Stablecoins, Sovereignty, and Currency Competition*

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Abstract

This study examines the competitive dynamics between stablecoins and sovereign currencies using a monetary search framework that explicitly models decentralized trade and currency choice. The rapid rise of stablecoins, exemplified by the USDT, and the expansion of decentralized finance (DeFi) present a direct challenge to traditional money, threatening monetary sovereignty, policy effectiveness, and seigniorage revenues. By integrating micro-level interactions between buyers' portfolio decisions and sellers' currency acceptance, this study captures how stablecoins alter currency usage in multi-currency environments. The analysis revealed three key findings. First, when a rival-currency-pegged stablecoin becomes widely accepted for transactions, it tightens the set of inflation policies under which the local currency is valued in steady state, with a stronger effect when the transaction fee g is small. Second, as g approaches zero, the stablecoin can displace the anchor currency itself from valued equilibria. Third, without regulatory intervention, the proliferation of stablecoins can lead to substantial seigniorage losses for the government, a loss that is attenuated when more sellers remain exclusive to the local currency. These results provide critical insights into the implications of private digital currencies for monetary stability and the governance of fiat money.

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1 Introduction

The increasing prevalence of stablecoins, particularly those pegged to the US dollar such as USDT (Tether), coupled with the rapid development of decentralized finance (DeFi), constitutes a significant challenge to established monetary systems. By offering speed, low transaction costs, and global accessibility, these digital instruments increasingly compete with sovereign currencies in performing the core function of money as a medium of exchange. This competition threatens to erode monetary sovereignty, constrain the effectiveness of domestic monetary policy, and shift seigniorage revenues from the state to private issuers. In this evolving environment, understanding how stablecoins alter currency choice and weaken the position of traditional money is essential for assessing both the financial stability and future of state-backed currencies.

While conventional fiat money has long been justified in terms of the medium of exchange, the digital transformation of money raises a more fundamental question: how do people choose which currency to use? Traditional models overlook this question by focusing on aggregates rather than transactions themselves. In contrast, a framework that explicitly accounts for decentralized trade and the frictions of money search can illuminate how competing currencies interact in practice. This perspective is especially valuable for understanding the new competitive pressures that stablecoins impose on sovereign currencies in a non-cooperative environment.

The third generation of monetary search framework by Lagos and Wright (2005) has become a fundamental tool for examining the role of money as a medium of exchange in decentralized markets. By integrating search frictions and decentralized trade, it offers rigorous microfoundations for understanding how agents decide which currencies to hold and utilize. Over the past two decades, this framework has been expanded to encompass the coexistence of multiple medium of exchanges, the introduction of central bank digital currency (CBDC) (Chiu et al. (2023)), and the implications of monetary policy for currency competition. Its broad applicability underscores its robustness as a theoretical lens for investigating environments in which multiple media of exchange interact.

The primary findings of this study are as follows. Firstly, when a rival currency-linked

stablecoin attains widespread transactional acceptance, it intensifies currency competition by reducing the range of inflation policies under which the local currency retains its value in steady-state equilibrium. This effect becomes more significant when the transaction fee associated with stablecoin payments, denoted by the gas fee g, is minimal. Secondly, the government's seigniorage from the local (weaker) currency is predominantly determined by the volume of sellers that remain exclusive to the local currency. Lastly, in the absence of effective regulation, the widespread adoption of stablecoins can result in a significant loss and reallocation of seigniorage-like revenues away from the local government. In dollarized contexts, USD-based stablecoins can further propagate dollarization through a novel payment rail, potentially substituting for physical dollar cash, thereby reshaping seignorage and incentives for both the local and anchor-currency valuation.

2 Model

Building upon the Lagos-Wright monetary search framework Lagos and Wright (2005), this study first incorporates a multi-currency system, following Zhang (2014), Yu (2023), and Fernández-Villaverde and Sanches (2019). The baseline model features three distinct fiat currencies: 1. Red currency (r) 2. Blue currency (b) 3. stablecoin. Red and blue currencies are issued by sovereign governments through debt issuance. A stablecoin company issues stablecoins in several stages. The process begins with collateral acquisition, where the company secures fiat currency or other assets to serve as its backing. Next, the company mints stablecoins on a blockchain platform, after which the collateral is deposited into the reserve accounts. The newly created stablecoins are then distributed through the cryptocurrency exchanges. Users can redeem these stablecoins for the underlying collateral, while the company manages reserves to maintain a one-to-one peg with the backing assets.

The concept of spatial transaction constraints in local currencies introduces complexity within economic systems. As explored by Chiu et al. (2023), Zhu and Hendry (2022), and Curtis and Waller (2000), these constraints allow for the maintenance of strategic parity among different currencies without establishing clear hierarchical dominance. This

equilibrium creates a unique market structure in which sellers are categorized based on their acceptance of specific currencies. Some sellers, classified as the first type, reject red currencies, while others choose not to accept blue currencies. This selective acceptance of currencies by different seller types contributes to the overall balance and functionality of the local currency systems. The introduction of a blue currency-based stablecoin offers a compelling solution to the limitations imposed by local currency constraints. This phenomenon is particularly evident when the blue country is the United States, and the stablecoin is USD-based, such as USDT issued by Tether.

The economy operates in discrete time periods t=0,1,2,3,..., each partitioned into two sequential sub-markets:

1. **Decentralized Market (DM):** Characterized by bilateral matching between households (buyers) and entrepreneurs (sellers) using Take-it-or-leave-it solutions for perishable good y exchange

2. **Centralized Market (CM):** A Walrasian market enabling frictionless trading of perishable good x and currency portfolio adjustment

The timeline progresses as $DM_t \to CM_t \to DM_{t+1}$, creating an infinite horizon dynamic optimization problem with periodic-portfolio adjustments.



In the centralized market, a representative household solves the following recursive Bellman equation:

$$W_t^H(r_t, b_t, e_t) = \max_{\substack{x_t, \ell_t, \\ \hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}}} \left\{ U(x_t) - \ell_t + \beta \mathbb{E}\left[V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}) \right] \right\} \tag{1}$$

$$\text{s.t.} \quad x_t = \underbrace{\phi_t(r_t - \hat{r}_{t+1})}_{\text{Red currency adjustment}} + \underbrace{\psi_t(b_t - \hat{b}_{t+1})}_{\text{Blue currency adjustment}} + \underbrace{\psi_t(e_t - \hat{e}_{t+1})}_{\text{stablecoin adjustment}} + \ell_t + T_t. \tag{2}$$

which can be written as

$$W_t^H(r_t, b_t, e_t) = \max_{\substack{x_t, \ell_t, \\ \hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}}} \Bigg\{ U(x_t) + \phi_t(r_t - \hat{r}_{t+1}) + \psi_t(b_t - \hat{b}_{t+1}) + \psi_t(e_t - \hat{e}_{t+1}) - x_t + T_t + \beta \mathbb{E}\left[V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}) \right] \Bigg\}.$$

Where $\hat{r}_{t+1}$ and $\hat{b}_{t+1}$ denote currency holdings carried forward to $t+1$, with hatted variables representing post-adjustment portfolios in the last period t.

Three fundamental results are derived as follows:

1. **Consumption Efficiency:** $U'(x_t) = 1$ emerges from the first-order condition for CM goods consumption

2. **Portfolio Homogeneity:** All households select identical currency portfolios $\{\hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}\}$.

3. **Envelope Condition:**

$$\frac{\partial W_t^H}{\partial r_t} = \phi_t \tag{3}$$
$$\frac{\partial W_t^H}{\partial b_t} = \psi_t \tag{4}$$
$$\frac{\partial W_t^H}{\partial e_t} = \psi_t \tag{5}$$

The first-order conditions are as follows:

$$\hat{r}_{t+1}: \ \phi_t \geq \beta \frac{\partial}{\partial \hat{r}_{t+1}} V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}) \tag{6}$$
$$\hat{b}_{t+1}: \ \psi_t \geq \beta \frac{\partial}{\partial \hat{b}_{t+1}} V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}) \tag{7}$$
$$\hat{e}_{t+1}: \ \psi_t \geq \beta \frac{\partial}{\partial \hat{e}_{t+1}} V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}) \tag{8}$$

Strict equality holds when $\hat{r}_{t+1} > 0$, $\hat{b}_{t+1} > 0$, and $\hat{e}_{t+1} > 0$ respectively. The portfolio adjustment mechanism allows households to optimally adjust $\{r_t, b_t, e_t\}$ to $\{\hat{r}_{t+1}, \hat{b}_{t+1}, \hat{e}_{t+1}\}$ incorporating expectations about future exchange opportunities in both DM and CM markets.

Consider a strategic scenario in which both currencies exhibit symmetric weakness, with a stablecoin that can be universally accepted in line with Ikeda (2022). This equilibrium configuration features three distinct seller types with heterogeneous currency acceptance policies:

- **Type 1 Sellers:** Accept Red currency and stablecoin
- **Type 2 Sellers:** Accept Blue currency and stablecoin
- **Type 3 Sellers:** Accept all currencies

Let $\alpha_j \in (0,1)$ denote the exogenous measure of type-j sellers, with $\Sigma_{j\in\{1,2,3\}}\alpha_j \leq 1$. Because sellers and buyers do not change their positions in different periods, sellers do not have the chance to purchase goods y. Therefore, sellers have no incentive to bring any currency to the next period's DM. Then, the portfolio change for sellers at the CM demonstrates their behavior in converting the medium of exchange that they obtained from the last DM into utility. The entrepreneur's centralized market optimization problem is as follows:

The DM value function for households that contain the three types of trade is as follows:

$$V_t^H(r_t, b_t, e_t) = \alpha_1\Big[u(y_t^1) + W_t^B(r_t - r_t^1,\ b_t,\ e_t - e_t^1 - g_t e_t^1)\Big] \tag{9}$$

$$+ \alpha_2\Big[u(y_t^2) + W_t^B(r_t,\ b_t - b_t^2,\ e_t - e_t^2 - g_t e_t^2)\Big] \tag{10}$$

$$+ \alpha_3\Big[u(y_t^3) + W_t^B(r_t - r_t^3,\ b_t - b_t^3,\ e_t - e_t^3 - g_t e_t^3)\Big] \tag{11}$$

$$+ (1 - \Sigma_{j\in\{1,2,3\}}\alpha_j)\, W_t^B(r_t, b_t, e_t). \tag{12}$$

In this context, $g \in [0,1)$ represents the gas fee, or transaction fee, associated with the transfer of the stablecoin, functioning as a service fee. By incorporating the approach of Chiu and Koeppl (2022) regarding the inclusion of gas fees in the model, I enhance it by recognizing that, in stablecoin transactions, it is actually the buyers, not the sellers, who are responsible for the gas fees. This fee is incurred when the coin is transferred to another

account for the purpose of purchasing goods, and it is borne by the coin sender, who is the buyer in this model. Using 3, 4 and 5, get

$$V_t^H(r_t, b_t, e_t) = \alpha_1\big[u(y_t^1) - \phi_t r_t^1 - (1+g_t)\psi_t e_t^1\big] + \alpha_2\big[u(y_t^2) - \psi_t b_t^2 - (1+g_t)\psi_t e_t^2\big] + \alpha_3\big[u(y_t^3) - \phi_t r_t^3 - \psi_t b_t^3 - (1+g_t)\psi_t e_t^3\big] + W_t^H(r_t, b_t, e_t). \tag{13}$$

The effective provision of liquidity that influences the terms of trade during period t depends on currency-specific present values at time t as follows:

$$\mathcal{L}_{1,t} = \underbrace{\phi_t r_t}_{\text{Red currency's liquidity}} + \underbrace{\frac{\psi_t}{1+g_t} e_t}_{\text{stablecoin' effective liquidity}} \tag{14}$$

$$\mathcal{L}_{2,t} = \underbrace{\psi_t b_t}_{\text{Blue currency's liquidity}} + \frac{\psi_t}{1+g_t} e_t \tag{15}$$

$$\mathcal{L}_{3,t} = \phi_t r_t + \psi_t b_t + \frac{\psi_t}{1+g} e_t \tag{16}$$

The term of trade y_t depends on whether the liquidity constraint is binding or not. If the buyer holds enough liquidity, the first-best DM solution, $u'(y^*) = c'(y^*)$, can be satisfied. Otherwise, the household spends all the liquidity they have at hand $\mathcal{L}_t$.

