Tariff Wars and Net Foreign Assets*

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Abstract

We examine whether and how international financial claims accumulated during a period of free trade can be settled without default once a tariff war erupts. Our key insight is that tariff-induced shifts in international relative prices revalue gross asset positions such that net financial claims can be zeroed out even without trade. In a multi-good model, when both countries hold gross claims on each other of the same sign, autarky is always an equilibrium for high enough tariffs. In a dynamic interpretation, domestic interest rates and the real exchange rate decouple under a severe tariff war: interest rates align with their autarkic values while international relative prices revalue net foreign assets to zero. We explore a two-good version of the model to show that when liabilities are denominated in domestic goods, autarky is the unique equilibrium for high enough tariffs, though a continuum of balanced-trade equilibria with positive trade flows may also exist at moderate tariff levels. When liabilities are denominated in foreign goods, sunspot-driven multiplicity arises: the direction of trade determines which country is impoverished. We provide a quantitative exploration of how the impact of a tariff war depends on the nature of ex-ante portfolios in a model calibrated to the US foreign asset position in 2023.

1 Introduction

The two eras of globalization before the First World War and after the 1970s witnessed large declines in trade costs and substantial increases in cross-border asset positions. In the United States, both eras eventually ended in protectionism, exemplified in the US by the Smoot-Hawley tariffs

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in 1930 and Trump tariffs in 2018-2025. This latter period, combining the implementation of substantial tariffs by the United States with the accumulation of unprecedented gross asset positions during the prior decades of financial globalization, makes understanding the interaction between trade wars and international balance sheets a crucial question in international macroeconomics.

In this paper, we study whether and how the stock of international financial claims accumulated during a period of relatively free trade can be settled once a trade war breaks out. Specifically, we analyze whether equilibria exist in which these claims are honored without explicit default, how international prices adjust to facilitate such settlement, and what welfare consequences follow for creditor and debtor nations.

Our key insight is that international relative prices serve as the primary revaluation mechanism in a trade war. When tariffs restrict trade, these prices adjust to revalue gross asset positions such that net financial claims can be zeroed out even without trade. This revaluation is possible when the gross positions are of the same sign. However, it can also create scope for multiple equilibria with dramatically different welfare outcomes depending on the composition of liabilities.

We formalize this in a two-country (Home and Foreign) multiple-good static exchange economy where households hold gross claims (positive or negative) on foreign goods. Our first main result determines when autarky is an equilibrium in a tariff war. In an autarky allocation, domestic relative prices, determined by endowments, must value foreign claims. We show that if both countries' external gross claims have the same sign when valued at these domestic prices, there exists a unique international relative price that zeros out these claims. While this price clears the financial market, high enough tariffs ensure there is no scope for goods arbitrage, rendering autarky an equilibrium. The result is robust to the presence of non-traded goods and the inclusion of more flexible general trade costs.

Because this result depends only on the "same-sign" condition at autarkic prices, the logic extends naturally to dynamic settings with time and uncertainty. In a severe tariff war, domestic interest rates and the real exchange rate decouple: interest rates align with autarkic values to aggregate domestic claims, while the real exchange rate is pinned down by the need to zero out cross-border assets. This provides a potential interpretation for why the recent trade war affected the dollar exchange rate but had no major discernible impact on interest rates.

We then specialize to a two-good setting to explore additional results. When liabilities are denominated in domestic goods, autarky is the unique equilibrium for high enough tariffs. At moderate tariff levels, though, a trade war can induce a revaluation of the gross asset positions to support a balanced-trade equilibrium with positive trade flows.

By contrast, when Home's liabilities are denominated in foreign goods, mirroring emerging markets with foreign-currency debt, we uncover a potential multiplicity of equilibria. If Foreign's

tariff is high enough, there exists an equilibrium where Home must export both goods, regardless of its free-trade net position. The mechanism is self-fulfilling: trade flowing to Foreign raises the valuation of Home's liabilities (payable in Foreign) via the tariff wedge, increasing the real debt burden and forcing Home to export more to satisfy its budget constraint. In this scenario, Home is immiserized at the expense of Foreign. Crucially, these conditions can apply to both countries simultaneously, making the outcome indeterminate. The equilibrium depends on self-fulfilling expectations about the direction of trade, reminiscent of the "debt dilemma" faced by emerging markets confronting a sharp dollar appreciation.

Across all of these two-good results, we use an Edgeworth box diagram to identify three competing effects on welfare. First, the net debtor at free trade prices benefits from debt revaluation while the creditor loses. Second, terms-of-trade movements have asymmetric effects depending on portfolio composition: if countries hold positive claims on the other's export good, the shift favors the creditor; if both hold short positions in the other's export, the shift favors the debtor. Third, both countries suffer from reduced gains from trade. The net welfare effect is therefore ambiguous and depends on the relative magnitude of these channels.

Finally, we extend the analysis to the case without gross positions. We show that autarky fails to be an equilibrium even for arbitrarily high tariffs, highlighting that gross positions are crucial to our results.

Most of our theoretical results apply to the extreme case of full autarky. We also quantify the valuation, terms of trade, and welfare effects of a trade war which falls short of imposing autarky, and show how these effects depend on the nature of the gross portfolio underlying a given net foreign asset position. To do so, we use a dynamic stochastic gravity model based on Fitzgerald (2025) calibrated to the US net foreign asset position and net exports in 2023. The quantitative results are consistent with the theoretical predictions.

Literature review. Our questions and the associated answers echo the Transfer Problem famously debated by Keynes (1929) and Ohlin (1929) in the aftermath of the first world war. That debate turned on whether a transfer by one country to another affected the terms-of-trade. With identical homothetic preferences, the answer is no, due to the fact that the distribution of wealth does not influence relative prices in such an environment. However, when the marginal dollar is spent differently, such transfers do affect the terms-of-trade. Our analysis is closest to the treatment in Samuelson (1952, 1954) with an environment in which preferences are identical and homothetic, but where there are trade frictions. Samuelson discusses how, when tariffs are present, the required transfer may have an adverse impact on the payer's terms-of-trade. Our question is the mirror image – if, under free trade, one country needs to make a net transfer to the other, can

the disruption to trade generate terms of trade that nullifies the need to make the payment.[1]

The role of valuation effects for understanding the behavior of the net foreign asset positions dates back to the pioneering work of Lane and Milesi-Ferretti (2001), Tille (2003), and Gourinchas and Rey (2007). This initial work focused in large part on the behavior of the dollar exchange rate to account for changes in asset valuations. Recent work by Atkeson, Heathcote, and Perri (2025) highlights that even if the deterioration of the US dollar accounted for the surprising stability of the US NFA position in the early 2000s (even in the presence of very large trade deficits), it is crucial to consider the spectacular performance of the US stock markets relative to foreign ones to account for the US NFA deterioration after 2008. Our paper argues that valuation effects through endogenous changes in international relative prices are also important in the context of a tariff war, showing that such price adjustments may be necessary to rebalance net foreign asset positions during trade disruptions.

Prompted by the recent policy events, there are several new papers on the interactions between tariffs and trade deficits.[2] Pujolas and Rossbach (2024) study the welfare effects of trade wars in an Armington model with trade imbalances, where countries have exogenous international net (but not gross) positions. Ignatenko et al. (2025) quantify the welfare effects of tariffs in a model with trade imbalances where net foreign asset positions are given but trade deficits endogenously respond to policy. Costinot and Werning (2025) analyze a dynamic model with fixed terms-of-trade and study the effects of a permanent increase in tariffs on the trade deficit by affecting the incentives of domestic households to save and consume. Obstfeld (2025) surveys the arguments for the determination of the U.S. trade deficit and argues that trade policies are not the main driver. In his discussion of this paper, Perri (2025) uses an international macro model to analyze the effects of tariff shocks, and their persistence, on the trade deficit, emphasizing also that the response of the economy to other shocks may be hampered by the presence of tariffs. Unlike these papers, our focus is on valuation effects and their role in determining equilibrium outcomes under tariff wars.

In a related and contemporaneous paper to ours, Itskhoki and Mukhin (2025) study the optimal unilateral tariff in the presence of valuation effects in an Armington model.[3] They identify a novel break in Lerner symmetry once gross positions are taken into account and obtain a formula for

[1] Epifani and Gancia (2017) study global imbalances in a trade model and discuss how the price indexes react differently in a trade model with increasing returns a la Krugman.

[2] There is previous work on global imbalances, trade deficits and trade frictions. See for example, Razin and Svensson (1983), Dekle, Eaton, and Kortum (2007), Fitzgerald (2012), Reyes-Heroles (2017), Kehoe, Ruhl, and Steinberg (2018), Alessandria, Bai, and Woo (2024), among others. There is also a large body of work on gross asset positions and diversification, see for example Stockman and Dellas (1989), Baxter and Jermann (1997), Heathcote and Perri, 2013, Tille (2003), Broner et al. (2013), Lee (2024), among many others.

[3] There is a classic and very general theory on optimal tariffs emphasizing the terms-of-trade effects, but that does not consider valuation effects. See Dixit and Norman (1980) and Dixit (1985) for comprehensive summaries.

the optimal tariff in the absence of export taxes. Different from them, we do not explore optimal policy, nor the role of export taxes. We consider instead arbitrary tariff policies that may or may not be optimal although we do discuss their welfare implications. Our focus is on characterizing the set of equilibria that arise, including conditions under which autarky emerges and where multiple equilibria coexist. We do not impose the Armington assumption, which allows us to consider situations where the pattern of trade reverses under high tariffs. This proves crucial for identifying equilibrium multiplicities in which the direction of trade is self-fulfilling.[4] There is also an important antecedent in Alvarez, Atkeson, and Kehoe (2009), a connection we discussed at the end of Section 3.

Paper structure. Section 2 presents relevant facts on international portfolios. Section 3 sets up the general environment and shows conditions for autarky to be an equilibrium when tariffs are high enough. This section also shows how the results extends to time and uncertainty and to the presence of trade costs. Section 4 analyzes in detail the two (traded) goods case, shows when autarky is the unique equilibrium and when multiple equilibria arise. It also discusses other balanced trade equilibria for tariffs that are not too high and the welfare implications. Section 5 provides a quantification of valuation, terms of trade effects and welfare in a trade war which falls short of imposing autarky. Section 6 concludes. All proofs are collected in the Appendix.

2 International portfolios

Since the collapse of the Bretton Woods system in the 1970s, international asset markets have become increasingly integrated. For the world as a whole, cross-border liabilities accounted for just over 20% of world GDP in 1970, but grew to more than 200% of world GDP in 2023.[5] Financial integration has increasingly taken the form of two-way asset trade. Summing net foreign assets across all countries for which net foreign assets are negative gives an indication of the minimum cross-border asset holdings necessary to account for observed net foreign asset positions (what is a liability for one country is an asset for another). In 1970, this accounted for 28% of all cross-border asset holdings. By 2023, this had fallen to just 15% of cross-border asset holdings, consistent with faster growth of gross than net positions.

For developed countries, gross foreign asset positions are particularly large, whether expressed relative to GDP, or relative to net foreign assets. Gross positions are smaller for de-

[4]Related to the focus on optimal policy responses, Bianchi and Coulibaly (2025) analyze the optimal monetary policy to an increase in tariffs. See also the recent contributions of Werning, Lorenzoni, and Guerrieri (2025) and Auclert, Rognlie, and Straub (2025). For previous work on the monetary response to tariffs, we refer the reader to Erceg, Prestipino, and Raffo (2023) and Bergin and Corsetti (2023).

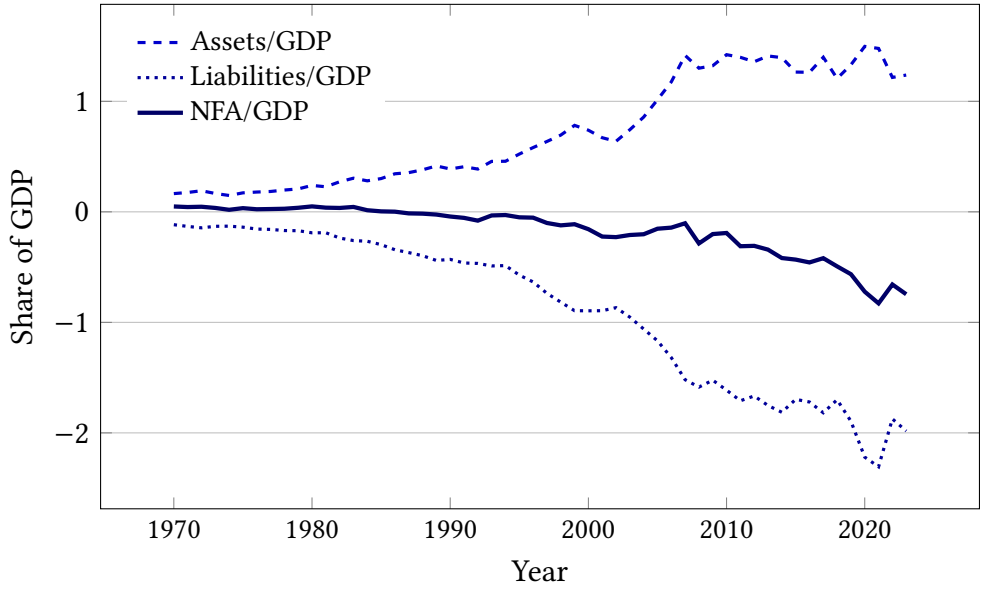
[5]Source: The External Wealth of Nations database; Lane and Milesi-Ferretti (2018).

Table 1: Gross and net foreign assets in OECD and non-OECD countries

	$\frac{\text{Assets} + \lvert\text{Liabilities}\rvert}{GDP}$		$\frac{\lvert NFA \rvert}{\text{Assets} + \lvert\text{Liabilities}\rvert}$		$\frac{\lvert NFA \rvert}{GDP}$	
	OECD	Non-OECD	OECD	Non-OECD	OECD	Non-OECD
25th percentile	2.41	0.91	0.06	0.14	0.26	0.20
50th percentile	3.88	1.32	0.10	0.34	0.42	0.40
75th percentile	6.61	2.24	0.23	0.51	0.58	0.60

Notes: Data from the External Wealth of Nations database. There are 31 non-tax-haven OECD countries with GDP greater than 0.1% of US GDP in 2023. There are 70 non-tax-haven non-OECD countries with GDP greater than 0.1% of US GDP in 2023.

Figure 1: US gross and net foreign assets relative to GDP, 1970-2023



Notes: Data from the External Wealth of Nations database.

veloping countries, but still exceed net positions. This is illustrated in Table 1, which reports the 25th, 50th and 75th percentile of gross foreign asset positions (assets + the absolute value of liabilities) relative to GDP in 2023 for non-tax-haven OECD and non-OECD countries whose 2023 GDP is greater than 0.1% of US GDP. It also reports the 25th, 50th and 75th percentile of the absolute value of net foreign assets relative to gross foreign assets for these countries.

Similar to other developed economies, the gross foreign asset position of the US is large relative to its GDP, and relative to its net foreign assets. Figure 1 plots foreign assets, foreign liabilities, and net foreign assets for the US from 1970-2023. The US has been a net debtor since 1987, but in 2023, the sum of US foreign assets and the absolute value of US foreign liabilities was 322% GDP, while US net foreign liabilities were just 75% of GDP.

For valuation effects, it matters not just that gross asset positions differ from net positions,

and are large, but it also matters the units in which foreign claims are denominated. Claims on foreign countries can be divided broadly into contingent claims such as FDI and equities, which are claims to flows of profits, and debt, which is a claim to a predetermined stream of payments denominated in a particular currency (financial derivatives also play a minor role). On both the asset and the liability side of the balance sheet, FDI and equities account for a substantially greater share of developed country portfolios than they do of developing country portfolios. This is illustrated in Table 2 for the sample of countries referred to above. Again, the US is typical of developed countries, though FDI and equities account for a larger share of its assets (65%) than its liabilities (54%).

Table 2: Portfolio composition in OECD and non-OECD countries

	$\frac{\text{FDI + Equity}}{\text{Assets}}$		$\frac{\text{FDI + Equity}}{\text{Liabilities}}$	
	OECD	Non-OECD	OECD	Non-OECD
25th percentile	0.36	0.08	0.34	0.31
50th percentile	0.50	0.19	0.44	0.52
75th percentile	0.60	0.35	0.60	0.60

Notes: Data from the External Wealth of Nations database. There are 31 non-tax-haven OECD countries with GDP greater than 0.1% of US GDP in 2023. There are 70 non-tax-haven non-OECD countries with GDP greater than 0.1% of US GDP in 2023.

Debt accounts for most of the remainder of international portfolios. The currency composition of debt assets and liabilities matters for valuation effects. Absent differential default probabilities, the value of a country's dollar debt assets relative to its dollar debt liabilities (or the value of its debt assets denominated in any other currency relative to its debt liabilities denominated in that same currency) does not depend on international relative prices. However the value of net debt in one currency relative to net debt in another currency is subject to valuation effects. Debt also has the property that a country may have a negative position denominated in terms of its import good rather than its export good.

For the US, both sides of the debt balance sheet are largely expressed in terms of dollars. At year-end 2023, 78% of US debt assets were dollar-denominated, with most of US foreign-currency debt assets being claims on developed countries (private and official). At mid-year 2023, 94% of US debt liabilities were dollar-denominated, with private sector liabilities expressed in developed country currencies accounting for most of the remainder.[6] The converse of this is that from the perspective of developing countries, a substantial fraction of debt assets and liabilities are expressed in terms of foreign currency.

[6]Source: U.S. Treasury.

3 Trade wars, autarky, and international relative prices

In this section, we introduce a general environment to analyze the role of trade wars in the presence of international gross asset positions. The generality of the model will be useful as it extends easily to a dynamic model with uncertainty, which is the most appropriate setting to analyze current account imbalances. We postpone this extension until the end of the section, once the core results have been established. We can also incorporate physical trade costs. The environment tracks textbook trade models closely (e.g., Dixit and Norman, 1980) although the existence of gross asset positions changes the analysis in interesting ways. More conveniently, it turns out that despite the presence of an arbitrary number of goods (both tradable and nontradable), the key equilibrating force is always a single relative price.

3.1 Environment

We start by considering a static exchange economy with two countries, Home and Foreign, and an arbitrary number of traded and non-traded goods. We let a star denote Foreign. Let Home have endowments of $N + M$ goods, denoted by the vector $\boldsymbol{y} = \{y_1, \ldots, y_N, y_{N+1}, \ldots, y_{N+M}\}$. Let $\mathcal{I} \equiv \{1, \ldots, N + M\}$. Let Foreign have endowments of $N + M^*$ goods, denoted by the vector $\boldsymbol{y}^* = \{y_1^*, \ldots, y_N^*, y_{N+1}^*, \ldots, y_{N+M^*}^*\}$, and let $\mathcal{I}^* \equiv \{1, \ldots, N + M^*\}$. The first $N \geq 1$ goods in each vector represent traded goods. Indices $N+1$ to $N+M$ for Home, and $N+1$ to $N+M^*$ for Foreign, denote non-traded goods.

The utility functions of the corresponding representative households in Home and Foreign are $u(\boldsymbol{c})$ and $u^*(\boldsymbol{c}^*)$, respectively, where $\boldsymbol{c} \equiv \{c_i\}_{i\in\mathcal{I}}$ and $\boldsymbol{c}^* \equiv \{c_i^*\}_{i\in\mathcal{I}^*}$ are the consumption vectors. Let $\boldsymbol{p} \equiv \{p_i\}_{i\in\mathcal{I}}$ and $\boldsymbol{p}^* \equiv \{p_i^*\}_{i\in\mathcal{I}^*}$ denote the domestic price vectors at Home and at Foreign, all quoted in a numeraire unit.

The households in each country start the period with gross claims on each other. It is important to be precise about *where* the claims are settled; that is, whether the goods are paid before or after import taxes are levied. For an asset that promises to deliver a unit of a tradable good i, for example, there are two possibilities. One is that payment is made in Home, and thus carries a payout of p_i; a second is that payment is made in Foreign, and thus carries a payout of p_i^*.[7] These payouts differ in value when tariffs induce a deviation from the law of one price for the respective good. For non-traded goods, we assume that the payouts are delivered in the prices of their respective countries. So, let $\boldsymbol{a} \equiv \{a_i\}_{i\in\mathcal{I}^*}$ denote Home ownership of assets (or liabilities,

[7] There is also a third possibility: that payment is made in what Dixit and Norman (1980) refer to as "border" prices, which is the price after any export taxes are levied but before any import tariffs. For example, if assets are settled in a global financial center outside both Home and Foreign's customs area. We do not pursue this alternative here.

if negative) in Foreign, denominated in each of the Foreign goods $i \in \mathcal{I}^*$ (potentially including claims indexed on non-traded goods). Similarly, we denote by $\boldsymbol{a}^* \equiv \{a_i\}_{i\in\mathcal{I}}$ Foreign ownership of assets denominated in goods at Home.

