What drives the greenium, and does it align with green innovations?

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Abstract

The greenium—the expected return spread between brown and green assets—arises from the brown sector's exposure to the risk of climate policy tightening and from investors' diversification motives. In addition, the representative investor has an altruistic preference for consuming green output, which makes it riskier to hold green assets when green goods are scarce. This generates a relative-supply hedging premium that masks part of the green premium. Although brown firms' investments in mitigation (green) technologies reduce the intensity of policy risk and further counteract the greenium, their effectiveness is limited, helping to explain the recent downtrend in green innovations. Taken together, policy risk and diversification are the primary drivers of the green premium; however, incorporating the relative-supply hedging channel and green innovation in a unified framework is crucial for understanding the forces shaping the greenium and the impacts of transitional policy risks on firms' real decisions.

Keywords: Green premium, Green consumption, ESG investing, Green innovation.

JEL codes: G11, Q50, Q51, Q55.

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1 Introduction

The increasing intensity of climate-induced disasters has prompted burgeoning studies on how these events affect financial markets, and how finance can play a role in mitigating such tail risks to the economy. One channel that receives much attention is the way investors price climate-related physical and transition risks, thereby driving a wedge between firms' costs of capital. A rise in the cost of capital should, in theory, incentivize brown firms to mitigate the risks and green their businesses. Since Bansal et al. (2016) published their pioneering work, scholars have utilized various advanced methods to quantify the magnitude of the difference in investors' return expectations between green and brown assets. The carbon return (or green premium/greenium) implied by equity prices emerges as a natural candidate for investigation. Although a strand of research is dedicated to estimating its true size, the spread in expected returns between green and brown equities still challenges financial economists. Is it true that a brown equity asset should carry a higher expected return, and if this is the case, how big is the difference, and what drives it?

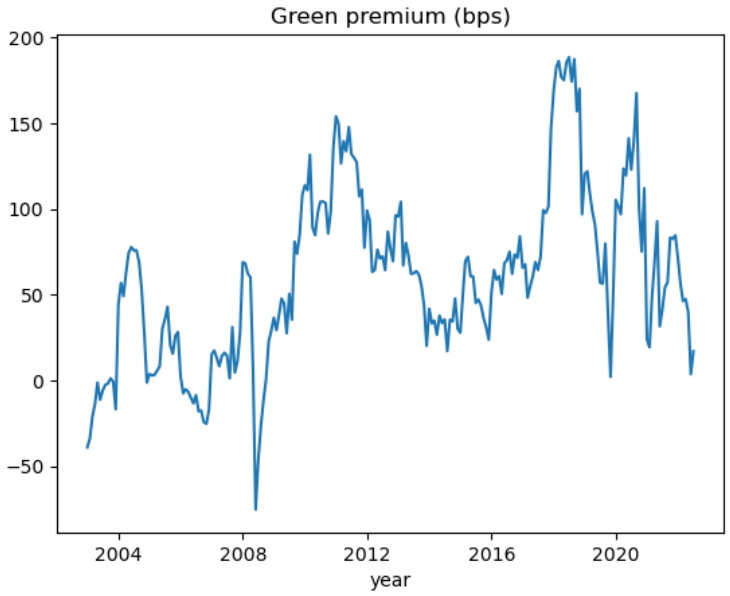
Using realized returns as the proxy for their expected counterparts, several studies document a positive spread in the performance of brown versus green equities, while others argue for the opposite[1]. This inconsistency arises because, as with many other economic quantities, the greenium has been dynamically evolving and is far from a stable state. If the climate-related risks tied to assets are still being continuously repriced by investors, thereby causing the realized returns to deviate from their expected values, it would be non-trivial to approximate the green premium. Indeed, during the last decade, institutional investors began to overweight green assets in their portfolios, which in turn resulted in the realized outperformance of green assets compared to brown ones (Ardia et al. (2023); Pastor et al. (2022)) and complicated the greenium debate.

Abstaining from the noisy realized return, Pastor et al. (2022) show that using an appropriate measure of expected return can reveal the true positive greenium. They estimate an annual average range of 48-144 bps spread between the expected returns of brown and green stocks from 2013 to 2020. Sautner et al. (2023b) use the forward-looking information from options to confirm an annual 50-100 bps green premium for the period 2011-2014. Most recently, Eskildsen et al. (2024) test the greenium in a comprehensive way by constructing a robust green score from various

[1]The branch of literature that documents positive realized brown-green return spread includes Bolton and Kacperczyk (2023, 2021); Hsu et al. (2023). Those that show the opposite are: Pastor et al. (2022); Giese et al. (2021); Ardia et al. (2023); Bauer et al. (2022); Zhang (2024); Karolyi et al. (2023).

Figure 1: **Green premium.**

Green premium. The following figure plots the expected return of the brown-minus-green factor from 2003 to 2023. The factor is a market-value-weighted long-short portfolio of U.S equities. By the end of each year t, firms are sorted within industries into terciles based on both their total and intensity of Scope 1 and Scope 2 emissions. The emission data is from Trucost and is lagged 12 months following Zhang (2024). Then, the terciles are combined across industries. Firms that fall in the top tercile based on both criteria are classified as brown, while those that belong to the bottom tercile are green. The difference in expected returns between brown and green firms is computed each month from July of year t+1 to June of year t+2. The expected return is the average of four implied cost of capital measures from Gebhardt et al. (2001), Claus and Thomas (2001), Easton (2004), and Ohlson and Juettner-Nauroth (2005). The data is provided by Eskildsen et al. (2024).



measures of greenness and deducing the brown-green expected return spread from a wide range of forward-looking expected return estimates. Their final results show an annual greenium of around 50 bps, demonstrate a time-varying pattern in which the green premium is deepening over time, but do not decompose the greenium into underlying components.

Although the advanced empirical evidence points to a positive green premium, the powerhouse behind it is as important as its magnitude in the path of transitioning toward a sustainable world. A thorough understanding of the forces that drive the brown-green premium spread helps policymakers, economists, and practitioners push for mechanisms that propel the nascent green economy forward. Studies have proposed different explanations for why investors are willing to accept a lower reward for holding green equities. There are two main strands in the literature regarding the potential causes. The first one relies on the warm-glow preference of investors for

holding green assets. Pedersen et al. (2021), Pastor et al. (2021, 2022), and Zerbib (2022) build this type of preference into investors' utility functions, and corroborate that it leads to a sacrifice of expected returns. Nevertheless, a recent survey of equity portfolio managers from both sustainable and traditional funds by Edmans et al. (2025) points to the opposite. Financial considerations, including downside risk mitigation, return enhancement, and volatility reduction, remain the dominant rationales for selecting stocks with strong environmental and social (E&S) performance. This result, in general, agrees with Krueger et al. (2020), who find that institutional investors regard climate risks as financially material for their portfolio firms and believe that such risks, particularly regulatory risks, have already begun to materialize. The second strand of literature, which is more closely aligned with these survey findings, focuses on the pricing of the foundational transition and physical risks induced by climate change. Bansal et al. (2016) explore how the long-run shift in temperature results in a positive risk premium in the capital market. They propose and calibrate the disaster impact of temperature rise on future output growth, which is the underlying reason behind the expected return spread between firms with high and low heat exposure. Most importantly, their study reconciles the doubling in the temperature-tied risk premium with the global warming trend over the last 80 years. Instead of focusing on physical risks, Sautner et al. (2023b) go a different route and back out an option-implied premium from firms' different vulnerability to climate policy uncertainty. Notably, Bolton and Kacperczyk (2023) document that climate transition risks have a pervasive impact on equity prices, showing cross-sectional return spreads reflect both policy shifts and technological change.

The implications of climate change on asset prices extend beyond physical and transition risks. Because the brown sector has traditionally dominated the economy, investors' wealth is heavily concentrated in brown equities, so shocks to these firms can generate large fluctuations in the market portfolio. This creates a strong demand for diversification to reduce overall portfolio volatility. Consequently, besides transition and physical risks, the diversification motive is instrumental in shaping investors' portfolio allocations and hence the green premium. This is consistent with Edmans et al. (2025), where many managers mentioned diversification besides returns as a top concern when facing E&S constraints in their stock selection process. Krueger et al. (2020) also report that portfolio diversification is among top three approaches to manage climate risks by institutional investors. On the theory side, Zerbib (2022) shows diversification affects the

efficacy of exclusion and integration on firms' cost of capital. Hambel et al. (2024) prove that the desire to mitigate climate risks and the diversification purpose are complements in the short term, but turn out to be trade-offs in the long run. In light of this discussion, it is crucial to account for the diversification force when explaining the green premium. Altogether, while various risk-based explanations of the greenium are available, the existing literature does not demonstrate how different factors interact within an integrated equilibrium framework, which is arguably closer to reality.

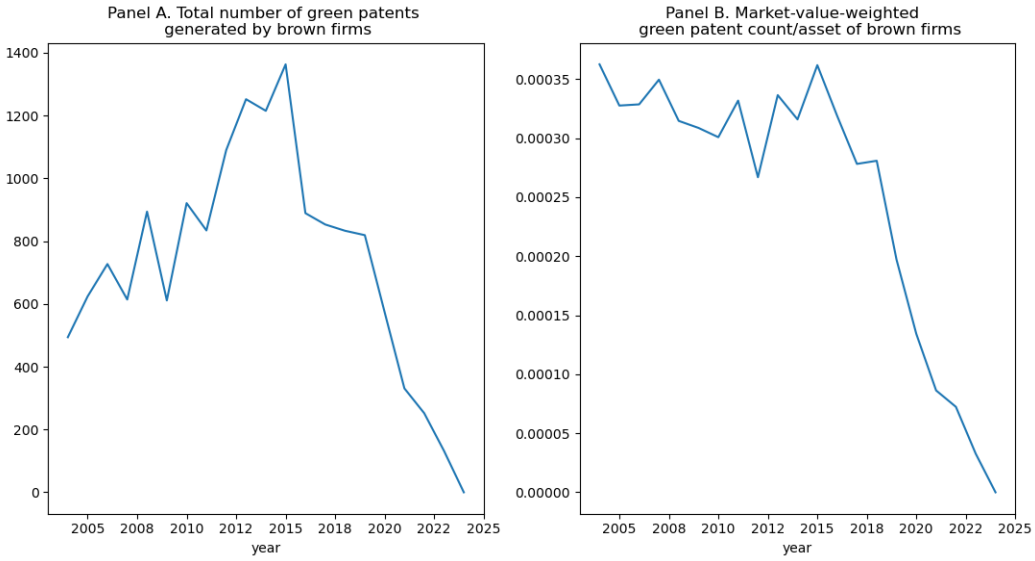
Moreover, within the economy, asset prices, investors' portfolio decisions, and consumption preferences are tightly interconnected. This is particularly true in the current discussion of sustainable investing. Empirical evidence (Meier et al. (2023)) has identified households' preference for the products of green firms, and this taste contributes a significant part to firms' cashflows, risks, and expected returns. While a stronger desire increases the demand for green firms' outputs, it also embeds a risk that affects firms' equity returns. Sauzet and Zerbib (2024) unveil that as investors exhibit a bias toward green consumption, the potential scarcity of green outputs makes green equities riskier to hold. As the consumption channel potentially affects assets' risks and returns, a natural question arises as to how it interacts and compares with other factors in unfolding the greenium.

On top of the narrative that climate-related risks play a salient role in financial markets, another pressing question is whether their manifestations in asset prices can translate meaningfully into effective risk mitigation. Even though there is clear evidence that climate-related risks are priced, for instance in the equity market through a green premium, the limited real effect of impact investing has been repeatedly raised by many scholars[2]. Nevertheless, to the best of my knowledge, the feedback effects and underlying mechanisms linking green innovations and the greenium in an equilibrium setting remain largely unexplored. The issue is critical because green technological innovations and firms' mitigation efforts more broadly are instrumental to the economy, not only through their interactions with asset prices but also through their contributions to climate-related risk mitigation and the green transition. The seminal work by Dietz, Gollier and Kessler (2018) shows that even though emissions abatement projects carry a positive climate beta (the elasticity

[2]See Berk and van Binsbergen (2025); Heath et al. (2023); Kolbel et al. (2020). This phenomenon is fundamentally different from the macro effect of carbon pricing, which Colmer et al. (2024), using evidence from the European Union Emissions Trading System, show to induce firms to reduce carbon emissions.

Figure 2: **Green innovation intensity.**

The following figure depicts the performance of U.S brown firms in generating green patents from 2004 to 2023. Panel A shows the total of green patents produced. Panel B plots the weighted average of patent count intensity as measured by the total number of patents/asset ratio. The updated data is provided by Kogan, Papanikolaou, Seru, and Stoffman (KPSS) following their work Kogan et al. (2017). The patents are categorized as "green" following the European Patent Office's Y02/Y04S tagging scheme.



of climate damages with respect to aggregate consumption) and hence a positive risk premium, the expected benefits they generate dominate the discount-rate effect, so that their net present values can increase with the climate beta itself. Hong et al. (2023a,b) derive the cross-sectional value of mitigation effort. They show that mitigation technologies delay the tipping point, increase welfare, and earn firms a lower required rate of return, which becomes even lower over time as the economy approaches the steady state. Leippold and Yu (2025) leverage green patent data to estimate the return spread between firms with high and low levels of innovation. Their results reiterate the positive return spread and point to the resilience of highly innovative firms to regulatory shocks. Despite the potential benefits of investing in mitigation technologies in an environment with high climate-related risks and increasingly stringent regulations, the level of green innovation in the United States has been trending downward since the Paris Agreement in 2015, as shown in Figure 2. This alarming paradox, which is reminiscent of the green paradox, underscores the urgent need for analytical work on the underlying dynamics, enabling policymakers and practitioners to identify effective solutions for greening the economy.

Motivated by the above discussions, this paper develops a unified model to quantify and explore the interaction of forces that generate the green premium, as well as to examine the feedback between the greenium and green innovation. The setting features two types of firms: green and brown. They differ in their exposure to climate policy risk because brown firms generally face heavier regulations from the Government in the course of transition. This is micro-founded by the work of Sautner et al. (2023a) that quantifies firm-level exposure to climate risk. The impact of policy risk reduces brown firms' output, thereby imposing a risk on holding their equities. In addition, consistent with literature showing dirty firms are innovating at a higher rate than green firms (Cohen et al. (2026), Hartzmark and Shue (2023)), the brown firm in this setting has the option to spend on mitigation technologies to reduce the externality generated and thus the intensity of policy shocks they face. As expected, because the brown firm bears greater policy risk, its equity commands a higher risk premium than the green firm's. However, the magnitude of this policy spread is smaller than that of the diversification component. Since the brown sector has been accruing for a long time, its equity value comprises a major share of investors' holdings and drives a large part of the market portfolio or total wealth volatility. Meanwhile, the green equity, whose value is less correlated with total wealth, is valuable for the purpose of diversification. These differential correlations with total wealth promote the brown-green return spread. The contributions of the diversification and policy risk components to the green premium are partially offset by the relative-supply hedging factor. The intuition parallels the previous discussion of green consumption preferences: a stronger taste for green goods makes green equity riskier to hold in the face of prospective scarcity. In summary, the model not only generates a 60-bps green premium at the current state but also explains the dynamic structure of the greenium, which deepens as the frequency of climate risk grows.

