Growing Together and Apart: Scale Economies and Labor Specialization in Global Value Chains *

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Abstract

This paper analyzes how division of labor along global value chains generates scale economies and productivity growth. We construct a novel dataset linking the universe of cross-border transactions between Danish and Swedish firms to administrative worker-firm data from both countries, providing detailed occupation information on every buyer and supplier. This allows us to construct a novel measure of occupational similarity between buyers and sellers in global value chains. Exploiting exogenous shifts in demand, we show that positive demand shocks lead to higher measured productivity while occupational similarity with foreign suppliers declines, consistent with greater specialization. We then propose and structurally estimate a parsimonious model with task specialization and outsourcing using our novel data. Our quantification and counterfactual show that scale and labor specialization with foreign suppliers are a quantitatively important economic mechanisms for generating higher labor productivity and lower input costs.

JEL: F10, F12, F16, D24, L11, L25
Keywords: international supply chain, outsourcing, scale economies, labor specialization, labor productivity, production networks, relationship specificity.

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1 Introduction

> "The greatest improvement in the productive powers of labour [...] seem to have been the effects of the division of labour." (Wealth of Nations, Chapter 1, 1776)

The division of labor - breaking down work into tasks so that workers can specialize in a subset of tasks - is believed by many economists to be key to economic progress, by providing efficiency gains when producing goods and services. In today's economy, this division occurs increasingly between firms as production is fragmented across borders in global value chains (GVC). Yet, the empirical evidence on the magnitude of these gains remains limited. In particular, we lack evidence on how the division of labor across firms in GVCs affect scale economies, labor productivity and input prices.

In this paper we address these gaps in the literature and make three main contributions. *First*, we assemble a new comprehensive dataset linking transactions between Danish and Swedish firms with matched employer-employee data. This dataset enables us to construct a novel measure of the division of labor across firms, the occupational similarity between buyers and suppliers, a dimension previously unobserved in GVC data. *Second*, we provide novel empirical evidence that positive exogenous demand shocks increase the buyers' scale and measured productivity while reducing the occupational similarity with suppliers, consistent with greater specialization in the GVC. *Third*, we propose a parsimonious task-trade model and structurally estimate it using simulated method of moments. This allows us to quantify the importance of labor specialization for scale economies, measured productivity and input prices. In the rest of this section we elaborate on each of these contributions.

One of the reasons for the lack of progress is that the division of labor is typically unobserved in standard data on firms and GVCs. In this paper, we make progress by building a comprehensive register-based dataset covering both Denmark and Sweden. Our key data source is VAT records with all goods and services transactions between Danish and Swedish firms. Because each transaction includes VAT identifiers of the buyer and supplier firms, we can merge the VAT data with other register data on firms and workers. The final dataset spans 2012-2018 and combines: (1) the universe of firm-to-firm trade flows between Denmark and Sweden, (2) detailed employer-employee data from both Denmark and Sweden on all workers and their characteristics, including narrowly defined occupations, and (3) detailed firm level data for both countries on firm characteristics, such as sales, costs, and employment.[1] This novel merged *and* linked data enables us to measure for the first time, the occupation similarity between buyers and their (foreign) suppliers, providing a direct measure of specialization across firms in the GVC. Specifically, for each buyer firm and its foreign suppliers, we measure occupation similarity by computing a cosine similarity index ranging from 0 (completely different occupational structure) to 1 (identical occupational structure).

[1] Relative to the previous studies that have used detailed employer-employee and firm-level data from a single country (e.g., Hummels et al., 2014) our data covers the workers and firms on both sides of international trade. Relative to those that have matched customs data from importing and exporting countries (e.g., Blum et al., 2010), our data provides detailed information on all the workers that buyer and seller firms employ.

The second contribution of this paper is to provide new evidence on how demand shocks shape the division of labor between buyers and suppliers in the value chain. Because our data is a short panel, we adopt the long-differencing specification and examine the change from the base period of 2012-13 to the end period of 2017-18. An empirical challenge is to find an exogenous demand shifter. Here, we turn to the world import demand (WID) shocks commonly used in the trade literature (Hummels et al., 2014). Specifically, in the base period, we observe the portfolio of products that buyer firm i exports to third countries, outside of Denmark and Sweden, as well as the shares of these products in i's third-country export. The demand shock for i is then the weighted average of the change in global imports of these products, from all other sources than Sweden and Denmark, where the weights are the base-period export shares.

Our empirical analysis shows that a positive demand shock raises the buyer's sales, employment and labor productivity, while reducing the value added to sales ratio and the labor share. As buyers expand, they also increase imports from their foreign suppliers in Denmark and Sweden. Importantly, using our unique data, we find that occupation similarity *declines* between the Swedish (Danish) buyers and their Danish (Swedish) suppliers. Therefore, buyers and their foreign suppliers grow together, in the sense that they trade more, but also grow apart - in the sense that they increasingly specialize in different tasks along the value chain.

The third contribution of the paper is to propose and estimate a parsimonious model of labor specialization and outsourcing (following e.g. Grossman and Rossi-Hansberg 2008, Costinot 2009). The buyer firm produces output by combining a unit continuum of tasks, each of which can either be produced in-house using labor or outsourced to a foreign supplier. Task productivity varies across tasks and firms, creating potential gains from specialization when in-house production is relatively inefficient (high unit costs). But outsourcing a task is also costly, due to e.g. transaction costs, monitoring costs or costs associated with reorganizing production. We model this as a task-specific fixed cost. The buyer firms will then optimally outsource tasks where the efficiency gains from doing so are relatively large. In this environment, a positive demand shock will lead to: (i) more specialization, i.e., more tasks being outsourced, and less task similarity between the buyer and its suppliers; (ii) a lower value added to sales ratio and labor share; and (iii) higher labor productivity (value added per worker). The economic intuition is straightforward: more sales and profits imply that the marginal gain from specialization increases, while the marginal transaction cost remains fixed. Importantly, the model predictions of (i)-(iii) are in line with the empirical results described above.

For the quantitative analysis, we estimate the parameters of the model using simulated method of moments (SMM). The quantification allows us to assess the importance of labor specialization along the supply chain in generating productivity growth and scale economies. The estimation results show that costs related to outsourcing are economically important; total fixed costs of outsourcing is about 31,400 USD, or 1.1 percent of the average firm's total labor cost. Furthermore, scale economies due to specialization along the supply chain are substantial. Specifically, the elasticity of output quantity to labor, or the returns to scale coefficient, is 1.05, implying that doubling employment yields 5% higher output per worker. We also find that labor

productivity for the 90th percentile firm is 30 percent higher than for the 10th percentile firm, confirming that specialization gives rise to substantial dispersion in labor productivity (in the model, labor productivity is equalized across firms in the absence of outsourcing).

Next, we ask whether the estimated model can explain our reduced-form findings. We simulate the WID shocks in the model and estimate the same regressions described above on simulated data. We find that all the simulated coefficient estimates have the same signs as the data coefficient estimates, and their magnitudes are 1/10th to 1/5th relative to the actual coefficient estimates.[2]

Finally, as a counterfactual, we simulate the scenario of prohibitively expensive outsourcing, in which the buyer firm would produce all tasks in-house and there would be no labor specialization across firms. We find that for the average buyer firm, labor productivity would be 0.29 log points or 34% lower in the counterfactual, and the input price index 0.096 log points or 10% higher. These results suggest that outsourcing and labor specialization significantly boost the firms' labor productivity and reduce their input costs, complementing previous work on the economic significance of supplier links in the GVC literature.[3]

This paper relates to the following distinct bodies of work. The first is the literature on global value chains (e.g., Antràs and Chor, 2013; Antràs et al., 2017; Baldwin and Venables, 2010; Blaum et al., 2018; Costinot et al., 2013; Grossman et al., 2023), which emphasizes the mechanism of supplier links.[4] In comparison, our model and quantification highlight the reorganization of tasks between buyer firms and their foreign suppliers and how this interacts with buyer firm scale.

Our stylized model draws from the (mostly) theoretical work on offshoring and task trade (e.g., Feenstra and Hanson, 1996; Grossman and Rossi-Hansberg, 2008), as well as labor specialization and trade (e.g., Becker et al., 2025; Chaney and Ossa, 2013; Costinot, 2009). We parameterize the distributions of task productivity, estimate these parameters using our unique data, and quantify the economic significance of the mechanism of labor specialization across firms linked in the GVC.

The third is the literature that shows that foreign sourcing affects measured productivity and marginal costs (e.g., Amiti and Konings, 2007; De Loecker et al., 2016; Goldberg et al., 2010; Gopinath and Neiman, 2014; Halpern et al., 2015), which emphasizes the mechanism through the number of imported products. Relative to this literature, we focus on task reorganization in GVCs.

More broadly, our study also relates to the empirical literature on the division of labor (e.g., Atalay et al., 2024; Baumgardner, 1988; Becker et al., 2025; Boehm and Oberfield, 2022; Duranton and Jayet, 2011; Garicano and Hubbard, 2009; Tian, 2021),[5] the literature on domestic

[2] In the data, there are likely other sources of scale economies operating at the same time, e.g. division of labor within the firm itself or learning-by-doing.

[3] For example, Antràs et al. (2017) shows that if all foreign supplier links were lost, the average U.S. buyer firm would experience a 9% increase in marginal cost.

[4] Antràs and Chor (2022) provide a recent survey.

[5] For example, Boehm and Oberfield (2022) uses data on plants' input and output mix to explore the relationship between fragmentation of production and growth, by focusing on plants' vertical spans of production.

production networks (e.g., Bernard et al., 2018, 2019, 2022; Demir et al., 2024; Dhyne et al., 2021, 2025; Lim, 2018)[6], and the literature on search and matching in international trade (e.g., Benguria, 2021; Blum et al., 2010; Eaton et al., 2025).

The rest of the paper is organized as follows. Section 2 presents the data, key data patterns and our measures of labor specialization between buyer firms and their foreign suppliers. Section 3 constructs the WID variables and shows how the buyer firm responds to exogenous demand shocks. Section 4 introduces the quantitative model, Section 5 estimates the model, and Section 6 provides model validation. Section 7 concludes.

2 Data

In this section, we describe the data and the construction of our main sample. We leave additional details to the Data Appendix. We also highlight the main data patterns and clarify our measures for labor specialization.

2.1 Data Sources

The cornerstone of our data is the universe of cross-border firm-to-firm relationships. All exporters in Denmark and Sweden are required to submit an EU sales form every month (for goods or joint export of goods and services) or quarter (for services) for exports to firms in other EU countries.[7] On that form, the exporter reports its own VAT registration number, its customer's VAT registration number, and the total value of exported goods, services or triangulated trade.[8] If the exporter sells to multiple customers in a month or quarter, it submits the VAT numbers of the various customers as different lines, one per customer. Before submitting the form, the exporter should verify the validity of the VAT numbers by using a VIES VAT number validation service. When the form is submitted, the format of the VAT numbers is checked to ensure it matches the structure in the destination. If the format of the VAT number is incorrect, an error will be shown asking for additional information. The validity of the numbers are, in addition, later checked separately in the country of the customer. If the number is invalid (e.g. the customer is not registered for VAT) this information is reported back to the local tax authorities in Denmark and Sweden that contacts the exporters.

We obtain data from these monthly or quarterly sales forms from both Danish and Swedish tax authorities for 2012-2018. The key for us in constructing the firm-to-firm match is that the data from Denmark (Sweden) will include Swedish (Danish) VAT numbers. We first combine these data on a server in Statistics Denmark. Next, having ensured that all VAT numbers are anonymized with the same key, we can link Swedish VAT numbers reported in Denmark to firm identifiers in other Danish register data (see below). Likewise, we can link Danish VAT numbers

[6] Bernard and Moxnes (2018) provide an early review of the literature.

[7] Small goods exporters may apply to file the sales list at the quarterly level.

[8] Triangulated trade refers to transactions when a firm sells goods as an intermediary between a supplier and the final customer. In these triangulated transactions, the goods are shipped directly from the supplier to the customer without entering the intermediaries country of origin.

reported in Sweden to other Swedish registers. We use annual aggregates of the sales-form data where each observation shows the export value by year, by (Swedish or Danish) supplier, by customer, and by types of exports (goods, services, or triangulation). For most of the analysis, we will focus on firm-to-firm relationships between Danish (Swedish) firms and their Swedish (Danish) suppliers.

We merge the VAT register data with the following administrative data for 2012-2018. First, we obtain data on firm characteristics from The Firm Statistics Register and the Firm Account Statistics from Denmark and the Structural Business Activity (Swedish: Företagens ekonomi, FEK) and the VAT Register (MOMS) from Sweden. These data sources cover the universe of private sector firms, and provide data on variables such as revenue, value added, employment, and industry affiliation. Second, we match workers to firms using The Employment Statistics for Employees and the Education Attainment Register from Denmark and the Longitudinal Integrated Database (LISA) from Sweden. These cover the population of private-sector workers, and provide data on worker characteristics such as salary, occupation, and education. Third, for each firm The Statistics for International Trade in Goods from Denmark and the Foreign Trade Register from Sweden provide imports (exports) by origin (destination) country by 8-digit Combined Nomenclature (CN) products. In all these administrative data, firms and employees are associated with unique identifiers.

While previous studies have used data comparable to *subsets* of our data, we are in the unique position to observe detailed characteristics of firms and their employees on *both* sides of cross-border transactions.[9] Such level of detail is essential for our analyses, as we show below.

In our data, the variable definitions and classification codes are largely comparable between Denmark and Sweden with one important exception: employees' occupation codes. Whereas Denmark has adopted the International Standard Classification of Occupation (ISCO08), prepared by the International Labor Organization, the Swedish classification system, SSYK12, is based on, but different from, ISCO08. As we will see, comparing occupation structure between a firm and its suppliers is crucial for our empirical analysis. We tackle this by aggregating 4-digit ISCO08 enough to ensure comparability and label this grouping 'ISCO+'.[10]

2.2 Sample Construction and Coverage

The aim of the reduced-form empirical analysis in Section 3 is to examine the effect of an exogenous demand shock (defined below) on the focal firm. As described below, we adopt a long-difference approach and use years 2012-13 as the initial period and 2017-18 as the end period. We thus trim our data in the following ways. First, we require that the focal firm reports

[9]For example, with Danish matched worker-firm data and customs data, Hummels et al. (2014) observe one side of international trade. Eaton et al. (2022) use firm-to-firm customs data from France but do not observe firm or labor market data for foreign customers.

[10]Statistics Sweden provides a many-to-many concordance file between SSYK12 and ISCO08. We construct ISCO+ as the most disaggregate grouping that ensures (a) there exists a many-to-one key between ISCO08 and ISCO+, and (b) there exists a many-to-one key between SSYK12 and ISCO+. This is akin to the so-called CN8+ product classification, suggested by Van Beveren et al. (2012), which ensures a many-to-one key between all year-specific CN classifications and CN8+. See Appendix B for further details.

Table 1: Sample coverage, 2012-2013 (%)

	All	Denmark	Sweden	Manufacturing	Wholesale/Retail
#Firms	6.45	8.54	5.45	25.8	13.9
Employment	43.4	48.4	41.7	80.3	51.3
Revenue	60.8	61.8	60.2	89.7	63.9
Imports from DK/SE	73.2	72.5	74.2	88.8	71.5
Exports to 3rd countries	94.7	94.1	95.1	97.3	86.1

positive total revenue and employment in both periods. This is necessary for studying changes within the focal firm. Second, the focal firm must have at least one Swedish/Danish supplier in both periods.[11] This is required in order to study how the focal firm has changed relative to its foreign suppliers. Third, to construct the exogenous demand shocks the focal firm must according to the customs data be an exporter to other countries than Sweden and Denmark (a "third-country exporter") in the initial period. As for the focal firm's foreign suppliers, we only impose the first restriction, but not the latter two.

Table 1 shows that our estimation sample has decent coverage. Although the sample contains a small fraction of Danish and Swedish firms (6.5%), it comprises 43% of employment, 61% of revenue, and 95% of third-country exports, and these shares are even larger among manufacturing firms.

Table 2 clarifies why our estimation sample has decent coverage. It shows that relative to the VAT-data population (in the initial period, 2012-13), the focal firms in our sample are larger in size, have higher labor productivity, and engage in larger volumes of international trade. For example, the average Swedish firm in the full population has 14 employees, while the average Swedish firm in the estimation sample has 103 employees.

