t+1}) = \mu_r^{(1)}(z_t) + e^{z_{2,t+1}/2}\varepsilon_{\text{M},t+1},$$

where $R^{(1)}_{f,t:t+1}$ is the return on a 1-period zero-coupon bond over the period, $\mu_r^{(1)}(z_t)$ is a function of the state, $z_{2,t+1}$ is the state variable controlling stochastic volatility, and $\varepsilon_{\text{M},t+1}$ is a stochastic shock. The state variable controlling equity volatility, $z_{2,t}$, satisfies the first-order autoregression:

$$z_{2,t} - \mu_2 = \phi_2(z_{2,t-1} - \mu_2) + \sigma_2 \varepsilon_{2,t} \tag{5}$$

in every period t, where μ_2, ϕ_2, and σ_2 are fixed parameters, and $\varepsilon_{2,t}$ is a shock.

For parsimony, we assume a constant spread between the log yields of the T-period bond and the 1-period bond: $\ln(1+z_{1,t})/12 - \ln(1+R^{(1)}_{f,t:t+1}) = \pi$. This avoids using a more complex model of the term structure and thus a higher dimensional state space. The log excess return on equity relative to the T-period bond yield, $r^e_{\text{M},t:t+1} = \ln(1+R_{\text{M},t:t+1}) - \ln(1+z_{1,t})/12$, satisfies

$$r^e_{\text{M},t:t+1} = \mu_r(z_t) + e^{z_{2,t+1}/2}\varepsilon_{\text{M},t+1}, \tag{6}$$

where $\mu_r = \mu_r^{(1)} - \pi$. In the empirical application, we estimate equation (6) directly, thereby avoiding the need to estimate the short yield process.

We close the specification of bond and equity returns by assuming that the vector of stochastic shocks, $\varepsilon_{t+1} = (\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, \varepsilon_{\text{M},t+1})'$, is independent through time. Furthermore, it is Gaussian with zero mean and variance-covariance matrix:

$$\Sigma = \begin{pmatrix} 1 & \rho_{1,2} & \rho_{1,\text{M}} \\ \rho_{1,2} & 1 & \rho_{2,\text{M}} \\ \rho_{1,\text{M}} & \rho_{2,\text{M}} & 1 \end{pmatrix} \tag{7}$$

in every period t.

Equations (3) to (7) fully specify the dynamics of bond and equity returns. Throughout the paper, we treat the vector of shocks, ε_t, the parameters μ_1, ϕ_1, σ_1, μ_r, μ_2, ϕ_2, σ_2, $\rho_{1,2}$, $\rho_{1,\text{M}}$, and $\rho_{2,\text{M}}$, and therefore the state vector reflecting market conditions, z_t, as exogenous inputs of the general equilibrium (GE) model.

Short put product. The SPP issued at t is illiquid until it reaches maturity at date $t+T$. Its return over this horizon, $R_{\text{SPP},\,t:t+T}$, depends on a barrier, B, set at origination, and is contingent on the performance of the equity index return, $R_{\text{M},t:t+T}$.

At maturity, the SPP pays a coupon rate, y_t, if the equity index return over the life of the product is above the barrier, and otherwise delivers the equity index return. More compactly, the net return on the SPP is given by:

$$R_{\text{SPP},\,t:t+T} = y_t \, \mathbb{1}_{\left\{R_{\text{M},\,t:t+T} \geq B\right\}} + R_{\text{M},\,t:t+T} \, \mathbb{1}_{\left\{R_{\text{M},\,t:t+T} < B\right\}},$$

where $\mathbb{1}_{\left\{R_{\text{M},\,t:t+T} \geq B\right\}}$ is an indicator equal to one if the equity return is at least B, and zero otherwise, and $\mathbb{1}_{\left\{R_{\text{M},\,t:t+T} < B\right\}} = 1 - \mathbb{1}_{\left\{R_{\text{M},\,t:t+T} \geq B\right\}}$.

In our model, the barrier B is treated as exogenous, while the SPP yield is endogenously determined by supply and demand forces. The coupon yield is therefore a function of the state:

$$y_t = Y(z_t), \tag{8}$$

where the mapping $Y(\cdot)$ is determined in GE. The SPP yield exceeds the risk-free rate over the period, $y_t > R_{f,t:t+T}$, whenever the bond return is strictly higher than the barrier, $R_{f,t:t+T} > B$ – as is the case for all SPP contracts in our sample.[13]

Puts. Investors can trade a set of plain-vanilla puts written on the equity index return, which are issued at t and mature at date $t+T$. Their strikes are exogenous and denoted by $K_1, \ldots, K_N$. A European put with strike K_n pays off $(K_n - 1 - R_{\text{M},\,t:t+T})_+$ at date $t+T$, where x_+ denotes the maximum of x and 0 for any real x. The prices of the European puts, $P_{\text{PUT},n,t}$, and their net returns, $R_{\text{PUT},n,t} = (K_n - 1 - R_{\text{M},\,t:t+T})_+ / P_{\text{PUT},n,t} - 1$, are endogenously determined in GE.

An important property of our setup is that the SPP can be only partially, not fully, hedged by trading the zero-coupon bond, the equity index, and European puts at date t. The reason is that the SPP payoff is discontinuous at the barrier, whereas the payoffs of the available hedging assets (bond, equity, puts) are continuous in the index. Put slightly differently, an SPP contract delivers the same payoff at maturity as a portfolio consisting of (i) a short position in one put option on the equity index with strike $1+B$, (ii) B units of the zero-coupon bond, and (iii) $(y_t - B)$ units of a digital call option that pays \$1 if the equity index return exceeds the barrier. Since digital calls are thinly traded and difficult to hedge in practice (Gallus, 1999), we assume that they are not traded in the model, so that the SPP is not a redundant asset.

[13]The condition $R_{f,t:t+T} > B$ is equivalent to the annualized yield satisfying $z_{1,t} > (1+B)^{12/T} - 1$. For instance, if $B = -40\%$ and T =36 months, this requires $z_{1,t} > 0.6^{1/3} - 1 = -15.7\%$, which holds in every period of our dataset. In simulations of the bond yield, we winsorize $z_{1,t}$ at a lower bound of -10%. Under the parameters reported in Table IX, the stationary distribution of $z_{1,t}$ has a first percentile of -4.4%, a 0.1th percentile of -6.5%, and a 0.01th percentile of -8.2%, so the -10% winsorization affects only a negligible fraction of simulated draws.

B. Agents

Every period t, financial markets are populated by three types of investors: a long-lived representative household, a representative dealer, and a group of investors with an inelastic demand for puts.

Households. The representative household at t trades and consumes at dates t, $t+T$, $t+2T, \ldots, \infty$. It can trade the riskless asset, the equity index fund, and the SPP, but importantly it does not have access to put markets.

The household's consumption-portfolio problem is defined as follows. The household starts period t with wealth W_t^H. It allocates C_t to consumption and invests the remaining wealth, $FW_t^H = W_t^H - C_t^H$, in financial assets through the dealer. The dealer receives a fraction ϕ of gross invested wealth, FW_t^H, as compensation for his services. The household's net invested wealth is therefore

$$X_t^H = (1-\phi)\, FW_t^H.$$

The household allocates a fraction $\alpha_{\text{M},t}^H$ of net invested wealth to the index fund, a fraction $\alpha_{\text{SPP},t}^H$ to the SPP, and the remaining fraction, $1-\alpha_{\text{M},t}^H-\alpha_{\text{SPP},t}^H$, to the zero-coupon bond. The financial wealth available to the household at the beginning of period $t+T$ is therefore

$$W_{t+T}^H = X_t^H \left(1 + R_{f,t:t+T} + \alpha_{\text{M},t}^H R_{\text{M},t:t+T}^e + \alpha_{\text{SPP},t}^H R_{\text{SPP},t:t+T}^e\right),$$

where $R_{\text{M},t:t+T}^e = R_{\text{M},t:t+T} - R_{f,t:t+T}$ and $R_{\text{SPP},t:t+T}^e = R_{\text{SPP},t:t+T} - R_{f,t:t+T}$ denote, respectively, the excess return on the index fund and the SPP over the period.

The household selects consumption and portfolio weights by maximizing the Epstein-Zin

recursive utility:

$$V^H(W_t^H; z_t, Y) = \max_{\{C_t^H, \alpha_{\text{M},t}^H, \alpha_{\text{SPP},t}^H\}} \left[(1-\delta^T)\,(C_t^H)^{1-1/\psi} + \delta^T\,(\mu_{t+T}^H)^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \tag{9}$$

where Y is the state-contingent SPP coupon yield defined in (8),

$$\mu_t^H = \left\{\mathbb{E}_t^{\mathbb{P}^*}[V^H(W_{t+T}^H)^{1-\gamma}]\right\}^{\frac{1}{1-\gamma}}$$

is the certainty equivalent of future consumption, and $\mathbb{P}^*$ is the subjective belief of the representative agent.

The subjective belief is defined as follows. Let $F_{\varepsilon_{\text{M}}}(\varepsilon_{\text{M},t})$ denote the cumulative distribution function of the shock, $\varepsilon_{\text{M},t}$, under the physical measure $\mathbb{P}$. The agent's subjective belief is characterized by

$$F_{\varepsilon_{\text{M}}}^*(\,\cdot\,) = w[F_{\varepsilon_{\text{M}}}(\,\cdot\,)],$$

where

$$w(p) = \exp\left[-b\,(-\ln p)^a\right] \tag{10}$$

for every $p \in (0,1]$, $a > 0$, and $b > 0$. This formulation modifies the marginal distribution of $\varepsilon_{\text{M},t}$, while preserving the dependence structure of the vector ε_t.[14]

The household is a price-taker viewing the state-contingent SPP yield and other asset prices as exogenous. We denote by $\alpha_{\text{SPP},t}^H = \alpha_{\text{SPP}}^H(z_t, Y)$ the optimal SPP weight in period t.

[14]Specifically, let $F_\varepsilon(x_1,x_2,x_3) = \mathbb{P}(\varepsilon_{1,t} \le x_1; \varepsilon_{2,t} \le x_2, \varepsilon_{\text{M},t} \le x_3)$ denote the cumulative distribution function of ε_t. We note that $F_\varepsilon(x_1,x_2,x_3) = C[\Phi(x_1),\Phi(x_2),\Phi(x_3)]$, where C denotes the copula of the multivariate Gaussian ε_t. Under the subjective probability $\mathbb{P}^*$, the cumulative distribution of ε_t is

$$F_\varepsilon^*(x_1,x_2,x_3) = C[\Phi(x_1),\Phi(x_2), w\circ\Phi(x_3)]. \tag{11}$$

This equation illustrates that the vector ε_t has the same copula under $\mathbb{P}$ and $\mathbb{P}^*$, and that $\varepsilon_{\text{M},t}^*$ has the marginal distribution $w\circ\Phi$. Furthermore, if (x_1,x_2,x_3) is sample from the distribution F, the vector $(x_1,x_2,\Phi^{-1}\circ w^{-1}\circ\Phi(x_3))$ is sampled from the modified distribution F_ε^*.

Dealers. The dealer available in period t has no initial wealth but receives from the household the management fee $X_t^D = \phi\, FW_t^H = \phi\, X_t^H/(1-\phi)$, or equivalently

$$X_t^D = X_t^H/\lambda^H,$$

where $\lambda^H = (1-\phi)/\phi$. The dealer allocates a fraction $\alpha_{\mathrm{M},t}^D$ of his wealth X_t^D to the index fund, a fraction $\alpha_{\mathrm{SPP},t}^D$ to the SPP, a fraction $\alpha_{\mathrm{PUT},n,t}^D$ to each put n, and the remaining fraction to the zero-coupon bond.

The dealer's wealth in period $t+T$ is $W_{t+T}^D = X_t^D\left(1+R_{t:t+T}^D\right)$, where

$$R_{t:t+T}^D = R_{f,\,t:t+T} + \alpha_{\mathrm{M},t}^D\, R_{\mathrm{M},\,t:t+T}^e + \alpha_{\mathrm{SPP},t}^D\, R_{\mathrm{SPP},\,t:t+T}^e + \sum_{n=1}^{N} \alpha_{\mathrm{PUT},n,t}^D\, R_{\mathrm{PUT},n,\,t:t+T}^e \tag{12}$$

denotes the dealer's portfolio return, and $R_{\mathrm{PUT},n,\,t:t+T}^e = R_{\mathrm{PUT},n,\,t:t+T} - R_{f,t:t+T}$ is the excess return of each put n over the period.

The dealer has constant absolute risk aversion (CARA) utility, $U_t = -\mathbb{E}_t^{\mathbb{P}}(e^{-A_t\, W_{t+T}^D})$. The CARA specification is commonly used in intermediary asset pricing (see, e.g., He and Krishnamurthy (2018)) because it permits the dealer to absorb the losses than can be caused by imperfect hedging or mismatches between supply and demand. We assume that the CARA coefficient in period t satisfies $A_t = \gamma^D/X_t^D$, which allows us to obtain stationary portfolio weights, as we now explain.

The dealer solves

$$\max_{\alpha_t^D} \mathbb{E}_t\left[-e^{-\gamma^D\,(W_{t+T}^D/X_t^D)}\right],$$

where $\alpha_t^D = (\alpha_{\mathrm{M},t}^D, \alpha_{\mathrm{SPP},t}^D, \alpha_{\mathrm{PUT},1,t}^D, \ldots, \alpha_{\mathrm{PUT},N,t}^D)$. Since $W_{t+T}^D/X_t^D = 1 + R_{t:t+T}^D$, the dealer's optimal portfolio is independent of the wealth level X_t^D. We denote the dealer's optimal portfolio by $\alpha^D[z_t; Y(z_t)]$. Since the dealer is short-lived, this choice depends only on the current market state, z_t, and the contemporaneous SPP coupon yield, $Y(z_t)$. This contrasts with the long-lived household, whose optimal portfolio in state z_t depends on the coupon

yield in *all* current and future states, as captured by the mapping Y.

Exogenous Demand for Puts. We assume that an unmodelled group of investors has an inelastic demand for puts, as in Gârleanu et al. (2009). Specifically, this group invests $d_n^E\, X_t^D$ units in each put n, where $d_n^E \in \mathbb{R}$. We take the number of puts demanded per unit of wealth, d_n^E, as exogenous. Importantly, the exogenous demand, $d_n^E\, X_t^D$, scales with the size of the economy, ensuring that the price impact of this demand pressure remains sizable over time.

C. Recursive General Equilibrium

Definition 1 (Recursive General Equilibrium): *A recursive general equilibrium consists of a state-contingent SPP yield, $Y(z_t)$, and state-contingent put prices, $P_{\text{PUT},n}(z_t)$, such that markets clear:*

$$\begin{aligned} \alpha^D_{\text{SPP},t}[z_t; Y(z_t)]\, X_t^D + \alpha^H_{\text{SPP},t}(z_t, Y)\, X_t^H &= 0 \\ \alpha^D_{\text{PUT},n}[z_t; Y(z_t)]\, X_t^D + d_n^E\, X_t^D\, P_{\text{PUT},n}(z_t) &= 0 \end{aligned}$$

in every period t and state z_t.

