International Currency Dominance*

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Abstract

We present a micro-founded monetary model of the world economy to study international currency competition. Our model features "unipolar" equilibria, with a single dominant international currency, and "multipolar" equilibria, in which multiple currencies circulate internationally. Long-run equilibria are highly history-dependent and tend towards the emergence of a dominant currency. Governments can compete to internationalize their currencies by offering attractive interest rates on their sovereign debt, but large economies have a natural advantage in ensuring the dominance of their currencies. We calibrate the model to assess the quantitative importance of these mechanisms and study the international monetary system's dynamics under several counterfactual scenarios.

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1 Introduction

Throughout history, international trade and finance have been dominated by a few currencies. In the colonial era, the Dutch florin and the French franc circulated widely until the British pound sterling emerged as the leading global currency. The dollar later assumed this role with the Bretton Woods system, though other reserve currencies, such as the pound, the Japanese yen, and eventually the euro, also played significant roles (Eichengreen, 2019).

While contingent events, like the Bretton Woods agreement, shaped the international monetary system, economic fundamentals and government policies mattered just as much. Issuers of dominant currencies have typically been large economies with substantial fiscal capacity, such as Victorian Britain or the postwar U.S. Even the small Dutch economy of the 17th and 18th centuries had financial markets that supported extensive private debt issuance. Once a currency becomes dominant, its international status typically persists for decades.

This paper examines how economic fundamentals, government policies, and historical contingencies shape the international circulation of currencies. In the long run, which countries' currencies are most likely to circulate globally? Can governments adopt fiscal or monetary policies to internationalize their currencies? Are international monetary regimes with multiple competing currencies stable, or does the world economy tend towards an equilibrium with a single dominant currency? Under what circumstances can a dominant currency lose its status?

To address these questions, we adapt the New Monetarist framework (Lagos and Wright, 2005) to a canonical multi-country setting (see also Matsuyama et al., 1993, and Wright and Trejos, 2001). The model emphasizes money's role as an international medium of exchange, with each country's government bonds potentially used for international trade.[1] The model incorporates three types of international monetary regimes: a "classical" equilibrium without an internationally accepted currency, a "dominant-currency" equilibrium where one country's bond circulates globally, and a "multipolar" equilibrium where multiple bonds coexist as international currencies.

In our model, the demand for a currency is determined by its *liquidity* in international trade (how widely accepted it is) and its *liquidity premium* (the cost of holding that currency relative to an illiquid asset). Our assumptions, described in greater detail below, imply that the liquidity of currencies may differ because agents always accept their domestic currency, but they do not necessarily accept foreign currency. We endogenize each currency's liquidity in international trade by assuming that foreign agents must pay a (realistically small) fixed cost to accept it. Strategic complementarities emerge naturally from the model: an increase in the global acceptability of a country's bonds enhances demand for that currency, increasing the benefit of accepting it. Hence, dominant-currency equilibria in which only one currency circulates internationally often arise in

[1] The study of which commodities serve as media of exchange dates back further. Kiyotaki and Wright (1989) introduced one of the first models examining how the medium of exchange is determined in equilibrium.

the long run. Governments can compete to internationalize their currencies by offering attractive real returns on their nominal debt (i.e., a low liquidity premium): to do so, they must back their debt with taxes rather than seigniorage. However, countries are not on equal footing in this competition: larger countries have a natural advantage, as foreign agents expect higher chances of trading with domestic agents who accept that currency.

Our benchmark model features several countries, each populated by two agent types: buyers and sellers. Time is continuous. Agents trade continuously in a centralized market (CM), where intermediate input goods sourced from each country and labor are aggregated to produce final output. Governments issue nominal bonds, set nominal interest rates, and levy taxes on domestic agents in the CM. The main departure from canonical models concerns trade in intermediate goods. Instead of being traded in Walrasian markets, they are traded in a search-and-matching market where money is essential (the *decentralized* market, or DM). Agents meet at random in bilateral, anonymous meetings, where sellers from each country produce input goods for buyers, who bring them back to their domestic CM. The assumption that money is essential is motivated by evidence that limits to contract enforcement in international trade create the need for cash in advance or bonds posted as collateral in trade finance (e.g., Antràs and Foley, 2015).

A critical assumption concerns the acceptability of currencies in DM trade. By default, sellers recognize only their domestic currency, which implies they are unwilling to accept foreign currency. Thus, a country's bonds are more frequently accepted when purchasing input goods from that country. Imperfect recognizability is a common abstraction in the literature (Lester et al., 2012) that captures various reasons individuals or firms may not accept foreign currencies, such as overhead costs of hiring staff to manage currency exposures, legal tender restrictions, or compliance costs. Furthermore, this assumption is key to resolving the usual Kareken–Wallace indeterminacy (Kareken and Wallace, 1981).[2]

These administrative costs of accepting foreign currencies motivate our second key assumption: sellers must incur a fixed cost to recognize a foreign currency. When doing so, they take into consideration the discounted future gains from trade they will earn by trading with buyers who hold that currency. All else equal, sellers will be more willing to pay this cost for currencies widely held internationally, since doing so offers greater future trade opportunities. Crucially, we do not assume that these costs are high: when we calibrate the model, we find that the costs of recognizing foreign currencies are on the order of a few basis points (bp) of total trade quantities.

We derive our analytical results in a special case of the model with two countries: Home and Foreign. We begin by characterizing buyers' portfolio decisions, taking the liquidity of each currency as given. Then, we demonstrate our key results on the complementarities in sellers' decisions and the stability of dominant-currency regimes.

[2]Alternative approaches include Gomis-Porqueras et al. (2017), who introduce counterfeiting threats in a two-country model to address indeterminacy.

The key forces determining buyers' demand for each type of bond are (i) their demand for each country's input good, (ii) the probability that each currency will be accepted in trade, and (iii) the *liquidity premium* on each currency (i.e., the real return on an illiquid bond minus the real return on the corresponding government's nominal bonds). Agents have a home bias in their demand for input goods, which naturally translates into a home bias in portfolio composition. Nevertheless, agents may tilt their portfolios towards a foreign currency if that currency has a low liquidity premium or is widely accepted (e.g., if the foreign country is large or its currency is widely recognized globally).

Three types of international monetary regimes may emerge. First, in the "classical" regime, most sellers in each country accept only domestic currency. All buyers then hold a diversified portfolio of bonds, since they wish to purchase both Home and Foreign inputs. This regime corresponds to the one studied in most cash-in-advance models (e.g., Lucas, 1982; Svensson, 1985). Second, there are "dominant-currency" equilibria in which one currency is widely accepted internationally, so that the currency's share in trade settlement exceeds the domestic economy's share of trade in goods (Gopinath and Stein, 2021). Agents in the dominant-currency country hold only their domestic currency, whereas agents in other countries hold the dominant currency and may also hold their own currency. Finally, there is a "multipolar" regime in which agents hold only their own currency (given home bias in portfolio holdings), and all currencies circulate internationally.

We first examine how the international monetary regime depends on economic fundamentals and government policies, specifically the relative size of the two countries and the liquidity premium on each currency. A lower liquidity premium (i.e., a higher real rate of return on its debt) can help to internationalize a currency: high rates of return induce foreign investors to substitute towards a currency to meet their liquidity needs. Governments can target a high real rate of return on their debt by setting a high nominal interest rate relative to the growth rate of nominal liabilities: the gap between the two pins down how much of the interest bill must be tax-financed rather than covered by seigniorage. A larger country size also favors the circulation of its currency. For example, when Foreign is small relative to Home, Foreign agents are more willing to hold Home bonds, anticipating future trade opportunities with Home agents. Conversely, Home agents see fewer opportunities to trade with Foreign agents. That is, large economies can issue a dominant currency despite a high liquidity premium on their debt, e.g., the U.S. today or Britain at the turn of the 20th century (Chen et al., 2025).

We then turn to sellers' decisions to accept non-domestic currencies in order to study the determinants of currency dominance in the long run. The key insight here is that there is a natural two-way strategic complementarity between buyers' decision to hold a currency and sellers' decision to accept it. Namely, the returns to accepting a foreign currency are higher when global holdings of that currency are larger, and global demand for it is greater when it

is more widely accepted. For example, a Foreign seller's decision to recognize Home currency enhances the liquidity of Home bonds, thereby increasing demand for them. In turn, greater global holdings of Home currency raise the benefits of recognizing it.

These strategic complementarities have crucial implications. We demonstrate that there are no stable equilibria in the "classical" regime where all agents hold a diversified portfolio of bonds (except for the trivial equilibrium where the cost of accepting non-domestic currency is so high that no agent chooses to do it). The only non-trivial stable equilibria are dominant-currency equilibria (if the cost of recognizing non-domestic currency is intermediate) or multipolar equilibria (if the cost is small). The instability of classical regime equilibria implies that the long-run outcome may be very sensitive to initial conditions: a small change in policy (e.g., a small decline in one currency's liquidity premium) can lead to a different dominant currency in the long run. Once a dominant currency has emerged, though, it is difficult to dethrone: dominant-currency equilibria are stable, so they cannot unravel from small changes in policies or fundamentals.

Finally, we calibrate the model to assess the quantitative importance of this mechanism and the dynamics of currency competition in counterfactual scenarios. We study an extended setting with three regions: the U.S., the euro area, and the rest of the world (RoW), focusing on sellers' decisions in the RoW to accept dollars or euros. We find that the forces driving the world economy towards a dominant-currency equilibrium survive quantitatively, even in the extended setting.

Our first counterfactual experiment considers whether a dominant currency may lose its status if the world economy enters a "trade war" in which the U.S. imposes tariffs on imports and other countries retaliate via reciprocal tariffs on U.S. exports. We find that even with high and persistent tariffs (e.g., a permanent 30% tariff), the dollar maintains its dominance. The reason is that, even as trade with the U.S. declines, agents in Europe and the RoW continue to hold dollars to trade with each other. The failure of tariffs to destabilize international monetary arrangements is consistent with historical evidence: dominant currencies have maintained their status during eras of protectionism and volatility in international trade. Moreover, even if the dollar were to lose its dominant status, the real effects on trade would be quite modest. Quantitatively relevant parameter values imply a liquidity premium of less than 50bp and international holdings of domestic bonds not exceeding 50% of GDP. Hence, the "exorbitant privilege" can finance a stream of imports equal to at most 0.25% of GDP.

The second counterfactual experiment considers the fiscal version of the "Triffin dilemma" (Bordo and McCauley, 2018), which dictates that as emerging economies grow, a single developed economy will not have the fiscal capacity to meet the entire world's demand for liquid assets, prompting a shift to a multipolar system. We assume that the RoW grows faster than the U.S. and the euro area. The dollar begins as the dominant currency, but the U.S. must lower the

interest rate paid to bondholders if the interest burden on its debt exceeds its capacity to tax domestic agents. We find that although growth in the rest of the world may push the U.S. up against fiscal constraints, the resulting increase in the dollar liquidity premium does not prompt a broad shift away from dollars and into euros. Network externalities are strong enough that the dollar remains dominant despite lower returns on dollar-denominated assets.

Our final quantitative experiment asks whether China's economic rise could reshape the international monetary system. We introduce China as a fourth region in the global economy with its own currency, the renminbi. Motivated by China's recent efforts to better integrate its financial system with the global economy, we first consider a policy under which China expands its payment network (as it has done with CIPS). In our model, this policy amounts to providing Chinese agents with faster matching technology. We find that, absent an improved matching technology, the introduction of the renminbi into global markets (i.e., capital account liberalization) is insufficient to significantly reduce the dollar's share of global trade settlement and international reserves. However, the expansion of China's payment system could increase the renminbi's share of global trade and reserves, making it a secondary international currency while the dollar remains dominant. Second, we consider a policy in which China's government attempts to compete by shifting out its bond supply curve, thereby cutting its liquidity premium in half. This strategy is more successful: the renminbi's share of international reserves climbs to rival the dollar's.

Our paper is related to the literature on how currencies compete internationally as media of exchange. Building on earlier models (Matsuyama et al., 1993; Zhou, 1997; Wright and Trejos, 2001; Head and Shi, 2003; Camera and Winkler, 2003), we develop a tractable two-country model with divisible currency and no restrictions on portfolio holdings. We show that the model can be extended to address various international finance issues. Zhang (2014) also presents a two-country model with divisible currencies. Baughman and Flemming (2023) build on Zhang (2014) to develop a model of backed international currencies to study the role of stablecoins in cross-border payments. Madison (2024) examines fiscal policy and currency substitution in a domestic economy. In these papers, our contribution is to characterize the dynamics of the international monetary system analytically and to evaluate quantitatively scenarios that could alter international payment arrangements.

Our work also engages with studies of the emergence of "dominant" currencies. Gopinath and Stein (2021) emphasize *unit of account* frictions, arguing that strategic complementarities in trade invoicing and private debt denomination drive dominance. Farhi and Maggiori (2018) and Clayton et al. (2024) analyze currency competition as *stores of value* through reputation games, whereas Pflueger and Yared (2024) study the interaction between reserve asset competition and great power conflict.

Our model instead focuses on long-term competition between currencies as *media of exchange*,

asking which fundamental forces lead a currency to dominate as a liquid asset. This focus reflects historical precedents such as the Venetian ducat and Florentine florin, which were favored for their ease of storage and transport. Two important papers make related points. Chahrour and Valchev (2022) demonstrate how strategic complementarities in the choice of collateral can lead to the emergence and persistence of a dominant means of trade finance and study the interactions between patterns of trade and the dominant currency. Coppola et al. (2023) present a stylized theoretical model in which thick market externalities can create a dominant currency for cross-border debt settlement, focusing mainly on the role of fiscal policy in determining the dominant currency. We view our contribution as complementary. By endogenizing the medium of exchange in an otherwise canonical international monetary model, we can examine how a range of counterfactuals related to trade, finance, and monetary policy shape the international monetary system and its dynamics in general equilibrium.

Finally, our paper is connected to the broader literature on the "exorbitant privilege" of the U.S. (Gourinchas and Rey, 2005; Gourinchas and Rey, 2022). Gourinchas and Rey (2007) document how low returns on U.S. debt sustain trade deficits. Caballero et al. (2008) and Maggiori (2017) model the emergence of exorbitant privilege in contexts of global financial imbalances. Jiang (2024) explains the cyclical behavior of dollar convenience yields and U.S. debt returns, while Jiang and Richmond (2023) study a global fiscal cycle of reserve currency competition. In our model, the larger country enjoys an exorbitant privilege by internationalizing its currency while paying low interest rates. It also sustains persistent trade deficits by raising seigniorage revenues from foreign agents.

The rest of the paper is organized as follows. Section 2 introduces the model, and Section 3 derives the equilibrium conditions. Section 4 restricts the model to a two-country environment and presents the main analytical results on international monetary regimes, comparative statics, and the stability of dynamic equilibria. Our calibration and counterfactual exercises are presented in Section 5. Section 6 concludes. The Appendix contains all proofs.

2 Model

This section introduces a general model of the international monetary system. Each country's government issues bonds that circulate as media of exchange both domestically and internationally. The key assumption that differentiates our model from canonical international monetary models is that agents cannot always recognize foreign-currency bonds. Instead, we endogenize the degree to which each currency is accepted internationally. We outline the model and derive its equilibrium, setting the stage for analyzing the conditions under which currencies circulate internationally.

Setting. Time is continuous and infinite, $t \in [0, \infty)$, and there is no aggregate uncertainty.

The global economy consists of J regions indexed by j. The global population is normalized to one, with a fraction ξ^j of agents residing in region j. Each region's population is split equally between two types of agents: *buyers* and *sellers*. There is a global centralized market (CM) in which trade occurs continuously in a Walrasian setting: goods are traded within regions, while assets can be traded across regions. International trade in goods occurs in pairwise meetings in which agents match at random, according to a Poisson process described below. We refer to these meetings collectively as the decentralized market (DM).[3]

There are $N \leq J$ governments, corresponding to regions $j \in \{1, \ldots, N\}$, that conduct fiscal and monetary policy. Governments levy taxes on domestic agents and issue bonds denominated in their domestic currencies. This structure permits monetary unions as well as situations in which a region lacks its own currency. For example, in our quantitative model, there is a region (the "rest of the world") in which agents choose whether to accept dollars or euros.

Technology. Production takes place in two stages. The first stage occurs in the DM, where bilateral meetings arrive according to a Poisson process with rate λ. Trading partners are drawn uniformly at random from the world population; so, conditional on a meeting, the probability that an agent's trading partner is from region j is ξ^j (the assumption of uniform matching probabilities is analytically convenient but not essential to the main results). In DM meetings, a region-i seller can produce a region-i *raw input* good z_i at a linear disutility cost. A region-j buyer can convert region-i raw inputs one-for-one into an *intermediate input* good x_i^j.

