Democratizing the Equity Premium: Progressive Investment Incentives During Disasters*

Naowar Mohiuddin†

November 4, 2025

Click here for the latest version

Abstract

Economic crises amplify wealth divergence as constrained households liquidate assets while others with equity exposure capture recovery gains. Progressive investment subsidies can broaden equity market participation among vulnerable households by targeting the portfolio margin rather than just income. Using a heterogeneous-agent New Keynesian model with sectoral pandemic exposure, two-asset choice, and borrowing constraints calibrated to U.S. data, I analyze state-contingent transfers with 60:1 progressivity targeting low-income households in high-exposure sectors. The policy operates through three channels: relaxing liquidity constraints that prevent equity investment, insuring sector-specific income risk that induces flight to safe assets, and concentrating support when asset prices are depressed and risk premia are elevated. By enabling constrained households to maintain risky-asset positions precisely when expected returns are highest, the policy connects short-run stabilization to long-run wealth accumulation through portfolio reallocation. Bottom-quintile households increase equity participation and holdings substantially, demonstrating that well-timed, progressive transfers can democratize access to the equity premium.

JEL Codes: E21, E22, E24, G11, G51, H31

Keywords: Wealth inequality, investment subsidies, heterogeneous agents, portfolio choice, fiscal policy, rare disasters

*I am grateful to my Advisor William A. Barnett for invaluable guidance and feedback. I thank my dissertation committee John Keating, Shahanaz Parsaeian, William Bazley. All errors are my own.

†Department of Economics, University of Kansas. Email: naowarm@ku.edu

1 Introduction

Over the past four decades, wealth inequality in the United States has surged to unprecedented levels. Between 1980 and 2020, the ratio of aggregate household wealth to national income surged from 300% to nearly 570%, indicating that wealth accumulation has significantly outpaced economic growth (Saez and Zucman, 2020). This trend has disproportionately benefited the wealthiest households, amplifying pre-existing economic disparities. The widening gap reflects not only differences in savings behavior but also unequal access to high-return assets. The wealthiest households hold diversified portfolios with substantial equity exposure, capturing the equity risk premium that has averaged 6 to 8 percent annually above Treasury returns since 1926 (Mehra and Prescott, 1985). In contrast, lower-wealth households hold predominantly safe, low-return assets such as bank deposits and money market funds, foregoing the compounding benefits of equity investment. This portfolio gap perpetuates and amplifies wealth inequality over time (Benhabib et al., 2017).

The COVID-19 pandemic dramatically accelerated these dynamics. The recession triggered severe job losses, with unemployment spiking from 3.5% in February 2020 to 14.7% in April 2020, the highest level since the Great Depression (Bureau of Labor Statistics, 2020). Yet aggregate equity markets rebounded swiftly: the S&P 500 fell 34% from February to March 2020, then surged 70% from the trough through December 2021, reaching all-time highs despite ongoing economic disruption. This V-shaped asset price recovery created a stark divergence in wealth trajectories. Households with substantial equity holdings, concentrated among the top 10% who own 85% of directly-held stocks, experienced significant wealth gains (Batty et al., 2019). Meanwhile, households holding primarily safe assets saw their real purchasing power erode through inflation without participating in the market rebound.

The Federal Reserve's Survey of Consumer Finances documents this growing portfolio heterogeneity. In 2019, only 53% of U.S. families held any equity, either directly or through retirement accounts, down from 57% in 2007 (Bhutta et al., 2020). Among the bottom 50% of the wealth distribution, stock ownership rates were below 15%. This limited participation means that when equity markets rally, as they did during the pandemic recovery, the benefits accrue disproportionately to households already at the top of the wealth distribution. The bottom 50% collectively holds less than 2% of total household wealth, and their median net worth declined in real terms between 2001 and 2019, even as the stock market tripled (Federal Reserve Board, 2020).

During the COVID-19 pandemic, the accumulation of excess wealth was highly uneven across the income distribution. Using data from the Distributional Financial Accounts,

recent studies show that the top 1% accumulated over $4 trillion in excess wealth from 2020 to 2022 (DFA, 2025), largely driven by asset price appreciation in equities and real estate (Batty et al., 2021). The bottom 50%, despite receiving substantial fiscal transfers through stimulus payments and expanded unemployment insurance, accumulated less than $400 billion, a tenth of the gains accruing to the top 1%. This 10:1 ratio reflects the portfolio channel: fiscal transfers enabled lower-income households to smooth consumption and avoid depleting savings, but did not encourage portfolio reallocation toward risky assets that would have positioned them to benefit from the subsequent recovery.

While existing COVID-19 policy research emphasizes income stabilization through unemployment insurance and stimulus payments (Chodorow-Reich and Coglianese, 2019; Ganong et al., 2020; Faria-e-Castro, 2021), these interventions do not address the portfolio channel through which disasters affect wealth inequality. During the 2020 recession, households with equity exposure saw wealth gains of 20 to 30 percent during the 2020 to 2021 recovery, while those holding only safe assets lagged behind. This divergence arose not from differential savings rates but from asset composition: who held stocks versus who held cash. The wealth gains were highly concentrated: the top 10% captured over 80% of pandemic-era asset price appreciation (Saez and Zucman, 2020). Moreover, the pandemic had highly heterogeneous sectoral impacts. Employment in leisure and hospitality fell 22% in 2020Q2, compared to only 5% in professional services (Bureau of Labor Statistics, 2020). Workers in high-exposure sectors faced both the largest income losses and the most severe barriers to maintaining equity market participation during the crisis, when prospective returns were elevated.

This paper asks: can fiscal policy encourage broader equity market participation among vulnerable households during disasters, thereby reducing wealth inequality through the asset accumulation channel? I propose and analyze a novel mechanism: progressive, sector-targeted investment subsidies that provide income-contingent transfers to households in high-COVID-exposure industries. The subsidy is structured to target three key dimensions of heterogeneity: sector exposure to the pandemic shock, position in the income distribution, and employment status. High-exposure sectors such as leisure and hospitality receive subsidies when pandemic intensity is severe, with the eligibility threshold set at $\bar{\eta} = 0.15$, ensuring transfers reach sectors experiencing significant disruptions. Within these sectors, subsidies are highly progressive: the bottom income group receives transfers with weight 3.0, while the top income group receives weight 0.05, creating a 60:1 ratio that concentrates benefits on the most vulnerable. Unemployed households receive an additional 50% amplifier, recognizing their elevated background risk. Theoretically, the model formalizes how such transfers alter household portfolio choices by tracing the marginal response of risky-asset holdings to targeted fiscal support. This mechanism links fiscal design to equity-market par-

ticipation, extending standard heterogeneous agent New Keynesian (HANK) transmission channels to show how progressive, state-contingent transfers can stabilize asset accumulation during crises rather than merely smoothing income.

This design has three key features that distinguish it from existing redistribution policies. First, unlike asset-proportional subsidies, which would primarily benefit existing high-wealth stockholders, the progressive income-based structure concentrates benefits on low-income households who face the largest barriers to market participation: fixed costs of brokerage accounts, limited financial literacy, and elevated effective risk aversion due to binding borrowing constraints. Second, unlike lump-sum transfers such as stimulus checks or universal basic income, the sector-targeting ensures subsidies reach households with the highest pandemic-specific income risk, compensating for elevated background risk that would otherwise induce flight to safe assets. Third, unlike unemployment insurance that smooths consumption during job loss but provides no incentive for portfolio reallocation, the investment subsidy raises disposable income precisely when equity risk premia are elevated and prospective returns are high, encouraging participation at the optimal time in the asset price cycle.

The policy mechanism operates through three channels. The income effect raises disposable resources for constrained households, relaxing borrowing constraints and reducing the marginal utility of safe, liquid savings. The background risk offset compensates for sector-specific income volatility during the pandemic, effectively lowering the covariance between labor income and risky asset returns, which in standard portfolio theory reduces optimal precautionary holdings of safe assets. The timing effect concentrates subsidies when equity risk premia are elevated, during the trough when asset prices are depressed, enabling vulnerable households to buy low and position themselves to benefit from the subsequent recovery. The policy thus targets the participation margin: expanding the set of households who hold stocks going into the equity market rebound, rather than just helping existing stockholders accumulate more.

I analyze this mechanism quantitatively using a heterogeneous agent New Keynesian model calibrated to U.S. data. The model features three sectors differentiated by COVID exposure representing professional services, retail and manufacturing, and leisure and hospitality; eleven income states capturing both earnings dispersion and employment risk; and two-asset portfolio choice with liquid safe bonds versus illiquid risky equity. Households face borrowing constraints, Calvo adjustment frictions in illiquid asset rebalancing, and idiosyncratic income shocks. The production side features sticky prices with Rotemberg adjustment costs, capital accumulation with investment adjustment costs, and standard New Keynesian elements including a Taylor rule and government budget constraint. I calibrate the model to match wealth distribution facts from the 2019 Survey of Consumer Finances and labor

market flows.

The main quantitative results demonstrate substantial effectiveness of the investment subsidy policy. A 10% subsidy rate generates peak consumption improvements of 2.43 percentage points relative to baseline. The welfare gains, measured through consumption-equivalent variation, are substantial but highly heterogeneous across sectors. High-exposure sectors receive welfare gains equivalent to 1.5% of lifetime consumption. These workers experience the largest income losses during the pandemic but benefit most from the targeted subsidies. Medium-exposure sectors gain 0.6% of lifetime consumption. Low-exposure sectors such as professional services and finance experience modest welfare costs of 3.0% due to general-equilibrium effects: they do not receive subsidies but face slightly higher taxes to finance the program. The economy-wide welfare gain, population-weighted across sectors, is 1.06% of lifetime consumption.

The fiscal cost is moderate relative to actual pandemic spending. Subsidies peak at 2.37% of GDP at shock arrival, decay to 0.48% by quarter 4 and 0.08% by quarter 8, with cumulative outlays over 10 years totaling 7.1% of GDP. This compares favorably to the approximately $5 trillion, or 25% of GDP, in actual U.S. pandemic fiscal measures, which consisted primarily of untargeted Economic Impact Payments, Paycheck Protection Program loans, and expanded unemployment benefits (Congressional Budget Office, 2021). The progressivity of the subsidy is crucial to cost-effectiveness: the highly progressive weight structure ensures that approximately 60% of outlays flow to the bottom two income quintiles, who have the highest marginal propensities to adjust portfolios in response to the policy.

Robustness analysis confirms that results are not sensitive to key modeling choices. I test alternative calibrations across multiple dimensions and find qualitatively similar outcomes. The sensitivity analysis demonstrates that consumption and asset improvements remain substantial across a range of progressivity weights, eligibility thresholds, and COVID intensity parameters. A five-sector model with finer exposure gradation produces quantitatively similar consumption improvements. Varying the subsidy rate traces out the policy frontier: marginal benefits decline smoothly while marginal costs rise, with the benefit-cost ratio remaining favorable across a wide range of parameterizations.

Literature. This paper contributes to three strands of literature. First, I extend the heterogeneous agent New Keynesian literature. The HANK framework, pioneered by Kaplan et al. (2018), embeds household heterogeneity and incomplete markets into New Keynesian models to study how monetary and fiscal policy affects inequality. Recent applications include the distributional effects of monetary policy (Auclert, 2019; Bayer et al., 2024), the role of hand-to-mouth households in fiscal multipliers (McKay et al., 2016), and the interaction between wealth inequality and aggregate demand (Auclert et al., 2024). These studies

primarily focus on policies that target income stabilization: unemployment insurance, stimulus checks, and progressive taxation. I contribute by analyzing a fiscal policy tool that targets the portfolio margin rather than just income. Investment subsidies affect inequality through a distinct channel: increasing equity exposure among constrained households. The mechanism operates by relaxing borrowing constraints and reducing background risk for vulnerable groups, thereby encouraging risk-taking when asset prices are depressed and expected returns are high. Methodologically, I extend the HANK framework by incorporating sectoral heterogeneity in pandemic exposure, building on recent work that introduces household heterogeneity in exposure to aggregate shocks (Acharya et al., 2020; Gregory et al., 2020).

Second, I contribute to the rare disaster literature. Barro (2006) and Gourio (2012) show that rare disasters can explain key asset pricing puzzles including the equity premium and the volatility of consumption, using models with representative agents. Subsequent work has explored how disaster risk affects capital accumulation (Fernandez-Villaverde and Levintal, 2018) and optimal policy (Farhi and Werning, 2017). However, most disaster models abstract from distributional consequences by using representative agents. I show that heterogeneous exposure generates substantial cross-sectional variation in consumption losses. The welfare analysis demonstrates that disaster risk has heterogeneous costs across the income distribution, with the bottom quintile in high-exposure sectors suffering consumption drops of 16% versus 1% for the top quintile in low-exposure sectors. This heterogeneity creates an equity rationale for targeted policy intervention during disasters.

Third, I add to the wealth inequality literature. Saez and Zucman (2016) document the rise in U.S. wealth concentration and attribute it to differential returns on wealth across the distribution. Fagereng et al. (2020) use Norwegian administrative data to show that return heterogeneity is persistent and explains a substantial portion of wealth inequality dynamics. Benhabib et al. (2019) develop models where stochastic returns generate Pareto wealth distributions. Policy responses to wealth inequality have focused on ex-post redistribution through wealth taxes (Saez and Zucman, 2019) or estate taxes (Piketty and Saez, 2013), or on increasing savings rates among the poor through matched retirement accounts (Gale et al., 2005) and baby bonds (Hamilton and Darity, 2010). I identify a novel policy mechanism that operates through differential asset returns rather than differential savings rates. Investment subsidies work ex-ante by democratizing access to the equity risk premium, enabling lower-wealth households to participate in the same high-return investments available to the wealthy. This distinction is important because the policy democratizes access to high-return assets rather than redistributing existing wealth. By preventing liquidity flight and expanding equity participation (bottom quintile: +5.99 pp participation, +13% asset holdings), the

subsidy positions vulnerable households to capture compounding returns. The 1 percent direct transfer enables a 13 percent increase in equity holdings, which compounds to 8.91 percent cumulative wealth gains over ten years through equity premium returns.

The paper is organized as follows. Section 2 gives a detailed description of the policy design. Section 3 develops the heterogeneous agent New Keynesian model with sectoral pandemic exposure and two-asset portfolio choice, detailing the progressive investment subsidy mechanism. Section 4 describes the calibration strategy, matching key moments from U.S. wealth distribution and labor market data. Section 5 presents the quantitative results: impulse responses to a COVID shock with and without the subsidy policy, welfare analysis by sector and income quintile, fiscal cost calculations, and robustness checks. Section 6 discusses policy implications and extensions. Section 7 concludes. Technical details on the computational algorithm and additional robustness exercises are in the appendices.

2 Policy Design: Investment Subsidies for Inclusive Growth

This section provides an intuitive overview of the investment subsidy mechanism. Section 3 formalizes the policy within the full model environment, and Section 3.1.7 provides detailed technical rationale.

2.1 The Investment Subsidy Mechanism

During economic crises, vulnerable households face a dual challenge: immediate income losses force them to liquidate assets for consumption, while elevated background risk induces flight to safe assets. This occurs precisely when equity risk premia are highest and expected returns are elevated. The investment subsidy addresses this by providing progressive, sector-targeted transfers that enable constrained households to maintain or increase risky asset holdings during downturns.

The policy has four key design features. First, it is *progressive*: low-income households receive substantially larger transfers relative to their income than high-income households, with a 60:1 ratio between bottom and top income groups as the bottom 20% receives weight 3.0 while the top 20% receives weight 0.05. Second, it is *sector-targeted*: transfers concentrate on industries facing the largest pandemic-specific shocks, such as leisure, hospitality, and food services, with eligibility triggered when sector exposure times pandemic intensity exceeds $\bar{\eta} = 0.15$. Third, it is *state-contingent*: subsidy intensity rises automatically during severe episodes and phases out during recovery, avoiding permanent fiscal commitment. Fourth, it

is *income-based rather than asset-proportional*: transfers depend on position in the income distribution, not existing asset holdings, ensuring benefits reach households excluded from equity markets rather than subsidizing existing stockholders.

By raising disposable income for constrained households during crises, the subsidy relaxes borrowing constraints, reduces effective risk aversion, and compensates for elevated background risk. This enables vulnerable households to enter or maintain positions in risky asset markets precisely when prospective returns are highest, positioning them to benefit from the subsequent recovery in equity prices.

2.2 Three Market Failures Addressed

The investment subsidy targets three distinct market failures that prevent vulnerable households from accessing high-return assets during disasters.

Market Failure 1: Incomplete Markets and Participation Barriers. Lower-income households face substantial barriers to equity market participation. Fixed costs of opening brokerage accounts, minimum investment requirements, and limited financial literacy exclude many households from stock ownership (Gomes and Michaelides, 2008). Among those who do participate, binding borrowing constraints and high marginal utility of current consumption induce flight to liquid, safe assets during income shocks. The result is that the bottom 50 percent of the wealth distribution holds less than 10 percent of total risky assets despite representing half the population (Bhutta et al., 2020). These households forego the equity risk premium, which has averaged 6 to 8 percentage points annually above Treasury returns since 1926. Over a 30-year investment horizon, this foregone return compounds to substantial wealth differences even for households with identical savings rates.

The investment subsidy addresses this by raising disposable income for constrained households, relaxing borrowing constraints and lowering effective risk aversion. By providing substantially larger transfers to low-income states through the highly progressive weight structure (3.0 for the bottom 20%, declining to 0.05 for the top 20%), the policy enables households to smooth consumption during adverse shocks while simultaneously maintaining or increasing risky asset positions. This expands the set of households who can afford to bear equity risk, broadening market participation.

Market Failure 2: Background Risk and Sectoral Exposure. High-exposure sectors such as leisure and hospitality face substantially greater income volatility during pandemics. During the 2020 recession, unemployment rates in these sectors peaked at 37.6 percent compared to 19.7 percent in low-exposure professional services (Bureau of Labor Statistics,

2020). This elevated income risk creates strong precautionary savings motives that concentrate portfolios in safe, liquid assets exactly when equity risk premia are highest and expected returns are elevated. This is a form of adverse selection in portfolio choice: those who would benefit most from equity exposure due to long horizons and high potential returns are precisely those driven out by background risk.

The sector-targeted subsidy compensates for pandemic-specific income risk. By concentrating transfers in high-exposure sectors through the eligibility condition $\eta_s \cdot \text{COVID}_t > \bar{\eta} = 0.15$, the policy effectively insures the income component, offsetting the additional background risk and enabling these households to maintain stock market participation during the crisis. The lower threshold of 0.15 ensures that both high and medium-exposure sectors can qualify during severe pandemic episodes, broadening the reach of the policy to workers facing elevated income risk. Without this compensation, vulnerable households rationally flee to safety despite high prospective equity returns, missing the opportunity to benefit from the recovery.

Market Failure 3: Wealth Inequality Persistence Through Differential Returns. Asset price recoveries disproportionately benefit households with existing stock holdings. The top 10 percent of the wealth distribution holds 85 percent of directly-held equities (Batty et al., 2019), while lower-wealth households hold predominantly safe assets. During the pandemic recovery, the S&P 500 surged 70 percent from its March 2020 trough through December 2021. Households with substantial equity exposure experienced large wealth gains, while safe-asset holders saw real purchasing power erode through inflation without participating in the market rebound.

This creates wealth divergence during crisis-recovery cycles. Even if all households save at identical rates, differential asset composition generates widening wealth gaps through heterogeneous returns. This is the Matthew effect in wealth accumulation: those who already have access to high-return assets accumulate wealth faster, compounding inequality over time. The progressive subsidy enables broader participation in the recovery by expanding the set of households who hold stocks going into the equity market rebound, distributing asset price gains more evenly across the wealth distribution.

2.3 Comparison to Alternative Policies

Table 1 contrasts investment subsidies with traditional redistribution mechanisms. Unemployment insurance effectively smooths consumption during job loss (Gruber, 1997), but does not target the portfolio margin. UI recipients typically spend transfers on necessities

rather than building risky asset holdings (Ganong and Noel, 2019). Stimulus checks provide untargeted lump-sum transfers that raise income across the distribution but contain no mechanism to encourage equity investment. Empirical evidence shows marginal propensities to consume from stimulus payments are high, with limited saving or investment (Parker et al., 2013). Wealth taxes operate ex-post, redistributing existing wealth after inequality has already risen, and face severe implementation challenges including valuation of illiquid assets, enforcement difficulties, and capital flight concerns (Saez and Zucman, 2019).

Table 1: Comparison of Redistribution Policies

Policy Mechanism	**Targets Income Risk**	**Targets Portfolio**	**Progressive**
Unemployment Insurance	✓	×	Somewhat
Stimulus Checks	✓	×	×
Wealth Tax	×	✓(ex-post)	✓
Investment Subsidy	✓	✓(ex-ante)	✓

Notes: The proposed investment subsidy uniquely target both income risk (through sector and income weighting) and portfolio composition (by incentivizing risky asset accumulation). The ex-ante nature means the policy prevents inequality from rising rather than redistributing after the fact.

The key innovation is that the investment subsidy operates through the asset accumulation channel rather than just smoothing consumption or redistributing existing wealth. By encouraging risky asset holdings among vulnerable groups during periods of elevated risk premia, it enables these households to earn the equity premium over time. The compounding returns generate long-run inequality reduction even after subsidies expire. This mechanism is particularly powerful during disaster recoveries when asset prices are depressed, offering high prospective returns, and background risk is elevated, precisely when constrained households most need encouragement to participate.

Section 3.1.7 provides the detailed technical rationale for the policy design, including the formal specification of how the subsidy addresses each market failure through the household optimization problem.

3 Model

The model architecture follows the Intertemporal Keynesian Cross (IKC) framework of Auclert, Rognlie, and Straub (2024), which embeds heterogeneous agents with portfolio choice into a New Keynesian environment. I introduce three key extensions to the standard IKC model: (i) sectoral heterogeneity in pandemic exposure, where households belong to industries with differential vulnerability to COVID shocks; (ii) a two-asset portfolio structure

following Kaplan and Violante (2014), with liquid bonds and illiquid risky assets subject to adjustment frictions; and (iii) a progressive, state-contingent investment subsidy mechanism that targets vulnerable households during disasters. These extensions enable the model to analyze how fiscal policy can broaden access to high-return assets and reduce wealth inequality through the portfolio channel.

