Designing redistribution with endogenous transfer take-up*

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February 10, 2026

Preliminary and incomplete draft – please do not circulate

Abstract

The existing literature on optimal income taxation ignores that, across the world, redistribution towards the poor mainly happens via welfare transfers with take-up rates that are endogenous and far below 100%. This paper provides the first comprehensive analysis of tax-transfer systems that are composed of mandatory income taxes and optional transfer programs. We develop a theoretical model that (a) allows for heterogeneity in labor productivity and take-up costs and (b) accounts for behavioral responses at the take-up margin as well as the intensive and extensive margin of labor supply. We use this model to derive empirically applicable formulas that specify (i) conditions for the existence of Pareto-improving reforms of taxes, transfers, or both, (ii) the inverse optimum weights of transfer recipients and non-recipients that would make an observed safety net optimal, and (iii) the optimal base transfer to non-working recipients as well as optimal tax rates and transfer phase-out rates. We apply these formulas to investigate the German and Swedish tax-transfer system and find that both countries' safety nets are inefficient: There exist transfer reforms that would allow to make all recipients better off while generating a net surplus for the government.

Keywords: Redistributive taxation; welfare transfers; incomplete take-up.

JEL Codes: H21; H23; I38.

*We thank Pierre Boyer, Antoine Ferey, Andreas Haller, Laurence Jacquet, Etienne Lehmann, Claire Leroy, Jonas Löbbing, Andreas Peichl, Charlie Rafkin, Karl Schulz, Stefan Staubli, and participants at the ZEW Public Finance 2025, the EEA Annual Congress 2025, the Paris Economics of Taxation Workshop, and seminars at LMU Munich and University of Barcelona for helpful comments. Rafael Pfeiffer and Rafael Zamudio Valdés provided excellent research assistance.

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1 Introduction

The design of redistributive taxation is one of the central themes of public finance. Numerous papers have studied the optimal non-linear income tax schedule in the tradition of Mirrlees (1971). The conventional wisdom is that marginal tax rates should have a u-shaped pattern: Very high at the bottom, then gradually decreasing towards the middle and eventually increasing towards the top, see Diamond (1998) and Saez (2001). It has often been argued that, when accounting for both income taxes and welfare transfers, the empirical effective marginal tax rates are broadly in line with this pattern. In particular, this is true because most transfer programs are phased out at very high rates with earned income. This argument abstracts from the empirical fact, however, that most welfare transfers are only taken up by 40 to 80% of the eligible persons.[1] As a result, there is not one u-shaped non-linear schedule but there are two co-existing ones with very different properties: (1) For transfer recipients, effective marginal tax rates are indeed very high at the bottom and then typically drop abruptly to much lower levels at the income threshold where transfers are fully phased out.[2] (2) For non-recipients, by contrast, effective marginal tax rates are very low at the very bottom and increase almost monotonically with income. Figure 1 illustrates this widespread pattern for the case of Germany. Moreover, eligible people can choose endogenously whether or not to take up the transfers they are eligible for. The decision arguably depends on the generosity and the design of available transfers as well as program complexity, application rules, awareness of the transfers, etc. Surprisingly, the enormous literature on the optimal design of redistributive taxation has never taken into account the co-existence of two schedules due to incomplete transfer take-up, nor the behavioral responses in take-up behavior to tax and transfer reforms. However, these aspects are in the focus of a separate literature on endogenous transfer participation, optimal program complexity, and the desirability of ordeals, which does not study the optimal design of transfer schedules or their interaction with non-linear income taxes.[3]

In this paper, we seek to fill this gap by providing the first comprehensive analysis of redistributive taxation with endogenous take-up of a welfare transfer supporting the poor. We develop a flexible but tractable theoretical framework that allows for mandatory non-linear income taxes, optional non-linear welfare transfers and behavioral responses at three arguably important margins: the intensive margin of labor supply (hours worked, intensity), the extensive margin of labor market participation, and the take-up margin. We also allow for multi-dimensional heterogeneity in skills, fixed costs of working, and take-up cost, and account for the plausible interdependence of earnings and take-up choices. To study the optimal tax and transfer schedules, we extend the standard tax pertubation approach building on Piketty (1997), Saez (2001), and Bierbrauer et al. (2023). Specifically, we consider small variations in the tax schedule, in the transfer schedule, or in both schedules at the same time.

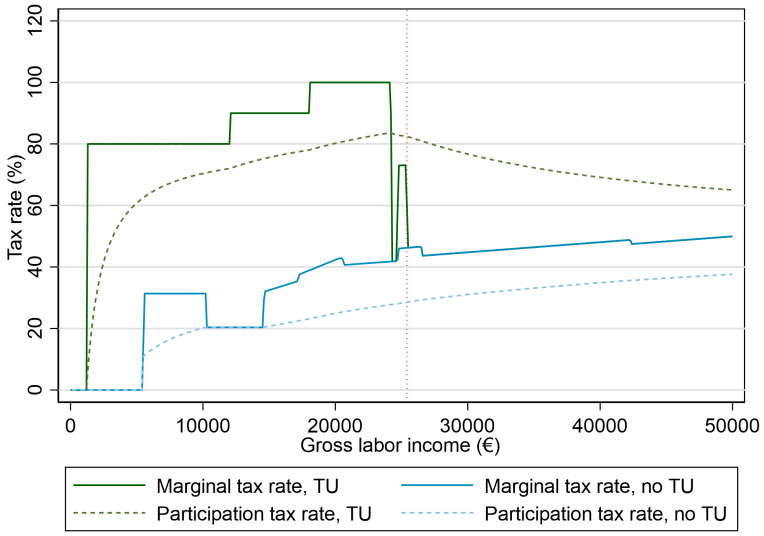
Our main contribution is to provide novel, widely applicable tools for the empirical analysis of redistribution systems that involve mandatory tax payments and optional welfare transfers with incomplete take-up. These tools are expressed in terms of sufficient statistics, i.e., empirically estimable objects such as tax and phase-out rates, income distributions, and behavioral elasticities. More specifically, we derive, first, a set of necessary conditions for an observed tax-transfer system to be Pareto-efficient. If one of these conditions is violated, there exist reforms that allow to make every agent better off. Second, we show how to determine the inverse optimum weights for transfer recipients and non-recipients, i.e., the social objective for which an observed tax-transfer system would be optimal. If these inverse optimum weights deviate from the extent to which society, or the government, cares for a subgroup of individuals, then this indicates that a reform of the system is desirable. Third, we provide ABC-type formulas for

[1] See Hernanz, Malherbet and Pellizzari (2004) for a cross-country comparison and Ko and Moffitt (2024) for a recent survey of the literature.

[2] Generous welfare transfers that are phased out at high rates can generate poverty traps that may even lead to severe inefficiencies, see Bierbrauer, Boyer and Hansen (2023).

[3] Influential papers include Nichols and Zeckhauser (1982), Besley and Coate (1992b), Kleven and Kopczuk (2011), and Rafkin, Solomon and Soltas (2023).

Figure 1: EMTRs and PTRs for single parents with one child, GER 2018.



Notes: This graph plots effective marginal tax rates (solid line) and participation tax rates (dashed line) for welfare recipients (green) and non-recipients (blue) against annual labor income. The vertical dotted line in red shows the point of full phaseout, i.e., the income level at which the household is no longer eligible for any of our three welfare transfer programs (social assistance, housing benefit, child supplement). The tax rates are based on a hypothetical single-parent household with one child living in a region with rent level 5. The recipient household always takes up the transfer program that is more advantageous.
Source: Authors' illustrations based on ifo Microsimulation model.

the optimal marginal tax rates, the optimal transfer phase-out rates, and the optimal transfer to non-working recipients. We then apply these tools to study the German tax-transfer system in 2018 and the Swedish tax-transfer system in 2023. This investigation shows that our tools are not only conceptually interesting, but also of practical relevance. We find that the German and Swedish welfare states violate our Pareto conditions, and that, specifically, the countries' transfer schedules can be reformed in a Pareto-improving way.

Theoretical results In our theoretical part, we provide three contributions. First, we derive empirically applicable sufficient-statistics formulas that capture the effects of small tax and transfer reforms on net tax revenue. These formulas transparently capture the interactions in the tax-transfer system and the relevance of all three behavioral margins. Second, we provide a set of necessary and sufficient conditions for the existence of small Pareto-improving reforms of the tax-transfer system. Our results do not only extend the results for an integrated schedule by Bierbrauer et al. (2023), they also provide novel insights. In particular, the richer set of instruments available implies additional, stricter conditions for Pareto efficiency, and allows to precisely pinpoint whether potential inefficiencies arise in the income tax or in the welfare transfers. Moreover, we can also provide an upper Pareto bound on the generosity of transfers to non-working people. Third, we show how to identify the inverse optimum weights along the income distribution, i.e., the marginal social welfare weights that make an observed tax-transfer system optimal. Specifically, we derive separate inverse optimum weights for transfer recipients and non-recipients with the same earnings levels, thereby identifying the degree of self-targeting that is required to make the current system ideal, see Rafkin et al. (2023). Fourth, we extend the classical results in optimal tax theory to provide optimal policy formulas for (a) the optimal phase-out rates for transfer recipients, (b) the optimal marginal tax rates for non-recipients, and (c) the optimal base transfer to recipients. These formulas have ABC-type representations in terms of empirically estimable objects such as income distributions, behavioral elasticities, and marginal social welfare weights.

Empirical application In our empirical part, we apply our theoretical insights to investigate the 2018 German tax-transfer system and the 2023 Swedish tax-transfer system for single parents (and childless singles). For this purpose, we combine information on the income tax schedule and the most important welfare transfers[4] with information on labor earnings and transfer take-up from the *German Socio-Economic Panel* and *linked Swedish registry data*, and empirical estimates on labor supply responses at the intensive and extensive margins. As the take-up elasticity has never been estimated for Germany or Sweden to the best of our knowledge, we simulate it with a state-of-the art microsimulation model that is combined with a structural labor supply model in the spirit of van Soest (1995).[5]

Equipped with these information, we first verify whether the German and Swedish tax-transfer systems are Pareto-efficient. We find that they are not. In particular, the German transfers are phased out in a highly inefficient way: Phase-out rates are excessively high and they fluctuate too much with earnings levels. The inefficiencies in the Swedish system are not as pronounced, but smoothing phase-out rates would still allow for Pareto-improvements. By contrast, the overall generosity of the German and Swedish safety net are not inefficiently high. There are no inefficiencies in the income tax schedules, except for a non-trivial interaction: At very low earnings levels, transfer entitlements are so generous that the introduction of a small automatic in-work benefit would be self-financing in both countries.

Second, we solve for the inverse optimum weights that would make the 2018 German and the 2023 Swedish tax-transfer system optimal. In both countries, we find that this would require marginal welfare weights on non-recipients to be negative at very low incomes and broadly increasing up to the median income. By contrast, the welfare weights on transfer recipients in Germany would have to fluctuate strongly with income and be very negative close to the highest incomes below the phase-out threshold. The weights on recipients in Sweden are much less volatile and consistently higher than the weights on non-recipients. In both countries, the inverse optimum weight on recipients without earned income, however, is four to seven times larger than the population average. Intuitively speaking, the German and Swedish system can only be optimal if the government cares much more for non-working recipients than for everyone else and is interested in making some people strictly worse off: non-recipients with low incomes and, in the German case, recipients with high incomes. It is hard to imagine that German or Swedish policymakers indeed have such objectives.

Next steps In ongoing work, we are also setting up a structural model that allows to investigate the effects of large reforms away from the current status quo policy and that can be calibrated to the German economy. We will use this model, first, to solve for the set of reforms that maximize net revenue gains to the government while eliminating any inefficiencies, i.e., bringing the German system to the Pareto frontier. Second, we will solve for the optimal tax and transfer schedules that maximize a welfare function with plausible properties.

In addition, we will extend our empirical analysis. As we shortly get access to admin data on the recipients of the German housing benefit, we will use this data to refine our estimates on take-up rates and income distributions and, in addition, estimate take-up responses to past reforms such as the one in 2023.

2 Literature

We contribute to two strands of literature. First, we contribute to the theoretical literature on redistributive taxation of labor earnings. Starting with Mirrlees (1971), a large set of papers studies the optimal

[4]For Germany, these are the social assistance (*Arbeitslosengeld II*), the housing benefit (*Wohngeld*), and the child supplement (*Kinderzuschlag*). For Sweden, we consider the social assistance and the housing allowance.

[5]In our sensitivity analysis, we also use take-up elasticities from recent work such as Leroy (2024) and Rafkin et al. (2023). In ongoing work, moreover, we will estimate take-up responses to a large reform of the German housing allowance in 2023.

(i.e., welfare-maximizing) design of income tax schedules. Among the most related papers are Diamond (1998), Saez (2001), Saez (2002), Jacquet, Lehmann and Van der Linden (2013), and Hansen (2021), where the three last ones account for labor supply responses both at the intensive and the extensive margin. A smaller set of papers study the potential for Pareto-improving tax reforms, see Stiglitz (1982), Werning (2007), Lorenz and Sachs (2016), and Bierbrauer et al. (2023). In a different vein, Jacobs, Jongen and Zoutman (2017) use the same formal framework to infer the inverse optimum weights, i.e., the function of marginal social welfare weights for which an empirically observable tax system would be optimal, see also Bourguignon and Spadaro (2012) and Hendren (2020). The focus of this literature is on the quantitative design of the tax schedule, in particular on which levels of marginal tax rates along the income distribution are desirable given the crucial tradeoff between equity and efficiency. While previous papers in this literature account for behavioral responses along several margins, they collectively ignore the empirical prevalence of welfare transfer programs with incomplete and endogenous take-up. They treat effective marginal tax rates as if both income taxes and transfer programs supporting the poor would be automatic, while in reality low-income workers need to choose between two schedules, one that involves transfers and one that does not. We contribute to this literature by studying the optimal and Pareto-efficient schedules and by identifying inverse optimum weights in a setting where mandatory income taxes and voluntary welfare transfers with endogenous take-up coexist.

Second, we contribute to a rapidly growing literature on the take-up of welfare transfers.[6] A first set of papers document incomplete participation in transfer programs across a large set of countries, typically with take-up rates between 40 and 80%, see, for example, Hernanz et al. (2004). Starting with Bhargava and Manoli (2015), a second strand of papers investigates the reasons underlying (non) take-up decisions, including hassle costs, limited awareness, misperceptions of eligibility rules and processes, or pychological costs and welfare stigma. Recent contributions include Finkelstein and Notowidigdo (2019), Linos, Prohofsky, Ramesh, Rothstein and Unrath (2022), Naik (2024), Leroy (2024), and Rafkin et al. (2023). There is also a theoretical literature on the design of transfer programs with incomplete participation, building on Akerlof (1978), Nichols and Zeckhauser (1982), and Moffitt (1983). Besley and Coate (1992a) and Besley and Coate (1992b) investigate the role of welfare stigma and work requirements in anti-poverty programs. Kleven and Kopczuk (2011) study the optimal level of program complexity and take-up rates in order to balance the tradeoff between unintended non-participation on the one hand and false applications on the other hand.

Closest to our paper is the one by Rafkin et al. (2023) on self-targeting in the US welfare system. In their theoretical framework, they study the effects of a marginal reform shifting poverty relief from a voluntary transfer program to an automatic income tax, holding fixed the characteristics of both programs. They derive a sufficient-statistics formula for the normative evaluation of such a reform, capturing self-targeting benefits and fiscal externalities from take-up and labor supply responses. We generalize their analysis, by endogenizing the optimal joint design of income tax and transfer schedules and by allowing for the interdependence of labor supply and take-up decisions. Our analysis demonstrates that not only the average generosity of voluntary programs matters, but also their specific design; in particular, the rates at which transfer entitlements are phased out with earned income. With a badly designed transfer program, the neediest recipients might end up in inefficient poverty traps.

3 The model

In the following, we present our theoretical framework. We start by describing the set of agents, their preferences, and the set of feasible tax-transfer systems. Then, we identify individually optimal behavior, the relation between type distributions and the implemented income distribution, and behavioral elasticities.

[6] See Ko and Moffitt (2024) for a recent survey.

3.1 Environment

Preferences. There is a continuum of individuals who choose consumption c, generate earnings y, and choose whether or not to take up a welfare transfer, $m \in \{0, 1\}$. They differ in a three-dimensional vector of characteristics $\theta = (\omega, \delta, \kappa)$, where ω is a measure of skill or individual productivity, δ represents a fixed cost of working, and κ is a fixed cost of taking up the transfer. The cross-section distribution of θ is atomless and represented by a cumulative distribution function F, with continuous density f. Individual preferences are captured by the utility function

$$U(c, y, m, \theta) = c - v(y, \omega) - \mathbb{1}_{y>0} \cdot \delta - m \cdot \kappa \,, \tag{1}$$

where v is an effort cost function that is strictly increasing and strictly convex in earnings y. A single-crossing condition holds with respect to skill ω: The cross-derivative $v_{y\omega}$ is strictly negative.[7]

The tax system. The status quo tax system is represented by two parameters z_0 and b_0, and two functions T_0 and B_0, which jointly define the individual budget set

$$C_0(y, m) = z_0 + y - T_0(y) + m\,[b_0 - B_0(y)] \ . \tag{2}$$

Parameter z_0 is an unconditional payment to all individuals (i.e., a universal basic income). T_0 is a non-linear tax function such that $T_0(0) = 0$, $T_0(y) \geq 0$ and $T_0'(y) \geq 0$. Parameter b_0 is an additional base transfer paid to people who take up transfers, and $B_0(y)$ is a non-linear transfer phase-out function. We assume that $B_0(0) = 0$ and that there is an income level $\bar{y}_B > 0$ such that $B_0(y) \in [0, b_0)$ and $B_0'(y) > 0$ for all $y < \bar{y}_B$, while $B_0(y) = b_0$ for $y \geq \bar{y}_B$. Hence, $\bar{y}_B$ is the income threshold at which transfers are fully phased out.

Reforms There are three types of reform. First, a *tax reform* implements a new tax function $T = T_0 + \tau \cdot h_t$, where τ is a scalar that measures the magnitude of reform and $h_t : y \to h_t(y)$ is a continuous function with $h_t(0) = 0$ that gives the direction of reform. That is, $h_t(y)$ gives the change in tax liability at income level y for a reform with magnitude $\tau = 1$. Second, a *transfer reform* implements a new phase-out function $B = B_0 + \beta \cdot h_b$, where the scalar β captures the magnitude and the function h_b the direction of reform. Third, a *base reform* implements a new base transfer $b = b_0 - \gamma$. We assume that the post-reform universal payment, denoted by z, is adjusted to absorb any change in net tax revenue (more on this below).

Consider the combination of a tax reform (τ, h_t), a transfer reform (β, h_b) and a base reform γ. The post-reform budget set is given by

$$C(y, m) = z + y - T(y) + m\,[b - B(y)] \ . \tag{3}$$

Let $y(\theta)$ denote the individually optimal earnings level and $m(\theta)$ denote the individually optimal take-up decision of a type θ agent after the combined reform. The corresponding pre-reform earnings are given by $y_0(\theta)$, and the pre-reform take-up choice is $m_0(\theta)$. Government net tax revenue is given by

$$\begin{aligned} R(\tau, \beta, \gamma) &= E\left[T\left(y(\theta)\right) - m(\theta)\left(b - B\left(y(\theta)\right)\right)\right] \\ &= E\left[T_0\left(y(\theta)\right) + \tau h_t\left(y(\theta)\right) - m(\theta)\left(b_0 - \gamma - B_0\left(y(\theta)\right) - \beta h_b\left(y(\theta)\right)\right)\right], \end{aligned} \tag{4}$$

where the expectation operator E refers to the type distribution F and we suppress the dependence on the reform directions h_t and h_b. We denote by $R_0 = R(0, 0, 0)$ the pre-reform net tax revenue. As mentioned above, we assume that the universal payment is adjusted to absorb any potential revenue gain or loss, $z = z_0 + R(\tau, \beta, \gamma) - R_0$. That is, we focus on revenue-neutral reforms throughout the analysis.

[7] The focus on quasi-linear utility is for simplicity; we can generalize the analysis to preferences such that the marginal rate of substitution between consumption and earnings is continuous and decreasing in ω. With quasi-linear utility, there are no income effects in labor supply or transfer take-up.

3.2 Individually optimal behavior

Consider an agent with type $\theta = (\omega, \delta, \kappa)$ who chooses income $y \geq 0$ and take-up $m \in \{0, 1\}$ to maximize her utility

$$U(C(y,m), y, m, \theta) = C(y,m) - v(y,\omega) - \mathbb{1}_{y>0} \times \delta - m\ \kappa\ . \tag{5}$$

To determine individually optimal earnings and take-up choices, consider the subset of agents with skill level ω. We proceed in three steps. First, we denote by $y_r(\omega)$ her individually optimal earnings conditional on working and *r*eceiving the transfer, and by $y_n(\omega)$ her optimal earnings conditional on working and *n*ot receiving the transfer. Both earnings levels are implicitly defined by the first-order conditions

$$v_y\left(y_r(\omega), \omega\right) = 1 - T'(y_r(\omega)) - B'(y_r(\omega))\ , \tag{6}$$

$$v_y\left(y_n(\omega), \omega\right) = 1 - T'(y_n(\omega))\ , \tag{7}$$

where we note that neither the fixed cost of working δ nor the take-up cost κ show up.

Second, we denote by $s_r(\omega)$ and $s_n(\omega)$ the participation surplus conditional on *r*eceiving the transfer and conditional on *n*ot receiving it, respectively. They are given by

$$s_r(\omega) = y_r(\omega) - T(y_r(\omega)) - B(y_r(\omega) - v\left(y_r(\omega), \omega\right)\ , \tag{8}$$

$$s_n(\omega) = y_n(\omega) - T(y_n(\omega)) - v\left(y_n(\omega), \omega\right)\ . \tag{9}$$

Again, these surpluses do neither depend on fixed costs of working nor on take-up costs.[8]

Third, we define the take-up surplus $\hat{\kappa}(\omega, \delta)$ as the utility gain that an agent with skill ω and fixed cost δ receives when she starts to take up the transfer and adjusts her earnings optimally. It is given by

$$\hat{\kappa}(\omega,\delta) \coloneqq \begin{cases} b + s_r(\omega) - s_n(\omega) & \text{if} \quad \delta < s_r(\omega)\ , \\ b - s_r(\omega) + \delta & \text{if} \quad \delta \in [s_r(\omega), s_n(\omega)]\ , \\ b & \text{if} \quad \delta > s_n(\omega)\ , \end{cases} \tag{10}$$

The following result characterizes individually optimal behavior. Taking up transfers is beneficial for agents with a take-up cost κ below the threshold $\hat{\kappa}(\theta, \delta)$, and detrimental in the opposite case. As a tie-breaking rule, we assume that agents participate in the labor market if they are indifferent between doing so and doing not, and that they do not take up transfers if they are indifferent between taking up and not taking up.