In GE, the dealer finds it optimal to offset the position of the household sector in the SPP market, and the exogenous demands of the other agents in the put markets. Dividing euilibrium equations by the dealer's wealth, we obtain that:

$$\alpha^D_{\text{SPP},t}[z_t; Y(z_t)] = -\lambda^H\, \alpha^H_{\text{SPP},t}(z_t, Y) \tag{13}$$

$$\alpha^D_{\text{PUT},n}[z_t; Y(z_t)] = -d_n^E\, P_{\text{PUT},n}(z_t). \tag{14}$$

in every state z_t.

In the appendix, we provide regularity conditions under which the following proposition

holds.

PROPOSITION 1 (Existence, Uniqueness, and Characterization of a Recursive GE): *There exists a unique stationary GE. In this equilibrium, the SPP yield, $Y(z_t)$, and the dealer's hedge, $\alpha_{\text{M}}^{D}[z_t, Y(z_t)]$, satisfy:*

$$\mathbb{E}\left(R^{e}_{\text{M},t:t+T}\, M_{t+T}\middle|\, z_t\right) = 0, \tag{15}$$

$$\mathbb{E}\left(R^{e}_{\text{SPP},t:t+T}\, M_{t+T}\middle|\, z_t\right) = 0, \tag{16}$$

where the excess return on the SPP satisfies

$$R^{e}_{\text{SPP},t+1} = Y(z_t)\,\mathbb{1}_{\left\{R_{\text{M},t:t+T} > B\right\}} + R_{\text{M},t:t+T}\,\mathbb{1}_{\left\{R_{\text{M},t:t+T} \leq B\right\}} - R_{f,t:t+T},$$

and the stochastic discount factor is given by:

$$M_{t+T} = e^{-\gamma^{D}\left\{\alpha^{D}_{\text{M}}[z_t, Y(z_t)]\, R^{e}_{\text{M},t:t+T} - \lambda^{H}\, \alpha^{H}_{\text{SPP}}(z_t, Y)\, R^{e}_{\text{SPP},t:t+T} - \sum_{n=1}^{N} d^{E}_{n}\,(K_n - 1 - R_{\text{M},t:t+T})_{+}\right\}}.$$

The price of each put n is given by:

$$P_{\text{PUT},n}(z_t) = \frac{\mathbb{E}_t\left[(K_n - 1 - R_{\text{M},t:t+T})_{+}\, M_{t+T}\middle|\, z_t\right]}{(1 + R_{f,t:t+T})\,\mathbb{E}\left(M_{t+T}\middle|\, z_t\right)}, \tag{17}$$

for every t and z_t.

We henceforth denote by $\mathcal{V}_n(z_t)$ the implied volatility of put n resulting from $P_{\text{PUT},n}(z_t)$.

V. Structural Estimation

A. *Estimation Strategy*

We turn to the estimation of our equilibrium model. This procedure has two objectives: (i) verify that SPP volume has a causal effect on put prices, and (ii) assess which household

preferences and beliefs are most consistent with observed SPP volumes. The estimation procedure must also overcome the limitations of our dataset.

To achieve these objectives, we reformulate the dealer's decision problem as follows. From the dealer's perspective, the demand for SPP in state z_t is itself a state variable that the dealer needs to provide. The dealer therefore has the expanded state vector $Z_t = (z_{1,t}, z_{2,t}, \lambda^H \alpha^H_{\text{SPP},t})$. Since the household's SPP share is not observed, we make the additional assumption that the empirical volume, $SPPVolume_t$, is proportional to the household's SPP weight, $\alpha^H_{\text{SPP},t}$:

$$\lambda\, SPPVolume_t = \lambda^H\, \alpha^H_{\text{SPP},t}.$$

For ease of interpretation, we scale volume by its historical maximum, so that $SPPVolume_t \in [0, 1]$. The SDF is then:

$$SDF_{t:t+T} = e^{-\gamma^D \left[\alpha^D_{\text{M},t}\, R^e_{\text{M},t:t+T} - \lambda\, SPPVolume_t\, R^e_{\text{SPP},t:t+T} - \sum_{n=1}^{N} d^E_n\, (K_n - 1 - R_{\text{M},t:t+T})_+\right]}.$$

The dealer computes the SPP yield, $y^D(z_t, SPPVolume_t)$, and the implied volatilities, $\mathcal{V}^D_n(z_t, SPPVolume_t)$.

We obtain parsimoniously specify the function $\mu_r(z_t)$ driving the log return as follows. We assume that

$$\mathbb{E}_t(r^e_{\text{M},t+1}) = \kappa\, Var_t(r^e_{\text{M},t+1}). \tag{18}$$

This specification is consistent with models of equity pricing. We also impose

$$\mathbb{E}_t(R_{\text{M},t:t+T} - R_{f,t:t+T}) = RP. \tag{19}$$

We show in the appendix that for any value of RP there exists a unique value of κ that satisfies equation (19). We set the risk premium RP to 15%, which corresponds to an annual risk premium of about 5% per year.

The estimation of our model is based on the following structural equations. The dynamics

of the interest rate and the equity index, which are exogenous to the model, are given by:

$$z_{1,t} = \mu_1 + \phi_1(z_{1,t-1} - \mu_1) + \sigma_1 \varepsilon_{1,t}, \tag{20}$$

$$z_{2,t} = \mu_2 + \phi_2(z_{2,t-1} - \mu_2) + \sigma_2 \varepsilon_{2,t}, \tag{21}$$

$$r^e_{\mathrm{M},t} = \mu_r(z_{t-1}) + e^{z_{2,t}/2}\varepsilon_{\mathrm{M},t} \tag{22}$$

The next set of equations characterize the dealer's ability to explain the SPP yield and the implied volatilities of put prices.

$$y_t = y^D(z_t, SPPVolume_t) + \sigma_{\mathrm{SPP}}\, u_t \tag{23}$$

$$IV_{n,t} = \mathcal{V}^D_n(z_t, SPPVolume_t) + \sigma_{\mathrm{IV}}\, \eta_{n,t} \tag{24}$$

Finally, the equation

$$SPPVol_t = \lambda^H\, \alpha^H_{\mathrm{SPP}}(z_t, y_t, Y) + \sigma_{\mathrm{VOL}}\, \nu_t \tag{25}$$

characterize the model's ability to explain household demand for the SPP contract.

The model is specified by the following parameters. The interest rate and equity index returns are parameterized by $\mu_1, \phi_1, \sigma_1, \mu_2, \phi_2$, and σ_2. By Proposition 1, the parameters γ^D, λ^H, and $d^E_1, \ldots, d^E_N$ are not identified, in the sense that $(\gamma^D, \lambda, d^E_1, \ldots, d^E_N)$ and $(\gamma^D/c, c\,\lambda, c\,d^E_1, \ldots, c\,d^E_N)$ are observationally equivalent. For this reason, we set $\gamma^D = 1$. In simulations, we have verified that the demand for the SPP, α exhibits limited variation with respect to the patience parameter, δ, and the elasticity of intertemporal substitution, ψ. For this reason, we calibrate δ and ψ and we estimate the model the coefficient of risk aversion, γ^H.

The structural estimation of the model has two objectives. First we wish to verify that SPP volume enters the stochastic discount factor and therefore impacts the implied volatility of put options. Second, we wish to explore the type of beliefs and preferences that can explain

the observed demand for SPPs and its sensitivity to market conditions.

A possible challenge is that a model's inability to explain household demand may impact the sensitivity of the SDF with respect to SPP volume. For this reason, we proceed in two steps. First, we use equations (20) to (24) to estimate

$$\theta^D = (\underbrace{\mu_1, \phi_1, \sigma_1, \mu_2, \phi_2, \sigma_2, \rho_{1,2}, \rho_{1,\mathrm{M}}, \rho_{2,\mathrm{M}}}_{\text{Interest rate and equity index parameters}}, \quad \underbrace{\lambda}_{\substack{\text{Sensitivity} \\ \text{of SDF to} \\ \text{SPP volume}}} \quad , \underbrace{d_1^E, \ldots, d_N^E}_{\substack{\text{Demand} \\ \text{for puts}}}, \quad \underbrace{\sigma_{\mathrm{SPP}}, \sigma_{\mathrm{IV}}}_{\substack{\text{Measurement} \\ \text{error}}} \quad).$$

The vector θ^D has $N + 12$ components, where N is the number of puts. Second, we use equation (25) to estimate the household parameters:

$$\theta^H = (\underbrace{\gamma}_{\substack{\text{Risk} \\ \text{aversion}}} , \underbrace{a, b}_{\substack{\text{Probability} \\ \text{weighting}}} , \underbrace{\lambda^H}_{\substack{\text{Scale} \\ \text{parameter}}} , \underbrace{\sigma_{\mathrm{VOL}}}_{\substack{\text{Measurement} \\ \text{error}}}).$$

The estimation proceeds by likelihood inference, as we explain below.

B. Does Higher SPP Volume Cause a Decline in Volatility Prices?

We estimate the parameter vector θ^D by likelihood-based inference. Since the stochastic volatiluty of the equity ijdex, $z_{2,t}$, is latent, we simulate the likelihood function by using a particle filter for $z_{2,t}$, as we explain in Appendix B.

INSERT TABLE IX

The results from the model estimation are provided in Table IX. The unrestricted estimate in column 1 corresponds to our baseline model. The restricted estimate in column2 corresponds to model where λ is set to zero, i.e. where household demand for SPP cannot play a role in the stochastic discount factor. The comparison between these two estimations

allows us to reject the null hypothesis that household demand for SPPs do not play a role in the stochastic discount factor.

C. Can we Explain Household Demand for the SPP?

We consider a version of the model under which investors neglect the probability that the equity index may generate a return lower than the barrier and the SPP may therefore pay less than the headline rate. The demand for the SPP is then substantial.
The SPP yield spread increase with the interest rate and equity volatility, which is consistent with the empirical evidence in Figure 5.

In Figure 9, we compare the implied volatility curve produced by the model estimation with the average one observed in the data, along SPP volumes and across moneyness. The model outputs closely match their empirical counterparts, which provides a validation for the framework.

INSERT FIGURE 9

Finally, we implement a counterfactual analysis to quantify the equilibrium impact of the SPP market on volatility prices. Our model enables us to estimate the implied volatility across different strikes in the absence of the SPP market. We perform this exercise for a range of realized volatilities at both 60% and 100% moneyness, with the results presented in Figure 10. The findings highlight the economically significant influence of the SPP market on volatility pricing, particularly at lower moneyness levels. This evidence reinforces the view that households, through their investment in exotic products, effectively act as insurers against tail risk by lowering volatility prices.

INSERT FIGURE 10

The estimation of θ requires the following. For a given θ, we compute the household's demand schedule for the SPP, $\alpha^{H}_{\text{SPP}}[z_t; Y(z_t)]$, by solving the Bellman equation (A3). We

then obtain the dealer's demand for the equity index and the SPP yield spread in each state z_t by solving the set of first-order conditions (A12) and (A13).

VI. Conclusion

This study uncovers how the development of Short Put Products (SPPs) has significant implications for volatility pricing and financial stability. By allowing households to implicitly sell deep out-of-the-money put options, SPPs create a persistent demand for left-tail risk exposure, which in turn affects its equilibrium pricing on the option market. Higher SPP issuance volumes suppress implied volatility and the volatility risk premium, particularly at the moneyness levels corresponding to common SPP barriers. These effects are amplified in periods of market stress, when early redemption of existing products becomes unlikely, reinforcing the role of SPPs in volatility price suppression.

To explain these empirical patterns, we develop a demand based asset pricing framework that captures the interactions between households, intermediaries, and option markets. Our estimation results suggest that household preferences?such as underweighting left-tail risk?drive demand for SPPs, which in turn influences equilibrium option prices through intermediaries? hedging behavior. Counterfactual analysis confirms that, in the absence of SPPs, implied volatility would be significantly higher, highlighting the role of these instruments in shaping volatility market dynamics.

These insights have broad implications for financial market participants and policymakers. First, by altering the pricing of downside protection, SPPs can impact risk management strategies for institutional investors. Second, our results suggest a trade-off between investor protection and financial stability: while regulators may seek to limit household exposure to complex financial products, restricting SPPs may also increase the cost of insuring against tail risk, as seen in the South Korean regulatory intervention. Finally, our study underscores the broader equilibrium effects of market innovation, showing that the introduction of new

financial instruments can reshape asset pricing by incorporating previously unaccounted-for investor preferences.

Overall, our research contributes to the growing literature on the macrostructure of financial markets by highlighting how financial innovation and household demand interact to influence option pricing and volatility dynamics, reinforcing the need to consider equilibrium effects when assessing whether a given financial innovation is beneficial or harmful.

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Appendix A. Proof of Proposition 1

A.1 Equilibrium Conditions

The dealer selects the portfolio weight of equity, $\alpha^D_{\mathrm{M},t}$, the weight of the SPP, $\alpha^D_{\mathrm{SPP},t}$ and weight of each put, $\alpha^D_{\mathrm{PUT},n,t}$, that maximizes expected utility. The corresponding first-order conditions are:

$$\begin{aligned}
\mathbb{E}_t\left[R^e_{\mathrm{M},\,t:t+T}\, e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,\,t:t+T}\right)}\right] &= 0,\\
\mathbb{E}_t\left[R^e_{\mathrm{SPP},t+1} e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},t+1}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},t+1}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,t+1}\right)}\right] &= 0,\\
\mathbb{E}_t\left[R^e_{\mathrm{PUT},n,t+1} e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},t+1}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},t+1}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,t+1}\right)}\right] &= 0,
\end{aligned}$$

for every $n \in \{1,\ldots,N\}$. Let

$$M^0_{t+T} = e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,\,t:t+T}\right)}. \tag{A1}$$

We infer from (A6) and (A7) that

$$M^0_{t+T} = e^{-\gamma^D\left[\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T}-\lambda^H\alpha^H_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T}-\sum_{n=1}^N d^E_n\,(K_n-1-R_{\mathrm{M},t:t+T})_+ +(1+R_{f,t:t+T})\sum_{n=1}^N d^E_n P_n\right]}. \tag{A2}$$

Since $(1+R_{f,t:t+T})\sum_{n=1}^N d^E_n P_n$ is known conditional on z_t, we infer that is also a stochastic discount factor.

The first-order condition for holdings of put n can be rewritten as:

$$\mathbb{E}_t\left[(K_n-1-R_{\mathrm{M}})_+\, M_{t+T}\right] = (1+R_{f,t})P_n\mathbb{E}_t\left[(K_n-1-R_{\mathrm{M}})_+\, M_{t+T}\right],$$

and we conclude that the proposition holds.

A.2 Existence and Uniqueness of One-Step Equilibrium

The consumption-portfolio problem of the household at date t depends both on (i) the value of the SPP yield spread on contracts issued at date t and (ii) the value of the SPP yield spread in future states, which determines the continuation value of future wealth. To prove the existence and uniqueness of a stationary GE, it is useful to distinguish between current and future spreads.

