Sustainable Investing in General Equilibrium

Xintong Li Giacomo Rondina*

December 2025

Abstract

We investigate the conditions under which sustainable investing – the investing practice that accounts for environmental/societal/governance factors – alters the allocation of productive capital across firms in a dynamic general equilibrium framework. We find that sustainable investing can generate both a *composition* and a *scale* impact on capital and output. We show that the timing of impact depends on the relative wealth of investors with sustainability motives, as well as the distribution of firms across sustainability metrics. While our framework predicts the eventual impact on capital allocation in favor of sustainable firms, our simulations indicate that the timing may lie far into the future, casting doubts on the effectiveness of sustainable investing as a market-driven mechanism to internalize environmental/societal/governance externalities.

Keywords: sustainable investing, ESG, portfolio choice, impact investing
JEL: E10, G11, O40, Q50

*Li: INSE - Peking University, xintongli@nsd.pku.edu.cn. Rondina: University of California – San Diego, grondina@ucsd.edu. We thank Shisham Adhikari for detailed feedback and participants at the Western Economic Association International (2023), the European Economic Association Conference (2023), CU Environmental & Resource Economics Workshop (2023), UCSC's Center for Analytical Finance Workshop (2024), the Conference on the Macroeconomic and Financial Aspects of Climate Change in Mexico City (2025) and various UCSD Macro Lunch presentations for their valuable comments and suggestions. All remaining errors are our own. (First version: February 15, 2023.)

I. Introduction

Sustainable investing refers to a range of strategies in which investors include environmental, social responsibility, and corporate governance considerations in their investment decisions and investor advocacy. Unlike traditional investing approaches that focus solely on maximizing risk-adjusted financial returns, investors and managers practicing ESG (environmental, social, and governance), SRI (socially responsible investing), and CSR (corporate social responsibility) investment typically also seek to align capital allocation with broader societal and environmental values (Starks, 2023).

Reflecting this shift in investor priorities, the financial sector has seen a remarkable growth in the number of and inflow toward funds with explicit sustainability considerations during the last decade (Azémar and Giroud, 2023). As shown in Figure 1, sustainable investing assets under management (AuM) across major financial markets grew from $12 trillion in 2014 to around $35 trillion as of 2024. With sustainable investments now constituting approximately a quarter to a third of global assets under management across major markets, this approach has evolved from a niche strategy to a mainstream financial practice.

The growing interest of the financial industry in sustainable investing has sparked a fast growing academic literature aiming at identifying and estimating the effect of sustainable investing practices on asset returns and corporate behavior.[1] However, little attention has been devoted so far to the question of *whether* and *how* sustainable investing can ultimately impact the allocation of productive capital when dynamics and general equilibrium are taken into account.[2] Our paper aims at filling this gap.

To do so, we develop a dynamic model of an economy where heterogeneous investors (households) hold their savings portfolios in companies with heterogeneous sustainability scores. Figure 2 illustrates the distribution of U.S. publicly traded firms by Refinitiv sustainability scores, which range from 0 to 1.[3] Panel (a) plots the distribution of total market capitalization, while panel (b) shows total employment, both binned into equal-width score intervals.

[1] A non-exhaustive list of recent contributions include Berk and van Binsbergen (2025); Gillan et al. (2021); Humphrey et al. (2021); Idzorek et al. (2021); Kedia et al. (2021); Atz et al. (2022); Berg et al. (2022a); Caramichael and Rapp (2022). Pástor et al. (2024) is an excellent introductory survey to the sustainable investing literature.

[2] Pástor et al. (2021) study the impact of non-pecuniary preferences for clean stocks on the capital allocation of clean firms in the context of the exponential utility two-period asset pricing model, but stop short of considering the implied dynamic general equilibrium adjustments. Early relevant contributions also include Beltratti (2005), Dam and Heijdra (2011), and Dam (2011).

[3] Eight major ESG rating agencies currently publish sustainability scores, including ISS ESG, Moody's, MSCI, Refinitiv, RepRisk, Sustainalytics, S&P Global CSA, and Truvalue Labs. The correlation across rating providers is far from perfect; for example, Berg et al. (2022b) estimate inter-provider correlations between 0.38 and 0.71.

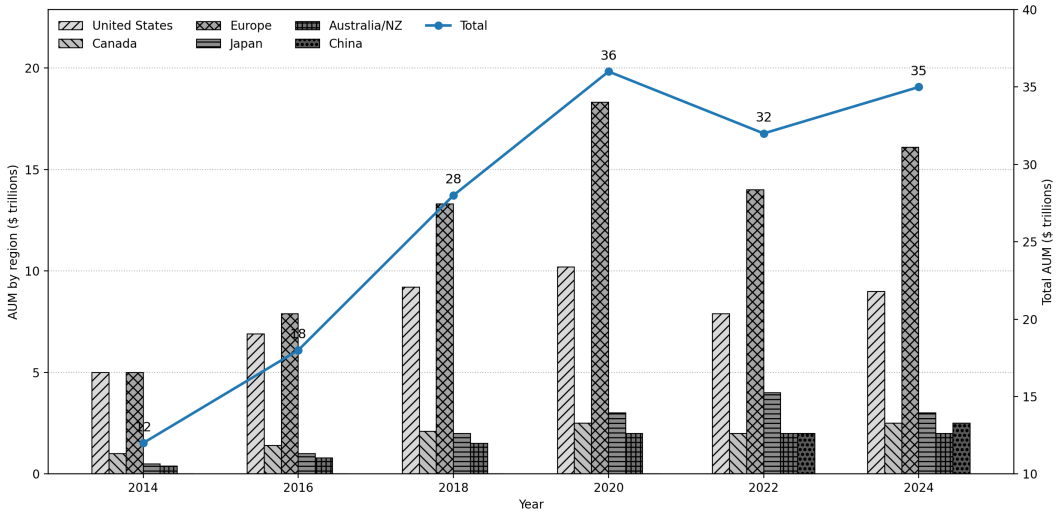


Figure 1: Sustainable Assets Under Management (2014-2024)

Notes: Figure 1 reports estimates of the asset under management for ESG-related funds – e.g. funds with explicit ESG goals –across global financial markets. Source: Bloomberg Intelligence.

In both cases, a substantial portion of firms display relatively high sustainability scores. The ultimate question we are after is when and how the surge of funds redirected towards stocks of sustainable companies, as documented in Figure 1, results into a reallocation of productive capital in favor of those companies.

In our model, there are two types of financial investors (households): *Advocate*, who care about environmental/social/governance values associated with their portfolio holdings in addition to their pecuniary returns, and *Benchmark*, who are only focused on pecuniary returns. Our key assumption is that Advocate households enjoy a non-pecuniary return in the form of higher utility when holding portfolios of *clean* (high-sustainability-score) firms and lower utility when holding portfolios of *dirty* (low-sustainability-score) firms.[4] As a consequence, they are willing to accept lower returns for holding clean stocks, while requiring higher returns to hold dirty stocks.[5]

[4]Advocate households do not necessarily seek for impact in their investment, but they essentially enjoy a "warm glow" knowing that their savings are invested in firms whose values align with theirs. This is sometimes referred to as a "non-consequentialist" modeling of households with stustainability preferences, see Pástor et al. (2024)

[5]Hong and Shore (2022) survey the recent evidence on non-pecuniary returns. Geczy et al. (2005) find that mutual funds investing in socially-responsible companies tend to slightly underperform standard mutual funds. In incentivized experiments, Riedl and Smeets (2017) find that "social preferences" and "social signaling" explain socially responsible investment (SRI) decisions more than financial motives. By examining a salience shock in the U.S. mutual fund market, Hartzmark and Sussman (2019) present causal evidence that investors marketwide value sustainability beyond what would be explained by their expectation of future performance or risk, consistent with non-pecuniary

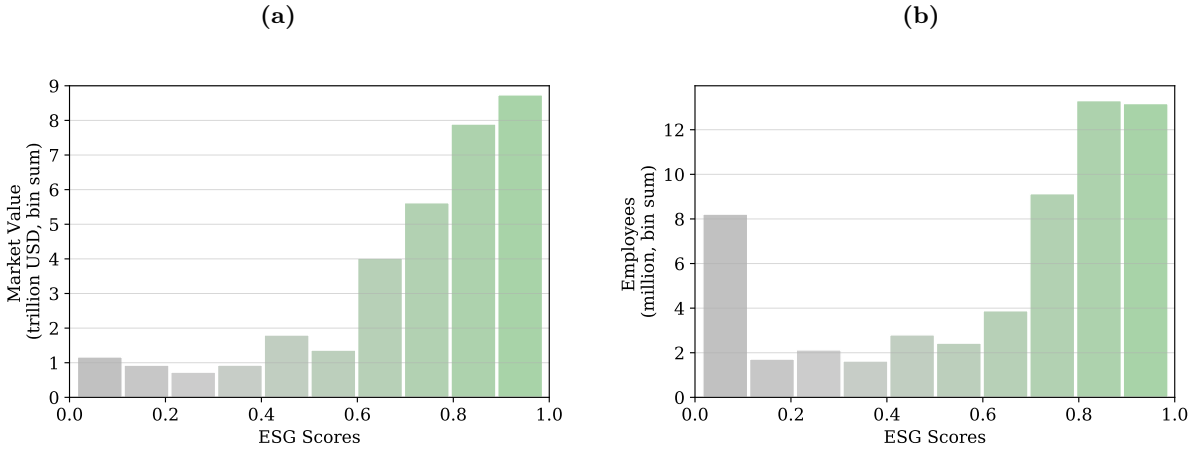


Figure 2: Distribution of U.S. Publicly Traded Companies by Sustainability Score

Notes: Figure 2 presents the distribution of market capitalization (Panel (a)) and total employment (Panel (b)) across U.S. publicly traded companies, according to their Refinitiv sustainability scores as of 2019. The illustration features all companies listed on the NYSE, NYSE American, NASDAQ, and NYSE Arca exchanges. Companies that do not have Refinitiv ESG ratings are assigned to the zero bin. The sustainability scores range from 0 to 1, with higher scores indicating better sustainability performance, as per Refinitiv's ESG Company Summary methodology.

We show that Advocate households impact the equilibrium allocation – via a lower cost of capital in the form of lower opportunity cost of retained earnings – only when they are the marginal investors in a company. In turn, whether Advocate households are marginal investors, depends on the relative size of their wealth in the economy. In particular, when the relative wealth of advocate investors is small, the marginal investor in clean companies are Benchmark households, and the impact of sustainable investing on capital allocation is limited. Advocate households, however, are willing to substitute current consumption for an increase in their holdings of clean firms. This amounts to a faster wealth accumulation compared to Benchmark, and a growing share of clean companies where Advocate are the marginal investors. Over time, the cost of capital of a larger share of clean firms decouples from that of the dirty

motives influencing investors' decisions. Barber et al. (2021) show that investors derive non-pecuniary utility from investing in impact-oriented venture capital funds and are willing to sacrifice pecuniary returns. Bauer et al. (2021) observe two-thirds of pension fund holders in their field survey are willing to expand the fund's engagement with companies based on the UN's Sustainable Development Goals, even when they expect engagement to hurt financial performance. Baker et al. (2022a) find that green municipal bonds are issued at a premium. Using a revealed preference approach, Baker et al. (2022b) find that investors are willing, on average, to pay 20 basis points more per annum for an investment in a fund with an ESG mandate as compared to an otherwise identical mutual fund without an ESG mandate. Pástor et al. (2022) find that the stocks of clean firms had negative alphas while the stocks of brown firms had positive alphas. Zerbib (2022) studies an asset pricing model with tastes for clean stocks and dislikes for sin stocks and finds that the taste effect ranges from -1.12% to +0.14% for the different industries, and the average exclusion effect is 2.79%, in terms of annual returns.

firms, driving a reallocation of physical capital in favor of the former.

We organize the impact of sustainable investing into a *composition* effect – how capital is allocated across firms – and a *scale* effect – how total output in the economy is affected. We show that along the transition dynamics and in the steady state, the composition effect is always in favor of clean firms, and the scale effect is always positive. However, we also show that the timing of both effects is sensitive to the initial size of wealth of Advocate households. In our numerical simulations, when the share of wealth of Advocate households in the economy is 33%, it takes between 5 to 35 years for the composition and scale impact to emerge, and about 85 to 100 years to reach a level close to the steady state. Remarkably, when the share of wealth is lower – 12% in our analysis – the timing of both effects is delayed by 80-90 years. While these calculations ought to be taken with a grain of salt since they are based on a stylized representation of sustainability preferences, financial markets, and output technologies, they represent, to the best of our knowledge, a first systematic attempt to quantify the ultimate impact of sustainable investing taking both dynamics and general equilibrium into account.

Our results have important policy implications. On the one hand, we show that market forces can indeed work to reallocate capital towards firms that are favored by investors whose preferences align with sustainability motives. On the other hand, for those forces to have the intended impact, investors with preferences of sustainability must be large enough, which might substantially delay their ultimate impact. To the extent that capital reallocation towards clean firms is socially desirable, our results suggest that policies that subsidize the holding of clean stocks for both advocate and benchmark investors might be socially desirable. Most importantly, sustainable investing seems unlikely to be an effective substitute for government policies aimed at internalizing environmental/societal/governance externalities.

