Distributional Monetary Transmission in Bank Lending under Abundant Reserves

Wentong Chen*

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Abstract

In the post-QE environment, reserves are abundant, unevenly distributed, and increasingly remunerated non-linearly. These features break the traditional view that a single policy rate determines a uniform marginal funding cost. This paper shows that when the return on reserves is tiered, monetary policy acts through distributional channels: it reallocates liquidity across financial institutions, shifts intermediation capacity in the interbank market, and reshapes the entire distribution of loan rates. I develop a general framework in which institutions differ in their exposure to low-remuneration reserves. A non-linear reserve pricing schedule, such as reserve tiering, creates asymmetric funding incentives. These asymmetries prompt institutions with excess reserves to supply interbank liquidity and those with scarce reserves to borrow and expand low-rate lending. The model predicts that monetary easing compresses medium-risk loan rates, disproportionately lowers rates for riskier borrowers, and generates distinct distributional changes in credit supply. Using data on reserve balances, interbank trades, and firm-bank lending in Japan, I provide empirical evidence consistent with these mechanisms. The framework offers new insight into monetary transmission under abundant reserves and guidance for designing reserve remuneration systems in both low-rate and overheating environments.

*Department of Economics, Leavey School of Business, Santa Clara University, Santa Clara, CA 95053, USA; E-mail: wchen10@scu.edu. Research support from the L.R. Red Wilson Excellence in Economics Medal at Cornell University is gratefully acknowledged. This paper was previously circulated as "The Power of Reserve Tiering: Financial Institution Heterogeneity and Monetary Policy Pass-Through". I am deeply grateful to my committee, Eswar Prasad (chair), Kristoffer Nimark, Ryan Chahrour, and Matthew Baron, for their continuous guidance and support. I also thank Isha Agarwal, Levon Barseghyan, Jaroslav Borovicka, Kaiji Chen, Federico Di Pace, Wenxin Du, Gaetano Gaballo, Malin Hu, Yang Jiao, Philipp Kircher, Yizhou Kuang, Juan Camilo Medellín (discussant), Paul Mizen, Gulcin Ozkan, Kenneth Rogoff, Mathieu Taschereau-Dumouchel, Judit Temesvary, and Wentao Zhou (discussant) for their thoughtful comments and feedback. I am grateful to participants at the BIS–CEPR–SCG–SFI Conference on Financial Intermediation 2024, the AFA Annual Meeting 2024, the 21st Macro Finance Society (MFS) Workshop, the Midwest Macroeconomics Conference 2023, and other conferences for their valuable input. All errors are my own.

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1 Introduction

Monetary policy today operates in a financial environment shaped by abundant reserves. After years of large-scale asset purchases, reserve balances are no longer marginal or uniform, but are instead unevenly spread across financial institutions. At the same time, the remuneration applied to these balances has become increasingly non-linear: central banks now use tiered reserve systems, quotas, and differentiated remuneration rates to protect bank margins, stabilize funding markets, and improve policy pass-through when deposit rates are downward-rigid. These developments fundamentally break the classical view that a single policy rate maps into a common marginal funding cost for all intermediaries. When reserve balances are heterogeneous and remunerated non-linearly, monetary policy interacts with a distribution of reserve exposures rather than a single balance sheet margin. This interaction creates asymmetric marginal incentives, generates endogenous reallocation of liquidity across institutions, and alters pricing behavior in loan markets.

A set of robust empirical regularities across jurisdictions highlights the limitations of the traditional framework. First, reserve balances are highly skewed, with some institutions holding large excess liquidity and others operating near binding thresholds. Second, when remuneration schedules introduce kinks, such as under tiering, interbank activity rises sharply even when aggregate reserves remain abundant, indicating that liquidity reallocates across institutions rather than simply expanding uniformly. Third, loan pricing responses to monetary easing are systematically heterogeneous: institutions with ample unused tiering capacity reduce loan rates aggressively, while those facing low remuneration adjust only modestly. Finally, these asymmetric adjustments reshape the

loan-rate distribution itself: low-rate segments expand, medium-rate segments contract, and risk premia on higher-risk borrowers compress. These patterns cannot be reconciled with models in which reserves are perfect substitutes and the policy rate is the universal marginal price.

This paper develops such a framework by formalizing a theory of *distributional monetary transmission in bank lending under abundant reserves*. When reserves are unevenly distributed and remunerated through a tiered system, monetary policy acts through the *distribution* of reserve exposures rather than through the aggregate quantity of reserves. Tiering first generates heterogeneous marginal funding incentives, the *reserve pricing channel*, because a non-linear remuneration schedule causes institutions at different points of the tiering schedule to experience different shifts in their marginal cost of liquidity. These heterogeneous incentives then activate the *intermediation reallocation channel*: institutions penalized by low remuneration seek to offload excess balances, while institutions with scarce reserves borrow more aggressively, producing endogenous liquidity flows in the interbank market that strengthen policy pass-through even when deposit rates cannot adjust. Finally, because institutions facing tighter reserve exposures disproportionately lend to riskier firms, these pricing and reallocation forces feed directly into loan pricing through a *distributional risk channel*. Monetary easing compresses loan rates most strongly in the medium- and high-risk segments of the market, altering not only the average loan rate but the entire distribution of credit terms offered to firms. The distribution of loan rates, across risk categories and borrower types, thus becomes a central object of monetary transmission under abundant reserves.

The empirical analysis uses Japan's reserve tiering policy, where reserve balances below a threshold earn a higher (positive or zero) rate, while those above it receive a lower, often negative, rate, as a particularly transparent setting in which these mechanisms can be observed at high frequency. Interbank data reveal that institutions with greater exposure to low-remuneration reserves systematically increase net interbank lending,

while banks with unused negative-rate capacity borrow more aggressively, precisely the liquidity flows predicted by the intermediation reallocation channel. Bank-level loan rates and granular bank–firm lending responses further confirm the reserve pricing channel: heterogeneous reserve exposures translate into heterogeneous marginal funding costs, leading low-exposure banks to cut loan rates more sharply and expand lending, while high-exposure banks adjust more weakly. Finally, changes in borrower composition and loan-portfolio risk provide evidence for the distributional risk channel: tiering compresses loan rates disproportionately in the lower and medium segments of the distribution and reallocates credit toward riskier firms, even when aggregate loan growth remains modest. Across all three dimensions, the BOJ's tiered remuneration system generates the reallocation, pricing, and distributional effects implied by the model, demonstrating that, under abundant but unevenly distributed reserves, monetary policy transmits primarily through the structure of reserve remuneration rather than through aggregate reserve quantities alone.

Finally, the framework yields clear policy implications for monetary design in an era of abundant reserves. Because tiered remuneration shapes marginal incentives rather than aggregate liquidity, central banks can use nonlinear reserve schedules to strengthen pass-through when the policy rate is constrained, selectively tighten credit when the economy overheats, and manage risk-taking by influencing which institutions face binding funding conditions. The structure of reserve remuneration, its thresholds, wedges, and asymmetries, thus becomes a first-order instrument for steering liquidity flows, shaping the distribution of loan rates, and preserving financial stability.

Related Literature. This paper contributes to two strands of literature. First, it relates to research on the effects of Negative Interest Rate Policy (NIRP) on bank lending. Empirical studies find that NIRP encourages banks to rebalance portfolios by expanding credit, particularly to riskier firms, to avoid holding negatively remunerated reserves (Demiralp et al., 2021; Basten and Mariathasan, 2020; Bittner et al., 2020). Other work highlights

adverse effects, such as reduced lending by deposit-dependent or undercapitalized banks (Heider et al., 2019; Eggertsson et al., 2019; Arce et al., 2021). NIRP also operates through the liquidity channel: Bottero, Minoiu, Peydró, Polo, Presbitero and Sette (2019) find that Italian banks with liquid balance sheets or short-term interbank positions shift from safe assets to loans, particularly to smaller, riskier firms, while Arseneau (2020) show that U.S. banks with large liquid holdings expect greater harm from negative rates. Most closely related is Hong and Kandrac (2022), which shows that Japanese banks more exposed to reserve tiering, identified via larger stock price declines, responded by increasing credit supply and risk-taking. This paper extends their analysis by identifying bank-level exposure to negative-rate balances and developing a theoretical framework that rationalizes the mechanism linking reserve tiering to risk reallocation.

Second, the paper contributes to research on monetary policy's effect on the interbank market. A small number of papers explore tiered policy impacts. Fuhrer, Jüttner, Wrampelmeyer, Zwicker et al. (2020) use Swiss data and a theoretical model to show that tiering boosts interbank activity. Altavilla, Boucinha, Burlon, Giannetti and Schumacher (2022) find that ECB tiering reduces market fragmentation via interbank trading. More broadly, this paper connects to models of the interbank market structure, either as a centralized Walrasian market, as in Fuhrer, Jüttner, Wrampelmeyer, Zwicker et al. (2020), Ennis (2018), and Bech and Keister (2017), or as an over-the-counter (OTC) market, as in Afonso, Armenter and Lester (2019), who use a random search model to capture dynamics under both scarce and abundant reserves.

The paper contributes by unifying these theoretical and empirical insights into a coherent account of monetary transmission when reserves are abundant. It shows that nonlinear reserve remuneration interacts with heterogeneous reserve exposures to produce systematic distributional adjustments in loan pricing. It develops the first model that generates the full set of distributional patterns observed in the data, explaining why loan rates respond unevenly across borrower risk categories and why interbank market

activity becomes central in an environment where reserves are plentiful. It documents micro-level evidence from Japan that aligns closely with these mechanisms. And it draws out the broader policy implications: central banks can shape the distribution of credit, across institutions and risk categories, by altering the structure of reserve remuneration, offering an additional tool for strengthening policy pass-through, managing risk-taking, and cooling overheating credit markets.

2 Stylized Facts

This section documents four empirical facts that motivate the theory: (1) reserve balances are highly unequally distributed across institutions; (2) interbank trading rises when reserve remuneration becomes tiered; (3) banks with greater exposure to low-remuneration reserves adjust lending differently from banks with lower exposure; and (4) loan-rate adjustments are strongly distributional, with low-rate segments expanding and medium-rate segments contracting. These facts jointly point to a distributional process of monetary transmission operating within the loan market.

2.1 Fact 1: Reserve balances are highly unequal across institutions

Reserve tiering in Japan provides a clear setting in which to observe the heterogeneity of reserve exposures across financial institutions. Under the three-tier system introduced in 2016, balances below a threshold earn a positive or zero rate, while balances above the threshold earn a negative rate. Because each institution's tier thresholds depend on prior reserve holdings, the effective remuneration rate differs sharply across banks. More institutional backgrounds are documented in Appendix B.

Figure 1 plots the share of reserves subject to the negative policy rate for each bank in March 2016. Large institutions, city banks and trust banks, hold significantly higher negative-rate balances, while most regional banks hold little or none. Some regional banks have substantial unused capacity below the policy-rate threshold, generating negative

values of the negative-rate ratio. Figure 2 shows that this heterogeneity has a strong and persistent geographical component: regional banks in some prefectures consistently remain far from the negative-rate tier, while others operate close to or within it. These patterns reveal a first key fact: *reserve exposures are highly unequal, creating asymmetric marginal incentives across institutions.*

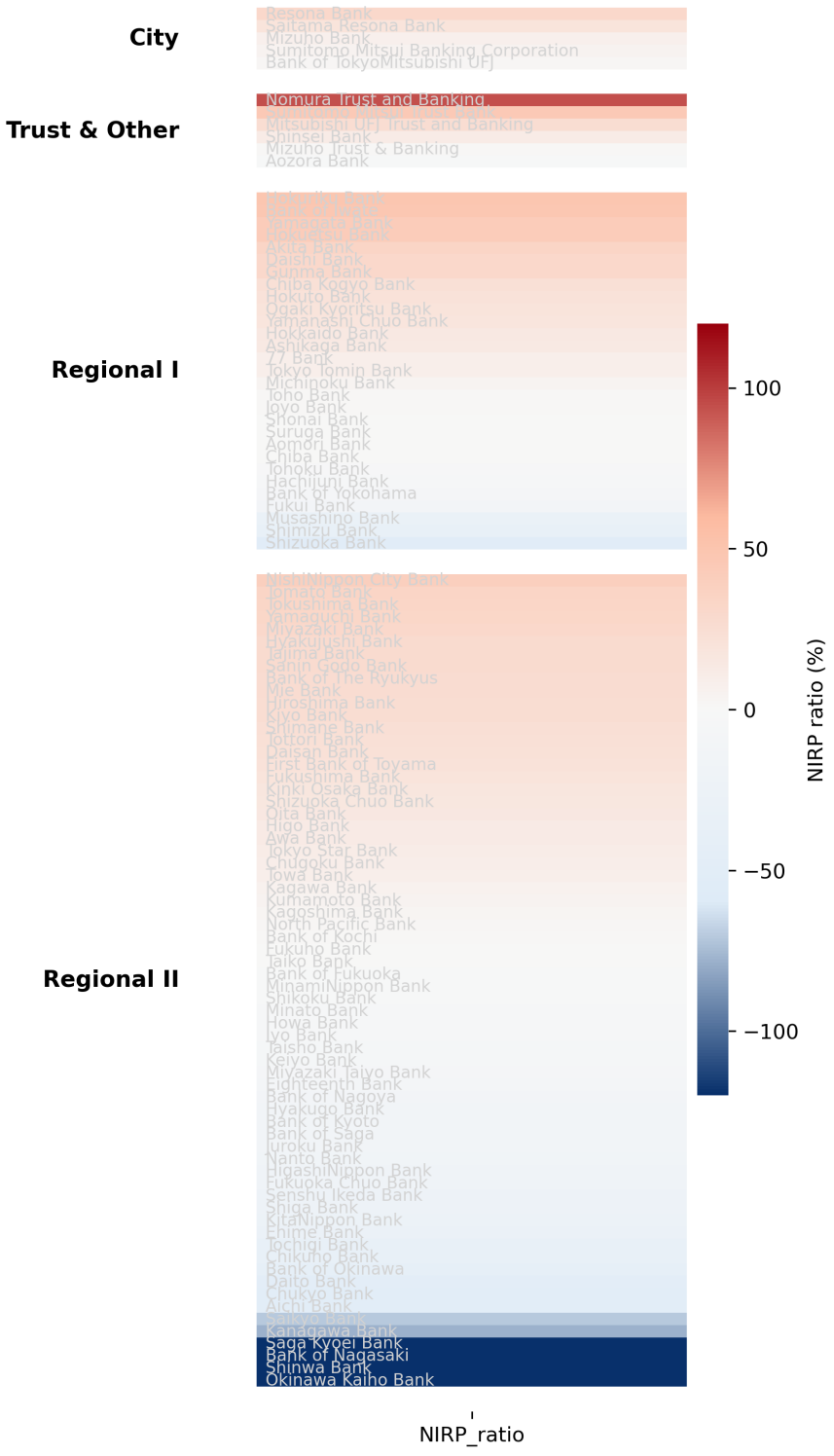


Figure 1: Negative Interest Rate Balance Ratio by Banks in March 2016

Notes: This figure shows the share of negative interest rate balances relative to total reserves for each bank in Japan as of March 2016.

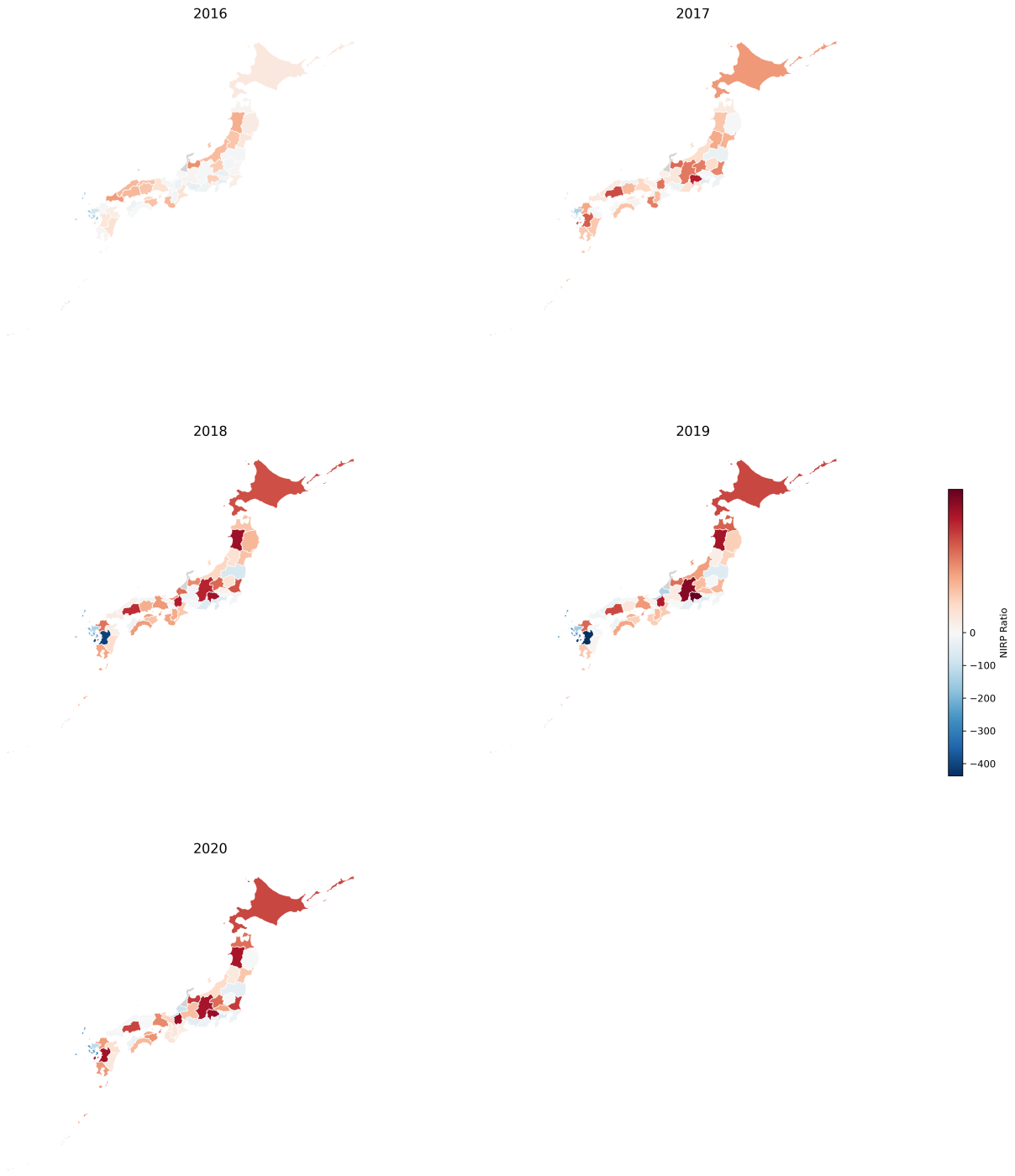


Figure 2: Geographical allocation of negatve interest rate exposure by region banks

Notes: This figure shows the geographical distribution of negative interest rate exposures among regional banks in Japan. Regions where banks hold a larger share of negative interest rate balances relative to total reserves are shaded in red, while those with higher unused negative rate quotas or no negative rate exposure are shaded in blue. Darker colors indicate larger absolute values.