Then, the money demand can be solved through the following equations,

$$\frac{\partial}{\partial r_t} V_t^H(r_t, b_t, e_t) = \big[\alpha_1 \lambda(\mathcal{L}_{1,t}) + \alpha_3 \lambda(\mathcal{L}_{3,t}) + 1\big]\phi_t \tag{17}$$

$$\frac{\partial}{\partial b_t} V_t^H(r_t, b_t, e_t) = \big[\alpha_2 \lambda(\mathcal{L}_{2,t}) + \alpha_3 \lambda(\mathcal{L}_{3,t}) + 1\big]\psi_t \tag{18}$$

$$\frac{\partial}{\partial e_t} V_t^H(r_t, b_t, e_t) = \psi_t + \frac{\psi_t}{1+g_t}\big[\alpha_1 \lambda(\mathcal{L}_{1,t}) + \alpha_2 \lambda(\mathcal{L}_{2,t}) + \alpha_3 \lambda(\mathcal{L}_{3,t})\big] \tag{19}$$

Formally, a utility function is determined with constant relative risk aversion (CRRA) as

$$u(y) = \frac{(y+\epsilon)^{1-\sigma} - \epsilon^{1-\sigma}}{1-\sigma}, \tag{20}$$

where y denotes consumption, $\sigma > 0$ is the coefficient of relative risk aversion, and $\epsilon > 0$ is a small constant that ensures a well-defined utility for $y \to 0$.

Then, in the take-it-or-leave-it setting, the liquidity premium is

$$\lambda(\mathcal{L}_t) = max\{u'(\mathcal{L}_t) - 1, 0\} = max\{(\mathcal{L}_t + \epsilon)^{-\sigma} - 1, 0\} \tag{21}$$

Insert 17, 18 and 19 to 6, 7 and 8, and the following Euler's equations can be obtained.

$$\phi_t \geq \beta\phi_{t+1}\big[\alpha_1\lambda(\mathcal{L}_{1,t+1}) + \alpha_3\lambda(\mathcal{L}_{3,t+1}) + 1\big], \tag{22}$$

$$\psi_t \geq \beta\psi_{t+1}\big[\alpha_2\lambda(\mathcal{L}_{2,t+1}) + \alpha_3\lambda(\mathcal{L}_{3,t+1}) + 1\big], \tag{23}$$

$$\psi_t \geq \beta\psi_{t+1} + \beta\frac{\psi_{t+1}}{1+g_{t+1}}\big[\alpha_1\lambda(\mathcal{L}_{1,t+1}) + \alpha_2\lambda(\mathcal{L}_{2,t+1}) + \alpha_3\lambda(\mathcal{L}_{3,t+1})\big]. \tag{24}$$

By the currency market cleaning, the net money growth rate satisfies

$$\mu_t^r = \frac{\hat{r}_{t+1}}{r_t} - 1, \mu_t^b = \frac{\hat{b}_{t+1}}{b_t} - 1, \mu_t^e = \frac{\hat{e}_{t+1}}{e_t} - 1. \tag{25}$$

And the opportunity cost of holding each type of currencies is defined as

$$i_t^r = \frac{1}{\beta}\frac{\phi_t}{\phi_{t+1}} - 1, \tag{26}$$

$$i_t^b = \frac{1}{\beta}\frac{\psi_t}{\psi_{t+1}} - 1, \tag{27}$$

$$i_t^e = \frac{1}{\beta}\frac{\psi_t}{\psi_{t+1}} - 1 = i_t^b. \tag{28}$$

Insert the money growth rate into the Euler's equations and get the following

$$(1+\mu_t^r)\phi_t r_t \geq \alpha_1\beta\lambda(\mathcal{L}_{1,t+1})\phi_{t+1}\hat{r}_{t+1} + \alpha_3\beta\lambda(\mathcal{L}_{3,t+1})\phi_{t+1}\hat{r}_{t+1} + \beta\phi_{t+1}\hat{r}_{t+1} \tag{29}$$

$$(1+\mu_t^b)\psi_t b_t \geq \alpha_2\beta\lambda(\mathcal{L}_{2,t+1})\psi_{t+1}\hat{b}_{t+1} + \alpha_3\beta\lambda(\mathcal{L}_{3,t+1})\psi_{t+1}\hat{b}_{t+1} + \beta\psi_{t+1}\hat{b}_{t+1} \tag{30}$$

$$(1+\mu_t^e)\psi_t e_t \geq \beta\psi_{t+1}\hat{e}_{t+1} + \beta\frac{\psi_{t+1}}{1+g_{t+1}}\Big[\alpha_1\lambda(\mathcal{L}_{1,t+1}) + \alpha_2\lambda(\mathcal{L}_{2,t+1}) + \alpha_3\lambda(\mathcal{L}_{3,t+1})\Big]\hat{e}_{t+1}. \tag{31}$$

Consider the steady state of the problem, where the inflation rate of currencies is steady and equals the growth rate, making the effective DM liquidity of currencies to be the same

over time. Besides, define $g_{t+1} = g_t = g$. Then,

$$r_t\phi_t = \mathcal{L}_r, \qquad b_t\psi_t = \mathcal{L}_b, \qquad \frac{\psi_t}{1+g_t}e_t = \mathcal{L}_e. \tag{32}$$

The effective provision of liquidity becomes

$$\mathcal{L}_1 = \mathcal{L}_r + \mathcal{L}_e, \mathcal{L}_2 = \mathcal{L}_b + \mathcal{L}_e, \mathcal{L}_3 = \mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e. \tag{33}$$

Insert 32 and 33 into 29 - 31 and get

$$(1+\mu^r)\mathcal{L}_r \geq \beta\Big[\alpha_1\lambda(\mathcal{L}_1) + \alpha_3\lambda(\mathcal{L}_3) + 1\Big]\mathcal{L}_r, \tag{34}$$

$$(1+\mu^b)\mathcal{L}_b \geq \beta\Big[\alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3) + 1\Big]\mathcal{L}_b, \tag{35}$$

$$(1+\mu^e)\mathcal{L}_e \geq \beta\left[1 + \frac{1}{1+g_t}\Big(\alpha_1\lambda(\mathcal{L}_1) + \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3)\Big)\right]\mathcal{L}_e. \tag{36}$$

Finally, the Euler's equations in steady state become,

$$i^r = \frac{1+\mu^r}{\beta} - 1 \geq \alpha_1\lambda(\mathcal{L}_1) + \alpha_3\lambda(\mathcal{L}_3), \tag{37}$$

$$i^b = \frac{1+\mu^b}{\beta} - 1 \geq \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3), \tag{38}$$

$$i^e = i^b = \frac{1+\mu^e}{\beta} - 1 \geq \frac{1}{1+g_t}\Big[\alpha_1\lambda(\mathcal{L}_1) + \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3)\Big]. \tag{39}$$

where the equalities hold iff $\mathcal{L}_j > 0$ for $j \in \{r, b, e\}$. This system of equations describes that currency holds value only when the benefits of holding it, which is the potential to purchase goods in the decentralized market, exceed the associated costs, which is the opportunity cost related to a type of currency's interest rate. Therefore, the following proposition can be formulated.

Proposition 1 (Stablecoin Entrance Equilibria)**.** *Then, the monetary equilibria when a stablecoin enters the environment are the following eight regimes: In each regime,* valued *currencies satisfy the binding equalities, and* non-valued *currencies satisfy strict inequalities against the appropriate threshold (the* $\bar{i}^{\cdot(\cdot)}$ *below are evaluated at that regime's* $\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e$*).*

(A) ***No currency valued*** $(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e) = (0,0,0)$*:*

$$i^r \geq (\alpha_1+\alpha_3)\lambda(0), \qquad i^b \geq \max\Big\{(\alpha_2+\alpha_3)\lambda(0),\ \frac{1}{1+g}(\alpha_1+\alpha_2+\alpha_3)\lambda(0)\Big\}.$$

(B) ***Only red valued*** $(\mathcal{L}_r > 0,\ \mathcal{L}_b = \mathcal{L}_e = 0)$*:*

$$i^r = (\alpha_1+\alpha_3)\lambda(\mathcal{L}_r), \qquad i^b > \max\Big\{\bar{i}^{b(r)},\ \frac{1}{1+g}\bar{i}^{e(r)}\Big\}.$$

(C) ***Only blue valued*** $(\mathcal{L}_b > 0,\ \mathcal{L}_r = \mathcal{L}_e = 0)$*:*

$$i^b = (\alpha_2+\alpha_3)\lambda(\mathcal{L}_b)\ >\ \frac{1}{1+g}\bar{i}^{e(b)}, \qquad i^r > \bar{i}^{r(b)}.$$

(D) ***Only stablecoin valued*** $(\mathcal{L}_e > 0,\ \mathcal{L}_r = \mathcal{L}_b = 0)$*:*

$$i^b = \frac{1}{1+g}(\alpha_1+\alpha_2+\alpha_3)\lambda(\mathcal{L}_e)\ >\ \bar{i}^{b(e)}, \qquad i^r > \bar{i}^{r(e)}.$$

(E) ***Red & Blue valued (no*** e***)*** $(\mathcal{L}_r > 0,\ \mathcal{L}_b > 0,\ \mathcal{L}_e = 0)$*:*

$$\begin{aligned} i^r &= \alpha_1\lambda(\mathcal{L}_r) + \alpha_3\lambda(\mathcal{L}_r+\mathcal{L}_b), \\ i^b &= \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_r+\mathcal{L}_b)\ >\ \frac{1}{1+g}\bar{i}^{e(rb)}. \end{aligned}$$

(F) ***Red & stablecoin valued (no*** b***)*** $(\mathcal{L}_r > 0,\ \mathcal{L}_e > 0,\ \mathcal{L}_b = 0)$*:*

$$\begin{aligned} i^r &= (\alpha_1+\alpha_3)\lambda(\mathcal{L}_r+\mathcal{L}_e), \\ i^b &= \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_r+\mathcal{L}_e) + \alpha_2\lambda(\mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r+\mathcal{L}_e)\Big]\ >\ \bar{i}^{b(re)}. \end{aligned}$$

(G) ***Blue & stablecoin valued (no*** r***)*** $(\mathcal{L}_b > 0,\ \mathcal{L}_e > 0,\ \mathcal{L}_r = 0)$*:*

$$\begin{aligned} &i^b = (\alpha_2+\alpha_3)\lambda(\mathcal{L}_b+\mathcal{L}_e) = \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_e) + \alpha_2\lambda(\mathcal{L}_b+\mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_b+\mathcal{L}_e)\Big], \\ &i^r > \bar{i}^{r(be)}. \end{aligned}$$

(H) ***All three valued*** $(\mathcal{L}_r > 0,\ \mathcal{L}_b > 0,\ \mathcal{L}_e > 0)$*:*

$$
\begin{aligned}
i^r &= \alpha_1\lambda(\mathcal{L}_r + \mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e),\\
i^b &= \alpha_2\lambda(\mathcal{L}_b + \mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e)\\
&= \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_r + \mathcal{L}_e) + \alpha_2\lambda(\mathcal{L}_b + \mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e)\Big].
\end{aligned}
$$

$$\bar{i}^{r(b)} = \alpha_1\lambda(0) + \alpha_3\lambda(\mathcal{L}_b), \tag{40}$$

$$\bar{i}^{r(e)} = (\alpha_1 + \alpha_3)\lambda(\mathcal{L}_e), \tag{41}$$

$$\bar{i}^{r(be)} = \alpha_1\lambda(\mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_b + \mathcal{L}_e), \tag{42}$$

$$\bar{i}^{b(r)} = \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r), \tag{43}$$

$$\bar{i}^{b(e)} = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_e), \tag{44}$$

$$\bar{i}^{b(re)} = \alpha_2\lambda(\mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_e), \tag{45}$$

$$\bar{i}^{e(r)} = \alpha_1\lambda(\mathcal{L}_r) + \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r), \tag{46}$$

$$\bar{i}^{e(b)} = \alpha_1\lambda(0) + \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_b), \tag{47}$$

$$\bar{i}^{e(rb)} = \alpha_1\lambda(\mathcal{L}_r) + \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b). \tag{48}$$

The proposition outlined above offers a comprehensive classification of steady-state valuation regimes within a tri-currency framework, associating each regime with a set of inequalities pertaining to opportunity costs and market masses. This taxonomy is instrumental in interpreting regime diagrams and elucidating which currencies can be concurrently supported as media of exchange. Nevertheless, the s ta b le co in payment friction in the model is dictated by the DM transfer fee g, which is typically minimal in practice (often below 0.1%). When g is sufficiently low, the fee wedge is insufficient to sustain regimes that necessitate valuing b while either preventing or delicately balancing stablecoin usage. The subsequent corollary formalizes this empirical constraint: regimes (C), (G), and (E) become obsolete for sufficiently low g, whereas the fully interior regime (H) becomes precarious and is thus unlikely to characterize current payment systems. Consequently, focus can be confined to (A), (B), (D), and (F) as the empirically pertinent cases.

