Asset Pricing with Dynamic and Static Investors

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ABSTRACT

I analytically characterize a tractable model with dynamic investors, whose asset allocation responds to news, and static investors, whose asset allocation remains stable over time. Static investors buy equities to meet pre-specified policy targets, regardless of news about earnings, creating price pressure. Dynamic investors, though unconstrained, do not regard such price pressure as an arbitrage opportunity. They rationally expect it to persist, because static investors adhere consistently to their strategy. In equilibrium, the aggregate valuation of equity assets depends on the level of wealth managed by static investors, over and above dividends, discount rates, and the wealth flows they receive.

JEL classification: G11, G12, G14, G23.

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1 Introduction

What determines the level of stock prices? Standard asset pricing theories explain stock prices as the outcome of agents' dynamic optimization—e.g., by households (Merton, 1973), firms (Cochrane, 1991), or intermediaries (He and Krishnamurthy, 2013). In the traditional view, stocks are priced at the present discounted value of their expected future dividends by agents engaging in dynamic asset allocation strategies who control their policies in response to news. Gabaix and Koijen (2023) offer a new perspective, by observing that investors adopting static asset allocation strategies keep their capital allocation across asset classes close to target weights that exhibit little to no variation in response to news. In this more recent view, wealth flows to the stock market lead to price pressure because there are no investors engaged in dynamic optimization able to accommodate the resulting flow-induced demand for equity assets.

Financial markets include investors who adopt dynamic asset allocation strategies as well as investors who adhere to static asset allocation strategies.[1] This coexistence suggests that, while both perspectives discussed above capture important forces, neither of the two is complete in isolation. This paper integrates them by developing and solving in closed form an Intertemporal Capital Asset Pricing Model (ICAPM) with heterogeneous agents. *Dynamic Investors*, such as sophisticated households and hedge funds, optimize expected utility from consumption by adjusting their asset allocation upon the arrival of news, as in Merton (1973). *Static Investors*, such as balanced mutual funds and exchange-traded funds (ETFs), are professionally managed institutions that adhere to a pre-specified policy of capital allocation across asset classes, as in Gabaix and Koijen (2023). Static investors receive stochastic wealth flows, which can be partly correlated with news about the economy, that originate from inertial households (Brunnermeier and Nagel, 2008), who buy fund shares and hold them over diverse periods of time in order to avoid continuous market observation.

To motivate the model, Figure 1 shows the share of the U.S. stock market held by 'static funds'—mutual funds and ETFs with stable equity weights over time, as defined in the caption. On average, static funds hold about 15% of the U.S. stock market capitalization, a sizeable but non-exhaustive market share. The figure also illustrates the realized Sharpe ratio on the U.S. stock market. These two time series exhibit a negative comovement over the past two decades. This stylized fact, while descriptive, suggests that static funds tend to hold a larger fraction of the stock market when its risk-adjusted returns deteriorate. The model reproduces this pattern, because static investors keep a stable proportion of capital in equity assets, whereas dynamic investors control their exposure to equity assets to time stock market risk-adjusted returns.

[1] Dynamic asset allocation strategies are observed among hedge funds (Fung and Hsieh, 1997), mutual funds that engage in market timing (Kacperczyk et al., 2014), and households that are wealthy (Giglio et al., 2021) and have higher cognitive ability (Grinblatt et al., 2012). Static asset allocation strategies are characteristic of, though not universal among, institutional investors such as pension funds (Andonov and Rauh, 2022) and balanced mutual funds (Dahlquist and Ibert, 2024), as frequently formalized in their prospectuses. They are also widely recommended by advisors to retail investors (Canner et al., 1997).

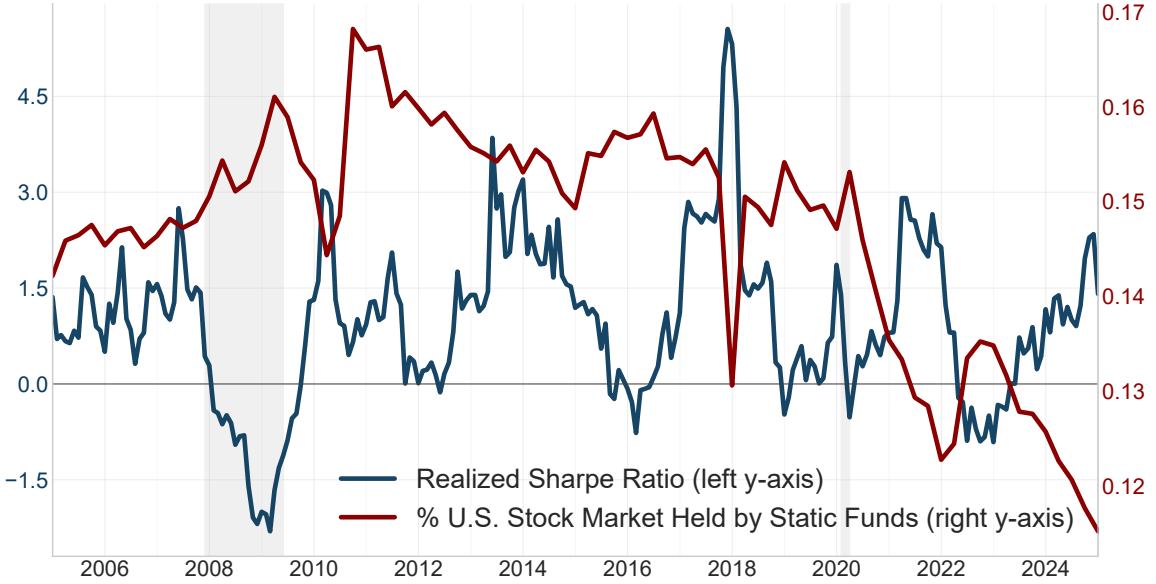


FIGURE 1: **U.S. stock market Sharpe ratio and static funds' ownership share**. This figure displays the realized annual Sharpe ratio of the U.S. stock market (left y-axis) alongside the fraction of U.S. stock market capitalization held by Mutual Funds and ETFs adhering to static asset allocation strategies (right y-axis). Funds are classified as static if their percentage wealth allocated to common equity has a standard deviation below 5%. To avoid look-ahead bias, the standard deviation is computed using only the fund's allocation history available up to each date. The two time series in the figure have a correlation of -0.21 in levels and -0.65 in first differences. Gray shaded areas denote NBER recessions. Source: quarterly 2005-2025 CRSP data. The data cleaning procedure is standard and described in Section 5.

The main contribution of the model is to show theoretically that the aggregate valuation of equity assets depends on the level of wealth managed by static investors, over and above dividends, discount rates, and the wealth flows they receive. The idea runs roughly as follows. Static investors maintain target portfolio exposures across asset classes, regardless of news—e.g., keeping a given fraction of wealth in stocks, regardless of aggregate earnings surprises. Hence, static investors are strongly predictable: they can be expected not to reduce their target equity exposure in bad times, nor to increase it in good times. Static investors implement this policy *consistently*, producing an entire time series of investments that is unrelated to the expectations of future dividends. Anticipating that static investors demand equity regardless of news about future dividends, dynamic investors revise their rational expectations of stock prices. Through this channel, the level of wealth invested under static asset allocation strategies generates aggregate price pressure, even in the presence of unconstrained dynamic investors engaging in intertemporal optimization.

For intuition, consider an investor who commits to allocating a given fraction of wealth to a fixed-supply perpetual asset, regardless of its fundamental value. Other investors may sell into that demand at a higher price than the asset's fundamental value, but cannot rationally expect the deviation to unwind, because that investor keeps demanding the asset persistently over time.

This asset pricing mechanism is consistent with the evidence on the aggregate effect of wealth flows into mutual funds on stock prices (e.g., Edelen and Warner, 2001; Da et al., 2018). However, it also points to wealth *flows* as part of a broader quantity that impacts asset prices: the *level* of wealth invested under static asset allocation strategies, which reflects wealth flows, capital revaluations, and dividend distributions. This distinction is important, because the level of wealth managed by static funds exhibits an upward trend, hovering around $7 trillion in 2025. The net wealth flows they receive, computed as in Lou (2012), are stationary and about two orders of magnitude smaller (see Appendix Table 1). The model suggests that, beyond the price impact of billions of dollars of flows, the stock market features persistent price pressure on the order of trillions. A quantitative assessment follows toward the end of the paper.

Strikingly, the price pressure from static investors is compatible with the presence of unconstrained dynamic investors in asset markets free of borrowing limits and short-sale restrictions. Indeed, the sole departure of the model from the frictionless market is that dynamic and static investors differ in their asset allocation strategy. In anticipation of static investors' pre-specified equity exposure, dynamic investors rationally price the predictable component of their demand. A rational asset price bubble emerges, defined as a difference between the equilibrium price of the equity asset class and the present discounted value of its dividends (Tirole, 1985).[2] The price pressure from static investors is thus compatible with arbitrage, rather than arising from limits to arbitrage common in the inelastic markets literature.

The equilibrium provides a new perspective on how aggregate stock prices respond to capital flows, a quantity labeled as the stock market macro-elasticity in Gabaix and Koijen (2023). Building on their paper, a growing literature estimates how the market responds when an investor sells $1 worth of bonds and buys $1 worth of stocks. Still, this approach treats all flows equally, without considering them as part of a broader intertemporal strategy. The model presented in this paper considers a flow into equities from static investors as part of a stable strategy that shapes rational expectations of future asset demand, generating price pressure. Nonetheless, a flow into equities from dynamic investors that is motivated by news should not be regarded as price pressure. The model suggests that before assessing how the stock market responds to a wealth flow, we should consider the *strategy* behind that investment. Otherwise, we might incorrectly interpret the stock price response to a wealth flow by dynamic investors reacting to earnings news as evidence of inelastic markets, rather than of efficient markets.

Having established that the equilibrium valuation of the equity asset class reflects the wealth managed by static investors, three interrelated questions remain to be addressed. How should dynamic investors account for the market presence of static investors? When does the wealth invested under static asset allocation strategies exert a larger price pressure in the time series? Which security prices in the cross-section are most influenced?

[2]Hence, the equilibrium violates the transversality condition that the discounted asset price converge to zero as the horizon tends to infinity.

Addressing the first question, I find that the presence of static investors induces a tilt in the portfolio choice of dynamic investors. Beyond the myopic component reflecting the contemporaneous trade-off between risk and return, the optimal portfolio of dynamic investors also contains an intertemporal hedging component, proportional to the number of equity shares held by static investors. Intuitively, when static investors become larger owners of the stock market, stock prices become more driven by their wealth rather than by dividends. This makes stocks less attractive as vehicles to finance future consumption, because high ownership from static investors tends to coincide with lower expected returns and greater exposure to price volatility. Thus, as the number of equity shares held by static investors rises, dynamic investors tilt their portfolios away from equity assets in order to better smooth consumption over time.

In the time series, static investors' wealth becomes relatively more important for equilibrium stock prices when expected risk-adjusted returns on stocks are low, at which point dynamic investors shift their portfolio toward the bond market. With dynamic investors taking flight to the safety of bonds, stock prices increasingly reflect static investors' wealth rather than their underlying dividend streams. In equilibrium, the risk-return profile of the equity asset class tracks how closely, on aggregate, stock prices align with the present discounted value of the dividend stream. In other words, static investors' wealth is more important for equilibrium stock prices in bad times than it is in good times.

At the aggregate level, static investors' wealth is a priced state variable. But which securities in the cross-section are most exposed to it? Reliable answers to this question require modeling not only static investors' asset allocation, but also their stock selection. To parsimoniously capture their approach to the cross-section of stocks, I divide static investors into two groups: *Index Trackers* and *Stock Pickers*. Both index trackers and stock pickers adopt static asset allocation strategies—they invest a pre-specified fraction of their capital in equity assets. Index trackers, such as passive balanced mutual funds, continuously replicate the returns of an index composed of a subset of stocks. Stock pickers, such as active balanced mutual funds, select optimally the mix of stocks in their portfolio at each point in time.

Having defined index trackers and stock pickers, I show that when static investors' wealth rises, two groups of stocks experience the largest valuation effects: index stocks, attracting investments from index trackers, and stocks with favorable risk-return profiles, attracting investments from stock pickers. Which of the two groups of stocks exhibits the strongest price responses depends on the relative importance of stock pickers and index trackers. This equilibrium implication adds to recent evidence on the cross-sectional sensitivity of stocks to aggregate mutual funds flows (Dou et al., 2025). A broader message from this theoretical exercise is that, despite optimal stock selection, even stock pickers exert aggregate price pressure by maintaining static exposures to the equity asset class.

At a conceptual level, this paper highlights a simple insight. As is well established, the in-

clusion of a stock in an index impacts its price, because the stock becomes part of the asset demand of investors tracking that index (Harris and Gurel, 1986; Shleifer, 1986). My findings suggest that a similar logic extends across asset classes. Namely, the classification of assets within an asset class impacts their valuation, insofar as static asset allocation strategies specify target exposures to that asset class. The basic idea is intuitive: just as cross-sectional constraints imposed by index-tracking policies affect the prices of stocks included in an index, the constraints imposed by static asset allocation strategies affect the prices of assets classified within target asset classes. For illustration, consider a set of securities forming an asset class, say, stocks. Stocks are part of the investment universe for static investors with target exposures to the equity asset class. In equilibrium, when the level of static investors' wealth or their target equity exposure to the equity asset class rises, the aggregate valuation of the equity asset class rises accordingly, because more capital is allocated to equity assets. This mechanism, which I term the *Asset Classification Effect*, rationalizes the evidence of aggregate price pressure in the stock market (Warther, 1995; Edelen and Warner, 2001; Hartzmark and Solomon, 2025).[3]

I examine the robustness of my theory to various considerations without qualitative changes to my findings. (i) Correlation between wealth flows and earnings news. Investments under static allocation strategies may partly reflect information about fundamentals. For example, as negative earnings news reach the market, static asset allocation funds with a higher exposure to stocks are more likely to experience outflows. However, to the extent that some of the fund shareholders are inertial (see, e.g., Calvet et al., 2009), wealth flows and earnings news are only partly correlated. I show that the results are robust to accounting for this effect. (ii) Preferences of dynamic investors. The results hold both when dynamic investors have constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) utility functions. (iii) Microfounding static investors. I endogenize the adoption of static asset allocation strategies as the outcome of a preference to avoid continuous market observation. (iv) Flexible asset allocation targets. Static asset allocation strategies may flexibly adjust their policy targets to changes in the investment opportunity set (Buffa et al., 2022). Yet, flexible asset allocation targets tend to only partially track changes in the opportunity set, which means there exists a residual degree of rigidity in asset allocation targets that preserves the validity of the results. (v) Survival of agents. Agents' survival and their price impact are two independent concepts (Kogan et al., 2006). In this model, dynamic investors are likely to accumulate wealth at a faster rate than static investors by timing their exposure to the stock market, but this very responsiveness means that they may exit equities at any moment in response to news. By contrast, static investors buy equities regardless of market conditions; because this demand is predictable, it is priced. Their price impact persists because it originates from the level of their wealth, not their wealth share.

[3] Da et al. (2018) show that the reallocation of wealth between investment funds with different static asset allocation targets in a setting plausibly unrelated to information about fundamentals generates price pressure on the aggregate stock market.

The remainder of this paper is organized as follows. Section 2 surveys the related literature. Section 3 presents an intertemporal asset pricing model with dynamic and static investors. Section 4 solves for the equilibrium in the cross-section of stocks. Section 5 examines the empirical relevance of the theory. Section 6 offers concluding remarks.

2 Literature Review

To the best of my knowledge, this paper is the first to show that the level of wealth invested under static asset allocation strategies leads to a rational bubble in the aggregate valuation of the stock market. The resulting price pressure is compatible with the presence of unconstrained investors engaging in dynamic optimization.

The effect of demand forces on asset prices was first explored in the cross-section of stocks by seminal articles on index inclusion (Harris and Gurel, 1986; Shleifer, 1986). Recent literature in this area includes Pavlova and Sikorskaya (2023), Greenwood and Sammon (2025), and Haddad et al. (2025). Relatedly, Boyer (2011) examines price pressure in the context of groups of stocks with the same style label. I contribute to this literature by exploring how price pressure in the cross-section of individual stocks relates to the aggregate demand for the equity asset class, a question that has received limited attention in prior work.

The effect of demand forces on the aggregate value of the stock market was first explored by Warther (1995) and Edelen and Warner (2001). Recent literature in this area includes Da et al. (2018), Parker et al. (2023), and Hartzmark and Solomon (2025). This paper offers a theoretical model consistent with the insights from this empirical literature. Interest in the effect of demand forces on the aggregate value of the stock market has been rekindled by Gabaix and Koijen (2023), who develop a Lucas tree model in which a representative household sector delegates its investments to professional funds facing asset allocation portfolio constraints. In that setup, wealth flows to the stock market give rise to aggregate stock price fluctuations, due to the absence of investors adopting dynamic optimization strategies ready to accommodate the resulting asset demand. This paper departs from this mechanism by showing that the effect of demand forces on the aggregate value of the stock market is consistent with the presence of investors adopting dynamic asset allocation strategies, such as hedge funds and mutual funds engaging in market timing. The heterogeneous-agent perspective presented in this paper also addresses how the presence of investors adopting static asset allocation strategies shapes the portfolio choice implied by dynamic asset allocation strategies.