The second stage of production takes place in the CM. In each region j, inputs purchased by region-j buyers from other regions are aggregated into a *composite intermediate* good X^j according to the CES production function

$$X_t^j = \left(\sum_{i=1}^{J} {\omega_i^j}^{\frac{1}{\sigma}} {x_{it}^j}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 0$ is the elasticity of substitution across intermediate inputs (analogous to an Armington elasticity) and $\omega_i^j \geq 0$ denotes region j's expenditure weight on region-i inputs, with $\sum_{i=1}^{J} \omega_i^j = 1$ for all j.[4]

Competitive firms then aggregate intermediate inputs and labor, supplied by domestic agents, into final output using a Cobb–Douglas production function:

$$y_t^j = {X_t^j}^{\alpha} {\ell_t^j}^{1-\alpha}, \tag{2}$$

where $1 - \alpha \in (0, 1)$ denotes labor's share of output. Goods cannot be traded across regions or

[3] The continuous-time version of the Lagos-Wright model is developed in Choi and Rocheteau (2020).

[4] When $\sigma = 1$, we take limits and set $X_t^j = \prod_{i=1}^{J} {x_{it}^j}^{\omega_i^j}$.

physically invested in the CM. As a result, the resource constraint in each region is

$$c_t^j = y_t^j, \tag{3}$$

where c_t^j denotes consumption in region j.

Preferences. Agents derive utility from consumption and disutility from labor in the CM and from producing raw inputs in the DM. As is standard in New Monetarist models, we assume that agents have CRRA preferences over consumption and suffer linear disutility from labor.

Let $\{T_n\}_{n=1}^{\infty}$ denote the (countable) set of times at which an agent enters the DM. The lifetime utility of a region-j agent is then:

$$U^j = \mathbb{E}_0\left[\int_0^{\infty} e^{-\rho t}\left(\frac{{c_t}^{1-\gamma}-1}{1-\gamma} - \ell_t\right)dt - \sum_{k=1}^{\infty} e^{-\rho T_k} z_{T_k}\right].$$

Here, z_{T_k} denotes raw inputs produced in the DM, c_t is individual consumption of region j's final good, and ℓ_t is labor supplied in the CM. The parameter $\rho > 0$ is the subjective discount rate, and $\gamma^{-1} > 0$ is the intertemporal elasticity of substitution.[5]

Assets and the CM. In the CM, there are Walrasian markets for labor, intermediate inputs, final output, and financial assets. Since goods cannot be traded across countries in the CM, input and final-good prices may differ across countries. Final output in region $j = 1$ is used as the numéraire, and q_t^j denotes the real exchange rate, defined as the price of region j's final output relative to that of region 1. In region j, region-i intermediate goods x_i^j trade at price $q_t^j \psi_{it}^j$, and wages are $q_t^j w_t^j$, so that ψ_{it}^j and w_t^j are input prices and wages measured in units of domestic output.

There are two types of financial assets in the world economy: nominal bonds denominated in currencies $n \in \{1, \ldots, N\}$ and privately issued illiquid, risk-free debt in zero net supply. Illiquid bonds can be viewed as privately issued financial claims in international markets, such as corporate debt or equity. A bond issued by government n pays an instantaneous nominal interest rate $i_t^n\, dt$. The price of currency-n bonds in the CM is denoted by φ_t^n, so the real return on currency-n debt is:

$$r_t^n = i_t^n + \frac{\dot{\varphi}_t^n}{\varphi_t^n},$$

where a dot denotes a time derivative. Illiquid bonds pay a market-clearing real interest rate $r_t\, dt$.

The DM. In the DM, buyers and sellers are anonymous, with private trading histories, making a medium of exchange essential. Buyers and sellers negotiate the terms of trade according to a Nash bargaining protocol, in which the buyer has a share $\theta \in [0, 1]$ of the bargaining power.

[5] When $\gamma = 1$, we take limits and set the utility of consumption equal to $\log(c_t)$.

Following Zhang (2014), we break the indeterminacy result of Kareken and Wallace (1981) by assuming that sellers always recognize their domestic currency but cannot always recognize non-domestic currencies. Specifically, only an endogenous fraction $\delta_{nt}^j \in [0,1]$ of region-j sellers can recognize currency n, with $\delta_{nt}^n = 1$ for all $n \leq N$ and all t. Under this assumption, currencies are imperfect substitutes and can therefore earn different rates of return: each currency is more liquid when purchasing the corresponding region's input good.

Formally, imperfect recognizability means that sellers cannot distinguish genuine units of non-domestic currency from counterfeits. This assumption captures a range of institutional frictions that make accepting foreign-currency payments costly, including compliance and legal costs, the need to manage currency risk, or, simply, a lack of familiarity with a foreign currency. As in Lester et al. (2012), we make the analytically convenient (though strong) assumption that counterfeit currency can be produced at no cost, which implies that sellers will never accept a currency they cannot recognize.[6]

We endogenize currency recognizability by assuming that sellers can make a fixed-cost investment to recognize a foreign currency in the future. Specifically, for each n, there is a technology that allows region $j \neq n$ sellers to recognize that currency. According to a Poisson process with rate ζ, a region-j seller receives an opportunity to pay a utility cost κ to obtain this technology, or to maintain an existing investment.[7] A seller who invests can recognize currency n until the next such event, at which point the existing investment expires. As a result, the fraction of region-j sellers who recognize currency n evolves according to

$$\dot{\delta}_{nt}^j = I_{nt}^j - \zeta \delta_{nt}^j, \tag{4}$$

where $I_{nt}^j dt \in [0, \zeta dt]$ denotes the measure of sellers who choose to incur the utility cost at time t. The recognizability friction applies only to sellers in the DM. Agents can trade currencies freely in the CM, and buyers can carry any currency into the DM without investing in the technology.

Importantly, in the calibration, we find that the utility cost κ required to generate meaningful results is realistically small. Thus, even modest administrative frictions to accepting foreign currencies are sufficient to generate stable dominant-currency regimes.

Governments. Governments service their debt by levying lump-sum taxes τ_t^n on domestic agents and by issuing nominal bonds B_t^n in the CM. Government policy is given by a path of bond issuances $\{B_t^n\}_{t=0}^{\infty}$ and a path of nominal interest rates $\{i_t^n\}_{t=0}^{\infty}$ on domestic-currency debt. The growth rate of the stock of currency-n debt is denoted by $\mu_t^n \equiv \dot{B}_t^n / B_t^n$.

[6] Lester et al. (2012) show that the results are qualitatively unchanged when counterfeiting is costly. However, such costs substantially complicate the analysis by inducing probabilistic strategies.

[7] These Poisson events are independently distributed across currencies n.

Government n's budget constraint, expressed in terms of the numéraire good, is:

$$\dot{b}_t^n = r_t^n b_t^n - q_t^n \tau_t^n, \tag{5}$$

where $b_t^n \equiv \varphi_t^n B_t^n$ denotes the real quantity of currency-n bonds outstanding at time t. The real rate on currency-n debt, r_t^n, is determined in equilibrium, and taxes $\{\tau_t^n\}_{t=0}^{\infty}$ adjust to satisfy equation (5) at each instant.

We assume that each government n uses its monetary and fiscal policy tools (i_t^n, μ_t^n) to target a path for the *liquidity premium* l_t^n on its debt, defined as the difference between the return r_t on an illiquid bond and the return on currency-n assets:

$$l_t^n \equiv r_t - r_t^n.$$

The liquidity premium is the cost, in forgone interest, that agents incur when holding a currency. In steady state, the real return on currency n is $i^n - \mu^n$ (since prices grow at rate μ^n), so the liquidity premium is simply $l^n = \rho - (i^n - \mu^n)$. Note that $i^n - \mu^n$ is the portion of the interest bill financed by taxes rather than by debt issuance. Hence, greater backing of the debt with taxes implies a lower liquidity premium.[8]

Remark 1 *In our model, governments are the sole issuers of liquid assets, which we refer to as "bonds." Ultimately, however, what matters for currency demand is the liquidity premium on each currency. In our model, one could think of cash as a liquid asset that pays no interest, i.e., $i^n = 0$. A metallic standard (such as the gold standard) could be thought of as a constraint that μ_t^n must adjust to target a particular path for the price level. Hence, our model can accommodate other types of publicly issued liquid assets.*

Remark 2 *We abstract from private liquidity (e.g., bank deposits) for simplicity. Given that governments elastically supply liquidity at a given price in our model, private issuance would not affect the liquidity premium. Of course, this would no longer be true if governments inelastically supplied liquid assets. However, private issuance would matter in one sense: the government would no longer capture the liquidity premium on the entire stock of domestic-currency debt, since private issuance would make up part of that stock.*

Remark 3 *In our model, all liquid assets are used for cross-region trade in goods. However, all that is required for our main comparative statics and analytical implications on the stability of*

[8] Governments have two instruments, the nominal rate and the rate of bond issuance, to target one variable, the liquidity premium. This leaves one degree of freedom. In Appendix B.2, we show that for any path of the nominal rate i_t^n (respectively, any path of bond issuance μ_t^n), there exists a path of μ_t^n (respectively, i_t^n) that implements the desired liquidity premium in equilibrium. However, for a given policy path, there may be multiple equilibria with different inflation rates. An analysis of how the government can rule out such equilibria using off-equilibrium policies is beyond the scope of this paper.

equilibria is some source of gains from trade that is decreasing in the quantities traded. Hence, we conjecture our results would continue to hold if liquid assets were used for other purposes, such as trade or settlement in financial markets, in which case the parameters ω_j^i and ξ^j could be reinterpreted as determinants of country j's centrality in financial trade.

3 Equilibrium

We now outline the model's equilibrium conditions. We first demonstrate the linearity of agents' Bellman equations, since this will be a key result in characterizing DM outcomes. Then, we solve the DM bargaining problem, which permits us to derive buyers' static portfolio decisions and sellers' dynamic decision to invest in recognizing foreign currencies. Finally, we present the goods and asset market clearing conditions in the CM and define an equilibrium.

3.1 Bellman equations

Consider the problem of an agent in the CM. The agent's individual state variables at time t are total financial asset holdings a_t and a type η_t indicating their region of residence and the set of currencies they recognize. Hence, we write $\eta_t = (j, B)$ if the agent is a region-j buyer and $\eta_t = (j, S_t)$ if the agent is a region-j seller, where $S_t \subset \{1, \ldots, N\}$ denotes the set of currencies that the agent recognizes at t. The agent chooses consumption c_t, labor ℓ_t, and asset holdings $\mathbf{b}_t = (b_{1t}, \ldots, b_{Nt}, b_{It})$ (where b_{it} denotes real holdings of country-i bonds and b_{It} denotes holdings of illiquid bonds), taking prices as given. These choices are subject to short-selling constraints on government bonds, and the portfolio allocation constraint

$$\sum_{n=1}^{N} b_{nt} + b_{It} = a_t, \tag{6}$$

and the flow budget constraint

$$\dot{a}_t = \sum_{n=1}^{N} r_t^n b_{nt} + r_t b_{It} + q_t^j \left(w_t^j \ell_t - c_t - \tau_t^j \right), \tag{7}$$

where τ_t^j is the tax levied by region j's government (if it has one) in the CM.

Let $W_t(a|\eta)$ represent the CM value function of an agent of type η with assets a, and let $V_t(\mathbf{b}|\eta)$ be the expected value upon entering the DM, conditional on bond portfolio $\mathbf{b}$. The value

function satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}\rho W_t(a|\eta) = \max_{c,\ell,\mathbf{b}} \frac{c^{1-\gamma}-1}{1-\gamma} - \ell + \lambda\big(V_t(\mathbf{b}|\eta) - W_t(a|\eta)\big) \\ + \tilde{\zeta}^{\eta}\big(\tilde{W}_t(a|\eta) - W_t(a|\eta)\big) + \frac{d}{dt}W_t(a|\eta) \\ \text{s.t. (6), (7), } b_{nt} \geq 0. \end{aligned} \tag{8}$$

The second line captures the change in a seller's value function when the seller has the opportunity to pay the cost to recognize currency n. The term $\tilde{\zeta}^{\eta}$ is the rate at which the agent gets an opportunity to invest in the technology to recognize a currency: it is equal to zero for buyers and $\sum_{n\neq j}\zeta$ for sellers, since an opportunity for each individual currency $n \neq j$ arrives at rate ζ. The term $\tilde{W}_t(a|\eta)$ is the seller's expected continuation value in this contingency, averaging over all $n \neq j$. We return to this term later, when we solve the sellers' problem.

Assuming utility is quasi-linear in labor, value functions become linear in asset values, with the coefficient on asset values equal to the inverse of the domestic wage.

Proposition 4 *There exists a constant $\hat{W}_t(\eta)$ for each t and η such that:*

$$W_t(a|\eta) = \hat{W}_t(\eta) + \frac{a}{q_t^j w_t^j}.$$

As is typical in the New Monetarist literature, this result is convenient because it implies we do not need to track the distribution of wealth. All agents in a region have the same marginal value of wealth, regardless of their previous trading histories. In turn, this implies all agents of the same type have identical demand curves and behave identically in the DM. Next, we solve for DM outcomes to derive the value functions $V(\mathbf{b}|\eta)$.

3.2 DM bargaining

To characterize bargaining outcomes in the DM, we begin by solving for the terms of trade. By Proposition 4, a region-i seller is willing to produce one unit of raw input goods (i.e., incur one unit of disutility) in exchange for a payment equal to their domestic wage $q_t^i w_t^i$. A region-j buyer values region-i raw input goods at $q_t^j \psi_{it}^j$. Therefore, there are gains from trade as long as $q_t^j \psi_{it}^j > q_t^i w_t^i$. Hence, let

$$\Psi_{it}^j \equiv \frac{q_t^j \psi_{it}^j}{q_t^i w_t^i}$$

be an indicator of the gains from trade between a region-j buyer and a region-i seller. The following proposition characterizes the Nash bargaining outcome in this case.

Proposition 5 *The price at which a region-i seller sells raw inputs to a region-j buyer in the DM is*

$$\phi_{it}^{j} = \frac{\Psi_{it}^{j}}{1+\theta(\Psi_{it}^{j}-1)} q_t^i w_t^i. \tag{9}$$

The buyer spends all bonds that the seller can recognize if $\Psi_{it}^{j} > 1$.

This solution to the Nash bargaining problem implies that the split of surplus realized between a region-j buyer with a portfolio of bonds $\mathbf{b} = (b_1, \ldots, b_N, b_I)$ and a region-i seller who recognizes a set of currencies S is

$$\text{DM surplus} = \left(\Psi_{it}^{j} - 1\right)^{+} \sum_{n \in S} b_n \times \begin{cases} \theta / q_t^j w_t^j & \text{buyer} \\ (1-\theta)/\Psi_{it}^{j} q_t^i w_t^i & \text{seller} \end{cases}$$

This total surplus is obtained by multiplying the per-unit surplus, $q_t^j \psi_{it}^{j} - q_t^i w_t^i$, by the quantity produced, $\sum_{n \in S} b_n / \phi_{it}^{j}$ (i.e., the quantity of the buyer's bonds the seller can recognize divided by the price).

Averaging over agents' possible trading partners yields the following characterization of DM value functions.

Proposition 6 *The expected value obtained by a type-η agent with bond portfolio* $\mathbf{b}$ *in the DM is*

$$V_t(\mathbf{b}|\eta) = W_t(a|\eta) + \frac{1}{q_t^j w_t^j} \times \begin{cases} \theta \sum_{i=1}^{J} \sum_{n=1}^{N} \xi^i \delta_{nt}^i \left(\Psi_{it}^{j} - 1\right)^{+} b_n & \textit{buyer} \\ (1-\theta) \sum_{i=1}^{J} \sum_{n \in S} \xi^i \left(1 - \frac{1}{\Psi_{jt}^{i}}\right)^{+} b_{nt}^i & \textit{seller} \end{cases} \tag{10}$$

where $a = \sum_{n=1}^{N} b_n + b_I$ *denotes total assets and* b_{nt}^i *denotes aggregate holdings of currency-n bonds by region-i buyers.*

This proposition yields two crucial results. First, a buyer's value function is linear in bond holdings of each type, which will allow us to derive independent Euler equations for each type of bond. Second, a seller's value function is additive in the set of currencies recognized in the DM, implying that from the perspective of an individual seller, decisions to recognize a currency are independent of past decisions to recognize other currencies. Both of these properties are useful in keeping individual decision problems analytically tractable. However, we will demonstrate that there are important strategic interactions between agents' decisions at the aggregate level.

3.3 Portfolio choices and currency recognizability

Having derived the DM value functions, we can proceed to solve for buyers' portfolio allocations and sellers' decisions regarding the recognition of foreign currencies.

Portfolio choices. The optimality conditions for liquid bond holdings reflect the tradeoff between the transaction benefits a currency yields in the DM and the cost of holding that currency. For an agent to hold currency n, the liquidity premium must be at least equal to the benefit of holding that currency, which from the Bellman equation (8) is $\lambda\frac{\partial(V_t(\mathbf{b}|\eta)-W_t(a|\eta))}{\partial b_n}$. Using the solution (10) for the DM value function, the Euler inequality determining region-j buyers' holdings of currency-n bonds can then be written as

$$l_t^n \geq \lambda\theta \sum_{i=1}^{J} \xi^i \delta_{nt}^i \big(\Psi_{it}^j - 1\big)^+ \quad \text{with equality if } b_{nt}^j > 0. \tag{11}$$

The demand for a currency, hence, depends on both its liquidity premium and its liquidity benefits in international markets. The right-hand side reflects that, all else equal, agents are more willing to hold a currency n if it is widely accepted (high δ_{nt}^i). A currency that is not frequently accepted may have small enough liquidity benefits that some agents choose not to hold it at all. Indeed, in our dominant-currency equilibria, the dominant currency will be the only currency liquid enough in the DM for foreign agents to hold.