We accordingly rewrite the household's decision problem as follows. Let $\mathcal{Z}$ denote the state space and let $Y_0 : \mathcal{Z} \to \mathbb{R}_+$ denote a continuous mapping providing the state-contingent SPP yield spread at dates $t+T, t+2T, \ldots, \infty$. The household selects consumption and portfolio weights at date t by maximizing the Epstein-Zin recursive utility:

$$V^H[W^H_t; y(z_t), Y_0, z_t] = \max_{\{C^H_t,\alpha^H_{\mathrm{M},t},\alpha^H_{\mathrm{SPP},t}\}} \left[(1-\delta^T)\,(C^H_t)^{1-1/\psi} + \delta^T\,(\mu^H_{t+T})^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \tag{A3}$$

where the excess return on the SPP is given by:

$$R^e_{\text{SPP},\,t:t+T} = y_(z_t)\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T}>B\right\}} + R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T}\leq B\right\}}, \tag{A4}$$

the wealth available at the beginning of date $t+T$ by:

$$W^H_{t+T} = X^H_t\,\left(1 + R_{f,t:t+T} + \alpha^H_{\text{M},t} R^e_{\text{M},t:t+T} + \alpha^H_{\text{SPP},t}\,R^e_{\text{SPP},t:t+T}\right),$$

and the certainty equivalent of future wealth by:

$$\mu^H_{t+T} = \left(\mathbb{E}^{\mathbb{P}}_t\left\{V^H\left[W^H_{t+T}; Y_0(z_{t+T}), Y_0, z_{t+T}\right]^{1-\gamma}\right\}\right)^{\frac{1}{1-\gamma}}.$$

We denote by

$$\alpha^H_{\text{SPP},t} = \alpha^H_{\text{SPP}}[y(z_t), Y_0, z_t]$$

the household's optimal SPP weight in period t. We know that $\alpha^H_{\text{SPP}}(y_t, Y_0, z_t)$ is contained in $[0,1]$ and is a continuous function of y_t, Y_0, and z_t. We do not need to reformulate the dealer's problem since her decisions do not depend on future investments opportunities.

More generally, we establish the existence of a one-step equilibrium under the following set of sufficient conditions.

CONDITION 1 (Sufficient conditions for existence of one-step equilibrium): *The state vector z_t is first-order Markov and contained in a compact subset, $\mathcal{Z}$, of a Euclidean space. The household share of the SPP,*

$$\alpha^H_{\text{SPP}}(y_t, Y_0, z_t), \tag{A5}$$

is contained in $[0,1]$, is continuous and increasing in y_t, and satisfies $\alpha^H_{\text{SPP}}(0, Y_0, z_t) = 0$ for every z_t.

The condition $\alpha^H_{\text{SPP}}(0, Y_0, z_t) = 0$ states that if the SPP yield spread is zero, the SPP is dominated by the bond and the household allocates no funds to the SPP. Let $\mathcal{C}_+$ denote the set of continuous mappings define on $\mathcal{Z}$ and taking values in $\mathbb{R}_+$. In the rest of the section, we derive the following result.

PROPOSITION 2 (Existence and uniqueness of one-step equilibrium): *Assume that Condition 2 holds. For every $Y_0 \in \mathcal{C}_+$, there exists a unique $Y_1 \in \mathcal{C}_+$ such that for every state $z_t \in \mathcal{Z}$, the dealer maximizes her utility and the SPP and put markets clear:*

$$\alpha^D_{\text{SPP},t}[z_t, Y_1(z_t)] = -\lambda^H\,\alpha^H_{\text{SPP},t}[Y_1(z_t), Y_0, z_t], \tag{A6}$$

$$\alpha^D_{\text{PUT},n}[z_t; Y_1(z_t)] = -d^E_n\,P_{\text{PUT},n}(z_t). \tag{A7}$$

We denote by $\mathcal{T}$ the transformation on $\mathcal{C}_+$ that maps the future yield spread, Y_0, to the current equilibrium yield spread: $Y_1 = \mathcal{T}(Y_0)$.

The proof of the proposition proceeds as follows. Consider the stochastic discount factor

$$M_{t+T}(a^D_{\text{M}}, y_t, a^H_{\text{SPP}}) = e^{-\gamma^D\left[a^D_{\text{M}}\, R^e_{\text{M},\, t:t+T} - \lambda^H\, a^H_{\text{SPP}}\, R^e_{\text{SPP},\, t:t+T}(y_t) - \sum_{n=1}^N d^E_n\,(K_n - 1 - R_{\text{M},t:t+T})_+\right]}$$

where $R^e_{\text{SPP},\, t:t+T}(y_t) = y_t\, \mathbb{1}_{\left\{R_{\text{M},\, t:t+T} > B\right\}} + R^e_{\text{M},\, t:t+T}\, \mathbb{1}_{\left\{R_{\text{M},\, t:t+T} \leq B\right\}}$ expresses the SPP's excess return as a function of the the current yield.

LEMMA 2 (Auxiliary function G): *For a given state, z_t, SPP yield spread, y_t, and household demand for the SPP, $\alpha^H_{\text{SPP},t}$, the dealer's first-order condition in z_t:*

$$\mathbb{E}_t\left[R^e_{\text{M},\, t:t+T}\, M_{t+T}(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t})\right] = 0. \tag{A8}$$

has a unique solution, $\alpha^D_{\text{M},t}$, which we denote by $G(y_t, \alpha^H_{\text{SPP},t}, z_t)$. Furthermore, the dealer's equity share, $\alpha^D_{\text{M},t} = G(y_t, \alpha^H_{\text{SPP},t}, z_t)$, is strictly increasing in the SPP yield spread, y_t, and in the household's demand for the SPP, $\alpha^H_{\text{SPP},t}$.

Proof. Consider the function $f(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t}) = \mathbb{E}_t\left[R^e_{\text{M},\, t:t+T}\, M_{t+T}(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t})\right]$, or equivalently

$$f(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t}) = \mathbb{E}_t\left\{R^e_{\text{M},\, t:t+T}\, e^{-\gamma^D\left[\alpha^D_{\text{M},t}\, R^e_{\text{M},\, t:t+T} - \lambda^H \alpha^H_{\text{SPP},t}\, R^e_{\text{SPP},\, t:t+T}(y_t) - \sum_{n=1}^N d^E_n\,(K_n - 1 - R_{\text{M},t:t+T})_+\right]}\right\}.$$

We note that

$$\frac{\partial f}{\partial \alpha^D_{\text{M},t}} = -\gamma^D\, \mathbb{E}_t\left[(R^e_{\text{M},\, t:t+T})^2\, M_{t:t+T}\right] < 0,$$

$$\frac{\partial f}{\partial y_t} = \gamma^D\, \lambda^H\, \alpha^H_{\text{SPP},t}\, \mathbb{E}_t\left(R^e_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M_{t:t+T}\right),$$

$$\frac{\partial f}{\partial \alpha^H_{\text{SPP},t}} = \gamma^D\, \lambda^H\, \mathbb{E}_t\left(R^e_{\text{M},\, t:t+T}\, R^e_{\text{SPP},\, t:t+T}\, M_{t+T}\right).$$

By the Implicit Function Theorem, the partial derivatives of G are given by:

$$\frac{\partial G}{\partial y_t} = -\frac{\partial f}{\partial y_t} \times \left(\frac{\partial f}{\partial \alpha^D_{\text{M},t}}\right)^{-1} = \lambda^H\, \alpha^H_{\text{SPP},t}\, \frac{\mathbb{E}_t\left(R^e_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M_{t:t+T}\right)}{\mathbb{E}_t\left[(R^e_{\text{M},\, t:t+T})^2\, M_{t+T}\right]}, \tag{A9}$$

$$\frac{\partial G}{\partial \alpha^H_{\text{SPP},t}} = -\frac{\partial f}{\partial \alpha^H_{\text{SPP},t}} \times \left(\frac{\partial f}{\partial \alpha^D_{\text{M},t}}\right)^{-1} = \lambda^H\, \frac{\mathbb{E}_t\left(R^e_{\text{M},\, t:t+T}\, R^e_{\text{SPP},\, t:t+T}\, M_{t+T}\right)}{\mathbb{E}_t\left[(R^e_{\text{M},\, t:t+T})^2\, M_{t+T}\right]}. \tag{A10}$$

Since $\mathbb{E}_t\left(R^e_{\text{M},\, t:t+T}\, M_{t+T}\right) = 0$, we know that

$$\mathbb{E}_t\left(R^e_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} M_{t+T}\right) = -\mathbb{E}_t\left[R^e_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t+T} \leq B\}} M_{t:t+T}\right] > 0. \tag{A11}$$

Hence the function G strictly increases in y_t.

Similarly, since $R^e_{\text{M},\, t:t+T}\, R^e_{\text{SPP},\, t:t+T} = (R^e_{\text{M},\, t:t+T})^2 \mathbb{1}_{\{R_{\text{M},\, t:t+T} \leq B\}} + y_t\, R^e_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}$, we

infer that

$$\begin{aligned}\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,R^e_{\mathrm{SPP},\,t:t+T}\,M_{t+T}\right) &= \mathbb{E}_t\left[(R^e_{\mathrm{M},\,t:t+T})^2\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}\le B\}}M_{t+T}\right]\\ &\quad + y_t\,\mathbb{E}_t\left[R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}M_{t+T}\right] > 0.\end{aligned}$$

Hence the function G strictly increases in $\alpha^H_{\mathrm{SPP},t}$. □

In a one-step equilibrium, the following first-order conditions hold:

$$\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,M_{t+T}\right) = 0, \tag{A12}$$

$$\mathbb{E}_t\left(R^e_{\mathrm{SPP},t+1}\,M_{t+T}\right) = 0. \tag{A13}$$

By equation (A4), the first-order condition (A13) can be rewritten as:

$$y_t\,\mathbb{E}_t\left(\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}>B\right\}}M_{t+T}\right) + \mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}\le B\right\}}\,M_{t+T}\right) = 0, \tag{A14}$$

or

$$y_t = -\frac{\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}\le B\right\}}\,M_{t+T}\right)}{\mathbb{E}_t\left(\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}>B\right\}}M_{t+T}\right)} > 0. \tag{A15}$$

We infer from equation (A12) that

$$y_t = \frac{\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M_{t+T}\right)}{\mathbb{E}_t\left(\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M_{t+T}\right)}. \tag{A16}$$

Since the return on the SPP is deterministic for $R_{\mathrm{M},\,t:t+T} > B$, this equation reduces to:

$$y_t = \frac{\mathbb{E}_t\left[R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M^*_{t+T}(\alpha^D_{\mathrm{M},t})\right]}{\mathbb{E}_t\left[\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M^*_{t+T}(\alpha^D_{\mathrm{M},t})\right]}, \tag{A17}$$

where

$$M^*_{t+T}(\alpha^D_{\mathrm{M},t}) = e^{-\gamma^D\left[\alpha^D_{\mathrm{M},t}\,R_{\mathrm{M},t:t+T} - \sum_{n=1}^N d^E_n\,(K_n - 1 - R_{\mathrm{M},t:t+T})_+\right]}. \tag{A18}$$

We now study the properties of the function on the right-hand side of equation (A17).

LEMMA 3 (Auxiliary function H): *The function $H(\alpha^D_{\mathrm{M},t}, z_t)$ strictly decreases in $\alpha^D_{\mathrm{M},t}$ for every state z_t.*

Proof. We let

$$H(a, z_t) = \frac{u(a, z_t)}{v(a, z_t)} - R_{f,t,t:T}, \tag{A19}$$

where

$$\begin{aligned}u(a, z_t) &= \mathbb{E}_t\left[R_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M^*_{t+T}(a)\right],\\ v(a, z_t) &= \mathbb{E}_t\left[\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M^*_{t+T}(a)\right].\end{aligned}$$

We note that

$$\frac{\partial u}{\partial a} = -\gamma^D \, \mathbb{E}_t \left[(R_{\text{M},\, t:t+T})^2 \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T} \right]$$

$$\frac{\partial v}{\partial a} = -\gamma^D \, \mathbb{E}_t \left[R_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T} \right] = -\gamma^D \, u_t(a).$$

We infer that

$$\frac{\partial H}{\partial a} = \gamma^D \, \frac{\left\{ \mathbb{E}_t \left[R_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T} \right] \right\}^2 - \left\{ \mathbb{E}_t \left[(R_{\text{M},\, t:t+T})^2 \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T} \right] \right\} \left\{ \mathbb{E}_t \left[\mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T} \right] \right\}}{\left\{ \mathbb{E}_t [\mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T}] \right\}^2}.$$

By the Cauchy Schwarz inequality, the numerator is negative.[15] Hence, H is strictly decreasing in a. We also note that

$$\left| \frac{\partial H}{\partial a}(a, z_t) \right| \leq \gamma^D \, \frac{\mathbb{E}_t \left[(R_{\text{M},\, t:t+T})^2 \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T} \right]}{\mathbb{E}_t [\mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+T}(a)]}.$$

for every a and z_t. □

We consider the function

$$\Phi(y_t, Y_0, z_t) = y_t - H \left\{ G[y_t, \alpha^H_{\text{SPP}}(y_t, Y_0, z_t), z_t], z_t \right\}.$$

Under Condition 2, the function α^H_{SPP} increases in y_t. By Lemmas 2 and 3, the function Φ therefore increases in y_t.

When $y_t = 0$, the household has zero demand for the SPP; $\alpha^H_{\text{SPP}}(0, Y_0, z_t) = 0$, as per Condition 2. Hence,

$$\Phi(0, Y_0, z_t) = -H \left[\hat{\alpha}(z_t), z_t \right],$$

where $\hat{\alpha}(z_t) = G(0, 0, z_t)$ denotes the dealer's optimal equity share in the absence of the SPP contract. The equity share, $\hat{\alpha}(z_t)$, satisfies the first-order condition:

$$\mathbb{E}_t \left\{ R^e_{\text{M},\, t:t+T} \, e^{-\gamma^D \left[\alpha^D_{\text{M},t} \, R^e_{\text{M},\, t:t+T} - \sum_{n=1}^N d^E_n \, (K_n - 1 - R_{\text{M},t:t+T})_+ \right]} \right\} = 0, \tag{A20}$$

or, more compactly, $\mathbb{E}_t \left\{ R^e_{\text{M},\, t:t+T} \, M^*_{t+1}[\hat{\alpha}(z_t)] \right\} = 0$. Since

$$\mathbb{E}_t \left\{ R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M^*_{t+1}[\hat{\alpha}(z_t)] \right\} = -\mathbb{E}_t \left\{ R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} \leq B\}} \, M^*_{t+1}(\hat{\alpha}(z_t)) \right\} > 0,$$

we infer that $H \left[\hat{\alpha}(z_t), z_t \right] > 0$ and therefore $\Phi(0, Y_0, z_t) < 0$.

[15] Let $X = R_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, [M^*_{t+T}(a)]^{1/2}$ and $Y = \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, [M^*_{t+T}(a)]^{1/2}$. By the Cauchy Schwarz inequality, $[E(XY)]^2 < E(X^2)E(Y^2)$. The inequality is strict because X and Y are not perfectly correlated.