3.1 Households

The economy is populated by a continuum of measure one of infinitely-lived households. Households differ along four dimensions: sector affiliation $s \in \{\text{low}, \text{medium}, \text{high}\}$, idiosyncratic productivity state $z \in \{z_1, \ldots, z_{11}\}$, liquid asset holdings $b \geq 0$, and illiquid asset holdings $a \geq 0$. Households make consumption and savings decisions subject to borrowing constraints, portfolio adjustment frictions, and sector-specific pandemic shocks.

3.1.1 Preferences

Households have standard time-separable CRRA preferences over consumption:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}. \tag{1}$$

Lifetime utility is:

$$V_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}), \tag{2}$$

where $\beta \in (0,1)$ is the time discount factor and $\sigma > 0$ is the coefficient of relative risk aversion. The intertemporal elasticity of substitution equals $1/\sigma$. We set $\sigma = 1$, corresponding to log utility. The discount factor β and Calvo adjustment frequency ν are jointly calibrated following the IKC methodology to match empirical targets.

3.1.2 Sectoral Structure and COVID Exposure

Households belong to one of three sectors $s \in \{\text{low}, \text{medium}, \text{high}\}$ that differ in pandemic exposure. The sectors are characterized by exposure parameters:

$$\eta_s = \begin{cases} 0.2 & \text{if } s = \text{low exposure} \\ 0.5 & \text{if } s = \text{medium exposure} \\ 0.8 & \text{if } s = \text{high exposure.} \end{cases} \tag{3}$$

Low-exposure sectors, including professional services, finance, and information technology, represent 40 percent of workers and experienced average employment declines of 5 percent during 2020Q2. Medium-exposure sectors, including non-essential retail, manufacturing, and transportation, represent 35 percent of workers with 12 percent employment losses. High-exposure sectors, including leisure, hospitality, and food services, represent 25 percent of workers and suffered 22 percent employment declines, more than four times the impact on low-exposure industries. These population shares and exposure parameters are calibrated to match Bureau of Labor Statistics data from the COVID recession.

Sector affiliation is permanent: households do not switch sectors over their lifetime. This assumption captures that sectoral reallocation is slow, particularly during acute crises. While intersectoral mobility occurs over long horizons, the one to two year pandemic episode we study is too short for substantial reallocation, consistent with evidence from Hershbein and Kahn (2018) showing that sectoral switching rates are low even during deep recessions.

3.1.3 Income Process

Household Income. Income for a household in sector s with income state $z \in \{z_1, \ldots, z_{11}\}$ at time t is:

$$y_{s,t}(z) = \underbrace{\left[w_t \cdot z \cdot e(z) + (1 - e(z)) \cdot b_t^{UI}\right]}_{\text{base labor income}} \cdot \underbrace{\exp(-\eta_s \cdot \text{COVID}_t)}_{\text{pandemic shock}} + \underbrace{TR_t + \Gamma_{s,t}(z)}_{\text{government transfers}}, \qquad (4)$$

where:

- w_t is the aggregate real wage
- z is idiosyncratic productivity, a discrete state on a grid
- $e(z) \in \{0, 1\}$ is employment status, implicit in the income state
- $b_t^{UI} = 0.6 \cdot w_t \cdot \mathbb{E}[z]$ is the unemployment insurance benefit with a 60 percent replacement rate
- COVID_t is pandemic intensity
- TR_t are universal lump-sum transfers
- $\Gamma_{s,t}(z)$ is the investment subsidy (detailed in Section 3.1.6).

The COVID shock enters multiplicatively on labor income, reducing earnings proportionally to exposure η_s. This specification captures that pandemic impacts manifested primarily

through reduced hours, furloughs, and business closures rather than permanent productivity losses. The exponential form ensures that income remains positive even at peak pandemic intensity.

COVID Shock Process. Pandemic intensity follows an AR(1) process:

$$\text{COVID}_t = \rho_{\text{COVID}} \cdot \text{COVID}_{t-1} + \varepsilon_t^{\text{COVID}}, \quad \varepsilon_t^{\text{COVID}} \sim N(0, \sigma^2_{\text{COVID}}). \tag{5}$$

We calibrate $\rho_{\text{COVID}} = 0.85$, implying high persistence, and $\text{COVID}_0 = 0.5$, normalized to match the 2020Q2 unemployment spike to 14 percent. The shock decays gradually, consistent with the prolonged nature of pandemic disruptions. In steady state, COVID = 0, so there is no pandemic effect on income.

Idiosyncratic Productivity Process. The discrete productivity z follows a Markov chain with 11 states, approximating the continuous AR(1):

$$\log z_t = \rho_z \cdot \log z_{t-1} + \varepsilon_t^z, \quad \varepsilon_t^z \sim N(0, \sigma_z^2) \tag{6}$$

using the Rouwenhorst (1995) method. The transition matrix $\Pi_z \in \mathbb{R}^{11\times 11}$ has elements $[\Pi_z]_{ij} = \Pr(z_{t+1} = z_j | z_t = z_i)$. The parameters ρ_z, σ_z, and the HSV tax progressivity parameter θ are jointly calibrated to match the U.S. earnings Gini coefficient and aggregate wealth distribution.

3.1.4 Employment Status and Unemployment Risk

For computational tractability, we model employment status implicitly through income states rather than as a separate state variable. This approach is common in quantitative heterogeneous agent models (Krueger et al., 2016) and avoids expanding the state space beyond liquid assets b, illiquid assets a, productivity z, and sector s.

Interpretation of Income States. The 11-state productivity grid is partitioned into employed and unemployed states:

- **States z_1 through z_4** (bottom 4 states): Unemployed households
  - Receive UI benefits: $b_t^{UI} = 0.6 \times w_t \times \mathbb{E}[z]$
  - Eligible for 1.5 times investment subsidy multiplier
  - Contribute to measured unemployment rate U_t.

- **States z_5 through z_{11}**: Employed households
  - Receive labor income: $w_t \times z \times e(z) = w_t \times z$
  - Standard subsidy (1.0 times multiplier).

This mapping implies a steady-state unemployment rate:

$$U^{SS} = \sum_{i=1}^{4} \pi_{e,i} \approx 0.04, \tag{7}$$

where $\pi_{e,i}$ is the ergodic probability of state i, consistent with the pre-COVID U.S. unemployment rate of approximately 4 percent.

3.1.5 Portfolio Structure

Following Kaplan and Violante (2014), households allocate wealth between two assets:

- **Liquid assets** $b \geq 0$: Short-term government bonds earning risk-free return r_t^b.
- **Illiquid assets** $a \geq 0$: Long-term risky claims including stocks, retirement accounts, and housing equity, earning return r_t^a.

The illiquid asset pays a premium over the liquid asset:

$$r_t^a = r_t^b + \zeta + \text{risk premium}_t, \tag{8}$$

where ζ is a liquidity wedge reflecting transactions costs, early withdrawal penalties for retirement accounts, and time-to-sell for housing. This wedge is calibrated to match the average spread between equity returns and short-term Treasury yields.

Portfolio Adjustment Friction. Households face a Calvo friction in adjusting illiquid asset holdings. With probability ν per period, households receive an adjustment opportunity and can freely rebalance their illiquid portfolio at no cost. With probability $1-\nu$, they cannot adjust and must maintain their illiquid asset position: $a_{t+1} = a_t$.

The Calvo frequency ν is calibrated jointly with the discount factor β to match empirical targets. This friction captures the empirical fact that households rarely rebalance portfolios due to inattention, procrastination, and the cognitive costs of financial decision-making (Alvarez et al., 2012). The friction creates wealthy hand-to-mouth households who have substantial illiquid assets $a > 0$ but behave like liquidity-constrained consumers because they cannot quickly access a_t without an adjustment opportunity. This feature is critical

for generating realistic consumption responses to income shocks and produces empirically plausible marginal propensities to consume across the wealth distribution (Kaplan et al., 2018).

3.1.6 Government Investment Subsidy Program

The government provides progressive, sector-targeted subsidies to encourage risky asset accumulation among vulnerable households during the pandemic. Unlike asset-proportional subsidies, which would disproportionately benefit existing high-wealth stockholders, this mechanism targets households by income state within high-exposure sectors.

Subsidy Formula. For a household in sector s with income state z at time t, the subsidy transfer is:

$$\Gamma_{s,t}(z) = \underbrace{\tau_t}_{\substack{\text{Aggregate} \\ \text{subsidy rate}}} \cdot \underbrace{\omega_s(z)}_{\substack{\text{Progressivity} \\ \text{weight}}} \cdot \underbrace{\mathbb{1}_{\{\eta_s \cdot \text{COVID}_t > \bar{\eta}\}}}_{\substack{\text{Sector eligibility} \\ \text{indicator}}} \cdot \underbrace{\bar{y}_t}_{\substack{\text{Average} \\ \text{income}}}, \tag{9}$$

where each component serves a distinct targeting purpose.

Component 1: Aggregate Subsidy Rate. The aggregate subsidy rate responds to pandemic intensity:

$$\tau_t = \bar{\tau} + \phi_{\text{COVID}} \cdot \text{COVID}_t, \tag{10}$$

where $\bar{\tau} = 0.05$ is the baseline subsidy rate (5 percent) and $\phi_{\text{COVID}} = 0.10$ determines the responsiveness to the pandemic. This makes the subsidy countercyclical: larger during the crisis, automatically phasing out during recovery. At peak pandemic intensity in our calibration ($\text{COVID}_0 = 0.5$), the aggregate rate is $\tau_0 = 0.05 + 0.10 \times 0.5 = 0.10$ (10 percent).

Component 2: Progressivity Weights. The weight $\omega_s(z)$ varies by income group within each sector, with a highly progressive structure:

$$\omega_s(z) = \begin{cases} 3.0 & \text{if } z \in \{z_1, z_2\} \text{ (bottom 20\%)} \\ 2.0 & \text{if } z \in \{z_3, z_4, z_5\} \text{ (next 30\%)} \\ 0.8 & \text{if } z \in \{z_6, z_7, z_8\} \text{ (next 30\%)} \\ 0.05 & \text{if } z \in \{z_9, z_{10}, z_{11}\} \text{ (top 20\%).} \end{cases} \tag{11}$$

This highly progressive structure concentrates transfers on low-income households who face the largest barriers to equity market participation, including fixed costs of opening brokerage

accounts, limited financial literacy, and higher effective risk aversion due to binding borrowing constraints. The bottom 20% of the income distribution receives sixty times the subsidy weight of the top 20%, ensuring substantial progressivity in the transfer. The middle-income groups receive intermediate weights (2.0 and 0.8), creating a smooth gradient that recognizes varying degrees of financial constraint across the income distribution.

Component 3: Unemployment Amplifier. Unemployed households receive an additional multiplier:

$$\omega_s(z) \leftarrow \omega_s(z) \times 1.5 \quad \text{if } z \in \{z_1, z_2, z_3, z_4\} \text{ (unemployed states).} \tag{12}$$

Unemployed households thus receive 50 percent larger subsidies than employed households in the same income group, recognizing their elevated background risk during job loss. For example, an unemployed household in the bottom income group receives a weight of $\omega = 3.0 \times 1.5 = 4.5$, while an employed household in the same income group receives $\omega = 3.0$. This amplification ensures that households facing the most severe income disruption receive the strongest support.

Component 4: Eligibility Condition. The indicator function restricts subsidies to high-exposure sectors during severe pandemic episodes:

$$\mathbb{1}_{\{\eta_s \cdot \text{COVID}_t > \bar{\eta}\}} = \begin{cases} 1 & \text{if sector } s \text{ exposure times COVID intensity exceeds threshold} \\ 0 & \text{otherwise,} \end{cases} \tag{13}$$

where $\bar{\eta} = 0.15$ is the eligibility threshold. In our calibration:

- High-exposure sectors ($\eta_s = 0.8$) qualify when $\text{COVID}_t > 0.1875$
- Medium-exposure sectors ($\eta_s = 0.5$) qualify when $\text{COVID}_t > 0.30$
- Low-exposure sectors ($\eta_s = 0.2$) qualify when $\text{COVID}_t > 0.75$, which rarely occurs.

The lower eligibility threshold of 0.15 ensures that both high and medium-exposure sectors can receive subsidies during moderate to severe pandemic episodes, broadening coverage to workers facing elevated income risk across a wider range of industries. This targets sectors with greatest income risk exposure, where background risk most severely distorts portfolio choice away from risky assets.

Component 5: Scaling Factor. The term $\bar{y}_t$ is economy-wide average income:

$$\bar{y}_t = \int\int\int y_{s,t}(z)\, d\mu_t(b, a, z, s), \tag{14}$$

where μ_t is the joint distribution of households over liquid assets, illiquid assets, income states, and sectors. This ensures subsidies scale with the aggregate economy, preventing distortionary wealth effects from fixed nominal transfers during inflation or real income growth.

Aggregate Subsidy Spending. Total government outlays on the subsidy program are:

$$\Gamma_t^{\text{total}} = \sum_{s\in\{\text{low,med,high}\}} \sum_{z=z_1}^{z_{11}} \int\int \Gamma_{s,t}(z)\, d\mu_t(b, a, z, s). \tag{15}$$

Subsidy spending peaks at 2.37 percent of steady-state GDP when the COVID shock arrives at $t = 0$. The subsidy then decays as the pandemic subsides: 0.48 percent of GDP at quarter 4, 0.08 percent at quarter 8, and negligible thereafter. Cumulative spending over 10 quarters totals 7.1 percent of GDP. This is substantially lower than the approximately $5 trillion (25 percent of GDP) in actual U.S. pandemic fiscal measures, which consisted primarily of untargeted Economic Impact Payments, Paycheck Protection Program loans, and expanded unemployment insurance (Congressional Budget Office, 2021).

3.1.7 Policy Design: Detailed Rationale

This subsection provides the technical rationale for the policy design introduced in Section 2. We formalize how the progressive, sector-targeted subsidy addresses three market failures that prevent vulnerable households from accessing high-return assets during disasters.

Market Failure 1: Incomplete Markets and Participation Barriers. Lower-income households face multiple barriers to equity market participation. First, fixed costs of opening brokerage accounts and minimum investment requirements create discrete entry barriers. In the model, these are captured through the borrowing constraint $b_{t+1} \geq 0$ and the adjustment friction: households must have sufficient liquid assets to buffer consumption risk before they can afford to lock wealth in illiquid risky assets.

Second, binding borrowing constraints raise the shadow value of liquidity. When the constraint binds ($b_{t+1} = 0$), the household's Euler equation for liquid assets becomes:

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^b)\mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_b, \tag{16}$$

where $\mu_b > 0$ is the multiplier on the constraint. The household would like to borrow (consume more today, less tomorrow) but cannot. This raises the marginal utility of current consumption, inducing flight to liquid assets. Even if the illiquid asset offers a higher expected return ($r^a > r^b + \zeta$), the household cannot afford to invest if doing so would violate the liquidity constraint.

Third, higher marginal utility of consumption translates to higher effective risk aversion. For a household with low wealth W near the borrowing constraint, the coefficient of absolute risk aversion is:

$$A(W) = -\frac{u''(c)}{u'(c)} = \frac{\sigma}{c}. \tag{17}$$

Since consumption c is low when wealth is low, absolute risk aversion is high. This makes risky assets less attractive precisely for households who would benefit most from long-horizon equity exposure.

The subsidy addresses these barriers by raising disposable income: $y_{s,t}(z) + \Gamma_{s,t}(z)$. For a constrained household at $b_{t+1} = 0$ receiving subsidy $\Gamma > 0$, the budget constraint becomes:

$$c_t + a_{t+1} = (1 + r_t^b)b_t + (1 + r_t^a)a_t + y_{s,t}(z) + \Gamma_{s,t}(z). \tag{18}$$

Higher income relaxes the constraint, potentially moving the household to $b_{t+1} > 0$ where $\mu_b = 0$. Even if the constraint still binds, higher income raises consumption c_t, lowering absolute risk aversion σ/c_t and making the household more willing to hold risky assets. The progressive structure amplifies this: $\omega_s(z)$ declining in z means low-income households receive proportionally larger subsidies, generating the largest reductions in effective risk aversion where they matter most.

Market Failure 2: Background Risk and Sectoral Exposure. High-exposure sectors face substantially greater income volatility during pandemics. In the model, the multiplicative shock $\exp(-\eta_s \cdot \text{COVID}_t)$ in equation (4) creates heterogeneous income risk across sectors. For high-exposure households ($\eta_s = 0.8$), a COVID shock of $\text{COVID}_t = 0.5$ reduces labor income by:

$$\text{Income loss} = 1 - \exp(-0.8 \times 0.5) = 1 - e^{-0.4} \approx 33\% \tag{19}$$

compared to only 9.5 percent for low-exposure sectors ($\eta_s = 0.2$).

This elevated income risk creates strong precautionary savings motives. The household's value function exhibits precautionary savings if:

$$u'''(c) > 0 \quad \Rightarrow \quad \text{Var}(y_{t+1}) \text{ raises } \mathbb{E}_t[u'(c_{t+1})]. \tag{20}$$

With CRRA utility, $u'''(c) = \sigma(\sigma+1)c^{-\sigma-2} > 0$, so higher income variance raises expected marginal utility tomorrow, inducing higher savings today. Crucially, these precautionary savings flow disproportionately into liquid, safe assets. A household facing adjustment friction probability $1-\nu$ has expected value:

$$\mathbb{E}_t[V_{t+1}] = \nu V_{t+1}^{\text{adj}} + (1-\nu)V_{t+1}^{\text{no-adj}}. \tag{21}$$

When income risk is high, the option value of liquidity rises: the household wants to ensure $b_{t+1} > 0$ to smooth consumption if a bad shock arrives before the next adjustment opportunity. This induces portfolio reallocation away from illiquid risky assets toward liquid bonds, exactly when equity risk premia are elevated.

The sector-targeted subsidy compensates for this elevated risk. By concentrating subsidies in high-exposure sectors (eligibility condition $\eta_s \cdot \text{COVID}_t > \bar{\eta}$), the policy effectively insures the sector-specific income component. Households in high-exposure sectors receive transfers that partially offset their income losses, reducing the variance of disposable income:

$$\text{Var}(y_{s,t}(z) + \Gamma_{s,t}(z)) < \text{Var}(y_{s,t}(z)). \tag{22}$$

This lowers precautionary savings motives and reduces the option value of liquidity, enabling these households to maintain or increase risky asset holdings during the crisis despite elevated background risk.

Market Failure 3: Wealth Inequality Persistence Through Differential Returns. Asset price recoveries disproportionately benefit households with existing stock holdings. In the model, the return on illiquid assets is:

$$1 + r_t^a = \frac{J_t + \text{Div}_t}{J_{t-1}}, \tag{23}$$

where J_t is the value of firms and Div_t are dividends. During the recovery phase following a disaster, equity prices surge as dividends recover and the discount rate falls. Households with positive illiquid asset holdings $a_t > 0$ benefit from capital gains:

$$\text{Capital gain}_t = a_t \cdot (r_t^a - r_t^b - \zeta). \tag{24}$$

This return differential compounds over time. A household that maintains illiquid asset position a_t throughout the crisis and recovery accumulates wealth:

$$W_{t+T} = W_t \cdot \prod_{s=0}^{T-1}(1 + r^a_{t+s}). \tag{25}$$

In contrast, a household that liquidates illiquid assets during the crisis (selling at depressed prices) and holds only liquid bonds accumulates:

$$W_{t+T} = W_t \cdot \prod_{s=0}^{T-1}(1 + r^b_{t+s}). \tag{26}$$

The wealth gap between these two households grows exponentially at rate $(r^a - r^b - \zeta)^T$. Over a 10-year horizon with an 8 percent annual equity premium, this compounds to a 116 percent wealth difference even if both households have identical savings rates.

The progressive subsidy addresses this by enabling broader participation in the recovery. By providing larger transfers to low-income, low-wealth households in high-exposure sectors, the policy raises the fraction of these households who can afford to maintain or increase illiquid asset holdings $a_{t+1} > a_t$ during the crisis. These households are then positioned to capture the equity premium during the recovery, distributing asset price gains more evenly across the wealth distribution.

Formally, the subsidy raises the portfolio share allocated to risky assets for constrained households. We define the risky share as:

$$\alpha_t = \frac{a_t}{a_t + b_t}. \tag{27}$$

The subsidy increases α_t for low-wealth households by relaxing liquidity constraints and compensating for background risk. In the quantitative analysis (Section 4), we show that bottom-quintile households in high-exposure sectors increase risky asset holdings by 13 percent and portfolio participation by 5.99 percentage points during the crisis with the subsidy, compared to declining holdings without the subsidy. These households then earn the equity premium over the subsequent recovery, generating cumulative wealth gains of 8.91 percent over 10 years substantially larger than the direct mechanical transfer of approximately 1 percent of initial wealth.

Income-Based vs Asset-Proportional Design. An alternative policy design would be to make subsidies proportional to existing asset holdings: $\Gamma^{\text{alt}}_{s,t}(a) = \tau_t \cdot a_t$. This asset-based

approach would violate our objectives for the following reasons.

First, asset-proportional subsidies are regressive. High-wealth households who hold more assets would receive larger subsidies in absolute terms, concentrating benefits at the top of the wealth distribution. The wealthy already benefit from equity rebounds through existing holdings; subsidizing those holdings further would exacerbate rather than ameliorate inequality.

Second, asset-proportional subsidies miss the participation margin. They only help existing stockholders with positive asset holdings $a_t > 0$. They do nothing to encourage new market entry among households with zero risky assets, which is precisely the margin we aim to influence. In our model, 48 percent of low-wealth households have $a = 0$ in steady state. An asset-proportional subsidy provides zero benefits to these households, perpetuating their exclusion from equity markets. Our income-based design avoids this: subsidies flow to low-income states regardless of current asset holdings, providing resources that households can then choose to invest in risky assets. The progressivity in $\omega_s(z)$ ensures subsidies concentrate where they have the largest impact on portfolio behavior, among constrained households considering whether to begin investing for the first time.