Home households maximize utility subject to their budget constraint:

$$\max_{\boldsymbol{c}} u(\boldsymbol{c}) \quad \text{subject to} \quad \boldsymbol{p}\cdot\boldsymbol{c} \leq \boldsymbol{p}\cdot\boldsymbol{y} + \boldsymbol{p}^*\cdot\boldsymbol{a} - \boldsymbol{p}\cdot\boldsymbol{a}^* + T. \tag{1}$$

where T is Home's tariff revenue, and the operator $\cdot$ denotes the dot product. Similarly, the Foreign households' problem is

$$\max_{\boldsymbol{c}^*} u^*(\boldsymbol{c}^*) \quad \text{subject to} \quad \boldsymbol{p}^*\cdot\boldsymbol{c}^* \leq \boldsymbol{p}^*\cdot\boldsymbol{y}^* + \boldsymbol{p}\cdot\boldsymbol{a}^* - \boldsymbol{p}^*\cdot\boldsymbol{a} + T^*. \tag{2}$$

where T^* is Foreign's tariff revenue.

We allow for u and u^* to be different and potentially non-homothetic. We only impose that they are concave and differentiable functions with finite derivatives at the autarkic allocations:

Assumption 1. *The utility functions u and u^* are strictly increasing, concave, differentiable, and have finite marginal utilities evaluated at the endowment allocation: $u_i(\boldsymbol{y}) < \infty$ and $u^*_j(\boldsymbol{y}^*) < \infty$ for $i \in \mathcal{I}$ and $j \in \mathcal{I}^*$, where u_i and u^*_j denote the partial derivatives of u and u^* with respect to good i and j, respectively.*

This requirement of finite derivatives will be satisfied if the endowment of each good is strictly positive in both countries, or if the marginal utilities are finite when consumption is zero for a good (for example, if the utility delivers a demand with a choke price). The key is that domestic relative prices for all goods are well defined (that is, neither zero nor infinity) in autarky.

Tariffs are set uniformly across all imported goods, and we let $\tau \geq 0$ and $\tau^* \geq 0$ denote the Home and Foreign tariffs. International goods arbitrage requires that for all traded goods, $i \in \{1, \ldots, N\}$, we have:

$$\frac{p_i}{p_i^*} = \begin{cases} (1+\tau) & \text{for Home imports: } c_i > Y_i \\ 1/(1+\tau^*) & \text{for Foreign imports: } c_i < Y_i \end{cases}$$
$$\text{with } p_i/p_i^* \in [1/(1+\tau^*), (1+\tau)] \text{ if } c_i = Y_i.$$

That is, if a good is imported into Home, its price is greater at Home than Foreign by the tariff factor, and vice versa if Foreign imports the good.

The governments' budget constraints is given by equating transfers to the tariffs raised on the

goods actively imported:

$$T = \tau \sum_{i=1}^{N} p_i^* \max\{c_i - Y_i, 0\}, \qquad \text{and} \qquad T^* = \tau^* \sum_{i=1}^{N} p_i \max\{c_i^* - Y_i^*, 0\}.$$

Finally, the resource constraints are:

$$\begin{aligned} c_i + c_i^* &= y_i + y_i^* && \text{for all } i \in \{1, \ldots, N\} \\ c_i &= y_i && \text{for all } i \in \{N+1, \ldots, N+M\} \\ c_i^* &= y_i^* && \text{for all } i \in \{N+1, \ldots, N+M^*\}, \end{aligned}$$

where the last two constraints follow from the non-traded nature of these goods.

We now define a competitive equilibrium for a given set of tariffs rates, $\{\tau, \tau^*\}$, which is standard:

Definition 1. *Given a pair of tariff rates, (τ, τ^*), an equilibrium is a pair of consumption allocations for Home and Foreign, $\boldsymbol{c}$ and $\boldsymbol{c}^*$, domestic prices in Home and Foreign, $\boldsymbol{p}$ and $\boldsymbol{p}^*$, such that: (i) households optimize their consumption levels subject to their budget constraints; (ii) prices are consistent with international goods arbitrage; (iii) the government budget constraints hold; and (iv) the aggregate resource constraints hold.*

The main difference of this setup and the classic analysis in Dixit and Norman (1980) is the presence of p^* in Home's budget constraint in (1) (and of p in Foreign's budget constraint in (2)), reflecting that Home (and Foreign) has ownership of claims *that are not located within its physical boundaries.*[8]

3.2 Free trade

Before turning to trade wars in the next subsection, it is useful to establish the free trade equilibrium as a benchmark. A free trade equilibrium, $\{\boldsymbol{c}_{FT}, \boldsymbol{c}_{FT}^*, \boldsymbol{p}_{FT}, \boldsymbol{p}_{FT}^*\}$ is the one that attains when

[8] In the classic analysis, the levels of p and p^* do not matter for equilibrium determination, only relative prices across goods within the locations are necessary. This is Lerner symmetry: there is an equivalence in the choices of export and import taxes. For example, if $\boldsymbol{a}$ and $\boldsymbol{a}^*$ were vectors of zeros in (1) and (2), respectively, then only domestic prices in each region would appear in the budget sets. In our environment, with p and p^* in the budget constraints, this no longer holds, and Lerner symmetry breaks. This important break of Lerner symmetry in the presence of cross-country asset positions was first noted and emphasized in Itskhoki and Mukhin (2025).

$\tau = \tau^* = 0$. In a free trade equilibrium, prices are equalized across countries for all traded goods:

$$p_i = p_i^* \text{ for all } i \in \{1, \ldots, N\}.$$

Prices and consumption levels are related through the optimality condition of the respective households. For the traded goods, it must be that

$$\frac{p_i}{p_1} = \frac{u_i(\boldsymbol{c}_{FT})}{u_j(\boldsymbol{c}_{FT})} = \frac{u_1^*(\boldsymbol{c}_{FT}^*)}{u_j^*(\boldsymbol{c}_{FT}^*)} \text{ for all } i \in \{1, \cdots, N\}.$$

For the non-traded goods in each country we have that

$$\frac{p_i}{p_1} = \frac{u_i(\boldsymbol{c}_{FT})}{u_1(\boldsymbol{c}_{FT})} \text{ for } i \in \{N+1, M\}, \quad \text{and} \quad \frac{p_i^*}{p_1^*} = \frac{u_i^*(\boldsymbol{c}_{FT}^*)}{u_1^*(\boldsymbol{c}_{FT}^*)} \text{ for } i \in \{N+1, M^*\}.$$

Note that if $\boldsymbol{p}_{FT} \cdot \boldsymbol{a} - \boldsymbol{p}_{FT}^* \cdot \boldsymbol{a}^* < 0$, then Home is a net debtor under free trade prices, and a net creditor for the reverse inequality.

A two-good economy. To provide diagrammatic intuition throughout the analysis, it is useful to consider the special case of two traded goods, which we label 1 and 2. Such an environment can be analyzed easily with an Edgeworth box, depicted in Figure 2. The dimensions of the box are $\overline{y}_1 \times \overline{y}_2$, where $\overline{y}_i \equiv y_i + y_i^*, i = 1, 2$ denote world endowments. Home's consumption of 1 and 2 is depicted from the southwest corner, with 1 on the horizontal axis and 2 on the vertical axis. Foreign consumption is the mirror image, with its origin in the northeast corner.

For exposition, we assume identical, homothetic preferences, such that the indifference curves are tangent along the diagonal. The diagonal thus outlines the efficient contract curve. The endowment point is depicted by the point $\boldsymbol{y}$. The point $F \equiv (y_1 + a_1 - a_1^*, y_2 + a_2 - a_2^*)$ depicts the endowment adjusted by initial asset positions.

As depicted, we have $a_1 < a_1^*$ and $a_2 > a_2^*$, which means that Home is short good 1 and long good 2. This places F to the northwest of the endowment point $Y \equiv \boldsymbol{y}$. We define the international relative prices (terms-of-trade) as

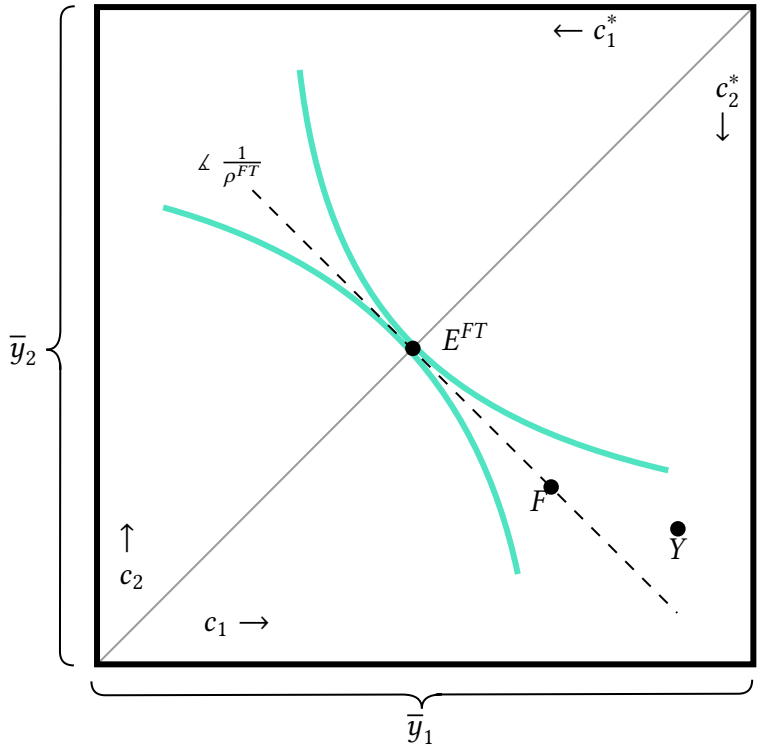
$$\rho^{FT} \equiv \frac{p_2^{FT}}{p_1^{FT}}.$$

The budget constraint for the representative household at Home is:

$$c_1 + \rho^{FT} c_2 = (y_1 + a_1 - a_1^*) + \rho^{FT}(y_2 + a_2 - a_2^*)$$

This budget constraint is represented by the line through point F with slope $-1/\rho^{FT}$. From market

Figure 2: Free Trade Equilibrium



clearing, the Foreign budget set can be read along the same line but from the opposite origin. The free-trade equilibrium is anchored where this budget line intersects the diagonal, E^{FT}. As depicted, E^{FT} is to the left and above Y, indicating that Home imports good 1 and exports good 2.

Note that free trade equilibrium can be characterized *as if* the endowment point were shifted by the initial net asset positions (for each good); that is, treating F as the endowment point. However, as we shall see, this characterization of equilibrium by a simple translation of the endowment point does not extend in general to the case with tariffs.

3.3 Autarky is an equilibrium in a trade war: a general result

Our research questions start here: suppose that the economy is initially at a free trade equilibrium with $\tau = \tau^* = 0$. But unexpectedly, there is a trade-war, and tariffs increase. What type of equilibria obtain? And in particular, what happens if there are global imbalances, that is gross positions that are non-zero, $\boldsymbol{a} \neq 0$ and $\boldsymbol{a}^* \neq 0$? In a main result, we show that if tariffs are sufficiently high (and there are gross positions denominated in different assets of the same sign), then autarky is an equilibrium. In such an equilibrium, the net foreign asset positions must rebalance to zero.

We define the *autarkic allocation* to be the one where each country's consumption corresponds

to their *physical* endowment. That is, $\boldsymbol{c} = \boldsymbol{y}$ and $\boldsymbol{c}^* = \boldsymbol{y}^*$. We proceed next to show when is this allocation an equilibrium.

Towards this, denote by $\boldsymbol{\rho}_{aut}$ and $\boldsymbol{\rho}^*_{aut}$ the vectors of relative prices with respect to good 1 consistent with autarky for both Home and Foreign. That is, $\boldsymbol{\rho}_{aut} = \{1, \rho_{aut,2}, \ldots, \rho_{aut,N+M}\}$ and $\boldsymbol{\rho}^*_{aut} = \{1, \rho^*_{aut,2}, \ldots, \rho^*_{aut,N+M^*}\}$ such that:

$$\rho_{aut,i} \equiv \frac{u_i(\boldsymbol{y})}{u_1(\boldsymbol{y})} \text{ for all } i \in \mathcal{I}, \qquad \text{and} \qquad \rho^*_{aut,j} \equiv \frac{u^*_j(\boldsymbol{y}^*)}{u_1(\boldsymbol{y}^*)} \text{ for all } j \in \mathcal{I}^*.$$

Then, we have the following result.

Proposition 1. *Suppose Assumption 1 holds, and that $\boldsymbol{a}$ and $\boldsymbol{a}^*$ are such that*

$$(\boldsymbol{\rho}_{aut} \cdot \boldsymbol{a}^*) \times (\boldsymbol{\rho}^*_{aut} \cdot \boldsymbol{a}) > 0. \tag{3}$$

Then there exists $\underline{\tau}$ and $\underline{\tau}^*$ such that for all tariffs such that $\tau \geq \underline{\tau}$ and $\tau^* \geq \underline{\tau}^*$, the autarky allocation $\boldsymbol{c} = \boldsymbol{y}$ and $\boldsymbol{c}^* = \boldsymbol{y}^*$ is an equilibrium.

Proof. In the text below. □

Condition (3) may hold independently of the sign of net foreign asset positions of the countries under free trade (that is, independently of whether Home is a net debtor or a net creditor under free trade). What is important is that the gross positions on each country are non-zero and of the same sign when evaluated at the autarkic relative prices. A simple case where this will occur is when all of the elements of $\boldsymbol{a}$ and $\boldsymbol{a}^*$ are of the same sign, with at least one of each non-zero.

To see how international relative prices rebalance the net foreign asset positions in a trade war, it is illustrative to follow the proof of the proposition, which is constructive. Given that the consumption allocations must equal the endowments, we know that equilibrium Home and Foreign consumption levels can be supported by domestic relative prices that equal the ratios of the marginal utilities in autarky:

$$\boldsymbol{p} = p_1 \times \boldsymbol{\rho}_{aut}, \qquad \text{and} \qquad \boldsymbol{p}^* = p^*_1 \times \boldsymbol{\rho}^*_{aut}.$$

for some finite non-zero scalars p_1 and p^*_1. Assumption 1 guarantees that these prices are well defined. Given that the endowments pin down relative prices in each region, the ratio p^*_1/p_1 determines the ratio of price *level* in Foreign versus Home. Hence, the ratio will be proportional

to the real exchange rate. For convenience, we therefore define

$$e \equiv p_1^*/p_1$$

to be our notion of a real exchange rate.

In an autarkic equilibrium the net foreign asset position of each country zeros out; there is no trade; and no tariff revenue, $T = T^* = 0$. Hence, given that the budget constraints of both Home and Foreign hold with equality for $\boldsymbol{c} = \boldsymbol{y}$ and $\boldsymbol{c}^* = \boldsymbol{y}^*$, it must be that $\boldsymbol{p} \cdot \boldsymbol{a}^* - \boldsymbol{p}^* \cdot \boldsymbol{a} = 0$. For the net foreign asset positions to zero out we need that:

$$\boldsymbol{p} \cdot \boldsymbol{a}^* = \boldsymbol{p}^* \cdot \boldsymbol{a} \quad \Rightarrow \quad p_1(\boldsymbol{\rho}_{Aut,2} \cdot a^*) = p_1^*(\boldsymbol{\rho}_{Aut,2}^* \cdot \boldsymbol{a}), \tag{4}$$

which will hold for a particular ratio of the real exchange rate:

$$e = \frac{p_1^*}{p_1} = \frac{\boldsymbol{\rho}_{Aut,2} \cdot \boldsymbol{a}^*}{\boldsymbol{\rho}_{Aut,2}^* \cdot \boldsymbol{a}} > 0, \tag{5}$$

where the inequality follows from the condition in the proposition. It is in this way that *asset positions pin down the real exchange rate* when tariffs are sufficiently high.

Concavity of the utility functions guarantee that, given that the optimality condition holds by the construction of $\boldsymbol{p}$ and $\boldsymbol{p}^*$, the autarkic consumption bundles solve the households' problem at Home and Foreign. Moreover, the autarky allocations satisfy the resource constraints by construction.

The only equilibrium condition left to check is the international goods arbitrage. For that, we need to ensure that the relative prices for each traded good across countries are consistent with no trade:

$$\frac{p_i}{p_i^*} = \frac{p_1}{p_1^*}\frac{u_i(\boldsymbol{y})}{u_1(\boldsymbol{y})}\frac{u_1^*(\boldsymbol{y}^*)}{u_i^*(\boldsymbol{y}^*)} \in \left[\frac{1}{1+\tau^*}, 1+\tau\right] \text{ for all } i \in \{1,\ldots,N\}.$$

The following (finite) lower bounds on tariffs ensure this is true for all goods:

$$\begin{aligned} \tau \geq \underline{\tau} &= \left(\frac{\boldsymbol{\rho}_{Aut,2}^* \cdot \boldsymbol{a}}{\boldsymbol{\rho}_{Aut,2} \cdot \boldsymbol{a}^*}\right) \max_{i\in\{1,\ldots,N\}} \left\{\frac{u_i(\boldsymbol{y})}{u_1(\boldsymbol{y})}\frac{u_1^*(\boldsymbol{y}^*)}{u_i^*(\boldsymbol{y}^*)}\right\} - 1, \\ \tau^* \geq \underline{\tau}^* &= \left(\frac{\boldsymbol{\rho}_{Aut,2} \cdot \boldsymbol{a}^*}{\boldsymbol{\rho}_{Aut,2}^* \cdot \boldsymbol{a}}\right) \max_{i\in\{1,\ldots,N\}} \left\{\frac{u_i^*(\boldsymbol{y}^*)}{u_1^*(\boldsymbol{y}^*)}\frac{u_1(\boldsymbol{y})}{u_i(\boldsymbol{y})}\right\} - 1. \end{aligned} \tag{6}$$

If tariffs satisfy these bounds, then the autarky allocation is an equilibrium.

It is instructive to consider a case where Assumption 1 fails, such as an Armington model with fully specialized endowments. If the marginal utility of the imported good approaches infinity

at zero consumption, Assumption 1 is violated, rendering autarky unattainable via finite tariffs. However, introducing a choke price (implying finite marginal utility at zero) restores the validity of Assumption 1.[9]

The two-good economy. To provide some intuition, we return to our two-good special case from Figure 2. Recall that we depicted the case where Home is relatively abundant in good 1. For simplicity, we assume now that $a_1 = a_2^* = 0$, so that each country holds a claim on the other country's relatively abundant good. Condition (5) reduces to

$$\boldsymbol{p} \cdot \boldsymbol{a}^* = p_1 a_1^* = p_2^* a_2 = \boldsymbol{p}^* \cdot \boldsymbol{a} \quad \Rightarrow \quad \frac{p_2^*}{p_1} = \frac{a_1^*}{a_2} \equiv \widehat{\rho}.$$

The corresponding equilibrium exchange rate, (5), can the be written as

$$e = \frac{p_1^*}{p_1} = \frac{p_1^*}{p_2^*}\frac{a_1^*}{a_2} = \frac{1}{\rho_{aut,2}^*}\frac{a_1^*}{a_2}$$

Given that assets and relative prices within each region are pinned down by primitives in autarky, this defines e uniquely. Moreover, we can immediately see the need for the condition for the asset positions to be of the same sign; otherwise, e would be negative.

We depict the equilibrium in Figure 3. The point E^{FT}, F, and Y correspond to the same variables as in the free-trade diagram Figure 2. The dashed line connecting E^{FT} to F corresponds to the free-trade budget line from the previous figure, which we include for reference. Point $Y = (y_1, y_2)$ is the endowment point, which corresponds to the autarkic equilibrium when tariffs are sufficiently high.

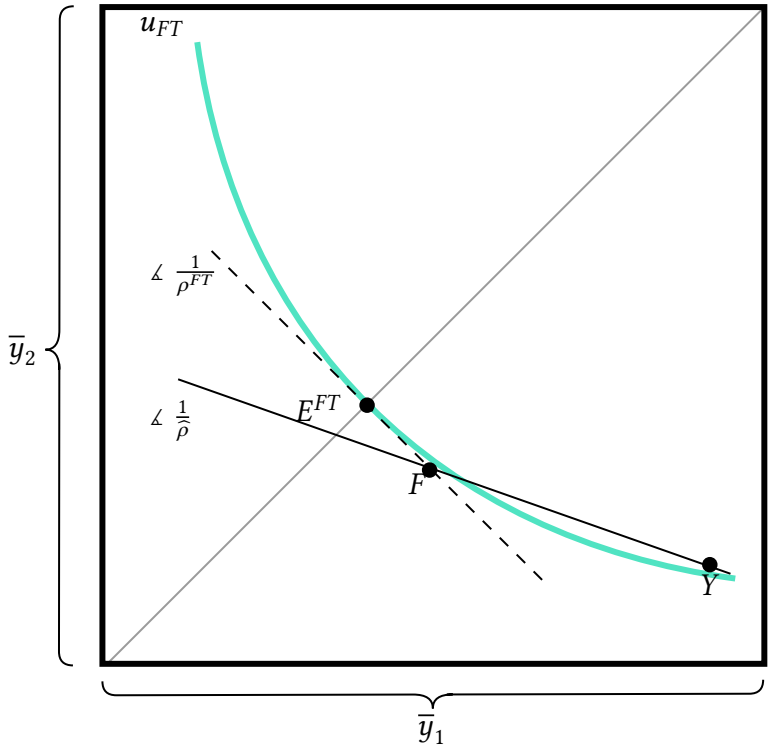
The figure also includes a line connecting points F and Y. As $a_1 = a_2^* = 0$, we have $F = (y_1 - a_1^*, y_2 + a_2)$, and hence $-1/\widehat{\rho} = -a_2/a_1^*$ represents the slope of the line that connects F and $Y = (y_1, y_2)$. This line has a useful interpretation away from autarky. Along this line, Home is exporting good 1 and Foreign is exporting good 2 until we reach the endowment point. Hence, $\widehat{\rho} = p_2^*/p_1$ is the terms-of-trade. With this relative price, Home's budget set can be written

$$c_1 + \widehat{\rho} c_2 = y_1 + \widehat{\rho} y_2,$$

where we have used $p_2^* a_2 = p_1 a_1^*$ and netted out the tariff paid by consumers and lump-sum

[9] A related question is whether the autarkic prices $\boldsymbol{p}$ and $\boldsymbol{p}^*$ are unique under Assumption 1. Provided all autarkic consumption levels are strictly positive, the differentiability of the utility functions ensures uniqueness. However, if consumption of some goods is zero and marginal utilities remain bounded, the supporting prices are not unique; any price exceeding the marginal valuation at zero would support autarky.