By recognizing the importance of investors' heterogeneous investment strategies for asset prices, this paper ties in well with the literature on the impact of asset demand on financial markets, a line of inquiry to which Koijen and Yogo (2019) renewed attention through their estimation of asset demand systems. Although the demand system approach to asset pricing has yielded fundamental insights, it has also raised points of debate. For instance, Fuchs et al.

(2023) caution against interpreting estimated demand elasticities as structural parameters, van Binsbergen et al. (2025) point out that the path of future trading prices represents an omitted asset characteristic, and He et al. (2025) emphasize that demand shocks might violate the exclusion restriction in a dynamic economy. By developing an ICAPM where investors follow heterogeneous asset allocation strategies, which specify their plan of action at each point in time, my paper incorporates key insights from the literature on asset demand systems while remaining robust to the aforementioned considerations.

Related literature on asset pricing theory in intertemporal settings and through models with heterogeneous agents includes Veronesi (1999), Basak and Chabakauri (2010), Chabakauri (2013), and Kimball et al. (2020). This literature provides tractable characterizations of intertemporal equilibria primarily in models where all agents engage in dynamic optimization.[4] This paper builds upon Basak and Pavlova (2013), who examine how institutional investors affect asset prices. Their analysis centers on benchmarking within the equity market; mine examines static allocation across asset classes and its implications for aggregate valuations. In Basak and Cuoco (1998), frictions prevent some agents from stock market participation. In this paper, frictions prevent some agents from dynamic portfolio choice, rather than from stock market participation. Agents facing high frictions access equity exposure by delegating to funds that implement static asset allocation strategies.

Notwithstanding the widespread adoption of the static asset allocation strategy, theoretical exploration of its impact on asset prices remains limited. Exceptions include Chien et al. (2011) and Chien et al. (2012), who show that heterogeneous asset allocation strategies help to match asset prices and the distribution of household wealth. However, agents in their frameworks trade state-contingent finite-maturity bonds, which rule out the presence of a rational asset price bubble that is instead central to this paper. Independent work by Duarte et al. (2025) examines the dynamics of the aggregate stock market elasticity within a Lucas economy with non-storable consumption and endogenous risk-free rate populated by active and passive investors. Their findings emphasize that unconstrained arbitrage eliminates price pressure, which arises only by introducing risk misallocation and leverage constraints. However, their analysis relies on a local perturbation around a stationary equilibrium, which abstracts from non-stationary rational bubbles. This paper does not aim to dissect the stock market elasticity; as discussed, it views market elasticity as potentially misleading when abstracted from the strategies through which wealth flows are invested. This paper shows analytically that, above and beyond the market impact of flows, the level of wealth managed by static investors is priced through a rational bubble compatible with no-arbitrage, even absent leverage or short-selling constraints.

[4] A related strand of this literature studies the interaction of newswatchers and trend followers in financial markets (Hong and Stein, 1999; Barberis and Shleifer, 2003; Barberis et al., 2015). Although dynamic investors resemble newswatchers, trend followers form beliefs about future price changes by extrapolating past price changes. In sharp contrast, static investors maintain a pre-specified exposure to the stock market even during time periods when bonds represent more efficient investment opportunities, generating entirely different market outcomes.

Overall, this paper provides a unified framework to examine price pressure from static asset allocation strategies both at the aggregate level and in the cross-section of stocks in an otherwise standard setting free of arbitrage opportunities.

3 The Model

Consider a continuous-time pure-exchange economy with an infinite horizon. The investors have rational expectations and identical information, and are grouped into two categories. Dynamic investors optimally adjust their asset allocation in response to news. Static investors maintain a constant asset allocation, without regard to the revelation of information. The environment is otherwise standard.

3.1 Assets

There is a risk-free bond and a set of risky stocks that distribute stochastic dividends. The risk-free bond is elastically supplied and yields a constant instantaneous real rate of return, r. The set of risky stocks forms the equity asset class, a portfolio that distributes stochastic dividends based on the aggregate earnings of the firms underlying the individual stocks. The ex-dividend price of the equity asset class, P_t, is the sum of the ex-dividend share prices of the individual stocks. Its drift and diffusion are denoted by μ_t and σ_t, respectively. The aggregate earnings of the firms in the economy, denoted by E_t and expressed in real terms, follow a stochastic differential equation with drift m and diffusion ω.

$$dE_t = mdt + \omega dB_t, \tag{1}$$

where B_t is a Brownian motion that generates the filtration $\{\mathcal{F}_t\}$. Equation (1) is motivated by the linear growth of real earnings. The aggregate dividend rate per share, net of rights issues, is proportional to aggregate earnings, $D_t = aE_t$, with the payout ratio normalized to $a = 1$.[5]

3.2 Dynamic Investors

Dynamic investors optimize the utility function $U(c_t) = -\exp(-\gamma c_t)$, where c_t denotes consumption, and δ and γ are respectively patience and risk aversion parameters. Their portfolio choice changes continuously as news reach the market, in the style of Merton (1973).[6]

Dynamic investors control their consumption and investment policies to maximize their derived utility from consumption over an infinite time horizon, subject to their budget constraint and transversality condition. They are *not* subject to any leverage and short-selling constraints.

[5] Rights issues influencing the value of shares without affecting their supply to the public can account for negative dividends.

[6] Chien et al. (2012) examine the asset pricing implications of dynamic but intermittent portfolio choice.

The wealth of dynamic investors is denoted by W_t, and their derived utility from consumption

$$J(W_t, V_t, t) \equiv \max_{\{c_t, X_t\}} \quad \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)} U(c_s)\, ds\right], \tag{2}$$

$$\text{s.t.} \quad dW_t = (rW_t - c_t)dt + X_t dY_t, \qquad \lim_{h\to\infty} \mathbb{E}\Big[J(W_{t+h}, V_{t+h}, t+h)\Big] = 0. \tag{3}$$

In the above, $dY_t = (D_t - rP_t)dt + dP_t$ is the return of a share of the equity asset class financed at the risk-free rate, and X_t is the number of such shares held by dynamic investors. This optimization program is similar to that in Veronesi (1999) and others, with the key distinction that in this paper dynamic investors interact with static investors, described below.

3.3 Static Investors

Static investors allocate their wealth in a fixed proportion across asset classes, without regard to the going and prospective market prices. To implement this asset allocation, a group of agents transfers their wealth into professional investment funds that continuously rebalance their portfolios to maintain target exposures across asset classes. (Appendix A micro-founds the investment approach of these agents through a preference for avoiding continuous market observation). Investment funds following static asset allocation strategies thus experience idiosyncratic wealth flows. Those investment funds are, for expositional simplicity, aggregated into a representative fund whose wealth, V_t, is allocated in a constant proportion, θ, to the risky asset class.[7] Throughout, I assume $\theta \in [0, 1)$, which is economically plausible: pure equity vehicles such as equity ETFs typically invest all their wealth in stocks, whereas balanced mutual funds only allocate a fraction to equities. Static investors collectively hold Q_t shares of the risky asset class, where

$$Q_t = \theta V_t / P_t. \tag{4}$$

Static investors present a downward sloping demand for the equity asset class, reminiscent of the downward sloping demand for individual securities in Harris and Gurel (1986) and Shleifer (1986). For simplicity, it is assumed that static investors reinvest the dividends distributed by equity assets. Their wealth dynamics are

$$dV_t = V_t\big[(1-\theta)r dt + \theta(dP_t + D_t dt)/P_t\big] + \pi dF_t \tag{5}$$

[7] For example, two funds with wealth $V_x = \$100$ and $V_y = \$200$ and static equity allocation $\theta_x = 0.5$ and $\theta_y = 0.75$ aggregate into a representative fund with wealth $V = \$300$ that allocates to equities the average of the allocation of the two funds weighted on their wealth, $\theta = 0.67$. The subsequent wealth flows into each of the funds are scaled equivalently.

where π is a loading parameter and F_t is a Brownian motion under the risk-adjusted measure and adapted to $\{\mathcal{F}_t\}$, which describes wealth flows to the static allocation fund net of share redemptions. Wealth flows are a reduced-form device, motivated by realistic considerations, that capture idiosyncratic retail investors' decisions.[8] The wealth flows are considered uncorrelated with economic fundamentals; this assumption is relaxed in Internet Appendix B. Equation (5) is analogous to Equation (3), except that portfolio shares are constrained by the static asset allocation strategy, θ, and wealth is subject to flow risk, πdF_t.

3.4 Market Clearing

The shares of the equity asset class are in fixed supply, $\overline{S}$, normalized to one without loss of generality.[9] The market clearing condition is

$$X_t(P_t, D_t, V_t) + Q_t(P_t, V_t) = \overline{S}. \tag{6}$$

3.5 Equilibrium

The equilibrium consists of a price, P_t, of the equity asset class such that the supply of stock shares, $\overline{S}$, is equal to their demand, $X_t + Q_t$. Dynamic investors maximize their indirect utility from consumption, given their wealth, corporate earnings, and market prices. Static investors receive stochastic wealth flows and allocate a fixed share θ of their wealth to the equity asset class. The following three equalities characterize the equilibrium.

$$X_t + Q_t = \overline{S}, \qquad 0 = \max_{\{c_t, X_t\}} U(c_t) + \frac{\mathbb{E}_t[dJ(W_t, V_t, t)]}{dt}, \qquad Q_t = \theta V_t / P_t. \tag{7}$$

The first equality imposes market clearing. The second solves the Hamilton-Jacobi-Bellman (HJB) equation for dynamic investors. The third is the static investors' asset allocation strategy.

For illustrative purposes, I begin by considering the equilibrium price of the risky asset class in the absence of static investors. Throughout the paper, $\mathbb{E}_t^*$ denotes conditional expectations taken with respect to the probability measure adjusted for the dynamic investors' preferences toward risk. Optimal policies are instead denoted with a star superscript; e.g., $X_t^\star$.

Lemma 1. Equilibrium without Static Investors.

$$P_t = \underbrace{\mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_s \, ds \right]}_{Fundamentals}.$$

[8] Following extremely negative wealth outflows, V_t could potentially turn negative, an event ruled out by assumption. Its dynamics near this barrier can be disciplined by a term ηdL, where η is the speed of reflection and L the local time of V_t at zero, without changing the economics.

[9] The economic mechanism emphasized in this paper sets it apart from standard models with stochastic supply. There, unlike in this model, assets typically become effectively worthless when, even temporarily, it is inefficient to hold them.

Proof. Special case of Proposition 1. See also Veronesi (1999).

Substituting the dividend process into Lemma 1 yields the price in closed form.

$$P_t = p_g + p_D D_t + p_m m, \tag{8}$$

where $p_g = -\frac{\gamma\omega^2}{r^2}$, $p_D = \frac{1}{r}$, and $p_m = \frac{1}{r^2}$.[10] Lemma 1 describes a standard dividend discount valuation model. The same result can alternatively be obtained by iterating forward the no-arbitrage condition, $P_t = \mathbb{E}_t^*[e^{-rdt}(P_{t+dt} + D_{t+dt})]$, and imposing

$$\lim_{h\to\infty} \mathbb{E}_t^*[e^{-rh} P_{t+h}] = 0. \tag{9}$$

Absent the transversality condition in Equation (9), the stock price would exceed the discounted dividend stream, leading to a rational bubble. A rational bubble is defined as a difference between an asset's price and the risk-adjusted present value of its expected cash flows that does not constitute an arbitrage opportunity. This definition follows the literature on rational expectations in present-value models (Tirole, 1985; Froot and Obstfeld, 1991; Santos and Woodford, 1997). It does not require that the asset's cash flows be replicable, but that it bears the same yield as alternative assets. When the asset's risk-adjusted expected return equals that on alternative assets, the asset price deviation from its fundamental value is merited (Stiglitz, 1990). This condition could be looked at another way. A rational bubble impacts both the current level of the asset price and its risk-adjusted expected future value equally.

In the equilibrium of Lemma 1, dynamic investors ensure that the price equals the discounted dividend stream, so the transversality condition holds. As in Tirole (1982), there are no rational bubbles when all of the agents optimize their actions dynamically. Static investors, by contrast, deviate from dynamic optimization—they *consistently* depart from dynamic asset allocation. The analysis of the equilibrium with dynamic and static investors begins by noting a useful fact.

Remark 1. Static investors' discounted equity exposure is a risk-adjusted martingale. Absent arbitrage opportunities, the risk-adjusted expected return on stocks equals the risk-free rate. Hence, the risk-adjusted conditional expectation of static investors' wealth evolution is

$$\mathbb{E}_t^*[dV_t] = rV_t dt. \tag{10}$$

[10] Throughout the paper, it is assumed that the risk correction is not too large, so that equilibrium prices remain strictly positive after the additive risk adjustment. An analogous restriction is maintained in Veronesi (1999). The opposite case corresponds to dynamic investors being sufficiently risk averse to attach a negative certainty-equivalent value to equity payoffs making them willing to pay to avoid their risk.

Thus, static investors' discounted equity exposure is a risk-adjusted martingale. Namely,

$$\theta V_t = \mathbb{E}_t^* \left[e^{-rdt} \theta V_{t+dt} \right] = \lim_{h \to \infty} \mathbb{E}_t^* [e^{-rh} \theta V_{t+h}]. \tag{11}$$

Investors cannot systematically earn abnormal returns or incur abnormal losses from their equity exposure in arbitrage-free markets. Remark 1 uses this fact to show that the discounted equity exposure of static investors is a risk-adjusted martingale. Hence, accommodating static investors' demand for equity assets by short-selling stocks yields no risk-adjusted expected excess returns. Consequently, any impact of static investors' equity exposure on the level of the equity asset price would neither be arbitraged away nor alter the dividend stream. I now establish that static investors' equity exposure does indeed affect equilibrium asset prices by solving for the equilibrium conditions in Equation (7).

Proposition 1. Equilibrium with Dynamic and Static Investors.

$$P_t = \underbrace{\mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_s \, ds \right]}_{Fundamentals} + \underbrace{\theta V_t}_{Asset\ Classification\ Effect}.$$

Proof. See Appendix B. The appendix also reports the portfolio choice and consumption level that maximize the HJB equation for dynamic investors.

Proposition 1 is the first contribution of the paper. Clearly, the aggregate price of the equity asset class reflects its *Fundamentals*, the risk-adjusted present value of its expected cash flows given the discount rates required by dynamic investors—the marginal agents, whose optimality conditions determine asset prices. Above and beyond its fundamentals, the aggregate price of the equity asset class reflects an *Asset Classification Effect*, corresponding to the equity exposure of static investors—a rational bubble due to an observable characteristic of the economy. Economically, the interpretation of this effect is that the aggregate value of equity assets rises with the level of wealth invested in that asset class under static asset allocation strategies.

Replacing the dividend process implied by Equation (1) into Proposition 1 yields a closed-form representation of the price of the equity asset class.

$$P_t = \text{PDV}_t(D_t) + \theta V_t, \tag{12}$$

where $\text{PDV}_t(D_t) = p_\gamma + p_D D_t + p_m m$ denotes the discounted value of the expected aggregate dividend stream, corrected for the risk adjustment, $p_\gamma = -\gamma(\frac{\omega^2}{r^2} + \theta^2 \pi^2)$, required by dynamic investors for the exposure of the equity asset class to earnings risk and flow risk. This adjustment for risk underscores that the presence of flow risk affects risk premia. The equity exposure of static investors, θV_t, exerts an impact on price of the risky asset class, above and beyond the

effect of the dividends, D_t, the risk-free rate, r, and the adjustment for risk required by dynamic investors, p_γ.[11]

Is static investors' price pressure compatible with arbitrage by dynamic investors? Indeed. Remark 1 shows that the equity exposure of static investors is a risk-adjusted martingale. Hence, unconstrained short-sellers attempting to force the asset price back to its fundamentals would not expect to earn any risk-adjusted returns.[12] The transversality condition offers an alternative lens to address this question.

Remark 2. Transversality Condition. The equilibrium price of the equity asset class in Proposition 1 violates the transversality condition:

$$\lim_{h\to\infty} \mathbb{E}_t^*[e^{-rh}P_{t+h}] = \lim_{h\to\infty} \mathbb{E}_t^*[e^{-rh}\text{PDV}_{t+h}(D_{t+h}) + e^{-rh}\theta V_{t+h}] = \theta V_t. \tag{13}$$

Here, the first equality uses Equation (12) and the second equality uses Remark 1.

