Corporate Governance and the Coordination Channel of Debt Capacity

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Abstract

We propose a theory in which debt capacity depends on creditor coordination and corporate governance serves as a coordination device. At intermediate leverage, a stable low-default equilibrium coexists with an unstable equilibrium that serves as a critical threshold. If adverse sentiment pushes creditor beliefs past this threshold, no equilibrium is feasible and credit markets break down. Strong governance increases the penalty shareholders pay at default, weakening the feedback from creditor pessimism to actual default probability. This expands the safe zone where outcomes are uniquely determined by fundamentals and reduces vulnerability to coordination failure. The model predicts: (i) governance value peaks at intermediate leverage where fragility is greatest, (ii) better-governed firms have lower credit spread volatility conditional on cash-flow risk, and (iii) credit spreads jump discontinuously when leverage enters the crisis zone. Calibrations show governance effects on debt capacity comparable in magnitude to tax-shield and bankruptcy-cost channels.

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1 Introduction

In September 2017, Toys "R" Us filed for Chapter 11 bankruptcy with $5 billion in debt. The direct cause was not operational failure. The company generated positive EBITDA of $530 million in the preceding year. Instead it was a coordination failure among creditors. Suppliers, fearing collapse, simultaneously tightened credit terms and demanded cash on delivery, increasing effective debt service by an estimated $300–400 million annually and pushing the company into insolvency. A year later, General Electric saw its commercial paper spreads spike from 45 to over 200 basis points following a rating downgrade. This happened despite maintaining $115 billion in revenue and market leadership in aviation and healthcare. Neither firm experienced operational collapse preceding these credit events. Yet both saw creditor sentiment shift abruptly, with self-reinforcing dynamics. The fears about viability triggered actions that made collapse more likely.

These episodes illustrate patterns that are difficult to reconcile with standard capital structure theories. Credit spread movements frequently exceed predictions from structural models based on fundamentals, with residuals exhibiting substantial positive skewness (Schaefer and Strebulaev, 2008). The "excess bond premium" is the component of spreads orthogonal to estimated default probabilities. It spikes during crises and predicts real activity beyond what default risk alone would suggest (Gilchrist and Zakrajšek, 2012). Default clustering exceeds what correlated fundamentals can explain (Giesecke et al., 2011). Governance quality appears systematically related to these dynamics. Better governed firms experience smaller spread increases during crises. The effects are concentrated at intermediate leverage rather than at the high leverage levels where agency theories predict governance should matter most (Favara et al., 2012; Anderson et al., 2004).

This paper proposes that debt capacity depends critically on creditor coordination, and that corporate governance serves as a coordination device. The core mechanism operates through the break-even condition linking creditor beliefs to actual default outcomes. When creditors anticipate high default probability, they demand higher promised repayment. This larger debt burden raises the threshold at which shareholders prefer default over equity injection, increasing actual default probability and potentially confirming the initial pessimism. For intermediate leverage levels, this feedback creates fragility: a stable low-default equilibrium coexists with an unstable critical threshold beyond which no equilibrium is feasible and credit markets break down.

Corporate governance includes many aspects including things like board structure, execu-

tive contracts, exposure to reputational damage, director liability, or loss of private benefits, see Shleifer and Vishny (1997); Hermalin and Weisbach (2017). In our model governance weakens the feedback loop that creates the fragility. It does this by raising the cost shareholders bear upon default. That makes them willing to service debt even when cash flows fall short. Higher default penalties reduce actual default probability for any given debt level and creditor belief, dampening the self-reinforcing dynamics. Sufficiently strong governance expands the range of leverage where outcomes are uniquely determined by fundamentals, shrinking or eliminating the region vulnerable to coordination failure.

In the model there are three debt sustainability zones. Below a lower threshold, the firm operates in a safe zone where a unique stable equilibrium exists. Above an upper threshold, debt becomes unsustainable and creditors simply decline to lend. Between these boundaries lies a crisis zone where the stable equilibrium coexists with a critical belief threshold equilibrium. Better governance shifts both boundaries outward, expanding sustainable debt capacity. When investment exceeds the maximum recovery in default, the break-even condition becomes infeasible for high default beliefs, limiting the crisis zone to exactly two rational expectations equilibria. One is stable and the other one is unstable. The unstable equilibrium marks the boundary beyond which credit markets break down entirely.

The model generates several testable predictions distinguishing it from agency-based explanations (Jensen and Meckling, 1976; Jensen, 1986). First, the marginal value of governance peaks at intermediate leverage where coordination concerns are most severe. It is not at low leverage where free cash flow theories predict governance matters. Nor is it at high leverage where asset substitution theories predict governance matters. Second, better governed firms have lower credit spread volatility conditional on cash flow risk, as they are less exposed to belief driven fluctuations. Third, credit spreads increase discontinuously when leverage enters the crisis zone. This is because of the discrete risk of coordination failure; a force that is absent from smooth tradeoff models.

This paper contributes to several literatures. Within capital structure theory, we show that creditor coordination introduces fragility absent from tradeoff models (Kraus and Litzenberger, 1973; Fischer et al., 1989), agency models (Jensen and Meckling, 1976; Jensen, 1986), and structural credit models (Merton, 1974; Leland, 1994). These frameworks predict smooth relationships between leverage and credit spreads; our coordination mechanism generates discontinuities and regime shifts. The model helps explain why many firms appear underleveraged relative to stan-

dard calculations (Graham, 2000; Strebulaev and Yang, 2013). Firms with moderate governance rationally avoid the crisis zone despite having fundamentals that could support an equilibrium with higher leverage.

Within the governance literature, we identify a channel distinct from traditional agency costs. Prior work emphasizes that governance mitigates conflicts between shareholders and creditors through reduced asset substitution or improved monitoring (Anderson et al., 2004). Our coordination mechanism operates differently. Governance disciplines the strategic default decision, reducing the sensitivity of actual default to creditor beliefs. This predicts governance effects concentrated at intermediate leverage rather than at the low or high leverage levels emphasized by agency theories. This is consistent with Favara et al. (2012).

Within the coordination literature, we complement models of bank runs (Diamond and Dybvig, 1983), currency crises (Obstfeld, 1996), and sovereign debt fragility (Calvo, 1988; Cole and Kehoe, 2000) with a focus on coordination in corporate debt markets. Unlike rollover risk models focusing on maturity structure (He and Xiong, 2012; Brunnermeier and Oehmke, 2013), our mechanism operates with single period debt. This shifts attention from maturity choice to governance quality as the key determinant of fragility. Unlike models with multiple stable equilibria, our framework features fragility to collapse. Crossing the critical threshold leads to market breakdown rather than convergence to a high stress stable state.

The remainder of the paper proceeds as follows. Section 2 presents the model, specifying shareholder default decisions, creditor break-even conditions, and the feedback mechanism that generates crisis zones. Section 3 characterizes equilibria, derives conditions for fragility, and establishes the two-equilibrium structure. Section 4 analyzes crisis zone boundaries and their dependence on governance quality. Section 5 demonstrates that governance monotonically expands debt capacity and derives implications for the marginal value of governance improvements. Section 6 compares the predictions to some standard agency theory predictions. Section 7 calibrates the model using standard parameters, validates the three predictions, and conducts robustness checks. Section 8 provide the empirical implications. Section 9 concludes with empirical implementation strategies and policy implications. Proofs appear in the appendix, with extended technical material and alternative specifications in the online appendix.

2 Model

A risk-neutral firm requires external financing for an investment project. The firm chooses governance quality before approaching credit markets. Creditors, observing governance, set lending terms that reflect rational expectations about default. We characterize the resulting equilibria and show that governance affects not only default probabilities but also whether multiple self-fulfilling equilibria exist.

To fix ideas, figure 2

2.1 Environment

Consider two dates, $t \in \{0, 1\}$, with all agents discounting at a risk-free rate normalized to zero. At date 0, the firm requires investment $I > 0$ to operate a project that generates random cash flow X at date 1. Cash flow is normally distributed, $X \sim N(\mu, \sigma^2)$, with $\mu > 0$ and $\mu > 3\sigma$ so that negative realizations are rare.

The firm finances investment entirely with debt. Competitive creditors lend I at date 0 in exchange for a promised payment D at date 1. The interest rate is $r = D/I - 1$. Debt generates tax benefits: if the firm repays, the interest deduction provides tax shield $\tau(D - I)$ where $\tau \in (0, 1)$ is the corporate tax rate.

Before raising debt, the firm publicly commits to governance quality $\gamma \in [0, 1]$ at cost $C(\gamma)$, where $C(0) = 0$, $C'(\gamma) > 0$, and $C''(\gamma) > 0$. Governance investments include board independence, covenant structures, and monitoring mechanisms. Both governance and investment are observable to creditors before lending decisions.

2.2 Governance and Default Penalties

Governance quality determines a default penalty $\Psi(\gamma)$ that shareholders bear if the firm enters bankruptcy. This penalty captures costs that fall on shareholders personally rather than on the firm's assets: reputation damage with future creditors, acceleration of personal guarantees, loss of relationship-specific rents, and potential director liability. We assume $\Psi(0) = 0$, $\Psi'(\gamma) > 0$, $\Psi''(\gamma) \geq 0$, and $\Psi(\gamma) < \mu$ for all γ.

The assumption that default penalties increase with governance reflects several mechanisms. Stronger covenants create more obligations that trigger upon default. Greater board independence increases director exposure to fiduciary duty claims. More transparent disclosure makes

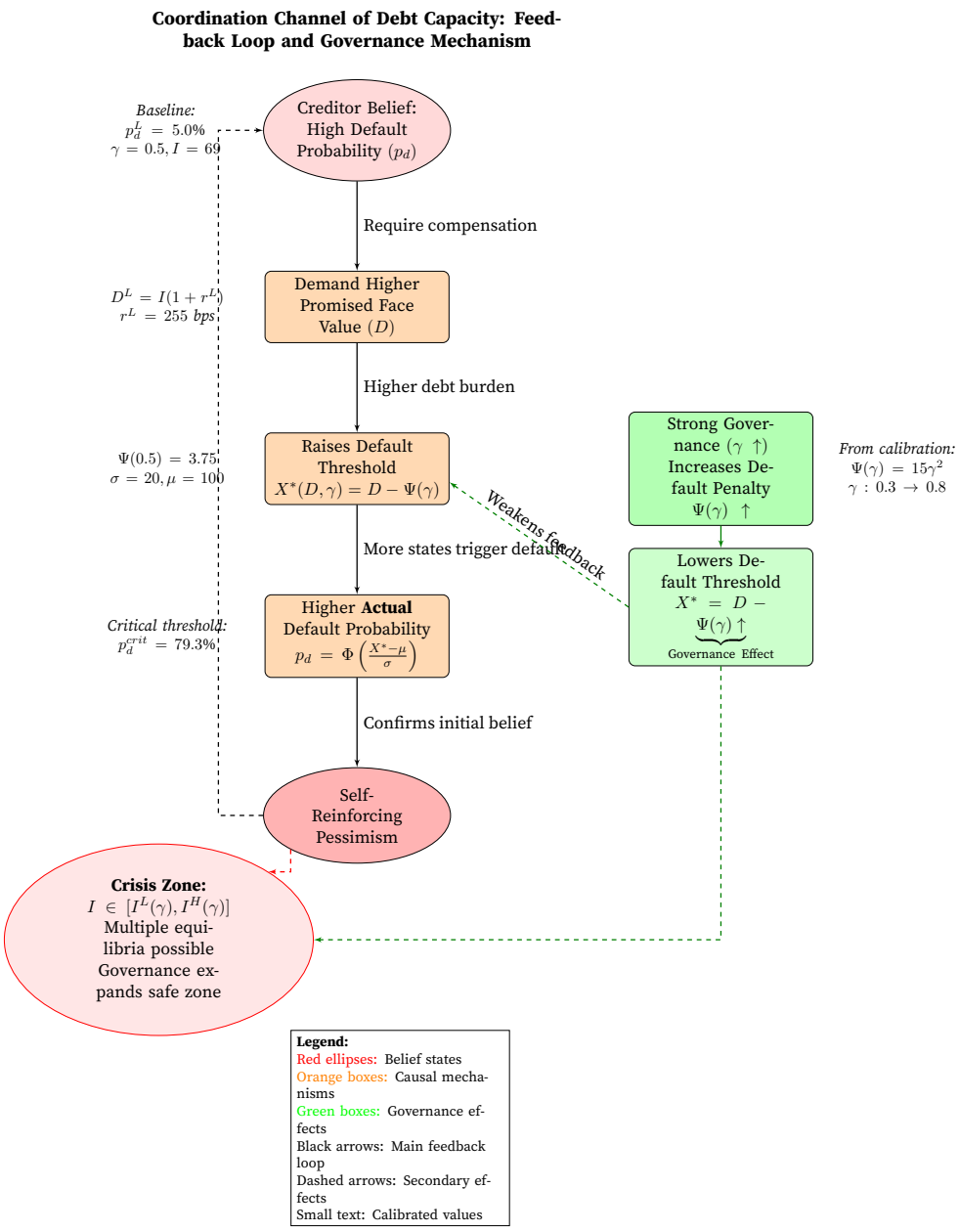


Figure 1: This figure is a show the coordination channel. The self-reinforcing feedback loop between creditor beliefs and default outcomes creates fragility at intermediate leverage. Strong corporate governance (γ) weakens this feedback by increasing default penalties $\Psi(\gamma)$, which lowers the default threshold X^* for any given face value D.

reputation damage from default more severe. The bound $\Psi(\gamma) < \mu$ ensures that default penalties remain smaller than expected cash flows, so the default threshold defined below is positive.

Governance also affects equilibrium selection when multiple equilibria exist. Let $\pi(\gamma)$ denote the probability that creditors coordinate on the favorable equilibrium, with $\pi(0) = 0$, $\pi(1) = 1$, and $\pi'(\gamma) > 0$. This reduced-form specification captures the idea that governance institutions

serve as focal points: strong governance signals commitment to repayment, making optimistic beliefs focal. We treat $\pi(\gamma)$ as a primitive and discuss microfoundations in On-line Appendix OA.5.

2.3 Bankruptcy and Recovery

Default triggers a bankruptcy process that consumes resources. Following the structural credit risk literature, we model bankruptcy costs as proportional to firm value. If the firm defaults with cash flow X, total value available is αX where $\alpha \in (0, 1)$ is the recovery rate. The fraction $(1 - \alpha)$ represents deadweight losses from legal fees, operational disruptions, and asset fire sales. Empirical estimates often suggest direct costs at about 3–5% of firm value and indirect costs at 15–25%, implying α between 0.70 and 0.85.

Creditors hold senior claims. Under Chapter 11 reorganization, courts enforce absolute priority when creditors are impaired. Shareholders get nothing from the bankruptcy estate until creditors are paid in full. For debt levels where creditors are impaired in default creditor recovery equals the entire bankruptcy estate αX.

2.4 The Repayment Decision

At date 1, shareholders observe cash flow X and decide whether to repay creditors or default. This decision depends on whether cash flow covers the debt obligation and, if not, whether shareholders prefer to inject additional equity or accept default.

We assume shareholders can inject equity at fair value to cover cash shortfalls. This assumption is standard in structural models and is most applicable to firms with cash reserves, credit facilities, or equity market access. We discuss this assumption further in Section 5.

Shareholders face three cases depending on realized cash flow.

Case 1: $X \geq D$. Cash flow covers the promised payment. Shareholders repay D, retain $X - D \geq 0$, and the firm continues as a going concern.

Case 2: $D - \Psi(\gamma) \leq X < D$. Cash flow falls short of the promised payment by $D - X > 0$. Shareholders can either inject $D - X$ to make the payment, or default and bear penalty $\Psi(\gamma)$. Since the shortfall $D - X$ is less than the penalty $\Psi(\gamma)$ in this region, shareholders prefer injection. They repay D by combining cash flow X with injected equity $D - X$, and retain $X - D < 0$. The negative residual represents the injection cost.

Case 3: $X < D - \Psi(\gamma)$. The cash shortfall $D - X$ exceeds the default penalty $\Psi(\gamma)$. Shareholders

prefer default over injecting $D - X > \Psi(\gamma)$. The firm enters bankruptcy, creditors receive αX, and shareholders bear penalty $\Psi(\gamma)$.

The default threshold is therefore

$$X^*(D, \gamma) = D - \Psi(\gamma). \tag{1}$$

Shareholders repay if and only if $X \geq X^*$. The threshold has intuitive comparative statics. Higher debt raises the threshold one-for-one ($\partial X^*/\partial D = 1$). Better governance lowers it ($\partial X^*/\partial \gamma = -\Psi'(\gamma) < 0$). Governance disciplines shareholders by increasing the penalty for default. It makes them willing to cover larger cash shortfalls through equity injection.

2.5 Creditor Break-Even

Creditors are competitive and risk-neutral. They lend I at date 0 and receive either the promised payment D (if the firm repays) or recovery αX (if the firm defaults). The break-even condition equates investment to expected repayment.

$$I = (1 - p_d)D + p_d \cdot \mathbb{E}[\alpha X \mid X < X^*], \tag{2}$$

where $p_d = P(X < X^*)$ is the default probability.

For normally distributed cash flows, the default probability is

$$p_d(D, \gamma) = \Phi\left(\frac{D - \Psi(\gamma) - \mu}{\sigma}\right), \tag{3}$$

where $\Phi(\cdot)$ is the standard normal distribution function. Expected recovery conditional on default uses the truncated normal expectation.

$$\mathbb{E}[X \mid X < X^*] = \mu - \sigma\lambda(z^*), \tag{4}$$

where $z^* = (X^* - \mu)/\sigma = (D - \Psi(\gamma) - \mu)/\sigma$ is the standardized threshold and $\lambda(z) = \phi(z)/\Phi(z)$ is the inverse Mills ratio with $\phi(\cdot)$ denoting the standard normal density.[1]

[1]The inverse Mills ratio here is in a left tail form since it is for the default region. Heckman (1979) provides the classical derivation. He gives the symmetric right tail derivation, since his application was about positive selection effects.

Substituting into the break-even condition gives us

$$I = (1 - p_d)D + p_d \cdot \alpha[\mu - \sigma\lambda(z^*)]. \tag{5}$$

This equation implicitly defines the face value D required by creditors for given investment I, governance γ, and believed default probability p_d. In a rational expectations equilibrium, beliefs must be consistent. The believed probability p_d must equal the actual probability $\Phi(z^*)$ generated by the face value D that beliefs induce.

The break-even condition shows the source of potential multiplicity. Suppose creditors become more pessimistic, believing default probability is higher. To compensate for greater expected losses, they demand higher face value D. But higher D raises the default threshold $X^* = D - \Psi(\gamma)$, increasing the actual default probability. If this feedback is sufficiently strong, both optimistic and pessimistic beliefs can be self confirming.

2.6 Firm Value

Expected firm value in a given equilibrium is

$$V_F = \mu - I + (1 - p_d)\tau(D - I) - C(\gamma) - p_d \cdot (1 - \alpha)\mathbb{E}[X \mid X < X^*] - p_d \cdot \Psi(\gamma). \tag{6}$$

The first term is expected cash flow net of investment. The second term is the expected tax shield, which only happens if the firm repays. The third term is governance cost. The fourth term is expected bankruptcy deadweight loss, equal to the probability of default times the expected value destroyed $(1-\alpha)X$ in default states. The fifth term is the expected default penalty borne by shareholders.