Sellers' decisions. We now turn to sellers' decisions to pay the cost to recognize foreign currencies. The (expected) value of a type $\eta = (j, S)$ seller who receives an opportunity to pay the cost to recognize some currency at t, $\tilde{W}_t(a|\eta)$ in (8), satisfies

$$\tilde{\zeta}^\eta \tilde{W}_t(a|j,S) = \sum_{n\neq j} \zeta \max\Big\{W_t(a|j, S\cup\{n\}) - \kappa,\ W_t(a|j,S\setminus\{n\})\Big\}. \tag{12}$$

With intensity ζ, the seller receives an opportunity to pay the cost κ to recognize currency n. A seller who pays the cost can recognize currencies $S\cup\{n\}$ thereafter, whereas a seller who does not pay the cost does not recognize n going forward. We must then derive the value $W_t(a|j,S\cup\{n\}) - W_t(a|j,S\setminus\{n\})$ of recognizing currency n.

From the Bellman equation, the flow value to a region-j seller of being able to recognize a currency is just the meeting rate, λ, times the value of being able to recognize that currency in the DM. Per (10), sellers' flow value in the DM is linear in the set of currencies accepted by the seller, where

$$v_{nt}^j = \frac{\lambda(1-\theta)}{q_t^j w_t^j} \sum_{i=1}^{J} \xi^i \big(1 - \frac{1}{\Psi_{jt}^i}\big)^+ b_{nt}^i \tag{13}$$

is the flow value of accepting currency n. The value of recognizing currency n is equal to a discounted sum of future flow values v_{nt}^j. In the Appendix, we show that sellers choose to pay the cost to accept currency n whenever the present value v_{nt}^j is sufficiently high.

Proposition 7 *It is optimal for region-j sellers to pay the cost to recognize currency n at time*

t if and only if

$$\kappa \leq \int_0^\infty e^{-(\rho+\zeta)s} v_{n,t+s}^j \, ds. \tag{14}$$

Note that the discount rate in the integral is $\rho+\zeta$, since sellers have to renew their investment with intensity ζ if they wish to continue accepting currency n. Intuitively, from (13), sellers find it worthwhile to accept a currency as long as (1) buyers hold large quantities of that currency, and (2) there are large gains from trade to be realized with such buyers. In our analytical two-country model, we demonstrate how this logic can lead to strategic complementarities in sellers' decisions.

3.4 Goods markets, asset markets, and equilibrium

We have solved buyers' and sellers' problems in terms of the equilibrium gains from trade Ψ_{it}^j, aggregate bond holdings b_{nt}^j, and real wages $q_t^j w_t^j$. Goods market clearing yields an additional equilibrium condition relating gains from trade to bond holdings. Equilibrium in international financial markets pins down wage-based real exchange rates.

Goods markets. Firms in region j purchase quantities $\{x_{it}^j\}_{i=1}^J$ of intermediate inputs and hire labor ℓ_t^j to maximize profits, taking the prices $\{\psi_{it}^j\}_{i=1}^J$ and w_t^j as given:

$$\max_{\{x_{it}^j\}_{i=1}^J, \ell_t^j} \; y_t^j - \sum_{i=1}^J \psi_{it}^j x_{it}^j - w_t^j \ell_t^j \quad \text{s.t.} \;\; (1), \; (2).$$

This optimization problem yields the usual first-order conditions, which state that the marginal product of intermediate inputs should be set equal to their price,

$$\psi_{it}^j = \frac{\alpha y_t^j}{X_t^j} \left(\frac{x_{it}^j}{\omega_i^j X_t^j} \right)^{-\frac{1}{\sigma}}, \tag{15}$$

and that the marginal product of labor should be set equal to the wage,

$$w_t^j = \frac{(1-\alpha) y_t^j}{\ell_t^j}. \tag{16}$$

The labor supply condition derived from the Bellman equation (8) equates the marginal utility of consumption times the wage to the marginal disutility of labor:

$$w_t^j = \left(c_t^j / \xi^j \right)^\gamma, \tag{17}$$

since c_t^j/ξ^j is per-capita consumption.

These equations yield one relationship between input quantities and prices coming from firm demand in the CM. The quantity of intermediate inputs supplied in the CM equals the quantity

purchased by buyers in the DM, which depends on their bond holdings. Specifically, region j buyers' aggregate expenditures on region-i inputs, $\phi_{it}^j x_{it}^j$, satisfy

$$\phi_{it}^j x_{it}^j \leq \lambda \xi^i \sum_{n=1}^{N} \delta_{nt}^i b_{nt}^j \quad \text{with equality if} \quad \Psi_{it}^j > 1. \tag{18}$$

By Proposition 5, actual expenditures are equal to this maximum whenever there are gains from trade ($\Psi_{it}^j > 1$), since buyers spend all available bonds in that case. When combined with the terms of trade condition (9), portfolio decisions (11), and wage-based real exchange rates $q_t^i w_t^i / q_t^j w_t^j$, the goods market clearing conditions can be used to pin down the gains from trade Ψ_{it}^j and bond holdings b_{nt}^j.

International financial market equilibrium. It remains to specify the equilibrium conditions that pin down exchange rates and the real interest rate on illiquid bonds.

Agents hold illiquid bonds only to smooth consumption over time, yielding a typical Euler equation derived from (8):

$$r_t = \rho + \frac{\dot{q}_t^j}{q_t^j} + \gamma \frac{\dot{c}_t^j}{c_t^j} \quad \forall\ j \in \{1, \ldots, J\}. \tag{19}$$

The Euler equation (19), along with agents' labor-leisure optimization condition (17), yield a Backus-Smith condition for the goods-based real exchange rate,

$$\frac{q_t^j {c_t^j}^\gamma}{{c_t^1}^\gamma} = \left(\frac{\xi^j}{\xi^1}\right)^\gamma \frac{q_t^j w_t^j}{w_t^1} = \text{constant},$$

which in this model is equivalent to a constant wage-based real exchange rate.

The real exchange rate adjusts in equilibrium to balance countries' intertemporal budget constraints. Region j's net foreign asset position n_t^j evolves according to

$$\dot{n}_t^j = r_t n_t^j + \underbrace{\sum_{i=1}^{J} (\phi_{jt}^i x_{jt}^i - \phi_{it}^j x_{it}^j)}_{\text{net exports}} + \underbrace{l_t^j \sum_{i=1}^{J} b_{jt}^i - \sum_{n=1}^{N} l_t^n b_{nt}^j}_{\text{liquidity premia}}. \tag{20}$$

That is, region j's net saving consists of interest income on assets, net exports, and liquidity premia paid by foreign agents to hold currency-j bonds, minus liquidity premia paid by domestic agents to hold foreign-currency bonds. The wage-based exchange rate adjusts so that the transversality condition holds for all countries:

$$\lim_{T \to \infty} \exp\left(- \int_0^T r_t \, dt \right) n_T^j = 0 \quad \forall\ j \in \{1, \ldots, J\}. \tag{21}$$

World equilibrium. A *world equilibrium* consists of paths of prices $\{\psi_{it}^j, \phi_{i,t}^j, l_t^n, r_t, w_t^j, q_t^j\}$, quantities $\{c_t^j, y_t^j, \ell_t^j, x_{it}^j, X_t^j, \tau_t^j\}$, bond portfolios $\{b_{nt}^j, b_{It}^j\}$, dynamics of the aggregate state variables $\{\delta_{nt}^j\}$, and net foreign asset positions $\{n_t^j\}$ such that equations (1)–(3), (5), (9)–(11), and (15)–(21) hold at all dates, and the dynamics of the aggregate state are consistent with conditions (4) and (14).

4 Monetary regimes and dominant currencies

In this section, we study a two-country, two-currency version of the model with convenient parametric restrictions to derive analytical results. These restrictions are relaxed below when we calibrate the model.

We refer to the two countries as Home (H) and Foreign (F). We assume log utility and a Cobb-Douglas production function, $\gamma = \sigma = 1$. Further, we impose home bias in trade: for each country $j \in \{H, F\}$, the expenditure weight on domestic goods is

$$\omega_j^j = \frac{\beta \xi^j}{\beta \xi^j + \xi^{-j}},$$

where $\beta > 1$ is the home bias parameter. Hence, each country's expenditure weight on its goods is greater than its share of the global population. Finally, for simplicity, in this section we let $\delta_{Ht} = \delta_{Ht}^F$ denote the fraction of Foreign sellers who recognize Home currency. Similarly, we let $\delta_{Ft} = \delta_{Ft}^H$ be the fraction of Home sellers who recognize Foreign currency.

We first characterize the different regimes of currency circulation that may arise and comparative statics with respect to governments' monetary policies. Then, we derive our key results on the strategic complementarities in sellers' decisions to accept non-domestic currency and the stability of dominant-currency regimes.

4.1 Currency circulation regimes

We characterize the possible monetary regimes by combining agents' Euler equations for liquid bonds with a goods market clearing condition in the market for internationally traded inputs.

Euler equations: The Euler equation for a country-j buyer with respect to Home bonds is

$$l_t^H \geq \lambda\theta\left(\xi^H\left(\Psi_{Ht}^j - 1\right)^+ + \xi^F \delta_{Ht}\left(\Psi_{Ft}^j - 1\right)^+\right), \tag{22}$$

whereas the Euler equation for Foreign bonds is

$$l_t^F \geq \lambda\theta\left(\xi^H \delta_{Ft}\left(\Psi_{Ht}^j - 1\right)^+ + \xi^F\left(\Psi_{Ft}^j - 1\right)^+\right), \tag{23}$$

where the superscript $(\cdot)^+$ denotes the positive part of the term in parentheses. Since currencies are imperfectly recognizable, they are not perfect substitutes. In particular, Home currency is more frequently accepted in transactions for Home inputs, whereas Foreign currency is more frequently accepted in transactions for Foreign inputs. Hence, both currencies can circulate even if they yield different rates of return.

The Euler equations may not hold with equality because agents may choose not to hold one kind of currency. These are precisely the conditions under which a dominant currency can arise (e.g., if Home buyers choose to conduct all transactions in Home currency). When does this occur?

Goods market clearing: To answer this question, we must consider a second key equilibrium condition that comes from goods market clearing. Per (18), expenditures by country-j buyers on country-i goods are bounded by their holdings of bonds that are accepted in country i:

Proposition 8 *Aggregate DM expenditures of country-j buyers on country-i inputs, $\phi^j_{it}x^j_{it}$, satisfy*

$$\phi^j_{it}x^j_{it} = \underbrace{\frac{\alpha\omega^j_i \times \xi^j q^j_t w^j_t}{1+\theta(\Psi^j_{it}-1)}}_{\text{expenditures on } i} \leq \underbrace{\lambda\xi^i\left(\delta^i_{Ht}b^j_{Ht} + \delta^i_{Ft}b^j_{Ft}\right)}_{\text{bonds accepted in } i} \quad \text{for } j,i \in \{H,F\}. \tag{24}$$

The right-hand side is the maximum quantity of bonds that country-j buyers can spend in transactions with country-i sellers per unit of time. The left-hand side represents their aggregate demand for country-i inputs, which has three parts: country j's expenditure share $\alpha\omega^j_i$ on country-i inputs, times a demand shifter $\xi^j q^j_t w^j_t$ (country size times real wages), divided by a term that is increasing in the gains from trade. When country j imports a large quantity of country-i inputs, then the price of i-inputs (and therefore the gains from trade) fall.

If country-j buyers hold Home currency only and spend all of their bonds in DM meetings, then the ratio of expenditures on Home versus Foreign goods will be $\phi^j_{Ht}x^j_{Ht}/\phi^j_{Ft}x^j_{Ft} = \xi^H/\xi^F\delta_{Ht}$, since a fraction ξ^H of meetings are with Home sellers, and a fraction $\xi^F\delta_{Ht}$ are with Foreign sellers who accept Home currency. Similarly, if country-j buyers hold only Foreign bonds, the ratio of expenditures is $\phi^j_{Ht}x^j_{Ht}/\phi^j_{Ft}x^j_{Ft} = \xi^H\delta_{Ft}/\xi^F$.

Hence, if country-j buyers choose to hold both bond types and there are gains from trade with both countries, the expenditure ratio must lie between these two extremes. On the other hand, if there are no gains from trade with Home ($\Psi^j_{Ht} = 1$), the ratio of expenditures can fall below $\xi^H\delta_{Ft}/\xi^F$, or if there are no gains from trade with Foreign ($\Psi^j_{Ft} = 1$), the expenditure ratio can rise above $\xi^H/\xi^F\delta_{Ht}$. Applying (18), we get the following proposition.

Proposition 9 *Country-j buyers hold both currencies if there exists a solution $(\Psi^j_{j,t}, \Psi^j_{-j,t})$ to*

the Euler equations (22)–(23) *with*

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)^+}{1+\theta(\Psi^j_{j,t}-1)^+} \in \left[\delta_{-j,t}, \frac{1}{\delta_{j,t}}\right].$$

Otherwise, they hold domestic currency only if the domestic-currency Euler equation holds with equality and

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)^+}{1+\theta(\Psi^j_{j,t}-1)^+} \begin{cases} = \frac{1}{\delta_{j,t}} & \Psi^j_{j,t}, \Psi^j_{-j,t} > 1 \\ > \frac{1}{\delta_{j,t}} & \Psi^j_{-j,t} = 1 \\ < \frac{1}{\delta_{j,t}} & \Psi^j_{j,t} = 1 \end{cases}. \tag{25}$$

Finally, they hold foreign currency only if the foreign-currency Euler equation holds with equality and

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)^+}{1+\theta(\Psi^j_{j,t}-1)^+} \begin{cases} = \delta_{-j,t} & \Psi^j_{j,t}, \Psi^j_{-j,t} > 1 \\ > \delta_{-j,t} & \Psi^j_{-j,t} = 1 \end{cases}. \tag{26}$$

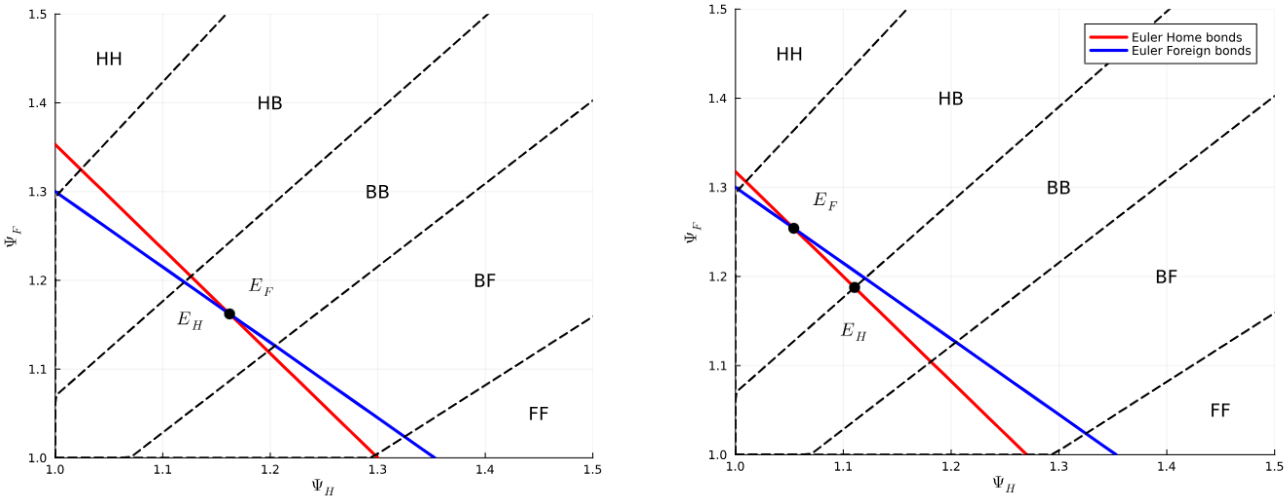


Figure 1: The solution to the Euler equations is plotted for two parameter configurations. Both panels have $\lambda = 0.2, \theta = 1.0, \xi = 0.5, \delta_H = \delta_F = 0.15, \beta = 1.1, l^F = 0.03$. The first letter in each region marks the currency held by Home, and the second marks the currency held by Foreign ("H" for Home, "B" for both, or "F" for Foreign). In the left panel, $l^H = 0.03$, and in the right panel, l^H decreases to 0.027. The equilibrium shifts from one in which both countries hold both currencies to one in which Home holds only its domestic currency. In the left panel, (Ψ^F_H, Ψ^F_F) is at the point E_F, whereas (Ψ^H_H, Ψ^H_F) is at the point E_H.

