Shotgun Wedding: The Unpleasant Monetarist Arithmetic and Modern Monetary Theory

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January 26, 2026

Abstract

Stabilization policy conducted through the interest rate on bank reserves and partial monetary finance of fiscal deficits is a potentially beneficial policy combination. Under the *unpleasant monetarist arithmetic*, a sale of bonds by the monetary authority raises steady state inflation due to the higher interest cost of the net debt. The same intuition suggests that monetary finance of fiscal deficits, the purchase of bonds by the monetary authority, as advocated by Modern Monetary Theorists, would lower inflation. While full monetary finance is not recommended, as it eliminates the bond market, partial monetary finance with monetary policy targeting interest on reserves could be beneficial for countries with existing debt in excess of the needs of financial markets.

This paper presents a shopping time model of money demand with a banking sector where deposits provide liquidity services but require bank reserves. Fiscal and monetary budget constraints are treated separately to allow for the study of monetary finance. The resulting consolidated budget constraint demonstrates that monetary seigniorage must cover the cost of the fiscal deficit, the interest on the debt, and the interest on reserves. Higher costs imply higher steady state inflation, and the model embodies the intuition about monetary finance and the unpleasant monetarist arithmetic. Alternatively, for a given inflation target there is a trade-off between the fiscal deficit and the steady state interest rate on reserves. A calibrated version of the model verifies these intuitions, shows that the underlying dynamic model is determinate, and that a minimal, direct response of the policy rate to inflation is required to achieve expectational stability.

Keywords: Unpleasant Monetarist Arithmetic, Modern Monetary Theory, Fiscal Theory, Fiscal-Monetary models, Expectational Stability

JEL classification: A22, E0, E52, E60

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[†]There are no relevant financial relationships or financial support to report.

1 Introduction

The *unpleasant monetarist arithmetic* (Sargent and Wallace 1981) conveys the idea that a sale of bonds by the monetary authority can increase inflation due to the higher interest costs of the net debt. If selling bonds is bad, isn't buying bonds good? Modern Monetary Theorists (MMT), such as Kelton (2020), advocate for full monetary finance, which is equivalent to the monetary authority buying all government debt.

Monetary finance has a bad reputation, as it is the cause of most hyperinflations, Sargent (1982). However, Japan is notable case where monetary finance has been used responsibly. One objection, particularly relevant before the Great Recession, is that full monetary finance impedes the conduct of open market operations, since it eliminates the bond market. Furthermore, Woodford (1990), Aiyagari and McGrattan (1998) and Brunnermeier, Merkel, and Sannikov (2024) demonstrate that government debt held by the public alleviates financial market frictions, so full monetary finance cannot be recommended.

However, the use of partial monetary finance while targeting the interest rate on reserves for short run stabilization is a policy combination that merits consideration, particularly for countries with existing debt in excess of that required by financial markets. Partial monetary finance means the monetary authority buys a fraction of the debt issued by the fiscal authority each period. Monetary finance does reduce the interest payments on the net debt, but the interest on reserves constitutes an additional cost.

The present work combines a shopping time model of money demand with a banking sector that includes reserves, while accounting for the fiscal and monetary budget constraints separately to allow for the study of monetary finance. The resulting steady state consolidated government budget constraint contains much of the intuition. Higher costs to the government imply higher inflation to generate the necessary seigniorage. Those costs include fiscal deficits, interest on the debt and interest on reserves.

The unpleasant monetarist arithmetic and the benefit of monetary finance are both apparent. The greater the fraction of debt purchased by the monetary authority, the lower the steady state inflation rate and vice versa. From another perspective, for a given inflation target and degree of monetary finance, there is a trade-off between the fiscal deficit and the interest on reserves. The higher the deficit, the lower the steady state interest rate on reserves must be to achieve the steady state inflation target.

For stabilization policy, an important issue is stability. The steady state results are derived from a dynamic model with expectations. The equilibria for calibrated versions of the model are determinate. For a pegged policy rate, the model is not expectationally stable as in Howitt (1992), Friedman (1968) and Evans and McGough (2018), but it is stable for an interest rate rule that has a direct response to inflation. However, the magnitude of

the required response is quite modest compared to the Taylor Principle, in contrast to the results for the New Keynesian model in Bullard and Mitra (2002).

Regarding the short term conduct of monetary policy, there is an inverse relationship between policy rates and subsequent inflation. In the steady state analysis, however, higher costs to the government cause inflation and nominal rates to rise together, as with the Neo-Fisherian policy in Cochrane (2023). The contrast between the short and long run intuitions echoes the interpretations of open market operations in the first paragraph. An open market sale of bonds increases the interest on the net debt, which leads to higher inflation in the steady state analysis. Such a sale is permanent. In the short run, an open market sale to lower the money supply can reduce inflation due to liquidity effects, but it is likely followed by a bond purchase in the future.

There are no countries where the fraction of bonds purchased by the monetary authority is an explicit target, but some countries do use monetary finance. The European Central Bank has made indirect purchases of government bonds to control spikes in yields of the bonds for some countries, as discussed in Attavila, Fache Rousova, and Lena (2015). The Bank of Japan currently holds 43% of the debt issued by the fiscal authorities, which helps to explain the limited impact of a gross debt well over 200% of GDP. Since 2010, Japanese policy has been de facto close to the policy proposed here. The net debt has stabilized around 120% of GDP, largely due to bond purchases by the Bank of Japan, and monetary policy is conducted using the interest rate on reserves. See Hansen and İmrohoroğlu (2023) and Waters (2025) for more detailed discussions.

The model builds on those of Ireland (2014) and Chapter 27 of Sargent and Lungqvist (2018) who both develop shopping time models of money demand. The former includes a banking sector to study monetary policy conducted through interest on reserves, which uses a model of deposits based on Chari, Christiano, and Eichenbaum (1995). Belongia and Ireland (2014) use a similar model to study the relationships between monetary aggregates. Sargent and Lungqvist (2018) focuses on fiscal-monetary interactions including the unpleasant monetarist arithmetic. Bassetto and Messer (2013) develop a model treating fiscal and monetary policy constraints independently to determine conditions for central bank profitability.