Despite having the option to invest in mitigation technologies to abate its policy risk exposure, the brown firm in this setting chooses not to do so when its size strongly dominates that of the green peer and the intensity of policy risk is too high. Essentially, in such an adverse state of the economy, the disaster effect of recurrent policy shocks raises the level of the stochastic discount factor (SDF) and hence the present value of each additional brown consumption unit. Therefore, the fall in the level of brown output following policy tightening is more than compensated by the increase in the present value of each unit, thereby rendering the brown firm's total value higher.

This explains why it is suboptimal for firms to invest in mitigation technologies even when they face intense policy shocks and get penalized by investors with higher costs of capital, shedding light on why the green innovation intensity has diminished recently.

Under calibrated/estimated parameters, the model produces an insightful structure of the policy, diversification, and hedging factors. Over time, as the economy transitions towards the greener spectrum, the policy factor attenuates, while the diversification component strengthens. However, overall, the policy factor still has a significant contribution to the green premium. While the first two are the main driving forces, the hedging component also represents a non-trivial role, masking part of the contributions from its counterparts. All in all, the structural decomposition highlights the evolving nature of the green premium, underscoring how shifting investor preferences and regulatory frameworks are increasingly influencing market dynamics.

The paper outline is as follows: Section 2 introduces the two-sector endowment economy with transitional policy risk and the representative household's preference over green and brown consumption. Section 3 outlines the investor's and brown firm's optimization problems, and characterizes the decentralized market equilibrium as well as the optimal choices of players. Section 4 discusses asset pricing implications and the greenium decomposition. Section 5 lays out the estimation procedure, and Section 6 concludes.

2 Model

2.1 Endowments

The setting being considered is a continuous-time endowment economy with an infinite horizon. The economy is populated with a large number of firms classified into two types or sectors: eco-friendly green firms (the green sector) and non-eco-friendly brown firms (the brown sector). I denote by $Y_{n,t}$ the output of type-n firm, where $n \in \{b, g\}$ with b standing for brown and g standing for green. The output processes are as follows:

$$\frac{dY_{b,t}}{Y_{b,t_-}} = \left(\mu_b - z_{t_-}\right) dt + \sigma_b dB_{b,t} - \zeta dN_{p,t}, \tag{2.1}$$

$$\frac{dY_{g,t}}{Y_{g,t_-}} = \mu_g dt + \sigma_g dB_{g,t}. \tag{2.2}$$

$N_{p,t}$ represents the transitional policy risk which follows a jump process with intensity λ_t, and $\zeta > 0$ is the fraction of brown output lost after each round of policy shock. I assume the Brownian shocks $B_{b,t}$ and $B_{g,t}$ are uncorrelated with the jump process $N_{p,t}$, and $cov\left(dB_{b,t}, dB_{g,t}\right) = \nu\, dt$. All stochastic processes are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Importantly, following Nguyen et al. (2024), the number of policy risk events follows a Poisson distribution, with the instantaneous probability being an increasing function of the fraction of aggregate output produced by the brown sector, i.e.

$$\lambda_t = \lambda_g + (\lambda_b - \lambda_g)\,\eta_t = \lambda_g + \eta_t \Delta_\lambda, \tag{2.3}$$

where $0 \leq \lambda_g \leq \lambda_b$ are two constants, $\Delta_\lambda = \lambda_b - \lambda_g$, and $\eta_t = \frac{Y_{b,t}}{Y_{b,t}+Y_{g,t}}$ represents the share of brown output at time t. The intuition behind this formulation is that the more predominant brown firms are in the economy, the more negative impacts on the environment resulting from climate change. This pushes Governments to adopt tighter environmental policies that disproportionately affect the brown sector.

In turn, brown firms can invest z_t into mitigation technologies (hereafter, green technologies) to reduce their contributions to negative climate externality and abate their exposure to the policy risk: $\lambda_t \longrightarrow \frac{\lambda_t}{1+\phi z_t}$. Essentially, when a firm develops environmentally friendly technologies, it is able to adapt better to stricter standards that Governments require. The cost of investments is $z_t Y_{b,t}$. I assume that these investments are observable by investors [3].

2.2 Preferences

The representative consumer has a recursive preference over his/her consumption streams, i.e.

$$U_t = \mathbb{E}\left[\int_t^\infty f\left(C_s, U_s\right) ds\right], \tag{2.4}$$

where $f(\cdot)$ is the instantaneous Epstein–Duffie aggregator. I consider the special case of unit IES in which the aggregator simplifies to

$$f\left(C_t, U_t\right) = \rho(1-\gamma)U_t\left[log\left(C_t\right) - \frac{1}{1-\gamma} log\left((1-\gamma)U_t\right)\right],$$

[3] For implications of non-observability on firms' green transitions, see Gupta and Starmans (2025).

in which $\rho > 0$ corresponds to the subjective discount factor, and γ is the degree of relative risk aversion. When $\gamma = 1$, the aggregator becomes $f(C_t, U_t) = \rho[log(C_t) - U_t]$. The representative consumer is endowed with claims on the output produced by both types of firms.

Following Sauzet and Zerbib (2024), the consumption bundle C_t is calculated using a modified Stiglitz-Dixit aggregator such that:

$$C_t = \left[(1-\theta)^{\frac{1}{\alpha}} C_{b,t}^{\frac{\alpha-1}{\alpha}} + \theta^{\frac{1}{\alpha}} C_{g,t}^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}}. \tag{2.5}$$

$C_{b,t}, C_{g,t}$ represent the consumptions of brown/green outputs at time t, and α controls the elasticity of substitution between them. This parameter α is, according to Acemoglu et al. (2012), as important as the discount rate when studying optimal responses to climate change. In addition, the reasonable assumption is $\alpha > 1$, consistent with Papageorgiou et al. (2017). The substitutability between green and brown output is validated by the estimation procedure in Section 5[4].

The parameter $0 \leq \theta \leq 1$ controls altruistic preferences for output of the green sector. Because the average investor prefers to consume green over brown output[5], $\theta > \frac{1}{2}$, which is also confirmed by the estimation results. Note that in the limiting case where $\alpha \longrightarrow \infty$ (i.e., perfect substitution) and $\theta = \frac{1}{2}$ (i.e., no altruistic preference for green output), the consumption bundle becomes:

$$C_t = C_{b,t} + C_{g,t}. \tag{2.6}$$

3 Competitive Market Equilibrium

I focus on the decentralized equilibrium instead of the social planner's solution. The representative agent (also, the household) and firms take as given equity as well as good prices and maximize the lifetime utility/firm values. Goods and equity markets clear, i.e., $C_{b,t} = Y_{b,t}, C_{g,t} = Y_{g,t}$, and the representative agent holds all the dividend claims to both sectors. The Markovian Nash equilibrium of this stochastic differential game can be defined as in Carmona (2016). The state variables are $Y_{b,t}, Y_{g,t}, \lambda_t$, and W_t, where W_t denotes the total wealth of the representative agent at time t. Following Cochrane et al. (2008) and Eberly and Wang (2025), I exploit the homogeneity of the

[4]Meanwhile, Yin (2024) shows that clean and dirty capital are complementary.

[5]Aghion et al. (2023) provide evidence that consumers' environmental preferences and product-market competition can jointly steer firms toward clean innovation, using panel data on automobile patents and measures of environmental willingness to pay.

model and reduce the dimension of the problem so that the only relevant state variable is η_t.

The evolution of the relative size of the brown sector, η_t, is given by

$$d\eta_t = \Sigma(\eta_{t_-})dt + \eta_{t_-}\left(1-\eta_{t_-}\right)\left[\sigma_b dB_{b,t} - \sigma_g dB_{g,t}\right] - \frac{\zeta\,\eta_{t_-}\left(1-\eta_{t_-}\right)}{\eta_{t_-}\left(1-\zeta\right)+\left(1-\eta_{t_-}\right)}dN_{p,t}, \tag{3.1}$$

with the expression for $\Sigma(\eta_{t_-})$ being given in the Appendix.

After each policy shock $N_{p,t}$, the state variable changes discretely to

$$\eta_t^J = \frac{\eta_{t_-}\left(1-\zeta\right)}{\eta_{t_-}\left(1-\zeta\right)+\left(1-\eta_{t_-}\right)} < \eta_{t_-}, \qquad \forall \eta_{t_-} \in (0,1)\,. \tag{3.2}$$

Let $m_{b,t}$ and $m_{g,t}$ be the prices of brown/green output. I normalize $\left[(1-\theta)\times m_{b,t}^{1-\alpha} + \theta \times m_{g,t}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$ to unity. With the normalization, the price of the aggregate consumption basket, $\frac{m_{b,t}C_{b,t}+m_{g,t}C_{g,t}}{C_t}$, is unity.

Let $\xi_t = \frac{m_{g,t}}{m_{b,t}}$ be the ratio of green/brown output prices. The output prices are written in terms of ξ_t as follows

$$m_{b,t} = \left[(1-\theta) + \theta {\xi_t}^{1-\alpha}\right]^{\frac{1}{\alpha-1}}, \tag{3.3}$$

$$m_{g,t} = \left[(1-\theta)\,{\xi_t}^{\alpha-1} + \theta\right]^{\frac{1}{\alpha-1}}. \tag{3.4}$$

Using the homogeneity property, the equity price/output ratios of the two sectors can be defined as

$$p_{b,t} = \frac{P_{b,t}}{Y_{b,t}},\; p_{g,t} = \frac{P_{g,t}}{Y_{g,t}}. \tag{3.5}$$

The equity returns of the brown/green firms are

$$dR_{b,t} = \Lambda_{b,t_-}dt + \Theta_{b,t_-}^T dB - \left[1 - \frac{(1-\zeta)\,p_b\left(\eta_t^J\right)}{p_b(\eta_{t_-})}\right]dN_{p,t}, \tag{3.6}$$

and

$$dR_{g,t} = \Lambda_{g,t_-}dt + \Theta_{g,t_-}^T dB - \left[1 - \frac{p_g\left(\eta_t^J\right)}{p_g(\eta_{t_-})}\right]dN_{p,t}, \tag{3.7}$$

where $dB = (dB_{b,t}, dB_{g,t})^T$,and the expressions for Λ_b, Λ_g, Θ_b, Θ_g are given in the Appendix.

The market equilibrium is characterized by the set of functions $\{p_b(\eta_t), p_g(\eta_t), \xi(\eta_t), r(\eta_t)\}$ that

respectively represent the ratios of equity price/output, the relative price of green to brown output, and the instantaneous risk-free interest rate. To search for the equilibria, I employ the PDE method proposed by Carmona (2016), which boils down to solving the system of Hamilton-Jacobi-Bellman equations derived from the brown firm's and household's problems. The system of equations is described in detail in the Appendix.

3.1 Household's Problem

At any point in time, the representative household takes as given the above equilibrium variables and chooses consumption streams C_b, C_g as well as portfolio holdings π_b, π_g to maximize his/her lifetime utility

$$V_t = \sup_{C_b, C_g, \pi_b, \pi_g} \mathbb{E}\left[\int_t^\infty f\left(C_s, V_s\right) ds\right], \tag{3.8}$$

subject to the budget constraint

$$\begin{aligned}\frac{dW_t}{W_{t_-}} = &\left[r_{t_-} - c_{b,t_-} m_{b,t_-} - c_{g,t_-} m_{g,t_-} + \pi_{b,t_-}\left(\Lambda_{b,t_-} - r_{t_-}\right) + \pi_{g,t_-}\left(\Lambda_{g,t_-} - r_{t_-}\right)\right] dt \\ &+ \left[\pi_{b,t_-}\Theta^T_{b,t_-} + \pi_{g,t_-}\Theta^T_{g,t_-}\right] dB \\ &+ \left\{\pi_{b,t_-}\left(1-\zeta\right)\frac{p_b\left(\eta^J_t\right)}{p_b(\eta_{t_-})} + \pi_{g,t_-}\frac{p_g\left(\eta^J_t\right)}{p_g(\eta_{t_-})} - \left(\pi_{b,t_-} + \pi_{g,t_-}\right)\right\} dN_{p,t}.\end{aligned} \tag{3.9}$$

In the Appendix, I show that the consumption of green/brown output is

$$C_{b,t} = \frac{\rho W_t}{m_{b,t}\left[1 + {\xi_t}^{1-\alpha}\left(\frac{\theta}{1-\theta}\right)\right]},\ C_{g,t} = \xi_t^{-\alpha}\left(\frac{\theta}{1-\theta}\right) C_{b,t}. \tag{3.10}$$

As a matter of fact which I verify in the Appendix, in the case of unitary EIS, the price-dividend ratio of the aggregate consumption claim is a constant, i.e.

$$\frac{m_{b,t}\, C_{b,t} + m_{g,t}\, C_{g,t}}{W_t} = \rho. \tag{3.11}$$

Due to the normalization in which the price of the aggregate consumption basket is unitary, it follows that

$$C_t = \rho W_t. \tag{3.12}$$

Exploiting the homogeneity of the problem, I show that the household's value function takes

the form

$$V_t = V\left(W_t, \eta_t\right) = \rho^{1-\gamma} G\left(\eta_t\right) \frac{W_t^{1-\gamma}}{1-\gamma}, \tag{3.13}$$

where the reduced-form G of the value function measures the (transformed) marginal value of wealth and satisfies the HJB Equation (6.11).

3.2 Brown Firm's Problem

At any time t, the brown firm takes prices, together with the dynamics of the stochastic discount factor (SDF), as given and maximizes the firm present value by choosing its investment intensity in mitigation technologies, i.e.

$$P_{b,t} = p_b(\eta_t) Y_{b,t} = \sup_{z \geq 0} \mathbb{E}\left[\int_t^\infty \frac{M_s}{M_t} Y_{b,s}\, m_{b,s} ds\right], \tag{3.14}$$

where M_t is the equilibrium stochastic discount factor that captures both the time value and risk premium.

As shown by Duffie and Skiadas (1994), the stochastic discount factor M_t is

$$M_t = e^{\int_0^t f_V(C_s, V_s) ds} f_C\left(C_t, V_t\right). \tag{3.15}$$

No arbitrage implies that

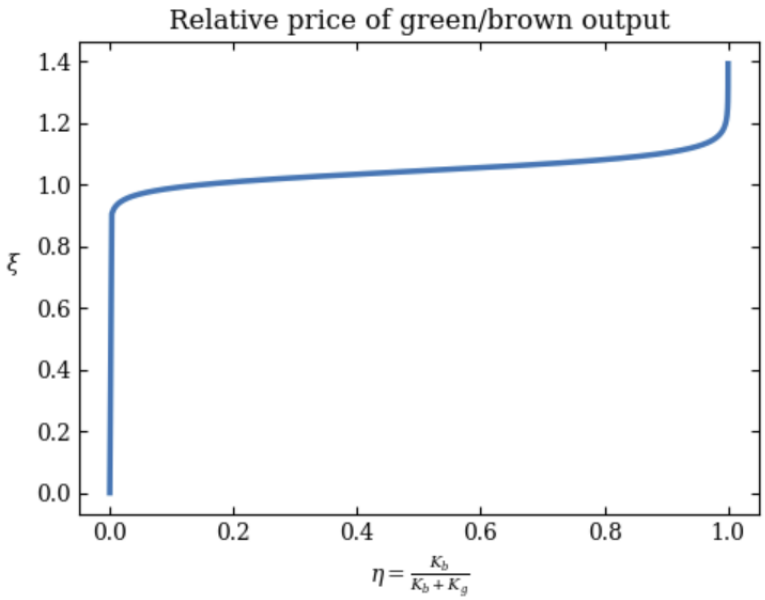
$$\begin{aligned} \frac{dM_t}{M_{t_-}} = & -r_{t_-} dt - \Phi_{b,t_-} dB_{b,t} - \Phi_{g,t_-} dB_{g,t} \\ & + \left(\kappa_{p,t} - 1\right)\left(dN_{p,t} - \frac{\lambda_g + \Delta_\lambda \eta_{t_-}}{1 + \phi z_{t_-}} dt\right), \end{aligned} \tag{3.16}$$

where $\kappa_p - 1$, Φ_b, Φ_g are endogenously determined market prices of the transitional policy risk and diffusion risks. Their expressions are derived in the Appendix. Especially, κ_p also measures the relative magnitude of the SDF after versus before each jump N_p, i.e. $\frac{M_t^J}{M_{t_-}} = \kappa_{p,t}$.