2.3 Key Data Patterns

The firms in our sample vary substantially in size and labor productivity. Figure 1 shows a binned scatter plot, with log sales on the horizontal axis and log value added per worker on the vertical axis. Two-digit industry (NACE) and country (Denmark/Sweden) fixed effects are included, so that cross-industry and cross-country variation is excluded from the analysis. The figure shows a strong positive correlation between size and labor productivity. The slope coefficient is 0.103 and the R^2 is 0.23. A firm at the 90th percentile of the sales distribution has 50 percent higher value added per worker than the one at the 10th percentile.

Figure 2 shows a binned scatter plot with log sales on the horizontal axis and the labor share, defined as the wage bill relative to total costs, on the vertical axis. Total costs are constructed as purchases by the firm (goods and services, foreign and domestic) plus the wage bill. Importantly, capital is not included, so that the labor share is unaffected by spending on capital. Again, two-digit industry and country fixed effects are included. The figure shows that large firms have

[11] E.g., a Swedish focal firm must have a least one Danish supplier in both periods (but not necessarily the same one).

Table 2: Firm characteristics, 2012-2013. Population vs. estimation sample.

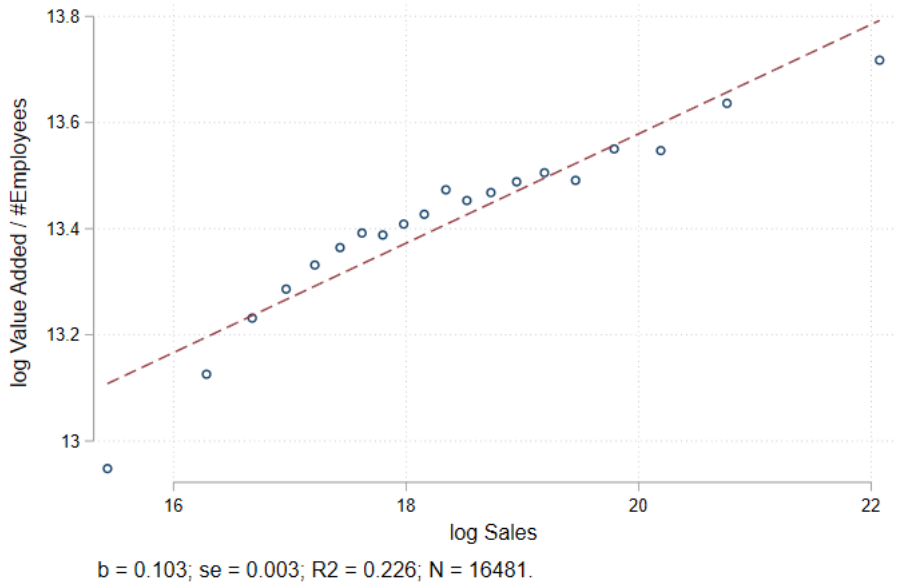
	Population			Estimation Sample		
	All	Denmark	Sweden	All	Denmark	Sweden
Employment						
- Mean	13.7	13.8	13.6	92.1	77.6	103.0
- Median	3.0	3.0	3.0	17.5	16.5	18.5
Sales						
- Mean	4.5	5.0	4.3	42.6	36.5	47.1
- Median	0.5	0.6	0.4	5.5	5.3	5.8
Value added per worker						
- Mean	79.4	95.7	73.0	144.0	207.7	95.9
- Median	62.6	68.8	59.8	80.5	82.8	78.4
Imports from DK/SE						
- Share >0	0.25	0.32	0.21	1.00	1.00	1.00
- Mean (cond. on >0)	0.24	0.31	0.18	0.67	0.85	0.52
- Median (cond. on >0)	<0.01	<0.01	<0.01	0.02	0.03	0.02
Exports to 3rd countries						
- Share >0	0.13	0.14	0.13	1.00	1.00	1.00
- Mean (cond. on >0)	4.7	4.9	4.6	9.2	7.5	10.5
- Median (cond. on >0)	0.04	0.10	0.02	0.20	0.28	0.15

Notes: Value added per worker is measured in thousands euros, while the remaining nominal variables are measured in million euros.

a substantially lower labor share compared to small firms. A firm at the 90th percentile of the sales distribution has a 12 percentage points lower labor share compared to a firm at the 10th percentile.

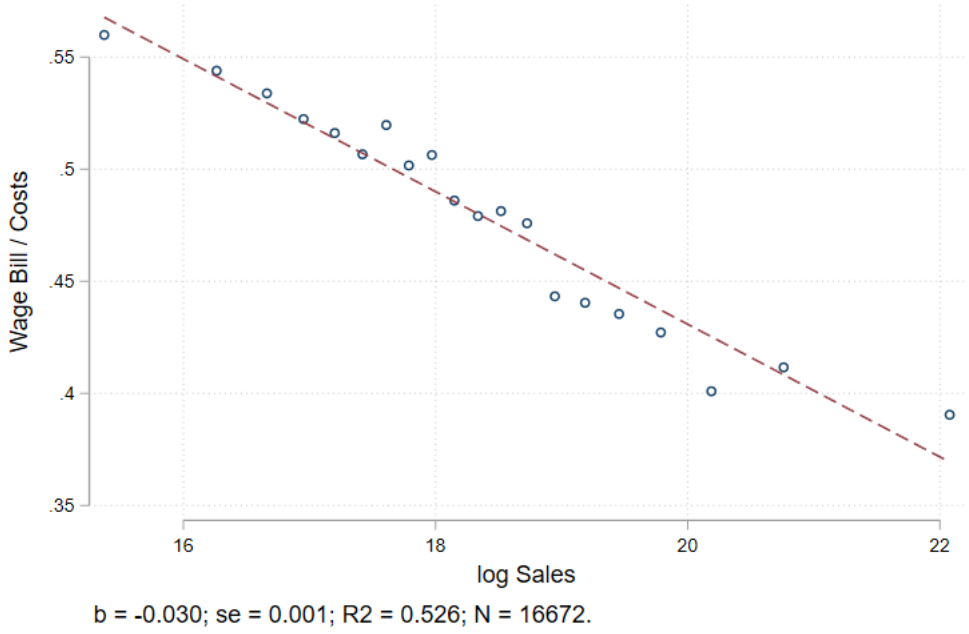
While these empirical patterns are merely correlations, they are nevertheless consistent with the central hypotheses we investigate in this paper. First, large firms may be able to reorganize production vis-à-vis their suppliers in a way that gives labor productivity improvements compared to small firms. Second, large firms may be able to outsource more tasks, which lowers their labor share compared to small firms.

Figure 1: Sales and Labor Productivity



Notes: The figure shows a binned scatter plot with log sales on the horizontal axis and log value added per worker on the vertical axis, both residualized by 2-digit industry (NACE) and country (Denmark/Sweden) fixed effects.

Figure 2: Sales and Labor Share



Notes: The figure shows a binned scatter plot with log sales on the horizontal axis and the wage bill-cost ratio on the vertical axis, both residualized by 2-digit industry (NACE) and country (Denmark/Sweden) fixed effects. Costs are constructed as the purchases by the firm (goods and services, foreign and domestic) plus the wage bill.

Next, we explore the importance of relationship-specificity in our sample by adapting Nunn

(2007)'s measure to our firms. For each focal firm, i, that is engaged in goods trade according to the customs data, we compute rs_i as the share of goods imports that are differentiated according to the Rauch (1999) classification.[12] rs_i range from 0 to 1. High values of rs_i indicate high relationship specificity, because it is more difficult to find alternatives for differentiated goods than for homogeneous goods, which are either traded on organized exchanges or have reference prices. For the focal firms in our sample, the mean value of rs_i is 0.82. Likewise, we calculate the relationship specificity measure for each foreign supplier j, rs_j. For the foreign suppliers, the mean rs_j is also 0.82. These results suggest that relationship specificity is high among Danish-Swedish firm-to-firm links.

2.4 Measures of Labor Specialization

Our first set of measures of labor specialization focuses on the focal firm's *top occupation*. Intuitively, if the top occupation has a large wage-bill share, the focal firm relies more heavily on a single occupation and so is more specialized. To be specific, define firm i's wage bill share of occupation o as $\theta_{io} = W_{io}L_{io}/\sum_{o'} W_{io'}L_{io'}$ where L_{io} is the number of employees and W_{io} is the average wage for occupation o in firm i.

We employ two definitions of *top occupation*. The first definition is based on the level of θ_{io}; i.e.

$$o_i^{*1} = \arg\max_o \{\theta_{io}\}. \tag{1}$$

Equation (1) says that the top occupation is simply the occupation with the highest wage bill share among the firm's own employees, and it gives us the first measure of labor specialization, $\theta(o_i^{*1})$.

The second definition is based on the value of θ_{io} relative to i's foreign suppliers. Let $\Theta_{io} = \sum_{i'} W_{i'o}L_{i'o}/\sum_{i',o'} W_{i'o'}L_{i'o'}$ be the wage bill share of occupation o of firm i and firm i's suppliers combined. The alternative definition of i's top occupation is then

$$o_i^{*2} = \arg\max_o \left\{\frac{\theta_{io}}{\Theta_{io}}\right\}. \tag{2}$$

In equation (2), o_i^{*2} is conceptually identical to revealed comparative advantage (RCA): If $\theta_{io} > \Theta_{io}$, firm i is said to have RCA in occupation o because it relies relatively more on occupation o than its suppliers, and o_i^{*2} is simply the occupation with the highest degree of RCA. Equation (2) gives us the second measure of labor specialization, $\theta(o_i^{*2})$.

Summarizing equations (1) and (2), large values of $\theta(o_i^{*1})$ and $\theta(o_i^{*2})$ indicate a high degree of labor specialization for the focal firm i. Note that both o_i^{*1} and o_i^{*2} are calculated in the initial period and so are not time-varying.

The second set of measures of specialization focuses on the similarity in occupational structure

[12]For this exercise, we condition on firms being goods importers (or exporters) according to the customs data in 2012-2013. In recent work, Cajal-Grossi et al. (2023)'s measure is based on the number of suppliers, while Martin et al. (2026) measures "relationship stickiness" using the average duration of buyer-supplier relationships.

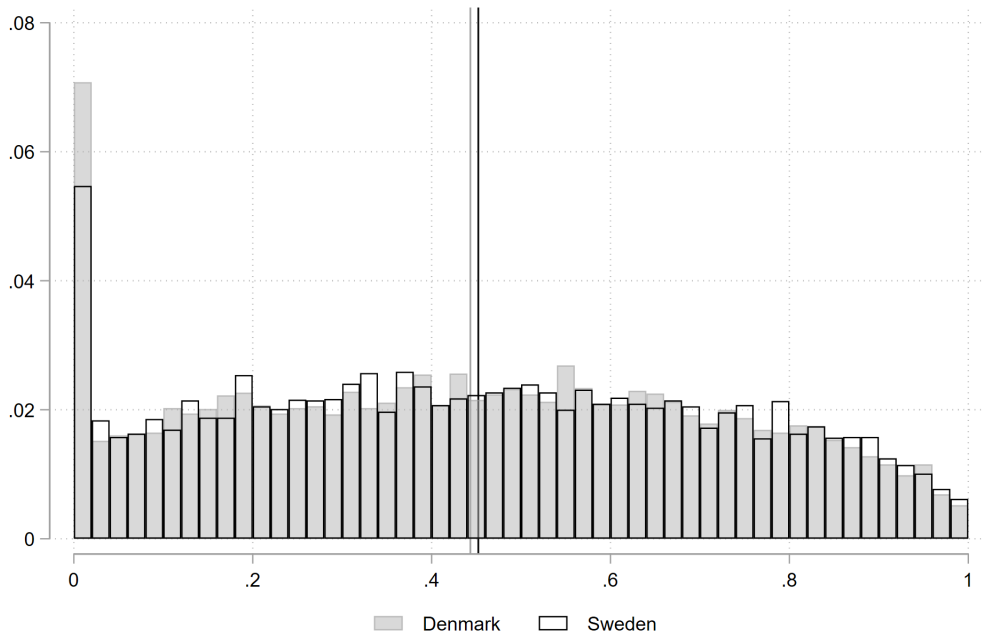
between the focal firm i and its foreign suppliers j, by comparing the set of wage bill shares across occupations. Specifically, we construct a similarity index sim_{ij} between the focal firm i and a single supplier j as the cosine similarity between their wage bill shares:

$$sim_{ij} = \frac{\sum_o \theta_{io}\theta_{jo}}{\left(\sum_o \theta_{io}^2 \sum_o \theta_{jo}^2\right)^{1/2}}, \tag{3}$$

which takes value zero if i and j have no occupations in common (i.e. no similarity), and one if they have identical wage bill shares for all occupations o (i.e. max similarity).[13] We obtain the third measure of labor specialization, sim_i, by averaging sim_{ij} across suppliers; i.e. $sim_i = \sum_j w_{ij} sim_{ij}$, where w_{ij} is the share of i's purchases from supplier j.

Figure 3 shows a histogram of sim_i for the initial period, 2012-13. The white bars with black frames are for firms in Sweden and the gray bars are for firms in Denmark. The bar heights sum to 1 for each country by construction. Figure 3 shows that sim_i varies substantially across focal firms.

Figure 3: Occupation Similarity



Notes: Vertical lines indicate means for Danish (gray line) and Swedish (black line) firms, respectively.

We supplement sim_{ij} with a similarity index where wage bill shares θ_{io} are replaced by dummy variables that take value one if firm i has at least one employee in occupation o:

$$bisim_{ij} = \frac{\sum_o \mathbf{1}\{\theta_{io} > 0\}\mathbf{1}\{\theta_{jo} > 0\}}{\left(\sum_o \mathbf{1}\{\theta_{io} > 0\}\sum_o \mathbf{1}\{\theta_{jo} > 0\}\right)^{1/2}} \tag{4}$$

[13] In the same way, Jaffe (1988) and Bloom et al. (2013) use cosine similarity of research effort across fields to measure technology proximity between firms.

Equation (4) gives us the fourth, and final, measure of labor specialization, $bisim_i = \sum_j w_{ij} bisim_{ij}$, where w_{ij} is, again, the share of i's purchases from supplier j. Summarizing equations (3) and (4), high values of sim_i and $bisim_i$ indicate high similarity between focal firm i's occupational structure relative to its foreign suppliers and so a *low* degree of labor specialization.

Table 3 provides summary statistics for all four measures of labor specialization for Danish and Swedish firms separately.

Table 3: Measures of Labor Specialization

	Denmark					Sweden				
	Mean	SD	P25	P50	P75	Mean	SD	P25	P50	P75
$\theta(o_i^{*1})$	.54	.21	.39	.51	.66	.54	.20	.39	.50	.66
$\theta(o_i^{*2})$	.27	.28	.05	.16	.41	.27	.28	.05	.17	.41
sim_i	.44	.27	.21	.44	.66	.45	.27	.22	.45	.67
$bisim_i$	.32	.14	.24	.32	.40	.32	.13	.24	.32	.40

3 Reduced-Form Model and Results

In this section, we explore how the focal firm responds to exogenous demand shocks, in terms of size, labor productivity, and labor specialization.

3.1 Specification and Identification Strategy

We start by spelling out our empirical specification, which is in long-differences:

$$\Delta \ln y_i = \alpha_{s(i)} + \gamma_{r(i)} + \beta z_i + \epsilon_i. \tag{5}$$

In equation (5), Δ is the change from period 0 (2012-13) to period 1 (2017-18), i indexes the focal firm, $\alpha_{s(i)}$ represents 2-digit industry fixed effects, and $\gamma_{r(i)}$ represents country fixed effect (Sweden or Denmark). The dependent variable, y_i, denotes various outcomes related to firm i. Some are readily available from the data sources, such as sales, employment, and value added, while others are constructed from the raw data, such as the labor specialization measures based on equations (1)-(4).

The main explanatory variable, z_i, is a measure of demand which is orthogonal to the error term, ϵ_i. We follow the trade literature and employ a so-called world import demand (WID) shock from (Hummels et al., 2014). Specifically, z_i is defined as:

$$z_i = \sum_{c,p} \omega_{icp} \Delta \ln imports_{cp}. \tag{6}$$

In equation (6), $imports_{cp}$ is country c's total purchases of (4-digit HS) product p from the world market, except from Sweden and Denmark, and ω_{icp} is the share of firm i's exports in period 0 allocated to country-product pair cp. Note that, when constructing this instrument, the weights

Table 4: Firm-level Outcomes.