2.2 Fact 2: Interbank trading rises when reserve remuneration becomes tiered

Heterogeneous reserve exposures immediately translate into distinct behaviors in the interbank market. Institutions heavily exposed to negative-rate balances, such as trust banks, foreign banks, and investment trusts, have strong incentives to lend out their excess reserves to avoid negative remuneration. Institutions with low exposure, especially regional banks, have incentives to borrow. Figure 3 shows outstanding positions in the uncollateralized call market by sector. Following the introduction of reserve tiering, lending by trust banks and investment trusts increases sharply, while borrowing by regional banks rises. City banks, more exposed to negative-rate balances, reduce borrowing substantially. Grouping institutions into "large" (city and trust banks) and "small" (regional banks), small banks become net borrowers from both large banks and non-depository institutions (NDIs). Despite abundant aggregate reserves, interbank activity increases. This yields the second fact: *the tiered remuneration schedule activates the interbank market, reallocating liquidity from institutions with excess reserves to those with reserve scarcity.*

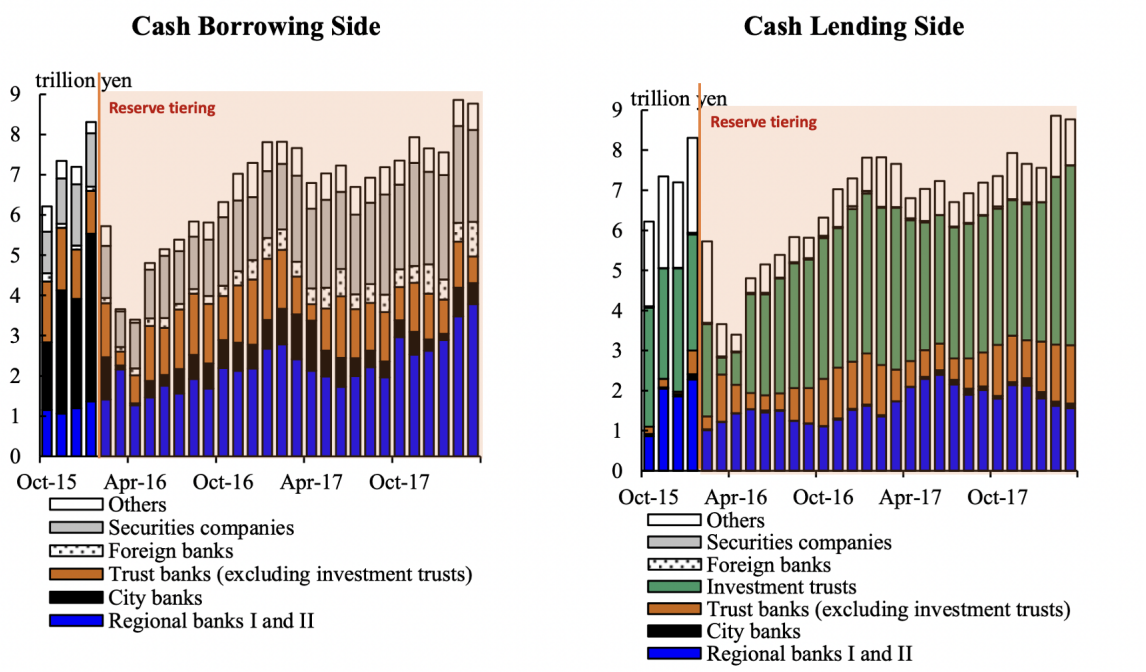


Figure 3: Outstanding Amounts in Japan's Uncollateralized Call Market by Sector

Notes: This figure shows outstanding amounts in the uncollateralized call market by sector in Japan, sourced from the Bank of Japan.

2.3 Fact 3: Banks with greater reserve exposure adjust lending differently

Differences in reserve exposure translate into clear, unconditional differences in how banks adjust their lending following the introduction of reserve tiering. Institutions with a large share of reserves in the negative-rate tier, principally city banks, trust banks, and certain foreign banks, face stronger incentives to shed or reallocate excess balances. Regional banks, by contrast, typically hold little or no negative-rate exposure and therefore face a much weaker marginal incentive.

These differences manifest visibly in both the interbank market and lending patterns. Figure 3 shows that heavily exposed institutions, trust banks and investment trusts, expand their net lending in the uncollateralized call market, while city banks reduce their borrowing. Regional banks, which remain far below the negative-rate threshold, increase borrowing sharply. This structural segmentation, large, exposed institutions

acting as lenders and small, low-exposure institutions acting as borrowers, mirrors the heterogeneity in reserve remuneration.

Loan composition adjusts in parallel. Figures 4 and 5 reveal that the expansion in low-rate lending is likely driven primarily by bank groups with low negative-rate exposure, such as regional banks. Institutions facing larger negative-rate balances do not expand low-rate loans to the same extent and instead show a relative shift toward lending at higher risk categories. These patterns arise even in simple group averages across bank types, before imposing controls or identification structure. In the empirical parts, I test this loan market changes formally.

This yields the third stylized fact: *lending responses are systematically non-uniform across institutions and closely aligned with their negative-rate exposure, providing direct evidence that monetary transmission operates through reserve heterogeneity rather than a uniform shift in funding costs.*

2.4 Fact 4: Loan-rate adjustments are distributional, not uniform

Finally, monetary easing under reserve tiering leaves a clear signature in the distribution of loan rates. Figure 4 shows that in the months following the introduction of the tiered system, outstanding loans with rates below 0.25% rise by roughly ¥22 trillion, while medium- and high-rate loans decline. Figure 5 reveals that the loan-rate distribution shifts sharply toward the lower end, with the mass of loans migrating from the 0.25–1% range into the near-zero range.[1]

These changes are too large to be explained by the negative policy rate alone, which

[1] This surge in low-interest-rate lending is significant even after accounting for special loan programs introduced around the same time. The BOJ provided ¥300 billion in 0% loans through its Fund-Supplying Operations to support institutions affected by the 2016 Kumamoto Earthquake (https://www.boj.or.jp/en/mopo/measures/mkt_ope/ope_u/index.htm/). Other programs included the Fund-Provisioning Measures to Stimulate Bank Lending and to Support Strengthening the Foundations for Economic Growth, as well as support for areas affected by the Great East Japan Earthquake. From January to June 2016, total new loan disbursement under these programs was approximately ¥9.8 trillion (https://www.boj.or.jp/en/mopo/measures/mkt_ope/len_b/index.htm/). Together, these explain only about half of the increase in low-interest-rate loans, even before accounting for repayments, suggesting additional contributing factors.

typically produces weak pass-through due to downward rigidity in deposit rates.(Heider et al., 2021; Arce et al., 2021; Basten and Mariathasan, 2020; Heider et al., 2019; Eggertsson et al., 2019) Instead, the patterns are consistent with reserve tiering inducing differential incentives across institutions, amplifying lending at the low end of the rate distribution while narrowing the middle of the distribution.

This yields the fourth fact: *loan-rate adjustments under abundant reserves are strongly distributional, altering the composition of lending across rate segments rather than shifting loan rates uniformly.*

Together, these four facts motivate the theory developed in Section **??**, which formalizes how non-linear reserve remuneration interacts with heterogeneous reserve exposures to generate distributional monetary transmission in the loan market.

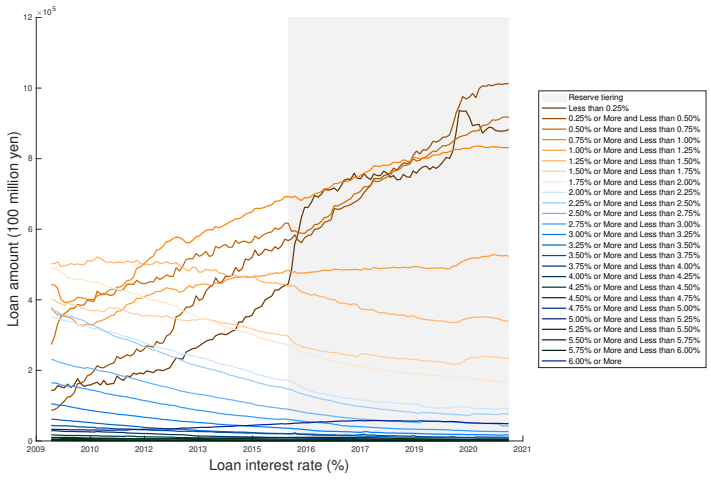
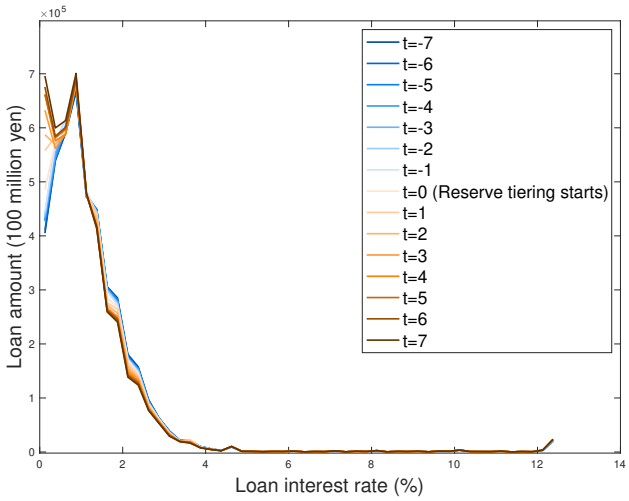


Figure 4: Loans and Discount Outstanding by Interest Rate in Japan

Notes: This figure presents the time series of outstanding loan and discount amounts by interest rate in Japan, sourced from the Bank of Japan.



Source: Bank of Japan

Figure 5: Loans and Discount Distribution by Interest Rate in Japan (t is monthly)

Notes: This figure shows the distribution of outstanding loans and discounts by interest rate in Japan, sourced from the Bank of Japan.

3 Baseline Model

In this section, I use a simple model to capture the monetary transmission mechanism of the tiered monetary policy. The model features six types of agents. Capital investors provide capital to banks and NDIs. Entrepreneurs take loans from banks and carry out risky projects. Depositors place deposits at banks. The central bank sets reserve requirements and tiering thresholds for each bank and NDI, imposes liquidity ratio requirements on banks, pays tiered interest on reserves, and offers lending facilities to financial institutions. There are two types of participants in the interbank market: banks and non-depository institutions (NDIs). Banks issue loans, receive deposits, face liquidity and reserve constraints, and trade in the interbank market. NDIs can trade with banks and are subject to reserve requirements, but they do not accept deposits or issue loans.[2]

[2]In Afonso, Armenter and Lester (2019), similar institutions are referred to as government-sponsored enterprises (GSEs). The model in Afonso, Armenter and Lester (2019) captures the fact that, today, activity in the federal funds market is largely driven by GSEs seeking yield on their overnight balances, rather than interbank trading among depository institutions.

This section introduces the timeline of market events and then describes the model setup.

3.1 Model Timeline

The model capturing interbank market activities builds on Fuhrer, Jüttner, Wrampelmeyer, Zwicker et al. (2020), with two key extensions. First, it incorporates NDI agents, who primarily act as lenders due to their negative policy rate balances. In contrast, banks are generally net borrowers, although some banks may lend depending on their initial balances. Second, the model endogenizes the loan and deposit markets, allowing loan and deposit amounts to be determined within the model. This feature helps replicate the loan market dynamics observed in the empirical analysis.

Central Bank: As shown in Figure 6, the central bank pays a positive interest rate r_M on required reserves and a negative rate r_R on reserves exceeding M^i for bank i and $\tilde{M}$ for NDIs. It sets the reserve requirement K^i for each bank and the liquidity ratio q. The central bank also provides lending facilities at rate r_X, allowing financial institutions to borrow to meet reserve requirements. The rates satisfy $r_R < r_M < r_X$.

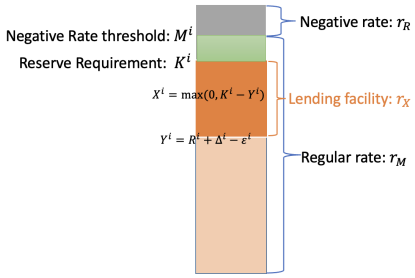


Figure 6: Central Bank Reserve Requirement

Notes: This figure illustrates the reserve balance of a financial institution at the central bank, as used in the baseline model in Section 3.

Banks: A continuum of banks indexed by i (measure one) maximizes expected profits. Each bank i starts with a balance sheet comprising reserves (R^i), deposits (D^i), loans (L^i), and capital (E^i). It faces a reserve requirement (K^i) and a negative interest rate threshold

(M^i). The loan rate r_L and deposit rate r_D are determined by aggregate loan and deposit amounts in the market.

Assets	Liabilities
Reserves (R^i)	Deposits (D^i)
Loans (L^i)	Capital (E^i)

Figure 7: Pre-shock balance sheet

Notes: This figure shows the pre-shock balance sheet of bank i as used in the baseline model in Section 3.

NDI: A continuum of non-depository institutions ("NDI") with measure Γ holds pre-determined central bank balances exceeding the reserve tiering threshold $\tilde{M}$, thus earning the negative policy rate r_R. These institutions operate to maximize profits or minimize losses in the market.

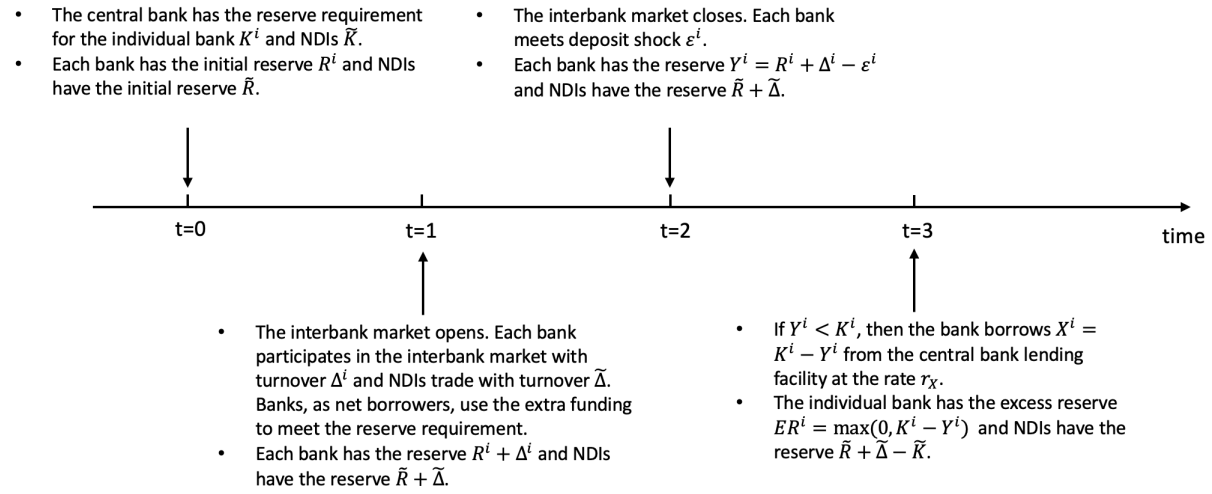


Figure 8: Model Timeline

Notes: This figure shows the timeline of the baseline model in Section 3.

There are four periods in the model timeline, as illustrated in Figure 8.

t=0 (Initial conditions): The central bank sets the reserve requirements for each bank (K^i) and for NDIs ($\tilde{K}$). Each bank starts with reserves R^i at the central bank, and NDIs begin with reserves $\tilde{R}$.