Figure 3: Tariff War Equilibrium With Balanced Trade



Note: Home is a net debtor, and its terms-of-trade worsen.

rebated back.[10] The line connecting F and Y contains the allocations that satisfy this condition, and hence trace out possibilities for equilibria that involve non-zero but balanced trade. We will study these in more detail in Section 4.

In the figure, we have depicted the case where $a_1^* > 0$ and $a_2 > 0$. It follows that the terms-of-trade have deteriorated for Home relative to free trade. To see this, Home's debtor status implies

$$\rho^{FT} a_2 - a_1^* < 0.$$

Using $a_2 > 0$, this implies

$$\rho^{FT} < a_1^*/a_2 = \widehat{\rho}.$$

If $a_2 < 0$ and $a_1^* < 0$, then the terms of trade would shift instead in the favor of Home's export. This case is shown in Appendix A.

[10] Specifically, consumers pay $\tau p_2^* c_2$ in tariffs. Hence, $p_2 c_2 - T = p_2^*(1+\tau)c_2 - \tau p_2^* = p_2^* c_2$.

3.4 Time and uncertainty

Trade imbalances are inherently a reflection of inter-temporal trade. In this subsection we extend our analysis to a dynamic, stochastic economy. As Dixit and Norman (1980) highlight, if the analysis contains an arbitrary number of goods, it is straightforward to also introduce intertemporal trade (and uncertainty). This just requires re-interpreting different goods as different times or states, and intra-temporal prices as inter-temporal ones, as in the classic Arrow-Debreu analysis.

The result of Proposition 1 then tells us that, in a dynamic environment, an increase in bilateral tariffs that is high enough makes autarky an equilibrium as long as the total value of the asset positions of each country have the same sign *when evaluated at the respective autarky prices.*[11]

To be more explicit about this, consider a deterministic dynamic economy starting at $t = 0$. For simplicity, we abstract from uncertainty, but the addition of one more index indicating the state of nature is straightforward. The households' utilities are given by:

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \quad \text{and} \quad \sum_{t=0}^{\infty} \beta_*^t u^*(C_t^*),$$

where C_t and C_t^* as consumption aggregates at every date. That is,

$$C_t = C(c_1(t), c_2(t), \cdots, c_{N+M}(t)), \quad \text{and} \quad C_t^* = C^*(c_1^*(t), c_2^*(t), \cdots, c_{N+M^*}^*(t)),$$

where $c_i(t)$ is the consumption of good i at time t at Home, and $c_j^*(t)$ for Foreign, and C and C^* are the respective aggregator functions. Let $y_i(t)$ and $y_j^*(t)$ be the endowments of good $i \in \mathcal{I}$ in Home and $j \in \mathcal{I}^*$ in Foreign at time t.

Let $p_i(t)$ be the time-zero price of good $i \in \mathcal{I}$ at Home at time t, and let $p_j^*(t)$, $j \in \mathcal{I}^*$ be the same for Foreign. Autarkic relative prices are pinned down by endowments. In particular, relative prices in autarky at Home satisfy

$$\frac{p_i(t)}{p_j(t')} = \beta^{t-t'} \frac{u_i(C_t)}{u_j(C_{t'})},$$

where $u_i(C_t) = \partial u(C(y_1(t), \cdots, y_{N+M}(t))/\partial c_i(t)$. A similar condition determines relative prices in Foreign.

Let $a_j(t)$ denote Home's initial claim to good $j \in \mathcal{I}^*$ in period t in Foreign, and $a_i^*(t)$ for $i \in \mathcal{I}$

[11] One thing that needs to hold, given that we now have an infinity of goods, is that the values of the endowments and the claims are finite in each country when evaluated at autarkic prices. We assume this the case.

to be Foreign's corresponding claims on Home. The condition corresponding to (4) is

$$\sum_{i\in\mathcal{I}}\sum_{t} p_i(t)a_j^*(t) = \sum_{j\in\mathcal{I}^*}\sum_{t} p_j^*(t)a_j(t). \tag{7}$$

As relative prices in each region are determined by endowments, equation (7) pins down the relative value of a numeraire in each country. It will be useful to consider as our numeraire the value of a basket consisting of one of each good in period zero in each location:

$$\begin{aligned}\overline{p}_0 &\equiv \sum_{i\in\mathcal{I}} p_i(0) = p_1(0)\sum_{i\in\mathcal{I}}\frac{p_i(0)}{p_1(0)}\\ \overline{p}_0^* &\equiv \sum_{j\in\mathcal{I}^*} p_j^*(0) = p_1^*(0)\sum_{j\in\mathcal{I}^*}\frac{p_j^*(0)}{p_1^*(0)},\end{aligned}$$

where the second equality in each line establishes that, as in the benchmark, there is only one degree of freedom given that relative prices are given by endowments. Let $q_0 \equiv \overline{p}_0^*/\overline{p}_0$ be the initial exchange rate (i.e., relative value of the numeraire baskets) that zeros out initial asset positions in (7).

Proposition 1 states that q_0 defined in this way supports an autarkic equilibrium for high enough tariffs. The initial exchange rate is such that at period zero each country finds that the value of their initial assets equals their initial liabilities. From this period on, the financial positions can be netted out and there is no further need for additional trades.

We can shed more light on the dynamic behavior of the exchange rate by analyzing the corresponding sequence formulation of the autarkic equilibrium. Consider two one-period risk-free bonds that promises at time t to pay one unit of each good at time $t+1$ in each country. The time-zero value of a such a claim at Home is $\overline{p}_t \equiv \sum_{i\in\mathcal{I}} p_i(t)$, and similarly for Foreign: $\overline{p}_t^* \equiv \sum_{j\in\mathcal{I}^*} p_j^*(t)$. The one-period interest rate at time t for each bond can be defined as the relative value of the promised basket for period $t+1$ relative to t:

$$R_t \equiv \frac{\overline{p}_{t+1}}{\overline{p}_t} \quad \text{and} \quad R_t^* \equiv \frac{\overline{p}_{t+1}^*}{\overline{p}_t^*}.$$

The interest rate on each bond ensures the residents in the respective countries do not wish to borrow or save in their own bonds. Defining the sequential exchange rate at time t as $q_t \equiv \frac{\overline{p}_t^*}{\overline{p}_t}$, we have

$$R_t = R_t^*\frac{q_{t+1}}{q_t},$$

which is the usual party condition relating the differences in rates of return across countries to the expected depreciation of the exchange rate. The exchange rate is such that neither country

wishes to borrow or save in the other country's bonds. Hence, the autarkic interest rates and the changes in the exchange rate ensure zero asset net positions after the initial period, while the *the level* of the initial exchange rate q_0 ensures that budgets balance in autarkic.[12]

This result provides an interpretation of why the recent trade war affected the dollar exchange rate, but had no major discernible impact on interest rates. In a severe enough trade war, the exchange rate and the interest rates are not connected but rather are balancing distinct equilibrium forces.

3.5 Trade costs.

Modern quantitative trade theory emphasizes the role of trade costs in generating the "gravity" relation between bilateral expenditure shares, economic sizes, and distance. It is straightforward to incorporate such costs into our environment. We focus now on an environment where all goods can potentially be traded (so $M = M^* = 0$). Let G be an "export" technology that takes as input a vector of goods in Home and generates a vector of goods in Foreign. Specifically, for $G(\boldsymbol{x})$ to reach Foreign, a vector of goods $\boldsymbol{x}$ needs to allocated. Let G^* be the analogous "export" technology for Foreign. These technologies capture all feasible methods of transporting goods across countries. We assume that G and G^* are weakly increasing, weakly concave, and differentiable functions in $\mathbb{R}^N_{\geq 0}$, both are constant returns to scale, and satisfy $\mathbf{0} = G(\mathbf{0}) = G^*(\mathbf{0})$. The case with no trade costs corresponds to $G(\boldsymbol{x}) = \boldsymbol{x}$ and $G^*(\boldsymbol{x}^*) = \boldsymbol{x}^*$. The standard case of good-specific iceberg trade costs corresponds to $G(\boldsymbol{x}) = \Lambda \boldsymbol{x}$ and $G^*(\boldsymbol{x}^*) = \Lambda^* \boldsymbol{x}^*$, where Λ and Λ^* are diagonal matrices with elements in $(0, 1]$, reflecting the survival rate of goods in transit (that is, the inverse of the iceberg trade costs).

The resource constraints are modified as follows:

$$\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{x} - G^*(\boldsymbol{x}^*), \quad \text{and} \quad \boldsymbol{y}^* = \boldsymbol{c}^* + \boldsymbol{x}^* - G(\boldsymbol{x}),$$

where $\boldsymbol{x} \geq 0$ is the vector of Home goods used as inputs (or lost in transit for the iceberg case) to produce Foreign goods (exports), and $\boldsymbol{x}^* \geq 0$ is the corresponding vector for Foreign.

Competitive firms operate these technologies. Their final outputs are taxed with tariffs at

[12] There is a clear antecedent in Alvarez, Atkeson, and Kehoe (2009). There, the authors analyze a two-country model where households consume only a non-traded good in each country, so by construction there cannot be international trade in goods. International financial markets are open, and thus rates of return equalization determines the required changes in exchange rates, just as here. The absence of trade in goods implies that the initial level of the exchange rate must be such that the initial net foreign asset position of each country is zero. Our result is related, with some distinctions. Our trade friction arises from the imposition of tariffs, and thus the absence of trade is not rule out by feasibility, but rather it is an equilibrium outcome. In addition, whether autarky is the unique equilibrium that emerges in the presence of high enough tariffs requires additional restrictions on initial asset positions, as we discuss below.

Home and in Foreign. The optimization problems of the representative exporting firms are:[13]

$$\max_{\{\boldsymbol{x}\}} \left\{ \frac{1}{1+\tau^*} \boldsymbol{p}^* \cdot G(\boldsymbol{x}) - \boldsymbol{p} \cdot \boldsymbol{x} \right\}, \quad \text{subject to: } \boldsymbol{x} \geq 0.$$
$$\max_{\{\boldsymbol{x}^*\}} \left\{ \frac{1}{1+\tau} \boldsymbol{p} \cdot G^*(\boldsymbol{x}^*) - \boldsymbol{p}^* \cdot \boldsymbol{x}^* \right\}, \quad \text{subject to: } \boldsymbol{x}^* \geq 0.$$

International goods arbitrage is encapsulated in these optimization problems. Constant returns to scale in the exporting technologies imply that exporting firms make zero profits in equilibrium, and thus the households' budget constraints remain as before. The governments' tariff revenues become:

$$T = \frac{\tau}{1+\tau} \boldsymbol{p} \cdot G^*(\boldsymbol{x}^*), \quad \text{and} \quad T^* = \frac{\tau^*}{1+\tau^*} \boldsymbol{p}^* \cdot G(\boldsymbol{x}).$$

The definition of equilibrium is analogous to the baseline definition, Definition 1, but includes $\boldsymbol{x}$ and $\boldsymbol{x}^*$ as equilibrium objects, have the updated resource constraints and government budget constraints, and requires that exporting firms optimize.

Consider the autarky allocation where $\boldsymbol{y} = \boldsymbol{c}$, $\boldsymbol{y}^* = \boldsymbol{c}^*$, and $\boldsymbol{x} = \boldsymbol{x}^* = \boldsymbol{0}$. Is this an equilibrium outcome for sufficiently high tariffs? This allocation satisfy the resource constraints. Domestic relative prices are determined by the respective autarkic allocations (endowments) exactly as in the frictionless case. Furthermore, the international relative price that zeros out the net foreign asset positions remains uniquely determined by the ratio of asset valuations at these autarkic prices, given that the households budget constraints are unchanged as there is no tariff revenue. This pins down $\boldsymbol{p}$ and $\boldsymbol{p}^*$.

The only remaining equilibrium condition to check is whether $\boldsymbol{x} = \boldsymbol{x}^* = \boldsymbol{0}$ is optimal for the exporting firms given these prices and tariffs. This holds if the marginal products of G and G^* are bounded above at $\boldsymbol{0}$, a natural restriction satisfied, for example, by linear iceberg costs. Under this restriction, a sufficiently high bilateral tariff pair (τ, τ^*) renders exporting unprofitable for any good, sustaining autarky as an equilibrium.[14]

[13] In our notation, domestic prices are measured inclusive of tariffs. As a result, the revenue of exporting firms must be scaled by $1/(1+\tau)$ and $1/(1+\tau^*)$ to reflect the pre-tariff revenue retained by firms.

[14] The exact condition, which follows from the concavity of the export technologies, is

$$\max_{j \in \{1,\ldots,N\}} \sum_{j=1}^{N} \frac{p_j}{p_i^*} \left. \frac{\partial G_j^*(\boldsymbol{x}^*)}{\partial x_i^*} \right|_{\boldsymbol{x}^*=\boldsymbol{0}} - 1 = \underline{\tau} \leq \tau, \quad \text{and} \quad \max_{j \in \{1,\ldots,N\}} \sum_{j=1}^{N} \frac{p_j^*}{p_i} \left. \frac{\partial G_j(\boldsymbol{x})}{\partial x_i} \right|_{\boldsymbol{x}=\boldsymbol{0}} - 1 = \underline{\tau}^* \leq \tau^*.$$

This collapses to (6) when $G(\boldsymbol{x}) = G^*(\boldsymbol{x}) = \boldsymbol{x}$, the case without trade frictions.

What's next. The result in Proposition 1 is quite general as it requires only a weak condition on asset holdings across countries. However, this result does not tell us whether autarky is the unique equilibrium when tariffs are high enough, whether other equilibria may also be possible, and what happens when condition (3) does not hold. It also doesn't clarify whether there are other equilibria with strictly positive trade across countries, and where the net foreign asset positions also zero out. We proceed next to restrict the analysis to the case where preferences are symmetric and homothetic, and there are only two (traded) goods. Under this restriction, we will answer the remaining questions, as well as provide a clean intuition for the results through the use of Edgeworth box diagrams.

4 A deeper dive into the two-good economy

The previous section established that a sufficiently severe trade war can generate an autarkic equilibrium when initial gross asset positions are of the same sign. In this section, we use the simple two-good model to dig deeper in the characterization of equilibrium outcomes. In particular, in this section:

- We find conditions under which the autarkic allocation is the unique equilibrium when tariffs are high enough.
- We show that, under a different set of conditions, an additional type of equilibria emerges when tariffs are high enough.
- We find that there are tariffs rates levels that lead to balanced but non-zero trade.
- We characterize the equilibrium outcome that arises for high enough tariff when the financial positions involved only a single asset (that is, no gross positions), as in many standard models.

We also use the two-good Edgeworth diagram to characterize the welfare consequences of a trade war.

We focus on the two-good special case introduced in Figure 2. For simplicity, we also assume that u and u^* are identical, homothetic, and satisfy Inada conditions. By homotheticity, we can define the Marginal Rate of Substitution (MRS) between goods 2 and 1 as a function of the ratio $m = c_2/c_1$:

$$g(m) \equiv \frac{u_2(1,m)}{u_1(1,m)},$$

where we make the standard assumption $g'(m) < 0$, and the Inada conditions imply $\lim_{m\to 0} g(m) = \infty$ and $\lim_{x\to\infty} g(m) = 0$. Let h denote the inverse of g, which is well defined given the strict

monotonicity of MRS:

$$h(m) \equiv g^{-1}(m).$$

In a free trade allocation, we must then have that

$$g(c_2/c_1) = g(c_2^*/c_1^*) = g(\overline{y}_2/\overline{y}_1) = \rho^{FT},$$

where the second equality follows from the strict monotonicity of g. Thus, in this case, the free trade relative price is independent of the financial positions.

We assume that all endowments are strictly positive. As in Figures 2 and 3, we assume that Home has a relative abundance in good 1, and we consider a case where under free trade Home is a net debtor and there are two-way trades (that is, Home is not so indebted that it exports both goods):

Assumption 2. *In the two (traded) goods economy, (i) the endowments are strictly positive* $y_1, y_2, y_1^*, y_2^* > 0$*; (ii) home is relatively 'in good 1:*

$$\frac{y_2^*}{y_1^*} > \frac{\overline{y}_2}{\overline{y}_1} > \frac{y_2}{y_1};$$

(iii) under free trade, there are two-way trades, $c_1^{FT} < y_1$ *and* $c_2^{FT} > y_2$*; and (iv) under free trade, home is a net debtor,* $c_1^{FT} + \rho^{FT} c_2^{FT} < y_1 + \rho^{FT} y_2$.

4.1 Characterizing equilibria

We first focus on the case where the countries liabilities are denominated in their own goods. That is, Home has liabilities only in their goods 1 and 2 to be paid in Home, and Foreign has liabilities only in their goods 1 and 2 to be paid in Foreign:

Assumption 3. *In the two (traded) goods economy, under free trade, each country has a non-negative claim in the foreign goods. That is,*

$$a_1 + \rho^{FT} a_2 \geq 0, \quad \text{and} \quad a_1^* + \rho^{FT} a_2^* \geq 0.$$

With this assumption, we can now state the following characterization result:

Lemma 1. *Suppose Assumptions 2 and 3 hold. Then any equilibrium is characterized by* $(c_1, c_2, c_1^*, c_2^*, \rho)$ *such that:*

(i) Home (weakly) exports good 1 *and (weakly) imports good* 2*:* $c_1 \leq y_2$ *and* $c_2 \geq y_2$*;*

(ii) Households optimize and goods arbitrage holds:

$$g(c_2/c_1) = \alpha\rho \quad \text{and} \quad g(c_2^*/c_1^*) = \rho/\alpha^*,$$

where $\alpha, 1/\alpha^* \in [1/(1+\tau^*), 1+\tau]$ *and* $\alpha = 1+\tau$ *if* $c_2 > y_2$ *and* $\alpha^* = (1+\tau^*)$ *if* $c_1 < y_1$*;*

(iii) And budget sets and the resource constraints are satisfied:

$$c_1 + \rho c_2 = y_1 + \rho y_2 - (a_1^* + \alpha\rho a_2^*) + \alpha^* a_1 + \rho a_2$$
$$c_1^* + \rho c_2^* = y_1^* + \rho y_2^* + (a_1^* + \alpha\rho a_2^*) - \alpha^* a_1 - \rho a_2,$$

and $c_1 + c_1^* = \overline{y}_1$, $c_2 + c_2^* = \overline{y}_2$.

The corresponding equilibrium prices are any (p_1, p_2, p_1^*, p_2^*) *such that*

$$\rho \equiv p_2^*/p_1, \quad p_2 = \alpha p_2^*, \quad \text{and} \quad p_1^* = \alpha^* p_1.$$

Proof. In Appendix B.1. □

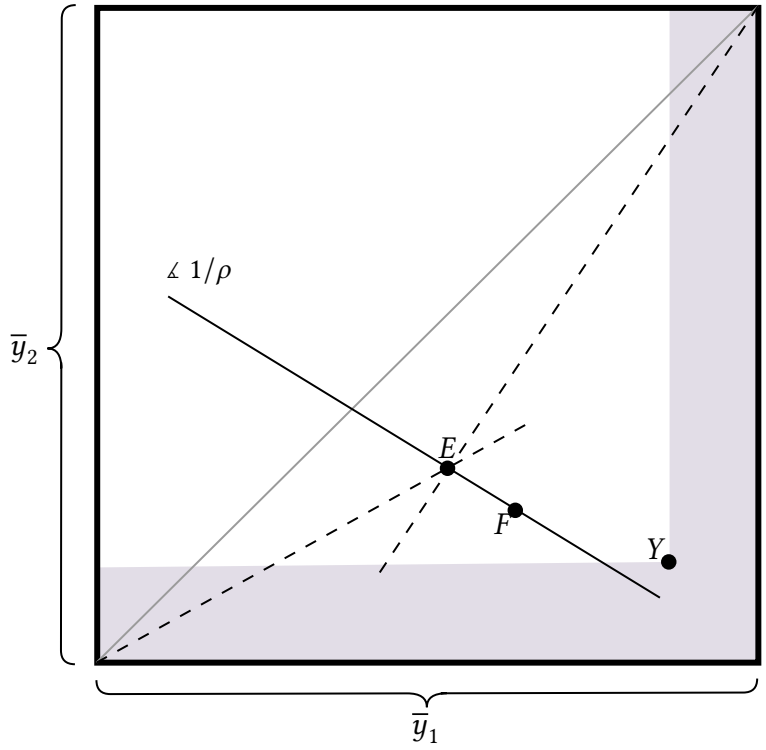
Unpacking Lemma 1: The case of $a_1 = a_2^* = 0$. To explore this lemma, we focus on the case where $a_1 = a_2^* = 0$ and make use of the Edgeworth box diagram, displayed in Figure 4. As before, $Y = (y_1, y_2)$ is the endowment point and $F = (y_1 - a_1^*, y_2 + a_2)$ is the endowment translated by the asset positions. Assumption 3 guarantees that F is not in the shaded region.