	(1) $\Delta \ln Sales$	(2) $\Delta \ln L$	(3) $\Delta \ln$ DK/SE Imp	(4) $\Delta \ln \frac{ValueAdded}{Sales}$	(5) $\Delta \ln LaborShare$	(6) $\Delta \ln \frac{ValueAdded}{L}$	(7) $\Delta \ln TFP$
z	0.121***	0.059***	0.14**	-0.028**	-0.045***	0.037***	0.029**
	(0.020)	(0.015)	(0.056)	(0.013)	(0.013)	(0.014)	(0.012)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	16,788	16,788	16,788	16,297	16,628	16,297	13,074

Note: Labor share is defined as the wage bill relative to total costs. TFP is computed using the Ackerberg et al. (2015) method ("acfest" in Stata), using deflated revenue, labor, capital and materials. Industry FE refers to NACE 2-digit fixed effects. Country FE is a dummy for whether the firm is Swedish or Danish. Standard errors in parentheses.

are calculated from the Swedish/Danish customs data, i.e. the sample is limited to firms that are third-country exporters (to other countries than Sweden/Denmark) of goods in period 0.

We estimate equation (5) with OLS and interpret the coefficient estimate, β, as a causal effect. The underlying identification assumption is that the change in country c's imports of product p from the world, minus Denmark and Sweden, is uncorrelated with the error term, ϵ_i. Intuitively, this assumption says that firm i, being part of the small open economies of Denmark or Sweden, does not have a significant influence over aggregate trade of Denmark or Sweden's trade partners.

3.2 Results: Firm-Level Outcomes

Table 4 presents the results and clarifies the ways in which the focal firm changes in response to exogenous demand shocks. Columns (1) and (2) show that the focal firm increases both sales and employment, but that sales grow at a faster rate. Column (3) shows that the focal firm also increases purchases from its Danish/Swedish suppliers, at a rate comparable to the increase in sales.

Next, we present results for firms' sourcing of intermediates. Column (4) shows the impact on the value added to sales ratio while column (5) shows the impact on the labor share, defined as wage costs relative to total costs. Both these measures decline, suggesting that firms increase their reliance on intermediates when they grow.

Finally, column (6) shows that a positive demand shock raises labor productivity, as measured by value added per worker. As a robustness, we compute the focal firm's TFP by two-digit industry and report the results in column (7). These results are suggestive of economies of scale in production.

In summary, Table 4 shows that, in response to positive demand shocks to the focal firm, sales, employment and labor productivity of the focal firm and its purchases from foreign suppliers grow together, while the value added to sales ratio and the labor share decline. Section 4 will interpret these responses through the lens of a parsimonious quantitative model.

3.3 Results: Between-Firm Outcomes

We now examine whether focal firms establish new foreign (Danish/Swedish) supplier links and report our results in Table 5. In columns (1) and (2), the dependent variable is the change in

the (log of the) number of foreign suppliers. In column (3), the dependent variable is a dummy variable for whether the number of foreign suppliers increases from period 0 to period 1. In column (4), it is a dummy for whether the focal firm has new foreign suppliers in period 1. While the coefficient estimates for all four measures are positive, they lack statistical significance. These results suggest that new foreign-supplier links are unlikely to be driving the increase in labor productivity shown in Table 4. Rather, they suggest that reorganization of production across existing suppliers is important.

Table 5: New Links

	(1) $\Delta \ln \#Suppliers$	(2) $\Delta \#Suppliers$	(3) $1\{\Delta \#Suppliers > 0\}$	(4) $1\{NewSupplier\}$
z	0.001	0.026	0.001	0.013
	(0.017)	(0.095)	(0.013)	(0.011)
Industry FE	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes
N	16,788	16,788	16,788	16,788

A unique feature of our data is that we observe detailed characteristics of the workers of both the focal firm and its foreign (Swedish/Danish) suppliers. We thus explore whether the occupational structure of the focal firm grows more similar, or less similar, as compared with that of its foreign suppliers.

Table 6: Between-Firm Outcomes.

	(1) $\Delta\theta\left(o^{*1}\right)$	(2) $\Delta\theta\left(o^{*2}\right)$	(3) Δsim	(4) $\Delta bisim$	(5) Δsim fix imp shares	(6) Δsim fix θ_j	(7) Δsim fix θ_i
z	0.014*	0.019***	-0.016**	-0.010**	-0.017***	-0.010*	-0.009**
	(0.008)	(0.007)	(0.007)	(0.004)	(0.006)	(0.005)	(0.004)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	16,788	16,788	16,788	16,788	16,788	16,788	16,788

Note: Standard errors in parentheses. Significance levels: $*p < 0.1, **p < 0.05, ***p < 0.01$.

In column (1) of Table 6, the dependent variable is the change in the wage-bill share of the focal firm's top occupation, as defined by equation (1). In column (2), we examine the change in the wage-bill share of the focal firm's top occupation relative to its suppliers, as defined by equation (2). The coefficient estimate is positive and statistically significant, suggesting that the focal firm's comparative-advantage occupations, relative to its foreign suppliers, become more important.

We compare the vector of the focal firm's occupational wage-bill shares with that of its foreign (Swedish/Danish) suppliers in column (3) of Table 6, by using the change in the similarity index constructed from equation (3), Δsim_i, as the dependent variable. The coefficient estimate is negative and significant. In column (4), we use the binary version of the similarity index, (4), instead, and obtain similar results. Columns (3) and (4) provide clear evidence that the focal firm and its foreign suppliers are growing apart, in terms of their occupational structures.

Columns (5)-(7) of Table 6 explore whether the change in the similarity index is due to changes of the focal firm or its foreign suppliers. In column (5), we fix the import shares w_{ij} at period-0 levels and recompute Δsim_i. The coefficient estimate is similar to that in column (3) and serves as a benchmark for columns (6) and (7). In column (6), we fix both the import shares and j's occupational wage-bill shares, θ_{jo}, at period-0 levels. The coefficient estimate remains negative and (marginally) significant, and is slightly more than half the size of the estimate in column (5). In column (7), we fix both the import shares and θ_{io} at period-0 levels, so that Δsim_i only captures in the suppliers' occupation structure, and obtain similar results. Columns (5)-(7) suggest that the changes in both the focal firm's and its foreign suppliers' occupational structures contribute to the decrease in the similarity index shown in column (3). In summary, Table 6 provides evidence that firm growth leads to increased labor specialization between the focal firm and its foreign suppliers.

3.4 Robustness and Heterogeneity

A potential concern is that the demand shock variable z might affect both the focal firm and its suppliers directly. If, for example, all firms in the supply chain are exporting similar products, then a demand shock to i is also a demand shock to j. We deal with this issue by directly controlling for the suppliers' demand shocks, see Appendix C.1. The coefficient estimates are very close to our baseline results.

Another concern may be that the demand shock variable might affect both the focal firm and its customers directly. To address this, we let the unit of analysis be a supplier (instead of a customer) and estimate the direct impact of z on the supplier, and the indirect impact of z from its customers. More details are provided in Appendix C.2.

In summary, our empirical results in this section establish the importance of labor specialization along international supply chains. Interestingly, we find a weak extensive margin response, suggesting that reallocation occurs within existing suppliers. Our empirical results also show that labor specialization goes hand in hand with labor productivity growth and lower labor shares. We now turn to a quantitative model to explore the importance of this mechanism.

4 A Quantitative Model

In this section, we present a stylized task-trade model broadly consistent with the empirical evidence outlined above that focuses on the buyer firms' make-vs-buy decision. Our model highlights how task outsourcing and specialization give rise to cost reductions and scale economies.

We first outline the main model elements, and then discuss main model features and clarify how they are related to the supply-side parameters of the model. The model also allows us to quantify the economic significance of these mechanisms, which we explore in the next section.

4.1 Economic Environment

We start with the following overview of the model. Buyer firms are heterogeneous and produce final goods by combining a unit continuum of tasks. Tasks can either be produced in-house or outsourced, and there is heterogeneity in task efficiency, varying both across tasks and firms. There are potential gains from outsourcing, or labor specialization, as in-house task production may not be very efficient. But outsourcing is costly, due to e.g. transaction costs, monitoring costs or costs associated with reorganizing production. We model this as a task-specific fixed cost. Therefore, buyer firms will outsource tasks where the efficiency gains from doing so are relatively large.

Notation wise, all the variables in our model vary across buyer firms or suppliers, except for the model parameters. We thus drop the buyer-firm and supplier subscripts for ease of notation. In addition, we relegate all detailed derivations and proofs to Appendix D. The specific model elements are as follows.

Final goods demand Final goods are differentiated and sold to consumers with CES preferences with an elasticity of substitution $\gamma > 1$. We assume that the market structure of the final goods is monopolistic competition. Final good sales, s, is then

$$s = \lambda p^{1-\gamma}, \tag{7}$$

where p is the final good price and λ is a demand shock that is exogenous to the buyer firm. In our model, λ measures the buyer firm's scale and the variation of λ across buyer firms is the main driver of their heterogeneity.[14]

Final goods production The buyer firms produce final goods y by combining a unit continuum of firm-specific tasks ω with the following CES production function:

$$y = \left[\int_0^1 v\left(\omega\right)^{(\sigma-1)/\sigma} d\omega\right]^{\sigma/(\sigma-1)},$$

where $\sigma > 1$ is the elasticity of substitution across tasks and $v\left(\omega\right)$ is the quantity of task ω. Buyer firms are heterogeneous in two dimensions. First, they face different demand shocks λ, as shown in equation (7), second, there is heterogeneity in the efficiency of producing tasks.[15]

[14] λ is a composite of the firm-specific demand shock, the final goods price index and aggregate spending. The latter two variables do not vary across buyer firms.

[15] The demand shock is isomorphic to heterogeneity in total factor productivity (TFP). To ease notation, we do not include TFP heterogeneity in the analysis.

Task production Labor is the only factor of production, and our numeraire is the wage that the buyer firms pay. Firms choose between producing tasks in-house or outsourcing them. If task ω is produced in-house, the production uses labor under constant-returns-to-scale technologies with the task productivity of $\phi^b(\omega)$ (b for buyer). This means that the buyer firm incurs the marginal cost of $c^b(\omega) = 1/\phi^b(\omega)$. If, instead, ω is outsourced, the buyer firm's marginal cost is $c^s(\omega) = 1/\phi^s(\omega)$, where $\phi^s(\omega)$ is the supplier's task productivity (s for supplier).[16] In addition, the buyer firm pays a fixed cost f, in terms of labor, for each outsourced task. Therefore, the final goods firm might choose to produce a task in-house even if $c^b(\omega) > c^s(\omega)$.

We assume that task productivities, $\phi^b(\omega)$ and $\phi^s(\omega)$, are drawn from the log normal distribution, with the joint distribution of

$$(\ln \phi^b, \ln \phi^s) \sim \mathcal{N}(\Gamma, \Omega), \Gamma = (0, \mu^s_{\ln \phi}), \Omega = \begin{pmatrix} \kappa^2_{\ln \phi} & \rho\kappa^2_{\ln \phi} \\ \rho\kappa^2_{\ln \phi} & \kappa^2_{\ln \phi} \end{pmatrix}, \tag{8}$$

where Γ is the mean and Ω is the covariance matrix. Equation (8) says that we have normalized the mean of the buyer firm's task productivity draws to 0, and that $\mu^s_{\ln \phi}$ captures the average productivity advantage of the foreign supplier. As a result, $\mu^s_{\ln \phi}$ represents the foreign supplier's absolute advantage in our model. Equation (8) also says that the standard deviation of task productivity draws is identical, $\kappa_{\ln \phi}$, for the buyer firm and the supplier. As a result, $\kappa_{\ln \phi}$ represents firms' comparative advantage. Finally, equation (8) says that the buyer firm's and supplier's task productivity draws, $\phi^b(\omega)$ and $\phi^s(\omega)$, might be correlated, summarized by the correlation coefficient ρ. This may happen, for example, if the buyer firm and foreign supplier are from the same industry.

Note that our model is more general than the parameterization of equation (8). We have chosen equation (8) mainly because it eases estimation in the next section, and that the lognormal distribution is empirically relevant.

4.2 Model Features

The previous sub-section showed that the set of supply-side parameters of our model is $\Upsilon_S = \left\{f, \mu^s_{\ln \phi}, \kappa_{\ln \phi}, \rho\right\}$ ("S" for supply-side).[17] In this sub-section, we discuss the main model features and how they are related to these model parameters.

The pattern of task trade Final goods firms are maximizing profits by selecting the set of tasks to produce in-house and the set of tasks to outsource. Define the foreign supplier's relative task productivity

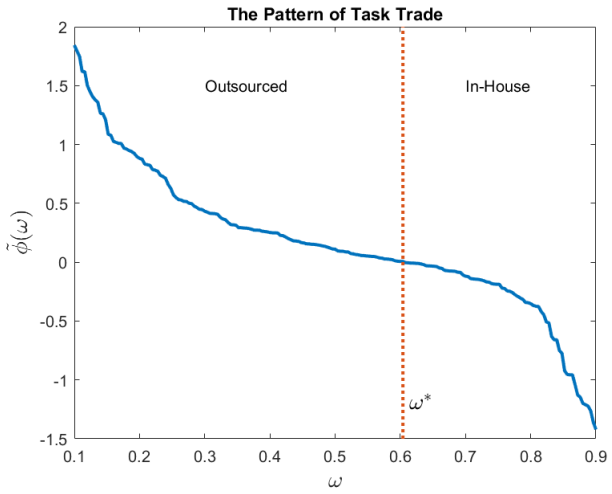
$$\tilde{\phi}(\omega) = \phi^s(\omega)^{\sigma-1} - \phi^b(\omega)^{\sigma-1}, \tag{9}$$

and without loss of generality, sort tasks such that $\partial\tilde{\phi}(\omega)/\partial\omega < 0$. We illustrate $\tilde{\phi}(.)$ in Figure 4 by plotting it against ω for a randomly selected buyer firm. The overall shape of the curve $\tilde{\phi}(.)$

[16] $\phi^s(\omega)$ is inclusive of variable trade costs, potential markups, and the wage that the foreign supplier pays relative to the one that the buyer firm pays.

[17] We parameterize the demand-side variable of λ in section 5.

Figure 4: The Pattern of Task Trade



Notes: The range of tasks is limited to $[0.1, 0.9]$ to improve visualization.

is given by the log normal parameterization of equation (8).

Figure 4 also shows that there is a cutoff task, ω^*. The buyer firm outsources the tasks in the interval $[0, \omega^*]$ because, intuitively, the foreign supplier has large relative task productivity for these tasks so that the benefits of outsourcing exceed the costs. The remaining tasks, in the interval $[\omega^*, 1]$, are then produced in-house. In our model, ω^* represents the extent of the buyer firm's labor specialization in task production with respect to the foreign supplier; i.e. the higher is ω^*, the more tasks are outsourced, and the higher is the degree of labor specialization.

The gains from outsourcing The pattern of task trade above implies that the buyer firm's CES input price index can be written as

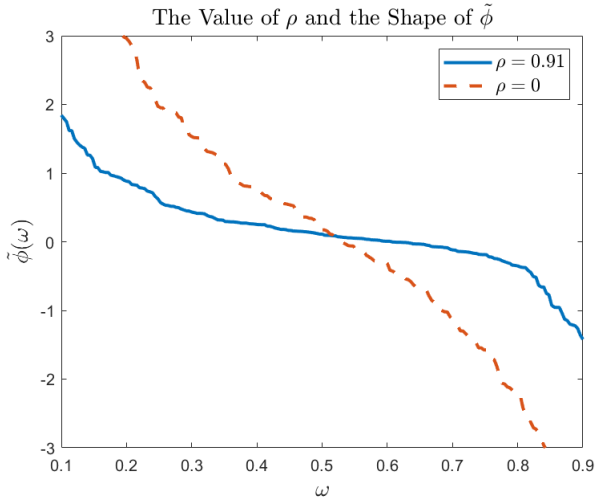
$$P^{1-\sigma} = \int_0^{\omega^*} \phi^s(\omega)^{\sigma-1} d\omega + \int_{\omega^*}^1 \phi^b(\omega)^{\sigma-1} d\omega. \tag{10}$$

On the other hand, in the counterfactual scenario of autarky, the same buyer firm's input price index would be $P_A^{1-\sigma} = \int_0^{\omega^*} \phi^b(\omega)^{\sigma-1} d\omega + \int_{\omega^*}^1 \phi^b(\omega)^{\sigma-1} d\omega$. Therefore,

$$P_A^{1-\sigma} - P^{1-\sigma} = \int_0^{\omega^*} (\phi^b(\omega)^{\sigma-1} - \phi^s(\omega)^{\sigma-1}) d\omega = -\int_0^{\omega^*} \tilde{\phi}(\omega) d\omega, \tag{11}$$

where $\tilde{\phi}(.)$ is the foreign supplier's relative task productivity, given by equation (9). With the help of Figure 4, we see that for the outsourced tasks of $[0, \omega^*]$, $\tilde{\phi}(.) > 0$, and so the right-hand side of equation (11) corresponds to the area between the $\tilde{\phi}(.)$ curve and the x-axis for these tasks. Therefore, equation (11) says that $P_A > P$, given $\sigma > 1$. Intuitively, the buyer firm would have a higher input price index if it were not allowed to outsource, and the difference between P_A and P provides a measure of the buyer firm's gains from outsourcing. Equation (11) also says that the magnitude of the gains from outsourcing, which is related to the its

Figure 5: ρ and the Gains from Outsourcing



Notes: XXXX We have limited the range of tasks to $[0.1, 0.9]$ on the x-axis and the scale of the y-axis to $[-3, 3]$ for good visualization. The scale of the y-axis is different from Figures 4 and **??** above. XXXX

left-hand side, it large if the difference in task productivity between the foreign supplier and the buyer firm, $\tilde{\phi}(.)$, is large for the outsourced tasks. Figure 5 illustrates how ρ, the correlation coefficient between the buyer firm's and supplier's task productivity draws, affects the buyer firm's gains from outsourcing. The solid $\tilde{\phi}(.)$ curve is the same as in Figure 4, and corresponds to $\rho = 0.91$, the estimated value that we obtain from section 5. The dashed $\tilde{\phi}(.)$ curve, on the other hand, has $\rho = 0$. Comparing the solid curve with the dashed one, we see that as ρ decreases, $\tilde{\phi}(.)$ rotates clockwise and becomes steeper, indicating large differences in task productivity between the foreign supplier and the buyer firm. Therefore, the buyer firm enjoys large gains from outsourcing; i.e. large reductions in the input price index relative to autarky.