Models of the fiscal theory of the price level (Leeper and Leith 2016, Cochrane 2022) show interactions of fiscal and monetary policy and their impact on prices as well. Bassetto and Sargent (2020) discuss the interpretation of the unpleasant monetarist arithmetic, the fiscal theory of the price level and the quantity theory of money in a single model. Eusepi and Preston (2018) summarize expectational stability results under various combinations of fiscal and monetary policy.

The paper is organized as follows. Section 2 derives the consolidated budget constraint

and discusses the results from the steady state version. Section 3 describes the micro-founded model with a banking sector that determines the demand for money and reserves. Section 4 details a calibrated version of the model with results on determinacy and expectational stability, as well as quantitative results about the policy implications. Section 5 concludes.

2 Budget Constraints

To examine monetary financing of fiscal deficits and interest on reserves together, consider the flow budget constraints for fiscal and monetary policy separately. The fiscal constraint is

$$P_t g_t = P_t \tau_t + r_t^{-1} B_t^T - B_{t-1}^T + RCB_t$$

where the price level is P_t, and real government spending and taxes are g_t and τ_t. One period discount bonds B_t^T are issued by the fiscal authority and paid off at a gross yield of r_t. The fiscal authority also receives a transfer RCB_t from the monetary authority.

The monetary authority pays gross interest r_t^n on reserves N_t and has the flow budget constraint

$$r_t^{-1} B_t^M - B_{t-1}^M + RCB_t = M_t - M_{t-1} - (r_t^n - 1)N_t$$

where bonds held by the central bank is B_t^M, and currency is M_t. The timing of the N_t term reflects the short term payments of interest on reserve deposits and matches the expression for bank profits.

Monetary financing is represented by the fraction Ω of the bonds issued that are purchased by the monetary authority. The bonds held by the public or net debt is defined as

$$B_t = B_t^T - B_t^M,$$

so the following hold.

$$B_t^M = \Omega B_t^T \tag{1}$$

$$B_t = (1 - \Omega) B_t^T \tag{2}$$

Using the above relation for B_t^M and substituting for RCB_t in the above fiscal and monetary constraints yields the following consolidated budget constraint.

$$M_t = M_{t-1} + P_t g_t - P_t \tau_t + (r_t^n - 1)N_t - (1 - \Omega)(r_t^{-1} B_t^T - B_{t-1}^T) \tag{3}$$

The MMT case of full monetary finance is equivalent to the monetary authority purchasing all the bonds issued so $\Omega = 1$ and $B_t^M = B_t^T$. Partial monetary finance means

$\Omega \in (0, 1)$.

The policy of purchasing bonds is a money supply rule. Given all the other values including the fraction purchased Ω, the monetary authority sets M_t according to the constraint (3). However, active monetary policy can still be conducted via interest on reserves. Let the interest rate rule for r_t^n depend on gross inflation

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

relative to steady state inflation Π as follows.

$$r_t^n = r_t^{\alpha_i} \left(\frac{\Pi_t}{\Pi} \right)^{b_\pi} \tag{4}$$

The interest rate is set below the market rate on bonds so the parameter α_i is such that $0 \leq \alpha_i < 1$. The policy rate is set to respond directly to inflation so the parameter b_π is non-negative, and the Taylor principle is satisfied when $b_\pi > 1$.

2.1 Real steady state consolidated budget constraint

The consolidated budget constraint can be written in real terms where real currency and real bonds are

$$m_t = \frac{M_t}{P_t},$$

and

$$b_t = \frac{B_t}{P_t}.$$

The constraint becomes

$$g_t - \tau_t + (r_t^n - 1)n_t = (1 - \Omega)(r_t^{-1} b_t^T - \Pi_t^{-1} b_{t-1}^T) + m_t - \Pi_t^{-1} m_{t-1}. \tag{5}$$

The steady state version of the consolidated budget constraint is the focus of this section. The existing real debt is denoted b_0^T.

$$g - \tau + n(r^n - 1) = (1 - \Omega) b_0^T (r^{-1} - \Pi^{-1}) + m(1 - \Pi^{-1}). \tag{6}$$

The following assumptions allow for interpretation of the constraint.

Assumption 1. *Policymakers act to maintain the existing steady state real debt, so b_0^T is fixed.*

Assumption 2. *The real rate given by* $R = r\Pi^{-1}$ *is fixed*[1].

Assumption 3. *The steady state demand for real currency* $m = L(\Pi)$ *is decreasing in inflation.*

Assumption 4. *The steady state quantity of real reserves* $n = J(\Pi)$ *is decreasing in inflation.*

Using the above and the steady state version of the interest rate rule (4), it is convenient to express the steady state budget constraint, showing the relationships to inflation as follows.

$$g - \tau + R^b(\Pi) + R^n(\Pi) = s^m(\Pi) \tag{7}$$

The real interest payment on the net debt is

$$R^b(\Pi) = (1 - \Omega)b_0^T(1 - R^{-1})\Pi^{-1},$$

the real interest payment on reserves is

$$R^n(\Pi) = J(\Pi)\left[(\Pi R)^{\alpha_i} - 1\right]$$

and the monetary seigniorage is

$$s^m(\Pi) = L(\Pi)(1 - \Pi^{-1}).$$

The monetary seigniorage must cover the fiscal deficit, interest on existing debt and interest on reserves.

The two sides of the constraint (7) are represented in Figure 1. The interest on the debt term $R^b(\Pi)$ is unambiguously non-increasing in inflation and the steady state deficit is constant, but the other two terms are more involved. The money demand $L(\Pi)$ function is decreasing with inflation, but the positive slope of the term $1 - \Pi^{-1}$ dominates for Π near one. As inflation rises the slope of the term $1 - \Pi^{-1}$ approaches zero, so there is a maximum level of seigniorage. Whether the seigniorage function $s^m(\Pi)$ ultimately declines depends on the money demand $L(\Pi)$. In Sargent and Wallace (1981) and Werning (2024), money demand is linear, which leads to a downward sloping portion of $s^m(\Pi)$ for large Π, but that is not guaranteed here[2].