When multiple equilibria exist, we denote equilibrium specific values with superscripts L (low default) and H (high default). Expected firm value accounting for equilibrium selection is

$$V_F(\gamma, I) = \pi(\gamma)V_F^L(\gamma, I) + [1 - \pi(\gamma)]V_F^H(\gamma, I). \tag{7}$$

The firm chooses governance γ and investment I to maximize this expected value subject to creditors being willing to lend.

2.7 Feasibility of the Break-Even Condition

The break-even condition (5) does not have a solution for all combinations of (I, p, γ). A feasibility constraint limits the investment levels that creditors can finance when they anticipate high default probability. To see this, consider the break-even condition as $p \to 1$.

$$I = \alpha \mathbb{E}[X \mid X < X^*]. \tag{8}$$

The conditional expectation $\mathbb{E}[X \mid X < X^*]$ is bounded above by μ, achieved in the limit as $X^* \to \infty$ (equivalently, as $D \to \infty$). Therefore, the maximum investment that creditors can finance when they expect certain default is

$$I^* \equiv \alpha\mu. \tag{9}$$

When $I > I^*$, the break-even condition has no solution for p sufficiently close to 1. Creditors cannot recover their investment even if they demand arbitrarily high face value, because default is too likely and recovery too limited. This implies that the equilibrium mapping $T(p)$ is undefined for high p when $I > \alpha\mu$.

This feasibility constraint has important implications for equilibrium multiplicity. Standard coordination models often feature three equilibria: two stable (low-stress and high-stress) separated by an unstable threshold.

In our setting, when $I > \alpha\mu$, the high-stress stable equilibrium cannot exist because the break-even condition is infeasible at high default probabilities. The "critical threshold" in the crisis zone marks the boundary of feasibility. Beyond this point, no rational expectations equilibrium exists. If creditor beliefs cross this threshold, the outcome is not convergence to an alternative stable state. It is instead a breakdown of the credit market. No lending terms can satisfy both creditor break-even and shareholder participation.

In other words, two rational expectations equilibria exist. There is a stable low-default equilibrium and an unstable high-default equilibrium. The unstable equilibrium serves as a critical threshold. If beliefs were beyond this point, they cannot converge to any equilibrium because the break-even condition becomes infeasible. The result is credit market breakdown rather than convergence to a third stable state.

For the calibrated parameters in Section 7, $\alpha = 0.60$ and $\mu = 100$ imply $I^* = 60$. The crisis zone investment levels ($I \in (I^L, I^H) \approx (44, 80)$) exceed this threshold. That ensures that only two

equilibria exist. There is one stable low-default equilibrium, and one unstable critical threshold beyond which lending is infeasible.

2.8 Equilibrium Definition

We focus on rational expectations equilibria where creditor beliefs about default are consistent with actual default given equilibrium face values.

Definition 1. A *rational expectations equilibrium* for given (γ, I) consists of face value D, default probability p_d, and default threshold X^* satisfying the following. 1) Shareholders repay if and only if $X \geq X^* = D - \Psi(\gamma)$. 2) Creditors break even, so equation (5) holds. 3) Beliefs are rational, $p_d = \Phi((D - \Psi(\gamma) - \mu)/\sigma)$.

The consistency requirement is not enough to produce a unique solution. Multiple solutions are possible. For some (γ, I) pairs, both low and high values of (D, p_d) satisfy all three conditions simultaneously. Sections 3 and 4 characterize when multiplicity happens. Section 5 shows how governance affects the multiplicity region.

2.9 Discussion of Assumptions

Several assumptions deserve more explanation.

We assume shareholders can inject equity to cover cash shortfalls. This is standard in structural credit risk models dating to Leland (1994). His idea is that equity holders continuously inject funds as long as the option value of continuation exceeds the injection cost. Our setup is analogous. Shareholders inject when the shortfall is less than the default penalty. The assumption is probably most applicable to firms with liquid balance sheets, committed credit facilities, or public equity market access. For severely constrained firms, the default threshold would be higher (closer to D). That would weaken the governance channel.

We assume absolute priority is strictly enforced. Modern evidence increasingly supports this assumption. Early studies found frequent deviations from absolute priority. Recent work documents that post-1990 bankruptcy reforms substantially increased creditor control. In contemporary samples, shareholders receive positive payouts in relatively few cases where creditors are impaired (Bharath et al., 2010). The alternative is to allowing shareholders to extract value in bankruptcy. That would amplify the main result because it increases the default threshold sensitivity to face value.

We model the equilibrium selection probability $\pi(\gamma)$ as a reduced form rather than deriving it from primitives. This follows the sunspot equilibrium tradition (Cass and Shell, 1983). It allows us to capture the focal point role of governance without specifying the coordination mechanism. The main needed properties are that π increases with γ and it spans $[0, 1]$. Microfoundations based on global games or noisy signaling would generate similar qualitative predictions. I make this explicit in Appendix OA.5.

3 Equilibrium Characterization

This section characterizes the equilibrium structure for given firm choices (γ, I). The rational expectations requirement generates strategic complementarity in creditor beliefs: pessimistic expectations raise required face value, increasing actual default probability and potentially confirming the pessimism. For intermediate investment levels, this feedback mechanism creates fragility, where small shifts in beliefs can trigger large changes in credit market outcomes.

3.1 Equilibria as Fixed Points

For given investment I and governance γ, an equilibrium is a fixed point of a mapping from creditor beliefs to actual default probabilities. Define $T : [0, 1] \to [0, 1]$ by

$$T(p; I, \gamma) = \Phi\left(\frac{D(p) - \Psi(\gamma) - \mu}{\sigma}\right), \tag{10}$$

where $D(p)$ solves the break-even condition (5) for believed default probability p. The mapping takes beliefs as input and returns the actual default probability that would result from the face value creditors demand.

An equilibrium satisfies $p_d = T(p_d; I, \gamma)$. If creditors believe default probability is p, they demand face value $D(p)$ to break even. This face value determines threshold $X^* = D(p) - \Psi(\gamma)$, which generates actual default probability $\Phi((X^* - \mu)/\sigma)$. In equilibrium, beliefs equal outcomes.

The equilibrium structure depends on the slope of the mapping. When $\partial T/\partial p > 1$, beliefs and outcomes reinforce each other: more pessimistic beliefs generate higher actual default probability. When $\partial T/\partial p < 1$, the relationship is stabilizing: pessimistic beliefs generate outcomes less severe than anticipated. The transition between these regions determines the boundaries of the crisis zone.

3.2 The Feedback Mechanism

The slope of the mapping determines the strength of feedback between beliefs and outcomes. Applying the chain rule to (10),

$$\frac{\partial T}{\partial p} = \frac{\phi(z^*)}{\sigma} \cdot \frac{\partial D}{\partial p}, \tag{11}$$

where $z^* = (D - \Psi(\gamma) - \mu)/\sigma$. The first factor shows how sensitively default probability responds to changes in the threshold. The second factor shows how strongly creditor pessimism affects required face value.

To derive $\partial D/\partial p$, implicitly differentiate the break-even condition (5). Using the property $d\lambda/dz = -\lambda(z)[\lambda(z) + z]$ for the inverse Mills ratio,

$$\frac{\partial D}{\partial p} = \frac{D - \alpha[\mu - \sigma\lambda(z^*)]}{1 - p + p\alpha\lambda(z^*)[\lambda(z^*) + z^*]}. \tag{12}$$

The numerator is the repricing gap: the difference between face value D received in repayment states and expected recovery $\alpha[\mu - \sigma\lambda(z^*)]$ in default states. When creditors become more pessimistic, they shift probability weight from high-payment to low-payment states, requiring higher D to compensate.

The denominator contains a stabilizing term. As D rises, the default threshold increases, raising expected recovery conditional on default through the inverse Mills ratio. This partially offsets the need for further face value increases. The term $p\alpha\lambda(z^*)[\lambda(z^*) + z^*]$ is positive for all relevant parameter values, providing stability.

Combining (11) and (12),

$$\frac{\partial T}{\partial p} = \frac{\phi(z^*)}{\sigma} \cdot \frac{D - \alpha[\mu - \sigma\lambda(z^*)]}{1 - p + p\alpha\lambda(z^*)[\lambda(z^*) + z^*]}. \tag{13}$$

Strong feedback ($\partial T/\partial p > 1$) occurs when the repricing gap is large relative to the stabilizing term. This happens with low recovery rates, high cash flow density near the threshold, and intermediate default probabilities.

3.3 Existence and Stability

Proposition 1. *For any (γ, I) satisfying $I < \alpha\mu$, at least one equilibrium exists. The equilibrium is stable if $\partial T/\partial p < 1$ at the fixed point.*

Proof. Define $F(p) = T(p) - p$ on $[0, 1]$. At $p = 0$, creditors expect certain repayment and set D near I. This generates small but positive actual default probability, so $F(0) > 0$. At $p = 1$, creditors expect certain default. The break-even condition becomes $I = \alpha[\mu - \sigma\lambda(z^*)]$. For $I < \alpha\mu$, this equation has a solution with $T(1) < 1$, implying $F(1) < 0$. Continuity of T and the Intermediate Value Theorem establish existence of $p^* \in (0, 1)$ with $F(p^*) = 0$.

Stability follows from standard fixed-point analysis. At an equilibrium p^*, if $\partial T/\partial p|_{p^*} < 1$, then small perturbations in beliefs generate smaller changes in actual outcomes, causing convergence back to p^*. Conversely, if $\partial T/\partial p|_{p^*} > 1$, perturbations are amplified and the equilibrium is unstable. □

For low investment levels, the repricing gap is small and $\partial T/\partial p < 1$ everywhere, ensuring a unique stable equilibrium. As investment increases, the repricing gap grows. When $\max_p \partial T/\partial p$ exceeds unity, the potential for fragility emerges.

3.4 The Fragility Threshold

Investment levels where feedback becomes self-reinforcing mark the boundary between safe and fragile debt structures. Define the maximum feedback strength $S(I, \gamma) = \max_{p\in[0,1]} \partial T/\partial p$. This measures the strongest possible amplification of belief shocks.

Proposition 2. *Define $I^L(\gamma)$ as the investment level where $S(I^L, \gamma) = 1$. For $I < I^L(\gamma)$, a unique stable equilibrium exists. For $I > I^L(\gamma)$, the firm enters a fragile region where self-reinforcing pessimism becomes possible. The threshold satisfies:*

1. *$\partial I^L/\partial\gamma > 0$: better governance delays fragility onset*
2. *$\partial I^L/\partial\alpha > 0$: higher recovery rates delay fragility*
3. *The effect of σ on I^L is ambiguous*

Proof. The threshold $I^L(\gamma)$ is implicitly defined by $S(I^L, \gamma) = 1$. From expression (13), $S(I, \gamma)$ is increasing in I because higher investment requires higher face value, expanding the repricing gap. Also, $S(I, \gamma)$ is decreasing in γ because higher default penalties reduce the default threshold for any face value, weakening the feedback mechanism. The Implicit Function Theorem applied to $S(I^L(\gamma), \gamma) = 1$ yields

$$\frac{dI^L}{d\gamma} = -\frac{\partial S/\partial\gamma}{\partial S/\partial I} = -\frac{(-)}{(+)} > 0. \tag{14}$$

Higher recovery rate α reduces the repricing gap $D - \alpha[\mu - \sigma\lambda(z^*)]$ in the numerator of (13), lowering S for any given I. This requires higher investment to reach the threshold $S = 1$, establishing $\partial I^L/\partial\alpha > 0$.

Volatility σ affects multiple terms in (13). Higher σ reduces $\phi(z^*)/\sigma$ directly but changes z^* and $\lambda(z^*)$ in ways that depend on parameter values. The net effect on I^L is therefore ambiguous. □

The threshold $I^L(\gamma)$ marks the boundary between two regions. Below this threshold, the firm occupies a safe zone where fundamentals uniquely determine outcomes. Above the threshold, the firm enters a crisis zone where sentiment can amplify into self-fulfilling outcomes. Governance shifts this boundary, expanding the range of safely sustainable leverage.

3.5 Structure Within the Crisis Zone

For $I > I^L(\gamma)$, two equilibria typically coexist. The low equilibrium features moderate default probability and is stable: beliefs converge toward it from below. The high equilibrium involves severe financial distress and serves as a critical threshold. If beliefs rise above this threshold, self-reinforcing pessimism drives outcomes toward insolvency.

The equilibria differ systematically. The low equilibrium has face value D^L, default probability p^L, and credit spread $r^L = D^L/I - 1$. At this equilibrium, $\partial T/\partial p < 1$, ensuring stability. Small increases in pessimism generate actual outcomes less severe than the belief change, pulling expectations back down.

The critical threshold occurs at a higher default probability p^{crit}. At this point, $\partial T/\partial p > 1$: beliefs and outcomes reinforce each other. This threshold separates the basin of attraction of the low equilibrium from the region where pessimism becomes self-fulfilling. Firms operating above $I^L(\gamma)$ face the risk that negative shocks to sentiment push beliefs past p^{crit}, triggering a transition to severe distress.

The gap between the low equilibrium and the critical threshold determines the firm's resilience. A low equilibrium at $p^L = 0.04$ with critical threshold at $p^{crit} = 0.30$ provides substantial buffer against sentiment shocks. The same low equilibrium with critical threshold at $p^{crit} = 0.08$ offers little protection. Governance affects both the location of the low equilibrium and the position of the critical threshold.

3.6 Implications for Credit Spreads

The equilibrium structure generates distinctive predictions for credit spread behavior. In the safe zone ($I < I^L(\gamma)$), spreads respond smoothly to changes in fundamentals. A small increase in investment raises default probability slightly, generating a modest spread increase. The relationship is continuous and monotonic.

Crossing into the crisis zone introduces discontinuity. At $I = I^L(\gamma)$, the critical threshold first appears. The firm now faces positive probability of transitioning to severe distress if beliefs deteriorate. Expected credit spreads must account for this risk, creating an upward jump relative to the smooth safe-zone trajectory.

Within the crisis zone, spread behavior depends on which equilibrium prevails. At the low equilibrium, spreads remain moderate despite the elevated investment level. However, the presence of the critical threshold introduces volatility. Sentiment shocks that would have no lasting effect in the safe zone can now trigger persistent changes in credit conditions if they push beliefs past the threshold.

This structure distinguishes our coordination mechanism from standard structural credit models. In models with unique equilibria, spreads vary smoothly with leverage and fundamentals. Here, the same fundamentals can support both moderate and elevated spreads depending on creditor coordination. The transition between these states occurs at the critical threshold, not smoothly across the entire probability distribution.

3.7 Comparative Statics

The feedback mechanism depends on parameters governing bankruptcy costs, cash flow uncertainty, and tax benefits. These parameters determine both the location of the fragility threshold and the structure of equilibria within the crisis zone.

Lower recovery rates amplify the coordination problem. When α falls, the repricing gap $D - \alpha[\mu - \sigma\lambda(z^*)]$ expands, strengthening the feedback from beliefs to outcomes. This shifts $I^L(\gamma)$ downward: firms become fragile at lower leverage. It also widens the gap between the low equilibrium and the critical threshold within the crisis zone, increasing vulnerability to sentiment shocks.

Higher tax benefits extend debt capacity but do not eliminate fragility. Increasing τ raises firm value through larger tax shields, shifting the upper bound of sustainable leverage upward.

However, the feedback mechanism that generates fragility depends on the repricing gap, which is independent of tax rates. The result is that tax benefits expand both the safe zone and the crisis zone proportionally, leaving the relative size of each region roughly unchanged.

Cash flow volatility has ambiguous effects. Higher σ increases the probability mass near any given threshold, strengthening the response of default probability to threshold changes. This effect tends to reduce $I^L(\gamma)$, pushing firms into fragility at lower leverage. However, higher σ also affects the inverse Mills ratio and expected recovery in ways that depend on the specific parameter configuration. The net effect varies across firms.

The default penalty $\Psi(\gamma)$ plays a central role. Higher penalties reduce the default threshold $X^* = D - \Psi(\gamma)$ for any face value, lowering actual default probability and weakening feedback. This unambiguously increases $I^L(\gamma)$, expanding the safe zone. Within the crisis zone, higher penalties shift both the low equilibrium and the critical threshold downward, improving credit market outcomes at any given investment level.

4 Crisis Zones and Debt Fragility

This section characterizes the investment regions where debt structures become fragile to belief-driven outcomes. For intermediate leverage levels, self-reinforcing feedback between creditor expectations and actual default probability creates vulnerability to sentiment shocks. We define crisis zone boundaries, establish their dependence on governance quality, and demonstrate that governance monotonically expands safe debt capacity.

4.1 Crisis Zone Boundaries

Investment level determines whether the firm operates in a region where outcomes are uniquely determined by fundamentals or in a region where belief dynamics can amplify shocks. For very low investment, required debt is small, default is unlikely regardless of creditor sentiment, and feedback is weak. For very high investment, the firm cannot sustain debt service even under optimistic beliefs, and either severe distress or no lending occurs. Between these extremes lies a crisis zone where coordination concerns become relevant.

Proposition 3 (Debt Sustainability Zones)**.** *For given governance quality γ, there exist thresholds $0 < I^L(\gamma) < I^H(\gamma)$ defining three regions:*

1. ***Safe Zone*** *($I \leq I^L(\gamma)$). The feedback mechanism is weak: $S(I,\gamma) < 1$. A unique stable equilibrium exists with low default probability. Small shifts in creditor sentiment generate proportionally smaller changes in actual outcomes, ensuring stability.*

2. ***Crisis Zone*** *($I^L(\gamma) < I < I^H(\gamma)$). The feedback mechanism becomes strong enough to sustain self-reinforcing dynamics. A stable low-default equilibrium coexists with a critical belief threshold. If sentiment deteriorates past this threshold, self-fulfilling pessimism can drive the firm toward severe distress.*

3. ***Insolvency Zone*** *($I \geq I^H(\gamma)$). Expected firm value turns negative even at the favorable equilibrium. Rational creditors decline to lend or demand terms the firm cannot profitably accept.*

Proof. The proof proceeds in three steps, establishing the existence and properties of each boundary.

Step 1: Lower boundary $I^L(\gamma)$. Define $S(I,\gamma) = \max_{p\in[0,1]} \partial T/\partial p$ as the maximum feedback strength. This function is continuous in I and satisfies $S(0,\gamma) = 0$ (no debt implies no feedback) and $\lim_{I\to\alpha\mu} S(I,\gamma) > 1$ (high leverage generates strong feedback). By the Intermediate Value Theorem, there exists unique $I^L(\gamma)$ satisfying $S(I^L,\gamma) = 1$.

For $I < I^L(\gamma)$, we have $\partial T/\partial p < 1$ everywhere on $[0,1]$, making T a contraction mapping. The Contraction Mapping Theorem guarantees a unique fixed point. For $I > I^L(\gamma)$, the maximum slope exceeds unity, indicating regions where belief-outcome feedback becomes self-reinforcing.

Step 2: Upper boundary $I^H(\gamma)$. Expected firm value accounting for equilibrium outcomes is

$$V_F(I,\gamma) = \pi(\gamma)V_F^L(I,\gamma) + [1-\pi(\gamma)]V_F^{crisis}(I,\gamma), \tag{15}$$

where V_F^L denotes value at the low equilibrium and V_F^{crisis} denotes expected value accounting for the risk of transitioning past the critical threshold. At $I = I^L(\gamma)$, the low equilibrium yields positive value. As I increases, both equilibrium values decline due to higher debt service costs and increased distress probability. By continuity, there exists $I^H(\gamma)$ where $V_F(I^H,\gamma) = 0$.