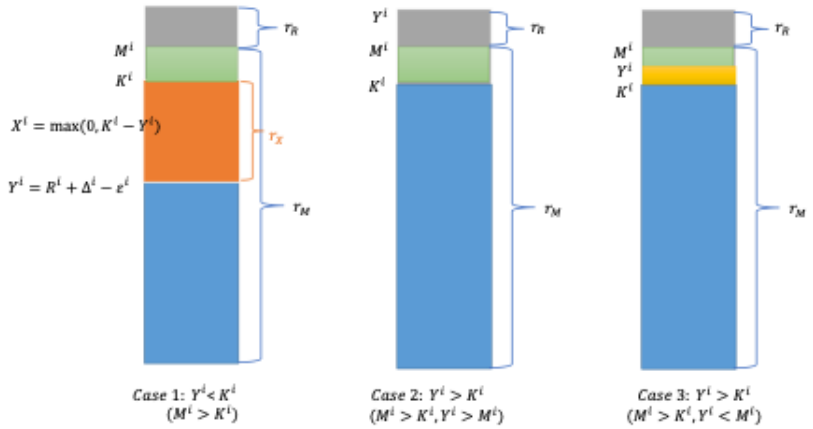


Figure 9: Three bank reserve cases

Notes: This figure illustrates the three reserve balance cases for banks in the baseline model in Section 3.

t=1 (Interbank market opens): Banks participate in the interbank market with net trading volume Δ^i (positive for borrowers, negative for lenders). Banks may trade with each other or with NDIs. NDIs are homogeneous, hold exogenous reserves $\tilde{R}$ above their negative-rate threshold $\tilde{M}$, and act solely as lenders. The interbank trading rate is r_Δ.

t=2 (Deposit shocks): After interbank trading closes, banks receive i.i.d. deposit shocks ε^i with cumulative distribution G. Post-shock reserves are $Y^i = R^i + \Delta^i - \varepsilon^i$.

t=3 (Central bank lending facility): If $Y^i < K^i$, bank i borrows $X^i = \max(0, K^i - Y^i)$ from the central bank at rate r_X. If $Y^i > K^i$, excess reserves $ER^i = \max(0, Y^i - K^i)$ are held at the central bank. The central bank pays rate r_M on reserves below M^i and charges $r_R < 0$ on reserves above M^i.

3.2 Model setup

3.2.1 Banks

In this model, banks face three possible reserve outcomes after shocks, illustrated in Figure 9. Bank i's reserve balance can fall (1) below the required reserve threshold K^i, (2) between K^i and the negative interest rate threshold M^i, or (3) above M^i. These cases correspond to

distinct profit outcomes, which can be summarized by the bank's profit functions:

$$\pi^i = -r_\Delta \Delta^i + r_M K^i - r_X \max(0, K^i - Y^i) + r_M \max(0, Y^i - K^i) + (r_R - r_M)\max(0, Y^i - M^i) \\ + r_L L^i - r_D(D^i - \varepsilon^i) - \chi(L^i) - r_E E^i$$

where r_L is the loan interest rate offered by the bank and is endogenous in this model. L^i is the loan supplied by the bank. The loan volume and the loan rate are the key variables of interest in my analysis. The loan market has a negative elasticity of demand. Thus L^i is negatively associated with the interest rate r_L, which will be derived in detail in the following part. When offering loans, banks have a cost $\chi(L^i)$, which is strictly increasing, strictly convex, and twice continuously differentiable. These assumptions imply that the bank has some monopoly power in the loan market as the loan volume increases. Also, there is a fixed cost of offering loans, i.e., $\lim_{L^i \to 0} = \underline{\chi} > 0$ and $\underline{\chi}$ is a constant. [3] r_D is the deposit rate with the zero lower bound $r_D \geq 0$.

The first term $-r_\Delta \Delta^i$ in the profit function captures the cost or gain associated with interbank trading. $r_M K^i$ is the interest income on the required reserve. $-r_X \max(0, K^i - Y^i)$ is the lending cost in the central bank discount window. $r_M \max(0, Y^i - K^i)$ is the profits of the balances exceeding the required reserve but lower than the threshold of negative interest rate. $(r_R - r_M)\max(0, Y^i - M^i)$ is the term to capture the cost of the negative interest rate balance and the adjustment of the benefits received on this part of the balance. $r_L L^i$ is the return on loans. And $\chi(L^i)$ is the loan cost. $-r_D(D^i - \varepsilon^i)$ is the cost associated with the deposits after shock. $r_E E^i$ is the cost for the capital.

[3] The assumption of the cost of loans follows Ennis (2018).

Assets	Liabilities
Reserves ($R^i + \Delta^i$)	Deposits ($D^i - \varepsilon^i$)
Loans (L^i)	Capital (E^i)
	Interbank borrowing (Δ^i)

(a) Balance sheet at stage 2 after borrowing ($\Delta^i > 0$)

Assets	Liabilities
Reserves ($R^i + \Delta^i$)	Deposits ($D^i - \varepsilon^i$)
Loans (L^i)	Capital (E^i)
Interbank lending ($-\Delta^i$)	

(b) Balance sheet at stage 2 after lending ($\Delta^i < 0$)

Figure 10: Balance Sheet After Interbank Shock

Notes: This figure shows the post-shock balance sheets of bank i in the baseline model in Section 3. Panel (a) corresponds to the case when the bank acts as a net borrower in the interbank market ($\Delta^i > 0$), and panel (b) shows the case when it is a net lender ($\Delta^i < 0$).

Due to the balance sheet accounting features, banks have the same balance sheet constraint before and after interbank trading. So I have

$$R^i + L^i = D^i - \varepsilon_i + E^i$$

Following Zarruk and Madura (1992) and Ennis (2018), banks are subject to a required capital-to-asset ratio q. Capital E^i for bank i must meet the condition $E^i \geq q(R^i + \Delta^i + L^i)$ for borrowers and $E^i \geq q(R^i + L^i)$ for lenders. Thus, banks face the following budget constraint on their post-interbank trading balance sheet. [4]

$$E^i \geq qR^i + qL^i + q\max(0, \Delta^i)$$

Meanwhile, banks face a minimum deposit constraint $D^i \geq \underline{D}^i$. Intuitively, this reflects depositors' cash hoarding costs, which incentivize them to hold deposits at banks even at

[4]This model excludes cash holdings on bank balance sheets to reflect the design of tiered monetary policy, which discourages cash hoarding as a means to bypass low-interest-rate balances. When the central bank observes such behavior, it adjusts the policy rate balance downward. For the same reason, the required reserve K^i is not defined as a fixed ratio of deposits, as in Ennis (2018).

zero interest. This constraint is also consistent with the zero lower bound on deposit rates, which will be further discussed in the depositor section.

Banks borrow from the central bank when $Y^i < K^i$, i.e., $\varepsilon^i > R^i + \Delta^i - K^i$. When reserves fall between the thresholds, $K^i < Y^i < M^i$ (i.e., $\varepsilon^i < R^i + \Delta^i - K^i$), banks earn interest at rate r_M. If $Y^i > M^i$, or equivalently $\varepsilon^i < R^i + \Delta^i - M^i$, banks earn rates r_M and r_R on respective reserve segments. The bank's expected payoff is thus:

$$E[\pi^i] = -r_\Delta \Delta^i + r_M K^i - r_D D^i + r_L L^i - \chi(L^i) - r_E E^i - r_X \int_{R^i+\Delta^i-K^i}^{\infty} (\varepsilon^i - (R^i + \Delta^i - K^i)) dG(\varepsilon^i) +$$
$$r_M \int_{-\infty}^{R^i+\Delta^i-K^i} ((R^i + \Delta^i - K^i) - \varepsilon^i) dG(\varepsilon^i) + (r_R - r_M) \int_{-\infty}^{R^i+\Delta^i-M^i} ((R^i + \Delta^i - M^i) - \varepsilon^i) dG(\varepsilon^i)$$

Therefore, the bank problem can be summarized as follows. Given r_M, r_X, r_R, r_E, K^i, M^i, r_Δ, r_L, r_D, bank i chooses the interbank trading amount Δ^i, loan amount L^i, deposit amount D^i, and capital holding E^i to maximize the expected payoff $E[\pi^i]$ subject to the liquidity constraint, the balance sheet constraint and the deposit constraint.

$$\max_{\Delta^i, L^i, D^i, E^i} E[\pi^i]$$

$$\text{s.t.} R^i + L^i = D^i - \varepsilon_i + E^i$$

$$E^i \geq qR^i + qL^i + q\max(0, \Delta^i)$$

$$D^i \geq \underline{D}^i$$

Substituting the expression of E^i using the first budget constraint back to the profit function and the second budget constraint, the Lagrange multiplier attached to the second constraint and the third constraint is λ_1 and λ_2, with $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. The FOCs for Δ^i, L^i, D^i respectively are as follows:

$$[\Delta^i] \quad -r_\Delta + r_X(1 - G(R^i + \Delta^i - K^i)) + r_M G(R^i + \Delta^i - K^i) + (r_R - r_M) G(R^i + \Delta^i - M^i) - \lambda_1 q 1_{(\Delta^i > 0)} = 0$$

$$[L^i] \quad r_L - \chi^{'}(L^i) - r_E + \lambda_1(1-q) = 0$$

$$[D^i] \quad -r_D + r_E - \lambda_1 + \lambda_2 = 0$$

3.2.2 NDIs

As discussed, non-depository institutions (NDIs) primarily act as lenders. In this model, each NDI starts with a balance $\tilde{R}$ and lends $-\tilde{\Delta} > 0$ at rate r_Δ. NDIs face a reserve requirement $\tilde{K}$ and satisfy $\tilde{R} + \tilde{\Delta} > \tilde{K}$. Assume NDIs always hold balances exceeding their negative tier $\tilde{M}$ if they do not trade, thus consistently earning the negative rate r_R. Each NDI has capital $\tilde{E}$, with reserves fixed as an exogenous share $\tilde{q}$ of capital. NDIs are assumed homogeneous. Given $\tilde{R}, \tilde{M}, r_R, r_M$, the NDI's payoff maximization problem is:

$$\max_{\tilde{\Delta}, \tilde{E}} \tilde{\pi} = r_R(\tilde{R} + \tilde{\Delta} - \tilde{M}) + r_M\tilde{M} - r_\Delta\tilde{\Delta} - r_E\tilde{E}$$

$$\text{s.t. } \tilde{R} = \tilde{q}\tilde{E}$$

$$\tilde{R} + \tilde{\Delta} \geq \tilde{M}$$

Substituting the first constraint into the objective function and assigning $\mu_1 \geq 0$ as the Lagrange multiplier for the second constraint, the first-order condition (FOC) with respect to $\tilde{\Delta}$ is:

$$[\tilde{\Delta}] \quad r_R - r_\Delta + \mu_1 = 0$$

In equilibrium, when the budget constraint binds ($\tilde{R} + \tilde{\Delta} = \tilde{M}$), i.e., when the NDIs lend out all their balances exceeding the budget constraint, the interest rates satisfy $r_R < r_\Delta$. Intuitively, it is beneficial for the NDIs to lend out current account balances in the interbank market, and demand in the interbank market is sufficient. On the contrary, when the budget constraint does not bind ($\tilde{R} + \tilde{\Delta} > \tilde{M}$), i.e., there is no sufficient demand in the interbank market for NDIs to lend out as much as they want. In this case, since the demand is less than the supply, the interbank trading interest rate r_Δ will be pushed down

until the central bank reserves top tier interest rate r_R. For NDIs, there is no difference between lending out the balance exceeding the low-interest-rate tier or putting the balance at the central bank. To accommodate the reality that the interbank rate is usually located within a corridor system of the reserve rate and the discount window rate, in the following parts, I assume the first case is true and $\mu_1 > 0$. Therefore, in the model, $r_R < r_\Delta$ holds. [5]

3.2.3 Entrepreneurs

Following Ennis (2018), there is a continuum of entrepreneurs in the economy. Each entrepreneur j borrows L^j from a bank at interest rate r_L at time t to invest in a project yielding return r_P in the next period. At $t+1$, the loan must be repaid. Projects are risky and vary in returns r_P, which follow a cumulative distribution function $H(r_P)$, with support $r_P \in [0, \bar{r}_P]$. An entrepreneur takes the loan and invests if $r_P > r_L$. The total loan demand is:

$$L^d(r_L) = \int_{r_L}^{\bar{r_P}} dH(r_P) \equiv 1 - H(r_L).$$

Therefore, total loan demand decreases with the loan rate, since $H(r_L)$ is a cumulative distribution function, consistent with the bank-side assumptions.

3.2.4 Depositors

Suppose there is a continuum of depositors in the economy. Each depositor places deposits D in banks and earns interest rate r_D. The total deposit is:

$$D^d(r_D) = \underline{D} + \kappa r_D$$

where $\kappa > 0$ and $\underline{D} > 0$. The deposit supply is assumed to be an increasing function of the deposit rate. Even when the deposit rate is zero ($r_D = 0$), a minimum amount of deposits

[5] NDIs are homogenous in this model, which can be modified to be heterogeneous if using the search model in the interbank market. The heterogeneous setting of the NDIs will help to figure out the order of the bank-to-bank trading or bank-to-NDI trading. A similar result is discussed in Afonso, Armenter and Lester (2019).

$\underline{D}$ remains in banks. This reflects the real-world observation that individuals prefer to deposit money in banks rather than hold cash, due to the costs associated with cash storage. Additionally, banks are typically reluctant to impose negative deposit rates and instead maintain $r_D \geq 0$. As a result, the model ensures that total deposits always satisfy $D \geq \underline{D}$.

3.2.5 Capital investors

Suppose there is a continuum of capital investors who supply capital inelastically to both banks and NDIs. These investors receive a return r_E on the capital they provide. To ensure that the capital constraint binds in equilibrium, the model assumes $r_E > r_D$.

3.2.6 Central bank

As discussed previously, the central bank determines the reserve requirement for each bank K^i and for NDIs $\tilde{K}$, as well as the tiered monetary policy thresholds M^i for banks and $\tilde{M}$ for NDIs. It pays a negative interest rate r_R on reserves exceeding M^i (or $\tilde{M}$), and a positive rate r_M on reserves below these thresholds. The central bank also offers lending facilities at rate r_X, which financial institutions can access to meet their reserve requirements. These rates are ordered such that $r_R < r_M < r_X$. Additionally, the central bank imposes a capital-to-asset ratio requirement q. Let R^i denote the reserve of bank i, and $\tilde{R}$ the reserve held by NDIs. The central bank imposes the following aggregate constraints on the reserve tiering threshold M, the aggregate reserve R, and the reserve requirement K.

$$\begin{aligned}\int_i M^i di + \Gamma \bar{M} &= M \\ \int_i R^i di + \Gamma \bar{R} &= R \\ \int_i K^i di + \Gamma \bar{K} &= K\end{aligned}$$

3.3 Equilirium

In equilibrium, the following market clearing conditions must hold.

$$\text{Interbank market:} \quad \int_i \Delta^i di + \Gamma\tilde{\Delta} = 0$$

$$\text{Deposit market:} \quad \int_i D^i di = D = D^d(r_D)$$

$$\text{Loan market:} \quad \int_i L^i di = L = L^d(r_L)$$

$$\text{Capital market:} \quad \int_i E^i di + \Gamma\tilde{E} = E$$

Definition 1. ***(Equilibrium)*** *The equilibrium is defined as a set of allocations* $\left\{\Delta^i, D^i, L^i, E^i, \tilde{\Delta}, \tilde{E}\right\}_{\forall i}$ *and prices* $\{r_\Delta, r_L, r_D\}$ *such that, given the policy rates* $\{r_M, r_R, r_X, r_E\}$ *and central bank allocations* $\left\{K^i, M^i, R^i\right\}_{\forall i}$ *and* $\left\{\tilde{K}, \tilde{M}, \tilde{R}\right\}$*, the optimization problems of banks, NDIs, depositors, and entrepreneurs are solved, and the interbank market, deposit market, loan market, and capital market all clear.*

The summary of the equilibrium solutions is shown in Appendix C.

4 Monetary Policy Pass-Through and Loan Rate Distribution

In an environment of abundant but unevenly distributed reserves, tiered reserve remuneration affects monetary transmission not by shifting a single marginal funding cost, but by interacting with a distribution of reserve exposures across institutions. To capture this structural feature, this section reorganizes the transmission logic into three unified mechanisms. First, a *reserve pricing channel* maps tiered remuneration into heterogeneous marginal funding incentives across institutions. Second, these heterogeneous incentives generate an *intermediation reallocation channel*, in which NDIs and banks endogenously reallocate liquidity through the interbank market, altering the aggregate supply of funding available for loan creation. Finally, a *distributional risk channel* emerges as banks with different risk exposures adjust their loan pricing in systematically different ways, reshaping

both the level and the distribution of loan rates. All propositions derived below remain valid; they are now interpreted within these three higher-level mechanisms.

4.1 Intermediation Reallocation Channel: Endogenous Liquidity Flows

Reserve tiering enhances monetary policy pass-through through liquidity reallocation. Faced with negative interest rates on excess reserves held at the central bank, non-depository institutions (NDIs) are incentivized to lend out their balances in the interbank market to avoid earning the low return. This behavior injects additional liquidity into the banking system, as banks obtain funding from NDIs through interbank transactions.

In equilibrium, the liquidity channel operates through the following propositions.

Proposition 4.1. ***(Loan Rate and NDIs' Tiering Threshold)*** *In equilibrium, the loan rate and the NDI tiering threshold satisfy the following relationship:*

$$\frac{\partial r_L^*}{\partial \tilde{M}} = \frac{qg\Gamma(r_M - r_R)}{(r_X - r_R)g\left((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\right) + q^2\alpha\left(\frac{1}{1-q} + 2h\right)} > 0$$

That is, when the central bank lowers the negative interest rate threshold $\tilde{M}$ for NDIs, effectively increasing the portion of reserves subject to the negative rate r_R, the equilibrium loan rate r_L^* declines.

Proof. See Appendix C.4. □

Proposition 4.2. ***The borrowing banks' trading volume and the NDI tiering threshold*** *In equilibrium, $\frac{\partial \beta}{\partial \tilde{M}} < 0$. The borrowing banks' total trading volume in the interbank market is negatively associated with the NDI reserve tiering threshold. Therefore, when the central bank cuts the negative interest rate balance threshold for the NDIs, borrowing banks have incentives to borrow more in the interbank market.*

Proof. See Appendix C.4. □

Intuitively, Propositions 4.1 and 4.2 suggest that a reduction in the reserve tiering threshold for NDIs incentivizes them to lend more aggressively in the interbank market. In turn, banks respond by increasing their borrowing to take advantage of the cheaper funding. This expanded interbank activity enhances bank liquidity, which can then be channeled into additional lending. Consequently, lowering the reserve tiering threshold for NDIs serves as an effective tool for the central bank to stimulate monetary policy pass-through via the liquidity channel.