Our model abstracts from many interesting dimensions that are likely to inject significant nuances into the results just described, especially in terms of the timing of impact and the policy implications highlighted above, such as endogenous technology choice, risk considerations in asset returns, preferences of sustainable consumption, etc. However, we believe that the core mechanism we isolate in the model—the notion that sustainable investing modifies the saving profiles of investors and leads to a time-varying impact on the scale and composition of capital allocation—will continue to play a central role also in the presence of additional channels.

The rest of the paper is organized as follows: Section II introduce a two-period model to fix ideas and highlight the key equilibrium mechanisms; Section III contains the full dynamic model and our main theoretical results; Section IV presents and discusses numerical simulations;

Section V concludes. The proofs of all the results are reported in the Appendix.

II. Two-Period Model

We begin by presenting a two-period model, with time $t = 0, 1$. Our goal is to highlight and discuss the key assumptions in modeling sustainable investing and to isolate key equilibrium mechanisms that will extend to the full dynamic model.

FIRMS. The economy is populated by two firms producing a homogeneous output good y using capital k, according to the technology $y = f(k)$, which is strictly concave, increasing, and differentiable. The two firms are identical, except for how they are labeled: one is labeled as "clean," the other is labeled as "dirty." In what follows, letters with a tilde, such as $\tilde{k}$, $\tilde{y}$, denote variables pertaining to the dirty firm.

Firms own the capital and use retained earnings to finance investment with the goal of maximizing the present discounted value of dividends.[6] The initial capital held by each firm is $k_0 > 0$, and capital fully depreciates when used in production. The problem for the firm can be expressed as

$$\max_{k} \quad \left\{ d_0 \; + \; \frac{d}{\theta} \right\} \tag{1}$$

subject to

$$d_0 = f(k_0) - k, \quad \text{and} \quad d = f(k), \tag{2}$$

where d_0, d are the per-share dividends paid at time 0 and 1, respectively, and k is investment in capital to carry into period 1. The discount factor $1/\theta$ is determined by the marginal investors in the firm's stocks, and it is taken as given by the firm. The optimal investment for the firm is determined by the familiar condition equating the marginal product of capital to its opportunity cost, formally,

$$f'(k) = \theta. \tag{3}$$

For the dirty firm a similar condition holds, namely $f'(\tilde{k}) = \tilde{\theta}$.

Firms cannot issue new shares or debt, so their only source of financing is retained earnings.

[6] We follow Brock and Turnovsky (1981) and Turnovsky (1990) in our modeling of the problem for the firm. The assumption that firms maximize shareholders' value is not without loss of generality when the firm can issue new stocks or bonds, but it corresponds to profit maximization under our maintained assumptions.

In particular, firms cannot sell or lend capital to other firms. This assumption limits the ability of firms to exploit possible arbitrage opportunities when they have access to a lower cost of capital and would want to sell/lend that capital to firms that have access to a higher cost of capital. In equilibrium, firms that operate with a different marginal product of capital can therefore coexist. Two arguments can justify our assumption. First, firms may have access to heterogeneous financial conditions because of technological features related to their clean and dirty labels that are not necessarily portable. Second, if a clean firm lends capital to a dirty firm, it might jeopardize its reputation as clean because some of its output, in the form of interest on the loan, is now generated by the dirty firm. The consequence could be a higher cost of capital for the clean firm, which would reduce the incentive to lend to the dirty firm.[7]

We finally note that the ex-dividend price of a clean firm share q at time $t = 0$ can be defined as

$$q = \frac{d}{\theta}, \tag{4}$$

with a similar expression for the price of a dirty stock, namely $\tilde{q} = \tilde{d}/\tilde{\theta}$.

HOUSEHOLDS. The economy is also populated by two households that receive income from holding stocks in the two firms, and decide how much to consume, save, and allocate savings between stocks in clean and dirty firms. Both households derive utility from consumption, but one household, which we denote as "Advocate," also derives utility (disutility) from holding clean (dirty) stocks. Preferences for the Advocate household are specified as

$$U^A = \ln c_0^A + \beta \ln c_1^A + vzq - \tilde{v}\tilde{z}\tilde{q} \tag{5}$$

where $\beta \in (0, 1]$, and z, $\tilde{z}$, represent the quantity of clean stocks and dirty stocks, respectively, held by the household into period 1. The total outstanding shares for each firm are normalized to 1 so that $z \in [0, 1]$ and $\tilde{z} \in [0, 1]$.

The term $v \geq 0$ represents the non-pecuniary return (in utility units) received by the advocate investor for each unit of savings held in clean stocks. Symmetrically, $\tilde{v} \geq 0$ represents the non-pecuniary penalty for each unit of savings held in dirty stocks.[8] Let W_0 denote the

[7] In reality, the cross-financing between clean and dirty firms can be substantial and hard to trace, with little repercussion on the sustainability reputation of firms. Our model assumes that no cross-financing takes place, and it should be considered as a best-case scenario for sustainable investing.

[8] The linear function form for the non-pecuniary return/penalty is made for analytical convenience, and it does not impact the qualitative features of our results.

total wealth in the economy at time 0, and let w_0 denote the share of wealth held by the advocate household at the beginning of time 0. The budget constraints can then be written as

$$c_0^A = w_0 W_0 - (zq + \tilde{z}\tilde{q}) \quad \text{and} \quad c_1^A = zd + \tilde{z}\tilde{d}. \tag{6}$$

where $(zq + \tilde{z}\tilde{q})$ corresponds to total savings. Note that period 1 consumption, c_1^A, is equal to the dividend payout of the household's portfolio. The second type of household, denoted as "Benchmark" has the standard preferences

$$U^B = \ln c_0^B + \beta \ln c_1^B, \tag{7}$$

with constraints

$$c_0^B = (1 - w_0) W_0 - ((1 - z)q + (1 - \tilde{z})\tilde{q}) \quad \text{and} \quad c_1^B = (1 - z)d + (1 - \tilde{z})\tilde{d}. \tag{8}$$

Note that we have already substituted the market-clearing condition for stocks in (8).

Portfolio Choice. The solution to the problem for the two households takes the form of optimal saving/portfolio conditions. For Advocate they are

$$[\text{clean}] \ : \ \frac{c_1^A}{c_0^A} = \beta\theta + v c_1^A \qquad \text{and} \qquad [\text{dirty}] \ : \ \frac{c_1^A}{c_0^A} \geq \beta\tilde{\theta} - \tilde{v} c_1^A \tag{9}$$

For Benchmark they are

$$[\text{clean}] \ : \ \frac{c_1^B}{c_0^B} \geq \beta\theta \qquad \text{and} \qquad [\text{dirty}] \ : \ \frac{c_1^B}{c_0^B} = \beta\tilde{\theta} \tag{10}$$

The inequalities indicate that corner solutions where $\tilde{z} = 0$ and/or $z = 1$ are possible. In particular, three portfolio outcomes can emerge in equilibrium:

1) $\{0 < z < 1, \tilde{z} = 0\}$: Advocate owns some clean stocks, fully disinvests in dirty.

2) $\{z = 1, \tilde{z} = 0\}$: Advocate owns all clean stocks, fully disinvests in dirty.

3) $\{z = 1, \tilde{z} > 0\}$: Advocate owns all clean stocks, owns some dirty.

In case 1) Benchmark holds both clean and dirty stocks, which requires both stocks to provide equal returns, hence in equilibrium $\theta = \tilde{\theta}$. It follows that capital is equally allocated between

clean and dirty firm since they face the same discount factor. This is consistent with the optimal portfolio conditions for the Advocate because the second equation in (9) can be slack. In case 2), Advocate holds all the clean stocks, and Benchmark holds all the dirty stocks. Because the first equation in (10) can hold with inequality, it is possible to have $\theta < \tilde{\theta}$, which results in more capital being allocated to the clean firm. Case 3) also allows for (10) to be slack, while the second equation in (9) holds tight, so $\theta < \tilde{\theta}$ is possible in that case, too, with similar implications for capital allocation.

Which case applies in equlibrium depends on the initial allocation of wealth w_0. When w_0 is low, Advocate has limited wealth and their savings may not be enough to hold the entire clean stocks, but they can still divest the dirty stocks entirely. For a higher w_0, Advocate can hold the entire clean stocks, which results in case 2), and, eventually, in case 3).

Equilibrium. Equilibrium requires (3), (6), (8)-(10), as well as market clearing for stocks and output good. Figure 3 reports the equilibrium solution for the special case of $v = 1$ and $\tilde{v} = 0$, so Advocate perceives utility for holding clean stocks, but no disutility for holding dirty stocks. The top panel shows the portfolio allocation as w_0 goes from 0 to 1. The thresholds separating the three cases outlined above are marked by the vertical dotted lines. The second panel shows the investment k and $\tilde{k}$ corresponding to the equilibrium for a given w_0. Investment is symmetric across clean and dirty up until Advocate becomes the sole owner of the clean stocks. At that point, the two diverge, and more capital is allocated to the clean firm. The gap keeps increasing as the equilibrium enters case 3). The example shows that sustainable investing has a *composition* effect on capital only when Advocate is the marginal investor in the clean stock, which happens only when w_0 is large enough.

The bottom panel shows the total output in equilibrium as a ratio of the output that would emerge with in an economy with $v = \tilde{v} = 0$. Interestingly, total output increases with w_0 for all three cases, resulting in what we call a positive *scale* effect. The intuition for the scale effect can be found in the differential saving behavior of Advocate and Benchmark. Denoting the saving rates of the two households by s^A and s^B, one can show that[9]

$$s^A = \frac{\beta + vzq}{1 + \beta + vzq}, \qquad \text{and} \qquad s^B = \frac{\beta}{1 + \beta}. \tag{11}$$

[9]To be precise, the saving rate for Advocate applies for Case 1) and under $\tilde{v} = 0$. The saving rate for Advocate in Cases 2) and 3) takes a slightly different form than the one reported in (11), but the ultimate impact of v remains the same. See Appendix A for details.

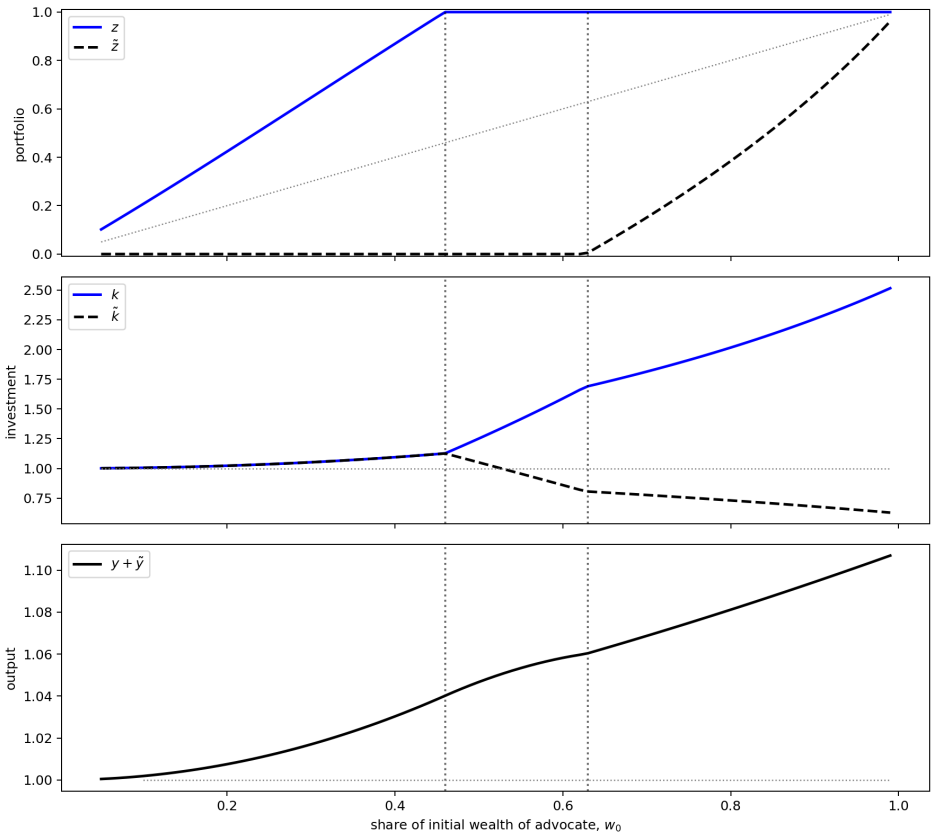


Figure 3: Composition and Scale Impact for the Clean and Dirty Firm

Notes: Figure 3 reports the equilibrium outcomes when $\beta = 1$, $\alpha = 0.33$, $v = 1$, and $\tilde{v} = 0$. The initial levels of capital is set at $k_0 = \tilde{k}_0 = \frac{\alpha\beta}{1+\alpha\beta}$. Investment and Output are reported as a ratio of the leves that would result in an economy with only Benchmark households (i.e with $v = \tilde{v} = 0$).

