Capacity Constraints, Encounter Frequency, and the Paradox of Tightening

Endre J. Reite

Norwegian University of Science and Technology, NTNU Business School

Abstract

In this paper, I examine how regulated agents organize a fixed volume of activity when regulatory interactions involve both within-encounter processing costs and per-encounter exposure costs. Screening capacity is allocated at the encounter level, which means that detectability declines with encounter size whenever screening resources do not scale proportionally. I show that higher per-encounter burdens lead agents to reduce the number of encounters and consolidate activity, even though the total level of underlying activity remains unchanged. This consolidation can weaken enforcement. When detection falls with encounter size, an increase in procedural stringency may reduce the overall probability of detection, creating a non-monotonic relationship between enforcement intensity and regulatory effectiveness. The framework also generates persistence in interaction patterns when exposure costs depend on past encounters. I use these results to highlight encounter organization as an important, and often overlooked, margin in enforcement design. The analysis clarifies when reforms that increase per-encounter requirements risk reducing observability unless accompanied by screening architectures that prevent consolidation.

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Email address: endre.j.reite@ntnu.no (Endre J. Reite)

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1. Introduction

Regulatory systems often rely on repeated administrative encounters such as filings, inspections, case reviews, or audit episodes. In many settings, agents retain considerable discretion over how these encounters are structured: how often they appear, how much activity they bundle, and when they choose to engage with the regulator. Although typically treated as background conditions, these organizational choices shape what regulators observe and how effective enforcement can be under limited administrative capacity. I examine how encounter organization responds to per–encounter burdens and fixed screening capacity, and how this interaction can generate counterintuitive enforcement outcomes.

I study enforcement regimes in which agencies interact with regulated agents through discrete encounters subject to binding time and processing constraints. When total activity is fixed, enforcement policy affects not the level of activity but how that activity is partitioned across encounters. This organizational margin matters because screening resources are allocated per encounter, so screening intensity per unit falls when activity is bundled. I show how agents respond by adjusting encounter frequency and size, and that standard levers such as higher penalties or per–encounter stringency can unintentionally reduce effective detection by inducing consolidation. Unlike deterrence models that treat encounter timing as fixed, the analysis shows that enforcement can reshape how economic activity is organized rather than how much of it occurs.

Stringency refers here to per–encounter procedural burden—the fixed overhead of appearing—and is distinct from screening resources. Screening capacity is captured separately by B, and the mechanism relies on the possibility that B does not scale proportionally with encounter size.

Scope and limits.. The model isolates an encounter–organization margin: how a fixed workload is distributed across encounters. I abstract from endogenous evasion/compliance choices and from optimal enforcement by the agency; these margins operate alongside encounter reorganization rather than replacing it.

The mechanism is most relevant when procedural burden is paid per encounter and when screening resources are allocated at the encounter level or scale sublinearly with scope.

Raising per–encounter burdens induces consolidation into fewer, larger encounters. In the baseline model, convex within–encounter processing costs and per–encounter exposure costs jointly determine an interior optimal encounter count, yielding a tradeoff between more frequent encounters and higher exposure. Two extensions amplify the mechanism. First, when exposure depends on past interactions, encounter patterns exhibit persistence even after scrutiny relaxes. Second, when detectability declines with encounter size due to congestion or complexity, consolidation can reduce total detection. Detection falls discretely at the thresholds where agents switch from n to $n-1$ encounters, generating a non–monotonic relationship between enforcement and detection. This produces a *paradox of tightening*: beyond a threshold, higher per–encounter burden can reduce detection when screening effectiveness declines with encounter size.

The contribution is conceptual and structural. The paper endogenizes encounter frequency and characterizes conditions under which enforcement reshapes the organization of activity across encounters. This delivers sharp comparative statics for consolidation thresholds, discontinuous detection responses, and persistence following temporary scrutiny shocks. These predictions differ from standard frameworks in which encounter structure is fixed and enforcement effects monotone. Administrative design implications follow once the mechanism is established, focusing on encounter architecture—scope, frequency, and within–encounter allocation—under binding screening capacity.

2. Related Literature and Contribution

This paper sits at the intersection of enforcement/compliance models and work on monitoring design. There is an extensive literature on how penalties, audit probabilities, and enforcement intensity affect compliance and underlying

activity, typically taking the timing or frequency of regulatory encounters as given. A complementary line of research emphasizes state dependence, targeting, or follow-up that evolves with past interactions, creating dynamic incentives that can generate persistence in observed outcomes. The present paper focuses on a distinct margin that becomes central whenever encounter timing or partitioning is discretionary: the organization of a fixed workload across encounters. I organize the discussion around encounter timing/frequency, administrative burden, monitoring design, and capacity constraints.

2.1. History-dependent Enforcement and Persistence

Much of the enforcement literature studies behavior *conditional on appearing* (e.g., reporting, compliance effort), while treating the timing of encounters as exogenous. In contrast, the present paper highlights a complementary margin: when scrutiny depends on past interactions (risk scoring, escalation, follow-ups), agents can respond by reducing encounter frequency and retiming activity, generating persistence in the encounter process even after the temporary shock dissipates.

This perspective is consistent with models of state-dependent enforcement and dynamic audit intensity, and aligns with hysteresis logic in which temporary shocks produce lasting changes because the state evolves with past actions. In this paper, persistence arises because accumulated scrutiny makes returning more costly, so agents rationally 'cool off' by delaying encounters until the scrutiny stock decays.

2.2. Administrative Burden and Modern Enforcement Delivery

A key premise of this study is that enforcement costs are not limited to expected fines. Encounters can impose real procedural burdens related to, for example, documentation, follow-up tasks, time, and cognitive managerial attention. The administrative-burden framework formalizes these interaction costs as learning, psychological, and compliance burdens that arise in encounters with the state (Moynihan, Herd, and Harvey, 2015).

I treat exposure costs in reduced form to isolate how procedural frictions reshape the encounter process rather than institution-specific details, consistent with enforcement systems that allocate scarce scrutiny using risk information and targeted capacity.

2.3. Monitoring Design, Thresholds, and Strategic Responses

The paper's monitoring-design results connect naturally to settings with discontinuities and size-based regimes. Evidence on tax and regulatory thresholds shows that kinks or notches can reshape observed distributions (bunching) and that behavioral responses need not map one-for-one to real activity changes (Kleven, 2016). Audit experiments and information-based enforcement further show that reporting and measured compliance clearly respond to audit probabilities and information trails (Kleven et al., 2011; Pomeranz, 2015). Size-dependent monitoring can induce strategic responses around eligibility cutoffs (Almunia and Lopez–Rodriguez, 2018; Kleven and Waseem, 2013).

Finally, the monitoring-redesign approach is consistent with evidence that size-contingent regulation distorts organizational scale and observed distributions (Garicano, Lelarge, and Van Reenen, 2016), reinforcing the broader point that the enforcement architecture can change behavior *and* the informational content of administrative data.

2.4. Processing Constraints and Capacity Dilution

The capacity-dilution mechanism in Subsection 3.10 is consistent with bounded-attention and processing-constraint foundations in which effective scrutiny is limited by scarce cognitive or organizational capacity (Sims, 2003; Matějka and McKay, 2015; Gabaix, 2014; Maćkowiak, Matějka, and Wiederholt, 2023).

Taken together with history-dependent scrutiny and size-sensitive detectability, the framework generates persistence in encounter patterns and identifies conditions under which stricter per-encounter requirements can reduce detection by inducing consolidation into fewer, larger encounters.