Corollary 1 (Empirically relevant regimes under low gas fees)**.** *In many practical payment environments, the proportional stablecoin transfer fee g is very small (e.g., below 0.1%). When g is sufficiently low, regimes that require valuing the blue currency b while avoiding (or finely balancing) stablecoin usage become infeasible or nongeneric in the pegged environment where $i^e = i^b$.*

In particular, if g is small enough (e.g., $g \leq 10^{-3}$ and $g \leq \alpha_1/(\alpha_2 + \alpha_3)$), then regimes (C) *and* (G) *cannot arise. Moreover, the exclusion condition for the stablecoin in the red–blue coexistence regime* (E) *requires a strictly positive wedge of order g to offset the stablecoin's additional acceptance mass; hence the admissible set for* (E) *collapses as $g \downarrow 0$ and disappears for sufficiently low g in the empirically relevant region. Finally, regime* (H) *is highly restrictive in the pegged case: it imposes an additional knife-edge restriction linking i^b to the fee-adjusted stablecoin benefit, and is therefore unlikely to describe the current world.*

Consequently, under low transfer fees, the regimes of primary empirical interest reduce to (A), (B), (D), *and* (F).

In addition to merely describing the equilibria involving stablecoin participation, I aim to illustrate the differences observed before and after the introduction of stablecoin e. This comparison seeks to determine whether the entry of stablecoin e modifies the conditions for competition within the local currency market scenario. Given that the monetary search framework incorporating dual currencies has become quite standard, I have moved the proof to Appendix. The following section delineates equilibria 1 to 4 within a dual-currency environment, where only the local Red and Blue currencies compete with each other, both of which encounter limitations in decentralized market interactions.

I. **No currency valued** $(\mathcal{L}_r, \mathcal{L}_b) = (0, 0)$:$i_r \geq (\alpha_1 + \alpha_3)\lambda(0), \quad i_b \geq (\alpha_2 + \alpha_3)\lambda(0)$

II. **Only red valued** $(\mathcal{L}_r > 0, \mathcal{L}_b = 0)$: $i_r = (\alpha_1 + \alpha_3)\lambda(\mathcal{L}_r) < (\alpha_1 + \alpha_3)\lambda(0)$, $i_b \geq \bar{i}_b = \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r)$

III. **Only blue valued** $(\mathcal{L}_r > 0, \mathcal{L}_b = 0)$: $i_b = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b) < (\alpha_2 + \alpha_3)\lambda(0)$, $i_r \geq \bar{i}_r = \alpha_1\lambda(0) + \alpha_3\lambda(\mathcal{L}_b)$

IV. **Both red and blue currencies valued** ($\mathcal{L}_r > 0, \mathcal{L}_b > 0$):

$$i_r = \alpha_1\lambda(\mathcal{L}_r) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b) < \bar{i}_r = \alpha_1\lambda(0) + \alpha_3\lambda(\mathcal{L}_b),$$
$$i_b = \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b) < \bar{i}_b = \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r)$$

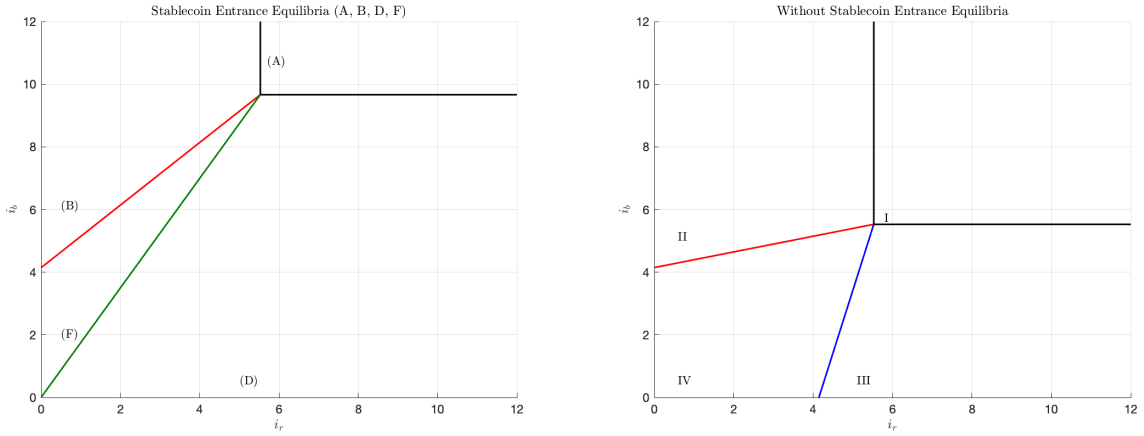


Figure 1: Regime comparison: with or without stablecoin

The figure 1 depicts the equilibria associated with the entrance of stablecoins (left) and the absence of stablecoin entrance (right). It is evident that the area representing the red currency-only regime contracts from (II) to (B). The region where the red currency retains its value diminishes from (II) and (IV) to (B) and (F). This phenomenon is primarily due to the presence of an approximately 45% degree line in the equilibria with stablecoins, indicated by the green line. This line suggests that stablecoins serve as an (almost) perfect substitute in all transactions when the gas fee g is sufficiently low. Consequently, only in scenarios where the holding cost of the red currency is lower than that of the stablecoin can the red currency maintain its value.

By analytically comparing these two environments, the conditions for the local currency to be valued can be obtained.

Proposition 2 (Competition Intensification)**.** *The stablecoin entry tightens the survival conditions for the local currency in the following sense:*

Holding i^b fixed and comparing the red-currency survival cutoff with and without the stablecoin,

$$\bar{i}^{r(be)} < \bar{i}^{r},$$

whenever the stablecoin is valued ($\mathcal{L}_e > 0$)

The analysis indicates that the introduction of a more efficient medium of exchange intensifies currency competition by reducing the range of inflation (or equivalently, money-growth) policies under which the local currency can maintain their value in equilibrium. Within the model, the entry of stablecoins lowers the survival threshold for a weak currency: given the opportunity cost of the rival currency, the domestic currency ceases to be valued at a significantly lower threshold than in the scenario without the stablecoin. This mechanism constrains the local central bank's policy space, particularly its capacity to rely on inflation taxation and seigniorage, thereby diminishing monetary sovereignty in a precise, equilibrium context. Concurrently, the availability of a widely usable alternative can discipline high-inflation outcomes (Nakano (2025)): when domestic inflation rises to a certain level, households shift transactions toward the alternative medium of exchange, thereby reducing their exposure to domestic inflation taxes. The effectiveness of this discipline is contingent upon the transaction wedge associated with the stablecoin. When the stablecoin payment friction, represented by the gas fee g, is minimal—as is often the case on netted or off-chain rails—the effective liquidity provided by the stablecoin is greater, and the tightening of domestic-currency survival conditions is correspondingly more pronounced.

These implications are particularly significant in dollarized economies. If the rival currency is the U.S. dollar, a USD-pegged stablecoin effectively extends dollarization by providing a dollar-pegged medium of exchange through a new payment rail, potentially accelerating the displacement of local currency usage in retail transactions. Simultaneously, stablecoin-based dollarization may reallocate the benefits associated with supplying a transaction medium: stablecoin adoption can substitute away from physical dollar cash and shift "seigniorage-like" revenues from the issuer of fiat liabilities toward stablecoin intermediaries and their balance-sheet structures. Consequently, the welfare and fiscal implications of stablecoin entry are not confined to the local economy; they also affect the incentives and revenue implications for the anchor-currency jurisdiction. These cross-border feedbacks underscore the necessity to evaluate stablecoin adoption not only as a domestic monetary-sovereignty issue but also as a form of payment-rail competition that

	i^r	i^b	g	α_1	α_2	α_3
$\mathcal{L}_r$	$-$	$+$	$+$	$+$	$-$	$+$
$\mathcal{L}_b$	$+$	$-$	$+$	$-$	$+$	$+$
$\mathcal{L}_e$	$+$	$+$	$-$	$+$	$+$	$+$
$\bar{i}^{r(be)}$	0^*	$+$	0^*	$+$	0^*	$+$
$\bar{i}^{b(er)}$	$+$	0^*	0^*	0^*	$+$	$+$
$\bar{i}^{e(rb)}$	$+$	$+$	0^*	$+$	$+$	$+$

Table 1: Comparative statics (rival-currency peg).

reshapes the incidence of monetary rents and the distribution of policy constraints across countries.

Then, comparative statics are calculated for the, where the stablecoin is priced in the rival currency ($i^e = i^b$), and we do not treat i^e as an independent comparative-statics direction. Instead, stablecoin-specific frictions are summarized by the DM payment wedge g, which affects the stablecoin's effective liquidity provision in the decentralized market. The results are shown in Table 1 (please see the proof in Appendix).

Note that entries 0^* indicate no direct term in the corresponding threshold formula; general equilibrium spillovers may still arise via $\mathcal{L}$. Formally, if a threshold

$$\bar{i}^{r(be)} = \alpha_1 \lambda(\mathcal{L}_e) + \alpha_3 \lambda(\mathcal{L}_b + \mathcal{L}_e)$$

has no g in its formula, then the *partial derivative at fixed liquidities* is

$$\frac{\partial \bar{i}^{r(be)}}{\partial g}\Big|_{\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e \text{ fixed}} = 0 \quad (= 0^*).$$

However, in equilibrium the liquidities $\mathcal{L} = (\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e)$ *do* move with g, so the *total effect* uses the chain rule:

$$\frac{d\bar{i}^{r(be)}}{dg} = \alpha_1 \lambda'(\mathcal{L}_e) \frac{d\mathcal{L}_e}{dg} \; + \; \alpha_3 \lambda'(\mathcal{L}_b + \mathcal{L}_e) \left(\frac{d\mathcal{L}_b}{dg} + \frac{d\mathcal{L}_e}{dg} \right).$$

The table implies the following patterns. The holding costs for the red and rival (blue) currencies, i^r and i^b, are negatively associated with their own currency balances, consistent with standard portfolio substitution. In addition, a larger DM payment wedge g reduces

the stablecoin's effective liquidity provision and hence lowers $\mathcal{L}_e$, while shifting liquidity demand toward the other currencies. Finally, for the red and blue currencies, a larger mass of sellers who *reject* the corresponding currency (i.e., lower acceptance probabilities governed by α_1 or α_2) depresses the equilibrium currency balance, with spillovers operating through the endogenous liquidity aggregator $\mathcal{L}$.

Remark 1 (Regime maps beyond rival-currency pegs)**.** *The environment in the main text is rival-currency-based, where the stablecoin is valued in terms of the rival currency, so that its valuation co-moves with the rival-currency pricing kernel (i.e., the pegged slice $\omega_t = \psi_t$). Here, I relax the peg restriction in a generalized specification where the stablecoin carries an independent CM valuation ω_t, which can be interpreted as pegging to a third-country currency or a commodity such as gold, and hence need not be tied to either domestic (red) or rival (blue) pricing kernels. Appendix provides the complete characterization of the eight regimes and the associated boundary conditions in this independent-ω case together with proof details. To summarize the equilibria under stablecoin entry beyond rival-currency pegs, regime visualizations are reported.*

Figures 2 and 3 illustrate equilibria (A) to (H), as described in 1. Figure 2 presents two-dimensional representations, as one of the three currencies is not valued in equilibrium. The thresholds for various scenarios are linear according to these equations. The second and third graphs in Figure 2 feature 45-degree lines, illustrating the condition that the holding cost of the red currency (i^r) or the blue currency (i^b) cannot surpass (i^e) for coexistence with the stablecoin. Consequently, the region where the stablecoin is exclusively valued is relatively large, benefiting from its universal acceptance. Figure 3 depicts two versions of the rotation of regime (H) in a 3-D plot. All currencies are valued within the green area, whereas the gray plane signifies the extension of the 45-degree threshold line in the two-dimensional plot shown in Figure 2.

Comparative statics are calculated as in Table 2 , with details and proof provied in the Appendix.