Proposition 1 *Under the status-quo tax system, the individually optimal behavior of an agent with type* $\theta = (\omega, \delta, \kappa)$ *is given by*

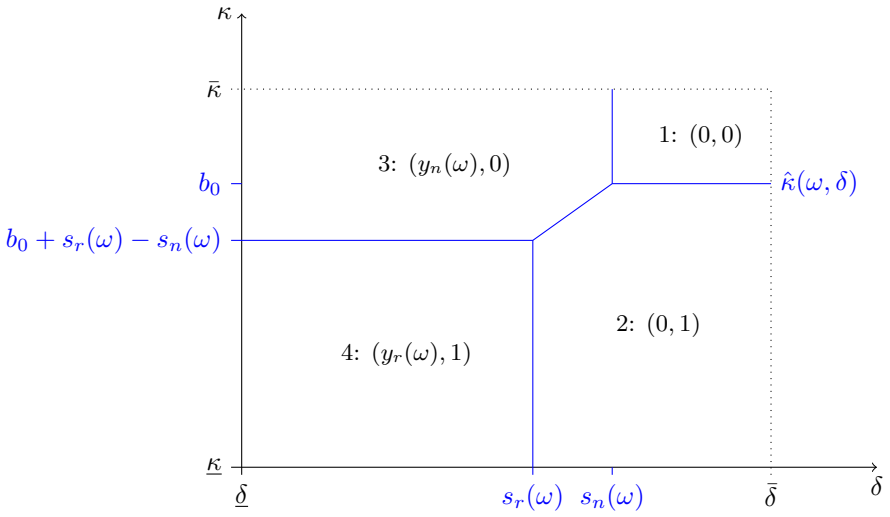
$$(y(\theta), m(\theta)) = \begin{cases} (0,0) & \textit{if} \quad \delta > s_n(\omega) \ \textit{and}\ \kappa \geq \hat{\kappa}(\omega,\delta)\ , \\ (0,1) & \textit{if} \quad \delta > s_r(\omega) \ \textit{and}\ \kappa < \hat{\kappa}(\omega,\delta)\ , \\ (y_n(\omega),0) & \textit{if} \quad \delta \leq s_n(\omega) \ \textit{and}\ \kappa \geq \hat{\kappa}(\omega,\delta)\ , \\ (y_r(\omega),1) & \textit{if} \quad \delta \leq s_r(\omega) \ \textit{and}\ \kappa < \hat{\kappa}(\omega,\delta)\ . \end{cases} \tag{11}$$

By Proposition 1, agents with the same skill type ω choose four different bundles, depending on their fixed costs δ and κ. Figure 2 below illustrates this result, separating four areas in the $\delta \times \kappa$ space. We denote by h_{0n} the share of agents within skill group ω who do neither work nor take up transfers (area 1), by $h_{0r}(\omega)$ the share of agents who do not work but take up transfers (area 2), by $h_{1n}(\omega)$ the share of agents who work and do not take up (area 3), and by $h_{1r}(\omega)$ the share of agents who work and receive transfers within skill group ω (in area 4).[9]

[8] By the single-crossing property, there is a unique skill threshold $\hat{\omega}_B$ such that $y_b(\omega)$ is below (above) $\bar{y}_B$ if and only if ω is below (above) $\hat{\omega}_B$. As $B_0'(y) > 0$ for all $y \leq \bar{y}_B$, we have $y_r(\omega) < y_n(\omega)$ if $\omega < \hat{\omega}_B$ and $y_r(\omega) = y_n(\omega)$ otherwise.

[9] Hence, $h_{0n}(\omega) + h_{0r}(\omega) + h_{1n}(\omega) + h_{1r}(\omega) = 1$ for each ω. Note that area 4 — where agents participate in the labor market and take up transfers — is empty for high skill types $\omega > \hat{\omega}_B$. Formally, for $\omega > \hat{\omega}_B$, we have $\hat{\kappa}(\omega, \delta) = 0$.

Figure 2: Individually optimal behavior under the status quo, given skill type ω.



Due to the single-crossing condition, the conditionally optimal income levels $y_r(\omega)$ and $y_n(\omega)$ are montonically increasing in an agent's skill type ω. With a continuous type distribution, any well-behaved tax-transfer system (T, B, z, b) implements an non-trivial income distribution. We denote by G_r the income distribution among (self-selected) transfer recipients and by G_n the income distribution among non-recipients. Both distributions may have a mass point at zero. We denote by g_r the pdf of the income distribution among recipients, and by g_n thee pdf of the distribution among non-recipients. We spell out the relation between the type distribution F and the implemented income distribution G in Appendix A.1.

Labor supply elasticities The previous discussion and Proposition 1 have made clear that reforms of the tax schedule T, the transfer phase-out schedule B, and the base transfer b will have effects on labor supply choices and on take-up choices within certain skill groups ω. In the following, we define a set of labor supply elasticities that we will use below.[10] First, we define two standard intensive-margin elasticities for agents that either do, or do not, take up transfers,

$$\varepsilon_r(y) = \frac{\partial y_r(\omega_r(y))}{\partial \left[1 - T'(y) - B'(y)\right]} \frac{1 - T'(y) - B'(y)}{y} \tag{12}$$

$$\varepsilon_n(y) = \frac{\partial y_n(\omega_n(y))}{\partial \left[1 - T'(y)\right]} \frac{1 - T'(y)}{y} . \tag{13}$$

Second, we define two participation elasticities. By $\pi_r(y)$, we denote the elasticity of the participation share $h_{1r}(\omega_r(y))$ among transfer recipients with income y with respect to net labor income, $y - T(y) - B(y)$,

$$\pi_r(y) = \frac{dh_{1r}(\omega_r(y))}{d\left[y - T(y) - B(y)\right]} \frac{y - T(y) - B(y)}{h_{1r}(\omega_r(y))} \tag{14}$$

The size of this elasticity depends on the share of agents who are just indifferent between participating in the labor market or not but strictly prefer to take up transfers, i.e., at the boundary between areas 2 and 4 in Figure 2.

[10] As we assumed quasi-linear utility in (1), there are no income effects in labor supply and transfer take-up. We can generalize these assumptions and define a set of income elasticities (i) at the intensive margin, (ii) in labor-market participation, and (iii) in transfer take-up.

Correspondingly, we denote by $\pi_n(y)$ the elasticity of the participation share $h_{1n}(\omega_n(y))$ among the subset of people with income y who do not take up transfers with respect to their net labor income $y - T(y)$. Formally, it is given by

$$\pi_n(y) = \frac{dh_{1n}(\omega_n(y))}{d[y - T(y)]} \frac{y - T(y)}{h_{1n}(\omega_n(y))}, \tag{15}$$

This elasticity can be decomposed into the behavioral changes by previously non-working people who either did (π_{rn}) or did not (π_{nn}) take up transfers before, such that $\pi_n(y) = \pi_{rn}(y) + \pi_{nn}(y)$. More specifically, these components are given by

$$\pi_{rn}(y) = \left.\frac{dh_{1n}(\omega_n(y))}{d[y - T(y)]}\right|_{rn} \frac{y - T(y)}{h_{1n}(\omega_n(y))} \tag{16}$$

$$\pi_{nn}(y) = \left.\frac{dh_{1n}(\omega_n(y))}{d[y - T(y)]}\right|_{nn} \frac{y - T(y)}{h_{1n}(\omega_n(y))}. \tag{17}$$

The responses captured by the first component depend on the mass of agents with (δ, κ) pairs at the boundary between areas 2 and 3, who are just indifferent between (i) either working and not taking up transfers or (ii) not working and taking up transfers. The second component depends on the mass of people with (δ, κ) pairs at the boundary between areas 1 and 3 in Figure 2, who are just indifferent between participating or not but unambiguously prefer not to take up transfers in any case.

Third, we define two take-up elasticities. By $\psi_1(y)$, we denote the elasticity of the share $h_{1r}(\omega_r(y))$ of agents who work (before and after the reform) and take up transfers with respect to the transfer level $b - B(y)$ they are entitled to,

$$\psi_1(y) = \frac{dh_{1r}(\omega_r(y))}{d[b - B(y)]} \frac{b - B(y)}{h_{1r}(\omega_r(y))} \tag{18}$$

This elasticity depends on the mass of agents who participate in the labor market and are indifferent between taking up transfers or not, i.e., with (δ, κ) pairs at the boundary between areas 3 and 4 in Figure 2.

Finally, we denote by $\bar{\psi}_{00}$ the elasticity of the share of non-working agents who take up transfers with respect to the base transfer b. It is given by

$$\bar{\psi}_{00} = \int_{\underline{\omega}}^{\bar{\omega}} \left.\frac{dh_{0r}(\omega)}{db}\right|_{00} \frac{b}{h_{0r}(\omega)} dF(\omega). \tag{19}$$

Note that this elasticity is defined to capture only the responses of people who did not work before the reform. The responses by people who worked without taking up transfers before the reform and take up transfers without working after the reform are already captured by the elasticity π_{rn}.

4 Conditions for Pareto efficiency

This section provides a number of conditions that are necessary and sufficient for the existence of Pareto-improving reforms of the tax and transfer system. The results in this section hold whenever agents are rational and maximize preferences that satisfy equation (1). To derive these results, we focus on tax reforms (τ, h_t) and phase-out reforms (β, h_b) that change the marginal tax (phase-out) rates in a single income bracket $(\hat{y}, \hat{y} + \ell)$. Formally, the reform directions are given by

$$h_t(y) = h_b(y) = h(y) = \begin{cases} 0 & \text{for} \quad y \leq \hat{y}, \\ y - \hat{y} & \text{for} \quad y \in (\hat{y}, \hat{y} + \ell), \\ \ell & \text{for} \quad y \geq \hat{y} + \ell. \end{cases} \tag{20}$$

For this class of reforms, denote the net tax revenue after a tax reform with magnitude τ and a phase-out reform with magnitude β by $R(\hat{y}, \tau, \beta, \ell)$. We concentrate on reforms that imply *small* changes of tax

(or phase-out) rates on *small* brackets, i.e., with ℓ and τ (or β) close to zero. We can approximate the effects of such small reforms by looking at second-order derivatives. For example, the effect of a *small tax reform* on net tax revenue $R(\hat{y}, \tau, 0, \ell) - R(\hat{y}, 0, 0, \ell)$ can be approximated by $\tau\ell \frac{\partial^2 R(\hat{y},0,0,0)}{\partial\tau\partial\ell}$. We shall express our formal results in terms of functions that capture

(i) the revenue effect of a small tax reform, $\mathcal{R}_t(y) = \frac{\partial^2 R(y,0,0,0)}{\partial\tau\partial\ell}$,

(ii) the revenue effect of a small phase-out reform, $\mathcal{R}_b(y) = \frac{\partial^2 R(y,0,0,0)}{\partial\beta\partial\ell}$,

(iii) the revenue effect of a small joint reform that combines a tax reform and phase-out reform with $\tau = -\beta$ at the same income y, denoted by $\Delta\mathcal{R}(y) = \mathcal{R}_t(y) - \mathcal{R}_b(y)$, and

(iv) the revenue effect of a small base reform, $\mathcal{R}_0 = \frac{dR(y,0,0,0)}{d\gamma}$.

The first result directly extends Theorem 1 in Bierbrauer, Boyer & Hansen (2023).

Proposition 2 *There exists a Pareto-improving tax reform unless function $\mathcal{R}_t$ is bounded from below by 0, bounded from above by 1, and monotonically decreasing.*

If $\mathcal{R}_t(y) < 0$ for some y, then there is a self-financing and Pareto-improving tax cut with one bracket around y. If $\mathcal{R}_t(y) > 1$ for some y, then there is a tax increase with one bracket that raises enough revenue to compensate even those agents who face the largest increase in their tax burden. If there is a pair of income levels y_a and $y_b > y_a$ such that $\mathcal{R}_t(y_a) < \mathcal{R}_t(y_b)$, then there is a self-financing two-bracket tax cut that makes everyone better off.

The second result provides corresponding conditions on the existence of transfer reforms.

Proposition 3 *There exists a Pareto-improving transfer reform unless*

(i) $\mathcal{R}_0 \in [0, 1]$, and

(ii) $\mathcal{R}_b$ is bounded from below by 0, bounded from above by $\mathcal{R}_0$, and monotonically decreasing.

Part (i) imposes conditions on the level of the base transfer b. If the derivative $\mathcal{R}_0$ is above 1, a reduction in b would create sufficient gains in net revenue to increase the sum $z + b$ and, hence, make everybody better off. This implies an upper bound on the base transfer b. If $\mathcal{R}_0$ is instead negative, then an increase in b would be self-financing and, hence, allow to further increase z. This condition thus implies a lower bound on b.

Part (ii) imposes conditions on the phase-out function B. If $\mathcal{R}_b$ is negative at some income y, a decrease in B is self-financing. If $\mathcal{R}_b(y)$ is above $\mathcal{R}_0$, then an increase in the phase-out rate in combination with an increase in the base transfer b makes transfer recipients with incomes below y better off while lowering the government's expenses. Finally, if $\mathcal{R}_b$ is non-monotonic, there is a self-financing reform that increases the transfer amount between two incomes y_a and y_b.

The third result establishes novel conditions on the joint tax-transfer system. For this purpose, we define the composite revenue function $\Delta\mathcal{R} \coloneqq \mathcal{R}_t - \mathcal{R}_b$. It measures the revenue effects of joint reforms that simultaneously increase the marginal tax rate and decrease the marginal transfer phase-out rate at income level y.

Proposition 4 *There exists a Pareto-improving joint reform of taxes and transfers unless function $\Delta\mathcal{R}$ is bounded from below by 0 and monotonically decreasing.*

By the first condition in this result, $\Delta\mathcal{R} > 0$ holds for every Pareto-efficient tax-transfer system, i.e., $\mathcal{R}_b(y) < \mathcal{R}_t(y)$ must be true at any income level y. Assume that this condition is violated, i.e., $\mathcal{R}_b(y) > \mathcal{R}_t(y)$ holds for some $\hat{y}$. Consider a reform that reduces the marginal tax in a small intervall $(\hat{y}, \hat{y} + \ell)$ by $\tau > 0$, and simultaneously increases the transfer phase-out rate in the same interval by

the same amount τ. The reform keeps the net income of all transfer recipients constant and makes non-recipients with incomes above $\hat{y}$ better off. For τ and ℓ close enough to 0, the reform yields a gain in net tax revenue by $\tau\ell\left[\mathcal{R}_b(\hat{y}) - \mathcal{R}_t(\hat{y})\right] > 0$. By using this net revenue gain for a uniform lump-sum payment to all agents, we can realize a strict Pareto improvement.

By the second condition, $\Delta\mathcal{R}$ must be decreasing, i.e., the average gradient of $\mathcal{R}_b$ must be below – more negative than – the one of $\mathcal{R}_t$ on each interval. Assume that this condition is violated so that, for some pair of incomes y_a and $y_b > y_a$, we have

$$\begin{aligned} \Delta\mathcal{R}(y_b) &> \Delta\mathcal{R}(y_a) \\ \Leftrightarrow \mathcal{R}_b(y_a) - \mathcal{R}_b(y_b) &> \mathcal{R}_t(y_a) - \mathcal{R}_t(y_b)\ . \end{aligned}$$

Then, we can simultaneously reduce transfer entitlements and tax liabilities at all incomes in the interval (y_a, y_b) by the same small amount. This does not affect the budget set of transfer recipients, but increases net incomes of non-recipients with earnings between y_a and y_b. Moreover, the joint reform allows to realize a net revenue gain that can be paid out lump-sum to all agents. Again, we have a Pareto improvement.

5 Sufficient-statistics formulas for the revenue functions

In this section, we provide formulas for the revenue functions in terms of sufficient statistics, i.e., empirically estimable objects. The formal derivation of these formulas can be found in Appendix A.2. We focus on the case of quasi-linear preferences to avoid overly long formal expressions.[11]

Lemma 1 *The revenue effect of a small transfer reform at income level $\hat{y}$ is given by*

$$\begin{aligned} \mathcal{R}_b(\hat{y}) \;=\; & G_r(\bar{y}_B) - G_r(\hat{y}) \\ & - \; g_r(\hat{y})\ \hat{y}\ \varepsilon_r(\hat{y}) \frac{T'(\hat{y}) + B'(\hat{y})}{1 - T'(\hat{y}) - B'(\hat{y})} \\ & - \int_{\hat{y}}^{\bar{y}_B} g_r(y)\ \pi_r(y) \frac{T(y) + B(y)}{y - T\left(y\right) - B\left(y\right)} dy \\ & + \int_{\hat{y}}^{\bar{y}_B} g_r(y)\ \psi_1(y) \frac{b - B(y) - T(y) + T\left(\tilde{y}_n(y)\right)}{b - B\left(y\right)} dy \end{aligned} \tag{21}$$

where $\tilde{y}_n(y) \coloneqq y_n(\omega_r(y))$ is the income that is optimal conditional on no take-up for an agent who previously took up transfers and chose income y.

In equation (21), the first line of the right-hand side captures mechanical effects of the transfer cut, while the second line tracks intensive-margin responses. The third line accounts for revenue effects due to transfer recipients leaving the labor market, and the last line accounts for revenue effects due to people who stop taking up transfers. Note that function $\tilde{y}_n$ depends on the marginal tax T' and marginal phase-out rate B' as well as the intensive-margin elasticity.

[11] Note that we can also provide generalized formulas that allow for income effects in optimal behavior.

Lemma 2 *The revenue effect of a small tax reform at income level $\hat{y}$ is given by*

$$\begin{aligned}\mathcal{R}_t(\hat{y}) &= 1 - G_n(\hat{y}) - G_r(\hat{y}) \\ &\quad -g_n(\hat{y})\ \varepsilon_n(\hat{y})\ \hat{y}\ \frac{T'(\hat{y})}{1-T'(\hat{y})} - g_r(\hat{y})\ \varepsilon_r(\hat{y})\ \hat{y}\ \frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})} \\ &\quad -\int_{\hat{y}}^{\infty} g_n(y)\ \pi_{nn}(y)\frac{T(y)}{y-T(y)}dy - \int_{\hat{y}}^{\infty} g_n(y)\ \pi_{bn}(y)\frac{T(y)+b}{y-T(y)}dy \\ &\quad -\int_{\hat{y}}^{\bar{y}_B} g_r(y)\ \pi_b(y)\frac{T(y)+B(y)}{y-T(y)-B(y)}dy \\ &\quad -\int_{\tilde{y}_n^{-1}(\hat{y})}^{\hat{y}} g_r(y)\ \psi_1(y)\frac{b-B(y)-T(y)+T(\tilde{y}_n(y))}{b-B(y)}dy\ . \end{aligned} \tag{22}$$

In equation (22), the first line of the right-hand side captures mechanical effects for non-recipients and recipients of transfers. Line 2 accounts for intensive-margin responses; lines 3 and 4 account for extensive-margin responses. Line 5 considers the revenue effects due to people who start taking up transfers with income $y_r(\theta) < \hat{y}$ due to the reform. Note that the latter group are agents with incomes $y_n(\theta) > \hat{y}$ before the reform.

Lemma 3 *The revenue effect of a small joint reform at income level $\hat{y}$ is given by*

$$\begin{aligned}\Delta\mathcal{R}(y) &= 1 - G_r(\bar{y}_B) - G_n(\hat{y}) \\ &\quad -\ g_n(\hat{y})\ \varepsilon_n(\hat{y})\ \hat{y}\ \frac{T'(\hat{y})}{1-T'(\hat{y})} \\ &\quad -\int_{\hat{y}}^{\infty} g_n(y)\ \pi_{nn}(y)\frac{T(y)}{y-T(y)}dy - \int_{\hat{y}}^{\infty} g_n(y)\ \pi_{bn}(y)\frac{T(y)+b}{y-T(y)}dy \\ &\quad -\int_{\tilde{y}_n^{-1}(\hat{y})}^{\bar{y}_B} g_r(y)\ \psi_1(\theta)\frac{b-B(y)-T(y)+T\left(\tilde{y}_n(y)\right)}{b-B(y)}dy\ . \end{aligned} \tag{23}$$

In equation (23), the first line on the right-hand side captures mechanical revenue effects, the second line revenue effects due to intensive-margin responses, and the third line revenue effects due to participation responses of transfer recipients. The fourth line captures revenue effects due to responses in transfer take-up.

Lemma 4 *The revenue effect of a small increase in the base transfer is given by*

$$\begin{aligned}\mathcal{R}_0 &= G_r\left(\bar{y}_B\right) + G_r(0)\ \bar{\psi}_{00} \\ &\quad +\int_0^{\infty} g_n(y)\ \pi_{bn}(y)\ \frac{T(y)+b}{y-T(y)}dy \\ &\quad +\int_0^{\bar{y}_B} g_r(y)\ \psi_1(y)\ \frac{b-B(y)-T(y)+T\left(\tilde{y}_n(y)\right)}{b-B(y)}dy\ , \end{aligned} \tag{24}$$

In equation (24), the first line on the right-hand side captures mechanical effects on net tax revenue and revenue effects due to take-up responses of non-working agents, the second line revenue effects due to joint responses in labor market participation and transfer take-up, and the third line effects due to responses in transfer take-up among previously working agents.

6 Welfare maximization

Assume that the government's objective is given by a Bergson-Samuelson welfare function of the type

$$W(\tau,\beta,\gamma) = E\left[\Psi\left[V(\theta,\tau,\beta,\gamma),\theta\right]\right] \tag{25}$$

where $V(\theta, \tau, \beta, \gamma)$ is the indirect utility function of a type θ agent after a combination of reforms with magnitudes (τ, β, γ), and function Ψ is strictly increasing and weakly concave in its first argument. We allow for a direct dependence on the type θ.