2.5. Batching and lumpy adjustment (background)

The benchmark partitioning problem mirrors classic batching and lumpy-adjustment logic: agents trade off convex within-batch costs against per-occurrence costs, generating discrete changes in frequency and scope. For additional references and discussion, see Appendix Appendix A.

2.6. Contribution relative to the literature

Taken together, the framework endogenizes encounter frequency and clarifies how enforcement can change what regulators observe by reshaping the encounter process itself. With history-dependent scrutiny and size-sensitive detectability, the model generates persistence in interaction patterns and identifies conditions under which stricter per-encounter requirements can lower equilibrium detection by inducing consolidation into fewer, larger encounters.

Appendix Appendix A provides an extended mapping and additional references.

3. Model

3.1. Environment

Consider a risk-neutral agent operating over a fixed horizon (one period). The agent must conduct a fixed total volume of economic activity, denoted by $Y > 0$, that is subject to regulatory oversight. Each time the agent engages with the regulator (for example, by filing a report, responding to an inquiry, or undergoing an inspection) constitutes an interaction.

The agent chooses the following:

- an integer number of interactions $n \in \{1, 2, 3, \ldots\}$, and
- an allocation of activity across interactions $(y_1, \ldots, y_n)$ with $y_i \geq 0$ for all i.

All activity must be processed or reported within the following horizon:

$$\sum_{i=1}^{n} y_i = Y. \tag{1}$$

The model isolates a margin that is often treated as exogenous in enforcement frameworks: the agent can choose how often to interact and, therefore, the size of each interaction.

Scope note. Holding total activity fixed isolates the organizational margin, how activity is distributed across regulatory encounters, rather than the level of that activity. This assumption is appropriate in settings where agents face fixed demand, mandated output, or contractual obligations. Allowing activity to be elastic would introduce standard deterrence effects; these would coexist with, rather than eliminate, the consolidation mechanism studied here.

3.2. Capacity Constraints in Administrative Enforcement

In many enforcement environments, screening resources are allocated at the encounter level and are subject to binding time and processing constraints. When activity is consolidated into larger encounters without proportional scaling of screening capacity, effective scrutiny per unit can fall. Encounter size y refers to the scope bundled into a single administrative interaction—most naturally, the number of issues/periods or line items examined within a tax audit (and in customs, the number of units/containers inspected)—while Y denotes total underlying activity over the relevant horizon.

My capacity-dilution assumption captures these settings by modeling a bounded screening capacity per encounter. The mechanism applies when per-encounter screening time is capped and when agents can reorganize activity across encounters. It is less relevant in regimes that automatically escalate large encounters into separate, uncongested processes or specialized units. In Section 4, I treat encounter design as a policy lever and describe how capacity constraints shape optimal administrative rules.

3.3. Cost Structure and Cumulative Exposure

Total cost has two components: an internal (batching) cost and an enforcement (exposure) cost. Interaction timing is assumed to be flexible. In many regulatory environments—such as voluntary filing and self-reporting regimes, discretionary transactions, and systems that allow extensions or amendments—agents have considerable leeway over when and how often they interact. The model abstracts from hard periodic mandates to isolate the extreme case of endogenous timing, clarifying what is possible when agents can bundle or delay interactions.

(i) Internal batching cost. Processing a large amount of activity in a single interaction is internally costly (administrative congestion, coordination, and complexity). Then let *g(y)* denote the internal cost of processing amount y in one interaction.

Assumption 1 (Convex internal cost)**.** $g(0) = 0$*,* g *is increasing, and* g *is strictly convex on* $\mathbb{R}_+$*.*

This captures the idea that making a single interaction very large becomes increasingly costly, reflecting the increase in internal challenges as activity is bundled.

(ii) Cumulative exposure cost. Each interaction creates regulatory exposure related to documentation, follow-up requests, managerial attention, and the risk of triggering enhanced scrutiny. Let *c(k)* denote the incremental exposure cost of the *k-th* interaction in the period. The total exposure cost from n interactions is:

$$C(n) = \sum_{k=1}^{n} c(k). \tag{2}$$

Assumption 2 (Escalating marginal exposure)**.** *For all* $k \geq 1$*,* $c(k+1) \geq c(k)$*.*

This allows later interactions to be weakly more burdensome than earlier ones.

Escalating exposure is plausible because encounters are not independent. Each interaction expands the record and can trigger a follow-up, update risk scores, and raise review intensity in later encounters. This captures routine enforcement practice in settings with targeted monitoring and state dependence (e.g., risk scoring, follow-ups, escalation). I treat (c(k)) as a reduced form for this history dependence.

3.4. Agent's Problem

The agent must carry out total activity of $Y > 0$ over the horizon. They choose an integer $n \in \mathbb{N}$ and a partition $\{y_i\}_{i=1}^n$ with $y_i \geq 0$ and $\sum_{i=1}^n y_i = Y$. The total cost is

$$\text{TotalCost}(n, \{y_i\}_{i=1}^n) = \sum_{i=1}^n g(y_i) + C(n), \tag{3}$$

where $g(\cdot)$ captures the within-encounter adjustment costs and $C(\cdot)$ captures the exposure costs that increase with the number of encounters. The agent solves

$$\min_{n \in \mathbb{N},\, y_i \geq 0} \sum_{i=1}^n g(y_i) + C(n) \quad \text{s.t.} \quad \sum_{i=1}^n y_i = Y. \tag{4}$$

In this problem, internal economies of scale are traded off against exposure: increasing n lowers the convex cost component $\sum_i g(y_i)$, but raises $C(n)$ by adding encounters.

3.5. Optimal Allocation Conditional on Interaction Count

Lemma 1 (Equal-split allocation)**.** *Fix $n \geq 1$. Under Assumption 1, the unique solution to*

$$\min_{\{y_i\}_{i=1}^n} \sum_{i=1}^n g(y_i) \quad \textit{s.t.} \quad \sum_{i=1}^n y_i = Y, \qquad y_i \geq 0,$$

is $y_i^\star = Y/n$ for all $i = 1, \ldots, n$. Consequently, the reduced-form objective is

$$\Phi(n) = n\, g\left(\frac{Y}{n}\right) + C(n). \tag{5}$$

Proof. By Jensen's inequality,

$$\frac{1}{n}\sum_{i=1}^n g(y_i) \;\geq\; g\left(\frac{1}{n}\sum_{i=1}^n y_i\right) = g(Y/n),$$

so $\sum_{i=1}^{n} g(y_i) \geq n\, g(Y/n)$. Strict convexity implies equality only if $y_1 = \cdots = y_n$, which, together with $\sum_i y_i = Y$ yields $y_i^\star = Y/n$ for all i. Substituting the optimizer into equation (3) gives equation (5). □

3.6. Existence and Static Comparative Statics

Assumption 3 (Coercivity)**.** *$C(n) \to \infty$ as $n \to \infty$.*

(This holds for many schedules, including constant c(k) > 0.)