The result in part (i) of the lemma shows that the presence of tariffs *cannot change* the pattern of trade, which is intuitive given that tariffs are levied on whichever good(s) is imported. If trade were reversed then import tariffs would fall on the relatively abundant good, reducing the incentive to import them compared to the free trade equilibrium.[15] This implies that the equilibrium cannot be in the interior of the shaded region of Figure 4. It also implies that we can focus on the ratio of $p_2^*/p_1 = \rho$ as the terms of trade in any equilibrium.

If trade is strictly positive in both goods, we have $p_2/p_1 = (1+\tau)\rho$ and $p_2^*/p_1^* = \rho/(1+\tau^*)$.

[15] However, a note of caution is required as Section 4.3 shows that this is not necessarily true without Assumption 3.

Figure 4: Tariff War Equilibrium: Simple Assets Case



This implies that the households' first-order conditions are:

$$\left(\frac{1}{1+\tau}\right) g\left(\frac{c_2}{c_1}\right) = \rho = (1+\tau^*) g\left(\frac{\overline{y}_2 - c_2}{\overline{y}_1 - c_1}\right).$$

In the Edgeworth box of Figure 4, the ray from Home's origin traces out $c_2/c_1 = h\left((1+\tau)\rho\right)$, where recall that h is the inverse of g. Similarly, the ray from Foreign's origin contains the locus $c_2^*/c_1^* = h(\rho/(1+\tau^*))$. Consistent with tariffs distorting Home's consumption toward good 1 and Foreign toward 2, we depict the rays below the free-trade diagonal.

The budget constraints in the lemma are now

$$\begin{aligned} c_1 + \rho c_2 &= y_1 + \rho y_2 - a_1^* + \rho a_2 \\ c_1^* + \rho c_2^* &= y_1^* + \rho y_2^* + a_1^* - \rho a_2, \end{aligned} \tag{8}$$

which are obtained by substituting T and T^* in the respective household budget sets using the governments' budget constraints. From (8) we can see that in any equilibrium the terms of trade, $\rho = p_2^*/p_1$, plays the dual role of being the international relative price of the goods, as well as being the relative price of the international claims. Equation (8) states that feasible consumption allocations for Home are those along the line with slope $-1/\rho$ passing through the point F. Using

the resource constraint, Foreign's allocations viewed from the opposite origin lie along the same line.

The equilibrium ρ is such that these rays intersect along the budget line, depicted as point E. This allocation satisfies both country's household optimality as well as the resource constraints. As drawn, E is to the northwest of F. This means $c_2 > y_2 + a_2$, and so Home's imports of 2 are greater than its asset. Similarly, $c_1 < y_1 - a_1^*$, and Home's exports of 1, $y_1 - c_1$, are greater in magnitude than its debt a_1. As $(1+\tau)(1+\tau^*)$ increases, the equilibrium is pushed further to the southeast, away from the free trade contract curve. If the equilibrium point lies below F, then $c_2 < y_2 + a_2$ and Home's imports of good 2 are less than its assets a_2. Similarly, if it lies to the right of F, then $c_1 > y_1 - a_1^\star$, and Home's exports of good 1 are less than its debt.

If trade is not strictly positive, the equilibrium will lie along the border of the shaded region. Such boundary points are supported in equilibrium by the appropriate α or α^* in the lemma. That is, if a good is not actively traded, the respective MRS and equilibrium prices can take values within the specified intervals spanned by the tariff wedges. The budget line continues to be defined by the international terms of trade, ρ.

4.2 When autarky is the unique equilibrium

We are ready to establish conditions that ensure autarky is the only equilibrium as tariffs become large:

Proposition 2 (Autarky is the Only Equilibrium)**.** *Suppose that Assumptions 2 and 3 hold. There exists finite $\underline{\tau}, \underline{\tau}^*$ such that for any $\tau \geq \underline{\tau}$ and $\tau^* \geq \underline{\tau}^*$, the autarkic allocation $(c_1 = y_1, c_2 = y_2, c_1^* = y_1^*, c_2^* = y_2^*)$ is the* unique equilibrium *if*

$$a_1 + \rho_{Aut,2}^* a_2 > 0, \quad \textit{and} \quad a_1^* + \rho_{Aut,2} a_2^* > 0.$$

Proof. In Appendix B.2. □

To understand the reason for this result, assume for simplicity that $a_1 = a_2^* = 0$. The conditions in Proposition 2 reduce to $a_2 > 0$ and $a_1^* > 0$. We can then use Lemma 1. As tariffs become very large, an equilibrium allocation cannot remain in the area where $c_1 < y_1$ and $c_2^* < y_2^*$ (the interior of the non-shaded region in Figure 4). In particular, if there is trade in both goods, household optimality requires $c_2/c_1 = h((1+\tau)\rho)$, which implies that as τ increases, $c_2/c_1 \to 0$, which is inconsistent with $c_2 > y_2$. A similar argument implies that $c_2^*/c_1^* = h(\rho/(1+\tau^*)) \to \infty$ as τ^* becomes large. Lemma 1 then implies that the only potential alternative to autarky is one-way

trade in a single good: either $c_1 < y_1$ and $c_2 = y_2$, or $c_1 = y_1$ and $c_2 > y_2$.

Consider the first potential alternative scenario. In this case, Home's budget constraint becomes:

$$c_1 - y_1 + \rho \underbrace{(c_2 - y_2)}_{=0} = \rho a_2 - a_1^* = (1+\tau^*) g\left(\frac{c_2^*}{c_1^*}\right) a_2 - a_1^*,$$

where $\rho = p_2^*/p_1$ and the last equality follows from the premise that Foreign is importing good 1 and therefore its marginal rate of substitution equals $p_2^*/p_1^* = \rho/(1+\tau^*)$. The premise that $c_1^* > y_1^*$ and $c_2^* = y_2^*$ also implies

$$\frac{\rho}{1+\tau^*} = g\left(\frac{c_2^*}{c_1^*}\right) > g\left(\frac{y_2^*}{y_1^*}\right) \equiv \rho_{Aut,2}^*.$$

Combining these two expressions and recalling that $a_2 > 0$, we have

$$c_1 - y_1 > (1+\tau^*)\rho_{Aut,2}^* a_2 - a_1^*.$$

As τ^* becomes large, given that $a_2 > 0$, $c_1 - y_1$ becomes arbitrarily positive, violating the premise that Home exports good 1. The reason is that, if Home is exporting to Foreign, a higher tariff in Foreign makes Home wealthier, as it appreciates its claim on Foreign goods. A similar argument in reverse shows that $c_1 = y_1$ with $c_2 > y_2$ cannot happen for sufficiently high τ. Thus the unique equilibrium outcome when tariffs are sufficiently high is autarky.

4.3 When autarky is not the unique equilibrium

In the previous section we show how Assumption 3 guaranteed that autarky is the unique equilibrium when tariffs are high enough. In practice, countries are often indebted in a foreign currency, and one way to interpret foreign currency liabilities in our setup is to consider the case where a country has liabilities in foreign goods, a situation that is ruled out by Assumption 3. As we will show now, relaxing this requirement opens the door to other type of equilibria when tariffs are high enough.

So assume now that Home has a net liability denominated in the Foreign goods, $a_1 + \rho^{FT} a_2 < 0$. We will show that in this case when Foreign tariffs are high enough an equilibrium emerges where Home exports both goods 1 and 2. In this equilibrium, the terms of trade are such that the real value of Home's liability increases relative to free trade. A similar argument applies to Foreign when it has a net liability denominated in Home goods. We have the following result:

Proposition 3. *Suppose that Assumption 2 holds.*

(i) ***(Home's Debt Dilemma)*** *If* $a_1 + \rho^{FT} a_2 < 0$ *and* τ^* *satisfies*

$$0 < \frac{(c_1^{FT} + \rho^{FT} c_2^{FT}) - (1/h(\rho^{FT}) + \rho^{FT}) y_2}{-(a_1 + \rho^{FT} a_2)} < \tau^* < \frac{c_1^{FT} + \rho^{FT} c_2^{FT}}{-(a_1 + \rho^{FT} a_2)},$$

then there exists a tariff equilibrium where $c_1 < y_1$, $c_2 < y_2$, $g(c_2/c_1) = \rho^{FT}$ *and*

$$c_1 - c_1^{FT} + \rho^{FT}(c_2 - c_2^{FT}) = \tau^*(a_1 + \rho^{FT} a_2).$$

(ii) ***(Foreign's Debt Dilemma)*** *If* $a_1^* + \rho^{FT} a_2^* < 0$ *and* τ *satisfies*

$$0 < \frac{(c_1^{*FT} + \rho^{FT} c_2^{*FT}) - (1 + \rho^{FT} h(\rho^{FT})) y_1^*}{-(a_1^* + \rho^{FT} a_2^*)} < \tau < \frac{c_1^{*FT} + \rho^{FT} c_2^{*FT}}{-(a_1^* + \rho^{FT} a_2^*)},$$

then there exists a tariff equilibrium where $c_1^* < y_1^*$, $c_2^* < y_2^*$, $g(c_2^*/c_1^*) = \rho^{FT}$ *and*

$$c_1^* - c_1^{*FT} + \rho^{FT}(c_2^* - c_2^{*FT}) = \tau(a_1^* + \rho^{FT} a_2^*).$$

Proof. In Appendix B.3. □

4.3.1 Debt dilemmas when $a_1 = a_2^* = 0$.

We now proceed to explain the intuition behind the result of Proposition 3. To simplify the exposition, we again focus on the cases where $a_1 = a_2^* = 0$.

Derivation of Home's debt dilemma result. Consider part (i) of Proposition 3. We can construct such an equilibrium, where Home exports both goods, via guess and verify. Goods arbitrage when $c_1 < y_1$ and $c_2 < y_2$ implies that $p_1^* = (1 + \tau^*)p_1$ and $p_2^* = (1 + \tau^*)p_2$. Hence, Foreign and Home households face the same relative prices, and those equate their marginal rates of substitution. That is, their indifference curves are tangent and the conjectured equilibrium allocation lies along the diagonal. We depict our candidate equilibrium as point E_1 in Figure 5. Note that this implies that

$$g(c_2/c_1) = g(c_2^*/c_1^*) = g(\overline{y}_2, \overline{y}_1) = \rho^{FT}$$

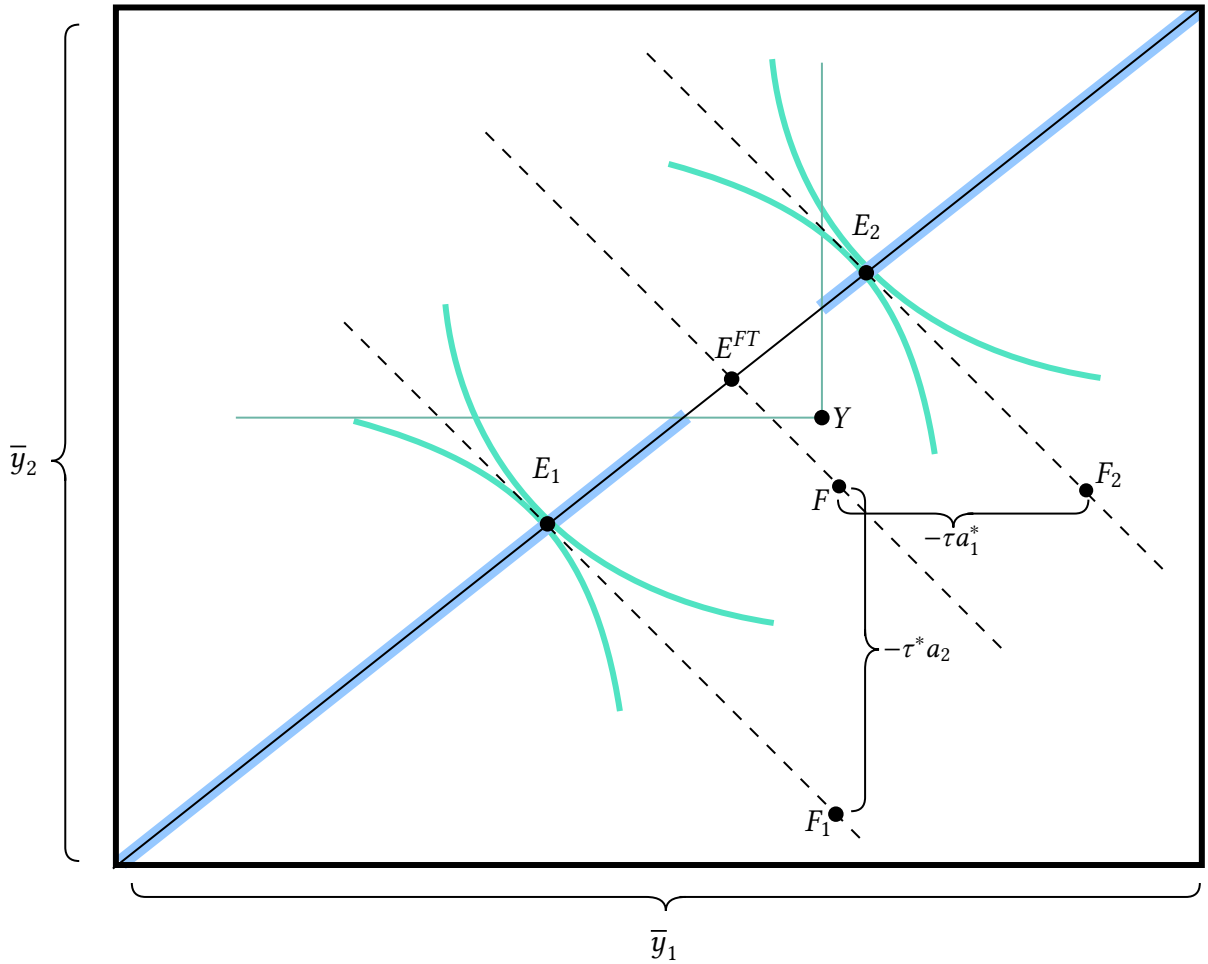
where the last equality follows from the definition of ρ^{FT}. Thus, households face the free trade relative price between goods 1 and 2. But their budget constraints differ from the ones under free

trade. Home's budget constraint in this case is:

$$
\begin{aligned}
& p_1 c_1 + p_2 c_2 = p_1(y_1 - a_1^*) + p_2(y_2 + (1+\tau^*)a_2) \\
& \Rightarrow c_1 + \rho^{FT} c_2 = y_1 - a_1^* + \rho^{FT}(y_2 + (1+\tau^*)a_2) \\
& \Rightarrow c_1 + \rho^{FT} c_2 = y_1 - a_1^* + \rho^{FT}(y_2 + a_2) + \rho^{FT}\tau^* a_2
\end{aligned}
$$

This is the free-trade budget line but translated down by $\tau^* a_2 < 0$. Define $F_1 \equiv (y_1 - a_1^*, y_2 + (1+\tau^*)a_2)$ as the translated financial position of Home in our conjectured equilibrium. Point E_1 in the figure must be at the intersection of the line with slope $-1/\rho^{FT}$ from F_1 and the diagonal.

Figure 5: Tariff War: Multiplicity with Short Selling



Note: Multiplicity of equilibria when $a_2 < 0$ and $a_1^* < 0$. E_1 and E_2 represent two tariff equilibria for the same tariff rates (τ, τ^*). The shaded segments in the contract curve represent allocations that can now be attained as an equilibrium outcome for some tariff combinations of (τ, τ^*).

The last thing to check is that E_1 is indeed in the region such that $0 < c_1 < y_1$ and $0 < c_2 < y_2$

as required for equilibrium. As we have moved along the diagonal from free trade to the left, we automatically have $c_1 < c_1^{FT} < y_1$. The lower bound on τ^* in the proposition ensures that F_1 is far enough to the left of E^{FT} such that $c_2 < y_2$. The upper bound on τ^* ensures that it is still feasible for Home to pay the additional $\tau^* \rho^{FT} a_2$ relative to free trade; that is, that consumption is not forced to be negative to satisfy the budget set.

As E_1 is on the diagonal and strictly below the free trade equilibrium, it is unambiguously the case that Home suffers in welfare terms from the foreign tariff relative to free trade and Foreign gains. As $a_2 < 0$, as τ^* increases, the burden of delivering good 2 to Foreign (in Foreign) increases for Home. It is "as if" Home must pay a liability in an appreciated foreign currency due to the foreign tariff. This immiserization forces Home to export both goods in order to service its debt.

Derivation of Foreign's debt dilemma result. A symmetric result holds for Foreign. This scenario is depicted by point E_2 in Figure 5. The point F_2 represents the position $(y_1-(1+\tau)a_1^*, y_2+a_2)$, which is the free trade position translated to the right by $-\tau a_1^*$. As Home's tariff increases, Foreign's liability in good 1 (delivered in Home) becomes more of a burden, and it must turn to exporting both goods. This pushes the equilibrium toward Foreign's origin, reducing Foreign's welfare and increasing Home's.

Importantly, note that τ plays no role in the condition for E_1 and τ^* plays no role in the condition for E_2. This is because both equilibria feature only one country with imports, and the other country's tariff is not relevant. This implies that conditions for both equilibria can be simultaneously satisfied. That is, either scenario can be an equilibrium *for a fixed set of tariffs* τ *and* τ^*.[16]

It is worth emphasizing that these equilibria are dramatically different. In one of them, Home is poor, and consumes strictly less than its endowment of both goods. In the other one, Foreign is poor, and consumes strictly less than its endowment of both goods. Which country has a "debt dilemma" is thus completely dependent on the coordination of beliefs (i.e., a sunspot). The multiplicity arises from the revaluation of net foreign asset positions due to the direction of trade. Under Assumption 3, these equilibria do not emerge. The reason is that in such cases, higher tariffs in Foreign make the Home wealthier (rather than poorer) when trade flows from Home to Foreign, invalidating the equilibrium conjecture that Home is poor enough. However, when a country is indebted in a "foreign currency" (i.e., a liability in its scarce good to be settled in the other country) it is vulnerable to a self-fulfilling welfare loss if tariffs are appropriately high.

[16]Note that if $a_1 = a_2^* = 0$, $a_2 < 0$, and $a_1^* < 0$, then Proposition 1 applies. If the upper thresholds in Proposition 3 exceed $\underline{\tau}$ and $\underline{\tau}^*$ in Proposition 1, then autarky becomes a third equilibrium for the tariffs high enough.

4.4 Balanced trade equilibria with positive trade

In the previous subsections we have focused on cases where tariffs are high enough so that autarky emerges as an equilibrium. We now show that for intermediate tariff levels, balanced trade is also attainable away from autarky.

To make this point, we restrict attention to the case where $a_1 = a_2^* = 0$ and where $a_1^* > 0$ and $a_2 > 0$. Thus, Lemma 1 applies. As we show in equation (8), with positive trade, the budget constraint for Home becomes:

$$c_1 + \rho c_2 = (y_1 - a_1^*) + \rho(y_2 + a_2)$$

where $\rho = p_2^*/p_1$ are the equilibrium terms-of-trade. Balanced trade occurs when $y_1 - c_1 + \rho(y_2 - c_2) = 0$. From the budget constraint, this requires the net foreign assets positions to be zero:

$$\rho a_2 = a_1^*.$$

Therefore, the terms-of-trade must be as in the autarky case, $\rho = \widehat{\rho} \equiv a_1^*/a_2$.

Let us define

$$\underline{\theta} \equiv \frac{\overline{y}_2 - h(\widehat{\rho})\left(y_1 - \widehat{\rho} y_2^*\right)}{h(\widehat{\rho})\widehat{\rho}\overline{y}_1 + y_1^* - \widehat{\rho} y_2}.$$

As we show below, $\underline{\theta}$ reflects the Foreign consumption ratio, c_2^*/c_1^*, that satisfies balanced trade and the resource conditions when Home's tariff is zero. Any Foreign relative consumption ratio that is weakly greater than this and less than y_2^*/y_1^* can be implemented with an appropriate weakly positive Home tariff. We have the following result:

Proposition 4 (Balanced Trade Equilibria)**.** *Suppose that Assumption 2 holds. And that*

$$a_1 = a_2^* = 0, \quad a_1^* > 0, \quad \text{and} \quad a_2 > 0.$$

Suppose also that $g(y_2^*/y_1^*) < \widehat{\rho} < g(y_2/y_1)$*. Let*

$$T^* \equiv [\mathbb{I}_{\{\underline{\theta}>0\}}(\widehat{\rho}/g(\underline{\theta}) - 1), \widehat{\rho}/g(y_2^*/y_1^*) - 1).$$

Then

(i) T^* *is non-empty.*

(ii) For any $\tau^* \in T^*$ *there exists a* $\tau \geq 0$ *such that there exists a tariff equilibrium with active*

trade such that trade is balanced:

$$y_1 - c_1 = -\widehat{\rho}(y_2 - c_2),$$

and Home exports good 1.

Proof. In Appendix B.4. □

The construction of the set of tariffs consistent with balanced and nonzero trade proceeds as in the characterization of the equilibrium of Figure 4, but in reverse. Specifically, we know *a priori* the required terms-of-trade and seek the associated tariffs. Balanced trade is thus associated with the budget line through F with slope $-1/\widehat{\rho}$, as shown in Figure 6. Recall that $F = (y_1 - a_1^*, y_2 + a_2)$; the fact that the slope of the budget line is $-a_2/a_1^*$ therefore implies it contains the endowment point $Y = (y_1, y_2)$, as well. That is, the value of the net foreign asset positions are zero along this line.