As shown in the Appendix, the brown firm chooses its green innovation intensity by balancing the benefit of lower exposure to policy risks and the cost of reduction in capital gain, i.e.

Figure 3: **Equilibrium relative price of the green/brown output.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



$$z_{t_-} = \arg\max_{z_{t_-} \geq 0} \underbrace{Y_{b,t_-} p_b(\eta_{t_-}) \left(1 - z_{t_-}\right)}_{\text{Capital Gain}} + \underbrace{Y_{b,t_-} \left(\frac{\lambda_g + \Delta_\lambda \eta_{t_-}}{1 + \phi z_{t_-}}\right) \left[\kappa_{p,t} \left(1 - \zeta\right) p_b \left(\eta_t^J\right) - p_b(\eta_{t_-})\right]}_{\text{Asset Stranding}}. \tag{3.17}$$

Therefore,

$$z_{t_-} = \max\left\{0, \frac{1}{\phi}\left(\sqrt{\frac{\phi\left(\lambda_g + \Delta_\lambda \eta_{t_-}\right)}{p_b(\eta_{t_-})}\left[p_b(\eta_{t_-}) - \kappa_{p,t}\left(1 - \zeta\right) p_b\left(\eta_t^J\right)\right]} - 1\right)\right\}. \tag{3.18}$$

3.3 Characterization of the Market Equilibrium

In this section, I characterize the equilibrium variables $p_b(\eta_t), p_g(\eta_t), \xi(\eta_t)$ and $r(\eta_t)$. Hereafter, I suppress the dependence on t when possible for the ease of notation. First, regarding ξ, the closed-form solution of the relative output price is derived in the Appendix as follows

$$\xi(\eta) = \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\alpha}} \left(\frac{\eta}{1-\eta}\right)^{\frac{1}{\alpha}} \tag{3.19}$$

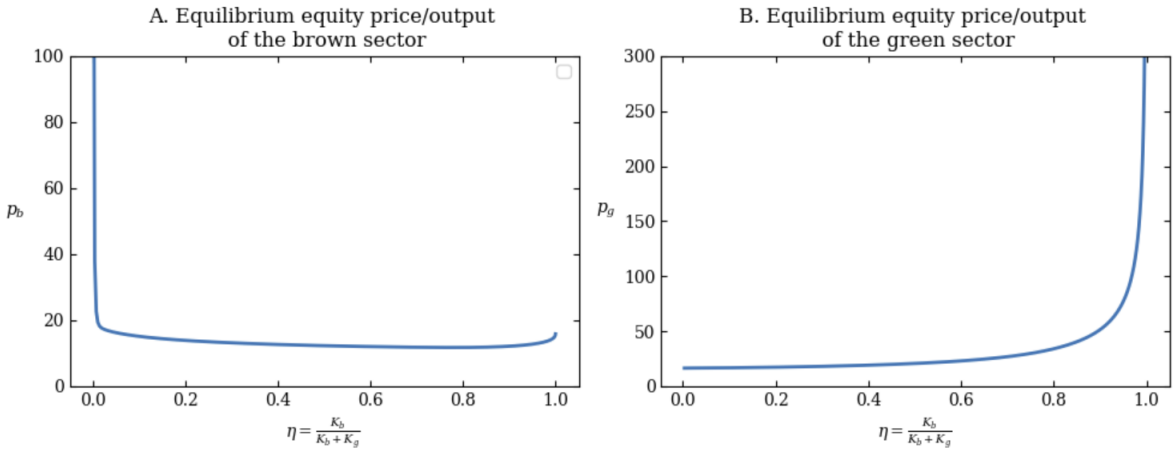
Figure 3 illustrates the relative output price ξ. Intuitively, due to the investor's CES consumption

basket, he/she wants to consume both goods. If either of the goods has a limited supply, the marginal value of its consumption moves up accordingly. When the green sector is small (high η), its output becomes scarce. The representative household, whose marginal utility of green consumption is high in such a state of the economy, has a strong desire for green goods while the supply is limited. The discrepancy between demand and supply spikes the price of green output compared to that of its brown peer. In contrast, in the low η state, the brown output is limited, and the marginal utility of its consumption inflates. Under such a circumstance, the price of brown output dominates that of the green, which then lowers ξ. In addition, holding η constant, the relative price of green output, ξ, is strengthened/attenuated by the bias towards green consumption θ/the degree of substitution between the two goods α. Essentially, the more the representative household values the green good, the higher the price he/she is willing to pay for it. In addition, when the brown output is a good substitute for the green one, the consumer is less sensitive to scarcity, and the price gap should be reduced.

The equity valuation ratios (equity price/output) of the brown and green sectors show a stark contrast, as in Figure 4. While p_b declines, p_g appreciates when the relative size of the brown sector ramps up. There are different forces that shape the valuation ratios of brown and green equities. The first one comes from the fact that the representative investor wants to hold a balanced portfolio to diversify away from idiosyncratic risks. When the green/brown sector dominates the economy (respectively left/right tail of η), most of the market portfolio is made up of the green/brown equity. Under such circumstances, the brown/green equity accounts for a small proportion of the market portfolio and its value co-moves less with wealth. This makes it valuable in helping the investor to diversify his/her portfolio. Thus, the investor is willing to accept a lower level of return and pay a higher price for the brown/green asset in exchange for the diversification benefit when η is low/high. Secondly, the policy risk is positively priced in equilibrium, and interestingly, both sectors load on that risk as observed in Equations (3.6)-(3.7). At the right tail of η when the risk arrives more frequently, both equity assets have positive loadings on the risk in equilibrium (see Figure 12). In such a state, the investor demands higher returns to compensate for the risk, which drives down firm valuations. The third factor lies in the household's preference for consuming green goods. As analyzed above, the relative price of the green output generally increases with the bias toward green consumption, thereby making the green tree more valuable. This effect of consumption preference

Figure 4: **Equilibrium valuation ratios.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



on the valuation ratio of the green equity is particularly stronger when the green output is scarce (high η), which further reinforces p_g at its right end.

Proposition 1. *The equilibrium interest rate has two components that reflect the precautionary saving motive with respect to the diffusion risks and policy risk*

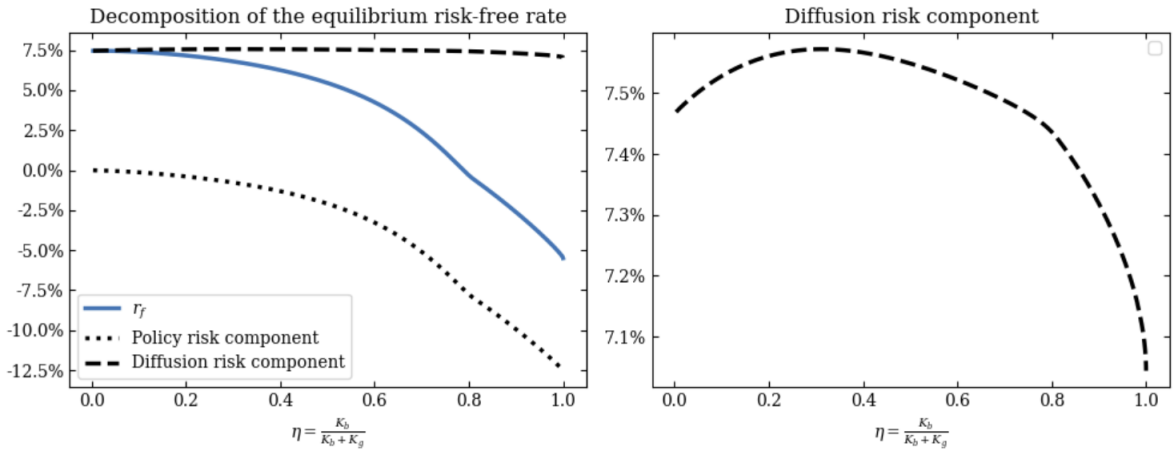
$$r(\eta) = \underbrace{\rho + \mu_c(\eta) - \gamma\sigma_c^2(\eta)}_{\text{Diffusion Risks}} + \underbrace{\left(\frac{\lambda_g + \eta\Delta_\lambda}{1 + \phi z(\eta)}\right)\kappa_p(\eta)\left[\pi_b(\eta)\frac{(1-\zeta)\,p_b\left(\eta^J\right)}{p_b(\eta)} + \pi_g(\eta)\frac{p_g\left(\eta^J\right)}{p_g(\eta)} - 1\right]}_{\text{Policy Risk}}, \tag{3.20}$$

where μ_c and σ_c are the drift and the volatility of the optimal consumption, which are both given in the Appendix.

Next, I turn to the risk-free interest rate. Equation (3.20) reveals two underlying components of the risk-free rate. The first one represents the precautionary saving motive in response to diffusion risks, which corresponds to the dashed line in Figure 5. This component is hump-shaped since the diversification effect is strongest when the volatilities of the two sectors balance each other. Essentially, the Brownian shocks B_b and B_g are sector-level risks, and their total impact on consumption can be reduced by holding a combination of brown and green assets. Diversification reduces total consumption volatility and the household's demand for the risk-free asset. In turn,

Figure 5: **Equilibrium risk-free rate.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



the interest rate has to go down to keep the risk-free asset in zero net supply. In contrast, the component of the risk-free rate with regard to the policy risk (dotted line) decreases monotonically in η. The probability of policy shock increases in η, and this shock, upon arrival, destroys a fraction of aggregate consumption. Therefore, the precautionary saving motive with respect to the policy risk also increases in η because households want to increase their savings as a way to smooth their consumption in the event of the shock. Hence, the disaster-like effect boosts the household's appetite for the risk-free asset, thereby sending the rate lower. It is worth noting that the second effect becomes less pronounced if the equilibrium innovation intensity z is higher.

3.4 Solutions to the Household's and Brown Firm's Problems

3.4.1 The Household

Proposition 2. *The optimal risky portfolio of the representative household is*

$$\begin{bmatrix} \pi_b(\eta) \\ \pi_g(\eta) \end{bmatrix} = \frac{1}{\gamma}\Sigma(\eta)^{-1}\left\{ \begin{bmatrix} \Lambda_b - r(\eta) \\ \Lambda_g - r(\eta) \end{bmatrix} + \begin{bmatrix} \left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi z(\eta)}\right)\kappa_p(\eta)\left[(1-\zeta)\frac{p_b(\eta^J)}{p_b(\eta)} - 1\right] \\ \left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi z(\eta)}\right)\kappa_p(\eta)\left[\frac{p_g(\eta^J)}{p_g(\eta)} - 1\right] \end{bmatrix} + \Omega(\eta)\frac{G'(\eta)}{G(\eta)} \right\}, \tag{3.21}$$

where Σ is the return variance-covariance matrix, and Ω is the covariance matrix between returns and η. Both expressions are provided in the Appendix.

Figure 6: **Optimal risky portfolio.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.

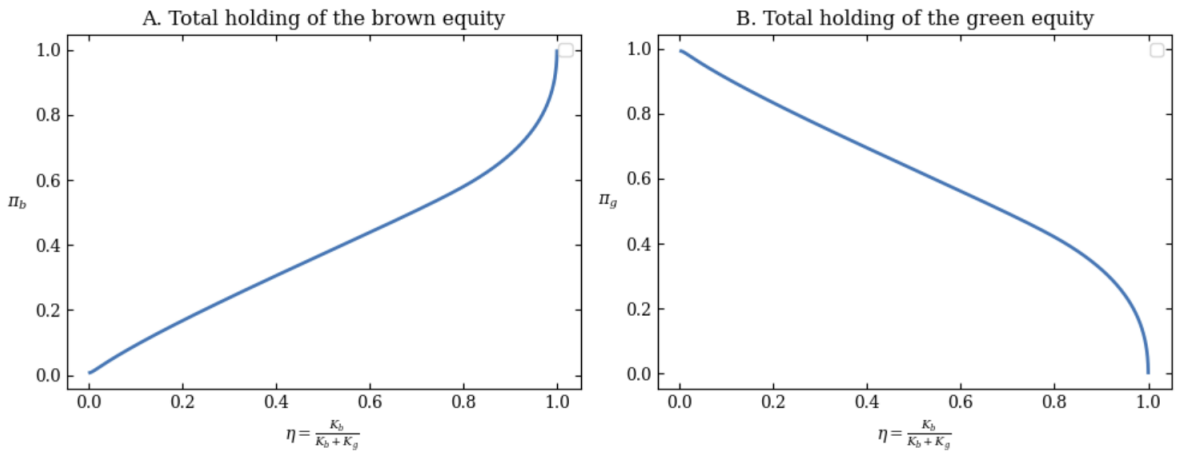


Figure 6 shows the optimal risky portfolio along different states. When the brown sector accounts for 90 percent of the total output in line with the recent estimate[6], the investor allocates around 70 percent of his/her wealth to the brown equity asset, and 30 percent to the green one. This is broadly consistent with empirical evidence from Global Sustainable Investment Alliance (2020). As our economy transitions toward a greener side thanks to intensive regulations, the investor gradually prioritizes green equity and underweights polluting assets in his/her portfolio. There is a strand of literature attributing the flow into green assets and the consequent expected return spread to the preference for green investing; however, those phenomena can be rationalized in this setting without relying on the warm-glow investing channel.

From Equation (3.21), the household's portfolio satisfies a three-fund separation property. The first component is the standard myopic part driven by the household's diversification motive to reduce consumption volatility. The second component corresponds to the tilt against policy risk. This term depends on the gain/loss in values of equity assets and the change in the SDF (mapped directly to the price of policy risk) when the risk materializes. In general, the household wants to tilt the risky portfolio toward/away from an asset if it becomes more valuable/stranded to him/her in the new state of the world when the Government tightens environmental policies. When these events happen, a fraction ζ of brown output is obliterated, causing a negative return from holding

[6]See Clements et al. (2024).

the brown equity. This loss is amplified through an increase in the SDF (equal to the increase in the marginal utility of consumption) by a factor κ_p. In addition, although the policy shock does not affect the level of output from the green firm directly, it alters the state variable, thereby having an indirect effect on the equity price/output ratio p_g. From Figure 4, it is clear that p_g declines after each policy shock when η jumps to the left; therefore, the green equity return is also negative following policy shocks. Thus, in general, both the values of brown and green equity assets deteriorate due to policy risks. That causes the investor to divert his/her wealth away from both risky assets if he/she is concerned about the Government enforcing stricter environmental regulations. The magnitude of the second component is attenuated by the equilibrium innovation intensity z. Put differently, if the intensity of policy shock is abated through mitigation efforts, investors should downsize the role of policy risk in their risky portfolios.