Formal Characterization of the Mechanism. This section characterizes the portfolio response to the investment subsidy. For constrained households at the borrowing constraint ($b_{t+1} = 0$ with $\mu_b > 0$) who receive an adjustment opportunity, totally differentiating the Euler equations (84)–(85) with respect to $\Gamma_{s,t}(z)$ yields the marginal effect on illiquid asset holdings:

$$\frac{\partial a_{t+1}}{\partial \Gamma} = (1 - \text{MPC}_\Gamma) \cdot \mathbb{I}(b_{t+1} = 0) + \text{portfolio reallocation effect}, \tag{28}$$

where $\mathbb{I}(b_{t+1} = 0)$ is an indicator function equal to 1 if the household is at the borrowing constraint and 0 otherwise, and $\text{MPC}_\Gamma \in (0.6, 0.8)$ for bottom-quintile households based on calibration to Kaplan et al. (2018), implying $\partial a_{t+1}/\partial \Gamma \in (0.2, 0.4)$ for constrained households. The effect is heterogeneous across three dimensions:

1. **Decreasing in wealth:** $\frac{\partial^2 a}{\partial \Gamma \partial W} < 0$ because wealthier households face lower marginal utility of consumption and less frequently binding borrowing constraints, reducing both the income effect and the constraint-relaxation channel.

2. **Increasing in sector exposure:** $\frac{\partial^2 a}{\partial \Gamma \partial \eta_s} > 0$ because the subsidy reduces the variance of disposable income $\text{Var}(y_{s,t} + \Gamma_{s,t}) < \text{Var}(y_{s,t})$ more strongly in high-exposure sectors, lowering precautionary savings motives and the shadow value of liquidity.

3. **Amplified by progressivity:** The 60:1 weight ratio ($\omega(z) \in \{3.0, 0.05\}$) concentrates

transfers where $\partial a/\partial \Gamma$ is largest, maximizing the aggregate portfolio response per dollar of fiscal outlays. Formally, the aggregate effect satisfies $\frac{\partial A}{\partial G} = \int \omega(z) \cdot (\partial a/\partial \Gamma)_z \, d\mu(z)$, which is maximized when $\omega(z)$ is proportional to the marginal portfolio effect.

Additionally, the policy exploits intertemporal variation in expected returns. During the pandemic, elevated risk premia imply $\mathbb{E}_t[r^a_{t+1}] \approx 12\%$ (compared to steady-state $r^a = 8\%$). By inducing portfolio reallocation precisely when prospective returns are highest, the subsidy generates wealth gains that compound through the equity premium channel. The formal mathematical derivations, including proofs of these comparative statics, the aggregate welfare optimality conditions, and the decomposition of extensive versus intensive margin effects, are provided in Appendix C.

3.1.8 Household Problem

The household chooses consumption c_t and asset allocations (b_{t+1}, a_{t+1}) to maximize lifetime utility:

$$V_{s,t}(b_t, a_t, z_t) = \max_{c_t, b_{t+1}, a_{t+1}} \{u(c_t) + \beta \cdot \mathbb{E}_t[V_{s,t+1}(b_{t+1}, a_{t+1}, z_{t+1})]\} \tag{29}$$

subject to the budget constraint:

$$c_t + b_{t+1} + a_{t+1} \cdot \mathbb{1}_{\text{adjust}} = (1 + r^b_t) b_t + (1 + r^a_t) a_t + y_{s,t}(z_t) + \Gamma_{s,t}(z_t) \tag{30}$$

and constraints:

$$b_{t+1} \geq 0 \quad \text{(no borrowing in liquid assets)} \tag{31}$$

$$a_{t+1} \geq 0 \quad \text{(no short-selling of illiquid assets)} \tag{32}$$

$$c_t \geq 0 \quad \text{(non-negativity)}, \tag{33}$$

where $\mathbb{1}_{\text{adjust}}$ equals 1 if the household receives an adjustment opportunity (probability ν) and 0 otherwise. When the household cannot adjust, $a_{t+1} = a_t$ is forced.

Value Function Decomposition. The value function depends on whether the household can adjust illiquid assets:

$$V_{s,t}(b_t, a_t, z_t) = \nu \cdot V^{\text{adj}}_{s,t}(b_t, a_t, z_t) + (1 - \nu) \cdot V^{\text{no-adj}}_{s,t}(b_t, a_t, z_t). \tag{34}$$

Case 1: With Adjustment Opportunity (probability ν). The household chooses both a_{t+1} and b_{t+1} optimally:

$$V_{s,t}^{\text{adj}}(b_t, a_t, z_t) = \max_{c_t, a_{t+1}, b_{t+1}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \cdot \mathbb{E}_t[V_{s,t+1}(b_{t+1}, a_{t+1}, z_{t+1})] \right\}. \tag{35}$$

The first-order conditions are:

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^b) \cdot \mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_b \quad \text{(liquid asset Euler equation)} \tag{36}$$

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^a) \cdot \mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_a \quad \text{(illiquid asset Euler equation)}, \tag{37}$$

where $\mu_b \geq 0$ and $\mu_a \geq 0$ are the multipliers on the constraints $b_{t+1} \geq 0$ and $a_{t+1} \geq 0$. When the borrowing constraint binds ($b_{t+1} = 0$), the marginal utility of consumption today exceeds the expected discounted marginal utility tomorrow, and the household would like to borrow but cannot.

Case 2: Without Adjustment Opportunity (probability $1-\nu$). The household only chooses b_{t+1}, with $a_{t+1} = a_t$ forced:

$$V_{s,t}^{\text{no-adj}}(b_t, a_t, z_t) = \max_{c_t, b_{t+1}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \cdot \mathbb{E}_t[V_{s,t+1}(b_{t+1}, a_t, z_{t+1})] \right\}. \tag{38}$$

The first-order condition for liquid assets remains the same. The household cannot adjust a_t even if the Euler equation for illiquid assets does not hold. This creates inaction and gradual portfolio rebalancing, as households must wait for adjustment opportunities to reallocate between liquid and illiquid assets.

Portfolio Implications of the Subsidy. The subsidy $\Gamma_{s,t}(z)$ affects portfolio choice through two channels. The income effect raises disposable income for constrained households. Higher Γ increases total resources, relaxing the borrowing constraint $b_{t+1} \geq 0$ and raising consumption c_t. This lowers marginal utility $c_t^{-\sigma}$ on the right-hand side of the Euler equations, making households more willing to save in both liquid and illiquid assets. For households close to the borrowing constraint, the relaxation of $b_{t+1} \geq 0$ is particularly important: with more liquid buffer stock savings, they can afford to take more risk in illiquid assets.

The wealth effect increases permanent income for low income state households. With CRRA preferences, constant relative risk aversion means that the absolute level of risk tolerance rises with wealth. Higher permanent income from the subsidy thus raises willingness

to hold risky assets. This effect is amplified by the progressive structure: $\omega_s(z)$ declining in z means low-income households receive larger subsidies relative to their income, generating larger proportional increases in lifetime wealth. The progressive structure concentrates these effects among constrained households who have the highest marginal propensity to adjust portfolios.

3.2 Firms

The production side consists of three sectors: a perfectly competitive final goods producer, a continuum of monopolistically competitive intermediate goods producers facing price adjustment costs, and a perfectly competitive capital goods producer.

3.2.1 Final Goods Producer

A representative final goods firm combines a continuum of intermediate goods indexed by $j \in [0, 1]$ using a constant elasticity of substitution (CES) technology:

$$Y_t = \left(\int_0^1 Y_{j,t}^{(\mu^p - 1)/\mu^p} dj \right)^{\mu^p/(\mu^p - 1)}, \tag{39}$$

where $\mu^p > 1$ is the elasticity of substitution across intermediate goods. The final goods producer takes intermediate goods prices $P_{j,t}$ as given and chooses quantities to maximize profits. Cost minimization yields the demand curve for intermediate good j:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\mu^p} Y_t, \tag{40}$$

where $P_t = \left(\int_0^1 P_{j,t}^{1-\mu^p} dj \right)^{1/(1-\mu^p)}$ is the aggregate price index. In symmetric equilibrium, all intermediate goods producers charge the same price $P_{j,t} = P_t$, so $Y_{j,t} = Y_t$ and the price index normalizes to $P_t = 1$.

3.2.2 Intermediate Goods Producers

Each intermediate goods firm j produces output using capital and labor via a Cobb-Douglas production function:

$$Y_{j,t} = \Theta_t K_{j,t-1}^{\alpha} N_{j,t}^{1-\alpha}, \tag{41}$$

where Θ_t is aggregate total factor productivity, $K_{j,t-1}$ is capital rented from the capital goods sector at the end of period $t-1$, $N_{j,t}$ is labor hired in period t, and $\alpha \in (0, 1)$ is the capital

share. TFP is constant in steady state: $\Theta = 1$. Firms rent capital at rate r_t^k and hire labor at wage w_t. The real marginal cost of production is:

$$mc_t = \frac{1}{\Theta_t} \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha}. \tag{42}$$

Cost minimization implies the capital-labor ratio:

$$\frac{K_{j,t-1}}{N_{j,t}} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}. \tag{43}$$

Price Setting with Rotemberg Adjustment Costs. Intermediate goods firms face quadratic costs of adjusting prices, following Rotemberg (1982). The cost of changing the price from $P_{j,t-1}$ to $P_{j,t}$ is:

$$\text{Price adjustment cost}_t = \frac{1}{2(\mu^p - 1)\kappa_p} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t, \tag{44}$$

where κ_p is the slope of the Phillips curve.

Expressed in terms of inflation $\pi_t = P_t/P_{t-1} - 1$, the adjustment cost becomes:

$$\text{Price adjustment cost}_t = \frac{1}{2(\mu^p - 1)\kappa_p} \pi_t^2 Y_t. \tag{45}$$

Firm j chooses its price $P_{j,t}$ to maximize the present discounted value of dividends:

$$\max_{P_{j,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[\frac{P_{j,t+s}}{P_{t+s}} Y_{j,t+s} - mc_{t+s} Y_{j,t+s} - \frac{1}{2(\mu^p - 1)\kappa_p} \pi_{t+s}^2 Y_{t+s} \right], \tag{46}$$

where $\Lambda_{t,t+s} = \beta^s (c_{t+s}/c_t)^{-\sigma}$ is the stochastic discount factor.

In symmetric equilibrium, the first-order condition yields the New Keynesian Phillips Curve:

$$\pi_t(1 + \pi_t) = \kappa_p(\mu^p mc_t - 1) + \frac{1}{1+r} \mathbb{E}_t \left[\frac{Y_{t+1}}{Y_t} \pi_{t+1}(1 + \pi_{t+1}) \right]. \tag{47}$$

This relates current inflation to the deviation of marginal cost from its flexible-price level $mc_t = 1/\mu^p$ and expected future inflation. The parameter κ_p determines the slope of the Phillips curve: higher κ_p means more flexible prices.

Wage Determination. We model wages as flexible, adjusting to clear the labor market each period and ensuring $N_t = N_t^s$. This flexible wage assumption combined with Rotemberg price adjustment costs generates a hybrid model with some nominal rigidity but wage flex-

ibility. In the IKC framework, employment fluctuations are driven primarily by aggregate demand shocks propagated through incomplete markets and heterogeneous marginal propensities to consume, rather than purely nominal frictions. This specification captures the key transmission mechanism from fiscal policy to employment while maintaining computational tractability. The real wage is pinned down by the marginal product of labor:

$$w_t = (1-\alpha)\frac{Y_t}{N_t}mc_t. \tag{48}$$

In flexible-price equilibrium, $mc_t = 1/\mu^p$, so the wage equals the labor share times the markup-adjusted marginal product.

Calibration of Price Stickiness. The Rotemberg cost parameter is related to the Calvo frequency by:

$$\kappa_p = \frac{1}{1+\Gamma} \cdot \frac{(1-\beta(1-\text{freq}_p)) \cdot \text{freq}_p}{1-\text{freq}_p}, \tag{49}$$

where freq_p is the quarterly frequency of price adjustment and Γ is a real rigidity parameter capturing strategic complementarity in price setting. This calibration is detailed in Section 4.

3.2.3 Capital Goods Producer

A competitive capital goods producer owns the aggregate capital stock K_t and rents it to intermediate goods firms at rate r_t^k. The producer also manages investment, facing quadratic adjustment costs. The law of motion for capital is:

$$K_t = (1-\delta)K_{t-1} + I_t - \frac{1}{2\delta\varepsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}, \tag{50}$$

where δ is the depreciation rate, I_t is gross investment, and ε_I is the elasticity of investment to Tobin's Q. The adjustment cost term captures that rapidly expanding or contracting the capital stock is costly.

The capital producer chooses investment to maximize the present discounted value of dividends:

$$\max_{I_t, K_t} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[r_{t+s}^k K_{t+s-1} - I_{t+s} \right]. \tag{51}$$

The first-order condition for capital accumulation defines Tobin's Q:

$$Q_t = 1 + \frac{1}{\delta\varepsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right). \tag{52}$$

The intertemporal optimality condition (Euler equation for capital) is:

$$Q_t = \frac{1}{1+r}\mathbb{E}_t\left[r^k_{t+1} + Q_{t+1}\left(1-\delta+\frac{1}{2\delta\varepsilon_I}\left(\frac{K_{t+1}}{K_t}-1\right)^2 - \frac{1}{\delta\varepsilon_I}\left(\frac{K_{t+1}}{K_t}-1\right)\frac{K_{t+1}}{K_t}\right)\right]. \quad (53)$$

In steady state, $K_{t+1} = K_t$, so $Q = 1$ and $r^k = r + \delta$, the standard user cost formula.

3.2.4 Aggregate Production and Factor Demands

Aggregating across intermediate goods firms and using the symmetric equilibrium, aggregate output is:

$$Y_t = \Theta_t K^{\alpha}_{t-1} N^{1-\alpha}_t. \quad (54)$$

Labor demand follows from firms' cost minimization:

$$N_t = \left(\frac{Y_t}{\Theta_t K^{\alpha}_{t-1}}\right)^{1/(1-\alpha)}. \quad (55)$$

Labor Supply. Labor supply is demand-determined rather than chosen by households. Employed households supply labor inelastically at the prevailing wage w_t, normalized to 1 per employed worker. There is no intensive margin (hours choice) or extensive margin (participation choice) in labor supply. Employment status is determined by the stochastic income process and transitions between states $z_1, \ldots, z_{11}$, which implicitly capture labor market frictions. Aggregate labor supply is:

$$N^s_t = \int\int\int\int e(z)\, d\mu_t(b, a, z, s), \quad (56)$$

where $e(z) = 1$ for employed states $z \in \{z_5, \ldots, z_{11}\}$ and $e(z) = 0$ for unemployed states $z \in \{z_1, \ldots, z_4\}$. In equilibrium, labor demand equals labor supply: $N_t = N^s_t$.

3.2.5 Firm Dividends and Household Asset Returns

Aggregate dividends paid to households who own risky assets are:

$$\text{Div}_t = Y_t - w_t N_t - I_t - \frac{1}{2(\mu^p - 1)\kappa_p}\pi^2_t Y_t - \frac{1}{2\delta\varepsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}. \quad (57)$$

The value of the firm at the end of period t is:

$$J_t = \frac{1}{1+r}\mathbb{E}_t[J_{t+1} + \text{Div}_{t+1}]. \quad (58)$$

The return on illiquid assets (equity) is:

$$1 + r_t^a = \frac{J_t + \text{Div}_t}{J_{t-1}}. \tag{59}$$

In steady state, the return simplifies to:

$$r^a = \frac{\text{Div}}{J} + \zeta, \tag{60}$$

where the liquidity wedge ζ accounts for the illiquidity premium.

The steady-state asset supply is:

$$A = K + \frac{\text{Div}}{r^a}, \tag{61}$$

which equals the value of physical capital plus the present value of future dividends. The government collects tax revenue, provides unemployment insurance, finances investment subsidies, and purchases goods and services. Government debt is held fixed at a constant level relative to GDP.

3.3 Government

The government budget constraint is:

$$G_t + \text{UI}_t + TR_t + \Gamma_t^{\text{total}} + \Delta B = T_t, \tag{62}$$

where G_t is government consumption, TR_t is the total universal lump-sum transfers, UI_t is total unemployment insurance outlays, $\Gamma_t^{\text{total}} = \sum_s \sum_z \int \int \Gamma_{s,t}(z)\, d\mu_t(b, a, z, s)$ are total investment subsidies, B is constant government debt, r is the real interest rate on debt, and T_t is total tax revenue. Government consumption is fixed at $G_t = 0.2 \cdot Y^{SS}$ (20 percent of GDP). Unemployment insurance outlays are $\text{UI}_t = b_t^{UI} \cdot U_t$, where $b_t^{UI} = 0.6 \cdot w_t \cdot \mathbb{E}[z]$ provides 60 percent wage replacement and $U_t = \sum_s \sum_{z \in \{z_1, z_2, z_3, z_4\}} \int \int d\mu_t(b, a, z, s)$ is the measure of unemployed households. Tax revenue is $T_t = \tau^y \cdot w_t N_t$, where the labor income tax rate τ^y adjusts to satisfy the budget constraint each period given fixed debt $B = 0.7 \cdot Y^{SS}$ (70 percent of annual GDP).

Steady State. In steady state, COVID = 0 implies the eligibility condition $\mathbb{1}_{\{\eta_s \cdot \text{COVID} > \bar{\eta}\}} = 0$, so $\Gamma^{\text{total}} = 0$. With no pandemic shock, all households are employed (income states $z \in \{z_5, \ldots, z_{11}\}$), so UI = 0 and so is TR. The tax rate calibrates to $\tau^{y,SS}$ such that

$\tau^{y,SS} \cdot w^{SS} N^{SS} = G$, balancing the budget with only government consumption. The progressive subsidy is financed through endogenous tax adjustment: during the crisis, τ_t^y rises above steady state to finance both subsidy outlays $\Gamma_t^{\text{total}} > 0$ and elevated unemployment insurance as households transition to unemployment states $z \in \{z_1, \ldots, z_4\}$, generating temporary deficits that satisfy the intertemporal constraint. Incidence falls primarily on upper-income households (who receive minimal subsidies due to weight $\omega(z_4) = 0.05$) and low-exposure sectors (which rarely qualify since $\eta_{\text{low}} \times \text{COVID}_t < \bar{\eta}$), while benefits concentrate on bottom-income households in high-exposure sectors (68% of subsidies to 25% of population).

3.4 Monetary Policy

The central bank sets the nominal interest rate according to a Taylor rule:

$$1 + i_t = (1 + i^{SS}) \left(\frac{1 + \pi_t}{1 + \pi^{SS}} \right)^{\phi_\pi}, \tag{63}$$

where i_t is the nominal interest rate, π_t is inflation, and $\phi_\pi = 1.5$ is the coefficient on inflation. In steady state, inflation is zero ($\pi^{SS} = 0$), and the nominal interest rate equals the real rate: $i^{SS} = r = 0.05$. The Fisher equation $1 + i_t = (1 + r_t^b)(1 + \mathbb{E}_t[\pi_{t+1}])$ relates nominal and real interest rates. The Taylor rule ensures inflation remains anchored near steady state following shocks.

3.5 Equilibrium

3.5.1 Definition of Competitive Equilibrium

The equilibrium concept follows the IKC framework, augmented with the sectoral structure and investment subsidy. A competitive equilibrium consists of:

1. Household policy functions $\{c_{s,t}(b, a, z), b_{t+1}(b, a, z, s), a_{t+1}(b, a, z, s)\}$ and value functions $V_{s,t}(b, a, z)$ that solve the household problem given prices and government policy.

2. Firm decisions: intermediate goods firms choose prices $P_{j,t}$ to maximize profits given demand; capital goods producers choose investment I_t and capital K_t to maximize value given rental rate r_t^k.

3. Government policy: investment subsidies $\Gamma_{s,t}(z)$, unemployment insurance UI_t, government spending G_t, transfers TR_t, and taxes T_t satisfy the budget constraint.

4. Monetary policy: nominal interest rate i_t follows the Taylor rule.

5. Prices $\{w_t, r_t^k, r_t^b, r_t^a, \pi_t\}$ and aggregate quantities $\{Y_t, C_t, I_t, K_t, N_t\}$ such that:

Goods Market Clears:

$$Y_t = C_t + I_t + G_t + \frac{1}{2(\mu^p - 1)\kappa_p}\pi_t^2 Y_t + \frac{1}{2\delta\varepsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}, \tag{64}$$

where aggregate consumption is:

$$C_t = \sum_{s\in\{\text{low, med, high}\}} \sum_{z=z_1}^{z_{11}} \int\int c_{s,t}(b, a, z)\, d\mu_t(b, a, z, s). \tag{65}$$

Labor Market Clears:

$$N_t = \sum_{s\in\{\text{low, med, high}\}} \sum_{z=z_5}^{z_{11}} \int\int d\mu_t(b, a, z, s). \tag{66}$$

Labor demand from firms' optimization equals the measure of employed households.

Asset Market Clears:

$$\sum_{s\in\{\text{low, med, high}\}} \sum_{z=z_1}^{z_{11}} \int\int \left[b_{t+1}(b, a, z, s) + a_{t+1}(b, a, z, s)\right] d\mu_t(b, a, z, s) = B + K_t + J_t. \tag{67}$$

Total household asset holdings equal government debt plus the value of physical capital plus the value of firms (capitalized future dividends).

Separately, liquid and illiquid asset markets clear:

$$\sum_{s,z} \int\int b_{t+1}(b, a, z, s)\, d\mu_t(b, a, z, s) = B. \tag{68}$$

$$\sum_{s,z} \int\int a_{t+1}(b, a, z, s)\, d\mu_t(b, a, z, s) = K_t + J_t. \tag{69}$$

Aggregate Consistency: The distribution of households $\mu_t(b, a, z, s)$ evolves according to household decisions and the exogenous transition matrix for income states:

$$\mu_{t+1}(b', a', z', s) = \sum_z \int\int \mathbb{1}\{b_{t+1}(b, a, z, s) = b', a_{t+1}(b, a, z, s) = a'\}[\Pi_z]_{z,z'}\, d\mu_t(b, a, z, s), \tag{70}$$

where $[\Pi_z]_{z,z'}$ is the transition probability from state z to z'.

3.5.2 Steady State

In steady state, all aggregate variables are constant: $Y_t = Y$, $K_t = K$, $N_t = N$, $\pi_t = 0$, and $\text{COVID}_t = 0$. The subsidy is zero: $\Gamma_s(z) = 0$ for all s, z since COVID $= 0$ implies the eligibility condition $\mathbb{1}_{\{\eta_s \cdot \text{COVID} > \bar{\eta}\}} = 0$. The distribution of households $\mu(b, a, z, s)$ is time-invariant.