While the real interest on the debt is decreasing with inflation, the slope of the real interest on reserves R^n is ambiguous. It is safe to assume that the slope of the payments on

[1] This is a "monetarist" assumption in Sargent and Wallace (1981)

[2] See Waters (2025) for a related example where $s^m(\Pi)$ has a maximum but does not decline.

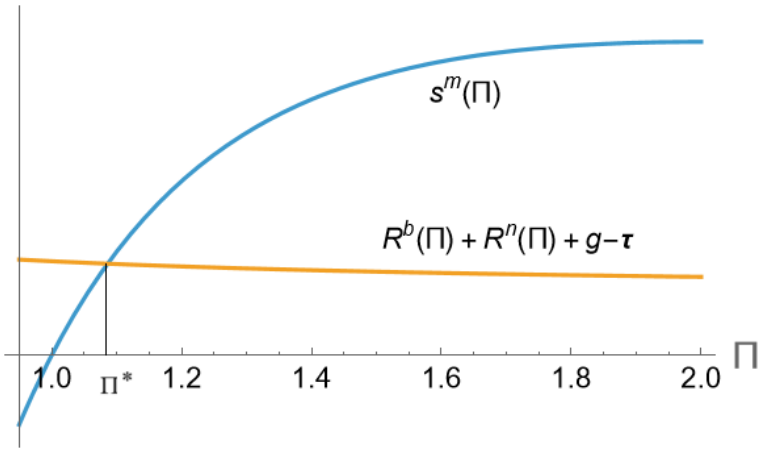


Figure 1: Representation of the steady state relation (7) as functions of inflation Π.

the left hand side of the steady state budget relation is less than that for seigniorage $s^m(\Pi)$ at the intersection for inflation near one.

2.2 Results

The steady state budget constraint (7) represented in Figure 1 extends some established results to a more general setting.

Result 1. *Higher deficits $g - \tau$ are associated with higher inflation, as shown by the $R^b + R^n + g - \tau$ curve shifting up, leading to a higher Π^*, but the curve has a maximum, so there is a limit to sustainable deficits.*

MMT advocates have suggested that higher deficits do not necessarily lead to higher interest rates, which contradicts the short run intuition from a loanable funds graph. Figure 1 shows that the claim is not correct for steady states either. Given a fixed real rate, nominal rates and inflation move together.

Result 2. ***The Unpleasant Monetarist Arithmetic**: Sargent and Lungqvist (2018) describe "one big open market operation" where the monetary authority sells bonds, raising the steady state net debt. On the graph, the fraction Ω falls, net debt b rises, and the $R^b + R^n + g - \tau$ curve shifts up, leading to a higher Π^*.*

Result 3. ***The Potentially Pleasant Modern Monetary Arithmetic**: Conversely, if the monetary authority commits to buying a fraction Ω of bonds, the $R^b + g - \tau$ curve shifts down, and inflation falls. Alternatively, for a given steady state inflation level Π, a higher fraction Ω of bonds purchased allows for a higher deficit $g - \tau$ (Waters 2025).*

Result 4. *The greater the interest payments on the debt, the greater the benefit to monetary finance. If the term R^b is small, a change in the parameter Ω has a modest impact. A low payment on the debt could arise due to a low level of the existing debt b_0^T or a low steady state real rate R.*

Secular stagnation, the long run fall in real rates, may be eroding the rationale for monetary finance.

The inclusion of interest on reserves in the constraint (7) allows for some further insights.

Result 5. *A higher value of α_i in the policy rule (4) raises the cost of interest on reserves, shifts the costs curve up, and leads to higher steady state inflation Π.*

Result 6. *For a given inflation target Π and fraction of bonds purchased by the monetary authority Ω, there is a trade-off between the costs to the government: the fiscal deficit $g - \tau$ and the interest on reserves determined by α_i. The higher the steady state fiscal deficit, the lower the interest on reserves must be and vice versa.*

It is important to note that these are all steady state results, and short run intuitions could be different. In the short run, a monetary authority that buys bonds or lowers the policy rate can easily stimulate inflation due to liquidity effects. The fiscal authority might deviate from the steady state deficit but compensate in the future, a Ricardian policy. These points highlight the importance of credibility. For monetary finance to be beneficial, institutions must be sufficiently trusted for steady state values to be meaningful. Japan could be interpreted as such a country, but cases where monetary finance led to hyperinflation, such as those discussed in Sargent (1982), are cautionary tales.

2.3 On the monetary policy rate

Most countries claim to have a policy of monetary dominance, where fiscal authorities control deficits so that central banks can target inflation independently. Typically, monetary policy is described by a Taylor rules that includes a steady state natural rate of interest *r-star* determined by the choice of inflation target, as in Platonov (2024). In the present model, the steady state rate of inflation Π and rate for interest on reserves r^n are jointly determined by the fraction of bonds purchased by the monetary authority Ω, the steady state deficit and the steady state rate of inflation according to the constraint (6), along with the parameter α_i.

The relationship between the steady state rate on reserves r^n and market rate r is governed by the policy parameter α_i according to the steady state version of the interest rate rule (4).

$$r^n = r^{\alpha_i} \tag{8}$$

The choices for the parameters Ω and α_i by the monetary policymaker impact the cost of the interest on the debt and the interest on reserves and, therefore, the steady state inflation rate. To achieve monetary dominance the fiscal authority would have to set the steady state deficit $g - \tau$ according those choices and the desired inflation target Π.

The interest rate rule (4) and the steady state government budget constraint (7) demonstrate the difference in short and long run dynamics with respect to inflation. The interest rate rate rule shows the short term conduct of policy where the policy would be raised in an attempt to reduce inflation above its steady state for $b_\pi > 0$, which is necessary for stability, see Section 5.3. However, the steady state policy and inflation rates can move together due to exogenous shifts. Hence, an increase in the parameter α_i would increase the steady state inflation rate, but the resulting increase in the policy rate could lead to a short run decrease in inflation.