We also note that

$$\frac{\partial \Phi(y_t, Y_0, z_t)}{\partial y_t} \geq 1. \tag{A21}$$

Hence $\Phi(y_t, Y_0, z_t)$ is positive for y_t sufficiently large. We infer that the equation $\Phi(y_t, Y_0, z_t) = 0$ has a unique solution, so that there exists a unique one-step equilibrium for every z_t.

We further note that

$$Y_1(z_t, Y_0) \leq | H [G(0, 0, z_t), z_t] | \tag{A22}$$

for every $z_t \in \mathcal{Z}$ and $Y_0 \in \mathcal{C}_+$.

A.3 Existence and Uniqueness of Recursive GE

Let $\mathcal{C}$ denote the set of continuous functions defined on $\mathcal{Z}$ and taking values on the real line. Let $\|f\|_\infty = \sup_{z \in \mathcal{Z}} |f(z)|$ denote the sup norm. We know that $(\mathcal{C}, \|f\|_\infty)$ is a Banach space.

Consider

$$\bar{y} = \max_{z_t \in \mathcal{Z}} |H [G(0, 0, z_t), z_t] | < \infty. \tag{A23}$$

We know that for every function $Y_0 \in \mathcal{C}$, the transform $Y_1 = \mathcal{T} Y_0$ takes values between 0 and $\bar{y}$, as equation (A22) implies. For this reason, we henceforth focus on the subset of functions:

$$\mathcal{K} = \{f \in \mathcal{C} : 0 \leq f(z) \leq \bar{y} \text{ for every } z \in \mathcal{Z}\}.$$

The set $\mathcal{K}$ is convex and closed.

Consider the function $f(y_t, a_{\text{SPP}}, z_t) = H [G(y_t, a_{\text{SPP}}, z_t), z_t]$. By the chain rule,

$$\frac{\partial f}{\partial a_{\text{SPP}}}(y_t, a_{\text{SPP}}, z_t) = \frac{\partial H}{\partial a_M} [G(y_t, a_{\text{SPP}}, z_t), z_t] \frac{\partial G}{\partial a_{\text{SPP}}}(y_t, a_{\text{SPP}}, z_t). \tag{A24}$$

The functions on the right-hand side are bounded and we let

$$K_1 = \sup \left\{ \left| \frac{\partial f}{\partial a_{\text{SPP}}}(y_t, a_{\text{SPP}}, z_t) \right| ; \ (y_t, a_{\text{SPP}}, z_t) \in [0, \bar{y}] \times [0, 1] \times \mathcal{Z} \right\}.$$

Importantly, we note that the upper bounds $\bar{y}$ and K_1 do not depend on the household demand for the SPP.[16]

CONDITION 2 (Sufficient conditions for existence of stationary equilibrium): *There exists* $k_{\text{SPP}} > 0$ *such that the household share of the SPP,* $\alpha^H_{\text{SPP}}(y_t, Y_0, z_t)$*, satisfies*

$$|\alpha^H_{\text{SPP}}(y_t, Y_0', z_t) - \alpha^H_{\text{SPP}}(y_t, Y_0, z_t)| \leq k_{\text{SPP}} \|Y_0' - Y_0\|_\infty$$

[16] Furthermore, we infer from and that

$$K_1 \leq \lambda^H \gamma^D \sup_{(y_t, a_{\text{SPP}}, z_t) \in [0, \bar{y}] \times [0,1] \times \mathcal{Z}} \frac{\mathbb{E}_t \left\{ R^e_{\text{M}, t:t+T} R^e_{\text{SPP}, t:t+T} M_{t+T}[G(a_{\text{SPP}}, z_t), y_t, z_t] \right\}}{\mathbb{E}_t \left\{ \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} M_{t+T}[G(a_{\text{SPP}}, z_t), y_t, z_t] \right\}}. \tag{A25}$$

for every $Y_0, Y_0' \in \mathcal{K}$, $y_t \in [0, \bar{y}]$, *and* $z_t \in \mathcal{Z}$. *Furthermore, the constant* k_{SPP} *satisfies* $k_{\text{SPP}} < 1/K_1$.

We consider $Y_0, Y_0' \in \mathcal{K}$. Let $Y_1 = \mathcal{T}Y_0$ and $Y_1' = \mathcal{T}Y_0'$. For a given state z_t, we let $y_t = Y_1(z_t)$ and $y_t' = Y_1'(z_t')$. Since $\Phi(y_t', Y_0', z_t) = \Phi(y_t, Y_0, z_t) = 0$, we infer that

$$\Phi(y_t', Y_0', z_t) - \Phi(y_t', Y_0, z_t) = \Phi(y_t, Y_0, z_t) - \Phi(y_t', Y_0, z_t). \tag{A26}$$

We infer from (A21)

$$|y_t' - y_t| \leq |\Phi(y_t, Y_0, z_t) - \Phi(y_t', Y_0, z_t)|. \tag{A27}$$

Sufficient Condition 2 implies

$$|\Phi(y_t', Y_0', z_t) - \Phi(y_t', Y_0, z_t)| \leq k_{\text{SPP}}\, K_1\, \|Y_0' - Y_0\|_\infty. \tag{A28}$$

We infer from (A26), (A27) and (A28) that

$$|y_t' - y_t| \leq k_{\text{SPP}}\, K_1\, \|Y_0' - Y_0\|_\infty.$$

Since this inequality holds for every state z_t, the transformation $\mathcal{T}$ is a contraction:

$$\|\mathcal{T}Y_0' - \mathcal{T}Y_0\|_\infty \leq k_{\text{SPP}}\, K_1\, \|Y_0' - Y_0\|_\infty.$$

Hence, the contraction mapping $\mathcal{T}$ has a unique fixed point.

Appendix B. Alternative Specification of the Dealer

The wealth of the dealer in period t is

$$X_t^D = \phi F W_t^H = \phi X_t^H / (1 - \phi).$$

We write

$$X_t^H = \lambda^H X_t^D,$$

where $\lambda^H = (1 - \phi)/\phi$.

The dealer's wealth in period $t + T$ is

$$W_{t+T}^D = X_t^D \left(1 + R_{t:t+T}^D\right).$$

where

$$R_{t:t+T}^D = R_{f,\,t:t+T} + \alpha_{\text{M},t}^D\, R_{\text{M},\,t:t+T}^e + \alpha_{\text{SPP},t}^D\, R_{\text{SPP},\,t:t+T}^e + \sum_{n=1}^{N} \alpha_{\text{PUT},n,t}^D\, R_{\text{PUT},n,\,t:t+T}^e \tag{B1}$$

denotes the dealer's return over the period.

The dealer has constant absolute risk aversion (CARA) utility:

$$\mathbb{E}_0^{\mathbb{P}} \sum_{t=1}^{\infty} \delta^t u_t(W_t^D)$$

where $u_{t+T}(W_{t+T}^D) = -e^{-A_t W_{t+T}^D}/A_t$ and $A_t = \gamma^D / X_t^D$.

Let $\alpha_t^D = (\alpha_{\mathrm{M},t}^D, \alpha_{\mathrm{SPP},t}^D, \alpha_{\mathrm{PUT},1,t}^D, \ldots, \alpha_{\mathrm{PUT},K,t}^D)$. Assume that to minimize attention costs etc., the dealer chooses the portfolio α_t^D and is then precluded from modifying it thereafter.

The dealer solves the optimization problem

$$\max_{\alpha_t^D} \mathbb{E}_t \left[-e^{-\gamma^D \left(W_{t+T}^D / X_t^D \right)} \right].$$

Since $W_{t+T}^D / X_t^D = 1 + R_{t:t+T}^D$, the solution to this problem is independent of the wealth level X_t^D. We denote by

$$\alpha^D[z_t; Y(z_t)]$$

the optimal portfolio of the dealer.

We can also assume that the dealer consumes X_t^D at date t and keeps the residual risk:

$$X_t^D \left(\alpha_{\mathrm{M},t}^D \, R_{\mathrm{M},\, t:t+T}^e + \alpha_{\mathrm{SPP},t}^D \, R_{\mathrm{SPP},\, t:t+T}^e + \sum_{n=1}^{N} \alpha_{\mathrm{PUT},n,t}^D \, R_{\mathrm{PUT},n,\, t:t+T}^e \right). \tag{B2}$$

Appendix C. Appendix to Section V

C.1 Equity Premium

We assume the following:

$$r_{\mathrm{M},t+1}^e = \mu_r(z_t) + e^{z_{2,t+1}/2} \varepsilon_{\mathrm{M},t+1} \tag{C1}$$

$$\mathbb{E}_t(r_{\mathrm{M},t+1}^e) = \lambda \, Var_t(r_{\mathrm{M},t+1}^e) \tag{C2}$$

$$\mathbb{E}_t(R_{\mathrm{M},t:t+T} - R_{f,t:t+T}) = RP, \tag{C3}$$

where RP is an exogenous constant. The values of $\mu_r(z_t)$ and λ are unique and can be computed as follows.

LEMMA 4: *The drift in each state is given by*

$$\mu_r(z_t, \lambda) = \lambda \, \mathbb{E}_t \left\{ \left[e^{z_{2,t+1}/2} \varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2} \varepsilon_{\mathrm{M},t+1}) \right]^2 \right\} - \mathbb{E}_t(e^{z_{2,t+1}/2} \varepsilon_{\mathrm{M},t+1}). \tag{C4}$$

The parameter λ is the unique solution to the equation:

$$\rho(\lambda) = RP, \tag{C5}$$

where

$$\rho(\lambda) = \mathbb{E}\left\{\prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right)\right] - (1+z_{1;t})^{T/12}\Bigg\} \tag{C6}$$

for every $\lambda \in \mathbb{R}$.

Proof. The system has the following properties. Since

$$\mathbb{E}_t(r^e_{\mathrm{M},t+1}) = \mu_r(z_t) + \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}),$$

we obtain that

$$Var_t(r^e_{\mathrm{M},t+1}) = \mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1})\right]^2\right\},$$

and therefore

$$\mu_r(z_t) = \lambda\,\mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1})\right]^2\right\} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}). \tag{C7}$$

Under the assumptions of the model, we obtain:

$$\begin{aligned}
1 + R_{\mathrm{M},t:t+T} &= (1+R_{f,t:t+1})\ldots(1+R_{f,t+T-1:t+T})\exp\left(r^e_{M,t:t+1} + \cdots + r^e_{\mathrm{M},t+T-1:t+T}\right) \\
&= \prod_{i=1}^{T}(1+R_{f,t+i-1:t+i})\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right] \\
&= \prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right].
\end{aligned}$$

and

$$1 + R_{f,t:t+T} = (1+z_{1,t})^{T/12}, \tag{C8}$$

Let

$$\rho(\lambda) = \mathbb{E}\left\{\prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right)\right] - (1+z_{1;t})^{T/12}\Bigg\} \tag{C9}$$

We obtain λ by solving □

C.2 Particle Filter

The observation vector contains the interest rate and the equity excess return, which are exogenous to the model, and the SPP yield, the implied volatilities of the puts, and the SPP volume, which are endogenous. We find it convenient to write it as:

$$X_t = (z_{1,t}, r^e_{\mathrm{M},t}, X^{(1)}_t), \tag{C10}$$

where

$$X_t^{(1)} = (y_t, IV_{1,t}, \ldots, IV_{n,t}, SPPVolume_t) \tag{C11}$$

contain equilibrium quantities.

The one-step probability density function (p.d.f.) of the observation vector, X_t, is given by:

$$f(X_t|X_{1:t-1}) = \iint h(X_t|X_{1:t-1}, z_{2,t-1}, z_{2,t}) f(z_{2,t-1}, z_{2,t}|X_{1:t-1})\, dz_{2,t-1}\, dz_{2,t}. \tag{C12}$$

We note that $h(X_t|X_{1:t-1}, z_{2,t-1}, z_{2,t}) = h(X_t|z_{1,t-1}, z_{2,t-1}, z_{2,t})$ and therefore

$$\begin{aligned} h(X_t|X_{1:t-1}, z_{2,t-1}, z_{2,t}) &= f(z_{1,t}|z_{1,t-1}, z_{2,t-1}, z_{2,t}) \times f(r^e_{\mathrm{M},t}|z_{1,t-1}, z_{2,t-1}, z_{2,t}, z_{1,t}) \\ &\quad \times f(X_t^{(1)}|z_{1,t-1}, z_{2,t-1}, z_{2,t}, z_{1,t}, r^e_{\mathrm{M},t}). \end{aligned}$$

Given the structure of the model, this reduces to

$$h(X_t|X_{1:t-1}, z_{2,t-1}, z_{2,t}) = f(z_{1,t}|z_{t-1}, z_{2,t})\, f(r^e_{\mathrm{M},t}|z_{t-1}, z_t)\, f(X_t^{(1)}|z_t). \tag{C13}$$

The conditional p.d.f.s $f(z_{1,t}|z_{t-1}, z_{2,t})$, $f(r^e_{\mathrm{M},t}|z_{t-1}, z_t)$ and $f(X_t^{(1)}|z_t)$ have closed-form expressions derived in Appendix C.3.

Since the volatility state $z_{2,t-1}$ is not observed directly, we sequentially approximate its distribution by a particle filter. We draw the initial filter from the stationary distribution of the volatility state. The filter is then recursively constructed as follows. Assume that we have constructed the filter $\{z_{2,t-1}^{(b)}\}_{b=1}^B$, which targets the distribution of $z_{2,t-1}$ conditional on $X_{1:t-1}$. We obtain the filter for date t as follows.

1. For every $b \in \{1, \ldots, B\}$, we simulate forward the particle $z_{2,t-1}^{(b)}$ and obtain the candidate particle pair $(z_{2,t-1}^{(b)}, \tilde{z}_{2,t}^{(b)})$. The resulting set $\{(z_{2,t-1}^{(b)}, \tilde{z}_{2,t}^{(b)})\}_{b=1}^B$ targets the distribution of $(z_{2,t-1}, z_{2,t})$ conditional on $X_{1:t-1}$.

2. By equation (C12), we approximate the one-step ahead likelihood by:

$$\hat{f}(X_t|X_{1:t-1}) = \frac{1}{B}\sum_{b=1}^{B} h(X_t|X_{t-1}, z_{2,t-1}^{(b)}, \tilde{z}_{2,t}^{(b)}).$$

3. We assign to each candidate particle pair $(z_{2,t-1}^{(b)}, \tilde{z}_{2,t}^{(b)})$ the normalized importance weight:

$$p_t^{(b)} = \frac{h(X_t|X_{t-1}, z_{2,t-1}^{(b)}, \tilde{z}_{2,t}^{(b)})}{\sum_{b'=1}^{B} h(X_t|X_{t-1}, z_{2,t-1}^{(b')}, \tilde{z}_{2,t}^{(b')})}$$

4. For every $b \in \{1, \ldots, B\}$, we sample the particle $z_{2,t}^{(b)}$ from the candidate particles $\tilde{z}_{2,t}^{(1)}, \ldots, \tilde{z}_{2,t}^{(B)}$ with probabilities $p_t^{(1)}, \ldots, p_t^{(B)}$.