Proposition 1 (Existence of an optimal interaction count)**.** *Under Assumptions 1 and 3, the reduced objective, equation* (5)*, attains a minimum at some finite $n^\star \in \mathbb{N}$.*

Proof. Because $g(\cdot) \geq 0$, equation (5) implies $\Phi(n) \geq C(n)$ for all n. Assumption 3 gives $C(n) \to \infty$ as $n \to \infty$, hence $\Phi(n) \to \infty$. Since Φ is defined on the positive integers, it attains a minimum on a finite set $\{1, 2, \ldots, N\}$ for some sufficiently large N, yielding a finite minimizer, $n^\star$. □

Definition 1 (Stricter exposure regime)**.** *Regime B is (weakly) stricter than regime A if $c_B(k) \geq c_A(k)$ for all $k \geq 1$, with strict inequality for at least one k.*

Proposition 2 (Consolidation under higher marginal exposure)**.** *Let g satisfy Assumption 1 and let $C_r(n) = \sum_{k=1}^{n} c_r(k)$ for regimes $r \in \{A, B\}$. If regime B is stricter than regime A in the sense of Definition 1, then the smallest minimizer*

$$\underline{n}_r \in \arg\min_{n \in \mathbb{N}} \left\{ n\, g\left(\frac{Y}{n}\right) + C_r(n) \right\}$$

satisfies $\underline{n}_B \leq \underline{n}_A$.

Proof. Define $H(n) \equiv n\, g(Y/n)$ and $\Phi_r(n) \equiv H(n) + C_r(n)$. For each regime r,

$$\Phi_r(n+1) - \Phi_r(n) = \big(H(n+1) - H(n)\big) + c_r(n+1).$$

By Assumption 1, g is strictly convex, which implies $H(n)$ is convex in n on $\mathbb{N}$ and, hence, the discrete difference $H(n+1) - H(n)$ is nondecreasing in n. If

$c_r(\cdot)$ is nondecreasing (Assumption 2), then $\Phi_r(n+1) - \Phi_r(n)$ is nondecreasing in n. Therefore, the minimizing set is an interval of integers and the smallest minimizer is characterized by the first index where the forward difference is nonnegative.

Since $c_B(k) \geq c_A(k)$ for all k, we have $\Phi_B(n+1) - \Phi_B(n) \geq \Phi_A(n+1) - \Phi_A(n)$ for all n, so the crossing to nonnegativity occurs slightly earlier under B, implying $\underline{n}_B \leq \underline{n}_A$. □

3.7. Static Benchmark Illustration

This partitioning problem mirrors classic inventory and transaction models in which agents trade off convex adjustment or processing costs against fixed or per-occurrence costs, generating endogenous batching and lumpy timing (Baumol, 1952; Tobin, 1956; Caballero and Engel, 1999). I make this trade-off concrete by considering a simple parameterization with total activity $Y = 100$ and internal processing cost $g(y) = y^2$. Compare two exposure schedules:

- A baseline schedule with moderate per-interaction exposure costs.
- A stricter schedule with higher exposure costs in which additional interactions are more heavily penalized.

For each candidate interaction count n, the reduced cost is:

$$\Phi(n) = n \cdot g\left(\frac{Y}{n}\right) + C(n). \tag{6}$$

3.8. Dynamic Scrutiny, Persistence, and Hysteresis

Next, I extend the benchmark by allowing regulatory exposure to depend on past interactions. The goal is not to add strategic behavior on the part of the regulator to the model, but to capture the empirically common feature that scrutiny is persistent: past contact affects the expected burden of future contact.

Time is indexed discretely by $t = 1, 2, \ldots, T$. Let $s_t \geq 0$ denote a scrutiny stock summarizing accumulated information, attention, or follow-up obligations

associated with the agent at the start of period t. Scrutiny evolves according to:

$$s_{t+1} = (1-\delta)s_t + \psi\, 1\{\text{interaction at } t\}. \tag{7}$$

where $\delta \in (0,1)$ governs decay ('cooling off') and $\psi > 0$ captures the incremental effect of an interaction on future scrutiny.

An interaction at time t imposes an exposure cost $c(s_t)$, where $c(\cdot)$ is increasing. Internal processing costs are as before. The agent allocates a fixed total activity Yacross interaction dates and sizes over the horizon, taking the law of motion for s_t as given.

This formulation nests the static benchmark as the special case $\psi = 0$ (or $\delta = 1$), but differs in one key respect: the marginal cost of an additional interaction depends on its timing relative to past interactions.

Proposition 3 (Persistence following a temporary scrutiny shock)**.** *Consider two periods. Let the agent's period-t objective be* $\Phi(n_t; s_t) = n_t\, g(Y/n_t) + C(n_t; s_t)$*, where* $C(\cdot;\cdot)$ *has increasing differences in* (n, s)*. Scrutiny follows*

$$s_{t+1} = (1-\delta)s_t + a_t, \qquad 0 \le \delta < 1, \tag{8}$$

with $a_1 > a_2$ *and* s_1 *given. Even if* a_t *reverts in period 2, the elevated state* s_2 *induced by* a_1 *weakly reduces the agent's optimal interaction count in period 2 relative to the no-shock path where* $a_1 = a_2$*.*

3.9. Monitoring Design and Strategic Encounter Avoidance

The persistence mechanism above takes scrutiny as exogenous to policy design. This subsection adds a minimal strategic layer to clarify how enforcement rules interact with endogenous interaction choices.

Consider a two-player game between a regulator and the agent. The regulator commits to a monitoring rule before activity commences. According to the baseline rule, scrutiny depends only on the following state:

$$a\,(s_t) \text{ or } d\,(s_t) \tag{9}$$

where $a(\cdot)$ denotes audit probability and $d(\cdot)$ denotes audit depth. The agent then chooses the timing and size of interactions, anticipating how scrutiny evolves as set out in Section 3.8.

Under this encounter-only monitoring regime, increasing the intensity of scrutiny has a clear behavioral effect.

Proposition 4 (Strategic consolidation with encounter monitoring)**.** *When monitoring intensity depends only on the scrutiny state s_t, an increase in per-encounter scrutiny weakly reduces encounter frequency and weakly increases average encounter size, holding total activity constant.*

The reason is mechanical. A higher expected burden per-encounter raises the shadow cost of appearing. Because total activity is fixed, the agent economizes on appearances by bundling activity into fewer interactions.

Next, I consider an alternative monitoring design. Suppose the regulator conditions scrutiny on both history *and* the size of the interaction:

$$a(s_t, y_t) \text{or} d(s_t, y_t), \tag{10}$$

with monitoring intensity increasing in y_t.

Proposition 5 (Size-contingent monitoring limits consolidation)**.** *Suppose size-contingent monitoring enters the agent's objective as an additional per-encounter burden that is increasing in encounter size. Specifically, let total cost be*

$$\sum_{i=1}^{n} g(y_i) \;+\; C(n) \;+\; \kappa \sum_{i=1}^{n} m(y_i),$$

where $m : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing and $\kappa \geq 0$ indexes the strength of size-contingent monitoring. Under Assumption 1, if m is convex, the equal-split allocation remains optimal conditional on n, and the reduced problem is

$$\min_{n \in \mathbb{N}} \Big\{ n\, g(Y/n) + C(n) + \kappa\, n\, m(Y/n) \Big\}.$$

Assume the discrete function $T(n) \equiv n\, m(Y/n)$ is decreasing and convex on $\mathbb{N}$. Let $\underline{n}(\kappa)$ denote the smallest minimizer of the reduced problem. Then $\underline{n}(\kappa)$ is weakly increasing in κ. In particular, $\underline{n}(\kappa) \geq \underline{n}(0)$ for all $\kappa \geq 0$.