Remark 2 shows that rational investors do not expect the market price to eventually converge to the fundamental, due to the infinite maturity of equity assets and the perpetual nature of the expected demand pressure. Dynamic investors are not subject to any portfolio constraint. Yet, maintaining a short position indefinitely without ever realizing any gains is not optimal for them. Static investors' equity exposure is thus compatible with the workings of arbitrage by dynamic investors.[13]

Formally, the equity exposure of static investors, θV_t, constitutes a rational bubble. Having established this, one may wonder whether this bubble is uniquely determined. As is standard in the stochastic control literature, the equilibrium price is obtained by ansatz and verification, which establishes the validity of the solution but does not guarantee its uniqueness. That said, the rational bubble resulting from the equity exposure of static investors has a clear interpretation as an objective feature of the economy. This sets it apart from an extraneous sunspot that has nothing to do with the economy, which Woodford (1987) argues are of questionable plausibility. In Proposition 1, the rational bubble term is uniquely determined by static investors' assets under management and target equity exposure. As such, it can be *measured* using portfolio holdings data. Accordingly, the bubble does not burst due to an extraneous sunspot, but rather if investors adopting static asset allocation strategies (such as balanced mutual funds, pension funds, and sovereign wealth funds) drop their equity allocation targets or are forced into liquidation by fund outflows.

[11] The Stochastic Discount Factor (SDF) is given by $M_t = e^{-\delta t}U'(c_t^\star)$ and evolves as $dM_t/M_t = -rdt - \gamma\omega dB_t - \gamma r\pi dF_t$.

[12] This argument continues to work when dynamic investors are risk-neutral. In this case, expected stock returns coincide with the risk-free rate. The price pressure from static investors drifts at the same rate and does not offer any arbitrage opportunities.

[13] Hugonnier (2012) also notes that portfolio constraints can generate a rational bubble, but their finite-horizon setup is markedly different. There, a bubble arises because prices may diverge further from fundamentals before terminal convergence, exhausting investors' capital. By contrast, in this setting, the bubble arises in infinite horizon from the predictability of static investors' demand.

Is Proposition 1 an artifact of restrictive assumptions? First, investment mandates often allow for limited flexibility (Buffa et al., 2022). Internet Appendix A allows static asset allocation targets to flexibly respond to the revelation of news. Second, wealth flows into investment funds following static asset allocation strategies are likely to be partly correlated with earnings news, as discussed in Internet Appendix B. Third, wealth flows may feature a drift term, for instance reflecting the shift from direct to delegated asset management. Appendix C extends the results to this case. This robustness exercise also makes clear that taking wealth flows to be a Brownian motion under the risk-adjusted measure entails no loss of generality, because a drift can be accommodated. Fourth, some of the static investors may adjust their holdings of individual stocks and potentially engage in market timing. The cross section of stocks is discussed in Section 4. Fifth, the CARA utility of dynamic investors prevents wealth effects. Appendix D extends the results to a case in which dynamic investors have CRRA utility. The asset classification effect remains robust to all of these considerations, suggesting that the economic force underneath it can be attributed to the static asset allocation strategy.

To see this mechanism in stark form, consider a thought experiment. Suppose that bonds are temporarily expected to outperform stocks. Dynamic investors then swiftly reallocate their entire wealth to the bond market. Absent static investors, stocks would be reduced to worthless pieces of paper—this is a serious limitation of the dividend discount model in Lemma 1. Bonds *are* sometimes expected to outperform stocks. For instance, Gómez-Cram (2022) documents that stock returns are predictably lower than the risk-free rate for several months after the onset of recessions. How can stocks have positive prices, when the risk-free asset is expected to outperform them and investors are risk-averse? Proposition 1 offers a way forward: stocks continue to price the abundance of cash invested under static asset allocation strategies.

Does the passage of time affect the relative wealth of dynamic and static investors? Because the portfolio choice of dynamic investors is relatively more efficient, their wealth share can be expected to rise over time. However, agents' survival and their price impact are two independent concepts (Kogan et al., 2006). Dynamic investors may accumulate wealth faster over time, but their asset demand is state contingent and thus cannot be anticipated. Static investors, by contrast, commit to a predetermined asset allocation—given their current wealth, their future demand for equities is predictable. It is precisely this predictability that allows static investors' asset demand to be reflected into current prices. Thus, the asset classification effect depends on the level of static investors' wealth, not on their share of aggregate wealth.

There is considerable evidence that wealth flows to investment funds impact equity asset prices (Coval and Stafford, 2007; Da et al., 2018). Proposition 1 is consistent with this evidence, but embeds wealth *flows* within a broader state variable: the *level* of wealth under management by static investors, whose evolution is described by Equation (5). In doing so, it raises the possibility that price pressure may be much larger than previously thought. Furthermore, it

offers a perspective consistent with the evidence of price pressure originating from dividend distributions (Hartzmark and Solomon, 2025) and portfolio rebalancing (Harvey et al., 2025), discussed in Appendix E.

A broader implication of the equilibrium is that the price impact of a trade depends on the intertemporal investment approach of its initiator. Stock purchases by dynamic investors are state contingent and do not generate any price pressure—they merely impound news about the investment opportunities into the equilibrium price. Stock purchases generate price pressure that cannot be justified by news if and only if they are initiated by static investors. This is because static investors are committed to maintaining a static asset allocation, regardless of the investment opportunities. This observation proposes a new perspective on the literature estimating the aggregate stock market elasticity, namely, the price increase resulting from each dollar of equity purchases (Gabaix and Koijen, 2023). My findings suggest that before assessing how the stock market responds to an investment, we should consider the *strategy* behind that investment.

Having characterized the equilibrium price, I turn to the equilibrium portfolio holdings. In Appendix B, it is shown that

$$X_t^\star(P_t, D_t, V_t) = \frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - (1-\theta)Q_t. \tag{14}$$

Dynamic investors hold a number of stock shares, $X_t^\star$, which increases with the expected returns per unit of variance and decreases with the risk-free rate. Beyond the instantaneous risk–return tradeoff, their optimal portfolio also contains an intertemporal hedging component, proportional to the number of equity shares held by static investors. When static investors become larger owners of the stock, prices become more driven by their wealth rather than by dividends. This makes the stock less attractive as a vehicle to finance future consumption, because high static ownership tends to coincide with lower future expected returns and greater exposure to non-fundamental price swings. Thus, as the number of equity shares held by static investors rises, dynamic investors tilt their portfolios away from equity assets in order to better smooth consumption over time.

Static investors hold a number of stock shares that adheres to their strategy; namely, $Q_t(P, V) = \theta V_t/P_t$. The market clearing condition in Equation (6), together with Equation (12) and the asset allocation strategy of static investors, implies that the equilibrium holdings of dynamic investors can be characterized as follows.

$$X_t^\star(P_t, D_t, V_t) = \frac{\text{PDV}_t(D_t)}{P_t}. \tag{15}$$

Equation (15) shows that, as the present discounted value of the dividend stream increases,

so does the fraction of the risky asset class held by dynamic investors. Thus, in good times, the asset classification effect represents a relatively lower fraction of the risky asset price. All else equal, when the risk-return ratio is higher, the scope for static investors' equity exposure to affect asset prices is more limited, because dynamic investors take comparatively more aggressive positions. The implication is that how closely the price of the risky asset class tracks the aggregate stream of dividends depends on its risk and expected return.[14] For stock prices to accurately reflect future dividends, it is essential that dynamic investors find it attractive to allocate resources to the equity asset class.[15] By contrast, when dynamic investors find it profitable to reduce their equity exposure, the influence of static investors' wealth becomes a relatively more important determinant of equity asset prices. Thus, in bad times, the asset classification effect represents a relatively larger fraction of the price of equity assets.

Overall, this section has demonstrated the presence of the asset classification effect under fairly general conditions. According to this effect, the aggregate market value of equity assets increases with the equity exposure of investors following static asset allocation strategies. In the time series, this effect becomes relatively more important when expected excess returns on stocks are low and volatile.

4 The Cross-Section of Stocks

The model presented thus far, which characterizes the aggregate stock market in the time series, opens the door to an important question: Which stocks exhibit the strongest price responses, when the wealth of static investors rises?

Before delving into this question, I distinguish investors based on their degrees of freedom in portfolio management. Dynamic investors are treated as unconstrained: they engage in both optimal asset allocation and optimal stock selection.[16] Static investors are instead divided into two groups: *Stock pickers*, which select stocks optimally while adopting a static asset allocation strategy; and *Index trackers*, which replicate the performance of an index composed by a subset of stocks and adopt a static asset allocation strategy.

4.1 Assets

There are I firms in the economy. The real earnings, E_{it}, of firm i at time t follow dynamics

$$dE_{it} = m_i dt + \omega_i dB_{it}. \tag{16}$$

[14] Indeed, under the earnings process of Equation (1), $\text{PDV}_t(D_t)$ is the best linear unbiased estimator of the dividend stream. Equation (15) shows that $P_t = \text{PDV}_t(D_t)$ when $X_t = 1$, at which point price forecast accuracy is maximized. More generally, the forecast accuracy of the price of the risky asset class for the aggregate stream of dividends rises when dynamic investors have stronger incentives to allocate resources to the equity asset class, which occurs as the expected return to risk ratio of equity assets rises.

[15] The effort to connect price forecast accuracy to observable market conditions is broadly related to Dávila and Parlatore (2023).

[16] For simplicity, I abstract from cross-sectional portfolio constraints on dynamic investors, treating all of them as unconstrained.

Here, m_i denotes the expected growth of earnings and ω_i their volatility, and B_{it} is a Brownian motion. The pairwise instantaneous correlation between earnings news is $dB_{it}dB_{jt} = \varrho_{ij}dt$. Firms distribute the real dividends per share $D_{it} = aE_{it}$ with a constant payout ratio $a = 1$. Stock shares trade at the real price P_{it}, with price drift and diffusion coefficients denoted by μ_{it} and σ_{it}, respectively. The covariance between the prices of stocks i and j is denoted by σ_{ijt}. To ensure that the real earnings of individual firms aggregate to the total earnings in the economy in Equation (1), impose the restrictions $m = \sum_{i=1}^{I} m_i$ and $\omega = \sqrt{\sum_i \sum_j \omega_i \omega_j \varrho_{ij}}$.

The number of shares outstanding for the i-th stock, $\overline{S}_i$, is constant over time and normalized to unity for every stock. The aggregate valuation of the equity asset class from Section 3 is thus

$$P_t = \sum_{i=1}^{I} P_{it}. \tag{17}$$

There is a stock market index consisting of a collection of $N \leq I$ stocks. The index level is

$$P_t^{\text{IDX}} = \sum_{i=1}^{I} N_i P_{it}. \tag{18}$$

The dummy variable N_i equals 1 if stock i is included in the index and 0 otherwise. As the supply of shares is held constant across stocks, float-adjusted market capitalization index weights coincide with price weights. Thus, each stock has a weight in the index that corresponds to its price relative to the index level and is denoted by $\lambda_{it} = N_i P_{it} / P_t^{\text{IDX}}$.

4.2 Dynamic Investors

The wealth of dynamic investors evolves as in Merton (1973).

$$dW_t = (rW_t - c_t)dt + \sum_{i=1}^{I} X_{it}[(D_{it} - rP_{it})dt + dP_{it}]. \tag{19}$$

Their derived utility function and related transversality condition were specified in Section 3.2. Dynamic investors optimally select the number of shares, X_{it}, in the cross-section of stocks. The aggregate number of shares of risky assets held by dynamic investors is $X_t = \sum_i X_{it}$.

4.3 Static Investors

Static investors allocate a proportion θ of their wealth in the stock market. There are two groups of static investors: *Stock Pickers* (active investors), and *Index Trackers* (passive investors). The wealth of static investors is the sum of the wealth of stock pickers and index trackers. Namely, $V_t = V_t^A + V_t^P$, where V_t^A and V_t^P denote the wealth of stock pickers and index trackers,

respectively. Aggregate wealth flows to static investors, dF_t, are split in two constant fractions: stock pickers receive π^A, and index trackers $\pi^P = \pi - \pi^A$. The number of shares of the i-th stock held by static investors is denoted by $Q_{it} = Q_{it}^A + Q_{it}^P$, where Q_{it}^A and Q_{it}^P denote the number of shares held by stock pickers and index trackers, respectively.

4.3.1 Stock Pickers

Stock pickers adopting a static asset allocation strategy have target allocation θ to equity assets. Their wealth evolves as follows.

$$dV_t^A = rV_t^A(1-\theta)dt + \sum_{i=1}^{I} Q_{it}^A \left(dP_{it} + D_{it}dt\right) + \pi^A dF_t. \tag{20}$$

Equation (20) makes clear that stock pickers do not regard the risk-free rate as the opportunity cost of their individual stock positions, as their allocation to equity assets is predetermined. Stock pickers select their cross-sectional stock holdings, Q_{it}, as a solution to the constrained mean-variance portfolio optimization problem:

$$\max_{\{Q_{it}\}} \mathbb{E}_t[dV_t^A] - 0.5\gamma\mathbb{E}_t[(dV_t^A)^2] \quad \text{s.t.} \quad \sum_{i=1}^{I} P_{it}Q_{it}^A = \theta V_t^A. \tag{21}$$

In Equation (21), the parameter capturing the degree of risk aversion of stock pickers, γ, is the same risk-aversion parameter used for dynamic investors.[17] Hence, stock pickers adopting a static asset allocation strategy differ from dynamic investors solely because the proportion of their wealth invested in stocks, θV_t^A, is fixed rather than sensitive to news. The optimal number of shares of the i-th stock held by stock pickers is denoted by $Q_{it}^{A^\star}$. The associated equity-sleeve weight is denoted by $q_{it}^\star$ and results from the accounting identity $Q_{it}^{A^\star} = q_{it}^\star \theta V_t^A / P_{it}$.

4.3.2 Index Trackers

Index trackers adopting a static asset allocation strategy have target allocation θ to equity assets. The number of shares of the i-th stock held by index trackers is proportional to the index weight of that stock, λ_{it}, and thus equal to $Q_{it}^P = \lambda_{it}\theta V_t^P / P_{it}$. Their wealth evolves as follows.

$$dV_t^P = rV_t^P(1-\theta)dt + \sum_{i=1}^{I} Q_{it}^P \left(dP_{it} + D_{it}dt\right) + \pi^P dF_t. \tag{22}$$

[17] This choice is a parsimonious normalization that aims to level the playing field between the preferences toward risk of dynamic investors and static stock pickers, so that differences in the portfolio choices of these agents can be attributed solely to their asset allocation strategies.

4.4 Market Clearing

There are I market clearing conditions, one for each individual stock. For the i-th stock,

$$\underset{\downarrow}{\text{Dynamic Investors}} \qquad \underset{\downarrow}{\text{Stock Pickers}} \qquad \underset{\downarrow}{\text{Index Trackers}}$$
$$X_{it} \quad + \quad Q_{it}^{A} \quad + \quad Q_{it}^{P} \quad = \quad \overline{S}_i. \tag{23}$$

The next proposition characterizes the cross-section of equilibrium stock prices by imposing the market clearing conditions and solving for the stock prices that equate agents' asset demands.

Proposition 2. The cross-section of Equilibrium Stock Prices.

$$P_{it} = \underbrace{\mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_{is} \, ds \right]}_{Fundamentals} + \underbrace{q_{it}^\star \theta V_t^A \quad + \quad \overbrace{\lambda_{it} \theta V_t^P}^{Index\ Inclusion}.}_{Asset\ Classification}$$

Proof. See Appendix F. The appendix also reports the optimal portfolio and consumption choices maximizing dynamic investors' HJB and solves stock pickers' mean-variance problem.

Proposition 2 is the second contribution of the paper. Clearly, the price of each stock reflects its *Fundamentals*, the expected cash flows and the discount rates required by dynamic investors—which adjust for the correlations with other stocks and flow risk, as discussed later. Above and beyond its fundamentals, the price of each stock reflects its *Asset Classification*, and the resulting price pressure from static investors—both stock pickers and index trackers. The importance of this effect for the price of each stock corresponds to its weight in the portfolio of static investors. Static investors hold two categories of stocks: (i) index stocks, attracting investments from index trackers; and (ii) stocks with favorable risk-return profiles, attracting investments from stock pickers.

There is little doubt that stocks included in an index tend to trade at high valuations relative to their fundamentals due to the price pressure from index trackers, since the seminal papers of Harris and Gurel (1986) and Shleifer (1986). In my model, the price pressure from *Index Inclusion* is due to the tracking constraint, λ_{it}, preventing index trackers from stock selection in the cross-section. Index trackers adhere to a policy of passively investing in each stock in proportion to its index weight, leading index stocks to trade at high valuations relative to non-index stocks with an equivalent dividend stream.

Proposition 2 finds that a similar constraint on portfolio choice impacts the price of a security due to its classification within the equity asset class. The price pressure from *Asset Classification* is due to the static asset allocation constraint, θ, preventing static investors from adjusting their exposure to equity assets over time. Static investors adhere to a policy of allocating a given fraction of their wealth in the cross section of equity assets, leading them to trade at high

valuations relative to their dividend stream (from the unconstrained perspective of dynamic investors). The asset classification encompasses all price pressure from static asset allocation strategies, of which the index inclusion is a special case applying to index stocks. Yet, the literature has only begun to characterize this effect.