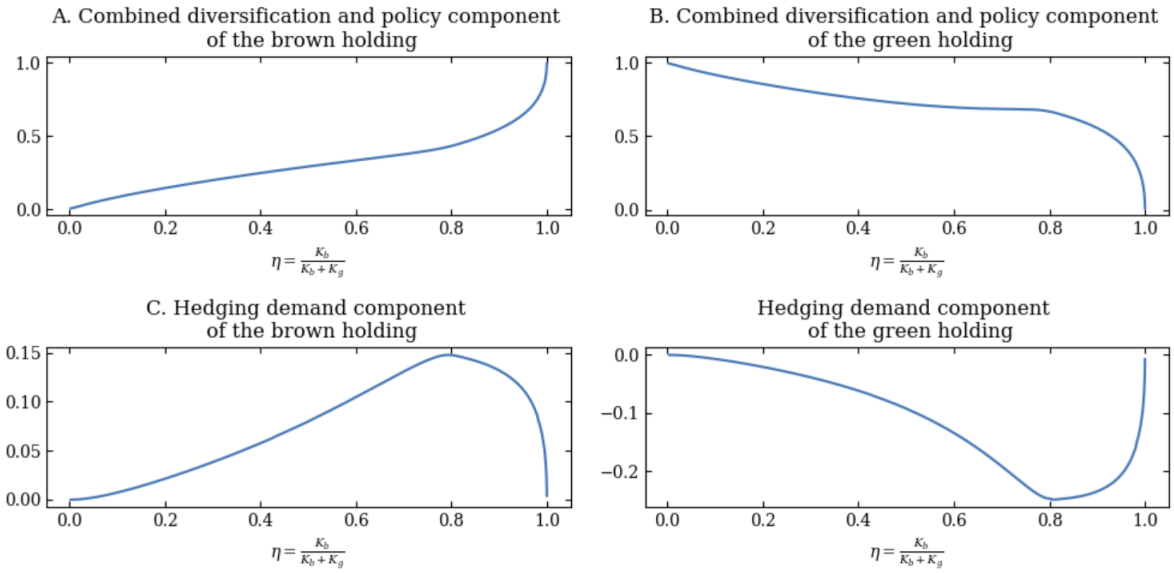
In combination, the diversification and policy components play a major role in the investor's holding of risky assets. In contrast to the tilt against policy risk that depresses it, the diversification motive generally draws the investment in risky assets. The diversification part is strong enough to override the negative policy component, as shown in Panels A and B of Figure 7. Once the green firm produces 10 percent of the total output, if not accounting for the third part—hedging demand—in panels C and D, an average investor would distribute around 50 percent of his/her wealth to the green equity. Nevertheless, the reason why the number is much smaller in equilibrium results from the hedging purpose described below.

The third component in Equation (3.21) reflects the hedging demand against changes in the state variable η, which measures the relative supply of green output. Since the household possesses an altruistic preference for the green good, he/she is averse to its scarcity in the state of high η. In that state, he/she needs to switch to the brown output, which is a good substitute for the green good. Thus, the hedging component drives down the holding of green and increases the holding of brown equity. This is confirmed by Figure 7, where the component is negative/positive for the green/brown holding. In aggregate, the hedging components of the two equity assets exactly offset each other, and thus do not impose any effect on the investor's total holding of risky assets beyond the reallocation between them.

The reduced-form value function G is a crucial driver of the marginal value of wealth for the household. The function has a reverse L shape with an elevated right tail depicted by Figure 8.

Figure 7: **Decomposition of the optimal risky portfolio.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



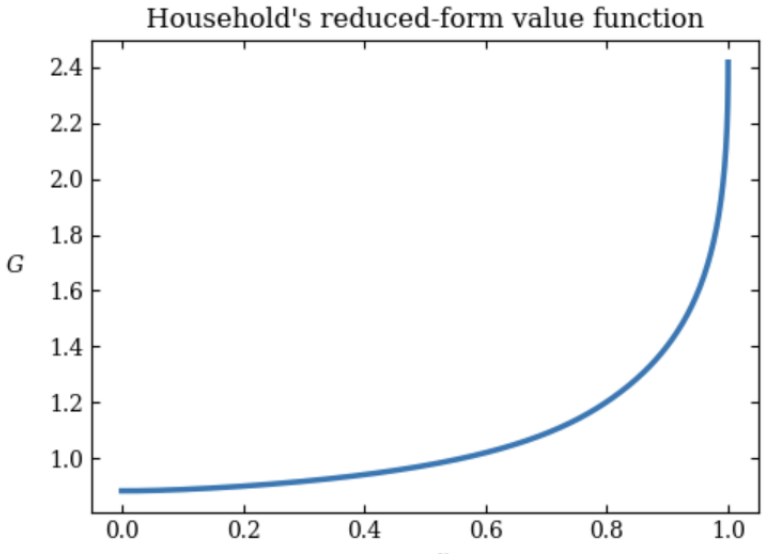
This is explained by the combination of three forces: diversification, taste for green consumption, and policy risk. Without the last two, G would be U-shaped due to the CES consumption basket. In that case, the household would prefer to consume a balanced mix and dislikes the scarcity of either type of good. Therefore, his/her marginal value of wealth, which is equal to the marginal value of consumption, would be high at both ends of η. However, in reality, in addition to the diversification effect, the household's taste for green consumption comes into play. Because he/she has a strong preference for green goods, it hurts when the goods become scarce. When the green sector is relatively small (high η), he/she cannot consume the favorite but has to switch to the less wanted substitute (i.e., the brown). Thus, the marginal value of consumption in this case outpaces its level at the other end of η where the brown supply is limited. Lastly, the policy risk kicks in at the right tail of η. The risk destroys part of the aggregate consumption and raises the household's desire for an extra unit of good to an excessive level.

3.4.2 The Brown Firm

Figure 9 displays the optimal mitigation effort for the brown firm. As shown previously in Section 3.2, the brown firm chooses its level of innovation in green technologies to abate its exposure

Figure 8: **Household's reduced-form value function.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.

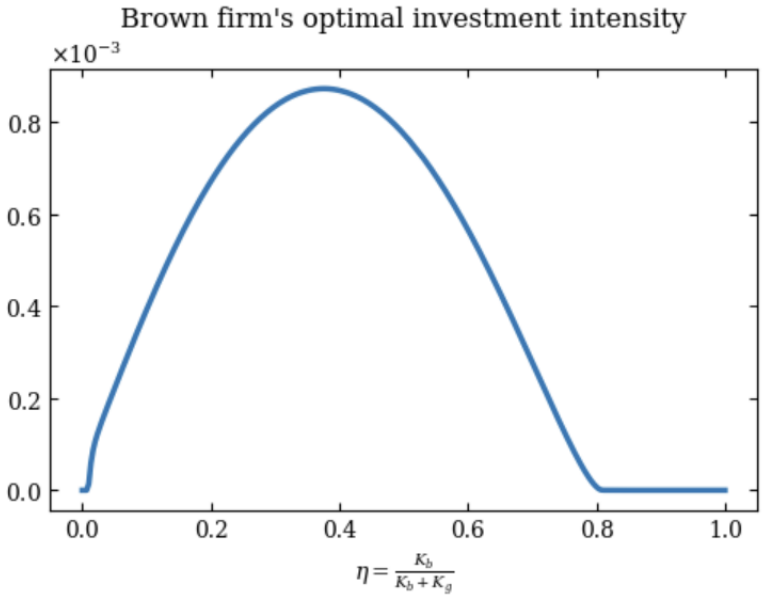


to policy risks and maximize firm value. By Equation (3.17), the brown firm trades off between the benefit from curbing asset stranding and the cost of lowering its equity's capital gain. When the policy risk materializes, the change in the brown firm's present value results from three sources: First, the policy tightening reduces the level of brown output. For instance, when the Government imposes a carbon tax on firm emissions, which is proportional to its output, the brown firm's cash flow shrinks accordingly. Second, the state variable η that captures the relative supply of brown output jumps discretely to the left (the degree of this jump is highest when the two sectors have a comparable size). This sudden change in the state variable leads to a revision in the firm valuation (price/output) ratio. Third, since the policy risk is priced positively as shown in Figure 11, the SDF - which maps directly to the marginal value of consumption - increases after such a disaster-like event arrives.

From Figure 4, one can observe that at the far right end of η, the brown firm's valuation ratio p_b may fall marginally when the policy risk realizes because η jumps just slightly to the left. However, when η is high, the drop in the valuation ratio, if any, would be masked totally by the increase in the SDF. As depicted in Figure 11, the modified price of policy risk κ_p, which gauges the SDF magnitude after a policy risk, is high when the brown sector strongly dominates. In this state, the

Figure 9: **Brown firm's optimal innovation intensity.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



policy shock materializes frequently. After such continual disaster-like events, a large amount of aggregate consumption is wiped out, rendering each additional unit of good highly valuable. Thus, this third effect on the SDF raises the present value of each brown output unit and, consequently, the firm's present value to the investor. This effect dominates any drop in the level of output Y_b as well as in the valuation ratio p_b. Because of that, in contrast to one's intuition that the brown firm value would plunge when it is hit by intensive policy tightening, the disaster effect actually lifts the firm's present value.

All in all, when η is too high, there is no benefit for the brown firm from curbing its exposure to asset stranding due to policy risk, and the firm is better off not investing in abatement efforts. It is only beneficial for the brown firm to invest in green technologies once the price of the policy risk is low enough for the drop in brown output to outweigh the effect of the SDF on the firm's present value. In contrast, at the far left tail of η, although the price of policy risk is low, the valuation ratio p_b of the brown firm rises sharply upon a contraction in relative brown output supply. This is due to the extraordinary diversification benefit that brown equity provides to the investor. The effect overrides a large part of the shrinkage in output level due to the policy shock, which now

occurs infrequently. As a consequence, the brown firm benefits only marginally from curbing the impact of policy risk when η is very low, and should invest less in mitigation technologies.

4 Asset Pricing Implications

4.1 Prices and Quantities of Risks

Proposition 3. *From the dynamic of the stochastic discount factor 3.16, the equilibrium prices of diffusion risks Φ_b, Φ_g and policy risk κ_p are obtained as*

$$\Phi_b(\eta) = \gamma\big[\Gamma_b\pi_b(\eta) + \Gamma_g\pi_g(\eta)\big] - \frac{G^{'}(\eta)}{G(\eta)}\eta\,(1-\eta)\,\sigma_b, \tag{4.1}$$

$$\Phi_g(\eta) = \gamma\big[-\Delta_b\pi_b(\eta) + \Delta_g\pi_g(\eta)\big] + \frac{G^{'}(\eta)}{G(\eta)}\eta\,(1-\eta)\,\sigma_g, \tag{4.2}$$

and

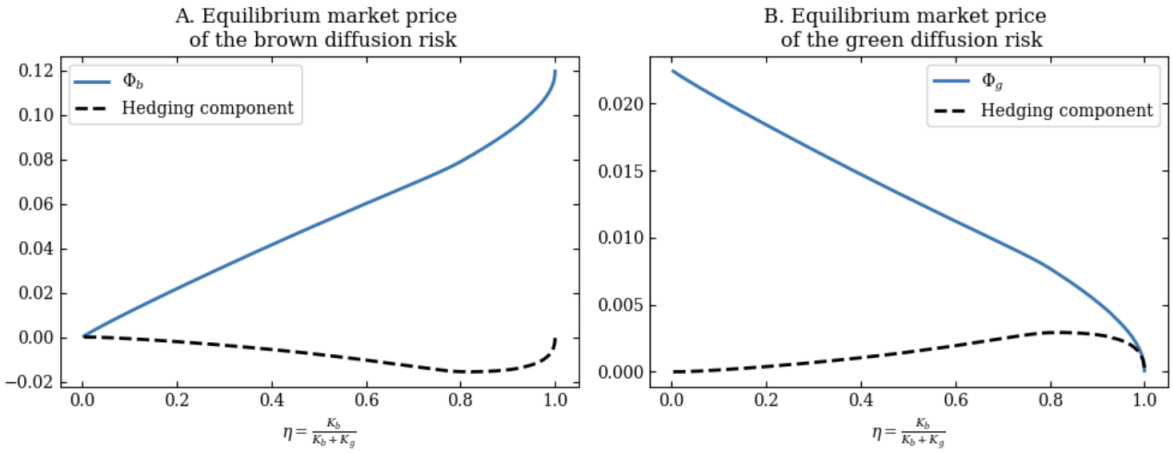
$$\kappa_p(\eta) = \left(\pi_b(\eta)\,(1-\zeta)\,\frac{p_b\left(\eta^J\right)}{p_b(\eta)} + \pi_g(\eta)\frac{p_g\left(\eta^J\right)}{p_g(\eta)}\right)^{-\gamma}\frac{G\left(\eta^J\right)}{G(\eta)}. \tag{4.3}$$

The prices of diffusion risk Φ_b, Φ_g contain two separate elements. The first one is the myopic (market) component capturing how much the investor requires for compensation from assets that co-move with his/her wealth. This market component of the price of brown/green diffusion risk moves in tandem with the relative share of the brown/green output. In Figure 10, the component corresponds to the gap between the solid and dashed lines. When a sector is big, it accounts for a majority of the market portfolio, and the shock to the sector's output causes a lot of volatility in the total wealth. The investor becomes averse to that sector-level risk and commands a high reward for bearing a unit of it. Vice versa, the other sector makes up less of the market portfolio, and its sector-level shock carries a low market-component price of risk.

