Climate Change, Demographics, and Economic Growth

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Abstract

Economic growth is driven by the discovery of new ideas by people. However, a larger population also results in greater greenhouse gas emissions. These raise a question: Is population growth beneficial? We construct a growth model with idea and environmental externalities and time allocation between labor supply and child-rearing. We find that the optimal population growth rate exceeds the competitive equilibrium rate under the baseline assumption where a degree Celsius increase reduces GDP by 3%. Since labor supply expands production and greenhouse gas emissions, the opportunity cost of child-rearing declines in the optimal allocation, sustaining high population growth.

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1 Introduction

Global warming has emerged as one of the most pressing threats to human welfare in the twenty-first century. Rising temperatures, extreme weather events, and the loss of biodiversity are already imposing substantial economic and social costs across the world. At the heart of this challenge lies the rapid accumulation of greenhouse gases, which is strongly tied to the scale of human activity. What matters for emissions is not only how much energy and output each person generates but also how many of us there are. A simple decomposition clarifies the point:

$$\text{Total emissions} \;=\; \text{emissions per capita} \times \text{population}.$$

Technologies, energy mixes, and policies that reduce emissions per person operate on the first term. Demographic change operates on the second. As populations grow, so too do energy use, production, and ultimately emissions. Figure 1 illustrates the tight correlation between global population size and CO_2 emissions.[1]

While the link between economic growth and the environment has been studied extensively, much less attention has been paid to the demographic dimension in spite of the fact that population is highly relevant to both economic growth and the environment. [2] Population growth is central in endogenous growth theory, where a larger population generates more ideas and sustains productivity growth. [3] Yet the same population growth intensifies environmental pressure, accelerating climate change and worsening its damages. This tension motivates the central question of this paper: is population growth ultimately beneficial for the economy once the environment is taken into account?

[1] Source: Our World in Data.
`https://ourworldindata.org/grapher/share-co2-emissions-vs-population`

[2] Notable exceptions are Budolfson et al. (2025), Kruse-Andersen (2023), and Eden and Kuruc (2024).

[3] Endogenous growth models such as Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) predict that the growth rate of GDP per capita increases with the level of population. Although this implication has been criticized empirically by studies such as Jones (1995), subsequent semi-endogenous growth models, including Jones (1995), instead predict that the growth rate of GDP per capita increases with the population growth rate. A common feature of both classes of models is that idea discovery by people is essential for achieving productivity growth.

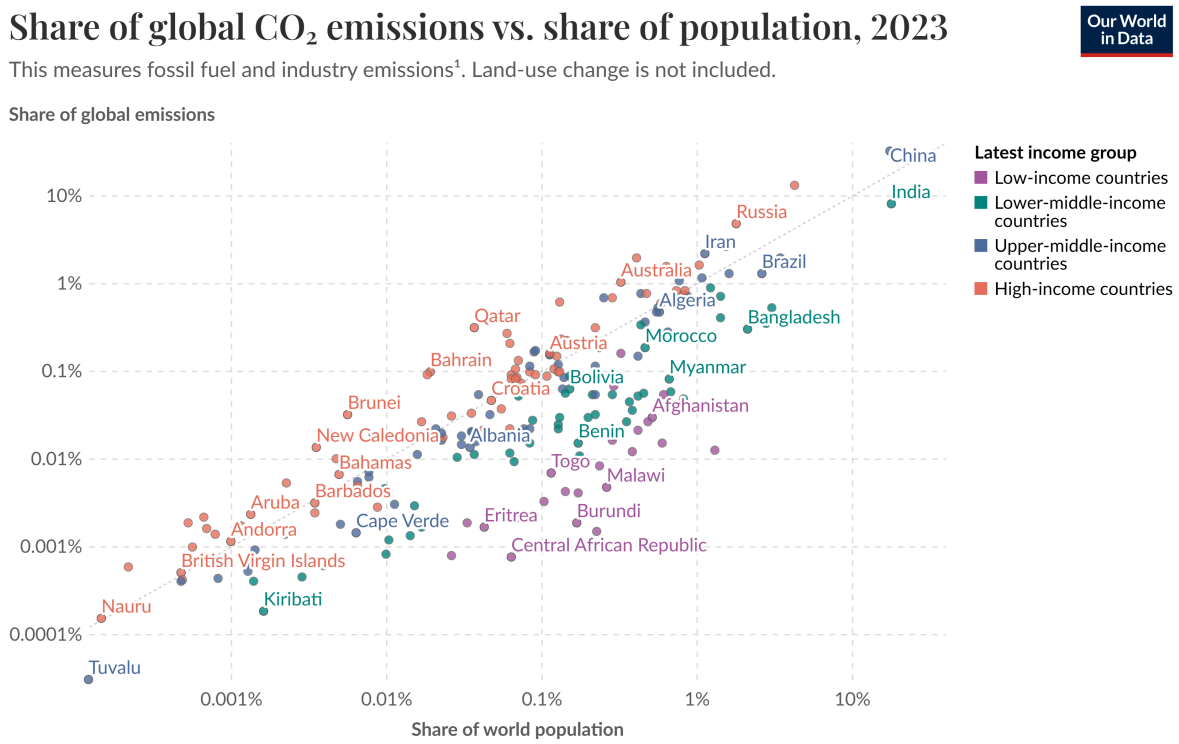


Figure 1: Population and CO2 Emissions.

We address this question by extending a growth model of Jones (2022), which incorporates time allocation between market work and child-rearing and endogenous productivity, to include environmental externalities. A positive population growth boosts the discovery of ideas and thus leads to productivity growth. On the other hand, expanding the economic activities threatens the environment, and it increases the disutility of households. The framework allows us to compare decentralized fertility choices with the socially optimal population when idea externality and climate damages are internalized.

The analysis yields quantitative results on optimal population growth in the presence of both idea externality and environmental externality. The optimal population growth rate exceeds the competitive equilibrium rate under the baseline calibration, in

which a 1-degree Celsius warming causes a 3% GDP loss. A larger population accelerates the discovery of new ideas, whereas a larger population size obviously leads to more greenhouse gas emissions, and the idea externality dominates the environmental externality in the baseline model. Specifically, a higher population growth rate is socially optimal unless the GDP loss from a 3-degree Celsius warming reaches a 11.7%.

Time allocation is critical to obtaining these quantitative results. Since labor supply increases both output and greenhouse gas emissions, the environmental externality depresses the marginal social return to labor. This wedge lowers the opportunity cost of child-rearing in the optimal allocation. As a result, time is reallocated from production toward fertility, allowing a higher population growth rate to persist even in the presence of substantial environmental damage. This mechanism holds regardless of whether we include leisure in the model. Without this time allocation channel, the threshold for GDP loss from a 3-degree Celsius warming falls to 6.4%. Whether time is reallocated between production and child-rearing, reflecting the reduced social return to labor, fundamentally alters the quantitative implications.

Finally, we investigate environmental policies to align the competitive equilibrium with the socially optimal allocation. Naturally, in the presence of severe environmental damage, implementing policies to curb emissions is expected. However, we find that a standard carbon tax generates a paradoxical result. In such high-damage economies, where the social planner prescribes a lower population growth rate, the tax unintentionally lowers wages and thus the opportunity cost of child-rearing, thereby increasing population growth. While this mechanism stimulates the creation of new ideas and boosts productivity growth, it paradoxically accelerates the accumulation of greenhouse gases, thereby widening the gap between the market outcome and the social optimum.

Literature Review

This paper connects and contributes to three strands of literature: the relationship between the environment and economic growth, the role of population in endogenous

growth theory, and a body of recent literature that explicitly links population dynamics to climate outcomes and research activities.

The first strand focuses on environmental sustainability and economic growth. Seminal contributions such as Acemoglu et al. (2012, 2016), Aghion et al. (2025), and Stokey (1998) have extensively analyzed the choice between clean and dirty technologies or between exhaustible and renewable resources. However, these studies primarily focus on greenhouse gas emissions per capita—the intensive margin of emissions—often treating population trends as exogenous. Other papers, such as Brock and Taylor (2010), de la Croix and Gosseries (2012), and Beulmann and Strulik (2025), explicitly include population dynamics but abstract from the endogenous feedback loop from the environment to the economy, or do not solve an optimal allocation problem. This paper departs from this tradition by focusing on the *extensive margin* of greenhouse gas emissions: we analyze the trade-off where a larger population stimulates innovation but simultaneously intensifies aggregate environmental damage. By endogenizing population dynamics, we bridge the gap between technological environmental solutions and demographic pressures.

The second strand examines the link between population and economic growth. In endogenous growth theory (e.g., Romer, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991) and semi-endogenous growth models (e.g., Jones, 1995; Kortum, 1997; Segerstrom, 1998), population is fundamental: more people bring more ideas, driving productivity growth. Reflecting this view, recent studies such as Jones (2022), Chang et al. (2025), and Boikos, Bucci, and Sequeira (2023) examine whether and how we can avoid stagnation caused by declining populations. Yet, these models typically abstract from environmental externalities. We contribute to this literature by integrating non-pecuniary environmental externalities into a semi-endogenous growth framework.[4] This allows us to re-evaluate whether positive population growth remains desirable when it acts as both an engine of innovation and a driver of climate change.

[4]Note that this paper differs from Malthusian growth papers such as Weil and Wilde (2009) in that we focus on greenhouse gas emissions, a non-pecuniary externality, whereas they study exhaustible resources, which are generally mediated by market prices.

Finally, our work relates to recent studies that explicitly connect population to innovation and environmental outcomes, such as Budolfson et al. (2025), Kruse-Andersen (2023), and Eden and Kuruc (2024). While these papers acknowledge the trade-off between innovation and environmental damage, they do not analyze how population growth is characterized, because they assume an exogenous population and do not account for time allocation. Our distinctive contribution lies in two areas. First, we provide a quantitative comparison between the competitive equilibrium population growth and the optimal one. Second, and crucially, we clarify the role of the *time allocation channel*. We show that environmental degradation endogenously lowers the marginal benefit of labor supply and thus the opportunity cost of child-rearing, creating an incentive to reallocate time from production to child-rearing. This mechanism is close to but contrasts with de la Croix and Gosseries (2012), in which time allocation changes are driven by policy interventions in a decentralized economy rather than by endogenous environmental feedback in the optimal allocation. Our analysis complements de la Croix and Gosseries (2012), which does not show the optimal allocation, by uncovering a policy paradox: In cases of severe environmental damage where the optimal population growth is lower than the competitive equilibrium rate, a carbon tax can unintentionally accelerate population growth in the decentralized economy, pushing the equilibrium further away from the social optimum.

The rest of the paper is as follows: Section 2 provides a competitive equilibrium, Section 3 discusses the optimal allocation, Section 4 generates the main results, Section 5 conducts policy experiments, and Section 6 concludes the paper. The appendix proposes additional model specifications.

2 Competitive Equilibrium

We extend an endogenous growth model with endogenous fertility of Jones (2003, 2022) to incorporate environmental externalities. The economy consists of representative altruistic households that allocate time to market work and child-rearing, per-

fectly competitive representative firms, the law of motion of productivity, and that of the stock of greenhouse gases. How productivity and the stock of greenhouse gases evolve are not internalized by agents in the decentralized economy.

2.1 Representative Household

Households are endowed with 1 unit of time and allocate it to child-rearing h_t and market work $1 - h_t$. Spending h_t units of time for child-rearing results in a fertility rate of $\bar{b}h_t$, and the population growth rate is $n_t = \bar{b}h_t - \delta$, where $\bar{b}$ is the productivity of child-rearing and δ is the mortality rate. People obtain utility from consumption c_t and the number of descendants N_t, while suffering from the stock of greenhouse gases S_t. The instantaneous utility takes the form:

$$u(c_t, N_t, S_t) = \frac{\left(c_t N_t^{\epsilon} S_t^{-\gamma}\right)^{1-\theta}}{1-\theta} \tag{1}$$

where $\epsilon > 0$ captures altruism towards descendants, $\gamma > 0$ represents the magnitude of environmental damage to utility, and $\theta > 0$ is the coefficient of relative risk aversion.[5]

People in the initial generation maximize lifetime utility:

$$\begin{aligned} \max_{\{h_t\}_{t\in[0,\infty)}} \quad & \int_0^{\infty} e^{-\rho t} u\left(c_t, N_t, S_t\right) \, dt \\ \text{subject to} \quad & c_t = w_t(1-h_t), \\ & \dot{N}_t = \left(\bar{b}h_t - \delta\right) N_t, \\ & \text{given } \{w_t, S_t\}_{t\in[0,\infty)}, \end{aligned}$$

where ρ is the discount rate. The discount rate $\rho = \bar{\rho} + \delta$ includes both the pure discount rate $\bar{\rho}$ and the mortality rate δ. Note that $\frac{N_t}{N_0}$ is the number of descendants, and $N_0 \equiv 1$ so that $N_t = \frac{N_t}{N_0}$ is the number of descendants, not only the population size. A

[5]The instantaneous utility function belongs to a class of instantaneous utility functions that satisfy the criteria of Acemoglu et al. (2012) if you define the environmental quality as $\frac{1}{S_t}$.

person receives a wage payment $w_t(1-h_t)$ for working $1-h_t$ units of time, which is used for consumption. We assume away the saving, but people can enjoy the utility in the future through having children, and they solve a dynamic optimization problem. The sequence of the stock of greenhouse gases $\{S_t\}_{t\in[0,\infty)}$ is also taken as given because a single person can't affect the aggregate environmental quality.[6]

2.2 Firms and Productivity

Firms operate in a perfectly competitive market with the production function $Y_t = A_t^{\sigma} L_t$, where $L_t = (1-h_t)N_t$ is the aggregate labor supply. In equilibrium, the wage is equal to the marginal product of labor, $w_t = A_t^{\sigma}$. The productivity A_t is the same as Jones (2022) and evolves according to:

$$\frac{\dot{A}_t}{A_t} = \frac{N_t^{\lambda_A}}{A_t^{\beta_A}} \tag{2}$$

More population N_t generate more ideas $N_t^{\lambda_A}$. The term $A_t^{\beta_A}$ means that ideas are getting harder to find, as in Bloom et al. (2020). This law of motion is somewhat exogenous in a decentralized economy, since every agent takes the productivity term as given.