The magnitude of the loan rate's response to changes in the reserve tiering threshold $\tilde{M}$ depends critically on the mass of NDIs, denoted by Γ. As Γ increases, banks gain access to a larger pool of interbank funding, enabling them to expand loan supply more readily. Additionally, the sensitivity of r_L to changes in $\tilde{M}$ is influenced by the riskiness of entrepreneurial projects, captured by the parameter h. When h is low, indicating higher project risk, each unit reduction in $\tilde{M}$ leads to a larger decline in the loan rate. This suggests that reserve tiering may introduce new financial stability concerns by compressing risk premiums, particularly for high-risk lending.

Therefore, compared to a conventional interest rate cut, tiered monetary policy enhances interest rate pass-through through the liquidity channel. First, by adjusting the tiering threshold, the central bank gains an additional policy lever to influence loan rates. Second, the interbank trading between NDIs and banks provides banks with a new and flexible source of liquidity. As a result, banks obtain additional funding from NDIs, enabling them to expand loan supply and reduce lending rates.

Moreover, by reshaping the bank's profit function, reserve tiering alters the trade-offs banks face and shifts their optimal response, from raising loan rates to lowering them.

Combining the first-order conditions for L^i and D^i with the market clearing conditions, the following proposition characterizes the effect of reserve tiering on bank interest margins:

Proposition 4.3. ***(Connection Between the Loan Rate and the Deposit Rate)*** *When the*

deposit rate is not binding, for a given bank i, the relationship between the loan rate r_L and the deposit rate r_D is given by:

$$r_L - \chi'(L) - r_E + (r_E - r_D)(1-q) = 0.$$

Consequently,

$$\frac{\partial r_D}{\partial r_L} = \frac{1 + \chi''(L)H'(r_L)}{1-q} \approx \frac{1+2h}{1-q} > 1,$$

indicating that the deposit rate and the loan rate move in the same direction.

Proof. See Appendix C.4. □

Proposition 4.3 is consistent with the empirical observation that banks tend to maintain their interest margin by lowering both loan and deposit rates in response to expansionary monetary policy. [6] Moreover, the model implies that banks reduce the deposit rate more than the loan rate, effectively widening their interest margin. This behavior arises due to capital requirements, loan issuance costs, and loan risk.

Intuitively, when capital requirements rise, banks rely more heavily on equity financing. In this context, lowering the deposit rate becomes less costly, as banks are less dependent on deposit funding. Additionally, assuming $\chi'''(L) > 0$, banks with larger loan volumes reduce the deposit rate more aggressively for each unit decrease in the loan rate. This aligns with empirical patterns in which larger banks, which typically issue more loans, tend to offer lower deposit rates. Furthermore, when entrepreneurial projects are less risky, reflected in a higher $H'(r_L)$, banks find it more attractive to reduce the loan rate and finance such projects. In turn, the model predicts a stronger reduction in the deposit rate, since the safer returns from these projects make it easier for banks to meet their obligations to depositors.

[6] In this model, banks choose the quantity of loans and deposits, while the corresponding interest rates are determined by market-clearing conditions. However, under a one-to-one matching assumption between banks and borrowers or depositors, choosing quantities or prices becomes equivalent. Many previous studies model banks as choosing interest rates directly, including Gertler and Karadi (2011), Ennis (2018), and Ravn and Sterk (2021).

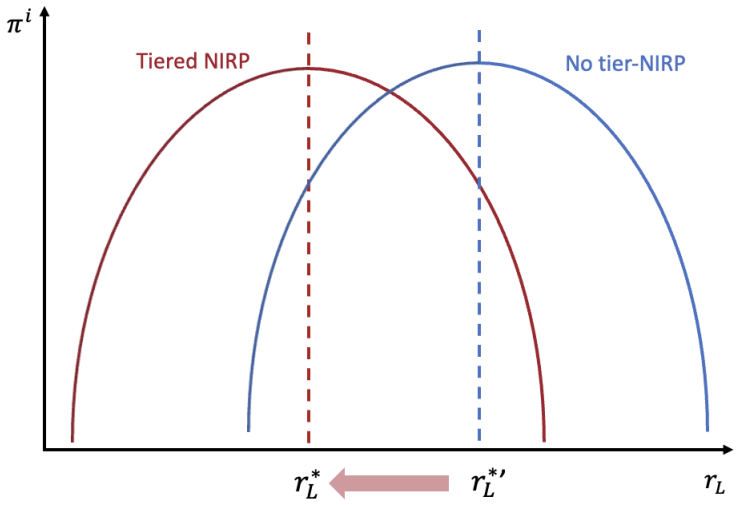


Figure 11: Bank loan rate cut mechanism

Notes: This figure illustrates how the bank loan rate changes following the implementation of the reserve tiering policy in the baseline model described in Section 3.

One key feature of a low or negative interest rate environment is the compression of bank interest margins, primarily due to the zero lower bound on deposit rates. A well-documented finding in the literature is that banks are generally unwilling to lower deposit rates below zero, even if doing so could help preserve their interest margins.

Proposition 4.4. ***(The Optimal Loan Rate Changes)*** *When the deposit rate is close to zero, the expected profit function of a bank can be approximated by a quadratic function of* r_L*:* $E(\pi^i) \approx -r_\Delta \Delta^i - r_D D^i + r_L L^i - \chi(L^i) - r_E L^i + C_1$*, where* C_1 *is a constant that has minor impact on the loan rate chosen. And then the optimal loam rate* $r_L^* \approx -[\frac{\alpha q}{(2+2h)h}(\frac{1+2h}{1-q})]\int_i \Delta^i di + \frac{2+r_E+\frac{1}{h}}{2+2h}$.

Proof. See Appendix D. □

Compared to a non-tiered monetary policy, the optimal loan interest rate r_L is reduced due to the first term in the expression, reflecting the fact that banks are net borrowers while NDIs act as lenders. As a result, the downward adjustment in the loan rate is primarily driven by borrowing banks. Moreover, the larger the borrowing volume, the stronger the downward pressure these banks exert on the loan rate. Intuitively, as illustrated in Figure

11, the optimal loan rate in the system declines from $r_L^{'}$ to a lower level r_L^* under the tiered monetary policy, as the profit function for borrowing banks shifts leftward.

4.2 Reserve Pricing Channel: Asymmetric Marginal Funding Costs

Tiered remuneration directly affects the marginal funding cost faced by different institutions. Reserve tiering affects the loan rate through its interaction with the interbank rate. This relationship is formalized in the following proposition.

Proposition 4.5. ***(Connection Between the Interbank Rate and the Loan Rate)*** *In equilibrium, the relationship between the loan rate and the interbank rate is given by:*

$$-r_\Delta + r_X(1-G(\delta_1)) + r_M G(\delta_1) + (r_R - r_M)G(\delta_2) = \alpha q r_E - \alpha \frac{q}{1-q}(r_L - \chi^{'}(L) - q r_E) \quad (1)$$

where $\delta_1 = R - \Gamma\tilde{M} - K + \Gamma\tilde{K}$ *and* $\delta_2 = R - M$. *Since* $\frac{\partial r_L}{\partial r_\Delta} = \frac{1-q}{\alpha q(1+\chi^{''}(L)H^{'}(r_L))} \approx \frac{1}{\alpha q(\frac{1+2h}{1-q})} > 0$, *there is a positive relationship between the loan rate and the interbank trading rate.*

Proof. See Appendix C.4. □

Proposition 4.5 demonstrates that an interest rate cut by the central bank transmits to the loan market through its impact on the interbank market. Moreover, the proposition implies the following comparative statics: $\frac{\partial r_\Delta}{\partial r_X} = 1 - G(\delta_1) > 0$, $\frac{\partial r_\Delta}{\partial r_R} = G(\delta_2)$, and $\frac{\partial r_\Delta}{\partial r_M} = G(\delta_1) - G(\delta_2) > 0$. These expressions indicate that the central bank can influence the interbank rate, and thereby the loan rate, by adjusting any of the three policy rates: the discount window rate r_X, the negative interest rate on excess reserves r_R, or the standard reserve rate r_M. Intuitively, when banks gain access to cheaper funding in the interbank market, they are more willing to reduce the loan rate, thereby facilitating monetary policy pass-through.

With tiered monetary policy, another important channel shaping the monetary pass-through is the gap between the reserve requirement K and the negative interest rate

threshold M. When the gap between K and M increases, monetary policy pass-through becomes more effective through cuts in the regular reserve rate r_M, as the magnitude of $\frac{\partial r_\Delta}{\partial r_M}$ increases. In contrast, cutting the discount window rate r_X or the negative reserve rate r_R may be less effective, since the magnitudes of $\frac{\partial r_\Delta}{\partial r_X}$ and $\frac{\partial r_\Delta}{\partial r_R}$ are lower due to a smaller share of reserves held below K or above M. On the other hand, when the gap between K and M narrows, changes in r_R and r_X tend to be more effective in stimulating the interbank rate and enhancing monetary pass-through. In conclusion, to optimize monetary policy pass-through through the interbank rate r_Δ, the central bank may choose a wider gap between K and M and cut the regular reserve rate r_M, or a narrower gap and cut the negative reserve rate r_R or the discount window rate r_X instead. Intuitively, the regular reserve rate r_M becomes more potent when the balance of reserves subject to r_M is larger, i.e., when M is low. Similarly, the discount window rate r_X has a stronger effect when more banks are borrowing from the central bank, i.e., when K is high. Therefore, increasing the gap between M and K raises the share of reserves earning r_M, enhancing the effectiveness of cutting r_M in stimulating pass-through.

Tiered monetary policy also enhances monetary policy pass-through through the interest rate. By adjusting the reserve requirements and the thresholds for tiered remuneration, the central bank can lower interest rates on different tiers of reserves to further stimulate pass-through via interbank trading. As a result, banks gain access to cheaper funding in the interbank market, which enables them to reduce loan rates more effectively.

4.3 Distributional Risk Channel: Uneven Compression of Risk Premia

An extended model with heterogeneous banks can capture the changes in the loan interest rate distribution, as illustrated in Figure 5, with appropriate modifications. Following the introduction of reserve tiering, the volume of low-interest-rate loans increases, while the volume of medium-interest-rate loans declines.

I introduce the following modifications to the baseline model. First, banks are het-

erogeneous in their risk preferences and are each matched with an entrepreneur whose project reflects that risk profile. Specifically, entrepreneur i receives a loan from bank i, and the project return follows a cumulative distribution function (CDF) $H^i(r_P)$, which is uniform on the support $r_P \in [0, \bar{r}P^i]$. Hence, the CDF takes the form $H^i(rP) = h^i r_P$ within the support, where $h^i = \frac{1}{\bar{r}P^i}$ captures the degree of risk aversion. Accordingly, each bank is characterized by a different $\bar{r}P^i$ and h^i. Second, in equilibrium, each bank sets a distinct loan rate r_L^i. Consequently, each bank faces an additional constraint linking its loan rate to loan demand: $1 - H^i(r_L^i) = L^i$.

Proposition 4.6. ***(The Deposit Rate and the Loan Rate:)*** *When the deposit rate is close to zero, the optimal loan rate for bank i is $r_L^i \approx \frac{(r_E+2+r_\Delta\frac{1-q}{q})+\frac{1}{h^i}}{2+2h^i}$. And $\frac{\partial r_L^i}{\partial h^i} = \frac{4(r_E+1+r_\Delta\frac{1-q}{q})}{(4+4h^i)^2} > 0$.*

Proof. See Appendix E.1. □

When reserve tiering is introduced, the loan rate set by each individual bank is primarily influenced by two factors: the bank's risk aversion parameter h^i and the interbank trading rate r_Δ. Banks with higher risk exposure (i.e., lower h^i) respond to reserve tiering by cutting loan rates more aggressively than banks with lower risk exposure. This differential response leads to a compression of risk premiums, particularly for high-risk loans, thereby potentially increasing systemic risk. In contrast, banks matched with lower-risk entrepreneurs (i.e., higher h^i) adjust their loan rates more cautiously, resulting in a more moderate reduction in loan pricing for safer borrowers.

Intuitively, a riskier bank tends to operate with higher leverage and a lower reserve balance. Consequently, it has more scope to borrow in the interbank market, benefiting from a larger inflow of cheap funding and enabling deeper cuts in its loan rate.

However, the central bank can mitigate the resulting increase in systemic risk, particularly in the event of a bank run. Suppose the risk of a bank run, or the size of the deposit shock, is proportional to the bank's underlying risk exposure; for instance, let $h^i = \frac{1}{\varepsilon^i}$. In this setup, banks with lower h^i (i.e., more risk-exposed) are more vulnerable to runs and

are more likely to rely on the central bank's lending facility. By raising the lending facility rate, the central bank can increase the marginal cost of funding for these risky banks, thereby discouraging excessive borrowing and helping contain systemic risk.

Tiered monetary policy affects not only the average level of loan rates but also the distribution of loan rates across banks. In particular, the policy tends to lower the interest rates offered by banks with higher risk exposure, while having a more limited impact on banks that lend to safer projects. This leads to a reallocation of credit conditions: the share of low-interest loans rises, while the share of medium-interest loans declines. As a result, the distribution of loan rates becomes more polarized, reflecting the heterogeneous responses of banks to the interbank funding environment under reserve tiering. This reallocation channel is especially important when assessing the broader financial stability implications of tiered monetary policy.

5 Empirical Results: Evidence from Bank of Japan (BOJ)

The empirical analysis is structured to mirror the three mechanisms developed in the theoretical section. The difference-in-differences design exploits heterogeneity in banks' exposure to the negative-interest-rate tier to identify causal effects of reserve tiering. First, changes in interbank positions test the *intermediation reallocation channel*, where asymmetric marginal incentives lead NDIs and banks to reallocate liquidity in the interbank market. Second, changes in bank-level loan rates test the *reserve pricing channel*, in which tiered remuneration differentially shifts marginal funding costs and thereby loan pricing. Finally, changes in bank-firm lending and portfolio risk test the *distributional risk channel*, which predicts that banks positioned differently along the tiering schedule adjust credit supply unevenly across firms of different risk profiles. Each empirical specification corresponds directly to one of these three transmission mechanisms.

5.1 Data and summary statistics

I use bank-level, loan-level, and firm-level data to formally test the monetary transmission mechanism described above.

Bank-level data: I use annual balance sheet and income statement data for Japanese banks from the Japanese Bankers Association (JBA) and S&P Capital IQ Pro. As these databases do not explicitly report reserve tiering information, I manually compute each bank's reserve tiering balance ratio, following the methodology outlined by the BOJ. A detailed explanation of this construction is provided in the next section. Data on interbank activity are derived by calculating the annual growth of the net position between call loans and call money, which proxy for short-term funds lent or borrowed in the interbank market. To control for bank-specific characteristics, I include variables commonly used in the bank lending channel literature, such as size (log of total assets), liquidity, and the capital-to-asset ratio (Kashyap and Stein, 1995, 2000; Agarwal, 2021). The sample is restricted to the 2013–2019 period to avoid distortions from COVID-era loan stimulus programs. Panel A of Table 1 details the data sources, while Table 2 presents summary statistics for the bank-level variables.

Loan-level data: I use granular loan data at both the bank-industry and bank-firm levels. Following the approach of Khwaja and Mian (2008), I implement a firm-year fixed effects strategy to examine how loan supply to the same firm varies across banks with different exposures to the negative interest rate policy. This allows me to compare changes in loan offerings between banks with higher versus lower negative interest rate ratios, holding firm demand constant.To analyze changes along the intensive margin, I also include bank-firm or bank-industry fixed effects, capturing how loan behavior evolves within a given relationship depending on the bank's exposure to NIRP. Panel B of Table 1 outlines the data sources, and Table 2 presents summary statistics for the loan-level data.

Firm-level data: Firm-level data are obtained from Worldscope and are primarily used to construct measures of firm risk, including the debt-to-capital ratio, debt-to-equity

ratio, interest coverage ratio, and the degree of combined leverage. These indicators help assess the financial health and risk exposure of firms in the sample. The data sources and summary statistics are presented in Panel C of Table 1 and Table 2.

Table 1: Data Sources

Variable	Definition	Source
A. Bank-level variables		
Interbank loan growth	Yearly growth rate of the difference between call loan and call money	Japanese Banker Association
Reserve	Cash balance with central banks	S &P Capital IQ Pro
	Cash and due from banks	Japanese Banker Association
Deposit	Deposits	Japanese Banker Association
Size	Log of total assets	Japanese Banker Association
Capital-to-assets	Ratio of total equity to total assets	Japanese Banker Association
Liquidity	Ratio of liquid assets to total assets	S &P Capital IQ Pro
	Ratio of sum of cash and due from banks and securities to total assets	Japanese Banker Association
Deposit Funding	Ratio of deposits to total assets	Japanese Banker Association
Leverage	Ratio of total loans to total deposits	Japanese Banker Association
Loan rate	Ratio of interest on loans and discount to total loans	Japanese Banker Association
B. Loan-level variables		
Industry-bank level loan		
Loan	Log loan	S &P Capital IQ Pro
Firm-bank level loan		
Loan	Log loan	Nikkei NEEDs database
C. Firm-level variables		
Debt to capital	Ratio of debt to captial	Worldscope
Debt to equity	Ratio of debt to equity	Worldscope
Interest coverage	Ratio of EBIT to total interest expense	Worldscope
Degree of combined leverage	Ratio of EPS growth rate to sales growth rate	Worldscope

Notes: This table provides the sources for all data used.