The non-pecuniary return v injects an extra incentive to save for Advocate compared to Benchmark. At the individual level, the extra saving demand is satisfied by a net transfer of clean stocks from Benchmark to Advocate. However, Benchmark's saving demand is a constant fraction of wealth, so in equilibrium, stock prices must increase to ensure market clearing via a valuation effect. The stock price increase reduces the required return on stocks and thus the cost of capital for firms leading to higher retained earnings and capital accumulation. The extra saving demand ultimately translates into higher investment via the "price signal" provided to the firms by the stock market. In Case 1), because the discount factor must be the same across firms, the extra investment is equally allocated across both clean and dirty, while in Cases 2) and 3), capital accumulation favors the clean firm. In all cases, the extra incentive to save is ultimately responsible for the observed scale effect in output.

Extensions. The numerical equilibrium reported above can be generalized in several direc-

tions. The most immediate is to consider the case with a disutility for holding dirty stocks, that is $\tilde{v} > 0$. Doing so does not change the qualitative features of Cases 1) and 2). However, the scale effect in Case 3) becomes non-monotonic, with a peak and a decline for high values of w_0. Intuitively, when the wealth held by Advocate investors is large, they are "forced" to hold dirty stocks, which provides an incentive to reduce their saving rate, and thus the overall scale impact on output. One can also consider the extreme case of Advocate only caring about disinvesting in dirty stocks, that is, setting $v = 0$ and $\tilde{v} > 0$. In this case, the scale effect would be completely reversed, and become negative in Cases 2) and 3). Appendix A provides details of the two extensions.

A second direction consists in introducing risk in the model in the form of uncertain dividends in period 1. Interestingly, introducing risk results in differential equilibrium returns, that is $\theta < \tilde{\theta}$, across all values of w_0. We sketch the intuition here and direct the reader to Appendix A for the details. In the absence of any sustainability considerations, households would want to hold the market portfolio to optimally diversify risk. However, when $v > 0$, Advocate tilts their portfolio towards clean stocks, forcing Benchmark to hold a portfolio tilted towards dirty stocks. To be compensated for the higher risk they hold in equilibrium, Benchmark requires a higher return on dirty stocks, which results in $\theta < \tilde{\theta}$ even for low values of w_0. Quantitatively, the differential return is small for values of w_0 in Case 1), so the qualitative feature of a significant composition effect emerging only when Advocate is the marginal investor of Figure 3 still holds.[10]

Discussion of Assumptions. We use the two-period model framework outlined above to discuss three important assumptions: sustainable wealth in the utility function, borrowing and short-sale constraints and limits to arbitrage opportunities for firms.

Sustainable Wealth in the Utility Function. The introduction of preferences for sustainable wealth can be thought to capture the internalization of some type of externality that is generated by firms in their operations. The simplest example is an economy in which firms produce an identical output good, but they do so by emitting different amounts of carbon dioxide to the atmosphere per unit produced. Carbon emissions, in turn, reduce the quality of the environment inhabited by households, thus reducing welfare.[11] A social planner in that economy

[10]The juxtaposition of investing motives based on "values"– represented by v and $\tilde{v}$ in our Advocate preferences, versus "value" – represented by different perceptions of risk exposure for clean and dirty firms, has been emphasized by Starks (2023) and constitutes an important open questions in the sustainability finance literature. Our model can accommodate both channels, and we systematically explore their interactions in separate work.

[11]The channel that we have in mind here is what is typically considered by the literature on the macroeconomics of

would allocate capital, and thus output production, across firms, taking into account the negative externality due to carbon emission. In the decentralized economy, if households have access to the information on carbon emissions for firms, and understand the negative impact, they might choose to internalize the externality by changing their holdings of firms according to their environmental impact.

A complementary way to model the preferences for sustainability is to assume that households can differentiate consumption depending on which firm has produced it. By directing their expenditure towards sustainable firms, households increase the marginal product of capital, and thus can change the allocation in equilibrium. We do not consider that channel in our model because we want to isolate the impact of the allocation of savings across firms, and thus obtain predictions on how wealth and portfolio allocation translate into the impact of sustainable investing.

Borrowing and Short-Sale Constraints. In our model, we have assumed that households cannot borrow from one another and cannot take short positions on stocks. Both assumptions are important for the existence of Case 1 because they prevent the Advocate household from holding all the stocks in the clean firm right away. Advocate would be willing to borrow from Benchmark at the gross interest rate $\tilde{\theta}$ and purchase all the stocks in the clean firm. Indeed, Advocate would be willing to do so until the difference between the interest rate on borrowing and the pecuniary return on clean stocks equals the difference between the marginal product of capital at the boundary of Case 2 and Case 3! Hence, with a non-binding borrowing constraint, only Case 3 would emerge in equilibrium. With ad-hoc borrowing constraints, both Case 1 and Case 2 are still part of the equilibrium. Our analysis assumes the strictest possible borrowing constraint by setting the borrowing limit to zero, but it can be generalized to less stringent constraints. The same argument applies to short-sale constraints.

Limits to Arbitrage Opportunities for Firms. Under the assumptions of our model, firms are restricted in the types of financial securities they can hold and issue. The implication is that there are arbitrage opportunities that remain unexploited. Let us take the equilibrium in Case 2, for example. The clean firm retains earnings at the opportunity cost $\theta < \tilde{\theta}$. The clean firm could, in principle, lend to the dirty firm at the gross interest rate $R \in [\theta, \tilde{\theta}]$ and thus reduce the cost of capital for the dirty firm. The equilibrium in this case would require $R = \theta = \tilde{\theta}$,

sustainability, for instance, see Heal (1998), where the stock of environmental assets is modeled as directly entering the utility function of the social planner. The goal is to capture in reduced form the many complex ways in which environmental assets contribute to welfare via their participation in the production process, and as amenities directly enjoyed by humans.

which implies that capital is equally distributed across firms, so the composition effect would not occur. Interestingly, the scale effect would still occur and extend to Case 2 as well, because of the saving demand of the Advocate household.

The extent of the composition effect thus depends on the extent of the limits to set up financial securities across firms that face different costs of capital. The example also indicates the intricacies of establishing, on the side of Advocate, a measure of sustainable wealth that is truly reliable. In the arbitrage example above, the clean firm would hold debt assets issued by the dirty firm, but still be considered as sustainable as before by Advocate. If that were not the case, the clean firm would no longer have access to the lower cost of capital θ. Hence, an appropriate measure of sustainable wealth should assess the sustainability of both a firm's productive and financial assets, adjusting the overall sustainability score accordingly. In the example above, this could mean that as soon as the clean firm lends to the dirty firm, it loses the status as a clean firm and access to cheaper capital. In reality, the indirect holdings of claims on firms' output by other firms are exceptionally complex, but this simple example suggests that closer scrutiny of the sustainability of productive activities behind financial assets is an important requirement for sustainability scores to translate into a composition effect on capital allocation.

Summary of Two Period Model. The two-period model provides two key insights. First, capital reallocation towards a company with a desirable sustainability profile emerges in equilibrium only when Advocate households become the marginal investors in that company. Second, sustainable investors can exhibit a stronger incentive to save if they value holding clean stocks. However, the scale effect can be reversed if they strongly value disinvesting in dirty stocks. Both mechanisms play a central role in shaping the equilibrium features in a dynamic model with infinite horizon and richer heterogeneity in sustainability scores. We turn to such model next.

III. Full Model

Our full model generalizes the two-period model in two directions: (i) we allow for infinitely lived households; (ii) we introduce a continuum of firms indexed by $g \in [0, 1]$ with density $\phi(g)$ to capture the heterogeneity in sustainability scores observed in the data. Our ultimate goal is to study when and how the dynamics of portfolio allocation, wealth, capital, and output are impacted by sustainability considerations. The details and proofs of all the derivations presented below can be found in Appendix B.

III.1. Framework

Preference for Sustainable Wealth. The heterogeneity in sustainability scores is embedded in the advocate households preferences by generalizing (5). Let $q(g)$ denote the price of a share in firm g at a given point in time, and let $z(g)$ denote the stock holdings in firm g by the household. We define a sustainable wealth index $\mathcal{G}$ takes the linear form:[12]

$$\mathcal{G} = \int_0^1 v(g)z(g)q(g)\phi(g)\mathrm{d}g. \tag{12}$$

The function $v(.)$ is assumed to be continuous, weakly increasing and bounded for $g \in [0,1]$. The value $v(g)$ represents the impact on the sustainability wealth index of a change in the value of the holdings of firms with score g, modulo $\phi(g)$. We assume that the function $v(.)$ implies the existence of a "neutral" sustainability score g_n such that

$$v(g_n) = 0 \quad \text{for some} \quad g_n \in [0,1]. \tag{13}$$

For firms with $g > g_n$, sustainable wealth is weakly increasing in their stock holdings, while for firms with $g < g_n$, sustainable wealth is weakly decreasing. In line with the terminology used in the two-period model, we will refer to the former as *clean* firms/stocks, and to the latter as *dirty*.

As in the two period model, there are two households in the economy, Advocate and Benchmark. We assume that the flow utility of Advocate takes the form

$$\ln c^A + \mathcal{V}(\mathcal{G}), \tag{14}$$

where $\mathcal{V}$ is weakly increasing and twice differentiable. For the rest of our analysis, we will set $\mathcal{V}$ to be linear for tractability.[13]

[12] The choice of a functional form for the index is made for analytical convenience, and it carries no substantial implications for the qualitative nature of our results. One key advantage of a linear index is allowing for zero stock holdings, i.e., $z(g) = 0$ for some g, while maintaining the optimization problem well-defined. Furthermore, because the holdings of stocks are bounded, the index is well-defined even when $z(g)$ is at the upper bound for all g's. A closely related preference representation is adopted in Chittaro et al. (2025).

[13] Note that preferences (14) are akin to the standard representation of preferences with wealth in the utility function, with the important difference that wealth is represented by $\mathcal{G}$, which corresponds to a measure of sustainable wealth.

HOUSEHOLDS. The objective function for Advocate is

$$\int_0^\infty e^{-\rho t} \left[\ln c^A(t) + \mathcal{G}(t)\right] \mathrm{d}t, \tag{15}$$

where $\rho > 0$ is the subjective discount rate. Sustainable wealth $\mathcal{G}(t)$ is now time-varying due to changes in portfolio composition, and is defined as

$$\mathcal{G}(t) = \int_0^1 v(g) z(g,t) q(g,t) \phi(g) \, \mathrm{d}g. \tag{16}$$

Total wealth for Advocate at time t is defined as

$$a^A(t) = \int_0^1 z(g,t) q(g,t) \phi(g) \, \mathrm{d}g, \tag{17}$$

which can be shown to evolve according to the differential equation:

$$\dot{a}^A(t) = \int_0^1 \left[d(g,t) + \dot{q}(g,t)\right] z(g,t) \phi(g) \, \mathrm{d}g - c^A(t). \tag{18}$$

Benchmark is assumed to possess the same subjective discount rate ρ but derive utility solely from consumption. Their total wealth at time t is given by:

$$a^B(t) = \int_0^1 [1 - z(g,t)] q(g,t) \phi(g) \, \mathrm{d}g, \tag{19}$$

with the corresponding law of motion:

$$\dot{a}^B(t) = \int_0^1 \left[d(g,t) + \dot{q}(g,t)\right] [1 - z(g,t)] \phi(g) \, \mathrm{d}g - c^B(t). \tag{20}$$

Total wealth in the economy at time t is, $W(t) = a^A(t) + a^B(t)$. The initial wealth $W(0)$ is assumed to be distributed with fraction w_0 to Advocate, and $1 - w_0$ to Benchmark, as in the two-period model.

FIRMS. As in the two-period model, the firm owns capital directly and chooses an investment strategy that maximizes its total stock market value. The firm produces output using the technology f, and capital depreciates at the instantaneous rate $\delta > 0$ when used in production. Investment in capital is financed by retained earnings, and the remaining revenues are

distributed as dividends, so that

$$d(g,t) = f(k(g,t)) - \dot{k}(g,t) - \delta k(g,t). \tag{21}$$

Let $\theta(g,t)$ denote the instantaneous return the firm's stock should provide in order to be held by households. The problem for firm g is to maximize

$$\int_0^\infty e^{-\int_0^t \theta(g,\tau)\mathrm{d}\tau} d(g,t)\mathrm{d}t, \tag{22}$$

by choosing a path for $d(g,t)$ that satisfies (21), and where $d(g,t) \geq 0$ for all $t \geq 0$.

