What Is the Carbon Premium a Premium On?*

Marcin Kacperczyk
Imperial College London

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Abstract

I develop an asset-pricing framework in which climate risk enters returns through a value-normalized mapping from firm fundamentals into cash-flow risk. In a complete-markets planner benchmark, the sustainable-discount-rate wedge collapses to emission intensity. With heterogeneous climate concern and unspanned damages, market incompleteness determines the equilibrium price of climate risk, while priced characteristics depend on how climate and policy shocks enter firm cash flows and need not collapse to emission intensity. I show that intensity sufficiency requires knife-edge scale invariance and derive implied testable restrictions. Both these scale and spanning conditions are rejected in an international panel, reconciling conflicting findings and clarifying the conceptual distinction between policy implementation and equilibrium pricing of climate risk.

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1 Introduction

Investors, regulators, and policy makers increasingly speak about a carbon premium or greenium in asset prices. Getting its measurement right matters: portfolio screens, regulatory taxonomies, and corporate discount rates all depend on correctly identifying which firm attributes carry climate risk. Yet there is little agreement on what this premium is a premium on. Empirical studies proxy climate exposure using absolute emissions (e.g., Bolton and Kacperczyk (2021, 2023)), emissions scaled by firm value or revenue (Pedersen (2025)), changes in emissions, sectoral classifications (Pastor et al. (2022)), or composite transition-risk scores (e.g., Sautner et al. (2023), Eskildsen et al. (2024)), while policy debates over sustainable discount rates often treat emission intensity as the natural benchmark for climate-adjusted costs of capital. The coexistence of these approaches leaves a basic issue unresolved: which firm characteristics actually govern climate risk premia in equilibrium?

This disagreement reflects a deeper model-selection problem. The climate finance literature has largely evaluated alternative characteristics—emission levels, intensity, changes, and composite scores—by asking which proxies predict returns. But expected returns are determined by how climate and policy shocks enter firm cash flows. Different economic mechanisms imply different cash-flow exposure functions and therefore different candidate sufficient statistics for pricing climate risk. Without a framework that links economic mechanisms to priced characteristics, empirical comparisons across alternative measures remain difficult to interpret.

A natural response to this proliferation is to aggregate multiple measures into composite scores. Such aggregation reduces measurement error only if the underlying signals correspond to the same economic object. When different measures instead originate from distinct economic mechanisms—policy exposure, technology choice, regulatory compliance, or investor preferences—aggregation obscures interpretation rather than improving it. In that case, it becomes unclear what economic exposure the resulting score is intended to proxy.

I address this model-selection problem by developing a framework that characterizes which firm characteristics can serve as sufficient statistics for pricing climate risk, using emission intensity as a leading and policy-relevant case. Climate and policy shocks affect firms through dollar cash-flow exposures summarized by exposure functions $g(Z_{i,t})$ that map firm characteristics into dividend sensitivities. Because returns are defined per unit of invested wealth, equilibrium risk premia load on value-normalized exposures $g(Z_{i,t})/V_{i,t}$, where $V_{i,t}$ denotes firm value.

Applying this framework yields a sharp result. Emission intensity $X_{i,t}/V_{i,t}$ is a sufficient statistic for pricing climate risk only under two joint restrictions: cash-flow exposures must be homogeneous of degree one in emissions and firm value, and priced exposures must be scale invariant across firms. Under these conditions, firms with the same emission intensity must carry identical priced exposure regardless of size. These restrictions generate falsifiable cross-sectional implications—spanning and scale invariance—that can be tested directly in the data. Yet this insight creates an apparent tension: if emission intensity is generally not sufficient for pricing climate risk, why does it play such a central role in policy discussions?

A useful benchmark comes from the complete-markets planner framework underlying the sustainable-discount-rate literature (e.g., Pedersen (2025)). With a representative agent, a well-defined social cost of carbon (SCC), and full spanning of climate risk, a social planner chooses emissions and consumption paths to maximize discounted welfare. In this setting, the planner's allocation can be decentralized through Pigouvian carbon taxes or through firm-specific sustainable discount rates that tilt the cost of capital toward cleaner firms. When climate damages enter firm cash flows proportionally to emission levels, the resulting implementation wedge between financial and sustainable discount rates is proportional to emission intensity. Intensity is therefore sufficient for implementing the planner's allocation. This is a statement about decentralizing a first best, rather than about which firm characteristics command risk premia in equilibrium. Distinguishing implementation from equilibrium pricing is therefore central, because intensity-based metrics arise naturally in policy design even when they need not summarize priced exposure.

In equilibrium asset pricing, priced characteristics are determined by how climate and policy shocks enter firms' cash flows through the exposure functions $g(Z_{i,t})$. Expected returns load on value-normalized exposures $g(Z_{i,t})/V_{i,t}$, so emission intensity appears as a priced characteristic only under the restrictive conditions that make value-normalized exposures collapse to $X_{i,t}/V_{i,t}$. Market structure—including the extent to which climate damages are spanned and the degree of heterogeneity in climate concern—primarily affects the equilibrium *price* of climate risk, not the mapping from firm fundamentals into priced exposure.

To study the equilibrium price of climate risk and connect it to the policy benchmark, I develop a tractable model with heterogeneous investors who face unspanned climate damages and differ in their concern about those damages. In a benchmark case, firms' dividends load on a climate or policy shock in proportion to their emission levels, generating a simple factor structure in which emission levels price exposure to transition risk. The equilibrium climate price of risk decomposes

into a component that compensates investors for bearing unhedgeable climate risk and a component driven by heterogeneous climate concern. In this environment, the equilibrium price of climate risk is not pinned down by the SCC, highlighting the distinction between policy implementation and equilibrium pricing.

To connect the planner and equilibrium results and derive general empirical implications, I embed both environments in a unified linear-factor framework for climate and transition risk. In this representation, expected excess returns take the form

$$E_t[R_{i,t+1} - R_{f,t}] \;=\; \sum_k \lambda_{k,t} \, \frac{g_k(Z_{i,t})}{V_{i,t}},$$

where $g_k(Z_{i,t})$ summarizes how factor k enters firm i's dollar cash flows. This representation provides a common structure for analyzing how policy design and technological responses shape exposure functions and delivers empirically testable restrictions for assessing when emission-based metrics can summarize priced exposure. Many economically natural mechanisms violate these restrictions, implying that intensity need not be sufficient in equilibrium.

One particularly sharp illustration arises from endogenous technology choice. In a dynamic production model with endogenous energy efficiency, a Jevons-type rebound effect can arise whereby efficiency improvements increase firms' exposure to priced climate shocks. Firms with lower emission intensity can therefore carry higher equilibrium climate risk, and efficiency improvements can raise rather than lower expected returns. This mechanism shows that intensity can produce qualitatively incorrect risk rankings even under optimal technology choice, and it persists in an infinite-horizon production economy with optimal investment and emissions choices.

Taken together, my paper makes three contributions. First, it reframes the measurement of climate risk premia as a problem of identifying sufficient statistics for priced exposure and links sustainable-discount-rate approaches to equilibrium asset pricing. Second, it derives testable cross-sectional restrictions that translate this conceptual distinction into empirical predictions. Third, it shows how economically natural policy and technology mechanisms generate exposure functions that violate these restrictions and lead to alternative priced characteristics.

I test these implications using an international panel of firms with Scope 1 emissions. Consistent with the model's predictions, emission intensity fails to span the cross-section of returns once emission levels are included, and its pricing varies with firm scale, violating scale invariance. Return-forecasting regressions show that absolute emissions are positively and robustly related

to future returns, while emission intensity becomes insignificant conditional on levels and adds no explanatory power. In joint specifications that condition on emission intensity, firm characteristics, and extensive country, industry, and time fixed effects—corresponding to the spanning restriction—a one-standard-deviation increase in log Scope 1 emissions predicts a 0.35% increase in next-month expected returns (about 4.2% annualized), with effects remaining significant over the subsequent year. This pricing asymmetry persists even within narrow size classes: conditional on emission intensity, firms with similar scale but different emission levels earn systematically different expected returns. Moreover, the pricing of intensity varies with firm scale, directly rejecting the scale-invariance restriction implied by intensity sufficiency.

Beyond their implications for asset pricing theory, these results clarify when intensity-based regulation is theoretically justified. The EU Taxonomy (Regulation 2020/852) and TCFD (2017) recommendations rely on intensity metrics for classifying sustainable activities and assessing climate risk. As established above, intensity is sufficient for pricing climate risk if and only if exposure functions are homogeneous of degree one in emissions and firm value and satisfy scale invariance. My empirical tests reject sufficiency. This suggests that while intensity may appropriately measure emissions per unit of economic activity—the relevant metric for regulatory targets—it systematically mismeasures the cross-sectional variation in climate risk premia that determines corporate discount rates. Regulatory frameworks conflating these objectives may produce taxonomies that appropriately target environmental outcomes yet fail to capture financial risk exposure.

Related Literature. This paper relates to several strands of the climate-finance and sustainable-investing literature. Theoretically, the framework relates to equilibrium models with heterogeneous ESG preferences. Pastor et al. (2021) develop a static equilibrium in which an exogenous greenness characteristic enters investors' utility and generates a green factor and greenium in expected returns. Pedersen et al. (2021) construct an ESG-efficient frontier and an ESG-adjusted CAPM with heterogeneously ESG-motivated investors, showing how ESG scores affect equilibrium Sharpe ratios and expected returns. Zerbib (2022) and De Angelis et al. (2023) analyze sustainable investing when some investors internalize environmental externalities, deriving taste and exclusion premia in which green investors' portfolio tilts raise brown firms' cost of capital. Dangl et al. (2025) develop a unified framework with different social preference types and establish that such preferences can have substantial real effects even when observed cost-of-capital gaps are small. These papers share the idea that heterogeneous ESG or climate preferences generate equilibrium premia, but typically take

the relevant ESG or environmental metric as given. My framework instead endogenizes green and brown classifications as equilibrium exposure functions–priced objects determined by how climate and policy shocks enter cash flows–rather than treating them as exogenous firm types.

A related literature examines sustainable-discount-rate and social-cost-of-carbon frameworks. Pedersen (2025) shows that in a complete-markets planner world, the sustainable-discount-rate wedge implementing the social optimum is proportional to firms' emission intensity. Chittaro et al. (2025) demonstrate that in a frictionless representative-agent economy, a carbon tax is equivalent to a return schedule increasing linearly in emission intensity. Empirically, Pastor et al. (2025a) introduce a firm's carbon burden–the present value of social costs of future emissions–and find that firms with higher carbon burdens have higher costs of capital. In my framework, such SCC-based burdens correspond to particular exposure functions $g(Z_i)$, and the analysis clarifies when SCC-based objects serve as priced characteristics in equilibrium versus planner implementation.

Empirically, a large literature documents correlations between carbon characteristics and expected returns. Studies use emission levels (Bolton and Kacperczyk, 2021, 2023), emission intensity (Pastor et al., 2022; Eskildsen et al., 2024), composite ESG scores, and text-based climate exposure measures (Sautner et al., 2023). Pastor et al. (2025b) survey the sustainable-investing literature and highlight as an open question which ESG attributes are actually priced in equilibrium. A related challenge is that ostensibly similar ESG characteristics can be constructed very differently across data providers (Berg et al., 2022). I take a structural approach rather than proposing new emission metrics: which emission-based characteristics can legitimately serve as sufficient statistics for exposure to priced climate factors? The framework delivers testable restrictions, scale invariance and spanning, that discipline when emission intensity suffices for equilibrium pricing and when alternative exposure measures are consistent with the data.

The remainder of the paper is organized as follows. Section 2 develops the incomplete-markets benchmark and shows that emission levels, not intensity, naturally emerge as the priced characteristic when dividends load proportionally on climate shocks. Section 3 presents the complete-markets planner benchmark and the sustainable-discount-rate representation. Sections 4 and 5 embed these benchmarks in a general factor framework and characterize the conditions under which intensity is sufficient. Sections 6 and 7 provide illustrative exposure specifications that violate these conditions. Section 8 discusses policy implications and presents the empirical tests.

2 A Simple Model with Heterogeneous Investors and Climate Risk

Consider a two-period economy, with $t = 0, 1$. Uncertainty is represented by a finite state space Ω with probability measure $\mathbb{P}$. The economy features a single physical consumption good that serves as numeraire, N firms indexed by $i = 1, \ldots, N$, and H households indexed by $h = 1, \ldots, H$. In addition to the N risky assets, a risk-free asset in zero net supply with gross return $R_f > 0$ is available.

2.1 Emissions, climate shock, and damages

Each firm i has a fixed level of emissions $X_i \geq 0$ at date 0, which determines its exposure to the climate shock realized at date 1. Aggregate emissions are

$$\bar{X} \equiv \sum_{i=1}^{N} X_i. \tag{2.1}$$

The economy is exposed to a one-dimensional aggregate climate or policy shock $\zeta : \Omega \to \mathbb{R}$ with

$$\mathbb{E}[\zeta] = 0, \qquad \mathrm{Var}(\zeta) = \sigma_\zeta^2 > 0. \tag{2.2}$$

I assume ζ is normally distributed and all payoffs are jointly normal.

Climate damages, Γ, are linear in ζ and proportional to aggregate emissions:

$$\Gamma(\zeta) \equiv \kappa \bar{X}\, \zeta, \qquad \kappa > 0. \tag{2.3}$$

Equation (2.3) is a reduced-form representation of the idea that the realization of the climate shock ζ scales damages in proportion to $\bar{X}$. The linear specification serves as a convenient benchmark, which I further relax in Internet Appendix A.

2.2 Firms and cash flows

Firm i issues a single share of equity in fixed supply. Its date-1 dividend in state $\omega \in \Omega$ is

$$D_i(\omega) = \mu_i + \eta X_i \zeta(\omega), \tag{2.4}$$

where $\mu_i \in \mathbb{R}$ is a deterministic component and $\eta > 0$ is a technology parameter common across firms. The shock ζ represents aggregate climate news and can be interpreted broadly as either transition or physical climate risk. When ζ captures transition risk—such as unexpected changes in carbon pricing, regulation, or clean technology—the linear loading on emissions has a direct interpretation: higher-emission firms experience larger cash-flow impacts from per-unit policy or technology shocks. When ζ instead reflects physical climate damages, the interpretation is more indirect. Physical damages affect firms primarily through aggregate productivity, demand, and policy responses rather than through firm-specific losses alone. Emissions therefore serve as a reduced-form proxy for exposure to sectors whose cash flows are more sensitive to climate-related macroeconomic or policy responses. Equation (2.4) should therefore be viewed as a benchmark specification that isolates a simple exposure channel in which cash-flow sensitivity is proportional to emissions. Later sections relax this assumption and allow the exposure function $g(Z_{i,t})$ to depend on richer firm characteristics, illustrating how alternative economic mechanisms generate different priced characteristics.

Let V_i denote the date-0 price of firm i's equity,[1] and define the gross return

$$R_i(\omega) \equiv \frac{D_i(\omega)}{V_i}. \tag{2.5}$$

2.3 Households, preferences, and market structure

Household h has date-0 financial wealth $W_0^h > 0$. At $t = 0$, the household chooses a portfolio $\theta^h = (\theta_1^h, \ldots, \theta_N^h)^\top$ in the N risky assets and invests an amount b^h in the risk-free asset, subject to the budget constraint

$$\sum_{i=1}^{N} V_i \theta_i^h + b^h = W_0^h. \tag{2.6}$$

Date-1 consumption of household h in state ω is then given by

$$C_1^h(\omega) = R_f\, b^h + \sum_{i=1}^{N} \theta_i^h D_i(\omega), \tag{2.7}$$

where R_f denotes the gross risk-free return.

[1] V_i denotes the equilibrium market value of claim i. Because returns are per unit of invested wealth, priced exposures appear in value-normalized form.

Households have exponential (CARA) preferences over climate-adjusted consumption,

$$U^h = \mathbb{E}\left[-\exp\left(-\alpha_h(C_1^h(\omega) - \phi_h\Gamma(\zeta))\right)\right], \tag{2.8}$$

where $\alpha_h > 0$ is the coefficient of absolute risk aversion and $\phi_h \geq 0$ indexes household h's climate concern. The term $\phi_h\Gamma(\zeta)$ captures disutility from aggregate climate damages $\Gamma(\zeta)$, with $\phi_h = 0$ corresponding to a purely financial (brown) investor and $\phi_h > 0$ to a climate-conscious (green) investor. Importantly, $\Gamma(\zeta)$ is not a direct financial loss borne uniformly through the household budget constraint. Rather, it is an aggregate damage state that enters utility through climate concern, capturing disutility from the realization of global climate outcomes even when the associated physical or economic losses are largely external to the household's own consumption stream.

An intuitive example of unspanned climate risk arises from global damages that affect households' welfare but are not directly traded in financial markets. Sea-level rise, biodiversity loss, or health damages from extreme weather may reduce welfare without mapping one-for-one into the cash flows of traded firms. Investors therefore care about these outcomes but cannot construct portfolios that perfectly hedge them. This wedge between welfare-relevant climate damages and traded payoffs generates an incomplete-markets pricing kernel.

Because these damages are non-tradable and investors differ in (α_h, ϕ_h), markets are incomplete with respect to climate risk and the stochastic discount factor that prices traded assets reflects a wealth-weighted aggregation of heterogeneous marginal rates of substitution rather than the planner's sustainable discount factor.[2] The resulting priced climate characteristic is therefore an equilibrium object, a function of firm exposures, and the associated premium admits a decomposition into a risk-based brown component and a preference-driven green component, as formalized in Section 4.1.

3 Complete-markets Benchmark

The model features an incomplete-markets setting in which climate damages are only partially internalized through heterogeneous portfolio choices. Before turning to equilibrium asset pricing in this environment, it is useful to present the normative benchmark that underlies much of the

[2] I use the term incomplete markets as shorthand for environments in which climate damages are non-tradable and heterogeneously valued across investors. Formally, the climate externality reflects a missing market, while heterogeneity in concern implies that the resulting damage risk is unspanned by traded assets, leading to an incomplete-markets pricing kernel.

policy and sustainable-discount-rate literature and provides a natural point of comparison for the equilibrium results that follow. I therefore develop a complete-markets planner framework in which a social cost of carbon (SCC) is well defined and the climate externality is fully internalized. In this environment, the planner's allocation can be decentralized through firm-specific discount-rate wedges that are proportional to emission intensity X_i/V_i. For transparency, I abstract from cross-sectional heterogeneity and financial risk and focus on deterministic cash flows and a representative decision maker.

3.1 Planner, social cost of carbon, and a Pigouvian tax

Consider again dates $t = 0, 1$ and a single consumption good. A representative agent has utility

$$U = u(C_0) + \beta\Big(u(C_1) - d(X_1)\Big), \tag{3.1}$$

where C_0 and C_1 denote aggregate consumption at dates 0 and 1, $\beta \in (0, 1)$ is the subjective discount factor, and $d : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing, convex damage function of aggregate emissions $X_1 \geq 0$. The function $d(\cdot)$ can be interpreted as the expected disutility from climate damages, for example $d(X_1) = \mathbb{E}[\Gamma(\zeta) \mid X_1]$ in a reduced-form sense. The per-period utility u is strictly increasing and concave.

Firm i produces a deterministic date-1 payoff $Y_i \geq 0$ per unit of scale and emits $X_i \geq 0$ per unit of scale. If I_i denotes the scale of firm i at date 0, we can aggregate emissions as

$$X_1 = \sum_{i=1}^{N} X_i I_i. \tag{3.2}$$

Let G_0 denote the date-0 endowment of the consumption good. The resource constraints are

$$C_0 + \sum_{i=1}^{N} I_i \leq G_0, \qquad C_1 = \sum_{i=1}^{N} Y_i I_i. \tag{3.3}$$

The technological mapping from I_i to (Y_i, X_i) is kept implicit. (Y_i, X_i) are treated as primitive characteristics of one unit of firm i, and the planner chooses the scale I_i.