The requirement that $g(y_2^*/y_1^*) < \widehat{\rho} < g(y_2/y_1)$ guarantees that in the absence of tariffs, when facing a hypothetical world price of $\widehat{\rho}$, Home will be an exporter of good 1, while Foreign will be an exporter of good 2. A tariff equilibrium cannot reverse the pattern of trade, so the equilibrium must remain to the north-west of the endowment point.

For a given foreign tariff rate τ^*, we obtain the slope of the ray from Foreign's origin:

$$\frac{c_2^*}{c_1^*} = h\left(\widehat{\rho}/(1+\tau^*)\right).$$

Where this intersects the budget line is the balanced-trade equilibrium candidate associated with τ^*. Given the c_2/c_1 associated with this intersection, we can recover the necessary Home tariff from Home's optimality condition:

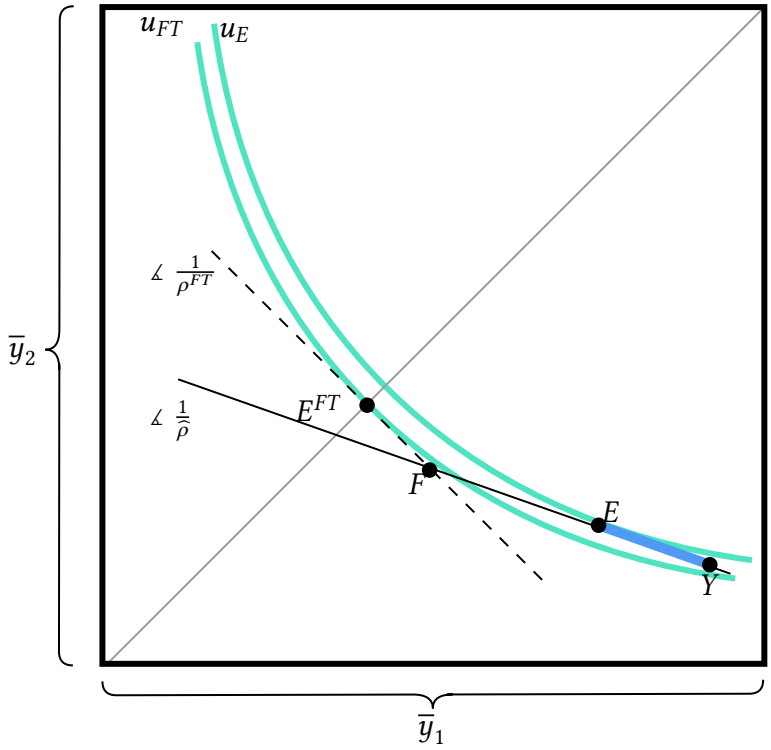
$$g\left(\frac{c_2}{c_1}\right) = (1+\tau)\widehat{\rho}.$$

The condition in Proposition that requires $\tau^* \geq \mathbb{1}_{\{\underline{\theta}>0\}}(\widehat{\rho}/g(\underline{\theta}) - 1)$ ensures that this allocation is consistent with a non-negative Home tariff.

Proceeding in this way, for each candidate τ^* large enough, we obtain a unique τ that balances trade. The proof of Proposition 4 derives this formally. Note that as tariffs increase, the equilibrium allocation approaches the autarkic outcome along the budget line. The condition that $g(y_2^*/y_1^*) < \widehat{\rho}$ guarantees that indeed the autarky limit is reached when $\tau^* = \widehat{\rho}/g(y_2^*/y_1^*) - 1$.

Figure 6 shows a tariff war that achieves balanced trade. It builds on Figure 3. Recall that the

Figure 6: Tariff War Equilibrium With Balanced Trade



Note: Home is a net debtor, and its terms-of-trade worsen.

dashed line is the budget line at free-trade prices, ρ^{FT}. The solid line represents the new budget line at prices $\widehat{\rho} > \rho^{FT}$. Let point E represent the point on this new budget line associated with a zero tariff at Home (which will contain the ray from Home's origin with slope $h(\widehat{\rho})$). At this point, an equilibrium with balanced trade is obtained when Foreign imposes the tariff that equates its marginal rate of substitution at point E's allocation to $\widehat{\rho}/(1+\tau^*)$.

As we increase Home's and Foreign's tariffs, the associated rays from the respective origins rotate away from the diagonal, tracing out the set of balanced trade equilibria (depicted by the segment connecting E to Y). As both tariffs increase, we eventually converge to the autarkic allocation.

Welfare consequences. Figure 6 can also speak to the welfare consequences of a trade war. Recall that free-trade equilibrium occurs where the original budget line with slope $1/\rho^{FT}$ intersects the diagonal, shown by point E^{FT} in the figure. Home's and Foreign's indifference curves are tangent at this point.

There are three effects of the trade war on each country's welfare. Recall that E is associated with zero Home tariff, and hence Home's indifference curve is tangent to the trade-war budget line at that point. As we move from E to Y, Home's welfare decreases. Similarly, at E, the steepness

of Foreign's indifference curve is $(1+\tau^*)/\widehat{\rho} \geq 1/\widehat{\rho}$.[17] Hence, as we move along the budget line with slope $-1/\widehat{\rho}$ from E to Y, Foreign's welfare falls. This reflects that the terms of trade are given by $\widehat{\rho}$, and the distortion in consumption generated by the tariffs is increasing as we move from E towards Y. That is, the movement from E to Y involves increases in τ and τ^* without any further change in the international terms of trade.

As Foreign's indifference curve is tangent to the free trade budget line at E^{FT}, and $\rho^{FT} < \widehat{\rho}$ (for $a_2 > 0$), then any point to the right of F (which includes Autarky) represents a lower welfare for Foreign. As Foreign is a net creditor under free trade, the loss reflects the decline in the value of its net assets (on top of the loss from the distortion to trade), and the associated shrinkage in its budget set. There is a potential positive for Foreign, which is the improvement in its terms of trade. For the points to the right of F, this is dominated by the other two factors. However, it may be the case that point E lies to the left of point F, opening the door to a potential welfare gain relative to free trade. Such gains are supported by the improved terms of trade dominating the asset and trade-distortion effects.

Home is a net debtor under free trade, and it is more intuitive that a trade-war can benefit it. Figure 6 is drawn in such way that even autarky is an improvement for Home when compared to the free trade outcome: as depicted, the free-trade indifference curve for Home u^{FT} lies below the autarky allocation Y. However, such an outcome is not a necessity: it can well be the case that all balanced trade outcomes generate a welfare loss in Home.[18]

One perspective on these welfare consequences is to evaluate the real debt burden faced by a country. Homotheticity of preferences imply that utility is a monotonic transformation of some composite commodity $C(x_1, x_2)$, where C is a constant returns to scale aggregator. This aggregator is common across countries due to symmetric preferences. Let $\mathcal{P}$ be the associated ideal price index, which inherits the constant-returns-to-scale property from C. In particular, Home and Foreign price indices are given by:

$$P \equiv \mathcal{P}(p_1, p_2) = p_1 \mathcal{P}(1, p_2/p_1)$$
$$P^* \equiv \mathcal{P}(p_1^*, p_2^*) = p_1^* \mathcal{P}(1, p_2^*/p_1^*).$$

With this notation, we can revisit the welfare consequences of the trade war in Figure 6, recalling that $\rho = \widehat{\rho} = a_1^*/a_2$. In particular, at point E, the real value of Home's claim on Foreign

[17] Specifically, $-dc_2^*/dc_1^* = 1/g(c_2^*/c_1^*) = (1+\tau^*)/\widehat{\rho} \geq 1/\widehat{\rho}$.

[18] While we do not consider the possibility of default, one could extend the analysis to limited commitment with autarky as the outside option. In that scenario, the respective indifference curves through point Y denote the bounds on welfare. The portion of the line segment $E - Y$ that lies between the two curves contain the set of tariff war equilibria that are consistent with no default.

is

$$\frac{p_2^* a_2}{P} = \frac{p_2^* a_2}{p_1 \mathcal{P}(1, \widehat{\rho})} = \frac{\widehat{\rho} a_2}{\mathcal{P}(1, \widehat{\rho})} = \frac{a_2}{\mathcal{P}(1/\widehat{\rho}, 1)} > \frac{a_2}{\mathcal{P}(1/\rho^{FT}, 1)},$$

where recall that at E we have $\tau = 0$ and $\widehat{\rho} > \rho^{FT}$. Hence, the trade war increases the real value of Home's claim on Foreign relative to free trade. Similarly, the real value of its liability falls:

$$\frac{p_1 a_1^*}{P} = \frac{a_1^*}{\mathcal{P}(1, \widehat{\rho})} < \frac{a_1^*}{\mathcal{P}(1, \rho^{FT})}.$$

Thus, for the debtor country, the trade war raises the real value of its assets and depresses the real value of its liabilities.

This discussion highlights the competing effects operating in the model: the welfare consequences of the valuation effects, the terms of trade movements in relation to free trade, and the adverse effects of tariff distortions as a trade war intensifies.

4.5 Net assets but no offsetting gross positions

We have studied the case where there are gross positions and show how in an equilibrium with high tariffs, the positions rebalance to zero due to changes in international prices. However if Condition (3) is not satisfied, then autarky cannot be an equilibrium outcome in a trade war as there is no strictly positive terms-of-trade that can rebalance the net foreign assets. Then what happens in this case when tariffs are high enough?

Suppose $a_1 = a_2 = a_2^* = 0$ but $a_1^* > 0$. This is a natural case where Home only has an initial liability in units of its abundant good, and maps into a common case in the international macro literature in which the net asset position is pinned down by a gross position in a single asset.

Note that the conditions in Lemma 1 hold, and we know that the pattern of trade cannot be reversed in any tariff equilibrium. So, let us conjecture then that when tariffs are high enough, the equilibrium is one where Home exports a_1^* units of its abundant good to Foreign, and imports nothing. In that case, we know that

$$c_1 = y_1 - a_1^*, \quad c_2 = y_2, \quad c_1^* = y_1^* + a_1^*, \quad \text{and} \quad c_2^* = y_2^*.$$

This consumption allocation satisfies part (i) and part (iii) of Lemma 1. We are just left to check part (ii). For this, the following must hold:

$$g(c_2^*/c_1^*) = \rho/(1+\tau^*) \quad \text{and} \quad g(c_2/c_1)/\rho = \alpha \in [1/(1+\tau^*), 1+\tau].$$

The first term above determines ρ, and to satisfy the second, it suffices that

$$\frac{g(c_2/c_1)}{(1+\tau^*)g(c_2^*/c_1^*)} \in \left[\frac{1}{1+\tau^*}, 1+\tau\right].$$

If $g(c_2/c_1) > g(c_2^*/c_1^*)$, then goods arbitrage for good B will be satisfied for sufficiently high τ. We can obtain a simple condition for $g(c_2/c_1) > g(c_2^*/c_1^*)$. This occurs if:

$$\frac{y_2}{y_1 - a_1^*} < \frac{y_2^*}{y_1^* + a_1^*}. \tag{9}$$

It is possible to show that this inequality follows from Assumptions 2.[19] Hence, the conjecture we have proposed satisfies all of the conditions in Lemma 1, and it is an equilibrium. We summarize this in the following Lemma:

Lemma 2. *Suppose Assumption 2 holds. Suppose that $a_1 = a_2 = a_2^* = 0$ and $a_1^* > 0$. Then the allocation $(c_1 = y_1 - a_1^*, c_2 = y_2, c_1^* = y_1^* + a_1^*, c_2^* = y_2^*)$ constitute an equilibrium for a home tariff τ high enough. In such an equilibrium, the terms-of-trade, ρ, is given by:*

$$\rho = (1+\tau^*)g(c_2^*/c_1^*).$$

Proof. In Appendix B.5. □

A curious reader may ask how Lemma 2 and Proposition 2 (for the case where $a_1 = a_2^* = 0$) can both hold, in particular, if we start from an initial $a_2 > 0$, and $a_1^* > 0$ and then take the limit as $a_2 \to 0$. For any $a_2 > 0$, Proposition 2 tells us that autarky is the unique equilibrium for high enough tariffs. But Lemma 2 tells us that when $a_2 = 0$, the equilibrium is no longer autarky, but rather one where $c_1 < y_1$. The resolution is by noticing that as a_2 goes to zero, the terms-of-trade that sustain autarky in Proposition 2, $\widehat{\rho}$, goes to infinity, and thus, the required tariff rates must also go to infinity.

[19] Basically, the assumption that good 1 is abundant at Home and that in a free trade equilibrium, Home consumes a level of good B higher than its endowment, guarantees that starting at an endowment point $(y_1 - a_1^*, y_2)$, Home most import the Foreign abundant good under free trade. Thus Home remains "abundant" in good 1, which is condition (9).

5 Quantifying valuation effects in a trade war

Our theoretical results apply to the extreme case of full autarky. We now quantify the valuation, terms of trade, and welfare effects of a trade war which falls short of imposing autarky, and show how these effects depend on the nature of the gross portfolio underlying a given net foreign asset position. To do so, we use a dynamic stochastic gravity model based on Fitzgerald (2025). The model has many features typical of the quantitative trade literature (such as trade costs and roundabout production) which allow it to match the pattern of gross trade. But in contrast to much of the quantitative trade literature, we allow productivity to be stochastic and do not impose balanced trade. The presence of uncertainty gives a rationale for gross foreign asset positions which are bigger than net positions. We assume that financial markets are complete and frictionless with respect to productivity shocks. We calibrate the model to the US and a rest of the world (ROW) aggregate in 2023 (including net foreign assets) under the assumption that no trade war is expected. We then use the calibrated model to calculate the impact of an unexpected trade war for four different asset portfolios, each of which is consistent with the same (data-driven) initial net foreign asset position.

5.1 Model

There are two countries, Home (the US) and Foreign (the rest of the world). Foreign variables are denoted by stars. In each period t, the world economy experiences one event, $s_t \in S$. Denote by s^t the history of events from date 0 to date t, $\{s_0, \ldots, s_t\}$. The probability at date 0 that history s^t will be realized at date t is given by $\pi\left(s^t\right)$, with $\sum_{s^t} \pi\left(s^t\right) = 1$. Since s_0 is known at date 0, $\pi\left(s_0\right) = 1$.

There is a representative agent in each country with CRRA expected utility. Date-0 utility in Home is given by:

$$U_0 = \theta C\left(s_0\right)^{1-\frac{1}{\sigma}} + \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) C\left(s^t\right)^{1-\frac{1}{\sigma}}$$

where $C\left(s^t\right)$ is Home consumption of an aggregate good at date t given s^t. The inter-temporal elasticity of substitution (and inverse risk aversion) is σ. The parameter β is the discount factor. The parameter θ_0 is a preference shifter for the initial period, which we use to target the initial period allocation in our calibration. Preferences in Foreign are similar, with $C^*\left(s^t\right)$ denoting Foreign consumption, and θ_0^* the Foreign preference shifter at date 0 (all other parameters are identical in Home and Foreign).

The representative agent in Home supplies exogenous labor L in all dates t and histories s^t. Labor supply in Foreign is L^*. At date t given s^t, Home labor is used to produce $y\left(s^t\right)$ units of

intermediate good 1:

$$y\left(s^t\right) = z\left(s^t\right)\left(L\right)^{1-\mu} M\left(s^t\right)^{\mu}$$

where $z\left(s^t\right)$ is productivity, $M\left(s^t\right)$ is material input, and μ is the materials share in production. Foreign labor is used to produce intermediate good 2 using a similar production function.

The intermediate goods are aggregated into a non-traded final good, X, which is used for consumption and materials:

$$X\left(s^t\right) = \left(x_1\left(s^t\right)^{\frac{\eta-1}{\eta}} + x_2\left(s^t\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

where $x_i\left(s^t\right)$ is absorption of intermediate i in Home at s^t. The aggregator function in Foreign is identical. The Armington elasticity of substitution between the intermediates (η) determines the static trade elasticity ($\eta - 1$). The resource constraint for the final good in Home is:

$$X\left(s^t\right) = C\left(s^t\right) + M\left(s^t\right)$$

There are resource costs of trading intermediates across countries, assumed to be fixed. In order for one unit of Home's intermediate (good 1) to arrive in Foreign, $d^* \geq 1$ units must be shipped from Home. In order for one unit of Foreign's intermediate (good 2) to arrive in Home, $d \geq 1$ units must be shipped from Foreign. The resource constraints for intermediate goods must take account of resource costs of trade:

$$y\left(s^t\right) = x_1\left(s^t\right) + d^* x_1^*\left(s^t\right)$$

$$y^*\left(s^t\right) = d x_2\left(s^t\right) + x_2\left(s^t\right)$$

Markets for labor and intermediate goods are perfectly competitive, so wages are equal to the marginal product of labor, and intermediate goods prices in the country of production are equal to marginal cost. Let $p_t\left(s^t\right)$ be the price of 1 unit of the the Home intermediate (good 1) in Home at date t given s^t in terms of date-t "dollars" (freely traded). Let $1 + \tau$ be the constant (gross) ad valorem tariff imposed by Home on imports from Foreign, and let $1 + \tau^*$ be the constant tariff imposed by Foreign on imports from Home. The price in terms of date-t dollars of 1 unit of the Home intermediate in Foreign at date t given s^t inclusive of tariffs is $\left(1 + \tau^*\right) d^* p_t\left(s^t\right)$. The price of 1 unit of the Foreign intermediate in Home is $\left(1 + \tau\right) d p_t^*\left(s^t\right)$. Denote by $P_t\left(s^t\right)$ the price in date-t dollars of 1 unit of the final good in Home at date t given s^t. Since the market for final

goods is perfectly competitive, $P_t\left(s^t\right)$ is given by:

$$P_t\left(s^t\right) = \left(p_t\left(s^t\right)^{1-\eta} + \left(\left(1+\tau\right) dp_t^*\left(s^t\right)\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

with a corresponding relationship between intermediate goods prices and the price of the final good in Foreign.

Tariff revenue in each country is rebated lump sum to agents in that country. Total Home tariff revenue in terms of date-t dollars at date t given s^t is:

$$T_t\left(s^t\right) = \tau dp_t^*\left(s^t\right) x_2\left(s^t\right)$$

with a similar expression for Foreign tariff revenue.

International Financial Positions. At date 0 agents can trade the full menu of Arrow-Debreu securities, which are claims to each of the intermediate goods, indexed by country of production and country of delivery. All agents face the same prices for these securities. Let $p_0\left(s^t\right)$ be the date-0 price of 1 unit of the Home intermediate good (good 1) in Home at date t given s^t, and let $p_0^*\left(s^t\right)$ be the price of the Foreign intermediate (good 2) in Foreign. Goods market arbitrage implies that the date-0 price of 1 unit of the Home intermediate in Foreign is $\left(1+\tau^*\right) d^* p_0\left(s^t\right)$, while the date-0 price of 1 unit of the Foreign intermediate in Home is $\left(1+\tau\right) dp_0^*\left(s^t\right)$. We denote Home tariff revenue in terms of date-0 dollars at date t given s^t by $T_0\left(s^t\right)$, while Foreign tariff revenue is $T_0^*\left(s^t\right)$.

The two countries may enter into date 0 with a portfolio of assets. Let $\alpha_0\left(s^t\right)$ denote Home's date-0 claim to the Foreign intermediate (good 2) delivered in Foreign at date t given s^t, expressed as a share of $y^*\left(s^t\right)$. In the notation used in the previous sections of the paper, $\alpha_0\left(s^t\right) = a_0\left(s^t\right)/y^*\left(s^t\right)$. Let $\alpha_0^*\left(s^t\right)$ denote Foreign's corresponding claim to a share of the Home intermediate (good 1) delivered in Home. Home's net foreign asset position at date 0 expressed in terms of date-0 dollars is then:

$$b_0 = \sum_{t=0}^{\infty}\sum_{s^t} p_0^*\left(s^t\right)\alpha_0\left(s^t\right) y^*\left(s^t\right) - \sum_{t=0}^{\infty}\sum_{s^t} p_0\left(s^t\right)\alpha_0^*\left(s^t\right) y\left(s^t\right)$$

which implies that the world net financial position at date 0 is zero:

$$b_0\left(s_0\right) + b_0^*\left(s_0\right) = 0$$

The date-0 budget constraint for Home is then:

$$\sum_{t=0}^{\infty}\sum_{s^t}\left(p_0\left(s^t\right)x_1\left(s^t\right)+\tau dp_0^*\left(s^t\right)x_2\left(s^t\right)\right)\leq b_0\left(s_0\right)+\sum_{t=0}^{\infty}\sum_{s^t}\left(p_0\left(s^t\right)y\left(s^t\right)+T_0\left(s^t\right)\right)$$

Equilibrium. A competitive equilibrium is given by sequences of date-0 prices, intermediate goods absorption, materials and consumption such that:

(i) Given prices, tariffs and trade costs, country i chooses absorption of each of the intermediate goods, materials, and consumption for each date and state, to maximize welfare, subject to the country-i final good resource constraint, and the country-i ex-ante budget constraint.

(ii) Firm input demand is optimal given wages and intermediate prices.

(iii) All intermediate goods resource constraints are satisfied, and markets clear.