Letting $q(g,t)$ denote the market price of a share of firm g, because the outstanding number of stocks is normalized to 1, the firm's total stock market value equals $q(g,t)$. The instantaneous stock return consists of the dividend flow and the capital gain, formally:

$$\theta(g,t) = \frac{d(g,t)}{q(g,t)} + \frac{\dot{q}(g,t)}{q(g,t)}. \tag{23}$$

The solution to this differential equation, under $\theta(g,t) > 0$, and with boundary condition $\lim_{t\to\infty} q(g,t) < \infty$, is (22). Intuitively, the firm g knows that the stock price must obey condition (23), and so it chooses a dividend path that implies a rate of capital accumulation that maximizes the firm's valuation resulting from (23). The optimal strategy corresponds to the condition

$$f'(k(g,t)) = \delta + \theta(g,t). \tag{24}$$

Note that equation (24) generalizes (3) to the full model.[14]

Equilibrium. An equilibrium consists of time paths for the allocation of consumption, $c^A(t), c^B(t)$, portfolio holdings, $z(g,t)$,$\tilde{z}(g,t)$, capital $k(g,t)$, $\tilde{k}(g,t)$, and time paths for prices $q(g,t)$, $\tilde{q}(g,t)$ for $g \in [0,1]$ and $t \geq 0$ such that households and firms solve their dynamic optimization problems and the markets for stocks and output clear, given the initial wealth distribution w_0. In particular, because investment is done directly by firms, aggregate con-

[14]The maximization problem allows for a solution where $d(g,t) = 0$ for some g and t. Note that this was the case for the two-period model as well, but in the presence of only two firms, and the Inada conditions of log utility, it was never optimal to set dividends to zero. With a continuum of firms, it is in principle possible for a subset of firms to choose zero dividends if the cost of capital $\theta(g,t)$ is low enough, without implying consumption going to zero, as long as other firms pay positive dividends. We proceed under the assumption that, in equilibrium, it is never optimal for any firm to do so given the path of $\theta(g,t)$.

sumption equals aggregate dividends, that is

$$c^A(t) + c^B(t) = \int_0^1 d(g,t)dg. \tag{25}$$

Aggregate saving is then equal to total output minus aggregate consumption, which corresponds to aggregate gross investment.

III.2. Equilibrium Characterization

We conjecture a dynamic equilibrium consisting of a threshold sustainability score, $g^*(t) \in [0,1]$, such that for $g \geq g^*(t)$, Advocate holds the entirety of stocks in those firms, which means $z(g,t) = 1$, while it holds zero stocks in all the remaining firms.[15]

Optimal portfolio allocation and the time-varying threshold equilibrium imply

$$\frac{\dot{c}^A}{c^A} = \theta(g^*) + v(g^*)c^A - \rho \tag{26}$$

and

$$\frac{\dot{c}^B}{c^B} = \theta(g^*) - \rho, \tag{27}$$

where we have suppressed time dependence for notational convenience. The difference between consumption growth for the two households is proportional to the non-pecuniary return of the threshold firm, $v(g^*)$. When $g^* > g_n$, $v(g^*) > 0$, so consumption growth for Advocate is larger than for Benchmark, which means that Advocate is saving relatively more and thus expanding their wealth. When $g^* < g_n$, on the other hand, $v(g^*) < 0$, and consumption growth for Advocate is now smaller than for Benchmark, suggesting that Advocate is saving less, thus reducing their wealth.

The dynamics for the equilibrium threshold are characterized in the following Lemma.

Lemma (Threshold Dynamics). *The threshold $g^*(t)$ evolves according to the differential equation*

$$\dot{g}^* = \frac{c^A - \int_{g*}^1 d(g)\phi(g)\mathrm{d}g}{q(g^*)\phi(g^*)}. \tag{28}$$

The numerator in (28) consists of the difference between Advocate's consumption and

[15] We formally state this equilibrium structure as Lemma 1 in Appendix B and provide its proof there.

income at time t, the latter consisting of the sum of dividend payments from its portfolio at time t. If the difference is negative, it means that some of the income is used to increase savings, which is only possible by increasing the holdings of firms with a lower score than the ones already in the household's portfolio, so that $\dot{g}^* < 0$. If the difference is positive, to sustain the higher consumption, the household is liquidating part of its portfolio, starting from the firms with lower sustainability scores, and so $\dot{g}^* > 0$. Finally, for a given difference in consumption and income, the rate of change of the threshold g^* is lower the higher the stock price of the threshold firm, $q(g^*)$, and the larger the mass of firms at that price, $\phi(g^*)$, reflecting the smaller need to move along the g score when price and density fill the required saving adjustment.

The optimal portfolio conditions for Advocate imply that

$$\theta(g) - \theta(g^*) = -\big(v(g) - v(g^*)\big)c^A \quad \text{for} \quad g \geq g^*, \tag{29}$$

and those for Benchmark imply

$$\theta(g) = \theta(g^*) \quad \text{for} \quad g < g^*. \tag{30}$$

Equations (29)-(30) outline the characterization of capital allocation in equilibrium that is analogous to Cases 1-3 in the two-period model. When equation (29) holds for a firm g, if g is above the neutral score g_n, capital allocation is tilted in its favor compared to the threshold firm g^*, while if g is below the neutral score, the tilt is reversed. The relevant reference for evaluating overall impact is thus whether g^* is higher or lower than the neutral score g_n. When $g^* > g_n$, equation (29) implies that all the firms with scores higher than g^* are positively impacted by Advocate's preferences, so we should expect both a composition and a positive scale effect. However, when $g^* < g_n$, the "clean" firms, those with $g > g_n$, register a positive composition impact, but the "dirty" firms register a negative composition impact. In this case, the scale impact is ambiguous along the dynamic path, and it depends on the location of g^* and the distribution of firms across sustainability scores.

III.3. Wealth Dynamics and Steady State

The full dynamic model allows us to investigate how the equilibrium changes over time, and where the economy is headed in the long run. Towards that goal, it is useful to compare

the evolution of wealth across the two households, described respectively by

$$\dot{a}^A = \theta(g^*)a^A - c^A - \big(\mathcal{G} - v(g^*)a^A\big)c^A, \tag{31}$$

and

$$\dot{a}^B = \theta(g^*)a^B - c^B. \tag{32}$$

For both households, wealth accumulation is driven by the return on their wealth minus consumption. For Benchmark, the combination of (27) and (32) implies the familiar result that consumption is a constant fraction of wealth, that is $c^B(t) = \rho a^B(t)$. It follows that wealth changes at the rate, $\theta(g^*) - \rho$, in equilibrium.

For Advocate the term, $\mathcal{G} - v(g^*)a^A$, injects a different wealth dynamics compared to Benchmark. Since

$$\mathcal{G} - v(g^*)a^A = \int_{g^*}^{1} \big(v(g) - v(g^*)\big)q(g)\phi(g)\mathrm{d}g > 0. \tag{33}$$

equation (31) shows that the Advocate's wealth grows at a return that is adjusted downward from $\theta(g^*)$. At the same time, equation (27) implies higher consumption growth whenever $v(g^*) > 0$ (equivalently $g^* > g_n$), which corresponds to lower consumption levels early along the optimal path, and thus faster wealth accumulation and faster consumption growth. Symmetrically, when $v(g^*) < 0$ (equivalently $g^* < g_n$), equation (27) implies lower consumption growth, and thus higher consumption levels early along the optimal path, and thus slower, or negative wealth accumulation. Taking all the equations together, the following proposition summarizes the characterization of the dynamic path.

Proposition 1 (Wealth Dynamics)**.** *Along the equilibrium path, When $g^*(t) > g_n$, Advocate accumulates wealth, which implies $\dot{g}^* < 0$. When $g^*(t) < g_n$, Advocate decumulates wealth, which implies $\dot{g}^* > 0$.*

Whether the initial threshold, $g^*(0)$, is above or below the neutral level g_n depends on the initial wealth distribution across households. In particular, if Advocate begins with a level of wealth that is relatively low compared to Benchmark, then the path with $\dot{g}^* < 0$ applies. Alternatively, if Advocate's initial wealth is relatively large, the path with $\dot{g}^* > 0$ applies. This implication has the same qualitative flavor as the equilibrium with two firms, where, depending on the initial wealth level of Advocate, one would see a wealth accumulation (Cases 1 and 2)

or decumulation (Case 3).

The natural question at this point is where the economy is headed in the long run, that is, what is the steady state of the dynamic system we just investigated? The following proposition states the result.

Proposition 2 (Steady-State)**.** *In the steady state of the equilibrium of the full dynamic model*

$$g^* = g_n \tag{34}$$

and the capital distribution $k_{ss}(g)$ *for* $g \in [0,1]$ *is determined by*

$$f'(k_{ss}(g)) = \rho + \delta - v(g)c^A_{ss}, \qquad for \qquad g \geq g_n, \tag{35}$$

where, $c^A_{ss} = \int_{g_n}^1 \left[f(k_{ss}(g)) - \delta k_{ss}(g)\right]\phi(g)dg$*, and*

$$f'(k_{ss}(g)) = \rho + \delta, \tag{36}$$

otherwise.

Recall that in an economy where only Benchmark households are present, the level of capital in the steady state would be the same across all firms and equal to $\hat{k}$, determined by the familiar efficiency condition

$$f'(\hat{k}) = \rho + \delta, \tag{37}$$

for all $g \in [0,1]$. In the steady state of an economy with Advocate households, things are different. Advocate holds a portfolio that consists of all the firms that have a score above or equal to the neutral score g_n. The rest of the stocks are held by Benchmark. For the firms above the neutral score, and held by Advocate, capital allocation is determined by (35). The term under the integral corresponds to one over Advocate's marginal utility of consumption in steady state, and it depends on the total portfolio dividend income. Note that the term is always positive, which implies that capital allocated to "clean" firms is always higher than what would be the case in economy with only Benchmark households. The capital composition depends on the distribution of firms above the neutral score $\phi(g)$ for $g \geq g_n$. The more the distribution is skewed towards firms that have a higher sustainability score, the more capital is allocated to sustainable firms overall. Intuitively, capital allocation is a consequence of

the savings of advocate investors in the long run. With more firms on the "clean" side of the sustainability distribution, advocate investors reduce consumption further early along the path, lowering the cost of capital and ensuring more earnings are retained to invest and accumulate capital, in line with the values expressed by their preferences. Over time, the saving behavior of Advocate Because Advocate is infinitely lived, they eventually save so that the capital scale is higher, the higher the mass of sustainable firms.

For firms below the neutral score, and held by Benchmark, long-run capital allocation is the same as what would emerge in a standard economy without Advocate households. In some respects, this is a surprising result, given that Advocate households harbor negative preferences for those firms, and we know from the two-period model that the scale effect can be negative when advocate investors really dislike dirty firms. However, both general equilibrium and long-run dynamics operate to completely neutralize the impact of preferences for disinvestment on dirty firms. General equilibrium implies that such firms must be held by some households and thus continue to operate. The long-run dynamics imply that Advocate gradually reduces their holding of dirty stocks, and if the equilibrium where $g^*(0) < g_n$ applies, they do so by reducing their saving, while Benchmark always saves a constant fraction of their wealth. Eventually, the marginal investor in dirty firms is Benchmark, and (36) holds.

Proposition 2 shows that aggregate capital and output in the steady state are a function of the preferences of Advocate households. If one thinks of $v(g)$ as modifying the patience of households into a degree of "effective patience," the result can be interpreted as the generalization of the familiar dependence of steady-state capital on the subjective discount rate ρ. Thus, when $v(g) > 0$, households are willing to permanently substitute consumption for capital $k(g)$, hence the higher steady-state capital. The combined result of equations (35) and (36) is that as long as $v(g) > 0$ for $g \geq g_n$, in steady state, there is always a composition effect in favor of firms with a score above the neutral score, and there is always a positive scale effect on output.

A perhaps surprising feature of the steady state allocation in Proposition 2 is that for the dirty firms, i.e., those with $v(g) < 0$, the level of capital in steady state is the same they would have accumulated in the absence of Advocate households. This result is in contrast with the equilibrium of the two-period model where for a higher share of wealth of advocate households, capital in the dirty firm was declining as capital in the clean firm was increasing. The reason is that Advocate's preferences do not matter for capital allocation in dirty firms in steady state. The result shows that if Advocate's preferences were to pursue only a disinvestment strategy in dirty firms, that is $v(g) < 0$ for $g < g_n$ and $v(g) = 0$ otherwise, dynamics and

general equilibrium would imply no impact on capital allocation our output! The presence of Advocate would ultimately only impact the portfolio allocation and relative consumption patterns.

Finally, an important corollary of Proposition 2 is that both the Advocate and Benchmark wealth are non-degenerate in steady state.

Corollary 1 (Wealth Convergence)**.** *In the steady state, Advocate's wealth converges to*

$$a_{ss}^{A} = \int_{g_n}^{1} \left(\frac{f(k_{ss}(g)) - \delta k_{ss}(g)}{\rho - v(g) c_{ss}^{A}} \right) \phi(g) dg \tag{38}$$

while Benchmark's wealth converges to

$$a_{ss}^{B} = \left(\frac{f(\hat{k}) - \delta \hat{k}}{\rho} \right) \Phi(g_n) \tag{39}$$

where $\Phi(g)$ is the cdf of g, and $\hat{k}$ is determined by (37).

Intuitively, in the accumulation equations (18) and (20), there are two forces that shape the evolution of wealth: the saving rate (negative of consumption rate out of wealth) and the return on existing wealth. For Benchmark, the consumption rate out of wealth, ρ and the return on existing wealth, $\theta(g)$, equate in steady state, which keeps wealth constant. For Advocate, the return on wealth is lower because of the non-pecuniary return $v(g)$, but the consumption rate is also lower (the saving rate is higher), and the two forces once again balance to keep Advocate wealth unchanged.[16]

IV. Numerical Simulations

We perform a number of numerical simulations to highlight the factors that drive the composition and scale effect in the full model. We first explore the impact of different parametric assumptions on the steady state, and then study their implications for the transition dynamics, and the timing of impact of sustainable investing under different scenarios.