3 The Model

The next two sections develop a micro-founded model to demonstrate that the budget constraint (5) and Assumptions 1-4 are consistent with household optimization. The model is a simplified version of a shopping time monetary model with a banking sector and interest on reserves as described in Ireland (2014) that also allows for fiscal deficits. The model is simplified in that there is only one stochastic shock, liquidity services for the household are given by bank deposits, and prices are flexible.

3.1 Household Problem

The household utility is determined by consumption and the disutility from working and shopping time, which is alleviated by the liquidity service provided by bank deposits D_t. Ireland (2014) includes a cash infusion each period that also provides liquidity services, but all transactions are electronic here. The household does hold currency as one source for deposits.

The household enters time t with currency held from the previous period M_{t-1}, bank loans taken L_t and the return from the bond holdings B_{t-1} with yield r_t, so

$$D_t = M_{t-1} + L_t + B_{t-1} - \frac{B_t}{r_t}. \tag{9}$$

In period t, the household pays gross interest r_t^l on the loans and receives gross interest r_t^d on the deposits. Real consumption, the price level, wage, hours worked and lump sum taxes are c_t, P_t, W_t, h_t, τ_t. The household receives bank profits E_t and firm profits F_t, though

they do not enter the optimality decisions. Hence, the household decision must also satisfy

$$P_t c_t = W_t h_t + r_t^d D_t - r_t^l L_t - M_t + F_t + E_t - P_t \tau_t. \tag{10}$$

The household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t \left[\log c_t - \eta \left((1+\psi)^{-1} h_t^{(1+\psi)} + h_t^s \right) \right], \tag{11}$$

where the discount factor is $\beta < 1$, and the preference shock $A_t = \exp a_t$ is such that

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \text{ for } \varepsilon_{a,t} \sim iid\,(0)\,. \tag{12}$$

Shopping time is given by

$$h_t^s = \chi^{-1} \left(\frac{v_a P_t c_t}{D_t} \right)^{\chi}. \tag{13}$$

The parameter v_a is positive and differentiates the disutility from working and disutility from shopping. Again, all transactions are electronic so liquidity services are provided by deposits, which reduces shopping time for $\chi > 0$.

Writing the budget constraints (9) and (10), in real terms using the convention $x_t = \frac{X_t}{P_t}$ yields

$$\begin{aligned} d_t &= m_{t-1}\Pi_t^{-1} + l_t + b_{t-1}\Pi_t^{-1} - b_t r_t^{-1} && (14) \\ c_t &= w_t h_t + r_t^d d_t - r_t^l l_t - m_t + \frac{F_t}{P_t} + \frac{E_t}{P_t} - \tau_t, && (15) \end{aligned}$$

where gross inflation is Π_t as above.

The objective (11) at is maximized substituting for h_t^s using equation (13), subject to the budget constraints (14) and (15), with Lagrange multipliers $\lambda_{1,t}$ and $\lambda_{2,t}$. The following are the first order conditions with respect to $c_t, h_t, d_t, l_t, b_t, m_t$.

$$\begin{aligned} \beta^t A_t \left(1 - \eta\chi h_t^s\right) &= \lambda_{2,t} c_t && (16) \\ \beta^t A_t \eta h_t^{\psi} &= \lambda_{2,t} w_t && (17) \\ \beta^t A_t \eta\chi h_t^s &= d_t \left(\lambda_{1,t} - \lambda_{2,t} r_t^d\right) && (18) \\ \lambda_{1,t} &= \lambda_{2,t} r_t^l && (19) \\ \lambda_{1,t} &= r_t E_t \left(\frac{\lambda_{1,t+1}}{\Pi_{t+1}} \right) && (20) \\ \lambda_{2,t} &= E_t \left(\frac{\lambda_{1,t+1}}{\Pi_{t+1}} \right) && (21) \end{aligned}$$

An immediate consequence of conditions (19), (20) and (21) is the arbitrage condition that the rate on bonds and the rate on loans must be the same.

$$r_t^l = r_t \tag{22}$$

3.2 Production

A competitive firm has only labor input. Production is

$$y_t = z h_t^g, \tag{23}$$

where output is y_t, labor in goods production is h_t^g, and productivity is the constant z. Firm profit is

$$P_t y_t - W_t h_t^g,$$

so the zero profit condition with the production function implies that the household receives

$$\frac{F_t}{P_t} = 0,$$

in constraint (15) and the real wage $w_t = W_t/P_t$ is given by productivity.

$$w_t = z \tag{24}$$

The resource constraint for the economy is

$$y_t = c_t + g_t. \tag{25}$$

3.3 Banking Sector

Banks take deposits and make loans while holding reserves. The technology for generating both deposits and reserves requires labor input following Chari, Christiano, and Eichenbaum (1995) and Ireland (2014).

Servicing real deposits $d_t = \dfrac{D_t}{P_t}$ requires both real reserves $n_t = \dfrac{N_t}{P_t}$ and labor h_t^d according to

$$x_a \left(x_n^{\frac{1}{\nu}} n_t^{\frac{\nu-1}{\nu}} + (1-x_n)^{\frac{1}{\nu}} h_t^{d\,\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} \geq d_t. \tag{26}$$

The parameter ν is the elasticity of substitution between the two inputs and the parameter x_n measures their relative importance.

Servicing reserves also requires labor h_t^n such that for a positive $\phi_n > 0$,

$$h_t^n \geq \phi_n n_t. \tag{27}$$

Banks maximize real profits where they generate returns from loans and reserves and have costs from wages and returns to depositors.

$$\frac{E_t}{P_t} = \left(r_t^l - 1\right) l_t + \left(r_t^n - 1\right) n_t - \left(r_t^d - 1\right) d_t - w_t h_t^b \tag{28}$$

The total labor employed by the bank h_t^b is the sum of those employed to service deposits and reserves.

$$h_t^b = h_t^d + h_t^n \tag{29}$$

The timing in the profits relation (28) is the same as with the household budget constraints (9) and (10) and the description of reserves after the budget constraint for the monetary authority (2).