2.3 Greenhouse Gas

Economic activity generates emissions. Emissions are proportional to aggregate output Y_t. The law of motion for the stock of greenhouse gases S_t follows Stokey (1998) and is:

$$\dot{S}_t = Y_t - \zeta S_t, \tag{3}$$

[6]You may wonder why an initial generation can determine the path of future actions $\{h_t\}_{t\in[0,\infty)}$. However, any generation $\tau > 0$ whose objective function is

$$\int_{\tau}^{\infty} e^{-\rho(t-\tau)} u\left(c_t, \frac{N_t}{N_\tau}, S_t\right) dt,$$

where $\frac{N_t}{N_\tau}$ is the number of descendants of a person in generation τ, chooses the same path of actions $\{h_t\}_{t\in[\tau,\infty)}$ because N_τ has already been realized at time τ, and multiplying the instantaneous utility function by a positive constant does not affect the solutions. In this sense, we can say that the model is time-consistent.

where ζ is the carbon depreciation rate. Unlike pecuniary externalities such as land and oil, greenhouse gas emissions cannot be adjusted through markets and prices. Golosov, Jones, and Tertilt (2007) show that the fertility choice in a competitive equilibrium is efficient in an economy with pecuniary externality. Therefore, we shed light on greenhouse gas emissions, not exhaustible resources such as land and oil, and non-pecuniary externality makes the optimal allocation differ from the competitive equilibrium allocation.

2.4 Equilibrium Characterization

Solving the household's problem yields the optimal time allocation. Set up the current-value Hamiltonian:

$$H = u(c_t, N_t, S_t) + v_t \left(\bar{b}h_t - \delta\right) N_t,$$

where v_t is the shadow price of an individual. The first-order condition (FOC) with respect to child-rearing h_t is

$$\underbrace{\bar{b}v_t N_t}_{\text{marginal benefit of child-rearing}} = \underbrace{w_t u_c(c_t, N_t, S_t)}_{\text{marginal cost of child-rearing}},$$

where u_x denotes the marginal utility of x. The amount of child-rearing h_t is chosen so that the marginal benefit of child-rearing, which comes from the increase in the number of children, is equal to the marginal cost of child-rearing, which is the reduction in consumption. An optimality condition with respect to the population size N_t yields

$$\rho v_t - \dot{v}_t = u_N(c_t, N_t, S_t) + v_t \left[\bar{b}h_t - \delta\right],$$

which is rewritten as

$$v_t = \frac{u_N(c_t, N_t, S_t)}{\rho - g_{v,t} - (\bar{b}h_t - \delta)},$$

where $g_{x,t}$ is the growth rate of x at time t, and the denominator is the discount rate of the shadow price v_t. The value of the population, which is defined as $\tilde{V}_{CE,t} \equiv$

$\frac{v_t N_t}{u_c(c_t, N_t, S_t)c_t}$, is represented by

$$\tilde{V}_{CE,t} = \frac{\varepsilon}{\rho - g_{vt} - (\bar{b}h_t - \delta)}.$$

While $\tilde{V}_{CE,t}$ is an auxiliary variable and does not correspond to any economic statistics, it plays a crucial role in helping us understand the model. $v_t N_t$, which is the value of the total population in terms of the marginal utility, is converted into the consumption units by dividing by the marginal utility u_c. It is further divided by c to yield a ratio to the per-person consumption. The numerator of $\tilde{V}_{CE,t}$ is ε, meaning that the value of the population increases with the degree of altruism, given the effective discount rate. The resulting allocation in the competitive equilibrium (CE) is characterized by:

$$h_t = 1 - \frac{1}{\bar{b}\tilde{V}_{CE,t}}, \quad 1 - h_t = \frac{1}{\bar{b}\tilde{V}_{CE,t}}$$

A higher value of population $\tilde{V}_{CE,t}$ leads to more time allocated to child-rearing h_t and less to work $1 - h_t$. As a result, we obtain the population growth rate

$$n_t = \bar{b} - \delta - \frac{1}{\tilde{V}_{CE,t}},$$

which is increasing in the value of the population $\tilde{V}_{CE,t}$.

2.5 Balanced Growth Path

For the growth rate of the productivity A_t to be constant on the balanced growth rate (BGP), the new ideas $N_t^{\lambda_A}$ and the ideas-are-getting-harder-to-find component $A_t^{\beta_A}$ must grow at the same rate because productivity is governed by the following law of motion:

$$\frac{\dot{A}_t}{A_t} = \frac{N_t^{\lambda_A}}{A_t^{\beta_A}}. \tag{2}$$

This implies that $\beta_A g_A = \lambda_A n$, and we obtain $g_A = \frac{\lambda_A}{\beta_A} n$. Variables such as n without the time subscript denote the variables on the BGP. Since consumption per capita is

given by $c_t = A_t^{\sigma} l_t$ and l_t does not have a growth trend, we obtain $g_c = \sigma g_A = \frac{\sigma\lambda_A}{\beta_A} n$. The aggregate output is consumption per capita times the population size, so $g_Y = n + g_c = \left(1 + \frac{\sigma\lambda_A}{\beta_A}\right) n$. Equation (3) tells that the growth rate of the stock of greenhouse gases S_t is given by

$$\frac{\dot{S}_t}{S_t} = \frac{Y_t}{S_t} - \zeta,$$

and we need $g_Y = g_S$ for the growth rate of S_t to be constant on the BGP, which gives $g_S = \left(1 + \frac{\sigma\lambda_A}{\beta_A}\right) n$. On the BGP, the value of the population $\tilde{V}$ has to be constant, and thus we have

$$g_v = g_u - n,$$

and the growth rate of an instantaneous utility function in Equation (1) is given by

$$g_u = (1-\theta)(g_c + \varepsilon n - \gamma g_S) = (1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right] n.$$

The effective discount rate $\rho - g_u$ increases with θ, a coefficient of CRRA utility function, as long as $g_c + \varepsilon n - \gamma g_S > 0$. The elasticity of intertemporal substitution, $\frac{1}{\theta}$, becomes smaller as θ increases. People with small elasticity of intertemporal substitution do not want to change the path of $c_t N_t^{\varepsilon} S_t^{-\gamma}$ too much even if high growth is realized in the future, and the future is then highly discounted.

We finally obtain the value of the population $\tilde{V}_{CE}$ and the population growth rate n_{CE} on the BGP at the intersection of the functions,

$$\tilde{V}_{CE}(n) = \frac{\varepsilon}{\rho - (1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right] n} \tag{4}$$

$$n_{CE}\left(\tilde{V}\right) = b - \delta - \frac{1}{\tilde{V}}. \tag{5}$$

We have the analytical BGP population growth rate

$$n_{CE} = \frac{\bar{b} - \delta - \frac{\rho}{\varepsilon}}{1 - \frac{(1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right]}{\varepsilon}}.$$

We assume the following:

Assumption 1.

$$1 - \frac{(1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right]}{\varepsilon} > 0.$$

This implies that the population growth rate in a Walrasian equilibrium is increasing in the productivity of child-rearing, $\bar{b}$ and the mortality rate δ. If this condition is violated, then the population growth rate decreases with the productivity of child-rearing and the mortality rate, which is really unrealistic. If the elasticity of intertemporal substitution is low, that is, a parameter θ is large, then the population growth rate n_{CE} is lowered because the substitution effect is sufficiently weak and is dominated by the wealth effect. Growth in the future implies that households do not want to reduce labor supply, be poor today, and have children.

3 Optimal Allocation

The optimal allocation internalizes two externalities: the positive knowledge spillover from population size to productivity and the negative environmental externality from production to utility. We use the same objective function as in a decentralized equilibrium, following Jones (2022), although we could use other welfare criteria.[7]Our objective function is the same as Alvarez (1999) and Jones and Schoonbroodt (2010) and satisfies $\mathcal{A}-$efficiency, a concept discussed in Golosov, Jones, and Tertilt (2007). In our setup, the difference between the competitive equilibrium allocation and the optimal allocation solely comes from externalities, not from preferences.

[7]Jones (2022) says, "With endogenous fertility, there is no unique criterion for social welfare. Instead, we consider the allocation that maximizes the dynastic utility of a representative generation."

3.1 Problem

The optimal allocation maximizes the same objective function subject to the aggregate resource constraints and laws of motion:

$$\begin{aligned}
\max_{\{h_t\}_{t\in[0,\infty)}} & \int_0^\infty e^{-\rho t} u(c_t, N_t, S_t) dt \\
\text{s.t.} \quad & u(c_t, N_t, S_t) = \frac{\left(c_t N_t^{\epsilon} S_t^{-\gamma}\right)^{1-\theta}}{1-\theta}, \\
& c_t = A_t^{\sigma}(1-h_t), \\
& \dot{N}_t = \left(\bar{b}h_t - \delta\right) N_t, \\
& \dot{A}_t = N_t^{\lambda_A} A_t^{1-\beta_A}, \\
& \dot{S}_t = A_t^{\sigma}(1-h_t)N_t - \zeta S_t.
\end{aligned}$$

The current-value Hamiltonian is:

$$\mathcal{H} = u(c_t, N_t, S_t) + v_t \left(\bar{b}h_t - \delta\right) N_t + \mu_{At} N_t^{\lambda_A} A_t^{1-\beta_A} - \phi_t \left[A_t^{\sigma}(1-h_t)N_t - \zeta S_t\right],$$

where μ_{At} is the shadow price of an idea and $-\phi_t$ is the shadow cost of pollution.

We define the following auxiliary variables:

$$\begin{aligned}
\tilde{\xi}_t &\equiv \frac{\phi_t Y_t}{u_c(c_t, N_t, S_t) c_t} && \text{Environmental damage} \\
z_{At} &\equiv \frac{\mu_{At} \dot{A}_t}{u_c(c_t, N_t, S_t) c_t} && \text{Value of productivity ideas} \\
\tilde{V}_{SP,t} &\equiv \frac{v_t N_t}{u_c(c_t, N_t, S_t) c_t} && \text{Social value of population}
\end{aligned}$$

$\dot{A}_t$ is the number of new ideas produced at time t and SP represents the Social Planner.

We have 4 optimality conditions that characterize the optimal allocation. First, the FOC with respect to child rearing h_t implies

$$\underbrace{\bar{b} v_t N_t}_{\text{marginal benefit of child-rearing}} = \underbrace{\frac{c_t u_c(c_t, N_t, S_t) - \phi_t Y_t}{1-h_t}}_{\text{marginal cost of child-rearing}}.$$

This condition equates the social marginal benefit of producing new descendants to the social marginal cost of withdrawing time from production. The right-hand side includes the marginal cost of pollution that each unit of labor supply brings, $\frac{\phi_t Y_t}{1-h_t}$, reflecting that labor supply generates not only output but also environmental damage.[8] The opportunity cost of child-rearing is smaller in the optimal allocation than in the decentralized economy.[9] By normalizing this condition, we obtain

$$\frac{1-\tilde{\xi}_t}{1-h_t} = \bar{b}\tilde{V}_{SP,t}.$$

Here, $\tilde{\xi}_t$ acts as an endogenous tax on market work, meaning the opportunity cost of child-rearing is effectively discounted by environmental damage. This incentivizes a reallocation of time from production to reproduction. The benefit of child-rearing also changes because the value of the population, $\tilde{V}_{SP,t}$, differs from the private value, as it incorporates both positive innovation spillovers and negative environmental externalities.

Second, an optimality condition of productivity A_t gives

$$z_{At} = \frac{\sigma g_{At}\left(1-\tilde{\xi}_t\right)}{\rho - g_{\mu_A t} - (1-\beta_A)g_{At}}.$$

The numerator of the value of ideas, z_{At}, increases in the productivity growth rate g_{At} but its value is discounted by the environmental damage term, $\tilde{\xi}_t$ because expanding economic activities through the increase in productivity lead to more greenhouse gas emissions.[10]

[8]One might argue that labor could be allocated to the service sector, which typically emits fewer greenhouse gases. Indeed, the share of the service sector is increasing in developed countries, and Aghion et al. (2025) insist that the service sector is a clean sector. However, an analysis in the Appendix demonstrates that the results are unchanged if the model incorporates a clean sector.

[9]Mothers' education is often negatively associated with fertility in reality, as in Fernández and Fogli (2006), for example.

[10]In the Appendix, we also analyze the development of clean technology, which abates the greenhouse gas emissions.

Third, an optimality condition of the population N_t generates

$$\tilde{V}_t = \frac{\varepsilon - \tilde{\xi}_t + \lambda_A z_{At}}{\rho - g_{vt} - \left(\bar{b} h_t - \delta\right)}.$$

Intuitively, the denominator of $\tilde{V}_t$ increases with the degree of altruism ε, the value of ideas z_{At}, and decreases with the environmental damage $\tilde{\xi}_t$.