[16] Using (31) one can also show that $a_{ss}^{A} = c_{ss}^{A} \frac{1+\mathcal{G}_{ss}}{\rho}$, where $\mathcal{G}_{ss} = \int_{g_n}^{1} v(g) q_{ss}(g) \phi(g) g$, and $q_{ss}(g)$ denotes the stock price of firm g in steady state. This expression confirms that Advocate's steady state wealth is positive, and it showcases how the consumption rate out of wealth is lower than ρ for Advocate because $\mathcal{G}_{ss} > 0$.

IV.1. Steady State

Figure 4 reports the numerical solution of the steady state equilibrium of Proposition 2 under two specifications for the sustainability score distribution $\phi(g)$: a uniform distribuion and the approximate empirical distribution of Figure 2. The non-pecuniary preferences $v(g)$, reported in the top panel of the figure, range from -0.05 to 0.05, with a neutral score $g_n = 0.5$. The Figure displays the distribution of output in steady state across sustainability scores, as a ratio of steady state output in an economy with only Benchmark households. For both distributions, output is unchanged for firms with $g \leq g_n$, while it is higher for those with $g > g_n$. In other words, the composition effect is in favor of clean firms, and the scale effect on output is unambiguously positive. In addition, the extent of capital allocation is directly dependent on the sustainability score of clean firms: the higher the score, the higher the positive capital difference. Importantly, the steady state is independent of the initial wealth distribution w_0, so the result holds irrespectively of the initial size of Advocate.

Contrasting the allocation across the two distributions, the compuation shows that a distribution skewed to the right results in a stronger allocation of capital for any given sustainability score above g_n. Equation (35) indicates that this is a consequence of marginal utility of consumption, $1/c^A_{ss}$. A larger mass of firms to the right of g_n requires a larger accumulation of wealth for Advocate to hold all of the clean stocks in steady state. Everything else equal, a larger wealth translates into a larger consumption, c^A_{ss}, and thus a lower marginal utility. The value for the non-pecuniary term $v(g)$ measured in marginal utility units is thus higher, which leads Advocate to accept a lower return on the stocks, and firms to accumulate more capital to ensure (35) holds.

Finally, the numerical results showcase that in the steady state, the extent of the negative non-pecuniary utility of the Advocate household has no impact on the allocation of capital and output of firms with a sustainability score below the neutral threshold, g_n (flat lines at 1 in both simulations). It is worth emphasizing again that the result is the consequence of both general equilibrium and dynamics. On the one hand, stocks of "dirty" companies sold by Advocate must be held by the rest of the market at some equilibrium price. On the other hand, the rest of the market implements an intertemporal substitution that sustains the accumulation of capital of dirty companies all the way to the level that would have resulted in the absence of households with sustainability motives. Effectively, the composition effect in steady state is the consequence of the positive scale effect for companies with $g > g_n$. In this sense, there is no reallocation taking place from dirty to clean firms in steady state, but the reallocation

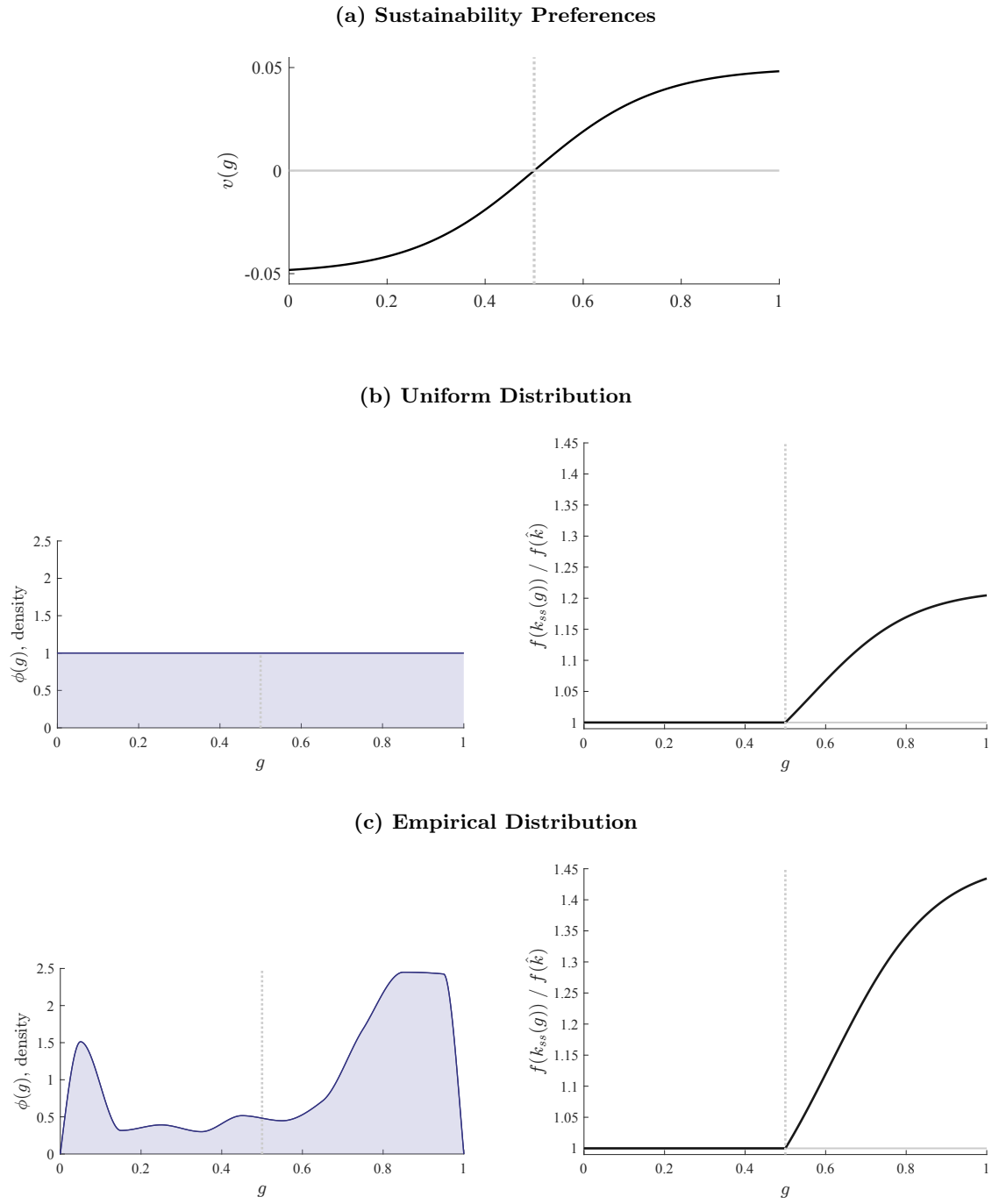


Figure 4: Output Composition and Scale Impact in Steady State

Notes: Figure 4 reports the steady-state composition impact on output across firms for a uniform score distribution (Panel (b)) and the empirical score distribution (Panel (c)) measured in terms of employment. Sustainability preferences $v(g)$ are specified as in Panel (a) with range $[-0.5, 0.5]$ and neutral score $g_n = 0.5$. Output technology is $f(k) = k^{\alpha}$, and parameters are set at $\alpha = 0.33$ and $\rho = 0.11$.

responsible for the larger capital in clean firms comes from the substitution between early and later consumption for Advocate households.

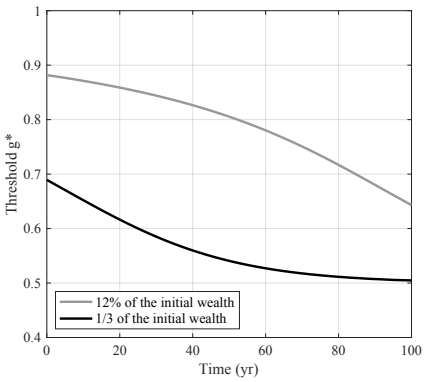
IV.2. Transition Dynamics

We next explore the transition dynamics of the economy. We imagine that the starting point is the steady state allocation of an economy with only Benchmark households. We then assume that a measure of households with wealth share $w_0 > 0$ adopt Advocate preferences permanently, and we simulate the dynamics of the equilibrium system from that point onward. We consider two possible values for w_0, 12% and 33%, which roughly represent the share of assets under management classified as sustainable, for the U.S. and for the global economy, respectively, according to the data in Figure (1) for 2024. Throughout the analysis, we set $\rho = 0.11$, and we interpret it as corresponding to an annual rate of 11.6% if we take a unitary time interval to correspond to a year.

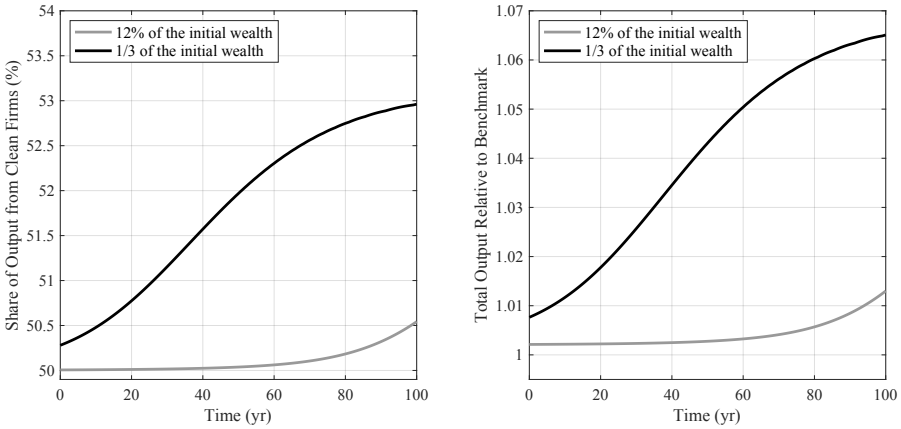
Figure 5 reports the transition dynamics for the equilibrium threshold $g^*(t)$ (Panel (a)), the share of output produced by clean firms (composition) and the total output (scale) (Panel (b)), and the consumption and wealth dynamics of Advocate and Benchmark (Panel (c)), for an economy with a uniform sustenability score distribution. While not shown in the diagram, we know from the steady state analysis that all the dynamics converge to the same steady state – independently of the initial wealth.

Focusing first on the case of $w_0 = 33\%$, we see that the equilibrium threshold starts at around 0.7, and declines steadily towards the neutral score $g_n = 0.5$. At the same time, the share of output produced by clean firms and the total output scale both increase steadily over time, starting within the first 5 years, and settle close to their steady state values after about 100 years. Along the transition dynamics, Advocate's wealth increases, while Benchmark's wealth decreases. Advocate's consumption path is low early on and then rises over time—a consequence of the stronger incentive to save in clean stocks. In contrast, Benchmark's consumption path is high early on and declines over time. To understand these dynamics, it is important to remember that the initial capital allocation to firms is the one that would obtain in the absence of Advocate. In particular, for dirty stocks, the capital allocation is already at the steady-state level they would reach in the presence of Advocate, so the marginal product of capital is relatively low. The decline in Benchmark's wealth is thus the consequence of both Advocate purchasing clean stocks from Benchmark and the relatively low return on dirty stocks. As Advocate saves early and purchases clean stocks from Benchmark, the payment in output is optimally allocated to Benchmark's consumption rather than to wealth, because the

(a) Threshold Dynamics



(b) Output Composition and Scale



(c) Wealth and Consumption

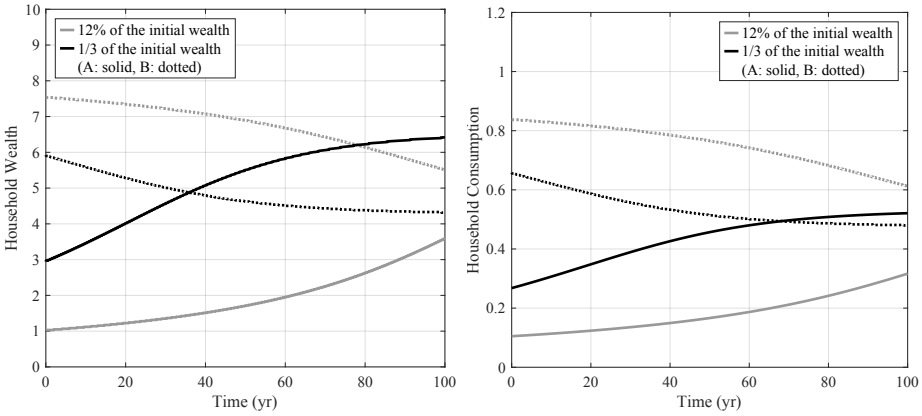


Figure 5: Transition Dynamics under Uniform Distribution

Notes: Figure 5 reports the transition dynamics for an economy with uniform sustainability score distribution. Sustainability preferences are specified as in Figure 4. The gray lines report the results for an economy with initial wealth share $w_0 = 12\%$, while the black lines correspond to an initial wealth share of $w_0 = 33\%$.