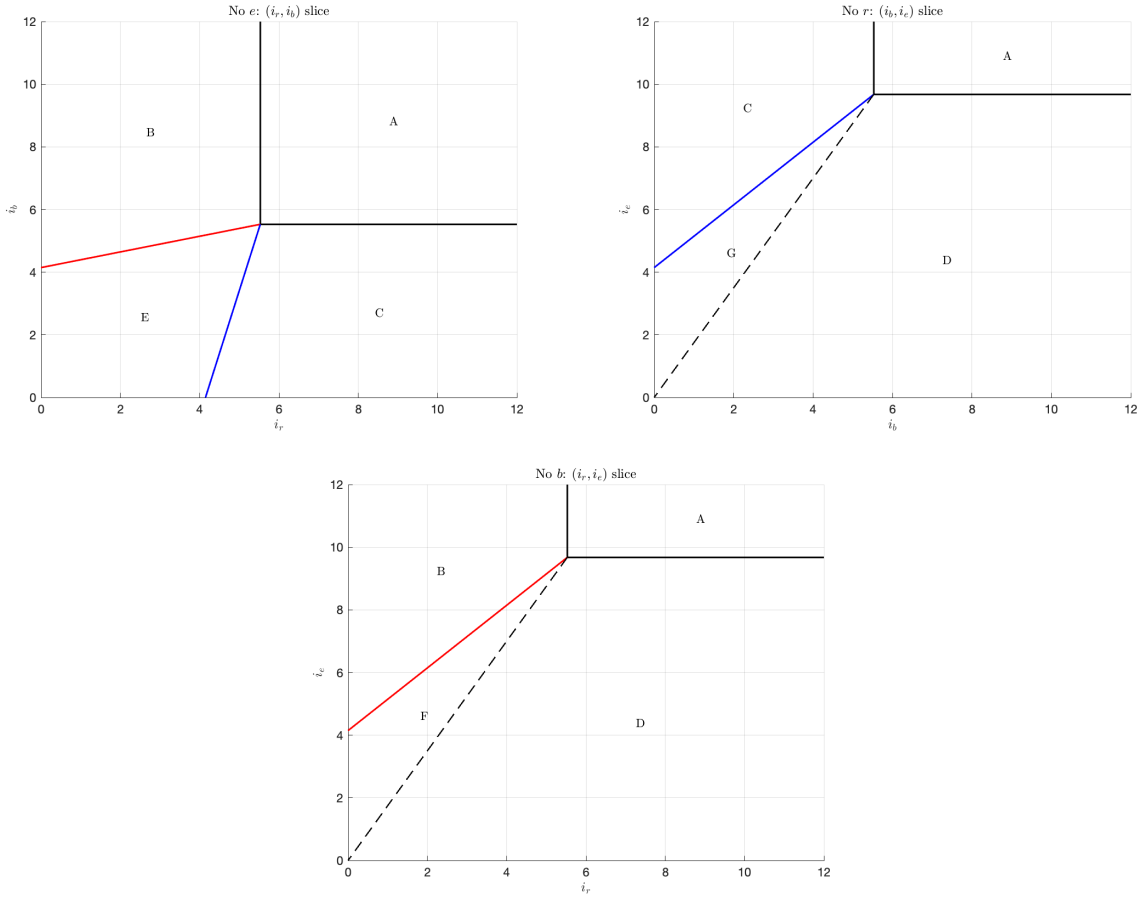


Figure 2: Regime maps beyond rival-currency pegs: 2D

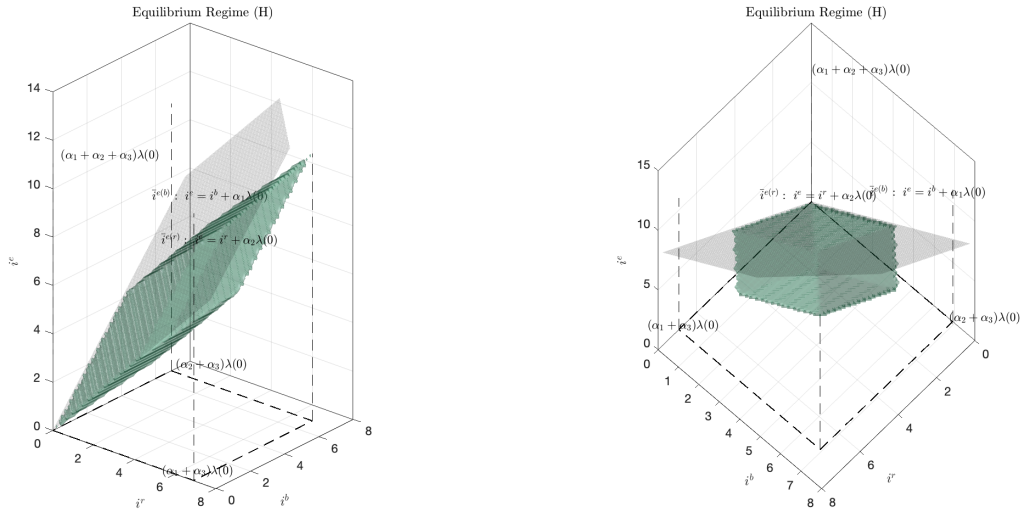


Figure 3: Regime maps beyond rival-currency pegs: 3D

	i^r	i^b	i^e	α_1	α_2	α_3
$\mathcal{L}_r$	$-$	$+$	$+$	$+$	$-$	$+$
$\mathcal{L}_b$	$+$	$-$	$+$	$-$	$+$	$+$
$\mathcal{L}_e$	$+$	$+$	$-$	$+$	$+$	$+$
$\bar{i}^{r(be)}$	0^*	$+$	$+$	$+$	0^*	$+$
$\bar{i}^{b(er)}$	$+$	0^*	$+$	0^*	$+$	$+$
$\bar{i}^{e(rb)}$	$+$	$+$	0^*	$+$	$+$	$+$

Table 2: Comparative statics.

Parameters	**Notation**	**Value**	**Calibration Targets**
Relative risk aversion	σ	0.9	Standard and constant
Smoothing term	ϵ	0.05	Standard and constant
Discount factor	β	0.98	Standard and constant
Blue currency /stablecoin's inflation rate	μ_b	2.7%	USD inflation rate
Non-pegged stablecoin's inflation rate	μ_e	3-5%	
probability of meeting type 1 sellers	α_1	30% (/50%)	
probability of meeting type 1 sellers	α_2	30% (/50%)	
probability of meeting type 1 sellers	α_3	10%	

Table 3: Calibration Parameters

3 Calibration

In this section, the parameters listed in Table 3 are used for calibration.

In the model, a liquidity premium following Chiu et al. (2023) is incorporated, denoted by $\lambda(z)$, which captures the additional value that agents require to hold a medium of exchange with limited liquidity. Conceptually, $\lambda(z)$ measures the gap between the marginal utility of consumption and the effective marginal benefit of holding a claim, considering the probability that the medium of exchange can be converted into consumption. This is analogous to the liquidity premium in financial markets, where less liquid assets require higher expected returns to compensate for their illiquidity. Then, based on the specification in 21, the following graph can be obtained:

Intuitively, when $\mathcal{L} < \mathcal{L}^*$, the liquidity premium $\lambda(\mathcal{L})$ is positive and increases as z decreases, indicating that agents require higher compensation for holding less liquid claims. Conversely, when $\mathcal{L} \geq \mathcal{L}^*$, the claim is sufficiently large such that liquidity constraints are nonbinding, resulting in $\lambda(\mathcal{L}) = 0$. This graph illustrates the strictly decreasing pattern of the liquidity premium function, which substantiates the proof of the first proposition.

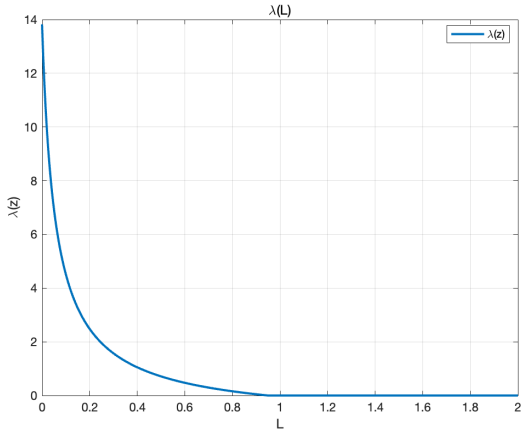


Figure 4: Liquidity Premium

Subsequently, the calibration aligns the monetary competition framework with a fiscal perspective, which is the seigniorage income of the local government. Seigniorage income, also referred to as inflation tax income, is an important and distinctive source of government funding, particularly for smaller nations where generating labor income requires additional expenditure on law enforcement. This revenue stream originates from the difference between the nominal value of currency and its production cost, which results in a financial benefit to the government from monetary expansion, given that the cost of producing fiat money is minimal.

To examine this mechanism, a seigniorage Laffer curve is then constructed.The Laffer curve, traditionally used to illustrate the relationship between tax rates and government revenue, can also be applied to elucidate the seigniorage revenue of a government. At low levels of money creation, the government accrues more revenue as it increases the money supply. However, a threshold exists beyond which excessive money printing results in high inflation. High inflation diminishes the value of money, thereby reducing the government's revenue. This phenomenon creates a curve analogous to the original Laffer curve. The curve indicates the optimal level of money creation to maximize government revenue. Beyond this point, further money creation becomes counterproductive to the government's financial position. The per-period seignorage laffer curve is calculated as:

$$S = i \times \mathcal{L} \tag{49}$$

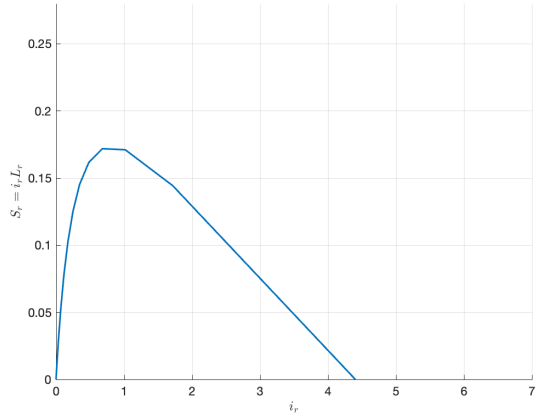


Figure 5: Laffer Curve with $\alpha_1 = \alpha_2 = 0.3$, and $\alpha_3 = 0.1$

By applying the seigniorage Laffer curve framework, this study demonstrates how the invasion of stablecoins negatively affects the capacity of governments to extract seigniorage, thereby clarifying the trade-off between inflation and revenue in an increasingly competitive monetary environment. I first apply the dual currency competition game: as in the environment in Appendix A, only two local currencies compete with each other. In this simpler environment, I compare how different acceptance rates of the local currency can alter the seignorage income. Then, I apply the triple currency model explained in the former chapter (two local weak currencies competing with a stablecoin (strong) currency), showing the seignorage lost by the local government.

Seigniorage revenue for the Red government is also related to the inflation of the blue currency μ^b. The real balances of the red and blue currencies are then solved by the following joint functions 59 and 60. The introduction of the stablecoin e disrupts this equilibrium through substitution effects. The real balance, red currency, blue currency, and stablecoin are then solved using the joint functions 29,30, and 31.

Figure 5 presents a symmetric monetary regime where transaction types excluding the Red or Blue currency occur with equal probability. In this equilibrium, both currencies face limitations during transactions, creating a hump-shaped Laffer curve. Then, I change the values of the acceptance rates α_1, α_2, and α_3 to highlight how shifts in sellers' currency preferences can alter the seigniorage revenue potential, providing insights into the interaction between currency acceptance patterns and optimal seigniorage extraction.