The second component relates to the hedging demand mentioned in Section 3.4.1 and stems from the sensitivity of the marginal value of wealth with respect to the state variable. From Figure 10, it is clear that the hedging component is negative/positive for the price of brown/green diffusion risk. In an economy where the representative household has a balanced CES consumption basket, he/she would like to consume an even mix of goods. In that world, any state with scarcity should be detrimental, and his/her marginal utility of consumption would be steep at both ends of η. The brown/green diffusion risk should then carry a positive hedging component in its price of

Figure 10: **Equilibrium price of diffusion risks.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



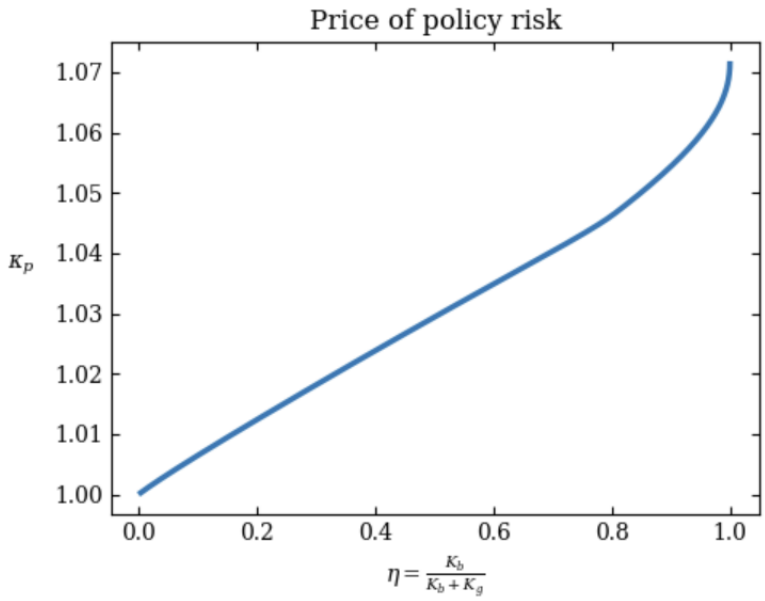
risk when η is low/high, and the sign would switch to negative once the relative supply scale tilts to the other side. However, in the current setting, the representative household derives a higher utility from consuming green goods and is more averse to the contraction in the supply of green output. The green asset is a poor hedge against that bad scenario, and hence, the green-sector shock that co-moves with the level of green output is penalized by the investor with a positive hedging component in the price of risk. On the contrary, the brown good can substitute for the green peer well in the adverse state of scarcity, thereby making the brown asset a good hedge. This explains why the hedging component of the brown-sector price of diffusion risk is negative.

Figure 11 shows a monotonic trend of the modified price of policy risk κ_p. This term plays a key role in driving both the risk-free rate, risk premia, and the brown firm's optimal innovation intensity. Given that the real price of policy risk is $\kappa_p - 1$, its magnitude is quite significant compared with the prices of diffusion risks. From Equations (4.3) and (3.13), it is straightforward to see that the price of policy risk is exactly equal to the jump in the marginal value of wealth after versus before the risk materializes. Essentially, the investor requires higher compensation for the policy risk when it causes a more abrupt surge in the marginal value of wealth.

The quantities of diffusion and policy risks obtained from Equations (6.23)-(6.24) show interesting patterns. Figure 12 depicts the degree to which equity assets are exposed to the policy risk. Both asset returns mostly possess a positive loading, which is surprising given the fact that only the

Figure 11: **Equilibrium price of policy risk.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



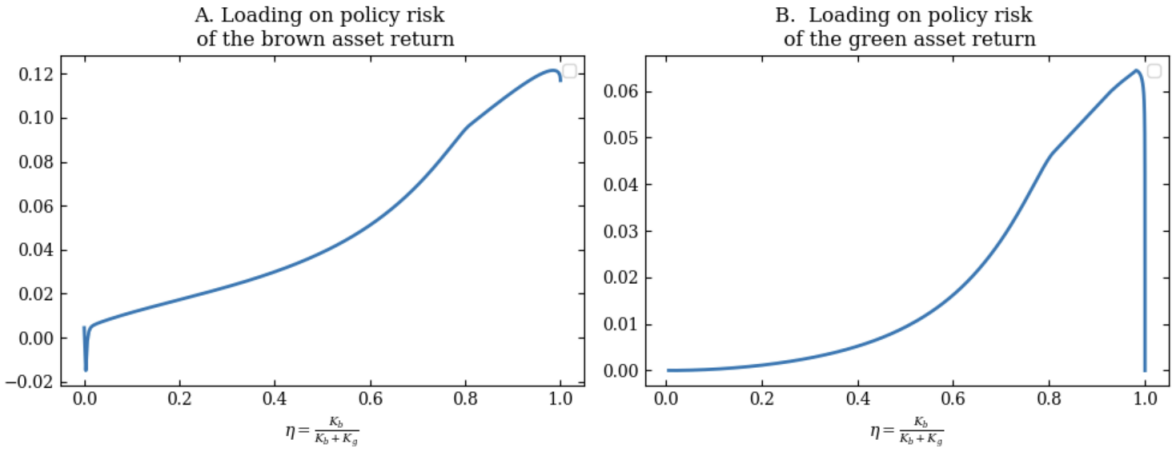
brown firm is directly targeted by the policy tightening. This is due to its impact on the relative size of firms. Once the brown sector shrinks after being hit by the policy shock, the green firm becomes relatively bigger, causing its valuation to change as shown in Figure 4. Under these circumstances, the combination of forces analyzed in Section 3.3 pulls down the green valuation ratio p_g, and hence its equity return.

Contrary to the similarity in their exposure to the policy shock, the returns of green and brown equity assets exhibit a stark difference in their loadings on diffusion risks. Figure 13 illustrates the distinct risk exposure patterns of the two assets. Although the diffusion risks are sector-specific, they spread out to the equity asset of the other sector through their impacts on the state variable, even in the case of no fundamental correlation[7]. Hence, both assets are exposed to dB_b and dB_g. In an economy with no fundamental correlation between sector shocks, equity returns generally load more on the shocks to their own sectors. However, it is not the case for the green equity under the estimated parameter. Its return loads more on the shock to the green output when the sector dominates, but more on the shock to the brown output once the brown tree becomes

[7]The phenomenon is also documented in Cochrane et al. (2008).

Figure 12: **Loadings of assets on the policy risk.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



overwhelming. This is due to a strong correlation between the two sector shocks and the lower green output volatility estimated in Section 5. Vice versa, the narrative changes for the brown equity return, which follows a standard case and has a higher exposure to the shock in its own output process. To compare the two assets, the brown (respectively green) equity carries a lower quantity of green (respectively brown) diffusion risk than its counterpart when η is low (high). This is exactly the time when the market component of the price of green (respectively brown) diffusion risk is surging, as shown in Figure 10 [8]. For that reason, the brown/green asset is favored by the investor to diversify the portfolio when the brown/green industry is nascent. The divergence between quantities and prices of diffusion risks is the main driver for the diversification component of the green-brown risk premium spread shown in panel B of Figure 14.

4.2 The Greenium

Proposition 4. *The spread between risk premia of the green and brown equity assets is*

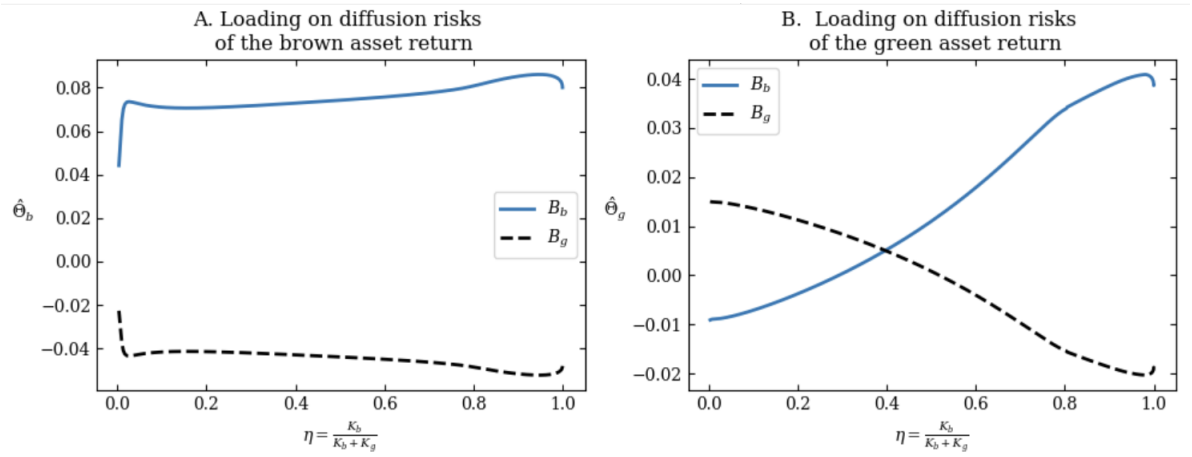
$$\begin{aligned}(E(R_g) - r) - (E(R_b) - r) = \gamma \left(\widehat{\Theta}_g - \widehat{\Theta}_b\right)^T \Theta_W - \left(\widehat{\Theta}_g - \widehat{\Theta}_b\right)^T \Theta_G \\ - \left(\frac{\lambda_g + \eta \Delta_\lambda}{1 + \phi\, z}\right)(\kappa_p - 1)\left[\frac{p_g\left(\eta^J\right)}{p_g(\eta)} - \frac{(1-\zeta)\, p_b\left(\eta^J\right)}{p_b(\eta)}\right]\end{aligned} \tag{4.4}$$

where $\widehat{\Theta}_b, \widehat{\Theta}_g, \Theta_W, \Theta_G$ *are derived in the Appendix.*

[8] The market components correspond to the gap between the total price of diffusion risks and the hedging elements.

Figure 13: **Loadings of assets on the diffusion risks.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



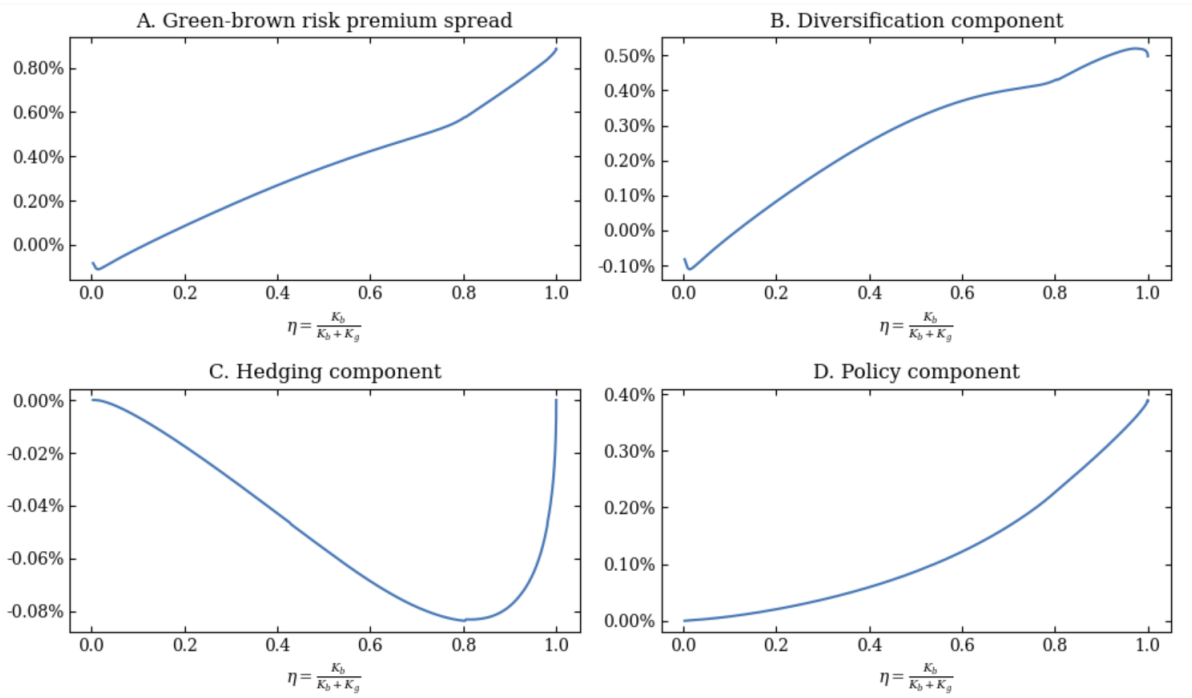
Panel A of Figure 14 shows that the green-brown risk premium spread (hereafter green premium or greenium) gradually declines as the brown sector grows. To gauge the size of the spread, when the relative share of brown output is around 80-90 percent, the green premium is approximately 60-70 bps. This magnitude is broadly in line with the robust estimate reported by Eskildsen et al. (2024). For the purpose of understanding the forces that shape the green premium, it is crucial to break down the spread into separate elements as in Equation (4.4).

The first term is a total wealth component driven by the diversification benefits that two assets provide. Whether the green or brown equity asset is more valuable for diversification depends on its size in the economy. When the economy tilts toward the brown/green side, the green/brown asset will command a lower premium from the diversification component because it co-moves less with total wealth and helps to diversify the representative investor's portfolio. This is confirmed by Panel B of Figure 14, which reiterates that this component is positive when the green sector strongly dominates at the far left end of η but gradually decreases and shifts to the negative territory along the increasing pathway of η. The diversification component accounts for a major share of the green premium, reaching about 40-50 bps as the relative share of brown output rises to 80-90 percent.

The second term is the relative-supply hedging premium originating from the representative investor's CES basket with an altruistic preference to consume green output. As the relative size of the brown sector increases, the green output becomes scarcer, and its price increases. In that

Figure 14: **Greenium Decomposition.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.

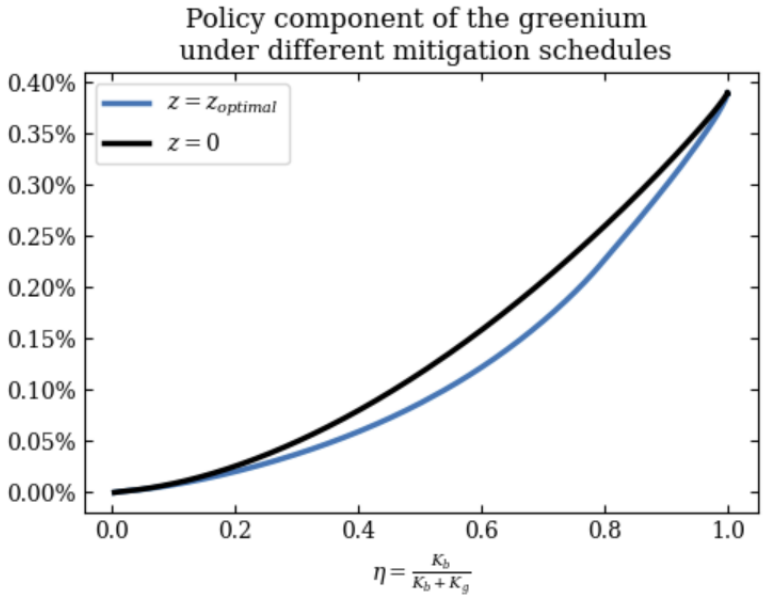


state, the representative investor's marginal value of consumption soars as he/she cannot have access to the green output that he/she favors. In Panel C of Figure 14, the hedging component keeps rising when the relative supply of the green output contracts. This component offsets part of the greenium, amounting to about 8 bps when the relative supply of green output falls to 10-20 percent.

The third term originates from the policy risk. Since the brown asset is more prone to being stranded when the Government enacts stringent climate policies, the investor demands a higher premium to hold such an asset. At a green output share of 10-20 percent, the policy component of the spread reaches 20-30 bps. Equation (4.4) shows that the policy component widens when the intensity of policy risk or the brown firm's exposure to it surges. Since the arrival rate of policy risk intensifies with the share of brown output, this term increases in proportion to η if the brown firm does not invest in abatement. From Equation (4.4), it is also clear that the policy component grows along with the price of policy risk κ_p. Because the price of policy risk increases in η, the expansion of the brown sector, through this channel, has an additional impact on the green premium above

Figure 15: **Policy component under different green-technology investment plans.**

Parameter values are $\gamma = 1.5$, $\rho = 0.06$, $\mu_b = 0.02$, $\sigma_b = 0.08$, $\mu_g = 0.015$, $\sigma_g = 0.015$, $\nu = -0.6$, $\theta = 0.85$, $\alpha = 40$, $\Delta_\lambda = 2.5$, $\lambda_g = 0.1$, $\zeta = 0.045$, $\phi = 600$.



and beyond the rise in policy risk intensity.