C.3 Likelihoods

LEMMA 5: *The p.d.f. of the interest rate, $z_{1,t}$, conditional on the lagged state vector, z_{t-1}, and the contemporaneous equity volatility state, $z_{2,t}$, is given by:*

$$f(z_{1,t}|z_{t-1}, z_{2,t}) = n\left[z_{1,t};\, m_{1,t},\, \sigma_1^2(1-\rho_{12}^2)\right],$$

where

$$m_{1,t} = \mu_1 + \phi_1(z_{1,t-1} - \mu_1) + \rho_{12}\,\frac{\sigma_1}{\sigma_2}\left[z_{2,t} - \mu_2 - \phi_2(z_{2,t-1} - \mu_2)\right]$$

in every period t.

Proof. Conditional on z_{t-1}, the state vector z_t is Gaussian with mean $(\mu_1 + \phi_1(z_{1,t-1} - \mu_1), \mu_2 + \phi_2(z_{2,t-1} - \mu_2))'$ and variance-covariance matrix

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\,\sigma_1\,\sigma_2 \\ \rho_{12}\,\sigma_1\,\sigma_2 & \sigma_2^2 \end{pmatrix}$$

The p.d.f. of $z_{1,t}$ conditional on $z_{2,t}$ and z_{t-1} is therefore Gaussian, with mean $m_{1,t}$ and variance $\sigma_1^2(1-\rho_{12}^2)$. □

The variance-covariance matrix of the Gaussian vector $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{\mathrm{M},t})'$ is the matrix Σ given by equation (7). We consider a Cholesky decomposition of Σ:

$$\Sigma = L\,L^\top$$

where $L = (L_{ij})$ is a lower triangular matrix. The random vector $u_t = L^{-1}\varepsilon_t$, is iid $\mathcal{N}(0, I)$. If the vectors z_{t-1} and z_t are known, then the innovations $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, as well as their linear transforms $u_{1,t}$ and $u_{2,t}$, are also known.

LEMMA 6: *The p.d.f. of the log equity index excess return, $r^e_{\mathrm{M},t}$, conditional on the lagged and current values of the state vector, z_{t-1} and z_t, is given by*

$$f(r^e_{\mathrm{M},t}|z_{t-1}, z_t) = n\left[r^e_{\mathrm{M},t};\, \mu_r(z_{t-1}) + e^{z_{2,t}/2}(L_{3,1}u_{1,t} + L_{3,2}u_{2,t}),\, L_{33}^2\,e^{z_{2,t}}\right], \tag{C14}$$

where

$$u_{1,t} = \varepsilon_{1,t}/L_{1,1}, \tag{C15}$$
$$u_{2,t} = (\varepsilon_{2,t} - L_{2,1}\,u_{1,t})/L_{2,2}. \tag{C16}$$

Proof. The vector of innovations satisfies $\varepsilon_t = L\,u_t$, or equivalently:

$$\begin{aligned} \varepsilon_{1,t} &= L_{1,1}\,u_{1,t} \\ \varepsilon_{2,t} &= L_{2,1}\,u_{1,t} + L_{2,2}\,u_{2,t} \\ \varepsilon_{\mathrm{M},t} &= L_{3,1}\,u_{1,t} + L_{3,2}\,u_{2,t} + L_{3,3}\,u_{3,t}. \end{aligned}$$

Hence equations (C15) and (C16) hold.

The log equity index excess return, $r^e_{\mathrm{M},t} = \mu_r(z_{t-1}) + e^{z_{2,t}/2}\,\varepsilon_{\mathrm{M},t}$, satisfies

$$r^e_{\mathrm{M},t} = \mu_r(z_{t-1}) + e^{z_{2,t}/2}(L_{3,1}\,u_{1,t} + L_{3,2}\,u_{2,t} + L_{3,3}\,u_{3,t}) \tag{C17}$$

Hence, it is Gaussian with conditional mean $\mu_r(z_{t-1}) + e^{z_{2,t}/2}(L_{3,1}u_{1,t} + L_{3,2}u_{2,t})$ and conditional variance $e^{z_{2,t}}L^2_{33}$. □

We consider the vector of errors

$$G(X_t^{(1)}, z_t) = \begin{bmatrix} y_t - y^D(z_t, SPPVolume_t) \\ IV_{1,t} - \mathcal{V}_1^D(z_t, SPPVolume_t) \\ \vdots \\ IV_{N,t} - \mathcal{V}_N^D(z_t, SPPVolume_t) \\ SPPVol_t - \lambda^H\,\alpha^H_{\mathrm{SPP}}(z_t, y_t, Y) \end{bmatrix}. \tag{C18}$$

Let Σ^* denote the variance-covariance matrix of the residuals $(u_t, \eta_{1,t}, \cdots \eta_{N,t}, \nu_t)'$.

LEMMA 7: *The conditional pdf of equilibrium outcomes, $X_t^{(1)}$, is given by:*

$$f(X_t^{(1)}|z_t) = n\left[G(X_t^{(1)}); 0, \Sigma^*\right]\left|1 - \lambda^H\,\frac{\partial \alpha^H_{\mathrm{SPP}}}{\partial y_t}\frac{\partial y^D}{\partial SPPVol_t}\right| \tag{C19}$$

Proof. We verify that the Jacobian matrix is:

$$DG = \begin{pmatrix} 1 & 0 & -\frac{\partial y^D}{\partial SPPVol_t} \\ 0 & I & -\frac{\partial \mathcal{V}^D}{\partial SPPVol_t} \\ -\lambda^H\,\frac{\partial \alpha^H_{\mathrm{SPP}}}{\partial y_t} & 0 & 1 \end{pmatrix} \tag{C20}$$

where $\mathcal{V}^D = (\mathcal{V}_1^D, \cdots, \mathcal{V}_N^D)'$. We verify that

$$|\det(DG)| = \left|1 - \lambda^H\,\frac{\partial \alpha^H_{\mathrm{SPP}}}{\partial y_t}\frac{\partial y^D}{\partial SPPVol_t}\right| \tag{C21}$$

We know that conditional on z_t, the vector $G(X_t^{(1)})$ is Gaussian

$$G(X_t^{(1)}) \sim \mathcal{N}(0, \Sigma^*). \tag{C22}$$

We infer that the conditional p.d.f. of $X_t^{(1)}$ is therefore

$$n\left[G(X_t^{(1)}); 0, \Sigma^*\right] \times |\det(DG)|.$$

We combine it with equation and conclude that the proposition holds. □

We can use a concentrated likelihood approach. The log likelihhod is optimal if

$$\Sigma^* = \frac{1}{T}\sum_{t=1}^{T} G(X_t^{(1)})G(X_t^{(1)})'$$

C.4 Likelihood (Step 2)

Let I_t denote the information set available at date t. Let $f_H(SPPVol_t|\theta^H, \theta^D, I_t)$ denote the conditional pdf of $SPPVol_t$ and the information set I_t. We note that

$$f_H(SPPVol_t|\theta^H, \theta^D, I_t) = \int f_H(SPPVol_t|\theta^H, \theta^D, z_t, I_t)\phi(z_t|I_t)dz_t \tag{C23}$$

We note that

$$f_H(SPPVol_t|\theta^H, \theta^D, z_t, I_t) = n\left[SPPVol_t;\, \lambda^H\, \alpha^H(z_t, y_t, \theta^H, \theta^D), \sigma^2_{\text{VOL}}\right]. \tag{C24}$$

We approximate the conditional pdf by

$$\frac{1}{B}\sum_{b=1}^{B} n\left[SPPVol_t;\, \lambda^H\, \alpha^H(z_t^{(b)}, y_t, \theta^H, \theta^D), \sigma^2_{\text{VOL}}\right] \tag{C25}$$

Table I
Summary Statistics on Short Put Products (2005-2019)

Panel A: Full Sample (N =178,161)							
	Mean	**S. D.**	**p10**	**p25**	**p50**	**p75**	**p90**
Maturity, in years	3.0	1.1	1.5	3.0	3.0	3.0	3.1
Headline Rate							
Yearly Coupon, in %	6.5	2.4	4.0	4.8	6.0	7.8	10.0
Spread to Benchmark, in %	4.5	2.0	2.2	3.0	4.2	5.6	7.2
Strike, in % of the index initial price	58.2	8.9	50.0	50.0	60.0	65.0	70.0
Ex-Post Returns							
Yearly Return, in %	5.5	8.6	1.1	3.9	5.4	7.2	9.5
Excess to 1-year Risk Free Rate, in %	4.9	4.4	1.2	3.2	5.1	6.7	8.5
Issuance Volumes							
In million $	5.1	15.3	0.1	0.6	2.0	5.4	12.3
Underlying Index, Market Share in %							
Eurostoxx 50	29.2						
S&P 500	18.2						
Nikkei 225	17.1						
Hang Seng China	13.6						
Kospi 200	10.5						
Hang Seng	2.8						
Russell	2.7						
FTSE	2.6						
Swiss Market Index	1.6						
CAC 40	0.8						
Dax	0.7						
Nasdaq	0.2						

Panel B: Monthly Panel across 12 Indices and 180 Months (N =2,160)							
	Mean	**S. D.**	**p10**	**p25**	**p50**	**p75**	**p90**
Market Parameters, in %							
3-year Swap Rate	2.4	1.6	0.6	1.3	2.1	3.4	4.6
Realized Volatility over 90Days	18.1	7.9	10.5	12.6	16.0	21.3	29.2
Implied Volatility - Three Main Indices, in %							
60% Strike	31.7	6.2	25.1	27.8	30.6	34.8	39.2
80% Strike	25.1	5.6	19.7	21.2	23.9	27.5	31.8
100% Strike	19.9	5.4	14.4	16.1	18.9	22.2	25.8
Implied Volatility - All Indices, in %							
80% Strike	25.0	6.7	18.3	20.7	23.6	27.5	32.8
90% Strike	22.8	6.8	16.2	18.6	21.4	25.2	30.4
100% Strike	21.0	6.8	14.3	16.6	19.7	23.2	28.5
Volumes							
Issuance Volumes, In billion $	0.5	0.8	0.0	0.0	0.1	0.7	1.4
Outstanding Volumes, In billion $	9.8	13.9	0.1	0.7	3.4	12.5	32.5
Open Interest, In billion $	67.3	116.1	2.1	4.1	12.5	53.8	267.9

This table reports summary statistics on our total sample of index-linked SPPs, as well as our aggregated monthly panel of SPP volumes, volatility prices, and market parameters. SPP data are from a dataset that covers the global issuance of SPPs over the 2005-2019 period. Volatility prices for the three main indices, S&P 500, Eurostox 50, and Nikkei 225, are from a proprietary dataset of a major investment bank. Volatility prices and open interest for all indices are from OptionMetrics. Open interest is the open interest of put options with strikes at 80% and maturity over three months.

Table II
Market Conditions and Product Design

	Headline Rate (spread to benchmark)					
	in %			in Log		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
90D realized volatility, in %	0.11***	0.05***	0.05***			
	(0.01)	(0.01)	(0.01)			
3Y Swap Rate	0.11	-0.32***	-0.72***			
	(0.10)	(0.08)	(0.11)			
90D realized volatility, Log				0.43***	0.19***	0.19***
				(0.05)	(0.04)	(0.03)
3 Year Swap Rate, Log				-0.08*	-0.26***	-0.59***
				(0.04)	(0.03)	(0.06)
Fixed Effects						
Underlying	Yes	Yes	Yes	Yes	Yes	Yes
Year	-	Yes	Yes	-	Yes	Yes
Design	-	-	Yes	-	-	Yes
Market Country	-	-	Yes	-	-	Yes
Observations	178,161	178,161	178,156	177,892	177,892	177,887
R^2	0.12	0.24	0.36	0.10	0.21	0.34

This table displays the coefficients of OLS regressions in which the dependent variable is the spread between the product headline rate and the benchmark interest rate. Headline rate is defined as the fixed yearly rate that the investor receives in the best possible scenario. The benchmark rate is the 3 year swap rate of the distribution country. The sample includes all SPP products issued over the 2005-2019 period with one or several indices as underlyings. Design fixed effects include fixed effects for maturity, strike, number of payoff features, number of scenari. Standard errors are clustered at the month and the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table III
Realized Volatility and SPP Volumes

	Net Issuances in Billion $				Outstanding Volumes in Billion $			
Sample	Indices		Period		Indices		Period	
	3 Main	**All**	**≤2011**	**>2011**	**3 Main**	**All**	**≤2011**	**>2011**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Volatility:								
- 2nd Quartile	0.02	0.04	0.01	0.02	1.87*	1.32**	2.01***	0.59
	(0.08)	(0.03)	(0.02)	(0.04)	(1.09)	(0.57)	(0.35)	(0.36)
- 3rd Quartile	0.25***	0.10***	0.03*	0.10***	3.58***	2.38***	2.69***	1.61***
	(0.08)	(0.04)	(0.02)	(0.04)	(1.28)	(0.84)	(0.51)	(0.45)
- 4th Quartile	0.26***	0.12***	0.04**	0.19***	3.31**	2.85**	3.19***	1.82**
	(0.08)	(0.05)	(0.02)	(0.04)	(1.52)	(1.35)	(0.67)	(0.71)
3Y Swap Rate	0.20***	0.03***	0.01	0.05***	-7.56***	-0.08	0.42***	-1.12***
	(0.04)	(0.01)	(0.01)	(0.02)	(0.64)	(0.31)	(0.13)	(0.14)
Fixed Effects								
Month					Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	540	2,136	1,008	1,128	540	2,136	1,008	1,128
R^2	0.13	0.05	0.34	0.12	0.81	0.77	0.78	0.88

This table displays the coefficients from regressing monthly index implied volatility on SPP issuance volumes with corresponding underlying. Monthly issuance volumes are scaled by the market cap of corresponding indices. The model includes month and index fixed effects, as well as strike fixed effects in Column 1. The sample includes monthly implied volatility data at 1 year for the 12 leading global indices over the 2005-2019 period: S&P500, Eurostoxx 50, Nikkey, Hang Send, Han Seng China Enterprise, Swiss Market Index, CAC 40, DAX, Nasdaq, Russel, FTSE, Kosip 200. Standard errors are clustered at the month and at the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table IV
Impact of SPP Volumes on Implied Volatility Surface:
S&P500, Eurostoxx 50, and Nikkei 225

	Dependent Variable : Implied Volatility, in %							
Strike	**All**		**50%**	**60%**	**70%**	**80%**	**90%**	**100%**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
SPP Volumes								
- In $	-0.43***							
	(0.12)							
- Scaled		-0.23***	-0.40***	-0.31***	-0.26***	-0.20***	-0.13*	-0.08
		(0.07)	(0.11)	(0.09)	(0.08)	(0.07)	(0.07)	(0.08)
90-Day Realized								
Volatility, in %	0.23***	0.23***	0.19***	0.20***	0.21***	0.23***	0.26***	0.29***
	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)
3-Year Swap								
Rate, in %	-0.05	-0.06	-0.05	-0.11	-0.15	-0.13	-0.06	0.12
	(0.17)	(0.17)	(0.25)	(0.21)	(0.20)	(0.19)	(0.17)	(0.16)
Fixed Effects								
Strike	Yes	Yes						
Month	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3,240	3,240	540	540	540	540	540	540
R^2	0.96	0.96	0.97	0.97	0.97	0.97	0.97	0.97