$\kappa_{\ln \phi}$, the standard deviation of the task productivity draws, has qualitatively similar effects as ρ. For example, suppose $\kappa_{\ln \phi}$ increases, indicating an increase in comparative advantage. Then the $\tilde{\phi}(.)$ curve also rotates clockwise, and so the buyer firm enjoys larger gains from outsourcing (see Appendix D for more details). Intuitively, both ρ and $\kappa_{\ln \phi}$ matter for differences in task productivity. $\kappa_{\ln \phi}$ matters more for the differences across tasks within firms, while ρ matters more for the differences between the buyer firm and foreign supplier.

Optimal outsourcing This section determines the optimal set of tasks to outsource: the ω^* cutoff. With monopolistic competition, final good prices are a constant mark-up over costs, $p = \bar{m}P$, where $\bar{m} = \gamma/(\gamma - 1)$ is the markup and P is given by equation (10). Gross profits are proportional to sales, s/γ, and therefore net profits are $\pi = s/\gamma - \omega^* f$. The buyer firm finds the ω^* that maximizes π. Under the assumption that $\gamma = \sigma$, it is straightforward to show that the first order condition is:

$$\lambda \tilde{\phi}(\omega^*) = \gamma \bar{m}^{\gamma - 1} f. \tag{12}$$

In the left-hand side of equation (12), $\tilde{\phi}(.)$ is the buyer firm's marginal gain from outsourcing and it is amplified by the buyer firm's scale, λ. The right-hand side of equation (12), meanwhile, depends on the per-task fixed cost of outsourcing, f.[18]

Equation (12) together with Figure 4 can help us visualize how the value of f affects the buyer firm's optimal choice of ω^*. Suppose f increases but λ and the other parameter values remain unchanged. Then the buyer firm's optimal choice moves to the left along the solid $\tilde{\phi}(.)$ curve and ω^* decreases, indicating fewer tasks outsourced and less labor specialization.

The impacts of demand shocks We present the following three propositions on the impact an exogenous increase in final-good demand, λ, on the buyer firm's labor specialization and labor productivity.

Proposition 1 *For $\gamma = \sigma$, a positive demand shock increases the cutoff task ω^*:* [19]

$$\frac{\partial \omega^*}{\partial \lambda} = -\frac{\tilde{\phi}\left(\omega^*\right)}{\left[\partial \tilde{\phi}/\partial \omega^*\right]\lambda} > 0. \tag{13}$$

Intuitively, higher demand implies that the marginal gains from outsourcing a task increases, as shown in equation (12), while the per-task fixed cost remains the same. Therefore, the cutoff task increases, indicating increased labor specialization.

Proposition 2 *For $\gamma = \sigma$, a positive demand shock decreases the ratio of value added, VA, to sales, s:*

$$\frac{\partial \left(VA/s\right)}{\partial \lambda} = -\frac{1}{\bar{m}} P^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda} \bar{\phi}\left(\omega^*\right) < 0, \tag{14}$$

where $\bar{\phi}\left(\omega^*\right)$ is a weighted average of buyer and supplier productivity at the cutoff point: $\bar{\phi}\left(\omega^*\right) = \left(1-\Pi\right)\phi^s\left(\omega^*\right)^{\sigma-1} + \Pi\phi^b\left(\omega^*\right)^{\sigma-1}$, where the weight Π is the cost of outsourced tasks relative to total variable costs.

Proposition 2 states that the value added to sales ratio, VA/s, a common measure of vertical (dis)integration, declines when demand is higher. This is a direct consequence of increased outsourcing, as shown in Proposition 1.

Let L denote the buyer firm's total labor.

Proposition 3 *For $\gamma = \sigma$, the elasticity of value added per worker, VA/L, is*

$$\frac{\partial \ln(VA/L)}{\partial \ln \lambda} = \theta_{L,F} + \theta_{L,F} \frac{\partial \ln \omega^*}{\partial \ln \lambda} A_1 - \theta_{L,F} \frac{\partial \ln \omega^*}{\partial \ln \lambda} A_2, \tag{15}$$

[18] Since $\tilde{\phi}\left(\omega^*\right)$ is a downward sloping function of ω^*, this equation yields a unique solution for ω^*.

[19] The propositions focus on the case where $\gamma = \sigma$ because it is difficult to obtain analytical results otherwise. However, additional numerical simulations of the model show that the sign of the propositions also hold more generally.

where $\theta_{L,F} = \omega^* f/L$ *is the share of outsourcing labor in total labor. Meanwhile,* $A_1 = \frac{(1-\gamma\omega^* f/s)}{(\bar{m}-\Pi)}(\frac{\phi^b(\omega^*)^{\sigma-1}}{\tilde{\phi}(\omega^*)})$ *and* $A_2 = \frac{(1-\gamma\omega^* f/s)}{(\bar{m}-\Pi)}(\frac{1}{\gamma-1})$*, where* s *is the buyer firm's sale, given in equation* (7)*, and* $\tilde{\phi}(.)$ *is the foreign supplier's relative task productivity, given in equation* (9)*.*

The intuition for the first term on the right-hand side of equation (15) is that, as λ increases, given ω^*, the buyer firm increases the value of imports of the outsourced tasks, $[0, \omega^*]$, because the fixed cost for these tasks has already been paid. This effect tends to increase VA/L, and its magnitude tends to be large, if the fixed cost of f is large. This effect is then amplified as the buyer firm increases ω^*, according to the second term on the right-hand side of equation (15), because the input price index, P, decreases, according to equation (10), further increasing VA/L. As ω^* increases, however, the buyer firm also increases outsourcing labor, and this effect tends to reduce VA/L, according to the third term on the right-hand side of equation (15). Overall, the first two effects dominate, and so VA/L increases.[20]

In summary, Propositions 1-3 say that a positive demand shock will lead to: (i) more labor specialization, more tasks being outsourced, and less task similarity between the buyer and foreign supplier, (ii) a lower value added to sales ratio, and (iii) a higher value added per worker.

Qualitatively, these model predictions match our empirical results from section XXX 3 XXXX. On the other hand, it is challenging to provide additional analytical results for the magnitude of the responses in Propositions 2-3 because a given model parameter typically affects multiple variables in equations (14) and (15).[21] Therefore, in the next section, we will simulate the model, estimate the buyer firm's response using simulated data, and then compare these estimates with our empirical estimates from section 3.

Discussion We now explicitly compare our modeling approach with the literature. First, our model builds upon previous studies on task trade (e.g. Grossman and Rossi-Hansberg 2008) and labor specialization and trade (e.g. Costinot 2009). The more novel elements of our work involve the parameterization of the task productivity using equation (8) and the estimation of these parameters in the next section.

Second, our model emphasizes the substitution between imported inputs and the buyer firm's own labor, or the intensive margin in GVC, but abstracts away from the extensive margin of which foreign supplier to choose,[22] because our empirical analyses show that the number of foreign suppliers in Denmark/Sweden is not significantly affected by the demand shocks in our data (Table 5). Clearly, our results do not take away the importance of the extensive margin in GVC in other contexts and studies, and so our model complements the existing GVC literature,[23] by highlighting the intensive margin mechanism that has been understudied there.

[20] Analytically, it is challenging to prove that $\frac{\partial \ln(VA/L)}{\partial \ln \lambda} > 0$ using equation (15), but this result holds in all our quantitative analyses.

[21] For example, equation (10) shows that ρ affects the input price index, P, on the right-hand side of equation (14), and Figure 5 shows that ρ also affects $\partial\omega^*/\partial\lambda$ by changing the shape of the $\tilde{\phi}(.)$ curve.

[22] In recent work in the GVC literature (e.g. Antras et al. 2017, Grossman et al. 2023), the buyer firm uses inputs but no labor in production and the analyses focus on which suppliers to choose.

[23] Our model is also compatible with the extensive margin. e.g. in principle, one could extend our model so that one subset of tasks is outsourced to foreign supplier 1 while a different subset is outsourced to supplier 2.

Third, because we do not explicitly model products or occupations,[24] the model variables of individual tasks, such as ω^* and the import value of an outsourced task ω, are not observable in the data. On the other hand, this modeling approach is compatible with our goal of studying labor specialization because it frees us from the need to map individual tasks in the model to individual occupations or products in the data, as we clarify in the next section.

Finally, our model also complements the literature on imported inputs and productivity, because task trade may represent one mechanism for buyer firms to change the number of imported product varieties. To clarify this point, we show that the number of imported products from Denmark/Sweden increases, whether we use 4-digit, 6-digit or 8-digit product codes (Appendix XXXX). Beyond these analyses, it is difficult to say more about the model predictions for imported products without knowing which tasks in the model map into which products in the data.

5 Structural Estimation and Counterfactual Exercises

In this section, we identify key parameters of the model using simulated methods of moments, and then simulate the model to quantify the economic significance of the intensive margin mechanism of labor specialization along the global supply chain. We lay out the setting and main assumption of our estimation procedure, present the estimated parameter values, and compare the model predicted variation of labor productivity and the model regression coefficients with their data counterparts. We then conduct the counterfactual with prohibitively costly outsourcing, and assess the impact on the buyer firms' labor productivity and input costs. Finally, we consider alternative model specifications and robustness analyses.

5.1 Setting and Key Parameters

We start by assuming that the buyer firm's demand shifter, λ, has the log normal distribution of,

$$\lambda = \epsilon^{\xi}, \ln \epsilon \sim \mathcal{N}(\mu_{\ln \epsilon}, \kappa_{\ln \epsilon}), \tag{16}$$

where $\mu_{\ln \epsilon}$ is the mean, $\kappa_{\ln \epsilon}$ is the standard deviation, and ξ is a scaling parameter. In equation (16), $\ln \epsilon$ corresponds to the exogenous demand shock of z_i that is used in our empirical analyses and given in equation (6), and so equation (16) translates the model variable of λ into the data variable of z_i.

Equation (16) says that the set of demand-side parameters of our model is $\Upsilon_D = \left\{\xi, \mu^s_{\ln \epsilon}, \kappa_{\ln \epsilon}\right\}$ ("D" for demand-side). The full set of parameters to estimate is thus $\Upsilon = \Upsilon_D \cup \Upsilon_S$, where $\Upsilon_S = \left\{\mu^s_{\ln \phi}, \kappa_{\ln \phi}, \rho, f\right\}$ is the set of supply-side parameters that we discussed in section 4. On the other hand, we set the substitution elasticity in demand, $\gamma = 4$, and set the substitution elasticity in production, $\sigma = \gamma$. All in all, we have 2 pre-determined parameters, γ and σ, and 7 endogenous parameters to estimate, in the set Υ. We list these 7 parameters in the 3rd column

[24] For example, Halpern et al. (2015) model products and Becker et al. (2025) model occupations.

Table 7: Identification of Model Parameters.

Targeted Moments	Data	Identifies	Par. Values
Demand-side:			
Regression of log sales on z	0.12	ξ	$\xi = 0.12$
mean(lnSales)	18.28	$\mu_{\ln \epsilon}$	$\mu_{\ln \epsilon} = 101.48$
stdev(lnSales)	1.67	$\kappa_{\ln \epsilon}$	$\kappa_{\ln \epsilon} = 13.62$
Supply-side:			
stdev($\ln \theta$) (mean)	1.61	$\kappa_{\ln \phi}$	$\kappa_{\ln \phi} = 0.50$
mean(*cosim* similarity index)	0.45	ρ	$\rho = 0.91$
mean(Labor share), top decile S	.42	$\mu^s_{\ln \phi}$	$\mu^s_{\ln \phi}$=0.046
mean(Labor share), bottom decile S	.58	$\mu^s_{\ln \phi}, f$	$f = 1851.26$

Notes: The demand-side parameters are given in equation (16), and the supply-side parameters of ρ, $\mu_{\ln \phi}$ and $\kappa_{\ln \phi}$ are given in equation (8). f is the fixed outsourcing cost per task. The data regression coefficient of log sales on z is given in Table 4, column (1).

of Table 7. Our estimation is the simulated method of moments, or SMM. Briefly speaking, we simulate 1,000 buyer firms by drawing their demand shifter, λ, using equation (16). For each buyer firm and its foreign supplier, we approximate the unit continuum of tasks with 250 discrete tasks[25] and then draw their task productivity using equation (8). We obtain the estimated parameter values by iterating until the model predicted moments match the data targets.

To implement the SMM estimation, we assume that the moments of the occupations in the data, such as the *cosim* index for occupation similarity, provide unbiased estimates of the corresponding moments of the tasks in the model.[26] This assumption clarifies that it is acceptable if the model predictions for *individual* tasks (e.g. ω^*) have no counterpart in the data and so are unobservable, as long as the model predictions for *moments* of tasks, which aggregate across individual tasks, match the corresponding *moments* of occupations in the data, which aggregate across individual occupations. Therefore, the main assumption of our estimation allows us to bypass the very difficult challenge of mapping the individual tasks in our model to individual occupations or products in the data.

5.2 Empirical Moments and Identification

Table 7 summarizes the data moments used for identification, their values and which parameters they help identify. While formally, all moments jointly identify all parameters, there is an intuitive mapping between moments and parameters.

We start with the demand-side parameters. Equation (7) says that log sales in the data is closely related to $\ln \lambda$ in the model, and as such the mean and standard deviation of log sales

[25] 250 is similar to the number of occupation categories in our data. Below, we show that the estimate of fixed outsourcing cost, f, has an intuitive interpretation (sub-section 5.3), and we also discuss the results with 10,000 tasks instead (sub-section 5.4).

[26] This assumption holds, e.g., if both the distribution of occupations in the data and the distribution of tasks in the model represent the same true distribution of tasks.

across buyer firms in the data[27] help identify $\mu_{\ln\epsilon}$ and $\kappa_{\ln\epsilon}$, which determine the mean and standard deviation of $\ln\lambda$ in the model. In addition, equation (16) says that ξ is the elasticity of $\ln\lambda$ with respect to $\ln\epsilon$, and so the regression coefficient of log sales on z_i, given in Table 4, column (1), helps identifies ξ.

We now move on to the supply-side parameters. In the model, for buyer firm i, the cost share of an in-house task ω relative to total in-house production cost is $c_i(\omega) = \phi_i^b(\omega)^{\sigma-1} / \int_{\omega^*}^1 \phi_i^b(\omega)^{\sigma-1} d\omega$, and so $\kappa_{\ln\phi}$, the standard deviation of $\ln\phi_i^b(.)$, determines the standard deviation of $\ln c_i(\omega)$ across tasks ω within i. On the other hand, $c_i(\omega)$ is the counterpart to the occupational wage-bill share of θ_{io} in the data, as given in equation (3). While we are unable to map individual $c_i(\omega)$ in the model into individual θ_{io} in the data, our main estimation assumption says that the mean, across i, of the standard deviation, across occupations o within i, of $\ln\theta_{io}$ provides an unbiased estimate of the mean of the standard deviation of $\ln c_i(\omega)$. As a result, the mean of the standard deviation of $\ln\theta_{io}$ helps identify $\kappa_{\ln\phi}$.