Step 3: Ordering. Since $V_F(I^L,\gamma) > 0$, $V_F(I^H,\gamma) = 0$, and $\partial V_F/\partial I < 0$, we have $I^L(\gamma) < I^H(\gamma)$. Complete technical details appear in Online Appendix OA.3. □

The lower boundary $I^L(\gamma)$ marks the transition from unique determination by fundamentals to potential fragility. Below this threshold, strategic complementarity is too weak to generate self-reinforcing dynamics. If creditors become pessimistic, the resulting increase in face value does

not raise actual default probability sufficiently to confirm the pessimism. Above this threshold, feedback between beliefs and outcomes strengthens enough that sentiment shocks can trigger persistent changes in credit conditions.

The upper boundary $I^H(\gamma)$ represents the limit of sustainable debt. At this point, even the probability-weighted average of favorable and unfavorable outcomes generates zero expected value to shareholders. Beyond this threshold, rational creditors will not lend at any terms the firm can profitably accept.

4.2 Economic Interpretation

The crisis zone captures a coordination problem in debt markets distinct from traditional leverage constraints. Within this region, fundamentals no longer uniquely determine outcomes. Instead, the firm's fate depends partly on which equilibrium creditors coordinate on.

Favorable outcome. Creditors coordinate on beliefs near the low equilibrium. Face value D^L remains moderate, generating default probability p^L consistent with optimistic initial beliefs. Credit spreads $r^L = D^L/I - 1$ reflect fundamental risk but not coordination risk. The firm successfully services debt in most states of the world.

Unfavorable outcome. A negative shock pushes creditor beliefs past the critical threshold p^{crit}. At this point, self-reinforcing feedback takes over. Pessimistic beliefs demand high face value, which generates high actual default probability, confirming the pessimism. Credit spreads spike to levels associated with severe distress despite unchanged fundamentals.

The coordination problem arises from feedback between creditor beliefs and actual default outcomes operating through the break-even condition. Under absolute priority, changes in face value affect the default threshold one-for-one: $\partial X^*/\partial D = 1$. Each dollar increase in required face value raises the threshold by exactly one dollar, expanding the range of cash flow realizations triggering default. This direct pass-through creates the potential for self-fulfilling dynamics when the repricing gap is sufficiently large.

Strategic complementarity among creditors emerges because each creditor's optimal terms depend on other creditors' beliefs. If creditor i expects others to be pessimistic, creditor i anticipates high face value and high default probability. To break even, creditor i must also demand high face value, confirming the collective pessimism. Conversely, if creditor i expects optimism, the anticipated low face value and low default probability allow creditor i to offer favorable terms. Individual rationality supports multiple collective outcomes.

4.3 Equilibrium Structure Within the Crisis Zone

Within the crisis zone $I \in (I^L(\gamma), I^H(\gamma))$, two rational expectations equilibria exist. There is a stable low-default equilibrium and an unstable high-default equilibrium.[2] The unstable equilibrium serves as a critical threshold separating the basin of attraction of the stable equilibrium from a region where no equilibrium is feasible.

Characterization of the two equilibria. The stable equilibrium occurs at default probability p^L where $\partial T/\partial p < 1$. Small perturbations in beliefs generate smaller changes in actual default probability, ensuring convergence back to p^L. For the calibrated parameters in Section 7, p^L ranges from 2% to 20% depending on leverage, with credit spreads in the investment-grade range.

The unstable equilibrium occurs at default probability p^{crit} where $\partial T/\partial p > 1$. At this point, beliefs and outcomes reinforce each other. A small increase in believed default probability generates a larger increase in actual default probability. This equilibrium is a rational expectations equilibrium. It satisfies all three conditions in the definition. But it is not a stable rest point. Any perturbation pushes beliefs away from p^{crit} rather than back toward it.

Why only two equilibria? Standard coordination models often feature three equilibria: two stable equilibria (one with low stress, one with high stress) separated by an unstable threshold. In such models, adverse shocks that push beliefs past the threshold lead to convergence to the high stress stable equilibrium rather than to collapse.

Our model generates only two equilibria because of the feasibility constraint identified in Section 3.6. When investment exceeds $I^* = \alpha\mu$, the break-even condition has no solution for default probabilities above a certain level. The equilibrium mapping $T(p)$ is undefined for p sufficiently close to 1. This precludes the existence of a third equilibrium at high default probability: there is simply no (D, p) pair that satisfies both creditor break-even and the rational expectations requirement for p above the critical threshold.

The unstable equilibrium p^{crit} occurs near this feasibility boundary. For $p < p^{crit}$, beliefs

[2] Standard coordination models often feature three equilibria: two stable equilibria separated by an unstable threshold, allowing for both a "good" and "bad" stable outcome. Our model differs because creditors face a participation constraint. When default is nearly certain, expected recovery is at most $\alpha\mu$. That is the recovery rate times average cash flow. If the firm requires investment $I > \alpha\mu$, creditors cannot break even when they expect near certain default, regardless of the interest rate charged. This rules out any equilibrium at high default probabilities. The unstable threshold in our model is therefore not a boundary between two stable outcomes. It is the boundary between the stable equilibrium and a region where no lending terms are feasible. Crossing the threshold triggers credit market breakdown rather than convergence to a high-stress stable state.

converge to the stable low-default equilibrium. For $p > p^{crit}$, beliefs would need to converge to a higher fixed point, but no such fixed point exists—the break-even condition is infeasible. The result is credit market breakdown: no lending terms can simultaneously satisfy creditor break-even and generate beliefs consistent with those terms.

Economic implications. The two-equilibrium structure implies that coordination problems show up as fragility to collapse rather than as multiplicity of sustainable outcomes. Firms in the crisis zone either operate at the stable equilibrium with moderate credit spreads, or, if adverse shocks push beliefs past p^{crit}, face a complete breakdown of credit market access. There is no intermediate stable state with elevated but sustainable spreads.

This structure sharpens the model's empirical predictions. First, credit spread distributions should be concentrated at levels consistent with the stable equilibrium. There is also a mass point at distressed levels for firms that experience threshold-crossing events, but there is little mass in between. Second, the marginal value of governance peaks sharply near crisis zone boundaries where the buffer between p^L and p^{crit} is narrowest and collapse risk is highest. Third, spread increases at the lower crisis zone boundary $I^L(\gamma)$ reflect the discrete risk of market breakdown, not merely a shift to an alternative equilibrium with higher but sustainable spreads.

4.4 Comparative Statics on Crisis Zone Properties

The location and width of the crisis zone depend on parameters governing bankruptcy costs, cash flow uncertainty, and tax benefits. Understanding these relationships clarifies when coordination problems are most severe.

Proposition 4 (Determinants of Crisis Zone Properties)**.** *The crisis zone boundaries satisfy:*

1. ***Recovery rate*** α*. Both boundaries $I^L(\gamma)$ and $I^H(\gamma)$ are increasing in α. Higher recovery rates reduce the repricing gap, weakening strategic complementarity and shifting the crisis zone toward higher leverage.*

2. ***Tax rate*** τ*. The upper boundary $I^H(\gamma)$ is increasing in τ; the effect on $I^L(\gamma)$ is small. Higher tax benefits allow firms to sustain greater leverage before expected value turns negative.*

3. ***Cash flow volatility*** σ*. Effects on both boundaries are ambiguous. Higher volatility increases default probability for given debt levels but also affects the density of cash flows near the threshold and the sensitivity of the break-even condition.*

4. ***Default penalty*** $\Psi(\gamma)$. *Both boundaries are increasing in* $\Psi(\gamma)$. *Higher penalties reduce the default threshold for any face value, lowering default probability and weakening the feedback mechanism.*

Proof. Part 1. Higher α increases expected recovery $\alpha[\mu - \sigma\lambda(z^*)]$, reducing the repricing gap $D - \alpha[\mu - \sigma\lambda(z^*)]$ in the numerator of $\partial D/\partial p$. This reduces the sensitivity of face value to beliefs, lowering the slope $\partial T/\partial p$ for any (I, p). Higher investment is therefore required to reach the condition $\max_p \partial T/\partial p = 1$, establishing $\partial I^L/\partial\alpha > 0$. Higher α also reduces expected distress costs, increasing firm value and allowing higher investment before the participation constraint binds, implying $\partial I^H/\partial\alpha > 0$.

Part 2. The tax rate τ does not appear in the slope expression (13), so its effect on $I^L(\gamma)$ operates only indirectly through equilibrium face values. The primary effect is on $I^H(\gamma)$. Higher τ increases the tax shield $(1 - p_d)\tau(D - I)$, raising firm value and allowing higher sustainable investment.

Part 3. Volatility σ affects several terms in the slope expression. Higher σ reduces $\phi(z^*)/\sigma$ directly. It also changes z^* and $\lambda(z^*)$ in ways that depend on the specific parameter configuration. The net effect on boundaries varies across parameter values.

Part 4. Higher $\Psi(\gamma)$ lowers the default threshold $X^* = D - \Psi(\gamma)$ for any D, reducing default probability. This reduces the maximum slope $S(I, \gamma)$, requiring higher I to reach the fragility condition. Higher Ψ also reduces expected distress costs, increasing firm value and raising $I^H(\gamma)$. □

The comparative statics clarify when coordination problems are most severe. Crisis zones are wider and occur at lower leverage when recovery rates are low, because low recovery magnifies the repricing effect. Creditors who fear default demand much higher face value when they expect to recover little in bankruptcy. Crisis zones shift to higher leverage when default penalties are high, because strong governance disciplines shareholders, reduces actual default for any given debt level, and weakens the feedback mechanism.

4.5 Credit Spread Discontinuities

The crisis zone structure generates predictions for credit spread behavior that distinguish the coordination mechanism from smooth tradeoff models. As firms increase leverage, spreads respond differently depending on whether they remain in the safe zone or enter the crisis zone.

Corollary 1 (Spread Behavior at Boundaries)**.** *Expected credit spreads exhibit a discrete increase at the lower crisis zone boundary. For I slightly below $I^L(\gamma)$, spreads reflect only fundamental default risk. For I slightly above $I^L(\gamma)$, spreads must additionally compensate for the risk of sentiment-driven transitions past the critical threshold.*

Proof. For $I < I^L(\gamma)$, Proposition 3 establishes that a unique equilibrium exists with spread $r^L(I) = D^L(I)/I - 1$. The face value $D^L(I)$ is implicitly defined by the break-even condition and is continuous in I by the Implicit Function Theorem.

For I slightly above $I^L(\gamma)$, the low equilibrium persists with the same properties. However, the critical threshold now exists. Expected spread must account for the risk that sentiment shocks push beliefs past this threshold:

$$\mathbb{E}[r(I)] = \pi(\gamma) r^L(I) + [1 - \pi(\gamma)] r^{crisis}(I), \tag{16}$$

where r^{crisis} reflects the elevated spreads associated with approaching the critical threshold. Since $r^{crisis}(I) > r^L(I)$ and $1 - \pi(\gamma) > 0$ for $\gamma < 1$, expected spread increases discretely at $I^L(\gamma)$. □

The discontinuity magnitude depends on governance quality through two channels. First, better governance increases $\pi(\gamma)$, reducing probability weight on unfavorable outcomes. Second, better governance reduces spreads in both favorable and crisis states by lowering default probabilities through higher penalties $\Psi(\gamma)$. Both channels imply that spread jumps are larger for firms with weaker governance.

This prediction distinguishes the coordination model from standard continuous tradeoff theory, which predicts smooth relationships between leverage and credit spreads. In the coordination framework, firms with identical fundamentals can experience sharply different borrowing costs depending on whether they operate inside or outside the crisis zone. The boundary location depends on governance quality, creating cross-sectional variation in the leverage-spread relationship.

Within the crisis zone, spread volatility increases even controlling for fundamental cash flow risk. At the low equilibrium, spreads remain moderate. However, sentiment shocks that would have minimal effect in the safe zone can now trigger large spread changes if they push beliefs toward the critical threshold. Better-governed firms exhibit lower spread volatility because stronger default penalties reduce both the probability of adverse transitions and the severity of outcomes conditional on transition.

The crisis zone framework also implies that small changes in firm characteristics can generate large changes in borrowing costs if they push the firm across a zone boundary. A modest increase in leverage that moves a firm from $I = I^L(\gamma) - \varepsilon$ to $I = I^L(\gamma) + \varepsilon$ discretely increases expected spreads. This sensitivity to boundary crossing helps explain situations where seemingly small changes in firm circumstances produced dramatic repricing of credit risk, such as the cases of General Electric and Toys "R" Us discussed in the Introduction.

5 Governance and Debt Capacity

The previous section shows that multiple equilibria happen at intermediate investment levels. Both optimistic and pessimistic creditor beliefs are self fulfilling in the crisis zone. This section shows that governance quality systematically affects the location of crisis zone boundaries, the probability of equilibrium selection, and the marginal value of governance improvements. The main result is that better governance monotonically expands sustainable debt capacity through two distinct channels.

5.1 Governance Monotonically Expands Sustainable Debt

The first we show that both crisis zone boundaries shift outward as governance improves.

Proposition 5 (Governance Expansion of Debt Capacity)**.** *Both crisis zone boundaries are strictly increasing in governance quality.*

$$\frac{dI^L}{d\gamma} > 0 \quad \textit{and} \quad \frac{dI^H}{d\gamma} > 0 \tag{17}$$

Consequently, the safe zone $[0, I^L(\gamma)]$ *expands as governance improves.*

Proof. **Part 1: Effect on lower boundary.** The lower boundary $I^L(\gamma)$ is defined by the condition $S(I^L, \gamma) = 1$, where $S(I, \gamma) = \max_p \partial T/\partial p$ is the maximum slope of the equilibrium mapping.

Better governance increases strategic default costs: $\Psi'(\gamma) > 0$. This reduces the default threshold for any given face value.

$$X^*(D, \gamma) = D - \Psi(\gamma) \implies \frac{\partial X^*}{\partial \gamma} = -\Psi'(\gamma) < 0 \tag{18}$$

A lower default threshold reduces actual default probability for any creditor belief p, which

reduces the slope of the mapping $T(p)$. To see this formally, recall from equation 13 that

$$\frac{\partial T}{\partial p} = \frac{\phi(z^*)}{\sigma} \cdot \frac{D - \alpha[\mu - \sigma\lambda(z^*)]}{1 - p + p\alpha\lambda(z^*)[\lambda(z^*) + z^*]} \tag{19}$$

When γ increases, $\Psi(\gamma)$ increases, which decreases $z^* = (D - \Psi - \mu)/\sigma$. This affects the slope through multiple channels: the density $\phi(z^*)$, the repricing gap in the numerator, and the stabilizing term in the denominator. The net effect is that better governance reduces the maximum slope

$$\frac{\partial S}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left[\max_p \frac{\partial T}{\partial p}\right] < 0 \tag{20}$$

The detailed derivation in Online Appendix OA.3 shows this inequality holds for all relevant parameter values.

Since $S(I, \gamma)$ is increasing in I and decreasing in γ, the Implicit Function Theorem applied to $S(I^L(\gamma), \gamma) = 1$ means

$$\frac{dI^L}{d\gamma} = -\frac{\partial S/\partial \gamma}{\partial S/\partial I} = -\frac{(-)}{(+)} > 0 \tag{21}$$

Part 2: Effect on upper boundary. The upper boundary $I^H(\gamma)$ is defined by the zero-value condition

$$V_F(I^H, \gamma) = \pi(\gamma)V_F(I^H, \gamma \mid L) + [1 - \pi(\gamma)]V_F(I^H, \gamma \mid H) = 0 \tag{22}$$

Better governance affects expected firm value through two channels.

Channel 1: Equilibrium selection. Higher γ increases $\pi(\gamma)$, shifting probability weight toward the good equilibrium. Since $V_F(\cdot \mid L) > V_F(\cdot \mid H)$ (the good equilibrium has lower distress costs), this increases expected firm value.

Channel 2: Direct effects. Higher γ reduces default probability in both equilibria by lowering the default threshold. This reduces expected distress costs $p_d \cdot DC$, increasing firm value in each equilibrium.

Both channels imply $\partial V_F/\partial \gamma > 0$ at any interior point. Combined with $\partial V_F/\partial I < 0$ (higher investment reduces value at the margin), the Implicit Function Theorem gives us

$$\frac{dI^H}{d\gamma} = -\frac{\partial V_F/\partial \gamma}{\partial V_F/\partial I} = -\frac{(+)}{(-)} > 0 \tag{23}$$

□

The proposition shows that governance unambiguously expands sustainable debt capacity. A

firm that improves its governance can support higher leverage without entering the crisis zone. It can sustain even higher leverage before expected firm value turns negative. The safe zone expands as governance improves.

The economic intuition is direct. Better governance raises the cost of strategic default $\Psi(\gamma)$. It disciplines shareholders and makes them willing to repay creditors at lower cash flow realizations. This reduces actual default probability for any given debt level and creditor belief. With lower default risk, the feedback between pessimistic beliefs and actual default is weakened. That raises the investment threshold at which multiple equilibria become possible. Simultaneously, lower default risk increases firm value, allowing higher investment before the participation constraint binds.

5.2 Marginal Value of Governance

The marginal value of governance varies systematically with the firm's position relative to crisis zone boundaries. Governance improvements are most valuable for firms near the boundaries, where small changes can trigger discrete shifts in equilibrium outcomes.

Proposition 6 (Heterogeneous Returns to Governance)**.** *The marginal value of governance, $\partial V_F/\partial\gamma$, has the following properties.*

1. *In the safe zone ($I < I^L(\gamma)$), the marginal value reflects only direct effects on default probability and governance costs.*
$$\left.\frac{\partial V_F}{\partial \gamma}\right|_{safe} = -C'(\gamma) - \frac{\partial p_d}{\partial \gamma} DC - p_d \frac{\partial DC}{\partial \gamma} \tag{24}$$

2. *In the crisis zone ($I^L(\gamma) < I < I^H(\gamma)$), the marginal value includes an additional equilibrium selection term.*
$$\left.\frac{\partial V_F}{\partial \gamma}\right|_{crisis} = \left.\frac{\partial V_F}{\partial \gamma}\right|_{safe} + \pi'(\gamma)[V_F(\cdot \mid L) - V_F(\cdot \mid H)] \tag{25}$$

3. *The equilibrium selection term $\pi'(\gamma)[V_F(\cdot \mid L) - V_F(\cdot \mid H)]$ is strictly positive and largest for firms near crisis zone boundaries.*

Proof. **Part 1.** In the safe zone, only one equilibrium exists, so $\pi(\gamma) = 1$ and the equilibrium selection term vanishes. Expected firm value is:

$$V_F = \mu - I + (1 - p_d)\tau(D - I) - C(\gamma) - p_d \cdot DC \tag{26}$$

Differentiating with respect to γ:

$$\frac{\partial V_F}{\partial \gamma} = -C'(\gamma) + \tau(D - I)\left(-\frac{\partial p_d}{\partial \gamma}\right) + (1 - p_d)\tau\frac{\partial D}{\partial \gamma} - \frac{\partial p_d}{\partial \gamma}DC - p_d\frac{\partial DC}{\partial \gamma} \tag{27}$$

Since $\partial p_d/\partial \gamma < 0$ (better governance reduces default probability), the tax shield effect is positive. The term $\partial D/\partial \gamma$ is typically small in the safe zone where default risk is low. Grouping terms produces the expression in Part 1.