Appendix F further shows that the i-th stock price admits the following representation:[18]

$$P_{it} = \text{PDV}_{it}(D_{it}) + p_{F_{it}} + \theta\left[q^{\star}_{it} V^A_t + \lambda_{it} V^P_t\right], \tag{24}$$

where $\text{PDV}_{it}(D_{it}) = p_{\gamma_i} + p_D D_{it} + p_m m_i$ denotes the present discounted value of the dividend stream of the i-th stock net of the risk adjustment $p_{\gamma_i} = -\frac{\gamma}{r^2}\sum_j \omega_i \omega_j \varrho_{ij}$. Each stock has a heterogeneous exposure to the risk of aggregate fund flows, which increases with its weight in the portfolio of static investors (Dou et al., 2025), and $p_{F_{it}} = -\frac{\gamma\theta}{r^2}(\pi^A q^{\star}_{it} + \pi^P \lambda_{it})$ is the corresponding risk adjustment. Together, p_{γ_i} and $p_{F_{it}}$ adjust the price of a stock for the variance of its dividends, its correlation with other stocks' dividends, and its exposure to aggregate flow risk.

The i-th stock's price thus exceeds the risk-adjusted present value of its dividends. This does not constitute an arbitrage opportunity, because the i-th stock is priced consistently with other assets. This remains true even if the cross-sectional weights $q^{\star}_{it}$ and λ_{it} are time varying, because investors' optimal portfolio choices ensure that each stock's risk return profile is equalized in the cross-section. Individual stocks contribute to the aggregate asset classification effect in proportion to their weight in the portfolio of static investors. This can be seen from the aggregation condition in Equation (17). Indeed,

$$\sum_i \left[\text{PDV}_{it}(D_{it}) + p_{F_{it}} + \theta\left[q^{\star}_{it} V^A_t + \lambda_{it} V^P_t\right]\right] = \text{PDV}_t(D_t) + \theta V_t. \tag{25}$$

Contrary to conventional wisdom, both groups of static investors exert pressure on stock prices. *Index trackers* are understood to do so, as they hold the index constituents in fixed proportions, regardless of the returns offered by non-index stocks or by the risk-free asset. *Stock pickers* also exert price pressure, even with their active approach to portfolio management, because they operate under a static asset allocation strategy—this may be surprising. Stock pickers may excel at identifying a relative mispricing in the cross-section of stocks, and they do use their stock selection to engage in market timing. However, they also maintain a static wealth allocation to stocks, without regarding the risk-free asset as an alternative investment opportunity. Relative to an unconstrained dynamic investor who would scale down equity in bad times, the static allocation constraint compels stock pickers hold too much equity—and

[18] The expression for $q^{\star}_{it}$ is provided in Equation (F.15). The right-hand sides of Proposition 2 and Equation (24) themselves depend on the equilibrium price through λ_{it}. Thus, these equations are fixed points rather than closed-form functions of the primitives. Nonetheless, they establish that an equilibrium exists and illustrate clearly that static investors induce a stock's price to exceed its risk-adjusted dividend stream.

thus, excessively large positions in their best stock ideas.

This is clear from investors' optimal portfolio choices. In Appendix F, it is shown that the portfolio choice maximizing the HJB for dynamic investors is

$$X_{it}^{\star} = \frac{\mu_{it} - rP_{it} + D_{it}}{r\gamma\sigma_{it}^2} - \frac{\sum_j X_{jt}^{\star}\sigma_{ijt}}{\sigma_{it}^2} - \frac{(1-\theta)P_{it}Q_t\sigma_t^2}{\sigma_{it}^2}. \tag{26}$$

The number of shares maximizing stock pickers' constrained mean-variance problem is

$$Q_{it}^{A\star} = \frac{\mu_{it} - \xi_t P_{it} + D_{it}}{\gamma\sigma_{it}^2} - \sum_{j\neq i} \frac{Q_{jt}^{A\star}\sigma_{ijt}}{\sigma_{it}^2}, \tag{27}$$

where ξ_t is the Lagrange multiplier on stock pickers' asset allocation constraint ensuring that

$$\sum_{i=1}^{I} Q_{it}^{A\star} P_{it} = \theta V_t^A. \tag{28}$$

Stock pickers may take short positions in some stocks. However, the constraint in Equation (28) is binding, because stock pickers are static investors. Increasing Q_{it}^A by one share necessarily forces an offsetting reduction in other equity holdings to satisfy the asset allocation target. Stock pickers thus regard ξ_t as the shadow price of investing in a stock. Economically, this corresponds to the marginal change in their mean–variance objective from relaxing Equation (28) by one unit of wealth. Even with a fixed equity allocation, stock pickers can time the market factor by forming long–short positions that reduce their portfolio's exposure to aggregate equity risk, while remaining net long equities in value.

Comparing Equations (26) and (27) shows that the portfolio choice of dynamic investors differs from that of stock pickers in two regards: (i) Dynamic investors compare the investment in each stock against the opportunity cost rP_{it}. Relatedly, dynamic investors' mean–variance demand is divided by r, reflecting their flexibility to invest in the risk-free asset. Stock pickers instead face a purely cross-sectional problem, and compare the investment in each stock against the opportunity cost $\xi_t P_{it}$.[19] (ii) Dynamic investors present a hedging demand that is proportional to the aggregate number of shares of static investors, Q_t. These aspects are sources of inefficiency. By adopting static asset allocation strategies, stock pickers end up exerting price pressure precisely on stocks they optimally select, leading these stocks to be priced coherently with other equity assets, but not relative to the risk-free asset.

The analysis of the cross section is kept simple to yield analytical tractability, but the key economic forces identified here hold more broadly. Stock pickers could care about their per-

[19]The Lagrange multiplier ξ_t generally differs from the risk-free rate r. When the target equity exposure forces stock pickers to hold more stocks than they optimally would, $\xi_t > r$. Conversely, when it forces them to hold fewer stocks than they optimally would, $\xi_t < r$. Equality $\xi_t = r$ obtains only when the target equity exposure corresponds to the unconstrained optimum.

formance relative to the index (Basak and Pavlova, 2013). They could also engage in style investing (Boyer, 2011), generating price pressure on a groups of stocks with those style label. There could be several indexes (Pavlova and Sikorskaya, 2023). The fraction of the aggregate flows directed to passive investors could feature a time trend due to the rise in passive investment (Jiang et al., 2025). These extensions would be natural and, I expect, would preserve the core findings.

This section has examined the relationship between stock-level price pressure and aggregate price pressure at the equity asset-class level. In equilibrium, aggregate price pressure has a stronger impact on index constituents, which attract the investment of index trackers, and on stocks with high and stable expected returns, which attract the investment of stock pickers.

5 Empirical Relevance

This section examines the empirical relevance of the model presented in this paper. Except for the presence of static investors, the model is standard. The empirical counterpart of static investors are institutional investment funds operating under static asset allocation strategies. Because these investors are relatively easy to measure, I focus on mutual funds and ETFs. Other institutional investors, such as pension funds and insurance companies, may also follow static allocation strategies but are harder to observe with comparable granularity. As a result, my estimates of the importance of static investors are likely to be conservative.

Static asset allocation strategies can mitigate the agency costs of delegated portfolio management (He and Xiong, 2013). However, they constrain investors from adjusting their asset allocation over time. By contrast, the performance of the stock market relative to the risk-free rate varies over time, and this variation is at least partly *ex-ante* predictable. Dynamic asset allocation strategies can thus generate large alphas by taking less risk when volatility is high (Moreira and Muir, 2017) and after the onset of recessions (Gómez-Cram, 2022).[20] Hence, static investors depart from dynamic optimization.

The data on mutual funds and ETFs are drawn from the CRSP Survivor-Bias-Free Mutual Fund Database Summary Files.[21] As this study focuses on the U.S. stock market, funds with CRSP objective codes beginning with "O" (Other), "I" (Fixed Income), "MT" (Mixed Target), or "EF" (Equity Foreign) are excluded.[22] The sample is further restricted to funds flagged as open to investment. To mitigate reporting errors, observations with Total Net Assets (TNA) below $5 million are excluded. CRSP reports fund data at the share-class level. Following standard practices, share-class data are aggregated into data on unique portfolios identified by

[20] The normative literature on dynamic portfolio choice is extensive (see, e.g., Campbell and Viceira, 1999; Brandt and Santa-Clara, 2006).

[21] CRSP Summary Files are not affected by the data quality problems documented by Schwarz and Potter (2016), which primarily pertain to CRSP Holdings Files prior to 2008.

[22] Target-date funds are excluded from the sample for consistency with the theoretical model, which does not explicitly account for deterministic glide paths in the proportion of the portfolio invested in stocks. This exclusion is again conservative.

the crsp_portno variable, available from 2004 onwards. Following Lou (2012), fund-level variables, such as the fraction of the portfolio allocated to common stocks, are computed as the TNA-weighted average of the underlying share classes. Quality screens are applied to mitigate reporting errors: observations are excluded from the sample if the sum of fixed income and equity allocations exceeds 110%, or if the total reported fraction of the portfolio allocated across asset classes falls outside the [75%, 125%] range. The resulting sample comprises 431,079 fund-quarter observations covering the period 2004:Q1–2024:Q4.

Funds are then classified based on the stability of their asset allocation strategies. To this end, the standard deviation of the allocation of each fund to common stocks (per_com) is computed. The computation uses only the fund's allocation history available up to that quarter; subsequent quarters are excluded to avoid look-ahead bias. To ensure reliable estimation and avoid incubation bias, the standard deviation is computed only when at least eight prior quarters of observations are available. As mentioned in the introduction, mutual funds and ETFs are classified as 'static' if the standard deviation of their asset allocation is less than or equal to 5%. At this threshold, equity weights exhibit very limited time-series variation, rather than being state-contingent, consistent with the static asset allocation strategy modeled in the paper. This classification identifies 6,789 unique funds below the threshold, which are classified as static funds, and 11,893 funds above the threshold, which display meaningful time-series variation in their equity allocations.

The sample is then restricted to static funds. Wealth flows to these funds are computed by comparing the change in each fund's TNA with the change implied by fund returns and mergers; any residual is attributed to net flows. This procedure closely follows Lou (2012). For each quarter, the net wealth flows to static funds, their TNA, and their equity exposure, are then aggregated across funds. The share of the U.S. stock market held by static investors is computed as the ratio of static funds' aggregate U.S. equity exposure to total U.S. stock market capitalization, measured using CRSP. Table 1 reports summary statistics for the sample of static funds. The data suggest that wealth flows are stationary, while the TNA of static funds exhibits an upward trend, consistent with the rise in delegated asset management over the sample period.

The model delivers a key prediction. It shows that the aggregate valuation of equity assets rises with equity investments operated under static asset allocation strategies (Proposition 1). The economic rationale of this effect is that stocks experience price pressure because they are classified as equity assets. Conceptually, the asset classification effect is reminiscent of the index inclusion effect, whereby stocks experience price pressure because of their inclusion in an index. However, testing for the asset classification effect is challenging, because exogenous events that influence asset classification are hard to find.

The literature on aggregate price pressure overcomes this challenge by exploiting variation in the level of wealth allocated to an asset class in settings plausibly unrelated to information about

TABLE 1: **Summary Statistics of Static Funds.** The table reports annual statistics for static funds from the CRSP Mutual Fund Database. Static funds are defined as mutual funds and ETFs with standard deviation of the fraction of the portfolio allocated to common stocks $\leq 5\%$ over the sample period. Wealth flows are computed closely following Lou (2012). TNA is the aggregate total net assets in trillions of dollars. U.S. Equity Exposure is the sum of TNA weighted on fund-specific Equity Allocation. Mkt Cap is total U.S. stock market capitalization. Fraction Held by Static Investors is the ratio of Static Investors' U.S. Equity Exposure to U.S. Stock Market Capitalization. Data are drawn from the CRSP Mutual Fund Summary Files for the period 2004–2024.

Model Counterpart	F_t	V_t	θV_t	P_t	Q_t
Year	Wealth Flows ($Bn)	TNA ($T)	U.S. Equity Exposure ($T)	U.S. Stock Market Cap ($T)	% U.S. Stock Market Held by Static Investors
2004	25.09	2.38	2.04	14.27	14.30
2005	50.48	2.43	2.16	14.80	14.63
2006	-13.75	2.65	2.42	16.46	14.72
2007	-9.76	2.73	2.50	16.49	15.13
2008	-23.49	1.72	1.56	9.98	15.67
2009	-14.11	2.13	1.92	12.56	15.32
2010	-28.35	2.81	2.42	14.48	16.69
2011	-38.10	2.70	2.26	14.04	16.07
2012	-25.55	3.00	2.46	15.72	15.67
2013	116.11	3.78	3.18	20.65	15.41
2014	-32.81	3.98	3.39	22.57	15.02
2015	-109.85	3.94	3.39	21.55	15.75
2016	-34.50	4.20	3.60	23.15	15.56
2017	-40.34	4.03	3.59	27.28	13.14
2018	-84.87	4.41	3.87	24.96	15.49
2019	-186.41	5.27	4.67	31.59	14.78
2020	-240.79	5.94	5.32	39.10	13.60
2021	35.29	6.69	6.05	49.04	12.34
2022	-23.90	5.75	5.14	37.95	13.54
2023	-128.53	6.50	5.89	46.63	12.63
2024	63.78	7.26	6.64	57.36	11.58
Average	-39.91	4.04	3.53	26.03	14.54

fundamentals. This literature presents strong evidence that unexpected mutual funds wealth flows are closely related to aggregate security returns (Warther, 1995; Edelen and Warner, 2001; Ben-Rephael et al., 2011). This source of exogenous variation corresponds to a shift in fund flows, dF_t. Da et al. (2018) take a different approach. They examine the reallocation of wealth between investment funds with different target equity allocation, such as a pension plan participant reallocating wealth from a 40% to a 60% equity mandate, documenting that it results in aggregate price pressure on the stock market. This source of exogenous variation corresponds to a shift in the equity target of static investors, θ. The model presented in this paper delivers both sets of findings: wealth flows and shifts in the equity target of static investors generate aggregate price pressure on the stock market.

The asset classification effect can also be viewed through the lens of a rational bubble gen-

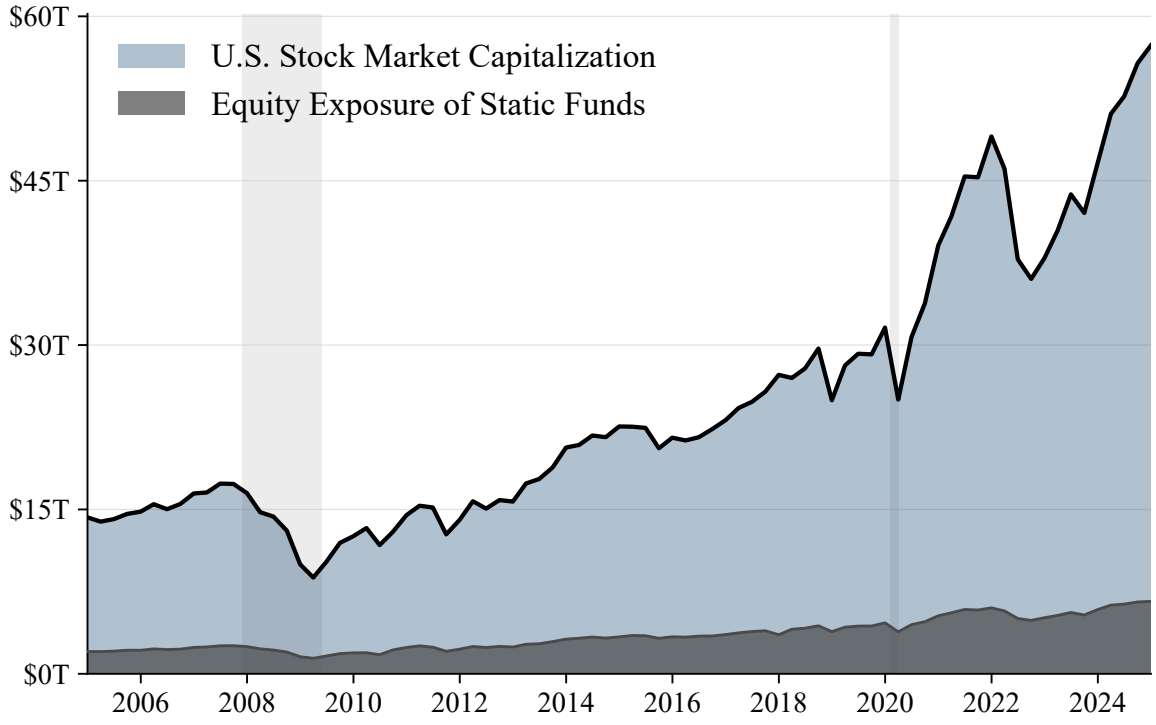


FIGURE 2: **U.S. stock market capitalization and equity exposure of static funds**. This figure plots the time series of aggregate U.S. stock market capitalization alongside the equity exposure of Mutual Funds and ETFs classified as following static asset allocation strategies. Funds are classified as static if their percentage wealth allocated to common equity has a standard deviation below 5%. To avoid look-ahead bias, the standard deviation is computed using only the fund's allocation history available up to each quarter. Gray shaded areas denote NBER recessions. Source: quarterly 2005-2025 CRSP data.

erated by investors adhering to static asset allocation strategies. Detecting a rational bubble in the data or rejecting its presence is no easy matter (Stiglitz, 1990).[23] Relative to much of the literature on rational bubbles, however, my model presents an advantage: the bubble term is pinned down by quantities that are observable in the data. In the model, static investors' TNA and target equity exposure map into a deviation between stock prices and discounted fundamentals. Under the assumptions maintained by the model, a rational bubble exists and the data can be used to gauge its magnitude. This approach is not a formal test for the presence of a bubble, but rather a model-implied quantification exercise.