This table displays the coefficients from regressing monthly index implied volatility on SPP issuance volumes with corresponding underlying. In Column 1, SPP volumes are monthly issuance volumes in billion USD, in Columns 2 to 8, SPP issuance volumes are scaled by market cap and expressed in basis points. The model includes month and index fixed effects, as well as strike fixed effects in Columns 1 and 2. The sample includes monthly implied volatility data at 1 year for the three main indices of the US, Europe and Asia: S&P500, Nikkei225, and Eurostoxx50. The sample covers the 2005-2019 period. Standard errors are clustered at the month and at the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table V
The Impact of SPP Volumes on Implied Volatility: Effects for All Indices across Strikes, Time, and Exposure

	Dependent Variable : Implied Volatility, in %							
	Strikes				**Time Period**		**Option Market Size**	
	All Strikes	**80%**	**90%**	**100%**	**<2011**	**≥2011**	**All Strikes**	**80%**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Scaled SPP Volumes	-0.34***	-0.39***	-0.35***	-0.30***	-0.39***	-0.13**	0.27	0.35
	(0.10)	(0.11)	(0.10)	(0.11)	(0.14)	(0.05)	(0.30)	(0.32)
Volumes × Medium Option Market							-0.54*	-0.62**
							(0.29)	(0.31)
Volumes× Small Option Market							-0.65**	-0.80***
							(0.28)	(0.31)
90-Day realized volatility, in %	0.41***	0.40***	0.41***	0.41***	0.41***	0.25***	0.41***	0.40***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.07)	(0.03)	(0.03)	(0.03)
3-Year Swap Rate	-0.21*	-0.27**	-0.20*	-0.16	-0.53***	-0.15	-0.21***	-0.27***
	(0.12)	(0.12)	(0.12)	(0.12)	(0.17)	(0.09)	(0.07)	(0.07)
Fixed Effects								
Strike	Yes				Yes	Yes	Yes	
Month	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6,398	2,128	2,135	2,135	983	1,145	6,398	2,128
R^2	0.92	0.91	0.92	0.92	0.91	0.92	0.92	0.91

This table displays the coefficients from regressing monthly index implied volatility on SPP issuance volumes with corresponding underlying. The scaled SPP volume is in BP. Monthly issuance volumes are scaled by the market cap of corresponding indices. The model includes month and index fixed effects, as well as strike fixed effects in Column 1. The sample includes monthly implied volatility data at 1 year for the 12 leading global indices over the 2005-2019 period: S&P500, Eurostoxx 50, Nikkei 2255, Hang Seng, Han Seng China Enterprise, Swiss Market Index, CAC 40, DAX, Nasdaq, Russel, FTSE, Kospi 200. Standard errors are clustered at the month and at the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table VI
SPP Volumes and Implied Volatility in Times of Market Stress

	Dependent Variable : Implied Volatility, in %									Realized Vol
Market Stress Indicator	**Yearly Returns**$< -15\%$			**Yearly Returns**$< -25\%$			**Yearly Returns**$< -35\%$			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
Market Stress	9.21***	14.02***	1.24	14.60***	21.97***	3.92**	17.34***	25.86***	7.72***	1.73
	(1.45)	(3.15)	(1.06)	(1.66)	(2.84)	(1.74)	(1.47)	(3.31)	(2.40)	(1.66)
SPP Volume Quartiles	-0.58***	-0.36**	-0.30***	-0.45***	-0.29*	-0.27***	-0.39**	-0.30*	-0.31***	-0.19*
	(0.17)	(0.17)	(0.07)	(0.16)	(0.16)	(0.06)	(0.16)	(0.16)	(0.07)	(0.11)
Market Downturn × Quartiles		-1.74**	-0.61*		-2.84***	-1.74***		-3.48***	-2.56***	-0.34
		(0.68)	(0.33)		(0.63)	(0.46)		(1.05)	(0.61)	(0.34)
3-Year Swap Rate	1.76***	1.73***	-0.56***	1.70***	1.68***	-0.56***	1.97***	1.96***	-0.54***	-0.43***
	(0.15)	(0.15)	(0.09)	(0.14)	(0.14)	(0.09)	(0.14)	(0.14)	(0.08)	(0.10)
Fixed Effects										
Month	-	-	Yes	-	-	Yes	-	-	Yes	Yes
Strike	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5,177	5,177	5,177	5,177	5,177	5,177	5,177	5,177	5,177	1,728
R^2	0.56	0.56	0.90	0.62	0.63	0.90	0.58	0.59	0.90	0.86

This table displays the coefficients from regressing monthly index implied volatility on an indicator dummy for market stress and its interaction with quartiles of SPP outstanding volumes. The model includes index and strike fixed effects, as well as month fixed effects in Columns 3, 6, 9 and 10. The sample includes monthly implied volatility data at 1 year for the 12 leading global indices over the 2005-2019 period: S&P500, Eurostoxx 50, Nikkei225, Hang Seng, Han Seng China Enterprise, Swiss Market Index, CAC 40, DAX, Nasdaq, Russel, FTSE, Kospi 200. Standard errors are clustered at the month level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table VII
Changes in HSCEI Implied Volatility, Before and After the South Korea Informal Ban of HSCEI index-linked SPPs

	Dependent Variable : Implied Volatility, in %							
	All	**80%**	**90%**	**100%**	**All**	**80%**	**90%**	**100%**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
HSCEI Index ×								
Post	1.92***	3.22***	2.05***	0.50	2.20***	3.49***	2.33***	0.80*
	(0.32)	(0.63)	(0.61)	(0.57)	(0.49)	(0.58)	(0.52)	(0.44)
90D Realized								
Volatility, in %					0.14***	0.14***	0.14***	0.14***
					(0.03)	(0.03)	(0.03)	(0.03)
3Y Swap Rate					0.01	-0.03	-0.01	0.07*
					(0.05)	(0.06)	(0.05)	(0.04)
Fixed Effects								
Strike	Yes				Yes			
Month	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	863	287	288	288	863	287	288	288
R^2	0.74	0.92	0.94	0.94	0.75	0.93	0.95	0.96

This table displays the coefficients from regressing monthly index implied volatility on a dummy equal to 1 on months after the informal bank for the HSCEI Index only. The model includes month and index fixed effects, as well as strike fixed effects in Columns 1 and 5. The sample is restricted to on year before and one year after the shock, i.. covers the period from September 2014 to September 2016. It includes monthly implied volatility data at 1 year for the 12 leading global indices : S&P500, Eurostoxx 50, Nikkei 2255, Hang Seng, Han Seng China Enterprise, Swiss Market Index, CAC 40, DAX, Nasdaq, Russel, FTSE, Kospi 200. Standard errors are clustered at the month level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table VIII
Maximum Likelihod Estimation of Structural Model

	Unrestricted		Restricted	
	Estimate (1)	Std Error (2)	Estimate (3)	Std Error (4)
Interest Rate				
Mean μ_1 (% per year)	1.9315	0.0584	1.9859	0.0121
Persistence parameter ϕ_1	0.9970	–	0.9970	–
Volatility σ_1 (% per year)	0.2138	0.0048	0.2151	0.0012
Equity Index				
Mean log volatility μ_2	-5.1656	0.0185	-5.1338	0.0109
Persistence parameter ϕ_2	0.9489	0.0067	0.9496	0.0013
Volatility of log volatility σ_2	0.3079	0.0051	0.3018	0.0021
Correlations				
ρ_{12}	-0.0401	0.0264	-0.0277	0.0078
ρ_{1M}	0.3650	0.0650	0.3411	0.0168
ρ_{2M}	-0.1055	0.0077	-0.0852	0.0090
Stochastic discount factor				
Sensitivity of SDF to SPP volume, λ_D	0.9747	0.0591	0	–
Payoff of Exogenous Put Portfolio:				
$S_T = 0.0$	0.7849	0.0102	0.8626	0.0112
$S_T = 0.5$	-1.2855	0.0242	-1.3594	0.0154
$S_T = 0.6$	-0.9409	0.0232	-0.9549	0.0135
$S_T = 0.7$	-0.8359	0.0265	-0.8031	0.0119
$S_T = 0.8$	-0.9033	0.0590	-0.8851	0.0162
$S_T = 0.9$	-0.5935	0.0238	-0.5974	0.0110
Household				
Risk aversion γ	7.2787	0.0701	7.8155	0.1866
Prelec coefficient a	1.9635	0.0773	1.9171	0.0709
Prelec coefficient b	3.5995	0.0737	3.9115	0.1222
Scaling coefficient, λ_H	0.4988	0.0764	0.4616	0.0139
Measurement errors				
Yield spread σ_{SPP} (%)	1.8529	0.0131	1.8988	0.0184
SPP volume σ_{VOL}	0.2371	0.0233	0.2173	0.0090
Correlation ρ_{SPP}	-0.6856	0.0201	-0.6847	0.0229
Implied volatility σ_{IV} (%)	0.6424	0.1048	0.6419	0.0119
Test of the restriction $\lambda_H = 0$				
Log likelihood	6309.04		6297.36	
LR statistic	23.39			
p value	$< 10^{-5}$			

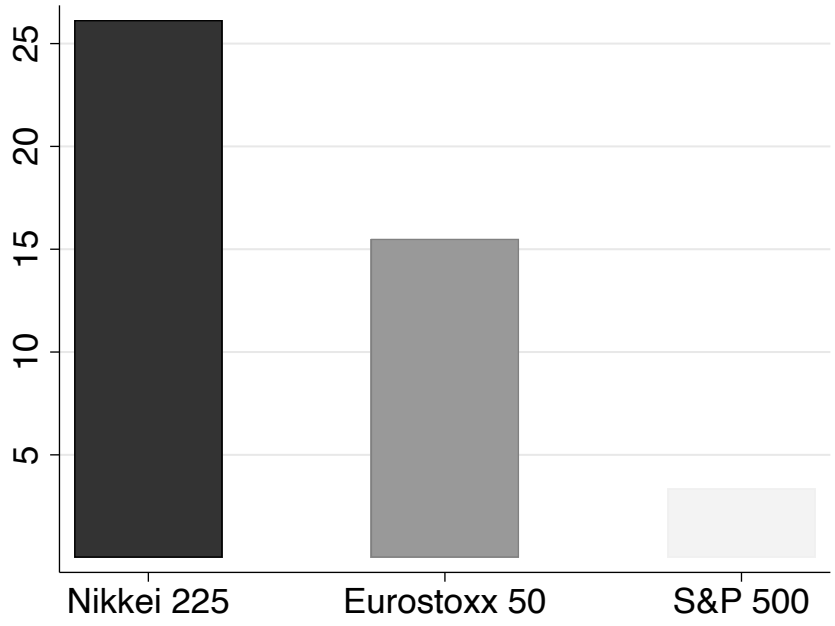
This table displays maximum likelihood estimates of the structural model defined in Section IV. Columns 1 and 2 report the point estimate and asymptotic standard error of the model under the restriction $\phi_1 \leq 0.997$. Because the constraint is binding, we do not report a standard error for ϕ_1. Columns 3 and 4 report the point estimate and asymptotic standard error under the additional restriction that the SPP volume does not impact the SDF ($\lambda_D = 0$).

Table IX
Maximum Likelihod Estimation
of Structural Model (OLD)

	Unrestricted Estimate (1)	Restricted Estimate (2)
Interest Rate		
Mean μ_1	0.01928	0.01924
Persistence parameter ϕ_1	0.99699	0.99676
Volatility σ_1	0.00212	0.00211
Equity Index		
Mean log volatility μ_2	-5.49322	-5.50605
Persistence parameter ϕ_2	0.93439	0.93319
Volatility of log volatility σ_2	0.33565	0.33668
Correlations		
ρ_{12}	0.10121	0.10291
ρ_{1M}	0.25972	0.28945
ρ_{2M}	-0.13300	-0.14236
Stochastic discount factor		
Sensitivity of SDF to SPP volume, λ_H	0.38810	0
Payoff of Exogenous Put Portfolio:		
$S_T = 0.0$	0.63072	0.53537
$S_T = 0.5$	-1.11587	-1.14221
$S_T = 0.6$	-0.82323	-0.82332
$S_T = 0.7$	-0.66744	-0.65700
$S_T = 0.8$	-0.88680	-0.86121
$S_T = 0.9$	-0.41482	-0.33398
Measurement errors		
Yield spread	0.01556	0.01615
Put prices	0.00629	0.00633
Test of the restriction $\lambda_H = 0$		
Log likelihood	6286.82283	6283.32066
LR statistic	7.00325	
p value	0.00814	

This table displays the ML estimates of the structural model outlined in Section IV.

Panel A. Scaled SPP Issuance Volumes across Indices



Panel B. Volatility Premium across Indices

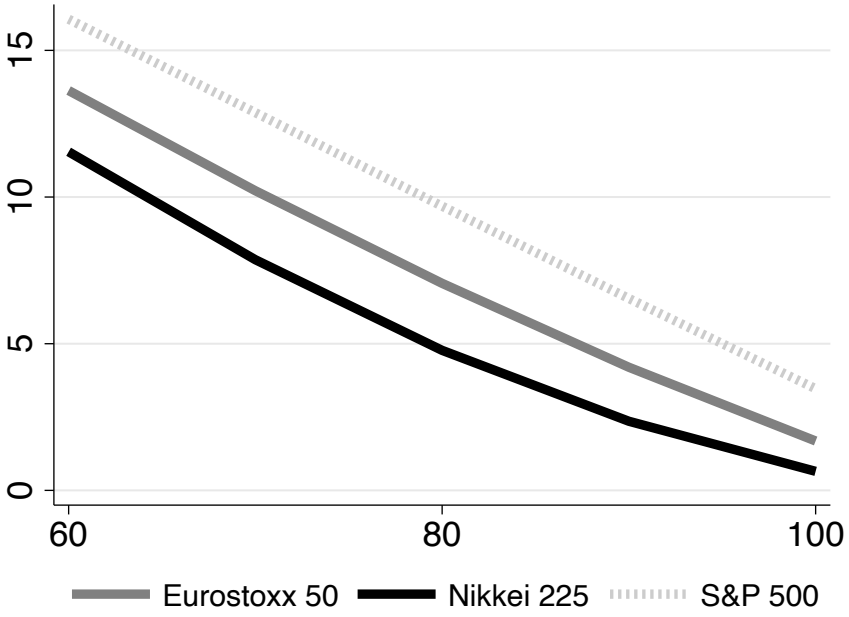


Figure 1. Average Scaled Volumes and Volatility Premium across Indices

Panel A plots the average yearly issuance volumes of SPP scaled by market capitalization across the three indices, S&P500, Nikkei225, and Eurostoxx50. Panel B plots the volatility premium, i.e. the difference between 1 year implied volatility and 90Day-realized volatility, at the 60, 70, 80, 90, and 100% strikes for the three indices, S&P 500, Nikkei 225, and Eurostoxx 50.