Figure 6 and 7 illustrate the variations in the seigniorage Laffer curve under different

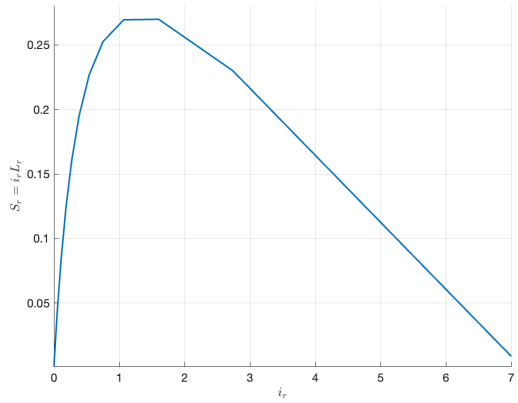


Figure 6: Laffer Curve when $\alpha_1 = 0.5$,$\alpha_2 = 0.3$, and $\alpha_3 = 0.1$

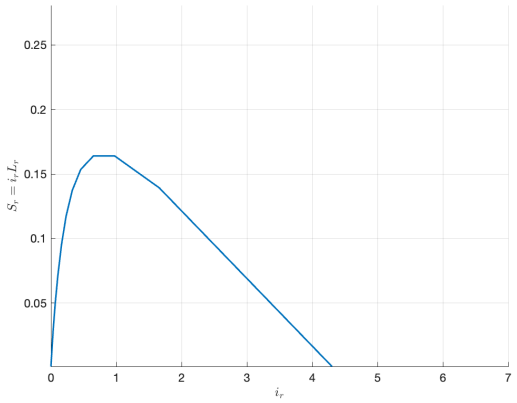


Figure 7: Laffer Curve when $\alpha_1 = 0.3$, $\alpha_2 = 0.5$ and $\alpha_3 = 0.1$

distributions of currency acceptance among sellers. In figure 6, the curve reflects a higher proportion of α_1, representing sellers who exclusively accept the red currency in transactions, while all other parameters are held constant, as in figure 5. Conversely, figure 7 depicts a scenario with a larger fraction of α_2, corresponding to sellers who only accept the blue currency, again maintaining the same parameter values as in figure 5.

A particularly important finding emerges from this comparison: the red government's seigniorage Laffer curve shifts upward only when α_1 increases, indicating that changes in the proportion of red currency–only sellers have a direct and positive impact on potential seigniorage revenue. In contrast, variations in α_2 do not produce a comparable shift in the spectrum. This result underscores the critical role of red-currency-only sellers in shaping revenue-maximizing policies for monetary authorities and illustrates how the distribution of currency acceptance can materially influence seigniorage potential.

Finally, the concluding figure illustrates the stablecoin-entrance model, wherein the stablecoin is pegged to the blue currency. In the absence of supply constraints from the stablecoin issuer, this model represents a distinctive application of the monetary search framework employed in this study, integrating both weak and strong currencies. Figure 8 reveals a significant reduction in seigniorage for the red currency, even when the acceptance rate parameters remain constant, as depicted in the initial figure. Furthermore, Figure 9 presents a series of Laffer curves, demonstrating the effects when the stablecoin's inflation rate is pegged to a third country's currency or assets, resulting in variations from 3% to 4%

or 5%. In this specific plot, the Y-axis has been enlarged to highlight the differences. Additionally, the figure identifies a cutoff point beyond which the red currency loses all value, illustrating a critical threshold in currency competition, consistent with the proposition from the preceding section. This finding provides a clear depiction of how the emergence of a robust stablecoin can undermine local monetary sovereignty, highlighting the potential challenges that such digital currencies pose to traditional government-issued currencies.

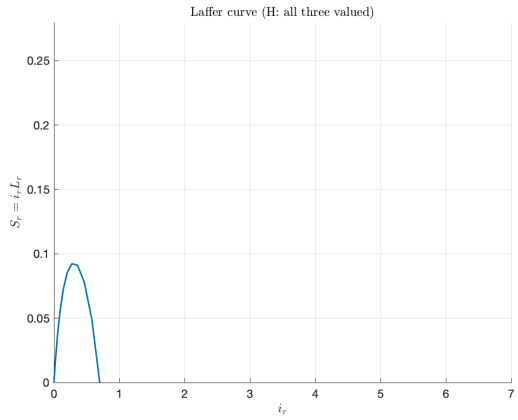


Figure 8: Laffer Curve with stablecoin entrance

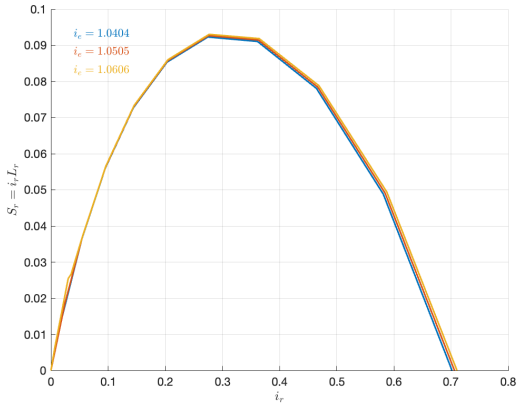


Figure 9: Laffer Curve with stablecoin entrance (3% to 5% inflation rate, enlarged Y-axis)

The findings from the calibration have important policy implications for monetary authorities facing the rise of stablecoins in the market. First, the asymmetric impact of sellers' currency preferences indicates that local governments should monitor the adoption of domestic currency relative to competing private currencies, as shifts in acceptance among key economic agents can significantly influence the seigniorage revenue. Second, the three-currency model highlights that the introduction of a strong stablecoin can drastically erode the value and revenue-generating capacity of a national currency, even when traditional acceptance rates are unchanged. This suggests that policymakers may need to consider regulatory measures, such as promoting the wider acceptance of domestic currency, enhancing the convenience and security of government-issued digital currencies (central bank digital currencies), or imposing usage frameworks for stablecoins to safeguard monetary sovereignty. Overall, the results emphasize that proactive management of currency competition is essential for maintaining fiscal stability and preserving the efficacy of seigniorage as a source of government revenue.

4 Conclusion

This study elucidates the strategic dynamics of currency competition from the fast-developing digital currency system in reshaping monetary sovereignty and fiscal capacities.The analysis indicates that the introduction of a stablecoin, which possesses the capability for universal transactional acceptance, fundamentally transforms the equilibrium conditions for existing currencies. The valuation thresholds for local (weak) currencies are more stringent compared to a regime without the introduction of stablecoins, indicating heightened competition due to potential currency substitution. Given that stablecoin issuers facilitate the holding of the US dollar, it is reasonable to assert that the prominent role delineated in this model is justified as a technical extension of the traditional dollarization issue. Consequently, it is imperative for governments to consider reducing the holding costs of sovereign currencies to ensure their continued valuation.

Given the increasing complexity of the global monetary landscape, the proposed framework serves as an essential instrument for analyzing the intensification of currency competition, encompassing both domestic and foreign currency dynamics, as well as the interplay between fiat money and stablecoins. Within this framework, the microfoundations clarifies how a blue-pegged stablecoin can penetrate trade networks and reshape traditional monetary hierarchies. By extending standard dual-currency monetary search frameworks with an additional entrant that is pegged on a rival currency while facing a DM-specific transaction fee g, the model provides a scalable way to quantify how widely accepted digital payment rails shift currency substitution and intensify international monetary competition. Importantly, when g is sufficiently small, transactional demand can migrate not only away from a weak local currency but also away from the underlying blue fiat, shrinking the blue real-balance base and thereby reducing the blue country's seigniorage; fiscal and welfare consequences may therefore arise on both sides of the currency hierarchy.

These findings underscore the need for policymakers to address the risks of digital currency substitution. A clear regulatory perimeter for stablecoins and private payment platforms, such as covering issuer authorization and supervision, high-quality reserve and redemption standards, disclosure or attestation, and compliance obligations, can mitigate

the likelihood of unchecked monetary displacement.[1] With well-designed oversight, digital currencies can be integrated in ways that safeguard monetary integrity while still allowing the benefits of financial innovation.

The analysis provides a foundation for studying welfare and policy design in multi-currency systems within a non-cooperative monetary framework. A natural next step is to quantify the welfare consequences of stablecoin entry which moves beyond a purely government-centric lens to capture broader household and firm outcomes. Moreover, endogenizing stablecoin issuance by incorporating the issuer's decision problem would deepen our understanding of public–private currency interactions and clarify the incentives, constraints, and equilibrium trade-offs that arise as monetary systems become increasingly digitalized.

[1] For example, the U.S. GENIUS Act establishes a federal framework for payment stablecoins.

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Appendix A: Dual-currency equilibria

Proof. [2]

Household in CM. Given balances (r_t, b_t), the representative household solves

$$W_t^H(r_t, b_t) = \max_{x_t, \ell_t, \hat{r}_{t+1}, \hat{b}_{t+1}} \left\{ U(x_t) - \ell_t + \beta \left[V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}) \right] \right\}$$

subject to

$$x_t = \phi_t(r_t - \hat{r}_{t+1}) + \psi_t(b_t - \hat{b}_{t+1}) + \ell_t + T_t.$$

The CM FOC for consumption implies $U'(x_t) = 1$; the envelope conditions are

$$\frac{\partial W_t^H}{\partial r_t} = \phi_t, \tag{50}$$

$$\frac{\partial W_t^H}{\partial b_t} = \psi_t. \tag{51}$$

Portfolio optimality (next-period post-adjustment holdings) gives

$$\phi_t = \beta \frac{\partial}{\partial \hat{r}_{t+1}} V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}), \tag{52}$$

$$\psi_t = \beta \frac{\partial}{\partial \hat{b}_{t+1}} V_{t+1}^H(\hat{r}_{t+1}, \hat{b}_{t+1}). \tag{53}$$

DM side, seller types, and effective liquidity. Buyers meet type-j sellers with probabilities $\alpha_j \in (0, 1)$, $j \in \{1, 2, 3\}$, accepting red only ($j = 1$), blue only ($j = 2$), or both ($j = 3$). The effective liquidities are

$$\mathcal{L}_{1,t} = \phi_t r_t, \quad \text{(red only)} \tag{54}$$

$$\mathcal{L}_{2,t} = \psi_t b_t, \quad \text{(blue only)} \tag{55}$$

$$\mathcal{L}_{3,t} = \phi_t r_t + \psi_t b_t.$$

Under the bargaining rule where buyers make take it or leave it offers to the sellers, the

[2] This proof follows the framework from Zhu and Hendry (2022).

DM value for a household can be written

$$V_t^H(r_t, b_t) = \alpha_1\big[u(y_t^1) - \phi_t r_t^1\big] + \alpha_2\big[u(y_t^2) - \psi_t b_t^2\big] + \alpha_3\big[u(y_t^3) - \phi_t r_t^3 - \psi_t b_t^3\big] + W_t^H(r_t, b_t). \tag{56}$$

DM marginal values and Euler equations. Differentiating (56) and using (50)–(51)yields

$$\frac{\partial V_t^H}{\partial r_t} = \alpha_1\,\lambda(\phi_t r_t)\,\phi_t + \alpha_3\,\lambda(\phi_t r_t + \psi_t b_t)\,\phi_t + \beta\,\phi_t, \tag{57}$$

$$\frac{\partial V_t^H}{\partial b_t} = \alpha_2\,\lambda(\psi_t b_t)\,\psi_t + \alpha_3\,\lambda(\phi_t r_t + \psi_t b_t)\,\psi_t + \beta\,\psi_t. \tag{58}$$

Insert (57)–(58) into (52)–(53) (at t+1). With $\mathcal{L}_{r,t} := \phi_t r_t$ and $\mathcal{L}_{b,t} := \psi_t b_t$, this gives

$$(1+\mu_{t+1}^r)\,\mathcal{L}_{r,t} = \alpha_1\beta\,\lambda(\mathcal{L}_{r,t+1})\,\mathcal{L}_{r,t+1} + \alpha_3\beta\,\lambda(\mathcal{L}_{r,t+1}+\mathcal{L}_{b,t+1})\,\mathcal{L}_{r,t+1} + \beta\,\mathcal{L}_{r,t+1}, \tag{59}$$

$$(1+\mu_{t+1}^b)\,\mathcal{L}_{b,t} = \alpha_2\beta\,\lambda(\mathcal{L}_{b,t+1})\,\mathcal{L}_{b,t+1} + \alpha_3\beta\,\lambda(\mathcal{L}_{r,t+1}+\mathcal{L}_{b,t+1})\,\mathcal{L}_{b,t+1} + \beta\,\mathcal{L}_{b,t+1}. \tag{60}$$

Stationary characterization and regimes. In a steady state with constant (μ^r, μ^b) and (z^r, z^b), define $i^r := \frac{1+\mu^r}{\beta} - 1$, $i^b := \frac{1+\mu^b}{\beta} - 1$, $\mathcal{L}_r := z^r$, $\mathcal{L}_b := z^b$. Then "valued" means the corresponding equality binds, while "not valued" means strict inequality.

$$\mathcal{L}_r > 0 \;\Rightarrow\; i^r = \alpha_1\lambda(\mathcal{L}_r) + \alpha_3\lambda(\mathcal{L}_r+\mathcal{L}_b), \quad \mathcal{L}_r = 0 \;\Rightarrow\; i^r > \alpha_1\lambda(0) + \alpha_3\lambda(\mathcal{L}_b),$$

$$\mathcal{L}_b > 0 \;\Rightarrow\; i^b = \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_r+\mathcal{L}_b), \quad \mathcal{L}_b = 0 \;\Rightarrow\; i^b > \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r).$$

These deliver the four cases: (i) no currency valued, (ii) only red valued, (iii) only blue valued, and (iv) both valued, as stated in the text. □

Appendix B: Proof of Proposition 1 and Corollary 1

The proof of the Proposition 1 is as the following.