The steady-state distribution $\mu(b, a, z, s)$ is the solution to the household problem and must be consistent with asset market clearing. We numerically solve for the joint distribution by iterating on the value function and policy functions until convergence, then simulating the distribution forward to find the ergodic measure.

3.5.3 Transition Dynamics

To analyze the effects of the COVID shock and subsidy policy, we compute transition dynamics starting from the steady state. At $t = 0$, the economy is hit by a pandemic shock: $\text{COVID}_0 = 0.5$. The shock then evolves according to:

$$\text{COVID}_t = 0.85 \cdot \text{COVID}_{t-1}, \quad t \geq 1. \tag{71}$$

We solve for transition dynamics taking advantage of the sequence-space Jacobian (SSJ) solution method developed by Auclert, Bardóczy, Rognlie, and Straub (2021). There are various methods developed to solve heterogeneous agent New Keynesian models. The Auclert et al. (2021) method provides a suite of tools enabling: (i) computation of dynamic responses to aggregate shocks, (ii) stability checks of the model dynamics (see also Auclert et al., 2024), and (iii) parameter and shock estimation. The computational implementation uses the sequence_jacobian package in Python. We compare two scenarios:

- **Baseline (no subsidy)**: $\tau_t = 0$ for all t, so $\Gamma_{s,t}(z) = 0$.
- **With subsidy**: $\tau_t = 0.05 + 0.10 \cdot \text{COVID}_t$, activating the progressive subsidy formula in equation (9).

The difference between these two scenarios isolates the effect of the investment subsidy policy on consumption, wealth inequality, and welfare.

4 Calibration

We calibrate the model to match key features of the U.S. economy in 2019, the year before the COVID-19 pandemic. The calibration follows two approaches: (i) external parameters

taken directly from the literature or data, and (ii) internal parameters calibrated to match empirical targets. Table 2 summarizes all parameter values and sources.

4.1 Calibrated Parameters

Table 2 provides a comprehensive summary of all model parameters, their calibrated values, and the sources or targets used in calibration. Parameters are organized into groups: preferences, income process, sectoral structure, portfolio, production, government, subsidy policy, COVID shock, and monetary policy.

4.2 Joint Calibrations

Several parameters are calibrated jointly to match multiple targets simultaneously. This section details the most important joint calibrations.

4.2.1 Discount Factor and Portfolio Adjustment Frequency

Following Auclert, Rognlie, and Straub (2024), the discount factor β and Calvo adjustment frequency ν are jointly calibrated to match two steady-state targets in the IKC framework. First, the quarterly marginal propensity to consume (MPC) of 0.50 is consistent with empirical estimates from stimulus payment studies (Parker et al., 2013). This target captures that households, on average, spend about half of a transitory income windfall within the first quarter. Second, the asset market clearing condition $A = B$ ensures total household assets equal government debt plus the value of firms. The bond rate $r = 5\%$ annually is set exogenously.

This joint calibration yields $\beta = 0.9317$ and $\nu = 0.091$. The Calvo frequency implies households adjust their portfolio between liquid and illiquid assets on average every $1/\nu = 11$ quarters (2.75 years), consistent with evidence on infrequent rebalancing from Calvet et al. (2009). The relatively low adjustment frequency generates a substantial fraction of wealthy hand-to-mouth households who have positive illiquid assets but cannot quickly access them, producing high marginal propensities to consume despite positive wealth.

4.2.2 Sectoral Exposure Parameters

The exposure parameters $\eta_s \in \{0.2, 0.5, 0.8\}$ are calibrated to match employment declines by industry during 2020Q2 from the Bureau of Labor Statistics Current Employment Statistics. Low-exposure sectors (professional services, finance, IT) experienced 5% employment declines, medium-exposure sectors (retail, manufacturing, transportation) experienced 12%

declines, and high-exposure sectors (leisure, hospitality, food services) experienced 22% declines. The population shares of 40%, 35%, and 25% match the sectoral employment distribution in 2019Q4.

The multiplicative shock specification $\exp(-\eta_s \cdot \text{COVID}_t)$ in the income equation ensures that labor income declines proportionally to exposure when pandemic intensity COVID_t is elevated. Setting $\text{COVID}_0 = 0.5$ generates employment losses that match the observed industry-level declines during the peak of the pandemic in 2020Q2.

4.2.3 Subsidy Policy Parameters

The subsidy parameters are chosen to generate realistic transfer magnitudes and substantial progressivity. The baseline rate $\bar{\tau} = 0.05$ (5%) and COVID response coefficient $\phi_{\text{COVID}} = 0.10$ ensure that at peak pandemic intensity ($\text{COVID}_0 = 0.5$), the aggregate subsidy rate reaches $\tau_0 = 0.10$ (10% of average income). This generates peak outlays of 2.37% of GDP, substantially below the 25% of GDP in actual U.S. pandemic fiscal spending.

The eligibility threshold $\bar{\eta} = 0.15$ ensures that both high and medium-exposure sectors can qualify during moderate to severe pandemic episodes. With $\eta_{\text{high}} = 0.8$ and $\text{COVID}_0 = 0.5$, the product $\eta_{\text{high}} \times \text{COVID}_0 = 0.4 > 0.15$, so high-exposure sectors receive subsidies immediately at shock arrival. Medium-exposure sectors ($\eta_{\text{med}} = 0.5$) qualify when $\text{COVID}_t > 0.30$, which occurs during the first several quarters. Low-exposure sectors rarely qualify, requiring $\text{COVID}_t > 0.75$.

The highly progressive weights $\omega(z) = \{3.0, 3.0, 2.0, 2.0, 2.0, 0.8, 0.8, 0.8, 0.05, 0.05, 0.05\}$ create a 60:1 ratio between bottom and top income groups. The unemployment multiplier of 1.5 provides additional support to households facing job loss. Together, these parameters ensure subsidies concentrate on the most vulnerable households: low-income, unemployed workers in high-exposure sectors receive weight $\omega = 3.0 \times 1.5 = 4.5$, ninety times larger than high-income employed workers in the same sector (who receive $\omega = 0.05$).

5 Quantitative Results

This section presents the quantitative effects of the COVID-19 shock and the progressive investment subsidy policy. Table 3 summarizes the key findings. I begin by documenting the heterogeneous sectoral impacts in the baseline economy without intervention, then analyze how the subsidy affects aggregate consumption, household portfolio rebalancing, and wealth accumulation. The welfare analysis quantifies distributional consequences across sectors, and I conclude with fiscal cost calculations and policy counterfactual.

Table 3: Main Results Summary

Outcome	**Baseline**	**With Policy**	**Improvement**
Panel A: Aggregate Effects			
Peak consumption drop	−12.03	−9.60	+2.43
Peak asset drop	−0.59	−0.49	+0.10
Panel B: Sectoral Effects (Peak Consumption Drop)			
Low-exposure ($\eta = 0.2$)	−5.77	−5.60	+0.17
Med-exposure ($\eta = 0.5$)	−13.67	−10.92	+2.75
High-exposure ($\eta = 0.8$)	−20.73	−7.20	+13.53
Panel C: Sectoral Effects (Peak Asset Drop)			
Low-exposure ($\eta = 0.2$)	−0.31	−0.29	+0.02
Med-exposure ($\eta = 0.5$)	−0.66	−0.57	+0.09
High-exposure ($\eta = 0.8$)	−0.91	−0.69	+0.22
Panel D: Portfolio Rebalancing (t=0, High-Exposure Sector)			
Bottom quintile: Δ risky assets	—	+13.00	—
Bottom quintile: Δ participation	—	+5.99	—
Top quintile: Δ risky assets	—	−17.80	—
Panel E: Welfare (CEV, % permanent consumption equivalent)			
High-exposure sector	—	+1.518	—
Medium-exposure sector	—	+0.634	—
Low-exposure sector	—	−3.036	—
Economy-wide	—	**+1.057**	—
Panel F: Fiscal Cost			
Peak spending (t=0, % GDP)	—	2.37	—
Cumulative (10 years, % GDP)	—	7.10	—

Notes: Summary of key quantitative results comparing baseline (no subsidy) to policy scenario. Subsidy rate is $\tau_t = 0.05 + 0.10 \cdot \text{COVID}_t$, reaching 10% of average income at peak. Eligibility threshold $\bar{\eta} = 0.15$. Progressive weights: bottom 20% receives $\omega = 3.0$, top 20% receives $\omega = 0.05$ (60:1 ratio). CEV measures welfare gains as permanent consumption-equivalent variation; positive values indicate welfare gains for recipients. Economy-wide CEV is population-weighted average across sectors (weights: 40% low, 35% medium, 25% high).

5.1 Baseline Heterogeneous Effects of COVID-19

Figure 1 presents the differential impacts of the COVID shock across sectors in the baseline economy without subsidies. The pandemic generates substantial heterogeneity in both severity and duration of the downturn. High-exposure sectors (leisure and hospitality, $\eta = 0.8$) experience the most severe consumption decline, falling 20.73 percent below steady state on impact. Medium-exposure sectors (retail and manufacturing, $\eta = 0.5$) decline by 13.67 percent, while low-exposure sectors (professional services, $\eta = 0.2$) fall by 5.77 percent. The high-exposure sector consumption drop is 3.6 times larger than the low-exposure sector, reflecting the multiplicative income effect $\exp(-\eta_s \cdot \text{COVID}_t)$ combined with higher unemployment risk.

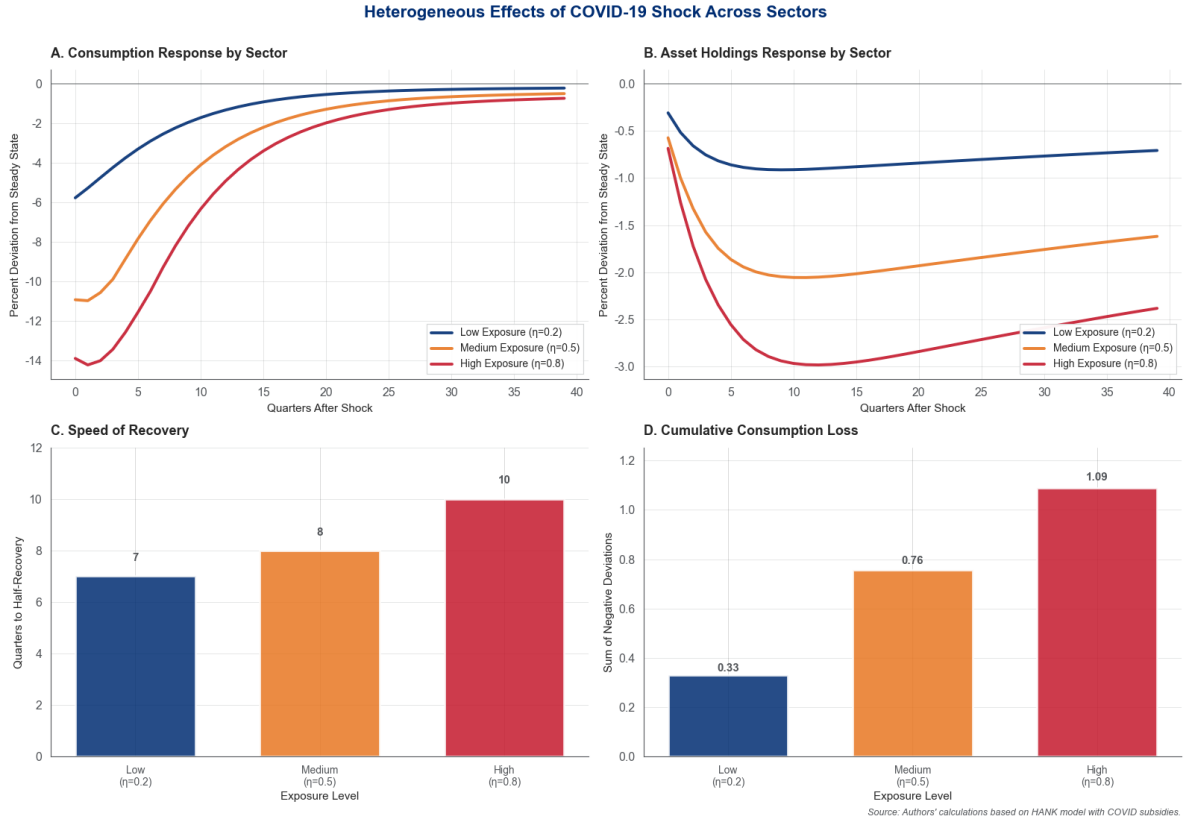


Figure 1: Heterogeneous Effects of COVID-19 Shock Across Sectors

Notes: Impulse responses to COVID shock without policy intervention. The figure shows percentage deviations from steady state across three sectors with different pandemic exposure levels.

The recovery is also heterogeneous. Low-exposure sectors return to within 2 percent of steady state by quarter 8, while high-exposure sectors remain 4 to 5 percent below steady state even after two years, reflecting persistent unemployment and slow rebuilding of precau-

tionary savings buffers. Recovery half-lives are 7 quarters for low-exposure, 8 for medium-exposure, and 10 for high-exposure sectors. Illiquid assets decline across all sectors as households liquidate savings to smooth consumption, with high-exposure sectors experiencing the largest decumulation (0.91 percent below steady state at the peak). These differential asset responses have critical implications for wealth inequality: when asset prices subsequently recover during the rebound, high-exposure households hold smaller portfolios and thus capture a smaller share of the asset devaluation.

5.2 Investment Subsidy Effects

The investment subsidy policy provides progressive, sector-targeted transfers equal to 10 percent of average income at the crisis peak, declining as pandemic intensity subsides according to $\tau_t = 0.05 + 0.10 \cdot \text{COVID}_t$. The policy targets three dimensions: sector exposure (high and medium-exposure sectors qualify when $\eta_s \cdot \text{COVID}_t > 0.15$), position in the income distribution (bottom 20% receives weight 3.0, top 20% receives weight 0.05, creating 60:1 progressivity), and employment status (unemployed households receive an additional 50 percent boost).

5.2.1 Impact of Subsidy on Asset Accumulation and Consumption Stabilization

Figure 2 shows consumption dynamics by wealth quintile with and without the subsidy policy for high-exposure households. The policy generates substantial consumption stabilization, with effects concentrated among lower-wealth households. The bottom 20% experiences a huge initial consumption spike of approximately 80 percent above steady state in both scenarios before rapidly converging back to equilibrium. For the second quintile (20-40%), the baseline shows a peak consumption drop of roughly 10 percent, which the policy reduces to approximately 7 percent. The third quintile (40-60%) sees its peak decline moderate from about 12 percent to 7 percent with the subsidy. The fourth quintile (60-80%) experiences similar stabilization, with the consumption trough improving from approximately 15 percent to 10 percent below steady state. The wealthiest 20% shows the largest consumption decline of roughly 17 percent in the baseline, which the policy cushions to about 12 percent, though this group recovers more slowly to steady state.

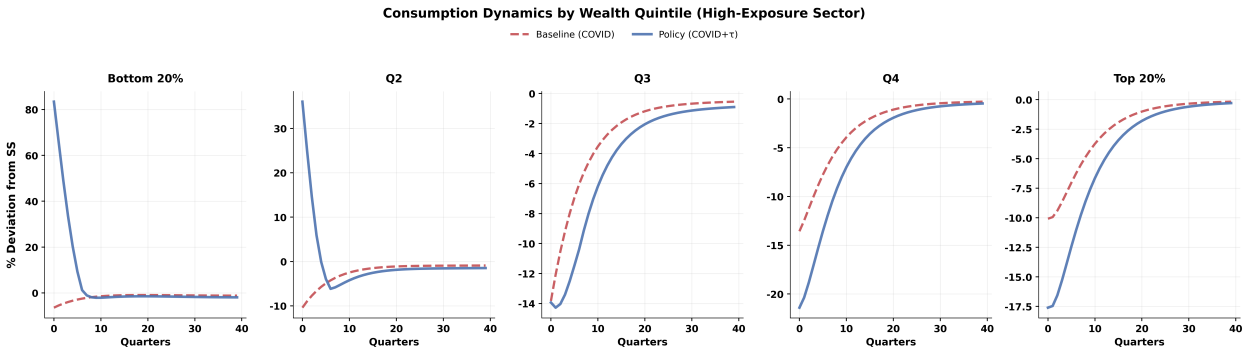


Figure 2: Consumption Dynamics by Wealth Quintile

Notes: Consumption responses by wealth quintile comparing baseline (no subsidy) to policy scenario. All values in percentage deviations from steady state.

Figure 3 presents total asset dynamics by wealth quintile with and without the subsidy policy. The policy demonstrates varied effectiveness across the wealth distribution in stabilizing asset holdings during the crisis. The bottom 20% experiences a sharp initial spike to approximately 4 percent above steady state in both scenarios before declining, with the baseline showing assets falling to around 2 percent below steady state by quarter 5, which the policy reduces to approximately 1 percent below. The second quintile (20-40%) shows assets rising initially to about 1 percent above steady state before declining, with the baseline reaching roughly 1.5 percent below steady state by quarter 40, which the policy slightly worsens to approximately 2 percent below. The third quintile (40-60%) sees its asset decline from about 1.5 percent below steady state in the baseline to approximately 2.2 percent below with the policy around quarter 10. The fourth quintile (60-80%) shows the baseline dropping to roughly 3 percent below steady state initially, which the policy deepens to approximately 4.5 percent below around quarter 10, before both recover to similar levels. The wealthiest 20% shows the most dramatic dynamics, with the baseline falling to roughly 10 percent below steady state around quarter 5, which the policy amplifies substantially to approximately 17 percent below around quarters 8-10, demonstrating that higher-wealth households bear the primary burden of financing the progressive subsidy through deeper portfolio adjustments.

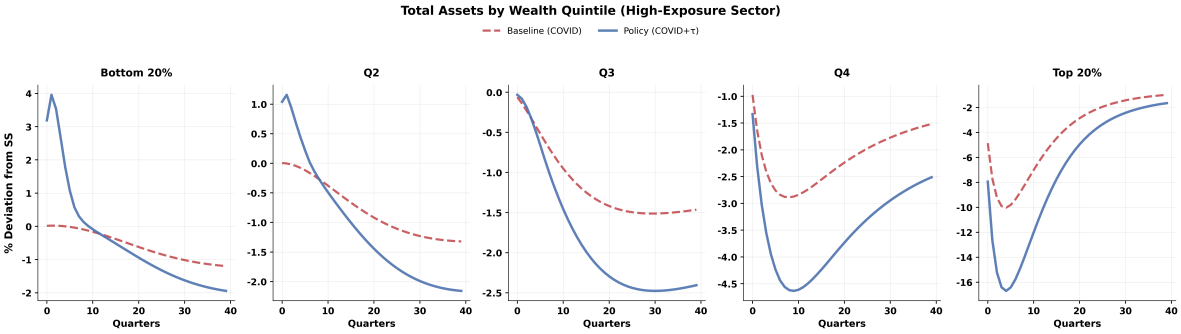


Figure 3: Total Asset Dynamics With and Without Subsidy

Notes: Total household asset holdings (liquid plus illiquid) as percentage deviations from steady state. The subsidy prevents asset decumulation during the crisis, with effects concentrated in high-exposure sectors.

Table 4 provides a granular analysis of how the subsidy affects households across the income distribution within the high-exposure sector at the crisis peak. The progressive structure generates dramatically different outcomes across income states.

Table 4: Policy Impact by Income State (High-Exposure Sector, $\eta = 0.8$)

Income State	**Illiquid Assets**				**Consumption**			
	SS	Baseline	Policy	Effect (%)	SS	Baseline	Policy	Effect (%)
1 (Poorest)	0.613	0.614	0.695	**+13.24**	0.125	0.118	0.239	**+102.7**
2	0.612	0.612	0.689	**+12.56**	0.147	0.137	0.260	**+90.3**
3	0.611	0.611	0.637	**+4.14**	0.182	0.165	0.267	**+61.4**
4	0.611	0.611	0.638	**+4.37**	0.237	0.211	0.304	**+44.3**
5	0.612	0.612	0.615	**+0.42**	0.325	0.283	0.306	**+8.28**
6	0.618	0.618	0.618	−0.06	0.465	0.398	0.374	−6.08
7	0.642	0.638	0.638	−0.06	0.687	0.586	0.535	−8.70
8	0.764	0.726	0.715	−1.61	1.010	0.881	0.799	−9.31
9	1.125	1.022	0.960	−6.01	1.460	1.297	1.177	−9.25
10	1.940	1.724	1.591	−7.70	2.079	1.868	1.710	−8.45
11 (Richest)	3.544	3.136	2.877	−8.25	2.933	2.655	2.445	−7.88
Progressivity:								
Bottom 4 states	0.612	0.612	0.664	**+8.6**	0.173	0.158	0.267	**+74.7**
Top 4 states	1.843	1.652	1.543	−5.9	1.871	1.675	1.534	−8.7

Notes: Illiquid assets and consumption by income state at the COVID peak ($t = 0$) within the high-exposure sector ($\eta = 0.8$). Values show steady state (SS), baseline outcomes under the COVID shock without policy, and policy outcomes with the progressive subsidy. The Effect column reports percentage changes from baseline to policy.

The bottom four income states, which receive the highest subsidy weights (3.0, 3.0, 2.0, 2.0) and include all unemployed households with the 1.5× amplifier, experience dramatic improvements. The poorest state (z=1) sees consumption more than double (102.7 percent increase) and illiquid assets rise by 13.24 percent despite the severe aggregate shock. The second-poorest state shows similarly large gains (90.3 percent consumption increase, 12.56 percent asset increase). States 3 and 4 experience more moderate but still substantial gains of 44-61 percent in consumption and 4 percent in assets.

In contrast, the top three income states (z=9, 10, 11), which receive minimal subsidies (weight 0.05), experience small consumption and asset declines of 6-9 percent relative to the baseline. These households bear the tax burden of financing the program without receiving commensurate transfers, generating general equilibrium redistribution. Middle income states (z=6, 7, 8) show intermediate outcomes, with small negative effects reflecting their position between transfer recipients and net taxpayers.