Green innovations by the brown firm can scale down the intensity of policy risk, thereby counteracting the policy component of the brown-green return spread. The brown firm's investment intensity in Figure 9 exhibits a nonlinear pattern because it has to balance between curbing the policy shocks to output and reaping the benefits from market valuation increases. While the brown firm chooses to substantially attenuate the policy risk exposure when η is low, it is no longer optimal to invest in mitigation technologies when the intensity of policy risk becomes too high, as discussed in section 3.4.2. Figure 15 demonstrates the impact of different abatement plans on the policy component of the greenium. The total effect of green innovations, both direct through the shock arrival rate and indirect through firm valuations, in shielding the brown firm from policy shocks is limited, as the firm acts at a time when the arrival intensity is relaxed, rather than when it is vigorous. Altogether, the brown sector's abatement effort and the hedging motive against the scarcity of green outputs mask part of the greenium, which contributes to explaining the modest robust estimates of greenium from Eskildsen et al. (2024).

Table 1: Parameter values

Parameter	Symbol	Value
Coefficient of risk aversion	γ	1.5
Rate of time preference	ρ	0.06
Average growth rate in brown output (normal time)	μ_b	0.02
Average growth rate in green output	μ_g	0.015
Volatility of brown output (normal time)	σ_b	0.08
Bias toward green consumption	θ	0.85
Base probability of policy risk	λ_g	0.1
Expected loss in brown output	ζ	0.045
Initial relative size brown output	η_0	0.87
Correlation between fundamental shocks	ν	-0.69
Sensitivity of policy risk probability	Δ_λ	2.397
Output elasticity of substitution	α	49.99
Mitigation benefit of green technologies	ϕ	509.5
Volatility of green output	σ_g	0.015

5 Model Estimation

Existing literature in climate finance has been trying to estimate the greenium as a whole without diving into the driving forces behind it. In contrast, the theoretical model in the previous section proposes a three-way decomposition of the green premium, which is based on transitional policy risk, diversification, and hedging motives. This section aims to explore the relative magnitudes and the time trends of the three major ingredients under the calibrated/estimated parameters.

5.1 Estimation Procedure

I run 1000 simulations, each of which contains a 20-year history of the economy from 2004 to 2023 with 300 time steps per year. I then convert the simulated variables to annual frequency. Finally, the constructed variables of each simulation are used to derive the 6 synthetic moments aiming to match their empirical counterparts. The details of the moments are described later in the section, and their values are reported in Table 2.

Simulating the model requires in total 14 parameters: $\gamma, \rho, \mu_g, \mu_b, \sigma_g, \sigma_b, \nu, \theta, \alpha, \Delta_\lambda, \lambda_g, \zeta, \phi$, and the initial value η_0. Out of those 14 parameters, 9 are fixed in line with previous literature. The other 5 parameters: $\alpha, \Delta_\lambda, \phi, \nu, \sigma_g$ are estimated using the Simulated Method of Moments (SMM) to match the targeted moments. Table 1 displays the values of all 14 parameters.

The base probability of policy risk follows Hong et al. (2023a), while the expected loss in brown

output closely aligns with one proposed by Pindyck and Wang (2013). Meanwhile, the bias toward green consumption follows Sauzet and Zerbib (2024), and the magnitude of the initial relative size of brown output broadly agrees with the estimate by Clements et al. (2024). Assuming linear (AK) production technologies, the diffusion volatility and the expected growth rate of brown output match the calibrations in Hong et al. (2023a) and Pindyck and Wang (2013), respectively. The rate of time preference is also comparable to one in Hong et al. (2023a). Regarding the degree of relative risk aversion, Pindyck (2012) points out that the range of estimates in the finance and macroeconomics literature spans from 1.5 to 6. I adopt the estimation from Albuquerque et al. (2016) and set the coefficient of relative risk aversion to 1.5.

The estimated elasticity of substitution between green and brown outputs corroborates that the two types of goods are indeed highly substitutable. Existing literature tends to assume a zero correlation between shocks to different sectors' output, with few exceptions. For example, Karydas and Xepapadeas (2019) calibrate the parameter to be 0.8, which demonstrates a positive correlation between the fundamental shocks. Nevertheless, my estimation shows that the correlation is deeply negative. This indicates a high potential for portfolio diversification benefits. In addition, the pattern is consistent with the view that the two sectors compete in the transition process and, combined with the fact that green and brown goods are good substitutes in consumption, generates significant relative-supply risk that is priced in equilibrium. The estimated sensitivity of the policy risk arrival rate to the brown-sector share is sizable, reflecting the strong policy movements seen in recent decades.

5.2 Estimation Results

During the estimation process, the first and second moments targeted are: the mean of brown firms' investment intensity in green technologies, the mean of the brown-green equity premium spread (greenium), the mean and variance of the brown/green sector's market capitalization ratio, and the mean and variance of the risk-free rate. The following are my construction of those empirical moments and their counterparts in the model.

First, to categorize firms into brown/green, I utilize firms' emission data from Trucost and sort them into terciles within their Fama-French 49 industries based on annual Scope 1 and Scope 2 total emissions, as well as emission intensity. A firm's emission intensity is defined as its total emissions

divided by its total assets. Firms in both top terciles (highest total emissions and emission intensity) are considered brown, and those in both bottom terciles are green. Bolton and Kacperczyk (2021) show that the green premium is closely associated with total emissions. However, because the model separates green and brown sectors based on the contributions to the climate change externality relative to their scales, I also control for firms' size and use emission intensity along with total emissions as criteria.

Second, I use the data provided by Kogan, Papanikolaou, Seru, and Stoffman (KPSS) following their work Kogan et al. (2017) for the list of firms' patents. The patents are then classified as "green" using the European Patent Office's Y02/Y04S tagging scheme. This scheme has been widely used in literature and is more restricted than the scheme developed by the Organization for Economic Co-operation and Development (OECD) (see Hascic and Migotto (2015)). Firms' investment intensity in mitigation technologies is measured as the ratio of their annual number of green patents to total assets. The sector-wide index is calculated as the market-value-weighted average of individual brown firms' intensity. Panel B of Figure 2 plots the time series of the index. In the model, the counterpart for this empirical moment is z, which is the optimal choice of the brown firm and given in Equation (3.18). Under the calibrated parameters, the model generates a similar first moment of investment intensity time series compared to the data.

The green premium referred to in this paper is the spread between the expected returns of brown versus green equities. Asset pricing literature traditionally uses realized returns as approximations for expected returns. In the context of greenium, the two measures can diverge significantly from each other. Pastor et al. (2022) demonstrate this phenomenon and provide the explanation based on the repricing of equities due to an increase in investors' attention to ESG matters. To derive the empirical green premium, I resort to the expected return measures detailed in Eskildsen et al. (2024). I follow their approach and calculate expected returns as the average of four implied cost of capital measures from Gebhardt et al. (2001), Claus and Thomas (2001), Easton (2004) and Ohlson and Juettner-Nauroth (2005). Next, I form the long-short portfolio of U.S. brown-green equities. The categorization is based on the emission lagged 12 months following Zhang (2024) to mimic the information set available to investors due to the delay in emission data release. The long-short portfolio returns are then tracked from July to June, and the monthly performance is depicted in Figure 1. From the model, the greenium is derived by taking the difference between

Table 2: Summary of targeted and simulated moments. The table reports the empirical and simulated values of moments used for the calibration procedure. The model-generated moments are based on 1,000 simulations of the economy for 20 years. The parameter values used in the calibrated model are shown in Table 1.

Moment	Data (Target)	Model (Simulated)
Investment intensity in green technologies (Mean)	0.00028	0.00023
Brown-Green equity premium spread (Mean)	0.0059	0.0053
Ratio of brown to green sector's market cap (Mean)	1.384	1.202
Ratio of brown to green sector's market cap (Variance)	0.1356	0.1073
Risk-free rate (Mean)	0.0136	0.0157
Risk-free rate (Variance)	0.00029	0.00037

brown/green equity premia given in Equations (6.23) and (6.24). From Table 2, one can observe that the model is able to match the first moment of the greenium time series closely.

Since I do not have the direct empirical counterpart for the price/output ratios p_b and p_g, it is natural to choose the ratio of the brown to green sector's market capitalization. The annual market cap for each firm is the total market value of common shares. The total market cap of each sector (brown/green) is the sum of the individual firm's value within that sector. From the model, the ratio can be expressed $\frac{\eta p_b(\eta)}{(1-\eta)p_g(\eta)}$. The simulation generates comparable first and second moments compared with the empirical counterparts.

The risk-free rate is computed by the return on 1-month Treasury bill. This is employed by Fama-French factor models and updated frequently on their website [9]. As shown in Table 2, the first and second moments of the risk-free rate are reasonably matched by the simulation under calibrated/estimated parameters.

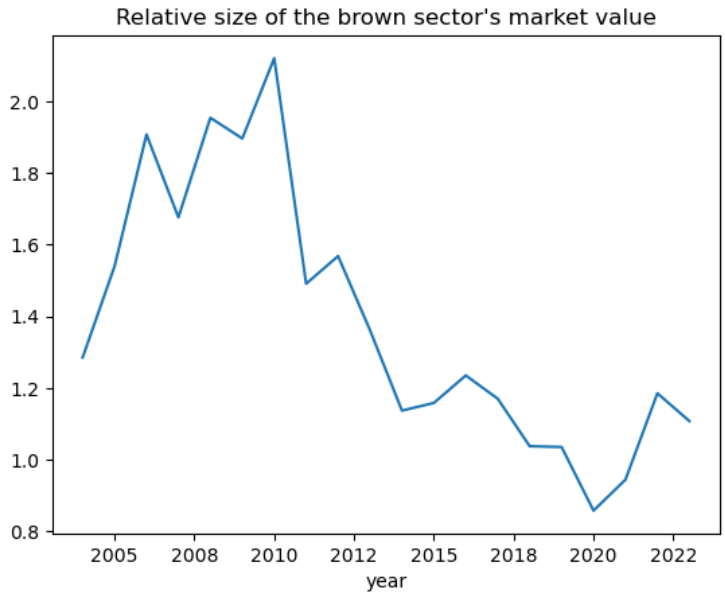
5.3 Decomposition of the Green Premium

The simulation under calibrated/estimated parameters generates a time-varying pattern of the greenium decomposition proposed in Equation (4.4). Figure 17 shows that the relative size of the diversification component dominates and is increasing over time. Because the brown sector has a long history, it is still the driving force of the economy. As the market portfolio is concentrated in the brown equity, the shock to the sector causes a big impact on investors' wealth. Thus, investors require a high risk premium to hold such assets in equilibrium. The nascent green assets help to

[9] See https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Figure 16: **Ratio of the brown to green sector's market capitalization.**

The following figure plots the ratio between the brown and green sectors' market capitalization. The annual market cap for each firm is the total market value of common shares. The total market cap of each sector is the sum of the individual firm's value within that sector.



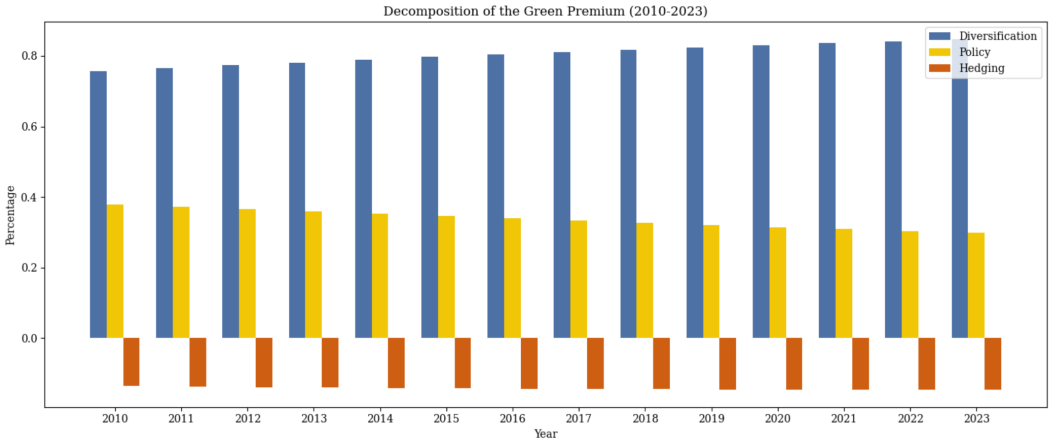
diversify investors' holdings, reduce the total volatility, and thus command lower risk premia.

The policy factor, which was about half the size of the diversification factor in 2010 and one-third in 2023, plays a significant role in shaping the greenium. Because of the pressure from society and governments on companies, the economy has been transitioning at a fast pace toward a greener spectrum over the past decade. Consequently, the dominance of the brown sector gradually shrinks. As the brown sector shrinks, the intensity of policy shocks diminishes, and so does the relative size of the corresponding factor that drives the green premium.

As analyzed in Section 4.2, the hedging component offsets a portion of the greenium contributed by the diversification motive and policy risks. Since the representative consumer features a strong preference for green consumption ($\theta = 0.85$), he/she dislikes the scarcity in green output. This is consistent with a growing number of customers who show a warm-glow preference for green products. When the supply of their preferred goods is limited and prices become more expensive, it is not a favorable state for them. A clear example comes from the electric vehicle industry in 2021-2022, when customers lamented a severe negative shock to the supply chain. Most recently, major technology firms have increasingly sought electricity from clean sources to power their

Figure 17: **Model-generated greenium decomposition over time.**

The figure shows the trend of relative sizes for three components that form the brown/green equity premium spread in Equation (4.4).



artificial intelligence initiatives. However, the available supply is still far from sufficient. In case green firms cannot fulfill the growing demand, their clients need to find alternatives (e.g., hybrid cars from traditional automakers). Green equities, which are negatively impacted by a shortage in the supply of green products, are riskier from this perspective. For instance, the share prices of many EV firms experienced substantial drawdowns in 2022. Meanwhile, brown firms that provide good substitutes will see their equities rewarded (e.g., traditional automakers that are expanding their hybrid lines). This rationalizes why investors place a penalty on green equities and reward brown peers for their hedging benefits against the shortages in their favored consumption.

6 Conclusion

Understanding the feedback between climate-related risks and financial markets is crucial for transitioning toward a sustainable economy. Among many issues, sustainable investing and its impacts have long been topics of debate due to their central roles in facilitating/hindering the progress. In particular, the spread in expected returns between green and brown equities has received substantial attention as many institutional investors have been overweighting green assets in their portfolios, leading to a repricing phase where the realized performance of equities deviates from their expected counterparts. Empirical evidence suggests a positive green premium, although the magnitude is wide-ranging. Existing studies also diverge on the forces that shape the greenium.