Figure 2 illustrates the model-implied bubble component, θV_t, which is measured in the data by static investors' aggregate equity exposure. This exposure rises from about $2 trillion in 2004 to $6.6 trillion in 2024, while U.S. stock market capitalization grows from $14.3 trillion to $57.4 trillion. Hence, the implied wedge between prices and discounted fundamentals is

[23] The pricing of claims on dividends at specific horizons could potentially offer a way forward. Unfortunately, claims on dividends are not available at long maturities. Giglio et al. (2024) develop a model to estimate the price of such claims, but their approach rules out rational bubbles by assumption, through the imposition of a transversality condition.

trending alongside the stock market. The model further implies that θV_t must be a martingale under the risk-adjusted measure (Remark 1). Appendix G evaluates this restriction in the data and cannot reject the null that the equity exposure of static investors is a risk-adjusted martingale (p-value = 0.32), consistent with the model.

6 Conclusion

This paper examines the importance of heterogeneous asset allocation strategies for the valuation of equity assets. The central insight from the paper is that static asset allocation strategies drive the aggregate value of stock prices above their discounted dividend stream, yet do not create any arbitrage opportunities. Analytical model solutions show that strategies that invest in static proportions across asset classes exert a price pressure on the securities within these asset classes. Given the persistence of this price pressure, attempting to reverse its price effects does not constitute an arbitrage opportunity. Unconstrained investors adopting dynamic asset allocation strategies regard the presence of assets priced above their fundamental values as consistent with arbitrage, under the rational expectation of high prices going forward. This tractable theory thus highlights a mechanism through which price pressure on perpetual assets is consistent with the absence of arbitrage.

The model presented in this paper offers a coherent theoretical framework that rationalizes existing empirical results on the aggregate price pressure of wealth flows to the stock market. Meanwhile, equilibrium analysis points to wealth flows as part of a broader quantity that impacts asset prices: the level of wealth invested under static asset allocation strategies. Furthermore, this paper explores how price pressure in the cross-section of individual stocks relates to the aggregate demand for the equity asset class. Policymakers appear inclined to harness the effects described in this paper to support domestic equity markets, as they often encourage investment vehicles under their influence to raise their domestic equity allocation targets in order to stimulate domestic financial conditions. A broader message from this paper is that asset allocation strategies matter for asset prices, as they reflect investors' intended course of action over time. The framework developed here opens avenues for future research on the effects of asset classification in other markets. A natural extension could examine how the capital allocated in target proportions to private equity and commodities shapes valuations in those asset classes.

Appendix

A Micro-founding Static Investors

Consider an economy populated by a continuum of retail investors. Each retail investor j is endowed with wealth W_{jt} and must choose between two permanent regimes: becoming a *Dynamic Investor* or a *Static Investor*. Dynamic investors continuously observe the state of the economy, defined by the investment opportunity set, and optimize their consumption and portfolio rules accordingly. However, maintaining such attentiveness entails a flow utility cost defined in consumption equivalent terms, ψ_j, representing the cognitive burden or the fee for sophisticated information technology required to process state variables continuously. Static investors, conversely, delegate their wealth to a fund that maintains a pre-specified asset allocation θ, thereby avoiding the observation cost, but accepting a static asset allocation strategy that ignores intertemporal optimality considerations.

The value function for an agent choosing the dynamic regime, denoted J^D, solves the standard HJB equation net of the observation cost:

$$J^D(W_{jt}, V_t, t) \equiv \max_{\{c_s, X_s\}} \quad \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)} U(c_s - \psi_j)\, ds\right], \tag{A.1}$$

$$\text{s.t.} \quad dW_{jt} = (rW_{jt} - c_t)dt + X_t dY_t, \qquad \lim_{h\to\infty} \mathbb{E}\Big[J(W_{jt+h}, V_{t+h}, t+h)\Big] = 0. \tag{A.2}$$

Conversely, the value function for an agent choosing the static regime, J^S, is determined by the constrained asset allocation $\theta W_j/P_t$, with no cost for continuous market observation:

$$J^S(W_{jt}, t) \equiv \max_{\{c_t\}} \quad \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)} U(c_s)\, ds\right], \tag{A.3}$$

$$\text{s.t.} \quad dW_{jt} = (rW_{jt} - c_t)dt + (\theta W_{jt}/P_t)dY_t, \qquad \lim_{h\to\infty} \mathbb{E}\Big[J(W_{jt+h}, V_{t+h}, t+h)\Big] = 0. \tag{A.4}$$

An agent optimally chooses to become a static investor if the welfare loss from suboptimal diversification is outweighed by the utility gain from avoiding the observation cost. This yields the selection condition:

$$J^D \geq J^S. \tag{A.5}$$

Agents are endowed with one of two observation-cost types, $\psi_j \in \{\psi_L, \psi_H\}$. Low-cost agents have $\psi_L = 0$, so that the selection condition in Equation (A.5) holds and they optimally choose to be dynamic investors. High-cost agents instead face a sufficiently large cost ψ_H such that Equation (A.5) does not hold, and therefore optimally choose to be static investors.

Let $\mathcal{D} \equiv \{j : \psi_j = \psi_L\}$ denote the set of low-cost agents who optimally choose the dynamic regime, and let $\mathcal{S} \equiv \{j : \psi_j = \psi_H\}$ denote the set of high-cost agents who optimally choose the static regime. Dynamic investors remain continuously engaged with financial markets and their aggregate wealth is obtained by integrating individual wealth across $\mathcal{D}$,

$$W_t = \int_{\mathcal{D}} W_{jt}\, dj. \tag{A.6}$$

Static retail investors become fund shareholders, but avoid continuously observing the fund because they dislike monitoring markets on a daily basis. Instead, they behave as buy-and-hold investors with heterogeneous investment horizons. Upon entering the static regime, investor $j \in \mathcal{S}$ delegates portfolio choice to the fund and holds fund shares with inertia until her horizon. When this horizon is reached, she redeems shares to finance consumption, while other investors with longer horizons remain invested and new investors enter the fund. Thus, even though each static investor follows a buy-and-hold strategy and does not adjust her investment in the fund in response to the state, the fund experiences time-varying net issuance and redemption induced by the staggered distribution of investor horizons. Aggregating these purchases and redemptions across investors generates net fund flows F_t and a corresponding flow component in the wealth managed by the static fund,

$$V_t = \int_{\mathcal{S}} W_{jt}\, dj + \pi F_t. \tag{A.7}$$

The aggregate fund flows F_t summarize net inflows into the fund, aggregating individual purchases and redemptions by retail investors that avoid continuous market observation, and may be correlated with innovations in earnings. Flow risk is an intrinsic feature of the delegated investment vehicle, stemming from the stochastic investment timing of inertial retail investors that delegate their portfolio choice to the fund due to a preference to avoid continuous market observation.

B Proof of Proposition 1

As is standard in the literature, the proof proceeds by postulating that the price function is as guessed and verifies that the HJB equation, the market clearing condition, and the transversality conditions are satisfied in equilibrium.

$$P_t = p_\gamma + p_D D_t + p_m m + \theta V_t, \tag{B.1}$$

where $p_\gamma = -\gamma\big(p_D^2\omega^2 + \theta^2\pi^2\big)$, $p_D = \frac{1}{r}$, and $p_m = \frac{1}{r^2}$. The state variables dynamics are:

$$dD_t = mdt + \omega dB_t, \tag{B.2}$$

$$dV_t = rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t. \tag{B.3}$$

The price dynamics are as follows.

$$dP_t = p_D dD_t + \theta\big[rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t\big].$$

The price drift is given by

$$\mu_t = \frac{\frac{m}{r} + \theta[rV_t(1-\theta) + Q_t D_t]}{1-\theta Q_t}. \tag{B.4}$$

The Brownian increments dB_t and dF_t are independent and the price diffusion is given by

$$\sigma_t = \frac{\sqrt{p_D^2\omega^2 + \theta^2\pi^2}}{1-\theta Q_t}. \tag{B.5}$$

The HJB equation is

$$\begin{aligned} 0 &= \max_{\{c_t, X_t\}} U(c_t) + \frac{\mathbb{E}_t[dJ]}{dt} \\ &= \max_{\{c_t, X_t\}} U(c_t) + J_t + J_W\mathbb{E}_t[dW_t] + J_V\mathbb{E}_t[dV_t] \\ &+ \frac{1}{2}J_{WW}\mathbb{E}_t[dW_t^2] + \frac{1}{2}J_{VV}\mathbb{E}_t[dV_t^2] + J_{WV}\mathbb{E}_t[dW_t dV_t]. \end{aligned} \tag{B.6}$$

The conditional moments of dynamic and static investors' wealth satisfy

$$\mathbb{E}_t[dW_t] = [rW_t - c_t + X_t\big(\mu_t - rP_t + D_t\big)]dt, \tag{B.7}$$

$$\mathbb{E}_t[dW_t^2] = \mathbb{E}_t[(X_t dP_t)^2] = (X_t\sigma_t)^2 dt, \tag{B.8}$$

$$\mathbb{E}_t[dV_t] = [rV_t(1-\theta) + Q_t(\mu_t + D_t)]dt, \tag{B.9}$$

$$\mathbb{E}_t[dV_t^2] = (Q_t^2\sigma_t^2 + \pi^2)dt, \tag{B.10}$$

$$\mathbb{E}_t[dV_t dW_t] = X_t Q_t \sigma_t^2 dt. \tag{B.11}$$

By substituting the above expressions in the HJB equation,

$$\begin{aligned}0 = \max_{\{c_t, X_t\}} & U(c_t) + J_t + J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] + \frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] \\ & + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t] \\ = \max_{\{c_t, X_t\}} & U(c_t) + J_t + J_W \big[rW_t - c_t + X_t(\mu_t - rP_t + D_t)\big] + J_V \big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big] \\ & + \frac{1}{2} J_{WW} X_t^2 \sigma_t^2 + \frac{1}{2} J_{VV}(Q_t^2 \sigma_t^2 + \pi^2) + J_{WV} X_t Q_t \sigma_t^2. \end{aligned} \tag{B.12}$$

The first order conditions (FOCs) are

$$U'(c_t^\star) = J_W, \tag{B.13}$$

$$X_t^\star = -\frac{J_W}{J_{WW}\sigma_t^2}(\mu_t - rP_t + D_t) - \frac{J_{WV}}{J_{WW}} Q_t. \tag{B.14}$$

As dynamic investors have CARA utility, I conjecture that the value function takes the form

$$J(W_t, V_t, t) = -e^{-\delta t - r\gamma W_t - g(V_t) - \beta} \tag{B.15}$$

with partial derivatives given by $J_t = -\delta J$, $J_W = -r\gamma J$, $J_{WW} = (r\gamma)^2 J$, $J_V = -g'(V_t)J$, $J_{VV} = \big(g'(V_t)^2 - g''(V_t)\big)J$, and $J_{WV} = r\gamma g'(V_t) J$. Therefore, the FOCs become

$$c_t^\star(W_t, V_t) = rW_t + \frac{1}{\gamma}\big(g(V_t) + \beta - \log r\big), \tag{B.16}$$

$$X_t^\star(P_t, D_t, V_t) = \frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - \frac{g'(V_t)}{r\gamma} Q_t. \tag{B.17}$$

At this stage, it is standard to replace in the HJB the FOCs paired with the market clearing condition, which is normally given by $X_t = 1$. However, the presence of static investors complicates the problem, as the market clearing conditions requires $X_t + Q_t = 1$. By replacing the expression for Q_t,

$$X_t^\star(P_t, D_t, V_t) = \frac{P_t - \theta V_t}{P_t}, \qquad Q_t(P_t, V_t) = \frac{\theta V_t}{P_t}, \tag{B.18}$$

replacing the guess for the equilibrium price yields the following expressions.

$$X_t^\star(P_t, D_t, V_t) = \frac{p_\gamma + p_D D_t + p_m m}{p_\gamma + p_D D_t + p_m m + \theta V_t}, \quad Q_t(P_t, V_t) = \frac{\theta V_t}{p_\gamma + p_D D_t + p_m m + \theta V_t}. \tag{B.19}$$

The equilibrium price must ensure consistency between the market clearing condition and the FOCs of the optimization program of the dynamic investors, requiring

$$\frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - \frac{g'(V_t)}{r\gamma}\frac{\theta V_t}{P_t} = \frac{P_t - \theta V_t}{P_t}, \tag{B.20}$$

hence,

$$\mu_t - rP_t + D_t = \frac{P_t - \theta V_t\big(1 - \frac{g'(V_t)}{r\gamma}\big)}{P} r\gamma\sigma_t^2. \tag{B.21}$$

In the constrained equilibrium, $\theta = 0$, and the Sharpe ratio equals to the supply of bonds normalized to 1 (see Veronesi, 1999). In general, however, a portion of investors may exert price pressure unrelated to fundamentals. Let me work out the left-hand side of Equation (B.21). After some tedious algebra,

$$\mu_t - rP_t + D_t = -\frac{P_t}{P_t - \theta^2 V_t} r p_\gamma, \tag{B.22}$$

which uses the relation $\frac{1}{1-\theta Q_t} = \frac{P_t}{P_t - \theta^2 V_t}$. Turning to the right-hand side of Equation (B.21),

$$\frac{P_t - \theta V_t\big(1 - \frac{g'(V_t)}{r\gamma}\big)}{P_t} r\gamma\sigma_t^2 = \frac{P_t}{P_t - \theta^2 V_t} r\gamma\left(\frac{\omega^2}{r^2} + \theta^2\pi^2\right), \tag{B.23}$$

if and only if $1 - \frac{g'(V_t)}{r\gamma} = \theta$. Thus, for Equation (B.21) to hold, ensuring that the FOC and the market clearing condition are satisfied, it must be the case that $1 - \frac{g'(V_t)}{r\gamma} = \theta$, satisfied by $g'(V_t) = (1-\theta)r\gamma$. Indeed, by replacing $g'(V_t)$, Equation (B.21) simplifies to the following expression, consistent with the definition of p_γ.

$$p_\gamma = -\gamma\left(\frac{\omega^2}{r^2} + \theta^2\pi^2\right). \tag{B.24}$$

In the equilibrium without static investors of Lemma 1, $\theta = 0$ and thus $p_\gamma = p_g$; with static investors, the compensation for risk, p_γ, incorporates flow risk, π. Let me replace the FOCs into the HJB.

$$\begin{aligned}
0 &= \frac{1}{J}U(c_t^\star) + \frac{1}{J}\frac{\mathbb{E}_t[dJ]}{dt} = r + \frac{1}{J}\frac{\mathbb{E}_t[dJ]}{dt} \\
&= r - \delta - r\gamma\left[\frac{1}{\gamma}(\log r - g(V_t) - \beta) + X_t^\star(\mu_t - rP_t + D_t)\right] - g'(V_t)\big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big] \\
&+ \frac{1}{2}(r\gamma\sigma_t X_t^\star)^2 + \frac{1}{2}\big(g'(V_t)^2 - g''(V_t)\big)(Q_t^2\sigma_t^2 + \pi^2) + r\gamma\sigma_t^2 g'(V_t)X_t^\star Q_t.
\end{aligned} \tag{B.25}$$

Substituting $U(c_t^\star)$ and $X_t^\star$,

$$
\begin{aligned}
0 = r - \delta - r\gamma\Bigg[\frac{1}{\gamma}(\log r - g(V_t) - \beta) + \left(\frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - \frac{g'(V_t)}{r\gamma}Q_t\right)(\mu_t - rP_t + D_t)\Bigg] \\
- g'(V_t)\big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big] + \frac{1}{2}\left[r\gamma\sigma_t\left(\frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - \frac{g'(V_t)}{r\gamma}Q_t\right)\right]^2 \\
+ \frac{1}{2}\big(g'(V_t)^2 - g''(V_t)\big)(Q_t^2\sigma_t^2 + \pi^2) + r\gamma\sigma_t^2 g'(V_t)\left(\frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - \frac{g'(V_t)}{r\gamma}Q_t\right)Q_t.
\end{aligned}
\tag{B.26}
$$