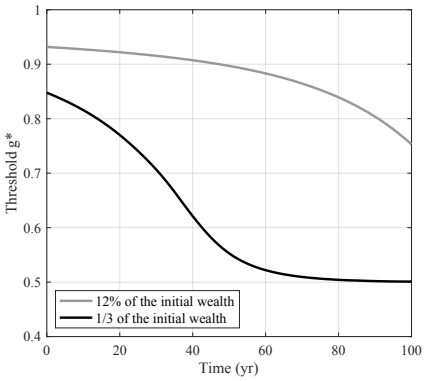
return on dirty stocks is relatively low, with capital already at its steady-state level.

For the case of $w_0 = 12\%$, the qualitative features of the dynamics are similar to what we just outlined, but the magnitudes are markedly different. Since the system is converging to the same steady state independently of w_0, the dynamics under $w_0 = 12\%$ would eventually correspond to the dynamics described above as soon as the relative wealth of Advocate in the economy reaches 33%. However, because the economy starts with a lower wealth for Advocate, the initial equilibrium threshold is just shy of 0.9, and it reaches 0.7 only after 85 years. Similarly, the output share of clean firms, and the total output, increase significantly only after 70 to 90 years. The same holds for the relative levels of wealth and for consumption across Benchmark and Advocate.

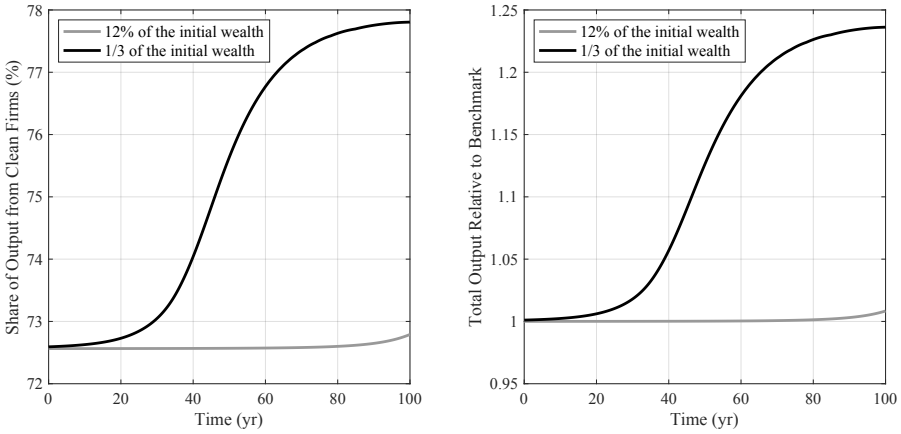
Figure 6 reports the transition dynamics under the empirical distribution of Figure 2 as approximated in Panel (c) of Figure 4. Recall that the empirical distribution features a larger fraction of firms to the right of the neutral score $g_n = 0.5$, with the mass increasing sharply between 0.75 and 1. The incentive to save in clean stocks is thus magnified compared to the uniform distribution case, as a larger amount of savings can be invested in firms with a higher sustainability score, and thus a higher non-pecuniary return. This creates a strong incentive to accumulate wealth in clean stocks with scores above 0.75. At the same time, a larger mass of clean firms also means that the initial share of clean output is already tilted in favor of clean – 72.5% for the empirical vs 50% under uniform distribution. The two features result in a slow impact for both $w_0 = 12\%$ and $w_0 = 33\%$ compared to the uniform case. With low initial wealth share, the output composition and the scale do not show a noticeable increase for about 90 years. For $w_0 = 33\%$, the increase is also slow initially, but it drastically accelerates between 35 and 60 years. As more wealth is accumulated in savings for firms with the highest scores, capital is increased beyond the initial level, with a visible impact only when the wealth share is large enough so that Advocate is the marginal investor in those firms. Because there is a larger mass of those firms under the empirical distribution, a higher wealth share is necessary to impact their cost of capital.

Finally, Figure 7 shows the transition dynamics for the distribution of capital across firms, under the uniform distribution. The gray screen represents the threshold for the neutral score $g_n = 0.5$. As expected, the impact on the distribution of capital in favor of clean firms is larger and faster with higher initial wealth (Panel (b)). Interestingly, the allocation of capital to dirty firms, those with $g < 0.5$, remains constant throughout the dynamics at the initial level – which corresponds to the steady state level in the economy with Advocate, as we know from

(a) Threshold Dynamics



(b) Output Composition and Scale



(c) Wealth and Consumption

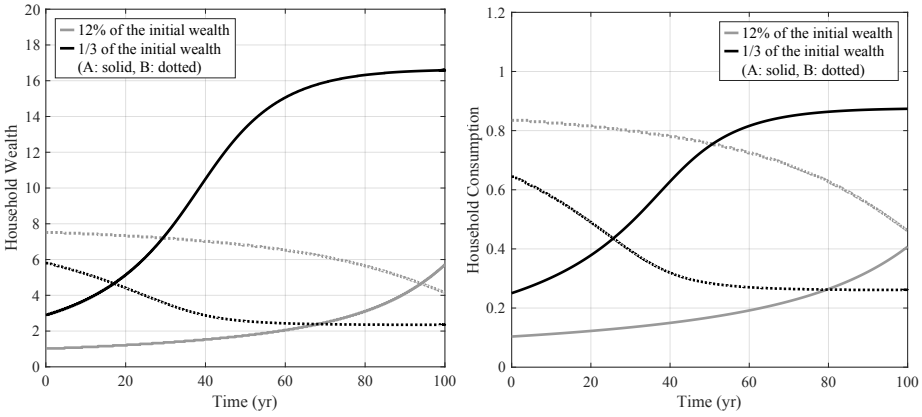
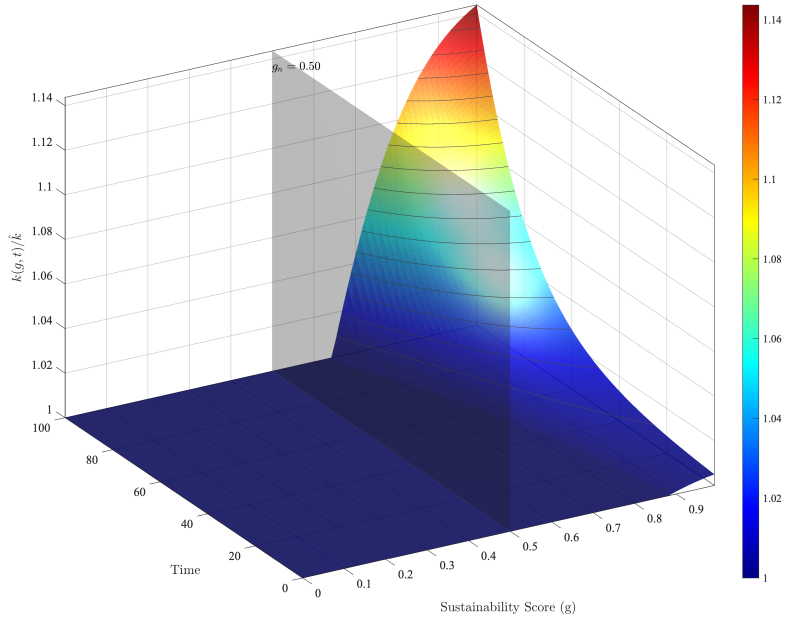


Figure 6: Transition Dynamics under Empirical Distribution

Notes: Figure 6 reports the transition dynamics for an economy with the empirical sustainability score distribution of Figure 2, Panel (b). Sustainability preferences are specified as in Figure 4. The gray lines report the results for an economy with initial wealth share $w_0 = 12\%$, while the black lines correspond to an initial wealth share of $w_0 = 33\%$.

(a) Initial Wealth $w_0 = 12\%$



(b) Initial Wealth $w_0 = 33\%$

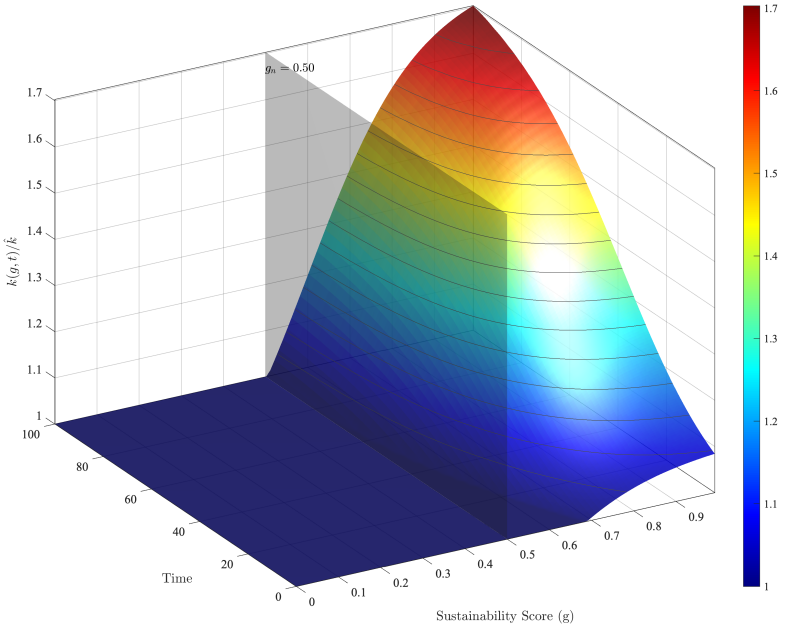


Figure 7: Capital Distribution in the Transition Dynamics

Notes: Figure 7 reports the transition dynamics for an economy with the empirical sustainability score distribution of Figure 2, Panel (b). Sustainability preferences are specified as in Figure 4. The gray lines report the results for an economy with initial wealth share $w_0 = 12\%$, while the black lines correspond to an initial wealth share of $w_0 = 33\%$.

Proposition 2. Remarkably, the result that negative non-pecuniary returns on dirty stocks do not impact the level of steady state capital accumulated by dirty firms, also holds along the transition dynamics, when the initial capital is already at the steady state level for dirty firms. In equilibrium, the presence of Advocate leads to higher capital allocation for clean firms, not because existing capital is reallocated away from dirty firms (via lower retained earnings by dirty firms), but because consumption is reallocated to savings by Advocate (via higher retained earnings by clean firms).

In summary, the numerical simulations show that the presence of Advocate eventually impacts the allocation of capital in favor of clean firms; however, the timing of the impact strongly depends on the initial share of wealth w_0. Under our parameterization, each percentage point of Advocate's initial wealth share w_0 corresponds to roughly a 4 year difference in where the economy would be along the transition dynamics.[17]

It is important to stress that these calculations are based on a stylized representation of sustainability preferences and financial markets. In addition, we have not explitictly modeled the link between the Advocate preferences and the impact of externalities on welfare. However, they represent, to the best of our knowledge, a first systematic attempt to model the impact of sustainable investing, taking both dynamics and general equilibrium into account. The message from this exercise is a somber one: for sustainable investing to be a market-based mechanism that can effectively and timely steer the allocation of productive capital towards uses that internalize environmental/societal/governance externalities, other mechanisms that act as magnifiers should be considered. Important examples are the adoption of existing sustainable technologies and R&D investment towards sustainable technological change.

V. Conclusion

We characterize under what conditions sustainable investing—wherein financial investors like (resp. dislike) holding assets that they believe have a positive (resp. negative) environmental/societal impact—alters the allocation of capital between clean and dirty firms in a dynamic general equilibrium framework. We show that advocate investors can have both a *scale* and a *composition* impact on capital allocation and aggregate output. We show how the value of the total wealth of advocate investors relative to total market capitalization, the shape of

[17] Taking 80 years as the average delay in the transition dynamics when $w_0 = 12\%$ vs $w_0 = 33\%$, and assuming a linear impact of each additional percentage of wealth for the sake of the calculation, one gets $80/(33-12) \approx 3.8$ years.

their preferences for non-pecuniary returns of holding clean and dirty stocks, and the size of firms across sustainability scores, interact in general equilibrium to determine the extent of the impact of sustainable investing. The model has rich predictions for the scale and composition effects, and suggests interesting avenues for the role of policy to address important sources of environmental and societal externalities in the presence of sustainable investing. We leave the study of such implications to future work.

References

Atz, U., Van Holt, T., Liu, Z.Z., Bruno, C.C., 2022. Does sustainability generate better financial performance? review, meta-analysis, and propositions. Journal of Sustainable Finance & Investment , 1–24.

Azémar, C., Giroud, A., 2023. World investment report 2022: International tax reforms and sustainable investment: United Nations conference on trade and development, geneva and new york, 2022, 219 pp. isbn: 978-9211130492.

Baker, M., Bergstresser, D., Serafeim, G., Wurgler, J., 2022a. The pricing and ownership of US green bonds. Annual Review of Financial Economics 14, 415–437.

Baker, M., Egan, M.L., Sarkar, S.K., 2022b. How Do Investors Value ESG? Working Paper. National Bureau of Economic Research.

Barber, B.M., Morse, A., Yasuda, A., 2021. Impact investing. Journal of Financial Economics 139, 162–185.

Bauer, R., Ruof, T., Smeets, P., 2021. Get real! individuals prefer more sustainable investments. The Review of Financial Studies 34, 3976–4043.