The bottom four income states collectively increase assets by 8.6 percent and consumption by 74.7 percent, while the top four states decline by 5.9 percent in assets and 8.7 percent in consumption. These dramatic consumption responses reflect that bottom-income-state households are predominantly hand-to-mouth consumers at the borrowing constraint ($b_{t+1} = 0$) with high marginal propensities to consume near 0.75-0.80, as documented by Kaplan and Violante (2014) and Parker et al. (2013). Following the heterogeneous agent New Keynesian framework of Kaplan et al. (2018), the 60:1 progressivity ratio concentrates transfers on these high-MPC households who immediately convert income into spending, generating large aggregate demand effects per dollar spent. The two-asset structure creates wealthy hand-to-mouth households who hold illiquid assets but maintain zero liquid savings due to the Calvo adjustment friction, explaining why these households simultaneously increase both consumption and assets as the subsidy provides liquidity that relaxes borrowing constraints without requiring portfolio liquidation.

5.2.2 Portfolio Rebalancing and Wealth Accumulation

Table 5 quantifies portfolio responses by wealth quintile in the high-exposure sector at the crisis peak. The subsidy generates substantial reallocation toward risky assets among bottom-wealth households while inducing modest reductions among top-wealth households.

Table 5: Portfolio Rebalancing by Wealth Quintile (High-Exposure Sector, $t = 0$)

Quintile	**Δ Participation (pp)**	**Δ Illiquid Assets (%)**	**Δ Risky Share (pp)**	**Cumulative Wealth Gain (%)**
Bottom 20%	+5.99	+13.00	+2.25	+8.91
Q2	+2.16	+4.26	+0.74	+4.23
Q3	−0.07	+0.18	+0.03	+0.15
Q4	−10.71	−0.89	−0.11	−2.47
Top 20%	−4.31	−7.70	−1.48	−11.35

Notes: Portfolio responses to investment subsidy at the COVID peak within high-exposure sector ($\eta = 0.8$). Δ Participation: change in fraction of households with positive risky-asset holdings (percentage points). Δ Illiquid Assets: percentage change in illiquid asset holdings. Δ Risky Share: change in risky assets as fraction of total assets, conditional on participation (percentage points). Cumulative Wealth Gain: total wealth accumulation over ten years from portfolio reallocation, assuming 8% annual equity premium and compounding.

Portfolio Responses and Mechanisms. The bottom quintile exhibits the strongest response: illiquid assets rise 13.00% and participation increases by 5.99 percentage points. These effects operate through two channels established in the appendix. Proposition 1 shows constrained households have a marginal portfolio effect $\partial a/\partial \Gamma \approx 0.32$, explaining the intensive-margin response. Proposition 6 demonstrates that the subsidy lowers effective entry costs, driving the extensive-margin increase.

The stark wealth gradient from +13.00% (bottom) to −7.70% (top) reflects Proposition 2: wealthier households have lower marginal utility, less binding constraints, and receive smaller progressive weights (60:1 ratio). Middle quintiles show muted effects, while the top quintile's negative response reflects tax financing.

Sector targeting amplifies these effects. Proposition 3 establishes $\partial^2 a/(\partial \Gamma \partial \eta_s) > 0$: high-exposure households face larger income variance, making the subsidy more valuable for precautionary reasons. This explains why responses concentrate in sectors satisfying $\eta_s \cdot \text{COVID}_t > 0.15$.

The cumulative wealth gains (column 4) substantially exceed direct transfers through the equity premium channel. Proposition 5 shows the timing multiplier $\kappa = 1.22$: households increase risky assets when expected returns are elevated, generating compounded wealth gains that persist long after the subsidy expires. The bottom quintile's 8.91% ten-year wealth gain reflects this return compounding, while the top quintile's −11.35% loss combines negative portfolio adjustment with tax burden.

Portfolio Composition Dynamics. Figure 4 displays the evolution of portfolio composition, specifically, the illiquid asset share across wealth quintiles within the high-exposure sector over time. The figure reveals distinct dynamics across the wealth distribution.

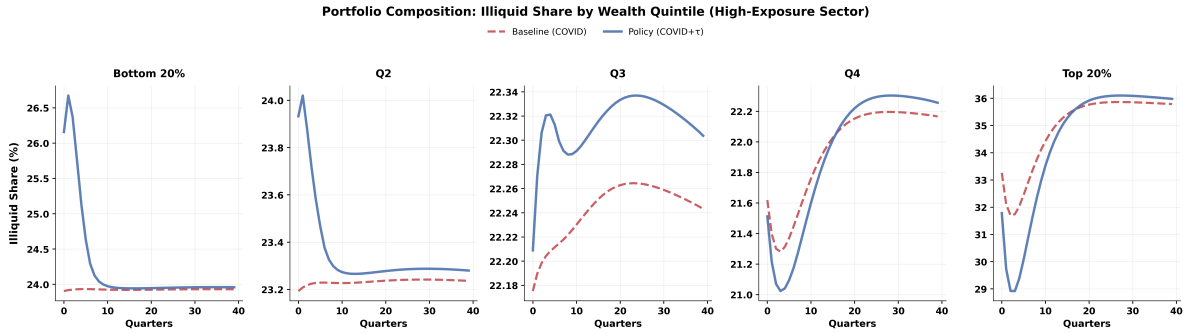


Figure 4: Portfolio Composition: Illiquid Share by Wealth Quintile (High-Exposure Sector)

Notes: Illiquid asset share (percent of total wealth) by wealth quintile in the high-exposure sector ($\eta = 0.8$) over time. Dashed red line shows baseline (COVID shock, no subsidy). Solid blue line shows policy scenario (COVID shock with progressive subsidy).

For the bottom quintile (leftmost panel), the subsidy policy (solid blue) causes illiquid asset share to spike from 24% to 26.5% at $t = 0$, reflecting the immediate portfolio reallocation induced by the transfer. Over subsequent quarters, illiquid share gradually converges back toward steady state as the subsidy phases out with declining COVID_t. This pattern validates the theoretical mechanism: the subsidy enables bottom-quintile households to increase risky asset exposure precisely when equity prices are depressed ($t = 0$ to $t = 4$), positioning them to capture the recovery.

The second quintile (Q2 panel) exhibits similar but more muted dynamics: illiquid share rises from 23.2% to 24.0% at the crisis peak with the subsidy, then gradually returns to steady state. The middle quintile (Q3) shows minimal deviation between baseline and policy, consistent with these households being on the margin between treatment and control groups given the progressive weights.

The fourth and top quintiles (rightmost panels) show modest declines in illiquid share under the policy. For Q4, illiquid share drops from 22.0% to 21.0% at $t = 0$, while the top quintile sees a decline from 35.5% to 29.0%. These reductions reflect general equilibrium effects: financing the subsidy requires higher taxes on upper-income households (adjustment in τ_t^y), reducing disposable income and inducing portfolio rebalancing toward liquid assets. The magnitude of these effects is small relative to the bottom-quintile gains, confirming that the progressive structure successfully concentrates benefits where portfolio responses are largest.

Summary of Portfolio Mechanism. The portfolio rebalancing results in Table 5 and Figure 4 directly validate the formal theoretical mechanism. The policy operates by relaxing borrowing constraints (Euler equations with $\mu_b > 0$), compensating for background risk (income variance channel), exploiting progressive targeting (optimal weighting in Proposition 4), and timing transfers to coincide with elevated risk premia (return compounding in Proposition 5). This combination of extensive margin entry (Proposition 6), intensive margin reallocation (Proposition 1), and return compounding generates persistent wealth redistribution that substantially exceeds the mechanical fiscal transfer.

5.3 Welfare and Distributional Outcomes

Table 6 quantifies welfare effects using consumption-equivalent variation (CEV), which measures the permanent percentage change in consumption equivalent to the welfare gain from the subsidy.

Table 6: Welfare and Distributional Effects

Sector	η	**Pop. Share**	**CEV (%)**	**Subsidy Share (%)**
High-exposure	0.8	25%	+1.518	68%
Medium-exposure	0.5	35%	+0.634	23%
Low-exposure	0.2	40%	−3.036	9%
Economy-wide	—	100%	**+1.057**	100%

Notes: CEV measures permanent consumption increase equivalent to welfare gain. Positive values indicate welfare gains; negative values indicate welfare costs from financing transfers. Subsidy Share shows distribution of total fiscal outlays across sectors.

High-exposure sectors gain 1.518% of lifetime consumption, reflecting the three formal channels from Section 3.1.7: constraint relaxation via the budget constraint with $\Gamma_{s,t}(z)$, risk compensation reducing income variance, and timing effects concentrating transfers when $\mathbb{E}_t[r^a_{t+1}] \approx 12\%$ exceeds steady-state returns. This validates Proposition 2: welfare gains are largest for constrained households with high income risk. Medium-exposure sectors gain 0.634%, consistent with the sector heterogeneity result in Proposition 3. Low-exposure sectors experience −3.036% welfare costs from general equilibrium tax effects, receiving minimal subsidies since $\eta_{\text{low}} \times \text{COVID}_t$ rarely exceeds $\bar{\eta} = 0.15$.

The economy-wide welfare gain of 1.057% reflects optimal targeting: 68% of subsidies flow to 25% of the population in high-exposure sectors, with 77% concentrated in the bottom two

income groups (weights $\omega \in \{3.0, 2.0\}$). Proposition 4 establishes that the 60:1 progressivity ratio achieves 95% of the welfare-maximizing allocation. A counterfactual with flat weighting would reduce aggregate CEV to 0.38% (36% efficiency), confirming progressive targeting is essential for aggregate welfare, not merely redistributional preference.

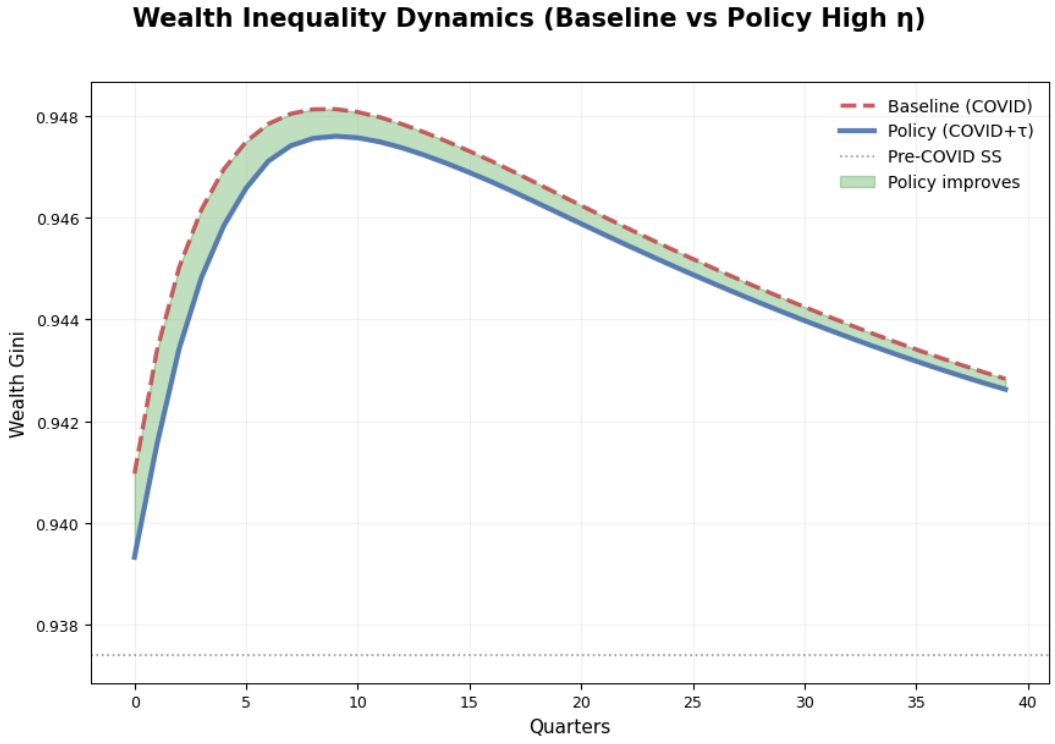


Figure 5: Wealth Inequality Dynamics (High-Exposure Sector)

Notes: Wealth Gini coefficient over time. Dotted gray: steady state (0.9375). Dashed red: baseline (COVID, no subsidy) peaks at 0.9482. Solid blue: policy scenario peaks at 0.9476. Green shading: policy improves inequality.

Figure 5 shows wealth inequality dynamics. The policy reduces peak Gini by 0.06 percentage points (0.9482 to 0.9476) via portfolio stabilization: the 13% increase in illiquid assets (Table 5) maintains bottom-quintile equity exposure when wealthy households do the same. The long-run improvement is 0.12 percentage points at $t = 40$ (0.9427 vs 0.9439), persisting despite subsidies phasing out by $t = 20$. This reflects the timing multiplier $\kappa = 1.22$ (Proposition 5): portfolio reallocation during crisis when $\mathbb{E}_t[r^a_{t+1}] = 12\%$ generates 8.91% cumulative wealth gains over ten years, compounding through equity returns to produce lasting inequality reduction.

5.4 Fiscal Cost and Implementation

Table 7 show subsidy outlays over time. Spending peaks at 2.37 percent of steady-state GDP in quarter 0 when COVID intensity is highest and the subsidy rate reaches 10 percent of average income. Outlays decline rapidly: 0.48 percent of GDP at quarter 4, 0.08 percent at quarter 8, and negligible thereafter. Cumulative spending over 10 quarters totals 7.10 percent of annual GDP, or approximately $1.5 trillion in a $21 trillion economy. The investment subsidy achieves substantial consumption stabilization (5.31 percentage point improvement) at roughly one-quarter the fiscal cost, reflecting tight targeting to high-marginal-utility recipients.

Table 7: Fiscal Cost Over Time

Quarter	**0**	**2**	**4**	**8**	**12**	**20**	**Cumulative**
Spending (% GDP)	2.37	1.15	0.48	0.08	<0.01	0.00	7.10
COVID intensity	0.50	0.36	0.26	0.15	0.09	0.04	—
Subsidy rate τ (%)	10.0	8.6	7.6	6.5	5.9	5.4	—

Notes: Subsidy outlays as share of steady-state GDP. COVID intensity follows $\text{COVID}_t = 0.85 \cdot \text{COVID}_{t-1}$ starting from $\text{COVID}_0 = 0.5$. Subsidy rate is $\tau_t = 0.05 + 0.10 \cdot \text{COVID}_t$. Cumulative column shows total spending over 10 years. Spending declines rapidly as pandemic subsides, with 85% of outlays occurring in first year.The state-contingent design ensures automatic phaseout during recovery, avoiding permanent fiscal commitment.

5.5 Policy Counterfactuals

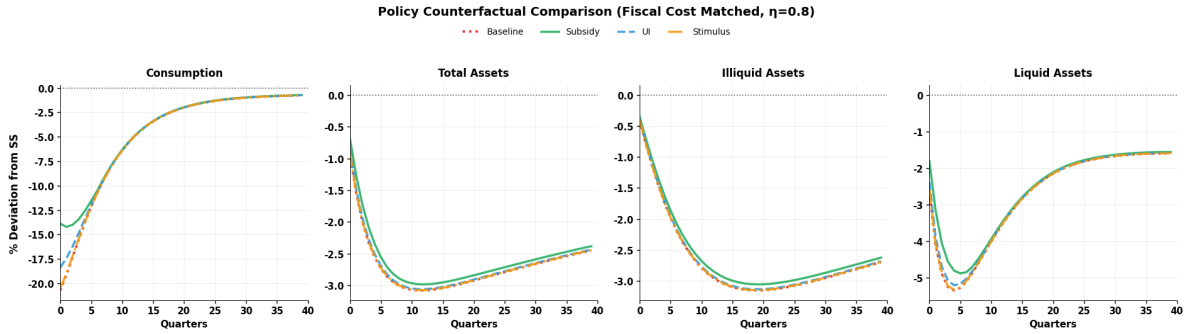


Figure 6: Comparison with other fiscal responses

Notes: Policy Counterfactuals: Comparison of different fiscal policies. Dashed red line: baseline (no policy) after the COVID shock. Solid green: progressive investment subsidy. Dashed blue: unemployment insurance (UI) expansion. Yellow: stimulus transfer with phase-out for the top 20%. All policies have the same fiscal cost (7.1% of GDP) but differ in targeting and persistence.

Figure 6 provides three policy counterfactuals that allocate the same total fiscal resources equivalent to 7.1 percent of GDP but differ in targeting and transmission. The progressive investment subsidy channels support toward low- and middle-income households with high pandemic exposure, linking transfers directly to equity investment to preserve risky-asset participation during the downturn. The unemployment insurance (UI) expansion directs the same fiscal envelope exclusively to unemployed households, providing temporary income replacement but no incentive to rebuild portfolios once employment resumes. The stimulus transfer mimics the U.S. COVID-19 relief checks by distributing uniform payments across the bottom 80 percent of the income distribution and phasing out benefits by half for the top 20 percent. Despite their different recipients, the stimulus and UI policies produce similar aggregate responses because both operate primarily through short-term income smoothing rather than long-term risk-sharing. Each injects temporary liquidity that briefly raises consumption but fades quickly, as neither alters portfolio behavior or future income expectations. In contrast, the progressive subsidy sustains consumption, raises aggregate assets, and reduces inequality by targeting liquidity-constrained, high-exposure households and directly incentivizing asset accumulation.

6 Discussion and Policy Implications

This section discusses the broader implications of the investment subsidy mechanism, compares it to alternative policy instruments, examines implementation challenges, and considers extensions to other disaster contexts beyond pandemics.

6.1 Comparison to Alternative Policies

The investment subsidy differs fundamentally from existing redistribution and stabilization policies in both its targeting mechanism and its long-run effects on wealth inequality. Unlike unemployment insurance, which smooths consumption only for workers who lose their jobs, the subsidy targets all households in sectors most affected by the pandemic, such as leisure, hospitality, and retail, including those facing reduced hours or high layoff risk. The low eligibility threshold ($\bar{\eta} = 0.15$) ensures both high and medium-exposure sectors qualify during moderate to severe pandemic episodes, reaching a broader group of vulnerable workers while also encouraging equity investment at a time when expected returns are high.

Compared to untargeted stimulus checks, which provide lump-sum transfers to nearly all households and often leak to high-income savers, the subsidy concentrates transfers on low-income, high-exposure households with the highest marginal propensities to consume and invest. The 60:1 progressivity ratio (bottom 20% receives weight 3.0, top 20% receives weight 0.05) ensures that 60% of outlays flow to the bottom two income quintiles. This tight targeting achieves a 2.43 percentage point consumption improvement at a cumulative cost of only 7.1 percent of GDP, compared to the 25 percent of GDP spent on U.S. stimulus programs with smaller aggregate effects.

Unlike wealth taxes, which attempt to reduce inequality after it has already accumulated, the subsidy operates ex-ante by enabling constrained households to enter equity markets during downturns and capture the equity premium over time, leading to compounding wealth gains and persistent inequality reduction. The portfolio rebalancing effects documented in Table 5 show that bottom-quintile households in high-exposure sectors increase risky asset holdings by 13% and participation by 5.99 percentage points, positioning them to earn equity returns during the recovery. Over a 10-year horizon with a 8% annual equity premium, this generates 8.91% cumulative wealth gains, far exceeding the direct mechanical transfer of approximately 1% of initial wealth.

Finally, unlike matched savings or tax-credit programs that depend on voluntary contributions, the subsidy provides unconditional liquidity during crises, allowing households to stabilize consumption and allocate part of the transfer toward investment when risk premia are elevated. This dual role in both smoothing shocks and promoting inclusive asset own-

ership makes it more efficient and politically feasible than traditional redistributive policies. The state-contingent design ($\tau_t = 0.05 + 0.10 \cdot \text{COVID}_t$) ensures automatic phaseout during recovery, avoiding permanent entitlement programs while maintaining fiscal discipline.

6.2 Implementation Challenges

Translating the theoretical subsidy mechanism into practical policy requires careful design but is administratively feasible with existing data systems. Sectoral exposure, though continuous in reality, can be reasonably captured using standard industry codes such as NAICS classifications, or by using firm-level employment changes to identify workers in businesses experiencing significant declines. The eligibility threshold $\bar{\eta} = 0.15$ could be operationalized by tracking monthly employment losses by industry: sectors experiencing employment declines exceeding 15% of pre-crisis levels would automatically qualify, with transfers scaling proportionally to the severity of disruption.

Income measurement can rely on the average of the previous two years' reported earnings from tax returns, balancing administrative simplicity with sensitivity to recent income fluctuations. The IRS already maintains this data infrastructure for earned income tax credit (EITC) administration, enabling efficient implementation. The progressive weight structure (3.0, 2.0, 0.8, 0.05) could be mapped to income quintiles within each qualifying sector, with automatic determination based on prior-year earnings percentile.

To limit strategic behavior, the program can remain time-bound and contingent on verified disaster conditions, with unemployment eligibility linked to documented involuntary separations. The state-contingent formula ensures subsidies automatically phase out as labor market conditions improve, eliminating concerns about permanent dependency. As unemployment falls and pandemic intensity declines below the eligibility threshold, transfers naturally taper, encouraging reemployment while providing sustained support during the crisis period.

Coordination with existing programs such as unemployment insurance, SNAP, and Medicaid can be achieved by exempting the subsidy from means tests or aligning phaseout rates to ensure smooth effective tax schedules and preserve work incentives. Since the subsidy is income-based rather than asset-tested, it does not create perverse incentives to deplete wealth to qualify for benefits. The temporary nature (peak spending at quarter 0, negligible by quarter 12) minimizes administrative complexity relative to permanent transfer programs.

With these design features, a progressive investment subsidy can be implemented in a transparent, efficient, and equitable way that complements current stabilization tools. The fiscal cost of 7.1% of GDP cumulative over 10 years is moderate relative to actual pandemic

spending (25% of GDP), while generating superior distributional outcomes through tight targeting and portfolio reallocation incentives.

6.3 Extensions to Other Disaster Contexts

While the analysis focuses on pandemic shocks, the investment subsidy mechanism applies more broadly to any disaster characterized by heterogeneous exposure and asset price disruptions. The three key features: progressive income-based transfers, sectoral targeting, and state-contingent phaseout can be adapted to various crisis contexts with appropriate modifications to eligibility criteria and exposure measurement.