In the model, for foreign supplier j, the cost share of task ω in task production is $c_j(\omega) = \phi_j^s(\omega)^{\sigma-1} / \int_0^1 \phi_j^s(\omega)^{\sigma-1} d\omega$. Because ρ, the correlation coefficient of $\ln\phi_i^b(.)$ and $\ln\phi_j^s(.)$, determines the similarity between $\phi_i^b(.)$ and $\phi_j^s(.)$, as illustrated in Figure 5, ρ also determines the $cosim_i$ index for $c_i(\omega)$ and $c_j(\omega)$ in the model. On the other hand, $c_i(\omega)$ and $c_j(\omega)$ correspond to the occupational wage-bill shares of θ_{io} and θ_{jo} in the data.[28] As a result, the mean of the $cosim_i$ index of occupational similarity in the data helps identify ρ.

Finally, in the model, both f, the fixed per-task outsourcing cost, and $\mu_{\ln\phi}^s$, the foreign supplier's absolute advantage, determine the buyer firm's pattern of task trade, summarized in ω^*. Because ω^* is not observable in the data, we examine the buyer firm's labor share, LS, which is the share of the wage bill in total cost,

$$LS = \frac{s/\bar{m} + \omega^* f - Imp}{s/\bar{m} + \omega^* f} = 1 - \frac{Imp}{s/\bar{m} + \omega^* f}. \tag{17}$$

In equation (17), $s/\bar{m}$, or sale divided by markup, is total variable cost, $\omega^* f$ is the total fixed outsourcing cost, and Imp is the value of imported inputs. Equation (17) says that according to the model, LS is closely related to the pattern of task trade, ω^*. For example, LS is low for high-sales buyer firms because they import a lot relative to sales, and vice versa. Equation (17) also shows that f and $\mu_{\ln\phi}^s$ affect LS in different ways for high- vs low-sales firms. On the one hand, as f increases, LS tends to increase, but this effect is muted for high-sales buyer firms, for which s is large relative to f. On the other hand, as $\mu_{\ln\phi}^s$ increases, LS tends to decrease, and this effect is strong for both high- and low-sales buyer firms because it works directly on Imp, the value of imported inputs. Therefore, the mean labor share for large firms, defined as the top

[27] In the data, log sales may vary across industries and countries for reasons unrelated to the model. We have experimented with regressing log sales on country and 2-digit industry fixed effects and then using the residual to compute the mean and standard deviation. The results are very similar to Table 7.

[28] In our computation, we approximate $cosim_i$ by using $c_i(\omega)$ only, i.e. we implicitly assume that i's fixed outsourcing labor does not affect $cosim_i$. This implicit assumption is consistent with our estimation results below, which say that fixed outsourcing cost is small relative to total labor cost.

decile of the firm size distribution,[29] helps identify $\mu^s_{\ln\phi}$. The mean labor share for small firms, defined as the bottom decile of the firm size distribution, then helps identify both $\mu^s_{\ln\phi}$ and f.

5.3 Structural Estimates and Economic Significance of Labor Specialization

Parameter Estimates and the Baseline Scenario Table 7 also displays the estimated parameter values, and they say that the productivity advantage of the foreign supplier is, on average, around 5 percent ($\mu^s_{\ln\phi} = 0.046$), and that the foreign supplier tends to be productive in the same tasks that the buyer firm is also productive ($\rho = 0.91$). In addition, the estimated parameter values allow our model to exactly match all the targeted moments listed in the 2nd column of Table 7, including the mean log sales of 18.28, which is about 86.9 million DKK or 13.6 million USD. Therefore, the fixed outsourcing cost is about 1850 DKK or 300 USD per task ($f = 1851.26$). This implies that for the average buyer firm, the total fixed outsourcing cost to Denmark/Sweden is about 200,000 DKK or 31,400 USD, accounting for 1.1 percent of the total labor cost.[30] Together, the parameter estimates allow us to compute the buyer firms' optimal choices, and these model predictions are the baseline scenario of our simulation.

We continue our quantitative analyses by comparing the variation in labor productivity across buyer firms in the baseline scenario with the variation in the data, and report our results in Table 8. We see that, according to our model, the $VA/worker$ for the 90th percentile buyer firm is 30 percent higher than the 10th percentile firm, and that in the data, this difference is 50 percent. We also see that that the model cross-buyer-firm regression of log $VA/worker$ on log sales has the coefficient estimate of 0.029, and the same data regression has the estimate of 0.10.[31] Finally, we obtain similar results for the mean of the value-added-to-sales ratio and the regression of labor share on log sales.

We also gauge the size of scale economies in the baseline scenario by regressing log output quantity on log employment using simulated data. The estimated coefficient is 1.05, i.e. a doubling of employment yields 5 percent higher output per worker due to reallocation of tasks through outsourcing.[32] This result, together, with the results in Table 8, provide the first piece of evidence that the intensive margin mechanism of labor specialization along the global supply chain is economically significant.

The Scenario with Demand Shocks Next, we simulate the buyer firms' response to demand shocks and compare them with our empirical estimates from Tables 4 and 6. To be specific, for each buyer firm i, we draw a new demand shock, ϵ_i', using equation (16) and hold all the other

[29]The labor share in the data is consistent with the model because it is computed as wage bill relative to total cost (see section 2).

[30]In recent work, Antras et al. (2017) estimate that the fixed sourcing cost is 10,000-56,000 USD per source country for the median U.S. buyer firm. Halpern et al. (2015) find that the average Hungarian buyer firm's total fixed import cost, for accessing all products and all source countries, accounts for 2.2 percent of total production cost. While these estimates are based on different data and model than our estimate, they are qualitatively similar to ours and help place ours into context.

[31]The data regression is also illustrated in Figure 1.

[32]We did not run a similar data regression because it is difficult to measure output quantity. XXXX Is there a benchmark for the number of 1.05 from previous work XXXX

Table 8: The Variation in Labor Productivity, Baseline Scenario.

Moment	Data	Model
VA/L, P90/P10	1.50	1.30
VA/Sales (mean)	.32	.58
Regression of labor share on ln sales	-0.029	-0.028
Regression of ln VA/L on ln sales	0.10	0.029

Notes: The coefficient estimate of the regression of log VA/L on log sales is displayed in Figure 1, and that of labor share on log sales displayed in Figure 2.

Table 9: The Simulated Effects of Demand Shocks on Scale and Labor Productivity

	(1) $\Delta \ln Sales$	(2) $\Delta \ln Imports$	(3) $\Delta \ln \frac{VA}{Sales}$	(4) $\Delta \ln LaborShare$	(5) $\Delta \ln \frac{VA}{L}$
Actual Data:					
z	0.12***	0.13***	-0.028**	-0.045***	0.037***
Simulated Data:					
$\ln \epsilon$	0.12***	0.14***	-0.004***	-0.007***	0.003***
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)

Notes: Significance levels: $*p < 0.1, **p < 0.05, ***p < 0.01$.. The data regression coefficients are taken from Table 4, and the standard errors are shown over there. The model regressions all have 1000 observations, and the standard errors of their coefficient estimates are in parentheses. The dependent variables of the model regressions are the changes of the scenario with demand shocks relative to the baseline scenario.

parameters fixed. We then compute i's optimal choices of outcome variables under this new scenario with demand shocks, y_i', and run the following regression using simulated data:

$$\Delta \ln y_i = \alpha + \beta \Delta \ln \epsilon_i + \nu_i, \tag{18}$$

where ν_i is an error term and Δ denotes the change of y_i' and ϵ_i' relative to the baseline scenario. The lower panels of Tables 9 and 10 report the β estimates of regression (18) and the upper panels recap the estimates of the corresponding data regression of (5).

Column (1) of Table 9 re-iterates that, because the model targets the data regression coefficient for log sales, it is the same as the model regression coefficient. Column (2) shows that the model regression coefficient for log import values is also very similar to the data regression coefficient. This result is re-assuring, because the value of imports is not used as a target in the SMM estimation of our model. The rest of Table 9 show that the model regression coefficients are qualitatively similar to the data regression coefficients for log value-added-to-sales ratio, log labor share, and log value added per worker. These results corroborate our earlier results in Table 8 above. Table 10 reports the response of buyer firms' labor specialization. Columns (1) and (2) show the wage-bill share and relative wage-bill share of the top occupation, as

Table 10: The Simulated Effects of Demand Shocks on Labor Specialization

	(1) $\Delta\theta\left(o_i^{1*}\right)$	(2) $\Delta\theta\left(o_i^{2*}\right)$	(3) $\Delta cosim_i$	(4) $\Delta bisim_i$
Actual Data:				
z	0.014*	0.019***	-0.016**	-0.010**
Simulated Data:				
$\ln \epsilon$	0.005***	0.005***	-0.001***	-0.006**
	(0.01)	(0.01)	(0.01)	(0.00)

Notes: Significance levels: $*p < 0.1, **p < 0.05, ***p < 0.01$.. The dependent variables in columns (1)-(4) are defined in equations (1), (2), (3) and (4), respectively. The coefficient estimates with actual data are taken from Table XXXX, and the standard errors are shown over there. The coefficient estimates with simulated data have 1000 observations, and their standard errors are in parentheses. The dependent variables of the model regressions are the changes of the scenario with demand shocks relative to the baseline scenario.

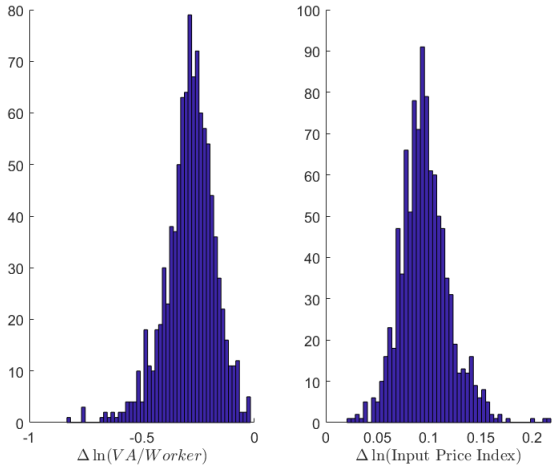
defined in equations (1) and (2), respectively. Column (3) shows the *cosim* index of occupation similarity, and column (4) shows the binary version of *cosim*, as defined in equation (4). Table 10 shows that the model regression coefficients are all qualitatively similar to the data regression coefficients.

Tables 9 and 10 also show that often times, the model regression coefficients are 1/10 to 1/5 the size of those from data regressions. Intuitively, this is because our model focuses on the single mechanism of labor specialization along the global supply chain and so abstracts away from other sources of labor specialization and labor productivity, which are captured in the data regression coefficients. Consider, for example, $\Delta cosim$, the change in the occupational similarity index, in column (3) of Table 10.[33] The data benchmark we have used, from column (5) of Table 6, is conservative, because the estimate from column (6) of the same table, -0.01, more closely matches the model setting by keeping the foreign suppliers' wage-bill shares unchanged. In addition, the buyer firm may also increase division of labor among its own in-house production tasks, which our model has abstracted away. Consider, as another example, the result for value added per worker, in column (5) of Table 9. The data regression coefficient likely captures other sources of scale economies, such as learning-by-doing, access to additional foreign suppliers from 3rd countries and access to more product varieties from 3rd countries, but these sources are not part of our model. Therefore, our interpretation of the results in Tables 9 and 10 is that they provide the second piece of evidence for the economic significance of the intensive margin mechanism of labor specialization along the global supply chain, but do not preclude the significance of other sources of labor specialization and scale economies outside of our model.

The Scenario with Autarky For the counterfactual analyses, we make outsourcing prohibitively expensive by setting $f = +\infty$ in the model and holding all the other parameters fixed.

[33] $\Delta cosim$ is a local property of the model, while the mean of *cosim*, which we discussed in Table 7 and section 5.2, is a global property.

Figure 6: The Scenario with Autarky and Gains from Outsourcing



Notes: The weighted mean uses employment as weights. The changes in log value added per worker and log input price index are the scenario with autarky relative to the baseline scenario.XXXX

In this new scenario with autarky, the buyer firms differ only in scale; there is no variation in labor productivity or input price index.

The left panel of Figure 6 shows the distribution of the change in log value added per worker relative to the baseline scenario, where the y-axis shows the number of buyer firms. Interestingly, the fall in efficiency is highly heterogeneous across firms. On the one hand, the change in labor productivity is small for firms initially not outsourcing many tasks. On the other hand, the change is big for firms highly exposed to outsourcing. These results corroborate our earlier result that there is substantial variation in value added per worker in the baseline scenario (e.g. Table 8). The upper panel of Table 11 shows that for the average buyer firm, log value added per worker would decrease by 0.29 log points, or 34 percent.

We next report the change in log input price index relative to the baseline scenario, which measures the buyer firms' gains from outsourcing, as discussed in sub-section 4.2 above. The right panel of Figure 6, the histogram of the outsourcing gains, shows that there is substantial heterogeneity across buyer firms. The upper panel of Table 11 shows that for the average buyer firm, the input price index would increase by 0.096 log points, or 10 percent.

Hence, our results point to relatively large efficiency gains due to labor specialization along the global supply chain, and provide the third piece of evidence for the economic significance of this mechanism. They also complement previous work on the economic significance of new supplier links in the GVC literature. For example, the counterfactual in Antras et al. (2017) shows that if all foreign supplier links were lost, the average U.S. buyer firm would experience a 9% increase in marginal cost.

Table 11: The Scenario with Autarky and Gains from Outsourcing

	Mean	Weighted mean	Median	St. dev.
Benchmark Model, $\rho = 0.91$				
$\Delta \ln(VA/worker)$	-0.29	-0.30	-0.28	0.11
$\Delta \ln$(input price index)	0.096	0.11	0.094	0.024
The Model with $\rho = 0$				
$\Delta \ln(VA/worker)$	-0.26	-0.30	-0.26	0.11
$\Delta \ln$(input price index)	0.24	0.25	0.24	0.062

Notes: The weighted mean uses employment as weights.XXXX

5.4 Alternative Specifications and Robustness

We end this section by performing the following "what if" analyses and comparing the results with the benchmark model from the previous sub-section. We discuss our results briefly below and relegate additional details to Appendix XXXX.

Zero Correlation To highlight the contribution of our unique data to our quantitative analyses, we suppose that we do not have the data for detailed occupations of both the buyer firm and their foreign suppliers. This means that the *cosim* similarity index is unavailable and we need to set the value of ρ, the correlation coefficient of the buyer firm's and foreign supplier's task productivity. We choose the specification of zero correlation, or $\rho = 0$.

We re-estimate the remaining six parameters in Table 7. The model predicted mean *cosim* index, 0.07, is very different from the data mean of 0.45, which is not surprising, but otherwise, the model exactly matches the other targeted moments in Table 7. We then repeat the autarky counterfactual analyses using the model with $\rho = 0$ and summarize the results in the lower panel of Table 11.

Relative to the benchmark model, with $\rho = 0.91$, the results for the change of log value added per worker are similar. Intuitively, the model with $\rho = 0$ also matches the mean labor shares of both high- and low-sales buyer firms, and so generates similar variation in labor productivity. The results for outsourcing gains, however, are very different. With $\rho = 0$, the model says that the average buyer firm's outsourcing gains would be 0.24 log points or 27 percent, and this is 2.5 times as large as the benchmark model. Intuitively, the assumption of $\rho = 0$ exaggerates the difference between the buyer firm's and foreign supplier's task productivity, as illustrated in Figure 5, and according to equation (11), this leads to exaggerated outsourcing gains. These results show that our unique data, including the *cosim* similarity index, play an important role in our quantitative analyses.

More Tasks in Simulation We now increase the number of tasks in the SMM estimation from 250 in the benchmark model to 10,000. We find that, while the estimate for fixed outsourcing cost changes to $f = 45.7$ DKK or 7.1 USD *per task*, the *total* outsourcing cost of the average buyer

firm remains similar, at about 193,000 DKK or 30,200 USD. In other words, the specification with 10,000 tasks substantially shrinks each individual task in the simulation but has small effects on the pattern of task trade. On the other hand, the values of the other model parameters are similar to the benchmark; e.g. $\rho = 0.93$ and $\kappa_{\ln \phi} = 0.51$. These results show that our quantitative analyses are robust to the number of tasks specified.

Fewer Model Parameters Finally, one might be concerned that we have too many parameters in the benchmark model. We thus experiment with a more parsimonious specification by setting $\mu_{\ln \epsilon} = 0$ in equation (16) in the SMM estimation and dropping the mean of log sales from the list of targeted moments. We find that $f = 0.009$ and the average buyer firm's total outsourcing cost is 0.98. These estimates are not intuitive because we don't know their units without matching the mean log sales in the data.[34] On the other hand, the values of the other model parameters are similar to the benchmark; e.g. $\mu_{\ln \phi} = 0.046$ and $\rho = 0.91$.