Proof. See Appendix Appendix B.2. □

This extension has policy implications but does not establish an additional mechanism. When encounters are endogenous, enforcement rules that shift scrutiny entirely to the act of appearing can reduce information flow. Allowing monitoring intensity to vary with interaction size, or maintaining low-burden frequent contact, mitigates strategic avoidance without weakening deterrence. Additional derivations are collected in Appendix Appendix B.

Table 2 summarizes the agent's optimal interaction frequency under encounter-only and size-contingent monitoring, holding total activity and internal processing costs constant. Figure 2 plots the total cost as a function of the interaction count under each monitoring rule. In the calibration shown, the discrete optimum shifts from $n^* = 5$ under encounter-only monitoring to $n^* = 6$ under size-contingent monitoring.

The comparison shows that conditioning scrutiny on interaction size raises the marginal cost of bundling activity and shifts the cost-minimizing interaction frequency upward.

3.10. Endogenous Observability and Information Collapse

The analysis thus far has treated regulatory encounters as costly but equally informative. In many settings, however, how activity is organized affects what regulators learn. Larger submissions, filings, or transactions can be harder to screen, audit, or interpret, even if the level of scrutiny per-encounter is high. I then show how the organizational responses studied above can, therefore, reduce both exposure and observability (Maćkowiak et al., 2023; Gabaix, 2014).

Suppose that a violation $v \in \{0, 1\}$ occurs with some exogenous probability. Conditional on an interaction of size y, the regulator detects a violation with probability $p(y)$. Detection probabilities are independent across interactions. Larger encounters can be harder to screen effectively. Auditors have limited attention and processing capacity: reviewing one massive submission may have diminishing returns relative to the review of several smaller ones. This 'haystack

effect' is consistent with rational inattention and observed audit failures in complex organizations, where scale overwhelms scrutiny.

Assumption 4 (Capacity-constrained detection)**.** *Each enforcement encounter is subject to a fixed screening capacity $B > 0$ (time, staff-hours, or processing resources) allocated at the encounter level. For an encounter of size $y > 0$, the screening intensity per unit is*

$$e(y) \equiv \frac{B}{y}.$$

Detection probability is a function of screening intensity,

$$p(y) \equiv \pi(e(y)) = \pi\left(\frac{B}{y}\right),$$

where $\pi'(\cdot) > 0$ and $\pi''(\cdot) < 0$.

Remark. (Sublinear scaling.) The dilution channel persists if encounter-level capacity scales *sublinearly* with scope, e.g. $B(y) = B_0 y^{\alpha}$ with $\alpha \in [0, 1)$, so that $e(y) = B(y)/y = B_0 y^{\alpha-1}$ declines in y. The knife-edge case $\alpha = 1$ corresponds to proportional scaling (no mechanical dilution), which removes the paradox channel.

Assumption 4 captures 'capacity dilution': holding the encounter-level capacity fixed, bundling activity into larger encounters lowers the screening intensity per unit, reducing detectability. The mechanism is most relevant when per-encounter screening time is capped and escalation thresholds are absent or binding.

Given n interactions of equal size $y = Y/n$, the total probability of detecting a violation is

$$P(n) = 1 - \left(1 - p\left(\frac{Y}{n}\right)\right)^n. \tag{11}$$

For a small $p(Y/n)$, $P(n) = 1 - (1 - p(Y/n))^n \approx n\, p(Y/n)$,

which can be approximated by $P(n) \approx n \cdot p(Y/n)$ when detection probabilities are small.

Theorem 1 (Paradox of tightening)**.** *Suppose Assumption 4 holds and the agent chooses an encounter frequency optimally, as in Section 3. Then there exist values of the exposure schedule at which a tightening (a higher per-encounter procedural burden) reduces the regulator's equilibrium probability of detection.*

Table 3 reports a numerical illustration. As the optimal encounter size rises from 20 to 25, the internal processing cost per encounter increases from $g(20) = 400$ to $g(25) = 625$ (under $g(y) = y^2$). In the illustration, the per-encounter exposure parameter increases from 0.40 to 0.50, inducing a discrete drop in the optimal encounter count from 5 to 4 and reducing total detection from 4.90% to 2.18%.

Interpretation. The mechanism operates through capacity-constrained screening: tighter regimes induce consolidation into fewer, larger encounters, which can reduce detectability (a capacity-dilution channel). When this size dependence is strong, detection can become non-monotonic in scrutiny; when it is weak, consolidation still counteracts stricter screening and attenuates marginal gains from increased scrutiny.

4. Enforcement Architecture Under Capacity Constraints

This section links the model to administrative design. Building on Assumption 4, screening capacity is bounded at the encounter level, so screening intensity per unit is $e(y) = B/y$ and detection probability is $p(y) = \pi(B/y)$. The regulator takes the total activity, Y, as given and chooses rules for how activity is partitioned across encounters.

4.1. Objective and Implementable Instruments

Let the agent allocate the total activity Y across n encounters with sizes $\{y_i\}_{i=1}^n$, $y_i > 0$, $\sum_{i=1}^n y_i = Y$. Given encounter-level detection probabilities $p(y_i)$, the probability of at least one detection over the horizon is

$$D(\{y_i\}_{i=1}^n) \equiv 1 - \prod_{i=1}^{n} \big(1 - p(y_i)\big). \tag{12}$$

I treat $D(\cdot)$ as the regulator's benchmark objective and focus on two administratively common instruments:

1. **Encounter size caps:** impose $y_i \leq \bar{y}$ for all i.
2. **Frequency requirements:** impose $n \geq n_{\min}$, where $n_{\min} \in \mathbb{N}$ is a policy parameter (equivalently, an upper bound on the reporting interval).

These instruments are distinct from reforms that increase penalties or raise per-encounter procedural requirements. In the model, higher per-encounter stringency raises exposure costs and can induce bundling, thereby reducing the number of encounters and increasing their size.

4.2. A Design Result: Encounter Caps and Detection

Proposition 6 below formalizes how encounter architecture can improve detection when screening capacity is bounded.

Proposition 6 (Encounter capping weakly increases detection)**.** *Suppose detection is capacity-constrained as in Assumption 4 and $p(y) \in (0,1)$ for all $y > 0$. Define*

$$\ell(y) \equiv -\log\big(1 - p(y)\big).$$

A standard sufficient condition for "risk reductions" to be concave in encounter size is that $\ell(\cdot)$ is concave on $(0,\infty)$. Fix total activity Y and consider any feasible allocation $\{y_i\}_{i=1}^n$ with $\sum_{i=1}^n y_i = Y$. Let $\bar{y} \in (0, Y]$ and construct $\{\tilde{y}_j\}_{j=1}^{\tilde{n}}$ by splitting any encounter with $y_i > \bar{y}$ into $\lceil y_i/\bar{y} \rceil$ encounters that sum to y_i and satisfy $\tilde{y}_j \leq \bar{y}$. Then

$$D(\{\tilde{y}_j\}_{j=1}^{\tilde{n}}) \;\geq\; D(\{y_i\}_{i=1}^n),$$

where $D(\cdot)$ is defined in (12). If $\ell(\cdot)$ is strictly concave and at least one encounter is split nontrivially, the inequality is strict.