Exploiting Lemma 1, we can write the objective as

$$\begin{aligned} W(\tau,\beta,\gamma) &= G_n(0)E\left[\Psi(z,\theta) \mid y(\theta)=0, m(\theta)=0\right] \\ &\quad + G_r(0)E\left[\Psi(z+b-\kappa,\theta) \mid y(\theta)=0, m(\theta)=1\right] \\ &\quad + \int_0^\infty g_n(y)E\left[\Psi\left(V(\theta,\tau,\beta,\gamma),\theta\right) \mid y(\theta)=y, m(\theta)=0\right]dy \\ &\quad + \int_0^\infty g_r(y)E\left[\Psi\left(V(\theta,\tau,\beta,\gamma),\theta\right) \mid y(\theta)=y, m(\theta)=1\right]dy \,, \end{aligned}$$

where E denotes the expectation given type distribution F.

At the margin, the government's redistributive preferences can be measured by two functions of marginal social welfare weights, $\alpha_r : y \to \alpha_r(y)$ and $\alpha_n : y \to \alpha_n(y)$ for agents taking up and not taking up transfers, respectively. In particular, we define

$$\alpha_r(y) \coloneqq \frac{E\left[\Psi'\left(V_0(\theta),\theta\right) \mid y(\theta)=y, m(\theta)=1\right]}{E\left[\Psi'\left(V_0(\theta),\theta\right)\right]} \,, \text{ and} \tag{26}$$

$$\alpha_n(y) \coloneqq \frac{E\left[\Psi'\left(V_0(\theta,\theta) \mid y(\theta)=y, m(\theta)=0\right]\right.}{E\left[\Psi'\left(V_0(\theta),\theta\right)\right]} \,, \tag{27}$$

where $V_0(\theta)$ is the indirect utility of type θ individuals at the status quo. Note that the marginal welfare weights measure by how much welfare goes up if the consumption of an agent with income y is raised by a marginal unit, relative to how much welfare goes up if the consumption of an average agent is raised. By construction, hence, the average marginal welfare weight equals 1.

6.1 Welfare effects of tax and transfer reforms

Again, we consider a set of marginal reforms of the tax-transfer system. For the sake of clarity, consider a one-bracket tax reform $(\tau, \ell, \hat{y})$. It increases the marginal tax rate by magnitude τ in an income bracket of length ℓ that starts at income $\hat{y}$. After this reform, the new tax schedule is given by $T = T_0 + \tau h$ with reform direction h defined by equation (20). As before, we assume that any potential gain or loss in net tax revenue is absorbed by the lump-sum payment, so that $z = z_0 + R(\tau, \ell, \hat{y})$. The same assumption is also imposed for the other types of reforms we consider below: transfer reforms $(\beta, \ell, \hat{y})$, joint reforms $(\tau, \beta, \ell, \hat{y})$, and reforms of the base transfer b.

First, the welfare effect of a small tax reform at income level $\hat{y}$ is given by

$$\frac{d^2W}{d\tau d\ell} = E\left[\Psi'\left(V_0(\theta),\theta\right)\right]\left\{\mathcal{R}_t(\hat{y}) - \int_{\hat{y}}^\infty g_n(y)\alpha_n(y)dy - \int_{\hat{y}}^\infty g_r(y)\alpha_r(y)dy\right\} \,. \tag{28}$$

Second, the welfare effect of a small transfer reform at $\hat{y}$ is given by

$$\frac{d^2W}{d\beta d\ell} = E\left[\Psi'\left(V_0(\theta),\theta\right)\right]\left\{\mathcal{R}_b(\hat{y}) - \int_{\hat{y}}^\infty g_r(y)\alpha_r(y)dy\right\} \,. \tag{29}$$

Third, the welfare effect of a small joint reform in taxes and transfers with $\tau = -\beta$ is given by

$$\frac{d^2W}{d\tau d\ell} - \frac{d^2W}{d\beta d\ell} = E\left[\Psi'\left(V_0(\theta),\theta\right)\right]\left\{\Delta\mathcal{R}(\hat{y}) - \int_{\hat{y}}^\infty g_n(y)\alpha_n(y)dy\right\} \,. \tag{30}$$

Finally, a marginal increase in the base transfer affects welfare by

$$\begin{aligned} \frac{dW}{d\gamma} &= E\left[\Psi'\left(V_0(\theta),\theta\right)\right]\left\{-\mathcal{R}_0 + G_r(0)\alpha(0) + \int_0^\infty g_r(y)\alpha_r(y)dy\right\} \\ &= E\left[\Psi'\left(V_0(\theta),\theta\right)\right]\left\{\mathcal{R}_0 - G_r(\hat{y}_B)\bar{\alpha}_r\right\} \,, \end{aligned} \tag{31}$$

where $\bar{\alpha}_r$ denotes the average marginal welfare weight across all transfer recipients.

With a given welfare function, we can identify the marginal welfare weights at all income levels in the status quo allocation. Equations (28) to (31) then allow us, in combination with the sufficient statistics that enter the revenue functions, to determine whether an increase in the marginal tax or phase-out rate is welfare-enhancing at any income level. We are currently working on a structural model that we will calibrate to the German economy in order to simulate the optimal tax-transfer system under alternative assumptions on labor supply elasticities and welfare weights for transfer recipients and non-recipients.

6.2 Inverse optimum weights

Building on the previous arguments, we can also identify the welfare weights for which the status-quo tax-transfer system would be optimal, i.e., locally welfare-maximizing. Put differently, we need to identify the welfare weights such that the marginal effects of all before-mentioned reforms equal zero. The literature refers to these implicitly defined welfare weights as *inverse optimum weights*.

Proposition 5 *For a given tax-transfer system (z_0, b_0, T_0, B_0), the inverse optimum weights are given by*

$$\tilde{\alpha}_r(y) = -\frac{1}{g_r(y)}\mathcal{R}_b'(y) \text{ , and} \tag{32}$$

$$\tilde{\alpha}_n(y) = -\frac{1}{g_n(y)}\Delta\mathcal{R}'(y) \tag{33}$$

at all income levels where $\mathcal{R}_b$ and $\Delta\mathcal{R}$ are continuously differentiable, respectively. The inverse optimum weight for transfer recipients without labor earnings is given by

$$\tilde{\alpha}_r(0) = \frac{1}{G_r(0)}\left[\mathcal{R}_0 - \lim_{y \overset{+}{\to} 0} \mathcal{R}_b(y)\right] , \tag{34}$$

and the one for non-recipients without labor earnings is given by

$$\tilde{\alpha}_n(0) = \frac{1}{G_n(0)}\left[1 - \mathcal{R}_0 - \lim_{y \overset{+}{\to} 0} \Delta\mathcal{R}(y)\right] . \tag{35}$$

Proposition 5 provides a complete set of inverse optimum weights for each subset of agents defined by identical income and take-up choices, (y, m). The inverse optimum weights at income y are derived by setting the marginal welfare effect given in (29) or (30) equal to zero, differentiating with respect to y, and then solving for $\alpha_r(y)$ or $\alpha_n(y)$, respectively.

We can also calculate the average inverse optimum weights for recipients (non-recipients) with incomes in the interval (y_j, y_k). They are given by

$$E\left[\tilde{\alpha}_r(y) \mid y \in (y_j, y_k)\right] = \frac{\mathcal{R}_b(y_j) - \mathcal{R}_b(y_k)}{G_r(y_k) - G_r(y_j)} , \tag{36}$$

$$E\left[\tilde{\alpha}_n(y) \mid y \in (y_j, y_k)\right] = \frac{\Delta\mathcal{R}(y_j) - \Delta\mathcal{R}(y_k)}{G_n(y_k) - G_n(y_j)} . \tag{37}$$

6.3 Optimal tax formulas

Alternatively, we can derive a set of first-order conditions for the typical optimal tax problem. A necessary condition for a tax-transfer system to be welfare-maximizing is that the marginal welfare effects of transfer cuts, joint reforms and increases in the base transfer simultaneously equal zero, i.e.,

$$\begin{aligned}
\mathcal{R}_b(\hat{y}) &= \bar{\alpha}_r(\hat{y})\left[G_r(\bar{y}_B) - G_r(\hat{y})\right] , \\
\Delta\mathcal{R}(\hat{y}) &= \bar{\alpha}_n(\hat{y})\left[1 - G_r(\bar{y}_B) - G_n(\hat{y})\right] , \\
\mathcal{R}_0 &= \bar{\alpha}_B G_r(\bar{y}_B) ,
\end{aligned}$$

where $\bar{\alpha}_B$ is the average welfare weight of all welfare recipients, $\bar{\alpha}_r(y)$ is the one of welfare recipients with incomes above y, and $\bar{\alpha}_n(y)$ is the one of non-recipients with incomes above y.

Using the sufficient-statistics formula (23) for the revenue effects of joint tax-transfer reforms, we can rearrange the second first-order condition to get an ABC-type formula for the optimal marginal tax rate faced by non-recipients,

$$\begin{aligned}\frac{T'(\hat{y})}{1-T'(\hat{y})} &= \frac{1}{\varepsilon_n(\hat{y})}\left(1-\alpha_n(\hat{y})\right)\frac{G_n(\bar{y})-G_n(\hat{y})}{g_n(\hat{y})\ \hat{y}} \\ &\quad -\frac{1}{\varepsilon_n(\hat{y})\ g_n(\hat{y})\ \hat{y}}\left\{\int_{\hat{y}}^{\infty} g_n(y)\pi_{nn}(y)\frac{T(y)}{y-T(y)}dy+\int_{\hat{y}}^{\infty} g_n(y)\pi_{nr}(y)\frac{T(y)+b}{y-T(y)}dy\right. \\ &\quad \left.+\int_{\tilde{y}_n^{-1}(\hat{y})}^{\bar{y}_B} g_r(y)\psi_1(y)\frac{b-B(y)-T(y)+T\left(\tilde{y}_n(\hat{y})\right)}{b-B(y)}dy\right\} \qquad (38)\end{aligned}$$

In the next step, using expression (21) for the revenue effects of transfer functions, we can derive a formula for the optimal effective marginal tax rate faced by transfer recipients,

$$\begin{aligned}\frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})} &= \frac{1}{\varepsilon_r(\hat{y})}\left(1-\alpha_r(\hat{y})\right)\frac{G_r(\bar{y}_B)-G_r(\hat{y})}{g_r(\hat{y})\ \hat{y}} \\ &\quad -\frac{1}{\varepsilon_r(\hat{y})\ g_r(\hat{y})\ \hat{y}}\left\{\int_{\hat{y}}^{\bar{y}_B} g_r(y)\pi_r(y)\frac{T(y)+B(y)}{y-T(y)-B(y)}dy\right. \\ &\quad \left.-\int_{\hat{y}}^{\bar{y}_B} g_r(y)\psi_1(y)\frac{b-B(y)-T(y)+T\left(\tilde{y}_n(\hat{y})\right)}{b-B(y)}dy\right\} \qquad (39)\end{aligned}$$

In both optimal tax formulas, the first term on the right-hand side has the typical ABC structure: It contains the inverse of the intensive-margin elasticity (A), the difference between the population average welfare weight of one and the average welfare weight of individuals with incomes above $\hat{y}$ (B), and the hazard rate of the income distribution (C), each for non-recipients and recipients of welfare transfers, respectively. Besides, in each case, the second line contains terms that capture the revenue effects due to responses in labor market participation, and the third line contains a term that captures the revenue effects due to responses in transfer take-up. Hence, the formulas nest as special cases applications in which there is (i) either complete take-up or (ii) incomplete but exogenous take-up, as well as cases (iii) without participation responses.

In the last step, we can use the expression for revenue effects of an increase in the base transfer b to provide a formular that implicitly defines the optimal base transfer,

$$\begin{aligned}b &= \frac{1}{\int_0^{\infty} g_n(y)\pi_{nr}(y)\frac{1}{y-T(y)}dy}\left\{G_r(\bar{y}_B)\left(\bar{\alpha}_B-1\right)-G_b(0)\bar{\psi}_{00}\right. \\ &\quad \left.-\int_0^{\infty} g_n(y)\tilde{\pi}_{nr}(y)\frac{T(y)}{y-T(y)}dy-\int_0^{\bar{y}_B} g_r(y)\tilde{\psi}_1(y)\frac{b-B(y)-T(y)+T\left(\tilde{y}_n(\hat{y})\right)}{b-B(y)}dy\right\}. \qquad (40)\end{aligned}$$

The formula clarifies that the optimal level of the base transfer depends on the share of transfer recipients and their marginal welfare weight, on the take-up elasticities of working and non-working agents, and on the participation responses of agents that only stop taking up transfers when working. The formula immediately clarifies that, with an increasing tax function, the base transfer can only be positive if the government considers welfare recipients more deserving than non-recipients, $\bar{\alpha}_B > 1$.

7 Empirical application

We apply our theoretical insights to study the 2018 German tax-transfer system and the 2023 Swedish tax-transfer system. This section first focuses on revenue functions to assess the potential for Pareto-improving reforms. In this context, we separately analyze tax, transfer and joint reforms and thus use Lemma 1 to 3 to implement our sufficient-statistics formulas (21), (22) and (23). Next, we present inverse optimum weights distinguishing between the groups of welfare recipients and non-recipients based on (36) and (37).

7.1 Setting

Germany The 2018 German tax-transfer system was characterized by three main ingredients: the personal income tax, social security contributions and three different welfare transfer programs.[12] While the social assistance (Arbeitslosengeld II) was mainly targeted towards the long-term unemployed and those with very little income, the housing benefit (Wohngeld) and the child supplement (Kinderzuschlag) could be primarily perceived as in-work benefits increasing low- and middle-income earners' disposable incomes. There also existed interaction between the three transfer programs: while a household could claim the housing benefit and child supplement in combination, the social assistance was only available if the household did receive neither of the other two transfers.[13] Each of these three transfer programs required the eligible population to proactively apply by filling out government forms and thus supplying information on, for instance, the personal living situation or current income and wealth levels. Importantly, the housing benefit and the child supplement had to be claimed independently even though households can be recipients of both programs at the same time.

Sweden For the 2023 Swedish tax and transfer system, we focus on the interaction of the tax system (consisting of municipal and state income tax)[14] with the two most important means-tested benefits: housing allowance, a means-tested subsidy for housing costs available to low-income families and young adults; and social assistance, the last-resort safety net for households whose resources fall below a subsistence level. Akin to the German case, recipients are required to apply for these programs. Unlike housing allowance, which is administered centrally by the Social Insurance Agency, social assistance is administered by municipal social services offices. Individuals can claim either only one of these programs or both at the same time – if they are eligible.

7.2 Calibration

To bring our theory to the data, we rely on state-of-the-art microsimulation models.

For Germany, we use the ifo Microsimulation model (ifo-MSM), which is widely used for policy advice and offers an accurate depiction of the 2018 German tax-transfer system (see Blömer and Peichl (2020) for details). The ifo-MSM builds on household microdata from the German Socio-Economic Panel (GSOEP). This cross-sectional sample of the German population covers about 30,000 individuals who live in 15,000 households.[15]

For the Swedish application, we follow two main steps. First, we construct a tax and transfer simulator based on the official SAS source files from the FASIT 2023 microsimulation model. It implements the 2023 Swedish tax code and benefit formulas, allowing us to compute theoretical tax liabilities and transfer entitlements across the income distribution. Second, we construct a population-wide data set merging several Swedish administrative registers. This data set contains individual-level records with demographics, income, housing costs, and observed transfer receipt. The combination of the two allows us to compare theoretical eligibility schedules against empirical distributions of transfer receipt.

Our analysis focuses on the subgroup of single parents with one child.[16] Table 1 shows the share of single-parent households with one child who are eligible for or take up a transfer by income level and

[12] We subsume the so-called solidarity surcharge under the personal income tax here and in the following.

[13] If households apply for a transfer program and the program authorities notice that the household would be better off taking up another transfer instead, the household will be redirected to apply for the more beneficial program.

[14] Municipal income tax applies at the local rate (averaging 32%, varying by municipality from about 29% to 35%) to all taxable earned income. State income tax applies at 20% to taxable income exceeding the threshold of 598,500 SEK in 2023).

[15] See Goebel, Grabka, Liebig, Kroh, Richter, Schröder and Schupp (2019) for details. For this version, the SOEP-Core v39 was used, see doi:10.5684/soep.core.v39eu.

[16] Results for working-age childless singles for the German case are available in Appendix Section A.3.3. Our Swedish application in that section focuses on young childless singles aged 18–28 since they are eligible for youth housing allowance, which provides a good counterpart to single parents' family housing allowance.

Table 1: Transfer eligibility and transfer take-up by income slice – in % of all single parents, 1 ch., GER 2018.

	all	>0	(0;phaseout]	0	(0;1000]	(1000;2000]	(2000;3000]	>3000
Any transfer								
Share eligible	46.21	36.21	47.95	81.20	72.90	85.09	41.88	0.13
Share take-up	28.41	17.27	22.87	67.38	51.99	35.24	12.82	0.03
Take-up rate	61.48	47.69	47.69	82.99	71.31	41.41	30.61	25.55
Social assistance								
Share eligible	20.30	8.55	11.33	61.39	47.28	6.98	0.27	0.00
Share take-up	17.27	6.36	8.42	55.43	37.03	3.76	0.10	0.00
Take-up rate	85.07	74.35	74.35	90.30	78.31	53.88	36.85	.
Housing benefit								
Share eligible	24.90	26.35	34.90	19.81	25.62	76.14	37.74	0.00
Share take-up	10.83	10.51	13.92	11.95	14.96	30.64	11.72	0.00
Take-up rate	43.51	39.90	39.90	60.33	58.39	40.24	31.06	.
Child supplement								
Share eligible	22.28	23.97	31.74	16.39	21.59	74.95	30.75	0.13
Share take-up	9.91	9.76	12.92	10.43	12.80	30.57	9.86	0.03
Take-up rate	44.46	40.71	40.71	63.63	59.27	40.79	32.05	25.55

Note: This table displays the share of people in households with eligibility for or take-up of means-tested transfer programs relative to all single-parent households with one child, in %. The different columns refer to different monthly income subsamples of the population of single-parent households with one child.
Source: ifo Microsimulation model based on GSOEP data.

welfare program for Germany in 2018. Almost half (46.21%) of these households is eligible for at least one of the major three transfer programs and 61.48% of the eligible households also take up at least one of the welfare transfers. The take-up rates differ significantly between transfer programs and income levels. While 85% of the eligible households take up the social assistance, this rate is halved for the other two transfer programs. This discrepancy can be partially explained by programs targeting populations with different income situations. The large majority of zero- and very-low-income households is eligible for the social assistance. In these income regions, both welfare entitlements are highest and transfer payments are needed the most – accordingly, the take-up rates are especially high. The housing benefit and child supplement primarily focus on the working poor with slightly higher incomes above 1,000 EUR per month. The take-up rates for these higher incomes are much lower and decline further as incomes increase and welfare entitlements shrink.

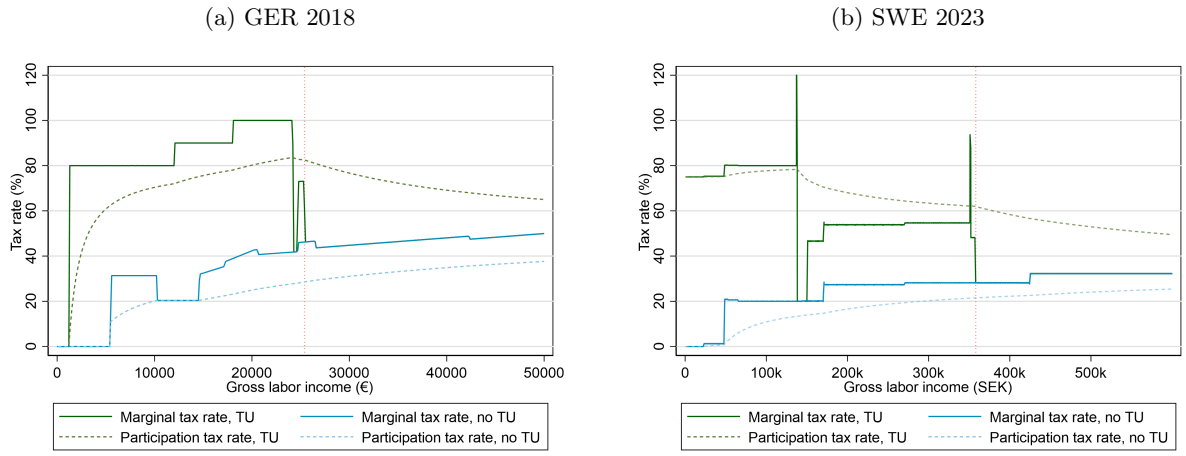
The following paragraphs describe how we estimate the ingredients to construct revenue functions for the group of single parents with one child.

Effective marginal tax rates Our effective marginal tax rates (EMTRs) for Germany are based on hypothetical data for a typical person. This person has no income sources other than labor (or self-employment) income. A household's possible transfer amount, especially in the housing benefit, depends on the rent level assigned to the household's place of residence. Throughout our analysis, we look at households living in a region with rent level 5.[17] We assume the rent of this household to be close to the maximum amount that is paid for by government transfers in a region with this rent level.

For the Swedish case, the simulations assume a 32% municipal tax rate, housing costs of 7,000

[17] Overall, there are seven rent levels in the German system, with Berlin being assigned rent level 4 and the most expensive cities such as Munich being in the highest level 7.

Figure 3: EMTRs and PTRs for single parents with one child.



Notes: This graph plots effective marginal tax rates (solid line) and participation tax rates (dashed line) for welfare recipients (green) and non-recipients (blue) against annual labor income for (a) the 2018 German tax-transfer system and (b) the 2023 Swedish tax-transfer sytem. The vertical dotted line in red shows the point of full phaseout, i.e., the income level at which the household is no longer eligible for any of the perceived welfare programs. The tax rates are based on a hypothetical single-parent household with one child living in a region with rent level 5. The recipient household always takes up the transfer program that is more advantageous.
Source: Authors' illustrations based on ifo Microsimulation model and FASIT microsimulation model.

SEK/month for single parents[18], small-municipality social assistance caps, zero capital income, and a school-age child (ages 7–10) for single parents. For social assistance, we assume at least 6 months of prior receipt, so the "job stimulus" (*jobbstimulans*) applies (reducing the implicit marginal tax rate on earnings from 100% (the standard withdrawal rate in social assistance) to 75%).