Fourth, an optimality condition of the stock of greenhouse gases S_t gives rise to

$$\tilde{\xi}_t = \frac{\gamma(g_{St} + \zeta)}{\rho + \zeta - g_{\phi t}}.$$

The denominator can be written as $\gamma \frac{Y_t}{S_t}$. The environmental damage is severe if the disutility of environmental damage γ is large and the today's greenhouse gas emissions are large relative to the stock of greenhouse gases.

3.2 Optimal BGP

We have the following equations

$$g_{\mu_A} = g_u - g_A, \qquad g_v = g_u - n, \qquad g_\phi = g_u - g_Y,$$

for $\tilde{V}_{SP,t}$, $z_{A,t}$, and $\tilde{\xi}_t$ to be constant on the BGP. As in the decentralized economy, the growth rates of variables are given by

$$g_A = \frac{\lambda_A}{\beta_A} n, \qquad g_c = \frac{\lambda_A \sigma}{\beta_A} n,$$
$$g_Y = g_S = \left(1 + \frac{\lambda_A \sigma}{\beta_A}\right) n, \qquad g_u = (1-\theta)(g_c + \varepsilon n - \gamma g_S).$$

Utilizing the above relationships, the optimality conditions on the BGP are rewritten as

$$h = 1 - \frac{1-\tilde{\xi}}{\bar{b}\tilde{V}_{SP}}, \qquad 1 - h = \frac{1-\tilde{\xi}}{\bar{b}\tilde{V}_{SP}}, \qquad n = \bar{b} - \delta - \frac{1-\tilde{\xi}}{\tilde{V}_{SP}},$$

$$z_A = \frac{\sigma g_A \left(1-\tilde{\xi}\right)}{\rho - g_u + \beta_A g_A}, \qquad \tilde{V}_{SP} = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A}{\rho - g_u}, \qquad \tilde{\xi} = \frac{\gamma\left(g_S + \zeta\right)}{\rho + \zeta - g_u + g_Y}.$$

The optimal population growth rate and the value of the population $\left(n_{SP}, \tilde{V}_{SP}\right)$ are determined by the intersection of the following two curves:

$$n_{SP}\left(\tilde{V}\right) = b - \delta - \frac{1-\tilde{\xi}}{\tilde{V}} \tag{6}$$

$$\tilde{V}_{SP}(n) = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A}{\rho - (1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right] n} \tag{7}$$

The difference between the optimal population growth rate $n_{SP} = b - \delta - \frac{1}{\tilde{V}_{SP}} + \frac{\tilde{\xi}}{\tilde{V}_{SP}}$ and the competitive equilibrium rate n_{CE} is determined by the interplay between the following two forces:

Value of Population Channel: The social planner internalizes both the positive externality of idea creation and the negative environmental externality. While the former increases the social value of the population, $\tilde{V}_{SP}$, the latter decreases it.

Time Allocation Channel: Environmental damages reduce the social marginal benefit of labor supply, effectively acting as a tax on market work. In other words, the opportunity cost of child-rearing is lower from the social planner's perspective. This wedge induces a reallocation of time from production toward child-rearing, exerting upward pressure on the population growth rate, while it is easy to argue that the population is a burden on the environment and less desirable.[11]

4 Results

We provide the quantitative results in the calibrated model.

[11] You may feel that this mechanism is a direct result of the assumption that households allocate time only to labor and child-rearing. However, this channel remains the same in a generalized model with leisure activities, as shown in the Appendix.

4.1 Calibration

4.1.1 Assigned Parameters

We choose parameter values for quantitative analysis. Many parameter values are assigned as in Table 1.

Table 1: Assigned Parameter Values

Parameter	Value	Explanation	Source
θ	3	coefficient of CRRA utility	Jones and Schoonbroodt (2010)
σ	1	degree of increasing returns in productivity	Jones (2022)
δ	0	mortality rate	A simplifying assumption
λ_A	0.75	people's role in generating ideas	Jones (2022)
β_A	2	degree of decreasing returns in idea	Bloom et al. (2020)
ζ	0.0313	greenhouse gas depreciation rate	Acemoglu et al. (2016)
ρ	$\frac{1}{90}$	discount factor	Jones (2022)

The key parameter is the coefficient of CRRA utility, θ. It is also the inverse of the elasticity of intertemporal substitution and is set to be 3. Jones and Schoonbroodt (2010) show that the model is likely to be consistent with the empirical findings if the elasticity of intertemporal substitution is $\frac{1}{3}$. In addition, assuming an elasticity of intertemporal substitution less than 1 is standard in the growth literature and other macroeconomics literature. While Jones (2022) adopts the log utility, sets $\theta = 1$, and analyzes the dynamics of the model with multiple BGPs in the optimal allocation, it is hard to examine the transitional dynamics of this model because our model is more complex than Jones (2022). We generally do not have multiple BGPs if the coefficient of CRRA utility function is large enough, because the value of the population $\tilde{V}(n)$ becomes decreasing in the population growth rate due to the weak substitution effect and the high effective discount rate, helping us obtain clear implications.

For simplicity, we assume that the mortality rate δ is zero. The purpose of the quantitative exercise is to compare the magnitudes of positive externality of innovation and negative externality of environmental damage, and innovation does not occur on an asymptotic BGP under the declining population. Therefore, the negative population growth rate is not a subject of interest and excluded.

The carbon depreciation rate ζ is from Acemoglu et al. (2016). The rate of decay of carbon concentration in Acemoglu et al. (2016) is essentially the same as the carbon

depreciation rate in our paper, although the law of motion of the greenhouse gases is similar to Stokey (1998), is much simpler than those in Golosov et al. (2014) and Acemoglu et al. (2016), and Stokey (1998) does not provide an estimate of the carbon depreciation rate.

4.1.2 Calibrated Parameters

We choose parameters $\bar{b}, \varepsilon, \gamma$ that match the empirical targets. The calibration result is summarized in Table 2. We first use the current average population growth rate among OECD countries, $n_{CE} = 0.006$. Next, we assume that the time spent child-rearing is one-eighth of the total time, $h_{CE} = \frac{1}{8}$, as in Jones (2022).

We target an average population growth rate among OECD countries, which is positive. One reason is that we believe that the growth rate of the world population is more important than that in a specific country to discuss climate change. Another reason is that the population size will keep expanding by around 2080 according to the projections of United Nations[12] and dealing with climate change is an urgent task that we need to deal with now. [13] Therefore, it is meaningful to construct a model with a positive population growth. At the same time, we do not use the world population growth rate but the OECD population growth rate because people in the developing countries are not very likely to contribute to producing ideas and expanding the scientific frontier.

There are many estimates of environmental damage, and these estimates are highly diverse. We follow the one in Nath, Ramey, and Klenow (2025) as a benchmark because its estimates are neither the smallest nor the largest. They say that a 1 °C increase in the world's average temperature reduces the world's GDP by 2.2% to 3.5%. Thus, we assume that a 1 °C increase in the temperature reduces the GDP by 3% in our model. Golosov et al. (2014) explain that "a doubling of the stock of atmospheric carbon leads to a 3-degree Celsius increase in the global mean temperature." These studies imply

[12] `https://population.un.org/wpp/` gives information about the population projections.

[13] `https://www.un.org/sustainabledevelopment/climate-change/` emphasizes the need to take urgent actions to stop climate change.

doubling the stock of greenhouse gases is equal to reducing GDP by 9%.[14] If we adjust consumption by the environmental damage term $S_t^{-\gamma}$ to replicate the GDP loss due to climate change, we have $c_t S_t^{-\gamma}$ in the instantaneous utility function. Writing down a formula consistent with these papers gives

$$(1 - 0.03 \times 3) c_t S_t^{-\gamma} = c_t (2S_t)^{-\gamma}.$$

we obtain $\gamma = -\frac{\log(1-0.09)}{\log(2)} = 0.136$.[15]

Table 2: Calibrated Parameter Values

Parameter	Description	Value	Target / Source
Environmental Parameter			
γ	Damage elasticity	0.136	3% GDP loss from 1 °C temperature rise
Preference and Demographic Parameters			
$\bar{b}$	Fertility parameter	0.048	Pop. growth $n_{CE} = 0.6\%$ (OECD average)
ε	Preference for children	0.446	Child-rearing $h_{CE} = 1/8$, Jones (2022)

4.2 Differences between Competitive Equilibrium and the Optimal Allocation

The difference between the optimal population growth rate n_{SP} and the competitive equilibrium rate n_{CE} is driven by two externalities. It is instructive to visualize these mechanisms separately as shown in Figure 2.

- **Idea Externality.** In a standard semi-endogenous growth model without environmental externalities, the social planner values population more than private agents due to knowledge spillovers ($\tilde{V}_{SP} > \tilde{V}_{CE}$). This shifts the valuation curve upward, leading to $n_{SP} > n_{CE}$.

[14] Note that the model has no saving and thus consumption per capita is equal to GDP per capita, although raising children is somewhat close to saving.

[15] We also treat an economy where the environmental damage is added to a production function, not to the household utility in the Appendix. Quantitative results are not significantly altered in a model with a damage function added to production.

- **Environmental Externality.** In an economy with only environmental externalities, the planner values population less ($\tilde{V}_{SP} < \tilde{V}_{CE}$). However, environmental degradation acts as an implicit tax on market work, shifting the cost of the child-rearing curve (the n curve) rightward. This time allocation channel exerts upward pressure on population growth.[16]

The net effect in our full model depends on which channel dominates under the calibrated parameters. We will see how the combination of two externalities affects

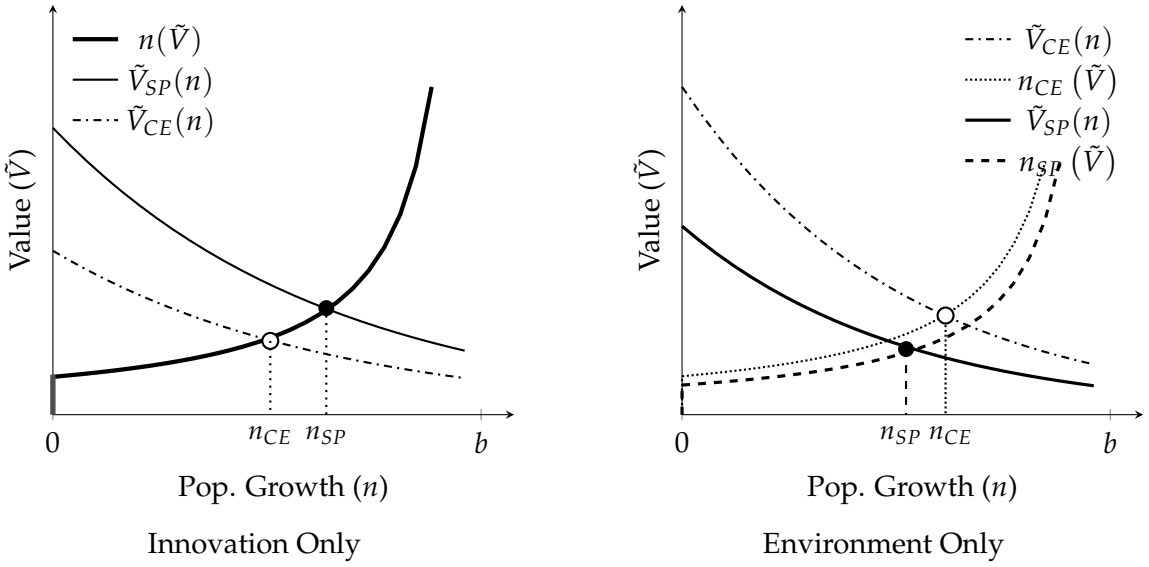


Figure 2: Innovation vs. Environment

each curve.

4.2.1 The population growth rate

Figure 3 shows the population growth rate as a function of the value of the population in two different allocations. We have

$$n_{CE}\left(\tilde{V}\right) = \bar{b} - \delta - \frac{1}{\tilde{V}} \tag{5}$$

$$n_{SP}\left(\tilde{V}\right) = \bar{b} - \delta - \frac{1-\tilde{\xi}}{\tilde{V}} \tag{6}$$

[16]We show in the Appendix that the time allocation channel even makes the optimal population growth rate higher than the competitive equilibrium rate in an economy only with non-pecuniary environmental externality under the assumptions of Barro and Becker (1989) and Jones and Schoonbroodt (2010).

Since the environmental damage $\tilde{\xi}$ lowers the incentive to work and increases the time available for child-rearing, the population growth rate is higher in the optimal allocation.

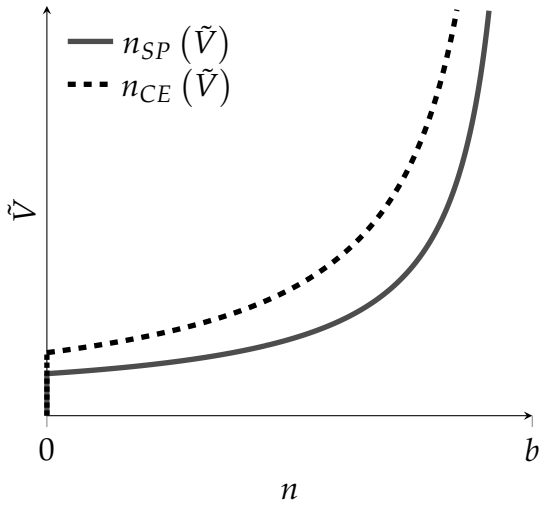


Figure 3: The comparison of the population growth rate $n\left(\tilde{V}\right)$.