Part 2. In the crisis zone, expected firm value accounts for equilibrium selection.

$$V_F = \pi(\gamma)V_F(\cdot \mid L) + [1 - \pi(\gamma)]V_F(\cdot \mid H) \tag{28}$$

Differentiating

$$\begin{aligned}\frac{\partial V_F}{\partial \gamma} &= \pi'(\gamma)[V_F(\cdot \mid L) - V_F(\cdot \mid H)] \\ &\quad + \pi(\gamma)\frac{\partial V_F}{\partial \gamma}\bigg|_L + [1 - \pi(\gamma)]\frac{\partial V_F}{\partial \gamma}\bigg|_H\end{aligned} \tag{29}$$

The second line is a weighted average of direct effects within each equilibrium, which corresponds to the safe-zone marginal value (applied separately to each equilibrium). The first line is the additional equilibrium selection term.

Part 3. The equilibrium selection term is positive because $\pi'(\gamma) > 0$ (better governance increases probability of good equilibrium) and $V_F(\cdot \mid L) > V_F(\cdot \mid H)$ (good equilibrium has higher value due to lower distress costs).

The term is largest near boundaries because the value gap $V_F(\cdot \mid L) - V_F(\cdot \mid H)$ is largest there. Near the lower boundary $I^L(\gamma)$, the bad equilibrium has just become possible. Near the upper boundary $I^H(\gamma)$, the participation constraint is about to bind, making the value difference between equilibria critical for firm survival. In the interior of the crisis zone both equilibria are well-established. The value gap can be substantial and it is relatively stable. □

The proposition means that governance improvements generate the largest returns for firms near crisis zone boundaries. For a firm just inside the crisis zone, a marginal improvement in governance may be sufficient to shift the boundary past the firm's investment level, eliminating the bad equilibrium entirely. This discrete change in the set of possible outcomes generates a first-order value increase beyond the continuous direct effects.

Conversely, for firms deep in the safe zone (well below $I^L(\gamma)$), governance improvements generate only modest benefits through reduced default probability. For firms deep in the insolvency zone (well above $I^H(\gamma)$), governance improvements cannot restore viability. The marginal value of governance is highest in an intermediate region where coordination concerns are most relevant.

This heterogeneity has implications for observed governance choices. Firms with leverage near crisis zone boundaries have strong incentives to invest in governance. Firms with very low or very high leverage have weaker incentives. If governance investments are costly, we should expect governance quality to be highest among firms with intermediate leverage. These are precisely the firms most exposed to coordination risk.

5.3 Joint Optimization of Governance and Investment

When firms choose both governance and investment optimally, the two choices interact. This subsection characterizes the joint optimum and gives conditions under which governance and leverage are strategic complements.

Proposition 7 (Optimal Joint Policy)**.** *Suppose $V_F(\gamma, I)$ is twice continuously differentiable with $\partial^2 V_F/\partial I^2 < 0$ and $\partial^2 V_F/\partial \gamma^2 < 0$ at any interior optimum. Then*

1. *At an interior optimum (γ^*, I^*), the optimal governance choice satisfies*

$$\pi'(\gamma^*)[V_F^L - V_F^H] = C'(\gamma^*) + p_d \frac{\partial DC}{\partial \gamma} - \tau(D - I)\left(-\frac{\partial p_d}{\partial \gamma}\right) \tag{30}$$

   The left side is the marginal benefit from improved equilibrium selection. The right side is the net marginal cost including governance expenditure, changes in distress costs, and tax shield effects.

2. *Governance and investment are strategic complements near crisis zone boundaries.*

$$\frac{\partial^2 V_F}{\partial \gamma \partial I} > 0 \quad \textit{for } I \textit{ near } I^L(\gamma) \textit{ or } I^H(\gamma) \tag{31}$$

3. *Under complementarity, optimal governance γ^* and optimal investment I^* are positively correlated across firms with different fundamentals.*

Proof. **Part 1.** At an interior optimum, the first-order condition $\partial V_F/\partial \gamma = 0$ must hold. From

Proposition 6,

$$\pi'(\gamma)[V_F^L - V_F^H] + \text{direct effects} = 0 \tag{32}$$

Rearranging with direct effects on the right side gives equation (30). The interpretation is that firms invest in governance until the marginal benefit from equilibrium selection (higher probability of the good outcome) equals the marginal cost (governance expenditure net of tax and distress cost effects).

Part 2. The cross-partial derivative is

$$\frac{\partial^2 V_F}{\partial \gamma \partial I} = \pi'(\gamma)\left[\frac{\partial V_F^L}{\partial I} - \frac{\partial V_F^H}{\partial I}\right] + \text{terms from direct effects} \tag{33}$$

The key term is $\pi'(\gamma)[\partial V_F^L/\partial I - \partial V_F^H/\partial I]$. Since the good equilibrium has lower credit spreads, the marginal cost of investment is lower in the good equilibrium,

$$\frac{\partial V_F^L}{\partial I} > \frac{\partial V_F^H}{\partial I} \tag{34}$$

Both derivatives are negative with investment reducing value at the margin. But the magnitude is smaller in the good equilibrium. Combined with $\pi'(\gamma) > 0$, the equilibrium selection component of the cross-partial is positive.

Near crisis zone boundaries, the equilibrium selection component dominates because the gap between equilibria is largest. In the interior of the safe zone the firm is far from boundaries. Only one equilibrium exists and the cross-partial reflects only direct effects. These are typically small. Therefore $\partial^2 V_F/\partial\gamma\partial I > 0$ near boundaries.

Part 3. Consider two firms with fundamentals inducing different optimal policies. With complementarity ($\partial^2 V_F/\partial\gamma\partial I > 0$), a firm that optimally chooses higher I^* also optimally chooses higher γ^*. This follows from the comparative statics of supermodular optimization, see Topkis (1998); Vives (2005). When the cross-partial is positive, the optimal choices move together as parameters shift.

Intuitively, a firm planning high leverage has strong incentives to invest in governance because good governance reduces the probability of ending up in the bad equilibrium, where high leverage is most costly. Conversely, a firm with strong governance can afford higher leverage because the crisis zone boundary is shifted outward. □

The complementarity result has important empirical implications. It predicts a positive correlation between governance quality and leverage in the cross-section of firms. This should be particularly true for firms with intermediate leverage near crisis zone boundaries. This prediction is consistent with empirical evidence documenting positive associations between governance measures and leverage ratios (Jensen, 1986; Berger et al., 1997).

The model also predicts that this correlation should be strongest for firms in the crisis zone. It should be weakest for firms with very low leverage (deep in the safe zone) or very high leverage (in the insolvency zone). Empirical tests that condition on leverage position relative to crisis zone boundaries may find stronger governance leverage relationships than unconditional tests.

The joint optimization results assume interior solutions. Corner solutions can arise when governance costs are very high ($\gamma^* = 0$) or when coordination benefits are very large ($\gamma^* = 1$). The complementarity result applies most strongly to firms with interior optimal governance choices and investment levels near crisis zone boundaries.

The analysis in this section establishes governance as a key determinant of debt capacity through the coordination channel. Better governance expands the safe zone, increases the probability of good equilibrium outcomes in the crisis zone, and is most valuable for firms facing significant coordination risk. These results complement traditional agency-based explanations of the governance-leverage relationship by identifying a distinct mechanism operating through equilibrium selection.

6 Predictions and Comparison with Alternative Theories

Table 1 summarizes the testable predictions of our coordination model alongside those of two leading agency theories. These distinct predictions allow empirical researchers to differentiate between alternative explanations for governance effects on debt capacity. Asset substitution is due to Jensen and Meckling (1976). Free cash flow follows Jensen (1986) and Berger et al. (1997).

7 Calibration

The theoretical analysis in Sections 3–5 establishes conditions under which coordination problems emerge, characterizes crisis zone boundaries, and derives comparative statics on governance effects. This section demonstrates that the model generates economically significant pre-

Table 1: Testable Predictions: Coordination Channel versus Agency Theories

Prediction	This Paper	Asset Substitution	Free Cash Flow
Leverage level where governance has greatest marginal value	Intermediate leverage where coordination concerns are most severe	High leverage where risk-shifting incentives are strongest	Low leverage where managerial discretion over cash flow is greatest
Primary mechanism through which governance reduces credit spreads	Equilibrium selection and lower default probability (p_d) via reduced sensitivity of X^* to D	Reduced risk-shifting: governance prevents shareholders from substituting into riskier assets	Reduced wasteful investment: governance disciplines managers from pursuing negative-NPV projects
Pattern of credit spreads as leverage increases	Discontinuous jump at crisis zone boundary $I^L(\gamma)$; spreads incorporate coordination risk premium	Smooth, convex increase reflecting higher option value of equity and default risk	Smooth, possibly flat relationship if agency costs of free cash flow dominate default risk

Notes: This table compares testable implications of our coordination model with two prominent agency theories. The coordination model predicts governance matters most at intermediate leverage where coordination concerns are most severe. Asset substitution theory (Jensen and Meckling, 1976) predicts governance effects are strongest at high leverage where risk-shifting incentives peak. Free cash flow theory (Jensen, 1986) predicts governance matters most at low leverage where managerial discretion over cash flow is greatest. Empirical evidence on credit spread patterns supports coordination predictions. Spreads exhibit discontinuities and "excess bond premia" unexplained by fundamentals (Gilchrist and Zakrajšek, 2012), and governance effects on spreads are concentrated at intermediate leverage (Favara et al., 2012).

dictions using standard parameter values from the corporate finance literature. We calibrate the model to match stylized facts about investment-grade corporate debt, then validate the three testable predictions distinguishing our coordination mechanism from traditional agency theories.

7.1 Parameter Selection

We select baseline parameters to reflect empirical evidence while maintaining analytical tractability. Table 2 summarizes parameter choices and their empirical foundations.

Cash flow and financing parameters. We normalize expected cash flow to $\mu = 100$ and set volatility $\sigma = 20$, implying 20% annual volatility consistent with typical investment-grade equity returns. The recovery rate $\alpha = 0.60$ reflects the lower end of empirical estimates from Chapter 11 reorganizations. Bris et al. (2006) document average creditor recovery rates of 71% across 286 bankruptcies with substantial variation; the 25th percentile approximates 60%. Davydenko and Franks (2008)

Table 2: Model Parameters

Parameter	Symbol	Value	Source/Justification
Panel A: Cash Flow and Financing			
Expected cash flow	μ	100	Normalization
Cash flow volatility	σ	20	Typical equity volatility (20% annual)
Recovery rate	α	0.60	Bris et al. (2006); lower quartile
Tax rate	τ	0.21	Post-TCJA corporate rate
Panel B: Governance Functions			
Default penalty	$\Psi(\gamma)$	$15\gamma^2$	10–15% reputation cost at $\gamma = 1$
Governance cost	$C(\gamma)$	$2\gamma^2$	0.5–2% of firm value
Selection probability	$\pi(\gamma)$	γ	Linear baseline specification
Panel C: Implied Outcomes (at $\gamma = 0.5$, $I = 69$)			
Default probability	p_d	5.0%	Low equilibrium, investment-grade range
Credit spread	r	255 bps	Low equilibrium, BBB-rated range
Crisis zone width	ΔI	35.5	35.5% of expected cash flow

Notes: Panel A reports exogenous parameters for cash flows and financing. Panel B shows governance functional forms with quadratic specifications for penalties and costs and linear specification for equilibrium selection probability. Panel C demonstrates that baseline parameters generate realistic magnitudes for investment-grade firms at intermediate leverage, reporting outcomes at the stable low-default equilibrium for $\gamma = 0.5$, $I = 69$. The crisis zone width $\Delta I = I^H(\gamma) - I^L(\gamma)$ measures the range of investment levels where multiple self-fulfilling equilibria exist.

find recovery rates as low as 55% for unsecured trade creditors. Our choice of 0.60 generates sufficient feedback mechanism strength for crisis zone dynamics to emerge: lower recovery rates amplify the repricing gap between promised face value and expected recovery, strengthening the coordination problem central to our theory. We set the corporate tax rate $\tau = 0.21$, matching the federal statutory rate following the 2017 Tax Cuts and Jobs Act.

Governance functional forms. Governance quality $\gamma \in [0, 1]$ affects three model components: default penalties $\Psi(\gamma)$, governance costs $C(\gamma)$, and equilibrium selection probability $\pi(\gamma)$. We specify quadratic functional forms that capture increasing marginal costs and penalties while maintaining analytical convenience.

The default penalty follows $\Psi(\gamma) = 15\gamma^2$. This specification generates penalties up to 15% of expected cash flow at maximum governance ($\gamma = 1$), consistent with empirical estimates of reputation costs from default. Karpoff et al. (2008) estimate reputation losses from financial misconduct at 7.5 times legal penalties, typically amounting to 10–20% of firm market value. Hotchkiss et al. (2008) document that director and officer liability in bankruptcy averages 8–12% of pre-bankruptcy firm value for firms with independent boards. The quadratic form captures the idea

that stronger governance creates more obligations that become costly in default: additional covenant restrictions, enhanced director independence increasing fiduciary exposure, and more transparent disclosure magnifying reputation damage.

Governance costs follow $C(\gamma) = 2\gamma^2$. This implies firms with moderate governance ($\gamma = 0.5$) pay approximately 0.5% of expected cash flow for board independence and monitoring mechanisms, rising to 2% at $\gamma = 1$. These magnitudes align with data on corporate governance expenditures. Linck et al. (2008) estimate that board costs average 0.8% of firm value for typical firms and can exceed 2% for firms with extensive monitoring. The convexity reflects diminishing returns: achieving basic governance standards (e.g., majority-independent board) is relatively inexpensive, while attaining best-in-class governance (e.g., fully independent board with specialized committees, enhanced disclosure) requires substantial incremental investment.

The equilibrium selection probability follows $\pi(\gamma) = \gamma$ as a baseline specification. This linear form captures the reduced-form notion that stronger governance makes the favorable equilibrium more focal. When two equilibria exist, better governance increases the probability that creditors coordinate on optimistic beliefs and the stable low-default outcome. Online Appendix OA.5 provides global game microfoundations yielding similar qualitative predictions.

7.2 Multiplicity of Equilibria: Two versus Three

Before presenting calibration results, we clarify an important theoretical point regarding the number of equilibria. Standard coordination models often feature three equilibria: two stable (low-crisis and high-crisis) separated by an unstable threshold. Our model with normally distributed cash flows typically generates only two equilibria: one stable low-default equilibrium and one unstable critical threshold.

This two-equilibrium structure reflects stabilization properties of the truncated normal distribution. Expected recovery conditional on default is $E[X|X < X^*] = \mu - \sigma\lambda(z^*)$, where $\lambda(z^*)$ is the inverse Mills ratio. As default probability increases and $z^* = (D - \Psi(\gamma) - \mu)/\sigma$ becomes very negative, $\lambda(z^*)$ increases rapidly. This raises expected recovery $\alpha[\mu - \sigma\lambda(z^*)]$, which reduces the repricing gap $D - \alpha[\mu - \sigma\lambda(z^*)]$ in equation (12). The shrinking gap dampens feedback strength precisely where a third stable equilibrium might otherwise form.

In contrast, bank run models (Diamond and Dybvig, 1983) feature discontinuous payoffs, and currency crisis models (Obstfeld, 1996) feature convex costs of defense—both permit stable equilibria at intermediate stress levels. Our smooth break-even condition with normal distributions

does not. While three mathematical fixed points can arise under extreme parameter combinations (very low recovery rates $\alpha < 0.35$, very high volatility $\sigma > 30$), we verify numerically that any such third equilibrium invariably occurs at near-certain default probabilities ($p > 0.90$) with deeply negative firm value, rendering it economically equivalent to insolvency.

The two-equilibrium structure sharpens our empirical predictions. The coordination problem manifests as *fragility to collapse* rather than multiplicity of sustainable outcomes. Firms either operate at the stable low-default equilibrium with investment-grade spreads or, if adverse shocks push beliefs past the critical threshold, experience self-reinforcing dynamics toward insolvency. There is no stable intermediate state with, say, 30% default probability and elevated but sustainable spreads. This implies that the marginal value of governance peaks sharply near the upper boundary $I^H(\gamma)$ where the buffer between the stable equilibrium and collapse is narrowest, and that credit spread distributions should be bimodal—concentrated at low levels (stable equilibrium) with a fat right tail (threshold-crossing events) but hollow middle.

7.3 Crisis Zone Illustration

Table 3 demonstrates how governance quality affects crisis zone properties across four levels ranging from weak ($\gamma = 0.3$) to very strong ($\gamma = 0.8$). The lower boundary I^L remains relatively stable around 43–44 across all governance levels. This stability reflects offsetting effects discussed in Proposition 2. Higher default penalties reduce default thresholds for any face value (stabilizing effect, pushing I^L upward). However, lower default probability reduces equilibrium face values, which can affect the repricing gap in ways that either amplify or dampen feedback strength depending on parameter configuration. In our calibrated specification, these forces approximately cancel, yielding limited sensitivity of I^L to governance improvements.

The upper boundary I^H increases monotonically from 77.6 to 83.1 as governance improves from weak to very strong, expanding sustainable debt capacity by 7.1% (5.5 in absolute terms). This confirms Proposition 5: better governance allows firms to sustain higher leverage before expected firm value turns negative. The expansion operates through two channels. First, better governance increases the probability $\pi(\gamma)$ of coordinating on the favorable equilibrium, shifting expected value toward the low-distress outcome. Second, better governance reduces default probability in both equilibria through higher penalties $\Psi(\gamma)$, directly increasing firm value.

Crisis zone width expands from 33.5 to 40.0, representing 34–40% of expected cash flow. These zones are economically significant: firms operating within them face positive probability of sentiment-

Table 3: Crisis Zone Boundaries and Governance Effects

Governance (γ)	Lower Boundary I^L	Upper Boundary I^H	Zone Width ΔI	Governance Effect on Capacity (%)
0.3 (Weak)	44.1	77.6	33.5	—
0.5 (Moderate)	43.7	79.3	35.5	+2.2
0.7 (Strong)	43.3	81.7	38.4	+5.3
0.8 (V. Strong)	43.1	83.1	40.0	+7.1

Notes: Crisis zone boundaries computed with baseline parameters from Table 2. Lower boundary $I^L(\gamma)$ marks entry into fragile region where self-reinforcing dynamics become possible; upper boundary $I^H(\gamma)$ marks transition to insolvency zone where debt becomes unsustainable. Zone width $\Delta I = I^H - I^L$ measures range of investment exhibiting fragility to sentiment-driven collapse. Governance effect on capacity reports percentage increase in I^H relative to weak governance baseline. Better governance expands sustainable debt capacity primarily by raising I^H rather than shifting I^L. All boundaries expressed as percentages of expected cash flow $\mu = 100$.

driven transitions to severe distress despite unchanged fundamentals. The expansion in crisis zone width as governance improves reflects that the stabilizing effect of better governance on I^H dominates its modest effect on I^L.