Figure 1 illustrates the different monetary regimes that can emerge. In the left panel, Home and Foreign bonds have equal liquidity premia. Agents from each country hold both bonds, and the marginal products of Home and Foreign inputs are equalized across countries ($\Psi^H_H = \Psi^F_H$ and $\Psi^H_F = \Psi^F_F$). The right panel shows the case where Home's liquidity premium is lower than Foreign's. Foreign agents continue to hold both types of bonds, while Home agents hold only domestic bonds. Since the liquidity premium on Home bonds is low, Home agents try to tilt their

expenditures towards domestic goods. Once their entire portfolio consists of domestic bonds, they cannot further decrease their expenditures on Foreign goods, and in equilibrium $\Psi_H^H > \Psi_H^F$ and $\Psi_F^H < \Psi_F^F$.

This analysis can be used to characterize how the international monetary regime depends on policies (l^H, l^F) and the model's primitives. Define

$$\tilde{l}_t^j \equiv \frac{l_t^j - \delta_{jt} l_t^{-j}}{\xi^j \left(1 - \delta_{Ht}\delta_{Ft}\right)}$$

to be the *effective* liquidity premium associated with purchasing country-j goods. This is a measure of the cost of carry for a bond portfolio that purchases one unit of country-j goods and zero units of the other country's goods, adjusting for the probability of matching with a country-j seller. The portfolio consists of a long position in currency j and a short position in the other currency. The effective liquidity cost depends both on currency j's liquidity premium and on the probability that currency j is accepted, which increases with country j's size and the fraction of foreign agents who recognize that currency.

If country-i agents hold both types of bonds, then $\Psi_{Ht}^i = \Psi_{Ht}^*$ and $\Psi_{Ft}^i = \Psi_{Ft}^*$, where

$$\Psi_{jt}^* \equiv 1 + \frac{\tilde{l}_t^j}{\lambda\theta} \tag{27}$$

Intuitively, the higher $\tilde{l}_t^j$, the more costly it is for buyers to purchase country-j goods, and the lower the quantity actually purchased. Then, Proposition 9 can be used to demonstrate the following.

Proposition 10 *Define*

$$G(\tilde{l}_t^H, \tilde{l}_t^F) = \frac{1 + \theta(\Psi_{Ft}^* - 1)}{1 + \theta(\Psi_{Ht}^* - 1)} = \frac{1 + \tilde{l}_t^F/\lambda}{1 + \tilde{l}_t^H/\lambda}.$$

Then

- *Home agents hold both bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) \in \left[\frac{\delta_{Ft}}{\beta}, \frac{1}{\beta\delta_{Ht}}\right]$*, hold only Home bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) > \frac{1}{\beta\delta_{Ht}}$*, and hold only Foreign bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) < \frac{\delta_{Ft}}{\beta}$*.*
- *Foreign agents hold both bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) \in \left[\beta\delta_{Ft}, \frac{\beta}{\delta_{Ht}}\right]$*, hold only Home bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) > \frac{\beta}{\delta_{Ht}}$*, and hold only Foreign bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) < \beta\delta_{Ft}$*.*

This proposition provides comparative statics demonstrating how patterns of international currency circulation depend on the model's parameters and government policies.

A country's bonds are more attractive when their liquidity premium is lower (lower l^j), when the issuing country is larger (higher ξ^j), or when its currency is more frequently accepted by

foreign sellers (higher δ_{jt}). These factors all contribute to a lower effective cost of holding a country's bonds. What matters to buyers is not only the forgone interest on liquid bonds, but also the probability that those bonds are widely recognized and accepted by sellers. Hence, one currency can dominate international commerce even if both currencies have similar liquidity premia. Figure 2 shows how the monetary regime depends on liquidity premia.

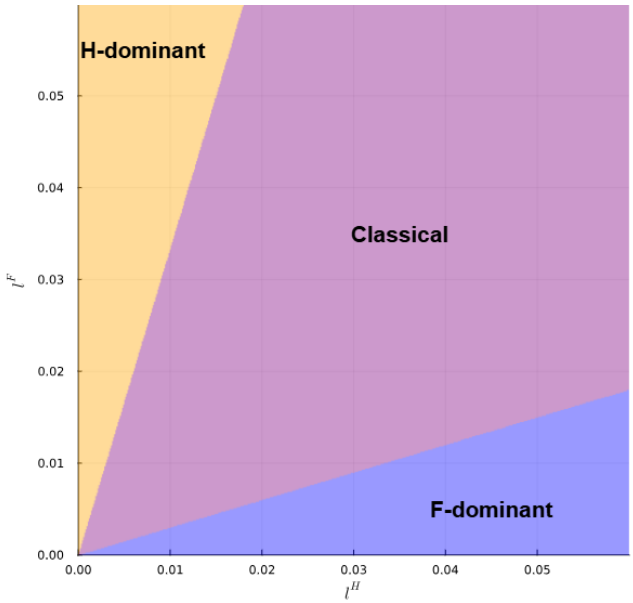


Figure 2: An illustration of how the international monetary regime depends on liquidity premia. We use parameters $\lambda = 0.2, \theta = 1.0, \xi = 0.5, \delta_H = \delta_F = 0.7, \beta = 1.3$.

Agents also have a greater incentive to hold foreign bonds when home bias is smaller (smaller β). When home bias is large, agents mainly want to purchase domestic inputs. Hence, they can nearly achieve their optimal expenditure on domestic and foreign goods by holding only domestic bonds. However, when home bias is smaller, this is no longer the case. One way to view this result is that when the global economy is more integrated, agents have greater incentives to hold a diversified portfolio of currencies.

Furthermore, as the fraction of agents who recognize foreign currencies goes to zero ($\delta_H, \delta_F \to 0$), all agents tend to hold both currencies. In this limit, the economy behaves like a traditional "cash-in-advance" model, where a country's currency is required to purchase its goods (Lucas, 1982; Svensson, 1985). Conversely, in the limit where all sellers accept both currencies ($\delta_H, \delta_F \to 1$), no agent holds both currencies. Instead, when the liquidity premia on the two currencies are sufficiently close, agents hold only their domestic currency because they recognize that foreign sellers will accept it anyway.

Given these results, portfolio holdings fall into six possible patterns, which we label by HH, HB, BB, BF, FF, and HF, where the first letter indicates the currency held by Home agents and the second indicates that held by Foreign agents (H for Home, F for Foreign, and B for both).

Thus, we can classify equilibria into four international monetary regimes (with two equilibria including two patterns each):

1. **"Classical" regime (BB):** All agents hold both currencies, just as in cash-in-advance models where each country's sellers accept only the domestic currency. In the limit where no agent accepts non-domestic currency, each currency's share of international trade settlement equals the corresponding country's export share.

2. **"Multipolar" regime (HF):** Agents hold only their domestic currency, and both currencies are accepted by a sufficiently large fraction of sellers in each country. In the limit where all sellers accept both currencies, each currency's share of international trade settlement equals the corresponding country's import share.

3. **"Home dominant" regime (HH, HB):** Home agents hold only domestic currency, whereas Foreign agents may hold both currencies. Home currency's share of global trade settlement exceeds Home's share of imports or exports.

4. **"Foreign dominant" regime (BF, FF):** Foreign agents hold only domestic currency, whereas Home agents may hold both currencies. Foreign currency's share of global trade settlement exceeds Foreign's share of imports or exports.

Our results above indicate that no single factor alone determines whether a currency is dominant. For example, our model does not predict that countries with a dominant currency will have low liquidity premia on their debt. In fact, there is some debate in the literature on whether dominant-currency bonds have high liquidity premia: Chen et al. (2025) defend the argument that they do, whereas Diamond and van Tassel (2025) argue that the "convenience yield" on U.S. debt is neither especially high nor especially low.

Instead, several factors determine whether a currency is dominant: the country's size, its monetary policy (as reflected in the real return on its government bonds), and its acceptability to foreigners in international trade. The empirical literature has highlighted factors that are quite similar to those in our model. For instance, Eichengreen et al. (2018) highlight country size, monetary policy "credibility" (i.e., the inflation rate), and a factor they call "inertia," intended to capture strategic complementarities and history-dependence in global firms' and banks' decisions to adopt a dominant currency.

So far, we have demonstrated how fundamental factors shape patterns of currency circulation. In the next section, we uncover strategic complementarities in sellers' decisions to accept non-domestic currencies that emerge naturally from our model.

4.2 The stability of dominant currencies

So far, we have studied buyers' portfolio choice at a given moment, treating sellers' decisions to accept non-domestic currencies as given. We have demonstrated that a currency tends to be dominant (all else equal) when it is more liquid than the other currency in international markets. In this section, we study the sellers' problem in detail. We characterize the conditions under which sellers' decisions to accept non-domestic currency are strategic complements, leading one currency to become more liquid internationally than the other. This allows us to show that dominant-currency equilibria, in which only one of the two currencies is accepted in international trade, are stable.

When deciding whether to invest in the technology to accept non-domestic currency, sellers consider the expected benefits of accepting foreign currency until the next Poisson event. Recall from (13) that the *flow* value of accepting currency $-j$ for country-j sellers at time t is

$$v^j_{-j,t} = \frac{\lambda(1-\theta)}{q^j_t w^j_t}\left(\xi^{-j}\big(1-\frac{1}{\Psi^{-j}_{j,t}}\big)b^{-j}_{-j,t} + \xi^{j}\big(1-\frac{1}{\Psi^{j}_{j,t}}\big)b^{j}_{-j,t}\right). \tag{28}$$

The seller invests if the expected discounted benefit of accepting foreign currency j, V^j_t, exceeds the investment cost κ, where

$$V^j_t \equiv \int_0^\infty e^{-(\rho+\zeta)s} v^j_{-j,t+s} ds.$$

At an *interior* steady state (δ_H, δ_F), $V^H = V^F = \kappa$. However, steady states where either V^H or V^F is at a corner are also possible. For instance, a steady state in which no Home agent recognizes Foreign currency has $V^H < \kappa$ and $\delta_F = 0$.

A dominant currency can emerge if decisions to accept foreign currency are strategic *complements*, that is, if one seller's decision to accept foreign currency enhances the incentives for all other sellers to do so as well. Strategic complementarities raise the possibility of self-fulfilling equilibria where sellers in one country all invest in the technology because they expect others to do so as well. By contrast, if decisions to accept foreign currency are strategic *substitutes*, the international monetary system will tend to feature "multipolar" equilibria: as long as both countries run similar monetary policies, both currencies will eventually be accepted internationally to some degree.

To understand the sources of strategic complementarity or substitutability in currency acceptance decisions, consider the value to Foreign sellers of recognizing Home currency at time t. From equation (28), there are two key considerations: (1) the gains from trade between buyers and Foreign sellers, and (2) global demand for Home bonds. We can characterize how each of these depends on the fraction of Foreign sellers who recognize Home currency.

Lemma 11 *The gains from trade between country-j buyers and country-i sellers, Ψ_{it}^{j}, are*

- *Increasing in the fraction of country-i sellers who recognize foreign currency j, δ_{jt}, if country-j buyers hold both currencies at t;*
- *Decreasing in δ_{jt} if country-j buyers hold only their domestic currency at time t.*

The intuition behind this lemma is straightforward. If, for example, Home buyers hold both currencies, an increase in the fraction of Foreign sellers who recognize Home currency will induce substitution from Foreign-currency to Home-currency bonds. These buyers then transact domestically more often, reducing the gains from trade in domestic meetings. For these buyers to continue to hold Foreign bonds at the same liquidity premium, the gains from trade with Foreign sellers must rise. On the other hand, if Home buyers hold only their domestic currency, an increase in the fraction of sellers who accept Home currency increases quantities purchased from Foreign sellers, reducing the marginal gains from trade.

The second component relevant to Foreign sellers' investment decision is the global demand for Home bonds. Naturally, both countries' demand for Home bonds is increasing as more Foreign sellers recognize them. Then, in the regime where both countries hold both currencies, decisions to accept foreign currency are strategic complements, since both the gains from trade and the bond demand increase with the fraction of sellers who invest.

Proposition 12 *Fix some level of the relative wage $q^F w^F / q^H w^H$ and $(l_t^H, l_t^F, \delta_{Ht}, \delta_{Ft})$ such that buyers in both countries hold both currencies. Then the value of accepting currency j at time t, v_{jt}^{-j}, is*

- *Locally increasing in the fraction of foreign sellers who recognize it, δ_{jt};*
- *Locally decreasing in the fraction of domestic sellers who recognize foreign currency, $\delta_{-j,t}$.*

Figure 3 illustrates these strategic complementarities. The arrows indicate the extent to which Home and Foreign sellers benefit from adopting the technology to accept foreign currency net of the cost. Small differences in initial conditions can lead to large differences in long-run outcomes: there is a long-run steady state in which Foreign agents accept Home currency, and another in which Home agents accept Foreign currency. The interior steady state, where each country accepts the other's currency, is unstable.

In fact, this instability is a general phenomenon. Well-known results in dynamical systems theory demonstrate that when such strategic complementarities are present (as they are in the "classical" monetary regime here), equilibria cannot be stable (Hirsch and Smith, 2006). The following proposition characterizes the possible types of stable long-run equilibria.

Proposition 13 *There are three possible types of stable long-run equilibria (δ_H, δ_F):*

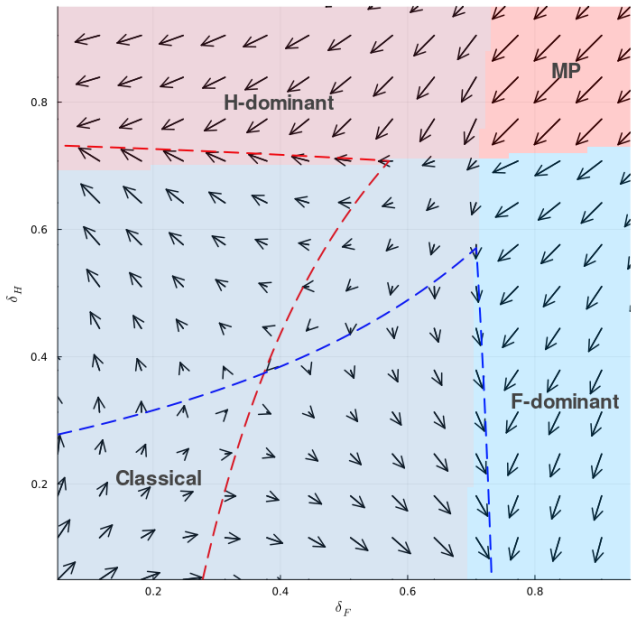


Figure 3: A phase diagram indicating the incentives to accept each currency as a function of (δ_F, δ_H). The arrows are of length $(v_F^H - \kappa, v_H^F - \kappa)$. The dashed blue line indicates the locus where $v_F^H = \kappa$, and the dashed red line indicates the locus where $v_H^F = \kappa$. The dark red indicates the multipolar regime, the gray region the classical regime, and the light red (blue) the H- (F-) dominant regime. We set $l^H = l^F = 0.02$, $\lambda = 0.2$, $\theta = 0.5$, $\xi^H = 0.5$, $\beta = 1.1$, and $\kappa = 0.01$.

1. *A trivial equilibrium in the classical regime with $\delta_H = \delta_F = 0$, where sellers in each country accept only their domestic currency;*

2. *An equilibrium where one currency j is dominant and no country-j seller recognizes non-domestic currency, $\delta_{-j} = 0$;*

3. *A multipolar equilibrium where agents in each country hold only domestic currency.*

The stability analysis also provides some indication of how the international monetary system is expected to evolve. During transitions to a long-run equilibrium, small changes in initial conditions or policies can have large effects on which currency is eventually selected as the dominant one. However, once a currency is established as dominant, small perturbations (such as a temporary change in policy or a moderate exogenous shock) are unlikely to alter the international monetary regime, since dominant-currency equilibria are stable. Instead, a large shock is likely required to dethrone a dominant currency if there is sufficient inertia in international payment systems (i.e., if the frequency ζ at which sellers can make their investment decisions is sufficiently low).

5 Quantitative exercises

Having laid out the theoretical case for the stability of dominant-currency regimes, we now quantify the model to assess the empirical relevance of this mechanism. We study a setting with three regions: the U.S. (US), the euro area (EU), and the rest of the world (RoW), which consists of a continuum of infinitesimally small countries. Since each country trades with itself only in a measure zero of trades, we can ignore the presence of specific local currencies and instead focus on a setting where the only two currencies are the dollar and the euro.[9]

We begin by describing our calibration and analyzing the model's steady-state properties. Then, we analyze the stability of international monetary regimes under various counterfactual scenarios. We consider perfect-foresight transition paths in which, at $t = 0$, agents learn of a change in the economy's parameters. We use a shooting algorithm to compute the evolution of the key state variables $\boldsymbol{\delta} = (\delta_{EU}^{US}, \delta_{US}^{EU}, \delta_{US}^{RoW}, \delta_{EU}^{RoW})$, i.e., the fraction of sellers in each region who recognize each non-domestic currency. For numerical stability, we assume that these state variables lie in a range $\delta_i^j \in [\delta_{min}, \delta_{max}] = [0.1, 0.9]$.[10] After finding the transition paths for the endogenous states, we compute economic aggregates of interest.

5.1 Calibration

We calibrate the model at an annual frequency, so that one unit of time equals a year. We split the parameter vector into two sets: *externally calibrated* parameters whose values are either standard in the literature or derived from straightforward empirical targets, and *internally calibrated* parameters that are jointly chosen to match a set of key moments in the data. We calibrate the model to a *dollar-dominant* steady state: for a given parameter vector, we find the corresponding steady state $\boldsymbol{\delta}$ such that neither U.S. nor RoW sellers pay the cost to recognize euros, $\delta_{EU}^{US} = \delta_{EU}^{RoW} = \delta_{\min}$.