Simplifying the expression yields

$$
0 = r - \delta - r(\log r - g(V_t) - \beta) - \frac{(\mu_t - rP_t + D_t)^2}{2\sigma_t^2} - g'(V_t)[rV_t(1-\theta) + rQ_tP_t + \frac{\pi^2}{2}].
\tag{B.27}
$$

Equivalently,

$$
\begin{aligned}
0 &= r - \delta - r(\log r - g(V_t) - \beta) - \frac{(\mu_t - rP_t + D_t)^2}{2\sigma_t^2} - g'(V_t)[rV_t + \frac{\pi^2}{2}] \\
&= r - \delta - r(\log r - g(V_t) - \beta) - \frac{1}{2}\gamma^2\left(\frac{\omega^2}{r^2} + \theta^2\pi^2\right) - g'(V_t)[rV_t + \frac{\pi^2}{2}].
\end{aligned}
\tag{B.28}
$$

We have used the fact that $(\mu_t - rP_t + D_t)^2/\sigma_t^2 = \gamma^2\big(\frac{\omega^2}{r^2} + \theta^2\pi^2\big)$, which can be derived from Equations (B.5), (B.22), and (B.24). We further know $g(V_t) = K + (1-\theta)r\gamma V_t$, which implies that $g'(V_t) = (1-\theta)r\gamma$ and $g''(V_t) = 0$. After replacing $\beta = \frac{(\gamma\omega)^2}{2r} + \frac{\delta}{r} + \log(r) - 1$,

$$
\begin{aligned}
0 &= r - \delta - r(\log r - g(V_t) - \beta) - \frac{\gamma^2}{2}\left(\frac{\omega^2}{r^2} + \theta^2\pi^2\right) - g'(V_t)[rV_t + \frac{\pi^2}{2}] \\
&= \frac{(\gamma\omega)^2}{2} - rK - \frac{\gamma^2}{2}\left(\frac{\omega^2}{r^2} + \theta^2\pi^2\right) - (1-\theta)r\gamma\frac{\pi^2}{2}.
\end{aligned}
\tag{B.29}
$$

Thus, the guess satisfies the requisite FOC and market clearing conditions for a suitable constant K. The transversality condition is also respected. Given Equation (B.15) and investors' wealth dynamics,

$$
\lim_{h\to\infty}\mathbb{E}\Big[J(W_{t+h}, V_{t+h}, t+h)\Big] = \lim_{h\to\infty}\mathbb{E}\Big[-e^{-\delta(t+h) - r\gamma W_{t+h} - K - r\gamma(1-\theta)V_{t+h} - \beta}\Big] = 0.
\tag{B.30}
$$

This concludes the proof of Proposition 1. Lemma 1 is obtained as a special case when $\theta = 0$, which prevents static investors from allocating their wealth into equity assets. Q.E.D.

C Wealth Flows with Drift

This appendix considers the robustness of a case in which wealth flows into the static fund have a constant drift term. Suppose that the flow process satisfies

$$dF_t = \nu\, dt + d\widetilde{F}_t, \tag{C.1}$$

where $\widetilde{F}_t$ is a Brownian motion specified under the risk-adjusted measure and adapted to $\{\mathscr{F}_t\}$, and ν is a constant drift term. We want to show that this environment is equivalent to the benchmark after a change of variables. Substituting Equation (C.1) into the static investors' wealth dynamics in Equation (5) yields

$$dV_t = rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi\nu dt + \pi d\widetilde{F}_t. \tag{C.2}$$

Hence, using Remark 1,

$$\mathbb{E}_t^*[dV_t] = rV_t\, dt + \pi\nu\, dt. \tag{C.3}$$

Define the auxiliary wealth process

$$\widetilde{V}_t \equiv V_t + \frac{\pi\nu}{r}. \tag{C.4}$$

Since $\frac{\pi\nu}{r}$ is constant, $d\widetilde{V}_t = dV_t$, and Equation (C.3) implies

$$\mathbb{E}_t^*[d\widetilde{V}_t] = r\widetilde{V}_t\, dt, \tag{C.5}$$

so that $e^{-rt}\widetilde{V}_t$ is a risk-adjusted martingale. Conjecture the equilibrium price

$$P_t = p_\gamma + p_D D_t + p_m m + \theta\widetilde{V}_t. \tag{C.6}$$

The verification in Appendix B carries over verbatim, with $\widetilde{V}_t$ in place of V_t. In particular, $d\widetilde{V}_t = dV_t$ ensures identical dynamics, and the martingale property in Equation (C.5) ensures that the auxiliary wealth process evolves as a rational bubble term. Expressing the solution in terms of V_t,

$$P_t = p_\gamma^\nu + p_D D_t + p_m m + \theta V_t, \qquad p_\gamma^\nu \equiv p_\gamma + \frac{\theta\pi\nu}{r}. \tag{C.7}$$

Setting $\nu = 0$ recovers the benchmark of Proposition 1. The asset classification effect remains unchanged, as captured by the term θV_t. Q.E.D.

D Preferences of Dynamic Investors

The baseline model of Section 3 assumes that dynamic investors' preferences are described by a CARA utility function. This section shows that the results continue to hold when preferences are described by a CRRA utility function. Assume that dynamic investors have utility

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}. \tag{D.1}$$

Internet Appendix C shows that the resulting equilibrium price of the equity asset class is

$$P_t = \bar{p} + \bar{p}_\gamma W_t + p_D D_t + p_m m + \theta V_t, \tag{D.2}$$

for two constant scalars $\bar{p} < 0$ and $\bar{p}_\gamma > 0$. Equation (D.2), where static investors' equity exposure, θV_t, enters additively, again indicates the presence of an asset classification effect. Thus, the key asset pricing mechanism of the model is robust to the specification of dynamic investors' preferences toward risk, and continues to hold under both CARA and CRRA utility. The asset classification effect arises because the equity exposure of static investors generates a rational asset price bubble, which is independent of the specific curvature of dynamic investors' utility function.

E Asset Price Dynamics

The dynamics of the price of the equity asset class described in Equation (12) are

$$dP_t = p_D dD_t + \theta dV_t. \tag{E.1}$$

Asset price dynamics are driven by the evolution of dividends and the wealth of static investors. By replacing Equation (5) in the above expression,

$$dP_t = p_D dD_t + \theta\big[rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t\big]. \tag{E.2}$$

From Equation (E.2), it is observed that the equity holdings of static investors, Q_t, give rise to price amplification effects.[24] The intuition for this effect is as follows. Price changes, dP_t, affect the wealth of both dynamic and static investors. Static investors reinvest into the risky asset class a proportion θ of their capital gains, exerting upward price pressure. This result contrasts with common wisdom, since static investors rebalance their portfolio in the opposite direction of price changes. To illustrate this mechanism, consider the example below.

[24] This wealth amplification effect shares similarities with Kyle and Xiong (2001) and Basak and Pavlova (2013), while stemming from a different group of investors following static asset allocation strategies.

Example 1. Suppose $P_t = \$100$, $X_t = Q_t = 0.5$, and $\theta = 0.4$. As firms report earnings, the present value of the dividend stream increases by \$20. Dynamic investors demand more shares and the price rises to $P_t + dP_t = \$120$. Dynamic investors realize capital gains $dW_t = X_t dP_t = \$10$. Static investors also realize capital gains $dV_t = Q_t dP_t = \$10$. Their actions can be understood through two alternative interpretations.

1. Static investors reinvest $\theta dV_t = \$4$ into the stock market leading to a price of \$124, which has a positive second-round effect on wealth and prompts further investment.

2. Static investors keep their capital gains in the stock market, leading to a price of \$130. Equities are now overweight in their portfolio and they divest $(1-\theta)dV_t = \$6$ from stocks, leading to a price of \$124. Equities are now cheaper, and thus slightly underweight, prompting further investment.

Under both interpretations, this process leads to an amplification of $\frac{1}{1-\theta Q_t} = 1.25$. In the new equilibrium, $P_t = \$125$, $X_t = 0.56$, and $Q_t = 0.44$. Following fundamental news of \$20 increase in the present value of dividends, the price changed by \$25. Absent static investors, the price would have closed at \$120.

Example 1 shows that as the price of the risky asset class increases, static investors sell some of their equity shares, trading counter-cyclically against the price change. The first interpretation is perhaps more intuitive, but the two are equivalent. Recently, Harvey et al. (2025) have highlighted the quantitative importance of this pattern in the data, documenting that portfolio rebalancing from institutional investors adopting static asset allocation strategies has a market-wide price impact. The model reproduces this mechanism through the amplification term implied by interpretation 2 of Example 1. Finally, this example clarifies that static investors' equity exposure in levels is pro-cyclical—even though their trades are contrarian—and highlights the counterfactual price that would obtain in the absence of static investors.

F Proof of Proposition 2

Before presenting the formal proof, let me present three illustrative benchmark cases.

First, consider the equilibrium with only *dynamic investors*, by setting $\theta = 0$. The optimality of dynamic investors implies that the price of the i-th stock takes the following standard form:

$$P_{it} = \mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_{is}\, ds \right] = p_{\gamma_i} + p_D D_{it} + p_m m_i. \tag{F.1}$$

There is no aggregate price pressure on the equity asset class.

Second, consider the equilibrium with only *static stock pickers*, by letting $r \to \infty$ to lure dynamic investors to the risk-free asset and by setting $N_i = 0$ for each stock, muting the

presence of index trackers. Then, by market clearing, the price of the i-th stock is

$$P_{it} = q_{it}^{\star} \theta V_t^A. \tag{F.2}$$

The aggregate price pressure on the equity asset class is given by $\sum P_{it} = \theta V_t^A$.

Third, consider the equilibrium with *static stock pickers and index trackers*. This benchmark can be obtained either by setting dynamic investors' wealth to zero, or by considering the limiting case in which the risk-free rate becomes arbitrarily high, so that dynamic investors optimally invest in the risk-free asset. By market clearing, the price of the i-th stock is

$$P_{it} = q_{it}^{\star} \theta V_t^A + \lambda_{it} \theta V_t^P. \tag{F.3}$$

The aggregate price pressure on the equity asset class is now given by $\sum P_{it} = \theta V_t$.

Formally, consider the equilibrium with *dynamic investors, stock pickers, and index trackers*. Guess that the price of the i-th stock takes the following form.

$$P_{it} = p_{\gamma_i} + p_{F_{it}} + p_D D_{it} + p_m m_i + \theta \left[q_{it}^{\star} V_t^A + \lambda_{it} V_t^P \right]. \tag{F.4}$$

The portfolio weights of index trackers are given by $\lambda_{it} = P_{it} N_i / P_t^{\text{IDX}}$. Stock pickers choose holdings $\{Q_{it}^A\}_{i=1}^I$ to solve:

$$\max_{\{Q_{it}^A\}} \sum_{i=1}^{I} Q_{it}^A (\mu_{it} + D_{it}) - \frac{\gamma}{2} \sum_{i=1}^{I} \sum_{j=1}^{I} Q_{it}^A Q_{jt}^A \sigma_{ijt} \tag{F.5}$$

subject to:

$$\sum_{i=1}^{I} Q_{it}^A P_{it} = \theta V_t^A. \tag{F.6}$$

The Lagrangian is:

$$\mathcal{L} = \sum_{i=1}^{I} Q_{it}^A (\mu_{it} + D_{it}) - \frac{\gamma}{2} \sum_{i=1}^{I} \sum_{j=1}^{I} Q_{it}^A Q_{jt}^A \sigma_{ijt} - \xi_t \left(\sum_{i=1}^{I} Q_{it}^A P_{it} - \theta V_t^A \right). \tag{F.7}$$

The FOC is:

$$\frac{\partial \mathcal{L}}{\partial Q_{it}^A} = (\mu_{it} + D_{it}) - \gamma \sum_{j=1}^{I} Q_{jt}^A \sigma_{ijt} - \xi_t P_{it} = 0. \tag{F.8}$$

Rearranging terms,

$$Q_{it}^{A\star} = \frac{\mu_{it} - \xi_t P_{it} + D_{it}}{\gamma \sigma_{it}^2} - \sum_{j \neq i} \frac{Q_{jt}^{A\star} \sigma_{ijt}}{\sigma_{it}^2}, \tag{F.9}$$

where ξ_t is a Lagrange multiplier such that:

$$\sum_{i=1}^{I} Q_{it}^{A\star} P_{it} = \theta V_t^A. \tag{F.10}$$

Stock pickers' problem can be reformulated in terms of the individual weights, $q_{it}^\star$. Let R_t be the vector of expected returns on the I stocks, Σ_t be the covariance matrix of their returns, $\mathbf{1}$ a $I \times 1$ vector of ones, and q_t a $I \times 1$ vector of individual portfolio weights. Stock pickers solve

$$\max_{q_t:} \quad q_t^\top R - \frac{\gamma}{2} q_t^\top \Sigma_t q_t \qquad s.t. \quad \mathbf{1}^\top q_t = 1. \tag{F.11}$$

The Lagrangian is

$$\mathcal{L} = q_t^\top R_t - \frac{\gamma}{2} q_t^\top \Sigma_t q_t - \chi_t \left(\mathbf{1}^\top q_t - 1 \right). \tag{F.12}$$

The first-order condition with respect to the vector q_t implies that

$$q_t^\star = \frac{1}{\gamma} \Sigma_t^{-1} \left(R_t - \chi_t \mathbf{1} \right). \tag{F.13}$$

Imposing the constraint $\mathbf{1}^\top q_t = 1$ yields the expression for the Lagrange multiplier χ_t,

$$\chi_t = \frac{\mathbf{1}^\top \left(\Sigma_t \right)^{-1} R_t - \gamma}{\mathbf{1}^\top \Sigma_t^{-1} \mathbf{1}}. \tag{F.14}$$

Substituting Equation (F.14) into (F.13), the optimal within-equity weights are

$$q_t^\star = \frac{1}{\gamma} \Sigma_t^{-1} R_t - \frac{1}{\gamma} \left(\frac{\mathbf{1}^\top \Sigma_t^{-1} R_t - \gamma}{\mathbf{1}^\top \Sigma_t^{-1} \mathbf{1}} \right) \Sigma_t^{-1} \mathbf{1}. \tag{F.15}$$

Stock pickers ensure that index stocks and non-index stocks are priced consistently in the cross-section. This greatly simplifies the problem of the dynamic investors, who can simply keep track of the aggregate wealth of static investors, V_t, as a state variable. The sum of the wealth of static stock pickers and index trackers is the wealth of static investors $V_t = V_t^A + V_t^P$, which follows Equation (5). Namely,

$$\begin{aligned} dV_t &= rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t \\ &= rV_t(1-\theta)dt + \sum_i Q_{it}(dP_{it} + D_{it}dt) + \pi dF_t. \end{aligned} \tag{F.16}$$

Equation (F.16) implies that $Q_t P_t = \sum Q_{it} P_{it}$ and $Q_t D_t = \sum Q_{it} D_{it}$. Analogously, Equation (19) implies that $X_t P_t = \sum X_{it} P_{it}$ and $X_t D_t = \sum X_{it} D_{it}$. These conditions state that the aggregate exposure of investors to the equity asset class is the sum of their exposures to individual

stocks. Finally, by definition of $P_t = \sum P_{it}$, we have $\mu_t = \sum \mu_{it}$ and $\sigma_t^2 = \frac{1}{dt}\mathbb{E}\big[\big(\sum dP_{it}\big)^2\big]$.