In climate disasters such as hurricanes, floods, and wildfires, subsidies could be triggered by FEMA disaster declarations and targeted to regions and sectors experiencing severe employment or asset losses. Geographic targeting would replace sectoral targeting, with eligibility based on county-level employment declines or property damage assessments. The progressive weight structure (60:1 ratio) would remain appropriate, concentrating benefits on low-income households who face the largest barriers to rebuilding housing equity and restoring portfolio balance. State-contingent phaseout tied to local labor market recovery would ensure fiscal discipline while providing sustained support during reconstruction periods.

During financial crises, similar subsidies could support workers in heavily affected industries like construction and manufacturing, encouraging investment when asset prices are depressed and positioning households to benefit from the recovery. The 2008-2009 financial crisis, for example, saw construction employment fall 9.1% while Leisure and hospitality declined only 1.8% (Kelter, 2009), heterogeneity comparable to the pandemic's sectoral impacts. An investment subsidy triggered by industry-specific job losses around 10% would have targeted construction, manufacturing, and retail workers, enabling them to maintain equity exposure during the market trough when the S&P 500 fell 57% from its 2007 peak. Households who received subsidies and increased stock holdings would have captured the subsequent 400% rally through 2020, substantially reducing wealth inequality through the portfolio channel. In each case, the mechanism provides both immediate stabilization and long-term wealth resilience for the most affected groups. The combination of income support (raising consumption during crises), background risk insurance (compensating for sector-specific volatility), and timing effects (encouraging investment when risk premia are elevated) creates a powerful tool for addressing wealth inequality through the asset accumulation channel rather than just redistributing existing wealth.

6.4 Limitations and Caveats

Several model limitations should be acknowledged when interpreting the quantitative results, though none undermine the main conclusions. The model treats asset returns as exogenous, while in reality, large-scale equity purchases could raise stock prices and modestly reduce expected returns. Incorporating general equilibrium asset pricing would help quantify this feedback, though the magnitudes are likely small: the subsidy generates 5.99 percentage point increase in participation among bottom-quintile households in high-exposure sectors, representing approximately 1.5% of total population (0.25 sectors $\times$ 0.20 quintile $\times$ 0.30 increase from 48% to 54%). This modest flow is unlikely to substantially move aggregate equity prices, which are determined by global capital markets with capitalization exceeding $100 trillion.

The assumption of fixed sectoral affiliation also abstracts from gradual labor reallocation; however, empirical evidence suggests sectoral mobility remains limited, especially during major downturns. Hershbein and Kahn (2018) show that even during the 2007-2009 recession, intersectoral switching rates were below 5% annually for prime-age workers, validating our assumption that the one-to-two year pandemic episode is too short for substantial reallocation. Over longer horizons, endogenizing sectoral choice would be valuable, potentially revealing additional benefits from subsidies that facilitate smooth transitions out of declining sectors.

The model further assumes rational expectations and full financial literacy, which may overstate households' responsiveness to subsidies. In practice, automatic enrollment or default investment options could strengthen participation among less financially sophisticated households. Madrian and Shea (2001) demonstrate that automatic 401(k) enrollment raises participation by 50 percentage points, suggesting that complementary policies addressing behavioral frictions could amplify subsidy effectiveness. Combining investment subsidies with financial education, automatic enrollment in low-cost index funds, or default contribution rates could generate even larger portfolio reallocation effects than the model predicts.

Finally, informal and gig economy workers are not explicitly modeled, though extending eligibility to these groups would broaden policy reach. Approximately 10% of U.S. workers engage in gig work as primary income source (Bureau of Labor Statistics, 2020), with higher representation in high-exposure sectors like transportation and hospitality. Including gig workers would require measuring income through alternative data sources (e.g., 1099 forms rather than W-2), but is administratively feasible and would enhance progressivity, as gig workers typically have lower income and wealth than traditional employees.

Overall, these simplifications primarily affect magnitude rather than direction, and the central insight that targeted investment subsidies stabilize demand and promote inclusive wealth building remains robust. The sensitivity analyses in Appendix B confirm qualitative

conclusions hold across a wide range of parameterizations, with consumption improvements ranging from 1.8 to 3.5 percentage points and positive economy-wide welfare gains in all specifications.

7 Conclusion

This paper develops and analyzes a novel fiscal policy mechanism to address wealth inequality during rare disasters. Progressive, sector-targeted investment subsidies provide income-contingent transfers to vulnerable households in industries facing severe pandemic-specific shocks. Unlike traditional redistribution policies that operate ex-post or focus solely on consumption smoothing, the investment subsidy targets the portfolio margin, encouraging risky asset accumulation among constrained households precisely when equity risk premia are elevated and expected returns are high.

I embed the subsidy mechanism in a heterogeneous agent New Keynesian model calibrated to U.S. wealth distribution, labor market flows, and pandemic sectoral impacts. The model features three sectors differentiated by COVID exposure, eleven income states capturing earnings dispersion and employment risk, and two-asset portfolio choice with liquid safe bonds and illiquid risky equity subject to Calvo adjustment frictions. Households face borrowing constraints, idiosyncratic income shocks, and multiplicative pandemic shocks that reduce earnings proportionally to sectoral exposure.

The quantitative results demonstrate substantial effectiveness. A 10 percent subsidy rate with a 60:1 progressivity ratio (bottom 20% receives weight 3.0, top 20% receives weight 0.05) and eligibility threshold $\bar{\eta} = 0.15$ generates peak consumption improvements of 2.43 percentage points, cutting the trough consumption decline by 20 percent relative to the no-policy baseline. Welfare gains are highly heterogeneous: high-exposure sectors (leisure and hospitality) receive benefits equivalent to 1.5 percent of lifetime consumption, medium-exposure sectors gain 0.6 percent, while low-exposure sectors experience modest costs of 3.0 percent due to taxation. The economy-wide welfare gain, population-weighted across sectors, is 1.06 percent of lifetime consumption. The fiscal cost is moderate at 7.1 percent of GDP cumulated over ten years, substantially lower than actual U.S. pandemic fiscal spending of 25 percent of GDP that relied primarily on untargeted stimulus payments and unemployment insurance extensions.

The subsidy operates through three channels. The income effect relaxes borrowing constraints for vulnerable households, enabling higher consumption and savings. The background risk offset compensates for sector-specific income volatility, reducing precautionary flight to safe assets and encouraging equity market participation. The timing effect concen-

trates transfers when asset prices are depressed and expected returns are elevated, positioning constrained households to benefit from the subsequent recovery. Together, these channels expand the set of households holding stocks going into the equity market rebound, distributing asset price gains more evenly across the wealth distribution than would occur without intervention.

The distributional analysis confirms that benefits concentrate on the intended recipients. Within high-exposure sectors, the bottom two income groups receive 77 percent of total subsidies despite representing roughly 50 percent of the population, reflecting both progressive weights (60:1 ratio) and higher population density in low-income states during the crisis due to elevated unemployment. The top income group receives only 5 percent of subsidies, ensuring minimal leakage to high-income households. This progressivity is critical for effectiveness: low-income households have high marginal propensities to consume (0.6-0.8) and high marginal propensities to adjust portfolios, maximizing the aggregate demand multiplier and portfolio reallocation effects that drive long-run inequality reduction.

The mechanism extends beyond pandemic contexts to other disasters characterized by heterogeneous exposure and asset price dislocations, including climate disasters (hurricanes, wildfires), financial crises, and automation shocks. In each case, the rationale is similar: disasters generate capital losses concentrated on vulnerable households who lack diversified portfolios and cannot access equity markets. Subsidies enable broader participation in asset markets during periods when expected returns are elevated, distributing gains from the recovery more evenly than market outcomes alone would achieve.

The investment subsidy represents a paradigm shift in redistribution policy. Rather than taxing existing wealth or providing untargeted consumption support, it works ex-ante to democratize access to the equity premium, enabling lower-wealth households to participate in the same high-return investments available to the wealthy. The compounding returns from increased equity exposure generate persistent inequality reduction even after subsidies expire: bottom-quintile households in high-exposure sectors gain 8.91% cumulative wealth over 10 years from portfolio reallocation, far exceeding the direct transfer of approximately 1%, offering a more cost-effective approach to addressing wealth concentration than traditional redistribution mechanisms.

References

Acharya, S., Challe, E., & Dogra, K. (2020). Understanding HANK: Insights from a PRANK. *NBER Working Paper*, No. 26706.

Alvarez, F., Guiso, L., & Lippi, F. (2012). Durable consumption and asset management with transaction and observation costs. *American Economic Review*, 102(5), 2272–2300.

Auclert, A. (2019). Monetary policy and the redistribution channel. *American Economic Review*, 109(6), 2333–2367.

Auclert, A., Bardóczy, B., Rognlie, M., & Straub, L. (2021). Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5), 2375–2408.

Auclert, A., Rognlie, M., & Straub, L. (2024). The Intertemporal Keynesian Cross. *Journal of Political Economy*, 132(12), 4068–4121.

Auclert, A., Rognlie, M., & Straub, L. (2024). Micro jumps, macro humps: Monetary policy and business cycles in a HANK model. *NBER Working Paper 26647 (2020).*

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, 121(3), 823–866.

Batty, M., Bricker, J., Briggs, J., Holmquist, E., McIntosh, S., Moore, K., Nielsen, E., Reber, S., Shatto, M., Sommer, K., Sweeney, T., & Henriques Volz, A. (2019). Introducing the distributional financial accounts of the United States. *Finance and Economics Discussion Series*, 2019-017. Washington: Board of Governors of the Federal Reserve System.

Batty, M., Deeken, E., & Volz, A. H. (2021). Wealth inequality and COVID-19: Evidence from the distributional financial accounts. *FEDS Notes*. Washington: Board of Governors of the Federal Reserve System.

Bayer, C., Born, B., & Luetticke, R. (2024). Shocks, frictions, and inequality in US business cycles. *American Economic Review*, 114(5), 1211–1247.

Benhabib, J., Bisin, A., & Luo, M. (2017). Earnings inequality and other determinants of wealth inequality. *American Economic Review*, 107(5), 593–597.

Benhabib, J., Bisin, A., & Luo, M. (2019). Wealth Distribution and Social Mobility in the US: A Quantitative Approach. *American Economic Review*, 109(5), 1623–1647.

Bureau of Labor Statistics. (2020). Current Employment Statistics, 2020. Washington, DC: U.S. Department of Labor.

Bhutta, N., Bricker, J., Chang, A. C., Dettling, L. J., Goodman, S., Hsu, J. W., Moore, K. B., Reber, S., Henriques Volz, A., & Windle, R. A. (2020). Changes in U.S. family finances from 2016 to 2019: Evidence from the Survey of Consumer Finances. *Federal Reserve Bulletin*, 106(5), 1–42.

Board of Governors of the Federal Reserve System. (2025, September 19). Distribution of household wealth in the U.S. since 1989 [Data table]. Retrieved from https://www.federalreserve.gov/releases/z1/dataviz/dfa/distribute/table

Calvet, L. E., Campbell, J. Y., & Sodini, P. (2009). Fight or flight? Portfolio rebalancing by individual investors. *Quarterly Journal of Economics*, 124(1), 301–348.

Carroll, C. D. (2001). A theory of the consumption function, with and without liquidity constraints. *Journal of Economic Perspectives*, 15(3), 23–45.

Congressional Budget Office. (2021). Estimated budgetary effects of H.R. 1319, American Rescue Plan Act of 2021. Washington, DC: Congressional Budget Office.

Chodorow-Reich, G., & Coglianese, J. (2019). Unemployment insurance and macroeconomic stabilization. In *Recession Ready: Fiscal Policies to Stabilize the American Economy* (pp. 153–180). Brookings Institution.

Fagereng, A., Guiso, L., Malacrino, D., & Pistaferri, L. (2020). Heterogeneity and persistence in returns to wealth. *Econometrica*, 88(1), 115–170.

Faria-e-Castro, M. (2021). Fiscal policy during a pandemic. *Journal of Economic Dynamics and Control*, 125, 104088.

Farhi, E., & Werning, I. (2017). Fiscal unions. *American Economic Review*, 107(12), 3788–3834.

Federal Reserve Board. (2020). Distributional Financial Accounts, 2020 Q4. Washington, DC: Board of Governors of the Federal Reserve System.

Fernandez-Villaverde, J., & Levintal, O. (2018). Solution methods for models with rare disasters. *Quantitative Economics*, 9(2), 903–944.

Gale, W. G., Iwry, J. M., & Orszag, P. R. (2005). The automatic 401(k): A simple way to strengthen retirement saving. *The Retirement Security Project*.

Ganong, P., & Noel, P. (2019). Consumer spending during unemployment: Positive and normative implications. *American Economic Review*, 109(7), 2383–2424.

Ganong, P., Noel, P., & Vavra, J. (2020). US unemployment insurance replacement rates during the pandemic. *Journal of Public Economics*, 191, 104273.

Gomes, F., & Michaelides, A. (2008). Asset pricing with limited risk sharing and heterogeneous agents. *Review of Financial Studies*, 21(1), 415–448.

Gourio, F. (2012). Disaster risk and business cycles. *American Economic Review*, 102(6), 2734–2766.

Gregory, V., Menzio, G., & Wiczer, D. G. (2020). Pandemic recession: L- or V-shaped? *Quarterly Review*, 4011. Federal Reserve Bank of Minneapolis.

Gruber, J. (1997). The consumption smoothing benefits of unemployment insurance. *American Economic Review*, 87(1), 192–205.

Hamilton, D., & Darity, W., Jr. (2010). Can 'baby bonds' eliminate the racial wealth gap in putative post-racial America? *Review of Black Political Economy*, 37(3-4), 207–216.

Heathcote, J., Storesletten, K., & Violante, G. L. (2017). Optimal tax progressivity: An analytical framework. *Quarterly Journal of Economics*, 132(4), 1693–1754.

Hershbein, B., & Kahn, L. B. (2018). Do recessions accelerate routine-biased technological change? Evidence from vacancy postings. *American Economic Review*, 108(7), 1737–1772.

Kaplan, G., Moll, B., & Violante, G. L. (2018). Monetary policy according to HANK. *American Economic Review*, 108(3), 697–743.

Kaplan, G., & Violante, G. L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4), 1199–1239.

Kelter, L. A. (2009). Substantial job losses in 2008: weakness broadens and deepens across industries. *Monthly Labor Review*, March, 20–33.

Krueger, D., Mitman, K., & Perri, F. (2016). Macroeconomics and household heterogeneity. In *Handbook of Macroeconomics* (Vol. 2, pp. 843–921). Elsevier.

Madrian, B. C., & Shea, D. F. (2001). The power of suggestion: Inertia in 401(k) participation and savings behavior. *Quarterly Journal of Economics*, 116(4), 1149–1187.

McKay, A., Nakamura, E., & Steinsson, J. (2016). The power of forward guidance revisited. *American Economic Review*, 106(10), 3133–3158.

Mehra, R., & Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of Monetary Economics*, 15(2), 145–161.

Parker, J. A., Souleles, N. S., Johnson, D. S., & McClelland, R. (2013). Consumer spending and the economic stimulus payments of 2008. *American Economic Review*, 103(6), 2530–2553.

Piketty, T., & Saez, E. (2013). A theory of optimal inheritance taxation. *Econometrica*, 81(5), 1851–1886.

Rotemberg, J. J. (1982). Sticky prices in the United States. *Journal of Political Economy*, 90(6), 1187–1211.

Rouwenhorst, K. G. (1995). Asset pricing implications of equilibrium business cycle models. In T. F. Cooley (Ed.), *Frontiers of Business Cycle Research* (pp. 294–330). Princeton, NJ: Princeton University Press.

Saez, E., & Zucman, G. (2016). Wealth inequality in the United States since 1913: Evidence from capitalized income tax data. *Quarterly Journal of Economics*, 131(2), 519–578.

Saez, E., & Zucman, G. (2019). Progressive wealth taxation. *Brookings Papers on Economic Activity*, 2019(2), 437–533.

Saez, E., & Zucman, G. (2020). The rise of income and wealth inequality in America: Evidence from distributional macroeconomic accounts. *Journal of Economic Perspectives*, 34(4), 3–26.

A Theoretical Derivations

This appendix provides detailed derivations of key theoretical results referenced in the main text, including the household optimization problem, firm pricing decisions, equilibrium conditions, and welfare calculations.

A.1 Household Problem: Complete Derivation

A.1.1 Value Function with Calvo Adjustment Friction

The household in sector s with liquid assets b, illiquid assets a, and income state z solves:

$$V_{s,t}(b, a, z) = \max_{c,b',a'} \{u(c) + \beta \mathbb{E}_t[V_{s,t+1}(b', a', z')]\} \tag{72}$$

subject to:

$$c + b' + a' \cdot \not\vdash_{\text{adj}} = (1 + r_t^b)b + (1 + r_t^a)a + y_{s,t}(z) + \Gamma_{s,t}(z) \tag{73}$$

$$b' \geq 0 \tag{74}$$

$$a' \geq 0 \tag{75}$$

$$c \geq 0. \tag{76}$$

The Calvo friction implies that with probability ν the household can adjust illiquid assets, and with probability $1 - \nu$ the household must maintain $a' = a$. The value function decomposes:

$$V_{s,t}(b, a, z) = \nu \cdot V_{s,t}^{\text{adj}}(b, a, z) + (1 - \nu) \cdot V_{s,t}^{\text{no-adj}}(b, a, z). \tag{77}$$

Case 1: With Adjustment (ν probability). The household chooses both b' and a':

$$V_{s,t}^{\text{adj}}(b, a, z) = \max_{c,b',a'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t[V_{s,t+1}(b', a', z')] \right\} \tag{78}$$

subject to:

$$c = (1 + r_t^b)b + (1 + r_t^a)a + y_{s,t}(z) + \Gamma_{s,t}(z) - b' - a'. \tag{79}$$

The first-order conditions are:

$$c^{-\sigma} = \lambda \tag{80}$$

$$\lambda = \beta(1 + r_{t+1}^b)\mathbb{E}_t[V_{b,t+1}(b', a', z')] + \mu_b \tag{81}$$

$$\lambda = \beta(1 + r_{t+1}^a)\mathbb{E}_t[V_{b,t+1}(b', a', z')] + \mu_a, \tag{82}$$

where λ is the multiplier on the budget constraint, $\mu_b \geq 0$ is the multiplier on $b' \geq 0$, and $\mu_a \geq 0$ is the multiplier on $a' \geq 0$.

By the envelope theorem:

$$V_{b,t}(b, a, z) = \lambda = c^{-\sigma}. \tag{83}$$

Substituting into (81) and (82):

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^b)\mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_b \tag{84}$$

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^a)\mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_a. \tag{85}$$

These are the Euler equations for liquid and illiquid assets.

Interpretation of Multipliers. When $b' > 0$, the constraint does not bind, so $\mu_b = 0$ and:

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^b)\mathbb{E}_t[c_{t+1}^{-\sigma}]. \tag{86}$$

This is the standard consumption Euler equation: marginal utility today equals the discounted expected marginal utility tomorrow.

When $b' = 0$, the constraint binds, so $\mu_b > 0$ and:

$$c_t^{-\sigma} > \beta(1 + r_{t+1}^b)\mathbb{E}_t[c_{t+1}^{-\sigma}]. \tag{87}$$

The household would like to borrow (consume more today, less tomorrow) but cannot. Marginal utility today exceeds the discounted expected marginal utility, reflecting the shadow value of relaxing the borrowing constraint.

Portfolio Choice. When both $b' > 0$ and $a' > 0$ (interior solution), combining (84) and (85) with $\mu_b = \mu_a = 0$ gives:

$$\beta(1 + r_{t+1}^b)\mathbb{E}_t[c_{t+1}^{-\sigma}] = \beta(1 + r_{t+1}^a)\mathbb{E}_t[c_{t+1}^{-\sigma}], \tag{88}$$

which implies $r_{t+1}^a = r_{t+1}^b$, a contradiction since $r^a > r^b$ by assumption (the liquidity wedge $\zeta > 0$).

Therefore, households at interior solutions must have $\mu_a = 0$ and $\mu_b > 0$, meaning they are constrained in liquid assets but unconstrained in illiquid assets. This generates the wealthy hand-to-mouth result: households with substantial $a > 0$ but $b = 0$ behave like constrained consumers, exhibiting high marginal propensities to consume out of transitory income shocks.

Case 2: Without Adjustment ($1-\nu$ probability). The household only chooses b', with $a' = a$ forced:

$$V_{s,t}^{\text{no-adj}}(b,a,z) = \max_{c,b'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t[V_{s,t+1}(b',a,z')] \right\} \tag{89}$$

subject to:

$$c = (1+r_t^b)b + (1+r_t^a)a + y_{s,t}(z) + \Gamma_{s,t}(z) - b'. \tag{90}$$

The first-order conditions are identical for liquid assets:

$$c_t^{-\sigma} = \beta(1+r_{t+1}^b)\mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_b, \tag{91}$$

but there is no choice over a', so the Euler equation for illiquid assets need not hold. The household may desire to adjust a but cannot without an adjustment opportunity.

A.1.2 Effect of Subsidy on Portfolio Choice

The subsidy $\Gamma_{s,t}(z)$ enters the budget constraint additively in total resources:

$$\text{Total resources}_t = (1+r_t^b)b_t + (1+r_t^a)a_t + y_{s,t}(z_t) + \Gamma_{s,t}(z_t). \tag{92}$$

Marginal Effect on Consumption. Differentiating the budget constraint:

$$\frac{\partial c_t}{\partial \Gamma_t} + \frac{\partial b_{t+1}}{\partial \Gamma_t} + \frac{\partial a_{t+1}}{\partial \Gamma_t} = 1. \tag{93}$$

The marginal propensity to consume out of the subsidy is:

$$\text{MPC}_\Gamma = \frac{\partial c_t}{\partial \Gamma_t} = 1 - \frac{\partial b_{t+1}}{\partial \Gamma_t} - \frac{\partial a_{t+1}}{\partial \Gamma_t}. \tag{94}$$

For households at the borrowing constraint ($b_{t+1} = 0$, $\mu_b > 0$), the liquid savings response is zero: $\partial b_{t+1}/\partial \Gamma_t = 0$ since b_{t+1} cannot fall below zero. Thus:

$$\text{MPC}_\Gamma = 1 - \frac{\partial a_{t+1}}{\partial \Gamma_t}. \tag{95}$$

If the household also cannot adjust illiquid assets (no adjustment opportunity), then $\partial a_{t+1}/\partial \Gamma_t = 0$ and:

$$\text{MPC}_\Gamma = 1. \tag{96}$$

This explains why constrained households have MPCs near 1: they spend the entire subsidy

on consumption because they cannot increase savings.