Total labor supplied h_t is the sum of that provided for goods production and banking.

$$h_t = h_t^g + h_t^b \tag{30}$$

Bank assets equal liabilities so

$$d_t = n_t + l_t. \tag{31}$$

By substituting for l_t and d_t with equations (26), (31) and (27), bank profits (28) can be written in term of n_t and h_t^d.

$$\begin{aligned} \frac{E_t}{P_t} = {} & (r_t^l - r_t^d) x_a \left[x_n^{1/\nu} n_t^{\frac{\nu-1}{\nu}} + (1 - x_n)^{1/\nu} (h_t^d)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \\ & - (r_t^l - r_t^n + \phi_n w_t) n_t - w_t h_t^d \end{aligned} \tag{32}$$

Maximizing bank profits, the first order conditions with respect to n_t and h_t^d are

$$n_t = (r_t^l - r_t^d)^\nu x_a^{\nu-1} x_n d_t (r_t^l - r_t^n + \phi_n z)^{-\nu} \tag{33}$$

$$h_t^d = (r_t^l - r_t^d)^\nu x_a^{\nu-1} (1 - x_n) d_t z^{-\nu}, \tag{34}$$

where the perfect competition condition $w_t = z$ is used to substitute for the wage.

The quantities n_t and h_t^d must be positive, which implies that the interest rate on loans must be greater than that on deposits, $r^l > r^d$, matching the usual intuition for the way banks make profits. See Ireland (2014) for a detailed discussion of the the optimal choices

by the bank.

3.4 Equilibrium Household Budget Constraints

Given the optimizing behavior for households, firms and banks, the two household budget constraints can be written in terms of currency m_t and reserves n_t. The combination of those two is equivalent to the consolidated government budget constraint (5).

In equilibrium, the real quantities b_t, m_t, and n_t, are equal to the corresponding quantities in the budget constraints in Section 2. Substituting for real bank profits (28) in the household budget constraint (15) and using the facts that firm profits are zero $F_t = 0$ and labor is split between goods production and banking (30) yields

$$c_t = w_t h_t^g + d_t - l_t - m_t + (r_t^n - 1)\, n_t - \tau_t.$$

Using the production function (23), the resource constraint (25) and the bank balance sheet condition (31), the above constraint becomes

$$m_t = g_t - \tau_t + r_t^n n_t. \tag{35}$$

Similarly, the first household budget constraint (14) with the bank condition (31) can be written

$$n_t = m_{t-1}\Pi_t^{-1} + b_{t-1}\Pi_t^{-1} - b_t r_t^{-1}. \tag{36}$$

Adding the above two equations (35) and (36) yields,

$$m_t - \frac{m_{t-1}}{\Pi_t} = g_t - \tau_t + \frac{b_{t-1}}{\Pi_t} - \frac{b_t}{r_t} + (r_t^n - 1)\, n_t, \tag{37}$$

which is equivalent to the consolidated budget constraint (5).

3.5 The Euler equation

An Euler equation is derived from the dynamic model, and the steady state version is consistent with Assumption 2. Solving out the Lagrange multipliers from the household optimization equations (17), (20), (19), and using the interest rate arbitrage relationship (22) with the wage specification (24) gives the following.

$$\beta E_t \left(\frac{r_{t+1}}{\Pi_{t+1}} \left(\frac{h_{t+1}}{h_t} \right)^{\psi} \frac{A_{t+1}}{A_t} \right) = 1 \tag{38}$$

The steady state version is

$$\beta r \Pi^{-1} = 1, \tag{39}$$

so the steady state real interest rate depends only on the constant discount factor $R = \beta^{-1}$, satisfying Assumption 2.

4 Quantitative Results

Calibration of the model allows for analysis of determinacy and expectational stability. In addition, this section shows quantitative counterparts of the steady state relations described in Section 2.

4.1 Model Summary

The model is simplified to describe the evolution of the 16x1 vector

$$X_t = (y_t, c_t, d_t, n_t, \Pi_t, r_t, r_t^d, r_t^n, h_t, h_t^g, h_t^d, h_t^n, h_t^s, b_t, m_t, a_t)'.$$

Throughout the following specification, the perfect competition condition $w_t = z$ is used to substitute for the wage w_t, the bank balance sheet condition (31) is used to substitute for l_t and the arbitrage condition $r_t = r_t^l$ from (22) is used to substitute for the interest rate on loans r_t^l.

The model equations include the specification for shopping time (13), the household optimality condition determined by equations (16) and (17)

$$\chi h_t^s + \frac{c_t}{z} h_t^\psi = \eta^{-1},$$

the condition determined by equations (16) and (17)

$$(r_t - r_t^d) h_t^\psi d_t = \chi z h_t^s,$$

and the Euler equation (38). The model also includes the production function (23) the resource constraint (25), as well as the banking conditions (26), (33), (34), (27) and the labor identity from combining (29) and (30).

$$h_t = h_t^g + h_t^d + h_t^n$$

The model is closed with the budget conditions (35), (36), the interest rate rule (4), and the specification for the demand shock (12). The fiscal authority rolls over the existing debt

each period, and the fraction Ω purchased by the monetary authority each period is fixed, so the gross debt b_0^T is constant, as in Assumption 1, as is the net debt specified by equation (2).

$$b_t = (1 - \Omega) b_0^T \tag{40}$$

Spending and taxes g, τ are constant as well.

4.2 Calibration

The model is calibrated to check for determinacy and expectational stability and examine steady state relationships. A number of the parameter values are taken from existing work. The household objective parameter values $\chi = 1.0$ and $\psi = 1.3$ are taken from the DSGE model of the Federal Reserve Bank of New York, Del Negro et al. (2023). Some banking sector values $\nu = 0.25$ and $\phi_n = 0.0005$ are from Ireland (2014). For the initial calibration, the value $\Omega = 0.0$ reflects current policy, since most central banks do not systematically purchase bonds.