Proof. Write the probability of no detection under allocation $\{y_i\}_{i=1}^n$ as

$$S(\{y_i\}) \equiv \prod_{i=1}^{n} \big(1 - p(y_i)\big), \qquad \text{so that} \qquad D(\{y_i\}) = 1 - S(\{y_i\}).$$

Consider splitting a single encounter of size $y > 0$ into $m \geq 2$ parts $\{y^{(1)}, \ldots, y^{(m)}\}$ with $\sum_{j=1}^{m} y^{(j)} = y$. Taking logs,

$$-\log\left(\prod_{j=1}^{m}(1 - p(y^{(j)}))\right) = \sum_{j=1}^{m} \ell(y^{(j)}).$$

By concavity of ℓ, for each j,

$$\ell(y^{(j)}) \;\geq\; \frac{y^{(j)}}{y}\,\ell(y),$$

so summing over j yields $\sum_{j=1}^{m} \ell(y^{(j)}) \geq \ell(y)$. Exponentiating implies

$$\prod_{j=1}^{m}(1 - p(y^{(j)})) \;\leq\; 1 - p(y).$$

Thus splitting weakly reduces the no-detection probability for that encounter. Applying the same argument to every encounter larger than $\bar{y}$ shows $S(\{\tilde{y}_j\}) \leq S(\{y_i\})$, hence $D(\{\tilde{y}_j\}) \geq D(\{y_i\})$.

If ℓ is strictly concave and at least one split is nontrivial, at least one inequality above is strict, so $S(\{\tilde{y}_j\}) < S(\{y_i\})$ and therefore $D(\{\tilde{y}_j\}) > D(\{y_i\})$. □

4.3. Architecture, Sampling, and Scalable Screening

Encounter caps and frequency requirements are extreme forms of a more general principle: screening intensity should be preserved per unit when activity is consolidated. Flexible implementation can be accomplished through *within-encounter sampling* or *size-contingent screening*. For large encounters, the regulator can apply a rule in terms of which a subset of subcomponents (e.g., line-item audits, random container checks, targeted deep dives) is inspected. This effectively replaces $p(y) = \pi(B/y)$ by

$$p(y) = \pi\big(B/y\big) \;\text{ with }\; B = B(y),$$

where $B(y)$ increases with encounter size because automated checks, analyst time, or inspection slots scale with volume. In this sense, scalable screening technologies and sampling rules mitigate the capacity-dilution problem without necessarily imposing hard caps on y.

4.4. Policy Tradeoff and Evaluation Criterion

The model highlights a tradeoff between (i) *procedural exposure*–the per-encounter burden that discourages frequent encounters—and (ii) *informational preservation*—the benefits of more frequent, smaller encounters when screening is capacity-constrained. A practical criterion for evaluating reforms follows: reforms that increase the per-encounter procedural burden should be paired with either (a) a reporting architecture that preserves encounter frequency (e.g., lightweight periodic micro-reporting with occasional deep dives), or (b) screening technologies that scale with encounter size, so that effective screening intensity per unit does not collapse under consolidation. Otherwise, stricter rules can mechanically shift activity into forms that are harder to screen. I discuss design implications and boundary conditions in Section 5, and empirical signatures and falsification strategies in Section 6.

4.5. Policy Counterfactuals Under Capacity Constraints

This subsection states the policy counterfactual that follows from the model when screening capacity is allocated at the encounter level (Assumption 4). The comparison is between reforms that increase per-encounter burdens and reforms that change encounter architecture, holding total activity Y fixed.

A standard tightening reform. Consider a reform that raises per-encounter procedural burden or scrutiny intensity, without scaling encounter-level screening resources. In the model, this is naturally represented as an upward shift in the exposure schedule $C(n)$. Proposition 2 implies weak consolidation: the agent reduces encounter frequency and increases average encounter size. Under capacity dilution, larger encounters receive lower screening intensity per unit, $e(y) = B/y$, which reduces per-encounter detectability $p(y) = \pi(B/y)$. At consolidation thresholds, this channel can dominate the mechanical increase in per-encounter burden, so equilibrium detection falls as tightening rises (Theorem 1). Table 3 provides a numerical illustration, and Figure 3 maps the parameter region where the detection loss occurs.

An architectural reform. Now consider reforms that restrict bundling directly rather than raising per-encounter burden. Two examples are encounter size caps, $y_i \leq \bar{y}$, and minimum frequency requirements, $n \geq n_{\min}$. These instruments operate on the encounter process itself and therefore act on the channel that generates capacity dilution. Proposition 6 shows that capping encounter size weakly increases detection when screening capacity per encounter is fixed, with strict gains under standard concavity conditions. A closely related design is within-encounter sampling: for large encounters, the regulator inspects a subset of components so that effective screening capacity scales with volume, as in Subsection 4.3. In the model, this corresponds to making B increasing in y, which attenuates the dilution channel and reduces the scope for the paradox.

Implication. Per-encounter stringency works through exposure and therefore strengthens incentives to consolidate along the encounter margin. Architectural reforms act on that margin directly by limiting bundling or preserving screening intensity per unit. When processing constraints bind, pairing higher per-encounter burdens with an architecture that constrains consolidation avoids a mechanical loss of observability.

5. Discussion: Administrative Design under Capacity Constraints

The analysis yields a policy-relevant design implication: when screening capacity is binding, regulators can improve detection by shaping encounter architecture rather than relying solely on higher penalties or stricter per-encounter requirements. I therefore characterize optimal encounter design under capacity constraints. In particular, I show that policies imposing an upper bound on encounter size or a minimum encounter frequency weakly increase detection and can strictly dominate increases in per-encounter stringency when detection is sufficiently concave in screening intensity. This provides an economic rationale for administrative rules such as filing frequency requirements, staggered reporting requirements, and shipment size caps in settings where enforcement agents face congestion and limited screening resources.

The model isolates a margin that is often treated as fixed in enforcement analyses, allowing its implications to be examined transparently. It holds total activity fixed and focuses on its organization across regulatory encounters. That choice isolates a margin that is easy to overlook: agents can adjust both what happens within a filing or inspection *and* how often those encounters occur. The framework employed here complements deterrence models by highlighting a margin, interaction frequency, that is typically taken as exogenous. When detectability depends on interaction size, this reorganization also affects what regulators can learn, not just how often agents appear.

Boundary conditions and scope. The mechanism is most relevant when regulated parties have discretion over the timing or partitioning of activity, and when the marginal burden of an additional encounter is nontrivial (procedural exposure, scrutiny, or follow-up costs). If encounter timing is fixed by statute or technologically constrained, enforcement operates primarily by reducing aggregate activity over the horizon rather than by reshaping encounter structure. However, many policy environments combine both margins: penalties affect the level of activity, while the procedural burden affects how activity is distributed across dates or channels, even when the particular frequency is mandated. The 'paradox' channel additionally requires that detectability declines sufficiently with encounter size or complexity; it is muted if screening resources scale proportionally with encounter size or if regulators can sample within large encounters such that effective detection does not fall mechanically with consolidation. Empirically, the mechanism is most plausible when enforcement shocks shift the joint distribution of encounters, encounter size, and spacing, not only aggregates. The key takeaway is that enforcement can change observability by changing the encounter process itself, and this can alter how enforcement effectiveness appears in administrative data.

6. Empirical Implications and Measurement

Empirical scope. Although the paper is theoretical, its main predictions concern objects routinely observed in administrative data: encounter counts, encounter sizes, and encounter timing. The cleanest tests exploit enforcement changes that shift per-encounter burden or targeting intensity while permitting longer-horizon totals to be tracked. Accordingly, filings and inspections should be interpreted as equilibrium outcomes of encounter design rather than direct measures of underlying activity. Many of the predictions are most relevant in environments with encounter-level screening and binding processing constraints (see Section 3.2).