Figure 3 separately shows the EMTR schedule for households with transfer take-up and households without transfer take-up.[19] In eligible income regions, the EMTRs in the German case for a household with take-up are 50 to 80 percentage points higher than for a household that refrains from taking up a transfer. For annual incomes around 20,000 EUR, the EMTR with take-up even reaches 100%. This large gap is also reflected in households' participation tax rates, which are important indicators for agents' extensive margin responses. The difference in marginal tax rates is not as extreme in the Swedish case. Starting at similarly high withdrawal rates in the social assistance, the effective marginal tax rates with take-up of the housing allowance are at 50 – 60% and, thus, much lower than in the German case. Consequently, the point of full phaseout is reached at a significantly higher income level than in Germany.

Income distributions and densities Income distributions are another key ingredient for the revenue functions. We focus on the working-age population, i.e., individuals aged 25 to 65. The densities are based on a kernel density estimation.[20] The procedure is done separately for each combination of household type and take-up status, i.e., we estimate a distribution for single parents with transfer take-up and a separate distribution for single parents with no transfer take-up.[21]

In the German application, we substitute the estimated distribution for households without transfer take-up by a Pareto tail at the top, since the SOEP data underrepresents top earners. The utilized

[18] This value is 6,000 SEK/month for young singles.

[19] The schedule for households with take-up living in regions of Germany with other rent levels can differ since this changes both the transfer entitlement and the relative attractiveness of the three transfer programs. This statement similarly holds for the two Swedish benefits.

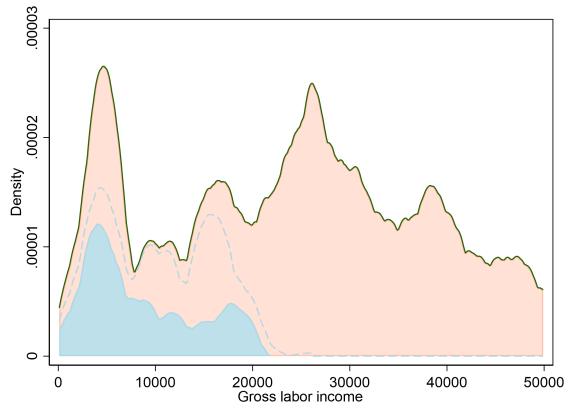
[20] After a broad set of tests analyzing typical bunching points, we have decided to use half the algorithm-based adaptive bandwidth (based on the *akdensity* command in Stata) for our baseline results.

[21] Unlike the case of EMTRs, we do not distinguish between empirically observable rent levels for the estimation of distributions.

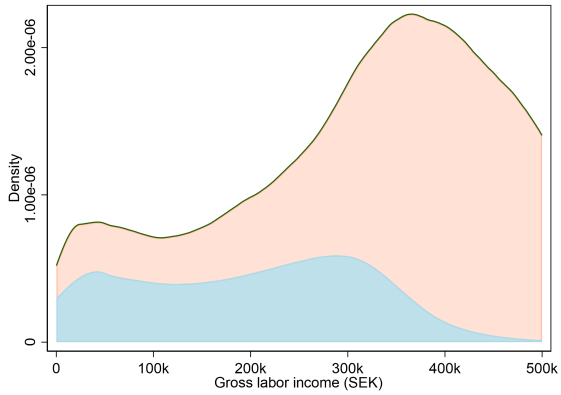
Pareto parameters are based on estimates by Drechsel-Grau, Peichl, Schmid, Schmieder, Walz and Wolter (2022).[22]

Figure 4: Density estimates for single parents with one child.

(a) GER 2018



(b) SWE 2023



Notes: This graph plots the density for single-parent households with one child in (a) Germany, 2018, and in (b) Sweden, 2023, who are welfare recipients (blue area) and non-recipients (red area) against gross labor income. The dashed blue line shows the mass of households who are eligible for any of the perceived welfare transfer programs. The estimates are based on half the algorithm-based adaptive bandwidth (derived from the *akdensity* command in Stata).
Source: Authors' illustrations based on ifo Microsimulation model with GSOEP data and Swedish registry data.

Figure 4 plots the resulting density estimates for single-parent households with one child split between those who take up (blue) and those who do not take up (red) a transfer. The eligibile households' density (dashed blue) in the German case confirms again that the take-up rate decreases with income.[23] Appendix Figure 14 reports the corresponding estimates of the cumulative distribution function.

Behavioral elasticities Our model includes three types of behavioral margins: an intensive margin, an extensive margin and a take-up margin. We refer to a rich empirical literature to select sensible values for the corresponding elasticities pinning down the magnitude of behavioral responses at the respective margins. We remain agnostic about possible differences in elasticities between Sweden and Germany and opt for the same parameterization in both applications.[24]

Our baseline results use an intensive-margin elasticity of $\varepsilon = 0.25$. This is in line with Saez et al. (2012) and close to the estimate of 0.33 from Chetty, Guren, Manoli and Weber (2013). For the moment, there is also no distinction in the magnitude of the intensive-margin elasticity between the group of agents who take up a transfer and the group who does not take up any transfer.[25]

We opt for a participation elasticity π that linearly declines from a value of 0.65 at zero income to a value of 0.25 at an income level of 30,000 EUR (300,000 SEK) per year for single parents.[26] For incomes above 30,000 EUR (300,000 SEK), it stays constant. This choice is in line with recent sufficient-statistics implementations such as Bierbrauer, Boyer, Peichl and Weishaar (2024). The same participation elasticity applies to those with (π_r) and those without (π_n) welfare transfer take-up. The term π_n is further

[22] Their estimates are only stratified by gender and not by household type. Their results are, therefore, mostly applicable to childless singles. Single parents only rarely appear at the top of the income distribution, which is why different Pareto parameters are tested for these households.

[23] We do not yet have information on potential transfer eligibility in the Swedish application. This is the subject of ongoing work.

[24] There might be good reason to believe that the magnitude of behavioral responses differs between these two countries (see e.g. Saez, Slemrod and Giertz (2012)). In a further robustness check, we intend to test whether our main conclusions hold with country-specific elasticity estimates.

[25] We also choose the same intensive-margin elasticity for the group of childless singles, see Appendix.

[26] The respective values for childless singles are 0.4 and 0.1.

split up into those who switch from working and not taking up transfers to not working and not taking up transfers (π_{nn}) as well as those who switch from working and not taking up transfers to not working and taking up transfers (π_{rn}).[27] We assume that π_n is distributed between π_{nn} and π_{rn} according to the baseline take-up rates among those with zero labor income of a perceived household type.[28]

There do not exist any reliable estimates for the take-up elasticities in the German or Swedish context as specified in our model. Therefore, we estimate these elasticities for the German application by using the ifo Microsimulation Model in conjunction with a discrete choice labor supply model in the spirit of van Soest (1995). To get at the take-up elasticity ψ_1 (the elasticity of the share of agents who work before and after the reform and take up transfers with respect to the transfer level they are entitled to), we simulate a shock to the transfer level and check its effect on the share h_{1r}. Importantly, movements of those not working prior to the shock into h_{1r} have to be discarded for the sake of this elasticity. Following this procedure, we estimate a take-up elasticity of $\psi_1 = 0.26$ for single parents with one child.[29] This is significantly larger than the value of 0.05 estimated in Leroy (2024) for France and slightly larger than the range of 0.09-0.18 estimated in Pukelis (2024) for SNAP in the US.

We similarly simulate a shock to the transfer level of all non-working agents to capture the second take-up elasticity parameter ψ_{00}. This elasticity captures how the share of non-working agents who take up transfers reacts to changes in the level of the base transfer. Our baseline estimate of ψ_{00} for single parents with one child is 0.77.[30] Akin to our choice for the other elasticities, we also use the estimates for take-up elasticities from the German context in the Swedish application.

7.3 Results

Putting all these ingredients together allows us to implement the formulas for the revenue functions (derived in Section 5) and the inverse optimum weights (derived in Section 6).

Revenue functions Figure 5 depicts the revenue functions for single parents with one child for three types of reforms: a tax increase (top row), a transfer cut (middle row) and a joint reform (bottom row) where taxes and transfers are increased jointly. The revenue effect of a base transfer reform $\mathcal{R}_0$, the fourth type of reform, is depicted as a red dot in each of the graphs. These graphs convey several key messages.

First, both the revenue functions for a tax reform $\mathcal{R}_t$ and for a transfer reform $\mathcal{R}_b$ exhibit negative values and non-monotonicities in the German case, thus allowing for Pareto-improving reforms as stipulated in Propositions 2 and 3. A small reduction of the marginal tax rate or the transfer phaseout rate for single parents with one child and annual incomes between 18,000 and 21,000 EUR would have been Pareto-improving. The 2023 Swedish tax-transfer system shows similar, but less severe, inefficiencies. The two negative spikes are due to small discontinuous reductions in transfer entitlements when individuals switch from social assistance to housing allowance or the housing allowance completely phases out, respectively. The Swedish system could be Pareto-improved with a two-bracket tax/transfer reform around the first negative spike such that EMTRs are lowered below the spike and increased above the spike.

Second, the fact that both $\mathcal{R}_t$ and $\mathcal{R}_b$ fall below 0 at very similar income levels underlines that the inefficiencies of both systems stem from the transfer phaseout function B. While a tax reduction

[27] Note that $\pi_n = \pi_{nn} + \pi_{rn}$.

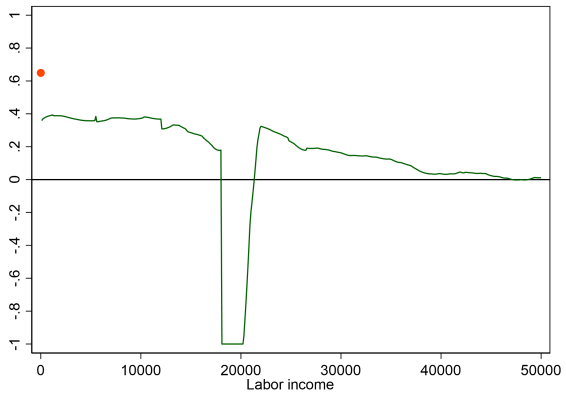
[28] If $\pi_n = 0.2$ at a certain income level and the take-up rate among childless singles with zero labor income is 20%, then $\pi_{nn} = 0.2 * 0.8 = 0.16$ and $\pi_{rn} = 0.2 * 0.2 = 0.04$.

[29] The respective elasticity for childless singles is estimated to be $\psi_1 = 0.19$. We also break down the take-up elasticity measured across all three types of transfer programs into elasticities that are specific to a certain transfer program. We find that the take-up elasticity is largest for the social assistance, followed by the housing benefit and the child supplement. Finally, we look at transfer-specific take-up elasticities by income interval. Take-up elasticities are highest when entitlements are largest, i.e., mostly among lower incomes.

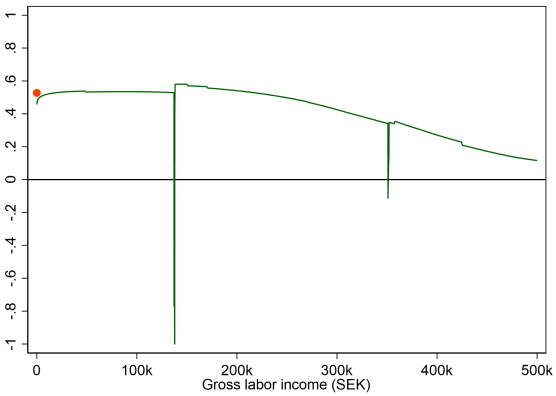
[30] For childless singles, we find $\psi_{00} = 0.71$.

Figure 5: Revenue functions $\mathcal{R}$ for single parents with one child by type of reform.

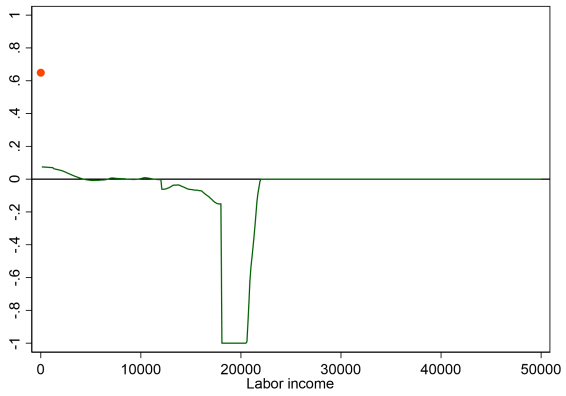
(a) Tax increase, $\mathcal{R}_t$, GER 2018



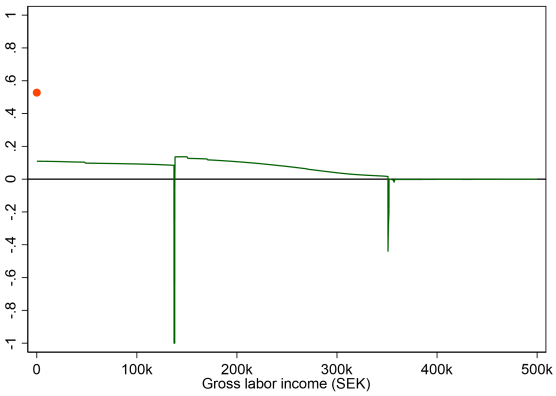
(b) Tax increase, $\mathcal{R}_t$, SWE 2023



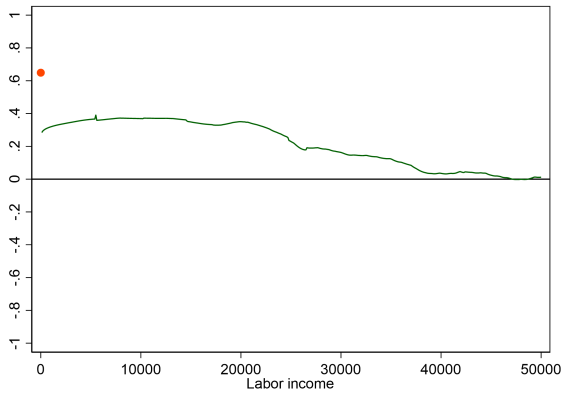
(c) Transfer cut, $\mathcal{R}_b$, GER 2018



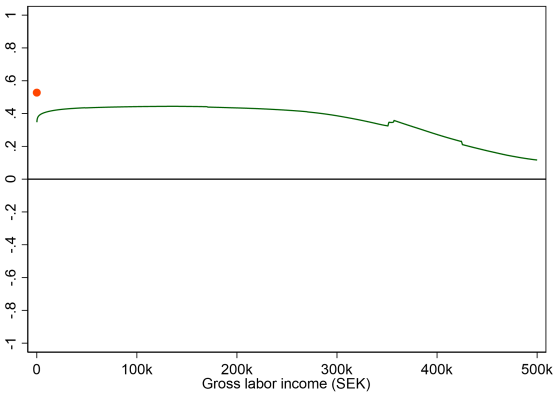
(d) Transfer cut, $\mathcal{R}_b$, SWE 2023



(e) Joint reform, $\Delta\mathcal{R}$, GER 2018



(f) Joint reform, $\Delta\mathcal{R}$, SWE 2023



Notes: This graph plots the revenue functions for different types of reforms of the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) faced by single parents with one child: a tax increase (top row), a transfer cut (middle row) and a joint reform (bottom row) based on the calibrations described in Section 7.2 and the formulas derived in Section 5. The red dot pins down the revenue effect of a base transfer reform, $\mathcal{R}_0$. The graph is truncated at values of -1 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model with GSOEP data and FASIT microsimulation model with Swedish registry data.

affecting both welfare recipients and non-recipients would be Pareto-improving, the inefficiency could be addressed by solely targeting the group of welfare recipients via a reduction of the transfer phaseout rate.[31]

Third, the revenue function for the joint reform $\Delta\mathcal{R}$ in both countries is monotonically decreasing for most income levels, but slightly increasing at the very bottom. Hence, the tax schedule for non-recipients is broadly Pareto-efficient, except for small inefficiencies at the bottom of the income distribution.

Fourth, there is no scope for Pareto improvements involving the level of the base transfer. The revenue effect of a base transfer decrease $\mathcal{R}_0$ lies at approximately 0.65 (GER) and 0.55 (SWE), respectively – clearly between the critical values of 0 and 1, but also well above the levels of the transfer cut revenue function $\mathcal{R}_b$.

This discussion demonstrates that our new tools allow us to study the (in)efficiency of a given tax-transfer system in a much more realistic way than in Bierbrauer et al. (2023). We can distinguish between inefficiencies arising from the tax schedule and inefficiencies arising from the transfer phaseout function. Taking into account imperfect and endogenous take-up also leads us to revise the recommendations for the optimal reform. While a revenue function under the assumption of full take-up exhibits inefficiently high transfer phaseout rates even for incomes below 18,000 EUR in the 2018 German tax-transfer system, these inefficiencies vanish when we account for agents' take-up behavior (see Appendix Figure 15). Similarly, accounting for imperfect and endogenous take-up in the 2023 Swedish tax-transfer system removes a considerable non-monotonicity at the point of full phaseout as compared to a revenue function based on the assumption of full take-up.

Inverse optimum weights Our model allows us to distinguish between the inverse optimum weights on recipients and non-recipients of welfare transfers. These inverse optimum weights can be deduced from the ratio of the slope of the revenue function relative to the density at a given income. We directly bring equations (36) and (37) to the data and thus reduce our results to one average weight per income bin – with bins of size 1,000 (10,000 for SWE).

Figure 6 shows the estimated inverse optimum weights for single-parent households with one child taking up transfers (top row) and not taking up transfers (bottom row). Transfer recipients in Germany exhibit a highly volatile and non-monotonic profile of weights, especially for annual incomes above 10,000 EUR. Rationalizing this welfare weight profile with seemingly arbitrary jumps from high positive weights to high negative weights is impossible with standard welfare criteria. The inverse optimum weights on German non-recipients also showcase non-monotonicities, but the volatility is much less pronounced than for transfer recipients. By and large, the weights for non-recipients follow an increasing trend for the first 15,000 EUR, starting with strongly negative weights at the lower end. For incomes between 15,000 EUR and 50,000 EUR, the welfare weight profile for German non-recipients fluctuates between weights of -1 to 2.

The inverse optimum weights derived from the 2023 Swedish tax-transfer system are significantly less volatile. The weights on recipients are consistently higher than those on non-recipients until the statutory point of full phaseout at 358,000 SEK. While recipients are almost exclusively assigned positive weights until the phaseout threshold, non-recipients exhibit strongly negative weights at low incomes, which continuously increase until they reach the population average of 1 for annual incomes 358,000 SEK.

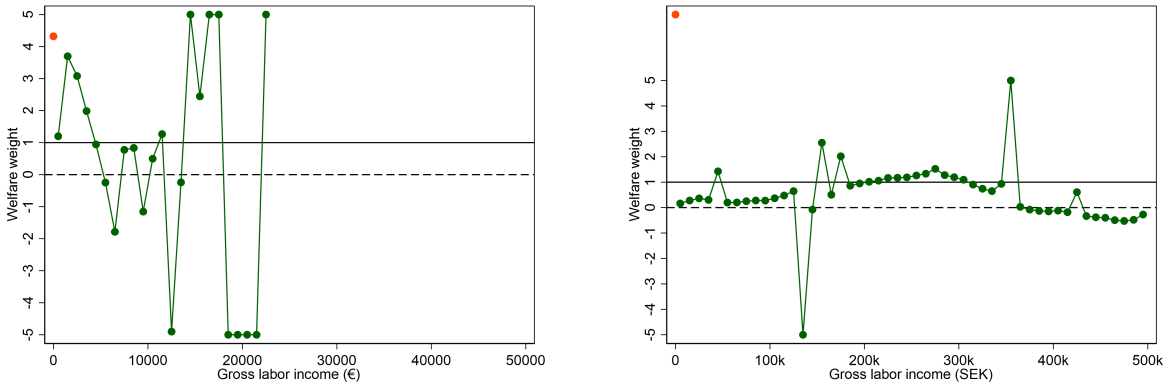
We interpret the results for Germany as indication for policymakers unwillingly creating a highly inefficient transfer system that does not reflect their actual social preferences. This interpretation is particularly relevant in a complicated system, such as the German one, with interactions between various transfer programs and local differences in inefficiencies due to varying rent levels. The Swedish welfare weight profiles, by contrast, are broadly rationalizable. Recipients being assigned higher weights than non-recipients, in particular, is in line with basic expectations.

[31] Effectively, a tax reform would even have no impact on welfare recipients in the German system since tax liabilities are fully accounted for in the transfer phaseout.

Figure 6: Inverse optimum weights for single parents with one child.

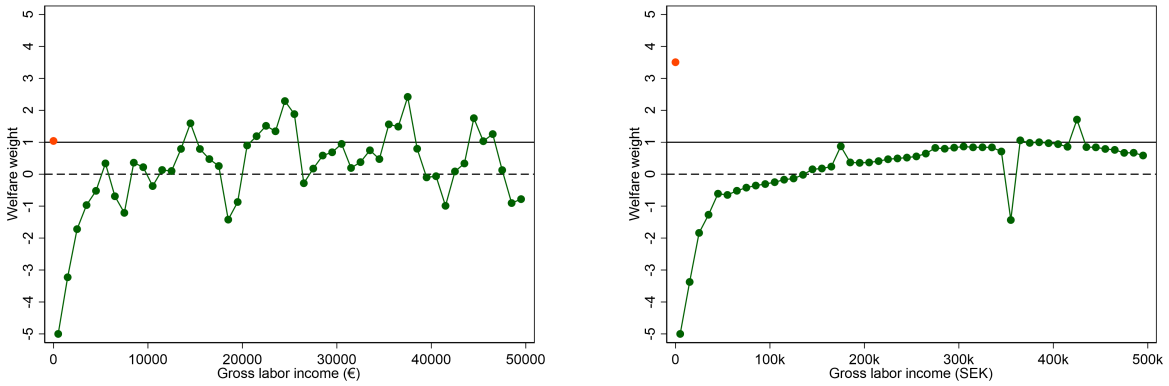
(a) Recipients, GER 2018

(b) Recipients, SWE 2023



(c) Non-recipients, GER 2018

(d) Non-recipients, SWE 2023



Notes: This graph plots the inverse optimum weights for single parents with one child facing the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) and distinguishes between individuals taking up welfare transfers (top row) and individuals not taking up welfare transfers (bottom row). The results are based on the calibrations described in Section 7.2 and the formulas for the weights of income bins derived in Section 6. The red dot pins down the inverse optimum weight of households with zero income. The graph is truncated at values of -5 and 5 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model with GSOEP data and FASIT microsimulation model with Swedish registry data.