The paper proposes a unified framework that explains the origin of the greenium, combining policy risk mitigation and diversification motives. The model is further enriched with heterogeneous outputs and households' desire for consumption of green goods, which results in a third component of the green premium–the hedging factor against output scarcity. The expected return spread between green and brown equity assets generated by the model is broadly consistent with recent robust estimates from the literature. Moreover, the decomposition analysis under the calibrated/ estimated parameters provides nuanced insights into the dynamics of the greenium over time.

The paper, through the general equilibrium effect of policy risks on the stochastic discount factor, also helps explain the puzzling decline in brown firms' investments in mitigation technologies over time. This unintended consequence of stricter regulations is an important consideration that policymakers need to factor in when designing mechanisms to facilitate the green transition.

Altogether, the work sheds light on the elements that govern the green premium and incentives to invest in green technologies. This not only contributes to our understanding of their current levels but also provides insights into their dynamic, time-varying evolution. Finally, given the increasing importance of the green consumption channel for the transition to sustainability, future studies may build on the framework to explore the impact of consumption preference on firms' innovation decisions.

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Appendix

A.1. Household's problem

By definition, $\eta = \frac{Y_b}{Y_b+Y_g}$. Applying Ito's lemma, the evolution of η follows

$$\begin{aligned}
d\eta =& \frac{Y_g\, dY_b}{(Y_b+Y_g)^2} + \frac{-Y_b\, dY_g}{(Y_b+Y_g)^2} + \frac{1}{2}\frac{-2Y_g\ (dY_b)^2}{(Y_b+Y_g)^3} + \frac{1}{2}\frac{2Y_b\ (dY_g)^2}{(Y_b+Y_g)^3} \\
&+ \frac{(Y_b)^2-(Y_g)^2}{(Y_b+Y_g)^4} dY_b\, dY_g + \left[\frac{Y_b\,(1-\zeta)}{Y_b\,(1-\zeta)+Y_g} - \frac{Y_b}{Y_b+Y_g}\right] dN_p \\
=& \underbrace{\eta\,(1-\eta)\left[\mu_b - z - \mu_g - \sigma_b^2 + (1-\eta)\,\sigma_g^2 + \nu\sigma_b\sigma_g\,(2\eta-1)\right]}_{\Sigma(\eta)}\, dt \\
&+ \eta\,(1-\eta)\left[\sigma_b dB_b - \sigma_g dB_g\right] - \frac{\zeta\eta\,(1-\eta)}{\eta\,(1-\zeta)+(1-\eta)} dN_p.
\end{aligned}$$

Given the dynamic of η, by Ito's lemma, the valuation ratios are obtained as

$$\begin{aligned}
dp_b(\eta) =& \left\{ p_b^{'}(\eta)\left[\mu_b - z - \mu_g - \eta\sigma_b^2 + (1-\eta)\,\sigma_g^2 + \nu\sigma_b\sigma_g\,(2\eta-1)\right] \right. \\
&\left. + \frac{1}{2} p_b^{''}(\eta)\,\eta^2\,(1-\eta)^2\left(\sigma_b^2+\sigma_g^2-2\nu\sigma_b\sigma_g\right)\right\}\eta\,(1-\eta)\,dt \\
&+ p_b^{'}(\eta)\,\eta\,(1-\eta)\,\sigma_b dB_b - p_b^{'}(\eta)\,\eta\,(1-\eta)\,\sigma_g dB_g \\
&+ \left[p_b(\eta^J) - p_b(\eta)\right] dN_p.
\end{aligned}$$

Firm valuations are obtained by the following calculation

$$\begin{aligned}
d(P_b) =& \, d\left[Y_b\, p_b(\eta)\right] = Y_b\, dp_b(\eta) + p_b(\eta)\, dY_b + dp_b(\eta)\, dY_b \\
=& \, Y_b \left\{ p_b^{'}(\eta)\,\eta\,(1-\eta)\left[\mu_b - z - \mu_g + (1-\eta)\left(\sigma_b^2+\sigma_g^2\right) + 2\nu\sigma_b\sigma_g\,(\eta-1)\right] \right. \\
&\left. + \frac{1}{2} p_b^{''}(\eta)\,\eta^2\,(1-\eta)^2\left(\sigma_b^2+\sigma_g^2-2\nu\sigma_b\sigma_g\right) + p_b(\eta)\,(\mu_b - z)\right\} dt \\
&+ Y_b\sigma_b\left[p_b + p_b^{'}(\eta)\,\eta\,(1-\eta)\right] dB_b - Y_b p_b^{'}(\eta)\,\eta\,(1-\eta)\,\sigma_g dB_g \\
&+ \left[Y_b\,(1-\zeta)\,p_b(\eta^J) - Y_b p_b(\eta)\right] dN_p.
\end{aligned}$$

Finally, the evolution of the return processes can be rewritten as

$$
\begin{aligned}
dR_b =& \frac{m_b(\eta)Y_b}{P_b} + \frac{dP_b}{P_b} \\
=& \underbrace{\left\{ \eta(1-\eta)\frac{p_b^{'}(\eta)}{p_b(\eta)}\left[\mu_b - z - \mu_g + (1-\eta)\left(\sigma_b^2+\sigma_g^2\right) + 2\nu\sigma_b\sigma_g(\eta-1)\right] \right.}_{\Lambda_b(\eta)} \\
& \underbrace{\left. + \frac{1}{2}\eta^2(1-\eta)^2\frac{p_b^{''}(\eta)}{p_b(\eta)}\left(\sigma_b^2+\sigma_g^2-2\nu\sigma_b\sigma_g\right) + \frac{m_b(\eta)}{p_b(\eta)} + \mu_b - z \right\}}_{\Lambda_b(\eta)} dt \\
& + \sigma_b\left[1 + \frac{p_b^{'}(\eta)}{p_b(\eta)}\eta(1-\eta)\right]dB_b - \frac{p_b^{'}(\eta)}{p_b(\eta)}\eta(1-\eta)\sigma_g dB_g \\
& + \left[\frac{(1-\zeta)p_b(\eta^J)}{p_b(\eta)} - 1\right]dN_p.
\end{aligned} \tag{6.1}
$$

Following similar steps, the return of green equity reads

$$
\begin{aligned}
dR_g =& \frac{m_g(\eta)Y_g}{P_g} + \frac{dP_g}{P_g} \\
=& \left\{ \eta(1-\eta)\frac{p_g^{'}(\eta)}{p_g(\eta)}\left[\mu_b - z - \mu_g - \eta\left(\sigma_b^2+\sigma_g^2\right) + 2\nu\sigma_b\sigma_g\eta\right] \right. \\
& \underbrace{\left. + \frac{1}{2}\eta^2(1-\eta)^2\frac{p_g^{''}(\eta)}{p_g(\eta)}\left(\sigma_b^2+\sigma_g^2-2\nu\sigma_b\sigma_g\right) + \frac{m_g(\eta)}{p_g(\eta)} + \mu_g \right\}}_{\Lambda_g(\eta)} dt \\
& + \sigma_b\frac{p_g^{'}(\eta)}{p_g(\eta)}\eta(1-\eta)dB_b + \left[1 - \frac{p_g^{'}(\eta)}{p_g(\eta)}\eta_t(1-\eta_t)\right]\sigma_g dB_g \\
& + \left[\frac{p_g(\eta^J)}{p_g(\eta)} - 1\right]dN_p.
\end{aligned} \tag{6.2}
$$

Therefore,

$$
\frac{dW}{W} = \left[r - c_b m_b - c_g m_g + \pi_b\left(\Lambda_b(\eta) - r\right) + \pi_g\left(\Lambda_g(\eta) - r\right)\right]dt \tag{6.3}
$$

$$
+ \left[\pi_b\Gamma_b(\eta) + \pi_g\Gamma_g(\eta)\right]dB_b + W\left[-\pi_b\Delta_b(\eta) + \pi_g\Delta_g(\eta)\right]dB_g \tag{6.4}
$$

$$
+ \left\{\pi_b(1-\zeta)\frac{p_b(\eta^J)}{p_b(\eta)} + \pi_g\frac{p_g(\eta^J)}{p_g(\eta)} - (\pi_b+\pi_g)\right\}dN_p, \tag{6.5}
$$

where

$$
\Gamma_b(\eta) = \sigma_b + \sigma_b\eta(1-\eta)\frac{p_b^{'}(\eta)}{p_b(\eta)},
$$

$$
\Delta_b(\eta) = \sigma_g\eta(1-\eta)\frac{p_b^{'}(\eta)}{p_b(\eta)},
$$

$$\Gamma_g(\eta) = \sigma_b \eta (1-\eta) \frac{p_g^{'}(\eta)}{p_g(\eta)},$$

$$\Delta_g(\eta) = \sigma_g - \sigma_g \eta (1-\eta) \frac{p_g^{'}(\eta)}{p_g(\eta)}.$$

I then formulate the HJB equation for the household's problem as follows

$$0 = \sup_{C_b, C_g, \pi_b, \pi_g} \left\{ \begin{array}{l} \rho (1-\gamma) V \left[\frac{\alpha}{\alpha-1} log \left((1-\theta)^{\frac{1}{\alpha}} C_b^{\frac{\alpha-1}{\alpha}} + \theta^{\frac{1}{\alpha}} C_g^{\frac{\alpha-1}{\alpha}} \right) - \frac{log((1-\gamma)V)}{(1-\gamma)} \right] \\ + V_W W \left[r - m_b \frac{C_b}{W} - m_g \frac{C_g}{W} + \pi_b (\Lambda_b(\eta) - r) + \pi_g (\Lambda_g(\eta) - r) \right] \\ + \frac{1}{2} V_{WW} W^2 \left[(\pi_b \Gamma_b(\eta) + \pi_g \Gamma_g(\eta))^2 + (-\pi_b \Delta_b(\eta) + \pi_g \Delta_g(\eta))^2 \right. \\ \left. + 2\nu (\pi_b \Gamma_b(\eta) + \pi_g \Gamma_g(\eta)) (-\pi_b \Delta_b(\eta) + \pi_g \Delta_g(\eta)) \right] \\ + V_\eta \Sigma(\eta) + \frac{1}{2} V_{\eta\eta} \eta^2 (1-\eta)^2 \left(\sigma_b^2 + \sigma_g^2 - 2\nu \sigma_b \sigma_g \right) \\ + V_{W\eta} W \eta (1-\eta) \left[\sigma_b (\pi_b \Gamma_b(\eta) + \pi_g \Gamma_g(\eta)) + \sigma_g (\pi_b \Delta_b(\eta) - \pi_g \Delta_g(\eta)) \right. \\ \left. + \nu \sigma_b (-\pi_b \Delta_b(\eta) + \pi_g \Delta_g(\eta)) - \nu \sigma_g (\pi_b \Gamma_b(\eta) + \pi_g \Gamma_g(\eta)) \right] \\ + \left(\frac{\lambda_g + \eta \Delta_\lambda}{1 + \phi z} \right) \left(V^J - V \right), \end{array} \right\}, \tag{6.6}$$

where $V^J = V\left(W^J; \eta^J\right)$, and $W^J = W\left[\pi_b (1-\zeta) \frac{p_b(\eta^J)}{p_b(\eta)} + \pi_g \frac{p_g(\eta^J)}{p_g(\eta)} + 1 - (\pi_b + \pi_g)\right]$is the household's wealth after each policy shock.

I guess (and subsequently verify) that

$$V(W; \eta) = \rho^{1-\gamma} G(\eta) \frac{W^{1-\gamma}}{1-\gamma}. \tag{6.7}$$

Replacing the guess into 6.6, and taking first-order conditions w.r.t $C_b$and C_g, I obtain

$$0 = \frac{\rho W (1-\theta)^{\frac{1}{\alpha}} C_b^{\frac{-1}{\alpha}}}{(1-\theta)^{\frac{1}{\alpha}} C_b^{\frac{\alpha-1}{\alpha}} + \theta^{\frac{1}{\alpha}} C_g^{\frac{\alpha-1}{\alpha}}} - m_b, \tag{6.8}$$

and

$$0 = \frac{\rho W \theta^{\frac{1}{\alpha}} C_g^{\frac{-1}{\alpha}}}{(1-\theta)^{\frac{1}{\alpha}} C_b^{\frac{\alpha-1}{\alpha}} + \theta^{\frac{1}{\alpha}} C_g^{\frac{\alpha-1}{\alpha}}} - m_g. \tag{6.9}$$

Solving for C_b and C_g from Equations (6.8)-(6.9) results in

$$C_b = \frac{\rho W}{m_b \left[1 + \xi^{1-\alpha}\left(\frac{\theta}{1-\theta}\right)\right]};\; C_g = \xi^{-\alpha}\left(\frac{\theta}{1-\theta}\right) C_b. \tag{6.10}$$

It is immediate from 6.10 that $\frac{m_b C_b + C_g m_g}{\left[(1-\theta)^{\frac{1}{\alpha}} C_b^{\frac{\alpha-1}{\alpha}} + \theta^{\frac{1}{\alpha}} C_g^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}} = 1$, and $m_b C_b + C_g m_g = \rho W$.

After replacing 6.10 and 6.7 into 6.6, the HJB equation becomes

$$0 = \sup_{\pi_b, \pi_g} \left\{ \begin{array}{l} -\rho\, log\left(G(\eta)\right) + (1-\gamma)\left[r - \rho + \pi_b(\eta)\left(\Lambda_b(\eta) - r\right) + \pi_g(\eta)\left(\Lambda_g(\eta) - r\right)\right] \\ +\frac{1}{2}(-\gamma)(1-\gamma)\left[\left(\pi_b\Gamma_b(\eta) + \pi_g\Gamma_g(\eta)\right)^2 + \left(-\pi_b\Delta_b(\eta) + \pi_g\Delta_g(\eta)\right)^2\right. \\ \left. +2\nu\left(\pi_b\Gamma_b(\eta) + \pi_g\Gamma_g(\eta)\right)\left(-\pi_b\Delta_b(\eta) + \pi_g\Delta_g(\eta)\right)\right] \\ +(1-\gamma)\frac{G'(\eta)}{G(\eta)}\eta(1-\eta)\left[\left(\sigma_b - \nu\sigma_g\right)\left(\pi_b\Gamma_b(\eta) + \pi_g\Gamma_g(\eta)\right)\right. \\ \left. +\left(\sigma_g - \nu\sigma_b\right)\left(\pi_b\Delta_b(\eta) - \pi_g\Delta_g(\eta)\right)\right] \\ +\frac{G'(\eta)}{G(\eta)}\Sigma(\eta) + \frac{1}{2}\frac{G''(\eta)}{G(\eta)}\eta^2(1-\eta)^2\left(\sigma_b^2 + \sigma_g^2 - 2\nu\sigma_b\sigma_g\right) \\ +\left(\frac{\lambda_g + \eta_t\Delta_\lambda}{1+\phi\, z}\right)\left[\left(\pi_b(1-\zeta)\frac{p_b(\eta^J)}{p_b(\eta)} + \pi_g\frac{p_g(\eta^J)}{p_g(\eta)} + 1 - (\pi_b + \pi_g)\right)^{1-\gamma}\frac{G(\eta^J)}{G(\eta)} - 1\right] \end{array} \right\}. \tag{6.11}$$

Taking first-order conditions w.r.t π_b, π_g from 6.11 give

$$\Sigma(\eta)\begin{bmatrix}\pi_b \\ \pi_g\end{bmatrix} = \frac{1}{\gamma}\left\{\begin{bmatrix}\Lambda_b(\eta) - r \\ \Lambda_g(\eta) - r\end{bmatrix} + \begin{bmatrix}\left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi\, z}\right)\kappa_p\left[(1-\zeta)\frac{p_b(\eta^J)}{p_b(\eta)} - 1\right] \\ \left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi\, z}\right)\kappa_p\left[\frac{p_g(\eta^J)}{p_g(\eta)} - 1\right]\end{bmatrix} + \Omega(\eta)\frac{G'(\eta)}{G(\eta)}\right\}, \tag{6.12}$$

where

$$\Sigma(\eta) = \begin{bmatrix}\Gamma_b^2 + \Delta_b^2 - 2\nu\Gamma_b\Delta_b & \Gamma_b\Gamma_g - \Delta_b\Delta_g + \nu\left(\Gamma_b\Delta_g - \Delta_b\Gamma_g\right) \\ \Gamma_b\Gamma_g - \Delta_b\Delta_g + \nu\left(\Gamma_b\Delta_g - \Delta_b\Gamma_g\right) & \Gamma_g^2 + \Delta_g^2 + 2\Delta_g\Gamma_g\end{bmatrix},$$

$$\Omega(\eta) = \begin{bmatrix}\eta(1-\eta)\left\{\left(\sigma_b - \nu\sigma_g\right)\Gamma_b + \left(\sigma_g - \nu\sigma_b\right)\Delta_b\right\} \\ \eta(1-\eta)\left\{\left(\sigma_b - \nu\sigma_g\right)\Gamma_g - \left(\sigma_g - \nu\sigma_b\right)\Delta_g\right\}\end{bmatrix}.$$

A.2. Brown firm's problem

To formulate the optimization problem of the brown firm, note that it takes the following dynamic of the SDF as given

$$\frac{dM}{M} = -r(\eta)\,dt - \Phi_b(\eta)\,dB_b - \Phi_g(\eta)\,dB_g + (\kappa_p(\eta) - 1)\,[dN_p - \lambda_p(\eta)dt]\,, \tag{6.13}$$

where the equilibrium prices of diffusion risks Φ_b, Φ_g and policy risk $\kappa_p$are derived in A.3.