Steady state output y and total hours $h + h^s$ are normalized to one. The banking parameter value $x_n = 0.95$ is set higher than in Ireland (2014) to achieve an value for labor hours in deposit creation h^d in line with the other labor hours steady state values. The rest of the parameters are chosen to achieve reasonable steady state annual values: $\Pi = 1.04$, $r = 1.07$, $r^d = 1.045$ and $r^n = 1.02$. The values are $\beta = 0.9568$, $v_a = 2.504$, $\eta = 1.529$, $\nu = 0.25$, $x_a = 5.253$. The persistence of the shock given by $\rho_a = 0.9$ is similar to that in Ireland (2014).

Since steady state output y is one, the chosen values for steady government spending $g = 0.2$ and fiscal deficit $g - \tau = 0.02$ are interpreted as fractions of GDP.

4.2.1 Determinacy and Expectational stability

The calibrated model is simulated with Dynare (Adjemian et al. 2011) to verify steady state values and determine the solution under rational expectations. For all specifications the model is determinate in that it satisfies the conditions in Blanchard and Kahn (1980). In contrast, the flexible price model in Platonov (2024) does not have a banking sector and requires a monetary policy response to inflation even stronger that given by the Taylor Principle to achieve determinacy for many parameter choices.

The model is not expectationally stable for a pegged policy rate $r_t^n = r^n$, following Friedman (1968), Howitt (1992), and Evans and McGough (2018). However, only a minimal response of the policy rate to inflation is required to achieve stability.

Expectational stability governs the stability of models for a wide range of non-rational

methods of forming expectations. The log-linearized model has the form

$$x_t = A_0 + A_1 E_{t-1} x_{t+1} + A_2 x_{t-1}, \tag{41}$$

where the vector x_t contains the log-linearized variables from X_t, including the shock a_t from (12). The minimum state variables rational expectations solution has the form

$$x_t = B_0 + B_1 x_{t-1}. \tag{42}$$

The results in Evans and Honkapohja (2001) (Chapter 10) determine the expectational stability of the model (41). Define the matrices DT_a and DT_b as follows.

$$DT_a = (I - A_1 B_1)^{-1} A_1 \tag{43}$$

$$DT_b = [(I - A_1 B_1)^{-1} A_2] \otimes [(I - A_1 B_1)^{-1} A_1] \tag{44}$$

Proposition 4.1 (Evans and Honkapohja (2001), Proposition 10.3)**.** *Let the value Λ be maximum of the real parts of the eigenvalues of DT_a and DT_b. The model (41) with minimum state variables, rational expectations solution (42) is expectationally stable if $\Lambda < 1$. It is not expectationally stable if $\Lambda > 1$.*

Remark 1. *A pegged policy rate is not expectationally stable for the calibrated model.*

For the case with no interest on reserves such that $\alpha_i = 0.0$, so the interest rate on reserves is one, $r_t^n = 1$, the maximum eigenvalue of matrices DT_a and DT_b is 11.72, indicating instability. A pegged rate where interest on reserves is paid is achieved with the interest rate rule (4) for the parameter values $\alpha_i = 0.3$ and $b_\pi = 0$. The value of Λ is essentially unchanged at 11.71, and the model remains expectationally unstable.

Remark 2. *Interest rate rules that have a minimally positive response to inflation yield expectational stability.*

For the value $b_\pi = 0.086$ along with $\alpha_i = 0.3$, the value of Λ is 0.94, indicating stability. Higher values for b_π also achieve stability.

So a direct policy rate response to inflation is required for expectational stability, which verifies the usual intuition for the short run response of the policy rate to inflation discussed in Section 2.4. Raising the policy rate acts to bring inflation down toward the target. For the steady state analysis, inflation and interest rates move together in response to parameter shifts (except for $\beta = R^{-1}$).

The Taylor Principle, which requires $b_\pi > 1$, if often a rough guide for achieving stability, see Waters (2024). To achieve stability in the present model, the required response of the

policy rate to inflation is much lower. An alternative calibration based on steady state values $\Pi = 1.02$, $r = 1.055$, $r^d = 1.015$ and $r^n = 1.01$ yields qualitatively equivalent results. In this case, a minimum value of the policy rate parameter to achieve stability is $b_\pi = 0.044$.

4.3 Steady State Relationships

This section reports results about the relationships described in the steady state budget constraint (7). The ultimate goal is to show quantitatively the trade-off between the steady state fiscal deficit $g - \tau$ and the monetary policy parameter α_i, which determines the cost of the interest on reserves, for a given inflation target Π, as described in Result 6.

4.3.1 Deficits versus Inflation

Figure 2 verifies the relationships between the deficit and inflation as in Result 1 and the relationship between inflation and money in Assumption 3. Values of the real tax τ range from 0.3 to 0.02, which translates into an increasing fiscal deficit $g - \tau$ on the horizontal axis. Inflation Π rises with the deficit while currency m falls. The analogous graphs for reserves is very similar, so the model is consistent with Assumptions 3 and 4. Steady state deposits are decreasing with inflation as well.

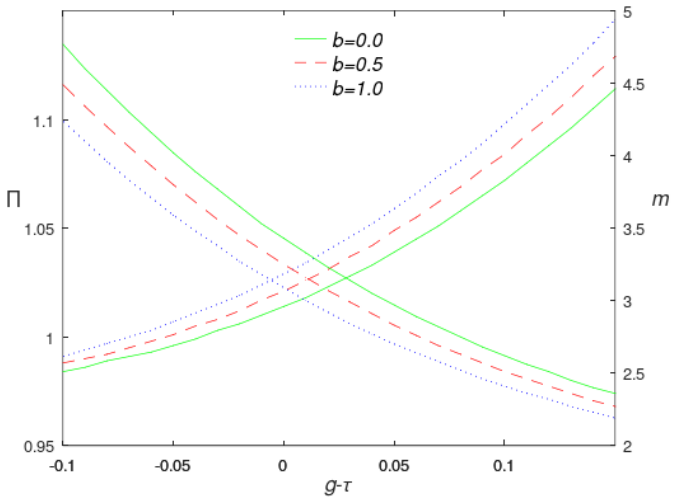


Figure 2: The upward sloping steady state inflation Π and downward sloping steady state currency m are functions of the fiscal deficit $g - \tau$. The net debt $b = (1 - \Omega)b_0^T$ takes values according to $\Omega = 0.0, 0.5, 1.0$ and $b_0^T = 1.0$.