Externally calibrated parameters. We set population shares to $\xi^{US} = \xi^{EU} = 0.25$ and $\xi^{RoW} = 0.5$ to match the facts that (1) U.S. and euro area output is roughly equal in PPP terms and (2) U.S. GDP is roughly 25% of global GDP.[11] The annual subjective discount rate is set to $\rho = 0.01$ to target a steady-state real interest rate of 1%. The intertemporal elasticity of substitution is set to unity ($\gamma^{-1} = 1$), a common choice in models with balanced growth. The elasticity of substitution among intermediate inputs is set to $\sigma = 2.0$, between conventional

[9] This simplification permits us to reduce the state space. Instead of assuming away local currencies, another possibility is to assume that local currencies account for some fraction of domestic trade but cannot be exchanged internationally (e.g., by setting κ to infinity for these currencies).

[10] We can formalize this restriction by assuming that a fraction δ_{min} of sellers always have access to the technology to recognize currency n, and a fraction $1 - \delta_{max}$ cannot acquire it at any cost.

[11] From the World Bank, https://data.worldbank.org/indicator/NY.GDP.MKTP.CD?locations=EU. Also, from the same source, global GDP was $111.33 trillion in 2024, and U.S. GDP was $29.18 trillion. See https://data.worldbank.org/indicator/ny.gdp.mktp.cd.

estimates of the Armington elasticity: Broda and Weinstein (2006) estimate an elasticity of 2.9, whereas Feenstra et al. (2018) find elasticities ranging from 0.87–4.08 across sectors.[12]

Two parameters do not have standard values. The bargaining power parameter, which has no obvious empirical counterpart, is set to $\theta = 0.5$ as in Drozd and Nosal (2012). The parameters governing sellers' decisions to adopt non-domestic currency do not have clear counterparts in the data either, since the stylized decision in our model captures several motives to accept domestic currency. The frequency at which sellers receive an opportunity to pay a cost to recognize a currency is set to $\zeta = 0.1$, so that sellers make this decision on average once every 10 years. Transitions between dominant-currency regimes will therefore have a half-life of about 7 years, roughly matching half the length of the transition from the pound sterling to the U.S. dollar during the interwar period. We calibrate the cost of investing in the technology, κ, below.

We also calibrate government policies externally. For our quantitative results, what matters is the steady-state liquidity premium $l^j = r^I - (i^j - \pi^j)$ on each currency, not the nominal rate and inflation individually. The empirical counterpart of the liquidity premium in our model is the "convenience yield" on a country's bonds: the difference between the interest rate on an illiquid bond and that on a government security. Using a novel methodology that constructs synthetic illiquid bonds via stocks and portfolios of options, Diamond and van Tassel (2025) demonstrate that the convenience yields on U.S. dollar and euro bonds are both roughly 30bp, so we set $l^j = 0.003$ for each country.

See Table 1 for a summary of these parameter values.

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
ξ^{US}	US share of global population	0.25
ξ^{EU}	euro area share of global population	0.25
ρ	Annual subjective discount rate	0.01
γ^{-1}	Intertemporal elasticity of substitution	1.0
σ	Elasticity of substitution (intermediate inputs)	2.0
θ	Buyer bargaining power	0.5
ζ	Seller investment frequency	0.1
l^j	Liquidity premium (both currencies)	0.003

Internally calibrated parameters. Next, we calibrate the remaining parameters to moments in the data. As in our analytical model, we assume that expenditure weights are pinned down by population shares and a home bias parameter, so that for $j \in \{US, EU\}$,

$$\omega_j^j = \frac{\beta\xi^j}{\beta\xi^j + \sum_{i\neq j}\xi^i}, \quad \omega_i^j = \frac{\xi^i}{\beta\xi^j + \sum_{i\neq j}\xi^i} \quad (\text{for } i \neq j).$$

[12]The literature on international trade typically finds higher values of the Armington elasticity (close to 5) than the international macroeconomics literature.

On the other hand, the expenditure weights for RoW are proportional to population shares, $\omega_i^{RoW} = \xi^i$, since RoW consists of individual small countries rather than a single large country. The home bias parameter is primarily identified by import/GDP ratios in the data.

We must also calibrate α, which pins down the share of intermediates in production. A larger α implies a greater amount of international trade, so we include global trade as a percentage of GDP as a moment in our calibration.

The parameter κ is the key determinant of the fraction of global sellers who recognize dollars in a dollar-dominant steady state. Thus, we target the weighted fraction of sellers who recognize dollars, $(\xi^{EU}\delta_{US}^{EU} + \xi^{RoW}\delta_{US}^{RoW})/(\xi^{EU} + \xi^{RoW})$. To ensure numerical stability, we assume that there is a small amount of dispersion in sellers' costs of investing to accept non-domestic currency, so that the law of motion of δ^j is a smooth approximation to the non-differentiable step function in (4). Formally, we set the distribution F of sellers' investment costs κ_i so that

$$\Pr(\kappa_i \leq x) = F(x) = \frac{1}{2}\left(1 + \tanh\left(\frac{x-\kappa}{\epsilon}\right)\right),$$

where $\epsilon = 5 \times 10^{-5}$ is chosen as small as possible so that our solution algorithm is stable.

Another important parameter is λ, the frequency with which agents meet in the DM. This parameter effectively governs bond velocity, thereby influencing bond demand. The government debt/GDP ratio is hence a natural target to include to identify λ.

Finally, we treat the relative wage $q^{w,RoW}$ as a parameter and set $q^{w,EU} = 1$ to ensure symmetry between the U.S. and the euro area. Recall that relative wages adjust in equilibrium so that countries' intertemporal budget constraints hold, and that they are key determinants of the terms of trade in the DM. Hence, relative wages will affect steady-state net foreign asset positions and the balance of trade.

We calibrate the parameters $\Theta = (\beta, \alpha, \kappa, \lambda, q^{w,RoW})$ to match five moments: (1) a U.S. debt/GDP ratio of 50%, near the average of its values over the post-war period; (2) a fraction of non-U.S. sellers who accept dollars of 70%, motivated by the finding in Berthou et al. (2022) that 70% of French export firms invoice trade in dollars; (3) a U.S. import/GDP ratio of 15%, near the middle of its range for the 21st century; (4) a U.S. trade deficit of 4%, also close to its 21st century average; and (5) a 40% trade/GDP ratio for the rest of the world, as in Chahrour and Valchev (2022).

In our calibration, we set the parameters to minimize a square loss function

$$L(\Theta) = (\hat{m}(\Theta) - m)'W(\hat{m}(\Theta) - m),$$

where $\hat{m}(\Theta)$ denotes the vector of simulated model moments under parameter vector Θ (in the dollar-dominant equilibrium), m denotes moments in the data, and W is a diagonal weighting matrix. We set the weight on all moments to 1, except for the share of RoW sellers who recognize

dollars, which we set to 10, since we are particularly interested in the model correctly capturing the dominance of the U.S. dollar at the start of each counterfactual.

Table 2 gives the value of our calibrated parameters, and Table 3 compares the simulated model moments to their counterparts in the data. Overall, our model closely matches the data moments.

Table 2: Internally Calibrated Parameters

Parameter	Description	Value
β	Home bias	5.0
α	Intermediate input share	0.36
λ	Meeting rate	3.4
κ	Cost of accepting non-domestic currency	5.3×10^{-4}
$q^{w,EU}$	EU relative wage	1.00
$q^{w,RoW}$	RoW Relative wage	0.62

Importantly, we find that the cost of accepting foreign currencies, parameterized by κ, is quite modest under our calibration. In the dollar-dominant steady state, the costs paid by EU and RoW sellers to accept dollars are less than 10bp of the total volume of dollars transacted by those sellers. Hence, our main results do not rely on unrealistically severe frictions to the acceptance of non-domestic currencies.

Table 3: Model and Data Moments

Moment	Model value	Target
U.S. Debt/GDP	51%	50%
δ_{US}^{RoW}	0.70	0.70
U.S. Imports/GDP	17%	15%
U.S. Trade Deficit/GDP	4.0%	4.0%
RoW Trade/GDP	36%	40%

5.2 The effects of a trade war

Our first experiment analyzes the effects of a trade war. We assume that in the CM, the U.S. levies a proportional tax, τ^I, on the sale of intermediate inputs purchased from all other regions, and, in retaliation, the euro area and the RoW levy symmetric tariffs on U.S. imports. These tariffs are assumed to be permanent (in this sense, our results will put an upper bound on the effects of a temporary trade war). The tariff is a tax on buyers' sales of intermediate inputs in the CM: for instance, U.S. buyers who import goods receive only $(1-\tau^I)\psi_{it}^{US}$ when selling inputs from region i. All proceeds from tariffs are distributed lump sum to domestic agents.

We solve for the economy's transition path to a new steady state for the range of values $\tau^I \in [0, 0.3]$, so the maximum tariff we consider is 30%. In all cases, we find that the tariff does

not trigger a switch to the euro as the world's dominant currency. Figure 4 illustrates the fraction of global sellers who accept dollars and euros along the transition path with $\tau^I = 0.3$.

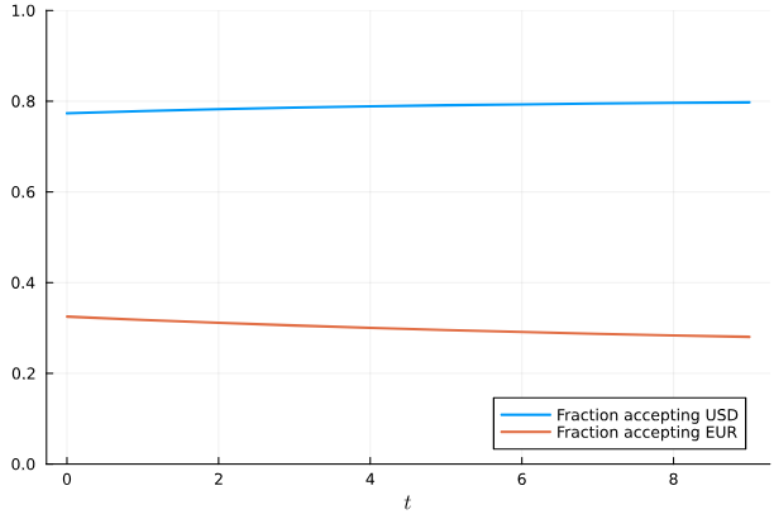


Figure 4: The fraction of sellers globally accepting dollars (blue) and euros (red) as a function of time in a trade war when import tariffs are set at $\tau^I = 0.3$.

A trade war could destabilize the dollar-dominant regime by reducing demand for dollar bonds; in particular, buyers from the euro area and the RoW might be expected to hold fewer dollar bonds due to tariffs on imports from the U.S. In turn, lower global dollar bond holdings would, in the long run, incentivize sellers to stop accepting dollars.

In our model, however, despite the strong effects of tariffs on trade flows, a trade war does not reduce global dollar bond demand. Under our calibration, most buyers do not hold dollar bonds at the margin because they wish to purchase U.S. goods; that is, most buyers hold enough dollar bonds to fully exhaust gains from trade with U.S. sellers, $\Psi^j_{US} = 1$. Instead, buyers' incentives to hold dollars primarily stem from trading with sellers in the RoW (where dollars are the only accepted currency) and the euro area. Thus, a trade war can actually slightly increase dollar demand, as expenditures switch from U.S. goods to RoW goods.

We perform a comparative statics exercise to highlight these points in Figure 5. We compute the log change in (1) dollar bond issuance and (2) U.S. imports between a steady state with zero tariffs and a steady state with tariff τ^I (for various levels of the tariff). Tariffs have large effects on real allocations: for a 30% tariff, U.S. imports are nearly 70% lower than in the no-tariff steady state. Nevertheless, dollar bond holdings are 6% higher in the steady state with 30% tariffs.

Our results on the effects of tariffs and trade wars broadly match the historical evidence. For example, the world economy underwent large shifts in trade flows and policies from the 1850s until the end of World War I (e.g., the McKinley tariffs of 1890 and naval blockades during the war). Nevertheless, the pound maintained its dominance throughout this period. Similarly, the dollar emerged as a competitor to the pound in the early 1930s, when the Smoot-Hawley tariffs

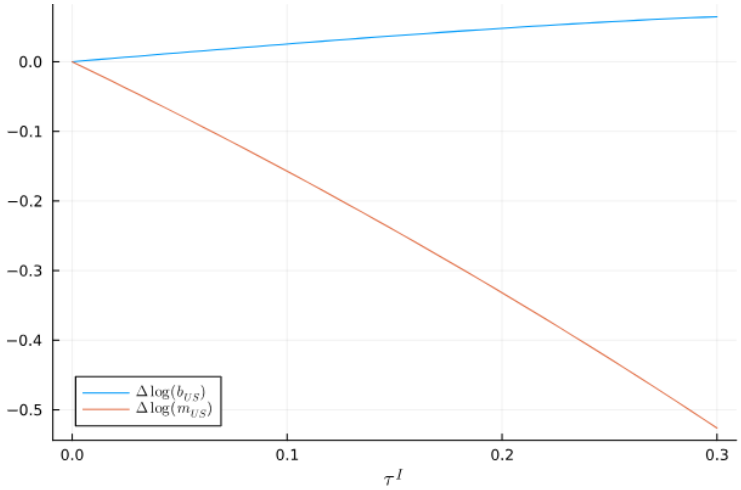


Figure 5: Comparative statics results from a trade war exercise. The blue line corresponds to the log change in steady-state dollar bond issuance from a steady state with zero tariffs to a steady state with tariff τ^I imposed on imports to and exports from the U.S. The red line plots the log change in U.S. imports between the two steady states.

were in place, and dollar dominance survived the more protectionist environment following the Bretton Woods agreement.

5.3 The "fiscal" Triffin dilemma

In recent years, the rapid growth of emerging market economies and their accumulation of dollar reserves have led observers to propose a "fiscal" version of the Triffin dilemma (Farhi et al., 2011; Bordo and McCauley, 2018). A dominant currency's status may become unstable because the issuing country cannot raise taxes to back the debt required to satisfy the world's growing demand for liquid assets. Hence, the international monetary system may need to become multipolar, with several countries issuing reserve assets in different currencies.

Since the primary concern is that growth in the rest of the world may outstrip the U.S.'s ability to supply a global safe asset, we design an experiment in which we introduce region-specific TFP growth, assuming that productivity grows in the RoW while remaining constant in the U.S. and the EU. Specifically, we let a_t^j denote labor-augmenting TFP in region j, so that the aggregate production function becomes $y_t^j = {X_t^j}^\alpha (a_t^j \ell_t^j)^{1-\alpha}$, and the utility cost of producing one unit of inputs in the DM is $1/a_t^j$. We assume that $a_0^j = 1$ in all regions initially, but in RoW, productivity grows linearly up to some value $\overline{a}^{RoW}$ over the course of $T = 20$ years.

The fiscal Triffin dilemma stems from the limited fiscal capacity of the U.S. and the euro area. Each region's government can raise at most $\overline{\tau}$ per capita in taxes. When a country's debt is about to exceed the fiscally sustainable level, the government targets a higher liquidity premium (i.e., a lower return on its debt) to ensure it can repay its debt. Specifically, if l^{j*} is a government's

target liquidity premium in the absence of fiscal constraints, it sets

$$l_t^j = \begin{cases} l^{j*} & \overline{\tau}^j \geq (r_t^I - l^{j*}) b_t^j \\ r_t^I - \frac{\overline{\tau}^j}{b_t^j} & \text{otherwise} \end{cases}$$

Fiscal capacity $\overline{\tau}$ is set so that the U.S. hits its fiscal constraint whenever its debt issuance is 50% greater than in the initial steady state. Our main interest is assessing whether the dollar can maintain its dominance even if its debt exceeds the point at which the initial rate of return is no longer sustainable.

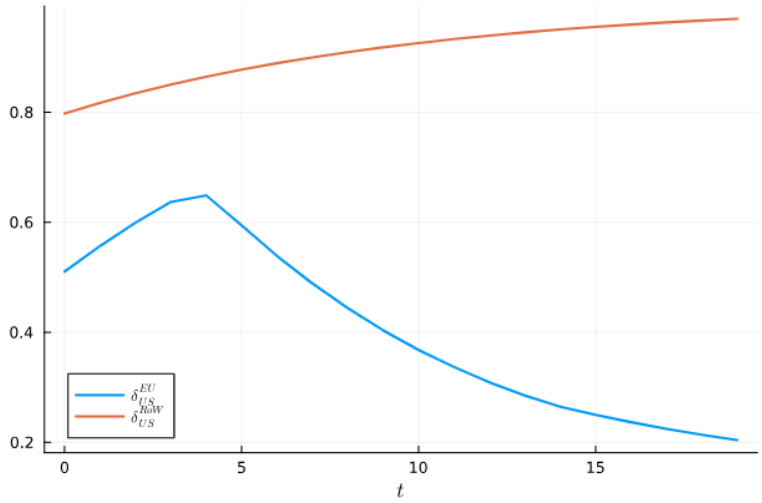


Figure 6: Dynamics in the "fiscal" Triffin experiment. The red (blue) line corresponds to the fraction of agents in RoW (EU) who accept dollars as a function of t.