4.2.2 The value of the population

The values of the population $\tilde{V}_{CE}(n)$ and $\tilde{V}_{SP}(n)$ are depicted in Figure 4. We have

$$\tilde{V}_{CE}(n) = \frac{\varepsilon}{\rho - (1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right] n} \tag{4}$$

$$\tilde{V}_{SP}(n) = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A}{\rho - (1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right] n} \tag{7}$$

$\tilde{V}_{SP}(n)$ is lower when the population growth rate is sufficiently low, but it surpasses the $\tilde{V}_{CE}(n)$ once n becomes high. Environmental damage persists even in the absence of population growth, whereas rapid technological development does not occur on the BGP without high population growth. Once the population grows sufficiently fast, technology also experiences progress, and the value of the population that internalizes externalities becomes large enough.

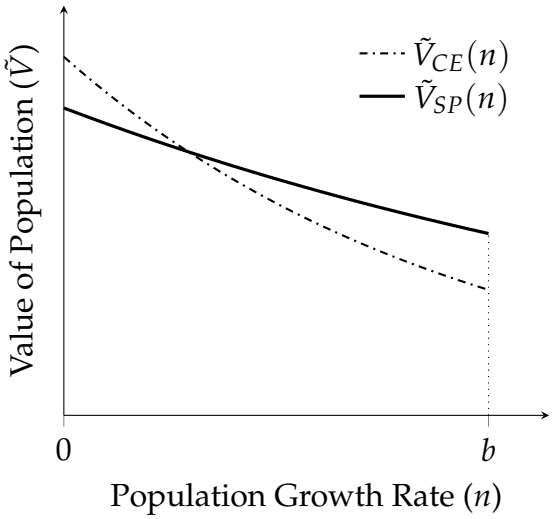


Figure 4: Comparison of the value of the population $\tilde{V}(n)$.

4.3 Quantitative Exercises

A change in the population growth rate in Figure 3 and a change in the value of the population in Figure 4 determine the optimal BGP population growth rate. Figure 5 shows the differences in the BGP population growth rates in the two allocations. Under the baseline parameters in Tables 1 and 2, we obtain $n_{SP} = 0.647\%$ and $n_{CE} = 0.60\%$. The world population growth rate should be higher by 7.8%, according to the model. If we did not have change in the population growth rate $n\left(\tilde{V}\right)$, which represents the time allocation channel, but had only a change in the value of the population, it would generate $n_{SP} < n_{CE}$, because the crossing point of $n_{CE}\left(\tilde{V}\right)$ and $\tilde{V}_{SP}(n)$ is on the left of n_{CE} in Figure 5. The time allocation channel critically changes the qualitative results of the baseline calibrated model. The change in time allocation is shown in Table 3. You can see that labor supply decreases and child-rearing increases in the optimal BGP allocation.

Without time reallocation in the optimal allocation, $n_{SP} > n_{CE}$ holds only when GDP loss from a 3-degree Celsius warming is less than 6.4%. Figure 6 shows the situation where $n_{CE}\left(\tilde{V}\right)$ and $\tilde{V}_{SP}(n)$ cross at the competitive equilibrium population growth rate n_{CE}, underscoring the quantitative impact of the time allocation channel.

We consider different magnitudes of environmental damage by varying the value of γ. Other parameters are calibrated at the baseline value of γ. Figure 7 shows the BGP

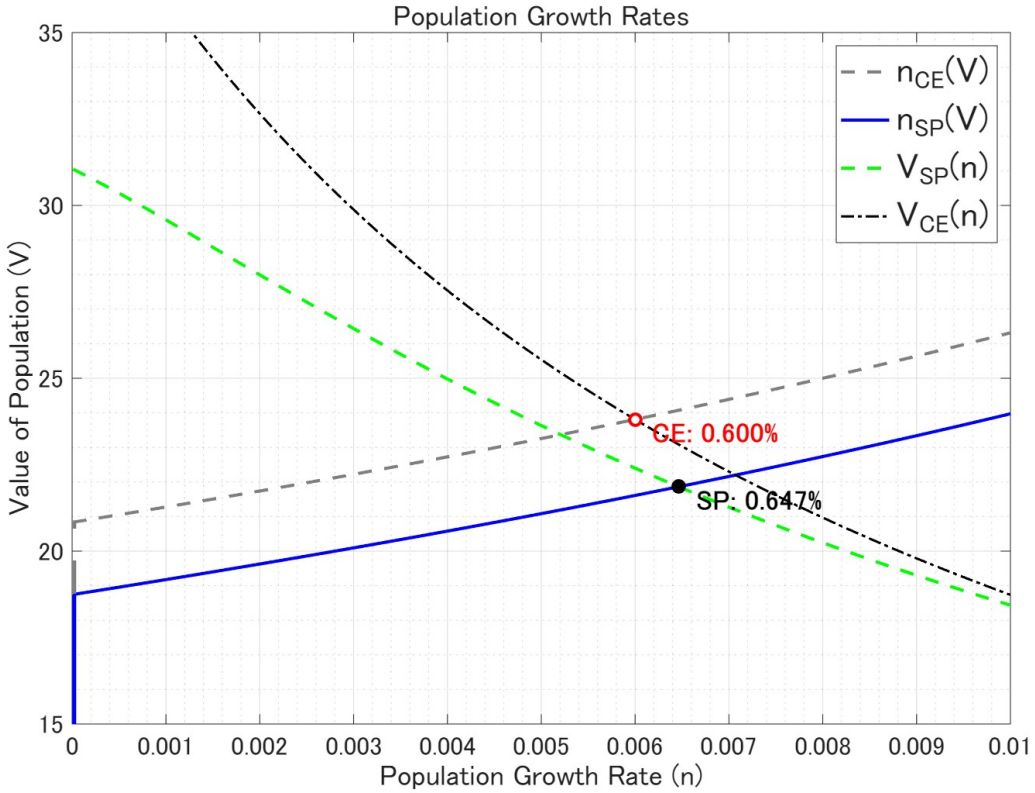


Figure 5: Determination of the BGP Population Growth Rates.

Table 3: BGP outcomes: Competitive Equilibrium vs. Optimal Allocation

	Competitive Equilibrium	Optimal Allocation
Population and Value		
Population growth rate (n)	0.600%	0.647%
Value of population $(\tilde{V})$	23.81	21.86
Time Allocation		
Market work $(1-h)$	0.875	0.865
Child-rearing (h)	0.125	0.135
Growth Rates		
Consumption per capita (g_c)	0.225%	0.242%
Aggregate output (g_Y)	0.825%	0.889%
Carbon stock (g_S)	0.825%	0.889%

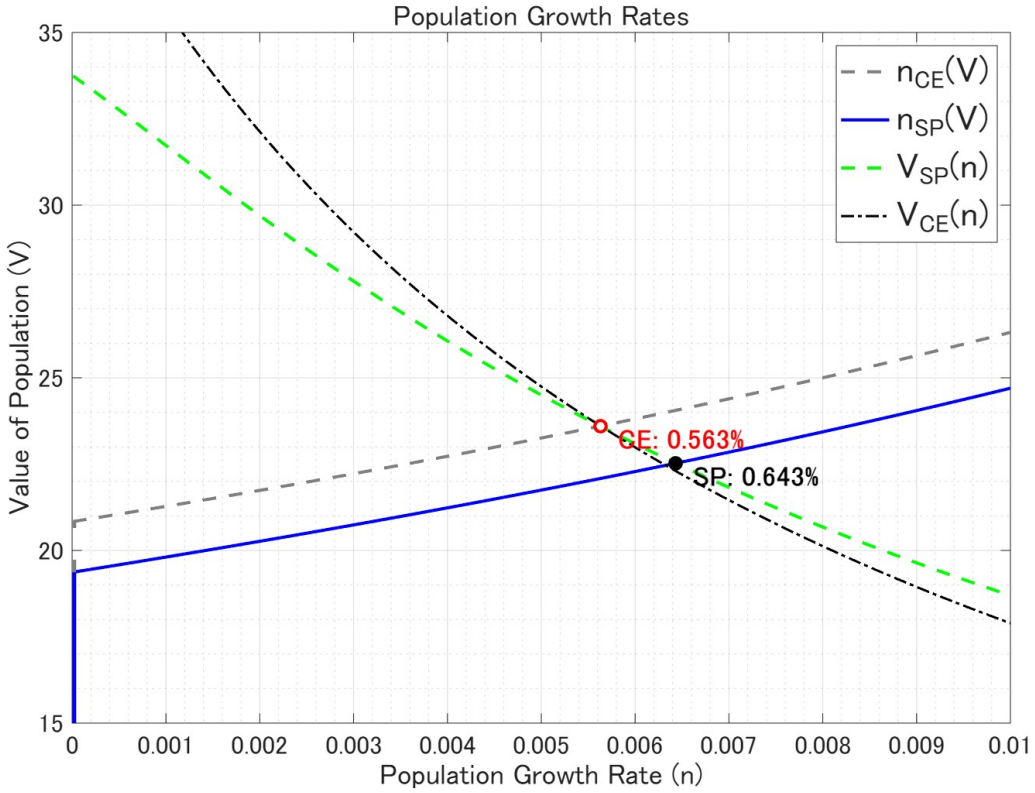


Figure 6: GDP loss from a 3-degree Celsius warming is 6.4%.

population growth rates in both allocations depending on the environmental damage, γ. You can see n_{CE} increases with γ. The effective discount rate, which is given by

$$\rho - (1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma\left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right] n,$$

decreases with γ in the case of $\theta > 1$ or equivalently EIS < 1. Large environmental damage implies that happiness in the future does not grow so much and that the marginal utility in the future is still high. Since $\theta > 1$ discounts the future growth of the input of the utility function $c_t N_t^{\varepsilon} S_t^{-\gamma}$, the environmental disutility $S_t^{-\gamma}$ lowers the discount rate. Hence, despite $\theta > 1$ making the households more impatient with respect to growth in the utility input, households cut today's consumption by reducing labor supply and then have children. Figure 7 demonstrates that the optimal population growth rate exceeds the competitive equilibrium rate as long as the GDP loss from a 3-degree Celsius warming does not reach 11.7%. Whether the actual damage falls

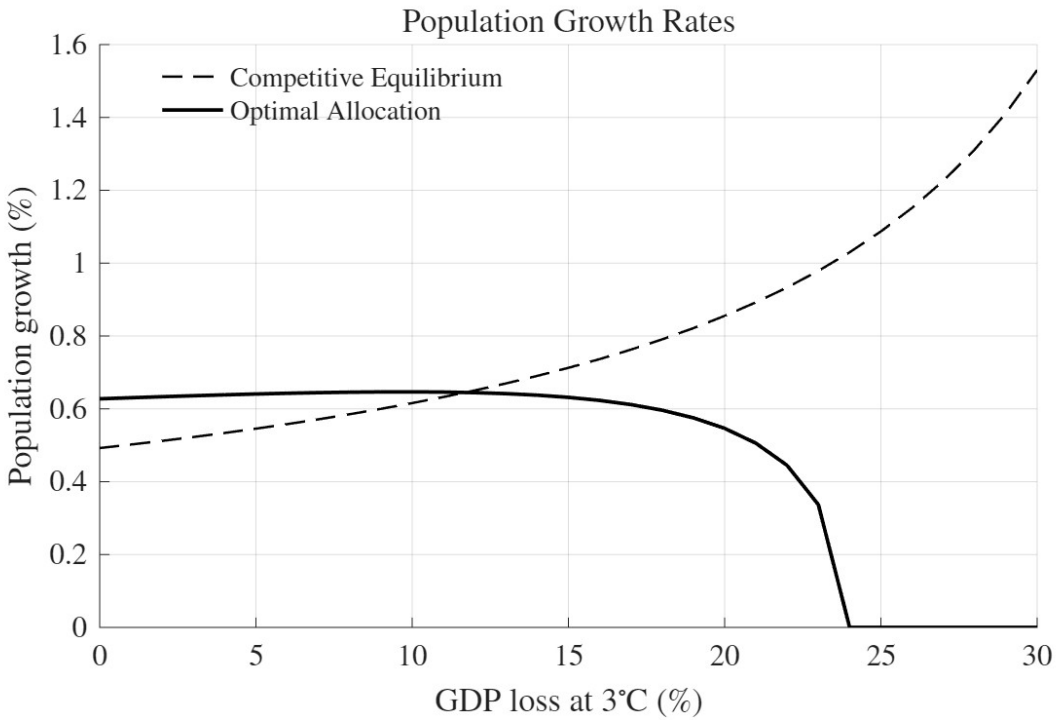


Figure 7: Population Growth Rates with Different Degrees of Environmental Damage

above or below this threshold is an empirical question, and estimates in the literature remain highly dispersed.[17] However, it is our contribution to provide a framework for evaluating the optimal population growth in a setup with endogenous fertility, externality of knowledge spillovers, and non-pecuniary environmental externality. It allows one to transparently translate updated environmental damage estimates into normative implications for optimal population growth.

5 Policy Experiment

As demonstrated in Section 4, the competitive equilibrium allocation generally deviates from the socially optimal allocation due to the presence of externalities. Thus, implementing environmental policies is crucial to address this market failure and achieve the socially optimal allocation.