The social planner chooses $(C_0, C_1, \{I_i\}_{i=1}^N)$ to maximize (3.1) subject to (3.2)-(3.3). Let λ_0 and

λ_1 denote the Lagrange multipliers on the two resource constraints. The planner's Lagrangian is

$$\mathcal{L} = u(C_0) + \beta\big(u(C_1) - d(X_1)\big) + \lambda_0\Big(G_0 - C_0 - \sum_i I_i\Big) + \lambda_1\Big(\sum_i Y_i I_i - C_1\Big).$$

The first-order conditions for C_0 and C_1 are

$$u'(C_0) = \lambda_0, \qquad \beta u'(C_1) = \lambda_1. \tag{3.4}$$

The first-order condition for I_i is

$$-\lambda_0 + \lambda_1 Y_i - \beta d'(X_1) X_i = 0. \tag{3.5}$$

Combining (3.4) and (3.5) yields

$$\frac{\lambda_0}{\lambda_1} = Y_i - \frac{\beta d'(X_1)}{\lambda_1} X_i = Y_i - \frac{d'(X_1)}{u'(C_1)} X_i. \tag{3.6}$$

The shadow cost of an additional unit of emissions in terms of the consumption numeraire is defined as the social cost of carbon,

$$S \equiv \frac{d'(X_1)}{u'(C_1)}. \tag{3.7}$$

Expression (3.6) can then be written as

$$\frac{\lambda_0}{\lambda_1} = Y_i - S X_i. \tag{3.8}$$

Equation (3.8) shows that, at the planner optimum, a marginal increase in scale I_i raises date-1 resources by Y_i but also increases the planner's marginal damage by SX_i. It is therefore convenient to interpret $Y_i - SX_i$ as the planner-implied net date-1 payoff (in consumption units) generated by one unit of firm i's scale.

To express this condition in present-value form, define the planner-implied intertemporal discount factor as

$$q \equiv \frac{\lambda_1}{\lambda_0} = \beta \frac{u'(C_1)}{u'(C_0)}. \tag{3.9}$$

Then (3.8) is equivalent to

$$1 = q(Y_i - S X_i). \tag{3.10}$$

Thus, the date-0 shadow value of one unit of scale is $q(Y_i - SX_i)$, the present value of its net date-1 payoff.

For later comparison, it is convenient to parameterize the planner-implied discount factor by a (gross) risk-free rate r via

$$q \equiv \frac{\lambda_1}{\lambda_0} = \frac{1}{1+r}. \tag{3.11}$$

A Pigouvian carbon tax τ per unit of emissions, levied uniformly on firms and rebated lump-sum to the representative household, decentralizes the planner's allocation if $\tau = S$. Under such a tax, firm i's date-1 net payoff per unit of scale is $Y_i - \tau X_i$, and its date-0 value is

$$V_i = q(Y_i - \tau X_i) = \frac{Y_i - \tau X_i}{1+r}. \tag{3.12}$$

If $\tau = S$, then (3.10) and (3.12) coincide, implying that the competitive equilibrium with tax $\tau = S$ reproduces the first-best allocation.

3.2 Sustainable discount rates and the emission-intensity wedge

The sustainable-discount-rate approach asks whether the same allocation that would be achieved by a Pigouvian carbon tax can instead be implemented by adjusting firms' discount rates while leaving their cash flows untaxed. In the one-period environment above, this equivalence can be made explicit and directly related to emission intensity.

Fix a firm i and suppose that, under the carbon tax τ, its date-0 value is V_i given by (3.12). Let r denote the financial discount rate that applies in the absence of any climate-related discount-rate adjustment. A sustainable discount rate r_i^x for firm i is defined implicitly by the condition that the present value of the taxed cash flow discounted at r equals the present value of the untaxed cash flow discounted at r_i^x, that is,

$$\frac{Y_i - \tau X_i}{1+r} = \frac{Y_i}{1+r_i^x}. \tag{3.13}$$

Solving (3.13) for r_i^x gives

$$1 + r_i^x = \frac{Y_i}{Y_i - \tau X_i}(1+r). \tag{3.14}$$

Equation (3.14) expresses the sustainable discount rate as a nonlinear function of the financial rate r, the tax τ, and the ratio $\tau X_i / Y_i$.

For small wedges and moderate discount rates, a first-order approximation in $\tau X_i / Y_i$ is informative. Writing $\varepsilon_i \equiv \tau X_i / Y_i$ and expanding the right-hand side of (3.14) to first order in ε_i

yields

$$1 + r_i^x \approx (1 + r)(1 + \varepsilon_i),$$

so that

$$r_i^x - r \approx \varepsilon_i = \tau \frac{X_i}{Y_i}. \tag{3.15}$$

Let V_i denote the firm's value under the tax as in (3.12). For small τ, we have $V_i \approx Y_i/(1+r)$, and hence

$$\frac{X_i}{Y_i} \approx \frac{X_i}{(1+r)V_i} \approx \frac{X_i}{V_i},$$

where the last approximation absorbs the factor $1 + r$ into the first-order approximation (and is numerically innocuous at annual rates). Substituting into (3.15) yields the familiar emission-intensity wedge,

$$r_i^x \approx r + \tau \frac{X_i}{V_i}. \tag{3.16}$$

If the tax τ coincides with the SCC S in (3.7), then (3.16) links the sustainable discount rate directly to the SCC and emission intensity:

$$r_i^x \approx r + S \frac{X_i}{V_i}. \tag{3.17}$$

More generally, suppose that the prevailing carbon tax is τ while the planner's SCC is S. The additional wedge required to move from the current tax to the first-best tax is $S - \tau$. Consider a small change $\mathrm{d}\tau$ in the tax and equate the induced change in firm value to that from a small change $\mathrm{d}r_i$ in the discount rate:

$$\mathrm{d}V_i^{(\tau)} = -\frac{X_i}{1+r}\mathrm{d}\tau, \qquad \mathrm{d}V_i^{(r)} = -\frac{V_i}{1+r}\mathrm{d}r_i.$$

Setting $\mathrm{d}V_i^{(\tau)} = \mathrm{d}V_i^{(r)}$ implies

$$\mathrm{d}r_i = \frac{X_i}{V_i}\mathrm{d}\tau.$$

Applying this with $\mathrm{d}\tau = S - \tau$ and linearizing around the initial allocation yields

$$r_i^x \approx r + (S - \tau)\frac{X_i}{V_i}. \tag{3.18}$$

Equations (3.7) and (3.18) formalize the sustainable-discount-rate representation developed by Pedersen (2025). The SCC S is the planner's shadow value of emissions in units of the consumption

numeraire, and, to a first-order approximation, the same allocation that would be achieved by adjusting the carbon tax from τ to S can be implemented by tilting each firm's discount rate in proportion to its emission intensity X_i/V_i. In this complete-markets benchmark, X_i/V_i is the sufficient statistic for how strongly a firm's cost of capital must be adjusted to implement the planner's allocation.

This is fundamentally an implementation result, not a restriction on equilibrium pricing. Equation (3.18) characterizes the deterministic discount-rate wedge needed to decentralize a first-best allocation under complete markets and a well-defined SCC. Once one relaxes the complete-markets and representative-agent assumptions–allowing for heterogeneous climate concern and incomplete risk sharing as in Section 2.3–the characteristics that govern equilibrium risk premia need not collapse to X_i/V_i and can instead depend on emission levels and more general exposure functions. Sections 4 and 5 make this distinction precise by characterizing when intensity remains a sufficient statistic for equilibrium returns (Proposition 3) and showing that the required conditions are knife-edge.

4 Equilibrium with Heterogeneous Investors and Unspanned Climate Risk

The benchmark models in the previous sections introduce two distinct objects: an incomplete-markets environment for asset pricing and implementation wedges in a complete-markets planner economy. What they do not yet provide is a general way to think about how climate and transition risk enter asset prices when firms are exposed through multiple, overlapping channels. In practice, emissions, technology, demand, and policy all shape how firms load on climate-related shocks, and no single emission measure need summarize these exposures a priori. This section introduces a general factor framework that makes these channels explicit and provides a unified representation for assessing when emission intensity is sufficient for pricing and when it is not.

4.1 Incomplete markets

The only traded risky payoffs are the N stocks with dividends given by (2.4). There is a single aggregate climate shock ζ, and I allow for additional idiosyncratic shocks in μ_i, but for the derivations below I focus on the one-factor structure (2.4).

Markets are incomplete in that there is no portfolio of traded assets whose payoff replicates the

damage term $\Gamma(\zeta)$. Equivalently, ζ is not fully spanned by the space of traded payoffs once the non-pecuniary climate term in preferences is taken into account. Differently, markets are complete for financial payoffs, but the climate damage term in utility is non-tradable, so the SDF is not the MRS of someone who internalizes the externality in a planner sense.

Nevertheless, equilibrium asset pricing remains well defined. There exists a unique stochastic discount factor m that prices all traded assets, but the incomplete spanning of $\Gamma(\zeta)$ implies that this pricing kernel cannot be interpreted as the marginal rate of substitution of a representative planner who internalizes the externality. This distinction from the complete-markets benchmark is central for understanding how climate risk is priced in equilibrium.

4.2 Mean-variance representation

Under CARA-exponential utility and joint normality, maximizing expected utility (2.8) is equivalent to maximizing a mean-variance objective. Substituting (2.3), (2.4), and (2.7), we have

$$C_1^h - \phi_h \Gamma(\zeta) = b^h R_f + \sum_{i=1}^N \theta_i^h \mu_i + \Big(\sum_{i=1}^N \theta_i^h \eta X_i - \phi_h \kappa \bar{X} \Big) \zeta.$$

Thus

$$C_1^h - \phi_h \Gamma(\zeta) = A_h + B_h \zeta,$$

where

$$A_h \equiv b^h R_f + \sum_{i=1}^N \theta_i^h \mu_i, \qquad B_h \equiv \sum_{i=1}^N \theta_i^h \eta X_i - \phi_h \kappa \bar{X}.$$

The mean-variance representation of (2.8) is

$$U^h = -\exp\Big(-\alpha_h \Big(\mathbb{E}[A_h] - \tfrac{\alpha_h}{2} \mathrm{Var}(B_h \zeta) \Big) \Big),$$

so household h chooses (θ^h, b^h) to maximize

$$\mathbb{E}[C_1^h - \phi_h \Gamma(\zeta)] - \frac{\alpha_h}{2} \mathrm{Var}[C_1^h - \phi_h \Gamma(\zeta)]. \tag{4.1}$$

Since b^h only enters linearly and serves to satisfy the budget constraint, the key choice is the stock vector θ^h.

Using $\mathbb{E}[\zeta] = 0$ and $\mathrm{Var}(\zeta) = \sigma_\zeta^2$, objective (4.1) becomes

$$\sum_{i=1}^{N} \theta_i^h (\mu_i - V_i R_f) \; - \; \frac{\alpha_h}{2} \sigma_\zeta^2 \Big(\sum_{i=1}^{N} \theta_i^h \eta X_i - \phi_h \kappa \bar{X} \Big)^2 .$$

4.3 First-order conditions and equilibrium prices

The first-order condition for θ_i^h is

$$\mu_i - V_i R_f - \alpha_h \sigma_\zeta^2 \Big(\sum_{j=1}^{N} \theta_j^h \eta X_j - \phi_h \kappa \bar{X} \Big) \eta X_i = 0, \qquad i = 1, \ldots, N. \tag{4.2}$$

Define

$$\delta_h \equiv \sum_{j=1}^{N} \theta_j^h \eta X_j - \phi_h \kappa \bar{X}.$$

Then (4.2) is equivalently

$$\mu_i - V_i R_f = \alpha_h \sigma_\zeta^2 \delta_h \eta X_i, \qquad i = 1, \ldots, N. \tag{4.3}$$

Since the left-hand side does not depend on h, equilibrium requires $\alpha_h \sigma_\zeta^2 \delta_h$ to be common across households with interior holdings.

Proposition 1 (One-factor structure and emission levels)**.** *In any competitive equilibrium with $\eta > 0$, there exists a scalar λ such that*

$$\mu_i - V_i R_f = \lambda \eta X_i, \qquad i = 1, \ldots, N, \tag{4.4}$$

and for every household h,

$$\delta_h = \frac{\lambda}{\alpha_h \sigma_\zeta^2}. \tag{4.5}$$

Proof is in Appendix A.1.

Equation (4.4) is a pricing relation in dollar units. Dividing by V_i yields the corresponding return representation,

$$\mathbb{E}[R_i] - R_f = \lambda \eta \frac{X_i}{V_i}, \tag{4.6}$$

where $R_i \equiv D_i / V_i$ and $\mathbb{E}[D_i] = \mu_i$. In this model, emission levels X_i govern the primitive cash-flow exposure in dividends, while expected returns load on the value-normalized exposure X_i / V_i. The

coefficient λ is the equilibrium price of climate-related risk.

4.4 Solving for the price of climate risk

Aggregating the definition of δ_h across households, multiplying by η, and imposing market clearing $\sum_{h=1}^{H} \theta_j^h = 1$ yields

$$\eta^2 \bar{X} \; - \; \kappa\eta\bar{X} \sum_{h=1}^{H} \phi_h \; = \; \eta \sum_{h=1}^{H} \delta_h. \tag{4.7}$$

Using (4.5), the right-hand side is $\eta \sum_{h=1}^{H} \delta_h = (\eta\lambda/\sigma_\zeta^2) \sum_{h=1}^{H} (1/\alpha_h)$. Solving for λ gives:

Proposition 2 (Price of climate risk)**.** *In the static model above, the equilibrium price of climate risk is*

$$\lambda = \frac{\sigma_\zeta^2 \bar{X}}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} \Big(\eta - \kappa \sum_{h=1}^{H} \phi_h\Big). \tag{4.8}$$

Proof is in Appendix A.1.

Under CARA preferences, λ depends on the distribution of risk aversion (α_h) and aggregate climate concern $\sum_h \phi_h$, and is independent of initial wealth (W_0^h).[3] Combining (4.8) with (4.6) shows how equilibrium expected returns reflect both the value-normalized exposure X_i/V_i and the aggregate price of climate risk λ, which depends on the distribution of risk aversion and climate concern across investors.

4.5 Decomposing transition risk and climate-concern hedging effects

Expression (4.8) admits a natural decomposition into a purely financial brown risk premium (transition-risk component) and a component associated with heterogeneous concern about non-tradable climate damages. To make this explicit, note the price of climate risk in the incomplete-markets economy derived above

$$\lambda_{X,t} = \frac{\sigma_\zeta^2 \bar{X}_t \eta}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} - \frac{\sigma_\zeta^2 \bar{X}_t \kappa}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} \sum_{h=1}^{H} \phi_h.$$

Define

$$\lambda_{X,t}^{\mathrm{TR}} \equiv \frac{\sigma_\zeta^2 \bar{X}_t \eta}{\sum_{h=1}^{H} \frac{1}{\alpha_h}}, \qquad \lambda_{X,t}^{\mathrm{H}} \equiv -\frac{\sigma_\zeta^2 \bar{X}_t \kappa}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} \sum_{h=1}^{H} \phi_h, \tag{4.9}$$

[3] In Internet Appendix B, I show that with CRRA utility the analogous expression features wealth weights through local risk tolerances.

so that

$$\lambda_{X,t} = \lambda_{X,t}^{\mathrm{TR}} + \lambda_{X,t}^{\mathrm{H}}. \tag{4.10}$$

The term $\lambda_{X,t}^{\mathrm{TR}}$ is the price of climate risk that would prevail if all households were climate-neutral in their preferences, that is, if $\phi_h = 0$ for all h. It compensates investors for bearing unhedgeable exposure to the aggregate climate or policy shock ζ_{t+1} through the technological channel $D_{i,t+1} = \mu_{i,t+1} + \eta X_{i,t}\zeta_{t+1}$. In this sense, $\lambda_{X,t}^{\mathrm{TR}}$ captures the transition-risk or physical-risk component of the brown premium. It depends on the volatility of the climate shock σ_ζ^2, aggregate emissions $\bar{X}_t$, the technological sensitivity η, and aggregate risk tolerance $\sum_h (1/\alpha_h)$, but it is independent of the climate-concern parameters $\{\phi_h\}_h$.

By contrast, the term $\lambda_{X,t}^{\mathrm{H}}$ is zero when $\phi_h = 0$ and is strictly decreasing in aggregate climate concern $\sum_h \phi_h$ whenever $\kappa > 0$. Differentiating (4.9) with respect to ϕ_h yields

$$\frac{\partial \lambda_{X,t}}{\partial \phi_h} = -\frac{\sigma_\zeta^2 \bar{X}_t \kappa}{\sum_{j=1}^{H} \frac{1}{\alpha_j}} < 0.$$

This result reflects a hedging-demand mechanism. The hedging interpretation depends on the economic interpretation of the climate shock ζ_{t+1}. When ζ_{t+1} captures aggregate damages or transition news that is positively correlated with non-tradable climate damages $\Gamma(\zeta_{t+1})$, assets with higher exposure provide a hedge against welfare-relevant climate outcomes. When ζ_{t+1} instead represents purely regulatory or carbon-price shocks, the sign of the hedging channel may reverse, but the decomposition of the price of risk into transition and hedging components remains unchanged.

Because climate damages are non-tradable, assets whose cash flows covary with the climate shock provide a hedge against these damages. In states where climate damages are high and positively related to ζ_{t+1}, firms with greater exposure perform relatively better and therefore serve as hedging assets. As climate concern rises, investors place greater value on this hedge and are willing to accept lower compensation for holding climate-exposed firms. Holding the shock process and technology block fixed, stronger aggregate climate concern therefore compresses the equilibrium price of climate risk.

Given the decomposition (4.10), the expected excess return on firm i can be written as

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx (\lambda_{X,t}^{\mathrm{TR}} + \lambda_{X,t}^{\mathrm{H}}) \frac{X_{i,t}}{V_{i,t}} = \lambda_{X,t}^{\mathrm{TR}} \frac{X_{i,t}}{V_{i,t}} + \lambda_{X,t}^{\mathrm{H}} \frac{X_{i,t}}{V_{i,t}}. \tag{4.11}$$

The first term compensates investors for bearing non-diversifiable transition (or physical) climate

risk. The second term captures hedging demand arising from heterogeneous concern about climate damages in incomplete markets.

An equivalent separation applies at the level of the aggregate stochastic discount factor. Writing the affine SDF as

$$m_{t+1} = \exp\big(-A_t - B_t\zeta_{t+1}\big), \qquad B_t = \frac{\lambda_{X,t}}{\sigma_\zeta^2},$$

and substituting (4.9) gives

$$B_t = B_t^{\mathrm{TR}} + B_t^{\mathrm{H}}, \qquad B_t^{\mathrm{TR}} = \frac{\bar{X}_t\eta}{\sum_{h=1}^{H}\frac{1}{\alpha_h}}, \quad B_t^{\mathrm{H}} = -\frac{\bar{X}_t\kappa}{\sum_{h=1}^{H}\frac{1}{\alpha_h}}\sum_{h=1}^{H}\phi_h.$$

The loading B_t^{TR} describes how the pricing kernel responds to the climate shock in an otherwise identical economy with $\phi_h = 0$. The adjustment B_t^{H} captures the additional tilt in the pricing kernel induced by heterogeneous concern about non-tradable climate damages.