Table 2: Summary statistics

Variable	mean	sd	p25	p50	p75	N
A. Bank-level variables						
Interbank loan growth	355.76	10321.19	-81.16	-21.71	42.17	905
Negative rate ratio	9.83	87.65	-2.75	24.42	55.65	822
Size	8.1	1.25	7.25	8.01	8.76	1415
Capital-to-assets	5.31	1.91	4.24	5.01	5.95	1415
Liquidity	33.6	10.85	26.87	32.45	39.09	1413
Deposit Funding	82.83	10.94	79.67	85.88	90	1415
Leverage	75.09	14.8	69.06	75.21	81.1	1415
Loan rate	1.46	1.16	1.03	1.28	1.61	1415
B. Loan-level variables						
Industry-bank level loan						
Loan	4.21	2.09	3.21	4.48	5.51	9,330
Firm-bank level loan						
Loan	6.47	1.74	5.3	6.42	7.6	178,619
C. Firm-level variables						
Debt to capital	0.41	0.23	0.24	0.41	0.58	2356
Debt to equity	1.19	1.98	0.31	0.7	1.37	2356
Interest coverage	107.83	1786.70	2.44	13.39	47	1653
Degree of combined leverage	-48.4	1100.09	-3.9	2.12	9.16	1403

Notes: This table reports summary statistics for all variables.

I proceed with a series of empirical tests to formally verify the mechanism outlined in the previous section. Specifically, I implement a difference-in-differences (DID) estimation strategy, leveraging variation in banks' exposure to negative interest rate balances over time to address endogeneity concerns. The use of micro-level bank data helps mitigate reverse causality, as the BOJ's policy decisions are unlikely to be influenced by the conditions of individual banks.

Since neither the BOJ nor individual banks publicly disclose bank-level exposure to negative interest rate balances, I construct this measure following the BOJ's published methodology, as illustrated in Figure 12. Specifically, I calculate the total negative interest rate balance (neg_{it}) for each bank b at time t, and normalize it by the bank's total reserve

balance ($R_{b,t}$). This yields the measure of negative interest rate exposure, defined as:

$$negexp_{b,t} \equiv \frac{neg_{b,t}}{R_{b,t}}.$$

This measure is positive when a bank holds reserves subject to the negative policy rate, and negative when the bank has unused capacity under the negative interest rate threshold. As shown in Figure 1, city banks, trust banks, and other banks typically exhibit higher negative interest rate balance ratios, while regional banks tend to have lower exposure or even negative values, indicating unused policy-rate balances. Moreover, as illustrated in Figure 2, these ratios vary over time. On average, as reported in Panel A of Table 2, banks held 9.83% of their total reserves at the BOJ in the negative interest rate tier.

To examine whether observable bank characteristics account for the heterogeneity in negative interest rate exposure, I estimate the following specification:

$$negexp_{b,t} = \alpha + \beta x_{b,t} + \epsilon_{b,t} \tag{2}$$

The dependent variable is the negative interest rate exposure ratio, while the independent variables include bank size, capital-to-asset ratio, liquidity, leverage, and deposit funding ratio. Table 3 shows results from univariate regressions for each variable, with the final column presenting a multivariate specification including all covariates. The findings indicate that larger and more liquid banks tend to have greater negative interest rate exposure. A 1% increase in bank size is linked to a 0.19 percentage point rise in the share of reserves subject to negative rates, while a 1% increase in liquidity corresponds to a 0.02 percentage point increase. Although statistically insignificant, banks with higher capital-to-asset ratios, lower leverage, and less reliance on deposit funding also appear more exposed. In the full specification, however, only bank size remains statistically significant.

Table 3: Negative Interest Rate Ratio

	$negexp_{b,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$Size_{b,t}$	19.27**					19.48**
	(5.76)					(7.22)
$Capital_{b,t}$		1.33				3.77
		(4.36)				(6.60)
$Liquidity_{b,t}$			1.98**			-1.51
			(0.68)			(3.12)
$Leverage_{b,t}$				-2.33		-3.01
				(1.21)		(3.08)
$Deposit_{b,t}$					-0.34	-0.27
					(0.86)	(1.76)
Constant	-162.72**	(13.84)	-71.86**	170.58	21.73	117.00
	(50.70)	(28.21)	(24.90)	(88.18)	(75.38)	(443.25)
R^2	0.06	0.00	0.03	0.06	0.00	0.12
N	418	418	418	418	418	418

Notes: This table reports estimates from equation (2) using bank-level data for Japanese banks from 2016 to 2019 (fiscal year ending in March). The dependent variable $negexp_{b,t}$ is bank b's negative interest rate balance as a share of total reserves at the central bank. The key independent variables from columns (1)–(5) are bank size, capital-to-asset ratio, liquidity, leverage, and deposit funding, respectively. Column (6) includes all variables. Standard errors are clustered at the bank level and reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

5.2 Intermediation Reallocation Channel: Effects on Interbank Positions

The theory predicts that tiered reserve remuneration creates heterogeneous marginal incentives that induce NDIs and banks to reallocate liquidity through the interbank market. Institutions facing large negative-rate exposures should supply more funds, while institutions with unused negative-rate capacity should borrow more aggressively. To test this reallocation mechanism, I examine whether banks with higher negative-rate exposure increase their net interbank lending relative to banks with lower exposure after the introduction of reserve tiering.

I estimate the following model using annual bank-level data from 2013 to 2019, a period chosen to avoid confounding effects from monetary policy responses during the

COVID-19 era (beginning in 2020):

$$intnet_{b,t} = \alpha + \beta post_t + \delta(post_t \times negexp_{b,t}) + \sum_{x \in controls} \mu_x x_{b,t-1} + \sum_{x \in controls} \eta_x post_t \times x_{b,t-1} + \epsilon_{b,t} \tag{3}$$

where $intnet_{b,t}$ is the annual growth rate of net interbank lending of bank b at time t. And the net interbank position is calculated as the difference between the interbank loan and interbank money. So a higher $intnet_{b,t}$ means the bank has a growing net lending in the interbank market. $post_t$ is a dummy variable that takes the value 1 for years after 2016 and 0 for years before 2015 since the data is as of March in each year. $post_t \times negexp_{b,t}$ is an interaction term between the post-shock period and the negative interest rate ratio. The set of control variables includes the lagged value of log total assets, the ratio of equity-to-asset, and bank liquidity, in line with the bank-lending channel of monetary policy. Also, the interaction terms between post dummy and bank characteristics ensure that variations in loan growth in the post reserve tiering period are absorbed by those terms. The main variable of interest is the coefficient on the interaction term δ, which estimates the differential impact of reserve tiering policy on banks with different levels of negative interest rate exposures. δ is expected to be positive. Intuitively, banks with a higher negative interest rate ratio in total reserves have incentives to lend out those balances to avoid the negative interest rate by the central bank. At the same time, banks with unused negative interest rate balances would like to take advantage of the cheaper funding source and borrow more in the interbank market.

Table 4 presents the results. Across all columns, the coefficient on the interaction between the post-treatment dummy and negative interest rate exposure is consistently positive. In column (5), a 10% increase in the negative rate balance ratio is associated with a 3.76% increase in annual net interbank lending. This suggests that banks with greater exposure to negative rates are incentivized to lend more in the interbank market. Simultaneously, banks with unused negative-rate capacity are more likely to borrow to

access cheaper funding.

Table 4: Impact of Reserve Tiering on Interbank Activities

	$intnet_{b,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$post_t$	-275.85	-200.54	-229.42			
	(248.38)	(255.98)	(268.52)			
$post_t \times negexp_{b,t}$	3.98***	3.38**	3.93***	3.23**	3.76***	3.50***
	(0.85)	(1.02)	(0.82)	(0.97)	(0.76)	(0.71)
$Size_{b,t-1}$		-4.74	-1000.23	-2.38	-368.19	-951.59
		(34.29)	(1342.04)	(35.33)	(839.98)	(1569.41)
$Capital_{b,t-1}$		-0.71	104.38	-2.2	115.93	49.9
		(28.12)	(124.03)	(28.68)	(119.01)	(96.37)
$Liquidity_{b,t-1}$		14.3	21.42	14.87	23.77	25.45
		(13.17)	(52.03)	(13.33)	(50.11)	(50.74)
$post_t \times Size_{b,t}$						82.42
						(124.17)
$post_t \times Capital_{b,t}$						80.58
						(90.69)
$post_t \times Liquidity_{b,t}$						19.61
						(31.31)
Constant	-28.79	-505.56	6950.83	-635.27	1450.88	5694.22
	(123.58)	(618.94)	(9333.38)	(726.37)	(5587.50)	(10334.21)
R^2	0.180	0.010	0.180	0.030	0.200	0.200
N	583	589	583	589	583	583
Bank FE	Y	N	Y	N	Y	Y
Time FE	N	N	N	Y	Y	Y
Bank Controls	N	Y	Y	Y	Y	Y
Bank Controls × post	N	N	N	N	N	Y

Notes: This table reports estimates from equation (3) using bank-level data for Japanese banks from 2012 to 2019 (fiscal year ending in March). The dependent variable is the annual growth rate of net interbank lending ($intnet_{b,t}$). $post_t$ is a dummy equal to 1 for years on or after 2016 (following the introduction of reserve tiering), and 0 otherwise. $negexp_{b,t}$ is bank b's negative interest rate balance as a share of its total reserves at the central bank. Bank-level controls include lagged log total assets, equity-to-asset ratio, and liquidity. Columns (1)–(3) sequentially add bank fixed effects, bank controls, and both. In addition to control bank controls, columns (4)–(5) control for year fixed effects only, and both year and bank fixed effects, respectively. Column (6) includes bank and year fixed effects, bank controls, and their interactions with $post_t$. Standard errors are clustered at the bank level and reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

5.3 Reserve Pricing Channel: Effects on Loan Supply

The theoretical reserve pricing channel states that tiered remuneration shifts marginal funding costs asymmetrically across banks, thereby generating differential adjustments in loan pricing and credit supply. Banks with unused negative-rate capacity, effectively facing

lower marginal funding costs, should expand lending more aggressively and reduce loan rates more sharply. I test this prediction by estimating how lending at the bank–industry and bank–firm levels responds to reserve tiering as a function of each bank's negative-rate exposure. I estimate the following specifications:

$$loan_{bi,t} = \alpha + \beta post_t + \delta(post_t \times negexp_{b,t}) + \sum_{x \in controls} \mu_x x_{b,t-1} + \sum_{x \in controls} \eta_x post_t \times x_{b,t-1} + \epsilon_{bi,t} \quad (4)$$

where the dependent variable is the log of loan amounts at either the bank-industry or bank-firm level.

Table 5: Impact of Reserve Tiering on Loan Offering by Firm-Industry

	$Loan_{bi,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$post_t$	0.1367***	0.1367***				
	(10.4435)	(10.4435)				
$post_t \times negexp_{b,t}$	0.0007**	0.0007**	0.0033*	0.0006*	0.0005*	0.0004*
	(2.6478)	(2.6478)	(2.3858)	(2.3311)	(2.1345)	(2.0423)
R^2	0.9862	0.9862	0.0045	0.9865	0.9886	0.9886
N	6492	6492	6493	6492	5662	5662
Industry × Year FE	Y	N	N	Y	Y	Y
Bank × Industry FE	N	Y	N	Y	Y	Y
Year FE	N	N	Y	Y	Y	Y
Bank Controls	N	N	N	N	Y	Y
Bank Controls × post	N	N	N	N	N	Y

Notes: This table reports estimates from equation (4) using bank-industry level data for Japanese banks from 2012 to 2019 (fiscal year ending in March). The dependent variable is the log of the loan amount for each bank-industry pair. $post_t$ is a dummy equal to 1 for years on or after 2016 (following the introduction of reserve tiering), and 0 otherwise. $negexp_{b,t}$ is bank b's negative interest rate balance as a share of its total reserves at the central bank. Bank-level controls include lagged log total assets, equity-to-asset ratio, and liquidity. Columns (1)–(3) sequentially add industry-year fixed effects, bank-firm fixed effects, and year fixed effects, respectively. Column (4) includes all three fixed effects. Column (5) further adds bank controls, and Column (6) includes both bank controls and their interactions with $post_t$. Standard errors are clustered at the bank-firm level and reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

Figure 5 presents results using bank-industry-level data. Lending increases across all specifications after reserve tiering, with banks holding more reserves subject to negative rates extending more credit. In column (5), a 10 percentage point higher exposure is associated with a 0.5% increase in lending to an industry, controlling for other factors. Two

mechanisms likely explain this pattern: greater exposure incentivizes banks to reallocate reserves into loans, while lower exposure allows banks to benefit from cheaper interbank funding. The evidence suggests the first channel is slightly more influential.

Table 6: Impact of Reserve Tiering on Loan Offering by Firm-Bank

	$Loan_{bi,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$post_t$	0.0304	-0.0058	-0.8200***	0.0361*	0.0382*	0.0391*
	(0.0442)	(0.0080)	(0.1224)	(0.0180)	(0.0175)	(0.0174)
$post_t \times negexp_{b,t}$	0.0010**	0.0007***	0.0023***	0.0007***	0.0006**	0.0006**
	(0.0003)	(0.0002)	(0.0005)	(0.0002)	(0.0002)	(0.0002)
R^2	0.6696	0.9288	0.0021	0.9637	0.9659	0.9659
N	79262	76581	80210	75631	61204	61204
Firm × Year FE	Y	N	N	Y	Y	Y
Bank × Firm FE	N	Y	N	Y	Y	Y
Year FE	N	N	Y	Y	Y	Y
Bank Controls	N	N	N	N	Y	Y
Bank Controls × post	N	N	N	N	N	Y

Notes: This table reports estimates from equation (4) using bank-firm level loan data for Japanese banks from 2012m1 to 2019m12. The dependent variable is the log of the loan amount for each bank-firm pair. $post_t$ is a dummy equal to 1 for years on or after 2016 (following the introduction of reserve tiering), and 0 otherwise. $negexp_{b,t}$ is bank b's negative interest rate balance as a share of its total reserves at the central bank. Bank-level controls include lagged log total assets, equity-to-asset ratio, and liquidity. Columns (1)–(3) sequentially add firm-year fixed effects, bank-firm fixed effects, and year fixed effects, respectively. Column (4) includes all three fixed effects. Column (5) adds bank controls, and Column (6) includes both bank controls and their interactions with $post_t$. Standard errors are clustered at the bank-firm level and reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

Using bank-firm linkage data from the Nikkei NEEDS database, I examine changes in lending at the bank-firm level. Figure 6 shows that banks with greater exposure to negative interest rates consistently increase firm lending, especially at the intensive margin. Reserve tiering appears to modestly boost credit availability by expanding firm-bank lending. In column (5), a 10 percentage point higher exposure is associated with a 38% increase in lending to a firm.

5.4 Distributional Risk Channel: Effects on Loan Rates and Portfolio Risk

The theory further predicts that heterogeneous funding incentives under tiered remuneration reshape not only the level of loan rates but also the distribution of credit supplied

across firms of different risk profiles. Banks with greater unused negative-rate capacity should cut loan rates more aggressively and expand credit disproportionately toward riskier borrowers, while banks with higher negative-rate exposure may shift lending toward higher-yield, higher-risk firms to compensate for funding cost disadvantages. To evaluate this distributional risk channel, I first estimate the effect of tiering on bank-level loan rates, and then examine whether banks' loan portfolios shift toward riskier firms, combining bank–firm lending data with firm-level risk measures.

I first focus on the changes in loan interest rates. The analysis employs the following specification to assess the influence of reserve tiering on bank-level loan rates:

$$loanrate_{b,t} = \alpha + \beta post_t + \delta(post_t \times negexp_{b,t}) + \sum_{x \in controls} \mu_x x_{b,t-1} + \sum_{x \in controls} \eta_x post_t \times x_{b,t-1} + \epsilon_{b,t} \tag{5}$$

where the dependent variable is loan interest rate at bank level, calculated as the ratio of interest on loans and discounts (as reported in the income statement) to the loans and bills discounted (as recorded on the balance sheet).

Table 7: The Impact of Reserve Tiering on Loan Rate

	$loanrate_{b,t}$			
	(1)	(2)	(3)	(4)
$post_t$	-0.3506***	-0.2487***		
	(0.0157)	(0.0364)		
$post_t \times negexp_{b,t}$	0.0003*	0.0003**	0.0003*	0.0002
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
R^2	0.9414	0.9563	0.9651	0.9664
N	900	776	776	776
Bank FE	Y	Y	Y	Y
Time FE	N	N	Y	Y
Bank Controls	N	Y	Y	Y
Bank Controls × post	N	N	N	Y

Notes: This table reports estimates from equation (5) using bank-level data for Japanese banks from 2012 to 2019 (fiscal year ending in March). The dependent variable is the loan interest rate ($loanrate_{b,t}$). $post_t$ is a dummy variable equal to 1 for years on or after 2016 (following the introduction of reserve tiering), and 0 otherwise. $negexp_{b,t}$ is bank b's negative interest rate balance as a share of its total reserves at the central bank. Bank-level controls include lagged log total assets, the equity-to-asset ratio, and liquidity. Columns (1)–(3) sequentially add bank fixed effects, bank controls, and time fixed effects. Column (4) additionally includes interactions between bank controls and $post_t$. Standard errors are clustered at the bank level and reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

Figure 7 presents results on how bank-level loan rates changed following reserve tiering. Across all specifications, loan rates decline after the policy's introduction. However, banks with a higher share of reserves subject to negative rates experience a smaller decline. A 10% increase in negative rate exposure corresponds to a 0.3 basis point smaller decline in loan rates, as indicated in column (3). This pattern reflects differences in funding dynamics: banks with lower exposure, or more unused negative-rate balances, benefit from cheaper interbank funding and can cut loan rates more aggressively. In contrast, higher-exposure banks face higher funding costs and adjust loan pricing less.

To further examine changes in loan portfolio risk, I use bank-firm linkage data from Nikkei NEEDS and firm-level data from Worldscope to estimate the following specifications:

$$risk_{b,t} = \alpha + \beta post_t + \delta(post_t \times negexp_{b,t}) + \sum_{x \in controls} \mu_x x_{b,t-1} + \sum_{x \in controls} \eta_x post_t \times x_{b,t-1} + \epsilon_{b,t}. \quad (6)$$

To assess the impact of reserve tiering on the risk level $risk_{b,t}$ of bank loan portfolios, I use four key metrics: the debt-to-capital ratio, debt-to-equity ratio, interest coverage ratio, and degree of combined leverage. Higher debt-to-capital and debt-to-equity ratios, along with greater combined leverage, indicate increased firm-level risk. In contrast, a higher interest coverage ratio signals lower risk, reflecting a firm's stronger ability to meet interest obligations. The overall portfolio risk is calculated as the weighted average of firm-level risk, with weights based on each firm's loan share in the bank's total monthly lending.