The benefit of the monetary authority purchasing bonds is apparent. The net debt $(1 - \Omega)b_0^T$ varies with the choice of Ω. The existing debt is set to $b_0^T = 1.0$, or 100% of

output, and the fraction purchased by the monetary authority is set to 0.0, 0.5 and 1.0. Higher values of Ω give lower net debt and lower steady state inflation rates, as in Result 3.

		$g-\tau$										
		-0.04	***-0.03***	***-0.02***	***-0.01***	***0.0***	***0.01***	***0.02***	***0.03***	***0.04***	***0.05***	***0.06***
Ω	***0.0***	1.07	1.49	1.93	2.40	2.91	3.44	4.01	4.61	5.24	5.91	6.61
	0.25	0.76	1.16	1.58	2.02	2.50	3.01	3.55	4.13	4.73	5.37	6.05
	0.5	0.47	0.84	1.24	1.66	2.12	2.60	3.12	3.67	4.25	4.86	5.51
	0.75	0.19	0.54	0.92	1.32	1.75	2.21	2.70	3.22	3.78	4.37	4.99
	1.0	-0.68	0.26	0.62	1.00	1.40	1.84	2.31	2.81	3.33	3.89	4.49

Table 1: Steady State inflation rates as simple annual percentages for given deficits $g-\tau$ and fractions Ω of debt purchased by the monetary authority. The parameter $\alpha_i = 0.3$.

Table 1 shows the annualized simple inflation rates $\Pi - 1$ for varying levels of the fraction Ω and the deficit $g-\tau$. Starting from a deficit of 0.02, or 2% of GDP, and no debt purchased by the monetary authority $\Omega = 0.0$ where inflation is 4%, the benefit of switching to an MMT policy $\Omega = 1.0$ would be a 1.7% drop in inflation. Alternatively, full monetary finance would allow for an increase in the deficit to over 5% of GDP while maintaining a 4% inflation rate.

4.3.2 Seigniorage, interest on the debt and interest on reserves versus inflation

Figure 1 in Section 2 represents the constraint (7), which contains much of the intuition. Figure 3 shows steady state seigniorage, interest on the debt and interest on reserves as functions of inflation for the calibrated model. Again, the driving factor is a decrease in taxes, so the deficit $g-\tau$ is not included on the graph. One can think of the curves in Figure 3 as intersections with an increasing deficit line as in Figure 1.

Figure 3 shows the case without debt purchases by the monetary authority $\Omega = 0.0$, so the interest on the net debt R^b is positive. It is declining with inflation as shown by the specification in Section 2, but the slope is barely discernible on the graph. The slope of the interest on reserves curve R^n is positive but less than the seigniorage curve s^m, as required. In the case under the MMT policy where the monetary authority buys all the bonds $\Omega = 1.0$, the graph is very similar, but the interest on the debt R^b is constant at zero, since the public holds none of the bonds.

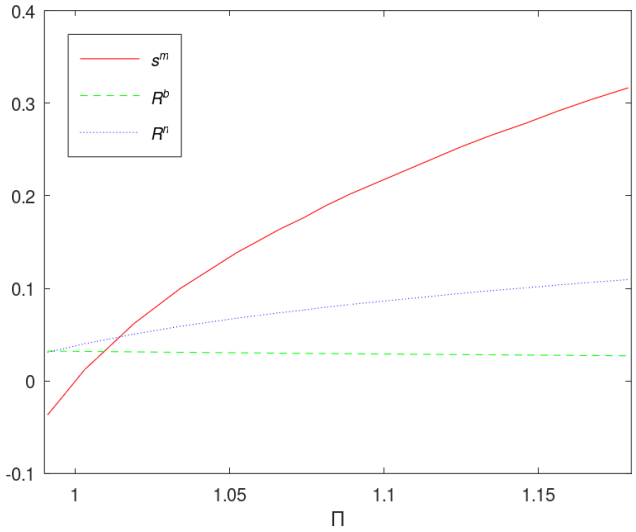


Figure 3: Steady state seigniorage s^m, interest on debt R^b and interest on reserves R^n as a function of inflation Π, for taxes τ varying from 0.3 to 0.0. The fraction of bonds purchased by the monetary authority is set to zero, $\Omega = 0.0$.

4.3.3 Targeting Inflation

Steady state inflation is increasing with any parameter that directly affects a cost to the government, including the deficit $g - \tau$ and the monetary policy parameters α_i, which determines the steady state interest rate on reserves r^n. Figure 4 shows the combinations $(g - \tau, \alpha_i)$ that give steady state simple inflation levels of 2% and 4%, for both cases $\Omega = 0.0, 1.0$. There is a non-linear trade-off between the cost of the deficit and the cost of interest on reserves, reflecting Result 6. For a given inflation target, the higher the deficit, the lower the steady state interest on reserves must be and vice versa. Monetary finance, a higher Ω, allows for higher costs for the same inflation target.

A simple example shows the maximum benefit of monetary finance. If the inflation target is 2% and a steady state rate on reserves of 1% (simple, annual for both) translates to the value $\alpha_i = 0.19$. Without any monetary finance $\Omega = 0.0$, a balanced budget is required, $g - \tau = 0.0$, represented by the large dot on the left in Figure 4. For the same inflation target and policy rate, switching to full monetary finance, where $\Omega = 1.0$, allows for a fiscal deficit $g - \tau = 0.03$. So the switch to full monetary finance allows for an increase in the steady state deficit of 3%, which is the saving from avoiding the interest on the debt, following Result 3.