These predictions can be examined using standard empirical designs. Under pure deterrence, reductions in the frequency of interactions must be accompanied by reductions in total activity. Under exposure–driven consolidation, interaction frequency falls while total activity remains stable, and per-interaction size rises. If interaction counts and per-interaction size both fall, the consolidation mechanism is inconsistent with the model.

Event studies of increases in monitoring intensity, documentation requirements, or review procedures can trace how interaction counts, sizes, and timing adjust relative to totals. A difference-in-differences design that exploits differential exposure to enforcement changes—such as risk-based targeting or procedural thresholds—can be used to test whether treated units exhibit consolidation and clustering. Policy changes that alter review or follow-up practices provide particularly clean settings, as they plausibly affect the encounter burden without directly changing statutory incentives (Choudhary and Gupta, 2025; Zhou et al., 2023).

The result concerning information collapse also has implications for measurement. When the probability of detecting a violation within an interaction declines with interaction size, consolidation affects both exposure and observability. In that case, organizational responses induced by enforcement can reduce the total information generated by regulatory encounters. Beyond a threshold

level of per-encounter stringency, further increases in enforcement can, therefore, lower overall detection probabilities, even as scrutiny intensifies.

Empirically, this implies a nonmonotonic relationship between enforcement intensity and detection outcomes. Detection per–encounter may rise as encounters are more heavily scrutinized, while detection per unit of underlying activity declines as activity is bundled into fewer, larger episodes that are harder to screen. This pattern distinguishes information collapse from standard deterrence or capacity constraints, which predict monotonic improvements in detection as enforcement increases (Maćkowiak et al., 2023).

A final implication concerns interpretation. Short aggregation windows can mechanically exaggerate declines in totals when activity is re-timed into fewer encounters, while declines in filing counts are often misread as reductions in underlying activity. My analysis clarifies why such patterns need not reflect weak enforcement or transitory responses. Instead, they may be the equilibrium outcome of rational agents reorganizing activity to economize on exposure and, when detectability is size-dependent, to reduce the informational value of each encounter. Empirical analyses that track encounter counts, sizes, timing, and detection outcomes jointly are therefore essential for assessing enforcement effects in environments with persistent scrutiny.

7. Conclusion

This paper has considered a simple margin that enforcement models often treat as fixed: how frequently regulated agents interact with authorities. In many settings, the main burden of enforcement is not the expected penalty per unit of activity, but the cost of repeated exposure in the form of preparation, documentation, review, and the possibility of follow-up. When past contact informs future scrutiny, a return encounter can be more costly than the first. Agents then have a clear incentive to reorganize activity to appear less often, even if this does not reduce the total activity.

The model formalizes this idea by letting an agent with fixed total activ-

ity choose how to partition that activity across encounters. Internal processing costs make very large batches costly, while exposure costs rise with the number of encounters. The resulting trade-off delivers a sharp prediction: when the marginal burden of an additional encounter increases, the agent chooses fewer encounters and larger average size per-encounter. If encounters are time-indexed, this mechanically generates retiming and clustering, with longer gaps followed by bursts. These outcomes arise under conditions of full rationality; temporary deterrence, biased beliefs, or other behavioral assumptions are not required.

This analysis has underscored a fundamental tension in modern enforcement systems: policies that increase scrutiny through repeated interactions may succeed in disciplining behavior, but they can also lead agents to reduce contact and consolidate activity, thereby complicating oversight. Treating encounter frequency as an endogenous choice helps explain lumpy post-enforcement patterns as rational adaptations and reveals how enforcement reshapes the organization of economic activity without necessarily altering its scale.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the author used ChatGPT 5.2 for code editing, structural assistance, and LaTeX formatting. After using this tool, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the published article.

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Tables and figures

List of Tables

1 Persistence of consolidation: illustrative two-period example . . . 31

2 Total cost by encounter count under encounter-only and size-contingent monitoring 31

3 The paradox of tightening: higher procedural burden can reduce detection (illustration) 31

List of Figures

1 Optimal interaction frequency and average interaction size over time 32

2 Total cost under encounter-only and size-contingent monitoring . 32

3 Paradox-of-tightening region and consolidation thresholds. 33

Table 1: Persistence of consolidation: illustrative two-period example

Period	Scrutiny level (s_t)	Optimal encounters (n_t^*)	Avg. size (Y/n_t^*)
1	High (s_H)	5	20.0
2	Low (s_L)	6	16.7

Notes: Total activity is normalized to $Y = 100$. Period 1 has higher scrutiny, inducing consolidation ($n_1^* = 5$). If scrutiny partially carries over, the period-2 optimum can remain below the low-scrutiny benchmark even when s_t reverts to s_L.

Table 2: Total cost by encounter count under encounter-only and size-contingent monitoring

n	Y/n	$\Phi_{\mathrm{EO}}(n)$	$\Phi_{\mathrm{SC}}(n)$
2	50.0	63.00	80.00
3	33.3	48.67	60.00
4	25.0	43.50	52.00
5	20.0	**42.00**	48.80
6	16.7	42.33	**48.00**
7	14.3	43.71	48.57
8	12.5	45.75	50.00
9	11.1	48.22	52.00
10	10.0	51.00	54.40

Notes: $Y = 100$. Encounter-only objective: $\Phi_{\mathrm{EO}}(n) = aY^2/n + c_0 n + \frac{\kappa}{2} n(n-1)$ with $a = 0.011$, $c_0 = 4$, $\kappa = 0$. Size-contingent objective adds $\tau\, n\, (Y/n)^{\beta}$ with $\tau = 0.0034$ and $\beta = 2$. Bold values denote the discrete minima (markers in Figure 2).

Table 3: The paradox of tightening: higher procedural burden can reduce detection (illustration)

Regime	Exposure cost s	n^*	Y/n^*	$p(Y/n^*)$	$D(n^*)$
A (moderate)	0.40	5	20.0	0.0100	0.0490
B (stricter)	0.50	4	25.0	0.0055	0.0218

Notes: Detectability $p(y)$ declines with encounter size y under capacity dilution. The tightening $s : 0.40 \to 0.50$ induces consolidation ($n^* : 5 \to 4$), increasing Y/n^* and lowering both $p(Y/n^*)$ and total detection $D(n^*)$.

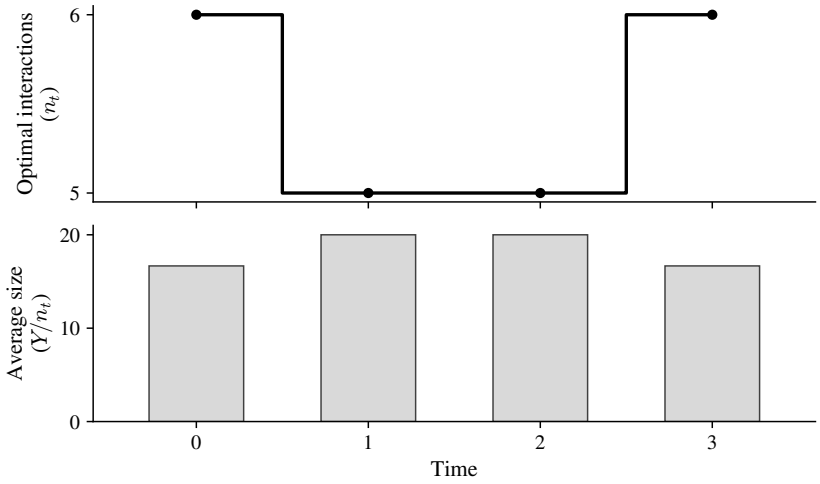


Figure 1: Optimal interaction frequency and average interaction size over time

Optimal interaction frequency (line) and average interaction size (bars) around a temporary scrutiny shock at $t = 1$.