This decomposition shows that transition risk and climate-concern hedging effects are conceptually distinct but jointly determine the observed carbon premium. The preference component in this framework reflects hedging demand arising from non-tradable climate damages rather than "warm-glow" preferences for holding green assets. Climate-concerned investors value assets that hedge adverse climate outcomes, which reduces the equilibrium compensation required to induce the market to bear climate risk. This mechanism differs from models in which investors directly derive utility from holding green assets, although both channels may coexist in practice.

4.6 When is emission intensity a unique sufficient statistic?

The previous subsection shows that expected excess returns are governed by firms' factor exposures, which may depend on a rich set of characteristics $Z_{i,t}$. There is therefore no general reason why any single primitive, such as emission levels $X_{i,t}$ or emission intensity $X_{i,t}/V_{i,t}$, must be sufficient for climate-related pricing. A key clarification is that, while economic mechanisms determine exposures in dollar units (cash-flow sensitivities), asset pricing maps these sensitivities into return premia by dividing by firm value: returns are measured per unit of invested wealth. This subsection formalizes and characterizes the knife-edge conditions under which emission intensity is a unique sufficient statistic for the priced exposure to a given climate or transition factor.

Definition 4.1 (Emission intensity as a sufficient statistic)**.** *Consider date t and a one-factor*

environment in which expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \Lambda_t \chi_{i,t},$$

for some scalar Λ_t *and some scalar characteristic* $\chi_{i,t}$ *that may depend on firm attributes. We say that* emission intensity *is a sufficient statistic at date* t *if for any two firms* i *and* j*,*

$$\frac{X_{i,t}}{V_{i,t}} = \frac{X_{j,t}}{V_{j,t}} \quad \Rightarrow \quad \chi_{i,t} = \chi_{j,t}.$$

We say that intensity is a unique *sufficient statistic if, in addition, any other characteristic* $\tilde{\chi}_{i,t}$ *that generates an equivalent one-factor representation must be proportional to* $X_{i,t}/V_{i,t}$ *in the cross-section, that is, there exists a scalar* ψ_t *such that*

$$\tilde{\chi}_{i,t} = \psi_t \frac{X_{i,t}}{V_{i,t}} \quad \textit{for all } i.$$

One-factor environment and value normalization. For clarity, let suppress time subscripts when no confusion arises. Let f_{t+1} denote a scalar transition or climate factor with $\mathbb{E}_t[f_{t+1}] = 0$, and consider dividends of the form

$$D_{i,t+1} = \mu_{i,t+1} + g(Z_{i,t}) f_{t+1}, \tag{4.12}$$

where $Z_{i,t}$ collects firm characteristics (including in particular $(X_{i,t}, V_{i,t})$), and $g(\cdot)$ is a deterministic function. The object $g(Z_{i,t})$ is a *cash-flow* exposure (in dividend units). Let the stochastic discount factor be

$$m_{t+1} = a_t + b_t f_{t+1} + \varepsilon_{t+1}, \tag{4.13}$$

with $a_t > 0$, b_t $\mathcal{F}_t$–measurable, and ε_{t+1} conditionally mean-zero and orthogonal to f_{t+1}. A standard log-linear approximation to the Euler equation implies that, to a first-order approximation in $\mathrm{Var}_t(f_{t+1})$, expected excess returns take the value-normalized form

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_t \frac{g(Z_{i,t})}{V_{i,t}}, \qquad \lambda_t = -\frac{b_t \mathrm{Var}_t(f_{t+1})}{\mathbb{E}_t[m_{t+1}]}, \tag{4.14}$$

regardless of whether markets are complete or incomplete. Market structure determines b_t and hence λ_t, but not the functional form of $g(\cdot)$.

Knife-edge conditions for intensity sufficiency. The next result characterizes when the return-relevant exposure $g(Z_{i,t})/V_{i,t}$ depends on firms only through emission intensity $X_{i,t}/V_{i,t}$, and when this dependence is unique up to scale. The result is model-agnostic and does not rely on the incomplete-markets structure developed earlier; it follows from the mapping between dollar cash-flow exposures and returns.

Proposition 3 (Conditions for emission intensity to be a unique sufficient statistic)**.** *Consider the one-factor environment* (4.12)-(4.14) *at a given date t. Suppose that:*

(i) The cross-section of firms is rich *in the sense that the set*

$$\mathcal{S}_t \equiv \left\{ \left(\frac{X_{i,t}}{V_{i,t}}, V_{i,t} \right) : i = 1, \ldots, N \right\}$$

exhibits independent variation in emission intensity and firm scale.[4]

(ii) The exposure function $g(\cdot)$ is homogeneous of degree one in $(X_{i,t}, V_{i,t})$ in the sense that, for all $\lambda > 0$ and all firms i,

$$g(Z_{i,t}; \lambda X_{i,t}, \lambda V_{i,t}) = \lambda \, g(Z_{i,t}; X_{i,t}, V_{i,t}).$$

(iii) For any two firms i and j with the same intensity, $X_{i,t}/V_{i,t} = X_{j,t}/V_{j,t}$, their value-normalized exposures coincide,

$$\frac{g(Z_{i,t})}{V_{i,t}} = \frac{g(Z_{j,t})}{V_{j,t}}.$$

Then there exists a (measurable) function $\varphi_t : \mathbb{R}_+ \to \mathbb{R}$ such that

$$\frac{g(Z_{i,t})}{V_{i,t}} = \varphi_t\left(\frac{X_{i,t}}{V_{i,t}} \right) \quad \text{for all } i. \tag{4.15}$$

If, in addition, $g(\cdot)$ is continuous on $\mathcal{S}_t$ and the mapping $v \mapsto \varphi_t(u)$ is linear in v for each fixed u, then there exists a scalar κ_t such that

$$\frac{g(Z_{i,t})}{V_{i,t}} = \kappa_t \frac{X_{i,t}}{V_{i,t}} \quad \text{for all } i, \tag{4.16}$$

[4] Formally, as $N \to \infty$, the support of $\mathcal{S}_t$ contains an open subset of $\mathbb{R}^2_+$, ruling out degenerate cases in which firms lie on a lower-dimensional set.

and $X_{i,t}/V_{i,t}$ is a unique sufficient statistic for expected excess returns in the sense of Definition 4.1. Equivalently, (4.16) holds if and only if

$$g(Z_{i,t}) = \kappa_t X_{i,t} \quad \textit{for all } i,$$

so that linear intensity sufficiency in returns corresponds to a level-proportional cash-flow exposure.

Proof is in Appendix A.1.

Proposition 3 makes explicit the identifying restrictions required for emission intensity to be uniquely sufficient in equilibrium. Two firms with the same ratio $X_{i,t}/V_{i,t}$ must have identical return-relevant exposure $g(Z_{i,t})/V_{i,t}$ regardless of their absolute scale, which requires both homogeneity of cash-flow exposures and scale invariance across firms. When they fail, additional characteristics beyond $X_{i,t}/V_{i,t}$ must enter priced exposure. Crucially, the restrictions have sharp empirical implications—spanning and scale invariance—that can be directly tested in the data.

The homogeneity and scale-invariance conditions are equivalent to requiring that the exposure function satisfies an additivity property under corporate reorganizations. Section 4.7 develops the economic foundations of these restrictions and identifies two distinct channels through which they fail in practice.

Notably, the analysis focuses on environments in which climate and transition shocks enter firm cash flows in dollar terms, even when shocks are large or fat-tailed. If transition risk instead operates through survival or default-type thresholds that generate losses proportional to firm value, then the homogeneity and scale-invariance conditions required for intensity sufficiency may hold. In that case, emission intensity would naturally emerge as the relevant priced characteristic. My framework accommodates this regime as a boundary case; the empirical results indicate that it has not dominated pricing over the sample period studied.

Finally, the proposition also clarifies what does and does not hinge on market completeness. Completeness affects the equilibrium *price* of risk λ_t in (4.14), but it does not impose the scale and homogeneity restrictions on $g(\cdot)$ needed for intensity sufficiency. These implications motivate the empirical sufficiency tests in Section 8.2, which evaluate the scale-invariance and spanning restrictions associated with Proposition 3.

4.7 Economic foundations of the sufficiency conditions

Proposition 3 shows that emission intensity is a sufficient statistic for priced exposure only when the exposure function $g(\cdot)$ is homogeneous of degree one in $(X_{i,t}, V_{i,t})$ and satisfies scale invariance. These are mathematical restrictions on the exposure function. This subsection provides their economic interpretation and explains why they are unlikely to hold in general.

The key economic property underlying homogeneity is *additivity* of exposure. Consider a firm with emissions $X_{i,t}$ and value $V_{i,t}$ that is split into n identical entities, each with emissions $X_{i,t}/n$ and value $V_{i,t}/n$. Additivity requires total dollar exposure to a climate shock to be preserved:

$$n \cdot g\left(\frac{X_{i,t}}{n}, \frac{V_{i,t}}{n}, Z_{-X,V}\right) = g(X_{i,t}, V_{i,t}, Z_{-X,V})\,. \tag{4.17}$$

where $Z_{-X,V}$ collects all other firm characteristics held fixed in the partition experiment.

To see why additivity implies homogeneity of degree one, consider two operations. First, splitting: partition a firm into n identical entities, each with emissions $(X_{i,t}/n)$ and value $(V_{i,t}/n)$. Additivity requires the total exposure of the n entities to equal that of the original firm, implying $[g\left(\frac{X_{i,t}}{n}, \frac{V_{i,t}}{n}\right) = \frac{g(X_{i,t},V_{i,t})}{n}.]$ Second, merging: combining n identical firms with emissions X and value V into a single firm with emissions nX and value nV implies $g(nX, nV) = ng(X, V)$. Together, these operations yield $g(\lambda X, \lambda V) = \lambda g(X, V)$ for all positive rational λ. Under continuity, the relation extends to all $\lambda > 0$, establishing homogeneity of degree one in $(X_{i,t}, V_{i,t})$. Consequently, value-normalized exposure $g(Z_{i,t})/V_{i,t}$ depends on firm characteristics only through emission intensity $X_{i,t}/V_{i,t}$.

Additivity holds when climate or policy shocks translate into cash-flow impacts that are mechanically proportional to emissions and independent of firm scale. A uniform carbon tax is the canonical example: a change $\Delta\tau$ generates a compliance cost $\Delta\tau\, X_{i,t}$, and splitting the firm preserves total exposure. The sustainable-discount-rate representation implicitly relies on exactly this structure. When the planner wedge takes the form $r^x_{i,t} \approx r_t + (S_t - \tau_t)X_{i,t}/V_{i,t}$, the underlying exposure is $g(Z_{i,t}) = \eta X_{i,t}$, which satisfies additivity by construction.

This property, however, can fail through two economically distinct channels that are both empirically relevant. First, many real-world climate policies do not operate on a per-ton basis. Regulatory thresholds, disclosure rules, and benchmark-based allocations often create fixed or level-based compliance costs that depend on absolute scale rather than proportional emissions. For example, firms above regulatory cutoffs must purchase allowances or install monitoring infrastructure,

while smaller firms may avoid these costs entirely even when aggregate emissions are unchanged. Similarly, net-zero benchmarks are typically allocated using sectoral budgets, installed capacity, or historical baselines rather than firm value, so two firms with identical emission intensity but different scale can face very different compliance gaps. In these settings, corporate structure affects total exposure and additivity fails at the level of the policy instrument.

Second, the property can fail even under a perfectly uniform carbon tax. While the policy shock is linear per ton, the cash-flow impact of that shock depends on firms' pricing power, input substitution possibilities, contractual structure, and adjustment costs. Firms differ in their ability to pass carbon costs through to output prices, implying that identical emissions can translate into very different cash-flow impacts across firms. Because these pass-through rates vary systematically with market structure and technology, the mapping from carbon price shocks to cash flows is generically firm-specific and scale-dependent. Achieving additivity therefore requires not only a per-unit instrument but also homogeneous economic incidence across firms—a joint restriction that is difficult to justify empirically.

The homogeneity and scale-invariance restrictions therefore require a joint set of conditions: climate exposure must operate through per-unit instruments, the incidence of those instruments must be homogeneous across firms, and regulation must not introduce thresholds, benchmarks, or trajectory-based constraints. Relaxing any of these assumptions allows exposure to depend on characteristics beyond emission intensity.

The mechanisms studied in later sections illustrate these channels. Trajectory-based regulation and net-zero benchmarks generate non-additive exposure through policy design, while endogenous technology and adjustment choices generate heterogeneous incidence even under uniform carbon pricing. These examples illustrate a general principle: whenever the incidence of climate risk varies systematically with firm scale, emission intensity cannot serve as a sufficient statistic for priced exposure.

More broadly, the exposure function $g(Z_{i,t})$ is generically multidimensional. Emissions determine the direct incidence of per-unit policy shocks, but the cash-flow impact of a given shock is mediated by firm-specific characteristics—pricing power, demand elasticity, contractual rigidity, input substitution, and adjustment costs—that vary across firms and are not summarized by any single emission metric. A sufficient statistic would require these mediating characteristics to be either irrelevant for cash-flow sensitivity or perfectly correlated with the chosen emission measure in the cross-section. Neither condition holds generically. The single-statistic benchmark in Sec-

tion 6.1, in which $g(Z_{i,t}) = \eta X_{i,t}$, should therefore be viewed as a tractable benchmark that isolates the direct policy channel by shutting down mediating heterogeneity, rather than as a description of the typical firm environment.

TThis argument applies symmetrically to any candidate emission characteristic, not only to intensity. Heterogeneous incidence implies that firms with identical emissions, identical intensity, and identical policy exposure can nevertheless have different cash-flow sensitivities to climate shocks. The empirical finding in Section 8.2 that levels dominate intensity in spanning tests therefore reflects which exposure mechanisms are quantitatively important in the data, not a theoretical claim that levels are universally sufficient. Identifying the full exposure function $g(Z_{i,t})$—incorporating incidence heterogeneity, pass-through, and technology—remains an open structural question to which this paper's selection framework provides the analytical foundation.

5 Transition Risk, Emission Intensity, and Priced Characteristics

In this section, I place emission-based climate metrics within a general linear-factor pricing environment. The key distinction is between (i) *cash-flow exposures* $g_k(Z_{i,t})$, which describe how climate and transition shocks enter dividends and are disciplined by technology and policy design, and (ii) *return exposures*, which are necessarily value-normalized because premia are measured per unit of invested wealth and therefore load on $g_k(Z_{i,t})/V_{i,t}$. The central question is thus when the value-normalized exposure can be summarized by emission intensity $X_{i,t}/V_{i,t}$ alone. Proposition 3 characterizes the knife-edge conditions for a one-factor environment. The remainder of this section extends the representation to multiple factors and clarifies how the planner-based sustainable-discount-rate wedge relates to the equilibrium pricing of stochastic transition risk.[5]

5.1 Sustainable discount rates and intensity in the complete-markets benchmark

In the complete-markets benchmark with a representative planner and a well-defined social cost of carbon $\{S_t\}_t$, a Pigouvian tax sequence $\{\tau_t\}_t$ or an equivalent cap-and-trade scheme decentralizes the planner's allocation. Let $V^*_{i,t}$ denote the value of firm i at date t along this allocation and let $X^*_{i,t}$ denote its emissions. For a given financial discount rate r_t, one can define firm-specific *sustainable* discount rates $r^x_{i,t}$ by equating the present value of taxed cash flows discounted at r_t

[5] I now adopt time subscripts. All static results in Sections 2-4 carry over to the dynamic setting under standard CARA-normal assumptions (see Internet Appendix D).

with the present value of untaxed cash flows discounted at $r^x_{i,t}$. Restating the planner-implied wedge derived in Section 3, a first-order expansion around a baseline with zero wedge yields

$$r^x_{i,t} \approx r_t + (S_t - \tau_t)\frac{X^*_{i,t}}{V^*_{i,t}}, \tag{5.1}$$

where $S_t - \tau_t$ is the gap between the social cost of carbon and the prevailing carbon price. In this benchmark, emission intensity summarizes the *deterministic* implementation wedge between financial and sustainable discount rates required to decentralize the planner's allocation.

Crucially, (5.1) is a statement about the *level* of the wedge needed for implementation at given S_t, not a restriction on the pricing of *stochastic* climate or policy risk in equilibrium. Intensity appears because cash-flow losses proportional to $X^*_{i,t}$ are being expressed on a per-unit-of-value basis.

5.2 Transition risk as stochastic exposure to policy or technology shocks

Consider now transition risk, defined as stochastic shocks to future climate policy, carbon prices, or technology that affect cash flows. To fix ideas, let τ_{t+1} denote a random carbon price at date $t+1$ and suppose that, holding quantities fixed at date t, firm i's date-$t+1$ dividend contains a term

$$D_{i,t+1} = \tilde{D}_{i,t+1} - \tau_{t+1} X_{i,t}, \tag{5.2}$$

where $\tilde{D}_{i,t+1}$ collects all other components orthogonal (conditionally) to τ_{t+1}. Let $\hat{\tau}_{t+1} \equiv \tau_{t+1} - \mathbb{E}_t[\tau_{t+1}]$ denote the innovation in the carbon price and write

$$D_{i,t+1} = \mu_{i,t+1} + g^{(\tau)}_{i,t}\hat{\tau}_{t+1}, \qquad g^{(\tau)}_{i,t} = -X_{i,t},$$

with $\mu_{i,t+1}$ conditionally independent of $\hat{\tau}_{t+1}$. In any equilibrium with a strictly positive SDF m_{t+1} that is conditionally square-integrable, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = -\frac{\mathrm{Cov}_t(m_{t+1}, D_{i,t+1})}{\mathbb{E}_t[m_{t+1}]\, V_{i,t}},$$

where $V_{i,t}$ denotes the date-t market value (price) of the claim to $D_{i,t+1}$. Under a standard log-linear approximation, and assuming that $\hat{\tau}_{t+1}$ is a factor in m_{t+1}, this yields a linear factor structure in

which the transition-risk premium associated with $\hat{\tau}_{t+1}$ takes the value-normalized form

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_t^{(\tau)} \frac{g_{i,t}^{(\tau)}}{V_{i,t}} = -\lambda_t^{(\tau)} \frac{X_{i,t}}{V_{i,t}}, \tag{5.3}$$

where the price of transition risk $\lambda_t^{(\tau)}$ is proportional to $-\mathrm{Cov}_t(m_{t+1}, \hat{\tau}_{t+1})$.

More generally, allowing for multiple transition channels, including demand shocks, stranded-asset risk, and technology shocks, dividends take the generic form

$$D_{i,t+1} = \tilde{D}_{i,t+1} + \sum_{k=1}^{K} g_k(Z_{i,t}) f_{k,t+1}, \tag{5.4}$$

where $f_{k,t+1}$ are mean-zero transition or climate factors and $g_k(Z_{i,t})$ are dollar exposure coefficients that depend on characteristics $Z_{i,t}$, which may include $X_{i,t}$, $V_{i,t}$, $X_{i,t}/V_{i,t}$, changes in emissions, efficiency measures, and so on. Asset pricing maps these dollar exposures into return premia through value normalization, so the empirically relevant objects are $g_k(Z_{i,t})/V_{i,t}$.

5.3 Complete versus incomplete markets and the priced characteristic

The distinction between complete and incomplete markets affects the prices of transition risk but not, for a given technology and shock structure, the mechanism-implied dollar exposure functions $g_k(Z_{i,t})$. To make this precise, consider the following general linear-factor environment.