In the theoretical section, we focused on zero trade, and thus required that autarky prices be well defined. In this case, the Armington assumption (complete specialization) together with the CES aggregator function imply that autarky prices are not well-defined, but this will not be an issue, as we focus on cases that do not lead to a complete trade shutdown.

5.2 Calibration

We treat the year 2023 as our period zero and the US and a composite of the rest of the world (ROW) as our two countries. The approach to calibrating preferences, the Armington parameters, and the productivity processes is similar to Fitzgerald (2025), and is detailed in Appendix C.

To calibrate initial gross asset positions, we use 2023 asset data from the External Wealth of Nations dataset (Lane and Milesi-Ferretti, 2002) and the US Treasury. We compute the gross positions of our two regions as follows. For the equity portion of US gross claims on ROW we sum the foreign direct investment (FDI) and equity positions held by US residents in foreign countries, expressed in 2023 US dollars. For debt claims, we distinguish by currency. Specifically, for the US asset position in debt instruments, we include non-dollar denominated foreign liabilities held by US residents minus US-issued non-dollar denominated liabilities held by the rest of the world, valued in 2023 US dollars. The ROW asset position is constructed in a symmetric manner. Hence, making 2023 nominal US GDP the numeraire, we define

$$a_0 \equiv \frac{\text{US FDI + equity assets + (US non-\$ debt assets} - \text{US non-\$ debt liab)}}{GDP^{US}_{2023}} = 0.83$$

$$a_0^* \equiv \frac{\text{ROW FDI + equity assets + (ROW \$ debt assets} - \text{ROW \$ debt liab)}}{GDP^{US}_{2023}} = 1.58.$$

For the US representative agent's problem, the initial net foreign asset position is simply the difference between US gross foreign assets minus ROW's assets:

$$b_0 = a_0 - a_0^* = -0.75.$$

This corresponds to the BEA net foreign asset position for the US, excluding financial derivatives and gold (which we ignore in the analysis). By market clearing, $b_0^* = -b_0$.

We calibrate to ensure that the initial period allocation in the model matches targeted macro aggregates observed in the 2023 data. Specifically, given b_0 as well as the structural parameters for preferences, production, and the productivity process estimated in Appendix C, we choose values for: period-0 relative productivity in ROW $\{z_0^*\}$ (normalizing $z_0 = 1$); the two trade costs, $\{d, d^*\}$; and the initial preference shifter for the ROW, θ_0^* (setting $\theta_0 = 1$); in order to match: the ratio of ROW nominal GDP to US GDP in 2023; US and ROW 2023 nominal exports as a fraction of US GDP; and the relative price levels in the two regions as measured by 2023 PPP adjustment factors. Details of the computational algorithm are contained in Appendix C. The calibrated parameters are reported in the same appendix.

We should note that the choice of θ_0^* is used to capture the fact that the US runs a large trade deficit in 2023 despite its negative net foreign asset position. In effect, a $\theta_0^* < 1$ induces a transitory "savings glut" in the rest of the world that raises relative absorption in the US in the initial period, a prominent feature of the data. The choice of $\theta_0 = 1$ is not without loss, as alternative values of this parameter would generate alternative inter-temporal prices, and hence shift the inter-temporal budget sets given observed initial asset positions.

Selecting the initial portfolios. As described above, we compute the benchmark equilibrium using the observed net foreign asset positions in 2023 expressed as a share of 2023 US GDP, b_0. In order to perform counterfactual exercises, we need to take a stand on the portfolio that underlies this net financial position. To illustrate the role of the portfolio in determining the impact of a given set of tariff policies, we propose four different candidate portfolios, each of which gives rise to the same b_0 in the free trade equilibrium.

The first, benchmark, portfolio assumes that initial assets are claims to a constant share of the other region's output such that the value of US foreign assets relative to US GDP and the value of US foreign liabilities relative to US GDP equal their values in the data. Hence, we find $\left\{\alpha_0^{bench}, \alpha_0^{*,bench}\right\}$ such that:

$$a_0 = \alpha_0^{bench} \left(\sum_{t=0}^{T} \sum_{s^t} p_0^* \left(s^t\right)^{FT} y^* \left(s^t\right)^{FT} \right)$$

$$a_0^* = \alpha_0^{*,bench} \left(\sum_{t=0}^{T} \sum_{s^t} p_0 \left(s^t\right)^{FT} y \left(s^t\right)^{FT} \right)$$

where the left-hand-side are the data described above, and the right-hand-side prices and quantities are those from the free trade equilibrium.

The second portfolio assumes that the rest of the world holds a claim to a constant share of US output, but the US holds no claim to rest of the world output, i.e. a net-only portfolio expressed in terms of the US intermediate. Hence we find $\alpha_0^{*,netus} > 0$ such that:

$$-b_0 = b_0^* = \alpha_0^{*,netus} \left(\sum_{t=0}^{T} \sum_{s^t} p_0 \left(s^t\right)^{FT} y \left(s^t\right)^{FT} \right)$$

while $\alpha_0^{netus} = 0$. Again, the left-hand-side is given by the data described above, while the right-hand-side prices and quantities are those from the free trade equilibrium.

The third portfolio assumes that the US owes a constant share of rest of the world output, but the rest of the world holds no claim to US output, i.e. a net-only portfolio expressed in terms of the rest of the world intermediate. Hence we find $\alpha_0^{netrow} < 0$ such that:

$$b_0 = -b_0^* = \alpha_0^{netrow} \left(\sum_{t=0}^{T} \sum_{s^t} p_0^* \left(s^t\right)^{FT} y^* \left(s^t\right)^{FT} \right)$$

while $\alpha_0^{*,netrow} = 0$.

The final portfolio (the "contract curve" portfolio) exactly delivers equilibrium absorption at free trade prices. That is, we choose the $\left\{\alpha_0\left(s^t\right), \alpha_0^*\left(s^t\right)\right\}$ in equation XXX such that:

$$dx_2 \left(s^t\right)^{FT} = \alpha_0 \left(s^t\right)^{cc} y^* \left(s^t\right)^{FT}$$

$$d^* x_1^* \left(s^t\right)^{FT} = \alpha_0^* \left(s^t\right)^{cc} y \left(s^t\right)^{FT}$$

5.3 Trade War Counterfactual

We assume that before any trade takes place at date 0, a permanent trade war is unexpectedly announced. The US announces tariff rate τ on all imports, current and future, while ROW announces tariff τ^* on all imports, current and future. We assume that the process for productivity is unaffected by the trade war, and all claims contingent on the history of realizations of productivity are honored despite the fact that tariffs have changed.

In our primary trade-war exercise, we assume that $\tau = \tau^* = 0.25$. We also consider what happens in the case of unilateral US tariffs ($\tau = 0.25$, $\tau^* = 1$), and in the case of unilateral ROW

tariffs ($\tau = 1$, $\tau^* = 0.25$).

Table 3 reports net foreign assets, net exports, the change in the terms of trade and consumption real exchange rate, and the change in welfare (in consumption-equivalent units) relative to free trade in 2023 for each tariff scenario. The first row is the benchmark equilibrium computed with no tariffs; the second panel is the baseline portfolio; the third panel is the net-only portfolio in terms of the US intermediate; the fourth panel is the net-only portfolio in terms of the rest of the world intermediate; the fifth panel is the contract curve portfolio. Within each panel, the first row is the symmetric trade war; the second row is a unilateral US tariff; and the final row is the unilateral ROW tariff.

Table 3: Trade War Scenarios

τ	τ^*	a	a^*	NX/GDP	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
Free Trade								
0	0	0.83	1.58	-2.9%	0	0	0	0
Baseline Portfolio								
25%	25%	0.86	1.55	-2.3%	-1.1%	-0.2%	−0.98 %	−0.18 %
0	25%	0.95	1.56	-2.4%	-11.4%	-11.0%	−0.93 %	0.13%
25%	0	0.75	1.56	-3.3%	12.4%	12.8%	0.20%	−0.21 %
Net-Only Portfolio: US good								
25%	25%	0.00	0.73	-2.1%	-1.5%	-0.2%	−1.17 %	−0.12 %
0	25%	0.00	0.74	-2.0%	-12.1%	-11.6%	−1.48 %	0.29%
25%	0	0.00	0.74	-3.6%	12.9%	13.1%	0.57%	−0.32 %
Net-Only Portfolio: ROW good								
25%	25%	0.00	0.77	-2.0%	-1.8%	-0.9%	−1.36 %	−0.07 %
0	25%	0.00	0.86	-1.5%	-12.7%	-12.1%	−2.03 %	0.45%
25%	0	0.00	0.66	-3.9%	13.5%	13.9%	0.94%	−0.43 %
Contract Curve Portfolio								
25%	25%	3.20	3.79	-2.6%	-0.2%	0.6%	−0.52 %	−0.32 %
0	25%	3.52	3.84	-3.4%	-9.9%	-9.5%	0.37%	−0.26 %
25%	0	2.85	3.84	-2.6%	11.1%	11.7%	−0.65 %	0.04%

Focusing first on the baseline portfolio, we see that in the symmetric trade war ($\tau = \tau^* = 0.25$),

net foreign asset positions shrink. This happens as the price of US output (in the US) relative to the price of ROW output (in ROW) falls modestly below its level under free trade. This raises the value of US claims on ROW relative to ROW claims on US. However the US loses marginally from this trade war in expected utility, as the benefit from the valuation effect is outweighed by losses from the adverse movement in the terms of trade. Examining the case of unilateral tariffs on one side or the other highlights that foreign tariffs reduce US net debt, while domestic tariffs increase it. Trade balances as a share of GDP shrink modestly in the trade war and unilateral ROW tariff scenarios, but rise in the case of unilateral US tariffs.

Examining the case of the net-only portfolio in terms of the US good, we see that the net foreign asset position of the US (as a share of US GDP) is insensitive to tariffs. The terms of trade move more than under the baseline portfolio, but this does not affect the net value of what the US owes relative to its ability to pay. As a result, the US loses more from a symmetric trade war than it does under the baseline portfolio. The US gains more from a unilateral US tariff, and loses more from a unilateral ROW tariff with this portfolio than with the gross portfolio that matches the data.

Examining the case of the net-only portfolio in terms of the ROW good, we see that the net foreign asset position of the US (as a share of US GDP) is sensitive to tariffs. Movements in the terms of trade are larger than under the baseline portfolio, or the net-only portfolio in terms of the US good. Because the adverse movement of the terms of trade in a symmetric trade war increases the value of what the US owes relative to its ability to pay, the US loses even more from a symmetric trade war than in the case of the net-only portfolio in terms of the US good. The US gains more from a unilateral US tariff, and loses more from a unilateral ROW tariff with this portfolio than with the gross portfolio that matches the data, or with the net-only portfolio in terms of the US good.

Finally, in the case of the contract curve portfolio, the terms of trade barely moves in a symmetric trade war, and welfare loss for the US is lower than under the baseline portfolio. However in this case, a unilateral tariff reduces rather than increases the welfare of the country that imposes it.

In Appendix C we report additional results with higher tariffs ($\tau = 2$, or a 200% tariff, and $\tau = 3$, or a 300% tariff). Under the baseline portfolio, high enough foreign tariffs can reverse who is a net debtor and who is a net creditor. But under a net-only portfolio in terms of the debtor country's good, net foreign debt as a share of GDP for the debtor country is insensitive to tariffs. Under a net-only portfolio in terms of the creditor country's good, high creditor tariffs can lead to very large welfare losses for the debtor country.

6 Conclusion

Using a two country model, we first provided a general result where autarky emerges as an equilibrium in a trade war when tariff rates are high enough. The main requirement for this is that each country has gross liabilities on each other of the same sign when evaluated at the autarky domestic prices. Using a two-good model with international asset positions, we have also shown that when countries hold foreign assets denominated in Foreign goods, then for high enough tariffs, autarky is the unique equilibrium. In this case, the terms of trade for the ex-ante debtor country worsen in a trade war. When countries hold more general asset positions, then multiple equilibria can arise for the same tariff rates. In such cases, the direction of trade affects the valuation of foreign assets and liabilities, leading to different possible equilibrium outcomes.

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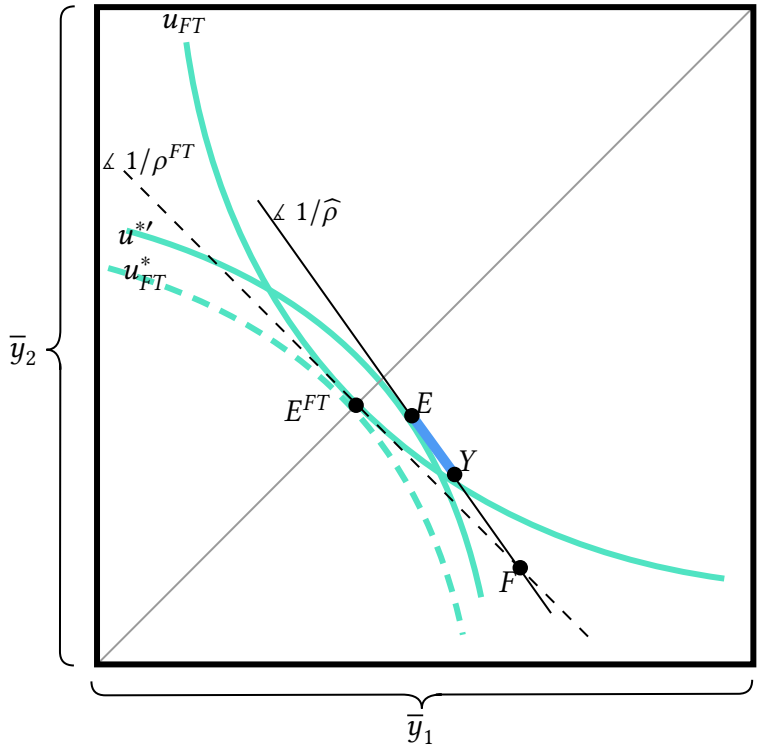
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A Home is a net debtor, and its terms-of-trade improve

Figure 7 shows the case where Home starts as a net debtor, but its terms of trade improve once a trade-war with balanced trade happens. We are drawing this for $a_1 = a_2^* = 0$ and $a_2 < 0$ and $a_1^* < 0$. The point E is now determined by the foreign zero tariff rate, represented by the tangency line of Foreign's indifference curve ($u^{*\prime}$) with the budget line given by $\widehat{\rho}$. The points from E to Y represent points that are attained in a tariff war where trade is balanced. As we move from E to Y, tariffs increase, eventually reaching the autarky equilibrium. Note that in this case, Foreign is always worse off. It started as a creditor, and the tariff war does not improve its terms-of-trade. Whether Home is better of or not depends on the location of the point E and the endowment Y. As drawn in the graph, Home benefits.

Figure 7: Tariff War Equilibrium With Balanced Trade



Note: Home is a net debtor, and its terms-of-trade improve.

B Proofs

B.1 Proof of Lemma 1

Proof. We start by ruling out some cases.

Case 1: $c_1 > y_1$ and $c_2 \leq y_2$. This case implies that

$$\frac{c_2}{c_1} < \frac{y_2}{y_1} < \frac{\overline{y}_2}{\overline{y}_1} < \frac{y_2^*}{y_1^*} < \frac{c_2^*}{c_1^*}.$$

This implies that

$$\frac{p_2}{p_1} = g(c_2/c_1) > g(c_2^*/c_1^*) = \frac{p_2^*}{p_1^*}.$$

Goods arbitrage implies that $p_1 = (1+\tau)p_1^*$. Using this and the above, we have that

$$\frac{p_2}{p_2^*} > (1+\tau).$$

But this is a contradiction of the good 1's arbitrage, which requires $p_2/p_2^* \in [1/(1+\tau^*), 1+\tau]$.

Case 2: $c_1 > y_1$ and $c_2 > y_2$. In this case, $p_1 = (1+\tau)p_1^*$ and $p_2 = (1+\tau)p_2^*$. The households optimality condition are then:

$$g(c_2/c_1) = g(c_2^*/c_1^*) = g(\overline{y}_2/\overline{y}_1) = \rho^{FT}.$$

Consider now the budget constraint for Foreign:

$$p_1^*(c_1^* + a_1) + p_2^*(c_2^* + a_2) = p_1^* y_1^* + p_2^* y_2^* + p_1 a_1^* + p_2 a_2^*$$

where we used that $T^* = 0$ given that $c_1 > y_1$ and $c_2 > y_2$ (and thus Home is importing both goods). Dividing by p_2^*, we get

$$c_1^* + \rho^{FT} c_2^* = y_1^* + \rho^{FT} y_2^* + (a_1^* + \rho^{FT} a_2^*) - (a_1 + \rho^{FT} a_2) + \tau(a_1^* + \rho^{FT} a_2^*)$$

But note that $y_1^* + \rho^{FT} y_2^* + (a_1^* + \rho^{FT} a_2^*) - (a_1 + \rho^{FT} a_2)$ is the total wealth available for consumption under free trade for Foreign.
In this conjectured equilibrium, Foreign has weakly higher wealth as $\tau(a_1^* + \rho^{FT} a_2^*) \geq 0$ by Assumption 3 and that $\tau \geq 0$. Given that Foreign households face the same prices as under free trade but have more resources, they cannot choose a level of consumption c_1^*, c_2^* that is lower than the free trade allocation $c_1^{*,FT}, c_2^{*,FT}$. It follows then that $c_1^* > c_1^{*,FT}$. But $c_1^{*,FT} > y_1^*$, generating a contradiction.

The contradictions in cases 1 and 2 imply that $c_1 \leq y_1$. A symmetric argument, using that $\tau^* \geq 0$ and Assumption 3 implies that $c_2^* \leq y_2^*$.
This shows the necessity of part (i). Letting $\alpha = p_2/p_2^*$ and $\alpha^* = p_1^*/p_1$, we obtain the necessity of part (ii) immediately from households optimality and goods arbitrage.

Finally, for the necessity of part (iii), let's start from Home's budget constraint:

$$p_1(c_1 - y_1) + p_2(c_2 - y_2) + p_1 a_1^* + p_2 a_2^* = p_A^* a_1 + p_2^* a_2 + T$$

where T is

$$T = \tau p_B^* \max\{(c_2 - y_2), 0\}$$

and where we have used that $\max\{(c_1 - y_1), 0\} = 0$ from part (i). Dividing by p_1 we obtain:

$$(c_1 - y_1) + \frac{p_2}{p_1}(c_2 - y_2) + a_1^* + \frac{p_2}{p_1} a_2^* = \frac{p_A^*}{p_1} a_1 + \frac{p_2^*}{p_1} a_2 + \tau \frac{p_B^*}{p_1} \max\{(c_2 - y_2), 0\}$$

Using the definition of ρ, α, and α^*, we can rewrite this as

$$(c_1 - y_1) + \frac{p_2}{p_2^*}\rho(c_2 - y_2) + a_1^* + \alpha\rho a_2^* = \alpha^* a_1 + \rho a_2 + \tau\rho \max\{(c_2 - y_2), 0\}$$

Note that if $c_2 = y_2$, this becomes:

$$(c_1 - y_1) + a_1^* + \alpha\rho a_2^* = \alpha^* a_1 + \rho a_2$$

And if $c_2 > y_2$, we have that $\frac{p_2}{p_2^*} = 1 + \tau$, and the budget constraints becomes:

$$(c_1 - y_1) + \rho(c_2 - y_2) + a_1^* + \alpha\rho a_2^* = \alpha^* a_1 + \rho a_2$$

Hence, a single budget constraint encompases both cases:

$$(c_1 - y_1) + \rho(c_2 - y_2) + a_1^* + \alpha\rho a_2^* = \alpha^* a_1 + \rho a_2$$

A symmetric argument shows the budget constraint for Foreign.
We have shown that a tariff equilibrium must satisfy conditions (i), (ii) and (iii). The reverse is straightforward: we can construct an equilibrium if conditions (i), (ii), and (iii) are satisfied. □

B.2 Proof of Proposition 2

Proof. We already know from Proposition 1 that autarky is an equilibrium when the tariff rates are high enough. The condition on Proposition 2 imply that Lemma 1 applies. Our approach is to rule all possible cases but autarky when tariff rates are high enough.