In scenarios where environmental damages are severe (i.e., a high γ), a government

[17] Barrage and Nordhaus (2023) say that the GDP loss from a 3-degree Celsius warming is just 1.6%. At the same time, Bilal and Känzig (2024) demonstrate that the GDP loss from a 3-degree Celsius global warming is, surprisingly, 36%. According to Nath, Ramey, and Klenow (2025), the GDP loss from a 3-degree Celsius warming is smaller than Bilal and Känzig (2024), raging from 6.6% to 10.5%.

generally has a strong incentive to regulate environmental externalities to curb emissions. This intervention is particularly critical in the high-damage regime identified in Section 4: when the GDP loss from a $3^{\circ}C$ warming exceeds the threshold of 11.7%, the decentralized economy generates excessive emissions, and the competitive population growth rate exceeds the social optimum ($n_{CE} > n_{SP}$).

To address this environmental market failure, we analyze the effect of introducing a carbon tax in the decentralized economy. We demonstrate below that a standard Pigouvian tax on production, intended to mitigate environmental damage, paradoxically has the opposite effect: it accelerates population and productivity growth and, consequently, the long-run accumulation of greenhouse gases.

Let $\tau \in [0, 1)$ denote a tax rate imposed on production (which is equivalent to a tax on emissions since emissions are proportional to output). The government collects the tax revenue and rebates it to households as a lump-sum transfer T_t, maintaining a balanced budget.

5.1 Firm's Problem and Factor Prices

Representative firms hire labor $L_t = (1 - h_t)N_t$ to produce output $Y_t = A_t^{\sigma} L_t$. The firm maximizes profit given the tax τ and the wage rate w_t:

$$\max_{L_t} \quad (1 - \tau) A_t^{\sigma} L_t - w_t L_t$$

The first-order condition equates the wage to the after-tax marginal product of labor:

$$w_t = (1 - \tau) A_t^{\sigma}$$

This equation shows that the carbon tax reduces the effective wage received by households.

5.2 Household's Decision

Households maximize their utility subject to the budget constraint, taking the tax rate τ, transfer T_t, and environmental path as given. The budget constraint is:

$$c_t = w_t(1 - h_t) + T_t$$

Since the tax revenue is rebated lump-sum ($T_t = \tau Y_t / N_t$), the aggregate resource constraint remains $c_t = A_t^{\sigma}(1 - h_t)$, but the marginal return to labor perceived by the household is distorted.

The first-order condition with respect to child-rearing h_t equates the opportunity cost of time to the marginal benefit of children:

$$\underbrace{w_t u_c}_{\text{marginal cost of child-rearing}} = \underbrace{\bar{b} v_t N_t}_{\text{marginal benefit of child-rearing}}$$

Substituting the wage $w_t = (1 - \tau)A_t^{\sigma}$, we obtain

$$(1 - \tau)A_t^{\sigma} u_c = v_t \bar{b} N_t.$$

Rearranging this to solve for the population growth rate $n(\tau)$ on the BGP, we find that the labor wedge $1 - \tau$ plays the same role as the environmental wedge $1 - \tilde{\zeta}$ in the optimal allocation:

$$n_{CE}\left(\tilde{V}; \tau\right) = \bar{b} - \delta - \frac{1 - \tau}{\tilde{V}}$$

Note that for a given value of population $\tilde{V}$, a higher tax rate τ reduces the opportunity cost term $\frac{1-\tau}{\tilde{V}}$. Consequently, an increase in τ shifts the $n(\tilde{V})$ curve upward (or to the right), as illustrated in Figure 8.

A crucial insight of this analysis is that the carbon tax does not alter the functional form of the value of population curve, $\tilde{V}_{CE}(n)$. Since the lump-sum rebate neutralizes the income effect, the structural relationship between the population growth rate n and the discounted value of descendants $\tilde{V}_{CE}$ remains unchanged. The tax τ affects the

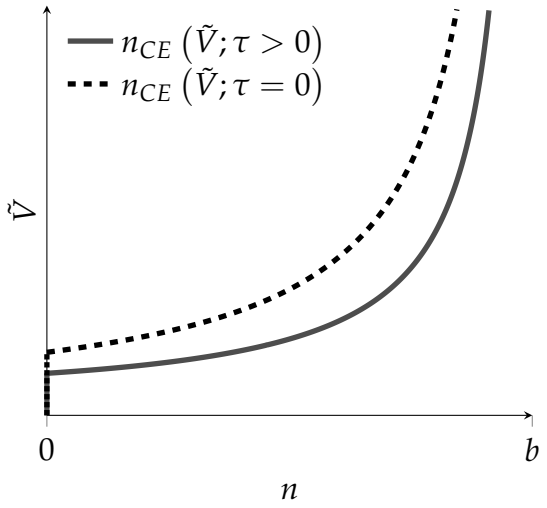


Figure 8: The population growth rate curve $n\left(\tilde{V}\right)$ and a carbon tax τ.

equilibrium $\tilde{V}$ only indirectly through the change in equilibrium $n_{CE}\left(\tilde{V}\right)$.

5.3 The Paradoxical Result

We can now analyze the comparative statics of the population growth rate with respect to the carbon tax τ. Since the term $(1-\tau)$ appears with a negative sign in the equation for $n(\tau)$, a higher tax rate increases the population growth rate. The carbon tax lowers the net wage $(1-\tau)A_t^{\sigma}$, which is the opportunity cost of child-rearing. Consequently, the policy moves the economy's population growth rate further away from the social optimum n_{SP}, exacerbating the distortion instead of correcting it.

This result contradicts the common intuition that environmental taxes, by penalizing production, necessarily stifle economic dynamism. In our endogenous growth framework, the tax lowers the net wage $(1-\tau)A_t^{\sigma}$, inducing a substitution from market work to child-rearing. Crucially, this demographic shift accelerates the creation of new ideas, as the growth rate of productivity is driven by population growth ($g_A = \frac{\lambda_A}{\beta_A} n$). Thus, we obtain a positive relationship between the tax rate τ and the productivity growth. Paradoxically, a tax designed to restrain economic activity ends up stimulating long-run productivity growth by reallocating time toward the production of people (and thus ideas). Consequently, the growth rate of the greenhouse gas stock, which follows $g_S = (1 + \frac{\lambda_A \sigma}{\beta_A}) n(\tau)$, also increases. This result highlights a green policy paradox.

In high-γ economies where the optimal population growth is low ($n_{SP} < n_{CE}$), policymakers are naturally inclined to impose strict environmental regulations. However, this very policy aimed at reducing the environmental burden inadvertently accelerates population growth and the long-run accumulation of greenhouse gases, exacerbating the deviation from the social optimum.

6 Conclusion

This paper addresses a fundamental question of our time: in the face of the climate crisis, is population growth a burden or a blessing? Endogenous growth theory champions the latter, positing that a larger population is the ultimate engine of innovation and rising living standards. Conversely, a smaller population seems desirable for mitigating greenhouse gas emissions and climate change, because aggregate emissions are positively associated with population size. To discuss the optimality of population growth, we extend a semi-endogenous growth model with endogenous fertility to incorporate a non-pecuniary environmental externality. The optimal population growth rate is determined by internalizing the trade-off that a larger population accelerates the discovery of new ideas while also increasing greenhouse gas emissions.

Quantitatively, our calibrated model demonstrates that under the baseline assumption of a 9% GDP loss from a 3-degree Celsius warming, the optimal population growth rate exceeds the competitive equilibrium rate by 7.8%. This ranking reverses only when the GDP loss from a 3-degree Celsius warming reaches a threshold of 11.7%.

We find that the time allocation channel plays an important role in generating these quantitative results. Because higher production intensifies environmental degradation, the marginal benefit of labor supply is reduced when environmental externalities are internalized. This implies that the opportunity cost of child-rearing declines in the optimal allocation. In the optimal allocation, the wedge to labor supply encourages a reallocation of time from production to child-rearing. This logic is also valid in a model in the Appendix, in which households can allocate time not only to labor and child-

rearing but also to leisure, which does not generate greenhouse gases. This mechanism supports higher population growth in an economy with negative environmental externality, counteracting the direct environmental costs of population growth. Absent the endogenous time-reallocation incentive arising from the labor supply penalty, the socially optimal population growth rate exceeds the competitive equilibrium rate only when the GDP loss from a 3-degree Celsius warming is below 6.4%.

Finally, our policy experiment uncovers a paradoxical result with significant implications for climate action. While standard carbon taxes are designed to curb emissions by penalizing production, we show that they unintentionally accelerate long-run pollution through demographic channels. By depressing the net return to labor, a carbon tax lowers the opportunity cost of child-rearing, inducing a shift from production to reproduction. This channel increases population growth and, consequently, greenhouse gas accumulation. This finding suggests that in scenarios of severe environmental damage, relying solely on production taxes may be counterproductive, underscoring the need for a more comprehensive policy approach that accounts for endogenous demographic responses.

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Appendix

In this appendix, we present the model setups and optimality conditions for five alternative specifications: (1) Damage to Production, (2) Without Leisure but With Clean Technology, (3) With Leisure but Without Clean Technology, (4) The General Model with both Leisure and Clean Technology, and (5) With the Service Sector. To highlight the fundamental trade-off between labor for production and time for child-rearing, we assumed for the baseline model that households allocate their time only between market work and child-rearing (i.e., no leisure). Furthermore, to focus on the demographic impact on emissions, we abstracted from clean technology in this baseline specification. We provide extended models below. We also demonstrate theoretically how the model behaves with only environmental externality.

A Damage to Production

We consider a setup in which environmental damage affects production rather than household utility. This setup is close to Golosov et al. (2014). Households gain utility from consumption and their descendants, but consumption is discounted by the environmental damage.

The optimal allocation solves the following problem:

$$
\begin{aligned}
\max_{\{h_t\}_{t\in[0,\infty)}} \quad & \int_0^\infty e^{-\rho t} u(c_t, N_t)\, dt \\
\text{subject to} \quad & u(c_t, N_t) = \frac{(c_t N_t^{\varepsilon})^{1-\theta}}{1-\theta}, \\
& c_t = A_t^{\sigma}(1-h_t)S_t^{-\gamma}, \\
& \dot{N}_t = \left(\bar{b}h_t - \delta\right) N_t, \\
& \dot{S}_t = A_t^{\sigma}(1-h_t)N_t S_t^{-\gamma} - \zeta S_t, \\
& \frac{\dot{A}_t}{A_t} = N_t^{\lambda_A} A_t^{-\beta_A}.
\end{aligned}
$$

Per capita consumption is given by $c_t = A_t^{\sigma}(1-h_t)S_t^{-\gamma}$, because a production function

incorporates a damage function $S_t^{-\gamma}$.

The BGP solution is characterized by:

$$n = \bar{b} - \delta - \frac{1-\tilde{\xi}}{\tilde{V}_{SP}}, \qquad z_A = \frac{\sigma g_A (1-\tilde{\xi})}{\rho - g_u + \beta_A g_A},$$

$$\tilde{V}_{SP} = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A}{\rho - g_u}, \qquad \tilde{\xi} = \frac{\frac{\gamma(g_S+\zeta)}{\rho+\zeta-g_u+g_Y}}{1+\frac{\gamma(g_S+\zeta)}{\rho+\zeta-g_u+g_Y}}.$$

The environmental damage $\tilde{\xi}$ is lower than in the baseline model by the denominator $1 + \frac{\gamma(g_S+\zeta)}{\rho+\zeta-g_u+g_Y}$. The greenhouse gas is less problematic than in the baseline setup because a larger stock of greenhouse gases lowers aggregate output, thereby reducing current emissions. The competitive equilibrium BGP allocation is obtained if you assume no idea externality $z_A = 0$ and no environmental externality $\tilde{\xi} = 0$. Note that on the BGP, the growth rates are given by $g_A = \frac{\lambda_A}{\beta_A} n$, $g_S = \frac{1+\frac{\lambda_A \sigma}{\beta_A}}{1+\gamma} n$, $g_Y = \sigma g_A + n - \gamma g_S$, and $g_c = \sigma g_A - \gamma g_S$. The results are shown in Table 5. The optimal population growth rate is 0.656%, whereas it is 0.647% in the baseline analysis. The results are not significantly altered in this specification. Parameters are calibrated as in Table 4 to match the competitive equilibrium objects to the targets.

Table 4: Calibrated Parameter Values (Damage to Production)

Parameter	Description	Value	Target / Source
Environmental Parameter			
γ	Damage elasticity	0.136	3% GDP loss from 1 °C temperature rise
Preference and Demographic Parameters			
$\bar{b}$	Fertility parameter	0.048	Pop. growth $n_{CE} = 0.6\%$ (OECD average)
ε	Preference for children	0.454	Child-rearing $h_{CE} = 1/8$, Jones (2022)

Figure 9 shows the BGP population growth rates under different estimates of environmental damage.