Beltratti, A., 2005. Capital market equilibrium with externalities, production and heterogeneous agents. Journal of Banking & Finance 29, 3061–3073.

Berg, F., Koelbel, J.F., Pavlova, A., Rigobon, R., 2022a. ESG confusion and stock returns: Tackling the problem of noise. Technical Report. National Bureau of Economic Research.

Berg, F., Kölbel, J.F., Rigobon, R., 2022b. Aggregate confusion: The divergence of ESG ratings. Review of Finance 26, 1315–1344.

Berk, J.B., van Binsbergen, J.H., 2025. The impact of impact investing. Journal of Financial Economics 164, 103972.

Brock, W.A., Turnovsky, S.J., 1981. The analysis of macroeconomic policies in perfect foresight equilibrium. International Economic Review , 179–209.

Caramichael, J., Rapp, A.C., 2022. The green corporate bond issuance premium. International Finance Discussion Paper .

Chittaro, L., Piazzesi, M., Sena, M., Schneider, M., 2025. Asset returns as carbon taxes .

Dam, L., 2011. Socially responsible investment in an environmental overlapping generations model. Resource and Energy Economics 33, 1015–1027.

Dam, L., Heijdra, B.J., 2011. The environmental and macroeconomic effects of socially responsible investment. Journal of Economic Dynamics and Control 35, 1424–1434.

Geczy, C., Stambaugh, R.F., Levin, D., 2005. Investing in socially responsible mutual funds. Available at SSRN 416380 .

Gillan, S.L., Koch, A., Starks, L.T., 2021. Firms and social responsibility: A review of ESG and CSR research in corporate finance. Journal of Corporate Finance 66, 101889.

Hartzmark, S.M., Sussman, A.B., 2019. Do investors value sustainability? a natural experiment examining ranking and fund flows. The Journal of Finance 74, 2789–2837.

Heal, G.M., 1998. Valuing the future: economic theory and sustainability. Columbia University Press.

Hong, H., Shore, E.P., 2022. Corporate Social Responsibility. Working Paper. National Bureau of Economic Research.

Humphrey, J., Kogan, S., Sagi, J., Starks, L., 2021. The asymmetry in responsible investing preferences. Technical Report. National Bureau of Economic Research.

Idzorek, T.M., Kaplan, P.D., Ibbotson, R.G., 2021. The popularity asset pricing model. Available at SSRN 3451554 .

Kedia, S., Starks, L.T., Wang, X., 2021. Institutional investors and hedge fund activism. The Review of Corporate Finance Studies 10, 1–43.

Pástor, L., Stambaugh, R.F., Taylor, L.A., 2021. Sustainable investing in equilibrium. Journal of Financial Economics 142, 550–571.

Pástor, L., Stambaugh, R.F., Taylor, L.A., 2022. Dissecting green returns. Journal of Financial Economics 146, 403–424.

Pástor, L., Stambaugh, R.F., Taylor, L.A., 2024. Sustainable investing. Working Paper .

Riedl, A., Smeets, P., 2017. Why do investors hold socially responsible mutual funds? The Journal of Finance 72, 2505–2550.

Starks, L.T., 2023. Presidential address: Sustainable finance and ESG issues—value versus values. The Journal of Finance .

Turnovsky, S.J., 1990. The effects of taxes and dividend policy on capital accumulation and macroeconomic behavior. Journal of Economic Dynamics and Control 14, 491–521.

Zerbib, O.D., 2022. A sustainable capital asset pricing model (s-capm): Evidence from environmental integration and sin stock exclusion. Review of Finance 26, 1345–1388.

Appendix A. Two-Period Model

Appendix A.1. Derivation of Optimal Portfolio Conditions

Let the preferences of Advocate be specified as

$$\ln c_0^A + \beta \ln c_1^A + vqz - \tilde{v}\tilde{q}\tilde{z}$$

To derive the optimal portfolio allocation under the budget constraints specified in (6), we set up the Lagrangian:

$$\mathcal{L}^A = \ln c_0^A + \beta \ln c_1^A + vqz - \tilde{v}\tilde{q}\tilde{z} + \lambda_0^A \left(w_0 W_0 - c_0^A - zq - \tilde{z}\tilde{q}\right) + \lambda_1^A \left(zd + \tilde{z}\tilde{d} - c_1^A\right) + \mu z + \tilde{\mu}\tilde{z}$$

where λ_0^A and λ_1^A are the Lagrange multipliers for the corresponding period budget constraints, while μ and $\tilde{\mu}$ are the multipliers for the short-sale constraint $z \geq 0$. The first-order conditions are

$$c_0^A: \quad \frac{1}{c_0^A} - \lambda_0^A = 0 \;\Rightarrow \lambda_0^A = \frac{1}{c_0^A}, \tag{A.1}$$

$$c_1^A: \quad \beta\frac{1}{c_1^A} - \lambda_1^A = 0 \;\Rightarrow \lambda_1^A = \beta\frac{1}{c_1^A}, \tag{A.2}$$

$$z: \quad -\lambda_0^A q + \lambda_1^A d + vq + \mu = 0 \tag{A.3}$$

$$\tilde{z}: \quad -\lambda_0^A \tilde{q} + \lambda_1^A \tilde{d} - \tilde{v}\tilde{q} + \tilde{\mu} = 0, \tag{A.4}$$

respectively. To satisfy the complementary slackness conditions, we need $\mu, \tilde{\mu} \geq 0$, and $\mu z = \tilde{\mu}\tilde{z} = 0$. Recalling that $\theta = d/q$ and $\tilde{\theta} = \tilde{d}/\tilde{q}$, and manipulating the first order conditions one obtains (9). The derivation of (10) for Benchmark follows similar steps but with $v = \tilde{v} = 0$.

Appendix A.2. Saving Rates

For a household with logarithmic utility over consumption in two periods, $\ln c_0 + \beta \ln c_1$, it is well known that the optimal saving strategy is to save a constant fraction $\frac{\beta}{1+\beta}$ of initial wealth, regardless of the return on savings since the substitution and income effect cancel one another. For Benchmark, this optimal saving strategy holds even in the presence of Advocate.

The savings of Benchmark in period 0 are chosen as the value of stocks they carry into

period 1:

$$q(1-z)+\tilde{q}(1-\tilde{z}). \tag{A.5}$$

Applying the optimal saving rule, the relationship

$$q(1-z)+\tilde{q}(1-\tilde{z})=\frac{\beta}{1+\beta}(1-w_0)W_0$$

must hold, regardless of the portfolio composition.

Advocate's saving behavior is modified by their non-pecuniary preferences for sustainable wealth in our model. The following details the derivation of the saving functions for Advocate across the three possible equilibrium cases.

Case 1: $0<z<1, \tilde{z}=0$. In this case, Advocate holds some but not all stocks of the clean firm and none of the dirty firm. Since Benchmark holds both types of stocks in Case 1, their optimal portfolio conditions imply

$$\theta=\tilde{\theta} \tag{A.6}$$

in equilibrium. Replace the equalized returns with θ in Advocate's Euler equation:

$$\frac{c_1^A}{c_0^A}=\beta\theta+vc_1^A, \tag{A.7}$$

in which

$$c_0^A=w_0W_0-qz \tag{A.8}$$

$$c_1^A=dz=\theta qz \tag{A.9}$$

Substituting these into the Euler equation, we obtain

$$\frac{qz}{w_0W_0-qz}=\beta+vqz \quad \Rightarrow \; qz=\underbrace{\frac{\beta+vqz}{1+\beta+vqz}}_{s^A}w_0W_0$$

after simplification, which is the saving function for Case 1. It shows that the presence of positive non-pecuniary motives ($v > 0$) tilts the saving function upward compared to the standard logarithmic case.

Case 2: $z = 1, \tilde{z} = 0$. In this case, Advocate holds all stocks of the clean firm ($z = 1$) and none of the dirty firm. The saving function remains the same as in Case 1, but with $z = 1$:

$$q = \underbrace{\frac{\beta + vq}{1 + \beta + vq}}_{s^A} w_0 W_0. \tag{A.10}$$

The key difference in Case 2 is that Advocate is now the marginal investor in the clean firm, allowing the required dividend yields (and thus capital allocation) to differ between clean and dirty firms:

$$\theta < \tilde{\theta}. \tag{A.11}$$

The optimal portfolio conditions for Advocate, as shown in equation (**??**), imply that

$$\frac{1}{\beta}(v + \tilde{v})c_1^A \geq \tilde{\theta} - \theta, \tag{A.12}$$

which characterizes the maximum pecuniary return differential Advocate is willing to tolerate.

Case 3: $z = 1, \tilde{z} > 0$. In this case,Advocate holds all stocks of the clean firm and some of the dirty firm. For household A to hold shares in dirty firms, the following optimal portfolio condition must hold with equality:

$$\frac{c_1^A}{c_0^A} = \beta\tilde{\theta} - \tilde{v}c_1^A, \tag{A.13}$$

Using the budget constrains this becomes

$$\frac{\theta q + \tilde{\theta}\tilde{q}\tilde{z}}{w_0 W_0 - q - \tilde{q}\tilde{z}} = \beta\tilde{\theta} - \tilde{v}\theta q - \tilde{v}\tilde{\theta}\tilde{q}\tilde{z} \tag{A.14}$$

The simplify the optimal saving strategy for Advocate, we first rescale both sides of equa-

tion (A.14) to

$$\frac{\theta q/\tilde{\theta} + \tilde{q}\tilde{z}}{w_0 W_0 - q - \tilde{q}\tilde{z}} = \beta\tilde{\theta} - \tilde{v}\theta q/\tilde{\theta} - \tilde{v}\tilde{q}\tilde{z}$$

which leads to

$$S^A - \left(1 - \frac{\theta}{\tilde{\theta}}\right) q = \left[w_0 W_0 - S^A\right] \left[\beta - \sigma(\tilde{v})\right],$$

where S^A stands for total savings, and in which we denote

$$\sigma(\tilde{v}) \equiv \tilde{v} q \frac{\theta}{\tilde{\theta}} + \tilde{v}\tilde{q}\tilde{z} = \frac{\tilde{v}}{\tilde{\theta}} c_1^A$$

After further simplification and rearrangement to isolate S^A, we arrive at the saving function:

$$S^A = \frac{\beta - \sigma(\tilde{v})}{1 + \beta - \sigma(\tilde{v})} w_0 W_0 + \frac{q\left(1 - \frac{\theta}{\tilde{\theta}}\right)}{1 + \beta - \sigma(\tilde{v})}. \tag{A.15}$$

Since Advocate holds both types of stocks in Case 3, the difference in returns, $\tilde{\theta} - \theta$, must exactly compensate them for the non-pecuniary utility difference between holding clean and dirty stocks. From the optimal portfolio conditions

$$\beta\theta + v c_1^A = \beta\tilde{\theta} - \tilde{v} c_1^A \tag{A.16}$$

must now hold with equality. Therefore,

$$c_1^A = \beta \frac{\tilde{\theta} - \theta}{v + \tilde{v}}, \tag{A.17}$$

which implies that the repeating component $\sigma(\tilde{v})$ defined in the saving function above can be equivalently expressed as

$$\sigma(\tilde{v}) = \frac{\tilde{v}}{\tilde{\theta}} c_1^A = \beta \underbrace{\frac{\tilde{v}}{v + \tilde{v}}}_{\in [0,1]} \underbrace{\left(1 - \frac{\theta}{\tilde{\theta}}\right)}_{>0}.$$

Compared with the standard discount factor β, $\sigma(\tilde{v})$ is proportional to the return differential

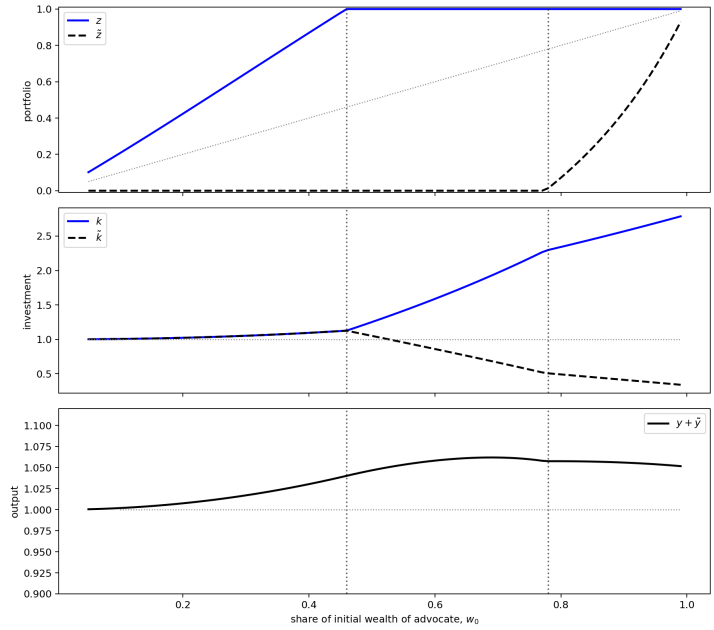


Figure A.8: Composition and Scale Impact for the Clean and Dirty Firm

Notes: Figure A.8 reports the equilibrium outcomes when $\beta = 1$, $\alpha = 0.33$, $v = 1$, and $\tilde{v} = 1$. The initial levels of capital is set at $k_0 = \tilde{k}_0 = \frac{\alpha\beta}{1+\alpha\beta}$. Investment and Output are reported as a ratio of the leves that would result in an economy with only Benchmark households (i.e with $v = \tilde{v} = 0$).

between clean and dirty firms, weighted by the relative strength of the negative non-pecuniary motive. This saving function for Case 3 shows how the negative non-pecuniary return of holding dirty stocks affects saving behavior through two competing effects: (1) reduced saving incentive due to being forced to hold dirty stocks, and (2) increased saving required to hold more expensive clean stocks due to their preferred capital allocation.