In summary, our quantitative analyses in this section have shown that the estimates of key model parameters are intuitive, that the mechanism of labor specialization along the global supply chain, or the intensive margin, is economically significant, and that our unique data play an important role in our results. By utilizing the moments of occupations in the data, our estimation procedure does not require the mapping of individual tasks in the model to individual occupations or products in the data. We hope that this approach might be useful for future work.

6 Additional Empirical Evidence

In this section, we use our novel linked worker-firm data from Denmark and Sweden to provide additional empirical evidence for our model. We first clarify that, because of high relationship specificity in our sample (section 2), the empirical results from section 3 are driven by relationship-specific buyer firms. We then show that these buyer firms' labor reallocation in response to exogenous demand shocks is consistent with the predictions of our model.

6.1 Relationship-specific Buyer Firms

Recall, from sub-section 2.3, that relationship specificity, based on Nunn (2007)'s measure, is high among the buyer-supplier relationships between Danish and Swedish firms. In this sub-section, we explore whether, and how, relationship specificity matters for the effects of exogenous demand shocks on the buyer firms' size, labor productivity, and labor specialization.

We split the sample used for Tables 4 and 6 as follows. The relationship-specific sub-sample consists of the buyer firms with $rs_i > 0.68$, the 75th percentile of the differentiated-goods share in Nunn (2007)'s data, and the generic-inputs sub-sample consists of the remaining buyer firms. We then estimate the same regressions as in Tables 4 and 6 by sub-sample, and report the

[34] The model predicted mean log sales is 6.05. Again, it is hard to interpret this result because we do not know its unit.

results in Tables 12 and 13, where panel A (B) shows the results for the relationship-specific (generic-inputs) sub-sample. We see that the vast majority, or more than four out of five, of the buyer firms in our sample are in the relationship-specific sub-sample.

Columns (1)-(2) of Table 12 show that the buyer firms in both sub-samples grow in sales and employment in response to positive exogenous demand shocks. Column (3) shows that the two sub-samples also exhibit similar responses in labor share, or the share of wage costs in total costs. Columns (4) and (5) show that the buyer firms in the relationship-specific sub-sample increase log value added per worker and log TFP when demand increases exogenously, and these results are similar to the results for the full sample in Table 4. On the other hand, the results for log value added per worker and log TFP are statistically insignificant for the generic-inputs sub-sample.

Table 12: Expansion and Scale Economies by Relationship Specificity Sub-samples

	(1) $\Delta \ln Sales$	(2) $\Delta \ln L$	(3) $\Delta \ln LaborShare$	(4) $\Delta \ln \frac{ValueAdded}{L}$	(5) $\Delta \ln TFP$
Panel A Relationship Specific ($rs_i > 0.68$)					
z	0.109***	0.054***	-0.043***	0.043***	0.024*
	(0.022)	(0.016)	(0.014)	(0.015)	(0.013)
Observations	13954	13954	13841	13547	10816
Panel B Generic Inputs ($rs_i < 0.68$)					
z	0.189***	0.091**	-0.053*	-0.001	0.024
	(0.061)	(0.043)	(0.031)	(0.033)	(0.031)
Observations	2671	2671	2641	2609	2246

Panel A of Table 13 shows that the relationship specific buyer firms increase the wage-bill shares of their top occupations and decrease the similarity of their occupational structure relative to their foreign suppliers, and these results are similar to Table 6. On the other hand, panel B of Table 13 shows that the coefficient estimates of the generic-inputs buyer firms have the same signs as in panel A, but they lack statistical significance.

Table 13: Between-Firm Labor Specialization by Relationship Specificity Sub-sample

	(1) $\Delta\ \theta(o_i^{1*})$	(2) $\Delta\ \theta(o_i^{2*})$	(3) Similarity	(4) Similarity (binary)
Panel A Relationship Specific ($rs_i > 0.68$)				
z	0.013	0.019***	-0.015**	-0.010**
	(0.009)	(0.007)	(0.008)	(0.004)
Observations	13954	13954	13954	13954
Panel B Generic Inputs ($rs_i < 0.68$)				
z	0.024	0.018	-0.017	-0.008
	(0.019)	(0.017)	(0.017)	(0.012)
Observations	2671	2671	2671	2671

Overall, the results in Tables 12 and 13 show that the earlier results from Tables 4 and 6 are driven by relationship-specific buyer firms, given that they account for most of the observations in our sample. These results clarify that our work fits in well with the GVC literature, where relationship specificity plays an important role.

6.2 Within-firm Labor Reallocation

Next, we note that a key mechanism of our quantitative model is that when output demand increases, the buyer firm allocates labor away from in-house production towards fixed outsourcing cost. In this sub-section, we explore whether these model predictions are consistent with our data.

We start with the model prediction that the buyer firm allocates labor away from in-house production. The management-layers literature (e.g. Caliendo et al. 2015) shows that the lowest-layer workers are present in almost all plants and firms, and it also provides these workers' occupation codes. We thus interpret these workers as primarily related to in-house production, and map their occupation codes to those in our data. We then estimate regression (5) with the change in the wage-bill or employment share of the production occupations as the dependent variable.

Column 1 of Table 14 reports the results for the change in wage-bill share and column 2 reports those for the change in employment share. In panel A, we include all observations, and obtain negative and statistically significant coefficient estimates for both wage-bill and employment shares. In panel B, we find that the coefficient estimates remain negative and significant for the relationship-specific sub-sample. Panel C shows that the coefficient estimates are statistically insignificant for the generic-inputs sub-sample. These results provide clear evidence that relationship-specific buyer firms allocate labor away from in-house production, as consistent with our quantitative model.

Table 14: Within-Firm Labor Reallocation: Production Occupations

	(1) Δ Wage-bill share	(2) Δ Employment share
Panel A: All		
z	-0.012**	-0.011**
	(0.006)	(0.005)
Observations	16788	16788
Panel B Relationship Specific ($rs_i > 0.68$)		
z	-0.016***	-0.016***
	(0.006)	(0.006)
Observations	13954	13954
Panel C Generic Inputs ($rs_i < 0.68$)		
z	0.009	0.017
	(0.013)	(0.013)
Observations	2671	2671

Table 15 examines the model prediction that the buyer firm allocates labor towards outsourcing costs. We measure outsourcing-costs labor by using college-educated workers, because previous studies, such as Matsuyama (2007), have hypothesized that trade costs, including outsourcing costs, are skilled-labor intensive relative to in-house production. Column 1 of Table 15 reports the results for the change in the wage-bill share of college-educated workers, and column 2 reports the change in employment share. With all observations included, in panel A, we obtain positive coefficient estimates, but their statistical significance is marginal at best. With relationship-specific buyer firms only, in panel B, however, we obtain statistical significance for the wage-bill share, and the coefficient estimates remain positive, and become larger in magnitude. In panel C, with generic-inputs buyer firms, we obtain statistically insignificant coefficient estimates. These results suggest that relationship-specific buyer firms allocate labor towards outsourcing costs, as consistent with our quantitative model.[35]

[35]Our results in Table 15 and earlier ones in Table 4 show that the buyer firms become more skilled-labor intensive and also achieves higher labor productivity. This positive correlation between changes in skill and productivity over time complements previous studies with the productivity-skill-complementarity hypothesis, such as Burstein and Vogel (2017).

Table 15: Within-Firm Labor Reallocation: College-Educated Workers

	(1) Δ Wage-bill share	(2) Δ Employment share
Panel A: All		
z	0.007*	0.005
	(0.004)	(0.004)
Observations	16788	16788
Panel B: Relationship Specific ($rs_i > 0.68$)		
z	0.009**	0.008*
	(0.004)	(0.004)
Observations	13954	13954
Panel C: Generic Inputs ($rs_i < 0.68$)		
z	-0.011	-0.016*
	(0.010)	(0.010)
Observations	2671	2671

Note that the in-house-production labor and outsourcing-cost labor in our model do not exactly map into the production workers and college educated workers in Tables 14 and 15. For example, college educated engineers might be involved in in-house production as well. As a result, unlike the regressions in Tables 4 and 6, those in Tables 14 and 15 do not have corresponding model regressions, and so the empirical evidence in Tables Tables 14 and 15 is qualitative in nature.

Summarizing this section, we have shown that in response to positive exogenous demand shocks, the relationship-specific buyer firms increase value added per worker, allocate labor away from in-house production towards outsourcing costs, and their occupational structures grow apart from their foreign suppliers'. These results are consistent with the key mechanisms of our quantitative model. In addition, while relationship specificity is a theoretical assumption that is commonly used in the literature of global supply chains, the empirical evidence for how relationship specificity matters at the micro level is more limited. Our results in this section provide novel empirical evidence that relationship specificity, as measured by Nunn (2007), matters for productivity, within-firm labor allocation, and between-firm labor specialization for the global supply links between Denmark and Sweden.

7 Conclusion

In this paper, we put together a novel dataset, where the universe of cross-border transactions between Danish and Swedish firms is merged with the administrative matched worker-firm data from both countries. This novel data allows us to explore labor specialization across firms, labor

productivity, relationship specificity, and within-firm labor allocation along international supply chains.

To be specific, we first provide novel empirical evidence that when output demand increases exogenously, the buyer firm experiences an increase in labor productivity but a decrease in occupational similarity with foreign suppliers, which suggests increased labor specialization. Second, we propose a simple model with task specialization and outsourcing that is broadly consistent with the evidence, in which the buyer firm substitutes its own labor used in in-house production for imported inputs when output demand increases exogenously. Third, we use our novel data to structurally estimate key model parameters, and our quantification and counterfactual show that for the buyer firms with foreign suppliers, labor specialization is a quantitatively important economic mechanism for higher labor productivity and lower input costs. For example, our model simulation accounts for a substantial portion of the cross-section variation of value-added per worker in the data, and that in the absence of labor specialization and outsourcing, the average buyer firm's labor productivity would be 34 percent lower and the input price price index 10 percent higher. Finally, we show that our main empirical results are driven by the relationship-specific buyer firms, which account for the majority of our sample. They also decrease the wage-bill and employment shares of production-related workers but increase the shares of college graduated workers, and these results provide additional empirical evidence for the key mechanisms of our quantitative model.

In our work, we have used data of international supply chains for our empirical analyses. Our quantitative model, on the other hand, is equally applicable to labor specialization with domestic suppliers, and so could be useful to the study of domestic supply chains, especially if the domestic-supply-chains data can be merged with matched worker-firm data in future research.

Finally, our study emphasizes the substitution margin between the buyer firm's own in-house production labor and imported inputs, and so complements the existing GVC literature's emphasis on the margin of which suppliers to choose. It could be fruitful for future research to incorporate both substitution margins in the same framework.

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Appendix

A Data

Table 16: Data Sources

	Denmark		Sweden	
Variable name (abb.)	Data source	Original variable name	Data source	Original variable name
Total sales (*Sales*)	Firm Statistics Register (FIRM)	gf_oms	The VAT Register (MOMS)	oms_exkl_moms
Employment in full-time equivalents (L)	Firm Statistics Register (FIRM)	gf_aarsv	Structural Business Statistics (FEK)	AntalAnstallda
Value added (*ValueAdded*)	Firm Statistics Register (FIRM)	gf_vtv	Structural Business Statistics (FEK)	ForadlingsVarde
Wage costs (-)	Firm Statistics Register (FIRM)	gf_lgagmv	Structural Business Statistics (FEK)	SummaPersonalkostnader
Total costs (-)	Firm Statistics Register (FIRM)	gf_kob + gf_lgagmv	The VAT Register (MOMS) & Structural Business Statistics (FEK)	M20_25_InkopVarEG+M21_InkopTjaEG+M22_InkopTjaUtEG+ M50_MomsUlagImport+M23_InkopVarSverige+M24_InkopTjaSverige+ SummaPersonalkostnader
Capital (-)	Firm Account Statistics (FIRE)	iaat+grby+atam+fmaa+aadi	Structural business statistics (FEK)	SummaEgetKapitalInklHBKBEnskilda
Materials (-)	Firm Account Statistics (FIRE)	krhe+fvv (-2016), frhe+fvv (2017-)	Structural business statistics (FEK)	KostnaderForRavaror
Industry (s)	Firm Statistics Register (FIRM)	gf_nace_db07 (first two digits)	Structural business statistics (FEK)	bransch_sni2007 (first two digits)
3rd-country exports (-)	Statistics for International Trade in Goods (uhdm)	vrd	Foreign trade - exports and imports of goods (UHV)	vrd
National occupation code (-)	Employment Statistics for Employees (BFL)	vmo_disco_kode	Longitudinal integrated database for health insurance and labour market studies (LISA)	Ssyk4_J16 (-2013), SSyk4_2012_J16 (2014-)
Occupation-specific wage bill (WL)	Employment Statistics for Employees (BFL)	ajo_smalt_loenbeloeb	Longitudinal integrated database for health insurance and labour market studies (LISA)	loneink
Workers' education (-)	The Attainment register (UDDA)	hfaudd (aggregated by level)	Longitudinal integrated database for health insurance and labour market studies (LISA)	Sun2000niva

Notes: Labor share is defined as Wage costs divided by Total costs. Capital and Materials are used for TFP estimation only. National occupation codes are used to construct ISCO+ codes as described in Appendix X.

B Occupation Codes

Occupation codes in Denmark and Sweden:

- Denmark: Danish occupation codes, DISCO-08, have been in use since 2010 (that is, during our full sample period). They are a 6-digit extension of 4-digit ISCO-08. Without further ado we collapse 6-digit DISCO-08 to 4-digit ISCO-08.
- Sweden: Swedish occupation codes changed between 2014 and 2015 from SSYK96 to SSYK12. The latter are still in use. SSYK96 (SSYK12) are 4-digit codes based on, but different from, ISCO-88 (ISCO-08).

To examine questions like "do Danish sellers share occupation codes with their Swedish buyers?" we need a set of occupation codes that are both consistent over time and comparable across Denmark and Sweden.

B.1 Aligning Swedish occupation codes over time

We wish to translate SSYK96 codes into SSYK12 codes. SCB provides an official many-to-many key. This means that for some SSYK96 we have multiple SSYK12 "candidates". We need a rule to choose between these candidates (in other words, we wish to collapse the many-to-many key to a many-to-one key). The rule is defined as follows:

1. Consider all employees working in the same firm in 2013 (the last year of SSYK96) and 2014 (the first year of SSYK12).
2. Disregard employees with missing occupation code in either year (after simple imputation using data for the adjacent years, 2012 and 2015). This means that all employees are associated with a SSYK96-SSYK12 pair.
3. Disregard employees with SSYK96-SSYK12 pairs not in accordance with the many-to-many key. For each SSYK96, choose the SSYK12 with the most observed SSYK96-SSYK12 pairs.

B.2 Aligning occupation codes across Denmark and Sweden

We wish to compare Danish (ISCO08) and Swedish (SSYK12) occupation codes. SCB provides an official many-to-many key. This means that for some SSYK12 we have multiple ISCO08 candidates. Unlike above, we have no data to help us form a rule between these candidates. Instead, we construct an occupation grouping, call it ISCO+, as the most dis-aggregate grouping that ensures

1. there exists at many-to-one key between ISCO08 and ISCO+, and
2. there exists at many-to-one key between SSYK12 and ISCO+.

This is akin to the so-called CN8+ product classification, suggested by Van Beveren et al. (2012), which ensures a many-to-one mapping between all year-specific CN classifications and CN8+.

Table 17: Aggregate statistics for occupation codes, Sweden

Digits	Four		Three		Two		One	
	t=0	t=1	t=0	t=1	t=0	t=1	t=0	t=1
Panel A: SSYK96								
#Occ codes	355	.	113	.	27	.	10	.
Non-mis share	94	.	94	.	94	.	94	.
Top share	6	.	8	.	13	.	22	.
Panel B: SSYK12								
#Occ codes	289	429	91	148	24	46	10	10
Non-mis share	94	91	94	91	94	91	94	91
Top share	6	6	8	7	16	10	22	20
Panel C: ISCO+								
#Occ codes	179	207	30	36	3	4	2	2
Non-mis share	94	91	94	91	94	91	94	91
Top share	33	32	80	83	100	100	100	100

Note: "Non-mis share" is share of workforce with observed occupation code (in percent). "Top share" is the share of the workforce in the largest occupation code (in percent).