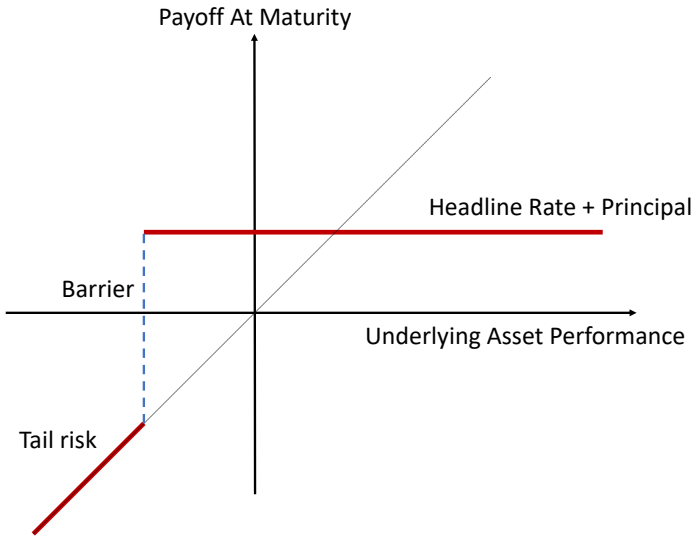
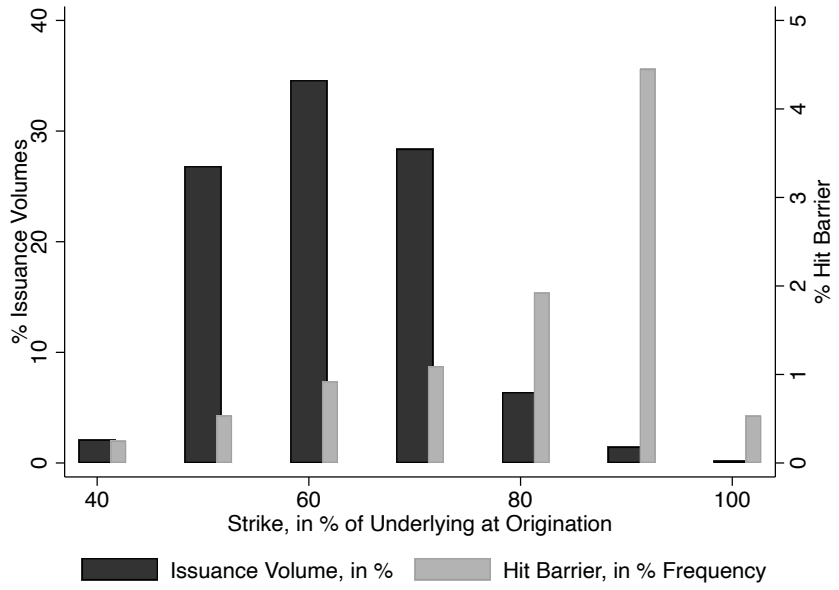


Figure 2. Design of a Simple Short Put Product

This figure shows the pay-off diagram of a typical Short Put Product.

Panel A. Histogram of SPP Strikes



Panel B. Histogram of SPP Ex Post Excess Returns

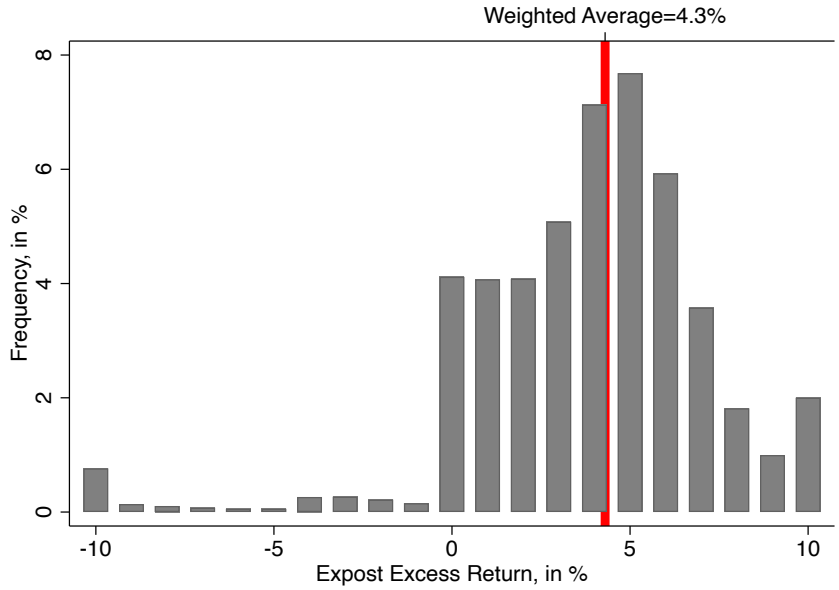


Figure 3. SPP Issuance volumes across Strikes and Ex-Post Excess Returns

Panel A presents the distribution of SPP issuance volumes by percentage strikes over the 2005-2019 period, along with the percentage of products that hit the barrier. Panel B shows the histogram of SPP ex-post excess returns over the same period. Ex-post excess returns are calculated as the difference between the SPP realized return at maturity and the corresponding 3-year swap rate. The sample consists of 100,000 SPPs issued globally since 2005, with indices as their underlying assets.

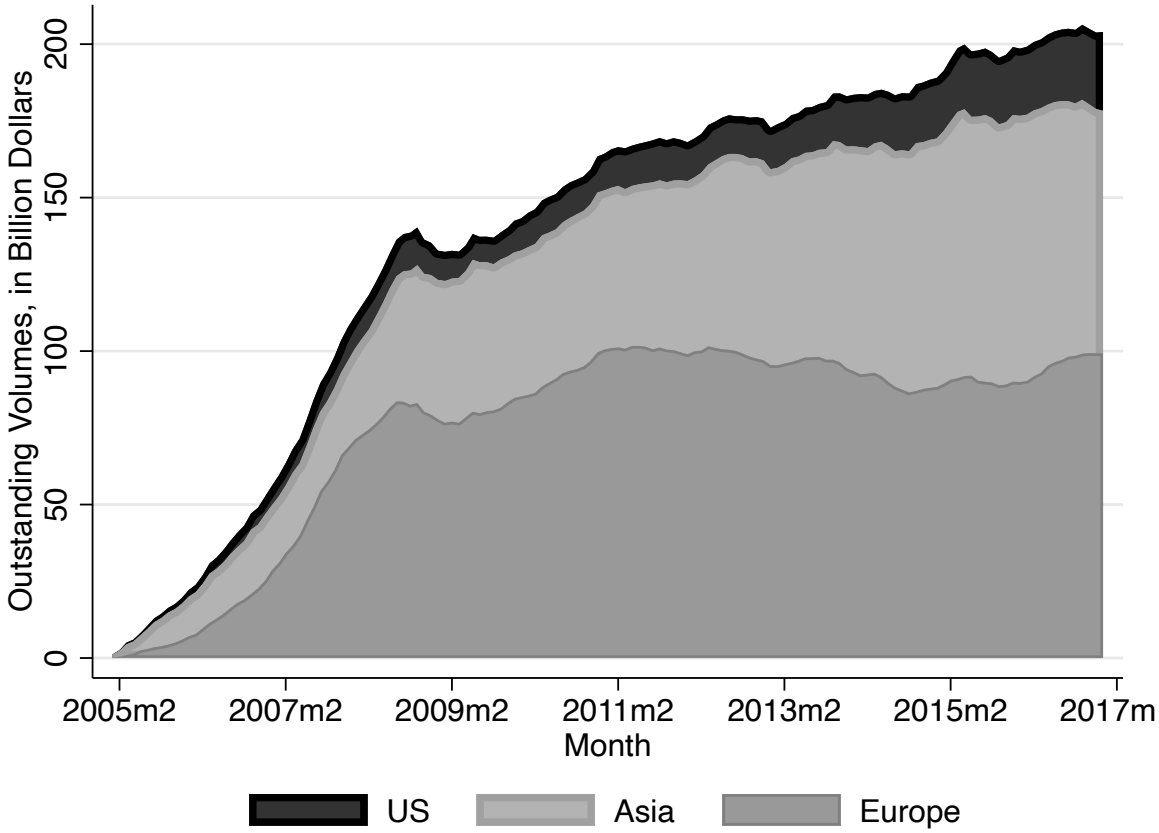


Figure 4. Outstanding Volumes of SPPs Across Regions

The figure plots the evolution of outstanding volumes of all SPPs across Europe, Asia and the U.S. in billion dollars from 2005 to 2016.

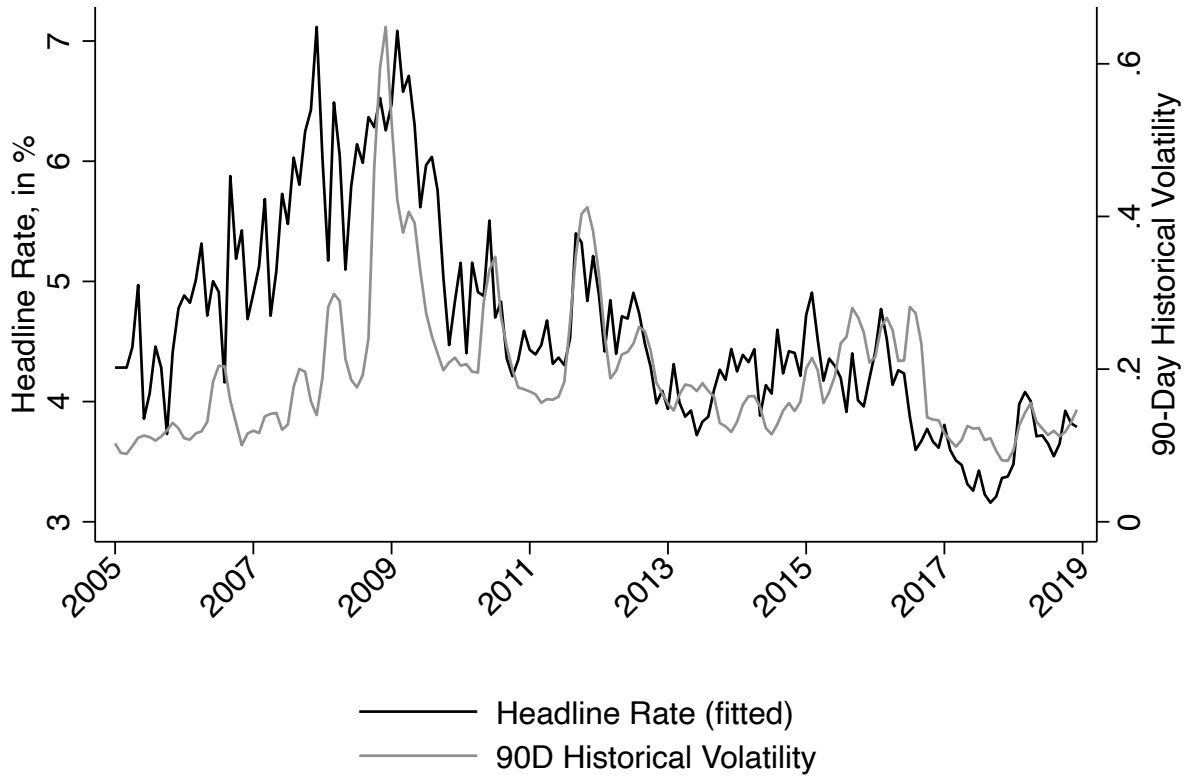


Figure 5. Eurostoxx SPP Headline Rates and realized volatility

This figure displays the fitted headline rates of SPPs issued globally from 2005 to 2019, with the Eurostoxx 50 as the underlying asset, alongside the 90-day realized volatility of the Eurostoxx 50. The sample is structured as a monthly panel, where realized volatility represents the average 90-day realized volatility for each month, and the fitted headline rate corresponds to the predicted headline rate for a simple SPP with a 3-year maturity and a 60% strike. We obtain the predicted headline rate by regressing the headline rates of 50,000 products in our sample on month, maturity, strike, the number of payoffs, the number of underlying assets (fixed effects), and dummies for key payoff features, such as worst-off, protection tracker, and knock-in.

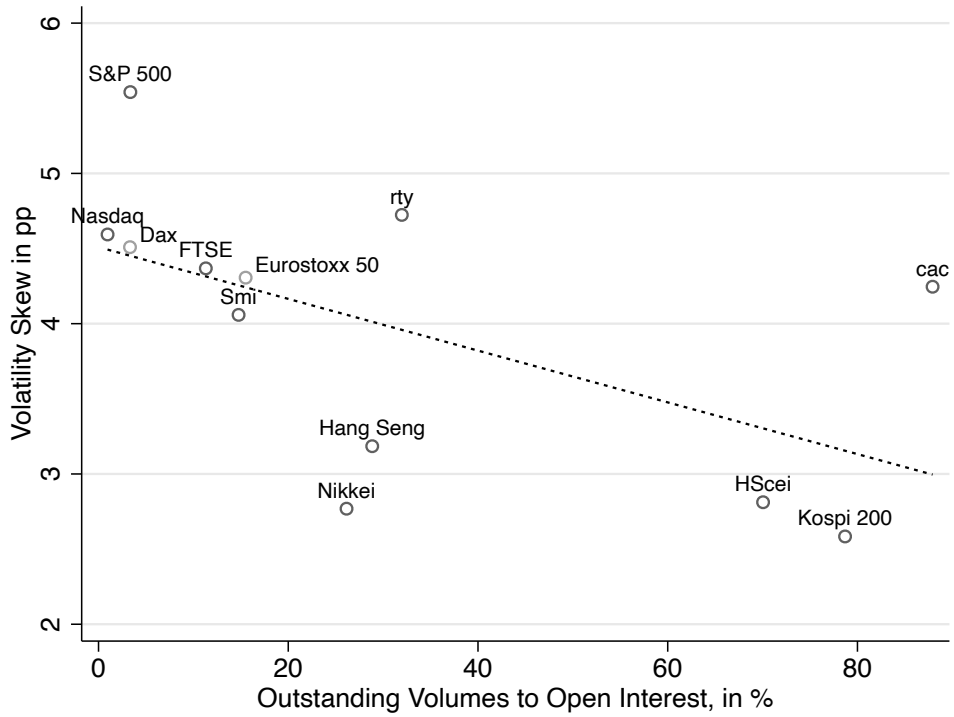


Figure 6. SPP Outstanding Volumes and Volatility Skewness

This figure displays the average volatility skewness over the 2005-2019 period versus outstanding volumes of SPP in percent of the open interest for options with a strike below 80% and a maturity above three months. The average volatility skewness is measured using monthly measure of the difference between mean implied volatility at the 80% strike and mean implied volatility at the 100% strike. Out sample focused on the 10 indices used as undrlyings for 99% of SPPs using indices as underlyings during the 2005-2019 period.