Table 8: The Impact of Reserve Tiering on Loan Risk

	Debt-to-capital	Debt-to-equity	Interest coverage	Degree of Combined Leverage
	(1)	(2)	(3)	(4)
$post_t$	0.0248	0.40	-7.1458	35.5970*
	(0.85)	(1.91)	(-1.3077)	(2.46)
$post_t \times negexp_{b,t}$	0.0002**	0.0017***	-0.0673	0.029
	(3.30)	(4.36)	(-1.3243)	(0.45)
R^2	0.29	0.18	0.04	0.02
N	5361	5361	5361	5361
Bank FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
Bank Controls	Y	Y	Y	Y

Notes: This table reports estimates from equation (6) using bank-firm level data for Japanese banks from January 2012 to December 2019. The dependent variable is the bank loan portfolio risk measure ($risk_{b,t}$), including the debt-to-equity ratio, interest coverage ratio, and degree of combined leverage. $post_t$ is a dummy equal to 1 for years on or after 2016 (following the introduction of reserve tiering), and 0 otherwise. $negexp_{b,t}$ is bank b's negative interest rate balance as a share of its total reserves at the central bank. Bank-level controls include lagged log total assets, equity-to-asset ratio, and liquidity. All columns include bank and year fixed effects. Standard errors are clustered at the bank level and reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

Figure 8 shows that reserve tiering systematically reshapes the composition of bank loan portfolios. Banks with lower negative-rate exposure, those holding unused negative-rate capacity, obtain cheaper interbank funding and therefore expand credit more aggressively following the policy change. This expansion is disproportionately directed toward higher-risk firms, consistent with compressed loan-rate premia and greater willingness to supply credit at the intensive margin. In contrast, banks with greater exposure to negative-rate balances face tighter balance sheet constraints and smaller declines in loan rates. To

preserve margins, these banks reallocate lending toward riskier firms with higher expected returns, rather than expanding total credit.

Together, these two adjustments, credit expansion by low-exposure banks and risk-shifting by high-exposure banks, generate an aggregate rise in portfolio risk. This pattern aligns with the distributional risk channel of the theoretical framework: tiered remuneration alters marginal funding incentives asymmetrically across banks, and these heterogeneous incentives translate into uneven compression of risk premia across borrowers. As a result, reserve tiering changes not only the average level of loan rates but also the distribution of credit across the risk spectrum, increasing the concentration of lending in medium- and high-risk segments of the market.

Taken together, the empirical results provide coherent support for the three transmission mechanisms articulated in the theoretical framework. First, the evidence on interbank positions confirms the intermediation reallocation channel: asymmetric reserve exposures activate liquidity flows between NDIs and banks, consistent with theory. Second, patterns in bank-level loan rates and bank–firm lending validate the reserve pricing channel, showing that tiered remuneration generates heterogeneous marginal funding incentives that translate into systematically different adjustments in credit supply. Third, changes in portfolio composition and borrower risk characteristics demonstrate the distributional risk channel, whereby tiered remuneration reshapes not only the average level of loan rates but also the distribution of credit across the risk spectrum. Across all three dimensions, the BOJ's reserve tiering policy produces precisely the reallocation, pricing, and distributional effects predicted by the model. These findings highlight how, in an environment of abundant but unevenly distributed reserves, monetary policy operates through the structure of reserve remuneration rather than through aggregate quantities alone, motivating the policy analysis that follows.

6 Policy Implications: Designing Nonlinear Reserve Remuneration

The framework developed in this paper highlights a central feature of monetary policy in the post-QE era: when reserves are abundant, heterogeneous, and remunerated nonlinearly, monetary transmission operates through distributional rather than purely aggregate channels. This shift has important implications for the design of reserve remuneration systems across different macroeconomic environments. In settings where traditional policy tools lose traction, either because the policy rate is constrained, deposit rates are downward-rigid, or reserves are so abundant that marginal changes in aggregate quantity no longer bind, a linear reserve rate becomes ineffective. By contrast, nonlinear reserve remuneration restores monetary traction by creating heterogeneous marginal incentives across financial institutions. These incentives activate the interbank market, reallocate liquidity across balance sheets, and ultimately reshape the distribution of loan rates offered to firms.

In an environment of abundant reserves, conventional interest rate changes do little to alter the marginal cost of holding reserves. The classical corridor system, in which the policy rate maps cleanly into the interbank rate and then into loan rates, becomes weak or entirely unresponsive. When balances held at the central bank are large and unevenly distributed, a uniform policy rate no longer anchors the marginal funding cost of the typical institution. The analysis shows that a tiered reserve remuneration fills this gap by generating marginal incentives that vary precisely with reserve exposure. Institutions facing a low remuneration rate on excess reserves have a strong incentive to lend in the interbank market, while those with limited reserves or unused tiering capacity have incentives to borrow. Monetary policy gains effectiveness not by shifting aggregate reserve levels, but by shaping the pattern of liquidity flows across institutions. The resulting redistribution of intermediation capacity, rather than a uniform shift in funding costs, becomes the primary force behind credit adjustments.

This mechanism can be harnessed in both easing and tightening environments. During periods of economic slack or when the policy rate is constrained by the zero lower bound, a descending tiering structure, where balances above a threshold earn a lower or negative interest rate, encourages institutions with excess reserves to push liquidity into the interbank market. Banks with low exposure to negative-rate balances expand low-rate lending, and credit supply grows disproportionately at the lower end of the loan-rate distribution. Because this response relies on cross-sectional heterogeneity, it remains potent even when aggregate deposit rates fail to adjust. Nonlinear remuneration therefore provides the central bank with a tool that strengthens pass-through at a time when conventional tools lose power.

The same logic applies in reverse when the central bank confronts an overheating economy. A tiering structure with ascending interest rates, where reserves above a threshold earn a higher interest rate, induces institutions with abundant reserves to hold these balances rather than lend them. Interbank liquidity becomes scarcer, the interbank rate rises, and banks that rely on interbank borrowing, often those with higher intrinsic risk, face tighter funding conditions. Loan rates increase more sharply for these institutions, and the distribution of credit tightens in a risk-sensitive manner. Because ascending tiering influences credit conditions through relative incentives rather than quantitative restrictions, it avoids the mechanical distortions associated with raising reserve requirements and allows the central bank to tighten credit selectively without impairing market functioning.

A key implication of this framework is that reserve heterogeneity is not a nuisance to be ignored but an essential policy lever. The effectiveness of tiering depends on differences in reserve exposures across institutions. By adjusting the tier threshold or the wedge between remuneration rates, the central bank can determine which institutions face the relevant marginal incentives and how strongly liquidity reallocates through the interbank market. The design of the remuneration schedule therefore becomes a central part of monetary

policy, with clear parameters that shape the strength of liquidity flows, the degree of pass-through, and the distribution of loan rates across firms.

The framework also clarifies how nonlinear reserve remuneration interacts with financial stability. Descending tiering compresses risk premia more strongly for high-risk borrowers, as riskier banks typically hold fewer reserves and borrow heavily in the interbank market. Policymakers who wish to strengthen pass-through without overstimulating risk-taking can raise the lending facility rate or adjust the tiering wedge to temper this effect. Conversely, ascending tiering naturally tightens credit conditions more sharply for high-risk institutions, improving the alignment between loan rates and underlying credit risk and reducing systemic vulnerability without resorting to more intrusive supervisory measures.

Taken together, these insights suggest that nonlinear reserve remuneration provides a flexible and powerful instrument for monetary policy in the post-QE environment. It allows the central bank to shape the distribution of credit, influence the composition of intermediation, and manage risk-taking through a market-based mechanism that complements traditional policy tools. Whether the challenge is weak pass-through at the effective lower bound, overheating following large fiscal expansions, or the need to balance credit support with financial stability, nonlinear reserve remuneration offers a design space well-suited to the structural realities of an economy with abundant reserves.

7 Conclusion

This paper develops a general framework for understanding distributional monetary transmission under abundant reserves, showing that when reserves are unevenly distributed and remunerated through a tiered system, monetary policy operates through heterogeneous exposures rather than through a single marginal funding cost. By linking banks and non-depository institutions in a unified model of the interbank and loan markets, the framework reveals three mechanisms, reserve pricing, intermediation reallocation, and

distributional risk, that jointly determine how credit conditions respond to tiered reserve remuneration.

The theory shows that a tiered or non-linear reserve schedule generates asymmetric marginal funding incentives across institutions. These asymmetric incentives activate a reallocation of liquidity in the interbank market, as institutions facing low remuneration seek to lend reserves to those with scarce liquidity. This intermediation reallocation amplifies monetary pass-through even when deposit rates are constrained by the zero lower bound. Because institutions with tighter reserve constraints are often those financing riskier firms, the resulting shifts in funding conditions transmit directly into loan pricing: loan rates compress most sharply in the medium- and high-risk segments, reshaping not only the average level of loan rates but the entire distribution of credit terms.

Empirical evidence from Japan provides a clean institutional setting to evaluate these mechanisms. The introduction of reserve tiering coincided with a sharp rise in low-rate lending, a decline in medium-rate loans, and a reallocation of interbank liquidity consistent with the model's predictions. Banks with greater exposure to negatively remunerated reserves expanded interbank lending, while less-exposed banks borrowed more and reduced loan rates more aggressively. Loan-level estimates confirm that credit supply shifts differentially across the risk spectrum, validating the distributional risk channel. Together, the results underscore that nonlinear reserve remuneration can materially reshape the allocation of credit, even when aggregate reserve levels remain unchanged.

These findings carry broader policy implications for economies facing abundant reserves, binding lower bounds on deposit rates, or overheating credit markets. Tiered reserve remuneration offers a flexible tool for steering credit conditions: lowering remuneration on marginal reserves can stimulate demand, while an ascending tier schedule, raising remuneration above a threshold, can tighten credit by increasing loan rates through all three channels. Relative to uniform rate changes, tiering operates through richer transmission forces, influencing not only average lending conditions but also their distribution

across institutions and borrowers. Compared to raising reserve requirements, tiering preserves market-based intermediation and can more directly target riskier lending without imposing broad liquidity costs. And unlike balance sheet policies such as quantitative easing, tiering can be implemented without large-scale asset purchases or distortions in longer-term yields.

Overall, the paper shows that when reserves are abundant, the structure of reserve remuneration, not their quantity, becomes the central object of monetary transmission. Designing remuneration schedules that account for heterogeneity in reserve exposures can improve the effectiveness, precision, and stability of monetary policy. As abundant reserves become a persistent feature of modern financial systems, understanding and optimizing these distributional channels will remain essential for both macroeconomic stabilization and financial stability.

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A Reserve Tiering Policy

Central banks typically impose reserve requirements on financial institutions to ensure they can meet liabilities during sudden withdrawals. At the same time, they pay a uniform interest rate on reserves to help control interbank rates. In response to oversized balance sheets, low inflation, and weakened monetary transmission due to the zero lower bound on deposit rates, several central banks, including the Swiss National Bank (SNB), European Central Bank (ECB), and Danmarks Nationalbank (DN), adopted negative interest rate policies with reserve tiering between 2014 and 2016, as shown in Table 9. Under this system, reserve balances below a threshold earn a higher (positive or zero) rate, while those above it receive a lower, often negative, rate.

Table 9: Overview of Countries with Reserve Tiering Policies

Central Bank	Tiers	Top Tier Rate	Bottom Tier(s) Rate
Denmarks Nationalbank	Two Tiers	-0.20% (Jul 6, 2012) -0.10% (Jan 25, 2013) +0.05% (Apr 25, 2014) -0.05% (Sep 5, 2014) -0.20% (Jan 20, 2015) -0.35% (Jan 23, 2015) -0.50% (Jan 30, 2015) -0.75 % (Feb 6, 2015) -0.65 % (Jan 8, 2016) -0.75% (Sep 13, 2019) -0.60% (Mar 20, 2020) -0.50% (Mar 19, 2021) -0.60% (Oct 1, 2021)	0.00% (Below threshold)
Bank of Japan (BoJ)	Three Tiers	-0.10% (Feb 16, 2016)	0.10% (Basic Balance) 0.00% (Macro Add-on Balance)
Swiss National Bank (SNB)	Two Tiers	-0.25% (Dec 18, 2014) -0.75% (Jan 15, 2015)	0.00% (Reserve requirement+ Exemption allowance)
European Central Bank (ECB)	Two Tiers	-0.10% (Jun 11, 2014) -0.20% (Sep 10, 2014) -0.30% (Dec 9, 2015) -0.40% (Mar 16, 2016) -0.50% (Sep 18, 2019)	0.00% (Exempt tier)

Notes: This table summarizes central banks that have implemented reserve tiering and their corresponding tier rates, based on Jobst, Lin and Pradhan (2016) and updates from official central bank websites.

B Institutional Background: The Reserve Tiering Policy in Japan

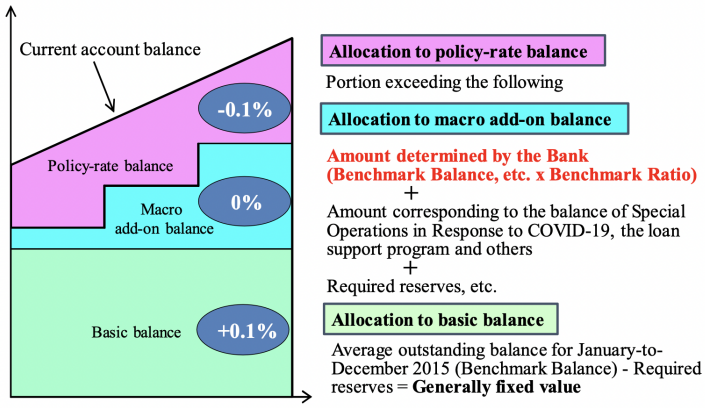


Figure 12: Framework of the Three-Tier System of NIRP

Notes: This figure illustrates the Bank of Japan's three-tier negative interest rate policy, sourced from the BOJ and Arao et al. (2022).

A prominent example of reserve tiering is the Bank of Japan (BOJ). While not the first to adopt negative interest rates or tiered remuneration, the BOJ was the first to introduce a three-tier reserve structure, applying different rates to portions of each financial institution's current account balance. On January 29, 2016, the BOJ announced its "Quantitative and Qualitative Monetary Easing (QQE) with a Negative Interest Rate" policy, effective February 16. The three tiers include: (1) the basic balance (+0.1%, approx. ¥210 trillion); (2) the macro add-on balance (0.0%, ¥40 trillion); and (3) the policy-rate balance (–0.1%, ¥10–30 trillion). Only reserves exceeding ¥250 trillion are subject to the negative rate. To mitigate the risk that financial institutions might shift into large cash holdings at a zero interest rate, the BOJ announced that any increases in cash holdings would be deducted from the tiers eligible for zero-interest remuneration. Over time, the BOJ has actively adjusted the benchmark ratio used to calculate the macro add-on balance, ensuring that only a small share of total reserves is exposed to the negative interest rate.[7]

Figure 13 classifies Japanese banks into six categories based on business functions

[7] Source: https://www.boj.or.jp/en/announcements/release_2016/k160129b.pdf

and historical background: city banks, regional banks, regional bank II, foreign banks, trust banks, other banks, and bank holding companies. City banks are large institutions headquartered in major cities or nearby suburbs. Regional banks are based in prefectural capitals and operate primarily within those areas, while regional bank II serve smaller businesses and individuals in more localized regions. Other banks include newer types of institutions, such as those focused on settlement services or internet banking, often established by non-financial firms.[8] Trust banks offer both traditional banking services and specialized trust services, managing client assets and corporate pensions. Their offerings include investment trusts, will and estate services, inheritance consulting, pension management, and asset liquidation. Trust banks typically operate with two separate balance sheets: one for their banking operations and another for their trust accounts.[9]

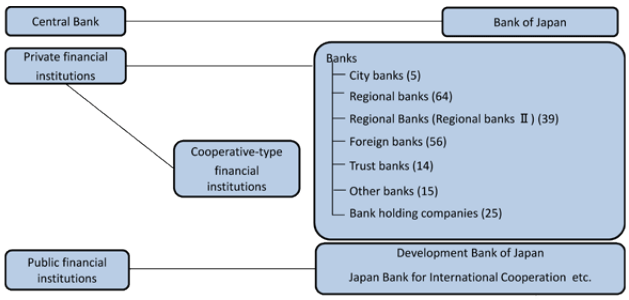


Figure 13: Bank Types in Japan

Notes: This figure shows the types of banks in Japan, sourced from the Japanese Bankers Association.

Under the three-tier reserve system, each financial institution faces a different effective interest rate based on the composition of its reserve holdings. As shown in Figure 12, the average outstanding balance from January to December 2015 is used to compute each institution's benchmark balance. The portion equal to the benchmark balance minus required reserves earns a +0.1% interest rate.[10] The second tier, earning 0% interest,

[8] Source: https://www.zenginkyo.or.jp/en/banks/financial-institutions/

[9] https://www.smth.jp/english/investors/individual/about_smth_group/about_trustbank

[10] According to the Bank of Japan, required reserve ratios vary by institution type and deposit category. For banks, the reserve ratio for time deposits ranges from 0.05% to 1.2%, and for other deposits, from 0.1% to 1.3%, depending on balance size. The reserve requirement for money held in trust is 0.1%. Source:

includes the required reserves, the amount calculated as the benchmark balance multiplied by the benchmark ratio, and any balance under the Loan Support Program. Reserves above these two tiers fall into the third tier, which is subject to a –0.1% interest rate.