. Use the equilibrium conditions established earlier in the household section. A currency is *valued* (positive steady holdings) if and only if its holding benefit equals its opportunity cost (binding equality), and it is *not valued* if and only if the opportunity cost is strictly higher than the benefit (strict inequality).

(A) No currency valued. At $(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e) = (0, 0, 0)$, the three benefits are

$$(\alpha_1 + \alpha_3)\lambda(0), \qquad (\alpha_2 + \alpha_3)\lambda(0), \qquad \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda(0),$$

so regime (A) is equivalent to the three non-valuation inequalities in the proposition. If any $\mathcal{L}_k > 0$, then at least one argument of $\lambda(\cdot)$ increases strictly, so the corresponding benefit falls strictly below its value at the origin; hence no other solution exists.

(B) Only red valued. Set $(\mathcal{L}_b, \mathcal{L}_e) = (0, 0)$ so $(\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3) = (\mathcal{L}_r, 0, \mathcal{L}_r)$. The red equality becomes

$$i^r = (\alpha_1 + \alpha_3)\lambda(\mathcal{L}_r),$$

which pins down a unique $\mathcal{L}_r > 0$ whenever $i^r < (\alpha_1 + \alpha_3)\lambda(0)$. The two strict inequalities that exclude valuation of b and e reduce to the single condition

$$i^b > \max\left\{\bar{i}^{b(r)},\ \frac{1}{1+g}\bar{i}^{e(r)}\right\},$$

as stated in (B).

(C) Only blue valued. Set $(\mathcal{L}_r, \mathcal{L}_e) = (0, 0)$ so $(\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3) = (0, \mathcal{L}_b, \mathcal{L}_b)$. The blue equality becomes

$$i^b = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b),$$

which pins down a unique $\mathcal{L}_b > 0$ whenever $i^b < (\alpha_2 + \alpha_3)\lambda(0)$. Excluding e is exactly

$$i^b > \frac{1}{1+g}\bar{i}^{e(b)},$$

and excluding r is $i^r > \bar{i}^{r(b)}$, giving (C).

(D) Only Stablecoin valued. Set $(\mathcal{L}_r, \mathcal{L}_b) = (0, 0)$ so $(\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3) = (\mathcal{L}_e, \mathcal{L}_e, \mathcal{L}_e)$. Valuing e means

$$i^b = \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda(\mathcal{L}_e),$$

which pins down a unique $\mathcal{L}_e > 0$ whenever $i^b < \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda(0)$. Excluding b and r gives $i^b > \bar{i}^{b(e)}$ and $i^r > \bar{i}^{r(e)}$, yielding (D).

(E) Red & Blue valued (no e). Set $\mathcal{L}_e = 0$ so $(\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3) = (\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_r + \mathcal{L}_b)$ and consider

$$\begin{aligned} i^r &= \alpha_1\lambda(\mathcal{L}_r) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b), \\ i^b &= \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b), \end{aligned} \qquad \mathcal{L}_r > 0,\ \mathcal{L}_b > 0.$$

For each fixed $\mathcal{L}_b \geq 0$, the first equation is strictly decreasing in $\mathcal{L}_r$ and hence pins down a unique $\mathcal{L}_r$; substituting into the second yields a continuous strictly decreasing equation in $\mathcal{L}_b$, so existence and uniqueness follow by IVT and strict monotonicity. Excluding e is

$$i^b > \frac{1}{1+g}\bar{i}^{e(rb)},$$

which is exactly (E).

(F) Red & Stablecoin valued (no b). Set $\mathcal{L}_b = 0$ so $\mathcal{L}_1 = \mathcal{L}_3 = \mathcal{L}_r + \mathcal{L}_e$ and $\mathcal{L}_2 = \mathcal{L}_e$. Valuing r gives

$$i^r = (\alpha_1 + \alpha_3)\lambda(\mathcal{L}_r + \mathcal{L}_e),$$

so $\mathcal{L}_r + \mathcal{L}_e$ is pinned down uniquely. Valuing e gives

$$i^b = \frac{1}{1+g}\Big[(\alpha_1 + \alpha_3)\lambda(\mathcal{L}_r + \mathcal{L}_e) + \alpha_2\lambda(\mathcal{L}_e)\Big] = \frac{1}{1+g}\Big[i^r + \alpha_2\lambda(\mathcal{L}_e)\Big],$$

so

$$\lambda(\mathcal{L}_e) = \frac{(1+g)i^b - i^r}{\alpha_2},$$

which pins down a unique $\mathcal{L}_e$ (when the RHS lies in $[0, \lambda(0)]$), and then $\mathcal{L}_r$ is unique. Excluding b is $i^b > \bar{i}^{b(re)}$, yielding (F).

(G) Blue & Stablecoin valued (no r). Set $\mathcal{L}_r = 0$ so $\mathcal{L}_1 = \mathcal{L}_e$ and $\mathcal{L}_2 = \mathcal{L}_3 = \mathcal{L}_b + \mathcal{L}_e$. Valuing b gives

$$i^b = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b + \mathcal{L}_e).$$

Valuing e gives

$$i^b = \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_e) + (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b + \mathcal{L}_e)\Big] = \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_e) + i^b\Big],$$

equivalently

$$g\, i^b = \alpha_1\lambda(\mathcal{L}_e),$$

which pins down $\mathcal{L}_e$ uniquely and then pins down $\mathcal{L}_b$ uniquely through the blue equality. Excluding r is $i^r > \bar{i}^{r(be)}$, yielding (G).

(H) All three valued. The three equalities are

$$\begin{aligned} i^r &= \alpha_1\lambda(\mathcal{L}_1) + \alpha_3\lambda(\mathcal{L}_3), \\ i^b &= \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3), \\ i^b &= \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_1) + \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3)\Big]. \end{aligned}$$

From the second and third equalities,

$$g\, i^b = \alpha_1\lambda(\mathcal{L}_1),$$

so $\mathcal{L}_1$ is pinned down uniquely. From $(1+g)$ times the third equality minus the first equality,

$$(1+g)i^b - i^r = \alpha_2\lambda(\mathcal{L}_2),$$

so $\mathcal{L}_2$ is pinned down uniquely. From the second equality,

$$i^b - \alpha_2\lambda(\mathcal{L}_2) = \alpha_3\lambda(\mathcal{L}_3),$$

so $\mathcal{L}_3$ is pinned down uniquely. Then

$$\mathcal{L}_e = \mathcal{L}_1 + \mathcal{L}_2 - \mathcal{L}_3, \qquad \mathcal{L}_r = \mathcal{L}_3 - \mathcal{L}_2, \qquad \mathcal{L}_b = \mathcal{L}_3 - \mathcal{L}_1,$$

so $(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e)$ is uniquely determined whenever the implied values are strictly positive.

For each $k \in \{r, b, e\}$, the conditions "$\mathcal{L}_k > 0$" and "$\mathcal{L}_k = 0$" correspond to a binding equality versus a strict inequality with the same strictly decreasing $\lambda(\cdot)$ terms. Hence two distinct regimes cannot hold simultaneously except on knife-edge boundaries where a non-valued currency satisfies its inequality at equality; such boundaries have measure zero and do not generate an additional interior equilibrium. □

The proof of the Corollary 1 is as the following.

Proof. Because $i^e = i^b$ in the CM, valuing b without (or with only a finely balanced amount of) e requires the DM fee wedge g to be large enough to offset the extra liquidity benefit provided by stablecoin acceptance.

Regime (C). With $(\mathcal{L}_r, \mathcal{L}_e) = (0, 0)$, the condition that e is not valued can be written as

$$i^b > \frac{1}{1+g}\bar{i}^{e(b)} \quad \Longleftrightarrow \quad g\, i^b > \alpha_1 \lambda(0).$$

But in (C), $i^b = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b) \leq (\alpha_2 + \alpha_3)\lambda(0)$, hence a necessary condition is

$$g(\alpha_2 + \alpha_3)\lambda(0) > \alpha_1 \lambda(0) \quad \Longleftrightarrow \quad g > \frac{\alpha_1}{\alpha_2 + \alpha_3}.$$

Therefore, if $g \leq \alpha_1/(\alpha_2 + \alpha_3)$ (in particular, for sufficiently small g), regime (C) is ruled out.

Regime (G). When $(\mathcal{L}_r = 0, \mathcal{L}_b > 0, \mathcal{L}_e > 0)$, imposing both equalities in (G) implies the same type of feasibility restriction: the small fee wedge must be large enough to prevent stablecoin liquidity from strictly dominating blue liquidity under $i^e = i^b$. This again requires g not to be too small; in particular, $g \leq \alpha_1/(\alpha_2 + \alpha_3)$ rules out (G).

Regime (E). In the red–blue coexistence regime with $\mathcal{L}_e = 0$, excluding stablecoin

valuation requires

$$i^b > \frac{1}{1+g}\bar{i}^{e(rb)} \quad \Longleftrightarrow \quad g\, i^b > \alpha_1 \lambda(\mathcal{L}_r),$$

where $\mathcal{L}_r > 0$ is pinned down by the red equation in (E). As $g \downarrow 0$, the left-hand side vanishes while the right-hand side remains strictly positive whenever $\alpha_1 > 0$ and $\mathcal{L}_r$ is finite; hence the admissible set for (E) shrinks and is absent for sufficiently low g in the empirically relevant region.

Regime (H). In the pegged case, (H) requires simultaneously satisfying the blue equality and the fee-adjusted stablecoin equality using the same opportunity cost i^b. This imposes an additional knife-edge restriction (of codimension one in (i^r, i^b)), making (H) non-robust as an empirical description.

Combining these observations yields the stated reduction in empirically relevant regimes.

□

Appendix C: Proof of Proposition 2

The proof of the proposition 2 is as the following

Proof. Throughout, use the fact that $\lambda(\cdot)$ is strictly decreasing (see 21).

- *Without stablecoin (dual-currency benchmark).* When only the blue currency is circulating, its liquidity $\mathcal{L}_b$ satisfies

$$i^b = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b).$$

The red-currency survival cutoff is

$$\bar{i}^r = \alpha_1\lambda(0) + \alpha_3\lambda(\mathcal{L}_b) = \alpha_1\lambda(0) + \frac{\alpha_3}{\alpha_2 + \alpha_3} i^b.$$

- *With stablecoin.* Consider the equilibrium in which the blue currency and the stablecoin are valued while the red currency is not. Let $\mathcal{L}_e > 0$ denote the stablecoin *effective* liquidity. The blue-currency Euler condition pins down the total liquidity relevant for blue acceptance:

$$i^b = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b + \mathcal{L}_e) \quad \Rightarrow \quad \lambda(\mathcal{L}_b + \mathcal{L}_e) = \frac{1}{\alpha_2 + \alpha_3} i^b.$$

The red-currency survival cutoff in this environment is

$$\bar{i}^{r(be)} = \alpha_1\lambda(\mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_b + \mathcal{L}_e) = \alpha_1\lambda(\mathcal{L}_e) + \frac{\alpha_3}{\alpha_2 + \alpha_3} i^b.$$

Since $\mathcal{L}_e > 0$ implies $\lambda(\mathcal{L}_e) < \lambda(0)$, we have $\bar{i}^{r(be)} < \bar{i}^r$.

Moreover, using $i^e = i^b$, the stablecoin Euler condition in the same (blue, stablecoin) equilibrium implies

$$i^b = \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_e) + (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b + \mathcal{L}_e)\Big] = \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_e) + i^b\Big],$$

so that

$$\alpha_1\lambda(\mathcal{L}_e) = g\, i^b \quad \Rightarrow \quad \bar{i}^{r(be)} = \Big(g + \frac{\alpha_3}{\alpha_2 + \alpha_3}\Big) i^b.$$

Hence

$$\bar{i}^{r} - \bar{i}^{r(be)} = \alpha_1 \lambda(0) - g\, i^{b},$$

which is strictly positive whenever $g < \alpha_1 \lambda(0)/i^{b}$. Since g is empirically small in many stablecoin payment settings, this provides a convenient sufficient condition for strict tightening.

□

Appendix D: Proof of Comparative Statistics

The proof of the comparative statistics in table 1 is as the following.