The HJB equation of dynamic investors is as follows.

$$\begin{aligned} 0 = \max_{\{c_t, X_t\}} & U(c_t) + J_t + J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] \\ & + \frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t], \end{aligned} \tag{F.17}$$

where

$$dW_t = (rW_t - c_t)dt + \sum_i X_{it}[(D_{it} - rP_{it})dt + dP_{it}]. \tag{F.18}$$

Thus,

$$\mathbb{E}_t[dW_t]/dt = \big[rW_t - c_t + \sum_i X_{it}\big(\mu_{it} - rP_{it} + D_{it}\big)\big], \tag{F.19}$$

$$\mathbb{E}_t[dV_t]/dt = [rV_t(1-\theta) + Q_t(\mu_t + D_t)]. \tag{F.20}$$

Substitute $X_t P_t = \sum X_{it} P_{it}$ and $Q_t P_t = \sum Q_{it} P_{it}$ into the wealth dynamics.

$$\mathbb{E}_t[dW_t^2]/dt = \mathbb{E}_t\left[\left(\sum_{i=1}^{I} X_{it} dP_{it}\right)^2\right] = \mathbb{E}_t[(X_t dP_t)^2] = X_t^2 \sigma_t^2, \tag{F.21}$$

$$\mathbb{E}_t[dV_t^2]/dt + \pi^2 = \mathbb{E}_t\left[\left(\sum_{i=1}^{I} Q_{it} dP_{it}\right)^2\right] + \pi^2 = \mathbb{E}_t[(Q_t dP_t)^2] + \pi^2 = Q_t^2 \sigma_t^2 + \pi^2, \tag{F.22}$$

$$\mathbb{E}_t[dW_t dV_t]/dt = \mathbb{E}_t\left[\left(\sum_{i=1}^{I} X_{it} dP_{it}\right)\left(\sum_{i=1}^{I} Q_{it} dP_{it}\right)\right] = X_t Q_t \mathbb{E}_t[dP_t^2] = X_t Q_t \sigma_t^2. \tag{F.23}$$

Again, guess that the value function is $J(W_t, V_t, t) = -e^{-\delta t - r\gamma W_t - g(V_t) - \beta}$. The HJB FOCs are

$$\begin{aligned} X_{it}^\star(P_{it}, D_{it}, V_t) &= -\frac{J_W}{J_{WW}}\big(\mu_{it} - rP_{it} + D_{it}\big) - \frac{\sum_j X_{jt}^\star \sigma_{ijt}}{\sigma_{it}^2} - \frac{J_{WV}}{J_{WW}} \frac{P_{it} Q_t \sigma_t^2}{\sigma_{it}^2} \\ &= \frac{\mu_{it} - rP_{it} + D_{it}}{r\gamma\sigma_{it}^2} - \frac{\sum_j X_{jt}^\star \sigma_{ijt}}{\sigma_{it}^2} - \frac{(1-\theta) P_{it} Q_t \sigma_t^2}{\sigma_{it}^2}, \end{aligned} \tag{F.24}$$

$$c_t^\star(W_t, V_t) = rW_t + \frac{1}{\gamma}\big(g(V_t) + \beta - \log r\big). \tag{F.25}$$

Replace FOCs, market clearing, $g(V_t)$, and state variables' conditional moments into the HJB.

$$
\begin{aligned}
0 &= \frac{1}{J}U(c_t^\star) + \frac{1}{J}\frac{\mathbb{E}_t[dJ]}{dt} = r + \frac{1}{J}\frac{\mathbb{E}_t[dJ]}{dt} \\
&= r - \delta - r\gamma\left[\frac{1}{\gamma}(\log r - g(V_t) - \beta) + \sum_i X_{it}^\star\big(\mu_{it} - rP_{it} + D_{it}\big)\right] + \frac{1}{2}(r\gamma\sigma_t X_t^\star)^2 \\
&- (1-\theta)r\gamma\big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big] + \frac{1}{2}(1-\theta)r\gamma(Q_t^2\sigma_t^2 + \pi^2) \\
&+ (r\gamma\sigma_t)^2(1-\theta)X_t^\star Q_t. \qquad \text{(F.26)}
\end{aligned}
$$

Equation (F.26) coincides with the HJB in Equation (B.25) if and only if

$$
\sum_i X_{it}^\star\big(\mu_{it} - rP_{it} + D_{it}\big) = X_t^\star(\mu_t - rP_t + D_t). \qquad \text{(F.27)}
$$

The relationship $\sum X_{it}P_{it} = X_tP_t$ directly implies $r\sum_i X_{it}^\star P_{it} = rX_t^\star P_t$. Moreover, as $\sum_i X_{it}^\star \mathbb{E}[dP_{it}] = X_t^\star \mathbb{E}[dP_t]$, we have $\sum_i X_{it}^\star \mu_{it} = X_t^\star \mu_t$. Finally, $\sum X_{it}D_{it} = X_tD_t$. Thus, Equation (F.27) holds. Consequently, Equation (F.26) corresponds to the HJB in Equation (B.25). The arguments of Appendix B apply, and Equation (F.26) is solved by the optimal choice of dynamic investors, the transversality condition is respected, and the guess for the price in Equation (24) is verified.

We now show that Equation (24) corresponds to the expression in Proposition 2. Using Equation (16) to replace the dividends process, we have

$$
\mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}D_{is}ds\right] = p_D D_{it} + p_m m_i. \qquad \text{(F.28)}
$$

Replacing this expression into Equation (24) yields

$$
P_{it} = p_{\gamma_i} + p_{F_{it}} + \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}D_{is}ds\right] + \theta\big[q_{it}^\star V_t^A + \lambda_{it}V_t^P\big], \qquad \text{(F.29)}
$$

where

$$
p_{\gamma_i} + p_{F_{it}} = -\frac{\gamma}{r^2}\left[\sum_j \omega_i\omega_j\varrho_{ij} + \theta\big(\pi^A q_{it}^\star + \pi^P\lambda_{it}\big)\right] \qquad \text{(F.30)}
$$

represents the compensation for risk, so that

$$
\mathbb{E}_t^*\left[\int_t^\infty e^{-r(s-t)}D_{is}\,ds\right] = p_{\gamma_i} + p_{F_{it}} + \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}D_{is}ds\right]. \qquad \text{(F.31)}
$$

Consequently, Equation (24) is equivalent to the expression in Proposition 2. Q.E.D.

G Martingale Restriction in the Data

Testing for the martingale restriction in Remark 1 requires estimating an SDF, M_t. I estimate a log-linear stochastic SDF, $M_t = \exp(b_0 - b_1' f_t)$, where f_t are the five Fama and French (2015) factors. These risk factors are not implied by my model's SDF, but they deliver a strong performance in the data. Following standard practices, b_0 is pinned down by the normalization $E[M_t(1+r)] = 1$, and b_1 is a vector of factor loadings that minimizes the squared average pricing errors across a standard set of test assets (25 Portfolios Formed on Size and Book-to-Market from the Kenneth French data library). The estimated SDF is consistent with economic intuition: it is strictly positive, with unconditional mean 0.973, standard deviation 0.221. Remark 1 implies the unconditional moment restriction $\mathbb{E}_t[M_t \theta V_t - \theta V_{t-1}] = 0$. I test this condition using sample averages and Newey–West standard errors with four quarterly lags. The resulting test yields a p-value of 0.32, so I cannot reject the null hypothesis that θV_t is a risk-adjusted martingale. This finding supports the specification of wealth flows as a Brownian motion under the risk-adjusted measure and is consistent with the economic intuition that investors cannot systematically earn abnormal returns from their investments in arbitrage-free markets.

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Internet Appendix to
"Asset Pricing with Dynamic and Static Investors"

Ruggero Jappelli

Internet Appendix A: Flexible Asset Allocation Targets

This section extends the environment of Section 3 by allowing static investors to adopt an asset allocation strategy with flexible asset allocation targets, defined as the convex combination of a fixed benchmark $\bar{\theta}$ and a state-contingent component $\hat{\theta}$. Namely,

$$\theta_t = (1-\tau)\bar{\theta} + \tau\hat{\theta}_t, \qquad \tau \in [0,1]. \tag{IA.A1}$$

The parameter τ governs flexibility over a continuum of cases, with $\tau = 0$ reproducing constant portfolio weights and $\tau = 1$ corresponding to fully state-contingent targets. For simplicity, the state-contingent asset allocation target, $\hat{\theta}$, is specified so as to replicate the optimal policy of dynamic investors. Namely,

$$\hat{\theta}_t = X_t^\star P_t / W_t. \tag{IA.A2}$$

This policy is always feasible, because there are no borrowing or short-selling constraints. Equation (IA.A2) implies that a proportion τ of the wealth of static investors is invested optimally, as is the wealth of the dynamic investors. In this case, it can be shown that the equilibrium price of the equity asset class is given by:

$$P_t = \underbrace{\mathbb{E}_t^*\left[\int_t^\infty e^{-r(s-t)} D_s \, ds\right]}_{\text{Fundamentals}} + \underbrace{(1-\tau)\bar{\theta} V_t}_{\text{Asset classification effect}}. \tag{IA.A3}$$

Thus, allowing mandates to adjust with states does not eliminate the asset-classification effect, which persists precisely to the extent that targets retain a fixed element, i.e., whenever $\tau < 1$.

Internet Appendix B: Wealth Flows and Earnings News

Thus far, wealth flows, dF_t, were assumed to be uncorrelated with earnings news, dB_t, and thus with changes in dividends, dD_t. (Recall that, since the payout ratio is constant and normalized to one, the dividend innovations are driven by earnings news). However, wealth invested under static allocation strategy investment funds may partly reflect information about earnings, as

some agents could use static funds interchangeably with direct exposure to stocks. For example, positive news about the stock market may attract wealth inflows into static investment funds; conversely, negative news may prompt outflows. To extend the model in this direction, consider the following specification:

$$dF_t = \rho dB_t + \sqrt{1-\rho^2} dZ_t, \tag{IA.B1}$$

where Z_t is a Brownian motion uncorrelated with earnings news. I assume that $0 < \rho < 1$, which is economically plausible, considering that some educated fund shareholders may time their exposure to the fund in response to earnings news, while other fund shareholders are inertial and unresponsive to earnings news.

Guess that the equilibrium price of the equity asset class is

$$P_t = p_\gamma + p_D D_t + p_m m + \theta V_t. \tag{IA.B2}$$

Using $dP_t = p_D\, dD_t + \theta\, dV_t$ and the static investors' wealth dynamics,

$$dV_t = rV_t(1-\theta)\, dt + Q_t(dP_t + D_t dt) + \pi\, dF_t, \tag{IA.B3}$$

and substituting Equation (IA.B1) yields

$$dP_t = p_D\, dD_t + \theta\big[rV_t(1-\theta)\, dt + Q_t(dP_t + D_t dt) + \pi \overbrace{(\rho dD_t + \sqrt{1-\rho^2} dZ_t)}^{dF_t}\big]. \tag{IA.B4}$$

Using Equation (1) for dD_t and collecting terms in dP shows that the price drift and diffusion coefficients are:

$$\mu_t = \frac{p_D m + \theta\big(rV_t(1-\theta) + Q_t D_t\big)}{1-\theta Q_t}, \tag{IA.B5}$$

$$\sigma_t = \frac{\sqrt{(p_D\omega + \theta\pi\rho)^2 + \theta^2\pi^2(1-\rho^2)}}{1-\theta Q_t}. \tag{IA.B6}$$

Comparing this environment with the benchmark of Proposition 1 shows that the only change is a reallocation of diffusion loadings across the two independent shocks: the loading on the earnings shock dB_t becomes $(p_D\omega + \theta\pi\rho)$, while the loading on the orthogonal flow shock dZ_t becomes $\theta\pi\sqrt{1-\rho^2}$. Define the auxiliary parameters

$$\widehat{\omega} \equiv \omega + \frac{\theta\pi\rho}{p_D}, \qquad \widehat{\pi} \equiv \pi\sqrt{1-\rho^2}. \tag{IA.B7}$$

Clearly, setting $\rho = 0$ yields $\widehat{\omega} = \omega$ and $\widehat{\pi} = \pi$, reverting the environment to the benchmark of Proposition 1. Moreover, the correlation coefficient only shows up in the diffusion coefficient in Equation (IA.B5). Such diffusion coefficient can be written in the same form as in Proposition 1, with ω replaced by $\widehat{\omega}$ and π replaced by $\widehat{\pi}$, as Z_t plays the role of the uncorrelated flow shock. With this adjustment, the problem is traced back to the baseline model derived in Proposition 1, and the derivations carry over verbatim. Hence, the guess is verified by Appendix B. Q.E.D.

Internet Appendix C: Dynamic Investors with CRRA Utility

The proof proceeds by verification that the conjectured price in Section D is an equilibrium. Differentiating the conjecture and using the dividend dynamics yields

$$dP_t = \bar{p}_\gamma dW_t + p_D dD_t + \theta dV_t. \tag{IA.C8}$$

Substituting the dynamics of dividends, dD_t, of dynamic investors' wealth, dW_t, and of static investors' wealth, dV_t, and collecting dP terms on the left-hand side yields

$$dP_t = \frac{\bar{p}_\gamma \left[(rW_t - c_t) + X_t(D_t - rP_t)\right] dt + p_D(mdt + \omega dB_t) + \theta[rV_t + Q_t(D_t - rP_t)]dt + \pi\theta dF_t}{(1 - \bar{p}_\gamma X_t - \theta Q_t)}.$$

The price drift coefficient is:

$$\mu_t = \frac{\bar{p}_\gamma \left[(rW_t - c_t) + X_t(D_t - rP_t)\right] + \frac{m}{r} + \theta \left[rV_t + Q_t(D_t - rP_t)\right]}{(1 - \bar{p}_\gamma X_t - \theta Q_t)}. \tag{IA.C9}$$

The price diffusion coefficient is:

$$\sigma_t = \frac{\sqrt{\frac{\omega^2}{r^2} + \theta^2\pi^2}}{(1 - \bar{p}_\gamma X_t - \theta Q_t)}. \tag{IA.C10}$$

For later use, it is convenient to derive the following expression from the conjectured price.

$$rP_t = r(\bar{p} + \bar{p}_\gamma W_t) + D_t + \frac{m}{r} + r\theta V_t, \tag{IA.C11}$$

so that

$$D_t - rP_t = -r(\bar{p} + \bar{p}_\gamma W_t) - \frac{m}{r} - r\theta V_t. \tag{IA.C12}$$

Substituting this expression into Equation (IA.C9) and simplifying gives the auxiliary equation

$$\mu_t - rP_t + D_t = -\frac{\bar{p}_\gamma c_t + r\bar{p}}{1 - \bar{p}_\gamma X_t - \theta Q_t}. \tag{IA.C13}$$

The HJB equation is

$$\begin{aligned} 0 = \max_{\{c_t, X_t\}} & e^{-\delta t} U(c_t) + J_t + J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] \\ & + \frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t]. \end{aligned} \tag{IA.C14}$$

The conditional moments of dV_t and dW_t are reported in Equation (B.7). Guess a homothetic value function of the form

$$J(W_t, V_t, t) = e^{-\delta t} \Phi\left(V_t + \phi W_t\right), \tag{IA.C15}$$

where Φ is a twice continuously differentiable, strictly increasing, and strictly concave function, and $\phi = \frac{1-\bar{p}_\gamma}{1-\theta}$ is a constant scalar. For ease of notation, define the positive state variable

$$s_t \equiv V_t + \phi W_t. \tag{IA.C16}$$

Under Equation (IA.C15), $J = e^{-\delta t}\Phi(s_t)$. The partial derivatives of the value function are

$$\begin{aligned} J_t = -\delta e^{-\delta t}\Phi(s_t), \quad J_W = e^{-\delta t}\phi\Phi'(s_t), \quad J_V = e^{-\delta t}\Phi'(s_t), \\ J_{WW} = e^{-\delta t}\left(\phi\right)^2\Phi''(s_t), \quad J_{VV} = e^{-\delta t}\Phi''(s_t), \quad J_{WV} = e^{-\delta t}\phi\Phi''(s_t). \end{aligned} \tag{IA.C17}$$

Differentiate the objective function in Equation (IA.C14) with respect to c_t, using the expression for $\mathbb{E}_t[dW_t]$. This yields

$$e^{-\delta t} U'(c_t) = J_W. \tag{IA.C18}$$

With CRRA utility, $U'(c_t) = c_t^{-\gamma}$, so the consumption policy is

$$c_t^\star = \left(\phi\Phi'(s_t)\right)^{-1/\gamma}. \tag{IA.C19}$$

Differentiate the objective in Equation (IA.C14) with respect to X_t, bearing in mind that X_t

now enters the expressions for μ_t and σ_t through the equilibrium pricing relation. This yields

$$0 = J_W \left(\mu_t - rP_t + D_t + X_t \frac{\partial \mu_t}{\partial X_t} \right) + J_V Q_t \frac{\partial \mu_t}{\partial X_t} + (J_{WW} X_t + J_{WV} Q_t)\sigma_t^2 + (J_{WW} X_t^2 + J_{VV} Q_t^2 + 2J_{WV} X_t Q_t)\sigma_t \frac{\partial \sigma_t}{\partial X_t}. \tag{IA.C20}$$

Replacing $J_V = \frac{1}{\phi} J_W$, $J_{VV} = \frac{1}{\phi^2} J_{WW}$, $J_{WV} = \frac{1}{\phi} J_{WW}$, and noting that $J_{WW} X_t + J_{WV} Q_t = J_{WW}(X_t + \frac{1}{\phi} Q_t)$,

$$0 = J_W \left[\mu_t - rP_t + D_t + \frac{\partial \mu_t}{\partial X_t} \big(X_t + \frac{1}{\phi} Q_t \big) \right] + \big(J_{WW} X_t + J_{WV} Q_t \big) \left[\sigma_t^2 + \big(X_t + \frac{1}{\phi} Q_t \big) \sigma_t \frac{\partial \sigma_t}{\partial X_t} \right].$$

Differentiating Equation (IA.C10) yields the partial derivative

$$\frac{\partial \sigma_t}{\partial X_t} = \frac{\bar{p}_\gamma \sigma_t}{1 - \bar{p}_\gamma X_t - \theta Q_t}. \tag{IA.C21}$$