If the status quo schedule was, indeed, set optimally, these weights are instructive for the quality of self-targeting. In such a world, the policymakers would have set the recipient schedule deliberately in such a way to target only the most needy households by unobservable characteristics such as lifetime income (see Rafkin et al. (2023)). However, given the estimated profile of inverse optimum weights for German recipients in Figure 6, this would imply poor self-targeting of households – as too many non-needy and responsive (in terms of elasticities) households have taken up transfers in these income regions.

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A Appendix

A.1 Relation between type distribution and income distribution

For a skill type ω, the shares of people in each of these four areas are given by

$$\begin{aligned}
h_{0n}(\omega) &= \int_{s_{n0}(\omega)}^{\bar{\delta}} 1 - F_\kappa\left(\hat{\kappa}(\omega,\delta) \mid \omega,\delta\right) dF_\delta(\delta \mid \omega) \,, \\
h_{0r}(\omega) &= \int_{s_{r0}(\omega)}^{\bar{\delta}} F_\kappa\left(\hat{\kappa}(\omega,\delta) \mid \omega,\delta\right) dF_\delta(\delta \mid \omega) \,, \\
h_{1n}(\omega) &= \int_{\underline{\delta}}^{s_{n1}(\omega)} 1 - F_\kappa\left(\hat{\kappa}(\omega,\delta) \mid \omega,\delta\right) dF_\delta(\delta \mid \omega) \,, \\
h_{1r}(\omega) &= \int_{\underline{\delta}}^{s_{r1}(\omega)} F_\kappa\left(\hat{\kappa}(\omega,\delta) \mid \omega,\delta\right) dF_\delta(\delta \mid \omega) \,.
\end{aligned}$$

In skill group ω, the share of agents taking up transfers is given by $G_r(\omega) \coloneqq h_{0r}(\omega) + h_{1r}(\omega)$. The share of agents with zero earnings is given by $h_0(\omega) \coloneqq h_{0n}(\omega) + h_{0r}(\omega)$. Finally, the share of agents with income $y_{r1}(\omega)$ – taking up transfers – is given by $h_{1r}(\omega)$, and the one of agents with income $y_{n1}(\omega)$ – not taking up transfers – is given by $h_{1n}(\omega)$.

We denote the income distribution under tax system (T, B_1, b_1, c_1) by G with density g. The share of agents with zero income is given by

$$G(0) = G_n(0) + G_r(0) = \int_{\underline{\omega}}^{\bar{\omega}} f(\omega) h_{0n}(\omega) d\omega + \int_{\underline{\omega}}^{\bar{\omega}} f(\omega) h_{0b}(\omega) d\omega \,.$$

Define $\omega_n(y) \coloneqq \omega \in \Omega : y_n(\omega) = y$ and $\omega_r(y) \coloneqq \omega \in \Omega : y_r(\omega) = y$. Then, the share of people with income y or below is given by

$$G(y) = G_n(y) + G_r(y) = G(0) + \int_{\underline{\omega}}^{\omega_n(y)} f(\omega) h_{1n}(\omega) d\omega + \int_{\underline{\omega}}^{\omega_r(y)} f(\omega) h_{1r}(\omega) d\omega \,.$$

The densities of the income distributions are given by

$$\begin{aligned}
g_n(y) &= \frac{dG_n(y)}{dy} = \frac{f\left(\omega_n(y)\right)\ h_{1n}\left(\omega_n(y)\right)}{y_{n\omega}(\omega)} = \frac{f\left(\omega_n(y)\right)\ h_{1n}\left(\omega_n(y)\right)}{\mu_n(\omega)} \frac{\omega_n(y)}{y} \,, \\
g_r(y) &= \frac{dG_r(y)}{dy} = \frac{f\left(\omega_b(y)\right)\ h_{1r}\left(\omega_b(y)\right)}{y_{r\omega}(\omega)} = \frac{f\left(\omega_r(y)\right)\ h_{1r}\left(\omega_r(y)\right)}{\mu_r(\omega)} \frac{\omega_r(y)}{y} \,, \\
g(y) &= g_n(y) + g_b(y) \,.
\end{aligned}$$

The overall share of people taking up transfers is given by

$$G_r(\bar{y}_B) = \int_{\underline{\omega}}^{\bar{\omega}} f(\omega) h_{0r}(\omega) d\omega + \int_{\underline{\omega}}^{\hat{\omega}_B} f(\omega) h_{1r}(\omega) d\omega \,.$$

A.2 Derivation of revenue functions

We start by defining additional elasticities that capture the relation between skill types and incomes. Specifically, we define a pair of elasticities, μ_n and μ_b, of income with respect to the skill type ω,

$$\begin{aligned}
\mu_n(\omega) &= \frac{dy_n(\omega)}{d\omega} \frac{\omega}{y_n(\omega)} > 0 \,, \text{ and} \\
\mu_b(\omega) &= \frac{dy_b(\omega)}{d\omega} \frac{\omega}{y_b(\omega)} > 0 \,.
\end{aligned}$$

By the single-crossing property, both elasticities are strictly positive. Note also that

$$\begin{aligned}
\frac{ds_n(\omega)}{d\omega} &= \frac{y_n(\omega)}{\omega^2}\ v'\left(\frac{y_n(\omega)}{\omega}\right) > 0 \,, \text{ and} \\
\frac{ds_b(\omega)}{d\omega} &= \frac{y_b(\omega)}{\omega^2}\ v'\left(\frac{y_b(\omega)}{\omega}\right) > 0 \,.
\end{aligned}$$

As a result, we can determine the relation between ω and skill-specific shares h_{0n} to h_{1b} under the assumption that, e.g., ω, δ, and κ are independently distributed.

Lemma 5 *Tax and transfer reforms affect income choices as follows*

$$\begin{aligned}
\frac{\partial y_n(\omega)}{\partial \beta} \mid_{\beta=0} &= 0\,, \\
\frac{\partial y_n(\omega)}{\partial \tau} \mid_{\tau=0} &= \begin{cases} -\frac{1}{\frac{1}{\omega^2} v''\left(\frac{y_n(\omega)}{\omega}\right)+T_0''(y_n(\omega))} & \text{for} \quad y_n(\omega) \in (\hat{y}, \hat{y}+\ell)\ , \\ 0 & \text{for} \quad y_n(\omega) \notin (\hat{y}, \hat{y}+\ell)\ , \end{cases}
\end{aligned}$$

and

$$\frac{\partial y_b(\omega)}{\partial \tau} \mid_{\tau=0} = \frac{\partial y_b(\omega)}{\partial \beta} \mid_{\beta=0} = \begin{cases} -\left[\frac{1}{\omega^2} v''\left(\frac{y_n(\omega)}{\omega}\right) + T_0''(y_n(\omega)) + B_0''(y_n(\omega))\right]^{-1} & \text{for} \quad y_b(\omega) \in (\hat{y}, \hat{y}+\ell)\ , \\ 0 & \text{for} \quad y_b(\omega) \notin (\hat{y}, \hat{y}+\ell)\ , \end{cases}$$

Reforms affect participation and take-up choices, moreover, as follows

$$\begin{aligned}
\frac{\partial s_n(\omega)}{\partial \beta} \mid_{\beta=0} &= 0\,, \\
\frac{\partial s_n(\omega)}{\partial \tau} \mid_{\tau=0} &= \begin{cases} 0 & \text{for} \quad y_n(\omega) \leq \hat{y}\,, \\ -\left[y_n(\omega) - \hat{y}\right] & \text{for} \quad y_n(\omega) \in (\hat{y}, \hat{y}+\ell]\ , \\ -\ell & \text{for} \quad y_n(\omega) > \hat{y}+\ell \end{cases} \\
\frac{\partial s_b(\omega)}{\partial \tau} \mid_{\tau=0} &= \frac{\partial s_b(\omega)}{\partial \beta} \mid_{\beta=0} = \begin{cases} 0 & \text{for} \quad y_b(\omega) \leq \hat{y}\,, \\ -\left[y_b(\omega) - \hat{y}\right] & \text{for} \quad y_b(\omega) \in (\hat{y}, \hat{y}+\ell]\ , \\ -\ell & \text{for} \quad y_b(\omega) > \hat{y}+\ell \end{cases} \\
\frac{\partial \hat{\kappa}(\omega,\delta)}{\partial \tau} \mid_{\tau=0} &= \begin{cases} -\left[y_n(\omega) - \hat{y}\right] & \text{for} \quad \delta \leq s_n(\omega) \text{ and } y_n(\omega) \in (\hat{y}, \hat{y}+\ell]\,, \\ -\ell & \text{for} \quad \delta \leq s_n(\omega) \text{ and } y_n(\omega) > \hat{y}+\ell, \\ 0 & \text{for} \quad \delta > s_n(\omega) \text{ or } y_n(\omega) < \hat{y}\ . \end{cases} \\
\frac{\partial \hat{\kappa}(\omega,\delta)}{\partial \beta} \mid_{\beta=0} &= \begin{cases} -\left[y_b(\omega) - \hat{y}\right] & \text{for} \quad \delta \leq s_b(\omega) \text{ and } y_b(\omega) \in (\hat{y}, \hat{y}+\ell]\,, \\ -\ell & \text{for} \quad \delta \leq s_b(\omega) \text{ and } y_b(\omega) > \hat{y}+\ell, \\ 0 & \text{for} \quad \delta > s_b(\omega) \text{ or } y_b(\omega) < \hat{y}\ . \end{cases}
\end{aligned}$$

The second-order derivatives with respect to τ (β) and ℓ follow as

$$\begin{aligned}
\frac{\partial^2 s_n(\omega)}{\partial \beta\ \partial \ell} \mid_{\beta=\ell=0} &= 0\,, \\
\frac{\partial^2 s_n(\omega)}{\partial \tau\ \partial \ell} \mid_{\tau=\ell=0} &= \begin{cases} 0 & \text{for} \quad y_n(\omega) \leq \hat{y}\,, \\ -1 & \text{for} \quad y_n(\omega) > \hat{y} \end{cases} \\
\frac{\partial^2 s_b(\omega)}{\partial \tau\ \partial \ell} \mid_{\tau=\ell=0} &= \frac{\partial^2 s_b(\omega)}{\partial \beta\ \partial \ell} \mid_{\beta=\ell=0} = \begin{cases} 0 & \text{for} \quad y_b(\omega) \leq \hat{y}\,, \\ -1 & \text{for} \quad y_b(\omega) > \hat{y} \end{cases} \\
\frac{\partial^2 \hat{\kappa}(\omega,\delta)}{\partial \tau\ \partial \ell} \mid_{\tau=\ell=0} &= \begin{cases} \frac{\partial^2 s_b(\omega)}{\partial \tau\ \partial \ell} \mid_{\tau=\ell=0} - \frac{\partial^2 s_n(\omega)}{\partial \tau\ \partial \ell} \mid_{\tau=\ell=0} & \text{for} \quad \delta \leq s_b(\omega), \\ -\frac{\partial^2 s_n(\omega)}{\partial \tau\ \partial \ell} \mid_{\tau=\ell=0} & \text{for} \quad \delta \in (s_b(\omega), s_n(\omega)]\,, \\ 0 & \text{for} \quad \delta > s_n(\omega)\ . \end{cases} \\
\frac{\partial^2 \hat{\kappa}(\omega,\delta)}{\partial \beta\ \partial \ell} \mid_{\beta=\ell=0} &= \begin{cases} \frac{\partial^2 s_b(\omega)}{\partial \beta\ \partial \ell} \mid_{\beta=\ell=0} & \text{for} \quad \delta \leq s_b(\omega)\,, \\ 0 & \text{for} \quad \delta > s_b(\omega)\ . \end{cases}
\end{aligned}$$

In the next step, we can determine the effect of tax and transfer reforms on the skill-specific shares h_{0n} (zero income, no take-up), h_{0b} (no income, take-up), h_{1n} (positive income y_n, no take-up), and h_{1b} (positive income y_b, take-up).

Tax reforms In general, a small tax reform makes it less attractive to work and more attractive to take up transfers. The effects on the skill-specific shares are given by

$$\begin{aligned}
\frac{\partial^2 h_{0n}(\omega)}{\partial\tau\ \partial\ell} &= -\frac{\partial^2 s_n(\omega)}{\partial\tau\partial\ell} f_\delta(s_n(\omega))\left[1 - F_\kappa(\hat\kappa(\omega, s_n(\omega)) \mid \omega, s_n(\omega))\right] - \int_{s_n(\omega)}^{\bar\delta} f_\kappa(\hat\kappa(\omega,\delta) \mid \omega,\delta)\frac{\partial^2\hat\kappa(\omega,\delta)}{\partial\tau\partial\ell} dF_\delta(\delta \mid \omega)\\
\frac{\partial^2 h_{0b}(\omega)}{\partial\tau\ \partial\ell} &= -\frac{\partial^2 s_b(\omega)}{\partial\tau\partial\ell} f_\delta(s_b(\omega)) F_\kappa(\hat\kappa(\omega, s_b(\omega)) \mid \omega, s_b(\omega)) + \int_{s_b(\omega)}^{\bar\delta} f_\kappa(\hat\kappa(\omega,\delta) \mid \omega,\delta)\frac{\partial^2\hat\kappa(\omega,\delta)}{\partial\tau\partial\ell} dF_\delta(\delta \mid \omega)\\
\frac{\partial^2 h_{1n}(\omega)}{\partial\tau\ \partial\ell} &= \frac{\partial^2 s_n(\omega)}{\partial\tau\partial\ell} f_\delta(s_n(\omega))\left[1 - F_\kappa(\hat\kappa(\omega, s_n(\omega)) \mid \omega, s_n(\omega))\right] - \int_{\underline\delta}^{s_n(\omega)} f_\kappa(\hat\kappa(\omega,\delta) \mid \omega,\delta)\frac{\partial^2\hat\kappa(\omega,\delta)}{\partial\tau\partial\ell} dF_\delta(\delta \mid \omega)\\
\frac{\partial^2 h_{1b}(\omega)}{\partial\tau\ \partial\ell} &= \frac{\partial^2 s_b(\omega)}{\partial\tau\partial\ell} f_\delta(s_b(\omega))\left[1 - F_\kappa(\hat\kappa(\omega, s_b(\omega)) \mid \omega, s_b(\omega))\right] + \int_{\underline\delta}^{s_b(\omega)} f_\kappa(\hat\kappa(\omega,\delta) \mid \omega,\delta)\frac{\partial^2\hat\kappa(\omega,\delta)}{\partial\tau\partial\ell} dF_\delta(\delta \mid \omega)
\end{aligned}$$

More specifically, these effects vary across skill types, depending on whether $\hat y$ is smaller or larger than $y_b(\omega)$ and $y_n(\omega)$ or, put differently, whether ω is smaller or larger than $\omega_n(\hat y)$ and $\omega_b(\hat y)$. For skill types $\omega < \omega_n(\hat y)$ such that $y_n(\omega) < \hat y$, a tax reform has no effect on any of the shares h_{0n} to h_{1b}.

For skill types such that $y_b(\omega) \le \hat y < y_n(\omega)$ or, equivalently, $\omega \in (\omega_n(\hat y), \omega_b(\hat y)]$, a reform makes it less attractive to work and more attractive to take up transfers. In particular, the effects of a small tax reform are given by

$$\begin{aligned}
\frac{\partial^2 h_{0n}(\omega)}{\partial\tau\ \partial\ell} &= f_\delta(s_n(\omega) \mid \omega)\left[1 - F_\kappa\left(\hat\kappa(\omega, s_n(\omega)) \mid \omega, s_n(\omega)\right)\right]\\
&= \pi_{nn}(\omega)\frac{h_{1n}(\omega)}{y_n(\omega) - T\left(y_n(\omega)\right)} > 0\ ,\\
\frac{\partial^2 h_{0b}(\omega)}{\partial\tau\ \partial\ell} &= \int_{s_b(\omega)}^{s_n(\omega)} f_\kappa(\hat\kappa(\omega,\delta) \mid \omega,\delta) dF_\delta(\delta \mid \omega)\\
&= \pi_{bn}(\omega)\frac{h_{1n}(\omega)}{y_n(\omega) - T\left(y_n(\omega)\right)} > 0\ ,\\
\frac{\partial^2 h_{1n}(\omega)}{\partial\tau\ \partial\ell} &= -f_\delta(s_n(\omega) \mid \omega)\left[1 - F_\kappa\left(\hat\kappa(\omega, s_n(\omega)) \mid \omega, s_n(\omega)\right)\right] - \int_{\underline\delta}^{s_n(\omega)} f_\kappa\left(\hat\kappa(\omega,\delta) \mid \omega,\delta\right) dF_\delta(\delta \mid \omega)\\
&= -\pi_n(\omega)\frac{h_{1n}(\omega)}{y_n(\omega) - T\left(y_n(\omega)\right)} - \psi_1(\omega)\frac{h_{1b}(\omega)}{b_0 - B\left(y_b(\omega)\right)} < 0\ ,\\
\frac{\partial^2 h_{1b}(\omega)}{\partial\tau\ \partial\ell} &= \int_{\underline\delta}^{s_b(\omega)} f_\kappa\left(\hat\kappa(\omega,\delta) \mid \omega,\delta\right) dF_\delta(\delta \mid \omega)\\
&= \psi_1(\omega)\frac{h_{1b}(\omega)}{b_0 - B\left(y_b(\omega)\right)} > 0\ .
\end{aligned}$$

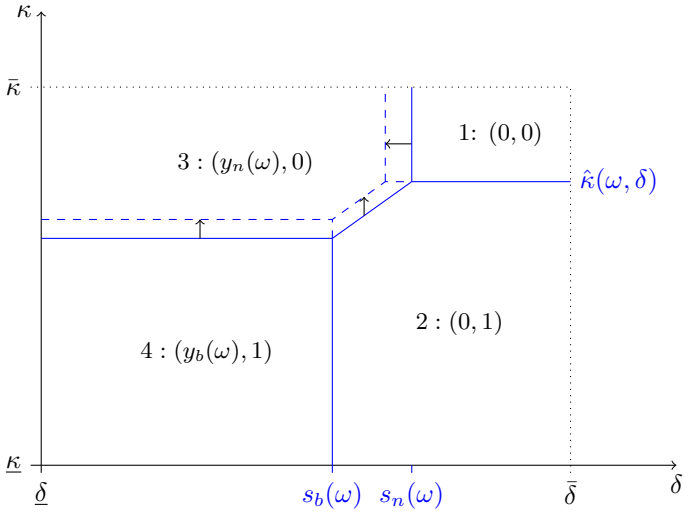
Hence, for $\hat y \in [y_b(\omega), y_n(\omega))$ or, equivalently, $\omega \in (\omega_n(\hat y), \omega_b(\hat y)]$, we have

$$\frac{d^2 h_{1n}(\omega)}{d\tau d\ell} = -\frac{d^2 h_{0n}(\omega)}{d\tau d\ell} - \frac{d^2 h_{0b}(\omega)}{d\tau d\ell} - \frac{d^2 h_{1b}(\omega)}{d\tau d\ell} < 0 \tag{41}$$

Figure 7 illustrates these effects.