Table 5: BGP outcomes (Appendix Model: Damage to Production)

	Competitive Equilibrium	Optimal Allocation
Population and Value		
Population growth rate (n)	0.600%	0.656%
Value of population ($\tilde{V}$)	23.81	22.09
Time Allocation		
Market work ($1-h$)	0.875	0.863
Child-rearing (h)	0.125	0.137
Growth Rates		
Consumption per capita (g_c)	0.126%	0.138%
Aggregate output (g_Y)	0.825%	0.902%
Carbon stock (g_S)	0.726%	0.794%

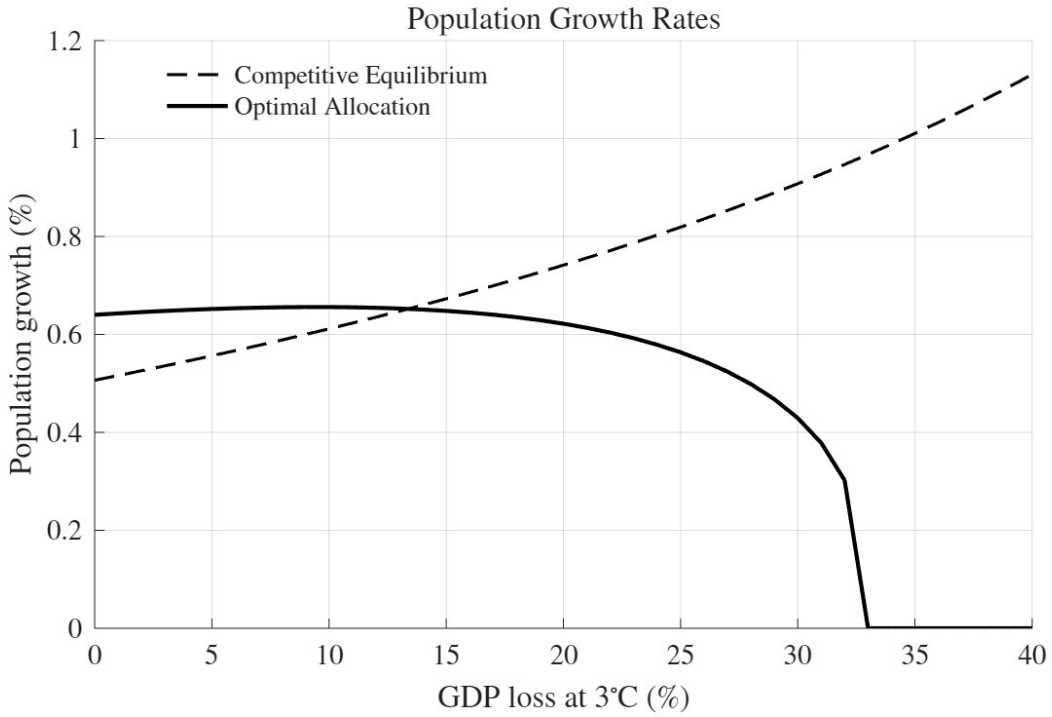


Figure 9: Population Growth Rates. Damage to Production.

B Clean Technology

In this specification, the economy has access to clean technology B_t that mitigates emissions. The law of motion for greenhouse gases includes clean technology B_t:

$$\dot{S}_t = \frac{Y_t}{B_t} - \zeta S_t$$

The clean technology B_t reduces greenhouse gas emissions from economic activity Y_t. Clean technology evolves endogenously as

$$\frac{\dot{B}_t}{B_t} = N_t^{\lambda_B} B_t^{-\beta_B}.$$

λ_B captures the contribution of ideas to technological progress in clean technology, and β_B determines how much ideas are getting harder to find.

B.1 Optimal Allocation

The optimal allocation solves the following problem:

$$\begin{aligned}
\max_{\{h_t\}_{t\in[0,\infty)}} \quad & \int_0^\infty e^{-\rho t} u(c_t, N_t, S_t)\, dt \\
\text{subject to} \quad & u(c_t, N_t, S_t) = \frac{\left(c_t N_t^{\varepsilon} S_t^{-\gamma}\right)^{1-\theta}}{1-\theta}, \\
& c_t = A_t^{\sigma}(1-h_t), \\
& \dot{N}_t = \left(\bar{b} h_t - \delta\right) N_t, \\
& \dot{S}_t = \frac{A_t^{\sigma}(1-h_t)N_t}{B_t} - \zeta S_t, \\
& \frac{\dot{A}_t}{A_t} = N_t^{\lambda_A} A_t^{-\beta_A}, \\
& \frac{\dot{B}_t}{B_t} = N_t^{\lambda_B} B_t^{-\beta_B}.
\end{aligned}$$

The current-value Hamiltonian is

$$H = u(A_t^{\sigma}(1-h_t), N_t, S_t) + v_t\left(\bar{b}h_t - \delta\right)N_t + \mu_{At}N_t^{\lambda_A}A_t^{1-\beta_A} + \mu_{Bt}N_t^{\lambda_B}B_t^{1-\beta_B} - \phi_t\left[\frac{A_t^{\sigma}(1-h_t)N_t}{B_t} - \zeta S_t\right],$$

where μ_{Bt} is the shadow price of an idea related to clean technology.

We define the following additional auxiliary variables:

$$\tilde{\xi}_t \equiv \frac{\phi_t \frac{Y_t}{B_t}}{c_t u_c\left(c_t, N_t, S_t\right)} \qquad \text{Environmental damage}$$

$$z_{Bt} \equiv \frac{\mu_{Bt}\dot{B}_t}{c_t u_c\left(c_t, N_t, S_t\right)} \qquad \text{Value of clean ideas}$$

The BGP optimal allocations satisfy the following conditions:

$$1-h = \frac{1-\tilde{\xi}}{\bar{b}\tilde{V}_{SP}}, \qquad n = b - \delta - \frac{1-\tilde{\xi}}{\tilde{V}_{SP}}, \qquad z_A = \frac{\sigma g_A\left(1-\tilde{\xi}\right)}{\rho - g_u + \beta_A g_A},$$

$$z_B = \frac{g_B\tilde{\xi}}{\rho - g_u + \beta_B g_B}, \quad \tilde{V}_{SP} = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A + \lambda_B z_B}{\rho - g_u}, \quad \tilde{\xi} = \frac{\gamma(g_S + \zeta)}{\rho + \zeta - g_u - g_B + g_Y}.$$

The numerator of the term z_B increases if the growth rate of clean technology is high and the environmental damage $\tilde{\xi}$ is large. You get the competitive equilibrium allocation on the BGP if you assume $z_A = z_B = \tilde{\xi} = 0$.

Parameters are set as in Table 6. Utilizing time series data of the growth rate of CO_2 in the atmosphere based on the Global Monitoring Laboratory, we also have $g_S = 0.623\%$, the average between 2015 and 2024. Also we get the average population growth rate of the OECD countries of the same time periods, 0.54%. The relationship $g_S = \left(1 + \frac{\lambda_A\sigma}{\beta_A} - \frac{\lambda_B}{\beta_B}\right)n$ implies

$$\frac{\lambda_B}{\beta_B} = 1 + \frac{\lambda_A\sigma}{\beta_A} - \frac{g_S}{n} = 0.221.$$

We assume $\lambda_B = 0.75$, so that the clean technology is comparable to productivity. Then, we obtain the ideas-are-getting-harder-to-find parameter $\beta_B = 3.39$. The comparison

between two allocations in the calibrated model appears in Table 7. The optimal population growth rate n_{SP} is higher than in the baseline model without clean technology. Figure 10 presents results for different levels of environmental damage.

Table 6: Calibrated Parameter Values (Clean Technology)

Parameter	Description	Value	Target / Source
Environmental and Technology Parameters			
γ	Damage elasticity	0.136	3% GDP loss from 1 °C temperature rise
λ_B	Clean tech efficiency	0.75	Assumption (Comparable to productivity)
β_B	Clean tech spillover	3.39	$g_S = 0.623\%$ and $n = 0.54\%$ (Average 2015-2024)
Preference and Demographic Parameters			
$\bar{b}$	Fertility parameter	0.048	Pop. growth $n_{CE} = 0.6\%$ (OECD average)
ε	Preference for children	0.404	Child-rearing time $h_{CE} = 1/8$

Table 7: BGP outcomes (With Clean Technology): Competitive Equilibrium vs. Optimal Allocation

	Competitive Equilibrium	Optimal Allocation
Population and Value		
Population growth rate (n)	0.600%	0.657%
Value of population $(\tilde{V})$	23.81	21.97
Time Allocation		
Market work $(1-h)$	0.875	0.863
Child-rearing (h)	0.125	0.137
Growth Rates		
Consumption per capita (g_c)	0.225%	0.246%
Aggregate output (g_Y)	0.825%	0.904%
Carbon stock (g_S)	0.692%	0.758%

C Leisure

In this specification, households value leisure. The instantaneous utility function includes leisure:

$$u(c_t, 1-l_t-h_t, N_t, S_t) = \frac{\left(c_t^{\omega}(1-l_t-h_t)^{1-\omega} N_t^{\epsilon} S_t^{-\gamma}\right)^{1-\theta}}{1-\theta},$$

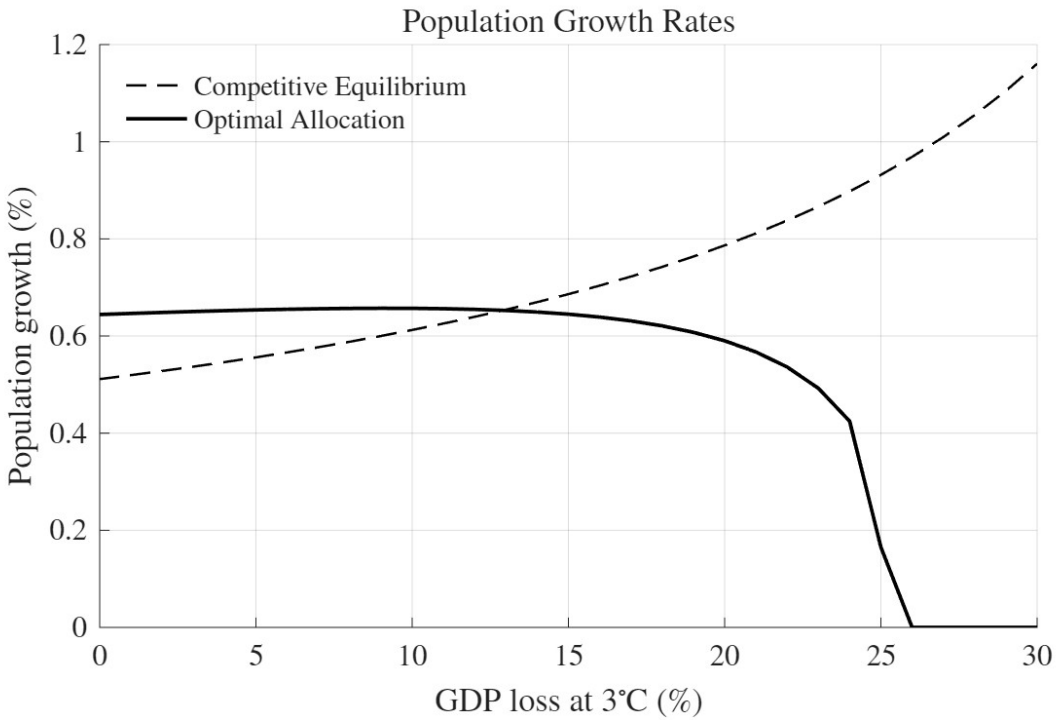


Figure 10: Population Growth Rates. With Clean Technology.

where ω is the weight on consumption relative to leisure, h_t is child-rearing, l_t is labor supply, and $1 - l_t - h_t$ is leisure. A utility function of the form $c_t^{\omega}(1 - l_t - h_t)^{1-\omega}$ is widespread and appears in numerous papers.

C.1 Optimal Allocation

The optimal allocation solves the following problem:

$$
\begin{aligned}
\max_{\{l_t, h_t\}_{t\in[0,\infty)}} \quad & \int_0^{\infty} e^{-\rho t} u(c_t, 1 - l_t - h_t, N_t, S_t)\, dt \\
\text{subject to} \quad & u(c_t, 1 - l_t - h_t, N_t, S_t) = \frac{\left(c_t^{\omega}(1 - l_t - h_t)^{1-\omega} N_t^{\varepsilon} S_t^{-\gamma}\right)^{1-\theta}}{1-\theta}, \\
& c_t = A_t^{\sigma} l_t, \\
& \dot{N}_t = \left(\bar{b} h_t - \delta\right) N_t, \\
& \dot{S}_t = A_t^{\sigma} l_t N_t - \zeta S_t, \\
& \frac{\dot{A}_t}{A_t} = N_t^{\lambda_A} A_t^{-\beta_A}.
\end{aligned}
$$

We redefine the following auxiliary variables since the utility function is somewhat different from the one in the baseline model:

$$\tilde{\xi}_t \equiv \frac{\phi_t Y_t}{(1-\theta)u_t} \qquad \text{Environmental damage}$$
$$z_{At} \equiv \frac{\mu_{At}\dot{A}_t}{(1-\theta)u_t} \qquad \text{Value of productivity ideas}$$
$$\tilde{V}_{SP,t} \equiv \frac{v_t N_t}{(1-\theta)u_t} \qquad \text{Social value of population}$$

The optimal BGP allocation are characterized by the following conditions:

$$l = \frac{\omega - \tilde{\xi}}{\bar{b}\tilde{V}_{SP}}, \qquad 1 - l - h = \frac{1-\omega}{\bar{b}\tilde{V}_{SP}}, \qquad n = \bar{b} - \delta - \frac{1-\tilde{\xi}}{\tilde{V}_{SP}},$$
$$z_A = \frac{\sigma g_A \left(\omega - \tilde{\xi}\right)}{\rho - g_u + \beta_A g_A}, \qquad \tilde{V}_{SP} = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A}{\rho - g_u}, \qquad \tilde{\xi} = \frac{\gamma(g_S + \zeta)}{\rho + \zeta - g_u + g_Y}.$$

In this model, the population growth rate as a function of the value of the population

$$n_{SP}\left(\tilde{V}\right) = \bar{b} - \delta - \frac{1-\tilde{\xi}}{\tilde{V}},$$

is unchanged and identical to that in the baseline model, confirming the robustness of the time allocation channel. The weight on consumption ω is less than 1, whereas $\omega = 1$ in the baseline analysis. Then, the marginal benefit of labor supply declines, and labor hours are reduced. However, that reduction is entirely offset by the introduction of leisure. The sum of labor and leisure is given by,

$$l + (1 - l - h) = \frac{1-\tilde{\xi}}{\bar{b}\tilde{V}_{SP}},$$

and this is equal to the labor supply in a model without leisure, given $\tilde{V}_{SP}$. The optimal labor supply is proportional to the weight $\omega - \tilde{\xi}$, while leisure corresponds to $1 - \omega$. The preference parameter ω governs the composition of these non-child-rearing activities but cancels out in their summation. You can also notice that non-child-rearing

activities incorporate the environmental damage term $\tilde{\xi}$. Therefore, the population growth rate $n_{SP}\left(\tilde{V}\right)$ remains unchanged and is influenced by the time allocation channel.