Proof. In the regime with $\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e > 0$,

$$i^r = \alpha_1\lambda(\mathcal{L}_r + \mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e),$$
$$i^b = \alpha_2\lambda(\mathcal{L}_b + \mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e),$$
$$(1+g)i^b = \alpha_1\lambda(\mathcal{L}_r + \mathcal{L}_e) + \alpha_2\lambda(\mathcal{L}_b + \mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e).$$

Hence

$$\alpha_1\lambda(\mathcal{L}_r + \mathcal{L}_e) = g\, i^b, \qquad \alpha_2\lambda(\mathcal{L}_b + \mathcal{L}_e) = (1+g)i^b - i^r, \qquad \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e) = i^r - g\, i^b.$$

Because $\lambda'(\cdot) < 0$, holding $(\alpha_1, \alpha_2, \alpha_3, g)$ fixed,

$$\frac{\partial(\mathcal{L}_r + \mathcal{L}_e)}{\partial i^b} = \frac{g/\alpha_1}{\lambda'(\mathcal{L}_r + \mathcal{L}_e)} < 0, \quad \frac{\partial(\mathcal{L}_b + \mathcal{L}_e)}{\partial i^b} = \frac{(1+g)/\alpha_2}{\lambda'(\mathcal{L}_b + \mathcal{L}_e)} < 0, \quad \frac{\partial(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e)}{\partial i^r} = \frac{1/\alpha_3}{\lambda'(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e)}$$

and

$$\frac{\partial(\mathcal{L}_b + \mathcal{L}_e)}{\partial i^r} = \frac{-1/\alpha_2}{\lambda'(\mathcal{L}_b + \mathcal{L}_e)} > 0, \quad \frac{\partial(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e)}{\partial i^b} = \frac{-g/\alpha_3}{\lambda'(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e)} > 0.$$

Using

$$\mathcal{L}_r = (\mathcal{L}_r+\mathcal{L}_b+\mathcal{L}_e)-(\mathcal{L}_b+\mathcal{L}_e), \quad \mathcal{L}_b = (\mathcal{L}_r+\mathcal{L}_b+\mathcal{L}_e)-(\mathcal{L}_r+\mathcal{L}_e), \quad \mathcal{L}_e = (\mathcal{L}_r+\mathcal{L}_e)+(\mathcal{L}_b+\mathcal{L}_e)-(\mathcal{L}_r+\mathcal{L}_b+\mathcal{L}_e),$$

the signs in (i)–(iii) follow.

(i) Own–rate effects.

$$\frac{\partial\mathcal{L}_r}{\partial i^r} < 0, \qquad \frac{\partial(\mathcal{L}_b + \mathcal{L}_e)}{\partial i^b} < 0, \qquad \frac{\partial\mathcal{L}_e}{\partial i^b} < 0.$$

(ii) Cross–rate effects.

$$\frac{\partial(\mathcal{L}_b + \mathcal{L}_e)}{\partial i^r} > 0, \qquad \frac{\partial \mathcal{L}_e}{\partial i^r} > 0, \qquad \frac{\partial \mathcal{L}_r}{\partial i^b} > 0.$$

(iii) Market–mass shifts (α's). Holding (i^r, i^b, g) fixed,

$$\frac{\partial(\mathcal{L}_r + \mathcal{L}_e)}{\partial \alpha_1} > 0, \qquad \frac{\partial(\mathcal{L}_b + \mathcal{L}_e)}{\partial \alpha_2} > 0, \qquad \frac{\partial(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e)}{\partial \alpha_3} > 0,$$

hence

$$\frac{\partial \mathcal{L}_e}{\partial \alpha_1} > 0,\ \frac{\partial \mathcal{L}_b}{\partial \alpha_1} < 0,\ \frac{\partial \mathcal{L}_r}{\partial \alpha_1} = 0; \qquad \frac{\partial \mathcal{L}_e}{\partial \alpha_2} > 0,\ \frac{\partial \mathcal{L}_r}{\partial \alpha_2} < 0,\ \frac{\partial \mathcal{L}_b}{\partial \alpha_2} = 0; \qquad \frac{\partial \mathcal{L}_r}{\partial \alpha_3} > 0,\ \frac{\partial \mathcal{L}_b}{\partial \alpha_3} > 0,\ \frac{\partial \mathcal{L}_e}{\partial \alpha_3} < 0.$$

(iv) Monotone comparative statics representation. Let

$$\lambda_1' := \lambda'(\mathcal{L}_r + \mathcal{L}_e) < 0, \quad \lambda_2' := \lambda'(\mathcal{L}_b + \mathcal{L}_e) < 0, \quad \lambda_3' := \lambda'(\mathcal{L}_r + \mathcal{L}_b + \mathcal{L}_e) < 0.$$

The Jacobian with respect to $(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e)$ is

$$J = \begin{bmatrix} \alpha_1\lambda_1' + \alpha_3\lambda_3' & \alpha_3\lambda_3' & \alpha_1\lambda_1' + \alpha_3\lambda_3' \\ \alpha_3\lambda_3' & \alpha_2\lambda_2' + \alpha_3\lambda_3' & \alpha_2\lambda_2' + \alpha_3\lambda_3' \\ \alpha_1\lambda_1' + \alpha_3\lambda_3' & \alpha_2\lambda_2' + \alpha_3\lambda_3' & \alpha_1\lambda_1' + \alpha_2\lambda_2' + \alpha_3\lambda_3' \end{bmatrix},$$

whose diagonal and off–diagonal entries are strictly negative. Moreover,

$$d\mathcal{L} = J^{-1} \begin{bmatrix} di^r \\ di^b \\ (1+g)\, di^b + i^b\, dg \end{bmatrix},$$

and $-J$ is a nonsingular M-matrix in the interior region. □

Appendix E: Beyond Rival-currency Pegged Stablecoin Entrance Equilibria

This appendix develops the generalized environment referenced in the remark, where I relax the peg restriction on the stablecoin. While the benchmark analysis in the main text focuses on a rival currency-pegged stablecoin (implemented as the special case $\omega = \psi$), here we allow the stablecoin to carry an independent CM valuation ω. This formulation accommodates stablecoins pegged to a third-country currency, a commodity such as gold, or other reference assets, so that the stablecoin valuation need not co-move with either the red- or the blue-currency pricing kernels (ϕ, ψ). Within this broader specification, we provide the complete equilibrium characterization, including the full set of eight valuation regimes (no currency valued; single-currency regimes; all pairwise two-currency regimes; and the three-currency regime), together with the corresponding boundary conditions and proof details.

(A) **No currency valued** $(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e) = (0, 0, 0)$:

$$i^r \geq (\alpha_1 + \alpha_3)\lambda(0), \quad i^b \geq (\alpha_2 + \alpha_3)\lambda(0), \quad i^e \geq \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda(0).$$

(Generically, these are strict; equality indicates knife-edge entry.)

(B) **Only red valued** $(\mathcal{L}_r > 0,\ \mathcal{L}_b = \mathcal{L}_e = 0)$:

$$i^r = (\alpha_1 + \alpha_3)\lambda(\mathcal{L}_r), \qquad i^b > \bar{i}^{b(r)}, \quad i^e > \bar{i}^{e(r)}(g).$$

(C) **Only blue valued** $(\mathcal{L}_b > 0,\ \mathcal{L}_r = \mathcal{L}_e = 0)$:

$$i^b = (\alpha_2 + \alpha_3)\lambda(\mathcal{L}_b), \qquad i^r > \bar{i}^{r(b)}, \quad i^e > \bar{i}^{e(b)}(g).$$

(D) **Only stablecoin valued** $(\mathcal{L}_e > 0,\ \mathcal{L}_r = \mathcal{L}_b = 0)$:

$$i^e = \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big), \qquad i^r > \bar{i}^{r(e)}(g), \quad i^b > \bar{i}^{b(e)}(g).$$

(E) **Red & Blue valued (no e)** ($\mathcal{L}_r > 0,\ \mathcal{L}_b > 0,\ \mathcal{L}_e = 0$):

$$
\begin{aligned}
i^r &= \alpha_1 \lambda(\mathcal{L}_r) + \alpha_3 \lambda(\mathcal{L}_r + \mathcal{L}_b), \\
i^b &= \alpha_2 \lambda(\mathcal{L}_b) + \alpha_3 \lambda(\mathcal{L}_r + \mathcal{L}_b), \\
i^e &> \bar{i}^{e(rb)}(g).
\end{aligned}
$$

(F) **Red & stablecoin valued (no b)** ($\mathcal{L}_r > 0,\ \mathcal{L}_e > 0,\ \mathcal{L}_b = 0$):

$$
\begin{aligned}
i^r &= (\alpha_1 + \alpha_3)\lambda\Big(\mathcal{L}_r + \frac{\mathcal{L}_e}{1+g}\Big), \\
i^e &= \frac{1}{1+g}\Big[\alpha_1 \lambda\Big(\mathcal{L}_r + \frac{\mathcal{L}_e}{1+g}\Big) + \alpha_2 \lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3 \lambda\Big(\mathcal{L}_r + \frac{\mathcal{L}_e}{1+g}\Big)\Big], \\
i^b &> \bar{i}^{b(re)}(g).
\end{aligned}
$$

(G) **Blue & stablecoin valued (no r)** ($\mathcal{L}_b > 0,\ \mathcal{L}_e > 0,\ \mathcal{L}_r = 0$):

$$
\begin{aligned}
i^b &= (\alpha_2 + \alpha_3)\lambda\Big(\mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big), \\
i^e &= \frac{1}{1+g}\Big[\alpha_1 \lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big) + \alpha_2 \lambda\Big(\mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3 \lambda\Big(\mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big)\Big], \\
i^r &> \bar{i}^{r(be)}(g).
\end{aligned}
$$

(H) **All three valued** ($\mathcal{L}_r > 0,\ \mathcal{L}_b > 0,\ \mathcal{L}_e > 0$):

$$
\begin{aligned}
i^r &= \alpha_1 \lambda\Big(\mathcal{L}_r + \frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3 \lambda\Big(\mathcal{L}_r + \mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big), \\
i^b &= \alpha_2 \lambda\Big(\mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3 \lambda\Big(\mathcal{L}_r + \mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big), \\
i^e &= \frac{1}{1+g}\Big[\alpha_1 \lambda\Big(\mathcal{L}_r + \frac{\mathcal{L}_e}{1+g}\Big) + \alpha_2 \lambda\Big(\mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3 \lambda\Big(\mathcal{L}_r + \mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big)\Big].
\end{aligned}
$$

where

$$\bar{i}^{r(b)} = \alpha_1\lambda(0) + \alpha_3\lambda(\mathcal{L}_b), \tag{61}$$

$$\bar{i}^{b(r)} = \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r), \tag{62}$$

$$\bar{i}^{r(e)}(g) = \alpha_1\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big), \tag{63}$$

$$\bar{i}^{b(e)}(g) = \alpha_2\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big), \tag{64}$$

$$\bar{i}^{r(be)}(g) = \alpha_1\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3\lambda\Big(\mathcal{L}_b + \frac{\mathcal{L}_e}{1+g}\Big), \tag{65}$$

$$\bar{i}^{b(re)}(g) = \alpha_2\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big) + \alpha_3\lambda\Big(\mathcal{L}_r + \frac{\mathcal{L}_e}{1+g}\Big), \tag{66}$$

$$\bar{i}^{e(r)}(g) = \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_r) + \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r)\Big], \tag{67}$$

$$\bar{i}^{e(b)}(g) = \frac{1}{1+g}\Big[\alpha_1\lambda(0) + \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_b)\Big], \tag{68}$$

$$\bar{i}^{e(rb)}(g) = \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_r) + \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b)\Big]. \tag{69}$$

. Use the equilibrium conditions established earlier in the household section: a currency $k \in \{r, b, e\}$ is *valued* (positive steady holdings) if and only if its holding benefit equals its opportunity cost (binding equality), and it is *not valued* if and only if the opportunity cost is strictly higher than the benefit (strict inequality).

(A) No currency valued. If

$$i^r \geq (\alpha_1 + \alpha_3)\lambda(0), \qquad i^b \geq (\alpha_2 + \alpha_3)\lambda(0), \qquad i^e \geq \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda(0),$$

then $(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e) = (0, 0, 0)$ satisfies all three non-valuation inequalities (note $\mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3 = 0$). Uniqueness holds: if any $\mathcal{L}_k > 0$, then at least one argument of $\lambda(\cdot)$ strictly increases, hence the corresponding benefit strictly falls below its value at the origin by strict monotonicity of λ, so it cannot equal the fixed i^k.