Let $\{f_{k,t+1}\}_{k=1}^{K}$ be conditionally mean-zero factors, and suppose that dividends satisfy (5.4):

$$D_{i,t+1} = \mu_{i,t+1} + \sum_{k=1}^{K} g_k(Z_{i,t}) f_{k,t+1},$$

with $\mu_{i,t+1}$ conditionally independent of $\{f_{k,t+1}\}_k$ and $Z_{i,t}$ $\mathcal{F}_t$–measurable. Assume that there exists an SDF of the form

$$m_{t+1} = a_t + \sum_{k=1}^{K} b_{k,t} f_{k,t+1} + \varepsilon_{t+1}, \tag{5.5}$$

where $a_t > 0$, $\{b_{k,t}\}_k$ are $\mathcal{F}_t$–measurable, and ε_{t+1} is a mean-zero residual orthogonal to all traded payoffs. Under complete markets, the SDF is unique and coincides with the representative agent's or planner's marginal rate of substitution. Under incomplete markets, the SDF is not unique in general, but in our CARA-normal environment a particular affine representation is pinned down by equilibrium.

Under a local linear approximation in which higher-order terms in the factors are negligible,

the Euler equation $\mathbb{E}_t[m_{t+1}R_{i,t+1}] = 1$ implies the following characterization.

Proposition 4 (Priced characteristics in a linear factor structure)**.** *Suppose that dividends are given by (5.4) and that the SDF admits the representation (5.5), with $\{f_{k,t+1}\}_k$ mutually uncorrelated conditional on $\mathcal{F}_t$. Then, up to a first-order approximation in the conditional variances of the factors,*

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \sum_{k=1}^{K} \lambda_{k,t} \frac{g_k(Z_{i,t})}{V_{i,t}}, \tag{5.6}$$

where

$$\lambda_{k,t} = -\frac{b_{k,t}\,\mathrm{Var}_t(f_{k,t+1})}{\mathbb{E}_t[m_{t+1}]}, \qquad k = 1, \dots, K, \tag{5.7}$$

are the factor prices of risk. In particular, for each factor k, the priced return exposure is the value-normalized object $g_k(Z_{i,t})/V_{i,t}$: two assets with identical $g_k(Z_{i,t})/V_{i,t}$ for all k have the same expected excess return in this approximation, regardless of how $g_k(\cdot)$ depends on primitives such as $(X_{i,t}, V_{i,t}, \dots)$.

The proof is immediate from substituting (5.4) and (5.5) into the Euler equation, using conditional mutual uncorrelatedness of the factors and discarding higher-order covariance terms. The result holds irrespective of whether markets are complete or incomplete. Completeness affects the determination of the coefficients $\{b_{k,t}\}_k$ and thus the prices of risk $\{\lambda_{k,t}\}_k$, but not the functional form of the dollar exposure coefficients $g_k(Z_{i,t})$, which are pinned down by technology and the way shocks enter cash flows.

The separation between market structure and the exposure function $g_k(\cdot)$ is exact when firm characteristics are taken as predetermined at date t. In richer general-equilibrium environments, firms may adjust emissions, technology, or production in response to expected policy and risk premia. In such settings, the equilibrium mapping $g_k(Z_{i,t})$ can itself depend on the price of risk through firms' optimal decisions. The linear representation in Proposition 4 should therefore be interpreted as describing the pricing of exposures conditional on the characteristics realized at date t. The dynamic production model in Appendix D provides an example in which exposures are endogenous, yet the value-normalized representation (5.6) still describes the cross-sectional pricing of realized exposures.

In the transition-tax example (5.2) with a single factor $f_{1,t+1} = \hat{\tau}_{t+1}$, Proposition 4 identifies $g_1(Z_{i,t})$ with $-X_{i,t}$ and thus the priced return exposure with $-X_{i,t}/V_{i,t}$. Once we allow for more general specifications in which $g_k(Z_{i,t})$ depends on levels, changes, technology, or policy allocations,

there is no reason to expect that a single statistic, such as $X_{i,t}/V_{i,t}$, will be sufficient to summarize transition exposure across models. The next section illustrates how alternative, economically motivated specifications of $g_k(\cdot)$ generate premia on emission levels, intensity, and changes through the value-normalized exposures $g_k(Z_{i,t})/V_{i,t}$, and how simple economic restrictions discipline admissible functional forms.

6 Illustrative Exposure Specifications

Section 4.7 showed that emission intensity is a sufficient statistic only under economically demanding homogeneity and scale-invariance restrictions. The goal of this section is complementary. Rather than asking when intensity is sufficient, I now construct exposure functions implied by concrete policy and technology mechanisms and derive the priced characteristics they generate. The incomplete-markets equilibrium in Section 4 relied on a simple benchmark in which dividends load linearly on emission levels. Realistic climate exposure, however, is richer. Policy shocks need not scale only with current emissions: they can depend on recent changes, distances from regulatory benchmarks, and technology choices that reshape how cash flows respond to climate risk. The analysis below therefore derives exposure functions directly from economically motivated mechanisms. The three specifications that follow–transition losses proportional to levels (the Section 4 benchmark), exposures depending jointly on levels and changes (motivated by trajectory-based regulation), and path-based exposures under net-zero commitments–illustrate a general lesson. Once exposure functions are disciplined by realistic policy design and technology, the priced characteristics that enter equilibrium returns need not collapse to emission intensity.

6.1 Exposure proportional to emission levels

I start with the simplest case: cash-flow exposure proportional to emission levels. For example, if a per-ton carbon price increases by \$10 and a firm emits 100 kilotons, its compliance cost rises by \$1 million. A reduced-form representation of this channel is $\theta_{i,t} = \eta X_{i,t}$, linear in emissions.[6] This specification is natural when policy operates through per-unit instruments–carbon taxes, permit obligations, or output restrictions that scale directly with total emissions.

Asset pricing converts dollar cash-flow exposure into a return premium by dividing by firm value. Under $\theta_{i,t} = \eta X_{i,t}$, the priced exposure is therefore $\eta X_{i,t}/V_{i,t}$, so emission intensity emerges

[6] To streamline notation, I denote the dollar exposure to the climate factor by $\theta_{i,t}$.

mechanically as a sufficient statistic for pricing. The key point, however, is that this sufficiency is not a generic implication of equilibrium pricing: it holds here only because the primitive cash-flow channel is linear in levels and does not depend on other scale objects. Once exposure depends on additional characteristics such as changes in emissions, compliance constraints, or distance from a regulatory path, two firms with the same $X_{i,t}/V_{i,t}$ can have different $g(Z_{i,t})/V_{i,t}$, and intensity no longer spans priced exposure.

Single-factor setup. To formalize this, consider a single climate factor ζ_{t+1} with $\mathbb{E}_t[\zeta_{t+1}] = 0$ and write firm i's dividend as

$$D_{i,t+1} = \tilde{D}_{i,t+1} + \theta_{i,t}\,\zeta_{t+1}, \tag{6.1}$$

where $\tilde{D}_{i,t+1}$ is orthogonal to ζ_{t+1}. The cash-flow exposure $\theta_{i,t}$ depends on firm characteristics $Z_{i,t}$ through

$$\theta_{i,t} = g(Z_{i,t}), \tag{6.2}$$

where $g : \mathcal{Z} \to \mathbb{R}$ maps characteristics–emissions, firm value, technology shifters, policy allocations–into dollar sensitivities. In the Section 4 benchmark, $Z_{i,t} = X_{i,t}$ and $g(z) = \eta z$, so $\theta_{i,t} = \eta X_{i,t}$.

Under the log-linear approximation in Proposition 4, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t \frac{g(Z_{i,t})}{V_{i,t}}, \tag{6.3}$$

where λ_t is the equilibrium price of factor risk. The priced object is the value-normalized exposure $g(Z_{i,t})/V_{i,t}$.

From mechanisms to empirical predictors. The functional form of $g(\cdot)$ is disciplined by economic structure: it reflects how climate shocks enter cash flows. If $g(Z_{i,t}) = \eta X_{i,t}$, equation (6.3) delivers

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t \eta \frac{X_{i,t}}{V_{i,t}},$$

so emission intensity becomes the return exposure even though the primitive cash-flow channel scales with levels. But if policy operates through adjustment costs, compliance constraints, or net-zero benchmarks, $g(Z_{i,t})$ naturally depends on additional characteristics–emission changes $\Delta X_{i,t}$, compliance constraints, and distances from regulatory paths or net-zero benchmarks. The priced return exposure remains $g(Z_{i,t})/V_{i,t}$, but this object need not collapse to intensity.

The theory thus disciplines measurement: an emission-based predictor is admissible only if it can be derived from a coherent cash-flow mechanism, and it is empirically sufficient if it spans the return variation associated with $g(Z_{i,t})/V_{i,t}$. Intensity is sufficient only when dollar exposures satisfy the homogeneity and scale-invariance conditions in Proposition 3–conditions that fail once $g(\cdot)$ depends on characteristics beyond $(X_{i,t}, V_{i,t})$.

Multi-factor extension. With multiple climate shocks $\boldsymbol{\zeta}_{t+1} = (\zeta_{1,t+1}, \ldots, \zeta_{K,t+1})^\top$, dividends become

$$D_{i,t+1} = \tilde{D}_{i,t+1} + \sum_{k=1}^{K} \theta_{i,t}^{(k)} \zeta_{k,t+1}, \qquad \theta_{i,t}^{(k)} = g_k(Z_{i,t}),$$

and expected returns inherit the structure

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \sum_{k=1}^{K} \lambda_t^{(k)} \frac{g_k(Z_{i,t})}{V_{i,t}}. \tag{6.4}$$

Each factor $\zeta_{k,t+1}$ is priced through its exposure function $g_k(Z_{i,t})/V_{i,t}$. Physical damage shocks, carbon-price innovations, and technology shocks can each generate distinct exposure functions, and there is no reason to expect a single characteristic such as intensity to span all of them.

Invariance restrictions on exposure functions. Not every function $g(\cdot)$ is economically coherent. Dollar exposures must satisfy basic invariance requirements. Splitting a firm into two identical copies with half the emissions and half the value should preserve total exposure (additivity). Holding technology fixed, higher emissions should not reduce exposure to a per-unit tax or damage shock (monotonicity). These restrictions ensure that exposure functions correspond to actual economic mechanisms rather than arbitrary functional forms.

Under these disciplines, emission levels, intensities, changes, and SCC-weighted measures are all admissible–each can be derived from a specific policy or technology channel. But they are empirically sufficient only if they span the priced exposure $g_k(Z_{i,t})/V_{i,t}$. When cash flows depend on characteristics beyond those captured by a given metric, that metric fails the spanning tests in Section 8.2.

6.2 Transition losses depending on levels and changes

Policy tightening doesn't just penalize current emissions–it targets firms moving in the wrong direction. A coal plant ramping up capacity faces different regulatory risk than one phasing down,

even if both have similar emission levels today. If enforcement responds to trajectories rather than stocks, cash-flow exposure depends on both $X_{i,t}$ and $\Delta X_{i,t}$, and intensity alone cannot span the priced risk.

To see this, consider a policy shock that arrives when regulators perceive insufficient progress toward transition goals. Firms increasing emissions signal resistance to decarbonization and face steeper penalties when enforcement tightens. Formally, let transition losses depend on both the current emission stock and recent changes:

$$\Gamma_{i,t} = \phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}, \tag{6.5}$$

where $\Delta X_{i,t} = X_{i,t} - X_{i,t-1}$ and $\phi_1, \phi_2 > 0$. The level $X_{i,t}$ captures baseline exposure; the change $\Delta X_{i,t}$ captures trajectory risk.

Policy-tightening shocks and transition losses. Let $J_{t+1} \in \{0, 1\}$ denote a binary tightening event with $\mathbb{E}_t[J_{t+1}] = p_t$, and define the innovation

$$\zeta_{t+1} = J_{t+1} - p_t.$$

The shock J_{t+1} captures unanticipated regulatory acceleration–stricter enforcement, faster phase-outs, or abrupt carbon-price increases. Firm i's dividend becomes

$$D_{i,t+1} = \bar{D}_{i,t+1} - \Gamma_{i,t} J_{t+1}, \tag{6.6}$$

where $\bar{D}_{i,t+1}$ is the dividend absent tightening and $\Gamma_{i,t}$ is the dollar loss when tightening occurs. This loss reflects stranded assets, accelerated abatement costs, or compliance penalties that hit when policy shifts earlier than expected.

Substituting (6.5) into (6.6) and using $\zeta_{t+1} = J_{t+1} - p_t$ yields

$$D_{i,t+1} = \bar{D}_{i,t+1} - (\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}) p_t - (\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}) \zeta_{t+1}.$$

The cash-flow exposure to the transition factor is

$$g_\zeta(Z_{i,t}) = -(\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}),$$

where $Z_{i,t}$ includes both levels and changes.

Equilibrium pricing with trajectory dependence. Under the log-linear approximation in Proposition 4, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t}] = \lambda_{\zeta,t} \frac{\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}}{V_{i,t}} + \text{other factor terms.} \tag{6.7}$$

Both levels and changes enter through the value-normalized exposure $(\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t})/V_{i,t}$. When $\phi_2 = 0$, the model collapses to the Section 6.1 baseline where exposure scales with levels alone. When $\phi_2 > 0$, firms ramping up emissions bear additional transition risk that cannot be captured by $X_{i,t}/V_{i,t}$ alone.

This has direct implications for intensity-based screens. Two firms with identical emission intensity can have different priced exposure if one is increasing emissions ($\Delta X_{i,t} > 0$) while the other is cutting ($\Delta X_{i,t} < 0$). The sufficiency conditions in Proposition 3 fail because the exposure function depends on an additional characteristic–recent emission changes–that is not summarized by the intensity ratio $X_{i,t}/V_{i,t}$.

Empirical implications. Equation (6.7) suggests that the appropriate empirical specification includes both $X_{i,t}/V_{i,t}$ and $\Delta X_{i,t}/V_{i,t}$ as predictors. A regression of returns on these characteristics is a reduced-form test of whether trajectory-based exposure is priced. The coefficients need not recover (ϕ_1, ϕ_2) separately, as they combine technological sensitivities with the equilibrium price of risk $\lambda_{\zeta,t}$, but the joint specification is the natural empirical analogue of the mechanism in (6.5).

The incomplete-markets structure, heterogeneous climate concern, and SDF decomposition from Section 4 remain unchanged. What changes is the exposure function $g_\zeta(Z_{i,t})$, which now depends on both stocks and flows. This illustrates the general point: different policy designs generate different priced characteristics, and intensity suffices only under the knife-edge restriction that exposure depends solely on $(X_{i,t}, V_{i,t})$.

6.3 Net-zero objectives and path-based exposure

Net-zero commitments change the regulatory game. A firm emitting 150 kilotons with a 100-kiloton benchmark faces different transition risk than a firm at 75 kilotons with a 50-kiloton benchmark–even though both are 50% above their targets. What matters is not just the intensity ratio but the

absolute distance from a mandated path, and that distance varies with scale even when $X_{i,t}/V_{i,t}$ is fixed.

This matters because net-zero benchmarks are typically allocated using level-based rules–sectoral budgets, capacity shares, historical baselines–not intensity targets. Two firms with identical emission intensity can have vastly different compliance trajectories if one is large and distant from its benchmark while the other is small and close. If enforcement tightens or phase-out schedules accelerate, exposure scales with the gap $(X_{i,t} - X_{i,t}^{NZ})$, not just with current emissions or their ratio to firm value. Because $X_{i,t}^{NZ}$ is not proportional to $V_{i,t}$ under level-based allocations, the normalized gap $(X_{i,t} - X_{i,t}^{NZ})/V_{i,t}$ varies across firms even when $X_{i,t}/V_{i,t}$ is held fixed. Intensity alone therefore cannot span priced exposure in (6.9).

Path-based exposure under net-zero regulation. Let $X_{i,t}^{NZ}$ denote firm i's net-zero-consistent benchmark at date t, constructed from sectoral pathways, regulatory allocations, or firm commitments. This benchmark is predetermined at t and reflects policy design or historical negotiations rather than equilibrium outcomes; it may be measured with error in practice. Let ζ_{t+1} denote net-zero transition news: an acceleration of phase-out schedules, stricter enforcement, or increases in marginal abatement costs along the transition path.

Cash-flow exposure to ζ_{t+1} reflects two channels. First, per-unit losses that scale with current emissions $X_{i,t}$ (permit obligations, tax exposure). Second, repricing and compliance risk tied to the distance from the benchmark, $(X_{i,t} - X_{i,t}^{NZ})$. Formally, let

$$g_\zeta(Z_{i,t}) = \vartheta_1 X_{i,t} + \vartheta_2 (X_{i,t} - X_{i,t}^{NZ}), \tag{6.8}$$

where $\vartheta_1, \vartheta_2 \geq 0$. The level $X_{i,t}$ captures baseline exposure, while the gap $(X_{i,t} - X_{i,t}^{NZ})$ captures path-deviation risk under net-zero regulation.[7]

Equilibrium pricing with path dependence. Under the log-linear approximation in Proposition 4, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t}] = \lambda_{\zeta,t} \frac{g_\zeta(Z_{i,t})}{V_{i,t}} + \text{other factor terms}. \tag{6.9}$$

[7] One can additionally allow adjustment-speed penalties proportional to $\Delta X_{i,t}$ as in Section 6.2.

The priced exposure is the value-normalized function

$$\frac{g_\zeta(Z_{i,t})}{V_{i,t}} = \vartheta_1 \frac{X_{i,t}}{V_{i,t}} + \vartheta_2 \frac{X_{i,t} - X_{i,t}^{NZ}}{V_{i,t}}.$$

Intensity $X_{i,t}/V_{i,t}$ alone cannot span this exposure. Even firms with identical intensity can have sharply different path deviations $(X_{i,t} - X_{i,t}^{NZ})/V_{i,t}$ because benchmarks are allocated by levels or capacity shares, not intensity ratios. A large firm at 100% above its benchmark carries fundamentally different compliance risk than a small firm at 100% above its benchmark—the absolute emission gap that must be closed differs by scale.

The sufficiency conditions in Proposition 3 require that priced exposure depend only on $(X_{i,t}, V_{i,t})$ and satisfy homogeneity and scale invariance. Net-zero exposures violate these conditions in two ways. First, they depend on an additional predetermined characteristic, $X_{i,t}^{NZ}$, that reflects policy design rather than current firm decisions. Second, because benchmarks are level-based, the gap $(X_{i,t} - X_{i,t}^{NZ})/V_{i,t}$ varies with scale even when intensity is fixed. Consequently, intensity-based screens systematically misclassify firms when compliance risk is path-dependent.

Empirical implications. Equation (6.9) suggests that empirical specifications motivated by path-based regulation include not only $X_{i,t}/V_{i,t}$ but also the normalized deviation $(X_{i,t} - X_{i,t}^{NZ})/V_{i,t}$. A spanning test that includes only intensity implicitly imposes $\vartheta_2 = 0$, that is, that transition risk is insensitive to distance from the benchmark. This restriction is economically strong in settings where compliance is assessed relative to net-zero pathways rather than current flow emissions alone.

Net-zero objectives thus provide a concrete illustration of the paper's central message: the appropriate emission-based predictor is determined by policy design, not by intensity a priori. When regulation operates through path constraints, the priced characteristic is the exposure function $g_\zeta(Z_{i,t})/V_{i,t}$, which depends on levels, gaps, and changes jointly. Intensity sufficiency is a knife-edge special case that holds only when transition risk ignores path deviations and adjustment dynamics—conditions unlikely to hold in environments where cumulative emissions or trajectory compliance drives enforcement.

To conclude this section, Table 1 summarizes the model's implications for empirical implementation. For each economic mechanism, formalized through an exposure function $g(Z_{i,t})$, the table reports the object that prices equilibrium returns and the corresponding variable implied for return regressions. The table serves as a compact reference, abstracting from regression controls and normalization that are common across specifications.