No arbitrage condition implies that $m_{b,t}M_tY_{b,t}dt + d\,(M_tP_{b,t})$ is a Martingale, which implies that its drift is zero under the physical measure. The condition is formulated as the following HJB equation

$$\begin{aligned}
r(\eta)p_b(\eta) = \sup_z\ & m_b(\eta) + p_b(\eta)\,(\mu_b - z) + p_b^{'}(\eta)\big[\Sigma(\eta) + \eta\,(1-\eta)\,\sigma_b\,(\sigma_b - \nu\sigma_g)\,\big] \\
& + \frac{1}{2}p_b^{''}(\eta)\,\eta^2\,(1-\eta)^2\left(\sigma_b^2 + \sigma_g^2 - 2\nu\sigma_b\sigma_g\right) - p_b(\eta)\,(\kappa_p(\eta) - 1)\,\lambda_p(\eta) \\
& + \left\{\sigma_b\left[p_b(\eta) + p_b^{'}(\eta)\,\eta\,(1-\eta)\right] - \nu\sigma_g p_b^{'}(\eta)\,\eta\,(1-\eta)\right\} \\
& \left\{\frac{G^{'}(\eta)}{G(\eta)}\eta\,(1-\eta)\,\sigma_b - \gamma\,(\pi_b(\eta)\Gamma_b(\eta) + \pi_g(\eta)\Gamma_g(\eta))\right\} \\
& + \left\{-\sigma_g p_b^{'}(\eta)\,\eta\,(1-\eta) + \nu\sigma_b\left[p_b(\eta) + p_b^{'}(\eta)\,\eta\,(1-\eta)\right]\right\} \\
& \left\{-\frac{G^{'}(\eta)}{G(\eta)}\eta\,(1-\eta)\,\sigma_g - \gamma\,(-\pi_b(\eta)\Delta_b(\eta) + \pi_g(\eta)\Delta_g(\eta))\right\} \\
& + \left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi\,z}\right)\left\{\kappa_p\,(1-\zeta)\,p_b(\eta^J) - p_b(\eta)\right\}
\end{aligned} \tag{6.14}$$

Therefore, the brown firm will take the price as given and choose its investment such that

$$z = \arg\max_{z\geq 0} p_b(\eta)\,(\mu_b - z) + \left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi\,z}\right)\left[\kappa_p(\eta)\,(1-\zeta)\,p_b(\eta^J) - p_b(\eta)\right], \quad n \in \{b, g\}, \tag{6.15}$$

which gives the optimal solution 3.17.

A.3. Competitive market equilibrium

To characterize the decentralized market equilibrium, I first derive the SDF dynamic. Since $f(C,V) = \rho(1-\gamma)V\left[log(C) - \frac{1}{1-\gamma}log((1-\gamma)V)\right]$ and $C = \rho W$, it follows that

$$f_C(C,V) = \frac{\rho(1-\gamma)V}{C} = \rho^{1-\gamma}G(\eta)W^{-\gamma},$$

and

$$f_V(C,V) = -\rho\left[1 + log(G(\eta))\right] = h(\eta).$$

Using Ito Lemma for $M_t = e^{\int_0^t f_V(C_s,V_s)ds} f_C(C_t,V_t)$:

$$\frac{dM}{M} = h(\eta)dt + \frac{df_C(C,V)}{f_C(C,V)}, \tag{6.16}$$

where

$$\begin{aligned}
\frac{df_C(C,V)}{f_C(C,V)} = \Bigg\{ & \frac{G^{'}(\eta)}{G(\eta)}\Sigma(\eta) + \frac{1}{2}\frac{G^{''}(\eta)}{G(\eta)}\eta_t^2(1-\eta_t)^2\left(\sigma_b^2 + \sigma_g^2 - 2\nu\sigma_b\sigma_g\right) \\
& -\gamma\left[r(\eta) - \rho + \pi_b\left(\Lambda_b(\eta) - r(\eta)\right) + \pi_g\left(\Lambda_g(\eta) - r(\eta)\right)\right] \\
& +\frac{1}{2}\gamma(\gamma+1)\left[\left(\pi_b\Gamma_b + \pi_g\Gamma_g\right)^2 + \left(\pi_g\Delta_g - \pi_b\Delta_b\right)^2 + 2\nu\left(\pi_b\Gamma_b + \pi_g\Gamma_g\right)\left(\pi_g\Delta_g - \pi_b\Delta_b\right)\right] \\
& -\gamma\frac{G^{'}(\eta)}{G(\eta)}\eta(1-\eta)\left[\left(\sigma_b - \nu\sigma_g\right)\left(\pi_b\Gamma_b + \pi_g\Gamma_g\right) + \left(\sigma_g - \nu\sigma_b\right)\left(\pi_b\Delta_b - \pi_g\Delta_g\right)\right]\Bigg\} dt \\
& -\left[\gamma\left(\pi_b\Gamma_b + \pi_g\Gamma_g\right) - \frac{G^{'}(\eta)}{G(\eta)}\eta(1-\eta)\sigma_b\right] dB_b \\
& -\left[\gamma\left(-\pi_b\Delta_b + \pi_g\Delta_g\right) + \frac{G^{'}(\eta)}{G(\eta)}\eta(1-\eta)\sigma_g\right] dB_g \\
& +\left[\left(\pi_b(1-\zeta)\frac{p_b(\eta^J)}{p_b(\eta)} + \pi_g\frac{p_g(\eta^J)}{p_g(\eta)}\right)^{-\gamma}\frac{G(\eta^J)}{G(\eta)} - 1\right] dN_p
\end{aligned} \tag{6.17}$$

Next, by combining 6.16 with 6.17 and grouping terms, I obtain the expression for the risk-free rate as following

$$
\begin{aligned}
r(\eta) = {} & \rho\left[1-\gamma+log\left(G(\eta)\right)\right]+\gamma\left[\pi_b\Lambda_b(\eta)+\pi_g\Lambda_g(\eta)\right] \\
& -\frac{G^{'}(\eta)}{G(\eta)}\Sigma(\eta)-\frac{1}{2}\frac{G^{''}(\eta)}{G(\eta)}\eta^2\left(1-\eta\right)^2\left(\sigma_b^2+\sigma_g^2-2\nu\sigma_b\sigma_g\right) \\
& -\frac{1}{2}\gamma\left(\gamma+1\right)\left[\left(\pi_b\Gamma_b+\pi_g\Gamma_g\right)^2+\left(\pi_g\Delta_g-\pi_b\Delta_b\right)^2+2\nu\left(\pi_b\Gamma_b+\pi_g\Gamma_g\right)\left(\pi_g\Delta_g-\pi_b\Delta_b\right)\right] \\
& +\gamma\frac{G^{'}(\eta)}{G(\eta)}\eta\left(1-\eta\right)\left[\left(\sigma_b-\nu\sigma_g\right)\left(\pi_b\Gamma_b+\pi_g\Gamma_g\right)+\left(\sigma_g-\nu\sigma_b\right)\left(\pi_b\Delta_b-\pi_g\Delta_g\right)\right] \\
& -\left(\frac{\lambda_g+\eta\Delta_\lambda}{1+\phi z(\eta)}\right)\left(\kappa_p(\eta)-1\right),
\end{aligned}
\tag{6.18}
$$

where $\kappa_p(\eta)=\left(\pi_b\left(1-\zeta\right)\frac{p_b(\eta^J)}{p_b(\eta)}+\pi_g\frac{p_g(\eta^J)}{p_g(\eta)}\right)^{-\gamma}\frac{G(\eta^J)}{G(\eta)}$ is the market price of policy risk.

Adding 6.18 to 6.11 gives the familiar formula for the risk-free rate as in the literature

$$
\begin{aligned}
r(\eta) = {} & \underbrace{\rho+\mu_c(\eta)-\gamma\sigma_c^2(\eta)}_{\text{Diffusion Risks}} \\
& \underbrace{+\left(\frac{\lambda_g+\eta\Delta_\lambda}{1+\phi z(\eta)}\right)\kappa_p(\eta)\left[\pi_b\frac{\left(1-\zeta\right)p_b\left(\eta^J\right)}{p_b(\eta)}+\pi_g\frac{p_g\left(\eta^J\right)}{p_g(\eta)}-1\right]}_{\text{Policy Risk}},
\end{aligned}
\tag{6.19}
$$

where

$$
\mu_c(\eta)=\left[\pi_b\Lambda_b(\eta)+\pi_g\Lambda_g(\eta)-\rho\right],
$$

$$
\sigma_c^2(\eta)=\left(\pi_b\Gamma_b+\pi_g\Gamma_g\right)^2+\left(\pi_g\Delta_g-\pi_b\Delta_b\right)^2+2\nu\left(\pi_b\Gamma_b+\pi_g\Gamma_g\right)\left(\pi_g\Delta_g-\pi_b\Delta_b\right).
$$

In addition, from Equations (6.16)-(6.17), the prices of diffusion risks conform to 4.1 and 4.2.

To derive the relative price of output ξ, I rely on the equilibrium market-clearing conditions

$$
C_b=Y_b, C_g=Y_g. \tag{6.20}
$$

From Equations (6.10) and (6.20), it is clear that

$$
\frac{C_b}{C_g}=\xi^\alpha\left(\frac{1-\theta}{\theta}\right)=\frac{\eta}{1-\eta},
$$

which is equivalent to 3.19.

Lastly, to nail down the market equilibrium, I jointly solve for $p_b(\eta)$, $p_g(\eta)$,and $G(\eta)$. 3.11 can be rewritten as

$$
\frac{Y_b p_b(\eta)+Y_g p_g(\eta)}{Y_b m_b(\eta)+Y_g m_g(\eta)}=\frac{1}{\rho},
$$

or

$$p_g(\eta) = \frac{1}{\rho}\left[\frac{\eta}{(1-\eta)}m_b(\eta) + m_g(\eta)\right] - \frac{\eta}{(1-\eta)}p_b(\eta)$$
$$= \frac{\eta\,(1-\theta)^{\frac{1}{\alpha-1}}}{\rho\,(1-\eta)}\left[1 + \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\alpha}}\left(\frac{\eta}{1-\eta}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} - \frac{\eta}{(1-\eta)}p_b(\eta). \tag{6.21}$$

By replacing 6.21 into 6.11 and 6.14, I obtain a system of delayed ordinary differential equations, which is then solved by the projection method under the following initial and boundary conditions

$$G(0) = e^{\frac{1}{2\rho}(1-\gamma)\left(2\mu_g - \gamma\sigma_g^2\right)},$$

$$G(1) = e^{\frac{1}{2\rho}\left\{2\lambda_b\left[(1-\zeta)^{1-\gamma}-1\right] + (1-\gamma)\left(2\mu_b - \gamma\sigma_b^2\right)\right\}},$$

$$p_b(0) = +\infty,\; p_b(1) = \frac{(1-\theta)^{\frac{1}{\alpha-1}}}{\rho}. \tag{6.22}$$

A.4. Asset prices

From 6.1, 6.2, and 6.13, the risk premia of the brown and green equity are

$$E(R_b) - r = \gamma\widehat{\Theta}_b^T\Theta_W + \widehat{\Theta}_b^T\Theta_G - \left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi z(\eta)}\right)(\kappa_p(\eta) - 1)\left[\frac{(1-\zeta)\,p_b\left(\eta^J\right)}{p_b(\eta)} - 1\right], \tag{6.23}$$

and

$$E(R_g) - r = \gamma\widehat{\Theta}_g^T\Theta_W + \widehat{\Theta}_g^T\Theta_G - \left(\frac{\lambda_g + \eta\Delta_\lambda}{1+\phi z(\eta)}\right)(\kappa_p(\eta) - 1)\left[\frac{p_g\left(\eta^J\right)}{p_g(\eta)} - 1\right], \tag{6.24}$$

where

$$\widehat{\Theta}_b = \left[\sigma_b + (\sigma_b - \nu\sigma_g)\frac{p_b^{'}(\eta)}{p_b(\eta)}\eta\,(1-\eta)\,, \nu\sigma_b + (\nu\sigma_b - \sigma_g)\frac{p_b^{'}(\eta)}{p_b(\eta)}\eta\,(1-\eta)\right]^T,$$

$$\widehat{\Theta}_g = \left[\nu\sigma_g + (\sigma_b - \nu\sigma_g)\frac{p_g^{'}(\eta)}{p_g(\eta)}\eta\,(1-\eta)\,, \sigma_g + (\nu\sigma_b - \sigma_g)\frac{p_g^{'}(\eta)}{p_g(\eta)}\eta\,(1-\eta)\right]^T,$$

$$\Theta_W = \left[\pi_b\Gamma_b + \pi_g\Gamma_g, -\pi_b\Delta_b + \pi_g\Delta_g\right]^T,$$

$$\Theta_G = \left[-\frac{G^{'}(\eta)}{G(\eta)}\eta\,(1-\eta)\,\sigma_b, \frac{G^{'}(\eta)}{G(\eta)}\eta\,(1-\eta)\,\sigma_g\right]^T.$$