(B) Only red valued. Suppose

$$i^r < (\alpha_1+\alpha_3)\lambda(0), \qquad i^b > \bar{i}^{b(r)} := \alpha_2\lambda(0)+\alpha_3\lambda(\mathcal{L}_r), \qquad i^e > \bar{i}^{e(r)}(g) := \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_r)+\alpha_2\lambda(0)+\alpha_3\lambda(\mathcal{L}$$

The binding equation $i^r = (\alpha_1 + \alpha_3)\lambda(\mathcal{L}_r)$ has a *unique* solution $\mathcal{L}_r > 0$ by the strict monotonicity and continuity of λ (IVT). Given $\mathcal{L}_b = \mathcal{L}_e = 0$, we have $\mathcal{L}_1 = \mathcal{L}_3 = \mathcal{L}_r$ and $\mathcal{L}_2 = 0$, so the strict inequalities rule out valuation of b and e, ensuring existence and uniqueness of this regime. The cases *(C) only blue valued* and *(D) only stablecoin valued* are symmetric (with e-case adjusted as below).

(D) Only stablecoin valued. Here $\mathcal{L}_r = \mathcal{L}_b = 0$ and $\mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3 = \mathcal{L}_e/(1+g)$. The valuation condition for e becomes

$$i^e = \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big),$$

which has a unique solution $\mathcal{L}_e > 0$ whenever $i^e < \frac{1}{1+g}(\alpha_1 + \alpha_2 + \alpha_3)\lambda(0)$. The inequalities $i^r > \bar{i}^{r(e)}(g)$ and $i^b > \bar{i}^{b(e)}(g)$ exclude valuation of r and b.

(E) Red & Blue valued (no e). Consider the system

$$\begin{aligned} i^r &= \alpha_1\lambda(\mathcal{L}_r) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b), \\ i^b &= \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda(\mathcal{L}_r + \mathcal{L}_b), \end{aligned} \qquad \mathcal{L}_r > 0,\ \mathcal{L}_b > 0,$$

with $\mathcal{L}_e = 0$ so that $\mathcal{L}_1 = \mathcal{L}_r$, $\mathcal{L}_2 = \mathcal{L}_b$, $\mathcal{L}_3 = \mathcal{L}_r + \mathcal{L}_b$. Fix $\mathcal{L}_b \geq 0$. The first equation is strictly decreasing in $\mathcal{L}_r$, hence determines a unique $\mathcal{L}_r = \mathcal{L}_r(\mathcal{L}_b)$ continuously in $\mathcal{L}_b$. Substitute into the second to define

$$\phi(\mathcal{L}_b) := \alpha_2\lambda(\mathcal{L}_b) + \alpha_3\lambda\big(\mathcal{L}_r(\mathcal{L}_b) + \mathcal{L}_b\big).$$

Then ϕ is continuous and strictly decreasing in $\mathcal{L}_b$, with $\phi(0) = \alpha_2\lambda(0) + \alpha_3\lambda(\mathcal{L}_r(0)) = \bar{i}^{b(r)}$ and $\lim_{\mathcal{L}_b\to\infty}\phi(\mathcal{L}_b) = 0$. Thus, for any $i^b \in (0, \bar{i}^{b(r)}]$, a unique $\mathcal{L}_b > 0$ exists with $i^b = \phi(\mathcal{L}_b)$; the corresponding $\mathcal{L}_r = \mathcal{L}_r(\mathcal{L}_b)$ is then uniquely determined. Finally, the condition $i^e > \bar{i}^{e(rb)}(g)$ excludes valuation of e, completing existence and uniqueness within regime (E).

(F) Red & stablecoin valued (no b). Let $X := \mathcal{L}_r + \frac{\mathcal{L}_e}{1+g}$ (so $\mathcal{L}_1 = \mathcal{L}_3 = X$ and $\mathcal{L}_2 = \frac{\mathcal{L}_e}{1+g}$ when $\mathcal{L}_b = 0$). The red equation gives

$$i^r = (\alpha_1 + \alpha_3)\lambda(X),$$

so X is uniquely determined by $X = \big[(\alpha_1+\alpha_3)\lambda\big]^{-1}(i^r)$. The stablecoin equation reads

$$i^e = \frac{1}{1+g}\Big[(\alpha_1+\alpha_3)\lambda(X) + \alpha_2\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big)\Big] = \frac{1}{1+g}\Big[i^r + \alpha_2\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big)\Big].$$

Hence

$$\lambda\Big(\frac{\mathcal{L}_e}{1+g}\Big) = \frac{(1+g)i^e - i^r}{\alpha_2}.$$

Whenever $\alpha_2 > 0$ and $(1+g)i^e \geq i^r$ (with the RHS also not exceeding $\lambda(0)$), this determines a unique $\mathcal{L}_e/(1+g)$ by monotonicity of λ, and hence a unique $\mathcal{L}_e$ and $\mathcal{L}_r = X - \mathcal{L}_e/(1+g)$. The strict inequality $i^b > \bar{i}^{b(re)}(g)$ rules out valuation of b. The case *(G) Blue & stablecoin valued (no r)* is analogous with roles of $(\alpha_1, i^r, \mathcal{L}_r)$ replaced by $(\alpha_2, i^b, \mathcal{L}_b)$.

(H) All three valued. Define

$$\begin{aligned}
F_r(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e) &:= \alpha_1\lambda(\mathcal{L}_1) + \alpha_3\lambda(\mathcal{L}_3),\\
F_b(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e) &:= \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3),\\
F_e(\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e) &:= \frac{1}{1+g}\Big[\alpha_1\lambda(\mathcal{L}_1) + \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3)\Big],
\end{aligned}$$

where $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ are as defined at the start of the proof. Consider $F_r = i^r$, $F_b = i^b$, $F_e = i^e$ on a large cube $[0, M]^3$. Each F_k is continuous and strictly decreasing in every argument that enters its $\lambda(\cdot)$ terms, with ranges from a finite maximum at the origin to 0 as holdings grow large. By the same elimination/IVT logic as in the two-currency case (solve $F_r = i^r$ for $\mathcal{L}_r$ given $(\mathcal{L}_b, \mathcal{L}_e)$, substitute into $F_b = i^b$ to obtain a one-dimensional strictly decreasing equation in $\mathcal{L}_b$ given $\mathcal{L}_e$, and finally solve $F_e = i^e$ for $\mathcal{L}_e$), a solution exists whenever $(i^r, i^b, (1+g)i^e)$ lies in the interior of the attainable ranges. Uniqueness within regime (H) follows from injectivity of the map $\mathbf{L} \mapsto (F_r, F_b, F_e)$ under strict monotonicity (a standard contradiction using coordinatewise maxima/minima as in the original argument).

The mutual exclusivity arguments follow the same contradiction logic as in the original proof, with two systematic substitutions: (i) whenever stablecoin liquidity enters $\lambda(\cdot)$, replace $\mathcal{L}_e$ by $\mathcal{L}_e/(1+g)$ inside $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$; (ii) whenever comparing the stablecoin valuation condition, replace the stablecoin benefit by its fee-adjusted form, i.e., i^e is compared to $\frac{1}{1+g}[\alpha_1\lambda(\mathcal{L}_1) + \alpha_2\lambda(\mathcal{L}_2) + \alpha_3\lambda(\mathcal{L}_3)]$ evaluated at the candidate liquidities. Since λ is strictly

decreasing, introducing any strictly positive $\mathcal{L}_e$ lowers all relevant $\lambda(\cdot)$ terms (through $\mathcal{L}_e/(1+g)$) and the extra factor $1/(1+g)$ further weakens the stablecoin benefit, so the same strict-inequality contradictions go through regime by regime.

Conclusion. For each regime, existence and uniqueness follow from the continuity and strict monotonicity of λ via single-variable elimination and intermediate value arguments (two-currency cases) or via a standard continuation argument (three-currency case). The ordering restrictions in any coexistence regime with $\mathcal{L}_e > 0$ follow from subtracting fee-adjusted binding equalities. Knife-edge boundaries, where a non-valued currency meets its threshold at equality, are transition sets of measure zero and do not generate an additional interior equilibrium. □

Then, comparative statics are calculated. The results are shown in Table 2. Note that entries 0^* indicate no direct term in the corresponding threshold formula; general equilibrium spillovers may still arise via $\mathcal{L}$. Formally, if a threshold $\bar{i}^{r(be)} = \alpha_1\lambda(\mathcal{L}_e) + \alpha_3\lambda(\mathcal{L}_b + \mathcal{L}_e)$ has no α_2 in its formula, then the *partial derivative at fixed liquidities* is

$$\frac{\partial \bar{i}^{r(be)}}{\partial \alpha_2}\Big|_{\mathcal{L}_r,\mathcal{L}_b,\mathcal{L}_e \text{ fixed}} = 0 \quad (= 0^*).$$

However, in equilibrium the liquidities $\mathcal{L} = (\mathcal{L}_r, \mathcal{L}_b, \mathcal{L}_e)$ *do* move with α_2, so the *total effect* uses the chain rule:

$$\frac{d\bar{i}^{r(be)}}{d\alpha_2} = \alpha_1\lambda'(\mathcal{L}_e)\frac{d\mathcal{L}_e}{d\alpha_2} \;+\; \alpha_3\lambda'(\mathcal{L}_b + \mathcal{L}_e)\left(\frac{d\mathcal{L}_b}{d\alpha_2} + \frac{d\mathcal{L}_e}{d\alpha_2}\right).$$

Proof of the competitive statistics for the three-currency equilibria is as the following.

Proof. **(i) Own–rate effects (law of demand).**

$$\frac{\partial \mathcal{L}_r}{\partial i^r} < 0, \qquad \frac{\partial \mathcal{L}_b}{\partial i^b} < 0, \qquad \frac{\partial \mathcal{L}_e}{\partial i^e} < 0.$$

(ii) Cross–rate substitution. An increase in the opportunity cost of any currency raises

the equilibrium holdings of the other two currencies.

$$\frac{\partial \mathcal{L}_b}{\partial i^r} > 0, \quad \frac{\partial \mathcal{L}_e}{\partial i^r} > 0; \qquad \frac{\partial \mathcal{L}_r}{\partial i^b} > 0, \quad \frac{\partial \mathcal{L}_e}{\partial i^b} > 0; \qquad \frac{\partial \mathcal{L}_r}{\partial i^e} > 0, \quad \frac{\partial \mathcal{L}_b}{\partial i^e} > 0.$$

(iii) Market–mass shifts (α's). Let α_1 weight the market that accepts (r, e), α_2 the market that accepts (b, e), and α_3 the market that accepts (r, b, e). Then

$$\begin{aligned}
&\frac{\partial \mathcal{L}_r}{\partial \alpha_1} > 0, \quad \frac{\partial \mathcal{L}_e}{\partial \alpha_1} > 0, \quad \frac{\partial \mathcal{L}_b}{\partial \alpha_1} < 0; \\
&\frac{\partial \mathcal{L}_b}{\partial \alpha_2} > 0, \quad \frac{\partial \mathcal{L}_e}{\partial \alpha_2} > 0, \quad \frac{\partial \mathcal{L}_r}{\partial \alpha_2} < 0; \\
&\frac{\partial \mathcal{L}_r}{\partial \alpha_3} > 0, \quad \frac{\partial \mathcal{L}_b}{\partial \alpha_3} > 0, \quad \frac{\partial \mathcal{L}_e}{\partial \alpha_3} > 0.
\end{aligned}$$

(iv) Monotone comparative statics representation. Writing J explicitly,

$$J = \begin{bmatrix}
\alpha_1\lambda_1' + \alpha_3\lambda_3' & \alpha_3\lambda_3' & \alpha_1\lambda_1' + \alpha_3\lambda_3' \\
\alpha_3\lambda_3' & \alpha_2\lambda_2' + \alpha_3\lambda_3' & \alpha_2\lambda_2' + \alpha_3\lambda_3' \\
\alpha_1\lambda_1' + \alpha_3\lambda_3' & \alpha_2\lambda_2' + \alpha_3\lambda_3' & \alpha_1\lambda_1' + \alpha_2\lambda_2' + \alpha_3\lambda_3'
\end{bmatrix},$$

whose diagonal and off–diagonal entries are strictly negative. The sign results in *(i)–(iii)* follow from $d\mathcal{L} = J^{-1}di$ and the fact that $-J$ is a nonsingular M-matrix in the interior region, so J^{-1} has a negative diagonal and nonnegative off-diagonal entries. □