We find that the rapid growth of the rest of the world does indeed lead the U.S. to run up against its fiscal capacity constraint. Nevertheless, the dollar remains the world's dominant currency, as sellers in the rest of the world continue to accept only dollars, not euros. Figure 6 illustrates how widely accepted the dollar is over time. The fraction of sellers in RoW who accept dollars, δ_{US}^{RoW}, actually increases over time, while the fraction in the EU first increases and then declines.

Dollar debt expands to roughly 125% of initial U.S. GDP in the long run. The liquidity premium on U.S. bonds thus doubles from 30 to 60bp to keep the debt sustainable. On the other hand, euro debt falls over time. See Figure 7.

What accounts for the increase in dollar bond demand that persists despite the increase in the dollar liquidity premium? News of future growth in RoW causes RoW's real exchange rate to appreciate vis-à-vis the U.S. and the euro area. In turn, this increases real wages in RoW, thereby expanding bond demand. The increase in bond demand actually causes the U.S. to hit its fiscal limit immediately at $t = 0$, raising the liquidity premium.

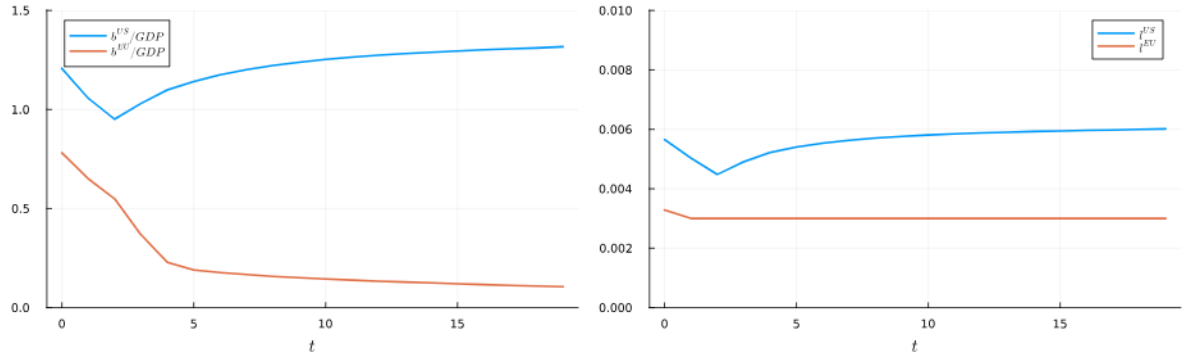


Figure 7: Dynamics of bond issuance and liquidity premia in each currency for the "fiscal" Triffin experiment. The left plot illustrates the quantity of bonds issued by the U.S. and the EU over time, normalized by each country's initial GDP. The right plot shows the liquidity premium for each bond type.

Despite the high initial liquidity premium on dollar bonds, buyers in the U.S. and RoW continue to prefer dollars because they are far more liquid in international markets: at $t = 0$, holding dollars is necessary for any buyer who wants to trade with the U.S. or RoW sellers. There is little additional incentive for any agent outside the euro area to increase euro bond holdings, since substituting dollars for euros reduces the cost of trading only with euro area sellers. Therefore, it remains valuable to accept dollars, and sellers do not switch to accepting euros. In this scenario, inertia is crucial to maintaining the dollar's dominance. It does not benefit buyers to switch to euros when the liquidity premium jumps, because, at that point, few sellers accept euros, and sellers have no incentive to incur the cost of moving to accept them (which, in the aggregate, could happen only gradually over time).

5.4 The rise of China

In recent years, observers have asked whether China's continued economic growth could lead to the renminbi's rise as an international currency. To pursue this goal, China has introduced several policies to integrate its trade and financial systems with those of other countries: it has invested in trade partnerships with regional partners, expanded its CIPS settlement system, and taken measures to liberalize its capital account.

To analyze whether China could successfully internationalize its currency, we introduce it into the model as a fourth country. Since China's economy is similar in size to those of the U.S. and Europe, we set its share of the global population equal to theirs, $\xi^{China} = 0.25$. Just like the U.S. and the euro area, China is assumed to have a home bias in goods. China's relative wage is assumed to be equal to that of RoW, as is the fraction of its sellers who recognize dollars and euros. We consider a scenario in which China has already fully liberalized its capital account; so RMB bonds can be traded freely across borders, and domestic and foreign investors earn identical

yields. We also assume that the liquidity premium on the renminbi is identical to that on other currencies, in line with the evidence in Diamond and van Tassel (2025) that most countries' government bonds have similar convenience yields (as we will show, capital account liberalization on its own is not sufficient to internationalize the renminbi.) When China is introduced into the global economy, only a fraction, $\delta_{\min}$, of sellers in other countries recognize its bonds.

We first consider an experiment in which China promotes renminbi settlement by expanding the use of its payment network. In our model, this roughly corresponds to increasing the rate at which agents from other countries meet agents from China in the DM. We assume that the intensity of meetings with agents from China is $\nu \times \lambda \xi^{China}$, where $\nu \geq 1$. We compare a situation in which China does not expand its payment network ($\nu = 1.0$) to one in which it does so aggressively ($\nu = 2.0$).

Figure 8 plots the shares of (1) global trade (solid lines) and (2) international reserves (dashed lines) in USD and RMB for a period of $T = 20$ years after China is introduced into the global economy. The introduction of China can be thought of as a liberalization of its capital account, so that foreigners can hold its bonds.

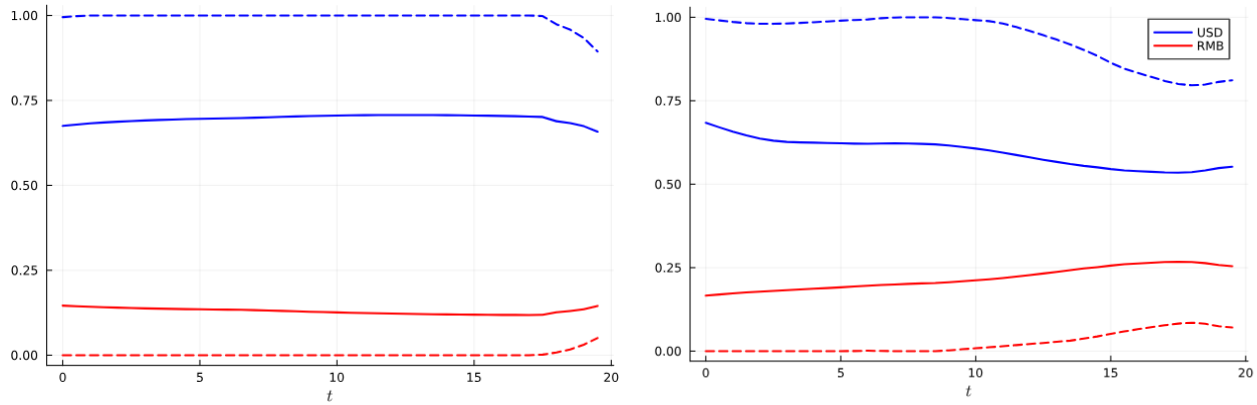


Figure 8: Shares of USD and RMB in global trade (solid lines) and international reserves (dashed lines) over time after China is introduced into the global economy. The left panel displays the results for the case $\nu = 1$, when China does not expand its international payment network, and the right panel displays the results for $\nu = 2$.

The introduction of RMB bonds to international markets, by itself, does not lead to broad international adoption of the renminbi. This is simply due to its lack of liquidity: only $\xi^{China} = 25\%$ of global sellers accept renminbi by default, and a large fraction of those also accept dollars. Therefore, buyers do not have strong incentives to hold Chinese bonds in their portfolios, as they rarely offer additional trade opportunities. Trade in renminbi is entirely driven by domestic trade and a small fraction of trade by Chinese buyers in other countries. International reserves are equal to zero because it is not optimal for foreign buyers to hold Chinese bonds. The fraction of sellers who recognize renminbi does not increase appreciably over time.

The renminbi gains greater international prominence as China expands its payment network.

When buyers meet Chinese sellers more frequently, they have stronger incentives to hold China's bonds. The inclusion of renminbi bonds in buyers' portfolios induces sellers to incur the cost of accepting them, gradually increasing the fraction of trade conducted in renminbi. In turn, buyers tilt their portfolios further towards China's bonds. It takes a long time for the transition to occur; however, even then, the dollar remains by far the dominant currency in international transactions: after 20 years, renminbi constitute only about 10% of international reserves, versus about 80% for the dollar.

Is there any way that the renminbi can become more competitive in international markets? Next, we consider a policy in which China's government targets a lower liquidity premium on its bonds. In our model, this corresponds to expanding the real supply of government debt. Hence, this policy could be viewed as corresponding to measures that expand the supply of Chinese debt to foreign investors. Specifically, we assume that at $t = 0$, China permanently halves its target liquidity premium to 15bp.

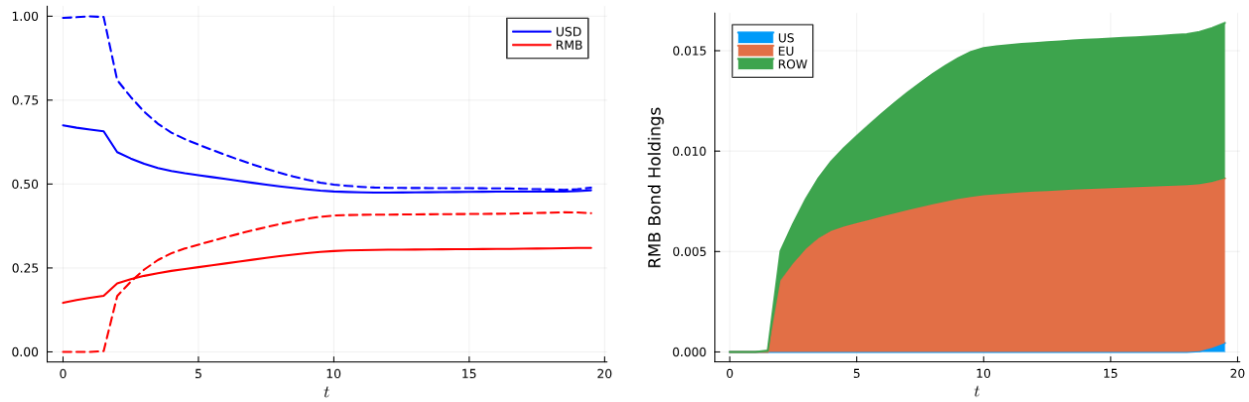


Figure 9: Results for the experiment in which China cuts its liquidity premium permanently to 15bp at $t = 0$. Left panel: The share of the dollar (blue) and renminbi (red) in global trade (solid lines) and international reserves (dashed lines) over time. Right panel: renminbi bonds held by the U.S. (blue), euro area (orange), and RoW (green).

Figure 9 plots the results. The dollar's share of both global trade and international reserves declines over time, while the renminbi rises as it becomes more widely accepted in international trade. In fact, after $T = 20$ years, the renminbi share of reserves is close to that of the dollar. Hence, a sustained decrease in the renminbi's liquidity premium can make it a competitor to the dollar. The right panel displays the composition of renminbi bond holdings. The increase in renminbi reserves is attributable to roughly equal increases in the euro area and the RoW, whereas buyers in the U.S. continue to hold dollars almost exclusively. There is a sense in which the international monetary system becomes fragmented: agents in the euro area and RoW hold dollars to trade with the U.S. and renminbi to trade with China, but neither the U.S. nor China holds significant quantities of the other's bonds.

6 Conclusion

We have developed a micro-founded monetary model to examine which assets serve as international media of exchange. Governments compete to internationalize their currencies by increasing the real interest rates on their bonds. Larger countries have a natural advantage in establishing themselves as dominant-currency issuers. However, this advantage may diminish as the large country's share of global trade declines. These model predictions align well with historical evidence from the 19th and 20th centuries.

Our model is well-suited to study transitions in the dominant-currency regime. When we endogenize the acceptability of currencies, we find that the "classical" equilibria studied by the early literature in international finance, in which a country's currency is required to purchase its own goods, are unstable. Instead, the economy often tends towards a dominant-currency regime. Small changes in initial conditions can have large effects on the structure of the international monetary regime, suggesting a significant role for history-dependence. However, once a dominant currency is established, it is unlikely to be dethroned by a small shock or policy change.

References

Antràs, P. and Foley, C. F. (2015). Poultry in motion: A study of international trade finance practices. *Journal of Political Economy*, 123(4):853–901.

Baughman, G. and Flemming, J. (2023). Global demand for basket-backed stablecoins. *Review of Economic Dynamics*, 48:244–264.

Berthou, A., Horny, G., and Mésonnier, J.-S. (2022). The real effects of invoicing exports in dollars. *Journal of International Economics*, 135.

Bordo, M. and McCauley, R. (2018). Triffin: Dilemma or myth? NBER Working Paper 24195.

Broda, C. and Weinstein, D. (2006). Globalization and the gains from variety. *Quarterly Journal of Economics*, 121(2):541–585.

Caballero, R., Farhi, E., and Gourinchas, P.-O. (2008). An equilibrium model of "global imbalances" and low interest rates. *American Economic Review*, 98(1):358–393.

Camera, G. and Winkler, J. (2003). International monetary trade and the law of one price. *Journal of Monetary Economics*, 50(7):1531–1553.

Chahrour, R. and Valchev, R. (2022). Trade finance and the durability of the dollar. *Review of Economic Studies*, 89(4):1873–1910.

Chen, Z., Jiang, Z., Lustig, H., van Nieuwerburgh, S., and Xiaolan, M. (2025). Exorbitant privilege gained and lost: Fiscal implications. *Journal of Political Economy*, 133(12):3713–3761.

Choi, M. and Rocheteau, G. (2020). New monetarism in continuous time: Methods and applications. *Economic Journal*, 131:658–696.

Clayton, C., dos Santos, A., Maggiori, M., and Schreger, J. (2024). International currency competition. Working paper.

Coppola, A., Krishnamurthy, A., and Xu, C. (2023). Liquidity, debt denomination, and currency dominance. NBER Working Paper 30984.

Diamond, W. and van Tassel, P. (2025). Risk-free rates and convenience yields around the world. *Journal of Finance*, (forthcoming).

Drozd, L. and Nosal, J. (2012). Understanding international prices: Customers as capital. *American Economic Review*, 102(1):364–395.

Eichengreen, B. (2019). *Globalizing Capital: A History of the International Monetary System*. Princeton University Press, 3rd edition.

Eichengreen, B., Mehl, A., and Chitu, L. (2018). *How Global Currencies Work: Past, Present, and Future*. Princeton University Press.

Farhi, E., Gourinchas, P.-O., and Rey, H. (2011). *Reforming the International Monetary System*. CEPR.

Farhi, E. and Maggiori, M. (2018). A model of the international monetary system. *Quarterly Journal of Economics*, 133(1):295–355.

Feenstra, R., Luck, P., Obstfeld, M., and Russ, K. (2018). In search of the Armington elasticity. *Review of Economics and Statistics*, 100(1):135–150.

Gomis-Porqueras, P., Kam, T., and Waller, C. (2017). Nominal exchange rate determinacy under the threat of counterfeiting. *American Economic Journal: Macroeconomics*, 9(2):256–273.

Gopinath, G. and Stein, J. (2021). Banking, trade, and the making of a dominant currency. *Quarterly Journal of Economics*, 136(2):783–830.

Gourinchas, P.-O. and Rey, H. (2005). From world banker to world venture capitalist: U.S. external adjustment and the exorbitant privilege. In Clarida, R., editor, *G7 Current Account Imbalances*. University of Chicago Press.

Gourinchas, P.-O. and Rey, H. (2007). International financial adjustment. *Journal of Political Economy*, 115(4):665–703.

Gourinchas, P.-O. and Rey, H. (2022). Exorbitant privilege and exorbitant duty. CEPR Discussion Paper No. DP16944.

Head, A. and Shi, S. (2003). A fundamental theory of exchange rates and direct currency trades. *Journal of Monetary Economics*, 50(7):1555–1591.

Hirsch, M. and Smith, H. (2006). Monotone dynamical systems. In Cañada, A., Drábek, P., and Fonda, A., editors, *Handbook of Differential Equations: Ordinary Differential Equations*, volume 2, pages 239–357. North-Holland.

Jiang, Z. (2024). Exorbitant privilege: A safe-asset view. Available at SSRN: https://ssrn.com/abstract=4600477.

Jiang, Z. and Richmond, R. (2023). Reserve asset competition and the global fiscal cycle. Available at SSRN: https://ssrn.com/abstract=4636643.

Kareken, J. and Wallace, N. (1981). On the indeterminacy of equilibrium exchange rates. *Quarterly Journal of Economics*, 96(2):207–222.

Kiyotaki, N. and Wright, R. (1989). On money as a medium of exchange. *Journal of Political Economy*, 97(4):927–954.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3):463–484.