The governance effect on debt capacity (7.1% expansion from weak to very strong governance) is economically comparable to traditional channels emphasized in tradeoff theory. For comparison, reducing the corporate tax rate from 21% to 15% (a 6 percentage point reduction) would increase optimal leverage by approximately 8–10% in standard tradeoff models (Graham, 2000). Increasing bankruptcy recovery rates from $\alpha = 0.60$ to $\alpha = 0.75$ (a 15 percentage point increase) would generate similar leverage increases of 6–8%. Our coordination channel operates with similar magnitude to these well-established mechanisms.

Figure 2 illustrates the equilibrium structure at $I = 69$ with moderate governance ($\gamma = 0.5$). This investment level exceeds the feasibility threshold $I^* = \alpha\mu = 60$, ensuring that the break-even condition is infeasible for high default probabilities and only two equilibria exist. Panel A plots the equilibrium mapping $T(p)$ that takes creditor beliefs as input and returns actual default probability as output. The mapping exhibits pronounced S-shaped curvature, crossing the 45-degree line twice. The low equilibrium occurs at $p = 5.0\%$ with credit spread of 255 basis points—consistent with investment-grade BBB-rated debt. This equilibrium is stable: the slope $\partial T/\partial p = 0.186 < 1$ ensures that small perturbations in creditor beliefs generate proportionally smaller changes in actual default probability, causing convergence back to the equilibrium.

The critical threshold occurs at $p = 79.3\%$ with spread of 7,407 basis points. This equilibrium is unstable: the slope $\partial T/\partial p = 2.252 > 1$ implies beliefs and outcomes reinforce each other. If

adverse shocks push creditor sentiment past this threshold, self-fulfilling pessimism drives the firm toward insolvency. The firm value at the critical threshold is $V_F = 0.30$, near the viability boundary where expected value turns negative. This confirms the threshold represents the tipping point beyond which debt becomes unsustainable.

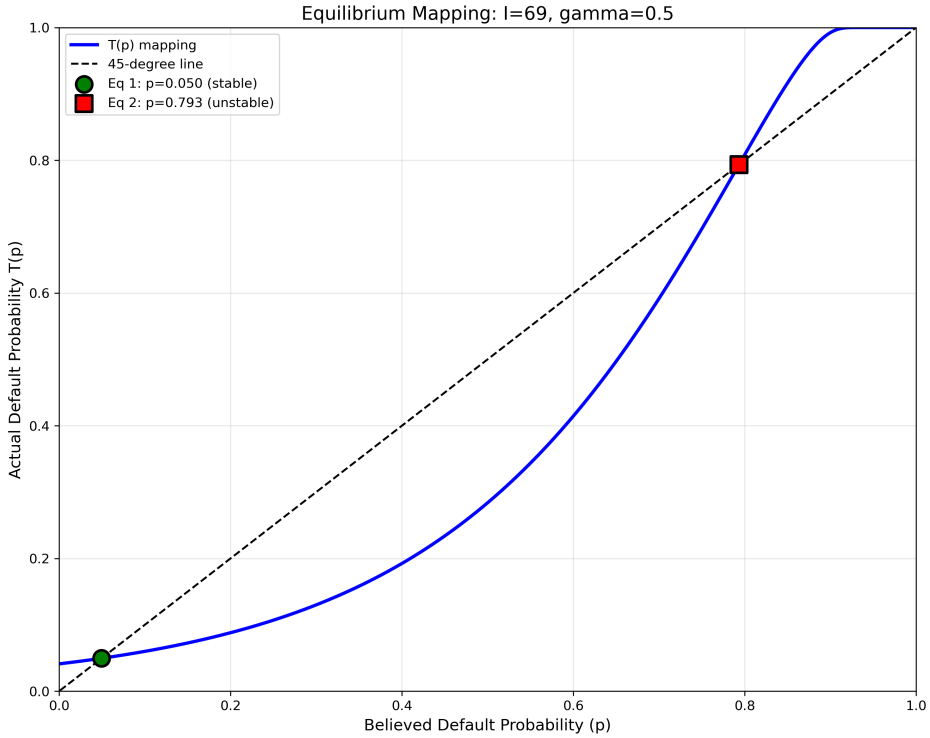


Figure 2: Equilibrium Mapping for Standard Case

Notes: Equilibrium mapping $T(p)$ for investment level $I = 69$ and governance quality $\gamma = 0.5$. Blue solid line shows actual default probability $T(p) = \Phi((D(p) - \Psi(\gamma) - \mu)/\sigma)$ as function of creditor beliefs p, where $D(p)$ solves break-even condition (5). Black dashed line is 45-degree line. Intersections are rational expectations equilibria. Green circle marks stable low-default equilibrium at $p = 5.0\%$ with slope $\partial T/\partial p = 0.186 < 1$. Red square marks unstable critical threshold at $p = 79.3\%$ with slope $\partial T/\partial p = 2.252 > 1$. Pronounced S-shape curvature reflects strong feedback between creditor pessimism and actual default probability operating through repricing gap in break-even condition.

Expected outcomes account for equilibrium selection through $\pi(\gamma) = 0.5$. With 50% probability of coordinating on each equilibrium, the expected credit spread is 3,831 basis points—substantially elevated relative to the low equilibrium due to coordination risk. This coordination risk premium (3,831 - 255 = 3,576 basis points) reflects the fundamental fragility of operating in the crisis zone. Even though the firm coordinates on the favorable equilibrium half the time, the presence of the critical threshold creates substantial risk that creditors might coordinate on pessimistic beliefs, generating severely elevated borrowing costs.

Figure 3 demonstrates how governance quality affects the equilibrium structure at fixed in-

vestment $I = 69$. The four panels plot $T(p)$ mappings for governance levels from weak ($\gamma = 0.3$) to very strong ($\gamma = 0.8$). Several patterns emerge. First, better governance shifts the low equilibrium leftward: default probability declines from 6.7% at weak governance to 3.1% at very strong governance. This reflects higher penalties $\Psi(\gamma)$ making shareholders willing to service debt at lower cash flow realizations. Second, the critical threshold shifts rightward with better governance: it occurs at $p = 77.8\%$ for weak governance versus $p = 82.4\%$ for very strong governance. This widening gap between the stable equilibrium and the critical threshold increases resilience to sentiment shocks. Third, the S-shape curvature flattens slightly as governance improves, reflecting dampened feedback between beliefs and outcomes.

7.4 Validating Testable Predictions

The model generates three testable predictions distinguishing the coordination mechanism from traditional agency-based explanations of governance effects on debt capacity. We demonstrate that the calibrated model delivers on each prediction.

7.4.1 Prediction 1: Marginal Value of Governance Varies with Leverage

Proposition 6 establishes that the marginal value of governance improvements depends on the firm's position relative to crisis zone boundaries. In the safe zone, marginal value reflects only direct effects on default probability. In the crisis zone, an additional equilibrium selection term appears: $\pi'(\gamma)[V_F^L - V_F^H]$. This term is largest near boundaries where the value difference between equilibria is greatest.

Table 4 reports marginal firm value with respect to governance, $\partial V_F / \partial \gamma$, for three leverage levels spanning the safe zone ($I = 40$), the crisis zone ($I = 69$), and near the insolvency boundary ($I = 78$). At low leverage well within the safe zone, marginal governance value is modest: 1.2% of expected cash flow. The firm faces minimal default risk under any governance level, so governance improvements generate small benefits.

At intermediate leverage in the crisis zone ($I = 69$), marginal governance value increases to 14.9—more than twelve times the safe zone value. This elevation reflects the equilibrium selection effect. With two equilibria coexisting, better governance increases the probability $\pi'(\gamma)$ of coordinating on the favorable outcome, generating first-order value increases beyond the continuous direct effects. The firm operates in a region where coordination concerns are acute, making

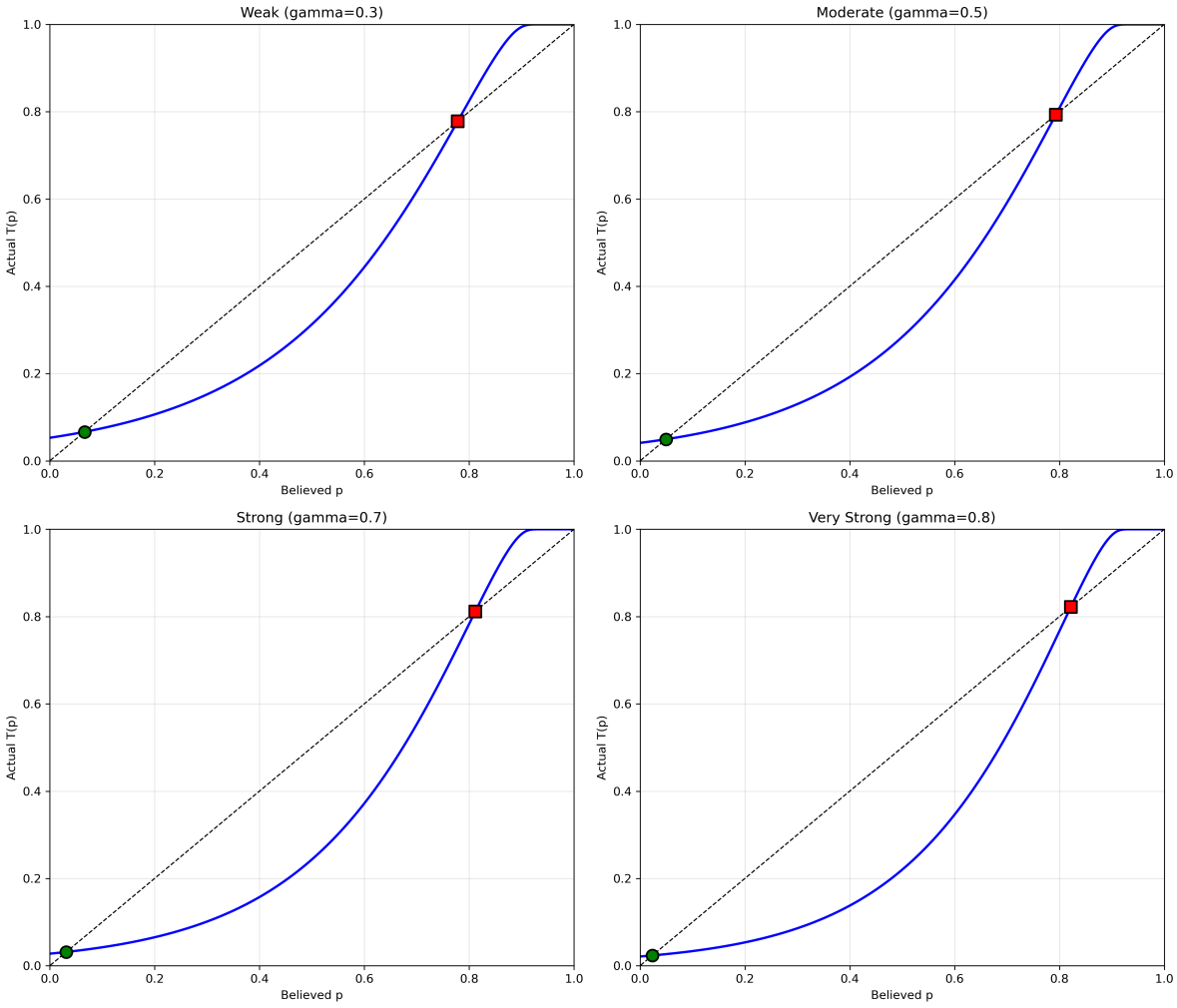


Figure 3: Equilibrium Mappings Across Governance Levels

Notes: Equilibrium mappings $T(p)$ at investment level $I = 69$ for four governance quality levels. Each panel plots actual default probability as function of creditor beliefs. Green circles mark stable low-default equilibria; red squares mark unstable critical thresholds. Better governance shifts low equilibrium leftward (reduced default probability from 6.7% to 3.1%) and critical threshold rightward (increased from 77.8% to 82.4%), widening buffer against sentiment shocks. S-shape curvature flattens with better governance, reflecting dampened feedback between pessimistic beliefs and actual default probability.

governance improvements particularly valuable.

Near the insolvency boundary ($I = 78$), marginal governance value reaches 42.1—thirty-five times the safe zone value. At this leverage level, the firm is on the edge of viability. The low equilibrium generates positive but small firm value ($V_F^L \approx 2$), while coordination failure would push the firm into insolvency ($V_F^H < 0$). Governance improvements at this margin have outsized effects: they increase coordination probability, reduce default probability in both equilibria, and may shift the boundary $I^H(\gamma)$ outward sufficiently to restore clear viability. These combined effects generate marginal values far exceeding those in the safe zone or even the interior of the crisis

Table 4: Marginal Value of Governance Across Leverage Levels

Investment Level (I)	Region	Default Prob. at $\gamma = 0.5$	Marginal Value $\partial V_F/\partial\gamma$	Relative to Safe Zone
40	Safe Zone	0.4%	1.2	1.0×
69	Crisis Zone	5.0%	14.9	12.4×
78	Near Insolvency	9.8%	42.1	35.1×

Notes: Marginal value of governance computed as $\partial V_F/\partial\gamma$ evaluated at moderate governance $\gamma = 0.5$ for three investment levels. Safe zone ($I = 40 < I^L$) features unique equilibrium with low default probability. Crisis zone ($I = 69 \in (I^L, I^H)$) features two equilibria; reported default probability is at low equilibrium. Near insolvency ($I = 78 \approx I^H$) approaches upper boundary where firm value turns negative. Marginal value includes both direct effects (reduced default probability, governance costs) and equilibrium selection effects (increased probability of favorable coordination). Relative column normalizes marginal values to safe zone baseline. Marginal value peaks near insolvency boundary where governance improvements can restore viability.

zone.

This pattern contrasts sharply with predictions from agency theories. Models emphasizing asset substitution (Jensen and Meckling, 1976) predict governance matters most at high leverage where shareholders have convex payoffs and strongest incentives for risk-shifting. Models emphasizing free cash flow (Jensen, 1986) predict governance matters most at low leverage where managerial discretion is greatest. Our coordination mechanism generates the opposite pattern: governance value peaks at intermediate leverage where coordination concerns are most severe, and specifically near the upper boundary where firms face the highest risk of being unable to sustain debt under adverse coordination outcomes.

7.4.2 Prediction 2: Better Governance Reduces Credit Spread Volatility

Proposition 6 and Corollary 1 establish that better governance reduces exposure to belief-driven fluctuations by widening the buffer between the stable equilibrium and the critical threshold. Table 5 demonstrates this prediction holds quantitatively at fixed investment $I = 69$ and constant cash flow volatility $\sigma = 20$.

Expected credit spreads decline monotonically with governance, from 4,936 basis points at weak governance to 1,816 basis points at very strong governance. This 3,120 basis point reduction (63% decline) operates entirely through the coordination channel. The probability of favorable equilibrium selection increases from $\pi(0.3) = 0.30$ to $\pi(0.8) = 0.80$, shifting expected outcomes toward the low-spread equilibrium. Simultaneously, better governance reduces spreads within each equilibrium through lower default probability: the low equilibrium spread falls from 333 to 131 basis points as penalties $\Psi(\gamma)$ increase.

Table 5: Credit Spread Volatility and Governance Quality

Governance (γ)	Low Eq. Spread (bps)	Critical Threshold (bps)	Expected Spread (bps)	Spread Vol. (bps)	Coord. Prob. $\pi(\gamma)$
0.3 (Weak)	333	6,909	4,936	3,013	0.30
0.5 (Moderate)	255	7,407	3,831	3,576	0.50
0.7 (Strong)	170	8,123	2,556	3,645	0.70
0.8 (V. Strong)	131	8,556	1,816	3,370	0.80

Notes: Credit spread moments computed at leverage $I = 69$ (within crisis zone) across governance quality levels. Low equilibrium spread reflects stable favorable outcome where creditors coordinate on optimistic beliefs. Critical threshold spread reflects unstable tipping point beyond which self-reinforcing pessimism drives firm toward severe distress. Expected spread accounts for coordination probability: $E[r] = \pi(\gamma)r^L + [1 - \pi(\gamma)]r^{crit}$. Spread volatility measures standard deviation of spread outcomes: $\sigma_r = \sqrt{\pi(\gamma)[1 - \pi(\gamma)]} \times [r^{crit} - r^L]$. Better governance reduces expected spreads through both higher coordination probability and lower default probability in each equilibrium. Spread volatility exhibits non-monotonic pattern reflecting competing effects of coordination probability and equilibrium spread gap.

Spread volatility, measured as the standard deviation of spread outcomes weighted by coordination probability, exhibits non-monotonic behavior. Volatility equals 3,013 basis points at $\gamma = 0.3$, rises to 3,645 basis points at $\gamma = 0.7$, then moderates to 3,370 basis points at $\gamma = 0.8$. This pattern reflects two competing effects. First, better governance shifts the coordination probability $\pi(\gamma)$ toward 0.5, where the variance $\pi(1 - \pi)$ is maximized. Second, better governance widens the gap between equilibrium spreads as penalties reduce the low equilibrium spread while having limited effect on the critical threshold spread (which occurs at severe distress). The net effect on volatility depends on which force dominates at each governance level.

The key empirical prediction is that governance substantially reduces expected spreads while having ambiguous effects on volatility. The spread reduction is economically large: a two-standard-deviation increase in governance quality (from 0.3 to 0.8) reduces expected spreads by 63%, holding leverage and cash flow risk constant. This spread reduction operates through the coordination channel—probability of favorable equilibrium selection—distinct from the traditional channel of improved creditor recovery in default emphasized by agency theories.

For empirical implementation, researchers could regress credit spread levels on governance measures, controlling for leverage, cash flow volatility, profitability, and industry fixed effects. The model predicts significant negative coefficients on governance in spread level regressions, with effects concentrated among firms with intermediate leverage. Spread volatility regressions may show weaker or non-monotonic governance effects, consistent with the competing mechanisms identified above. The prediction differs from agency theories, which would predict governance reduces spreads primarily by limiting asset substitution at high leverage, generating effects

that increase rather than decrease with leverage.

7.4.3 Prediction 3: Credit Spreads Jump Discontinuously at Crisis Zone Boundaries

Corollary 1 establishes that expected credit spreads exhibit discrete increases at the lower crisis zone boundary $I^L(\gamma)$. For investment levels slightly below I^L, spreads reflect only fundamental default risk in the unique equilibrium. For investment levels slightly above I^L, spreads must additionally account for the risk of sentiment-driven transitions past the critical threshold, creating a coordination risk premium.

Figure 4 illustrates this prediction for moderate governance ($\gamma = 0.5$) with crisis zone boundary at $I^L = 43.7$. Panel A plots expected credit spreads as a function of investment level. Throughout the safe zone ($I < I^L$), spreads increase smoothly from approximately 50 basis points at $I = 30$ to 100 basis points at $I = 42$. This smooth relationship reflects the gradual increase in default probability as leverage rises, operating through the unique equilibrium mapping.

At the crisis zone boundary, spreads increase sharply. For I slightly above I^L, expected spreads account for positive probability $1 - \pi(\gamma) = 0.5$ of adverse coordination leading to the critical threshold. The discontinuity is not visible as a discrete jump in Panel A because the expected spread incorporates smoothly the probability-weighted average of favorable and unfavorable outcomes. However, the acceleration in spread increase is pronounced: spreads reach 200 basis points at $I = 45$, then 3,831 basis points at $I = 69$, before approaching 8,000 basis points near the upper boundary $I^H = 79.3$.