For skill types ω such that $y_b(\omega) > \hat y$ or, equivalently, $\omega > \omega_b(\hat y)$, a small tax reform has the following

Figure 7: Effects of a small tax increase on behavior, skill type $\omega \in (\omega_n(\hat{y}), \omega_b(\hat{y})]$.



effects on the shares:

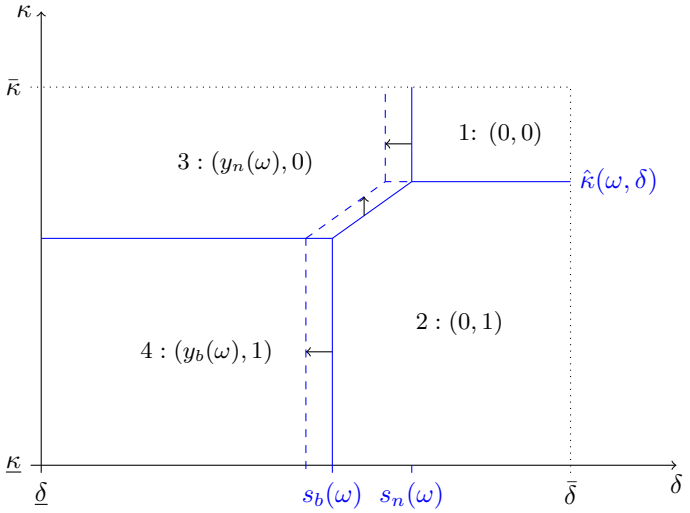
$$
\begin{aligned}
\frac{\partial^2 h_{0n}(\omega)}{\partial\tau\ \partial\ell} &= f_\delta(s_n(\omega) \mid \omega)\left[1 - F_\kappa\left(\hat{\kappa}(\omega, s_n(\omega)) \mid \omega, s_n(\omega)\right)\right] \\
&= \pi_{nn}(\omega)\frac{h_{1n}(\omega)}{y_n(\omega) - T\left(y_n(\omega)\right)} > 0\ , \\
\frac{\partial^2 h_{0b}(\omega)}{\partial\tau\ \partial\ell} &= f_\delta(s_b(\omega) \mid \omega)\ F_\kappa\left(\hat{\kappa}(\omega, s_b(\omega)) \mid \omega, s_b(\omega)\right) + \int_{s_b(\omega)}^{s_n(\omega)} f_\kappa(\hat{\kappa}(\omega,\delta) \mid \omega, \delta) dF_\delta(\delta \mid \omega) \\
&= \pi_{bn}(\omega)\frac{h_{1n}(\omega)}{y_n(\omega) - T\left(y_n(\omega)\right)} + \pi_b(\omega)\frac{h_{1b}(\omega)}{y_b(\omega) - T\left(y_b(\omega)\right) - B\left(y_b(\omega)\right)}\ , \\
\frac{\partial^2 h_{1n}(\omega)}{\partial\tau\ \partial\ell} &= -\left[1 - F_\kappa\left(\hat{\kappa}(\omega, s_n(\omega)) \mid \omega, s_n(\omega)\right)\right] f_\delta(s_n(\omega) \mid \omega) - \int_{s_b(\omega)}^{s_n(\omega)} f_\kappa\left(\hat{\kappa}(\omega,\delta) \mid \omega, \delta\right) dF_\delta(\delta \mid \omega) \\
&= -\pi_n(\omega)\frac{h_{1n}(\omega)}{y_n(\omega) - T\left(y_n(\omega)\right)} < 0\ , \\
\frac{\partial^2 h_{1b}(\omega)}{\partial\tau\ \partial\ell} &= -f_\delta(s_b(\omega) \mid \omega)\ F_\kappa\left(\hat{\kappa}(\omega, s_b(\omega)) \mid s_b(\omega), \delta\right) \\
&= -\pi_b(\omega)\frac{h_{1b}(\omega)}{y_b(\omega) - T\left(y_b(\omega)\right) - B\left(y_b(\omega)\right)}\ .
\end{aligned}
$$

As illustrated by Figure 8, for $\omega > \omega_b(\hat{y})$, we have

$$
\frac{d^2 h_{1n}(\omega)}{d\tau d\ell} + \frac{d^2 h_{1b}(\omega)}{d\tau d\ell} = -\frac{d^2 h_{0n}(\omega)}{d\tau d\ell} - \frac{d^2 h_{0b}(\omega)}{d\tau d\ell} < 0\ .
$$

A small increase in the transfer phase-out makes it less attractive to take up transfers and, conditional on take up, to work for all types such that $y_b(\omega) > \hat{y}$ or, equivalently, $\omega > \omega_b(\hat{y})$. Correspondingly, for

Figure 8: Effects of a small tax increase on behavior, skill type $\omega > \omega_b(\hat{y})$.



these types, the effects of a small transfer reform are given by

$$\begin{aligned}
\frac{\partial^2 h_{0n}(\omega)}{\partial\beta\ \partial\ell} &= 0\ , \\
\frac{\partial^2 h_{0b}(\omega)}{\partial\beta\ \partial\ell} &= f_\delta(s_b(\omega)\mid\omega)\ F_\kappa\left(\hat{\kappa}(\omega,s_b(\omega))\mid\omega,s_b(\omega)\right) \\
&= \pi_b(\omega)\frac{h_{1b}(\omega)}{y_b(\omega)-T\left(y_b(\omega)\right)-B\left(y_b(\omega)\right)}\ , \\
\frac{\partial^2 h_{1n}(\omega)}{\partial\beta\ \partial\ell} &= \int_{\underline{\delta}}^{s_b(\omega)} f_\kappa\left(\hat{\kappa}(\omega,\delta)\mid\omega,\delta\right)dF_\delta(\delta\mid\omega) \\
&= \psi_1(\omega)\frac{h_{1b}(\omega)}{b_0-B\left(y_b(\omega)\right)}\ , \\
\frac{\partial^2 h_{1b}(\omega)}{\partial\beta\ \partial\ell} &= -f_\delta(s_b(\omega)\mid\omega)\ F_\kappa\left(\hat{\kappa}(\omega,s_b(\omega))\mid s_b(\omega),\delta\right)-\int_{\underline{\delta}}^{s_b(\omega)} f_\kappa\left(\hat{\kappa}(\omega,\delta)\mid\omega,\delta\right)dF_\delta(\delta\mid\omega) \\
&= -\pi_b(\omega)\frac{h_{1b}(\omega)}{y_b(\omega)-T\left(y_b(\omega)\right)-B\left(y_b(\omega)\right)}-\psi_1(\omega)\frac{h_{1b}(\omega)}{b_0-B\left(y_b(\omega)\right)}\ .
\end{aligned}$$

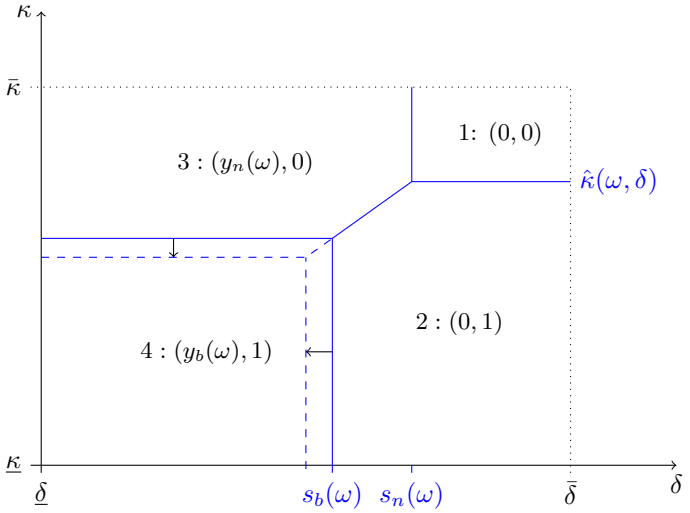
As illustrated by Figure 9, all changes in the shares h_{0n} to h_{1b} can be unambiguously signed for all cases such that $\hat{y}$ is below $y_b(\omega)$. Then, we have

$$\frac{\partial^2 h_{1b}(\omega)}{\partial\beta\ \partial\ell}=\frac{\partial^2 h_{0b}(\omega)}{\partial\beta\ \partial\ell}+\frac{\partial^2 h_{1n}(\omega)}{\partial\beta\ \partial\ell}<0\ .$$

If instead $\hat{y} > y_b(\omega)$, the reform does not affect the type-specific shares h_{0n} to h_{1b}.

To complete this section, we can also consider a small increase in the base transfer b_0, which makes taking up transfers more attractive both for working and non-working agents. In particular, we assume that $b_1 = b_0 + \beta\ell$. The reform has no effect on the income functions y_n and y_b, nor on the participation surpluses s_n and s_b. For all skill types below the threshold $\hat{\omega}_B$, it increases the shares of people taking

Figure 9: Effects of a small transfer cut on behavior, skill type $\hat{y} < y_b(\omega)$.



up transfers, however, as follows:

$$\begin{aligned}
\frac{d^2h_{0n}(\omega)}{d\beta d\ell} &= -\int_{s_n(\omega)}^{\bar{\delta}} f_\kappa\left(\hat{\kappa}(\omega,\delta)\mid\omega,\delta\right)dF_\delta(\delta\mid\omega)\\
&= -\psi_{00}\frac{h_{0b}(\omega)}{b_0}\ ,\\
\frac{d^2h_{0b}(\omega)}{d\beta d\ell} &= \int_{s_b(\omega)}^{\bar{\delta}} f_\kappa\left(\hat{\kappa}(\omega,\delta)\mid\omega,\delta\right)dF_\delta(\delta\mid\omega)\\
&= \psi_{00}\frac{h_{0b}(\omega)}{b_0}+\psi_{01}\frac{h_{0b}(\omega)}{b_0}\\
&= \psi_{00}\frac{h_{0b}(\omega)}{b_0}+\pi_{bn}\frac{h_{1n}(\omega)}{y_n(\omega)-T\left(y_n(\omega)\right)}\ ,\\
\frac{d^2h_{1n}(\omega)}{d\beta d\ell} &= -\int_{\underline{\delta}}^{s_b(\omega)} f_\kappa\left(\hat{\kappa}(\omega,\delta)\mid\omega,\delta\right)dF_\delta(\delta\mid\omega)-\int_{s_b(\omega)}^{s_n(\omega)} f_\kappa\left(\hat{\kappa}(\omega,\delta)\mid\omega,\delta\right)dF_\delta(\delta\mid\omega)\\
&= -\psi_1\frac{h_{1b}(\omega)}{b_0-B\left(y_b(\omega)\right)}-\psi_{01}\frac{h_{0b}(\omega)}{b_0}\\
&= -\psi_1\frac{h_{1b}(\omega)}{b_0-B\left(y_b(\omega)\right)}-\pi_{bn}\frac{h_{1n}(\omega)}{y_n(\omega)-T\left(y_n(\omega)\right)}\ ,\\
\frac{d^2h_{1b}(\omega)}{d\beta d\ell} &= \int_{\underline{\delta}}^{s_b(\omega)} f_\kappa\left(\hat{\kappa}(\omega,\delta)\mid\omega,\delta\right)dF_\delta(\delta\mid\omega)\\
&= \psi_1\frac{h_{1b}(\omega)}{b_0-B\left(y_b(\omega)\right)}\ .
\end{aligned}$$

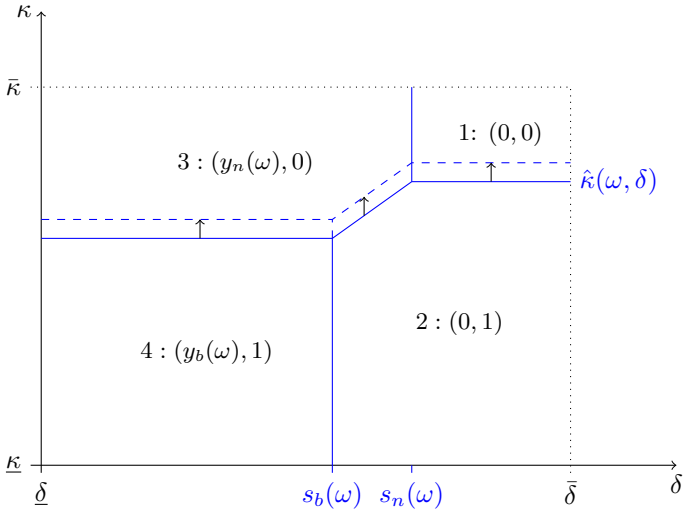
Figure 10 illustrates these effects and shows that $\frac{dh_{0b}(\omega)}{db_0}+\frac{dh_{1b}(\omega)}{db_0}=-\frac{dh_{0n}(\omega)}{db_0}-\frac{dh_{1n}(\omega)}{db_0}>0$.

Consider the subset of agents with skill type ω. The net tax revenue collected from this subset is given by

$$\begin{aligned}
R(0,0,0,\omega) &= f(\omega)\left\{h_{1n}(\omega)\left[T(y_n(\omega))-c_0\right]+h_{1b}(\omega)\left[T(y_b(\omega))+B\left(y_b(\omega)\right)-z_0-b_0\right]-h_{0b}(\omega)(b_0+c_0)-h_{0n}(\omega)z_0\right\}\\
&= f(\omega)\left\{h_{1n}(\omega)T(y_n(\omega))+h_{1b}(\omega)\left[T(y_b(\omega))+B\left(y_b(\omega)\right)-b_0\right]-h_{0b}(\omega)b_0-z_0\right\}\ .
\end{aligned}$$

Note that for agents with $\omega>\hat{\omega}_B$, $h_{1b}(\omega)=0$.

Figure 10: Effects of a small increase in the base transfer b_0 on behavior, skill type $\omega < \hat{\omega}_B$.



Transfer reforms Consider a small transfer cut $(\hat{y}, \beta, \ell)$ with $\hat{y} < y_b(\hat{\omega}_B)$, $\beta \to 0$ and $\ell \to 0$. The skill-specific revenue effects of this reform depend on the relation of ω to $\omega_b(\hat{y})$.

$$\frac{d^2R(\hat{y},\omega)}{d\beta d\ell} = \begin{cases} 0 & \text{if} \quad \omega < \omega_b(\hat{y}) \,, \\ -f(\omega)h_{1b}(\omega)\frac{\varepsilon_b(\omega)}{\mu_b(\omega)}\omega\frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})} & \text{if} \quad \omega = \omega_b(\hat{y}) \,, \\ f(\omega)h_{1b}(\omega)\left\{1-\pi_b(\omega)\frac{T(y_b(\omega))+B(y_b(\omega))}{y_b(\omega)-T(y_b(\omega))-B(y_b(\omega))}\right. & \\ \left. +\psi_1(\omega)\frac{b_0-B(y_b(\omega))-T(y_b(\omega))+T(y_n(\omega))}{b_0-B(y_b(\omega))}\right\} & \text{if} \quad \omega > \omega_b(\hat{y}) \,, \end{cases}$$

Collecting terms across all skill types, $\mathcal{R}_b(y) = \int_{\underline{\omega}}^{\bar{\omega}} \frac{d^2R(\hat{y},\omega)}{d\beta d\ell} d\omega$, the total revenue effects of a small transfer cut are given by

$$\begin{aligned} \mathcal{R}_b(y) \quad = \quad & \int_{\omega_b(\hat{y})}^{\bar{\omega}} f(\omega)h_{1b}(\omega)d\omega \\ & -f(\omega_b(\hat{y}))\ h_{1b}(\omega_b(\hat{y}))\ \omega_b(\hat{y})\frac{\varepsilon_b(\omega_b(\hat{y}))}{\mu_b(\omega_b(\hat{y}))}\frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})} \\ & -\int_{\omega_b(\hat{y})}^{\bar{\omega}} f(\omega)h_{1b}(\omega)\pi_b(\omega)\frac{T(y_b(\omega))+B\left(y_b(\omega)\right)}{y_b(\omega)-T\left(y_b(\omega)\right)-B\left(y_b(\omega)\right)}d\omega \\ & +\int_{\omega_b(\hat{y})}^{\bar{\omega}} f(\omega)h_{1b}(\omega)\psi_1(\omega)\frac{b_0-B(y_b(\omega))-T(y_b(\omega))+T(y_n(\omega))}{b_0-B\left(y_b(\omega)\right)}d\omega \,. \end{aligned}$$

The first line of the right-hand side captures mechanical effects of the transfer cut, while the second line tracks intensive-margin responses. The third line accounts for revenue effects due to transfer recipients leaving the labor market, and the last line accounts for revenue effects due to people who stop to take up transfers.

We can also rewrite this reform in terms of the income distribution, yielding

$$\begin{aligned} \mathcal{R}_b(\hat{y}) \quad = \quad & G_b(\bar{y}_B) - G_b(\hat{y}) - g_b(\hat{y})\ \hat{y}\ \tilde{\varepsilon}_b(\hat{y})\frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})} \\ & -\int_{\hat{y}}^{\bar{y}_B} g_b(y)\tilde{\pi}_b(y)\frac{T(y)+B(y)}{y-T\left(y\right)-B\left(y\right)}dy \\ & +\int_{\hat{y}}^{\bar{y}_B} g_b(y)\tilde{\psi}_1(y)\frac{b_0-B(y)-T(y)+T\left(\tilde{y}_n(y)\right)}{b_0-B\left(y\right)}dy \end{aligned} \tag{42}$$

where $\tilde{y}_n(y) \coloneqq y_n(\omega_b(y))$ is the income that is optimal conditional on no take-up for an agent who previously took up transfers and chose income y. Function $\tilde{y}_n$ depends on the marginal tax T' and

phase-out B' as well as the intensive-margin elasticity. To simplify notation, we write $\tilde{\pi}_b(y) = \pi_b(\omega_b(y))$ and correspondingly for the other elasticies.

Tax reforms Consider a small tax reform $(\hat{y}, \tau, \ell)$ with $\hat{y} < y_b(\omega_b)$, $\tau \to 0$ and $\ell \to 0$. The effects of this reform depend on the relation of ω to $\omega_n(\hat{y})$ and $\omega_b(\hat{y})$, again.

$$\frac{d^2R(\hat{y},\omega)}{d\tau d\ell} = \begin{cases} 0 & \text{if} \quad \omega < \omega_n(\hat{y}) \ , \\ -f(\omega)h_{1n}(\omega)\frac{\varepsilon_n(\omega)}{\mu_n(\omega)}\omega_n(\hat{y})\frac{T'(\hat{y})}{1-T'(\hat{y})} & \text{if} \quad \omega = \omega_n(\hat{y}) \ , \\ +f(\omega)\Big\{h_{1n}(\omega) - h_{1n}(\omega)\pi_{nn}(\omega)\frac{T(y_n(\omega))}{y_n(\omega)-T(y_n(\omega))} - h_{1n}(\omega)\pi_{bn}(\omega)\frac{T(y_n(\omega))+b_0}{y_n(\omega)-T(y_n(\omega))} & \\ \quad -h_{1b}(\omega)\psi_1(\omega)\frac{b_0-B(y_b(\omega))-T(y_b(\omega))+T(y_n(\omega))}{b_0-B(y_b(\omega))}\Big\} & \text{if} \quad \omega \in (\omega_n(\hat{y}),\omega_b(\hat{y})) \ , \\ f(\omega)\left[h_{1n}(\omega) - h_{1b}(\omega)\frac{\varepsilon_b(\omega)}{\mu_b(\omega)}\omega\frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})}\right] & \text{if} \quad \omega = \omega_b(\hat{y}) \ , \\ f(\omega)\Big\{h_{1n}(\omega) + h_{1b}(\omega) - h_{1n}(\omega)\pi_{nn}(\omega)\frac{T(y_n(\omega))}{y_n(\omega)-T(y_n(\omega))} & \\ \quad -h_{1n}(\omega)\pi_{bn}(\omega)\frac{T(y_n(\omega))+b_0}{y_n(\omega)-T(y_n(\omega))} - h_{1b}(\omega)\pi_b(\omega)\frac{T(y_b(\omega))+B(y_b(\omega))}{y_b(\omega)-T(y_b(\omega))-B(y_b(\omega))}\Big\} & \text{if} \quad \omega > \omega_b(\hat{y}) \ , \end{cases}$$

Collecting all terms, the revenue function follows as

$$\begin{aligned} \mathcal{R}_t(\hat{y}) \quad = \quad & \int_{\omega_n(\hat{y}))}^{\bar{\omega}} f(\omega)h_{1n}(\omega)d\omega + \int_{\omega_b(\hat{y}))}^{\bar{\omega}} f(\omega)h_{1b}(\omega)d\omega \\ & -f(\omega_n(\hat{y}))\ h_{1n}(\omega_n(\hat{y}))\frac{\varepsilon_n(\omega_n(\hat{y}))}{\mu_n(\omega_n(\hat{y}))}\omega_n(\hat{y})\frac{T'(\hat{y})}{1-T'(\hat{y})} \\ & -f(\omega_b(\hat{y}))\ h_{1b}(\omega_b(\hat{y}))\frac{\varepsilon_b(\omega_b(\hat{y}))}{\mu_b(\omega_b(\hat{y}))}\omega_b(\hat{y})\frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})} \\ & -\int_{\omega_n(\hat{y})}^{\bar{\omega}} f(\omega)h_{1n}(\omega)\pi_{nn}(\omega)\frac{T(y_n(\omega))}{y_n(\omega)-T\left(y_n(\omega)\right)}d\omega \\ & -\int_{\omega_n(\hat{y})}^{\bar{\omega}} f(\omega)h_{1n}(\omega)\pi_{bn}(\omega)\frac{T(y_n(\omega))+b_0}{y_n(\omega)-T\left(y_n(\omega)\right)}d\omega \\ & -\int_{\omega_b(\hat{y})}^{\bar{\omega}} f(\omega)h_{1b}(\omega)\pi_b(\omega)\frac{T(y_b(\omega))+B(y_b(\omega))}{y_b(\omega)-T\left(y_b(\omega)\right)-B\left(y_b(\omega)\right)}d\omega \\ & -\int_{\omega_n(\hat{y})}^{\omega_b(\hat{y})} f(\omega)h_{1b}(\omega)\psi_1(\omega)\frac{b_0-B(y_b(\omega))-T(y_b(\omega))+T(y_n(\omega))}{b_0-B\left(y_b(\omega)\right)}d\omega \ . \end{aligned}$$

The first line of the right-hand side captures mechanical effects for non-recipients and recipients of transfers. Lines 2 and 3 account for intensive-margin responses; lines 4 to 6 account for extensive-margin responses. Line 7 considers the revenue effects due to people who start taking up transfers with income $y_b(\omega) < \hat{y}$ due to the reform. Note that the latter group are agents with incomes $y_n(\omega) > \hat{y}$ before the reform.

In terms of the income distribution, revenue function $\mathcal{R}_t$ can be rewritten as

$$\begin{aligned} \mathcal{R}_t(\hat{y}) \quad = \quad & 1 - G_n(\hat{y}) - G_b(\hat{y}) \\ & -g_n(\hat{y})\ \tilde{\varepsilon}_n(\hat{y})\ \hat{y}\ \frac{T'(\hat{y})}{1-T'(\hat{y})} - g_b(\hat{y})\ \tilde{\varepsilon}_b(\hat{y})\ \hat{y}\ \frac{T'(\hat{y})+B'(\hat{y})}{1-T'(\hat{y})-B'(\hat{y})} \\ & -\int_{\hat{y}}^{\infty} g_n(y)\ \tilde{\pi}_{nn}(y)\frac{T(y)}{y-T(y)}dy - \int_{\hat{y}}^{\infty} g_n(y)\ \tilde{\pi}_{bn}(y)\frac{T(y)+b_0}{y-T(y)}dy \\ & -\int_{\hat{y}}^{\bar{y}_B} g_b(y)\ \tilde{\pi}_b(y)\frac{T(y)+B(y)}{y-T(y)-B(y)}dy \\ & -\int_{\tilde{y}_n^{-1}(\hat{y})}^{\hat{y}} g_b(y)\ \tilde{\psi}_1(y)\frac{b_0-B(y)-T(y)+T(\tilde{y}_n(y))}{b_0-B(y)}dy \ . \end{aligned} \tag{43}$$

Joint reforms Consider a joint tax reform that involves a small increase in the marginal tax rate by $\tau > 0$ in an interval $(y, y+\ell)$ and a small reduction in the transfer phase-out rate by the same amount

in the same interval. For $\tau \to 0$ and $\ell \to 0$, the effect of this joint reform is given by the joint revenue function $\Delta\mathcal{R} = \mathcal{R}_t - \mathcal{R}_b$. At income level $\hat{y}$, the value of the joint revenue function is given by

$$\begin{aligned}
\Delta\mathcal{R}(y) &= \int_{\omega_n(\hat{y}))}^{\bar{\omega}} f(\omega) h_{1n}(\omega) d\omega \\
&\quad - f(\omega_n(\hat{y}))\ h_{1n}(\omega_n(\hat{y})) \frac{\varepsilon_n(\omega_n(\hat{y}))}{\mu_n(\omega_n(\hat{y}))} \omega_n(\hat{y}) \frac{T'(\hat{y})}{1 - T'(\hat{y})} \\
&\quad - \int_{\omega_n(\hat{y})}^{\bar{\omega}} f(\omega) h_{1n}(\omega) \pi_{nn}(\omega) \frac{T(y_n(\omega))}{y_n(\omega) - T\left(y_n(\omega)\right)} d\omega \\
&\quad - \int_{\omega_n(\hat{y})}^{\bar{\omega}} f(\omega) h_{1n}(\omega) \pi_{bn}(\omega) \frac{T(y_n(\omega)) + b_0}{y_n(\omega) - T\left(y_n(\omega)\right)} d\omega \\
&\quad - \int_{\omega_n(\hat{y})}^{\bar{\omega}} f(\omega) h_{1b}(\omega) \psi_1(\omega) \frac{b_0 - B(y_b(\omega)) - T(y_b(\omega)) + T(y_n(\omega))}{b_0 - B\left(y_b(\omega)\right)} d\omega \ .
\end{aligned}$$

In comparison to the revenue function $\mathcal{R}_t$ for a tax reform, the joint revenue function does neither involve mechanical effects nor intensive- or extensive-margin responses for transfer recipients. For those agents taking up transfers both before and after the reform with incomes above $\hat{y}$, the joint reform does not affect the budget set. By contrast, for non-recipients, it affects the budget set and, hence, leads to mechanical effects (first line), behavioral effects at the intensive margin (second line) and participation effects (third and fourth line). It also makes transfer take-up more attractive and, as a result, yields an increase in the take-up rate (fifth line).