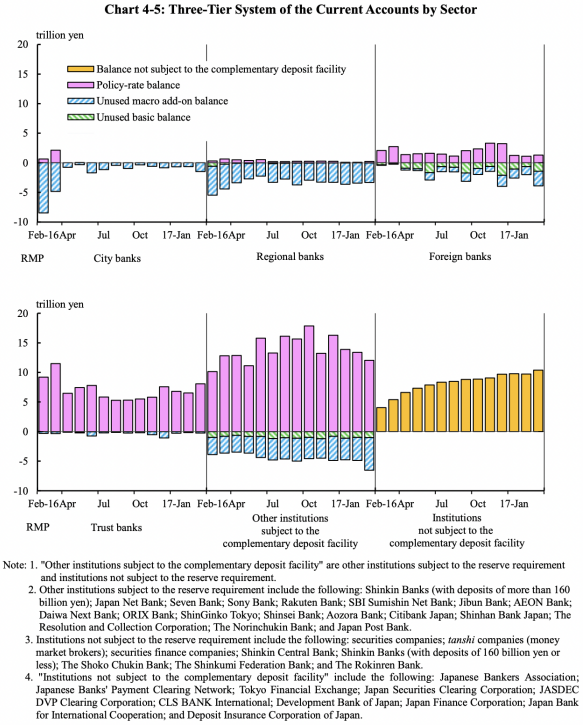
Not all financial institutions with current account balances at the Bank of Japan (BOJ) are exposed to the negative policy rate. As shown in Figure 14, foreign banks, trust banks, and other institutions subject to the Complementary Deposit Facility, including those categorized as "other banks", are the main holders of negative interest rate balances. These institutions face negative rate exposure for different underlying reasons, reflecting their distinct balance sheet compositions and reserve behaviors.

According to the BOJ, many of these institutions accumulated idle funds, driven by rising stock prices and expectations of lower interest rates, and increased their cash holdings in anticipation of purchasing Japanese government bonds (JGBs). To discourage such behavior, the BOJ deducts increases in cash holdings from the zero-interest-rate tier, shifting more reserves into the negative-interest-rate tier. As a result, these institutions are more likely to hold policy-rate balances subject to the negative rate. Some foreign banks willingly hold negative-rate balances when doing so remains profitable, particularly when yen funding costs in the FX swap market are lower than the BOJ's negative rate. Trust banks also tend to hold negative-rate balances, as money reserve funds (MRFs) and other investment trusts, unable to lend at positive rates, transfer idle funds to trust banks via a mechanism known as "lending to banking accounts." However, this pattern declined sharply after the April 2016 reserve maintenance period. The BOJ implemented a policy that counted current account balances equivalent to the outstanding amount of MRFs entrusted to trust banks toward their macro add-on balances, reducing their exposure to the negative rate. In response, trust banks began charging investment trusts a fee for holding idle funds. As a result, although investment trusts are not directly subject to BOJ reserve requirements, they now effectively bear the cost of negative interest rates through

https://www.boj.or.jp/en/statistics/boj/other/reservereq/junbi.htm/

their relationships with trust banks.[11]

In contrast, city banks and regional banks hold relatively small amounts of negative-rate balances, though they, along with trust banks, are key providers of consumer loans. Among them, city banks, as larger institutions, tend to hold more negative-rate balances than regional banks. Under the three-tier system, larger banks face higher reserve requirements, receive a smaller share of basic balances, and hold larger portions in the macro add-on and policy-rate tiers. As a result, they earn lower average interest rates on their reserves compared to smaller banks.



Note: 1. "Other institutions subject to the complementary deposit facility" are other institutions subject to the reserve requirement and institutions not subject to the reserve requirement.
2. Other institutions subject to the reserve requirement include the following: Shinkin Banks (with deposits of more than 160 billion yen); Japan Net Bank; Seven Bank; Sony Bank; Rakuten Bank; SBI Sumishin Net Bank; Jibun Bank; AEON Bank; Daiwa Next Bank; ORIX Bank; ShinGinko Tokyo; Shinsei Bank; Aozora Bank; Citibank Japan; Shinhan Bank Japan; The Resolution and Collection Corporation; The Norinchukin Bank; and Japan Post Bank.
3. Institutions not subject to the reserve requirement include the following: securities companies; *tanshi* companies (money market brokers); securities finance companies; Shinkin Central Bank; Shinkin Banks (with deposits of 160 billion yen or less); The Shoko Chukin Bank; The Shinkumi Federation Bank; and The Rokinren Bank.
4. "Institutions not subject to the complementary deposit facility" include the following: Japanese Bankers Association; Japanese Banks' Payment Clearing Network; Tokyo Financial Exchange; Japan Securities Clearing Corporation; JASDEC DVP Clearing Corporation; CLS BANK International; Development Bank of Japan; Japan Finance Corporation; Japan Bank for International Cooperation; and Deposit Insurance Corporation of Japan.

Figure 14: BOJ Current Accounts by Bank Types

Notes: This figure shows how the Bank of Japan's current accounts are divided into tiers by financial institution type, sourced from the Bank of Japan.

Using bank balance sheet data from the Japanese Bankers Association (JBA) and S&P Capital IQ Pro, I calculate each bank's reserve balances across the three BOJ reserve tiers.

[11] Source: BOJ Market Operations in Fiscal 2016, https://www.boj.or.jp/en/research/brp/ron_2017/data/ron170714a.pdf

Details on data sources and methodology are provided in the Data section. Figure 1 shows the distribution of negative interest rate ratios, defined as the share of reserves subject to negative rates as a percentage of total reserves, for individual banks as of March 2016. Positive values indicate banks holding reserve balances exposed to the BOJ's negative interest rate policy (NIRP), while negative values reflect banks with unused capacity below the policy-rate threshold. The figure, organized into three panels, highlights heterogeneity across bank types: the top panel shows that city banks and trust banks were most affected, with a larger share of reserves under negative rates. In contrast, the bottom panels reveal that regional banks, especially smaller ones, faced much less exposure, often maintaining sizable unused policy-rate balances. These patterns underscore the uneven transmission of NIRP across Japan's banking system, shaped by differences in bank size, business models, and reserve composition.

Since the Bank of Japan (BOJ) actively adjusts the benchmark ratio used to calculate the macro add-on balance, each financial institution's negative interest rate ratio, the share of reserves subject to negative rates, fluctuates over time. Given that regional banks primarily serve local markets, Figure 2 shows the evolution of average negative interest rate ratios across regional banks by geographical area. In the figure, shades of red indicate regions with greater exposure to negative rates, while shades of blue represent areas with unused capacity below the policy-rate threshold. The results reveal substantial regional variation in NIRP effects, reflecting differences in reserve composition and liquidity needs. Despite changes in exposure over time, the geographical pattern remains relatively stable, suggesting persistent structural differences in how the policy transmits across regions.

C Equilibirum Solutions for the Baseline Model

Suppose there is a mass of α banks are borrowers in the interbank market. i.e. $\int_{i\,|\,\Delta^i>0} di = \alpha$. And the total trading volume of the borrowers are $\int_{i\,|\,\Delta^i>0} \Delta^i = \beta \int_i \Delta^i di$, where $\beta > 1$ following the condition that banks are net borrowers in the interbank market. There are

eight endogenous variables $\left\{\beta, \int_i \Delta^i di, \tilde{\Delta}, D, L, r_\Delta, r_L, r_D\right\}$ in the model.

C.1 The conditions for banks' lending and borrowing choices

- When a bank is a borrower in the interbank market, i.e., $\Delta^i > 0$, taking the equilibrium interest rates r_Δ, r_L, r_D as given, then its equilibrium conditions are:

  1. Banks' balance sheet conditions:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i + q\Delta^i \tag{7}$$

  2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1 - G(R^i + \Delta^i - K^i)) + r_M G(R^i + \Delta^i - K^i) + (r_R - r_M)G(R^i + \Delta^i - M^i) = q(r_E - r_D) \tag{8}$$

  3. The relationship between the loan rate and the deposit rate:

$$r_L - \chi^{'}(L^i) - r_E + (r_E - r_D)(1 - q) = 0 \tag{9}$$

- When a bank is a lender in the interbank market, i.e., $\Delta^i < 0$, taking the equilibrium interest rates r_Δ, r_L, r_D as given, then its equilibrium conditions are:

  1. Banks' balance sheet conditions:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i \tag{10}$$

  2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1 - G(R^i + \Delta^i - K^i)) + r_M G(R^i + \Delta^i - K^i) + (r_R - r_M)G(R^i + \Delta^i - M^i) = 0 \tag{11}$$

  3. The relationship between the loan rate and the deposit rate:

$$r_L - \chi^{'}(L^i) - r_E + (r_E - r_D)(1 - q) = 0 \tag{12}$$

To simplify the expressions, assume the following conditions are true.

Assumption C.1. *The exogenous shock ε^i follows a uniform distribution $G(\varepsilon^i) = \frac{1}{\bar{\varepsilon}}\varepsilon^i = g\varepsilon^i$ in the interval $[0, \bar{\varepsilon}]$.*

Assumption C.2. *The project return r_P follows a uniform distribution $H(r_L) = \frac{1}{\bar{r_P}} r_L = h r_L$ in the interval $[0, \bar{r_P}]$.*

Assumption C.3. *The loan cost function $\chi(L) = L^2 + \underline{\chi}$ in the interval $(0, \infty)$ and is strictly increasing, strictly convex, and twice continuously differentiable function. And $\underline{\chi} > 0$.*

Under those assumptions, from equation 8, the borrowing bank trading volume $\Delta_b^i(R^i, K^i, r_E, r_D, r_D)$ is

$$\Delta_b^i(R^i, K^i, r_E, r_D, r_D) = \frac{r_\Delta - r_X + q(r_E - r_D) + (r_X - r_R)gR^i - (r_X - r_M)gK^i + (r_R - r_M)gM^i}{-(r_X - r_R)g}. \tag{13}$$

And from equation 11, the lending bank trading volume $\Delta_l^i(R^i, K^i, r_\Delta)$ is

$$\Delta_l^i(R^i, K^i, r_\Delta) = \frac{r_\Delta - r_X + (r_X - r_R)gR^i - (r_X - r_M)gK^i + (r_R - r_M)gM^i}{-(r_X - r_R)g} \tag{14}$$

Since $\Delta_b^i > 0$, I can derive that banks will be borrowers in the interbank market if

$$R^i - \frac{r_X - r_M}{r_X - r_R}K^i - \frac{r_M - r_R}{r_X - r_R}M^i < -\frac{r_\Delta - r_X}{(r_X - r_R)g} - \frac{(r_E - r_D)q}{(r_X - r_R)g}. \tag{15}$$

Intuitively, a bank is more likely to borrow in the interbank market if it has a lower initial reserve R^i, a higher reserve requirement K^i, or a higher reserve tiering threshold M^i.

Since $\Delta_l^i < 0$, I can derive that banks will be lenders in the interbank market if

$$R^i - \frac{r_X - r_M}{r_X - r_R}K^i - \frac{r_M - r_R}{r_X - r_R}M^i > -\frac{r_\Delta - r_X}{(r_X - r_R)g}. \tag{16}$$

Intuitively, a bank is more likely to lend in the interbank market if it has a higher initial reserve R^i, a lower reserve requirement K^i, or a lower reserve tiering threshold M^i.

C.2 Aggregate conditions

There are seven aggregate equilibrium equations.

1. Banks' balance sheet condition:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i + q\max(0, \Delta^i)$$

Aggregating across all banks yields the following equation:

$$(1-q)R-(1-q)\Gamma\tilde{R}+(1-q)L = D-E(\varepsilon)+q\int_{i\,|\,\Delta^i>0}\Delta^i di$$
$$= D-E(\varepsilon)+q\beta\int_i \Delta^i di$$

2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1-G(R-\Gamma\tilde{R}+\int_i \Delta^i di - K+\Gamma\tilde{K}))+r_M G(R-\Gamma\tilde{R}+\int_i \Delta^i di - K+\Gamma\tilde{K})$$
$$+(r_R-r_M)G(R-\Gamma\tilde{R}+\int_i \Delta^i di - M+\Gamma\tilde{M}) = \alpha q(r_E-r_D)$$

3. The relationship between the loan rate and the deposit rate:

$$r_L-\chi^{'}(L)-r_E+(r_E-r_D)(1-q)=0$$

4. The trading condition of the NDIs:

$$\tilde{R}+\tilde{\Delta}=\tilde{M}$$

5. The loan market clears:

$$1-H(r_L)=L$$

6. The deposit market clears:

$$\underline{D}+\kappa r_D = D$$

7. The interbank market clears:

$$\int_i \Delta di+\Gamma\tilde{\Delta}=0$$

C.3 An additional equation of β

Given $\int_{i\,|\,\Delta^i>0}\Delta^i di = \beta\int_i \Delta^i di$ and $\int_{i\,|\,\Delta^i>0}\Delta^i di + \int_{i\,|\,\Delta^i<0}\Delta^i di = \int_i \Delta^i di$, combining with equation 14, I can derive that

$$(1-\beta)\int_i \Delta^i di = \frac{r_\Delta - r_X + (r_X - r_R)g\int_{i\,|\,\Delta^i<0} R^i di - (r_X - r_M)g\int_{i\,|\,\Delta^i<0} K^i di + (r_R - r_M)g\int_{i\,|\,\Delta^i<0} M^i di}{-(r_X - r_R)g}$$

$$\Rightarrow \beta = 1 - \frac{r_\Delta - r_X + (r_X - r_R)gR^l - (r_X - r_M)gK^l + (r_R - r_M)gM^l}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})} \tag{17}$$

where $R^l = \int_{i\,|\,\Delta^i<0} R^i di$, $K^l = \int_{i\,|\,\Delta^i<0} K^i di$, and $M^l = \int_{i\,|\,\Delta^i<0} M^i di$. In the model calibration part, by setting the distributions of R^i, K^i, and M^i and α, β aims to match the data from Japan.

Let $\mathcal{A} \equiv -\frac{1}{(r_X - r_R)g\Gamma(\tilde{M}-\tilde{R})}$ and $\mathcal{B} \equiv 1 - \frac{-r_X + (r_X - r_R)gR^l - (r_X - r_M)gK^l + (r_R - r_M)gM^l}{(r_X - r_R)g\Gamma(\tilde{M}-\tilde{R})}$, then equation 17 can be rewritten as $\beta^* = \mathcal{A}r^*_\Delta + \mathcal{B}$.

When combining all eight equations, the eight endogenous variables can be solved.

C.4 Proofs of Propositions

C.4.1 Proposition 4.3

Proof. From the relationship between the loan rate and the deposit rate, it can be derived that

$$\begin{aligned} r_D &= \frac{1}{1-q}(r_L - \chi^{'}(L) - qr_E) \\ &= \frac{1}{1-q}(r_L - 2(1 - H(r_L)) - qr_E) \\ &= (\frac{1+2h}{1-q})r_L - \frac{(qr_E + 2)}{1-q} \end{aligned} \tag{18}$$

Plugging back the loan market clearing condition and the deposit market clearing condition back to the aggregate banks' balance sheet condition and substituting the equation (18),

$$(1-q)R - (1-q)\Gamma\tilde{R} + (1-q)(1 - H(r_L)) = \underline{D} + \kappa(\frac{1}{1-q}(r_L - \chi^{'}(L) - qr_E)) - E(\varepsilon) - q\beta\Gamma(\tilde{M} - \tilde{R})$$

$$\Rightarrow (1-q)H(r_L^*) + \frac{\kappa}{1-q}(r_L^* - \chi^{'}(L^*) - qr_E) = (1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\beta\Gamma(\tilde{M} - \tilde{R})$$

$$\Rightarrow r_L^* = \frac{\beta^* q\Gamma(\tilde{M} - \tilde{R}) + (1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \tag{19}$$

Taking derivative w.r.t. $\tilde{M}$ on both sides, it can be derived that

$$\frac{\partial r_L}{\partial \tilde{M}} = \frac{\beta^* q\Gamma}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} > 0$$

Therefore, $\frac{\partial r_L}{\partial \tilde{M}} > 0$.

Let $\mathcal{C} \equiv \frac{q\Gamma(\tilde{M}-\tilde{R})}{(1-q)h+\frac{\kappa}{1-q}+2\kappa h}$ and $\mathcal{D} \equiv \frac{(1-q)R-(1-q)\Gamma\tilde{R}+1-q-\underline{D}+E(\varepsilon)+q\kappa r_E+2\kappa}{(1-q)h+\frac{\kappa}{1-q}+2\kappa h}$, then equation 19 can be rewritten as $r_L^* = \mathcal{C}\beta^* + \mathcal{D}$.