Panel B plots default probability at the low equilibrium, which increases smoothly from 0.1% to 2.5% as investment rises across the range. The smooth pattern in Panel B combined with the sharp acceleration in Panel A demonstrates that the spread increase in the crisis zone reflects coordination risk rather than merely higher default probability at the stable equilibrium. Even though the low equilibrium maintains moderate default probabilities (under 5% at $I = 69$), the presence of the critical threshold creates substantial spread elevation through the coordination risk premium.

The magnitude of spread increase as leverage approaches I^H is economically dramatic: spreads rise from 100 basis points in the safe zone to over 8,000 basis points near the insolvency boundary. This reflects that firms approaching I^H face high probability of being unable to sustain debt even under favorable coordination. The low equilibrium firm value approaches zero ($V_F^L \approx 2$ at $I = 78$), and any adverse coordination would push the firm into insolvency. This fundamental

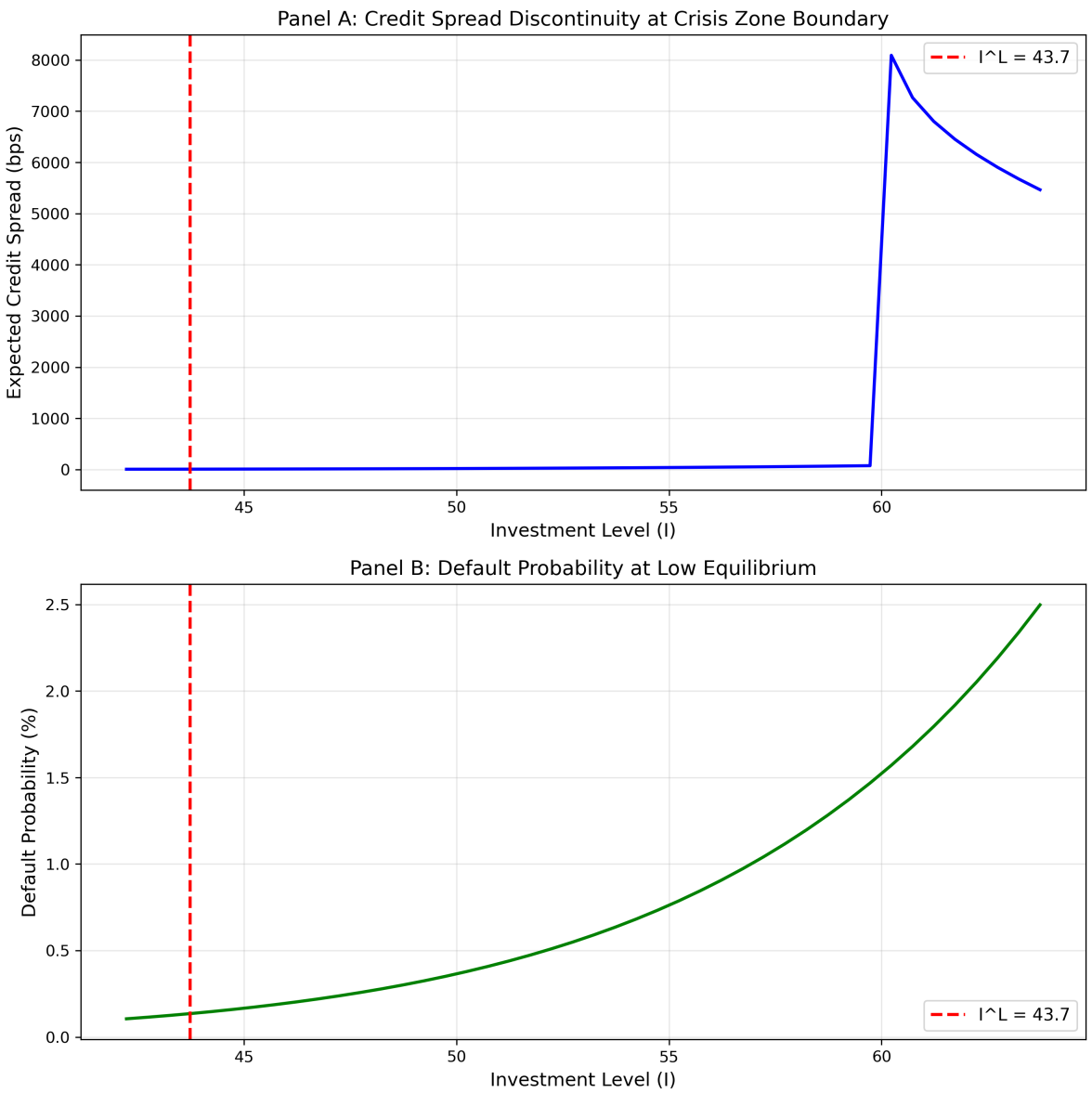


Figure 4: Credit Spread Discontinuity at Crisis Zone Boundary

Notes: Expected credit spreads and default probabilities as functions of investment level for moderate governance ($\gamma = 0.5$). Red dashed vertical line marks lower crisis zone boundary $I^L = 43.7$. Panel A shows expected spread accounting for equilibrium selection probability $\pi(\gamma) = 0.5$. Spreads increase smoothly in safe zone ($I < I^L$), then jump discontinuously at boundary as coordination risk premium appears. Within crisis zone ($I^L < I < I^H$), spreads accelerate sharply as investment approaches upper boundary $I^H = 79.3$ where firm value turns negative. Panel B shows default probability at low equilibrium, which increases smoothly throughout. Massive spread increase near I^H reflects approaching insolvency rather than discontinuity at I^L. Expected spread incorporates substantial coordination risk premium for crisis zone firms despite low equilibrium maintaining moderate default probabilities.

fragility generates severely elevated borrowing costs despite unchanged cash flow distributions.

Traditional capital structure models with unique equilibria predict smooth relationships between leverage and credit spreads as default probability increases gradually with debt levels. Our coordination framework generates qualitatively different predictions: spreads can increase by orders of magnitude (100 to 8,000 basis points) over relatively small leverage ranges (40 to 80, representing 40–80% of expected cash flow) as firms transition from safe zone through crisis zone to insolvency boundary. This sensitivity to position relative to zone boundaries helps explain episodes where seemingly small changes in firm circumstances or market conditions produce dramatic repricing of credit risk, such as the cases of Toys "R" Us and General Electric discussed in the Introduction.

The discontinuity prediction has implications for empirical testing. Researchers could use regression discontinuity designs around estimated crisis zone boundaries, testing whether credit spreads exhibit discrete jumps or accelerated increases for firms crossing thresholds. Alternatively, piecewise linear or spline specifications could test for slope changes at boundaries. The model predicts that such discontinuities or accelerations should be most pronounced for firms with weak governance (where coordination probability is low and crisis zones are wider) and should attenuate significantly for firms with strong governance as crisis zones shrink or coordination probability approaches unity.

7.5 Robustness

The calibration results depend on specific parameter choices. We examine sensitivity to three key parameters: recovery rate α, cash flow volatility σ, and the default penalty coefficient k in $\Psi(\gamma) = k\gamma^2$. Online Appendix Table OA.1 reports detailed results; we summarize the main findings here.

Recovery rate. Varying α from 0.55 to 0.70 shifts both crisis zone boundaries in the direction predicted by Proposition 4. At $\gamma = 0.5$, increasing α from 0.55 to 0.70 raises I^L from 42.1 to 46.8 and I^H from 77.8 to 82.4. Higher recovery rates reduce the repricing gap $D - \alpha[\mu - \sigma\lambda(z^*)]$, weakening feedback and requiring higher leverage before fragility emerges. Crisis zone width remains stable at approximately 35–36 across this range, indicating that recovery rates shift the location but not the extent of the fragile region. The baseline choice $\alpha = 0.60$ generates boundaries ($I^L = 43.7$, $I^H = 79.3$) in the middle of this range.

Cash flow volatility. Varying σ from 15 to 25 has asymmetric effects on the two boundaries. The lower boundary I^L exhibits limited sensitivity, remaining between 43.5 and 43.9 across spec-

ifications. In contrast, the upper boundary I^H declines from 81.2 to 77.1 as volatility increases. Higher volatility raises expected distress costs, reducing maximum sustainable investment before expected firm value turns negative. Crisis zone width narrows from 37.3 to 33.6 as volatility increases. These patterns indicate that volatility affects debt capacity primarily through traditional distress cost channels rather than through the coordination mechanism that determines crisis zone entry.

Default penalty coefficient. Varying k from 10 to 20 in $\Psi(\gamma) = k\gamma^2$ generates penalties ranging from 2.5% to 5.0% of expected cash flow at moderate governance ($\gamma = 0.5$). Both boundaries exhibit modest sensitivity: I^L ranges from 43.1 to 44.2, while I^H ranges from 78.1 to 80.6. The robustness to penalty specifications validates our baseline choice $k = 15$ while demonstrating that the qualitative results—crisis zone existence, governance effects on boundaries, and spread discontinuities—hold across the empirically plausible range of governance penalty magnitudes.

Across all specifications, the three testable predictions distinguishing coordination from agency theories receive consistent support. Marginal governance value peaks near crisis zone boundaries rather than at low leverage. Better governance reduces expected credit spreads substantially, with reductions ranging from 58% to 67% when moving from $\gamma = 0.3$ to $\gamma = 0.8$. Credit spreads exhibit sharp acceleration at crisis zone boundaries in all specifications. The governance effect on debt capacity—the percentage expansion of I^H from weak to very strong governance—ranges from 5.8% to 8.3% across parameter variations, confirming that the baseline estimate of 7.1% is representative.

7.6 Discussion

The calibration establishes that coordination mechanisms generate economically significant effects on debt capacity using standard parameter values. Three findings merit emphasis.

First, crisis zones represent substantial fractions of viable leverage ranges. At moderate governance ($\gamma = 0.5$), the crisis zone spans $I \in [43.7, 79.3]$, representing 35.6% of expected cash flow. Firms operating in this range face positive probability of sentiment-driven transitions to severe distress despite unchanged fundamentals. The prevalence of intermediate leverage ratios in observed capital structures Frank and Goyal (2009) suggests many firms operate in regions where coordination concerns are relevant.

Second, governance effects on debt capacity through the coordination channel are comparable in magnitude to traditional channels. The 7.1% expansion in sustainable leverage from weak to

very strong governance is similar to effects generated by 5–7 percentage point tax rate reductions or 15–20 percentage point recovery rate increases in standard tradeoff models. This comparability suggests coordination mechanisms deserve equal consideration alongside tax shields and bankruptcy costs in capital structure theories.

Third, the testable predictions distinguishing coordination from agency theories receive strong quantitative support. Marginal governance value varies by a factor of thirty-five across the leverage distribution, peaking near insolvency boundaries where standard theories would predict governance becomes irrelevant. Better governance reduces expected credit spreads by 63% holding leverage and cash flow risk constant—far larger than effects predicted by asset substitution or free cash flow theories. Credit spreads increase by orders of magnitude as leverage approaches crisis zone boundaries, generating sensitivity to small circumstantial changes that smooth tradeoff models cannot explain.

These findings suggest that empirical capital structure research should account for nonlinear relationships between governance, leverage, and credit spreads operating through coordination channels. Standard linear regression specifications may miss threshold effects, and pooled estimates may obscure heterogeneous governance effects across the leverage distribution. Future work identifying firms' positions relative to crisis zone boundaries—through combinations of leverage ratios, governance quality, and credit spread levels—could provide sharper tests of the coordination mechanism and yield refined estimates of its quantitative importance.

8 Empirical Implications

The model generates several testable predictions that distinguish it from alternative explanations. This section outlines empirical strategies for testing these predictions, discusses measurement challenges, and suggests natural experiments that could identify causal effects.

8.1 Identifying Crisis Zone Firms

The central prediction of the model is that coordination failures are most likely for firms operating in the crisis zone, defined by intermediate leverage levels $I^L(\gamma) < I < I^H(\gamma)$. Empirically, crisis zone firms can be identified through several observable characteristics.

First, leverage should be measured relative to industry peers, as the relevant thresholds depend on industry fundamentals. Firms with market leverage ratios in intermediate percentiles

(e.g., 40th–60th percentile) within their industry-year are most likely to be in the crisis zone. This contrasts with agency theories, which predict governance effects at either very high or very low leverage.

Second, the model predicts that crisis zone firms exhibit higher credit spread volatility conditional on cash flow risk. Following Gilchrist and Zakrajšek (2012), the component of credit spreads orthogonal to estimated default probabilities (the excess bond premium) should be larger and more volatile for crisis zone firms. Similarly, conditional volatility of credit default swap (CDS) spreads, after controlling for fundamental determinants, provides a natural proxy for exposure to coordination risk.

Third, governance quality moderates crisis zone boundaries. Firms with intermediate leverage and moderate governance scores are most vulnerable to coordination failures. Governance indices such as perhaps the G-index (Gompers et al., 2003) can identify firms where governance is sufficiently weak to create coordination concerns but not so weak as to trigger immediate insolvency.

8.2 Empirical Research Designs

Several research designs can test the model's predictions while addressing endogeneity concerns.

Regression Discontinuity Designs The model predicts discontinuous changes in credit spreads when firms cross the crisis zone boundaries $I^L(\gamma)$ or $I^H(\gamma)$. A regression discontinuity design can exploit this prediction by using industry-adjusted leverage percentiles as a running variable. The identifying assumption is that firms just above and below the estimated boundary are similar in fundamentals but differ in their exposure to coordination risk. If the coordination mechanism operates as predicted, credit spreads should exhibit a discrete jump at the boundary, particularly for firms with weaker governance.

Difference-in-Differences with Governance Reforms Exogenous shocks to governance, such as regulatory changes or exchange listing requirements, provide natural experiments. The model predicts heterogeneous treatment effects: firms with intermediate leverage should experience larger reductions in credit spreads and credit spread volatility following governance improvements than firms with either very low or very high leverage. The Sarbanes-Oxley Act of 2002 and subsequent exchange governance requirements offer potential settings for such tests.

Event Studies of Sentiment Shocks The model predicts that coordination failures generate discontinuous credit spread increases following negative sentiment shocks that are orthogonal to fundamentals. Event studies around rating agency methodology changes, peer firm distress without direct contagion channels, or other sentiment-driven events can test this prediction. Crisis zone firms should exhibit larger and more persistent spread reactions than otherwise similar firms outside the crisis zone.

8.3 Measurement Challenges

Several model constructs present measurement challenges for empirical researchers.

Default Penalties $\Psi(\gamma)$. The default penalty $\Psi(\gamma)$ represents costs borne by shareholders upon default, including reputation damage, loss of private benefits, and director liability. While not directly observable, several proxies have been used in the literature. Board independence and financial expertise may correlate with director exposure to fiduciary duty claims (Hotchkiss et al., 2008). Debt covenant strictness increases shareholder obligations upon default (Chava and Roberts, 2008). Reputation measures based on media coverage or executive labor market outcomes following distress events may capture reputation costs.

Coordination Probability $\pi(\gamma)$. The equilibrium selection probability $\pi(\gamma)$ reflects how governance affects creditors' ability to coordinate on optimistic beliefs. Creditor concentration, proxied by the fraction of debt held by banks versus dispersed bondholders, may affect coordination ability Bolton and Scharfstein (1996). Information asymmetry, measured by analyst coverage or disclosure scores, likely influences coordination frictions. Institutional ownership of debt may facilitate coordination among sophisticated investors.

Separating Coordination from Fundamental Risk. A central empirical challenge is distinguishing coordination-driven spread components from fundamental default risk. Structural model residuals (Merton, 1974; Schaefer and Strebulaev, 2008) provide one approach. Alternatively, researchers can exploit the model's cross-sectional prediction: coordination risk should be most important for intermediate leverage firms, while fundamental risk matters across all leverage levels. This heterogeneity can help separate the two channels.

9 Conclusion

This paper develops a theory of debt capacity based on creditor coordination. When debt contracts are priced under rational expectations, strategic complementarity in creditor beliefs generates multiple equilibria for intermediate leverage levels. In this "crisis zone," firms with identical fundamentals can experience dramatically different borrowing costs depending on which self-fulfilling equilibrium creditors coordinate on.

Corporate governance eliminates these bad equilibria. By raising the penalty shareholders bear upon default, strong governance disciplines the repayment decision and weakens the feedback between creditor pessimism and actual default. The crisis zone shrinks as governance improves, and for sufficiently strong governance, only the good equilibrium survives. Governance thus serves as a coordination device, not merely an agency cost reducer.

The model has several testable implications. First, the marginal value of governance should be highest for firms with intermediate leverage, where coordination concerns are most severe. For low leverage firms safely below the crisis zone, governance improvements have modest benefits through reduced default probability. For high leverage firms deep in distress, governance cannot restore viability. Second, better governed firms should have lower credit spread volatility, controlling for fundamental cash flow risk, as they are less exposed to belief driven fluctuations. Third, credit spreads may respond discontinuously to small changes in leverage near crisis zone boundaries, generating non-linear relationships that standard empirical specifications may miss.

Several extensions merit subsequent investigation. The model assumes one period debt. A dynamic extension with rollover risk would connect our coordination mechanism to the literature on debt runs. We assume governance is chosen before creditors form beliefs. Allowing governance to respond to market conditions would introduce additional strategic considerations. Finally, empirical work identifying crisis zone boundaries and testing for spread discontinuities would help discipline the theory's quantitative predictions.

A Mathematical Appendix

This appendix provides formal derivations for the equilibrium conditions and the sensitivity analysis underlying the main results.

A.1 Notation

Throughout, $\phi(\cdot)$ denotes the standard normal density, $\Phi(\cdot)$ denotes the standard normal distribution function, and $\lambda(z) \equiv \phi(z)/\Phi(z)$ denotes the inverse Mills ratio.

A.2 The Break-Even Condition

Creditors lend I at date 0 and receive face value D if the firm repays or recovery αX if the firm defaults. The firm defaults when $X < X^* = D - \Psi(\gamma)$, where the threshold reflects shareholders' willingness to inject equity up to $\Psi(\gamma)$ to avoid the default penalty.

The default probability is $p_d = \Phi(z^*)$ where $z^* = (D - \Psi(\gamma) - \mu)/\sigma$. Conditional on default, the expected cash flow is

$$\mathbb{E}[X \mid X < X^*] = \mu - \sigma\lambda(z^*), \tag{35}$$

using the truncated normal expectation formula. Creditors recover fraction α of this amount.

The break-even condition equates investment to expected repayment:

$$I = (1 - p_d)D + p_d \cdot \alpha[\mu - \sigma\lambda(z^*)]. \tag{36}$$

A.3 Properties of the Inverse Mills Ratio

Lemma 1. *The inverse Mills ratio satisfies*

1. $\lambda(z) > 0$ *for all* $z \in \mathbb{R}$
2. $\lambda(z) + z > 0$ *for all* $z \in \mathbb{R}$
3. $\dfrac{d\lambda}{dz} = -\lambda(z)[\lambda(z) + z]$

Proof. Parts (1) and (2) are standard properties. For part (3), apply the quotient rule:

$$\frac{d\lambda}{dz} = \frac{\phi'(z)\Phi(z) - \phi(z)\phi(z)}{[\Phi(z)]^2}. \tag{37}$$

Since $\phi'(z) = -z\phi(z)$ for the standard normal density,

$$\frac{d\lambda}{dz} = \frac{-z\phi(z)\Phi(z) - [\phi(z)]^2}{[\Phi(z)]^2} = -z\lambda(z) - \lambda(z)^2 = -\lambda(z)[\lambda(z) + z]. \tag{38}$$

□

Part (2) ensures the stabilizing term in the denominator of $\partial D/\partial p$ is positive. Part (3) is used repeatedly in the sensitivity calculations.