Case 1: $c_1 < y_1$ and $c_2 = y_2$. We start by noticing that

$$g(c_2^*/c_1^*) = \frac{\rho}{1 + \tau^*}, \quad \text{and} \quad g(c_2/c_1) = \alpha\rho,$$

for $\alpha \in [1/(1+\tau^*), 1+\tau]$. The budget constraint for Home implies that:

$$\begin{aligned} c_1 - y_1 &= (1+\tau^*)a_1 - a_1^* + \rho(a_2 - \alpha a_2^*) \\ &= (1+\tau^*)(a_1 + g(c_2^*/c_1^*)a_2) - (a_1^* + g(c_2/c_1)a_2^*) \end{aligned}$$

Given $c_1 < y_1$ and $c_2 = y_2$, we have that $c_2^*/c_1^* < y_2^*/y_1^*$ and $c_2/c_1 > y_2/y_1$, and thus

$$g(c_2^*/c_1^*) > g(y_2^*/y_1^*) = \rho_{Aut,2}^*, \quad \text{and} \quad g(c_2/c_1) < g(y_2/y_1) = \rho_{Aut,2}.$$

Now from household's optimality we have that

$$g(c_2^*/c_1^*) = \frac{1}{\alpha(1+\tau^*)} g(c_2/c_1).$$

Note that $\alpha \in [1/(1+\tau^*), 1+\tau]$, and thus $\alpha(1+\tau^*) \geq 1$. This implies that

$$c_2^*/c_1^* \geq c_2/c_1 \Rightarrow y_2^*/c_1^* \geq y_2/(\overline{y}_1 - c_1^*) \Rightarrow y_2^*\overline{y}_1 - y_2^* c_1^* \geq y_2 c_1^* \Rightarrow y_2^*\overline{y}_1 \geq \overline{y}_2 c_1^*.$$

and thus given that $c_2 = y_2$,

$$c_2^*/c_1^* \geq \overline{y}_2/\overline{y}_1.$$

A similar argument then shows that

$$c_2/c_1 \leq \overline{y}_2/\overline{y}_1.$$

Taken together, we have then that

$$\rho_{Aut,2}^* = g(y_2^*/y_1^*) < g(c_2^*/c_1^*) \leq g(\overline{y}_2/\overline{y}_1) = \rho^{FT} \leq g(c_2/c_1) < g(y_2/y_1) = \rho_{Aut,2}.$$

Now, if a_2 is positive, we have that

$$a_1 + g(c_2^*/c_1^*)a_2 \geq a_1 + \rho_{Aut,2}^* a_2.$$

If a_2 is negative, then,

$$a_1 + g(c_2^*/c_1^*)a_2 \geq a_1 + \rho_{FT} a_2.$$

Together, this means that

$$a_1 + g(c_2^*/c_1^*)a_2 \geq \min\{a_1 + \rho_{FT} a_2, a_1 + \rho_{Aut,2}^* a_2\} > 0.$$

where the inequality follows from the conditions in the Proposition.
Similarly, we can show that

$$a_1^* + g(c_2/c_1)a_2^* \leq \max\{a_1^* + \rho_{FT} a_2^*, a_1^* + \rho_{Aut,2} a_2^*\}.$$

Returning to the budget constraint, we have then that:

$$c_1 - y_1 \geq (1+\tau^*) \underbrace{\min\{a_1 + \rho_{FT} a_2, a_1 + \rho^*_{Aut,2} a_2\}}_{>0} - \max\{a^*_1 + \rho_{FT} a^*_2, a^*_1 + \rho_{Aut,2} a^*_2\}.$$

It follows that $c_1 > y_1$ for τ^* large enough, generating a contradiction and ruling Case 1 out.

Case 2: $c_1 = y_1$ and $c_2 > y_2$. This case can be ruled out with a symmetric argument to the one used for Case 1, and showing that for τ sufficiently large, it cannot be that $c^*_2 < y^*_2$.

Case 3: $c_1 < y_1$ and $c_2 > y_2$. From optimality and goods arbitrage we have that

$$g(c_2/c_1) = (1+\tau)\rho, \quad \text{and} \quad g(c^*_2/c^*_1) = \rho/(1+\tau^*).$$

It follows then that

$$g(c_2/c_1) = (1+\tau)(1+\tau^*) g(c^*_2/c^*_1).$$

Now, $c_1 = y_1$ and $c_2 > y_2$ imply that

$$c_2/c_1 > y_2/y_1 \Rightarrow g(c_2/c_1) < g(y_2/y_1),$$

and

$$c^*_2/c^*_1 < y^*_2/y^*_1 \Rightarrow g(c^*_2/c^*_1) > g(y^*_2/y^*_1).$$

Taken together we have that

$$g(y_2/y_1) > g(c_2/c_1) = (1+\tau)(1+\tau^*) g(c^*_2/c^*_1) > (1+\tau)(1+\tau^*) g(y^*_2/y^*_1).$$

If that tariffs are sufficiently high so that

$$(1+\tau)(1+\tau^*) > g(y_2/y_1)/g(y^*_2/y^*_1),$$

then we have a contradiction. Thus, this case can also be ruled out for tariff rates high enough.

According to Lemma 1, the only case left is autarky.

□

B.3 Proof of Proposition 3

Proof. We will prove the part (i) of the proposition. Part (ii) follows by symmetry.

Suppose that τ^* satisfies the condition in part (i).

Let Home prices be such that $p_2/p_1 = \rho^{FT}$, and Foreign prices be such that $p_2^*/p_1^* = \rho^{FT}$. Let $p_1 = p_1^*/(1+\tau^*)$ and $p_2 = p_2^*/(1+\tau^*)$. Note that with these prices, goods arbitrage holds.

Let c_1 and c_2 be such that $g(c_2/c_1) = \rho^{FT}$ and Home budget constraint holds with equality. This guarantees that Home's household optimality conditions hold. Home's budget constraint can be rewritten as:

$$c_1 + \rho^{FT} c_2 = y_1 + \rho^{FT} y_2 + p_1^*/p_1(a_1 + \rho^{FT} a_2) - (a_1^* + \rho^{FT} a_2^*).$$

Using the fact that $p_1^*/p_1 = (1+\tau^*)$, we can rewrite this as

$$c_1 + \rho^{FT} c_2 = y_1 + \rho^{FT} y_2 + (1+\tau^*)(a_1 + \rho^{FT} a_2) - (a_1^* + \rho^{FT} a_2^*).$$

And using the fact that $c_1^{FT} + \rho^{FT} c_2^{FT} = y_1 + \rho^{FT} y_2 + (a_1 + \rho^{FT} a_2) - (a_1^* + \rho^{FT} a_2^*)$, we can rewrite this once more as

$$c_1 + \rho^{FT} c_2 = c_1^{FT} + \rho^{FT} c_2^{FT} + \tau^*(a_1 + \rho^{FT} a_2).$$

Note that $\tau^* < (c_1^{FT} + \rho^{FT} c_2^{FT})/(-(a_1 + \rho^{FT} a_2))$ by the condition in the Proposition. This implies that $c_1 + \rho^{FT} c_2 > 0$. Hence c_1 and c_2 are non-negative.

Next, we need to check that $c_1 < y_1$ and $c_2 < y_2$. Note that $c_1 = c_2/h(\rho^{FT})$. Thus, from the budget constraint, we have that

$$c_2(1/h(\rho^{FT}) + \rho^{FT}) = c_1^{FT} + \rho^{FT} c_2^{FT} + \tau^*(a_1 + \rho^{FT} a_2).$$

For $c_2 < y_2$, it suffices to show that:

$$y_2(1/h(\rho^{FT}) + \rho^{FT}) > c_1^{FT} + \rho^{FT} c_2^{FT} + \tau^*(a_1 + \rho^{FT} a_2).$$

But this is the condition in Proposition that imposes $\frac{(c_1^{FT} + \rho^{FT} c_2^{FT}) - (1/h(\rho^{FT}) + \rho^{FT}) y_2}{-(a_1 + \rho^{FT} a_2)} < \tau^*$.

For $c_1 < y_1$, recall from Assumption 2 that $c_1^{FT} < y_1$ and $c_2^{FT} > y_2$. Given that we have shown that $c_2 < y_2$, it follows that $c_2 < c_2^{FT}$. Then, $c_1 < c_1^{FT} < y_1$, by homotheticity.

Using the resource constraints, we can obtain c_1^* and c_2^*. Note that $c_1^* = \overline{y}_1 - c_1 > 0$ and $c_2^* = \overline{y}_2 - c_2 > 0$.

Foreign optimality condition holds as $g(c_2^*/c_1^*) = g(c_2/c_1) = g(\overline{y}_2/\overline{y}_1) = \rho^{FT}$. We need to check that Foreign's budget constraint holds with equality. This follows from Walras law.

We have thus constructed an equilibrium with the properties in part (i) of the Proposition.

A final thing to check for part (i) follows by noticing that

$$y_2(1/h(\rho^{FT}) + \rho^{FT}) < c_2^{FT}(1/h(\rho^{FT}) + \rho^{FT}) = c_1^{FT} + \rho^{FT} c_2^{FT}$$

where the first inequality follows by $c_2^{FT} > y_2$ and the equality from $c_1^{FT} = c_2^{FT}/h(\rho^{FT})$. This implies that indeed

$$0 < \frac{(c_1^{FT} + \rho^{FT} c_2^{FT}) - (1/h(\rho^{FT}) + \rho^{FT}) y_2}{-(a_1 + \rho^{FT} a_2)}.$$

Part (ii) follows by symmetric argument. □

B.4 Proof of Proposition 4

Proof. Let $t_1 = \mathbb{I}\left(\underline{\theta} > 0\right)\left(\frac{\widehat{\rho}}{g(\underline{\theta})} - 1\right)$ and $t_2 = \frac{\widehat{\rho}}{g(y_2^*/y_1^*)} - 1$.
The first thing to check is that $t_1 < t_2$. Note that $t_2 > 0$ by the assumption that $g(y_2^*/y_1^*) < \widehat{\rho}$. So if $\theta \leq 0$, it follows that $t_1 < t_2$. For $\theta > 0$, doing some algebra, we get that

$$\underline{\theta} < \frac{y_2^*}{y_1^*} \Leftrightarrow y_2(y_1^* + \widehat{\rho} y_2^*) < g^{-1}(\widehat{\rho}) y_1 (y_1^* + \widehat{\rho} y_2^*) \Leftrightarrow y_2 < g^{-1}(\widehat{\rho}) y_1$$

which holds by $g(y_2/y_1) > \widehat{\rho}$.
Consider a $\tau^* \in [t_1, t_2)$, and let us construct a tariff equilibrium for some $\tau \geq 0$. Given that there is trade in equilibrium, $g(c_2^*/c_1^*) = \widehat{\rho}/(1+\tau^*)$. Define

$$\theta^* \equiv g^{-1}\left(\frac{\widehat{\rho}}{1+\tau^*}\right) = \frac{c_2^*}{c_1^*} < \frac{y_2^*}{y_1^*}, \tag{10}$$

where the last equality follows from condition (i): $g(y_2^*/y_1^*) < \widehat{\rho}/(1+\tau^*)$. Using the fact that $a_1^* - \widehat{\rho} a_2 = 0$ by definition of $\widehat{\rho}$, the Foreign budget constraint at world prices implies

$$c_1^* = \frac{y_1^* + \widehat{\rho} y_2^*}{1 + \widehat{\rho}\theta^*},$$

where we have used $c_2^* = \theta^* c_1^*$. The proposed equilibrium allocation for Foreign is thus:

$$(c_1^*, c_2^*) = \left(\frac{y_1^* + \widehat{\rho} y_2^*}{1 + \widehat{\rho}\theta^*}, \frac{\theta^*\left(y_1^* + \widehat{\rho} y_2^*\right)}{1 + \widehat{\rho}\theta^*}\right). \tag{11}$$

Note that $c_1^* > 0$ and $c_2^* > 0$. From the resource conditions, we can obtain the implied consumption allocation for Home:

$$\begin{aligned} c_1 &= \overline{y}_1 - c_1^* = \frac{y_1 + \widehat{\rho}(\theta^* \overline{y}_1 - y_2^*)}{1 + \widehat{\rho}\theta^*} = y_1 - \frac{\widehat{\rho}(y_2^* - \theta^* y_A^*)}{1 + \widehat{\rho}\theta^*} \\ c_2 &= \overline{y}_2 - c_2^* = \frac{\overline{y}_2 - \theta^* y_1^* + \widehat{\rho}\theta^* y_2}{1 + \widehat{\rho}\theta^*} = y_2 + \frac{y_B^* - \theta^* y_1^*}{1 + \widehat{\rho}\theta^*}. \end{aligned} \tag{12}$$

As $y_2^* > \theta^* y_1^*$ from (10), we have $c_2 > y_2 > 0$ and $c_1 < y_1$ as required. That is, Home exports good A.

To ensure that $c_1 \geq 0$, we require

$$0 \leq y_1 + \widehat{\rho}(\theta^* \overline{y}_1 - y_2^*) \iff \frac{\widehat{\rho} y_2^* - y_1}{\widehat{\rho}\overline{y}_1} \leq \theta^* = g^{-1}\left(\frac{\widehat{\rho}}{1+\tau^*}\right).$$

If $\widehat{\rho} y_2^* \leq y_1$, then this condition is satisfied immediately. Otherwise, the following condition ensures $c_1 \geq 0$:

$$\tau^* \geq \frac{\widehat{\rho}}{g\left(\frac{\widehat{\rho} y_2^* - y_1}{\widehat{\rho}\overline{y}_1}\right)} - 1. \tag{13}$$

Next, we need to construct a tariff τ such that the allocation in (12) is optimal for Home. Consider a candidate τ given by

$$\tau = \frac{g\left(c_2/c_1\right)}{\widehat{\rho}} - 1,$$

where c_1 and c_2 are given by (12). with this tariff rate, the proposed allocation would be optimal for Home but we need to verify $\tau \geq 0$:

$$\tau \geq 0 \iff (1+\tau)\widehat{\rho} \geq \widehat{\rho} \iff \frac{c_2}{c_1} = g^{-1}((1+\tau)\widehat{\rho}) \leq g^{-1}(\widehat{\rho}).$$

Substituting for c_2/c_1 using (12), we require

$$g^{-1}(\widehat{\rho}) \geq \frac{\overline{y}_2 + \widehat{\rho}\theta^* y_2 - \theta^* y_1^*}{y_1 + \widehat{\rho}(\theta^* \overline{y}_1 - y_2^*)}$$

$$\iff$$

$$\left(g^{-1}(\widehat{\rho})\widehat{\rho}\overline{y}_1 + y_1^* - \widehat{\rho} y_2\right)\theta^* \geq \overline{y}_2 - g^{-1}(\widehat{\rho})\left(y_1 - \widehat{\rho} y_2^*\right). \tag{14}$$

Note that the left hand side coefficient is strictly positive:

$$g^{-1}(\widehat{\rho})\widehat{\rho}\overline{y}_1 + y_1^* - \widehat{\rho} y_2 > (y_2/y_1)\widehat{\rho}\overline{y}_1 + y_1^* - \widehat{\rho} y_2 = \widehat{\rho} y_2(\overline{y}_1/y_1 - 1) + y_1^* > 0$$

where the first inequality follows from $g^{-1}(\widehat{\rho}) > y_2/y_1$ (a premise in the lemma).
Dividing through (14) by this coefficient, we obtain the following condition as requirement for the Home tariff to be non-negative:

$$\theta^* \geq \frac{\overline{y}_2 - g^{-1}(\widehat{\rho})\left(y_1 - \widehat{\rho} y_2^*\right)}{g^{-1}(\widehat{\rho})\widehat{\rho}\overline{y}_1 + y_1^* - \widehat{\rho} y_2} \equiv \underline{\theta}. \tag{15}$$

If $\overline{y}_2 - g^{-1}(\widehat{\rho})\left(y_1 - \widehat{\rho} y_2^*\right) \leq 0$, and thus $\underline{\theta} \leq 0$, then $\theta^* \geq \underline{\theta}$ and the condition holds.

Otherwise, if $\underline{\theta} > 0$, using $g(\theta^*) = \widehat{\rho}/(1+\tau^*)$, the condition is equivalent to

$$\tau^* \geq \frac{\widehat{\rho}}{g(\underline{\theta})} - 1. \tag{16}$$

And thus $\tau^\star \in T^*$ implies that the Home tariff is non-negative.
Finally, let us come back to the restriction we needed for $c_1 \geq 0$, (13). Note that,

$$\frac{\widehat{\rho} y_2^* - y_1}{\widehat{\rho}\overline{y}_1} < \underline{\theta}.$$

Thus if $\underline{\theta} \leq 0$, we have that $\widehat{\rho} y_2^* - y_1 < 0$. And if $\underline{\theta} > 0$, we have that

$$\frac{\widehat{\rho}}{g(\underline{\theta})} - 1 > \frac{\widehat{\rho}}{g\left(\frac{\widehat{\rho} y_2^* - y_1}{\widehat{\rho}\overline{y}_1}\right)} - 1$$

and (13) is implied by (16). And thus $\tau^\star \in T^*$ is sufficient to guarantee $c_1 \geq 0$.
Summarizing this: for any $\tau^* \geq [t_1, t_2)$ condition (15) holds, and thus there exists a $\tau \geq 0$ such that the allocation in (11) and (12) is a balanced trade equilibrium. In this equilibrium, Home exports good A.

□

B.5 Proof of Lemma 2

Proof. The proof follows from Lemma 1, which can be applied as $a_1 = a_2 = a_2^* = 0$ and $a_1^* \geq 0$. We just need to show that inequality (9) follows from Assumptions 2.
Note that under free trade, Home's budget constraint is:

$$c_1^{FT} + \rho^{FT} c_2^{FT} = y_1 + \rho^{FT} y_2 - a_1^*.$$

From the household first order condition under free trade:

$$g(c_2/c_1) = \rho^{FT} \Rightarrow c_2^{FT} = h(\rho^{FT}) c_1^{FT}.$$

Using this in the budget constraint leads to:

$$(1/h(\rho^{FT}) + \rho^{FT}) c_2^{FT} = y_1 - a_1^* + \rho^{FT} y_2$$

Assumption 2 requires that $c_2^{FT} > y_2$. Using this in the above equation, we get that:

$$(1/h(\rho^{FT}) + \rho^{FT}) y_2^{FT} < y_1 - a_1^* + \rho^{FT} y_2 \Rightarrow \frac{y_2}{y_1 - a_1^*} < h(\rho^{FT}) = \frac{\overline{y}_2}{\overline{y}_1}.$$

A symmetric argument for Foreign shows that

$$h(\rho^{FT}) = \frac{\overline{y}_2}{\overline{y}_1} < \frac{y_2^*}{y_1^* + a_1^*}.$$

Taken together these last two imply condition (9). The rest of the proof follows from argument in the body of the text. □

C Quantification appendix

C.1 Calibration of Armington parameters and productivity processes

The model has four parameters, $\{\beta, \mu, \eta, \rho\}$. We set $\beta = 0.96$ (discounting), $\mu = 0.5$ (materials share), $\eta = 4$ (Armington elasticity), and $\rho = 0.99$ (inverse of risk aversion, so utility close to log). We then need to take a stand on labor supply, trade costs, tariffs, productivity and foreign asset portfolios in date 0, as well as the process for labor supply, trade costs, tariffs and productivity going forward from date 0. We also need to calibrate the preference shifter θ_0^*.

We first recover historical productivity and trade costs consistent with the model for the period 1970-2023 using the approach of Fitzgerald (2025).[20] This requires data on labor supply (we use population), national income accounts and expenditure PPPs for the US and ROW aggregate. Given these data and $\{\mu, \eta\}$, model inversion yields time series for $\{z_t, z_t^*\}$ and $\{d_t, d_t^*\}$ for 1970-2023 such that the model exactly matches the data. This procedure is invariant to intertemporal preferences and the nature of the asset market. Details of data construction and the model inversion procedure are below.

Going forward from date 0, we assume that labor supply and iceberg trade costs are fixed at their levels in 2023 (i.e. $L = L_{2023}$, $L^* = L^*_{2023}$, $d = d_{2023}$, $d^* = d^*_{2023}$). In the baseline, we set tariffs permanently equal to zero (i.e. $\tau = \tau^* = 0$) consistent with how we recover productivity and trade costs. We use the 54-year panel of data on productivity for the US and ROW to estimate a process for productivity. We assume the US is the world productivity leader, and that $\Delta \ln z_t$ follows:

$$\Delta \ln z_t = g + \varepsilon_t$$

where

$$\varepsilon_t = \psi \varepsilon_{t-1} + \sigma \nu_t$$

[20] We assume that there are only resource costs of trade, and no tariffs throughout this period. Given data on total tariff revenues for the US, and ROW revenues from tariffs levied on imports from the US for the period 1970-2023, it would be straightforward to allow for tariffs.

and $v_t \sim N(0,1)$. Meanwhile $\Delta \ln z_t^*$ follows:

$$\Delta \ln z_t^* = g + \gamma^* \left(\ln z_{t-1} - \ln z_{t-1}^* \right) + \varepsilon_t^*$$

where

$$\varepsilon_t^* = \psi^* \varepsilon_{t-1}^* + \chi^* \varepsilon_{t-1} + \sigma^* v_t^* + \omega^* v_t$$

and $v_t^* \sim N(0,1)$. We estimate this process by maximum likelihood. Parameter estimates are reported in Table 4. Our estimates imply a similar long-run growth rate for the US and ROW

Table 4: Parameters of estimated productivity process

Parameter		Value	95% CI
US			
US long-run growth rate	g	0.026	[0.020, 0.034]
Autocorr. of deviations from trend, US	ψ	0.58	[0.30, 0.70]
SD of innovations to deviations from trend, US	σ	0.012	[0.009, 0.014]
ROW			
Rate of convergence to US	γ^*	0.0024	[−0.002, 0.008]
Autocorr. of deviations from trend	ψ^*	0.29	[−0.03, 0.44]
Corr. with lagged US deviations from trend	χ^*	0.28	[−0.04, 0.71]
SD of innovations to deviations from trend	σ^*	0.017	[0.012, 0.020]
Corr. with US innovations to deviations from trend	ω^*	0.009	[0.004, 0.015]

Notes: ROW coefficients are estimated by maximum likelihood conditional on coefficients for US. Bootstrapped 95% confidence intervals based on 100 draws with replacement.

(ROW convergence is very slow), but that productivity is more volatile in ROW than in the US. In making use of this estimated process, we condition on $\{\ln \mathbf{z}_0, \Delta \ln \mathbf{z}_0\} = \{\ln \mathbf{z}_{2023}, \Delta \ln \mathbf{z}_{2023}\}$.