□

C.4.2 Proposition 4.5

Proof. When plugging back equation (18) into the relationship between the interbank rate and the deposit rate, it can be derived that:

$$\begin{aligned} -r_\Delta + r_X(1 - G(R - \Gamma\tilde{R} + \int_i \Delta^i di - K + \Gamma\tilde{K})) + r_M G(R - \Gamma\tilde{R} + \int_i \Delta^i di - K + \Gamma\tilde{K}) \\ +(r_R - r_M)G(R - \Gamma\tilde{R} + \int_i \Delta^i di - M + \Gamma\tilde{M}) = \alpha q(r_E - \frac{1}{1-q}(r_L - \chi^{'}(L) - qr_E)) \end{aligned}$$

$$\Rightarrow -r_\Delta + r_X(1 - G(\delta_1)) + r_M G(\delta_1) + (r_R - r_M)G(\delta_2) = \alpha q r_E - \alpha\frac{q}{1-q}(r_L - \chi^{'}(L) - qr_E)$$

$$\begin{aligned} \Rightarrow r_\Delta^* = \alpha q(\frac{1}{1-q} + 2h)r_L^* + r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} \\ +g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q \end{aligned} \tag{20}$$

where $\delta_1 = R - \Gamma\tilde{M} - K + \Gamma\tilde{K}$ and $\delta_2 = R - M$. Taking derivative w.r.t. r_Δ on both sides, it can be derived that

$$\frac{\partial r_L}{\partial r_\Delta} = \frac{1}{\alpha q(\frac{1+2h}{1-q})} > 0.$$

Let $\mathcal{S} \equiv \alpha q(\frac{1}{1-q} + 2h)$ and $\mathcal{T} \equiv r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q$ and the equation 20 can be rewritten as $r_\Delta^* = \mathcal{S}r_L^* + \mathcal{T}$. □

C.4.3 Proposition 4.1

Proof. Combining equations 17, 19 and 20, I can find that:

$$\beta^* = \frac{\mathcal{A}\mathcal{D}\mathcal{S} + \mathcal{A}\mathcal{T} + \mathcal{B}}{1 - \mathcal{A}\mathcal{C}\mathcal{S}} \tag{21}$$

$$r_L^* = \frac{\mathcal{A}\mathcal{C}\mathcal{T} + \mathcal{B}\mathcal{C} + \mathcal{D}}{1 - \mathcal{A}\mathcal{C}\mathcal{S}} \tag{22}$$

$$r_\Delta = \frac{\mathcal{B}\mathcal{C}\mathcal{S} + \mathcal{C}\mathcal{T} + \mathcal{D}}{1 - \mathcal{A}\mathcal{C}\mathcal{S}}. \tag{23}$$

And I can derive that

$$\begin{aligned}\mathcal{A}\mathcal{C}\mathcal{S} &= -\frac{1}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})}\frac{q\Gamma(\tilde{M} - \tilde{R})}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h}\alpha q(\frac{1}{1-q} + 2h) \\ &= -\frac{q^2\alpha(\frac{1}{1-q} + 2h)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}\end{aligned}$$

$$\begin{aligned}\mathcal{A}\mathcal{C}\mathcal{T} = -\frac{1}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})}\frac{q\Gamma(\tilde{M} - \tilde{R})}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h}\Big(&r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} \\ &+ g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q\Big)\end{aligned}$$

$$= -\frac{q\Big(r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q\Big)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}$$

$$\begin{aligned}\mathcal{B}\mathcal{C} &= \Big(1 - \frac{-r_X + (r_X - r_R)gR^l - (r_X - r_M)gK^l + (r_R - r_M)gM^l}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})}\Big)\frac{q\Gamma(\tilde{M} - \tilde{R})}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \\ &= \frac{q\Big((r_X - r_R)g\Gamma(\tilde{M} - \tilde{R}) + r_X - (r_X - r_R)gR^l + (r_X - r_M)gK^l - (r_R - r_M)gM^l\Big)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}\end{aligned}$$

So I can derive that

$$r_L^* = \frac{\mathcal{A}\mathcal{C}\mathcal{T} + \mathcal{B}\mathcal{C} + \mathcal{D}}{1 - \mathcal{A}\mathcal{C}\mathcal{S}}$$

$$= \frac{qg\Gamma(r_M - r_R)\tilde{M} + g(r_X - r_R)R + qg(r_R - r_M)M - qg(r_X - r_M)K + qg\Gamma(r_X - r_M)\tilde{K} - (r_X - r_R)g\Gamma\tilde{R} + \alpha q^2(1+q)r_E + 2\alpha q^2 - q + (r_X - r_R)g(1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big) + q^2\alpha(\frac{1}{1-q} + 2h)}$$

Therefore, $\frac{\partial r_L^*}{\partial \tilde{M}} = \frac{qg\Gamma(r_M - r_R)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big) + q^2\alpha(\frac{1}{1-q} + 2h)} > 0.$ □

C.4.4 Proposition 4.2

Proof. Following the proof of Proposition 4.1, I can derive the formula parts for β.

$$\mathcal{ADS} = -\frac{1}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})} \frac{(1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \alpha q(\frac{1}{1-q} + 2h)$$

$$= -\frac{\alpha q(\frac{1}{1-q} + 2h)\Big((1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa\Big)}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}$$

$$\mathcal{AT} = -\frac{\Big(r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q\Big)}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})}$$

So I can derive that:

$$\beta = \frac{\mathcal{ADS} + \mathcal{AT} + \mathcal{B}}{1 - \mathcal{ACS}}$$

$$= \frac{-\mathcal{M}g\Gamma(r_X - r_M)\tilde{M} + [\Gamma\alpha q + 2h\alpha q(1-q)\Gamma - \mathcal{M}(r_X - r_R)g\Gamma]\tilde{R} + [-\alpha - 2\alpha qh(1-q) + g(r_R - r_X)\mathcal{M}]R + \mathcal{M}g(r_R - r_M)M - \mathcal{M}g(r_X - r_M)K + \mathcal{M}g\Gamma(r_X - r_M)\tilde{K} + \mathcal{N}}{\Gamma(\tilde{M} - \tilde{R})[(r_X - r_R)g\mathcal{M} - q^2\alpha(\frac{1+2h}{1-q})]}$$

where $\mathcal{M} \equiv (1-q)h + \frac{\kappa}{1-q} + 2\kappa h$ and $\mathcal{N} = -\alpha q(\frac{1+2h}{1-q})(1 - q + \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa) + \mathcal{M}[\alpha q(1 + q)r_E + 2\alpha q + r_X - (r_X - r_R)R^l - (r_X - r_M)gK^l + (r_R - r_M)gM^l]$.

I can further derive that:

$$\frac{\partial \beta}{\partial \tilde{M}} = \frac{[-\mathcal{M}g\Gamma(r_M - r_R) + \Gamma\alpha q + 2h\alpha q(1-q)\Gamma]\tilde{R} + [-\alpha - 2\alpha qh(1-q) + g(r_R - r_X)\mathcal{M}]R + \mathcal{M}g(r_R - r_M)M - \mathcal{M}g(r_X - r_M)K + \mathcal{M}g\Gamma(r_X - r_M)\tilde{K} + \mathcal{N}}{\{\Gamma(\tilde{M} - \tilde{R})[(r_X - r_R)g\mathcal{M} - q^2\alpha(\frac{1+2h}{1-q})]\}^2} < 0$$

since the term $\mathcal{M}g(r_R - r_M) + \mathcal{N}$ is the dominant factor and is negative. □

C.5 The interbank trading rate r_Δ^*

The model solution is based on the assumption that the interbank trading rate $r_\Delta \geq r_R$.

Proof. Following the proof of Proposition 4.1, I can derive the formula parts for r_Δ.

$$r_\Delta^* = \mathcal{S}r_L^* + \mathcal{T}$$

$$= \alpha q(\frac{1}{1-q} + 2h)r_L^* + r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q$$

$$= \alpha q(\tfrac{1}{1-q}+2h)\frac{qg\Gamma(r_M-r_R)\tilde{M}+g(r_X-r_R)R+qg(r_R-r_M)M-qg(r_X-r_M)K+qg\Gamma(r_X-r_M)\tilde{K}-(r_X-r_R)g\Gamma\tilde{R}+\alpha q^2(1+q)r_E+2\alpha q^2-q+(r_X-r_R)g(1-q-\underline{D}+E(\varepsilon)+q\kappa r_E+2\kappa)}{(r_X-r_R)g\Big((1-q)h+\frac{\kappa}{1-q}+2\kappa h\Big)+q^2\alpha(\frac{1}{1-q}+2h)}$$

$$+r_X+g(r_R-r_X)R-g(r_R-r_M)M+g\Gamma(r_X-r_M)\tilde{M}+g(r_x-r_M)K-g\Gamma(r_X-r_M)\tilde{K}-\alpha q(1+q)r_E-2\alpha q$$

And I can further derive that

$$\frac{\partial r^*_\Delta}{\partial \tilde{M}} = \alpha q(\frac{1+2h}{1-q})\frac{\partial r^*_L}{\partial \tilde{M}} + g\Gamma(r_X - r_M)$$

$$= \frac{\alpha(\frac{1}{1-q}+2h)q^2 g\Gamma(r_M-r_R)}{(r_X-r_R)g\Big((1-q)h+\frac{\kappa}{1-q}+2\kappa h\Big)+q^2\alpha(\frac{1}{1-q}+2h)} + g\Gamma(r_X-r_M) > 0$$

Therefore, as the central bank adjusts down the bank reserve tiering threshold for the NDIs, the interbank lending rate r^*_Δ declines accordingly.

Also, it can be derived that

$$\frac{\partial r^*_\Delta}{\partial R} = \frac{\alpha q(\frac{1}{1-q}+2h)(r_x-r_R)g}{(r_X-r_R)g[(1-q)h+\frac{\kappa}{1-q}+2\kappa h]+q^2\alpha(\frac{1+2h}{1-q})} - g(r_X-r_R).$$

$$= g(r_X-r_R)[\frac{\alpha q(\frac{1}{1-q}+2h)}{(r_X-r_R)g[(1-q)h+\frac{\kappa}{1-q}+2\kappa h]+q^2\alpha(\frac{1+2h}{1-q})} - 1]$$

□

D Bank Interest Margin Channel

D.1 Proposition 4.4

The proof for Proposition 4.4 is as follows.

Proof. The expected profit of the bank can be approximated by

$$E(\pi^i) \approx -r_\Delta\Delta^i - r_D D^i + r_L L^i - \chi(L^i) - r_E L^i + C_1 \approx -r_\Delta\Delta^i - r_D D^i + (r_L - r_E)L^i - \chi(L^i) + C_1$$

where C_1 is a constant that has minor impact on the loan rate chosen. Aggregating across all banks, the expected profit is

$$E(\pi) \approx -r_\Delta\int_i \Delta^i di - r_D D + r_L L - \chi(L) - r_E L + C_1 \approx -r_\Delta\int_i \Delta^i di - r_D D + (r_L - r_E)L - \chi(L) + C_1$$

Taking the F.O.C. with respect to L when r_D is close to zero.

$$-r_\Delta \frac{\partial \int_i \Delta^i di}{\partial L} - \frac{\partial r_\Delta}{\partial L} \int_i \Delta^i di + (r_L - r_E) + \frac{\partial r_L}{\partial L}(1 - hr_L) - \chi^{'}(L) = 0 \tag{24}$$

And I can derive that

$$\frac{\partial r_\Delta}{\partial r_L} = \frac{\partial r_\Delta}{\partial L}\frac{\partial L}{\partial r_L}$$

$$\Rightarrow \frac{\partial r_\Delta}{\partial L} = \frac{\frac{\partial r_\Delta}{\partial r_L}}{\frac{\partial L}{\partial r_L}} = \frac{\alpha q(\frac{1+2h}{1-q})}{-h} = -\frac{\alpha q}{h}(\frac{1+2h}{1-q})$$

Also, using the interbank market aggregate condition, I can derive that

$$\frac{\partial \int_i \Delta^i di}{\partial L} = \frac{\partial - \Gamma(\tilde{M} - \tilde{R})}{\partial L} = 0$$

Also using $1 - hr_L = L$, I can derive that

$$\frac{\partial r_L}{\partial L} = -\frac{1}{h}$$

So the equation 24 is

$$\Rightarrow \frac{\alpha q}{h}(\frac{1+2h}{1-q}) \int_i \Delta^i di + r_L - r_E - \frac{1}{h} + r_L - 2(1 - hr_L) = 0$$

$$\Rightarrow r_L^* \approx -[\frac{\alpha q}{(2+2h)h}(\frac{1+2h}{1-q})] \int_i \Delta^i di + \frac{2 + r_E + \frac{1}{h}}{2+2h}$$

E Equilibrium Solutions for the Extended Model

Suppose there is a mass of α banks are borrowers in the interbank market and $\int_{i \mid \Delta^i > 0} = \alpha$. In the extended model, suppose each bank i offers loans only to one entrepreneur at the rate r_L^i. And the endogenous variables are $\left\{\Delta^i, \tilde{\Delta}, D^i, L^i, r_\Delta, r_L^i, r_D\right\}_{\forall i}$. The equilibirum conditions are as follows.

- When a bank is a borrower in the interbank market, i.e., $\Delta^i > 0$, taking the equilibrium interest rates r_Δ, r_L^i, r_D as given, then its equilibrium conditions are:

    1. Banks' balance sheet conditions:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i + q\Delta^i \tag{25}$$

2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1-G(R^i+\Delta^i-K^i)) + r_M G(R^i+\Delta^i-K^i) + (r_R-r_M)G(R^i+\Delta^i-M^i) = q(r_E-r_D) \tag{26}$$

3. The relationship between the loan rate and the deposit rate:

$$r_L^i - \chi^{'}(L^i) - r_E + (r_E - r_D)(1-q) = 0 \tag{27}$$

4. The relationship between the loan volume and loan rate:

$$1 - H^i(r_L^i) = L^i \tag{28}$$

- When a bank is a lender in the interbank market, i.e., $\Delta^i < 0$, taking the equilibrium interest rates r_Δ, r_L^i, r_D as given, then its equilibrium conditions are:

1. Banks' balance sheet conditions:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i \tag{29}$$

2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1-G(R^i+\Delta^i-K^i)) + r_M G(R^i+\Delta^i-K^i) + (r_R-r_M)G(R^i+\Delta^i-M^i) = 0 \tag{30}$$

3. The relationship between the loan rate and the deposit rate:

$$r_L^i - \chi^{'}(L^i) - r_E + (r_E - r_D)(1-q) = 0 \tag{31}$$

4. The relationship between the loan volume and loan rate:

$$1 - H^i(r_L^i) = L^i \tag{32}$$

Additionally, I can also get the following conditions.

- The trading condition of the NDIs:

$$\tilde{R} + \tilde{\Delta} = \tilde{M} \tag{33}$$

- The deposit market clears:

$$\underline{D} + \kappa r_D = D \tag{34}$$

- The interbank market clears:

$$\int_i \Delta^i di + \Gamma\tilde{\Delta} = 0 \tag{35}$$

Suppose assumption C.1 and assumption C.3 still hold. And the assumption C.2 can be revised as follows.

Assumption E.1. *Each entrepreneur j has different risk exposure and gets loans from a different bank i, the project return r_P follows a uniform distribution $H(r_L) = \frac{1}{\bar{r_P}^i} r_L = h^i r_L$ in the interval $[0, \bar{r_P}^i]$.*

Assumption E.1 corresponds to the condition that each bank has a different risk tolerance level and is exposed to entrepreneurs with different risks.

Combing equation 25, equation 29 and the aggregate trading conditions of NDIs and the interbank market clearing condition, I can get the following condition between r^*_Δ and r^*_D:

$$r^*_\Delta = r_X(1-G(R-\Gamma\tilde{M}-K+\Gamma\tilde{K}))+r_M G(R-\Gamma\tilde{M}-K+\Gamma\tilde{K})+(r_R-r_M)G(R-M)-\alpha q(r_E-r^*_D) \quad (36)$$

The equilibrium can be solved using the following algorithm:

1. Apply bisection with bounds $\bar{r_D} - \epsilon_D$ and $\bar{r_D} + \epsilon_D$.
2. At iteration n:
   (a) Use $r_D^{(n)}$ and equation 36 to solve for $r_\Delta^{(n)}$
   (b) Given $(r_D^{(n)}, r_\Delta^{(n)})$, given $\{R^i, K^i, M^i, \tilde{R}, \tilde{K}, \tilde{M}, \varepsilon^i\}_{\forall i}$ solve for $\{\Delta^i, D^i, L^i, r_L^i, \tilde{\Delta}\}_{\forall i}$
   (c) Compute the aggregate deposit supply $\int_i D^i di$ and the aggregate demand $\underline{D}+\kappa r_D$ for convergence. Following the Walras law, if the deposit market clears, the interbank market also clears.

E.1 Proposition 4.6

Proof. If focusing on the individual bank i,

$$E(\pi^i) \approx -r_\Delta\Delta^i - r_D(D^i-\varepsilon^i)+r_L^i L^i-\chi(L^i)-r_E L^i+C_1 \approx -r_\Delta\Delta^i - r_D(D^i-\varepsilon^i)+(r_L^i-r_E)L^i-\chi(L^i)+C_1$$

Given r_Δ and r_D, taking F.O.C. with respect to L^i,

$$-r_\Delta\frac{\partial\Delta^i}{\partial L^i} - r_D\frac{\partial D^i}{\partial L^i} + \frac{\partial r_L^i}{\partial L^i}L^i + r_L^i - r_E - \chi'(L^i) = 0 \quad (37)$$

When the bank is a borrower, taking partial derivative with respect to L^i,

$$1 = \frac{\partial D^i}{\partial L^i} + q + q\frac{\partial\Delta^i}{\partial L^i}$$

$$\Rightarrow \frac{\partial\Delta^i}{\partial L^i} = -\frac{1}{q}\frac{\partial D^i}{\partial L^i} + \frac{1-q}{q}$$

Also, using $1-H^i(r_L^i) = 1-h^i r_L^i = L^i$, I have

$$\frac{\partial r^i}{\partial L^i} = -\frac{1}{h^i}$$

When the deposit rate is close to zero lower bound, using the loan market clearing condition, the aggregate loan volume is close to $\underline{D}$ and is fixed for each individual bank. Therefore, I have $\frac{\partial D^i}{\partial L^i} \approx 0$.

Plugging back to the equation 37,

$$-r_\Delta(-\frac{1}{q}\frac{\partial D^i}{\partial L^i} + \frac{1-q}{q}) - r_D\frac{\partial D^i}{\partial L^i} - \frac{1}{h^i}(1-h^i r_L^i) + r_L^i - r_E - 2(1-h^i r_L^i) = 0$$

$$\Rightarrow -r_\Delta\frac{1-q}{q} - \frac{1}{h^i}(1-h^i r_L^i) + r_L^i - r_E - 2(1-h^i r_L^i) = 0$$

$$\Rightarrow r_L^i \approx \frac{(r_E + 2 + r_\Delta\frac{1-q}{q}) + \frac{1}{h^i}}{2+2h^i}$$

$$\frac{\partial r_L^i}{\partial h^i} = \frac{4(r_E + 1 + r_\Delta\frac{1-q}{q})}{(4+4h^i)^2} > 0$$

□