7 Jevons-type Efficiency Effects

This section shows how endogenous energy efficiency can undermine emission intensity as a sufficient statistic for priced transition exposure. The goal is not to build a full production model, but to isolate a simple channel through which a technological choice shifts energy demand and therefore reshapes the priced characteristics in the linear factor structure. A Jevons-type rebound can arise in which higher efficiency expands energy use and emissions, so that firms that appear greener by intensity measures can have higher equilibrium premia by increasing their effective exposure to priced climate or policy shocks. Intuitively, improvements in energy efficiency lower marginal production costs, which induces firms to expand output. The expansion of production increases total energy use and emissions, potentially raising exposure to climate or policy shocks even as emissions per unit of output fall. As a result, efficiency improvements can increase rather than decrease the value-relevant exposure to priced climate risk. While the section is purely illustrative, the mechanism has empirical relevance, as discussed at the end of the section.

Efficiency choice and rebound. At date t, firm i uses energy $E_{i,t}$ and chooses an efficiency parameter $a_{i,t} > 0$ to produce an energy service

$$S_{i,t} = a_{i,t}E_{i,t}, \qquad X_{i,t} = \gamma E_{i,t},\ \gamma > 0,$$

so that $a_{i,t}$ increases service per unit of energy (and per unit of emissions). Let P_t^E denote the real energy price and suppose operating profits admit the reduced form

$$\Pi_{i,t} = \pi_i(S_{i,t}) - P_t^E E_{i,t} - \Psi_i(a_{i,t}),$$

with π_i increasing and concave and Ψ_i the private cost of efficiency. For given $(P_t^E, a_{i,t})$, optimal energy demand $E_{i,t}^*(a_{i,t})$ satisfies

$$a_{i,t}\pi_i'(a_{i,t}E_{i,t}^*) = P_t^E.$$

A Jevons-type rebound region is characterized by

$$\frac{\partial E_{i,t}^*}{\partial a_{i,t}} \geq 0,$$

so that higher efficiency does not reduce, and may increase, energy use and therefore emissions $X_{i,t} = \gamma E^*_{i,t}(a_{i,t})$. Because emissions are induced by the equilibrium energy choice, any shock sensitivity expressed in the primitives $(E^*_{i,t}, a_{i,t}, Z_{i,t})$ is equivalently a function of $(X_{i,t}, a_{i,t}, Z_{i,t})$, and its comparative statics with respect to efficiency need not be negative in the rebound region. Empirically, this implies that intensity can improve mechanically while priced exposure rises if efficiency shifts the firm into the rebound region, so intensity-based greenness need not be informative about the relevant $g(Z)$.

Pricing with efficiency-dependent exposures. Let ζ_{t+1} denote an aggregate climate or policy shock and let $\varepsilon_{E,t+1}$ denote an energy-price (or energy-demand) shock. Write dividends as

$$D_{i,t+1} = \tilde{D}_{i,t+1} + g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t})\,\zeta_{t+1} + g_E(X_{i,t}, a_{i,t}, Z_{i,t})\,\varepsilon_{E,t+1}, \tag{7.1}$$

where $Z_{i,t}$ collects other characteristics and $\tilde{D}_{i,t+1}$ is orthogonal (conditionally) to $(\zeta_{t+1}, \varepsilon_{E,t+1})$. The Jevons mechanism matters because efficiency affects energy demand and therefore both emissions and shock sensitivity, so the exposure coefficients in (7.1) generically depend on $(X_{i,t}, a_{i,t})$ jointly.

Under the same linear-factor pricing logic as in Proposition 4 and Section 4, expected excess returns admit the approximation

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t^{(\zeta)}\, g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t}) + \lambda_t^{(E)}\, g_E(X_{i,t}, a_{i,t}, Z_{i,t}), \tag{7.2}$$

with equilibrium prices of risk $(\lambda_t^{(\zeta)}, \lambda_t^{(E)})$ determined by preferences, risk sharing, and the aggregate exposure of cash flows and (where relevant) unspanned damages to $(\zeta_{t+1}, \varepsilon_{E,t+1})$. Equation (7.2) makes clear that the priced characteristics are the exposure functions $g_\zeta(\cdot)$ and $g_E(\cdot)$, which depend on efficiency and emissions jointly. Once $a_{i,t}$ shifts energy use and hence shock sensitivity, there is no generic reason for premia to collapse to a function of $X_{i,t}/V_{i,t}$ (or to $X_{i,t}$ alone).

A simple example with reversed comparative statics

To see how a rebound can raise premia, consider

$$\pi_i(S) = pS - \frac{c}{2}S^2, \qquad p > 0,\ c > 0,$$

and ignore $\Psi_i(a)$ for the moment. The optimal energy choice is

$$E^*_{i,t}(a_{i,t}) = \frac{a_{i,t}p - P^E_t}{ca^2_{i,t}} \quad \text{for } a_{i,t}p > P^E_t,$$

so that

$$\frac{\partial E^*_{i,t}}{\partial a_{i,t}} = \frac{2P^E_t - a_{i,t}p}{ca^3_{i,t}}.$$

Hence, for $a_{i,t} \leq 2P^E_t/p$, higher efficiency increases energy use and emissions. Suppose the climate shock enters dividends through emission levels,

$$g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t}) = \eta X_{i,t} = \eta\gamma E^*_{i,t}(a_{i,t}), \qquad g_E \equiv 0,$$

and the equilibrium price of climate risk satisfies $\lambda^{(\zeta)}_t > 0$. Then (7.2) reduces to

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda^{(\zeta)}_t \eta\gamma E^*_{i,t}(a_{i,t}),$$

and therefore, in the rebound region,

$$\frac{\partial}{\partial a_{i,t}}\left(\mathbb{E}_t[R_{i,t+1}] - R_{f,t}\right) = \lambda^{(\zeta)}_t \eta\gamma \frac{\partial E^*_{i,t}}{\partial a_{i,t}} \geq 0,$$

with strict inequality for $a_{i,t} < 2P^E_t/p$. Thus, higher efficiency can increase equilibrium premia by raising emissions and hence exposure to the priced climate factor. At the same time, the efficiency increase improves intensity mechanically by raising service per unit of energy, so intensity-based rankings can move in the opposite direction from level-based priced exposure in rebound regions.

The sign reversal has direct implications for ESG ratings and sustainable investment. Intensity-based screens can systematically favor firms in rebound regions with higher transition-risk exposure. Bolton et al. (2025) document that firms with higher rates of green innovation do not necessarily exhibit lower absolute emissions, consistent with rebound effects operating in practice.

Relation to the planner wedge and to intensity sufficiency. In the complete-markets planner benchmark, Jevons-type responses are fully internalized in the optimal paths for $(a^*_{i,t}, E^*_{i,t})$ and therefore affect the levels of optimal emissions and the social cost of carbon. They do not, however, alter the sustainable-discount-rate representation itself. The planner's implementation wedge remains a deterministic adjustment expressed per unit of firm value, proportional to $X^*_{i,t}/V^*_{i,t}$, along

the optimal allocation. By contrast, in equilibrium asset pricing with heterogeneous agents and incomplete spanning, Jevons operates by reshaping the exposure functions $g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t})$ and $g_E(X_{i,t}, a_{i,t}, Z_{i,t})$ that enter (7.2). Because these exposures depend jointly on emissions and efficiency and need not satisfy the homogeneity and scale-invariance restrictions characterized in Proposition 3, emission intensity $X_{i,t}/V_{i,t}$ typically fails to be a unique sufficient statistic for expected returns. The Jevons mechanism therefore reinforces the paper's central distinction between planner implementation, where intensity wedges are natural, and equilibrium pricing, where priced characteristics are model-dependent exposure functions, and it implies that efficiency improvements can be associated with higher required returns for some firms.

8 Empirical Implications

In this section I implement the sufficiency tests implied by Proposition 3 for emission intensity, the leading policy-relevant candidate. Importantly, the goal is not to determine which characteristic delivers the strongest return predictability. Instead, the tests evaluate the two knife-edge restrictions that must hold if emission intensity is a sufficient statistic for priced climate exposure. First, the spanning restriction implies that intensity should capture the cross-section of expected returns once emission levels are included. Second, the scale-invariance restriction implies that the pricing of intensity should not vary systematically with firm size. Rejection of either restriction is sufficient to reject intensity sufficiency. The empirical results below evaluate both restrictions directly.

The preceding analysis implies that intensity-based implementation wedges differ fundamentally from equilibrium risk premia, which depend on priced exposures $g(Z_{i,t})$ and need not reduce to emission intensity. This distinction guides my empirical design. Rather than interpreting return predictability as evidence of a particular equilibrium model, I examine whether the restrictions required for intensity sufficiency are supported by the data. These tests are not tests of market completeness per se. Market completeness affects the equilibrium price of risk λ_t, but emission intensity can be a sufficient statistic only if the exposure function $g(\cdot)$ satisfies the homogeneity and scale-invariance restrictions derived in Proposition 3. Section 8.1 synthesizes the theoretical restrictions, and Section 8.2 implements the corresponding tests using an international panel of firms.

8.1 Implementation vs Equilibrium Pricing

The framework developed in Sections 2-5 establishes a sharp conceptual distinction that is central to interpreting the empirical evidence below. Emission intensity appears naturally in two different economic settings, but it plays fundamentally different roles in each.

The planner's implementation problem. In the complete-markets benchmark of Section 3, a social planner with a well-defined social cost of carbon S_t chooses allocations to maximize welfare. Standard results show that the planner's first-best allocation can be decentralized either through a Pigouvian tax $\tau_t = S_t$ or, equivalently, through firm-specific sustainable discount rates that adjust the cost of capital in proportion to emission intensity $X_{i,t}/V_{i,t}$ (equation (3.18)). In this setting, intensity is the natural sufficient statistic because it summarizes how strongly each firm's discount rate must be tilted to implement the optimal allocation. This is fundamentally a statement about *policy design*: how to achieve a normative objective through price signals.

Equilibrium pricing with incomplete markets. The incomplete-markets framework of Sections 4-5 addresses a different question: which firm characteristics govern equilibrium risk premia when climate damages are unspanned and investors differ in climate concern? Here, the stochastic discount factor reflects a wealth-weighted aggregation of heterogeneous marginal rates of substitution rather than a planner's valuation. Expected returns load on value-normalized exposure functions $g_k(Z_{i,t})/V_{i,t}$ (equation (5.6)), where the functional forms of the $g_k(\cdot)$ are determined by how climate and policy shocks enter cash flows—not by intensity *a priori*. Proposition 3 shows that intensity $X_{i,t}/V_{i,t}$ is sufficient for pricing only under knife-edge restrictions: exposure functions must be homogeneous of degree one in $(X_{i,t}, V_{i,t})$ and satisfy scale invariance, so that firms with the same intensity carry identical priced exposure regardless of size.

Welfare implications of market incompleteness. Carbon premia reflect second-best equilibrium prices of transition exposure, not normative welfare wedges. Welfare comparisons require an explicit planner benchmark with a well-defined social cost of carbon and cannot be inferred from observed premia alone. Internet Appendix C formalizes the consumption-equivalent welfare loss from incomplete markets and shows it decomposes into risk-sharing and internalization components.

Why this matters for empirical work. The distinction is not merely conceptual—it has direct implications for how we interpret carbon premia in asset prices. Intensity-based approaches implicitly assume that equilibrium pricing inherits the structure of the planner's implementation problem. This assumption is valid under complete markets but breaks down once climate risk is

imperfectly shared. Under incomplete markets, the characteristics that govern equilibrium returns are exposure functions disciplined by technology and policy design. Section 6 shows that these functions may depend on emission levels (when per-unit policies scale cash-flow losses with $X_{i,t}$), emission changes (when regulation responds to trajectories), net-zero gaps (when compliance is path-dependent), or efficiency choices (when technology affects shock sensitivity).

The conflicting findings in the empirical literature—with some studies documenting premia on levels, others on intensity, and still others on changes or composite scores—reflect this deeper economic point. Different emission metrics capture different exposure channels, and no single metric is universally correct. Rather, the appropriate predictor depends on which economic mechanisms dominate priced exposure in a given setting.

More fundamentally, multifactor pricing does not imply that emission intensity must be one of the priced factors. Whether intensity enters equilibrium pricing depends on the microfoundations of how climate and policy shocks enter cash flows; in many economically relevant environments, priced exposure depends on emission levels, trajectories, or net-zero gaps rather than on intensity.

Testable implications. Proposition 3 translates the sufficiency conditions into two empirical restrictions. First, *scale invariance*: if intensity is sufficient, the premium per unit of $X_{i,t}/V_{i,t}$ must not vary with firm size (Table 3). Second, *spanning*: conditional on intensity, emission levels should carry no incremental explanatory power for expected returns (Tables 4 and 5). These tests discipline the interpretation of intensity-based metrics by evaluating whether they actually summarize equilibrium pricing or whether they reflect only one component of a richer exposure structure.

The empirical strategy below tests both restrictions directly. The goal is not to advocate for levels over intensity, but to assess whether the knife-edge conditions required for intensity sufficiency hold in the data. Rejection of these conditions clarifies that the debate in the literature reflects distinct economic questions—optimal policy implementation versus equilibrium risk measurement—rather than competing choices among observationally equivalent metrics.

8.2 Empirical evidence on sufficiency

I now test both restrictions using an international panel of firms. The contribution of the empirical analysis is not to estimate the magnitude of a carbon premium per se, but to evaluate the cross-sectional restrictions implied by the theory. First, *scale invariance*: if priced transition exposure depends only on $X_{i,t}/V_{i,t}$, then the premium per unit of intensity must be invariant to firm scale.

Second, *spanning*: if intensity is sufficient, it must span emission levels in return regressions, so that conditional on $X_{i,t}/V_{i,t}$, emission levels $X_{i,t}$ carry no incremental explanatory power.

I implement a direct test of scale invariance by allowing the intensity premium to vary across size bins and testing the implied cross-bin equality restriction. I then conduct asymmetric spanning tests that distinguish whether intensity subsumes levels or, conversely, whether levels subsume intensity. Finally, I report within-size-bin spanning tests that address the concern that pooled results reflect residual slope heterogeneity associated with firm scale.

The tests use an international panel of 13,373 firms representing 92 countries and 438 Trucost industries over 2005-2022. I collect panel data on stock returns $R_{i,t+1}$, the risk-free rate $R_{f,t}$, firm-level Scope 1 emissions $X_{i,t}$, firm value $V_{i,t}$, and standard firm characteristics $W_{i,t}$ (such as market capitalization, market-to-book ratio, book leverage, profitability, investment, momentum, age, PPE, and volatility). The emissions data come from S&P Trucost while all other information is from Capital IQ. Following Pedersen (2025), I define emission intensity as $X_{i,t}/V_{i,t}$, where V is proxied by the book value of equity, and I define the percentage change in emissions as $\Delta X_{i,t} = (X_{i,t} - X_{i,t-1})/X_{i,t-1}$. To mitigate the influence of outliers I winsorize intensity and emissions growth at the 2.5% level. Table 2 reports summary statistics for the key empirical variables.

A direct test of scale invariance. A central implication of the knife-edge intensity-sufficiency conditions is a scale-invariance restriction: firms with the same intensity but different scale must have the same priced transition exposure. If this restriction holds, the premium per unit of intensity should not vary systematically with firm scale. To test it, each period I sort firms by size into five equal-count bins (baseline: book value of equity) and estimate the pooled regression

$$R_{i,t+1} - R_{f,t} = \alpha_t + \sum_{b=1}^{B} b_b \, D_{b,i,t}\Big(\frac{X_{i,t}}{V_{i,t}}\Big) + \gamma_t^{\top} W_{i,t} + \varepsilon_{i,t+1},$$

where $D_{b,i,t}$ indicates membership in size bin b. The specification includes $W_{i,t}$, industry×time fixed effects, and country fixed effects. Standard errors are double-clustered by industry and time. The null implied by Proposition 3 is $H_0 : b_1 = \cdots = b_B$, which I test using a Wald statistic based on the clustered covariance matrix.

Table 3 reports the results. Panel A shows that the estimated intensity premium varies materially across size bins. Panel B reports formal Wald tests of the cross-bin equality restrictions and shows that the rejection is not specific to a particular definition of scale. The same restric-

tions are rejected (or remain economically close to rejection) when bins are formed using market capitalization, sales, or assets instead of book equity. The key implication for Proposition 3 is the robust rejection of cross-bin equality, providing direct evidence against the knife-edge hypothesis that emission intensity summarizes priced transition exposure in a scale-invariant manner.

Spanning tests: does intensity subsume levels? I complement the scale-invariance tests with asymmetric spanning tests that evaluate whether intensity spans levels, as required under intensity sufficiency, or whether the opposite spanning relation is more consistent with the data. Guided by the incomplete-markets benchmark and the general factor framework, I estimate pooled regressions of excess returns on emission levels and emission intensity (along with controls),

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{X,t} X_{i,t} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}, \tag{8.18}$$

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{I,t} \frac{X_{i,t}}{V_{i,t}} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}, \tag{8.19}$$

and

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{X,t} X_{i,t} + b_{I,t} \frac{X_{i,t}}{V_{i,t}} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}. \tag{8.20}$$

Equations (8.18)-(8.20) are interpreted as *spanning restrictions*, not as a symmetric "horse race." If emission intensity $X_{i,t}/V_{i,t}$ is sufficient in the sense of Definition 4.1, then it must span the return variation associated with the underlying exposure: conditional on $X_{i,t}/V_{i,t}$, emission levels $X_{i,t}$ should have no incremental explanatory power ($H_0 : b_{X,t} = 0$ in (8.20)). Conversely, if priced exposure contains a scale component, then levels can span intensity, so that conditional on $X_{i,t}$, the ratio $X_{i,t}/V_{i,t}$ adds little information ($H_0 : b_{I,t} = 0$). I report Wald tests of both restrictions based on the clustered covariance matrix.

Table 4 provides evidence consistent with the latter asymmetry. Emission levels are positively and statistically significantly related to both one-month- and twelve-month-ahead returns when entered on their own, and they remain positive and highly significant when intensity is included. By contrast, conditional on emission levels, the incremental coefficient on intensity is economically small and statistically indistinguishable from zero, and the explanatory power of the specification is essentially unchanged. In the language of spanning, these estimates suggest that levels span intensity in this reduced-form relation, whereas intensity does not span levels, which is inconsistent with intensity sufficiency.

These restrictions are not only statistically sharp but economically consequential. In the joint

specification in Table 4, the coefficient of log(Scope 1) measures the return effect of emission scale holding intensity fixed. A one-standard-deviation increase in log(Scope 1) predicts a 0.35% increase in next-month expected returns (about 4.2% annualized). At the twelve-month horizon, the corresponding effect remains economically meaningful. A one-standard-deviation increase in emissions predicts a 0.11% increase in average monthly returns over the subsequent year (about 1.33% annualized). Thus, firms with similar $X_{i,t}/V_{i,t}$ but different emission scales earn systematically different expected returns, contradicting the scale-invariance and spanning restrictions implied by Proposition 3.

Within-size-bin spanning. A remaining concern is that the spanning asymmetry in Table 4 reflects slope heterogeneity associated with firm scale. If the price of transition exposure varies with size, then emission levels may appear to add explanatory power in pooled regressions even when intensity is the relevant exposure measure within scale classes. To address this concern directly, I re-estimate the joint levels-intensity specification (8.20) separately within size quintiles, using the same controls and fixed effects. By construction, firm scale is approximately fixed within each regression, so this design isolates whether emission levels add return-relevant information beyond intensity even among firms of comparable size.

Table 5 reports the results. Emission levels remain positively and significantly related to expected returns within every size bin, while the coefficient of intensity is economically small and statistically weak. This within-bin evidence strengthens the spanning conclusion in two ways. First, it rules out the interpretation that the pooled spanning results are driven primarily by residual size differences or generic size-related slope heterogeneity. Second, it directly targets the key implication of intensity sufficiency: if $X_{i,t}/V_{i,t}$ were a uniquely sufficient statistic for priced transition exposure, then conditioning on intensity should eliminate systematic return differences associated with emission scale even within narrow scale classes. The data reject this implication.