Table 18: Aggregate statistics for occupation codes, Denmark

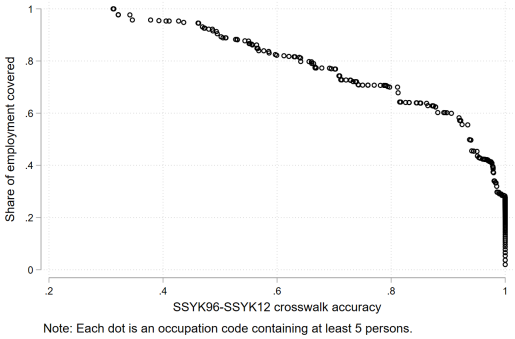
Digits	Four		Three		Two		One	
	t=0	t=1	t=0	t=1	t=0	t=1	t=0	t=1
Panel A: ISCO08								
#Occ codes	539	499	172	164	52	52	10	10
Non-mis share	95	96	95	96	95	96	95	96
Top share	10	10	11	10	13	15	16	17
Panel B: ISCO+								
#Occ codes	202	204	36	36	4	4	2	2
Non-mis share	93	96	94	96	95	96	95	96
Top share	33	34	73	76	100	100	100	100

Note: "Non-mis share" is share of workforce with observed occupation code (in percent). "Top share" is the share of the workforce in the largest occupation code (in percent).

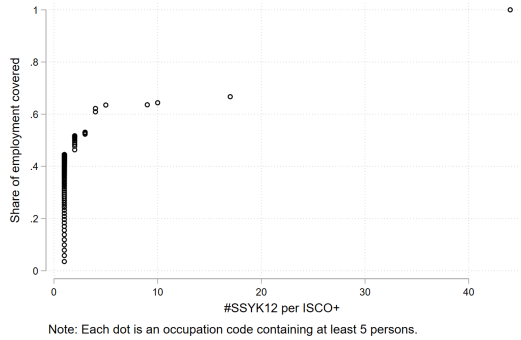
Figure 7: Occupation Code Concordance



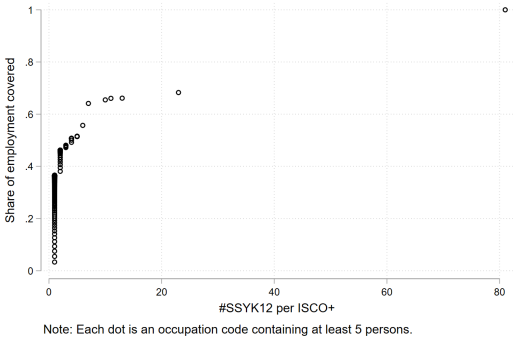
(a) SSYK96 to SSYK12



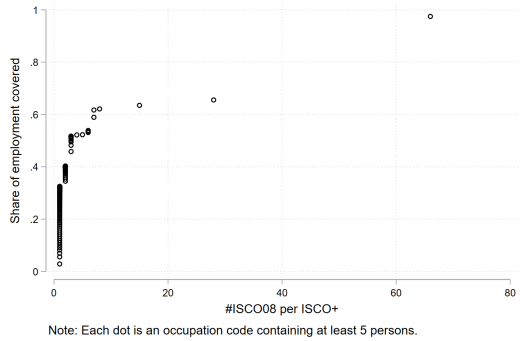
(b) SSYK96 to SSYK12



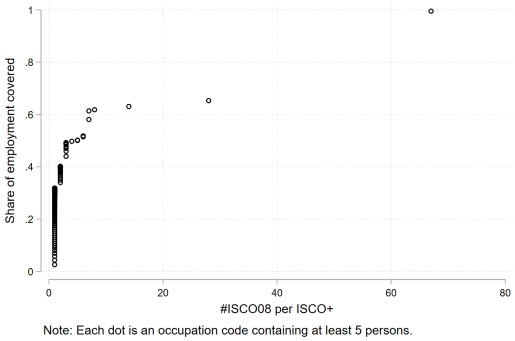
(c) SSYK12 to ISCO+, t=0



(d) SSYK12 to ISCO+, t=1



(e) ISCO08 to ISCO+, t=0



(f) ISCO08 to ISCO+, t=1

Notes: In all subfigures, except subfigure (b), occupation codes are ordered according to the number of "observed occupation codes" per "imputed occupation code", and we plot the empirical CDF of employment. In subfigure (b), we rank SSYK12 occupation codes according to the share of employees with the chosen SSYK12 among the candidate SSYK12's, and we plot the empirical CDF of employment.

Table 19: Top ISCO08 codes, Denmark

Rank	ISCO08 code	Employment share (%)	Description
Period t=0			
1	5223	11.5	Shop Sales Assistants
2	9334	5.7	Shelf Fillers
3	4110	3.6	General Office Clerks
4	3322	2.8	Commercial Sales Representatives
5	8160	2.7	Food and Related Products Machine Operators
Period t=1			
1	5223	10.7	Shop Sales Assistants
2	9334	4.0	Shelf Fillers
3	4110	3.8	General Office Clerks
4	3322	3.5	Commercial Sales Representatives
5	5230	3.5	Stall and Market Salespersons

Table 20: Top SSYK12 codes, Sweden

Rank	SSYK12 code	Employment share (%)	Description
Period t=0			
1	5223	5.1	shop sales, specialty stores
2	3322	4.7	Commercial sales representatives
3	4322	3.7	Warehouse and terminal Staff
4	4119	3.7	Office clerks not elsewhere classified
5	9120	2.8	Car reconditioning labourers, window cleaners and other hand cleaning workers
Period t=1			
1	5223	5.1	shop sales, specialty stores
2	4322	5.0	Warehouse and terminal Staff
3	3322	4.3	Commercial sales representatives
4	7223	3.3	Machine-tool operators
5	9629	2.6	Other service workers not elsewhere classified

Table 21: Top ISCO+ codes, Denmark

Rank	ISCO+	Emp. share (%)	#ISCO08	Top ISCO08	Emp. share (%), top ISCO08	Description, top ISCO08
Period t=0						
1	A	33.2	67	5223	34.7	Shop Sales Assistants
2	B	5.6	3	9334	98.1	Shelf Fillers
3	C	5.3	7	3322	53.4	Commercial Sales Representatives
4	D	2.7	1	8160	100	Food and Related Products Machine Operators
5	E	2.6	28	9313	18.9	Building Construction Labourers
Period t=1						
1	A	34.8	68	5223	30.7	Shop Sales Assistants
2	C	6.3	7	3322	55.2	Commercial Sales Representatives
3	B	4.2	3	9334	96.6	Shelf Fillers
4	E	2.8	28	9313	15.8	Building Construction Labourers
5	F	2.5	7	2512	52.4	Software Developers

Table 22: Top ISCO+ codes, Sweden

Rank	ISCO+	Emp. share (%)	#SSYK12	Top SSYK12	Emp. share (%), top SSYK12	Description, top SSYK12
Period t=0						
1	A	33.9	44	5223	15.0	shop sales, specialty stores
2	B	6.6	4	3322	71.7	Commercial sales representatives
3	C	3.1	17	7111	20.0	Woodworkers, carpenters
4	D	2.7	1	2512	100	Software- and system developers
5	E	2.4	1	7232	100	Aircraft mechanics and repairers
Period t=1						
1	A	32.6	81	5223	15.5	shop sales, specialty stores
2	B	7.0	7	3322	61.1	Commercial sales representatives
3	E	3.3	1	7232	100	Aircraft mechanics and repairers
4	D	3.3	6	2512	63.3	Software- and system developers
5	C	2.8	23	7111	18.0	Woodworkers, carpenters

C Additional Results

C.1 Controlling for WID shocks to suppliers

The following table is identical to Table 4 in the main text, but add suppliers' (buyers') average WID shock as a control: $\overline{z}_i = \sum_j w_{ij,t=0} z_j$, where $w_{ij,t=0}$ is export (import) share to supplier (from buyer) j. For suppliers and buyers that are not 3rd country exporters, we assign $z_j = 0$. For focal firms without buyers, we assign $\overline{z}_i = 0$.

Table 23: Same as Table 4 in main text, but controlling for suppliers' and buyers' average WID shock

	(1) Δ ln Sales	(2) Δ ln L	(3) Δ log(imports from DK/SE)	(4) Δ ln ValueAdded / Sales	(5) Δ ln(LaborShare)	(6) Δ ln ValueAdded / L	(7) $\Delta \ln TFP$
Panel A: Controlling for average shock to suppliers							
z	0.120***	0.059***	0.141**	-0.027**	-0.045***	0.037***	0.030**
	(0.020)	(0.015)	(0.056)	(0.013)	(0.013)	(0.014)	(0.012)
$\overline{z}$	0.042	0.012	-0.142	-0.051**	0.010	-0.014	-0.022
	(0.030)	(0.028)	(0.098)	(0.022)	(0.021)	(0.027)	(0.024)
Panel B: Controlling for average shock to buyers							
z	0.119***	0.059***	0.139**	-0.029**	-0.045***	0.034**	0.030**
	(0.020)	(0.015)	(0.056)	(0.013)	(0.013)	(0.014)	(0.012)
$\overline{z}$	0.059*	0.016	-0.042	0.039	-0.002	0.083**	-0.018
	(0.033)	(0.027)	(0.109)	(0.030)	(0.021)	(0.035)	(0.031)
Observations	16788	16788	16788	16297	16628	16297	13074

C.2 Supplier-level regressions

Consider the regression equation

$$\Delta \ln y_j = \alpha_{s(j)} + \gamma_{r(j)} + \delta z_j + \theta \bar{z}_j + \varepsilon_j \tag{19}$$

This differs from equation (5) in the main text in two regards. First, the unit of observation is a supplier j rather than a focal firm i. Second, we add an additional regressor $\bar{z}_j = \sum_i w_{ij} z_i$, where w_{ij} is the share of j's DK/SE exports in period 0 allocated to i and z_i is the focal firm's WID shock as defined in the main text. Hence, $\bar{z}_j$ measures the shock received by the supplier through its buyers (the focal firms). z_j is j's WID shock and enters as a control variable. Note we haven't restricted suppliers to be 3rd country exporters. If j was not a 3rd country exporter in period 0 then it didn't receive a WID shock. In this case, we assign $z_j = 0$.

In the draft, the only sampling requirement we impose on suppliers is that they report positive total sales and employment in both periods 0 and 1. In Table 1, however, we impose an additional requirement: Suppliers must have some DK/SE buyers in period 0. This is necessary for constructing $\bar{z}_j$.

Table 24: Supplier-level regressions. Suppliers are required to have DK/SE buyers in period 0.

	(1) Δ ln Sales	(2) Δ ln L	(3) Δ log(exports to DK/SE)	(4) Δ ln ValueAdded / Sales	(5) Δ ln(LaborShare)	(6) Δ ln ValueAdded / L
$\bar{z}$	0.099***	0.046**	0.218***	0.051***	0.007	0.101***
	(0.025)	(0.021)	(0.080)	(0.017)	(0.015)	(0.021)
z	0.105***	0.054**	0.052	-0.021	-0.071***	0.056***
	(0.029)	(0.022)	(0.082)	(0.018)	(0.020)	(0.020)
Observations	18118	18118	13069	17568	17935	17568

Note from the previous table that columns (1) and (2) are based on more observations than column (3) (the slightly smaller number of observations in 4-6 are due to some missing outcome, similar to the main text). In the next table, we restrict the sample to suppliers with DK/SE buyers in both periods 0 and 1. Now columns 1-3 are based on the same sample.

Table 25: Supplier-level regressions. Suppliers are required to have DK/SE buyers in both periods 0 and 1.

	(1) Δ ln Sales	(2) Δ ln L	(3) Δ log(exports to DK/SE)	(4) Δ ln ValueAdded / Sales	(5) Δ ln(LaborShare)	(6) Δ ln ValueAdded / L
$\bar{z}$	0.089***	0.031	0.218***	0.063***	0.001	0.119***
	(0.025)	(0.022)	(0.080)	(0.020)	(0.017)	(0.024)
z	0.140***	0.071***	0.052	-0.018	-0.087***	0.069***
	(0.028)	(0.020)	(0.082)	(0.022)	(0.018)	(0.023)
Observations	13069	13069	13069	12756	12959	12756

C.3 Robustness: Excluding smallest firms

Table 26: Same as Table 4 in main text, but dropping 5% smallest firms

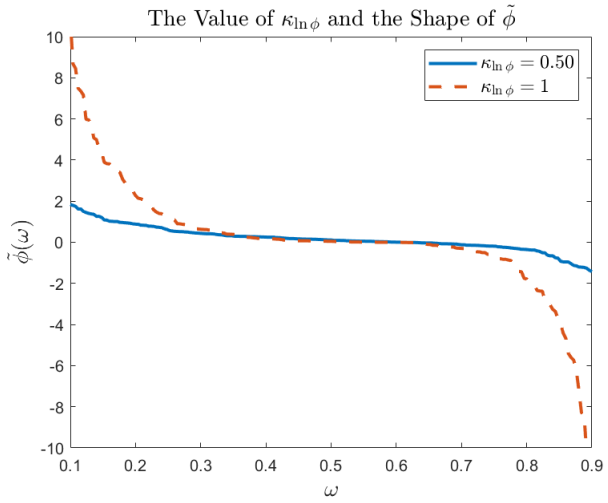
	(1) Δ ln Sales	(2) Δ ln L	(3) Δ ln DK/SE Imp	(4) Δ ln ValueAdded / Sales	(5) Δ ln(LaborShare)	(6) Δ ln ValueAdded / L	(7) $\Delta \ln TFP$
z	0.120***	0.067***	0.174***	-0.028**	-0.045***	0.029**	0.021*
	(0.021)	(0.015)	(0.057)	(0.013)	(0.013)	(0.014)	(0.012)
Observations	15750	15750	15750	15314	15612	15314	12355

Notes: For Danish and Swedish firms, separately, we drop the 5% smallest firms measured by the number of (full-time equivalent) employees measured in period 0.

D Theory

$\kappa_{\ln\phi}$ and the Shape of the $\tilde{\phi}(.)$ Curve Figure 8 illustrates how $\kappa_{\ln\phi}$ affects the shape of the $\tilde{\phi}(.)$ curve. The solid $\tilde{\phi}(.)$ curve is the same as in Figure 5 in the text, and corresponds to $\kappa_{\ln\phi} = 0.50$, the estimated value that we obtain from section 5. The dashed $\tilde{\phi}(.)$ curve, on the other hand, has $\kappa_{\ln\phi} = 1$. Comparing the solid curve with the dashed one, we see that as $\kappa_{\ln\phi}$ increases, the $\tilde{\phi}(.)$ curve rotates clockwise. This feature is the same as in Figure 5. In addition, we also see that the rotation is very prominent on both tails of the range of tasks but muted in the middle. This feature is distinct from Figure 5.

Figure 8: $\kappa_{\ln\phi}$ and the Shape of the $\tilde{\phi}(.)$ Curve



Notes: XXXX We have limited the range of tasks to $[0.1, 0.9]$ on the x-axis and the scale of the y-axis to $[-10, 10]$ for good visualization. The scale of the y-axis is different from Figures 4, **??** and 5 in the text. XXXX

Proof of Proposition 1 Assume that equation (12) has an interior solution such that $0 < \omega^* < \bar{\omega}$, where $\tilde{\phi}(\bar{\omega}) = 0$, i.e., not all offshorable tasks are outsourced.

Differentiate equation (12),

$$\frac{\partial\tilde{\phi}}{\partial\omega^*}\frac{\partial\omega^*}{\partial\lambda} = -\gamma f\left(\bar{m}\right)^{\gamma-1}\lambda^{-2}$$
$$\frac{\partial\omega^*}{\partial\lambda} = -\frac{\tilde{\phi}\left(\omega^*\right)}{\frac{\partial\tilde{\phi}}{\partial\omega^*}\lambda} > 0,$$

which is equation (13). $\frac{\partial\omega^*}{\partial\lambda} > 0$ because $\frac{\partial\tilde{\phi}}{\partial\omega^*} < 0$ and $\tilde{\phi}\left(\omega^*\right) > 0$ at the optimum. This completes the proof.