Lester, B., Postlewaite, A., and Wright, R. (2012). Information, liquidity, asset prices, and monetary policy. *Review of Economic Studies*, 79(3):1209–1238.

Lucas, R. E. (1982). Interest rates and currency prices in a two-country world. *Journal of Monetary Economics*, 10(3):335–359.

Madison, F. (2024). A microfounded approach to currency substitution and government policy. *Journal of Economic Theory*, 219(105847).

Maggiori, M. (2017). Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107(10):3038–3071.

Matsuyama, K., Kiyotaki, N., and Matsui, A. (1993). Toward a theory of international currency. *Review of Economic Studies*, 60(2):283–307.

Pflueger, C. and Yared, P. (2024). Global hegemony and exorbitant privilege. NBER Working Paper 32775.

Svensson, L. E. (1985). Currency prices, terms of trade, and interest rates: A general equilibrium asset-pricing cash-in-advance approach. *Journal of International Economics*, 18(1-2):17–41.

Wright, R. and Trejos, A. (2001). International currency. *B.E. Journal of Macroeconomics*, 1(0).

Zhang, C. (2014). An information-based theory of international currency. *Journal of International Economics*, 93:286–301.

Zhou, R. (1997). Currency exchange in a random search model. *Review of Economic Studies*, 64(2):289–310.

Supplemental Appendix to

"International Currency Dominance"

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A Proofs

Proof of Proposition 4. The first-order condition of the HJB equation (8) with respect to labor ℓ is

$$1 = \nu_t^{j,\eta} q_t^j w_t^j,$$

where $\nu_t^{j,\eta}$ is the Lagrange multiplier on the budget constraint (7). But $\nu_t^{j,\eta}$ is just the marginal value of wealth $W_t'(a|\eta)$. Hence,

$$W_t'(a|\eta) = \frac{1}{q_t^j w_t^j},$$

which immediately implies the linear form of the value function in the proposition, as desired. ■

Proof of Proposition 5. A buyer who purchases z_i inputs worth $q_t^j \psi_{it}^j$ gets utility $\frac{\psi_{it}^j}{w_t^j}$. On the other hand, the seller incurs disutility z_i. Let m be the transfer made from the buyer to the seller, resulting in a utility cost $\frac{m}{q_t^j w_t^j}$ for the buyer and a benefit $\frac{m}{q_t^i w_t^i}$ for the seller. If the buyer has a total quantity of bonds $\overline{m}$ that the seller recognizes, the Nash bargaining solution satisfies

$$\max_{z_i, m} \ \theta \log\left(\frac{\psi_{it}^j}{w_t^j} z_i - \frac{m}{q_t^j w_t^j}\right) + (1-\theta)\log\left(\frac{m}{q_t^i w_t^i} - z_i\right) \quad \text{s.t. } z_i \geq 0, \ m \leq \overline{m}.$$

After simplifying, the first-order condition with respect to z_i yields

$$z_i = \theta \frac{m}{q_t^i w_t^i} + (1-\theta)\frac{m}{q_t^j \psi_{it}^j}.$$

Hence, the terms of trade $\phi_{it}^j \equiv m/z_i$ satisfy equation (9), since $\Psi_{it}^j = q_t^j \psi_{it}^j / q_t^i w_t^i$. ■

Proof of Proposition 7. From Proposition 6, the flow value obtained by a country-j seller in the DM who can accept currency n is given by equation (28). Use equation (8) to subtract $W(a|(j, S \setminus \{n\}))$ from $W(a|(j, S \cup \{n\}))$ and obtain

$$(\rho+\zeta)\big(W(a|(j,S\cup\{n\})) - W(a|(j,S\setminus\{n\}))\big) = v_{nt}^j + \frac{d}{dt}\big(W(a|(j,S\cup\{n\})) - W(a|(j,S\setminus\{n\}))\big),$$

[12] † Federal Reserve Bank of Philadelphia, ‡ University of Pennsylvania.

noting that savings and consumption decisions are identical regardless of the set of currencies that the seller accepts. This equation can be integrated forward to obtain

$$W(a|(j, S \cup \{n\})) - W(a|(j, S \setminus \{n\})) = \int_0^\infty e^{-(\rho+\zeta)s} v^j_{n,t+s}\, ds.$$

Combining this result with the maximization problem in (12), we obtain the decision rule (14). ■

Proof of Proposition 8. See Appendix B.1, where we derive the result in the more general model used for calibration and counterfactuals. ■

Proof of Proposition 9. Given $\Psi^j_{j,t}$ and $\Psi^j_{-j,t}$, equation (18) implies that the ratio of expenditures on domestic versus foreign inputs is

$$\frac{\phi^j_{j,t} x^j_{j,t}}{\phi^j_{-j,t} x^j_{-j,t}} = \frac{\omega^j}{1-\omega^j} \frac{1+\theta(\Psi^j_{-j,t}-1)}{1+\theta(\Psi^j_{jt}-1)}.$$

If country j buyers hold both types of bonds, this ratio cannot be greater than $\frac{\xi^j}{\delta_{jt}\xi^{-j}}$, since they are able to spend domestic bonds in a fraction ξ^j of meetings and foreign bonds in a fraction $\xi^{-j}\delta_{jt}$. Similarly, the ratio cannot be less than $\frac{\delta_{-j,t}\xi^j}{\xi^{-j}}$. Given that $\omega^j = \beta\xi^j/\left(\beta\xi^j + \xi^{-j}\right)$, this restriction on the ratio of expenditures is equivalent to

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)}{1+\theta(\Psi^j_{jt}-1)} \in \left[\delta_{-j,t}, \frac{1}{\delta_{jt}}\right]$$

if country-j buyers hold both bonds.

Next, suppose there is no solution to the Euler equation in which country-j buyers hold both types of bonds. If they hold only domestic bonds, there are three possible cases. In the first case, country j has not exhausted all possible gains from trade with either country, so $\Psi^j_{jt} > 1$ and $\Psi^j_{-jt} > 1$. Thus, buyers spend all of their bond holdings in each meeting, meaning the ratio of expenditures on domestic versus foreign inputs is exactly $\xi^j/\delta_{jt}\xi^{-j}$. On the other hand, if gains from trade with foreign sellers are equal to zero ($\Psi^j_{-j,t} = 1$), then this ratio is higher, since not all bonds are spent in meetings with foreign sellers, but all bonds are spent in meetings with domestic sellers. Finally, if gains from trade with domestic sellers are equal to zero ($\Psi^j_{j,t} = 1$), the ratio is lower. Therefore, again applying equation (18) to write aggregate expenditures in terms of gains from trade and the definition of ω^j, we obtain condition (25).

A similar logic yields equation (26). The only difference is that there is no case in which gains from trade with domestic agents are equal to zero ($\Psi^j_{j,t} = 1$), but gains from trade with foreign agents are not ($\Psi^j_{-j,t} > 1$). Suppose otherwise. The locus of possible combinations ($\Psi^j_{j,t}$, $\Psi^j_{-j,t}$)

such that an agent holds foreign bonds only satisfies

$$\Psi^j_{-j,t} - 1 = \frac{\delta_{-j,t}}{\beta}(\Psi^j_{j,t} - 1) - \frac{1}{\theta}\left(1 - \frac{\delta_{-j,t}}{\beta}\right). \tag{A.1}$$

Suppose there is a solution to the Euler equation when $\Psi^j_{j,t} = 1$. Then $\Psi^j_{-j,t} < 0$, since $\delta_{-j,t} \leq 1$ and $\beta \geq 1$ by assumption. Therefore, such a solution is not possible. ■

Proof of Proposition 10. Define $\tilde{l}^H_t, \tilde{l}^F_t$ as in the main text. Suppose that equations (22)–(23) both hold for country-j buyers. Observe that these equations can be rearranged to obtain

$$\tilde{l}^H_t = \lambda\theta(\Psi^j_{Ht} - 1)^+, \quad \tilde{l}^F_t = \lambda\theta(\Psi^j_{Ft} - 1)^+,$$

which in turn implies $\Psi^j_{it} = \Psi^*_{it}$, as defined in equation (27).

Proposition 9 demonstrated that if $G(\tilde{l}^H_t, \tilde{l}^F_t) < \frac{\delta_{Ft}}{\beta}$, then there does not exist a solution of the Euler equation in which Home agents hold both bonds. Instead, we look for a solution to the Euler equation in which Home agents hold only Foreign bonds. Proposition 9 showed that we either have $\Psi^H_{Ft} = 1$, in which case the Euler equation implies

$$\Psi^H_{Ht} = 1 + \frac{l^F_t}{\lambda\theta\xi},$$

or we have $\Psi^H_{Ht} > 1$ and $\Psi^H_{Ft} > 1$, and equation (A.1) holds with $j = H$. In this case, the solution to the Euler equation has

$$\Psi^H_{Ht} - 1 = \frac{\beta}{\delta_{Ft}}(\Psi^H_{Ft} - 1) + \frac{1}{\theta}\left(\frac{\beta}{\delta_{Ft}} - 1\right),$$

$$l^F_t = \lambda\theta\Big(\xi\delta_{Ft}(\Psi^H_{Ht} - 1) + (1 - \xi)(\Psi^H_{Ft} - 1)\Big).$$

The solution to the Euler equation can be found by solving these two equations for the two unknowns Ψ^H_{Ht}, Ψ^H_{Ft}.

Similar arguments apply to all of the other inequalities in the proposition. ■

Proofs of Lemma 11 and Proposition 12. We will analyze Home agents' demand for bonds in each regime. By a symmetry argument, this will also permit us to characterize properties of Foreign agents' bond demand.

Some preliminaries: Here we derive some preliminary results that will be useful throughout the analysis. First, define $D = 1 - \delta_H\delta_F$.

We now present some derivatives: for $j \in \{H, F\}$,

$$\frac{\partial \tilde{l}^j}{\partial \delta_{-j}} = \frac{\delta_j \tilde{l}^j}{D},$$
$$\frac{\partial \tilde{l}^j}{\partial \delta_j} = -\frac{\xi^j}{\xi^{-j}} \frac{\tilde{l}^{-j}}{D},$$
$$\frac{\partial D}{\partial \delta_{-j}} = -\delta_j.$$

Also,

$$\frac{\partial \Psi_H^*}{\partial \tilde{l}^H} = \frac{1}{\lambda\theta\xi}, \quad \frac{\partial \Psi_H^*}{\partial \tilde{l}^F} = 0,$$
$$\frac{\partial \Psi_F^*}{\partial \tilde{l}^H} = 0, \quad \frac{\partial \Psi_H^*}{\partial \tilde{l}^F} = \frac{1}{\lambda\theta(1-\xi)}.$$

Case 1: Home agents hold both bonds: In this regime, we have

$$\Psi_H^H = \Psi_H^* \text{ and } \Psi_F^H = \Psi_F^*.$$

Bond demand is given by

$$b_H^H = \frac{1}{\lambda D}\left(\frac{\phi_H^H x_H^H}{\xi} - \delta_F \frac{\phi_F^H x_F^H}{1-\xi}\right),$$
$$b_F^H = \frac{1}{\lambda D}\left(\frac{\phi_F^H x_F^H}{1-\xi} - \delta_H \frac{\phi_H^H x_H^H}{\xi}\right),$$

where

$$\phi_H^H x_H^H = \frac{\alpha\omega^H}{1+\theta(\Psi_H^* - 1)}, \quad \phi_F^H x_F^H = \frac{\alpha(1-\omega^H)}{1+\theta(\Psi_F^* - 1)}$$

denote expenditures on Home and Foreign goods, respectively.

Derivatives with respect to δ_F: We have

$$\frac{\partial \Psi_H^*}{\partial \delta_F} = \frac{\partial \Psi_H^*}{\partial \tilde{l}^H}\frac{\partial \tilde{l}^H}{\partial \delta_F} = \frac{1}{\lambda\theta}\frac{\delta_H \tilde{l}^H}{D} \geq 0,$$
$$\frac{\partial \Psi_F^*}{\partial \delta_F} = \frac{\partial \Psi_F^*}{\partial \tilde{l}^F}\frac{\partial \tilde{l}^F}{\partial \delta_F} = -\frac{1}{\lambda\theta}\frac{\tilde{l}^H}{D} \leq 0.$$

To obtain the derivative of bond demand, we first differentiate expenditures:

$$\begin{aligned}\frac{\partial \phi_H^H x_H^H}{\partial \delta_F} &= \frac{\partial \phi_H^H x_H^H}{\partial \Psi_H^*}\frac{\partial \Psi_H^*}{\partial \delta_F} \\ &= -\frac{\alpha \omega^H \theta}{(1+\theta(\Psi_H^*-1))^2}\frac{1}{\lambda\theta}\frac{\delta_H \tilde{l}^H}{D} \\ &= -\frac{\delta_H}{D}\frac{\tilde{l}^H/\lambda}{1+\tilde{l}^H/\lambda}\phi_H^H x_H^H < 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial \phi_F^H x_F^H}{\partial \delta_F} &= \frac{\partial \phi_F^H x_F^H}{\partial \Psi_F^*}\frac{\partial \Psi_F^*}{\partial \delta_F} \\ &= -\frac{\alpha(1-\omega^H)\theta}{(1+\theta(\Psi_F^*-1))^2}\frac{1}{\lambda\theta}\big(-\frac{\tilde{l}^H}{D}\big) \\ &= \frac{1}{D}\frac{\tilde{l}^H/\lambda}{1+\tilde{l}^F/\lambda}\phi_F^H x_F^H > 0.\end{aligned}$$

Then, the derivative of bond demand is

$$\begin{aligned}\frac{\partial b_H^H}{\partial \delta_F} &= -\frac{\partial D/\partial \delta_F}{D}b_H^H + \frac{\partial \phi_H^H x_H^H/\partial \delta_F}{\lambda\xi D} \\ &\quad - \frac{1}{\lambda(1-\xi)D}\big(\phi_F^H x_F^H + \delta_F \frac{\partial \phi_F^H x_F^H}{\partial \delta_F}\big) \\ &= \frac{\delta_H}{D}b_H^H - \frac{\delta_H}{D}\frac{\tilde{l}^H/\lambda}{1+\tilde{l}^H/\lambda}\frac{\phi_H^H x_H^H}{\lambda\xi D} \\ &\quad - \left(\frac{\phi_F^H x_F^H}{\lambda(1-\xi)D} + \frac{\delta_F}{D}\frac{\tilde{l}^H/\lambda}{1+\tilde{l}^F/\lambda}\frac{\phi_F^H x_F^H}{\lambda(1-\xi)D}\right).\end{aligned}$$

To conclude that $\frac{\partial b_H^H}{\partial \delta_F} \leq 0$, it suffices to show that $\phi_F^H x_F^H \geq \lambda(1-\xi)\delta_H b_H^H$. In this regime, we have

$$\frac{\phi_H^H x_H^H}{\phi_F^H x_F^H} \leq \frac{\xi}{(1-\xi)\delta_H}$$

from the results in the paper. Then, from the definition of b_H^H,

$$\begin{aligned} b_H^H &\leq \frac{1}{\lambda D}\left(\frac{\phi_F^H x_F^H}{(1-\xi)\delta_H} - \delta_F \frac{\phi_F^H x_F^H}{1-\xi}\right) \\ &= \frac{1}{\lambda(1-\xi)\delta_H}\phi_F^H x_F^H \\ \Leftrightarrow \lambda(1-\xi)\delta_H b_H^H &\leq \phi_F^H x_F^H,\end{aligned}$$

as desired.

Next, take derivatives of Home agents' demand for Foreign bonds:

$$\begin{aligned}\frac{\partial b_F^H}{\partial \delta_F} &= -\frac{\partial D/\partial \delta_F}{D} b_F^H + \frac{\partial \phi_F^H x_F^H/\partial \delta_F}{\lambda(1-\xi)D} - \delta_H \frac{\partial \phi_H^H x_H^H/\partial \delta_F}{\lambda \xi D} \\ &= \frac{\delta_H}{D} b_F^H + \frac{1}{D}\frac{\tilde{l}^H/\lambda}{1+\tilde{l}^F/\lambda}\frac{\phi_F^H x_F^H}{\lambda(1-\xi)D} + \frac{\delta_H^2}{D}\frac{\tilde{l}^H/\lambda}{1+\tilde{l}^H/\lambda}\frac{\phi_H^H x_H^H}{\lambda \xi D} \\ &> 0.\end{aligned}$$

We conclude that in this regime,

$$\begin{aligned}\frac{\partial}{\partial \delta_F}\big(1-\frac{1}{\Psi_F^H}\big) b_H^H < 0, \\ \frac{\partial}{\partial \delta_F}\big(1-\frac{1}{\Psi_H^H}\big) b_F^H > 0.\end{aligned}$$

Derivatives with respect to δ_H: This case is completely symmetric. We have

$$\begin{aligned}\frac{\partial}{\partial \delta_H}\big(1-\frac{1}{\Psi_F^H}\big) b_H^H > 0, \\ \frac{\partial}{\partial \delta_H}\big(1-\frac{1}{\Psi_H^H}\big) b_F^H < 0.\end{aligned}$$

Case 2: Home agents hold Home bonds only: In this regime, there are two sub-cases based on whether $\beta\delta_H$ is greater than or less than one. Of course, regardless of which sub-case applies, we have $b_F^H = 0$.