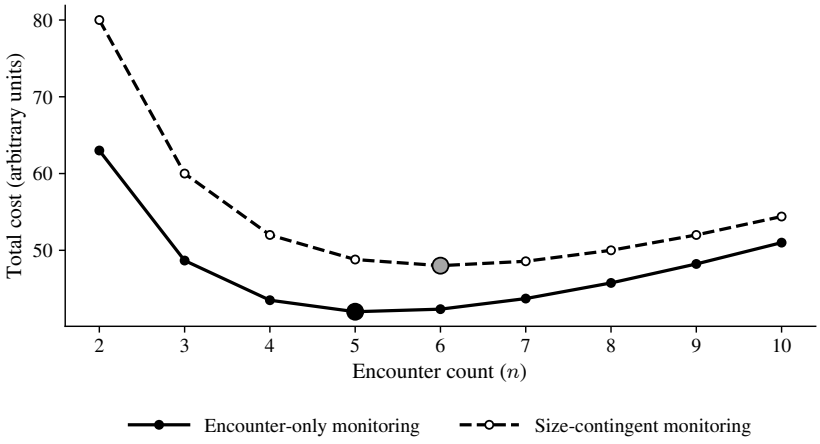


Figure 2: Total cost under encounter-only and size-contingent monitoring

Total cost as a function of the interaction count under encounter-only (solid) and size-contingent (dashed) monitoring. Markers denote optimal choices.

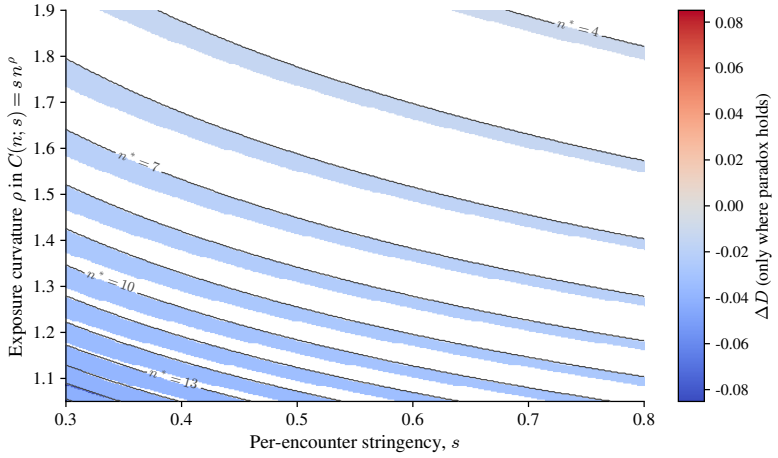


Figure 3: Paradox-of-tightening region and consolidation thresholds.

Detection $D(n) = 1 - (1 - p(Y/n))^n$; shading shows ΔD only where both n^* falls and detection declines under a tightening $s \to s + \Delta s$ (with $\Delta s = 0.05$ in the illustration).

Appendix

Supplementary Material

This appendix collects supplementary material supporting the main text. Appendix Appendix A provides an extended mapping, examples, and discussion of alternative exposure schedules. Appendix Appendix B collects auxiliary results and derivations referenced in the text.

Appendix A. Extended Framework Mapping and Examples

Appendix A.1. Additional example: targeted capacity under resource constraints

In the main text, exposure costs are treated in reduced form to isolate how procedural frictions reshape the organization of regulatory encounters rather than institution-specific details. This is consistent with enforcement systems that allocate scarce scrutiny using risk information under binding resource constraints. For example, in environmental regulation, inspection allocation can be framed as a capacity-constrained targeting problem, and data-driven tools can be used to prioritize high-risk facilities (Zhou et al., 2023).

Appendix A.2. Extended mapping: discrete regimes and reorganization around thresholds

A central implication of the paper is that monitoring design can induce reorganization of encounters around discrete regimes, altering the mapping from observed administrative events to underlying activity. When targeting relies on risk scores or cutoffs constructed from reported information, agents may change encounter timing, scope, or frequency in ways that shift measured risk without commensurate changes in underlying behavior. Choudhary and Gupta (2025), for instance, study dynamic responses around a size-based registration threshold; while their setting differs, the shared implication is that monitoring rules can change observables through reorganization and retiming.

Appendix A.3. Robustness to alternative exposure-cost schedules

The consolidation comparative static in Proposition 2 does not rely on a narrow functional form for the exposure schedule. To make this explicit, let the marginal exposure cost of the kth encounter be $c(k)$, so that $C(n) = \sum_{k=1}^{n} c(k)$. The key requirement is that $c(k)$ is nondecreasing over the relevant range (Assumption 2). The following stylized cases illustrate common institutional patterns:

1. **Constant marginal exposure.** $c(k) = \bar{c} > 0$ implies $C(n) = \bar{c}\, n$.
2. **Linear escalation.** $c(k) = c_0 + b(k-1)$ with $b > 0$ increases the marginal burden of additional encounters and shifts the optimum toward fewer encounters $n^\star$ and larger average size $Y/n^\star$.
3. **Convex escalation.** A convex $c(\cdot)$ produces sharper (discrete) reductions in $n^\star$ as policy increases the slope of marginal exposure around the pre-change optimum.
4. **Concave escalation / saturation.** $c(k) = c_0 + b(1 - e^{-\lambda(k-1)})$ with $\lambda > 0$ captures early escalation followed by saturation. Consolidation persists as long as $c(\cdot)$ is nondecreasing over the relevant range.
5. **Threshold escalation (trigger rules).** $c(k) = c_L$ for $k \leq K$ and $c(k) = c_H$ for $k > K$ with $c_H > c_L$ captures institutional triggers (repeat amendments, follow-ups, escalation rules) and implies discontinuous shifts in $n^\star$ and potentially strong bunching in encounter counts.

Magnitude. The size of the consolidation response depends on how much the marginal exposure schedule changes *around the pre-change optimum.* In discrete-choice problems, small policy changes can have large effects when they move the optimum across a switching threshold.

Appendix A.4. Batching logic and "ordeals" (background)

The benchmark mechanism—partitioning a fixed total Y across a chosen number of encounters—is closely related to classic batching and lumpy-adjustment

logic. In inventory, cash-management, and adjustment-cost models, agents trade off fixed or convex costs against the benefits of smoothing, generating discrete changes in frequency and batch size (Baumol, 1952; Tobin, 1956; Caballero and Engel, 1999). The present model applies this logic to administrative interactions, interpreting them as an endogenous *encounter process* rather than a fixed schedule.

A related conceptual link is "ordeal" design: when costs attach to repeated interactions, policies can screen or deter by increasing procedural burdens rather than monetary penalties. This paper differs in emphasis by focusing on equilibrium reorganization of encounters and the resulting implications for what regulators observe.

Appendix B. Mathematical Appendix

Appendix B.1. Monotone comparative statics for encounter consolidation

This appendix records a lattice-theoretic monotonicity condition that delivers the weak consolidation result invoked in the main text.

Let $s \in \mathcal{S} \subseteq \mathbb{R}$ index policy stringency and allow the exposure cost to depend on s. The agent chooses $n \in \mathbb{N}$ to minimize

$$V(n; s) \equiv n\, g(Y/n) + C(n; s). \tag{B.1}$$

Assume:

(i) (*Existence*) For each s, $V(n; s) \to \infty$ as $n \to \infty$.