Again, we can as well write this function in terms of the income distribution,

$$\begin{aligned}
\Delta\mathcal{R}(y) &= G_n(\bar{y}) - G_n(\hat{y}) - g_n(\hat{y})\ \tilde{\varepsilon}_n(\hat{y})\ \hat{y}\ \frac{T'(\hat{y})}{1 - T'(\hat{y})} \\
&\quad - \int_{\hat{y}}^{\infty} g_n(y)\ \tilde{\pi}_{nn}(y) \frac{T(y)}{y - T(y)} dy \\
&\quad - \int_{\hat{y}}^{\infty} g_n(y)\ \tilde{\pi}_{bn}(y) \frac{T(y) + b_0}{y - T(y)} dy \\
&\quad - \int_{\tilde{y}_n^{-1}(\hat{y})}^{\bar{y}_B} g_b(y)\ \tilde{\psi}_1(\omega) \frac{b_0 - B(y) - T(y) + T\left(\tilde{y}_n(y)\right)}{b_0 - B(y)} dy \ . \qquad (44)
\end{aligned}$$

Reforms of the base transfer Consider an increase in the base transfer, $b_1 = b_0 + \beta\ell$. The marginal revenue effect is given by

$$\begin{aligned}
-\mathcal{R}_0 = \frac{d^2 R(0,0,0)}{d\beta d\ell} &= - \int_{\underline{\omega}}^{\hat{\omega}_B} f(\omega) \left[h_{0b}(\omega) + h_{1b}(\omega)\right] d\omega \\
&\quad - \int_{\underline{\omega}}^{\bar{\omega}} f(\omega) h_{0b}(\omega) \psi_{00}(\omega) d\omega \\
&\quad - \int_{\underline{\omega}}^{\bar{\omega}} f(\omega) h_{1n}(\omega) \pi_{bn}(\omega) \frac{T\left(y_n(\omega)\right) + b_0}{y_n(\omega) - T\left(y_n(\omega)\right)} d\omega \\
&\quad - \int_{\underline{\omega}}^{\hat{\omega}_B} f(\omega) h_{1b}(\omega) \psi_1(\omega) \frac{b_0 - B\left(y_b(\omega)\right) - T\left(y_b(\omega)\right) + T\left(y_n(\omega)\right)}{b_0 - B\left(y_b(\omega)\right)} d\omega \ ,
\end{aligned}$$

where the first line captures mechanical effects on net revenue, the second line captures take-up responses by people who neither work before nor after the reform, the third line captures responses by people who, due to the reform, leave the labor market and start taking up transfers, and the fourth line captures take-up responses of agents who continue working after the reform.

As before, we can rewrite this function in terms of the income distributions g_n and g_b,

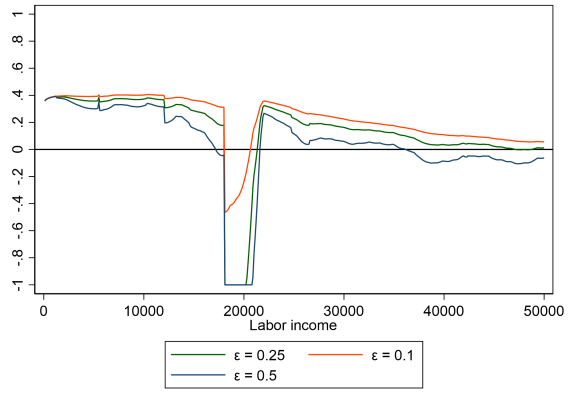
$$\begin{aligned}\mathcal{R}_0 &= G_b\left(\bar{y}_B\right) + G_b(0)\ \bar{\psi}_{00} \\ &\quad + \int_0^{\infty} g_n(y)\ \tilde{\pi}_{bn}(y)\ \frac{T(y)+b_0}{y-T(y)}dy \\ &\quad + \int_0^{\bar{y}_B} g_b(y)\ \tilde{\psi}_1(y)\ \frac{b_0 - B(y) - T(y) + T\left(\tilde{y}_n(y)\right)}{b_0 - B(y)}dy\ , \end{aligned} \tag{45}$$

A.3 Further empirical results

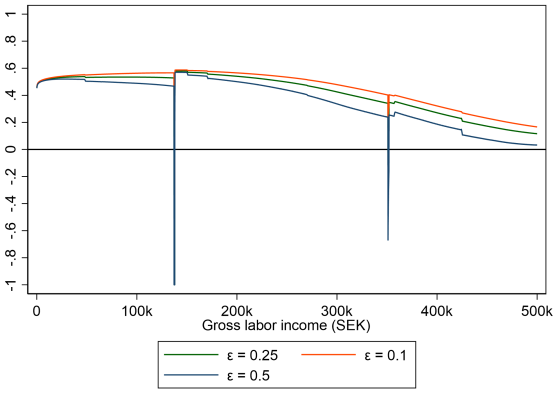
A.3.1 Robustness

Figure 11: Revenue functions by type of reform and choice of ε.

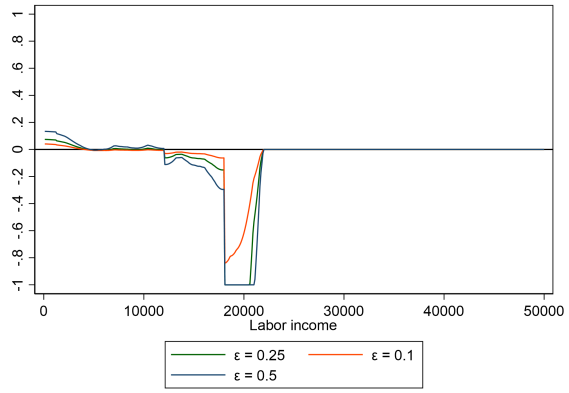
(a) Tax increase, GER 2018



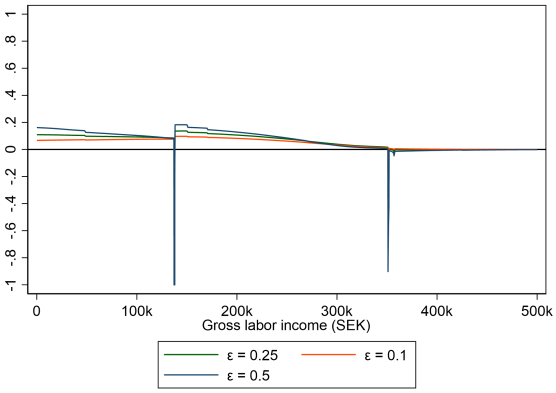
(b) Tax increase, SWE 2023



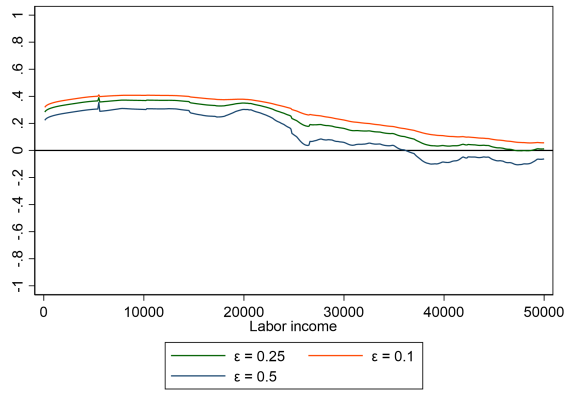
(c) Transfer cut, GER 2018



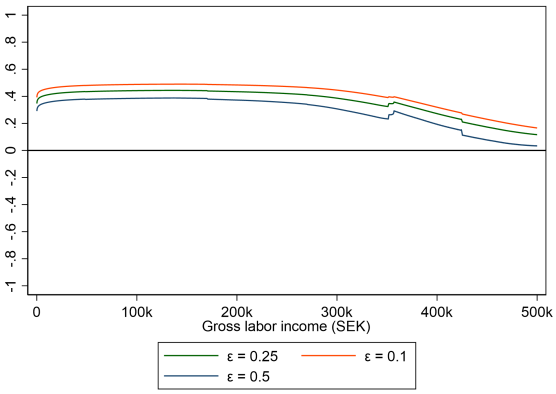
(d) Transfer cut, SWE 2023



(e) Joint reform, GER 2018



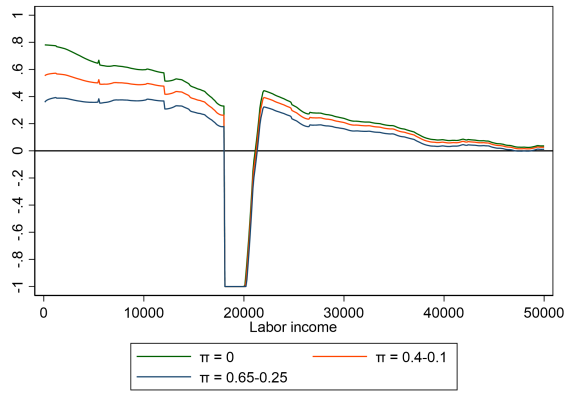
(f) Joint reform, SWE 2023



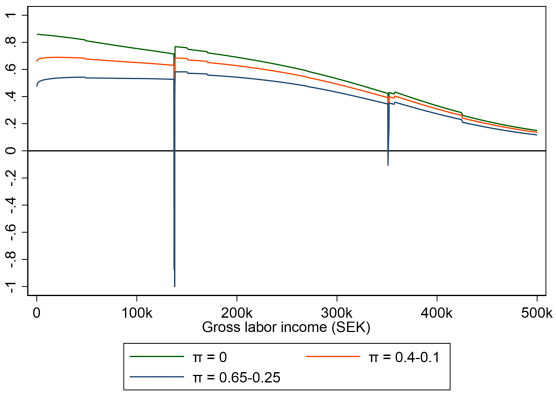
Notes: This graph plots the revenue functions for different types of reforms of the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) faced by single parents with one child: a tax increase (top row), a transfer cut (middle row) and a joint reform (bottom row) based on the calibrations described in Section 7.2 and the formulas derived in Section 5. The red dot pins down the revenue effect of a base transfer reform. The graph is truncated at values of -1 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.

Figure 12: Revenue functions by type of reform and choice of π.

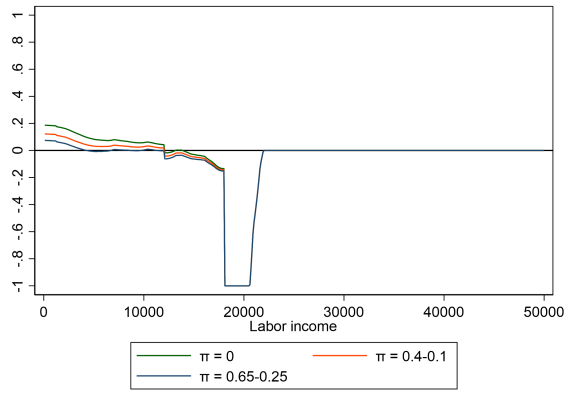
(a) Tax increase, GER 2018



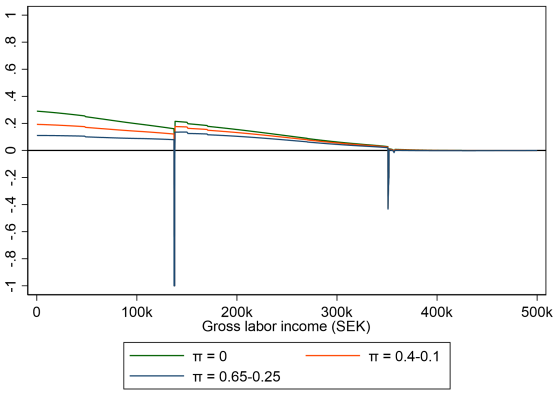
(b) Tax increase, SWE 2023



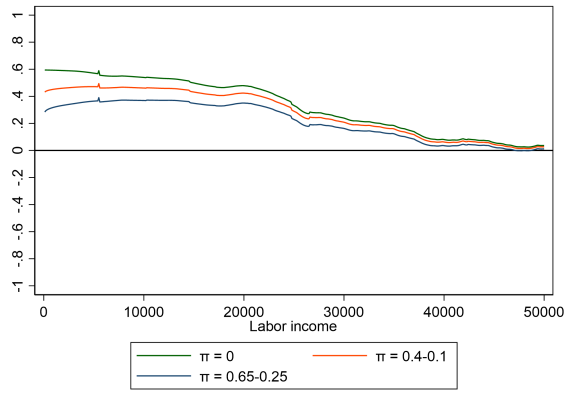
(c) Transfer cut, GER 2018



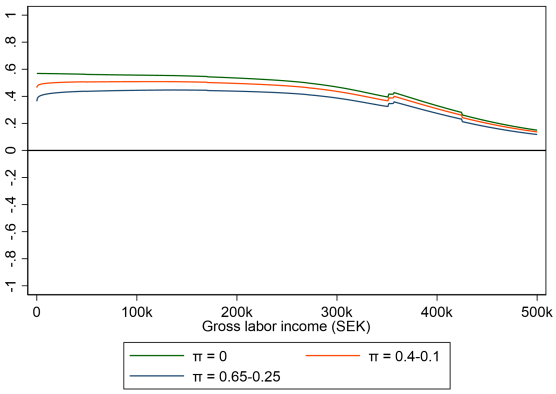
(d) Transfer cut, SWE 2023



(e) Joint reform, GER 2018



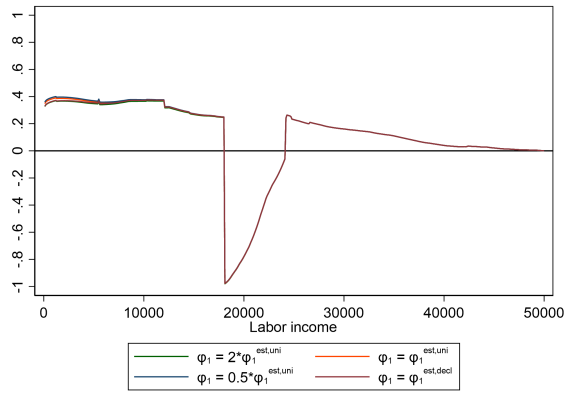
(f) Joint reform, SWE 2023



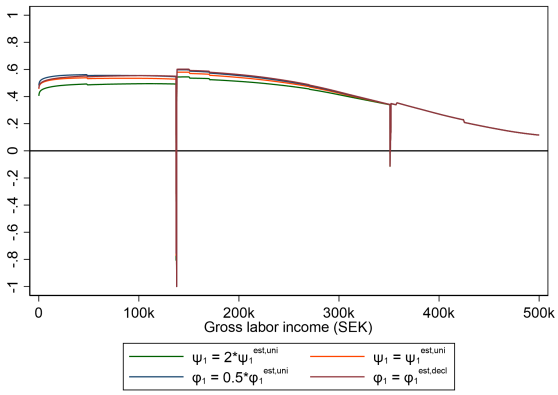
Notes: This graph plots the revenue functions for different types of reforms of the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) faced by single parents with one child: a tax increase (top row), a transfer cut (middle row) and a joint reform (bottom row) based on the calibrations described in Section 7.2 and the formulas derived in Section 5. The red dot pins down the revenue effect of a base transfer reform. The graph is truncated at values of -1 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.

Figure 13: Revenue functions by type of reform and choice of ψ_1.

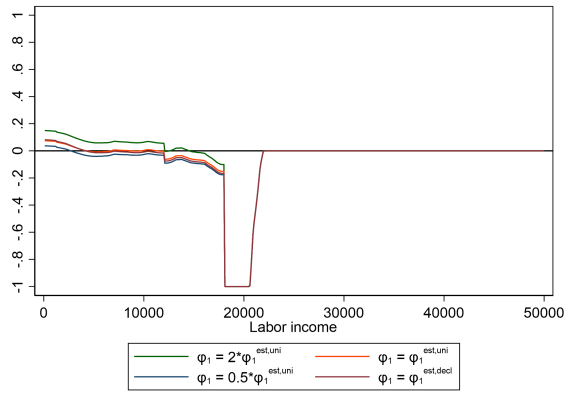
(a) Tax increase, GER 2018



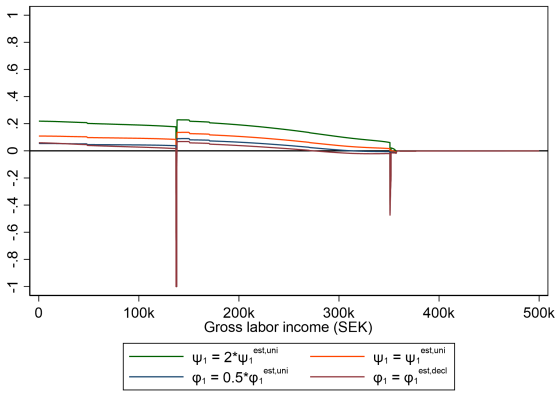
(b) Tax increase, SWE 2023



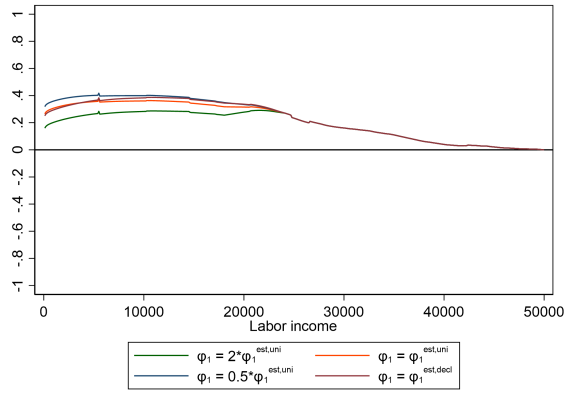
(c) Transfer cut, GER 2018



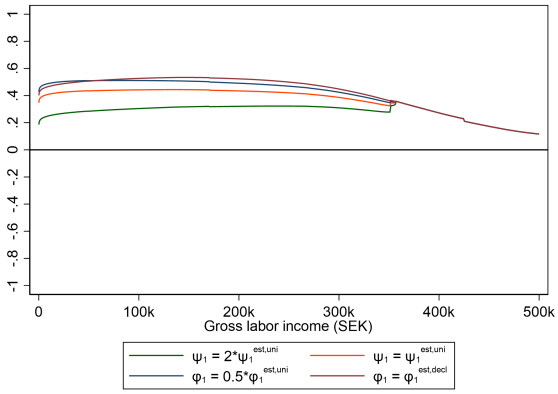
(d) Transfer cut, SWE 2023



(e) Joint reform, GER 2018



(f) Joint reform, SWE 2023

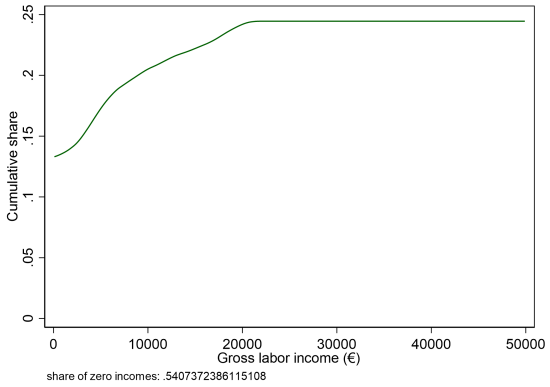


Notes: This graph plots the revenue functions for different types of reforms of the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) faced by single parents with one child: a tax increase (top row), a transfer cut (middle row) and a joint reform (bottom row) based on the calibrations described in Section 7.2 and the formulas derived in Section 5. The red dot pins down the revenue effect of a base transfer reform. The graph is truncated at values of -1 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.

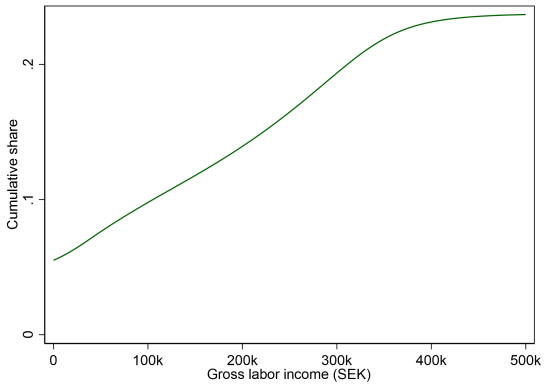
A.3.2 Single parents with one child

Figure 14: CDF estimates for single parents with one child.