Concerns that the joint specification merely reflects mechanical multicollinearity are mitigated by the correlation structure of the data. Emission intensity and log emissions are only moderately correlated, and intensity is close to orthogonal to firm size. These magnitudes indicate that levels, intensity, and scale capture distinct dimensions of firm heterogeneity, so that the asymmetric spanning and within-bin evidence is economically meaningful.

The asymmetric spanning result is also robust to alternative intensity definitions using assets and market capitalization. In contrast, scaling emissions by sales yields a sign reversal. Sales-

based intensity enters with a negative coefficient, which becomes stronger and more significant once emission levels are included, even after controlling for ROE and industry fixed effects. This behavior indicates that sales-based intensity loads on a different priced dimension, reflecting pricing power or pass-through embedded in revenues, rather than serving as a sufficient statistic for exposure per unit of firm value.

Finally, the rejection of intensity sufficiency in this section does not rely on long-horizon predictability or on the magnitude of estimated premia. The objects being tested are cross-sectional restrictions implied by Proposition 3, scale invariance and spanning, which must hold at any horizon if emission intensity is a uniquely sufficient statistic for priced transition exposure. Horizon-dependent attenuation affects the interpretation of the time-varying price of risk, but it does not affect the validity of these sufficiency tests.

Changes in emissions. To connect more directly to the transition-risk channel in Section 6, I also consider a specification that allows both the level and the recent change in emissions to enter the exposure function. Specifically, I estimate

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{X,t} X_{i,t} + b_{\Delta X,t} \Delta X_{i,t} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}, \tag{8.21}$$

and report the results in Internet Appendix OA.E. Because $\Delta X_{i,t}$ is inherently more sensitive to timing and transitory noise than $X_{i,t}$, I view these estimates as supplementary. The appendix results indicate that emission changes can be associated with return predictability at short horizons, while the level effect is more stable across specifications and horizons.

Overall, the evidence rejects both knife-edge restrictions implied by intensity sufficiency. Scale-invariance tests (Table 3) reject homogeneity across firm size, while spanning tests (Tables 4-5) show that emission levels remain strongly priced conditional on intensity and within narrow size classes. Intensity becomes redundant once levels are included, even among firms of similar scale. These findings therefore reject intensity sufficiency in the sense of Proposition 3. The results do not establish that emission levels are universally sufficient; rather, they demonstrate that the knife-edge conditions required for intensity sufficiency do not hold in the data, underscoring the need for a richer exposure-based characterization of climate risk premia. Developing and empirically disciplining such exposure functions is therefore a central task for future research.

9 Conclusion

This paper characterizes the conditions under which emission intensity summarizes climate risk premia. I show that intensity is sufficient only when exposure functions satisfy restrictive scale-invariance and homogeneity conditions—restrictions that hold in complete-markets planner implementations but fail under incomplete markets with unspanned climate damages. The analysis clarifies that intensity-based sustainable discount rates serve as implementation tools for first-best allocations, while equilibrium risk premia under incomplete markets depend on exposure functions $g(Z_{i,t})$ that may load on emission levels, changes, technology choices, and path deviations from regulatory benchmarks. Empirical tests reject the sufficiency restrictions, establishing that the debate in the literature reflects distinct economic questions—optimal policy design versus equilibrium asset pricing—rather than competing measurement approaches.

A key contribution of the dynamic analysis is the identification of a technological sign reversal. Through a Jevons-type rebound effect, I demonstrate that firm-level efficiency improvements can endogenously increase absolute emissions and, consequently, equilibrium risk premia. This result cautions against the use of emission intensity as a singular proxy for climate risk exposure, even when it moves in the direction conventionally associated with greenness.

Beyond metric choice, the framework provides a micro-foundation for the green and brown classifications central to the climate-finance literature (e.g., Pastor et al. (2021)). Rather than treating these types as exogenous labels, the analysis endogenizes them as equilibrium exposure functions shaped by technology and fundamentals. In this sense, a firm's climate-risk profile is a dynamic object: changes in efficiency, production structure, or policy exposure can alter priced risk even when conventional ESG metrics improve.

The framework provides a disciplined approach to empirical work. Emission-level specifications are not arbitrary alternatives to intensity but emerge naturally when climate shocks enter cash flows through per-unit exposures and markets are incomplete. The empirical strategy—testing scale-invariance and spanning restrictions—offers a template for evaluating when any climate-risk metric is theoretically coherent. Future research should structurally estimate the exposure functions $g_k(Z_{i,t})$ implied by specific policy and technology channels, quantify the relative importance of transition risk versus preference-driven greenium components, and extend the framework to incorporate dynamic investment and abatement decisions.

The framework has immediate implications for sustainable finance practice. Regulatory tax-

onomies and disclosure standards often emphasize emission intensity, implicitly assuming the knife-edge conditions under which intensity is sufficient for pricing. My analysis establishes a distinction between policy implementation and risk measurement: intensity-based metrics naturally implement normative objectives under complete markets, while equilibrium risk assessment under incomplete markets requires exposure functions that depend on absolute emissions, technology choices, and firm-specific transition vulnerabilities. This distinction explains why portfolios satisfying intensity-based regulatory criteria can nonetheless carry substantial climate transition risk.

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Tables

Table 1: Using the model: priced objects under alternative exposure mechanisms

Economic mechanism	**Exposure structure** $g(Z_{i,t})$	**Object priced in equilibrium**	**Regression object**
Per-unit transition risk (taxes, permits, linear policy shocks)	$g(Z_{i,t}) = \eta X_{i,t}$	Emission scale, conditional on firm value	$X_{i,t}$
Planner implementation (complete markets, SCC-based policy)	Deterministic loss $S_t X_{i,t}$	Firm-specific discount-rate wedge	$X_{i,t}/V_{i,t}$
Scale-invariant equilibrium (homogeneous exposure benchmark)	$g(Z_{i,t}) = \kappa_t X_{i,t}$	Priced exposure identical for firms with equal intensity	$X_{i,t}/V_{i,t}$
Trajectory-based regulation (policy responds to changes or trends)	$g(Z_{i,t}) = \phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}$	Emission levels and changes	$(X_{i,t},\, \Delta X_{i,t})$
Path-based constraints / net-zero (benchmarks, phase-out schedules)	$g(Z_{i,t}) = \vartheta_1 X_{i,t} + \vartheta_2 (X_{i,t} - X_{i,t}^{NZ})$	Emissions and distance to targets	$(X_{i,t},\, X_{i,t} - X_{i,t}^{NZ})$

Table 2: **Summary statistics**

The sample includes listed companies from 92 countries over 2005-2022. *Returns* are monthly stock returns (in percent). Scope 1 emission levels (X) are in metric tons of CO_2 equivalent. Book value of equity (V) is in millions of US dollars. Emission intensity (X/V) is the ratio of emissions to book value of equity. Scope 1 growth rate is the year-over-year percentage change in emissions. Statistics reported are mean, median, and standard deviation across all observations.

Variable / Statistic	Mean	Median	St. dev.
Returns (in %)	0.890	0.348	12.134
Scope 1 levels (X)	1,696,164	26,141	11,700,000
Book value of equity (V)	333,602	4,967	12,900,000
Intensity (X/V)	1.233	0.048	3.619
Log(Scope 1)	10.356	10.174	2.850
Scope 1 growth rate	0.086	0.023	0.432
Number of firms: 13,373			
Number of countries: 92			
Number of industries: 438			

Table 3: **Predicting Stock Returns with Emission Intensity and Tests of Scale Invariance**

The sample is at the firm-month level and includes listed companies from 92 countries over 2005-2022. The dependent variable is monthly stock returns ($Returns_{i,t+1}$). Each period, firms are sorted by book value of equity into quintiles ($Size\ bin$). Emission intensity (X/V) is the ratio of Scope 1 emissions to book value. All regressions include industry × time fixed effects and country fixed effects. Standard errors (in parentheses) are double-clustered by industry and time. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. Panel B reports F-statistics and p-values from Wald tests of the null hypothesis that coefficients on intensity are equal across all five size bins, based on the specification in Panel A, column (2). The baseline uses book value of equity for scaling; alternative specifications use market capitalization, sales, or total assets.

	(1)	(2)
VARIABLES	Returns	
Panel A: Predicting stock returns with emission intensity		
Intensity X/V	0.032**	0.042**
	(0.014)	(0.016)
Size bin 2	-0.017	0.123
	(0.182)	(0.147)
Size bin 3	-0.130	0.048
	(0.193)	(0.142)
Size bin 4	-0.181	0.006
	(0.208)	(0.130)
Size bin 5	-0.339	-0.021
	(0.217)	(0.109)
Size bin 2 × Intensity X/V	-0.029**	-0.038***
	(0.010)	(0.011)
Size bin 3 × Intensity X/V	-0.017	-0.027*
	(0.014)	(0.015)
Size bin 4 × Intensity X/V	-0.014	-0.026
	(0.014)	(0.015)
Size bin 5 × Intensity X/V	0.002	-0.022
	(0.023)	(0.026)
Log(Size)		-0.193***
		(0.051)
Log(M/B)		-0.260***
		(0.071)
Book leverage		-0.115
		(0.216)
Momentum		0.486***
		(0.154)
Investment/Assets		-0.383
		(0.879)
Log(PPE)		0.082**
		(0.030)
Volatility		2.000
		(3.117)
ROE		0.870***
		(0.267)
Age		0.840
		(0.896)
Constant	0.990***	1.932***
	(0.158)	(0.421)
Industry × time FE	Yes	Yes
Country FE	Yes	Yes
Observations	1,005,645	1,005,645
R-squared	0.235	0.236

Panel B: Tests of scale invariance (based on Panel A, col. 2)

Specification	F-stat	p-value
Baseline	5.09	0.007
Market-cap scaling	4.15	0.017
Sales scaling	2.59	0.076
Assets scaling	2.91	0.050

Table 4: **Predicting Stock Returns with Emission Levels and Intensity**

The sample is at the firm-month level and includes listed companies from 92 countries over 2005-2022. The dependent variable is monthly stock returns ($Returns_{i,t+1}$) over the next month (columns 1-3) or next 12 months (columns 4-6). Emission intensity (X/V) is the ratio of Scope 1 emissions to book value of equity. All regressions include industry × time fixed effects and country fixed effects. Standard errors (in parentheses) are double-clustered by industry and time. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
	Returns					
	1 month ahead			12 months ahead		
Intensity X/V	0.019**		-0.001	0.006		-0.001
	(0.009)		(0.011)	(0.010)		(0.011)
Log(Scope1 Emissions)		0.123***	0.123***		0.039***	0.039***
		(0.019)	(0.024)		(0.009)	(0.012)
Log(Size)	-0.200***	-0.269***	-0.269***	-0.139***	-0.162***	-0.162***
	(0.049)	(0.052)	(0.052)	(0.037)	(0.037)	(0.037)
Log(M/B)	-0.250***	-0.193**	-0.193**	-0.159*	-0.141*	-0.141*
	(0.072)	(0.068)	(0.072)	(0.076)	(0.072)	(0.077)
Book leverage	-0.060	-0.189	-0.189	-0.117	-0.156	-0.156
	(0.240)	(0.241)	(0.242)	(0.174)	(0.174)	(0.176)
Momentum	0.472***	0.458**	0.458**	-0.394*	-0.397*	-0.397*
	(0.160)	(0.161)	(0.161)	(0.190)	(0.190)	(0.190)
Investment/Assets	-0.408	-0.241	-0.241	0.099	0.153	0.153
	(0.865)	(0.877)	(0.891)	(0.738)	(0.752)	(0.757)
Log(PPE)	0.078**	0.041	0.041	0.056**	0.045**	0.045**
	(0.028)	(0.026)	(0.027)	(0.020)	(0.020)	(0.021)
Volatility	1.995	1.995	1.995	0.408	0.403	0.403
	(3.151)	(3.147)	(3.150)	(1.525)	(1.520)	(1.524)
ROE	0.842***	0.770**	0.770**	0.548	0.527	0.527
	(0.259)	(0.268)	(0.270)	(0.313)	(0.315)	(0.317)
Age	0.881	0.626	0.626	-0.442	-0.513	-0.513
	(0.935)	(0.959)	(0.972)	(0.940)	(0.971)	(0.990)
Constant	2.055***	1.738***	1.738***	1.791***	1.689***	1.689***
	(0.445)	(0.469)	(0.472)	(0.292)	(0.296)	(0.299)
Industry×time FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,006,908	1,006,023	1,006,023	862,872	862,251	862,251
R-squared	0.231	0.231	0.231	0.247	0.247	0.247

Robust standard errors in parentheses

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 5: **Within-size-bin Spanning Tests**

The sample is at the firm-month level and includes listed companies from 92 countries over 2005-2022. The dependent variable is monthly stock returns ($Returns_{i,t+1}$). The table reports spanning regressions estimated separately within firm size quintiles, where size is measured by book value of equity. Each column estimates the regression in Table 4 using firms in the corresponding size bin. All regressions include the same set of controls as those in Table 4 as well as industry × time fixed effects and country fixed effects. Standard errors (in parentheses) are double-clustered by industry and time. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Firm size quintile (book equity)				
	(1) Smallest	(2)	(3)	(4)	(5) Largest
log(Scope 1 Emissions)	0.122**	0.181***	0.142***	0.105***	0.125***
	(0.055)	(0.030)	(0.025)	(0.027)	(0.029)
Intensity X/V	0.032	−0.034**	−0.020*	−0.017	0.015
	(0.020)	(0.016)	(0.011)	(0.017)	(0.024)
Controls	Yes	Yes	Yes	Yes	Yes
Industry × time FE	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes
Observations	181,064	182,439	187,002	188,801	189,660
R^2	0.324	0.355	0.347	0.333	0.352

Appendix

A.1 Proofs and derivations

Proof of Proposition 1. By (4.3), for any h that holds a positive amount of stock i we have

$$\frac{\mu_i - V_i R_f}{\eta X_i} = \alpha_h \sigma_\zeta^2 \delta_h.$$

Since this ratio must be equalized across all stocks i and all households h with interior holdings, it must equal a common scalar λ:

$$\frac{\mu_i - V_i R_f}{\eta X_i} = \lambda \quad \forall i,$$

which yields (4.4). Substituting back gives $\alpha_h \sigma_\zeta^2 \delta_h = \lambda$, which yields (4.5). □

Proof of Proposition 2. In the static CARA-normal environment, aggregate consumption net of damages is $C_{h,1} - \phi_h \Gamma_1 = C_{h,1} - \phi_h \kappa \bar{X}\, \zeta_1$, so each household's SDF is exponential-affine in the climate shock,

$$m_{h,1} = \beta \frac{u_c^h(C_{h,1} - \phi_h \Gamma_1)}{u_c^h(C_{h,0} - \phi_h \Gamma_0)} \propto \exp\big(\alpha_h \phi_h \kappa \bar{X}\, \zeta_1\big),$$

where the proportionality absorbs β and all terms measurable at date 0. With mean-variance/CARA aggregation, the equilibrium (aggregate) SDF that prices all traded assets is the risk-tolerance-weighted average of individual SDF loadings, hence

$$m_1 \propto \exp\big(B\, \zeta_1\big), \qquad B = \kappa \bar{X} \frac{\sum_{h=1}^H \frac{\phi_h}{\alpha_h}}{\sum_{h=1}^H \frac{1}{\alpha_h}}.$$

Firm i's payoff (dividend) loads on the same shock with exposure ηX_i in the baseline specification, so $D_i = \tilde{D}_i + \eta X_i \zeta_1$ with $\tilde{D}_i \perp \zeta_1$. Using the Euler equation $1 = \mathbb{E}[m_1 R_i]$ and a second-order (mean-variance) expansion yields the standard linear pricing relation

$$\mathbb{E}[R_i] - R_f = -\operatorname{Cov}(R_i, \log m_1) = \sigma_\zeta^2\, (\eta X_i)\, \Big(1 - B\Big),$$

where we used $\log m_1 = \text{const} + B\zeta_1$ and $\operatorname{Cov}(\zeta_1, \zeta_1) = \sigma_\zeta^2$. Aggregating across firms gives the climate factor price

$$\lambda = \frac{\sigma_\zeta^2 \bar{X}}{\sum_{h=1}^H \frac{1}{\alpha_h}} \Big(\eta - \kappa \sum_{h=1}^H \phi_h\Big),$$

which is (4.8). □

Proof of Proposition 3. Condition (iii) implies that, for any fixed intensity level $u \in \mathbb{R}_+$, all firms with $X_{i,t}/V_{i,t} = u$ share the same exposure $g(Z_{i,t})/V_{i,t}$. Define $\varphi_t(u)$ to be this common value:

$$\varphi_t(u) \equiv \frac{g(Z_{i,t})}{V_{i,t}} \quad \text{for any firm } i \text{ with } \frac{X_{i,t}}{V_{i,t}} = u.$$

Condition (i) ensures that intensities span an open subset of $\mathbb{R}_+$, so that φ_t can be defined on an interval rather than on a finite set. This establishes (4.15).

To obtain (4.16), fix a firm with characteristics $(X_{i,t}, V_{i,t})$ and intensity $u = X_{i,t}/V_{i,t}$. By homogeneity of degree one in $(X_{i,t}, V_{i,t})$, for any $\lambda > 0$,

$$g(Z_{i,t}; \lambda X_{i,t}, \lambda V_{i,t}) = \lambda\, g(Z_{i,t}; X_{i,t}, V_{i,t}).$$

Dividing both sides by $\lambda V_{i,t}$ yields

$$\frac{g(Z_{i,t}; \lambda X_{i,t}, \lambda V_{i,t})}{\lambda V_{i,t}} = \frac{g(Z_{i,t}; X_{i,t}, V_{i,t})}{V_{i,t}}.$$

The left-hand side is the value-normalized exposure of a firm whose intensity remains u (since $(\lambda X_{i,t})/(\lambda V_{i,t}) = u$). By condition (iii), it depends only on u and therefore equals $\varphi_t(u)$, while the right-hand side equals $\varphi_t(u)$ by definition. Hence

$$\frac{g(Z_{i,t})}{V_{i,t}} = \varphi_t\left(\frac{X_{i,t}}{V_{i,t}}\right),$$

which restates (4.15).

Under the additional assumption in the proposition that the mapping is linear in size (equivalently, that $g(Z_{i,t})/V_{i,t}$ is proportional to $X_{i,t}/V_{i,t}$ on the relevant support), there exists a scalar κ_t such that for all firms,

$$\frac{g(Z_{i,t})}{V_{i,t}} = \kappa_t \frac{X_{i,t}}{V_{i,t}}.$$

Multiplying both sides by $V_{i,t}$ yields $g(Z_{i,t}) = \kappa_t X_{i,t}$ and establishes (4.16). Uniqueness in the sense of Definition 4.1 follows because any one-factor representation observationally equivalent to (4.14) must use a characteristic proportional to the priced characteristic $g(Z_{i,t})/V_{i,t}$, hence proportional to $X_{i,t}/V_{i,t}$ under (4.16). □

A Internet Appendix: Robustness to nonlinear damages and multiple climate shocks

This section shows that the main asset-pricing implications of our framework are robust to nonlinear damage functions and to the presence of multiple climate shocks (for example, physical, policy, and technology shocks). In particular, I show that under standard small-shock approximations the pricing kernel remains affine in the climate factors, expected excess returns remain linear in firm-specific exposure coefficients, and the relevant priced characteristics are generic functions $g_k(Z_{i,t})$ of firm characteristics rather than a uniquely determined emission-intensity statistic $X_{i,t}/V_{i,t}$.