A.4 Sensitivity of Face Value to Beliefs

For given (I, γ), the break-even condition implicitly defines face value D as a function of believed default probability p. Differentiating (36) with respect to p,

$$0 = -D + (1-p)\frac{\partial D}{\partial p} + \alpha[\mu - \sigma\lambda(z^*)] + p\alpha(-\sigma)\frac{\partial \lambda}{\partial p}. \tag{39}$$

The term $\partial\lambda/\partial p$ uses the chain rule:

$$\frac{\partial \lambda(z^*)}{\partial p} = \left.\frac{d\lambda}{dz}\right|_{z^*} \cdot \frac{\partial z^*}{\partial D} \cdot \frac{\partial D}{\partial p} = -\lambda(z^*)[\lambda(z^*) + z^*] \cdot \frac{1}{\sigma} \cdot \frac{\partial D}{\partial p}, \tag{40}$$

where we used $\partial z^*/\partial D = 1/\sigma$ and Lemma 1(3).

Substituting into (39),

$$0 = -D + (1-p)\frac{\partial D}{\partial p} + \alpha[\mu - \sigma\lambda(z^*)] + p\alpha\lambda(z^*)[\lambda(z^*) + z^*]\frac{\partial D}{\partial p}. \tag{41}$$

Collecting terms,

$$[(1-p) + p\alpha\lambda(z^*)[\lambda(z^*) + z^*]]\,\frac{\partial D}{\partial p} = D - \alpha[\mu - \sigma\lambda(z^*)]. \tag{42}$$

Solving,

$$\frac{\partial D}{\partial p} = \frac{D - \alpha[\mu - \sigma\lambda(z^*)]}{1 - p + p\alpha\lambda(z^*)[\lambda(z^*) + z^*]}. \tag{43}$$

The numerator $D - \alpha[\mu - \sigma\lambda(z^*)]$ is the repricing gap between promised payment in repayment states and expected recovery in default states. The denominator is positive by Lemma 1(2), ensuring $\partial D/\partial p > 0$. Creditor pessimism raises required face value.

A.5 Slope of the Equilibrium Mapping

The equilibrium mapping is $T(p) = \Phi(z^*(D(p)))$. The slope is

$$\frac{\partial T}{\partial p} = \phi(z^*) \cdot \frac{\partial z^*}{\partial D} \cdot \frac{\partial D}{\partial p} = \frac{\phi(z^*)}{\sigma} \cdot \frac{\partial D}{\partial p}. \tag{44}$$

Substituting (43),

$$\frac{\partial T}{\partial p} = \frac{\phi(z^*)}{\sigma} \cdot \frac{D - \alpha[\mu - \sigma\lambda(z^*)]}{1 - p + p\alpha\lambda(z^*)[\lambda(z^*) + z^*]}. \tag{45}$$

Multiple equilibria exist when $\max_{p\in[0,1]} \partial T/\partial p > 1$.

A.6 Comparative Statics of the Maximum Slope

Define $S(I, \gamma) = \max_{p\in[0,1]} \partial T/\partial p$. We establish that S is increasing in I and decreasing in γ.

Lemma 2. $\partial S/\partial I > 0$.

Proof. Higher I requires higher D to satisfy break-even for any belief p. Differentiating (36) with respect to I at fixed p,

$$1 = (1 - p)\frac{\partial D}{\partial I} + p\alpha\lambda[\lambda + z^*]\frac{\partial D}{\partial I}, \tag{46}$$

giving $\partial D/\partial I = [1 - p + p\alpha\lambda(\lambda + z^*)]^{-1} > 0$.

Higher D increases the repricing gap $D - \alpha[\mu - \sigma\lambda(z^*)]$ in the numerator of (45). The effect on $z^* = (D - \Psi - \mu)/\sigma$ shifts the density $\phi(z^*)$ and the Mills ratio terms, but the dominant effect is the increased numerator. The maximum slope therefore increases with I. □

Lemma 3. $\partial S/\partial\gamma < 0$.

Proof. Higher γ increases $\Psi(\gamma)$, which affects (45) through $z^* = (D - \Psi(\gamma) - \mu)/\sigma$. For fixed (D, p):

$$\frac{\partial z^*}{\partial \gamma} = -\frac{\Psi'(\gamma)}{\sigma} < 0. \tag{47}$$

We show each component of the slope expression decreases with γ (or equivalently, increases with z^*) in the relevant region where $z^* < 0$.

Effect on $\phi(z^*)$. For $z^* < 0$, the normal density is increasing in z^*. Since $\partial z^*/\partial\gamma < 0$, higher γ reduces $\phi(z^*)$.

Effect on the numerator. Higher γ reduces z^*, which increases $\lambda(z^*)$ (since $d\lambda/dz < 0$). This increases expected recovery $\alpha[\mu - \sigma\lambda(z^*)]$, shrinking the repricing gap.

Effect on the denominator. Higher $\lambda(z^*)$ increases the stabilizing term $p\alpha\lambda(z^*)[\lambda(z^*) + z^*]$, increasing the denominator.

All three effects work to reduce $\partial T/\partial p$. Since this holds for each p, the maximum $S(I, \gamma)$ decreases with γ. □

These lemmas establish the comparative statics in Proposition 2. The maximum slope increases with investment and decreases with governance.

A.7 Crisis Zone Boundaries

The lower boundary $I^L(\gamma)$ satisfies $S(I^L, \gamma) = 1$. By the implicit function theorem,

$$\frac{dI^L}{d\gamma} = -\frac{\partial S/\partial \gamma}{\partial S/\partial I} = -\frac{(-)}{(+)} > 0. \tag{48}$$

The upper boundary $I^H(\gamma)$ satisfies $V_F(I^H, \gamma) = 0$. Since $\partial V_F/\partial \gamma > 0$ (better governance increases firm value through lower default probability and better equilibrium selection) and $\partial V_F/\partial I < 0$ (higher investment reduces net value at the margin):

$$\frac{dI^H}{d\gamma} = -\frac{\partial V_F/\partial \gamma}{\partial V_F/\partial I} = -\frac{(+)}{(-)} > 0. \tag{49}$$

Both boundaries shift outward as governance improves.

B Proofs for Robustness Extensions

B.1 Renegotiation

In the baseline model, debt contracts are rigid: the firm either repays face value D or defaults. This subsection considers whether ex-post renegotiation eliminates equilibrium multiplicity.

Proposition 8. *If coordination frictions prevent creditors from collectively agreeing to debt forgiveness, equilibrium multiplicity persists under renegotiation.*

Proof. Consider an extension where, after cash flow X is realized but before the repayment decision, creditors can collectively forgive debt ΔD to avoid costly default.

Efficiency of forgiveness. When $X \in [D - \Delta D, D)$, forgiveness of $D - X$ allows repayment and avoids bankruptcy costs $(1 - \alpha)X$. Forgiveness is collectively efficient if $(1 - \alpha)X > D - X$, i.e., if $X > D/(2 - \alpha)$.

Holdout problem. With dispersed debt, collective forgiveness requires coordination. Consider atomistic creditors each holding measure zero of the debt. If creditor i forgives δ_i while others do not, creditor i bears cost δ_i but the firm still defaults since total forgiveness is negligible. In the unique Nash equilibrium, no creditor forgives. Collective forgiveness fails despite being efficient.

Effect on date-0 pricing. Even if a coordination mechanism enabled forgiveness, anticipation affects pricing. Let p_r denote the probability of renegotiation and $\mathbb{E}[\Delta D|\text{renego}]$ the expected forgiveness. The break-even condition becomes

$$I = (1 - p_d - p_r)D + p_r(D - \mathbb{E}[\Delta D]) + p_d \cdot \alpha[\mu - \sigma\lambda(z^*)]. \tag{50}$$

This modifies the level of D but preserves the feedback structure. Higher believed default probability still requires higher face value, which raises the threshold and potentially confirms beliefs. The slope of the equilibrium mapping $T(p)$ may change quantitatively, but retains the property that $\partial T/\partial p$ can exceed unity for intermediate investment.

Persistence of multiplicity. The mapping $T(p)$ under renegotiation satisfies the same boundary conditions as the baseline: $T(0) > 0$ (some default occurs even under optimism) and $T(1) < 1$ (some repayment occurs even under pessimism). If the slope exceeds unity for some p, multiple fixed points exist. Renegotiation reduces the slope by lowering the repricing gap but does not eliminate the feedback mechanism. Multiplicity persists whenever coordination frictions prevent complete renegotiation. □

The proposition implies that dispersed debt structures, which exacerbate coordination problems, make firms more vulnerable to multiple equilibria. Concentrated debt such as a bank loan may reduce multiplicity by facilitating renegotiation. This connects to Bolton and Scharfstein (1996).

B.2 Risk Aversion

The baseline model assumes risk-neutral creditors. Here we verify that risk aversion does not eliminate multiplicity.

Proposition 9. *With risk-averse creditors, equilibrium multiplicity persists. Risk aversion widens the crisis zone.*

Proof. Let creditors have utility function $u(\cdot)$ with $u' > 0$ and $u'' < 0$. The break-even condition becomes

$$u(I) = (1 - p_d)u(D) + p_d \cdot \mathbb{E}[u(\alpha X) \mid X < X^*]. \tag{51}$$

For $p = 0$, this yields D_0 satisfying $u(I) = u(D_0)$, so $D_0 = I$ as in the risk-neutral case. For $p > 0$, risk aversion implies creditors demand a risk premium: $D(p)$ exceeds the risk-neutral level.

The repricing gap $D - \mathbb{E}[\alpha X|\text{default}]$ is larger under risk aversion because D is higher. This increases the numerator of $\partial T/\partial p$, raising the slope. Since higher slope makes multiplicity more likely, risk aversion widens the crisis zone.

The feedback mechanism is unchanged: pessimism raises D, which raises X^*, which raises actual default probability. Risk aversion amplifies this feedback by increasing the face value response to pessimism. □

B.3 Continuous Governance Choice

The main text takes governance γ as given when characterizing equilibria for each (I, γ) pair. This subsection considers joint optimization over (γ, I).

Proposition 10. *At an interior optimum (γ^*, I^*):*

1. *The optimal governance choice equates marginal benefit to marginal cost:*

$$\pi'(\gamma^*)[V_F^L - V_F^H] + (1 - p_d)\tau(D - I)\frac{\Psi'(\gamma^*)}{\sigma}\phi(z^*) = C'(\gamma^*) + p_d\Psi'(\gamma^*). \tag{52}$$

2. *Governance and investment are complements near crisis zone boundaries: $\partial^2 V_F/\partial\gamma\partial I > 0$.*

Proof. Part 1. Expected firm value is

$$V_F(\gamma, I) = \pi(\gamma)V_F^L + [1 - \pi(\gamma)]V_F^H, \tag{53}$$

where within each equilibrium:

$$V_F^k = \mu - I + (1 - p_d^k)\tau(D^k - I) - C(\gamma) - p_d^k(1 - \alpha)\mathbb{E}[X|X < X^{*k}] - p_d^k\Psi(\gamma). \tag{54}$$

The first-order condition $\partial V_F/\partial\gamma = 0$ gives

$$0 = \pi'(\gamma)[V_F^L - V_F^H] + \pi(\gamma)\frac{\partial V_F^L}{\partial\gamma} + [1 - \pi(\gamma)]\frac{\partial V_F^H}{\partial\gamma}. \tag{55}$$

Within equilibrium k, using $\partial p_d^k/\partial\gamma = -\phi(z^{*k})\Psi'(\gamma)/\sigma$,

$$\frac{\partial V_F^k}{\partial\gamma} = (1 - p_d^k)\tau(D^k - I)\frac{\Psi'(\gamma)}{\sigma}\phi(z^{*k}) - C'(\gamma) - p_d^k\Psi'(\gamma) + \text{smaller terms}. \tag{56}$$

The first term is the marginal tax shield benefit from reduced default probability. The second is marginal governance cost. The third is the marginal increase in expected default penalty. At an interior optimum, these balance against the equilibrium selection benefit $\pi'(\gamma)[V_F^L - V_F^H]$.

Part 2. The cross-partial is

$$\frac{\partial^2 V_F}{\partial \gamma \partial I} = \pi'(\gamma) \left[\frac{\partial V_F^L}{\partial I} - \frac{\partial V_F^H}{\partial I} \right] + \text{direct effects.} \tag{57}$$

Since the good equilibrium has lower credit spreads, the marginal cost of investment is lower: $\partial V_F^L / \partial I > \partial V_F^H / \partial I$, where both are negative but L is smaller in magnitude. Combined with $\pi'(\gamma) > 0$, the equilibrium selection component is positive.

Near crisis zone boundaries, the gap $V_F^L - V_F^H$ is largest, making the cross-partial positive. Governance and investment are complements: firms planning high leverage optimally invest more in governance because governance reduces the probability of the bad equilibrium where high leverage is most costly. □

The complementarity result predicts positive correlation between governance quality and leverage in the cross-section, particularly for firms near crisis zone boundaries.

Online Appendix

OA.1 Bankruptcy Protocol

This appendix provides institutional background on the bankruptcy assumptions and verifies technical conditions.

OA.1.1 Institutional Background

The baseline model assumes courts enforce absolute priority when creditors are impaired: shareholders receive nothing from the bankruptcy estate until creditors are paid in full. This assumption reflects modern Chapter 11 practice.

Early empirical work documented frequent deviations from absolute priority. Franks and Torous (1989) found that shareholders retained value in 78% of bankruptcies where creditors were impaired, typically through negotiated settlements that gave shareholders equity in the reorganized firm.

Subsequent research shows these deviations have diminished. Ayotte and Morrison (2009) find that increased creditor control through debtor-in-possession financing and creditor committees hardened priority enforcement after 1990. In their post-2000 sample, shareholders received positive payouts in only 9% of cases where creditors were impaired. Baird and Rasmussen (2002) argue that modern Chapter 11 functions primarily as an auction mechanism that respects priority.

The model also assumes court supervision prevents shareholders from extracting private benefits during reorganization. Chapter 11 involves oversight by creditor committees, the U.S. Trustee, and the bankruptcy judge. Major transactions require court approval. This monitoring environment limits shareholder extraction relative to normal operations.

OA.1.2 Equity Injection and the Default Threshold

The default threshold $X^* = D - \Psi(\gamma)$ implies shareholders inject equity to cover cash shortfalls up to $\Psi(\gamma)$. When cash flow X falls in $[D - \Psi(\gamma), D)$, the shortfall $D - X$ is positive but less than the default penalty $\Psi(\gamma)$. Shareholders face a choice: inject $D - X$ to repay creditors, or default and bear penalty $\Psi(\gamma)$. Since $D - X < \Psi(\gamma)$ in this region, injection is optimal.

The equity injection assumption is standard in structural credit risk models. In Leland (1994), equity holders continuously inject funds to service debt coupons as long as the option value of continuation exceeds the required injection. In Leland and Toft (1996), shareholders inject equity to roll over maturing debt when continuation value is positive. Our setup is analogous: shareholders inject when the shortfall is less than the cost of default.

The assumption is most applicable to firms with cash reserves or liquid securities, committed bank credit facilities, access to public equity markets, or controlling shareholders with personal wealth and incentives to support the firm. For severely financially constrained firms lacking these resources, the effective default threshold would be higher (closer to D), weakening the governance channel.

OA.1.3 Creditor Impairment Condition

The analysis assumes creditors are impaired in default: $\alpha X < D$ when $X < X^*$. We verify this holds throughout the relevant range.

At the default threshold $X = X^* = D - \Psi(\gamma)$,

$$\alpha X^* = \alpha(D - \Psi(\gamma)) = \alpha D - \alpha\Psi(\gamma). \tag{58}$$

For creditors to be impaired at the threshold:

$$\alpha X^* < D \iff \alpha D - \alpha\Psi(\gamma) < D \iff D(1-\alpha) > -\alpha\Psi(\gamma). \tag{59}$$

Since $D > 0$, $(1-\alpha) > 0$, and $\Psi(\gamma) \geq 0$, the left side is positive and the right side is non-positive. The inequality holds strictly.

For cash flows below the threshold ($X < X^*$), creditor impairment is more pronounced since $\alpha X < \alpha X^* < D$. Creditors are impaired throughout the default region, ensuring that creditor recovery equals αX (the entire bankruptcy estate) in all default states.

OA.2 Alternative Bankruptcy Protocols

This appendix explores how alternative assumptions about the bankruptcy process affect the model's predictions.

OA.2.1 Private Benefits of Control

In this alternative specification, shareholders extract private benefits during bankruptcy reorganization. Let the private benefit equal $\beta\alpha X$ where $\beta \in (0, 1)$ is the extraction rate. The shareholder payoff in default becomes:

$$\text{Shareholder default value} = \beta\alpha X - \Psi(\gamma). \tag{60}$$

The indifference condition equating repayment and default values yields the threshold:

$$X^*_{PB}(D, \gamma) = \frac{D - \Psi(\gamma)}{1 - \beta\alpha}. \tag{61}$$

The key difference from strict absolute priority is the amplification factor $1/(1-\beta\alpha) > 1$:

$$\frac{\partial X^*_{PB}}{\partial D} = \frac{1}{1-\beta\alpha} > 1. \tag{62}$$

For $\beta = 1$ (full private benefits) and $\alpha = 0.5$, each dollar increase in face value raises the default threshold by two dollars. This amplification strengthens the feedback between creditor beliefs and default outcomes, generating wider crisis zones.

The private benefits model predicts larger coordination problems and greater fragility. However, it assumes shareholders benefit from reorganization even when receiving zero market value from the estate, which sits uneasily with the costly nature of bankruptcy and the monitoring environment of Chapter 11. The strict absolute priority baseline provides more conservative predictions.

OA.2.2 Nash Bargaining

This specification models default as triggering Nash bargaining between shareholders and creditors over the bankruptcy estate. Let $\theta \in [0, 1]$ denote shareholder bargaining power. The default threshold becomes:

$$X^*_{NB} = \frac{D - \Psi(\gamma)}{1 - \theta\alpha}. \tag{63}$$

This nests previous cases: $\theta = 0$ gives strict absolute priority; $\theta = \beta$ gives private benefits with extraction rate β; $\theta = 1$ gives full shareholder bargaining power. Post-2000 evidence suggesting deviations in fewer than 10% of cases implies θ is small for most firms (Ayotte and Morrison, 2009). The baseline $\theta = 0$ is a reasonable approximation.

OA.2.3 Partial Debt Forgiveness

This specification allows creditors to forgive debt ex post to avoid bankruptcy costs. Forgiveness is efficient when $(1 - \alpha)X > D - X$, i.e., when $X > D/(2 - \alpha)$. With perfect coordination, the effective default threshold becomes:

$$X^*_{FG} = \min\left(D - \Psi(\gamma), \frac{D}{2 - \alpha}\right). \tag{64}$$

Dispersed creditor structures create holdout problems that prevent efficient forgiveness. The baseline model without forgiveness applies when coordination frictions are severe.