Sub-case 1: $\beta\delta_H \geq 1$**.**

In this case, the gains from trade in each type of meeting are

$$\begin{aligned}\Psi_F^H &= 1 + \frac{\big(l^H - \lambda\xi\big(\beta\delta_H - 1\big)\big)^+}{\lambda\theta(\beta\xi + 1 - \xi)\delta_H}, \\ \Psi_H^H &= 1 + \frac{\max\left\{\frac{\beta\xi}{\beta\xi+1-\xi} l^H + \frac{1-\xi}{\beta\xi+1-\xi}\lambda\xi\big(\beta\delta_H - 1\big), l^H\right\}}{\lambda\theta\xi}.\end{aligned}$$

Bond demand is

$$b_H^H = \frac{\phi_H^H x_H^H}{\lambda\xi} = \frac{\alpha\omega^H}{\lambda\xi}\frac{1}{1+\theta(\Psi_H^H - 1)}.$$

It is immediately apparent from these expressions that $\frac{\partial \Psi_F^H}{\partial \delta_H} \leq 0$, $\frac{\partial \Psi_H^H}{\partial \delta_H} \geq 0$, and hence $\frac{\partial b_H^H}{\partial \delta_H} \leq 0$. Moreover, none of these quantities depend on δ_F (since Home agents do not hold Foreign bonds).

Therefore,

$$\frac{\partial}{\partial \delta_H}\big(1-\frac{1}{\Psi_F^H}\big) b_H^H \leq 0, \tag{A.2}$$

$$\frac{\partial}{\partial \delta_F}\big(1-\frac{1}{\Psi_F^H}\big)b_H^H = 0. \tag{A.3}$$

Sub-case 2: $\beta\delta_H < 1$.

This sub-case is similar to the previous one with minor alterations:

$$\Psi_H^H = 1 + \frac{\big(l^H - \lambda(1-\xi)\big(\frac{1}{\beta}-\delta_H\big)\big)^+}{\lambda\theta\big(\xi+\frac{1-\xi}{\beta}\big)},$$
$$\Psi_F^H = 1 + \frac{\max\left\{\frac{1-\xi}{\beta\xi+1-\xi}l^H + \frac{\beta\xi}{\beta\xi+1-\xi}\lambda(1-\xi)\big(\frac{1}{\beta}-\delta_H\big), l^H\right\}}{\lambda\theta(1-\xi)\delta_H}.$$

Now we have

$$b_H^H = \frac{\phi_F^H x_F^H}{\lambda(1-\xi)\delta_H} = \frac{\alpha(1-\omega^H)}{\lambda(1-\xi)\delta_H}\frac{1}{1+\theta(\Psi_F^H-1)}.$$

Again, it is clear that $\frac{\partial\Psi_F^H}{\partial\delta_H} \le 0$, $\frac{\partial\Psi_H^H}{\partial\delta_H} \ge 0$, and $\frac{\partial b_H^H}{\partial\delta_H} \le 0$. We therefore obtain equations (A.2) and (A.3).

Case 3: Home agents hold Foreign bonds only: In this regime, we have $b_H^H = 0$, and quantities do not depend on δ_H, the fraction of Foreign agents who accept Home bonds.

The gains from trade are

$$\Psi_F^H = 1 + \frac{\big(l^F - \lambda\xi\big(\beta-\delta_F\big)\big)^+}{\lambda\theta(\beta\xi+1-\xi)},$$
$$\Psi_H^H = 1 + \frac{\max\left\{\frac{\beta\xi}{\beta\xi+1-\xi}l^F + \frac{1-\xi}{\beta\xi+1-\xi}\lambda\xi\big(\beta-\delta_F\big), l^F\right\}}{\lambda\theta\xi\delta_F}.$$

Bond holdings are:

$$b_F^H = \frac{\phi_H^H x_H^H}{\lambda\xi\delta_F} = \frac{\alpha\omega^H}{\lambda\xi\delta_F}\frac{1}{1+\theta(\Psi_H^H-1)}.$$

It is immediate that $\frac{\partial\Psi_F^H}{\partial\delta_F} \ge 0$, $\frac{\partial\Psi_H^H}{\partial\delta_F} \le 0$, and $\frac{\partial b_F^H}{\partial\delta_F} \le 0$. Thus, we have

$$\frac{\partial}{\partial\delta_H}\big(1-\frac{1}{\Psi_H^H}\big)b_F^H = 0, \frac{\partial}{\partial\delta_F}\big(1-\frac{1}{\Psi_H^H}\big)b_F^H \le 0.$$

This concludes the proof. ■

Proof of Proposition 13. We proceed by classifying the possible equilibria in each regime. In what follows, it will be useful to define the state variable $\boldsymbol{\delta} = (\delta_H, \delta_F)$ and write the law of motion (4) as

$$\dot{\boldsymbol{\delta}} = \zeta\begin{pmatrix}\mathbf{1}\{v_H^F(\boldsymbol{\delta})/(\rho+\zeta) \ge \kappa\} - \delta_H \\ \mathbf{1}\{v_F^H(\boldsymbol{\delta})/(\rho+\zeta) \ge \kappa\} - \delta_F\end{pmatrix}. \tag{A.4}$$

	Classical	H-dominant	F-dominant	Multi-polar
$\frac{\partial v_H^F}{\partial \delta_H}$	$+$	?	≥ 0	≤ 0
$\frac{\partial v_H^F}{\partial \delta_F}$	$-$	≤ 0	≤ 0	0
$\frac{\partial v_F^H}{\partial \delta_H}$	$-$	≤ 0	≤ 0	0
$\frac{\partial v_F^H}{\partial \delta_F}$	$+$	≥ 0	?	≤ 0

Table A.1: Sign pattern of $\partial v_{-j}^j/\partial \delta_{-j}$ and $\partial v_{-j}^j/\partial \delta_j$ implied by the proof of Proposition 12.

Define the smooth approximation to the law of motion

$$\dot{\boldsymbol{\delta}}_t^{approx} = \zeta \begin{pmatrix} F(v_H^F(\boldsymbol{\delta}) - (\rho+\zeta)\kappa|\epsilon) - \delta_H \\ F(v_F^H(\boldsymbol{\delta}) - (\rho+\zeta)\kappa|\epsilon) - \delta_F \end{pmatrix},$$

where

$$F(x|\epsilon) = \frac{1}{2}\big(1 + \tanh(x/\epsilon)\big)$$

is a smooth approximation to the law of motion (A.4) as $\epsilon \to 0$. Near a steady state, the Jacobian of this approximate law of motion is proportional to that of (v_H^F, v_F^H):

$$\boldsymbol{J}^v = \begin{pmatrix} \partial v_H^F/\partial \delta_H & \partial v_H^F/\partial \delta_F \\ \partial v_F^H/\partial \delta_H & \partial v_F^H/\partial \delta_F \end{pmatrix}.$$

Stability of an interior steady state then requires, as usual, $\text{tr}(\boldsymbol{J}^v(\boldsymbol{\delta})) < 0$ and $\det(\boldsymbol{J}^v(\boldsymbol{\delta})) > 0$.

In what follows, we will also use the fact that the proof of Proposition 12 characterizes the sign of $\partial v_{-j}^j/\partial \delta_{-j}$ and $\partial v_{-j}^j/\partial \delta_j$ in each regime, as summarized in Table A.1. Given the expression for v_{-j}^j in (13), each element of the table for v_{-j}^j can be found via the signs of the derivatives of $\big(1 - 1/\Psi_j^{-j}\big)b_{-j}^{-j}$ and $\big(1 - 1/\Psi_j^j\big)b_{-j}^j$. (The proof derives the signs for Home agents' bond demand and gains from trade, but a symmetric argument applies for Foreign.)

Classical regime: Consulting Table A.1, if $\boldsymbol{\delta}^*$ is an interior steady state in this regime, then $\text{tr}(\boldsymbol{J}^v(\boldsymbol{\delta}^*)) > 0$. Hence, $\boldsymbol{\delta}^*$ cannot be stable, and the only possible stable equilibrium is $\boldsymbol{\delta}^* = (0,0)$. (Clearly, this point is an equilibrium for sufficiently large κ.)

Dominant-currency regime: We analyze the Home-dominant regime. The proof for the Foreign-dominant regime is exactly analogous.

Suppose by way of contradiction that $\boldsymbol{\delta}^*$ is a stable interior equilibrium in the Home-dominant regime. From Table A.1, if $\text{tr}(J^v(\boldsymbol{\delta}^*)) < 0$, it must be that $\partial v_H^F/\partial \delta_H < 0$. In turn, the sign pattern then yields $\det(J^v(\boldsymbol{\delta}^*)) \leq 0$, contradicting the assumption that $\boldsymbol{\delta}^*$ is stable. Any stable equilibrium must then lie along the boundary $\delta_F = 0$.

■

B Derivations of key equations

B.1 Goods market equilibrium

In this section, we derive equation (24) in a generalized version of the model. Just as in our counterfactual exercises, we allow for tariffs τ_{it}^j levied on country-j buyers who sell country-i inputs in the CM and time-varying productivity a_t^j, where the aggregate production function is

$$y_t^j = {X_t^j}^\alpha (a_t^j \ell_t^j)^{1-\alpha}, \tag{B.1}$$

Furthermore, sellers' utility cost of production in the DM is $1/a_t^j$. (Productivity a_t^j is permitted to follow an arbitrary deterministic path.) A key quantity is the sellers' marginal cost of production in the DM, defined as

$$mc_t^j \equiv q_t^j w_t^j / a_t^j.$$

With differential productivity, the gains from trade become $\Psi_{it}^j \equiv q_t^j \psi_{it}^j / mc_t^i$.

With a tariff, the terms of trade in the DM also change. They are given by

$$\begin{aligned}\phi_{it}^j &= \frac{(1-\tau_{it}^j)\Psi_{it}^j}{1+\theta((1-\tau_{it}^j)\Psi_{it}^j - 1)} mc_t^i, \\ &= \frac{\frac{mc_t^i}{mc_t^j}(1-\tau_{it}^j)\Psi_{it}^j}{1+\theta((1-\tau_{it}^j)\Psi_{it}^j - 1)} mc_t^j,\end{aligned}$$

and buyers spend all of their available bonds whenever the gains from trade net of tariffs are positive, $(1-\tau_{it}^j)\Psi_{it}^j \geq 1$

The first-order conditions for intermediate goods (15) can be combined to obtain, via the usual CES algebra,

$$\psi_{Xt}^j = \frac{\alpha y_t^j}{X_t^j} \quad \text{where} \quad \psi_{Xt}^j \equiv \left(\sum_{i=1}^J \omega_i^j {\psi_{it}^j}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{B.2}$$

Then, multiply equation (16)) raised to the power $1-\alpha$ with equation (B.2) raised to the power α, use the production function (B.1), and multiply by q_t^j on both sides to obtain

$$\begin{aligned}\alpha^\alpha (1-\alpha)^{1-\alpha} q_t^j {a_t^j}^{1-\alpha} &= q_t^j \Big(\sum_{i=1}^J \omega_i^j {\psi_{it}^j}^{1-\sigma} \Big)^{\alpha/(1-\sigma)} {w_t^j}^{1-\alpha} \\ &= {mc_t^j}^\alpha (q_t^j w_t^j)^{1-\alpha} \Big(\sum_{i=1}^J \omega_i^j \big(\frac{mc_t^i}{mc_t^j} \Psi_{it}^j\big)^{1-\sigma} \Big)^{\alpha/(1-\sigma)} \\ &= {a_t^j}^{1-\alpha} mc_t^j {\Psi_{Xt}^j}^\alpha,\end{aligned}$$

where

$$\Psi_{Xt}^{j} \equiv \Big(\sum_{i=1}^{J} \omega_i^j \big(\frac{mc_t^i}{mc_t^j}\Psi_{it}^j\big)^{1-\sigma}\Big)^{1/(1-\sigma)}.$$

This expression can be rearranged to obtain

$$w_t^j = \frac{\alpha^\alpha (1-\alpha)^{1-\alpha} a_t^j}{{\Psi_{Xt}^j}^{\alpha}},$$

which can be combined with the resource constraint (3) and the labor-leisure optimization condition (17) to obtain

$$y_t^j = \xi^j \left(\frac{\alpha^\alpha (1-\alpha)^{1-\alpha} a_t^j}{{\Psi_{Xt}^j}^{\alpha}}\right)^{\frac{1}{\gamma}}.$$

Then, note that the quantity of intermediates purchased by region j from region i can be rewritten as

$$\begin{aligned}
x_{it}^j &= \omega_i^j X_t^j \big(\frac{\psi_{it}^j}{\psi_{Xt}^j}\big)^{-\sigma} \\
&= \omega_i^j \frac{\alpha q_t^j y_t^j}{q_t^j \psi_{Xt}^j} \big(\frac{q_t^j \psi_{it}^j}{q_t^j \psi_{Xt}^j}\big)^{-\sigma} \\
&= \omega_i^j \frac{\alpha q_t^j y_t^j}{mc_t^j \Psi_{Xt}^j} \big(\frac{\frac{mc_t^i}{mc_t^j}\Psi_{it}^j}{\Psi_{Xt}^j}\big)^{-\sigma} \\
&= \frac{\alpha q_t^j y_t^j}{mc_t^j} \times \omega_i^j {\Psi_{Xt}^j}^{\sigma-1} \big(\frac{mc_t^i}{mc_t^j}\Psi_{it}^j\big)^{-\sigma} \\
&= \frac{\alpha q_t^j w_t^j}{mc_t^j} \frac{y_t^j}{w_t^j} \times \omega_i^j {\Psi_{Xt}^j}^{\sigma-1} \big(\frac{mc_t^i}{mc_t^j}\Psi_{it}^j\big)^{-\sigma} \\
&= \frac{\alpha \xi^j q_t^j w_t^j}{mc_t^j} \times \left(\alpha^\alpha (1-\alpha)^{1-\alpha} a_t^j\right)^{\frac{1}{\gamma}-1} {\Psi_{Xt}^j}^{\sigma-1-\alpha(\frac{1}{\gamma}-1)} \times \omega_i^j \big(\frac{mc_t^i}{mc_t^j}\Psi_{it}^j\big)^{-\sigma} \\
&= \frac{K_t^j}{mc_t^j} \times \omega_i^j \big(\frac{mc_t^i}{mc_t^j}\Psi_{it}^j\big)^{-\sigma}.
\end{aligned}$$

where

$$K_t^j \equiv \alpha \xi^j q_t^j w_t^j \times \left(\alpha^\alpha (1-\alpha)^{1-\alpha} a_t^j\right)^{\frac{1}{\gamma}-1} \times {\Psi_{Xt}^j}^{\sigma-1-\alpha(\frac{1}{\gamma}-1)}.$$

Putting this expression together with the terms of trade, we obtain

$$\phi_{it}^j x_{it}^j = K_t^j \omega_i^j \frac{(1-\tau_{it}^j)\big(\frac{mc_t^i}{mc_t^j}\Psi_{it}^j\big)^{1-\sigma}}{1+\theta((1-\tau_{it}^j)\Psi_{it}^j - 1)}.$$

B.2 Implementation of a liquidity premium target

In this subsection, we demonstrate how a government can use its fiscal and monetary tools (the nominal rate i_t^n and the rate of bond issuance μ_t^n) to implement a given path of the liquidity premium.

Let $\{\tilde{l}_t^n\}_{n=1}^N$ denote the set of liquidity premium targets for each currency. Let $\{\tilde{b}_t^n\}_{n=1}^N$ denote real demand for bonds in each currency at time t, and let $\tilde{r}_t$ denote the real interest rate, in some equilibrium in which liquidity premia are equal to their target values. We construct a set of policies $\{i_t^n, \mu_t^n\}_{n=1}^N$ such that there exists an equilibrium in which $l_t^n = \tilde{l}_t^n$ and $b_t^n = \tilde{b}_t^n$ for all n and t, and $r_t = \tilde{r}_t$ for all t.

Recall that $b_t^n = \phi_t^n B_t^n$, where B_t^n is nominal debt in currency n and ϕ_t^n is the real price of a currency-n bond. If such an equilibrium exists,

$$\mu_t^n = \frac{\dot{\tilde{b}}_t^n}{\tilde{b}_t^n} - \frac{\dot{\phi}_t^n}{\phi_t^n}.$$

Furthermore, from the definition of the liquidity premium,

$$\tilde{l}_t^n = \tilde{r}_t - \left(i_t^n - \frac{\dot{\phi}_t^n}{\phi_t^n}\right),$$

These are the only two equations that must hold in order for an equilibrium of the desired form to exist (maintaining the assumption that taxes can always adjust to satisfy governments' budget constraints). Combining these relationships, such an equilibrium exists whenever

$$i_t^n - \mu_t^n = \tilde{r}_t - \tilde{l}_t^n - \frac{\dot{\tilde{b}}_t^n}{\tilde{b}_t^n}.$$

Only the difference between i_t^n and μ_t^n is pinned down by equilibrium conditions, so there exists a μ_t^n (resp. i_t^n) satisfying these conditions for any given path of i_t^n (resp. μ_t^n), as claimed.