A.1 Nonlinear damages in a single climate factor

I first extend the static damage specification to allow for nonlinear dependence on the climate shock. For expositional clarity, I work in a one-period environment. The dynamic case follows by applying the same local arguments period by period.

Let ζ be a scalar climate factor with $\mathbb{E}[\zeta] = 0$ and $\mathrm{Var}(\zeta) = \sigma_\zeta^2 \in (0, \infty)$, and let aggregate emissions be $\bar{X} = \sum_i X_i$ as before. Instead of the linear damage function $\Gamma = \kappa \bar{X} \zeta$, suppose that damages are given by

$$\Gamma = h(\bar{X}, \zeta), \tag{A.1}$$

where $h : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable and satisfies $h(\bar{X}, 0) = 0$. A quadratic specification, for example,

$$h(\bar{X}, \zeta) = \kappa_1 \bar{X} \zeta + \frac{1}{2} \kappa_2 \bar{X} \zeta^2, \tag{A.2}$$

is included as a special case.

Household h has CARA preferences over climate-adjusted consumption,

$$U^h = \mathbb{E}\Big[u^h\big(C_1^h - \phi_h \Gamma\big)\Big], \qquad u^h(y) = -\exp(-\alpha_h y),$$

with $\alpha_h > 0$ and $\phi_h \geq 0$. Writing

$$Y_h \equiv C_1^h - \phi_h h(\bar{X}, \zeta),$$

and using a second–order Taylor expansion of h around $(\bar{X}, 0)$, we obtain

$$h(\bar{X}, \zeta) = h_\zeta(\bar{X}, 0)\zeta + \frac{1}{2} h_{\zeta\zeta}(\bar{X}, 0)\zeta^2 + o(\zeta^2), \tag{A.3}$$

where h_ζ and $h_{\zeta\zeta}$ denote the first and second partial derivatives with respect to ζ. Substituting (A.3) into Y_h yields

$$Y_h = \tilde{C}_1^h - \phi_h h_\zeta(\bar{X}, 0)\zeta - \frac{1}{2}\phi_h h_{\zeta\zeta}(\bar{X}, 0)\zeta^2 + o(\zeta^2),$$

where $\tilde{C}_1^h$ collects terms independent of ζ. To isolate the stochastic components, define the centered square

$$\tilde{\zeta}^2 \equiv \zeta^2 - \mathbb{E}[\zeta^2],$$

and rewrite Y_h as

$$Y_h = \bar{Y}_h + \beta_h^{(1)}\zeta + \beta_h^{(2)}\tilde{\zeta}^2 + o(\zeta^2), \tag{A.4}$$

for suitable constants $\bar{Y}_h, \beta_h^{(1)}, \beta_h^{(2)}$ that depend on $(\alpha_h, \phi_h, \bar{X})$ and on the moments of ζ. Under the assumption that ζ is conditionally normal and that higher-order terms are negligible, the pair $(\zeta, \tilde{\zeta}^2)$ can be treated as a two-dimensional factor vector with mean zero and finite second moments.

The CARA-normal structure implies that each household's portfolio problem again admits a mean-variance representation in terms of Y_h, now with respect to the joint distribution of $(\zeta, \tilde{\zeta}^2)$. Aggregating across households and solving for asset prices leads to an affine SDF of the form

$$m = \exp\big(-A - B^{(1)}\zeta - B^{(2)}\tilde{\zeta}^2\big), \tag{A.5}$$

with factor loadings $B^{(1)}$ and $B^{(2)}$ determined by the distribution of (α_h, ϕ_h) and by the coefficients $h_\zeta(\bar{X}, 0)$ and $h_{\zeta\zeta}(\bar{X}, 0)$.

Suppose now that firm i's dividend takes the reduced-form specification

$$D_i = \mu_i + g_1(Z_i)\zeta + g_2(Z_i)\tilde{\zeta}^2, \tag{A.6}$$

where Z_i collects firm characteristics and g_1, g_2 are measurable functions. A log-linear approximation to the Euler equation $\mathbb{E}[mR_i] = 1$ then yields

$$\mathbb{E}[R_i] - R_f \approx \lambda^{(1)} g_1(Z_i) + \lambda^{(2)} g_2(Z_i), \tag{A.7}$$

for factor prices of climate risk $\lambda^{(1)}, \lambda^{(2)}$ that are linear functions of $(B^{(1)}, B^{(2)})$ and of the conditional second moments of $(\zeta, \tilde{\zeta}^2)$. Expression (A.7) shows that allowing for quadratic (or more

general smooth) damages in ζ does not alter the linear factor structure: it simply enlarges the set of priced characteristics from a single function $g(Z_i)$ to a vector $(g_1(Z_i), g_2(Z_i))$, corresponding to exposure to the linear and quadratic components of the climate factor. In particular, if the technological channel through which ζ enters cash flows remains proportional to emission levels, as in

$$g_1(Z_i) = \eta_1 X_i, \qquad g_2(Z_i) = \eta_2 X_i,$$

then (A.7) implies that expected excess returns are proportional to X_i with a combined price of risk $\lambda^{(1)}\eta_1 + \lambda^{(2)}\eta_2$, and the qualitative conclusion that emission levels act as the relevant exposure remains unchanged. More generally, the quadratic damage term induces an additional priced characteristic $g_2(Z_i)$ whose functional form is determined by the way ζ^2 enters the technology and damage processes, but there is no mechanism that generically restores emission intensity X_i/V_i as a unique sufficient statistic.

A.2 Multiple climate shocks: physical, policy, and technology factors

I next allow for multiple climate-related shocks. Let $f_{t+1} = (f_{1,t+1}, \ldots, f_{K,t+1})^\top$ be a vector of K mean-zero factors that may represent, for example, a physical damage factor, a policy or carbon-price factor, and a technology factor. As in Section 5, suppose that firm i's dividend can be written as

$$D_{i,t+1} = \mu_{i,t+1} + \sum_{k=1}^{K} g_k(Z_{i,t}) f_{k,t+1}, \tag{A.8}$$

where $Z_{i,t}$ includes $(X_{i,t}, V_{i,t})$ and any other relevant characteristics, and $g_k(\cdot)$ are exposure functions determined by technology, market structure, and the incidence of policy. Climate damages are now given by a general function

$$\Gamma_t = H(\bar{X}_{t-1}, f_t), \tag{A.9}$$

where H is twice continuously differentiable in its arguments. Under CARA preferences $u^h(y) = -\exp(-\alpha_h y)$, household h's climate-adjusted consumption takes the form

$$Y_{h,t+1} = \tilde{C}_{h,t+1} - \phi_h H(\bar{X}_t, f_{t+1}).$$

A second-order Taylor expansion of H around a baseline $(\bar{X}_t, f_{t+1}) = (\bar{X}_t, 0)$ yields

$$H(\bar{X}_t, f_{t+1}) = H(\bar{X}_t, 0) + \sum_{k=1}^{K} H_k(\bar{X}_t, 0) f_{k,t+1} + \frac{1}{2} \sum_{k,\ell=1}^{K} H_{k\ell}(\bar{X}_t, 0) f_{k,t+1} f_{\ell,t+1} + o(\|f_{t+1}\|^2), \quad \text{(A.10)}$$

where H_k and $H_{k\ell}$ denote the first and second partial derivatives of H with respect to its factor arguments.

Under the maintained small-shock assumption, the leading-order contribution of the quadratic terms in (A.10) to the mean-variance approximation can be summarized by a finite set of additional factors constructed from products $f_{k,t+1} f_{\ell,t+1}$, for example their centered versions $f_{k,t+1} f_{\ell,t+1} - \mathbb{E}_t[f_{k,t+1} f_{\ell,t+1}]$. Aggregating across households and solving the mean-variance portfolio problem then yields an affine SDF of the form

$$m_{t+1} = \exp\Big(- A_t - \sum_{k=1}^{K} B_{k,t} f_{k,t+1} - \sum_{k,\ell=1}^{K} C_{k\ell,t}\, \tilde{f}_{k\ell,t+1} \Big), \quad \text{(A.11)}$$

where $\tilde{f}_{k\ell,t+1}$ are centered quadratic factor terms and the coefficients $B_{k,t}, C_{k\ell,t}$ are functions of (α_h, ϕ_h) and of the derivatives $H_k(\bar{X}_t, 0)$ and $H_{k\ell}(\bar{X}_t, 0)$. Substituting (A.8) and (A.11) into the Euler equation and applying the same linearization arguments as in Proposition 4 yields

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \sum_{k=1}^{K} \lambda_{k,t} g_k(Z_{i,t}) + \sum_{k,\ell=1}^{K} \lambda_{k\ell,t} \tilde{g}_{k\ell}(Z_{i,t}), \quad \text{(A.12)}$$

for suitable factor prices of risk $\lambda_{k,t}, \lambda_{k\ell,t}$ and exposure functions $\tilde{g}_{k\ell}(Z_{i,t})$ associated with the quadratic factor terms. Expression (A.12) is the natural multi-factor generalization of (5.6): each physical, policy, or technology factor $f_{k,t+1}$, as well as their relevant quadratic combinations, is associated with a priced characteristic given by its exposure function in the dividend and damage processes.

The key observation is that neither the presence of nonlinear damages nor the presence of multiple factors alters the basic structure of our earlier results: to leading order, expected excess returns are linear in a vector of exposure coefficients $(g_1(Z_{i,t}), \ldots, g_K(Z_{i,t}), \tilde{g}_{k\ell}(Z_{i,t}))$, and these coefficients are model-specific functions of firm characteristics $Z_{i,t}$. In particular, if policy shocks enter cash flows through a term of the form $-\tau_{t+1} X_{i,t}$, the corresponding exposure function is naturally $g_k(Z_{i,t}) = -X_{i,t}$, even in the presence of additional physical or technology factors. Conversely, if physical shocks enter via damages that are nonlinear in $\bar{X}_t$ and $f_{k,t+1}$, the resulting priced

characteristics will typically involve nonlinear functions of emissions and other firm attributes. In neither case is there a general reason for the emission intensity $X_{i,t}/V_{i,t}$ to emerge as a unique sufficient statistic for premia. The qualitative conclusions of Sections 4.6 and 5 are therefore robust to quadratic damages and to multiple climate shocks: the priced objects are exposure functions $g_k(Z_{i,t})$ determined by technology, policy, and preferences, and these need not collapse to a single ratio such as $X_{i,t}/V_{i,t}$.

B CRRA preferences and incomplete markets

This appendix shows that the main pricing structure derived in the CARA-normal benchmark carries over, in local approximation, to an economy with CRRA preferences and incomplete markets. The key difference is that the aggregate price of climate risk is now governed by wealth-weighted (or, more precisely, consumption-weighted) risk tolerance rather than by the sum of inverse CARA coefficients. Throughout, it is useful to keep the distinction between a *price* (dollar) equation and a *return* equation: the primitive exposure is level-based in dividends, while expected returns load on the value-normalized exposure.

B.1 Setup

Time is $t = 0, 1$. Uncertainty is driven by a scalar climate shock ζ with

$$\mathbb{E}[\zeta] = 0, \qquad \mathrm{Var}(\zeta) = \sigma_\zeta^2 \in (0, \infty),$$

and all random variables are jointly normally distributed conditional on date-0 information.

There are N firms indexed by $i = 1, \ldots, N$ and H households indexed by $h = 1, \ldots, H$. Firm i has a fixed level of emissions $X_i \geq 0$ and a date-1 dividend given by

$$D_i = \mu_i + \eta X_i \zeta, \tag{B.1}$$

where $\mu_i \in \mathbb{R}$ is a deterministic component and $\eta > 0$ is a technology parameter common across firms. Aggregate emissions are

$$\bar{X} \equiv \sum_{i=1}^{N} X_i.$$

Climate damages at date 1 are

$$\Gamma(\zeta) = \kappa \bar{X} \zeta, \qquad \kappa > 0. \tag{B.2}$$

The scalar ζ is therefore interpreted as a shock to the per-unit cost or value of emissions, in the reduced-form sense discussed in the main text.

Household h has date-0 wealth $W_0^h > 0$ and trades the N stocks and a risk-free asset with gross return $R_f > 0$. Let V_i denote the date-0 price of stock i, and let θ_i^h denote the number of shares of stock i held by household h. Let b^h denote the holding of the risk-free asset. The budget constraint at $t = 0$ is

$$\sum_{i=1}^{N} V_i \theta_i^h + \frac{1}{R_f} b^h = W_0^h. \tag{B.3}$$

Date–1 consumption of household h is

$$C_1^h(\zeta) = b^h R_f + \sum_{i=1}^{N} \theta_i^h D_i(\zeta). \tag{B.4}$$

Household h experiences disutility from climate damages with intensity $\phi_h \geq 0$ and has CRRA preferences over climate-adjusted consumption:

$$U^h = \mathbb{E}\left[u_h\big(C_1^h - \phi_h \Gamma(\zeta)\big)\right], \qquad u_h(x) = \frac{x^{1-\gamma_h}}{1-\gamma_h}, \quad \gamma_h > 0. \tag{B.5}$$

Preferences are thus heterogeneous in risk aversion γ_h and climate concern ϕ_h. Damages are not tradable, so markets are incomplete: the non-pecuniary climate risk cannot be fully hedged with traded assets.

We focus on equilibria in which the shocks are small relative to the level of climate-adjusted consumption for each household. This allows for a local mean-variance approximation of expected utility around a deterministic reference point.

B.2 Local mean-variance approximation under CRRA

Define climate-adjusted consumption of household h as

$$Y_h(\zeta) \equiv C_1^h(\zeta) - \phi_h \Gamma(\zeta). \tag{B.6}$$

Using (B.1), (B.2), and (B.4), we can write

$$\begin{aligned} Y_h(\zeta) &= b^h R_f + \sum_{i=1}^{N} \theta_i^h \mu_i + \left(\sum_{i=1}^{N} \theta_i^h \eta X_i - \phi_h \kappa \bar{X} \right) \zeta \\ &= A_h + B_h \zeta, \end{aligned} \tag{B.7}$$

where

$$A_h \equiv b^h R_f + \sum_{i=1}^{N} \theta_i^h \mu_i, \qquad B_h \equiv \sum_{i=1}^{N} \theta_i^h \eta X_i - \phi_h \kappa \bar{X}.$$

Let $\bar{Y}_h$ denote a deterministic reference level for Y_h; for the local approximation we treat $\bar{Y}_h$ as fixed and assume that the stochastic component $B_h \zeta$ is small relative to $\bar{Y}_h$. A second-order Taylor expansion of $u_h(\cdot)$ around $\bar{Y}_h$ yields

$$u_h(Y_h) \approx u_h(\bar{Y}_h) + u_h'(\bar{Y}_h)(Y_h - \bar{Y}_h) + \frac{1}{2} u_h''(\bar{Y}_h)(Y_h - \bar{Y}_h)^2.$$

Taking expectations and using $\mathbb{E}[\zeta] = 0$ implies

$$\mathbb{E}[u_h(Y_h)] \approx u_h(\bar{Y}_h) + \frac{1}{2} u_h''(\bar{Y}_h) \mathrm{Var}(Y_h).$$

Since $\mathrm{Var}(Y_h) = B_h^2 \sigma_\zeta^2$, expected utility is approximately

$$\mathbb{E}[u_h(Y_h)] \approx u_h(\bar{Y}_h) + \frac{1}{2} u_h''(\bar{Y}_h) B_h^2 \sigma_\zeta^2.$$

Define the local absolute risk aversion of household h at $\bar{Y}_h$ by

$$\tilde{\alpha}_h \equiv -\frac{u_h''(\bar{Y}_h)}{u_h'(\bar{Y}_h)}. \tag{B.8}$$

For CRRA utility,

$$u_h'(x) = x^{-\gamma_h}, \qquad u_h''(x) = -\gamma_h x^{-\gamma_h - 1},$$

so

$$\tilde{\alpha}_h = \frac{\gamma_h}{\bar{Y}_h}. \tag{B.9}$$

Dividing the approximation for $\mathbb{E}[u_h(Y_h)]$ by $u_h'(\bar{Y}_h) > 0$ (which does not affect the argmax over

portfolios) yields the equivalent objective

$$\mathbb{E}[Y_h] - \frac{\tilde{\alpha}_h}{2}\mathrm{Var}(Y_h) = (A_h - \bar{Y}_h) + \bar{Y}_h - \frac{\tilde{\alpha}_h}{2}B_h^2\sigma_\zeta^2. \tag{B.10}$$

The constant terms are irrelevant for portfolio choice. Using $Y_h = C_1^h - \phi_h\Gamma(\zeta)$ and $\mathbb{E}[\zeta] = 0$, maximizing (B.10) is thus equivalent, in the local approximation, to maximizing

$$\mathbb{E}[C_1^h - \phi_h\Gamma(\zeta)] - \frac{\tilde{\alpha}_h}{2}\mathrm{Var}[C_1^h - \phi_h\Gamma(\zeta)]. \tag{B.11}$$

Expression (B.11) has the same form as the CARA-normal mean-variance objective in the main text, with the constant absolute risk aversion coefficient α_h replaced by the local coefficient $\tilde{\alpha}_h = \gamma_h/\bar{Y}_h$.

B.3 First-order conditions and equilibrium price of climate risk

The mean-variance objective (B.11) implies that household h chooses $\{\theta_i^h\}_{i=1}^N$ to maximize

$$\sum_{i=1}^N \theta_i^h(\mu_i - V_i R_f) - \frac{\tilde{\alpha}_h}{2}\sigma_\zeta^2\delta_h^2,$$

where

$$\delta_h \equiv \sum_{j=1}^N \theta_j^h \eta X_j - \phi_h\kappa\bar{X}. \tag{B.12}$$

The first-order condition with respect to θ_i^h is

$$\mu_i - V_i R_f - \tilde{\alpha}_h\sigma_\zeta^2\delta_h\eta X_i = 0, \qquad i = 1, \ldots, N. \tag{B.13}$$

The left-hand side of (B.13) does not depend on h, so equilibrium requires that the product $\tilde{\alpha}_h\sigma_\zeta^2\delta_h$ coincide across all households h that hold stock i. Hence there exists a scalar λ such that

$$\mu_i - V_i R_f = \lambda\eta X_i, \qquad \delta_h = \frac{\lambda}{\tilde{\alpha}_h\sigma_\zeta^2}. \tag{B.14}$$

Equation (B.14) is the *price* (dollar) risk-premium relation. Dividing by V_i and using $R_i \equiv D_i/V_i$ yields the corresponding *return* representation,

$$\mathbb{E}[R_i] - R_f = \lambda\eta\frac{X_i}{V_i}. \tag{B.15}$$

Thus, as in the CARA benchmark, emission *levels* X_i govern the primitive cash-flow exposure in dividends, while expected *returns* load on the exposure X_i/V_i.

To solve for λ, multiply the definition of δ_h in (B.12) by η and sum over h:

$$\sum_{h=1}^{H}\sum_{j=1}^{N}\theta_j^h\eta^2 X_j - \kappa\bar{X}\eta\sum_{h=1}^{H}\phi_h = \eta\sum_{h=1}^{H}\delta_h. \tag{B.16}$$

Using market clearing $\sum_{h=1}^{H}\theta_j^h = 1$ for each j, the left-hand side becomes

$$\sum_{j=1}^{N}\eta^2 X_j - \kappa\bar{X}\eta\sum_{h=1}^{H}\phi_h = \eta^2\bar{X} - \kappa\bar{X}\eta\sum_{h=1}^{H}\phi_h.$$

The right-hand side of (B.16), using (B.14), is

$$\eta\sum_{h=1}^{H}\delta_h = \eta\sum_{h=1}^{H}\frac{\lambda}{\tilde{\alpha}_h\sigma_\zeta^2} = \frac{\eta\lambda}{\sigma_\zeta^2}\sum_{h=1}^{H}\frac{1}{\tilde{\alpha}_h}.$$

Equating both sides and dividing by η yields

$$\eta\bar{X} - \kappa\bar{X}\sum_{h=1}^{H}\phi_h = \frac{\lambda}{\sigma_\zeta^2}\sum_{h=1}^{H}\frac{1}{\tilde{\alpha}_h}.$$

Solving for λ gives the CRRA counterpart to Proposition 2.