The 3% increase in the deficit made possible by full monetary finance is the real interest

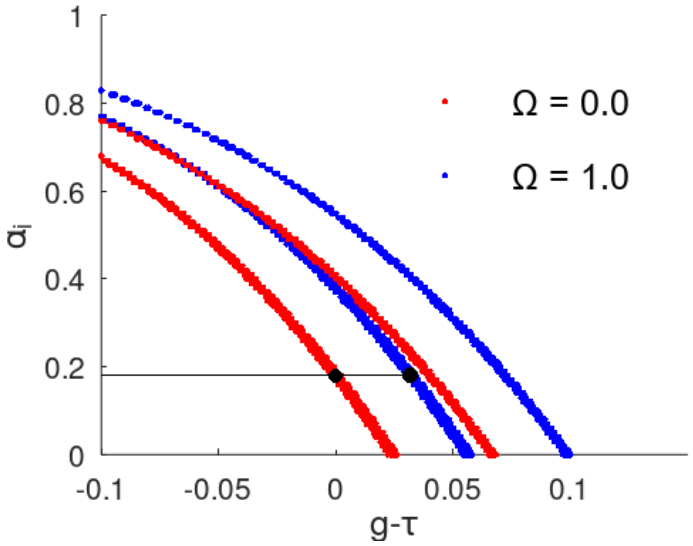


Figure 4: The plots are combinations $(g-\tau, \alpha_i)$ that give steady state simple inflation levels of 2% and 4% in the cases of full monetary finance and no monetary finance. The two dots represent the 2% inflation case where $\alpha_i = 0.19$, which corresponds to $r^n = 1.01$.

on the debt R^b in equation 7 and Figure 1. If the fraction of bonds Ω changes from zero to one, the R^b term is eliminated. To maintain the same steady state inflation rate Π, the deficit $g - \tau$ must rise by $R^b(\Pi) = b_0(1 - R^{-1})\Pi^{-1}$.

4.4 Discussion

MMT advocates raise an important point about the benefits of monetary finance. However, there are a number of reasons for caution, particularly concerning their focus on full monetary finance, which would eliminate the debt held by the public. Partial monetary finance in combination with targeting the interest rate on reserves is potentially beneficial, particularly for countries with a high burden from the interest on their debt.

Monetary finance leads to lower inflation or allows higher sustainable government cost, but it must be emphasized that these results come from steady state analysis. Suggestions that the interest on the debt can be eliminated (Globerman (2021)) immediately are not correct. Decades of experience with monetary policy conducted through open market operations indicate that, if the Federal Reserve were to buy all the outstanding U.S. debt over a short period of time, higher inflation would almost certainly result to due to liquidity effects. A switch to monetary financing should not be done quickly.

The existing U.S. debt is the largest stock of safe assets and is an important part of the

world financial market. Full monetary finance eliminates the bond market. MMT advocates say that the government should still issue bonds (Mitchell, Wray, and Watts 2019), but there is little discussion about the quantity or the use of the funds raised. As Woodford (1990), Aiyagari and McGrattan (1998) and Brunnermeier, Merkel, and Sannikov (2020) show, government debt provides safe assets that help overcome financial market imperfections. A government is not required to pay off the existing debt and may have good reason not to. For example, Norway maintains a debt of over 30% of GDP though its sovereign wealth fund could buy all of it.

Aiyagari and McGrattan (1998) compute the optimal debt level to be about two-thirds output. Consider the U.S. values[3] in 2025: an inflation target of 2%, net debt of 0.966 of output, and an interest rate on the debt of 3.3%., which implies a real rate of 1.27%. **If partial monetary finance is used to reduce the net debt from 0.966 to 0.666 of output, the fraction purchased is $\Omega = 0.31$. The savings on the interest on debt, and hence the increase in the steady state deficit that could be sustained for the U.S., is 0.0032 of output.**

4.5 Extensions

The DSGE model described here could provide the basis for empirical analysis of countries that engage in monetary finance, such as Japan and Argentina. Future work should extend the stability results to a model with price stickiness and interest rate rules that include a response to output and interest rate smoothing.

Reconciling the short run and long run impacts of policy for the present model is an important goal. For example, a permanent tax cut would increase the deficit putting upward pressure on inflation. The interest rate rule indicates that the monetary authority must raise rates to fight inflation while lowering the steady state rate on reserves to compensate for the cost to the government of the higher deficits. The proper path of the policy rate over the transition that accounts for both considerations is a non-trivial issue.

Another question the present work raises but cannot answer is the the optimal choice of α_i. The model suggests that lower values reduce the cost of interest on reserves, allowing for higher deficits, and nothing rules out negative values of α_i. In practice, the zero lower bound rules them out, and the results above indicate stability would be a problem when $r_t^n = 0$. Hence, the inflation target and the steady state interest on reserve r^n are assumed to be positive, so the value of α_i must be positive as well. A lower value reduces the cost of interest on reserves, but means the zero lower bound is more likely to bind.

The values of $g-\tau, \alpha_i, \Omega, b_0^T, \Pi$ are all interdependent policy choices. The ultimate goal is

[3]Values come from the U.S. Treasury Fiscal Data (2025)

to develop an empirically robust model that reflects the welfare implications of these choices and compute the optimum values given the budget constraint (7).

5 Conclusion

Partial monetary finance and stabilization policy conducted through the interest rate on bank reserves is a policy combination deserving attention. Full monetization of government debt is not recommended, since the bond market has an important role in the financial system. However, for countries with debt in excess of that needed for financial markets, partial monetary finance could alleviate some of the cost of the interest on the debt. Monetary finance could interfere with open market operations but not monetary policy conducted through the interest rate on bank reserves, which is current practice in many countries.

The steady state consolidated budget constraint in the present model shows that monetary seigniorage must cover the costs from the interest on the debt, the fiscal deficits and the interest on reserves. There is an inverse relationship between the fraction of bonds purchased by the monetary authority and the steady state inflation rate, which demonstrates the benefit of monetary finance and the unpleasant monetarist arithmetic. Alternatively, for a given inflation target, there is a policy trade-off between the fiscal deficit and the steady state interest rate on reserves.

That these are steady state results emphasizes the importance of credibility. They do not justify the monetary finance episodes that led to accelerating inflation in countries such as Argentina. In those cases, bond purchases by the monetary authorities increased with higher fiscal deficits and the interest on the debt, so there were no meaningful steady state values for those costs. In contrast, the trust in Japanese institutions has allowed them to use monetary finance as a tool for stability. For countries without such trust, the partial monetary finance policy described here may require an independent fiscal authority with the power to control deficits.

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