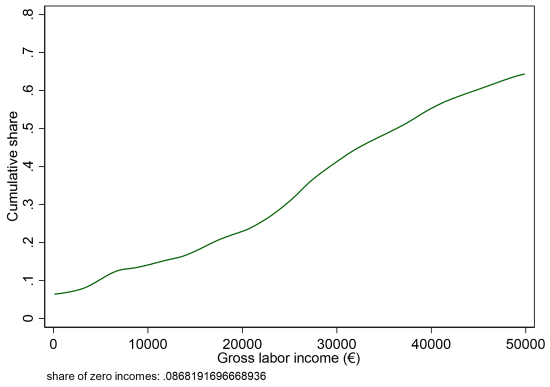
(a) Recipients, GER 2018



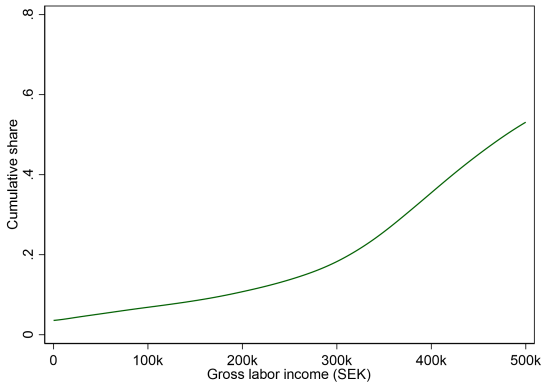
(b) Recipients, SWE 2023



(c) Non-recipients, GER 2018



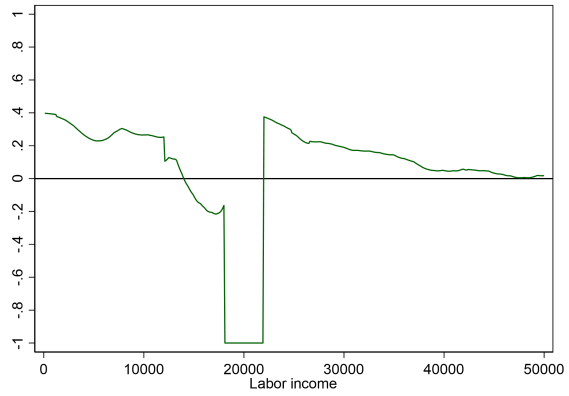
(d) Non-recipients, SWE 2023



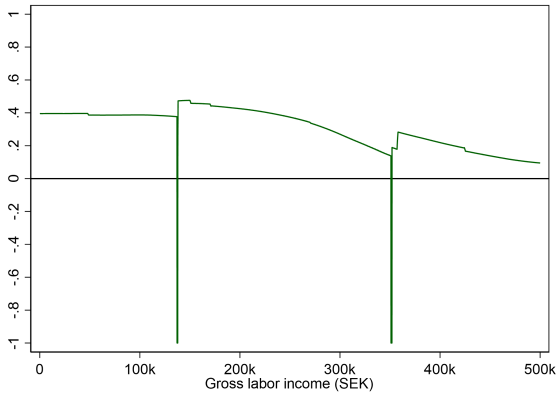
Notes: This graph plots the cumulative distribution function for single-parent households with one child who are (a) welfare recipients or (b) non-recipients against gross labor income. Recipients are defined as taking up any of the perceived welfare transfer programs. The estimates are based on half the algorithm-based adaptive bandwidth (derived from the *akdensity* command in Stata).
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.

Figure 15: Revenue functions $\mathcal{R}_t$ with full vs. incomplete and endogenous take-up.

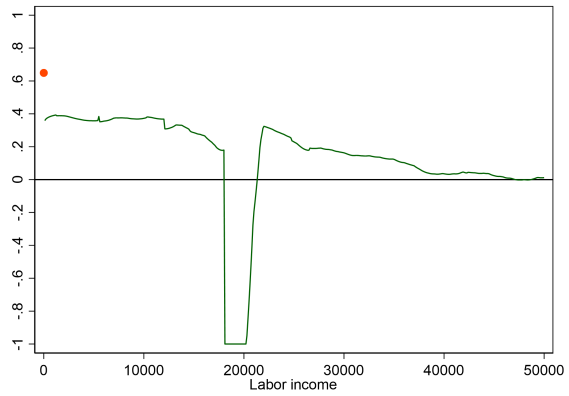
(a) Full take-up, GER 2018



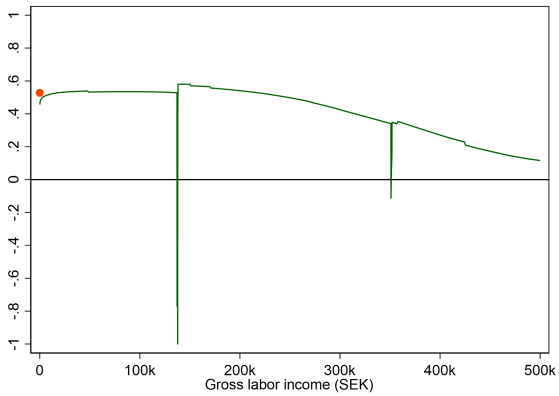
(b) Full take-up, SWE 2023



(c) Incomplete + endogenous take-up, GER 2018



(d) Incomplete + endogenous take-up, SWE 2023



Notes: This graph plots the revenue functions for a tax reform of the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) faced by single parents with one child: under the assumption of full take-up as in Bierbrauer et al. (2023) (top row) and taking into account imperfect and endogenous take-up based on the calibrations described in Section 7.2 and the formulae derived in Section 5 (bottom row). The red dot pins down the revenue effect of a base transfer reform, $\mathcal{R}_0$. The graph is truncated at values of -1 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.

A.3.3 Childless singles

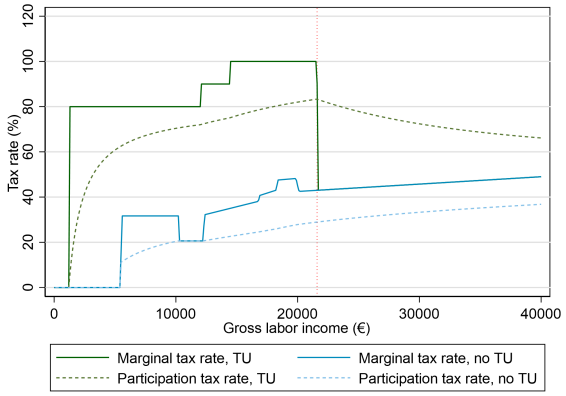
Table 2: Transfer eligibility and transfer take-up by income slice – in % of all childless singles, GER 2018.

	all	>0	(0;phaseout]	0	(0;1000]	(1000;2000]	(2000;3000]	>3000
Any transfer								
Share eligible	32.42	21.62	60.25	71.34	80.29	74.58	8.64	0.00
Share take-up	15.97	7.08	19.73	48.00	32.34	21.44	1.95	0.00
Take-up rate	49.25	32.74	32.74	67.29	40.28	28.75	22.57	.
Social assistance								
Share eligible	15.29	7.46	20.79	43.51	45.57	17.19	0.20	0.00
Share take-up	10.90	3.59	10.00	37.25	22.79	7.70	0.08	0.00
Take-up rate	71.28	48.09	48.09	85.61	50.02	44.77	39.55	.
Housing benefit								
Share eligible	17.13	14.16	39.46	27.83	34.72	57.39	8.44	0.00
Share take-up	5.07	3.49	9.73	10.76	9.55	13.74	1.87	0.00
Take-up rate	29.59	24.65	24.65	38.65	27.51	23.95	22.18	.
Child supplement								
Share eligible	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Share take-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Take-up rate	.	.	.	.	.	.	.	.

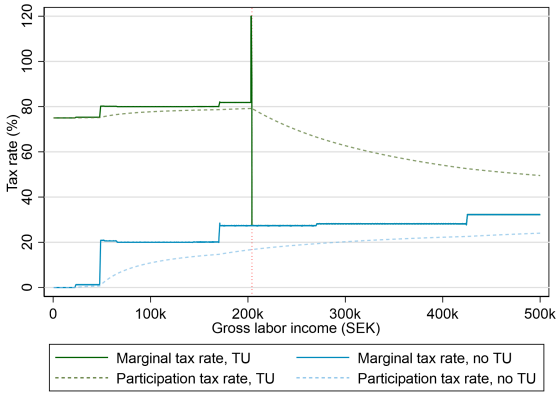
Note: This table displays the share of people in households with eligibility for or take-up of means-tested transfer programs relative to all childless singles households, in %. The different columns refer to different monthly income subsamples of the population of childless single households.
Source: ifo Microsimulation model.

Figure 16: EMTRs and PTRs for childless singles.

(a) GER 2018



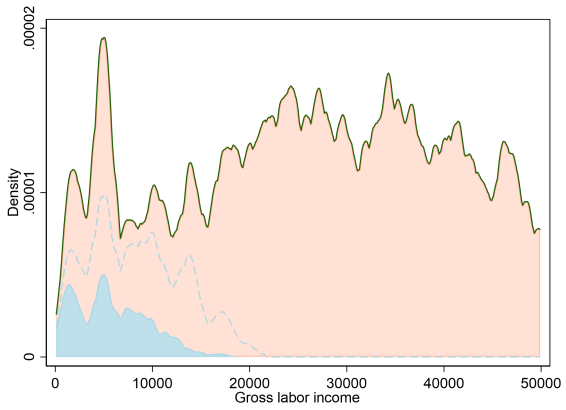
(b) SWE 2023



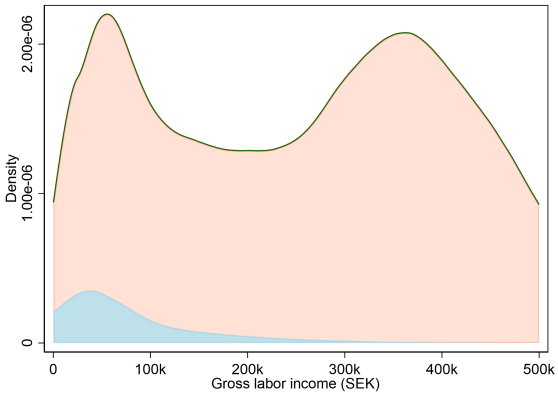
Notes: This graph plots effective marginal tax rates (solid line) and participation tax rates (dashed line) for welfare recipients (green) and non-recipients (blue) against annual labor income for (a) the 2018 German tax-transfer system and (b) the 2023 Swedish tax-transfer sytem.. The vertical dotted line in red shows the point of full phaseout, i.e., the income level at which the household is no longer eligible for any of the perceived welfare transfer programs. The tax rates are based on a hypothetical single household without children living in a region with rent level 5. The recipient household always takes up the transfer program that is more advantageous.
Source: Authors' illustrations based on ifo Microsimulation model.

Figure 17: Density estimates for childless singles.

(a) GER 2018



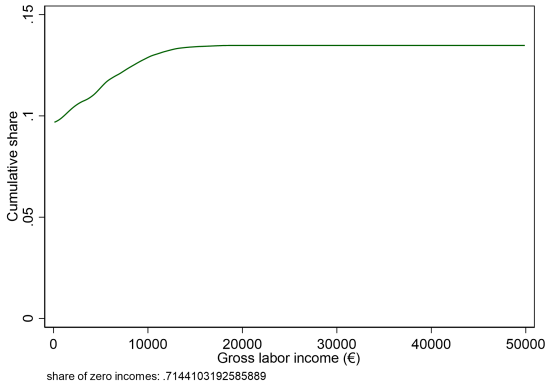
(b) SWE 2023



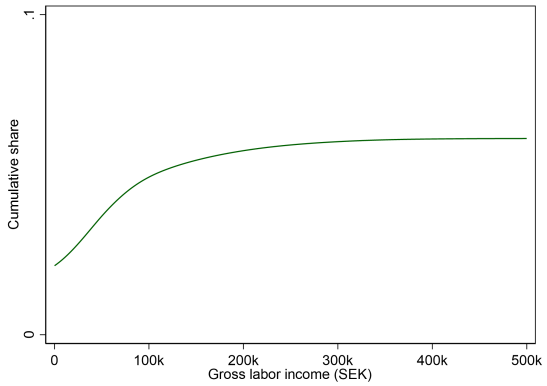
Notes: This graph plots the density for single households without children in Germany, 2018 (a), and in Sweden, 2023 (b), who are welfare recipients (blue area) and non-recipients (red area) against gross labor income. The dotted blue line in the German case shows the mass of households who are eligible for any of the perceived welfare transfer programs. The estimates are based on half the algorithm-based adaptive bandwidth (derived from the *akdensity* command in Stata).
Source: Authors' illustrations based on ifo Microsimulation model and GSOEP data.

Figure 18: CDF estimates for childless singles, 2018.

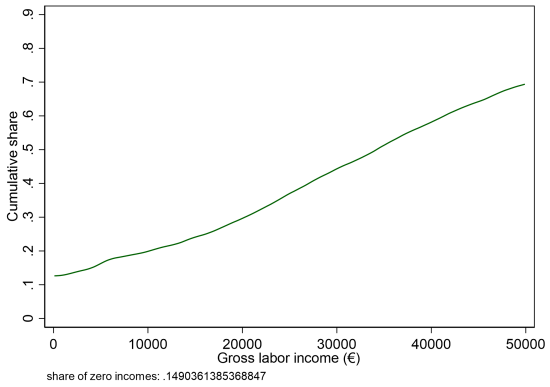
(a) Recipients, GER 2018



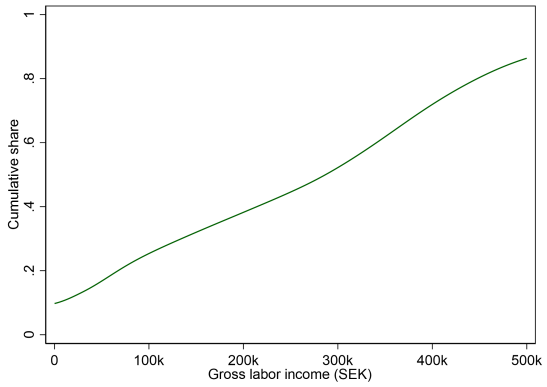
(b) Recipients, SWE 2023



(c) Non-recipients, GER 2018



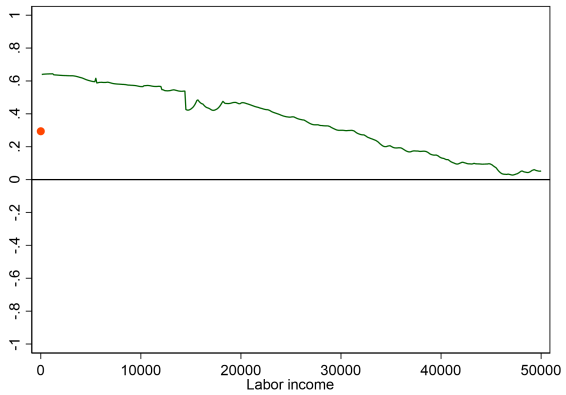
(d) Non-recipients, SWE 2023



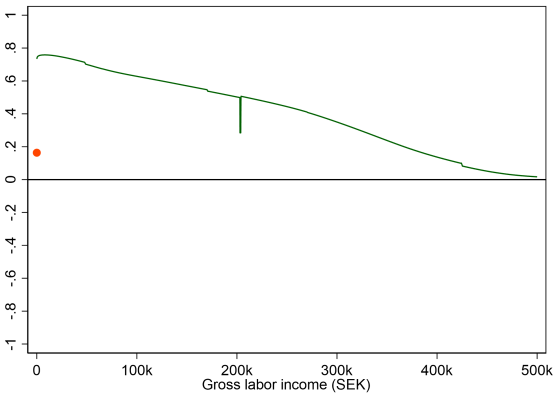
Notes: This graph plots the cumulative distribution function for childless single households with one child in Germany, 2018 (left column), and in Sweden, 2023 (right column), who are welfare recipients (top row) or non-recipients (bottom row) against gross labor income. Recipients are defined as taking up any of the perceived welfare transfer programs. The estimates are based on half the algorithm-based adaptive bandwidth (derived from the *akdensity* command in Stata).
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.

Figure 19: Revenue functions $\mathcal{R}$ for childless singles by type of reform.

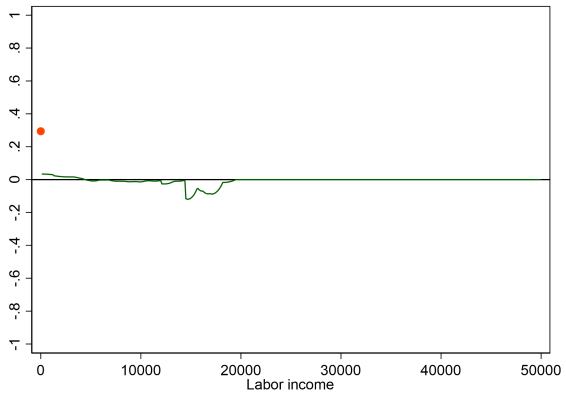
(a) Tax increase, $\mathcal{R}_t$, GER 2018



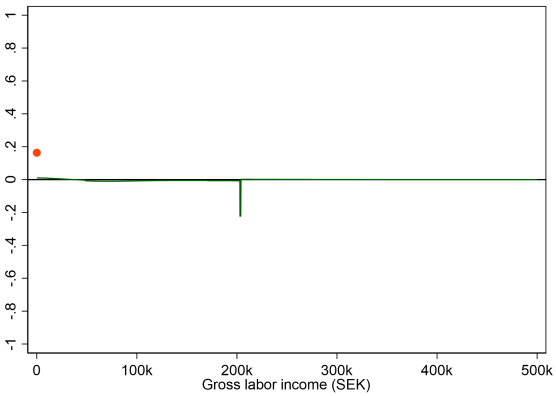
(b) Tax increase, $\mathcal{R}_t$, SWE 2023



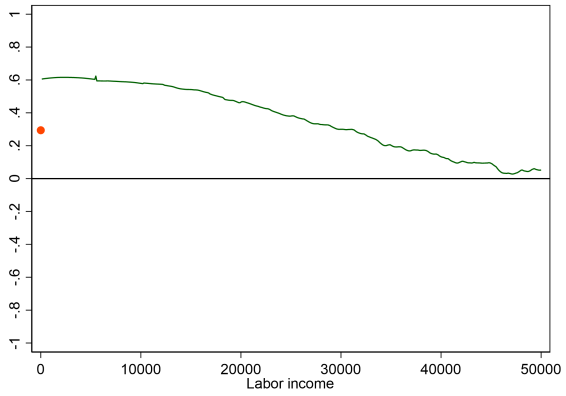
(c) Transfer cut, $\mathcal{R}_b$, GER 2018



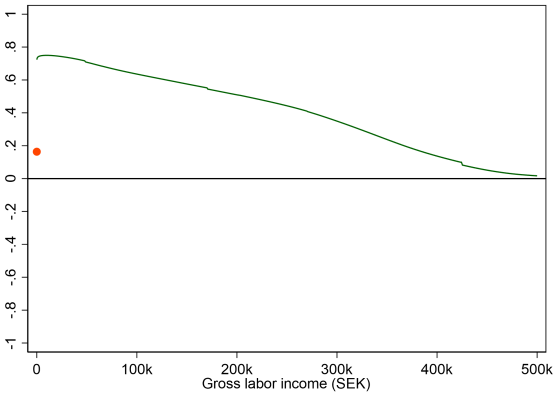
(d) Transfer cut, $\mathcal{R}_b$, SWE 2023



(e) Joint reform, $\Delta\mathcal{R}$, GER 2018



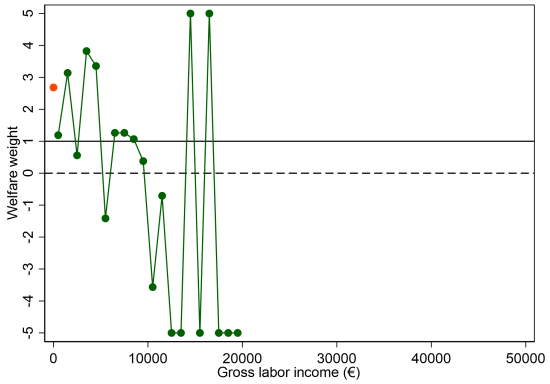
(f) Joint reform, $\Delta\mathcal{R}$, SWE 2023



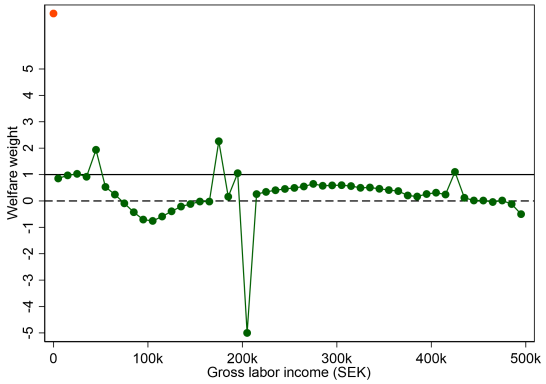
Notes: This graph plots the revenue functions for different types of reforms of the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) faced by childless singles: a tax increase (top row), a transfer cut (middle row) and a joint reform (bottom row) based on the calibrations described in Section 7.2 and the formulae derived in Section 5. The red dot pins down the revenue effect of a base transfer reform, $\mathcal{R}_0$. The graph is truncated at values of -1 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.

Figure 20: Inverse optimum weights for childless singles.

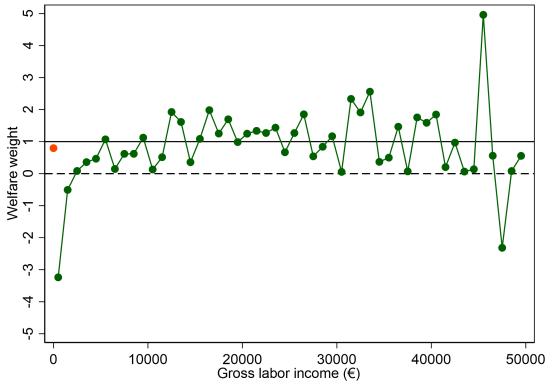
(a) Recipients, GER 2018



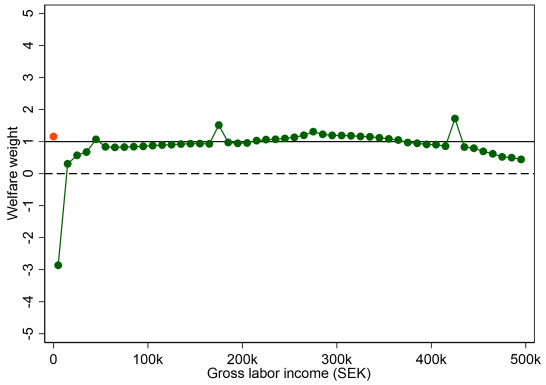
(b) Recipients, SWE 2023



(c) Non-recipients, GER 2018



(d) Non-recipients, SWE 2023



Notes: This graph plots the inverse optimum weights for childless singles facing the 2018 German tax-transfer schedule (left column) and the 2023 Swedish tax-transfer schedule (right column) and distinguishes between individuals taking up welfare transfers (top row) and individuals not taking up welfare transfers (bottom row). The results are based on the calibrations described in Section 7.2 and the formulae for the weights of income bins derived in Section 6. The red dot pins down the inverse optimum weight of households with zero income. The graph is truncated at values of -5 and 5 to allow for better graphical representation.
Source: Authors' calculations based on ifo Microsimulation model and GSOEP data.