Note that equation (13) can be re-written as the following equation about the elasticity of ω^*

$$\frac{\partial\ln\omega^*}{\partial\ln\lambda} = -\frac{1}{e_{\tilde{\phi}}(\omega^*)}, e_{\tilde{\phi}}(\omega^*) = \frac{\partial\ln\tilde{\phi}(\omega^*)}{\partial\ln\omega^*}. \tag{20}$$

Proof of Proposition 2 The cost share of outsourced tasks relative to variable costs is

$$\Pi = P^{\sigma-1}\int_0^{\omega^*}\phi^S\left(\omega\right)^{\sigma-1}d\omega$$

Therefore,

$$\begin{aligned}
\frac{\partial \Pi}{\partial \lambda} &= (\sigma-1) P^{\sigma-2} \frac{\partial P}{\partial \lambda} \int_0^{\omega^*} \phi^S(\omega)^{\sigma-1} d\omega + P^{\sigma-1}\left[\phi^S(\omega^*)^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda}\right] \\
&= (\sigma-1) \frac{1}{P} \frac{\partial P}{\partial \lambda} \Pi + P^{\sigma-1} \phi^S(\omega^*)^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda} \\
&= (\sigma-1) \frac{1}{P}\left(\frac{1}{1-\sigma} P^{\sigma} \tilde{\phi}(\omega^*) \frac{\partial \omega^*}{\partial \lambda}\right) \Pi + P^{\sigma-1} \phi^S(\omega^*)^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda} \\
&= P^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda}\left(\phi^S(\omega^*)^{\sigma-1} - \Pi \tilde{\phi}(\omega^*)\right) \\
&= P^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda}\left((1-\Pi)\phi^S(\omega^*)^{\sigma-1} + \Pi \phi^b(\omega^*)^{\sigma-1}\right) \qquad (21) \\
&= P^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda} \bar{\phi}(\omega^*) > 0
\end{aligned}$$

where $\bar{\phi} = (1-\Pi)\phi^S(\omega^*)^{\sigma-1} + \Pi\phi^b(\omega^*)^{\sigma-1}$.

In the derivation, we used the following results. First,

$$\frac{\partial P}{\partial \lambda} = \frac{1}{1-\sigma} P^{\sigma} \tilde{\phi}(\omega^*) \frac{\partial \omega^*}{\partial \lambda} = \frac{\partial P}{\partial \omega^*} \frac{\partial \omega^*}{\partial \lambda}. \qquad (22)$$

Second,

$$\begin{aligned}
\phi^S(\omega^*)^{\sigma-1} - \Pi \tilde{\phi}(\omega^*) &= \phi^S(\omega^*)^{\sigma-1} - \Pi\left(\phi^S(\omega^*)^{\sigma-1} - \phi^b(\omega^*)^{\sigma-1}\right) \\
&= (1-\Pi)\phi^S(\omega^*)^{\sigma-1} + \Pi\phi^b(\omega^*)^{\sigma-1} > 0.
\end{aligned}$$

Because $\frac{V\lambda}{s} = 1 - \frac{\Pi}{\bar{m}}$,

$$\frac{\partial (VA/s)}{\partial \lambda} = -\frac{1}{\bar{m}} \frac{\partial \Pi}{\partial \lambda} = -\frac{1}{\bar{m}} P^{\sigma-1} \frac{\partial \omega^*}{\partial \lambda} \bar{\phi}(\omega^*) < 0,$$

which is equation (14). This completes the proof.

Proof of Proposition 3 We start by describing the elasticity of sales. Equations (7), (12) and (22) imply that

$$\frac{\partial \ln s}{\partial ln\lambda} = 1 + (1-\gamma)\frac{\partial lnP}{\partial ln\lambda} = 1 + (1-\gamma)\frac{P^{\sigma-1}}{1-\sigma}\tilde{\phi}(\omega^*)\omega^* \frac{\partial ln\omega^*}{\partial ln\lambda} = 1 + \theta_{GF}\frac{\partial ln\omega^*}{\partial ln\lambda}, \qquad (23)$$

where the ratio of fixed labor to sales is

$$\theta_{GF} = \frac{\gamma f \omega^*}{s}. \qquad (24)$$

Note that $\theta_{GF} \leq 1$ because the buyer firm's variable profit of $VarProfits = s/\gamma$ exceeds its fixed labor cost of $FixLabor = \omega^* f$.

We next write down the components of value added and total labor, using the notation that

VC is the buyer firm's total variable cost with $VC = s/\bar{m}$.

$$VA = VarProfits + VarLabor$$

$$L = VarLabor + FixLabor = VarLabor + f\omega^*$$

$$VarLabor = (1-\Pi)VC = (1-\Pi)s/\bar{m}$$

$$VarProfits = s - VC = (1-1/\bar{m})s = s/\gamma$$

We also introduce the following notations about shares. First, the share of variable profits in value added is

$$\theta_{VA,V} = 1 - \frac{(1-\Pi)s/\bar{m}}{s/\gamma + (1-\Pi)s/\bar{m}} = 1 - \frac{(1-\Pi)/\bar{m}}{1/\gamma + (1-\Pi)/\bar{m}} = 1 - \frac{(1-\Pi)(\gamma-1)}{1+(1-\Pi)(\gamma-1)} \tag{25}$$

Second, the share of fixed labor in total labor is

$$\theta_{L,F} = \frac{\theta_{GF}s/\gamma}{\theta_{GF}s/\gamma + (1-\Pi)s/\bar{m}} = \frac{\theta_{GF}}{\theta_{GF} + (1-\Pi)(\gamma-1)}. \tag{26}$$

Using these results, we get

$$\begin{aligned}
\frac{\partial ln(VA)}{\partial ln\lambda} - \frac{\partial ln(L)}{\partial ln\lambda} &= \theta_{VA,V}\frac{\partial ln(VarProfits)}{\partial ln\lambda} + (1-\theta_{VA,V})\frac{\partial ln(VarLabor)}{\partial ln\lambda} \\
&\quad -(1-\theta_{L,F})\frac{\partial ln(VarLabor)}{\partial ln\lambda} - \theta_{L,F}\frac{\partial ln(FixLabor)}{\partial ln\lambda} \\
&= \theta_{VA,V}\frac{\partial ln(s)}{\partial ln\lambda} - (\theta_{VA,V}-\theta_{L,F})\frac{\partial ln(VarLabor)}{\partial ln\lambda} - \theta_{L,F}\frac{\partial ln(\omega^*)}{\partial ln\lambda} \\
&= (\theta_{VA,V}-\theta_{L,F})(\frac{\partial ln(s)}{\partial ln\lambda} - \frac{\partial ln(VarLabor)}{\partial ln\lambda}) + \theta_{L,F}(\frac{\partial ln(s)}{\partial ln\lambda} - \frac{\partial ln(\omega^*)}{\partial ln\lambda})
\end{aligned} \tag{27}$$

We now simplify the right-hand side of equation (27). First,

$$\theta_{VA,V} - \theta_{L,F} = 1 - \frac{(1-\Pi)(\gamma-1)}{1+(1-\Pi)(\gamma-1)} - \frac{\theta_{GF}}{\theta_{GF}+(1-\Pi)(\gamma-1)} = \frac{(1-\Pi)(\gamma-1)}{\theta_{GF}+(1-\Pi)(\gamma-1)} - \frac{(1-\Pi)(\gamma-1)}{1+(1-\Pi)(\gamma-1)}$$

$$= \frac{(1-\Pi)(\gamma-1)(1-\theta_{GF})}{[\theta_{GF}+(1-\Pi)(\gamma-1)][1+(1-\Pi)(\gamma-1)]} = \frac{(1-\Pi)(1-\theta_{GF})}{[\theta_{GF}+(1-\Pi)(\gamma-1)](\bar{m}-\Pi)} > 0. \tag{28}$$

Second, because $\frac{\partial ln(VarLabor)}{\partial ln\lambda} = \frac{\partial ln(1-\Pi)}{\partial ln\lambda} + \frac{\partial ln(s)}{\partial ln\lambda}$, we have

$$\frac{\partial ln(s)}{\partial ln\lambda} - \frac{\partial ln(VarLabor)}{\partial ln\lambda} = -\frac{\partial ln(1-\Pi)}{\partial ln\lambda} = \frac{\partial \Pi}{\partial \lambda}\frac{\lambda}{1-\Pi}$$

$$= \frac{\lambda}{1-\Pi}P^{\sigma-1}\frac{\partial \omega^*}{\partial \lambda}\bar{\phi}(\omega^*) = \frac{\partial ln\omega^*}{\partial ln\lambda}\omega^*\frac{P^{\sigma-1}}{1-\Pi}\bar{\phi}(\omega^*),$$

where we have used equation (21). We can further simplify $\frac{1}{1-\Pi}\bar{\phi}(\omega^*)$ using equations (21) and

(12):

$$\frac{1}{1-\Pi}\bar{\phi}\left(\omega^*\right)=\phi^S\left(\omega^*\right)^{\sigma-1}+\frac{\Pi}{(1-\Pi)}\phi^b\left(\omega^*\right)^{\sigma-1}=\tilde{\phi}\left(\omega^*\right)+\phi^b\left(\omega^*\right)^{\sigma-1}+\frac{\Pi}{(1-\Pi)}(\phi^b\left(\omega^*\right)^{\sigma-1})$$

$$=\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{1-\Pi}+\frac{\gamma f}{\lambda}\left(\bar{m}\right)^{\gamma-1}.$$

This implies that

$$\frac{P^{\sigma-1}}{1-\Pi}\bar{\phi}\left(\omega^*\right)=\frac{\lambda\left(\bar{m}\right)^{1-\gamma}}{s}\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{1-\Pi}+\frac{\gamma f}{s}=\frac{\lambda\left(\bar{m}\right)^{1-\gamma}}{s}\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{1-\Pi}+\frac{\theta_{GF}}{\omega^*}.$$

Combining these expressions, we get

$$\frac{\partial ln(s)}{\partial ln\lambda}-\frac{\partial ln(VarLabor)}{\partial ln\lambda}=\frac{\partial ln\omega^*}{\partial ln\lambda}\omega^*(\frac{\lambda\left(\bar{m}\right)^{1-\gamma}}{s}\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{1-\Pi}+\frac{\theta_{GF}}{\omega^*}). \tag{29}$$

Plugging equations (23), (26), (28) and (29) into equation (27), we have

$$\frac{\partial ln(VA)}{\partial ln\lambda}-\frac{\partial ln(L)}{\partial ln\lambda}=\frac{(1-\Pi)(1-\theta_{GF})}{[\theta_{GF}+(1-\Pi)(\gamma-1)](\bar{m}-\Pi)}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{\theta_{GF}A\left(\bar{m}\right)^{1-\gamma}}{\gamma f}\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{1-\Pi}+\theta_{GF})$$

$$+\theta_{L,F}-(1-\theta_{GF})\theta_{L,F}\frac{\partial ln\omega^*}{\partial ln\lambda}$$

$$=\theta_{L,F}+\frac{(1-\Pi)(1-\theta_{GF})\theta_{L,F}}{(\bar{m}-\Pi)}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{\lambda\left(\bar{m}\right)^{1-\gamma}}{\gamma f}\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{1-\Pi}+1)-(1-\theta_{GF})\theta_{L,F}\frac{\partial ln\omega^*}{\partial ln\lambda}$$

$$=\theta_{L,F}+(1-\theta_{GF})\theta_{L,F}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{(1-\Pi)}{(\bar{m}-\Pi)}(\frac{\lambda\left(\bar{m}\right)^{1-\gamma}}{\gamma f}\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{1-\Pi}+1)-1)$$

$$=\theta_{L,F}+(1-\theta_{GF})\theta_{L,F}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{(1-\Pi)}{(\bar{m}-\Pi)}(\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{(1-\Pi)\tilde{\phi}\left(\omega^*\right)}+1)-1)$$

$$=\theta_{L,F}+(1-\theta_{GF})\theta_{L,F}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{(\bar{m}-\Pi)\tilde{\phi}\left(\omega^*\right)}+\frac{(1-\Pi)}{(\bar{m}-\Pi)}-1)$$

$$=\theta_{L,F}+\frac{(1-\theta_{GF})\theta_{L,F}}{(\bar{m}-\Pi)}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{\tilde{\phi}\left(\omega^*\right)}+(1-\Pi)-(\bar{m}-\Pi))$$

$$=\theta_{L,F}+\frac{(1-\theta_{GF})\theta_{L,F}}{(\bar{m}-\Pi)}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{\tilde{\phi}\left(\omega^*\right)}-\frac{1}{\gamma-1})$$

$$=\theta_{L,F}[1+\frac{(1-\theta_{GF})}{(\bar{m}-\Pi)}\frac{\partial ln\omega^*}{\partial ln\lambda}(\frac{\phi^b\left(\omega^*\right)^{\sigma-1}}{\tilde{\phi}\left(\omega^*\right)}-\frac{1}{\gamma-1})],$$

which is equation (15). This completes the proof.

Results for Labor Share

Proposition 4 *For $\gamma = \sigma$, the elasticity of labor share is*

$$\frac{\partial \ln(LS)}{\partial \ln \lambda} = \frac{(1-LS)^2}{LS}(-\delta_{F,M} - \frac{\partial \omega^*}{\partial \lambda}\lambda A_3 - \frac{\delta_{F,M}}{e_{\tilde{\phi}}(\omega^*)}), \tag{30}$$

where $\delta_{F,M} = \omega^* f/(\Pi s/\bar{m})$ is the ratio of fixed outsourcing cost to the cost of imported inputs, and $e_{\tilde{\phi}}(\omega^*) = \frac{\partial \ln \tilde{\phi}(\omega^*)}{\partial \ln \omega^*} < 0$ is the elasticity of the supplier's cost advantage, $\tilde{\phi}(.)$, at the cut-off task of ω^*. Meanwhile, $A_3 = \frac{(\frac{1-\Pi}{\Pi}+\delta_{F,M})\phi^s(\omega^*)^{\sigma-1}+\phi^b(\omega^*)^{\sigma-1}}{\int_0^{\omega^*}\phi^s(\omega)^{\sigma-1}d\omega} > 0$.

To see the intuition of equation (30), suppose λ increases. First, the buyer firm increases the imports of infra-marginal tasks, those below ω^*. This effect shows up as the first term in the bracket on the right-hand side of equation (30), and tends to reduce the labor share. Second, the buyer firm also increases ω^*, or the range of outsourced tasks. This effect further increases the value of imports and tends to reduce the labor share, and corresponds to the second term in the bracket on the right-hand side of equation (30). Finally, the increase in ω^* implies an increase in outsourcing cost. This effect tends to increase the labor share, and corresponds to the last term in the bracket on the right-hand side of equation (30). Overall, labor share decreases because the first two terms dominate.[36]

To prove Proposition 4, we start by re-writing equation (17) as

$$LS = \frac{(1-\Pi)VC + \omega^* f}{VC + \omega^* f}, \tag{31}$$

where Π is the share of imported inputs in VC, as defined in Proposition 2, and again, VC is the total variable cost with $VC = s/\bar{m}$. Equation (31) implies $1 - LS = \frac{\Pi VC}{VC+\omega^* f}$, and so

$$\frac{1}{1-LS} = \frac{1}{\Pi} + \frac{\omega^* f \bar{m}}{\Pi s},$$

which implies that

$$\frac{\partial(\frac{1}{1-LS})}{\partial \lambda} = -\frac{1}{\Pi}\frac{\partial \ln \Pi}{\partial \lambda} + \delta_{F,M}(\frac{\partial \ln \omega^*}{\partial \lambda} - \frac{\partial \ln \Pi}{\partial \lambda} - \frac{\partial \ln s}{\partial \lambda}). \tag{32}$$

Equations (21) and (23) imply that

$$\frac{\partial \ln \omega^*}{\partial \lambda} - \frac{\partial \ln \Pi}{\partial \lambda} - \frac{\partial \ln s}{\partial \lambda} = \frac{\partial \omega^*}{\partial \lambda}(\frac{1}{\omega^*} - \frac{P^{\sigma-1}\bar{\phi}(\omega^*)}{\Pi} - P^{\sigma-1}\tilde{\phi}(\omega^*)) - \frac{1}{\lambda}$$

$$= \frac{\partial \omega^*}{\partial \lambda}(\frac{1}{\omega^*} - \frac{(P\phi^S(\omega^*))^{\sigma-1}}{\Pi}) - \frac{1}{\lambda} = \frac{\partial \omega^*}{\partial \lambda}(\frac{1}{\omega^*} - \frac{\phi^S(\omega^*)^{\sigma-1}}{\int_0^{\omega^*}\phi^s(\omega)^{\sigma-1}d\omega}) - \frac{1}{\lambda}. \tag{33}$$

Meanwhile,

$$\frac{\partial(\frac{1}{1-LS})}{\partial \ln \lambda} = \frac{LS}{(1-LS)^2}\frac{\partial \ln LS}{\partial \ln \lambda}. \tag{34}$$

Plugging equations (20), (21), (33) and (34) into equation (32), we obtain equation (30).

[36] This result is difficult to prove analytically but it holds in all our quantitative analyses.

This completes the proof.

XXXX Numerical simulations for $\gamma \neq \lambda$??? XXXX XXXX

XXXX Extension of the Model to Have Capital???? XXXX XXXX