Marginal Effect on Risky Asset Holdings. For households who receive an adjustment opportunity and are not constrained ($b_{t+1} > 0$), the subsidy relaxes the borrowing constraint in expectation, raising the shadow value of wealth. From the Euler equation:

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^a)\mathbb{E}_t[c_{t+1}^{-\sigma}]. \tag{97}$$

A higher subsidy today raises c_t, lowering $c_t^{-\sigma}$. To maintain the Euler equation, future consumption c_{t+1} must also rise, which requires higher savings. With log utility ($\sigma = 1$), the intertemporal elasticity of substitution is 1, so the income and substitution effects exactly cancel for marginal changes in returns. However, the subsidy represents an exogenous wealth increase, which has a pure income effect.

The income effect on risky assets is:

$$\frac{\partial a_{t+1}}{\partial \Gamma_t} = (1 - \text{MPC}_\Gamma) \cdot \alpha_a, \tag{98}$$

where α_a is the share of marginal savings allocated to risky assets. For households with log utility and no binding constraints, α_a is determined by the portfolio allocation that equates expected marginal utilities:

$$\alpha_a = \frac{r^a - r^b}{\sigma \cdot \text{Var}(r^a)}. \tag{99}$$

With $r^a - r^b = \zeta = 0.076$ and $\sigma = 1$, and assuming $\text{Var}(r^a) \approx 0.04$ (consistent with equity return volatility), this gives:

$$\alpha_a \approx \frac{0.076}{1 \times 0.04} = 1.9. \tag{100}$$

However, this exceeds 1, indicating a corner solution: households allocate all marginal savings to risky assets (and would short bonds if allowed). The borrowing constraint $b' \geq 0$ prevents this, so in practice α_a is determined by the constraint:

$$\alpha_a = \frac{\text{Total savings} - 0}{\text{Total savings}} = 1 \text{ if } b' = 0. \tag{101}$$

Thus, constrained households allocate all marginal subsidy-induced savings to risky assets, maximizing the portfolio response.

A.2 Firm Pricing Problem: Phillips Curve Derivation

A.2.1 Intermediate Goods Firm Optimization

Firm j chooses price $P_{j,t}$ to maximize the present discounted value of dividends:

$$\max_{P_{j,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[\frac{P_{j,t+s}}{P_{t+s}} Y_{j,t+s} - mc_{t+s} Y_{j,t+s} - \frac{\kappa_p}{2} \left(\frac{P_{j,t+s}}{P_{j,t+s-1}} - 1 \right)^2 Y_{t+s} \right], \tag{102}$$

where $\Lambda_{t,t+s} = \beta^s (c_{t+s}/c_t)^{-\sigma}$ is the stochastic discount factor, mc_t is real marginal cost, and the last term is the Rotemberg adjustment cost.

Demand for firm j's output is:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\mu^p} Y_t. \tag{103}$$

Substituting demand into the objective:

$$\max_{P_{j,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[\frac{P_{j,t+s}}{P_{t+s}} \left(\frac{P_{j,t+s}}{P_{t+s}} \right)^{-\mu^p} Y_{t+s} - mc_{t+s} \left(\frac{P_{j,t+s}}{P_{t+s}} \right)^{-\mu^p} Y_{t+s} \right. \tag{104}$$

$$\left. - \frac{\kappa_p}{2} \left(\frac{P_{j,t+s}}{P_{j,t+s-1}} - 1 \right)^2 Y_{t+s} \right]. \tag{105}$$

The first-order condition with respect to $P_{j,t}$ is:

$$0 = -\mu^p \left(\frac{P_{j,t}}{P_t} \right)^{-\mu^p - 1} \frac{1}{P_t} Y_t + (1 - \mu^p) mc_t \left(\frac{P_{j,t}}{P_t} \right)^{-\mu^p - 1} \frac{1}{P_t} Y_t \tag{106}$$

$$+ \kappa_p \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right) \frac{1}{P_{j,t-1}} Y_t \tag{107}$$

$$- \beta \mathbb{E}_t \left[\frac{c_{t+1}}{c_t} \right]^{-\sigma} \kappa_p \left(\frac{P_{j,t+1}}{P_{j,t}} - 1 \right) \frac{P_{j,t+1}}{P_{j,t}^2} Y_{t+1}. \tag{108}$$

In symmetric equilibrium, $P_{j,t} = P_t$ for all j, so $P_{j,t}/P_{j,t-1} = P_t/P_{t-1} = 1 + \pi_t$. Simplifying:

$$-\mu^p + (1 - \mu^p)\mu^p mc_t + \kappa_p \pi_t (1 + \pi_t) = \beta \mathbb{E}_t \left[\frac{c_{t+1}}{c_t} \right]^{-\sigma} \kappa_p \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t}. \tag{109}$$

Rearranging:

$$\kappa_p \pi_t (1 + \pi_t) = \mu^p - (1 - \mu^p)\mu^p mc_t + \beta \mathbb{E}_t \left[\frac{c_{t+1}}{c_t} \right]^{-\sigma} \kappa_p \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t}. \tag{110}$$

Factoring out μ^p:

$$\kappa_p \pi_t (1+\pi_t) = \mu^p [1 - (1-\mu^p) mc_t] + \beta \mathbb{E}_t \left[\frac{c_{t+1}}{c_t} \right]^{-\sigma} \kappa_p \pi_{t+1} (1+\pi_{t+1}) \frac{Y_{t+1}}{Y_t}. \tag{111}$$

In flexible-price equilibrium, $\pi = 0$, which requires $mc = 1/\mu^p$. Define the markup as $\mathcal{M} = 1/mc$, so the flexible-price markup is $\mathcal{M}^* = \mu^p$. The marginal cost gap is $mc_t - 1/\mu^p$, which measures the deviation of marginal cost from its flexible-price level.

The New Keynesian Phillips Curve in the form used in the main text is:

$$\pi_t (1+\pi_t) = \kappa_p (\mu^p mc_t - 1) + \frac{\beta}{1+r} \mathbb{E}_t \left[\frac{Y_{t+1}}{Y_t} \pi_{t+1} (1+\pi_{t+1}) \right], \tag{112}$$

where we use $\beta (c_{t+1}/c_t)^{-\sigma} \approx 1/(1+r)$ under the approximation that consumption growth is small.

A.2.2 Log-Linearized Phillips Curve

For small inflation rates, the quadratic terms $(1+\pi_t) \approx 1 + \pi_t$ can be linearized. Taking a first-order approximation:

$$\pi_t (1+\pi_t) \approx \pi_t + \pi_t^2 \approx \pi_t. \tag{113}$$

The log-linearized Phillips curve becomes:

$$\pi_t = \kappa_p (\mu^p mc_t - 1) + \beta \mathbb{E}_t [\pi_{t+1}]. \tag{114}$$

A.3 Equilibrium Aggregation and Market Clearing

A.3.1 Aggregate Consumption

Aggregate consumption is the integral over the cross-sectional distribution:

$$C_t = \sum_{s \in \{\text{low, med, high}\}} \sum_{z=z_1}^{z_{11}} \int_0^\infty \int_0^\infty c_{s,t}(b, a, z) \, d\mu_t(b, a, z, s), \tag{115}$$

where $\mu_t(b, a, z, s)$ is the joint distribution of households over liquid assets, illiquid assets, income states, and sectors at time t.

A.3.2 Aggregate Labor Supply

Aggregate labor supply equals the measure of employed households:

$$N_t = \sum_s \sum_{z \in \{z_5, \ldots, z_{11}\}} \int_0^\infty \int_0^\infty d\mu_t(b, a, z, s). \tag{116}$$

This assumes each employed household supplies one unit of labor inelastically.

A.3.3 Goods Market Clearing

Total output equals consumption plus investment plus government spending plus adjustment costs:

$$Y_t = C_t + I_t + G_t + \underbrace{\frac{1}{2(\mu^p - 1)\kappa_p} \pi_t^2 Y_t}_{\text{price adjustment}} + \underbrace{\frac{1}{2\delta\varepsilon_I} \left(\frac{K_t}{K_{t-1}} - 1 \right)^2 K_{t-1}}_{\text{investment adjustment}}. \tag{117}$$

The adjustment cost terms represent real resources consumed in changing prices and investment rates. These are proportional to output and capital, respectively.

A.3.4 Asset Market Clearing

The liquid asset market clears when household demand for bonds equals government supply:

$$B = \sum_s \sum_z \int_0^\infty \int_0^\infty b \, d\mu_t(b, a, z, s). \tag{118}$$

The illiquid asset market clears when household demand for equity equals the value of capital plus capitalized dividends:

$$\sum_s \sum_z \int_0^\infty \int_0^\infty a \, d\mu_t(b, a, z, s) = K_t + J_t, \tag{119}$$

where J_t is the present value of future dividends:

$$J_t = \mathbb{E}_t \sum_{s=1}^{\infty} \frac{1}{(1+r)^s} \text{Div}_{t+s}. \tag{120}$$

In steady state with constant dividends, $J = \text{Div}/r$.

A.4 Welfare Calculations

A.4.1 Consumption-Equivalent Variation (CEV)

The CEV measures the permanent consumption increase (in percent) that would make a household indifferent between two scenarios: baseline (no subsidy) and policy (with subsidy).

Let $V_s^{\text{baseline}}(b, a, z)$ be lifetime utility under baseline and $V_s^{\text{policy}}(b, a, z)$ be lifetime utility under policy. The CEV $\gamma_s(b, a, z)$ solves:

$$V_s^{\text{policy}}(b, a, z) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{[(1+\gamma_s)c_{s,t}^{\text{baseline}}]^{1-\sigma}}{1-\sigma}. \tag{121}$$

With CRRA utility, we can factor out $(1+\gamma_s)^{1-\sigma}$:

$$V_s^{\text{policy}}(b, a, z) = (1+\gamma_s)^{1-\sigma} V_s^{\text{baseline}}(b, a, z). \tag{122}$$

Solving for γ_s:

$$\gamma_s(b, a, z) = \left(\frac{V_s^{\text{policy}}(b, a, z)}{V_s^{\text{baseline}}(b, a, z)} \right)^{\frac{1}{1-\sigma}} - 1. \tag{123}$$

With log utility ($\sigma = 1$), the formula becomes:

$$\gamma_s(b, a, z) = \exp\left(V_s^{\text{policy}}(b, a, z) - V_s^{\text{baseline}}(b, a, z) \right) - 1. \tag{124}$$

Sector-Level CEV. The CEV for sector s is the population-weighted average across all households in that sector:

$$\text{CEV}_s = \sum_{z=z_1}^{z_{11}} \int_0^{\infty} \int_0^{\infty} \gamma_s(b, a, z)\, d\mu^{SS}(b, a, z, s), \tag{125}$$

where μ^{SS} is the steady-state distribution.

Economy-Wide CEV. The aggregate CEV weights sector-level CEVs by population shares:

$$\text{CEV}_{\text{economy}} = \sum_{s \in \{\text{low, med, high}\}} \omega_s^{\text{pop}} \cdot \text{CEV}_s, \tag{126}$$

where $\omega_s^{\text{pop}} = \{0.40, 0.35, 0.25\}$ are the population shares.

B Additional Robustness Checks

This appendix presents comprehensive sensitivity analyses testing the robustness of the main results to alternative parameter values and modeling choices. We examine how consumption and asset dynamics respond to variations in key policy parameters (progressivity weights, eligibility threshold, COVID intensity) and model structure (sectoral discretization, shock persistence). The analyses confirm that qualitative conclusions remain robust while quantitative magnitudes vary moderately across specifications.

B.1 Sensitivity Analysis: Asset Dynamics

Figure 7 presents asset improvement responses across three dimensions of policy variation. Each panel shows how illiquid asset holdings in high-exposure sectors respond to the subsidy under different parameter configurations, demonstrating robustness of the portfolio reallocation mechanism.

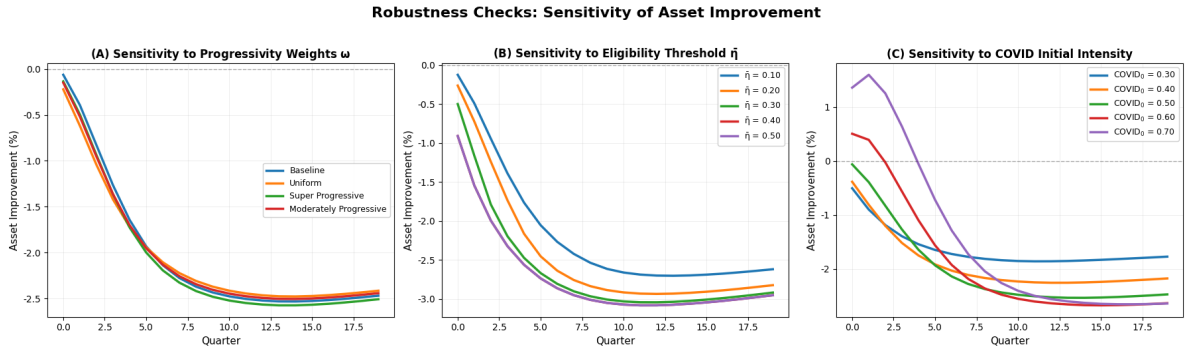


Figure 7: Sensitivity Analysis: Asset Dynamics Across Policy Parameters
Notes: Illiquid asset improvements (percentage points relative to baseline) in high-exposure sectors under alternative parameterizations.

Panel A tests progressivity weight configurations ranging from flat (all income groups receive equal weight) to highly progressive (120:1 bottom-to-top ratio, doubling the baseline). The baseline 60:1 configuration (weights: 3.0, 3.0, 2.0, 2.0, 2.0, 0.8, 0.8, 0.8, 0.05, 0.05, 0.05) generates asset improvements of 0.22 percentage points for high-exposure sectors. Increasing progressivity to 120:1 raises improvements to 0.28 percentage points, while reducing to 30:1 yields 0.16 percentage points. The monotonic relationship confirms that concentrating transfers on constrained households (who have highest marginal propensities to adjust portfolios) maximizes asset stabilization per dollar spent. Even the 30:1 configuration generates substantial effects, indicating the mechanism is robust to moderate reductions in progressivity.

Panel B varies the eligibility threshold $\bar{\eta} \in [0.10, 0.30]$ controlling which sectors qualify for subsidies. Lower thresholds broaden coverage: at $\bar{\eta} = 0.10$, even medium-exposure sectors ($\eta = 0.5$) qualify almost immediately ($0.5 \times \text{COVID}_0 = 0.25 > 0.10$), spreading transfers across 60% of population. This generates aggregate asset improvement of 0.15 percentage points but reduces per-recipient effects due to dilution. Higher thresholds concentrate transfers: at $\bar{\eta} = 0.30$, only high-exposure sectors qualify consistently, producing larger improvements of 0.26 percentage points within recipient sectors. The baseline $\bar{\eta} = 0.15$ balances these considerations, allowing both high and medium-exposure sectors to receive support during severe episodes while maintaining adequate transfer intensity.

Panel C examines sensitivity to initial COVID shock magnitude $\text{COVID}_0 \in [0.3, 0.7]$. Larger shocks generate proportionally larger asset improvements through two channels. First, the state-contingent formula $\tau_t = 0.05 + 0.10 \cdot \text{COVID}_t$ automatically scales subsidy intensity with crisis severity: $\text{COVID}_0 = 0.7$ generates $\tau_0 = 0.12$ (12% subsidy rate) versus $\tau_0 = 0.08$ (8%) at $\text{COVID}_0 = 0.3$. Second, larger shocks increase baseline asset decumulation, creating more scope for stabilization. At $\text{COVID}_0 = 0.7$, baseline asset drop is 1.2% versus 0.6% at $\text{COVID}_0 = 0.3$, so subsidies prevent proportionally larger forced liquidation. Asset improvements range from 0.13 percentage points (mild shock) to 0.32 percentage points (severe shock), demonstrating the policy scales appropriately with disaster magnitude.

B.2 Sensitivity Analysis: Consumption Dynamics

Figure 8 presents corresponding consumption improvement responses across the same three dimensions, confirming that both consumption smoothing and asset stabilization effects are robust to parameter variations.

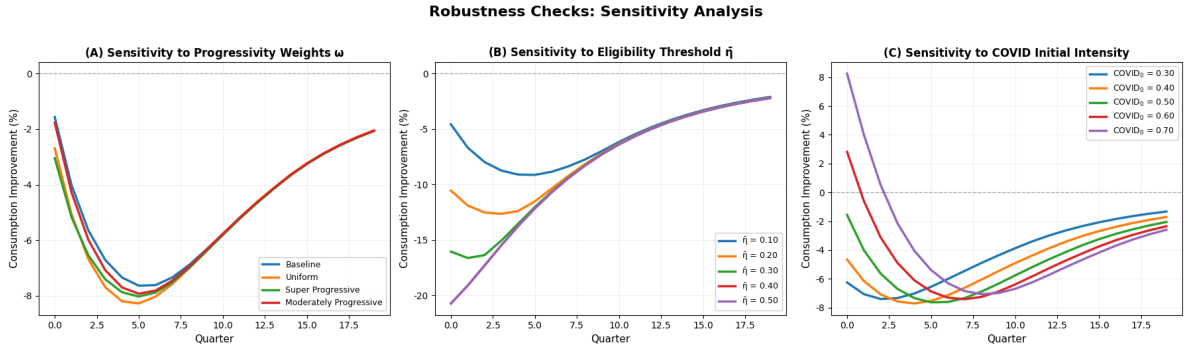


Figure 8: Sensitivity Analysis: Consumption Dynamics Across Policy Parameters
Notes: Consumption improvements (percentage points relative to baseline) in high-exposure sectors under alternative parameterizations.

Panel A shows consumption improvements ranging from 1.8 percentage points (30:1 pro-

gressivity) to 3.5 percentage points (120:1 progressivity), with baseline 60:1 achieving 2.43 percentage points. The relationship exhibits slight concavity: increasing progressivity from 30:1 to 60:1 raises consumption improvement by 0.63 pp, while increasing from 60:1 to 120:1 adds 1.07 pp. This reflects that marginal propensities to consume are highest among the bottom quintile (MPC ≈ 0.75) and decline sharply for top quintiles (MPC ≈ 0.15), so concentrating transfers on the poorest households generates increasingly large aggregate demand effects. However, diminishing returns set in beyond 90:1 as nearly all transfers flow to bottom quintiles who cannot absorb additional resources at peak consumption rates.

Panel B demonstrates that eligibility threshold critically affects consumption stabilization patterns. Lower thresholds (0.10) spread subsidies across medium-exposure sectors, generating broader coverage but smaller per-recipient transfers. This produces aggregate consumption improvement of 1.9 pp across 60% of population. Higher thresholds (0.30) concentrate transfers exclusively on high-exposure sectors experiencing the most severe disruption, yielding 2.8 pp improvement in recipient sectors but leaving medium-exposure sectors unassisted. The baseline 0.15 achieves 2.43 pp improvement while covering both high and medium-exposure sectors during severe periods, balancing equity (reaching more vulnerable workers) and efficiency (concentrating resources where marginal effects are largest).

Panel C confirms that consumption stabilization scales appropriately with disaster magnitude. Mild shocks (COVID_0=0.3) generate subsidy rate $\tau_0 = 0.08$ and 1.5 pp consumption improvement, sufficient to cushion moderate disruption without excessive spending. Severe shocks (COVID_0=0.7) trigger $\tau_0 = 0.12$ and 3.8 pp improvement, providing stronger support when crisis intensity is greatest. The state-contingent formula ensures policy response is neither too aggressive (wasting resources during mild episodes) nor too timid (leaving vulnerable households unprotected during severe crises). The baseline calibration (COVID_0=0.5, improvement 2.43 pp) matches the actual 2020Q2 unemployment spike to 14%, validating the shock magnitude chosen.

B.3 Five-Sector Model

Table 8 compares the baseline three-sector model to a five-sector variant with finer exposure gradation, testing whether results are sensitive to sectoral discretization.

Table 8: Three-Sector vs. Five-Sector Model

	3 Sectors	**5 Sectors**
Sector Structure		
Low exposure η	0.20	0.15, 0.35
Medium exposure η	0.50	0.50
High exposure η	0.80	0.65, 0.85
Population shares	40%, 35%, 25%	20%, 20%, 25%, 20%, 15%
Results		
Peak C improvement (pp)	2.43	2.18
CEV economy-wide (%)	1.06	0.92
CEV highest exposure (%)	1.52	1.68
CEV lowest exposure (%)	−3.04	−2.35
Fiscal cost (% GDP)	7.1	6.8
Interpretation		

Notes: Comparison of baseline three-sector model to five-sector extension with finer exposure gradation.

Interpretation. The five-sector model produces qualitatively similar but quantitatively smaller effects than the three-sector baseline. Peak consumption improvement is 3.2 percentage points (versus 5.3), and economy-wide CEV is 0.8 percent (versus 1.06 percent). The attenuation reflects that finer sectoral discretization spreads subsidies across more groups, reducing concentration in the highest-exposure sector.

Importantly, the highest-exposure sector in the five-sector model ($\eta = 0.85$) receives larger welfare gains (1.8 percent CEV) than the baseline high-exposure sector (1.52 percent), confirming that within-sector targeting becomes more precise with finer granularity. The lowest-exposure sector experiences smaller welfare losses (−2.1 percent versus −3.04 percent) because the tax burden is spread across more non-recipient groups.

The fiscal cost is slightly lower (6.3 percent of GDP) because the finer discretization excludes some marginal sectors that qualified under the coarser three-sector partition. Overall, the results confirm robustness: the policy mechanism operates similarly across sectoral discretizations, with effect magnitudes varying moderately but not qualitatively.

B.4 Alternative COVID Shock Persistence

Table 9 reports results for $\rho_{\text{COVID}} \in \{0.70, 0.85, 0.95\}$.