Table 9 gives the baseline results, and the competitive equilibrium is matched to the targets by setting parameter values as in Table 8. γ is smaller than the baseline value. We have $c_t^{\omega} S_t^{-\gamma} = \left(c_t S_t^{-\frac{\gamma}{\omega}}\right)^{\omega}$ in the instantaneous utility function. The formula

$$(1 - 0.03 \times 3) c_t S_t^{-\frac{\gamma}{\omega}} = c_t (2S_t)^{-\frac{\gamma}{\omega}}$$

implies $\gamma = -\omega \frac{\log(1-0.09)}{\log(2)} = 0.0518$.

Table 8: Calibrated Parameter Values (Leisure)

Parameter	Description	Value	Target / Source
Environmental Parameter			
γ	Damage elasticity	0.518	3% GDP loss from 1 °C temperature rise
Preference and Demographic Parameters			
$\bar{b}$	Fertility parameter	0.048	Pop. growth $n_{CE} = 0.6\%$ (OECD average)
ω	Preference for consumption	0.381	Child-rearing time $h_{CE} = 1/8$, Jones (2022)
ε	Preference for children	0.399	Working hours $l_{CE} = 1/3$

Table 9: BGP outcomes (With Leisure)

	Competitive Equilibrium	Optimal Allocation
Population and Value		
Population growth rate (n)	0.600%	0.619%
Value of population ($\tilde{V}$)	23.81	23.05
Time Allocation		
Market work (l)	0.333	0.311
Child-rearing (h)	0.125	0.129
Leisure ($1 - l - h$)	0.542	0.560
Growth Rates		
Consumption per capita (g_c)	0.225%	0.232%
Aggregate output (g_Y)	0.825%	0.852%
Carbon stock (g_S)	0.825%	0.852%

Figure 11 depicts the BGP population growth rates in two different allocations.

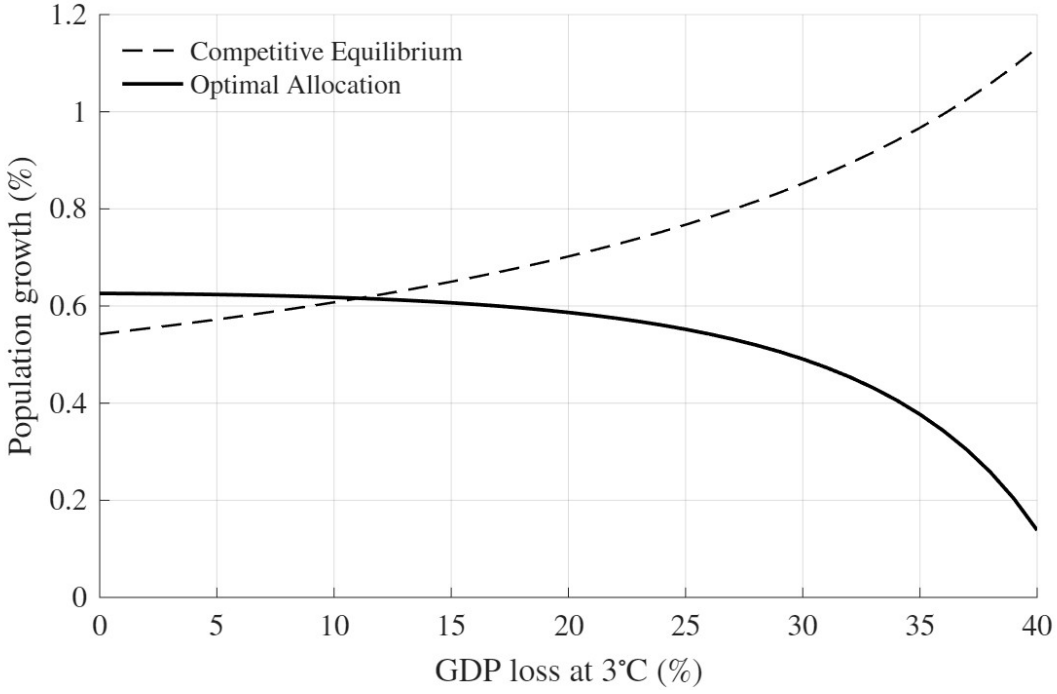


Figure 11: Population Growth Rate and Environmental Damage. With Leisure.

D Clean Technology and Leisure

This section presents the fully generalized model, incorporating both endogenous leisure and clean technology.

D.1 Optimal Allocation

The optimal allocation solves the following problem:

$$
\begin{aligned}
\max_{\{l_t,h_t\}_{t\in[0,\infty)}} \quad & \int_0^\infty e^{-\rho t} u(c_t, 1-l_t-h_t, N_t, S_t)\, dt \\
\text{subject to} \quad & u(c_t, 1-l_t-h_t, N_t, S_t) = \frac{\left(c_t^\omega (1-l_t-h_t)^{1-\omega} N_t^\varepsilon S_t^{-\gamma}\right)^{1-\theta}}{1-\theta}, \\
& c_t = A_t^\sigma l_t, \\
& \dot{N}_t = \left(\bar{b} h_t - \delta\right) N_t, \\
& \dot{S}_t = \frac{A_t^\sigma l_t N_t}{B_t} - \zeta S_t, \\
& \frac{\dot{A}_t}{A_t} = N_t^{\lambda_A} A_t^{-\beta_A},
\end{aligned}
$$

$$\frac{\dot{B}_t}{B_t} = N_t^{\lambda_B} B_t^{-\beta_B}.$$

We redefine

$$z_{Bt} \equiv \frac{\mu_{Bt}\dot{B}_t}{(1-\theta)u_t} \text{Value of productivity ideas.}$$

The optimality conditions on the BGP are written as

$$l = \frac{\omega - \tilde{\xi}}{\bar{b}\tilde{V}_{SP}}, \qquad n = \bar{b} - \delta - \frac{1-\tilde{\xi}}{\tilde{V}_{SP}}, \qquad z_A = \frac{\sigma g_A(\omega - \tilde{\xi})}{\rho - g_u + \beta_A g_A},$$

$$z_B = \frac{g_B\tilde{\xi}}{\rho - g_u + \beta_B g_B}, \quad \tilde{V}_{SP} = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A + \lambda_B z_B}{\rho - g_u}, \quad \tilde{\xi} = \frac{\gamma(g_S + \zeta)}{\rho + \zeta - g_u - g_B + g_Y}.$$

The BGP in the competitive equilibrium corresponds to the case with $z_A = z_B = \tilde{\xi} = 0$.

Table 10 shows the baseline results.

Table 10: BGP outcomes (With Leisure and Clean Technology): Competitive Equilibrium vs. Optimal Allocation

	Competitive Equilibrium	Optimal Allocation
Population and Value		
Population growth rate (n)	0.600%	0.625%
Value of population ($\tilde{V}$)	23.81	23.10
Time Allocation		
Market work (l)	0.333	0.311
Child-rearing (h)	0.125	0.130
Leisure ($1-l-h$)	0.542	0.558
Growth Rates		
Consumption per capita (g_c)	0.225%	0.235%
Aggregate output (g_Y)	0.825%	0.860%
Carbon stock (g_S)	0.692%	0.722%

Figure 12 provides the results with different estimates of environmental damages. Calibration result is summarized in Table 11.

E Service Sector

In this section, we extend the baseline model by introducing a service sector. The economy consists of two sectors: the goods sector (dirty) and the service sector (clean).

Table 11: Calibrated Parameter Values (Leisure and Clean Technology)

Parameter	Description	Value	Target / Source
Environmental and Technological Parameters			
γ	Damage elasticity	0.518	3% GDP loss from 1 °C temperature rise
λ_B	Clean tech efficiency	0.75	Assumption (Comparable to productivity)
β_B	Clean tech spillover	3.39	$g_S = 0.623\%$ and $n = 0.54\%$ (Average 2015-2024)
Preference and Demographic Parameters			
$\bar{b}$	Fertility parameter	0.048	Pop. growth $n_{CE} = 0.6\%$ (OECD average)
ω	Preference for consumption	0.381	Child-rearing time $h_{CE} = 1/8$, Jones (2022)
ε	Preference for children	0.399	Working hours $l_{CE} = 1/3$

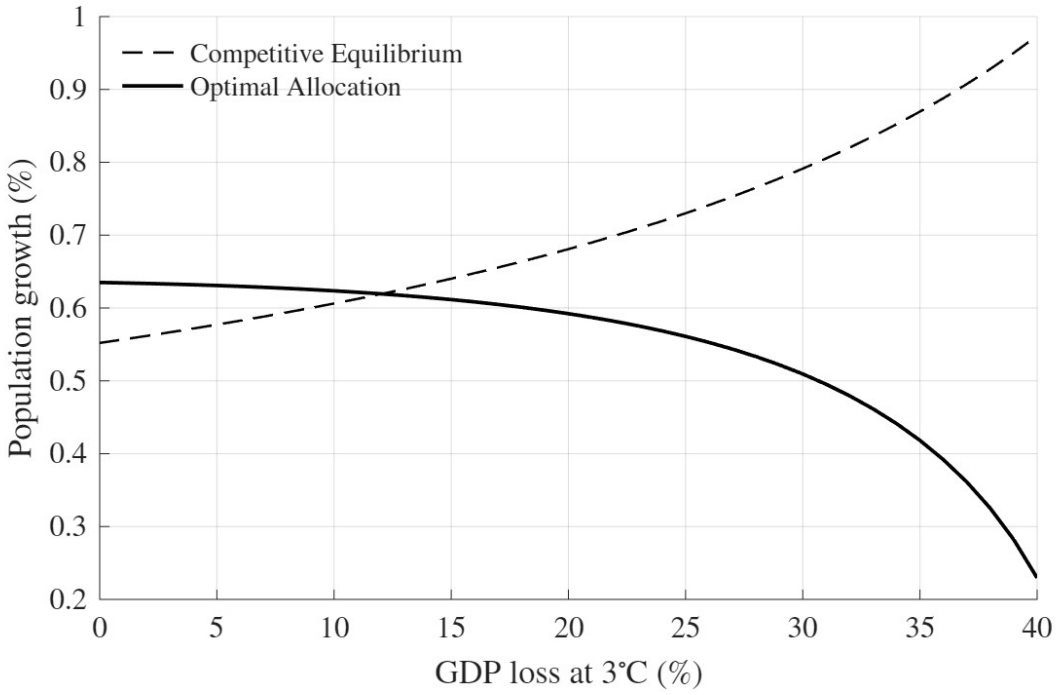


Figure 12: Population Growth Rates with Leisure and Clean Technology.

This extension allows us to analyze the shift of labor from dirty production to clean services as a possible mechanism to mitigate environmental damage.

Both sectors share the same endogenous productivity A_t. The goods sector (g) emits greenhouse gases, while the service sector (s) does not.

$$Y_{g,t} = A_t^{\sigma} l_{g,t} N_t \quad \text{(Dirty Goods)}$$
$$Y_{s,t} = A_t^{\sigma} l_{s,t} N_t \quad \text{(Clean Services)}$$

where $l_{g,t}$ and $l_{s,t}$ denote the labor hours allocated to each sector.

Households consume a bundle of goods and services. For simplicity, we assume a standard CES aggregator without preference weights:

$$c_t = \left[c_{g,t}^{\rho} + c_{s,t}^{\rho} \right]^{\frac{1}{\rho}}$$

where $\rho \leq 1$, and the elasticity of substitution is given by $\frac{1}{1-\rho}$.

Environment. Emissions are generated only by the production of goods. The stock of greenhouse gases S_t evolves according to:

$$\dot{S}_t = Y_{g,t} - \zeta S_t = A_t^{\sigma} l_{g,t} N_t - \zeta S_t$$

Time Constraint. The household's time endowment is normalized to 1. It is allocated between child-rearing h_t and labor in two sectors:

$$h_t + l_{g,t} + l_{s,t} = 1$$

E.1 Optimal Allocation

The optimal allocation maximizes the dynastic utility subject to the resource constraints and laws of motion. The current-value Hamiltonian H is:

$$H = \frac{\left(c_t N_t^{\epsilon} S_t^{-\gamma}\right)^{1-\theta}}{1-\theta} + v_t \left(\bar{b} h_t - \delta\right) N_t + \mu_{At} N_t^{\lambda_A} A_t^{1-\beta_A} - \phi_t \left[N_t A_t^{\sigma} l_{g,t} - \zeta S_t\right]$$

where $v_t, \mu_{At}, -\phi_t$ are the shadow prices of population, ideas, and pollution, respectively.