Differentiating Equation (IA.C9), after some algebra, yields

$$\frac{\partial \mu_t}{\partial X_t} = \frac{\bar{p}_\gamma (\mu_t - rP_t + D_t)}{1 - \bar{p}_\gamma X_t - \theta Q_t}. \tag{IA.C22}$$

Substituting these partial derivatives and collecting terms yields

$$0 = \left[1 + \frac{\bar{p}_\gamma \big(X_t + \frac{1}{\phi} Q_t \big)}{1 - \bar{p}_\gamma X_t - \theta Q_t} \right] \left[J_W \left(\mu_t - rP_t + D_t \right) + J_{WW} X_t \sigma_t^2 + J_{WV} Q_t \sigma_t^2 \right]. \tag{IA.C23}$$

As the prefactor in square brackets is different from zero, the portfolio FOC coincides with the standard portfolio FOC that would obtain treating μ_t and σ_t as exogenous to X_t. Namely,

$$0 = J_W (\mu_t - rP_t + D_t) + \sigma_t^2 (J_{WW} X_t + J_{WV} Q_t). \tag{IA.C24}$$

Use Equation (IA.C17) to rewrite Equation (IA.C24) as

$$0 = \Phi'(s_t)(\mu_t - rP_t + D_t) + \Phi''(s_t)\sigma_t^2 \left[\phi X_t + Q_t \right]. \tag{IA.C25}$$

Substitute Equations (IA.C10) and (IA.C13) into (IA.C25), obtaining

$$\frac{\Phi''(s_t)}{\Phi'(s_t)} = \frac{(1 - \bar{p}_\gamma X_t - \theta Q_t) \left[\bar{p}_\gamma c_t + r\bar{p} \right]}{\left(\frac{\omega^2}{r^2} + \theta^2 \pi^2 \right) \left[\phi X_t + Q_t \right]}. \tag{IA.C26}$$

Impose the market clearing condition, $X_t + Q_t = 1$, and recall that $\phi = \frac{1-\bar{p}_\gamma}{1-\theta}$. Thus,

$$\frac{1 - \bar{p}_\gamma X_t - \theta Q_t}{\phi X_t + Q_t} = 1 - \theta. \tag{IA.C27}$$

Substituting Equation (IA.C27) into (IA.C26), and replacing the expression for $c_t^\star$ from Equation (IA.C19), yields the differential equation for Φ':

$$\frac{\Phi''(s_t)}{\Phi'(s_t)} = \frac{\kappa\gamma}{r\bar{p}} \left[\bar{p}_\gamma \left(\phi \Phi'(s_t) \right)^{-\frac{1}{\gamma}} + r\bar{p} \right], \tag{IA.C28}$$

where κ is a constant scalar given by $\kappa \equiv \frac{1-\theta}{\frac{\omega^2}{r^2} + \theta^2 \pi^2} \frac{r\bar{p}}{\gamma}$. The solution to Equation (IA.C28) is

$$\Phi'(s_t) = \left(C \exp\left(\kappa s_t\right) - \frac{\bar{p}_\gamma}{r\bar{p}} (\phi)^{-\frac{1}{\gamma}} \right)^\gamma, \tag{IA.C29}$$

for a constant C chosen so that the term in parentheses is positive. To verify that Equation (IA.C29) solves Equation (IA.C28), define $\Psi(s_t) \equiv C \exp(\kappa s_t) - \frac{\bar{p}_\gamma}{r\bar{p}} (\phi)^{-\frac{1}{\gamma}}$ so that $\Phi'(s_t) = [\Psi(s_t)]^\gamma$. Differentiating yields $\Phi''(s_t) = \gamma [\Psi(s_t)]^{\gamma-1} C\kappa \exp(\kappa s_t)$. Thus, $\Phi(\cdot)$ is concave Φ is concave if and only if $C\kappa < 0$. For $C > 0$, this is equivalent to $\kappa < 0$, which holds because $r > 0$ and $\bar{p} < 0$. Moreover, the left-hand side of Equation (IA.C28) equals to

$$\frac{\Phi''(s_t)}{\Phi'(s_t)} = \frac{\gamma C \kappa \exp(\kappa s_t)}{\Psi(s_t)}. \tag{IA.C30}$$

Since $\left(\phi \Phi'(s_t)\right)^{-\frac{1}{\gamma}} = [\Psi(s_t)]^{-1} \phi^{-\frac{1}{\gamma}}$, the right-hand side of Equation (IA.C28) becomes

$$\frac{\kappa\gamma}{r\bar{p}} \left[\bar{p}_\gamma [\Psi(s_t)]^{-1} \phi^{-\frac{1}{\gamma}} + r\bar{p} \right] = \frac{\kappa\gamma}{r\bar{p}} \Psi(s_t)^{-1} \left[\bar{p}_\gamma \phi^{-\frac{1}{\gamma}} + r\bar{p}\Psi(s_t) \right]. \tag{IA.C31}$$

Equating these two conditions and multiplying both sides by $\Psi(s_t)$ yields

$$\gamma C \kappa \exp(\kappa s_t) = \frac{\kappa\gamma}{r\bar{p}} \left[\bar{p}_\gamma \phi^{-\frac{1}{\gamma}} + r\bar{p} C \exp(\kappa s_t) - \bar{p}_\gamma \phi^{-\frac{1}{\gamma}} \right], \tag{IA.C32}$$

which is verified, confirming that Equation (IA.C29) satisfies the differential Equation (IA.C28). Thus, Equation (IA.C29) pins down Φ' (and hence Φ'') given the conjectured constants $\bar{p}$ and $\bar{p}_\gamma$. I now verify that the candidate value function satisfies the HJB. Evaluate Equation (IA.C14) at the controls $(c_t^\star, X_t^\star)$ and substitute the conditional moments of dW_t and dV_t and the partial

derivatives of the value function.

$$0 = e^{-\delta t}U(c_t^\star) - \delta e^{-\delta t}\Phi(s_t) + e^{-\delta t}\phi\Phi'(s_t)rW_t - e^{-\delta t}\phi\Phi'(s_t)c_t^\star + e^{-\delta t}rV_t\,\Phi'(s_t)$$
$$+ e^{-\delta t}(\phi X_t + Q_t)\Phi'(s_t)\left(\mu_t - rP_t + D_t\right) + \frac{1}{2}e^{-\delta t}\Phi''(s_t)\sigma_t^2(\phi X_t + Q_t)^2 + \frac{1}{2}e^{-\delta t}\Phi''(s_t)\pi^2. \tag{IA.C33}$$

The portfolio FOC in Equation (IA.C25) becomes:

$$0 = \Phi'(s_t)(\mu_t - rP_t + D_t) + \Phi''(s_t)\sigma_t^2(\phi X_t + Q_t). \tag{IA.C34}$$

Multiply (IA.C34) by $(\phi X_t + Q_t)$ to obtain

$$(\phi X_t + Q_t)\Phi'(s_t)(\mu_t - rP_t + D_t) = -\Phi''(s_t)\sigma_t^2(\phi X_t + Q_t)^2. \tag{IA.C35}$$

Substituting Equation (IA.C35) into (IA.C33) yields

$$0 = U(c_t^\star) - \delta e^{-\delta t}\Phi(s_t) + e^{-\delta t}\phi\Phi'(s_t)\left(rW_t - c_t^\star\right) + e^{-\delta t}rV_t\,\Phi'(s_t)$$
$$- \frac{1}{2}e^{-\delta t}\Phi''(s_t)\sigma_t^2(\phi X_t + Q_t)^2 + \frac{1}{2}e^{-\delta t}\Phi''(s_t)\pi^2. \tag{IA.C36}$$

Using Equations (IA.C10) and (IA.C27),

$$\sigma_t^2(\phi X_t + Q_t)^2 = \frac{\frac{\omega^2}{r^2} + \theta^2\pi^2}{(1-\theta)^2}. \tag{IA.C37}$$

Substitute Equation (IA.C37) into (IA.C36), and replace $c_t^\star$ using the consumption FOC in Equation (IA.C19). This yields the HJB expressed solely in terms of $\Phi(s_t)$.

$$0 = \frac{\gamma}{1-\gamma}\left(\phi\Phi'(s_t)\right)^{\frac{\gamma-1}{\gamma}} - \delta\Phi(s_t) + rs_t\Phi'(s_t) + \frac{1}{2}\Phi''(s_t)\left[\pi^2 - \frac{\frac{\omega^2}{r^2} + \theta^2\pi^2}{(1-\theta)^2}\right]. \tag{IA.C38}$$

Equation (IA.C38) determines the *level* of Φ once its *slope* is pinned down by Equation (IA.C28). To see this, define

$$G(s_t) \equiv \frac{\gamma}{1-\gamma}\left(\phi\Phi'(s_t)\right)^{\frac{\gamma-1}{\gamma}} + rs\Phi'(s_t) + \frac{1}{2}\Phi''(s_t)\left[\pi^2 - \frac{\frac{\omega^2}{r^2} + \theta^2\pi^2}{(1-\theta)^2}\right]. \tag{IA.C39}$$

Equation (IA.C38) is equivalent to the linear equation

$$\delta\Phi(s_t) = G(s_t). \tag{IA.C40}$$

Equation (IA.C28) determines $\Phi'(s_t)$, and hence $\Phi''(s_t)$, by requiring that the FOC and market

clearing hold simultaneously. Once $\Phi'(s_t)$ is fixed, $\Phi(s_t)$ is determined up to a constant, C_0. Namely,

$$\Phi(s_t) = C_0 + \int_0^{s_t} \Phi'(u)du. \tag{IA.C41}$$

Equation (IA.C38) is the condition that makes the evaluated HJB equal to zero. Once $\Phi'(s_t)$ is fixed by Equation (IA.C28), we choose the unique additive constant C_0 in $\Phi(s_t)$ so that $\delta\Phi(s_t) = G(s_t)$ holds for all s, implying that the right-hand side of the HJB vanishes. Therefore, there exists a function $\Phi(s_t)$, with $\Phi'(s_t) > 0$ and $\Phi''(s_t) < 0$, satisfying Equation (IA.C38). This completes the verification, showing that the candidate value function solves the HJB under the conjectured equilibrium price. Q.E.D.

It remains to verify that the candidate value function satisfies the transversality condition. Under the candidate value function, $J(W_t, V_t, t) = e^{-\delta t}\Phi(s_t)$, with $s_t = V_t + \phi W_t$, it suffices to show that

$$\lim_{h\to\infty} \mathbb{E}_t\left[e^{-\delta(t+h)}\Phi(s_{t+h})\right] = 0. \tag{IA.C42}$$

Using the budget dynamics for V_t and W_t and the definition $s_t \equiv V_t + \phi W_t$, we have

$$ds_{t+h} = dV_{t+h} + \phi\, dW_{t+h}. \tag{IA.C43}$$

Taking conditional expectations,

$$\mathbb{E}_t[ds_{t+h}] = \mathbb{E}_t[dV_{t+h}] + \phi\, \mathbb{E}_t[dW_{t+h}]. \tag{IA.C44}$$

From the static investors' wealth dynamics,

$$\frac{1}{dh}\mathbb{E}_t[dV_{t+h}] = rV_{t+h} + Q_{t+h}(\mu_{t+h} - rP_{t+h} + D_{t+h}) \tag{IA.C45}$$

Similarly, from the dynamic investors' budget equation,

$$\frac{1}{dh}\mathbb{E}_t[dW_{t+h}] = rW_{t+h} - c_{t+h} + X_{t+h}(\mu_{t+h} - rP_{t+h} + D_{t+h}) \tag{IA.C46}$$

Combining these identities yields

$$\begin{aligned}\frac{d}{dh}\mathbb{E}_t[s_{t+h}] &= \frac{d}{dh}\mathbb{E}_t[V_{t+h}+\phi W_{t+h}] = \frac{1}{dh}\mathbb{E}_t[dV_{t+h}]+\phi\frac{1}{dh}\mathbb{E}_t[dW_{t+h}]\\ &= r\,\mathbb{E}_t[V_{t+h}]+r\phi\,\mathbb{E}_t[W_{t+h}]-\phi\,\mathbb{E}_t[c_{t+h}]\\ &\quad+\mathbb{E}_t[(Q_{t+h}+\phi X_{t+h})(\mu_{t+h}-rP_{t+h}+D_{t+h})]\,. \end{aligned} \tag{IA.C47}$$

By definition of s,

$$\frac{d}{dh}\mathbb{E}_t[s_{t+h}] = r\,\mathbb{E}_t[s_{t+h}]-\phi\,\mathbb{E}_t[c_{t+h}]+\mathbb{E}_t[(\phi X_{t+h}+Q_{t+h})(\mu_{t+h}-rP_{t+h}+D_{t+h})]\,. \tag{IA.C48}$$

By Equations (IA.C13) and (IA.C27),

$$(\phi X_{t+h}+Q_{t+h})(\mu_{t+h}-rP_{t+h}+D_{t+h}) = -\frac{\bar{p}_\gamma c_{t+h}+r\bar{p}}{1-\theta}. \tag{IA.C49}$$

Substituting this identity into the conditional drift of s derived above yields

$$\frac{d}{dh}\mathbb{E}_t[s_{t+h}] = r\,\mathbb{E}_t[s_{t+h}]-\phi\,\mathbb{E}_t[c_{t+h}]-\frac{\bar{p}_\gamma}{1-\theta}\mathbb{E}_t[c_{t+h}]-\frac{r\bar{p}}{1-\theta}. \tag{IA.C50}$$

Recalling that $\phi=\frac{1-\bar{p}_\gamma}{1-\theta}$, it follows that $\phi+\frac{\bar{p}_\gamma}{1-\theta}=\frac{1}{1-\theta}$. Equation (IA.C50) simplifies to

$$\frac{d}{dh}\mathbb{E}_t[s_{t+h}] = r\mathbb{E}_t[s_{t+h}]-\frac{1}{1-\theta}\mathbb{E}_t[c_{t+h}]-\frac{r\bar{p}}{1-\theta}\le r\mathbb{E}_t[s_{t+h}]-\frac{r\bar{p}}{1-\theta}, \tag{IA.C51}$$

where the inequality uses $c_{t+h}\ge 0$. Let $y(h)\equiv\mathbb{E}_t[s_{t+h}]$ and define the shifted process

$$\widetilde{y}(h)\equiv y(h)-\frac{\bar{p}}{1-\theta}. \tag{IA.C52}$$

Equation (IA.C51) implies

$$\frac{d}{dh}\widetilde{y}(h) = y'(h)\le ry(h)-\frac{r\bar{p}}{1-\theta} = r\widetilde{y}(h). \tag{IA.C53}$$

Since the function $\widetilde{y}(\cdot)$ is absolutely continuous and satisfies the differential inequality

$$\frac{d}{dh}\widetilde{y}(h)\le r\widetilde{y}(h). \tag{IA.C54}$$

Grönwall's inequality implies that, for all $h\ge 0$,

$$\widetilde{y}(h)\le e^{rh}\widetilde{y}(0). \tag{IA.C55}$$

Using $\widetilde{y}(0) = y(0) - \frac{\bar{p}}{1-\theta} = s_t - \frac{\bar{p}}{1-\theta}$ and $y(h) = \widetilde{y}(h) + \frac{\bar{p}}{1-\theta}$ yields

$$\mathbb{E}_t[s_{t+h}] \leq e^{rh} s_t - \frac{\bar{p}}{1-\theta}\left(e^{rh} - 1\right), \qquad h \geq 0. \tag{IA.C56}$$

In particular, this implies the simpler inequality

$$\mathbb{E}_t[s_{t+h}] \leq e^{rh}\left(s_t + \frac{|\bar{p}|}{1-\theta}\right), \qquad h \geq 0. \tag{IA.C57}$$

Next, since Φ is increasing and concave, for any $s_t \geq 0$,

$$\Phi(s_t) \leq \Phi(0) + \Phi'(0)\, s_t, \tag{IA.C58}$$

Thus,

$$\begin{aligned}\mathbb{E}_t\left[e^{-\delta(t+h)}\Phi(s_{t+h})\right] &\leq e^{-\delta(t+h)}\left(\Phi(0) + \Phi'(0)\mathbb{E}_t[s_{t+h}]\right) \\ &\leq e^{-\delta t}\left(e^{-\delta h}\Phi(0) + \Phi'(0)e^{-(\delta-r)h}\left(s_t + \frac{|\bar{p}|}{1-\theta}\right)\right).\end{aligned}$$

Hence, provided that the standard impatience condition in infinite-horizon continuous-time economies holds, namely $\delta > r$,

$$\lim_{h\to\infty} \mathbb{E}_t\left[e^{-\delta(t+h)}\Phi(s_{t+h})\right] = 0. \tag{IA.C59}$$

This guarantees that the transversality condition is respected and proves that the candidate value function is admissible and solves the HJB under the conjectured equilibrium price. Q.E.D.