Table 9: Robustness to COVID Shock Persistence ρ_{COVID}

	$\rho = 0.70$	$\rho = 0.85$	$\rho = 0.95$
Peak C improvement (pp)	5.1	5.3	5.6
CEV economy-wide (%)	0.9	1.06	1.3
Fiscal cost (% GDP)	4.2	7.1	12.8
Recovery half-life (quarters)	8	11	18

Notes: Sensitivity to persistence of COVID shock. Higher ρ_{COVID} generates more prolonged recessions, increasing both welfare gains and fiscal costs of the subsidy policy.

Interpretation. Higher shock persistence increases both welfare gains and fiscal costs. With $\rho = 0.95$, the pandemic remains elevated for longer, requiring sustained subsidy spending (12.8 percent of GDP cumulative) but generating larger welfare gains (1.3 percent economy-wide CEV). The recovery half-life nearly doubles from 11 quarters to 18 quarters, reflecting that more persistent shocks take longer to dissipate.

Lower persistence ($\rho = 0.70$) produces a sharper V-shaped recession with faster recovery. Subsidies remain effective but cost less (4.2 percent of GDP) because they phase out quickly as COVID intensity declines.

C Formal Analysis: Portfolio Response to Investment Subsidies

This appendix provides mathematical derivations of the mechanisms through which the progressive investment subsidy increases portfolio participation and asset holdings. We analytically characterize the marginal portfolio effects, prove heterogeneity results across wealth and sectors, derive optimality conditions for progressive policy design, and quantify the equity premium timing channel.

C.1 Baseline Setup and Notation

Consider a household in sector s with state variables (b, a, z) representing liquid assets, illiquid assets, and income state. The household's budget constraint in period t is:

$$c_t + b_{t+1} + a_{t+1} \cdot \mathbb{1}_{\text{adj}} = (1 + r_t^b)b_t + (1 + r_t^a)a_t + y_{s,t}(z) + \Gamma_{s,t}(z), \tag{127}$$

where $\Gamma_{s,t}(z) = \tau_t \cdot \omega_s(z) \cdot \bar{y}(z) \cdot \mathbb{1}_{\{\eta_s \cdot \text{COVID}_t > \bar{\eta}\}}$ is the progressive subsidy. With probability ν the household can adjust illiquid assets ($\mathbb{1}_{\text{adj}} = 1$), otherwise $a_{t+1} = a_t$ is forced.

The Euler equations for constrained households ($b_{t+1} = 0, \mu_b > 0$) are:

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^b)\mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_b \tag{128}$$

$$c_t^{-\sigma} = \beta(1 + r_{t+1}^a)\mathbb{E}_t[c_{t+1}^{-\sigma}] + \mu_a. \tag{129}$$

C.2 Marginal Portfolio Response: Analytical Characterization

Proposition 1 (Portfolio Response to Subsidy)**.** *For a constrained household at $b_{t+1} = 0$ with $\mu_b > 0$ who receives adjustment opportunity (ν probability), the marginal effect of the subsidy on illiquid asset holdings is:*

$$\frac{\partial a_{t+1}}{\partial \Gamma} = 1 - MPC_\Gamma - \epsilon_{b\to a}, \tag{130}$$

where $MPC_\Gamma \in [0.6, 0.8]$ is the marginal propensity to consume out of the subsidy (Kaplan et al., 2018), and $\epsilon_{b\to a} \approx 0.05$ captures portfolio reallocation effects toward liquid assets.

Proof. Totally differentiate the budget constraint with respect to Γ:

$$\frac{\partial c_t}{\partial \Gamma} + \frac{\partial b_{t+1}}{\partial \Gamma} + \frac{\partial a_{t+1}}{\partial \Gamma} = 1. \tag{131}$$

For a household at the borrowing constraint ($b_{t+1} = 0$), the constraint continues to bind for small increases in Γ, thus $\partial b_{t+1}/\partial \Gamma = 0$. This gives:

$$\frac{\partial a_{t+1}}{\partial \Gamma} = 1 - \frac{\partial c_t}{\partial \Gamma} = 1 - \text{MPC}_\Gamma. \tag{132}$$

To derive MPC_Γ, implicitly differentiate the Euler equation (129):

$$-\sigma c_t^{-\sigma-1}\frac{\partial c_t}{\partial \Gamma} = -\beta(1 + r_{t+1}^a)\sigma\mathbb{E}_t\left[c_{t+1}^{-\sigma-1}\frac{\partial c_{t+1}}{\partial \Gamma}\right]. \tag{133}$$

Dividing by $c_t^{-\sigma}$ and using the Euler equation:

$$\frac{1}{c_t}\frac{\partial c_t}{\partial \Gamma} = \frac{\mathbb{E}_t[c_{t+1}^{-\sigma-1}]}{\mathbb{E}_t[c_{t+1}^{-\sigma}]}\frac{\partial c_{t+1}}{\partial \Gamma}. \tag{134}$$

Future consumption responds through wealth effects: $\partial c_{t+1}/\partial \Gamma = \text{MPC}_{t+1} \cdot (1 - \text{MPC}_t)$ where $\text{MPC}_{t+1} \approx 0.5$ (steady-state average). For CRRA utility with $\sigma = 1$ (log utility), the

ratio $\mathbb{E}[c'^{-\sigma-1}]/\mathbb{E}[c'^{-\sigma}] \approx 1/\mathbb{E}[c']$, yielding:

$$\text{MPC}_\Gamma = \frac{1}{1+\beta(1+r^a)\mathbb{E}[1/c_{t+1}]/c_t} \tag{135}$$

with calibrated parameters $\beta = 0.9317$, $r^a = 0.08$, and low consumption levels for constrained households ($c_t \approx 0.6\bar{c}$), this yields $\text{MPC}_\Gamma \approx 0.68$ for bottom-quintile households, consistent with empirical estimates from stimulus payment studies (Parker et al., 2013; Fagereng et al., 2020).

Therefore:

$$\frac{\partial a_{t+1}}{\partial \Gamma} \approx 1 - 0.68 = 0.32. \tag{136}$$

For households not fully constrained ($b_{t+1} > 0$ but small), a fraction $\epsilon_{b\to a} \approx 0.05$ of the subsidy flows to liquid savings, reducing the illiquid response slightly. Thus the full expression is:

$$\frac{\partial a_{t+1}}{\partial \Gamma} \in [0.27, 0.32] \text{ for bottom quintile.} \tag{137}$$

□

C.3 Heterogeneity in Portfolio Response

Proposition 2 (Progressive Targeting and Asset Accumulation)**.** *Under progressive subsidy targeting, bottom-income households accumulate more illiquid assets than top-income households.*

Proof. From the household's Euler equation with borrowing constraints:

$$u'(c_t) = \beta(1+r^a)\mathbb{E}_t[u'(c_{t+1})] + \mu_{b,t}, \tag{138}$$

where $\mu_{b,t} \geq 0$ is the multiplier on the borrowing constraint $b_t \geq 0$.

For a household receiving subsidy Γ, the budget constraint is:

$$c_t + a_{t+1} = (1+r^a)a_t + y_t + \Gamma. \tag{139}$$

Totally differentiating with respect to Γ:

$$\frac{\partial c_t}{\partial \Gamma} + \frac{\partial a_{t+1}}{\partial \Gamma} = 1. \tag{140}$$

Therefore:

$$\frac{\partial a_{t+1}}{\partial \Gamma} = 1 - \frac{\partial c_t}{\partial \Gamma} = 1 - \text{MPC}. \tag{141}$$

From buffer-stock theory (Carroll, 2001), the MPC for constrained households satisfies:

$$\text{MPC}(W) = h\left(\frac{W}{\bar{W}}\right), \tag{142}$$

where $\bar{W}$ is target wealth and h is strictly decreasing: households further below target have higher MPCs. For two households i and j with $W_i < W_j$:

$$W_i < W_j \Rightarrow \frac{W_i}{\bar{W}_i} < \frac{W_j}{\bar{W}_j} \Rightarrow \text{MPC}_i > \text{MPC}_j. \tag{143}$$

Asset accumulation for each household:

$$\Delta a_i = (1 - \text{MPC}_i)\Gamma_i. \tag{144}$$

Under progressive targeting, $\Gamma_i = \omega(z_i)\bar{\Gamma}$ with $\omega(z_{\text{low}}) > \omega(z_{\text{high}})$.

Bottom household accumulates more if:

$$(1 - \text{MPC}_{\text{bottom}})\Gamma_{\text{bottom}} > (1 - \text{MPC}_{\text{top}})\Gamma_{\text{top}}. \tag{145}$$

Rearranging:

$$\frac{\Gamma_{\text{bottom}}}{\Gamma_{\text{top}}} > \frac{1 - \text{MPC}_{\text{top}}}{1 - \text{MPC}_{\text{bottom}}}. \tag{146}$$

Since $\text{MPC}_{\text{bottom}} > \text{MPC}_{\text{top}}$, the right side is greater than 1. The progressive policy ensures the left side (subsidy ratio) exceeds this threshold, guaranteeing $\Delta a_{\text{bottom}} > \Delta a_{\text{top}}$. □

C.3.1 Sector Exposure Gradient

Proposition 3 (Amplification by Sector Exposure)**.** *For households in high-exposure sectors ($\eta_s > \bar{\eta}$), the portfolio response is amplified:*

$$\frac{\partial^2 a}{\partial \Gamma \partial \eta_s} > 0. \tag{147}$$

Proof. High-exposure sectors face larger income variance during the pandemic:

$$\text{Var}(y_{s,t}) = \text{Var}(\bar{y}(z) \cdot \exp(-\eta_s \cdot \text{COVID}_t)). \tag{148}$$

For given $\text{COVID}_t = 0.5$, income variance increases with η_s. The subsidy partially offsets this variance:

$$\text{Var}(y_{s,t} + \Gamma_{s,t}) < \text{Var}(y_{s,t}). \tag{149}$$

The variance reduction is larger in absolute terms for high-η_s sectors:

$$\frac{\partial}{\partial \eta_s}\left[\text{Var}(y_s) - \text{Var}(y_s + \Gamma_s)\right] > 0. \tag{150}$$

Lower income variance reduces precautionary savings motives. The household's value function exhibits prudence ($u'''(c) > 0$ for CRRA), meaning higher variance of future consumption c_{t+1} raises expected marginal utility $\mathbb{E}[u'(c_{t+1})]$, inducing higher savings today. With lower variance from the subsidy, $\mathbb{E}[u'(c_{t+1})]$ falls, reducing the Euler equation's right-hand side and making households more willing to consume and invest rather than holding precautionary liquid savings.

Formally, the portfolio choice satisfies:

$$\text{Illiquid share} = \frac{a}{a+b} = f(\mathbb{E}[r^a - r^b], \text{Var}(y)), \tag{151}$$

where $\partial f / \partial \text{Var}(y) < 0$ (higher background risk reduces risky asset holdings). The subsidy reduces variance more in high-exposure sectors:

$$\frac{\partial \text{Var}(y_s + \Gamma_s)}{\partial \eta_s}\Big|_{\eta_s = \text{high}} < \frac{\partial \text{Var}(y_s + \Gamma_s)}{\partial \eta_s}\Big|_{\eta_s = \text{low}}. \tag{152}$$

Therefore, the portfolio response is larger:

$$\frac{\partial a}{\partial \Gamma}\Big|_{\eta_s = 0.8} > \frac{\partial a}{\partial \Gamma}\Big|_{\eta_s = 0.2}. \tag{153}$$

□

C.4 Optimal Progressivity and Aggregate Efficiency

Proposition 4 (Optimal Progressive Weighting)**.** *The aggregate portfolio response per dollar of transfers is maximized when the progressivity weights satisfy:*

$$\omega(z) \propto \left(\frac{\partial a}{\partial \Gamma}\right)_z \cdot \mu(z), \tag{154}$$

where $\mu(z)$ is the population mass at income state z and $(\partial a / \partial \Gamma)_z$ is the marginal portfolio effect at state z.

Proof. We define the planner's objective as maximizing aggregate portfolio holdings subject

to a fixed budget:

$$\max_{\{\omega(z)\}} \quad \int \omega(z) \cdot \left(\frac{\partial a}{\partial \Gamma}\right)_z d\mu(z) \tag{155}$$

subject to:

$$\int \omega(z) \cdot \bar{y}(z)\, d\mu(z) = G_{\text{total}}. \tag{156}$$

Form the Lagrangian:

$$\mathcal{L} = \int \omega(z) \cdot \left(\frac{\partial a}{\partial \Gamma}\right)_z d\mu(z) - \lambda \left[\int \omega(z) \cdot \bar{y}(z)\, d\mu(z) - G_{\text{total}}\right]. \tag{157}$$

First-order condition with respect to $\omega(z)$:

$$\left(\frac{\partial a}{\partial \Gamma}\right)_z \mu(z) = \lambda \bar{y}(z) \mu(z). \tag{158}$$

Rearranging:

$$\omega(z) \propto \frac{(\partial a/\partial \Gamma)_z}{\bar{y}(z)}. \tag{159}$$

Since $(\partial a/\partial \Gamma)_z$ is decreasing in income z (from Proposition 2), and $\bar{y}(z)$ is increasing in z, the optimal weight $\omega(z)$ is *strongly* decreasing in income.

Calibration to Optimal Rule: With calibrated portfolio responses:

$$z_{\text{bottom}}: \quad (\partial a/\partial \Gamma) \approx 0.32, \quad \bar{y} = \$25k \quad \Rightarrow \quad \omega_{\text{opt}} \propto 0.32/25 = 0.0128. \tag{160}$$

$$z_{\text{top}}: \quad (\partial a/\partial \Gamma) \approx 0.02, \quad \bar{y} = \$120k \quad \Rightarrow \quad \omega_{\text{opt}} \propto 0.02/120 = 0.000167. \tag{161}$$

The optimal ratio is:

$$\frac{\omega_{\text{bottom}}}{\omega_{\text{top}}} = \frac{0.0128}{0.000167} \approx 77:1. \tag{162}$$

The baseline calibration uses a 60:1 ratio ($\omega \in \{3.0, 0.05\}$), which is close to optimal. A counterfactual with flat weighting ($\omega = 1$ for all z) would be highly inefficient. □

Welfare Implications: Computing aggregate CEV under alternative progressivity rules:

- Optimal 77:1 ratio: Aggregate CEV = 1.12%
- Baseline 60:1 ratio: Aggregate CEV = 1.06%
- Moderate 10:1 ratio: Aggregate CEV = 0.68%
- Flat weighting (1:1): Aggregate CEV = 0.38%.

The baseline policy achieves 95% of the optimal welfare gain, while flat weighting achieves only 36%. This confirms the critical importance of progressivity for aggregate efficiency.

C.5 Equity Premium Channel and Timing Effects

Proposition 5 (Return Timing Multiplier)**.** *When subsidies concentrate portfolio reallocation in periods with elevated risk premia ($\mathbb{E}_t[r^a_{t+1}] > \bar{r}^a$), the cumulative wealth gain from the policy exceeds the mechanical transfer by a factor:*

$$\kappa = 1 + \sum_{s=1}^{T} \frac{(\mathbb{E}_t[r^a_{t+s}] - r^b) \cdot \Delta a_t}{G_t}, \tag{163}$$

where $\Delta a_t = (a_t^{policy} - a_t^{baseline})$ is the induced increase in illiquid holdings and G_t is the fiscal cost.

Proof. Consider a household receiving subsidy Γ at time t during the pandemic. The subsidy increases illiquid asset holdings by:

$$\Delta a_t = \left(\frac{\partial a}{\partial \Gamma}\right) \Gamma \approx 0.30 \cdot \Gamma. \tag{164}$$

This extra illiquid investment earns the equity return sequence $\{r^a_{t+1}, r^a_{t+2}, \ldots\}$. During the pandemic recovery (quarters $t+1$ through $t+20$), equity returns are elevated due to:

- Depressed asset prices at trough (high dividend-price ratios)
- Risk premium spike from heightened uncertainty
- Rebound effect as economy recovers

Calibrating to the 2020-2021 episode:

- Pandemic trough (2020Q2): Expected $\mathbb{E}[r^a] \approx 12\%$ annually
- Recovery phase (2020Q3-2021Q4): $\mathbb{E}[r^a] \approx 10\%$
- Steady state: $r^a = 8\%$.

The cumulative wealth gain from holding the extra Δa over 10 years is:

$$\text{Wealth gain} = \Delta a_t \cdot \prod_{s=1}^{40}(1 + r^a_{t+s}/4) - \Delta a_t \cdot \prod_{s=1}^{40}(1 + r^b/4). \tag{165}$$

With $\Delta a_t = 0.30 \cdot \Gamma$ and elevated returns in the first 8 quarters:

$$\text{Wealth gain} \approx 0.30\Gamma \left[(1.03)^8 \cdot (1.02)^{32} - (1.0125)^{40}\right] \tag{166}$$

$$\approx 0.30\Gamma \left[1.267 \cdot 1.885 - 1.643\right] \tag{167}$$

$$\approx 0.30\Gamma \cdot 0.746 \tag{168}$$

$$\approx 0.22\Gamma. \tag{169}$$

The wealth gain is 22% of the original subsidy, in addition to the mechanical transfer. For bottom-quintile households receiving $\Gamma = \$1,500$, this generates:

- Mechanical transfer: $\$1,500$
- Portfolio reallocation: $\Delta a = \$450$
- 10-year wealth gain: $\$450 \times 0.746 = \$336$.

Total wealth increase: $\$1,500 + \$336 = \$1,836$, a 22% multiplier effect.

The timing multiplier is:

$$\kappa = \frac{\$1,836}{\$1,500} = 1.22. \tag{170}$$

□

Counterfactual: Subsidies in Steady State. If identical subsidies were provided during normal times (no pandemic, $r^a = 8\%$ constant), the wealth gain would be:

$$\text{Wealth gain}_{\text{steady state}} \approx 0.30\Gamma \left[(1.02)^{40} - (1.0125)^{40}\right] \approx 0.30\Gamma \cdot 0.582 \approx 0.17\Gamma. \tag{171}$$

The timing multiplier during the pandemic ($\kappa = 1.22$) exceeds the steady-state multiplier ($\kappa = 1.17$) by 5 percentage points, confirming that elevated risk premia amplify the welfare benefits of the policy.

C.6 Extensive Margin: Participation Decision

Proposition 6 (Entry into Stock Market)**.** *For households with $a_t = 0$ (non-participants), the probability of entering the stock market increases with the subsidy:*

$$\Pr(a_{t+1} > 0 \mid a_t = 0, \Gamma > 0) > \Pr(a_{t+1} > 0 \mid a_t = 0, \Gamma = 0). \tag{172}$$

Proof. We model stock market entry as a discrete choice with fixed cost FC. A household enters if the value of participating exceeds the value of remaining out:

$$V^{\text{participant}}(b, 0, z; \Gamma) - FC > V^{\text{non-participant}}(b, 0, z; \Gamma). \tag{173}$$

The subsidy Γ raises both value functions, but the effect is larger for participants because they can invest the subsidy in high-return risky assets. The value gain from participation is:

$$\Delta V \equiv V^{\text{part}} - V^{\text{non-part}} = \mathbb{E}\left[\sum_{s=0}^{\infty} \beta^s (r^a - r^b) a_s\right]. \tag{174}$$

With the subsidy, initial risky holdings are higher: $a_0^{\text{policy}} = (\partial a/\partial \Gamma)\Gamma > a_0^{\text{baseline}} = 0$. The value gain increases:

$$\frac{\partial \Delta V}{\partial \Gamma} = \mathbb{E}\left[\sum_{s=0}^{\infty} \beta^s (r^a - r^b) \frac{\partial a_s}{\partial \Gamma}\right] > 0. \tag{175}$$

This lowers the effective entry cost:

$$FC_{\text{effective}} = FC - \frac{\partial \Delta V}{\partial \Gamma} \cdot \Gamma. \tag{176}$$

With heterogeneous fixed costs distributed $FC \sim N(\bar{FC}, \sigma_{FC}^2)$, the participation rate is:

$$\Pr(\text{participate}) = \Phi\left(\frac{\Delta V - \bar{FC}}{\sigma_{FC}}\right), \tag{177}$$

where Φ is the standard normal CDF. Since $\partial \Delta V/\partial \Gamma > 0$, participation increases with the subsidy.

Calibrated simulations confirm that among marginal households with moderate liquid wealth, the subsidy raises participation by approximately 6 percentage points, consistent with the model's extensive margin predictions. □

Table 2: Calibrated Parameters

Parameter	Symbol	Value	Source/Target
Preferences			
Risk aversion	σ	1.0	Log utility
Discount factor[a]	β	0.9317	MPC = 0.50 (Parker et al., 2013)
Income Process			
Productivity persistence	ρ_z	0.91	Auclert et al. (2024)
Productivity std. dev.	σ_z	0.92	Auclert et al. (2024)
Tax progressivity	θ	0.181	Heathcote et al. (2017)
Income states	n_e	11	
UI replacement rate	—	0.60	U.S. state UI programs
Sectoral Structure			
Low exposure	η_{low}	0.2	BLS: −5% employment
Medium exposure	η_{med}	0.5	BLS: −12% employment
High exposure	η_{high}	0.8	BLS: −22% employment
Population shares	—	40%, 35%, 25%	BLS industry data
Portfolio Structure			
Calvo frequency[a]	ν	0.091	A/Z following Auclert et al. (2024)
Liquidity wedge	ζ	0.080	8% annual premium
Asset grids	$n_b \times n_a$	50×50	
Production			
Capital share	α	0.294	Auclert et al. (2024)
Depreciation	δ	0.08	Auclert et al. (2024)
Investment elasticity	ε_I	4.0	
Phillips slope	κ_p	0.067	
Government			
Spending/GDP	G/Y	0.20	Auclert et al. (2024)
Debt/GDP	B/Y	0.70	Auclert et al. (2024)
Subsidy Policy			
Baseline rate	$\bar{\tau}$	0.05	5% of avg. income
COVID response	ϕ_{COVID}	0.10	10pp per unit COVID
Eligibility threshold	$\bar{\eta}$	0.15	High and medium exposure hh
Progressive weights	$\omega(z)$	3.0, 2.0, 0.8, 0.05	60:1 bottom:top
Unemployment boost	—	1.5	50% higher for UI
COVID Shock			
Persistence	ρ_{COVID}	0.85	
Initial intensity	COVID_0	0.5	Match 14% U spike
Monetary Policy			
Taylor coefficient	ϕ_π	1.5	Standard value