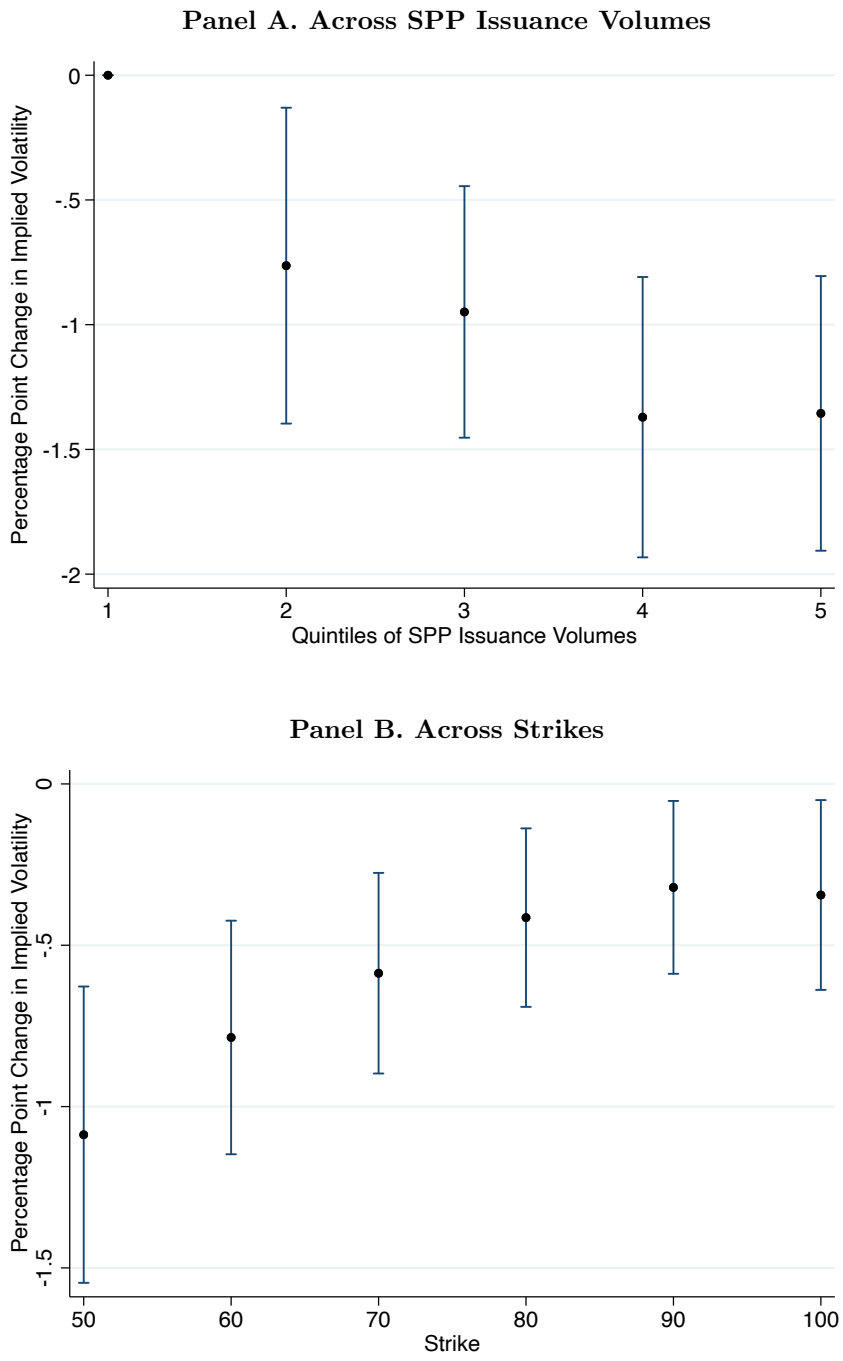


Figure 7. Impact of SPP Issuance Volumes on Implied Volatility: Coefficient Estimates

This figure plots the coefficient estimates of dummies for quintiles of issuance volumes in Panel A, and strikes, in Panel B in a regression where the dependent variable is the 1 year implied volatility. The sample is a monthly panel of the three indices, S&P500, Nikkei225, and Eurostoxx50. The regression includes month and underlying fixed effects.

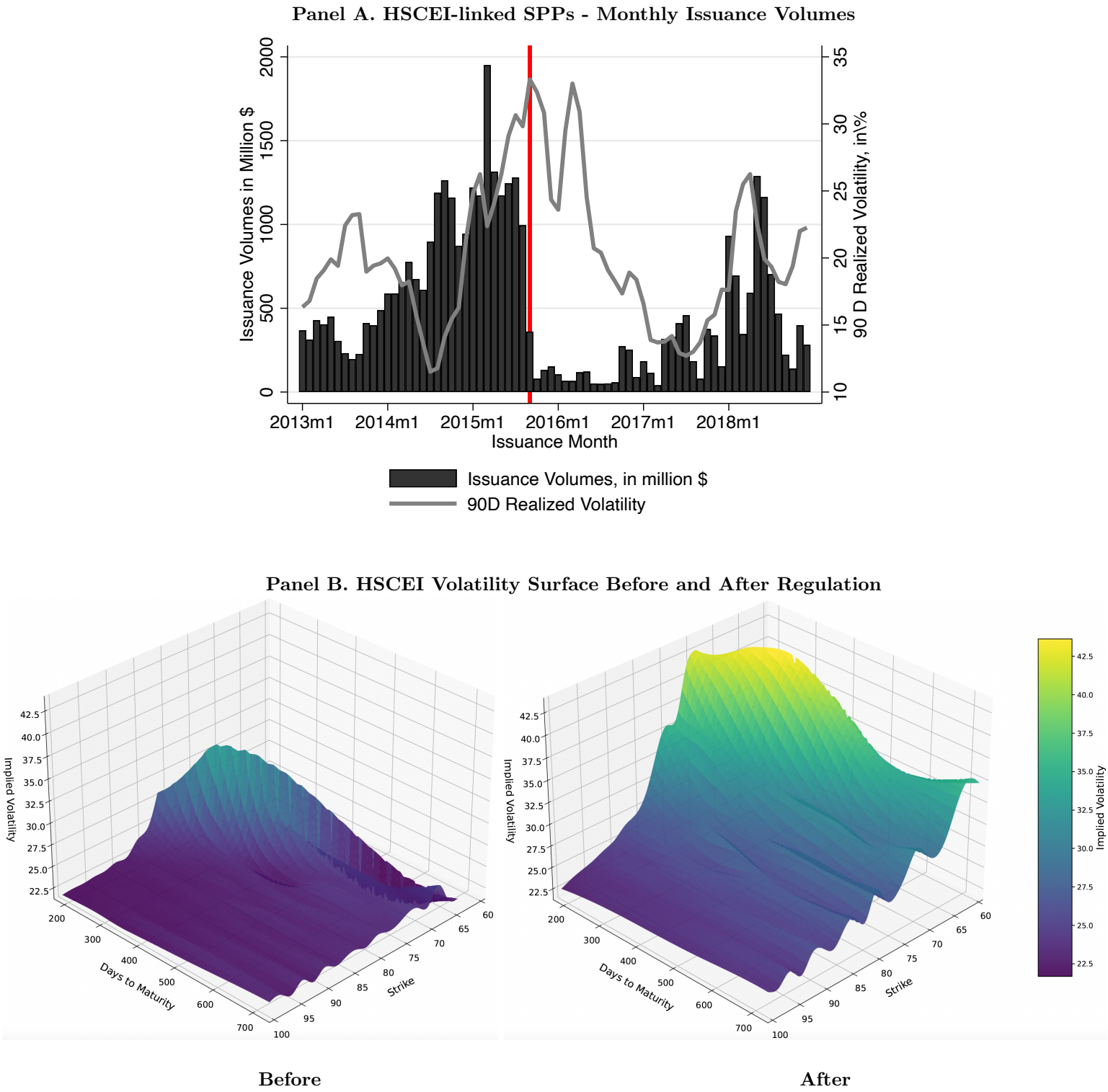
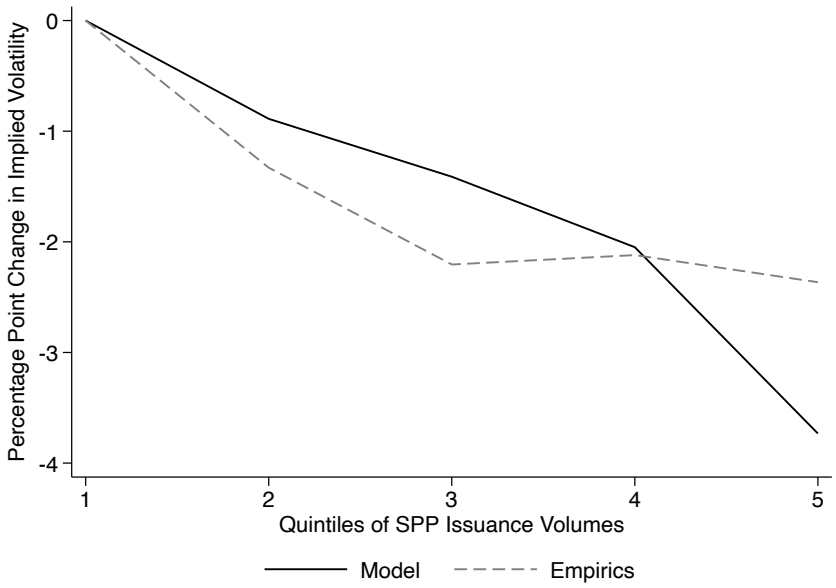


Figure 8. Natural Experiment: 2015 South Korean Ban on HSCEI-Linked SPPs

Panel A plots the monthly issuance volumes of HSCEI-linked SPPs over the 2005-2019 period. The vertical line indicates the date of the informal ban of HSCEI-linked SPPs by the South Korean market authority. Panel B plots the average volatility surfaces in the year before and the year after the ban.

Panel A. Across SPP Issuance Volumes



Panel B. Across Strikes

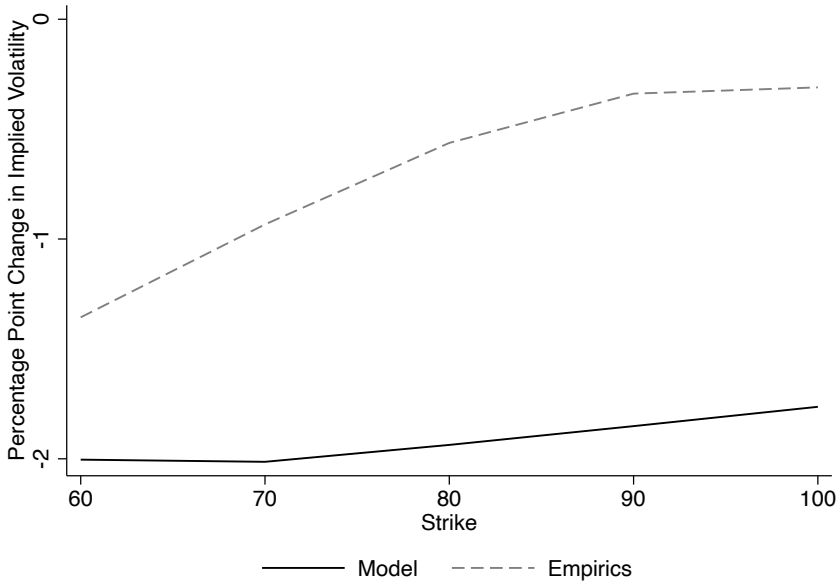


Figure 9. Impact of SPP Issuance Volumes on Implied Volatility: Model versus Empirics

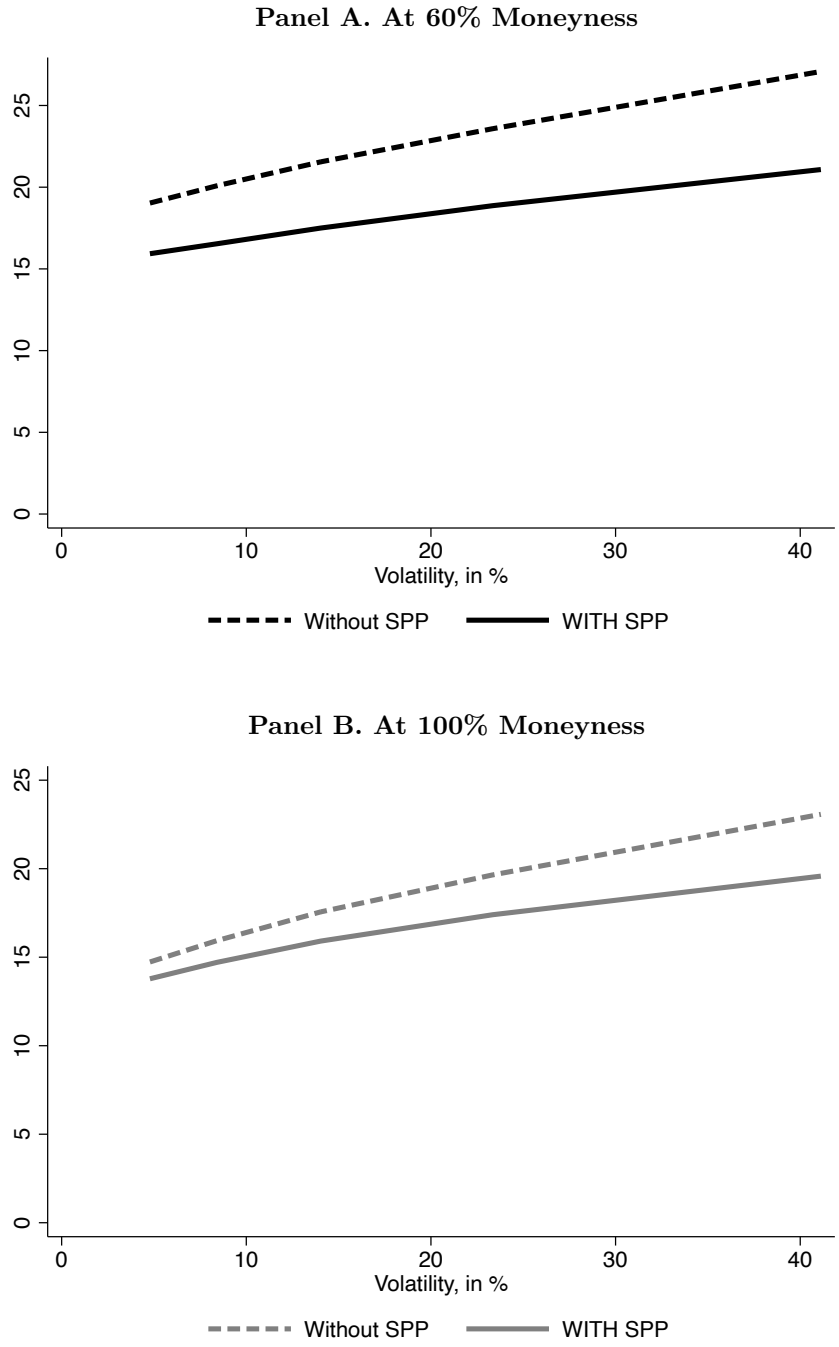


Figure 10. Implied Volatility and Realized Volatility – with and without SPP market