(ii) (*Increasing differences / single crossing*) For all n and $s_2 > s_1$,

$$\Delta(n; s) \equiv V(n+1; s) - V(n; s) \;\text{ satisfies }\; \Delta(n; s_2) \geq \Delta(n; s_1), \tag{B.2}$$

with strict inequality for at least one n for any nontrivial increase in s.

Proposition 7 (Monotone comparative statics (weak consolidation))**.** *Let* $n^\star(s) \in \arg\min_{n\in\mathbb{N}} V(n; s)$. *Under* ***(i)****–****(ii)****, the smallest selection from* $\arg\min_n V(n; s)$ *is weakly decreasing in* s. *In particular, higher stringency weakly reduces the optimal number of encounters.*

Proof. Condition **(ii)** is increasing differences on the lattice $\mathbb{N} \times \mathcal{S}$ in discrete-increment form. By Topkis' monotonicity theorem for minimization on lattices, the smallest minimizer is weakly decreasing in s (Topkis, 1978). □

Appendix B.2. Size-contingent monitoring limits consolidation

Proof of Proposition 5. Fix $\kappa \geq 0$. Under strict convexity of g and convexity of m, the Jensen argument used for Lemma 1 applies to $g + \kappa m$, so conditional on n the unique minimizer is $y_i = Y/n$ for all i. The reduced objective is

$$V(n;\kappa) = H(n) + C(n) + \kappa T(n), \quad \text{where} \quad H(n) \equiv ng(Y/n),\ \ T(n) \equiv nm(Y/n).$$

Define the forward difference $\Delta(n;\kappa) \equiv V(n+1;\kappa) - V(n;\kappa)$:

$$\Delta(n;\kappa) = \underbrace{\big(H(n+1) - H(n)\big) + c(n+1)}_{\Delta_0(n)} + \kappa \underbrace{\big(T(n+1) - T(n)\big)}_{\Delta_T(n)}.$$

By Assumption 1, $H(n)$ is convex in n on $\mathbb{N}$, so $H(n+1) - H(n)$ is nondecreasing in n. By Assumption 2, $c(n+1)$ is nondecreasing in n, hence $\Delta_0(n)$ is nondecreasing. By the assumed convexity of $T(n)$ on $\mathbb{N}$, $\Delta_T(n)$ is also nondecreasing in n. Therefore, for each κ, $\Delta(n;\kappa)$ is nondecreasing in n, implying the set of minimizers is an interval and the smallest minimizer $\underline{n}(\kappa)$ is characterized by the first index where $\Delta(n;\kappa) \geq 0$.

Since $T(n)$ is decreasing, $\Delta_T(n) = T(n+1) - T(n) \leq 0$ for all n. Thus for any fixed n, $\Delta(n;\kappa)$ is weakly decreasing in κ. Increasing κ shifts the sequence $\{\Delta(n;\kappa)\}_n$ downward, so the crossing point where it becomes nonnegative occurs at a weakly larger n. Hence $\underline{n}(\kappa)$ is weakly increasing in κ, and in particular $\underline{n}(\kappa) \geq \underline{n}(0)$ for all $\kappa \geq 0$. □

Appendix B.3. Benchmark where encounter frequency is irrelevant

A useful benchmark clarifies why many models treat encounter timing and frequency as exogenous. Suppose expected enforcement costs are linear in total activity and independent of encounter history, so expected enforcement cost

equals λY with $\lambda \geq 0$. Then expected enforcement cost is invariant to the partition $\{y_i\}_{i=1}^n$ and to the encounter count n. Encounter frequency becomes relevant only when exposure depends on the encounter process itself, e.g., through per-encounter overhead, history dependence, or state-dependent scrutiny.

Appendix B.4. Parametric illustration of the consolidation channel

To make the magnitude channel explicit, consider $g(y) = y^2$ and $C(n) = \alpha n^\rho$ with $\alpha > 0$ and $\rho \geq 1$. Then

$$V(n) = n\, g(Y/n) + C(n) = \frac{Y^2}{n} + \alpha n^\rho.$$

Treating n as continuous for approximation, the first-order condition yields

$$n^\star \approx \left(\frac{Y^2}{\alpha\rho}\right)^{\frac{1}{\rho+1}}.$$

Thus higher α or ρ lowers $n^\star$ and increases average encounter size $Y/n^\star$. In the discrete problem, the magnitude of the shift depends on how exposure increments change around the pre-change optimum.

Appendix B.5. Microfoundation for state-dependent exposure

A simple reduced-form microfoundation rationalizes escalating exposure costs. Let $s(t)$ denote a scrutiny state that summarizes accumulated information and attention about the agent. Between encounters, scrutiny decays as

$$\dot{s}(t) = -\delta s(t), \qquad \delta > 0.$$

Each encounter at time t_j increases scrutiny,

$$s(t_j^+) = s(t_j^-) + \eta, \qquad \eta > 0.$$

Let the per-encounter exposure cost be

$$c(t_j) = c_0 + \kappa s(t_j^-), \qquad c_0 > 0,\ \kappa > 0.$$

Then repeated encounters (especially in close succession) raise expected exposure costs even if statutory penalties do not change.

Appendix B.6. When equal splitting fails: order- and size-dependent exposure

In the baseline, exposure depends only on the number of encounters n, so equal splitting follows from convexity of $g(\cdot)$. If exposure depends on sequence position or encounter size, equal splitting need not hold. For example, let the exposure cost of encounter i be $c(i)+\kappa y_i$, where $c(i)$ is increasing in i and $\kappa \geq 0$. Then the agent may optimally front-load activity, with $y^1 \geq y^2 \geq \cdots \geq y^n$, because shifting activity from later encounters (higher fixed exposure) to earlier encounters (lower fixed exposure) reduces total cost. This yields an additional empirical implication: under escalating scrutiny, activity may shift earlier in the encounter cycle even when total activity is unchanged.

Appendix B.7. Proof of Theorem 1

Proof. Let the agent choose $n \in \mathbb{N}$ to minimize $\Phi(n; s) = ng(Y/n) + C(n; s)$, where $C(n; s)$ is increasing in s and satisfies increasing differences in (n, s). By Proposition 7, higher s weakly reduces the optimal n. Because n is discrete, $n^\star(s)$ is a step function in s with switching thresholds.

Let per-encounter detectability be $p(y) = \pi(B/y)$ as in Assumption 4. For a given n, total detection over the horizon is

$$D(n) \equiv 1 - \big(1 - p(Y/n)\big)^n.$$

When n falls from n to $n-1$, encounter size rises from Y/n to $Y/(n-1)$ and, under capacity dilution, per-encounter detectability weakly falls: $p(Y/(n-1)) \leq p(Y/n)$. Whether total detection falls depends on the tradeoff between fewer independent draws and lower per-draw detectability. A drop at a switching threshold occurs when

$$D(n-1) < D(n),$$

equivalently,

$$\big(1 - p(Y/(n-1))\big)^{n-1} > \big(1 - p(Y/n)\big)^n.$$

Thus there exist values of s at which a tightening that induces a discrete reduction in n reduces equilibrium detection, yielding the paradox-of-tightening pattern. □

Remark. (Boundary condition.) If screening resources scale proportionally with encounter size so that $p(y)$ does not decline with y, the dilution channel disappears and detection is monotone in s.