We define the following auxiliary variable representing the social cost of environmental damage relative to consumption utility:

$$\tilde{\zeta}_t \equiv \frac{\phi_t Y_{g,t}}{u_c(c_t, N_t, S_t) c_t}.$$

The optimality conditions on the BGP are the same as in the baseline model, and are written as

$$h = 1 - \frac{1-\tilde{\xi}}{\bar{b}\tilde{V}_{SP}}, \qquad 1-h = \frac{1-\tilde{\xi}}{\bar{b}\tilde{V}_{SP}}, \qquad n = \bar{b} - \delta - \frac{1-\tilde{\xi}}{\tilde{V}_{SP}},$$
$$z_A = \frac{\sigma g_A \left(1-\tilde{\xi}\right)}{\rho - g_u + \beta_A g_A}, \qquad \tilde{V}_{SP} = \frac{\varepsilon - \tilde{\xi} + \lambda_A z_A}{\rho - g_u}, \qquad \tilde{\xi} = \frac{\gamma\left(g_S + \zeta\right)}{\rho + \zeta - g_u + g_Y}.$$

Here, we also need to solve for the optimal allocation of labor between the two sectors ($l_{g,t}$ and $l_{s,t}$). The first-order conditions with respect to $l_{s,t}$ and $l_{g,t}$ are

$$\frac{\partial H}{\partial l_{s,t}} = u_c(c_t, N_t, S_t) c_t \frac{l_{s,t}^{\rho-1}}{l_{s,t}^{\rho} + l_{g,t}^{\rho}} - v_t \bar{b} N_t = 0,$$
$$\frac{\partial H}{\partial l_{g,t}} = u_c(c_t, N_t, S_t) c_t \frac{l_{g,t}^{\rho-1}}{l_{s,t}^{\rho} + l_{g,t}^{\rho}} - \phi_t N_t A_t^{\sigma} - v_t \bar{b} N_t = 0$$

Note that the marginal opportunity cost of working in the clean sector is equal to the marginal benefit of child-rearing ($v_t \bar{b} N_t$). For the dirty sector, the optimal allocation adds the marginal environmental cost ($\phi_t N_t A_t^{\sigma}$).

Combining both equations, we obtain the equation that determines the labor ratio $x_t \equiv l_{s,t}/l_{g,t}$ as a function of environmental wedge $\tilde{\zeta}_t$,

$$\tilde{\zeta}_t = \frac{1 - x_t^{\rho-1}}{1 + x_t^{\rho}}.$$

This equation implies the working hours in the clean sector is longer than in the manufacturing sector, $x_t > 1$, if the environmental externality is internalized, $\tilde{\zeta}_t > 0$. Intuitively, the clean sector is preferable as environmental damage becomes more severe.

The total working hours is determined by

$$1 - h_t = \frac{1 - \tilde{\zeta}_t}{\bar{b}\tilde{V}_{SP,t}}.$$

This equation is identical to that in the baseline analysis. Because time is allocated among child-rearing, (dirty) manufacturing labor $l_{g,t}$, and (clean) service labor $l_{s,t}$, optimality requires the same marginal utility payoff of an additional unit of time across all uses. As a result, the environmental externality lowers the common marginal return to *any* market work at the margin and therefore acts like a uniform wedge on aggregate labor, so the macro time-allocation condition remains $1 - h_t = \frac{1-\tilde{\zeta}_t}{\bar{b}\tilde{V}_{SP,t}}$ as in the baseline; the clean service sector only affects the within-work composition $x_t \equiv l_{s,t}/l_{g,t}$, not the aggregate labor wedge.

The competitive equilibrium (CE) can be characterized as a special case of the optimal allocation where agents ignore externalities. Specifically, by setting the shadow prices of environmental damage and ideas to zero ($\phi_t = 0 \implies \tilde{\zeta} = 0$ and $\mu_{At} = 0 \implies z_A = 0$) in the optimal conditions derived above, we immediately recover the CE allocation.

Substituting $\tilde{\zeta} = 0$ into equation $\tilde{\zeta}_t = \frac{1-x_t^{\rho-1}}{1+x_t^{\rho}}$ yields

$$0 = \frac{1 - x_{CE}^{\rho-1}}{1 + x_{CE}^{\rho}} \implies x_{CE}^{\rho-1} = 1 \implies x_{CE} = 1$$

Thus, labor is allocated equally ($l_g = l_s$) in the CE, failing to shift towards the clean

sector.

Substituting $\tilde{\zeta} = 0$ and $z_A = 0$ into the population equations yields

$$n_{CE} = \bar{b} - \delta - \frac{1}{\tilde{V}_{CE}}, \quad \text{where } \tilde{V}_{CE} = \frac{\epsilon}{\rho - g_u}.$$

F Environmental Externality

We consider an economy with only an environmental externality, without productivity, to clarify the difference between knowledge spillovers of endogenous growth models and non-pecuniary environmental externality, and to display the importance of time allocation channel.

The parameters in the calibrated model satisfy the following assumptions, and we assume them for theoretical analysis:

Assumption 2.

$$\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma \left(1 + \frac{\lambda_A \sigma}{\beta_A}\right) > 0.$$

This assumption says that the growth rate of an input of the CRRA utility function is positive, which we generally expect to hold. We also assume the following:

Assumption 3. *For any* $n \in [-\delta, b - \delta]$,

$$\rho - (1-\theta)\left[\frac{\lambda_A \sigma}{\beta_A} + \varepsilon - \gamma \left(1 + \frac{\lambda_A \sigma}{\beta_A}\right)\right] n > 0.$$

This ensures that the effective discount rate is positive and the lifetime utility of an initial generation does not explode. The above assumptions also suggest that households do not devote all their time to child-rearing, $l_{SP} < 1$.

The population growth rate is determined by

$$n = \bar{b} - \delta - \frac{1 - \tilde{\xi}(n)}{\tilde{V}_{SP}(n)} = \bar{b} - \delta - \frac{1 - \tilde{\xi}(n)}{\frac{\varepsilon - \tilde{\xi}(n)}{\rho - (1-\theta)(\varepsilon - \gamma)n}}.$$

in the optimal allocation. We also have $\varepsilon - \tilde{\xi}(n) > 0$ and $1 - \tilde{\xi}(n) > 0$ by assumptions. In a Walrasian equilibrium, on the other hand, we have

$$n = \bar{b} - \delta - \frac{1}{\tilde{V}_{CE}(n)} = \bar{b} - \delta - \frac{1}{\frac{\varepsilon}{\rho - (1-\theta)(\varepsilon-\gamma)n}},$$

and $n_{CE} = \frac{\bar{b} - \delta - \frac{\rho}{\varepsilon}}{1 - \frac{(1-\theta)(\varepsilon-\gamma)}{\varepsilon}}$.

An optimal population growth rate differs from that in a competitive equilibrium in two ways. First, n_{SP} increases due to the disutility of labor supply, which is captured by the term $\frac{\tilde{\xi}}{\tilde{V}_{SP}}$. This time allocation channel is in stark contrast to positive knowledge spillovers, which do not affect time allocation but increase the value of the population.[18]

Second, the value of the population $\tilde{V}_{SP}(n)$ is smaller than that of $\tilde{V}_{CE}(n)$ because a larger population emits more greenhouse gases, thereby lowering the optimal population growth rate. We formally state the condition under which the positive time allocation effect dominates the negative population value effect in the following theorem.

[18]You can also consider the time for research activities, and the time for research lowers the optimal population growth rate. However, we choose not to include it for three reasons. First, the time for R&D activities is minimal compared to leisure, child-rearing, and (standard) labor, and thus it is not essential in reality. Second, because we do not model the R&D sector for simplicity as in Jones (2022), the time spent on research must be exogenous in a decentralized economy, which makes it challenging to compare allocations. Third, whereas incorporating the time for research gives an additional incentive to spend time on non-child-rearing activities, it does not imply that the optimal population growth rate is likely to be lower than the competitive equilibrium rate. Suppose the law of motion of productivity is given by

$$\frac{\dot{A}_t}{A_t} = (r_t N_t)^{\lambda_A} A_t^{-\beta_A},$$

where r_t is the time spent for research. The optimal population growth rate in an economy with only productivity and without environmental damage, is given by

$$n_{SP} = \bar{b} - \delta - \frac{1 + \lambda_A z_A}{\tilde{V}_{SP}}.$$

However, the competitive equilibrium population growth rate is given by

$$n_{CE} = \bar{b}(1 - r) - \delta - \frac{1}{\tilde{V}_{CE}},$$

where r is the exogenous time spent for R&D. Introducing an additional activity that households can allocate time to automatically reduces time for child-rearing, and thus it is not clear whether the time for R&D reduces the gap between n_{SP} and n_{CE}. A sufficient condition for $n_{SP} > n_{CE}$ in this setup is $\varepsilon < 1$, which is satisfied in our parameter set. Thus, the time for research does not affect the qualitative results, and we abstract from it for simplicity.

Theorem 1. *Suppose the BGP population growth rate n_{CE} in a competitive equilibrium is an interior solution. Then, the optimal population growth rate n_{SP} is weakly greater than n_{CE} if $\varepsilon \geq 1$.*

Proof. Since $n_{CE} = \frac{\bar{b}-\delta-\frac{\rho}{\varepsilon}}{1-\frac{(1-\theta)(\varepsilon-\gamma)}{\varepsilon}}$ is a solution to the following equation

$$\bar{b} - \delta - \frac{\rho}{\varepsilon} = n_{CE} - \frac{(1-\theta)(\varepsilon-\gamma)n_{CE}}{\varepsilon},$$

we have

$$n - \left[\bar{b} - \delta - \frac{\rho}{\varepsilon} + \frac{(1-\theta)(\varepsilon-\gamma)n}{\varepsilon}\right] = \left[1 - \frac{(1-\theta)(\varepsilon-\gamma)}{\varepsilon}\right](n - n_{CE}).$$

We also have

$$n_{SP} - \left[\bar{b} - \delta - \frac{\rho}{\varepsilon} + \frac{(1-\theta)(\varepsilon-\gamma)n_{SP}}{\varepsilon}\right] = [\rho - (1-\theta)(\varepsilon-\gamma)n_{SP}]\left[\frac{1}{\varepsilon} - \frac{1-\xi(n_{SP})}{\varepsilon-\xi(n_{SP})}\right],$$

by applying

$$n_{SP} = b - \delta - \frac{1-\tilde{\xi}(n_{SP})}{\frac{\varepsilon-\tilde{\xi}(n_{SP})}{\rho-(1-\theta)(\varepsilon-\gamma)n_{SP}}}.$$

If $\varepsilon \geq 1$, the term $\frac{1}{\varepsilon} - \frac{1-\xi^*(n_{SP})}{\varepsilon-\xi^*(n_{SP})}$ is positive, and we have

$$n_{SP} - \left[\bar{b} - \delta - \frac{\rho}{\varepsilon} + \frac{(1-\theta)(\varepsilon-\gamma)n_{SP}}{\varepsilon}\right] \geq 0,$$

because Assumption 3 ensures $\rho - (1-\theta)(\varepsilon-\gamma)n_{SP} > 0$. Then, we have

$$n_{SP} - \left[\bar{b} - \delta - \frac{\rho}{\varepsilon} + \frac{(1-\theta)(\varepsilon-\gamma)n_{SP}}{\varepsilon}\right] = \left[1 - \frac{(1-\theta)(\varepsilon-\gamma)}{\varepsilon}\right](n_{SP} - n_{CE}) \geq 0.$$

This implies $n_{SP} \geq n_{CE}$ because Assumption 1 gives $1 - \frac{(1-\theta)(\varepsilon-\gamma)}{\varepsilon} > 0$. Similarly, we obtain $n_{SP} < n_{CE}$ if $\varepsilon < 1$. □

The above theorem shows the strength of the time allocation channel. Assumptions in Barro and Becker (1989) and Jones and Schoonbroodt (2010) satisfy the condition $\varepsilon \geq$

1. Surprisingly, the optimal population growth rate exceeds the one in a competitive equilibrium with only negative environmental externality under their assumptions.

When $\varepsilon \geq 1$, households (and thus the planner) place a sufficiently large weight on descendants in utility. The population's value is then sufficiently high, and it exhibits a rigidity in terms of percentage against environmental damage. Consequently, the labor wedge created by environmental externalities becomes relatively stronger, making labor socially less attractive, so it is optimal to reduce labor supply. Given a fixed time endowment, this reallocation increases the feasibility of child-rearing time, thereby mechanically raising fertility. Under $\varepsilon \geq 1$, the time-allocation effect dominates in the benchmark model.

Conversely, when $\varepsilon < 1$, the marginal utility from additional descendants is sufficiently weak that the planner primarily responds to the population-value effect, leading to $n_{SP} < n_{CE}$. The calibrated model has $\varepsilon < 1$, and this result holds under our parameter set. While our baseline model has $n_{SP} < n_{CE}$ without technology, the decrease in the marginal benefit of labor supply still provides the countervailing force.