Proposition A.1 (Price of climate risk under CRRA)**.** *Under the assumptions of this appendix, in a competitive equilibrium with CRRA preferences and incomplete markets, the equilibrium price of climate risk is*

$$\lambda = \frac{\sigma_\zeta^2\bar{X}}{\sum_{h=1}^{H}\frac{1}{\tilde{\alpha}_h}}\Big(\eta - \kappa\sum_{h=1}^{H}\phi_h\Big), \qquad \tilde{\alpha}_h = \frac{\gamma_h}{\bar{Y}_h}. \tag{B.17}$$

Accordingly, expected excess returns satisfy the return representation (B.15)*,*

$$\mathbb{E}[R_i] - R_f = \lambda\eta\frac{X_i}{V_i},$$

and the corresponding price (dollar) relation in (B.14) *can be written equivalently as*

$$\mathbb{E}[D_i] - P_i R_f = \lambda\eta X_i.$$

The denominator in (B.17) can be written as

$$\sum_{h=1}^{H} \frac{1}{\tilde{\alpha}_h} = \sum_{h=1}^{H} \frac{\bar{Y}_h}{\gamma_h},$$

which is the sum of local risk tolerances $\bar{Y}_h/\gamma_h$. Thus, relative to the CARA case, the price of climate risk is governed by aggregate risk tolerance rather than by $\sum_h 1/\alpha_h$. The term in parentheses, $(\eta - \kappa \sum_h \phi_h)$, is unchanged: as in the CARA benchmark, greater aggregate climate concern $\sum_h \phi_h$ reduces, and can in principle overturn, the purely financial component of the brown premium.

B.4 Discussion and relation to the CARA benchmark

Proposition A.1 shows that, in a CRRA-incomplete-markets setting, the local linear *price* relation

$$\mathbb{E}[D_i] - V_i R_f = \lambda \eta X_i$$

continues to hold, with a price of climate risk λ given by (B.17). The structure is identical to the CARA-normal benchmark: the primitive cash-flow exposure is proportional to the emission level X_i, the numerator scales with $\sigma_\zeta^2 \bar{X}$, and the aggregate climate concern $\sum_h \phi_h$ enters linearly in the term $(\eta - \kappa \sum_h \phi_h)$. The only change is that the denominator is the sum of local risk tolerances, $\sum_h \bar{Y}_h/\gamma_h$, rather than the sum of inverse CARA coefficients.

Translating this into a statement about *returns* requires dividing by the asset price: from the price relation one obtains

$$\mathbb{E}[R_i] - R_f = \frac{\mathbb{E}[D_i] - V_i R_f}{V_i} = \lambda \eta \frac{X_i}{V_i}.$$

Under the local approximation, the economic interpretation of the incomplete-markets equilibrium is unchanged. Heterogeneous risk aversion and climate concern, combined with unhedgeable climate damages, generate a nontrivial price of climate risk λ and a one-factor representation in which the primitive exposure is level-based in dividends while return premia load on the value-normalized exposure. The CARA and CRRA formulations therefore lead to qualitatively identical asset-pricing implications within the class of small-shock, linear-exposure environments considered in the main text.

C Welfare Losses from Market Incompleteness

This appendix provides the formal derivation of the welfare-loss measure summarized in Section **??**. The purpose is to make precise how welfare comparisons are conducted in the presence of incomplete markets and heterogeneous climate concern, and to clarify the sense in which equilibrium carbon premia differ from planner implementation wedges. As emphasized in the main text, the analysis here is not intended to map observed carbon premia into a social cost of carbon, but rather to characterize the allocation-level efficiency cost that arises when climate risk is imperfectly shared.

C.1 Planner welfare benchmark

Let $W(\cdot)$ denote the planner welfare functional introduced in Section 3. For any feasible aggregate allocation (C_0, C_1, X_1),

$$W(C_0, C_1, X_1) \equiv u(C_0) + \beta(u(C_1) - d(X_1)),$$

where $d(X_1)$ denotes aggregate climate damages. Let $(C_0^{FB}, C_1^{FB}, X_1^{FB})$ denote the first-best allocation that solves the planner's problem under complete markets, and define

$$W^{FB} \equiv W(C_0^{FB}, C_1^{FB}, X_1^{FB}).$$

C.2 Incomplete-markets allocation

Let $(C_0^{IM}, C_1^{IM}, X_1^{IM})$ denote the aggregate allocation induced by the competitive equilibrium in the incomplete-markets economy described in Section 4, with the same technology, damage function, and shock distribution as in the planner benchmark. The planner evaluation of this second-best allocation is

$$W^{IM} \equiv W(C_0^{IM}, C_1^{IM}, X_1^{IM}).$$

Because climate damages are unspanned and households differ in climate concern, the allocation $(C_0^{IM}, C_1^{IM}, X_1^{IM})$ generally does not satisfy the planner's first-order conditions, even though asset prices in equilibrium reflect well-defined carbon premia.

C.3 Consumption-equivalent welfare loss

The welfare loss from market incompleteness is defined in consumption-equivalent units. Specifically, let $\Delta c_1 \geq 0$ denote the unique scalar (on the relevant domain) that satisfies

$$u(C_0^{FB}) + \beta\big(u(C_1^{FB} - \Delta c_1) - d(X_1^{FB})\big) \;=\; W^{IM}.$$

The quantity Δc_1 measures the reduction in date-1 consumption that makes the planner indifferent between the first-best allocation and the incomplete-markets allocation.

For reporting purposes, we also express the welfare loss as a fraction of first-best date-1 consumption,

$$\Delta W \;\equiv\; \frac{\Delta c_1}{C_1^{FB}}.$$

C.4 Interpretation and decomposition

The welfare loss ΔW captures two distinct channels. First, incomplete markets prevent full insurance against aggregate climate risk, leading to inefficient consumption smoothing relative to the planner benchmark even in the absence of preference heterogeneity. Second, heterogeneity in climate concern implies that the equilibrium stochastic discount factor reflects a wealth-weighted aggregation of private marginal rates of substitution rather than the planner's marginal valuation of climate damages. As a result, equilibrium investment and emission choices do not internalize the social cost of carbon.

Importantly, ΔW is an allocation object and need not be monotone in the magnitude of equilibrium carbon premia. Carbon premia summarize second-best pricing of exposure to climate and policy shocks, whereas ΔW is pinned down by the gap between allocations under complete and incomplete risk sharing evaluated under a common welfare criterion. This distinction clarifies why observed carbon premia should not be interpreted as sufficient statistics for welfare or for optimal climate policy.

D Infinite-horizon production economy

This section embeds the static incomplete-markets structure into a dynamic production economy. The goal is to show formally that, under standard CARA-normal assumptions, the stochastic discount factor (SDF) admits an exponential-affine representation with a climate-shock loading

that is directly linked to the cross-section of (α_h, ϕ_h), and that the resulting dynamic pricing relation preserves the emission-based structure derived in the static model.

Technology, capital, and emissions Time is discrete, $t = 0, 1, 2, \ldots$. Each firm $i \in \{1, \ldots, N\}$ chooses capital $K_{i,t}$ and an emission scale $q_{i,t}$ at each date t. Output at t is given by

$$Y_{i,t} = F_i(K_{i,t}, q_{i,t}) - \Psi_i(q_{i,t}), \tag{D.1}$$

where $F_i : \mathbb{R}_+^2 \to \mathbb{R}_+$ is strictly concave and twice continuously differentiable, and $\Psi_i : \mathbb{R}_+ \to \mathbb{R}_+$ is convex.

Emissions of firm i at date t are

$$X_{i,t} = \chi_i q_{i,t}, \qquad \chi_i > 0, \tag{D.2}$$

so that aggregate emissions are $\bar{X}_t = \sum_{i=1}^N X_{i,t}$. Capital evolves according to

$$K_{i,t+1} = (1 - \delta_i)K_{i,t} + I_{i,t}, \qquad \delta_i \in (0, 1). \tag{D.3}$$

Climate risk and dividends Let $\{\zeta_t\}_{t \geq 1}$ be an i.i.d. climate or policy shock with $\mathbb{E}[\zeta_{t+1}|\mathcal{F}_t] = 0$ and $\mathrm{Var}[\zeta_{t+1}|\mathcal{F}_t] = \sigma_\zeta^2$. Climate damages at $t + 1$ are

$$\Gamma_{t+1} = \kappa \bar{X}_t \zeta_{t+1}, \qquad \kappa > 0. \tag{D.4}$$

Firm i's dividend at date $t + 1$ is

$$D_{i,t+1} = Y_{i,t+1} - I_{i,t+1} + \eta X_{i,t} \zeta_{t+1} + \tilde{\varepsilon}_{i,t+1}, \tag{D.5}$$

where $\tilde{\varepsilon}_{i,t+1}$ is idiosyncratic. As in the static model, the primitive cash-flow exposure to climate risk is proportional to the *level* of emissions $X_{i,t}$.

Households, incomplete markets, and dynamic mean–variance representation There are H households indexed by h. Household h has preferences

$$U^h = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u^h(C_{h,t} - \phi_h \Gamma_t)\right], \qquad u^h(x) = -\exp(-\alpha_h x), \tag{D.6}$$

with heterogeneity in (α_h, ϕ_h). Climate damages Γ_t are not traded, so markets are incomplete.

Define climate-adjusted consumption

$$\tilde{C}_{h,t} = C_{h,t} - \phi_h \Gamma_t.$$

Under CARA utility and joint conditional normality, the household's dynamic problem reduces at each date t to

$$\max \ \mathbb{E}_t[\tilde{C}_{h,t+1}] - \frac{\alpha_h}{2}\mathrm{Var}_t(\tilde{C}_{h,t+1}), \tag{D.7}$$

which is the exact dynamic analogue of the static mean-variance problem.

Aggregate stochastic discount factor and its climate loading Let $\theta^h_{i,t}$ denote household h's holdings of firm i at date t. Using (D.4)-(D.5), the innovation to climate-adjusted consumption can be written as

$$\tilde{C}_{h,t+1} - \mathbb{E}_t[\tilde{C}_{h,t+1}] = \Big(\sum_{i=1}^{N} \theta^h_{i,t}\eta X_{i,t} - \phi_h \kappa \bar{X}_t\Big)\zeta_{t+1} + \tilde{\varepsilon}_{h,t+1}. \tag{D.8}$$

The household's optimal portfolio choice implies a payoff-premium condition,

$$\mathbb{E}_t[D_{i,t+1}] - (1 + R_{f,t})V_{i,t} = \alpha_h \sigma_\zeta^2 \delta_{h,t}\eta X_{i,t}, \qquad \delta_{h,t} = \sum_{j=1}^{N} \theta^h_{j,t}\eta X_{j,t} - \phi_h \kappa \bar{X}_t. \tag{D.9}$$

Since the left-hand side does not depend on h, market clearing implies the existence of a scalar λ_t such that

$$\lambda_t = \frac{\sigma_\zeta^2 \bar{X}_t}{\sum_{h=1}^{H} 1/\alpha_h}\Big(\eta - \kappa \sum_{h=1}^{H} \phi_h\Big). \tag{D.10}$$

SDF representation and pricing implications The aggregate SDF between t and $t+1$ admits the exponential-affine form

$$m_{t+1} = M_t \exp(-B_t \zeta_{t+1}), \tag{D.11}$$

with $\mathbb{E}_t[m_{t+1}] = 1/(1 + R_{f,t})$. A log-linear approximation to the Euler equation implies the payoff risk-premium relation

$$\mathbb{E}_t[D_{i,t+1}] - (1 + R_{f,t})V_{i,t} \approx B_t \sigma_\zeta^2 \eta X_{i,t}. \tag{D.12}$$

Comparing (D.12) with (D.9) and using (D.10) identifies the SDF loading,

$$B_t = \frac{\lambda_t}{\sigma_\zeta^2} = \frac{\bar{X}_t}{\sum_{h=1}^{H} 1/\alpha_h}\Big(\eta - \kappa \sum_{h=1}^{H} \phi_h\Big). \tag{D.13}$$

Dividing (D.12) by firm value yields the expected excess return,

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx B_t \sigma_\zeta^2 \eta \frac{X_{i,t}}{V_{i,t}}. \tag{D.14}$$

The pricing kernel loading B_t summarizes the aggregate price of climate risk. Although B_t is common across firms, equilibrium emissions $X_{i,t}$ and firm values $V_{i,t}$ are jointly determined by firm-specific technologies, capital stocks, and adjustment costs. Consequently, comparisons based solely on contemporaneous emission intensity $X_{i,t}/V_{i,t}$ need not preserve the ranking of firms by equilibrium exposure to climate risk once emission policies are endogenous and firm values reflect continuation payoffs under optimal adjustment.

Dynamic asset pricing relation Combining (D.14) and (D.13) yields

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_{X,t} \frac{X_{i,t}}{V_{i,t}}, \qquad \lambda_{X,t} \equiv \lambda_t \eta. \tag{D.15}$$

Primitive cash-flow exposure depends on emission levels $X_{i,t}$, while expected returns load on the corresponding value-normalized exposure $X_{i,t}/V_{i,t}$. To relate the dynamic pricing relation to Proposition 3, note that equation (D.15) can be written in the one-factor form $\mathbb{E}_t[R_{i,t+1} - R_{f,t}] = \lambda_t\, g(Z_{i,t})/V_{i,t}$ with a reduced-form exposure $g(Z_{i,t})$ proportional to emissions $X_{i,t}$. However, in the dynamic economy, $g(Z_{i,t})$ reflects firms' equilibrium emission policies, which respond endogenously to the climate shock through optimal investment and adjustment decisions. These responses depend on firm-specific state variables and are therefore not scale-invariant across firms with the same emission intensity. As a result, emission intensity $X_{i,t}/V_{i,t}$ fails to be a sufficient statistic for expected excess returns, despite the apparent one-factor representation.

Firms' dynamic investment and emission choices Let $V_{i,t}$ denote the ex-dividend market value of firm i at date t. Shareholders value the stream of dividends using the aggregate stochastic discount factor:

$$V_{i,t} = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\left(\prod_{u=t+1}^{s} m_u\right) D_{i,s}\right]. \tag{D.16}$$

Firm i chooses sequences $\{K_{i,t+1}, q_{i,t}\}_{t\geq 0}$ to maximize $V_{i,0}$, taking prices and the SDF process $\{m_{t+1}\}$ as given.

The Bellman equation for firm i is

$$V_{i,t} = \max_{K_{i,t+1},\, q_{i,t}} \left\{ \mathbb{E}_t[m_{t+1} D_{i,t+1}] + \mathbb{E}_t[m_{t+1} V_{i,t+1}] \right\}, \tag{D.17}$$

subject to (D.1), (D.3), and (D.5).

The first-order condition with respect to the emission scale $q_{i,t}$ is

$$\mathbb{E}_t\left[m_{t+1} \frac{\partial D_{i,t+1}}{\partial q_{i,t}} \right] + \mathbb{E}_t\left[m_{t+1} \frac{\partial V_{i,t+1}}{\partial q_{i,t}} \right] = 0. \tag{D.18}$$

Under the maintained assumption that $q_{i,t}$ affects future firm value only through next-period dividends, the second term in (D.18) vanishes. Using (D.5), the first-order condition simplifies to

$$\mathbb{E}_t\left[m_{t+1} \left(\frac{\partial F_i(K_{i,t}, q_{i,t})}{\partial q_{i,t}} - \Psi_i'(q_{i,t}) + \eta \chi_i \zeta_{t+1} \right) \right] = 0. \tag{D.19}$$

Using (D.11) and normality of ζ_{t+1},

$$\mathbb{E}_t[m_{t+1}\zeta_{t+1}] = -B_t \sigma_\zeta^2 \, \mathbb{E}_t[m_{t+1}],$$

so that (D.19) can be written as

$$\mathbb{E}_t[m_{t+1}] \left(\frac{\partial F_i(K_{i,t}, q_{i,t})}{\partial q_{i,t}} - \Psi_i'(q_{i,t}) - \eta \chi_i B_t \sigma_\zeta^2 \right) = 0. \tag{D.20}$$

Dividing by $\mathbb{E}_t[m_{t+1}]$ and using $B_t = \lambda_t / \sigma_\zeta^2$ yields

$$\frac{\partial F_i(K_{i,t}, q_{i,t})}{\partial q_{i,t}} - \Psi_i'(q_{i,t}) = \eta \chi_i \lambda_t. \tag{D.21}$$

Equation (D.21) shows that firms equate marginal private benefits of emissions to a climate-risk pricing wedge governed by the same equilibrium price of risk λ_t that prices returns in (D.15). In contrast to a representative-agent benchmark, this wedge reflects equilibrium asset pricing under incomplete markets rather than a planner's Pigouvian tax based on emission intensity (violating Proposition 3 (ii)–(iii)).

Although expected returns admit the reduced-form representation $\mathbb{E}_t[R_{i,t+1} - R_{f,t}] \approx \eta B_t X_{i,t} / V_{i,t}$,

this expression should be interpreted as a pricing relation conditional on equilibrium emission choices. In the dynamic economy, both emissions $X_{i,t}$ and firm values $V_{i,t}$ are endogenous objects determined by optimal investment and adjustment decisions that depend on firm-specific state variables. As a result, the scale-invariance condition required for emission intensity to serve as a robust sufficient statistic becomes dynamically fragile: intensity-based comparisons need not be stable once equilibrium policies and continuation values are taken into account (Proposition 3).

E Additional Tables

Table A.1: Scope 1 Emissions Growth and Future Stock Returns

VARIABLES	(1)	(2)	(3)	(4)
	Stock returns			
	1 month ahead		12 months ahead	
Scope 1 Emissions Growth	0.491***	0.442***	-0.031	-0.056
	(0.057)	(0.054)	(0.034)	(0.034)
Log(Scope 1 Emissions)		0.090***		0.044***
		(0.016)		(0.010)
Log(Size)	-0.212***	-0.259***	-0.140***	-0.163***
	(0.050)	(0.051)	(0.037)	(0.037)
Log(M/B)	-0.258***	-0.218***	-0.157*	-0.138*
	(0.071)	(0.068)	(0.074)	(0.071)
Book leverage	-0.061	-0.166	-0.112	-0.159
	(0.246)	(0.247)	(0.176)	(0.174)
Momentum	0.437**	0.429**	-0.392*	-0.393*
	(0.162)	(0.163)	(0.188)	(0.188)
Investment/Assets	-0.628	-0.497	0.136	0.190
	(0.841)	(0.858)	(0.738)	(0.750)
Log(PPE)	0.088***	0.059**	0.056**	0.042**
	(0.028)	(0.027)	(0.020)	(0.020)
Volatility	1.931	1.932	0.418	0.408
	(3.161)	(3.155)	(1.516)	(1.515)
ROE	0.911***	0.849***	0.546	0.515
	(0.257)	(0.264)	(0.313)	(0.312)
Age	1.128	0.905	-0.436	-0.540
	(0.925)	(0.946)	(0.927)	(0.968)
Constant	2.091***	1.834***	1.800***	1.676***
	(0.449)	(0.464)	(0.297)	(0.294)
Industry×time fixed effects	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes
Observations	1,006,227	1,005,779	862,397	862,073
R-squared	0.232	0.232	0.247	0.247

Robust standard errors in parentheses

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$