C.2 Data used to recover historical productivity and trade costs

We use World Development Indicators (WDI) data on US population to proxy for US labor supply. We construct ROW labor supply by subtracting US population from world population from WDI. We take the value of US GDP expressed in current dollars from WDI. We construct ROW GDP by subtracting US GDP from world GDP. US imports from ROW are given by total US imports in current dollars from WDI, and ROW imports from the US are given by total US exports. We use Penn World Tables (PWT) data on expenditure PPPs to construct a ROW PPP by weighting expenditure PPPs for all countries other than the US by their share in ROW gross expenditure (using the WDI data to construct the weights). For this purpose, we augment the WDI national

income accounts data using OECD national income accounts data, and national income accounts data from the Taiwanese statistical agency as described in Fitzgerald (2025).

C.3 Model inversion to recover historical productivity and trade costs

Gross output in US is (ROW is analogous):

$$OUT_t = \frac{GDP_t}{1-\mu}$$

Expenditure on domestic production for the US is (ROW is analogous):

$$IMS_t = OUT_t - EX_t$$

where IMS_t is expenditure on domestic production and EX_t is total exports. Gross expenditure (absorption) for the US is (ROW is analogous):

$$ABS_t = OUT_t - EX_t - IM_t$$

where IM_t is total imports. The expenditure PPP for ROW is constructed as a weighted average of PPPs in all countries other than the US:

$$PPP_t^* = \sum_{i=1, i\neq US}^{N} \frac{ABS_t^i}{ABS_t^{ROW}} \frac{PPP_t^i}{PPP_t^{US}}$$

where

$$ABS_t^* = \sum_{i=1, i\neq US}^{N} ABS_t^i$$

Then the model tells us:

$$\left(\frac{p_t}{p_t^*}\right)^{1-\eta} = \frac{IMS_t}{ABS_t}\frac{ABS_t^*}{IMS_t^*}\left(\frac{PPP_t}{PPP_t^*}\right)^{1-\eta}$$

and

$$(d_t)^{1-\eta} = \frac{IMP_t}{ABS_t}\frac{ABS_t^*}{IMS_t^*}\left(\frac{PPP_t}{PPP_t^*}\right)^{1-\eta}$$

where IMP_t is US imports from ROW, and

$$\left(d_t^*\right)^{1-\eta} = \frac{EXP_t}{ABS_t}\frac{ABS_t^*}{IMS_t^*}\left(\frac{PPP_t}{PPP_t^*}\right)^{1-\eta}$$

where EXP_t is US exports to ROW.

Time-series variation in the US output price is given by:

$$\frac{p_t}{p_{1970}} = \frac{OUT_t/Realout_t}{OUT_{1970}/Realout_{1970}}$$

Normalize $p_{1970} = 1$. The output prices used to calculate ROW productivity are:

$$\left(\hat{p}_t^*\right)^{1-\eta} = \left(\frac{p_t^*}{p_t}\frac{p_t}{p_{1970}}\right)^{1-\eta}$$

Expenditure prices used to calculate US productivity are (ROW is analogous):

$$\hat{P}_t = \left(\frac{\left(\hat{p}_t\right)^{1-\eta} + \left(d\hat{p}_t^*\right)^{1-\eta}}{\left(\hat{p}_{1970}\right)^{1-\eta} + \left(d\hat{p}_{1970}^*\right)^{1-\eta}}\right)^{\frac{1}{1-\eta}}$$

Real output in the US is then (ROW is analogous):

$$y_t = \frac{OUT_t}{\hat{p}_t}$$

Materials inputs to production in the US are (ROW is analogous):

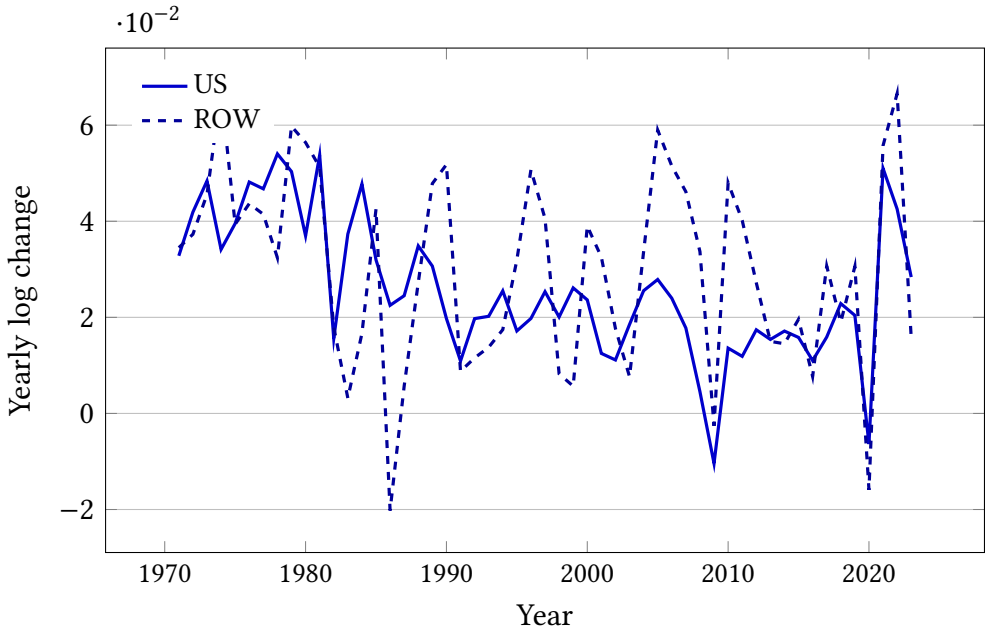
$$M_t = \mu\frac{\hat{p}_t y_t}{\hat{P}_t}$$

and productivity in the US is (ROW is analogous):

$$z_t = \frac{y_t}{\left(L_t\right)^{1-\mu}\left(M_t\right)^{\mu}}$$

Figure 8 plots the log change in the resulting productivity series for the US and ROW.

Figure 8: Log change in productivity



C.4 Construction of portfolio shares

US foreign assets a_0 are constructed as follows:

$$a_0 = \left(\frac{\text{US FDI \& equity assets}}{\text{US GDP}} + \frac{\text{US debt assets}}{\text{US GDP}} \frac{\text{US non-\$ debt assets}}{\text{US debt assets}} - \frac{\text{US debt liab}}{\text{US GDP}} \frac{\text{US non-\$ debt liab}}{\text{US debt liab}} \right) \frac{\text{US GDP}}{\text{ROW GDP}}$$

$$\begin{aligned} a_0 &= (0.8074408 + (0.3495999 \times (1 - 0.784)) - (0.8224106 \times (1 - 0.9405))) \times 0.349927 \\ &= 0.2918 \end{aligned}$$

And ROW foreign assets a_0^* are constructed as follows:

$$a_0^* = \frac{\text{US FDI \& equity liab}}{\text{US GDP}} + \frac{\text{US debt liab}}{\text{US GDP}} \frac{\text{US \$ debt liab}}{\text{US debt liab}} - \frac{\text{US debt assets}}{\text{US GDP}} \frac{\text{US \$ debt assets}}{\text{US debt assets}}$$

$$\begin{aligned} a_0^* &= 1.07999 + (0.8224106 \times 0.9405) - (0.3495999 \times 0.784) \\ &= 1.5794 \end{aligned}$$

US FDI & equity assets and liabilities relative to US GDP and US debt assets and liabilities relative to US GDP are taken from the External Wealth of Nations data. ROW assets are set equal to US liabilities, and ROW liabilities are set equal to US assets. The dollar share of US debt assets is

taken from the US Treasury's International Capital (TIC) annual report on U.S. Portfolio Holdings of Foreign Securities (Table 14 in report dated December 29, 2023). The dollar share of US debt liabilities is from the US Treasury's International Capital (TIC) annual report on Foreign Portfolio Holdings of U.S. Securities (constructed from Table 12 in report dated June 30, 2023). The ratio of US to ROW GDP used to rescale US assets as shares of ROW GDP is from WDI.

C.5 Computation of competitive equilibrium without tariffs

Set $\theta_0 = 1$. Fix a value for θ_0^*. Then guess a vector of output prices (i.e. $p_0\left(s^t\right)_0^{FT}$ and $p_0^*\left(s^t\right)_0^{FT}$ for all t, s^t), normalizing one value to 1. Calculate consumption prices using:

$$P_0\left(s^t\right)_0^{FT} = \left(\left(p_0\left(s^t\right)_0^{FT}\right)^{1-\eta} + \left(dp_0^*\left(s^t\right)_0^{FT}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

(and similarly for $P_0^*\left(s^t\right)_0^{FT}$). Then output is

$$y\left(s^t\right)_0^{FT} = L\left(z\left(s^t\right)\right)^{\frac{1}{1-\mu}}\left(\frac{\mu p_0\left(s^t\right)_0^{FT}}{P_0\left(s^t\right)_0^{FT}}\right)^{\frac{\mu}{1-\mu}}$$

and net foreign assets are

$$b_0\left(s_0\right)_0^{FT} = \left(\chi_0 - \chi_0^*\right)(1-\mu)\, p_0\left(s_0\right)_0^{FT} y\left(s_0\right)_0^{FT}$$

Define

$$\Pi_0\left(s_0\right)_0^{FT} =$$

$$\left(\frac{1-\beta}{\theta_0(1-\beta)+\beta\left(1-\beta^T\right)}\right)^{\frac{\rho}{1-\rho}}\left[\begin{array}{c}(\theta_0)^\rho\left(P_0\left(s_0\right)_0^{FT}\right)^{1-\rho} + \\ \sum_{t=1}^T\sum_{s^t}\left(\beta^t\pi\left(s^t\right)\right)^\rho\left(P_0\left(s^t\right)_0^{FT}\right)^{1-\rho}\end{array}\right]^{\frac{1}{1-\rho}}$$

Then

$$M\left(s^t\right)_0^{FT} = \mu^{\frac{1}{1-\mu}}\left(\frac{p_0\left(s^t\right)_0^{FT} z\left(s^t\right)}{P_0\left(s^t\right)_0^{FT}}\right)^{\frac{1}{1-\mu}} L^i$$

and

$$C\left(s_0\right)_0^{FT} =$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{FT}\theta_0}{P_0\left(s_0\right)_0^{FT}}\frac{1-\beta}{\theta_0(1-\beta)+\beta\left(1-\beta^T\right)}\right)^\rho\left(\frac{b_0\left(s_0\right)_0^{FT} + \sum_{t=0}^T\sum_{s^t}\left((1-\mu)\, p_0\left(s^t\right)_0^{FT} y\left(s^t\right)_0^{FT}\right)}{\Pi_0\left(s_0\right)_0^{FT}}\right)$$

and for $t \geq 1$

$$C\left(s^t\right)_0^{FT} =$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{FT}\beta^t\pi_{0,t}\left(s^t\right)}{P_0\left(s^t\right)_0^{FT}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\rho}\left(\frac{b_0\left(s_0\right)_0^{FT}+\sum_{t=0}^{T}\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{FT}y\left(s^t\right)_0^{FT}\right)}{\Pi_0\left(s_0\right)_0^{FT}}\right)$$

so

$$X\left(s^t\right)_0^{FT} = C\left(s^t\right)_0^{FT} + M\left(s^t\right)_0^{FT}$$

$$x_1\left(s^t\right)_0^{FT} = \left(\frac{P_0\left(s^t\right)_0^{FT}}{p_0\left(s^t\right)_0^{FT}}\right)^{\eta} X\left(s^t\right)_0^{FT}$$

and

$$x_2\left(s^t\right)_0^{FT} = \left(\frac{P_0\left(s^t\right)_0^{FT}}{dp_0^*\left(s^t\right)_0^{FT}}\right)^{\eta} X\left(s^t\right)_0^{FT}$$

and excess demand for the US good is given by (ROW is analogous):

$$ED\left(s^t\right)_0^{FT} = x_1\left(s^t\right)_0^{FT} + d^*x_1^*\left(s^t\right)_0^{FT} - y\left(s^t\right)_0^{FT}$$

Use excess demand to inform a new guess of output prices, i.e. $p_0\left(s^t\right)_1^{FT}$: If excess demand is positive, increase the output price, if it is negative, reduce the output price. Iterate to convergence. This gives market-clearing output prices conditional on the value of θ_0^*.

Next, solve for the value of θ_0^* that exactly matches observed data at date 0. For a guess of θ_0^*, check the date-0 US ratio of net exports to GDP:

$$\frac{NX_0\left(s_0\right)^{FT}}{GDP_0\left(s_0\right)^{FT}} = \frac{p_0\left(s_0\right)^{FT}y\left(s_0\right)^{FT} - P_0\left(s_0\right)^{FT}X_0^{i,FT}}{\left(1-\mu\right)p_0\left(s_0\right)^{FT}y\left(s_0\right)^{FT}} = \kappa_0$$

If κ_0 is greater than the target, increase θ_0^*. If κ_0 is less than the target, reduce θ_0^*. Iterate to convergence.

C.6 Computation of trade war equilibrium

Guess a vector of output prices (i.e. $p_0\left(s^t\right)_0^{TW}$ and $p_0^*\left(s^t\right)_0^{TW}$ for all t, s^t), normalizing one value to 1. Calculate consumption prices using:

$$P_0\left(s^t\right)_0^{TW} = \left(\left(p_0\left(s^t\right)_0^{TW}\right)^{1-\eta} + \left(\left(1+\tau\right)dp_0^*\left(s^t\right)_0^{TW}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

(and similarly for $P_0^* \left(s^t\right)_0^{TW}$). Then output is:

$$y\left(s^t\right)_0^{TW} = L\left(z\left(s^t\right)\right)^{\frac{1}{1-\mu}} \left(\frac{\mu p_0\left(s^t\right)_0^{TW}}{P_0\left(s^t\right)_0^{TW}}\right)^{\frac{\mu}{1-\mu}}$$

and

$$M\left(s^t\right)_0^{TW} = \mu^{\frac{1}{1-\mu}} \left(\frac{p_0\left(s^t\right)_0^{TW} z\left(s^t\right)}{P_0\left(s^t\right)_0^{TW}}\right)^{\frac{1}{1-\mu}} L^i$$

and net foreign assets are:

$$b_0\left(s_0\right)_0^{TW} = \sum_{t=0}^{\infty} \sum_{s^t} p_0^*\left(s^t\right)_0^{TW} \alpha_0\left(s^t\right) y^*\left(s^t\right)_0^{TW} - \sum_{t=0}^{\infty} \sum_{s^t} p_0\left(s^t\right)_0^{TW} \alpha_0^*\left(s^t\right) y\left(s^t\right)_0^{TW}$$

given the portfolio $\left\{\alpha_0\left(s^t\right), \alpha_0^*\left(s^t\right)\right\}$. Now define

$$\Pi_0\left(s_0\right)_0^{TW} =$$

$$\left(\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\frac{\rho}{1-\rho}} \left[\begin{array}{c} \left(\theta_0\right)^{\rho}\left(P_0\left(s_0\right)_0^{TW}\right)^{1-\rho} + \\ \sum_{t=1}^{T}\sum_{s^t}\left(\beta^t \pi\left(s^t\right)\right)^{\rho}\left(P_0\left(s^t\right)_0^{TW}\right)^{1-\rho} \end{array}\right]^{\frac{1}{1-\rho}}$$

Guess that $C\left(s^t\right)_0^{TW} = 0$ for all t, s^t, i (i.e. demand only comes from materials), and similarly for ROW. Use this guess to calculate tariff revenue in the US (and similarly for $T_0^*\left(s^t\right)_{00}^{TW}$):

$$T_0\left(s^t\right)_{00}^{TW} = \frac{\left(C\left(s^t\right)_{00}^{TW} + M\left(s^t\right)_0^{TW}\right)}{\left(P_0\left(s^t\right)_0^{TW}\right)^{-\eta}} \left(\tau\left(\left(1+\tau\right) dp_0^*\left(s^t\right)_0^{TW}\right)^{1-\eta}\right)$$

Then given the tariff revenue, calculate updated consumption:

$$C\left(s_0\right)_{01}^{TW} =$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{TW}\theta_0}{P_0\left(s_0\right)_0^{TW}} \frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\rho} \left(\frac{b_0\left(s_0\right)_0^{TW} + \sum_{t=1}^{\infty}\sum_{s^t}\left(\left(1-\mu\right) p_0\left(s^t\right)_0^{TW} y\left(s^t\right)_0^{TW} + T_0\left(s^t\right)_{00}^{TW}\right)}{\Pi_0\left(s_0\right)_0^{TW}}\right)$$

and for $t \geq 1$

$$C\left(s^t\right)_{01}^{FT} =$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{TW}}{P_0\left(s^t\right)_0^{TW}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\rho}\left(\frac{b_0\left(s_0\right)_0^{TW}+\sum_{t=1}^{\infty}\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{TW}y\left(s^t\right)_0^{TW}+T_0\left(s^t\right)_{00}^{TW}\right)}{\Pi_0\left(s_0\right)_0^{TW}}\right)$$

and similarly for ROW. Iterate until $C\left(s^t\right)_{0n}^{TW}=C\left(s^t\right)_{0n-1}^{TW}=C\left(s^t\right)_0^{TW}$. Then:

$$X\left(s^t\right)_0^{TW}=C\left(s^t\right)_0^{TW}+M\left(s^t\right)_0^{TW}$$

and

$$x_1\left(s^t\right)_0^{TW}=\left(\frac{P_0\left(s^t\right)_0^{TW}}{p_0\left(s^t\right)_0^{TW}}\right)^{\eta}X\left(s^t\right)_0^{TW}$$

and

$$x_2\left(s^t\right)_0^{TW}=\left(\frac{P_0\left(s^t\right)_0^{TW}}{\left(1+\tau\right)dp_0^*\left(s^t\right)_0^{TW}}\right)^{\eta}X\left(s^t\right)_0^{TW}$$

with similar expressions for ROW. Excess demand for the US good is given by (ROW is analogous):

$$ED\left(s^t\right)_0^{TW}=x_1\left(s^t\right)_0^{TW}+d^*x_1^*\left(s^t\right)_0^{TW}-y\left(s^t\right)_0^{TW}$$

Use excess demand to inform a new guess of output prices, i.e. $p_0\left(s^t\right)_1^{TW}$: If excess demand is positive, increase the output price, if it is negative, reduce the output price. Iterate to convergence.

C.7 Additional results

Tables 5 and 6 contain the results for counterfactual tariffs set at 200% and 300%, respectively.

Table 5: Net foreign assets, terms of trade and welfare: tariffs of 200%

τ	τ	a	a^*	NX/GDP	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
Free trade								
0%	0%	-0.76	0.27	-2.9%	0%	0%	0%	0%
Baseline Portfolio								
200%	200%	1.22	1.50	0.4%	-30%	-28%	-2.66%	-1.55%
200%	0%	1.49	1.51	-1.5%	-43%	-42%	-0.97%	-1.26%
0%	200%	0.58	1.52	0.7%	48%	50%	-3.9%	-0.26%
Net-only Portfolio: US good								
200%	200%	0.00	0.71	1.4%	-45%	-44%	-4.47%	-0.66%
200%	0%	0.00	0.72	0.7%	-51%	-50%	-4.02%	-0.22%
0%	200%	0.00	0.72	-0.2%	57%	59%	-2.75%	-0.66%
Net-only Portfolio: ROW good								
200%	200%	0.00	1.79	4.2%	-57%	-56%	-8.65%	1.06%
200%	0%	0.00	1.87	4.1%	-59%	-58%	-8.66%	1.38%
0%	200%	0.00	0.45	-1.3%	70%	72%	-1.24%	-1.17%
Contract Curve Portfolio								
200%	200%	3.55	3.62	-0.2%	-9%	-7%	-1.66%	-2.06%
200%	0%	4.51	3.68	-4.7%	-29%	-28%	3.17%	-2.52%
0%	200%	2.38	3.74	2.0%	36%	38%	-5.86%	0.42%

Table 6: Net foreign assets, terms of trade and welfare: tariffs of 300%

τ	τ	a	a^*	NX/GDP	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
Free trade								
0%	0%	-0.76	0.27	-2.9%	0%	0%	0%	0%
Baseline Portfolio								
300%	300%	1.36	1.49	0.2%	-36%	-35%	-2.22%	-1.93%
300%	0%	1.65	1.50	-1.5%	-48%	-47%	-0.52%	-1.79%
0%	300%	0.57	1.52	1.4%	51%	54%	-4.72%	-0.24%
Net-only Portfolio: US good								
300%	300%	0.00	0.71	1.5%	-58%	-58%	-4.54%	-0.66%
300%	0%	0.00	0.72	1.1%	-61%	-60%	-4.29%	-0.41%
0%	300%	0.00	0.71	0.7%	62%	65%	-3.62%	-0.67%
Net-only Portfolio: ROW good								
300%	300%	0.00	2.68	6.6%	-71%	-71%	-12%	2.24%
300%	0%	0.00	2.71	6.6%	-72%	-71%	-12%	2.35%
0%	300%	0.00	0.42	-0.4%	82%	85%	-2.09%	-1.25%
Contract Curve Portfolio								
300%	300%	3.59	3.61	-0.1%	-10%	-8%	-1.67%	-2.28%
300%	0%	4.60	3.66	-4.8%	-30%	-30%	3.5%	-2.9%
0%	300%	2.35	3.73	2.6%	38%	40%	-6.55%	0.45%