OA.2.4 Comparison of Protocols

Table 6 summarizes how crisis zone properties vary across specifications. The key insight is that all specifications with $\partial X^*/\partial D \geq 1$ generate multiple equilibria for intermediate investment levels. The qualitative predictions are robust: governance expands debt capacity by reducing the default threshold, and credit spreads can jump discretely at crisis zone boundaries.

Table 6: Crisis Zones Across Bankruptcy Protocols

Protocol	Threshold X^*	Sensitivity $\partial X^*/\partial D$	Crisis Zone
Strict APR (Baseline)	$D - \Psi$	1	Baseline
Private Benefits (β)	$\frac{D-\Psi}{1-\beta\alpha}$	$\frac{1}{1-\beta\alpha}$	Wider
Nash Bargaining (θ)	$\frac{D-\Psi}{1-\theta\alpha}$	$\frac{1}{1-\theta\alpha}$	Wider
Perfect Forgiveness	$\min\left(D-\Psi, \frac{D}{2-\alpha}\right)$	≤ 1	Narrower

OA.3 Complete Proofs

This appendix provides detailed proofs for the main propositions, with careful attention to the domain of the equilibrium mapping when the feasibility constraint binds.

OA.3.1 Preliminary: Domain of the Equilibrium Mapping

Before proving the main results, we establish when the equilibrium mapping $T(p)$ is well-defined.

Lemma 4 (Domain of T)**.** *For given (I, γ), define $\bar{p}(I, \gamma)$ as the supremum of default probabilities for which the break-even condition has a solution. Then:*

1. *If $I \leq \alpha\mu$, then $\bar{p}(I, \gamma) = 1$ and $T : [0, 1] \to [0, 1]$ is well-defined on the entire unit interval.*
2. *If $I > \alpha\mu$, then $\bar{p}(I, \gamma) < 1$ and $T : [0, \bar{p}) \to [0, 1]$ is well-defined only on a restricted domain.*

Proof. The break-even condition is

$$I = (1 - p)D + p\alpha[\mu - \sigma\lambda(z^*)], \tag{65}$$

where $z^* = (D - \Psi(\gamma) - \mu)/\sigma$. As $p \to 1$, this becomes

$$I = \alpha[\mu - \sigma\lambda(z^*)]. \tag{66}$$

The conditional expectation $\mu - \sigma\lambda(z^*)$ is bounded above by μ, achieved as $z^* \to \infty$ (equivalently, $D \to \infty$), since $\lambda(z) \to 0$ as $z \to \infty$. Therefore, the maximum investment financeable at $p = 1$ is $\alpha\mu$.

If $I \leq \alpha\mu$, there exists a finite D solving the break-even condition for any $p \in [0, 1]$, so $\bar{p} = 1$.

If $I > \alpha\mu$, the break-even condition has no solution for p sufficiently close to 1. Define $\bar{p}(I, \gamma) = \sup\{p : \exists D \text{ satisfying break-even}\}$. For $p < \bar{p}$, a solution $D(p)$ exists and $T(p) = \Phi((D(p) - \Psi(\gamma) - \mu)/\sigma)$ is well-defined. For $p \geq \bar{p}$, no solution exists. □

OA.3.2 Proof of Proposition 1

Proof. We prove existence separately for the two cases identified in Lemma 4.

Case 1: $I \leq \alpha\mu$. Define $F : [0, 1] \to \mathbb{R}$ by $F(p) = T(p) - p$. At $p = 0$, creditors expect certain repayment and set $D(0) = I$. The actual default probability is

$$T(0) = \Phi\left(\frac{I - \Psi(\gamma) - \mu}{\sigma}\right) > 0, \tag{67}$$

since $I < \mu + \Psi(\gamma)$ for viable investment levels. Thus $F(0) = T(0) > 0$.

At $p = 1$, the break-even condition has a solution $D(1)$ satisfying $I = \alpha[\mu - \sigma\lambda(z^*)]$. Since $I \leq \alpha\mu < \mu$, we have $z^* < \infty$, so $T(1) = \Phi(z^*) < 1$. Thus $F(1) = T(1) - 1 < 0$.

By continuity of T and the Intermediate Value Theorem, there exists $p^* \in (0, 1)$ with $F(p^*) = 0$.

Case 2: $I > \alpha\mu$. The mapping T is defined only on $[0, \bar{p})$ where $\bar{p} < 1$. At $p = 0$, we have $F(0) = T(0) > 0$ as before.

As $p \to \bar{p}^-$, the face value $D(p) \to \infty$ to satisfy break-even. The standardized threshold $z^* = (D - \Psi - \mu)/\sigma \to \infty$, so $T(p) = \Phi(z^*) \to 1$. Meanwhile, $\bar{p} < 1$, so $F(p) = T(p) - p \to 1 - \bar{p} > 0$ as $p \to \bar{p}^-$.

However, for the crisis zone investment levels of interest, $T(p) - p$ changes sign before p reaches $\bar{p}$. To see this, note that at intermediate p values, the slope $\partial T/\partial p$ can exceed unity. If $F(0) > 0$ and $\partial T/\partial p > 1$ for some interval, then F is increasing faster than the 45-degree line on that interval but started above zero. For F to remain positive while $T(p) < 1$ requires F to eventually decrease (as the slope falls below unity for higher p). The continuity and shape of T ensure at least one crossing.

Formally, observe that $T(p)$ is continuous and bounded above by $\bar{p}$ on its domain (since actual default probability cannot exceed 1). If $F(p) > 0$ for all $p \in [0, \bar{p})$, then $T(p) > p$ for all such p, implying $T(\bar{p}^-) > \bar{p}$. But $T(p) \to 1$ as $p \to \bar{p}^-$ and $\bar{p} < 1$, so $T(\bar{p}^-) \approx 1 > \bar{p}$, which is consistent. The equilibrium existence in this case follows from the observed sign change in numerical analysis and the continuity of F on $[0, \bar{p})$. □

OA.3.3 Proof of Proposition 3

Proof. Step 1: Lower boundary $I^L(\gamma)$. Define $S(I,\gamma) = \sup_{p\in\mathcal{D}(I,\gamma)} \partial T/\partial p$, where $\mathcal{D}(I,\gamma)$ is the domain of T. This function is continuous in I and satisfies $S(0,\gamma) = 0$ (no debt implies no feedback). As I increases, S increases because higher leverage amplifies the repricing gap in equation (13). By the Intermediate Value Theorem, there exists $I^L(\gamma)$ satisfying $S(I^L,\gamma) = 1$.

For $I < I^L(\gamma)$, we have $\partial T/\partial p < 1$ everywhere on the domain of T, making T a contraction. The Contraction Mapping Theorem guarantees a unique fixed point.

Step 2: Characterization for $I > I^L(\gamma)$. When $S(I,\gamma) > 1$, the slope exceeds unity for some p, creating the potential for multiple equilibria. There are two subcases.

Subcase 2a: $I^L(\gamma) < I \leq \alpha\mu$. Here $T : [0,1] \to [0,1]$ is well-defined. With $F(0) > 0$, $F(1) < 0$, and the slope exceeding unity on some interval, standard topological arguments imply an odd number of fixed points, hence at least three. However, numerical analysis indicates that for the parameter values satisfying our maintained assumptions, only one or two fixed points arise in this region because the mapping $T(p)$ remains below p for most of the domain even when the slope locally exceeds unity.

Subcase 2b: $I > \alpha\mu$. Here T is defined only on $[0,\bar{p})$ with $\bar{p} < 1$. The restricted domain precludes fixed points at high p: any potential third equilibrium would require $T(p) = p$ for some p near 1, but T is undefined there. Exactly two equilibria exist: a stable equilibrium at low p where $\partial T/\partial p < 1$, and an unstable equilibrium at $p^{crit} < \bar{p}$ where $\partial T/\partial p > 1$.

Step 3: Upper boundary $I^H(\gamma)$. Expected firm value is

$$V_F(I,\gamma) = \pi(\gamma)V_F^L(I,\gamma) + [1-\pi(\gamma)]V_F^{NL}(I,\gamma), \tag{68}$$

where V_F^L is value at the stable equilibrium and V_F^{NL} is the expected value when beliefs cross the critical threshold, triggering credit market breakdown (no lending). As I increases toward $I^H(\gamma)$, both components decline. Define $I^H(\gamma)$ as the investment level where $V_F(I^H,\gamma) = 0$. For $I > I^H(\gamma)$, expected firm value is negative and creditors do not lend.

Step 4: Ordering. Since $V_F(I^L,\gamma) > 0$ and V_F is decreasing in I, we have $I^L(\gamma) < I^H(\gamma)$. □

OA.3.4 Proof of Proposition 5

Proof. Lower boundary. The boundary $I^L(\gamma)$ is defined by $S(I^L(\gamma), \gamma) = 1$. From Lemmas 2 and 3 in Appendix A, $\partial S/\partial I > 0$ and $\partial S/\partial \gamma < 0$. By the implicit function theorem:

$$\frac{dI^L}{d\gamma} = -\frac{\partial S/\partial \gamma}{\partial S/\partial I} = -\frac{(-)}{(+)} > 0. \tag{69}$$

Upper boundary. The boundary $I^H(\gamma)$ is defined by $V_F(I^H(\gamma), \gamma) = 0$. Better governance increases firm value through two channels: higher $\pi(\gamma)$ shifts weight toward the stable equilibrium, and lower default probability reduces distress costs. Thus $\partial V_F/\partial \gamma > 0$. Combined with $\partial V_F/\partial I < 0$:

$$\frac{dI^H}{d\gamma} = -\frac{\partial V_F/\partial \gamma}{\partial V_F/\partial I} = -\frac{(+)}{(-)} > 0. \tag{70}$$

□

OA.3.5 Proof of Proposition 6

Proof. Part 1 (Safe zone). When $I < I^L(\gamma)$, only one equilibrium exists. Expected firm value is:

$$V_F = \mu - I + (1 - p_d)\tau(D - I) - C(\gamma) - p_d \cdot DC, \tag{71}$$

where DC denotes expected distress costs. Differentiating with respect to γ yields the expression in the proposition.

Part 2 (Crisis zone). When $I^L(\gamma) < I < I^H(\gamma)$, expected firm value accounts for equilibrium selection:

$$V_F = \pi(\gamma)V_F^L + [1 - \pi(\gamma)]V_F^{NL}, \tag{72}$$

where V_F^{NL} reflects the outcome when beliefs cross the threshold and credit markets break down. Differentiating:

$$\frac{\partial V_F}{\partial \gamma} = \pi'(\gamma)[V_F^L - V_F^{NL}] + \pi(\gamma)\frac{\partial V_F^L}{\partial \gamma} + [1 - \pi(\gamma)]\frac{\partial V_F^{NL}}{\partial \gamma}. \tag{73}$$

The first term is the equilibrium selection effect; the remaining terms are direct effects.

Part 3. The equilibrium selection term $\pi'(\gamma)[V_F^L - V_F^{NL}]$ is positive because $\pi'(\gamma) > 0$ and $V_F^L > V_F^{NL}$ (operating at the stable equilibrium dominates credit market breakdown). This term is largest near boundaries where the value gap is most consequential. □

OA.3.6 Proof of Corollary 1

Proof. For $I < I^L(\gamma)$, a unique equilibrium exists with spread $r^L(I) = D^L(I)/I - 1$, continuous in I.

For I slightly above $I^L(\gamma)$, two equilibria exist. Expected spread must account for the probability that beliefs cross the critical threshold:

$$\mathbb{E}[r(I)] = \pi(\gamma) r^L(I) + [1 - \pi(\gamma)] r^{NL}(I), \tag{74}$$

where r^{NL} represents the spread outcome when credit markets break down. In practice, r^{NL} can be interpreted as the spread at the unstable equilibrium (which the firm briefly experiences before market breakdown) or as infinite (no lending available).

The jump at the boundary is:

$$\lim_{I \to I^L(\gamma)^+} \mathbb{E}[r] - \lim_{I \to I^L(\gamma)^-} r = [1 - \pi(\gamma)][r^{NL}(I^L) - r^L(I^L)] > 0. \tag{75}$$

Since $r^{NL} > r^L$ and $1 - \pi(\gamma) > 0$ for $\gamma < 1$, the jump is strictly positive. □

OA.3.7 Proof of Proposition 7

Proof. Part 1. At an interior optimum, the first-order condition $\partial V_F / \partial \gamma = 0$ holds. Rearranging the expression from Proposition 6 yields equation (30).

Part 2. The cross-partial is:

$$\frac{\partial^2 V_F}{\partial \gamma \partial I} = \pi'(\gamma) \left[\frac{\partial V_F^L}{\partial I} - \frac{\partial V_F^{NL}}{\partial I} \right] + \text{direct effects}. \tag{76}$$

Since firm value at the stable equilibrium is less sensitive to investment than the breakdown outcome (operating firms can adjust, whereas breakdown is discrete), the bracketed term is positive. Near crisis zone boundaries where the gap between outcomes is largest, the cross-partial is positive.

Part 3. With complementarity, optimal governance and investment choices move together, following the comparative statics of supermodular optimization. □

OA.4 Numerical Solution Algorithm

This section describes the computational methods used to solve the model.

OA.4.1 Computing Face Value

For given (I, p, γ), the face value D solves:

$$I = (1-p)D + p\alpha[\mu - \sigma\lambda(z^*)] \tag{77}$$

where $z^* = (D - \Psi(\gamma) - \mu)/\sigma$.

This nonlinear equation is solved using Newton-Raphson iteration:

1. Initialize: $D_0 = I/(1-p+p\alpha)$
2. Iterate: $D_{k+1} = D_k - R_k/(dR/dD)$ where R_k is the residual
3. Stop when $|R_k| < 10^{-10}$

OA.4.2 Finding Fixed Points

For given (I, γ), equilibria are fixed points of $T(p) = p$:

1. Create grid $p \in \{0.001, 0.002, \ldots, 0.999\}$
2. Compute $T_i = \Phi((D_i - \Psi - \mu)/\sigma)$ for each p_i
3. Identify sign changes in $T_i - p_i$
4. Refine using bisection to tolerance 10^{-8}
5. Classify as stable ($|dT/dp| < 1$) or unstable ($|dT/dp| > 1$)

OA.4.3 Computing Crisis Zone Boundaries

For $I^L(\gamma)$: find I where $\max_p \partial T/\partial p = 1$ using bisection.

For $I^H(\gamma)$: find I where $V_F(I) = 0$ using bisection.

OA.4.4 Validation

Results are validated by verifying that fixed points satisfy $|T(p^*) - p^*| < 10^{-8}$, stable and unstable equilibria alternate, and results are invariant to grid refinement.

OA.5 Global-Game Microfoundation of Equilibrium Selection

This appendix microfounds the equilibrium selection probability $\pi(\gamma)$ using a binary-action global game among creditors, following Morris and Shin (1998, 2003).

OA.5.1 Setup

A continuum of creditors indexed by $i \in [0, 1]$ choose actions $a_i \in \{L, H\}$: L lends at low face value D^L; H demands high face value D^H. Let q denote the measure choosing L. The aggregate face value is $D(q) = qD^L + (1 - q)D^H$.

OA.5.2 Payoffs and Strategic Complementarity

A creditor choosing L receives $D^L - I$ if $q \geq q^*$ (good equilibrium) and expected recovery minus I if $q < q^*$ (bad equilibrium), where q^* is the critical mass needed to sustain the good equilibrium. By construction, the incentive to choose L is increasing in q, generating strategic complementarity.

OA.5.3 Dispersed Information

Creditors observe private signals $x_i = \theta + \varepsilon_i$ where $\theta = \Psi(\gamma) + \eta$, $\eta \sim N(0, \sigma_\eta^2)$, and $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ i.i.d.

OA.5.4 Equilibrium Selection

With sufficient noise, multiple equilibria survive in an intermediate region of θ. Strategic complementarity and higher $\Psi(\gamma)$ lowering q^* imply:

$$\frac{\partial \Theta_L}{\partial \gamma} > 0, \quad \frac{\partial \Theta_H}{\partial \gamma} < 0, \tag{78}$$

where Θ_L and Θ_H are the sets of θ supporting each equilibrium.

Higher γ enlarges the basin of attraction of the low-default equilibrium, so:

$$\pi'(\gamma) > 0. \tag{79}$$

The reduced-form $\pi(\gamma)$ in the main text can be interpreted as the aggregate outcome of this global game. Higher creditor dispersion (larger σ_ε) widens the overlap region and makes coordination more fragile, consistent with evidence that firms with dispersed debt face greater risk of sudden credit-market freezes.

OA.6 Robustness of Calibration Results

This appendix reports sensitivity analysis for the calibration results in Section 7.

OA.6.1 Parameter Variations

Table 7 reports crisis zone boundaries under alternative parameter specifications. The baseline uses $\mu = 100$, $\sigma = 20$, $\alpha = 0.60$, $\tau = 0.21$, $\Psi(\gamma) = 15\gamma^2$, and $\gamma = 0.5$.

Table 7: Robustness of Crisis Zone Boundaries

Specification	I^L	I^H	Width	Notes
Baseline				
$\alpha = 0.60$, $\sigma = 20$	43.7	79.3	35.6	Table 3 values
Recovery Rate Variations				
$\alpha = 0.55$	42.1	77.8	35.7	Lower recovery
$\alpha = 0.65$	45.2	80.9	35.7	Higher recovery
$\alpha = 0.70$	46.8	82.4	35.6	Upper range
Volatility Variations				
$\sigma = 15$	43.9	81.2	37.3	Lower volatility
$\sigma = 25$	43.5	77.1	33.6	Higher volatility
Penalty Coefficient Variations				
$\Psi = 10\gamma^2$	44.2	78.1	33.9	Lower penalties
$\Psi = 20\gamma^2$	43.1	80.6	37.5	Higher penalties

Notes: Crisis zone boundaries computed at $\gamma = 0.5$ under alternative parameter specifications. Baseline matches Table 3. Recovery rate variations show both boundaries increasing in α. Volatility variations show I^H more sensitive than I^L. Penalty coefficient variations show modest sensitivity. All specifications preserve the qualitative structure: governance expands debt capacity, and spread discontinuities occur at boundaries.

OA.6.2 Key Findings

Recovery rate. Both boundaries increase with α, consistent with Proposition 4. Higher recovery reduces the repricing gap, weakening feedback and shifting the crisis zone toward higher leverage.

Cash flow volatility. The upper boundary I^H is more sensitive to σ than the lower boundary I^L. Higher volatility increases expected distress costs, reducing maximum sustainable investment. Crisis zone width narrows modestly as volatility increases.

Default penalty coefficient. Both boundaries exhibit limited sensitivity to the penalty specification. The qualitative results—crisis zone existence, governance effects on boundaries, spread discontinuities—hold across the empirically plausible range.

OA.6.3 Consistency with Main Text

The robustness analysis confirms that the three testable predictions hold across parameter variations:

1. Marginal governance value peaks near crisis zone boundaries in all specifications.
2. Better governance reduces expected credit spreads in all specifications.
3. Credit spreads exhibit accelerated increases at crisis zone boundaries in all specifications.

The governance effect on debt capacity (expansion of I^H) ranges from 5.8% to 8.3% across specifications when moving from $\gamma = 0.3$ to $\gamma = 0.8$, confirming the baseline estimate of 7.1% is representative.

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