Appendix A.3. Extensions

Figures A.8 and A.9 report the numerical solution to the two-period model for the case where Advocate has both utility in holding clean stocks and disutility in holding dirty stocks, that is $v = \tilde{v} = 1$, and for the case where Advocate only perceives disutility in holding dirty stocks, that is $v = 0, \tilde{v} = 1$, respectively. From Figure A.8 the dampening of the scale effect due to the disutility of holding dirty stock is evident. From Figure A.9 the positive scale effect disappears entirely and the only effect is a negative one.

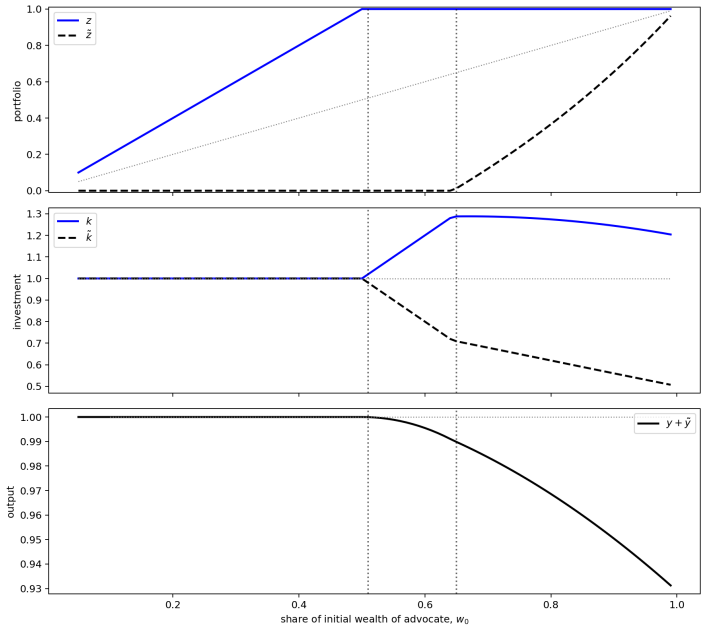


Figure A.9: Composition and Scale Impact for the Clean and Dirty Firm: $v = 0$, $\tilde{v} = 1$

Notes: Figure A.9 reports the equilibrium outcomes when $\beta = 1$, $\alpha = 0.33$, $v = 0$, and $\tilde{v} = 1$. The initial levels of capital is set at $k_0 = \tilde{k}_0 = \frac{\alpha\beta}{1+\alpha\beta}$. Investment and Output are reported as a ratio of the leves that would result in an economy with only Benchmark households (i.e with $v = \tilde{v} = 0$).

Appendix A.4. Two-Period Model with Risk

We present here a two-period model where stocks have uncertain returns. The preferences for Advocate in the presence of uncertain returns on stocks are

$$\ln c_0^A + \beta E\left(\ln c_1^A\right) + vzq - \tilde{v}\tilde{z}\tilde{q}$$

where

$$c_0^A = w_0 W_0 - (zq + \tilde{z}q)$$

and

$$c_1^A = zq\left(\frac{d}{q} + \varepsilon\right) + \tilde{z}\tilde{q}\left(\frac{\tilde{d}}{\tilde{q}} + \tilde{\varepsilon}\right)$$

where ε, $\tilde{\varepsilon}$ are i.i.d. (bounded) random variable with mean zero and variances σ^2 and $\tilde{\sigma}^2$, respectively.[18] From small values of σ^2 and $\tilde{\sigma}^2$, the objective function can be approximated

[18] Allowing for different variances can capture the "value" channel of sustainable investing as opposed to the "values" motive captured by v, whereas clean firms might be better equipped at reducing uncertainty around their realized returns.

via a second-order Taylor expansion as:

$$\ln c_0^A + \beta \ln E(c_1^A) - \beta \frac{Var(c_1^A)}{2E(c_1^A)^2} + vzq.$$

Substituting the expressions above the objective function can be written as

$$\ln\left(w_0 W_0 - (zq + \tilde{z}\tilde{q})\right) + \beta \ln\left(zd + \tilde{z}\tilde{d}\right) - \beta \frac{1}{2}\left(\frac{\sigma^2 (zq)^2 + \tilde{\sigma}^2 (\tilde{z}\tilde{q})^2}{(zd + \tilde{z}\tilde{d})^2}\right) + vzq$$

which is to be maximized with respect to z and $\tilde{z}$, with $z \geq 0$ and $\tilde{z} \geq 0$. The first order conditions are:

$$\begin{aligned}
\frac{E(c_1^A)}{c_0^A} &\geq \beta\theta - \beta\left(\sigma^2 zq\tilde{\theta} - \tilde{\sigma}^2 \tilde{z}\tilde{q}\theta\right)\frac{\tilde{z}\tilde{q}}{E(c_1^A)^2} + v, \quad = \text{ if } z > 0, \\
\frac{E(c_1^A)}{c_0^A} &\geq \beta\tilde{\theta} + \beta\left(\sigma^2 zq\tilde{\theta} - \tilde{\sigma}^2 \tilde{z}\tilde{q}\theta\right)\frac{zq}{E(c_1^A)^2} - \tilde{v}, \quad = \text{ if } \tilde{z} > 0.
\end{aligned}$$

The Figure below reports the equilibrium in the two period model for the case of $v = 1$, $\tilde{v} = 0$, and setting $\sigma = \tilde{\sigma} = 0.25$. The overall pattern is similar to the case presented in the main text without uncertainty, but the required returns of clean and dirty firms are different across the entire range for w_0. Quantitatively, the difference is small when Advocate is not the marginal investor in the clean firms. Intuitively, Benchmark would like to hold an evenly split portfolio between clean and dirty firms to optimally diversify risk. However, because Advocate tilts their portfolio towards clean stocks due to $v > 0$, Benchmark is forced to hold a portfolio that is heavier on dirty firms. To be compensate for holding the additional risk, Benchmark requires a higher return on dirty firms, and thus there is a gap in the expected return across the two firms even when Advocate is not the marginal investor.

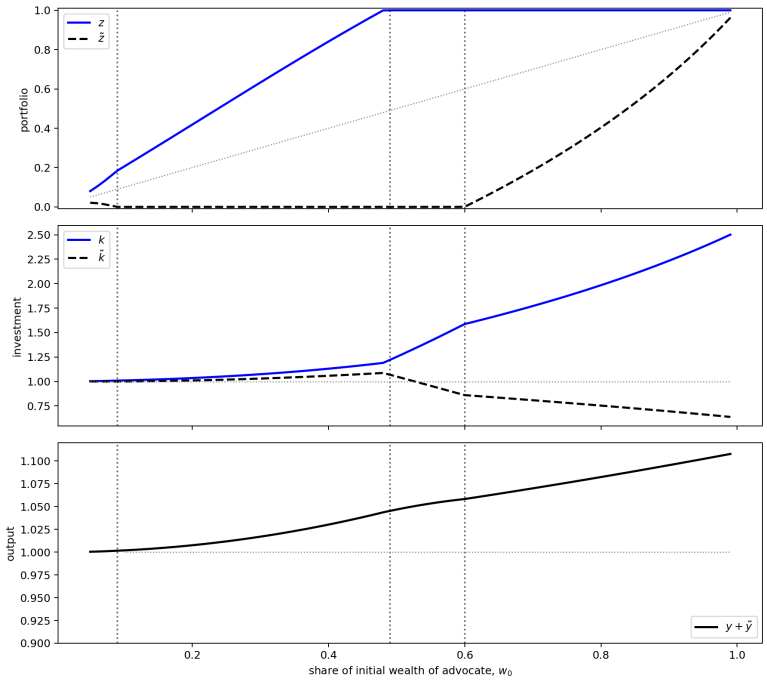


Figure A.10: Composition and Scale Impact for the Clean and Dirty Firm with Uncertainty: $v = 1$, $\tilde{v} = 0$

Notes: Figure A.10 reports the equilibrium outcomes when $\beta = 1$, $\alpha = 0.33$, $v = 1$, $\tilde{v} = 0$. and stock returns are uncertain with $\sigma = \tilde{\sigma} = 0.25$. The initial levels of capital is set at $k_0 = \tilde{k}_0 = \frac{\alpha\beta}{1+\alpha\beta}$. Investment and Output are reported as a ratio of the leves that would result in an economy with only Benchmark households (i.e with $v = \tilde{v} = 0$).

Appendix B. Full Dynamic Model Details

Appendix B.1. Derive Firms' Optimal Investment Strategy

Consider a representative firm indexed by g with capital stock $k(g,t)$ at time t. The firm produces output $f\big(k(g,t)\big)$, invests in capital, and faces physical depreciation at rate $\delta > 0$. Let $i(g,t)$ denote gross investment. The capital accumulation equation is

$$\dot{k}(g,t) = i(g,t) - \delta k(g,t). \tag{B.1}$$

The firm chooses its investment trajectory $\{i(g,t)\}_{t\geq 0}$ to maximize the present discounted value of its net dividend payouts $\{d(g,t)\}_{t\geq 0}$, which correspond precisely to the cash flow available to equity holders after deducting all reinvestment expenditures necessary to sustain or expand firm operations:

$$\max_{\{i(g,t)\}} \int_0^\infty e^{-\int_0^t \theta(g,\tau)\mathrm{d}\tau} \left[f(k(g,t)) - i(g,t)\right] \mathrm{d}t. \tag{B.2}$$

In equation (B.2), future dividends are discounted at the firm's cost of equity capital $\theta(g,t)$, the appropriate discount rate that reflects the return required by equity holders to hold the firm's shares.

Let us begin by formally justifying the objective function presented in equation (22) and (B.2). Given that the number of outstanding shares is normalized to 1 for each firm g, maximizing the market value of the firm $V(g,t) = 1 \times q(g,t)$ is equivalent to maximizing $q(g,t)$. It is straightforward to show that when firms maximize their stock price by optimally choosing investment and dividend policies, they are effectively maximizing the present value of dividends, discounted at the instantaneous required return $\theta(g,t)$. To explicitly connect $q(g,t)$ with the firm's objective function used throughout the paper, we solve for $q(g,t)$ from the first-order linear differential equation:

$$\theta(g,t) = \frac{d(g,t)}{q(g,t)} + \frac{\dot{q}(g,t)}{q(g,t)} \iff \dot{q}(g,t) - \theta(g,t)q(g,t) = -d(g,t)$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}[e^{-\int_0^t \theta(g,\tau)\mathrm{d}\tau} q(g,t)] = -e^{-\int_0^t \theta(g,\tau)\mathrm{d}\tau} d(g,t) \qquad \text{(use integrating factor method)}$$

$$\Rightarrow [e^{-\int_0^s \theta(g,\tau)\mathrm{d}\tau} q(g,s)]_t^\infty = \int_t^\infty -e^{-\int_0^s \theta(g,\tau)\mathrm{d}\tau} d(g,s)\mathrm{d}s$$

$$\Rightarrow 0 - e^{-\int_0^t \theta(g,\tau)\mathrm{d}\tau} q(g,t) = -\int_t^\infty e^{-\int_0^s \theta(g,\tau)\mathrm{d}\tau} d(g,s)\mathrm{d}s \qquad \text{(impose boundary condition)}$$

Further simplification yields the equivalence

$$q(g,t) = \int_t^\infty e^{-\int_t^s \theta(g,\tau)\mathrm{d}\tau} d(g,s)\,\mathrm{d}s \tag{B.3}$$

in the notation of our baseline model.

We use the Pontryagin maximum principle to derive the firm's optimal investment strategy. Define the current-value Hamiltonian as

$$\mathcal{H}(k,i,\psi,t) = f(k(g,t)) - i(g,t) + \psi(g,t)\left[i(g,t) - \delta k(g,t)\right], \tag{B.4}$$

where $\psi(g,t)$ is the current-value costate variable representing the shadow price of production capital. The evolution of the costate variable follows

$$\dot{\psi}(g,t) = \theta(g,t)\psi(g,t) - \left[f'(k(g,t)) - \delta\psi(g,t)\right]. \tag{B.5}$$

Differentiating the Hamiltonian with respect to investment $i(g,t)$ gives the first-order condition:

$$\psi(t) = 1, \tag{B.6}$$

which implies constant $\psi(t)$ and thus $\dot{\psi}(t) = 0$. The costate equation now simplifies to

$$f'(k(g,t)) = \theta(g,t) + \delta. \tag{B.7}$$

Condition (B.7) captures the fundamental principle of firm investment: at the optimum, the marginal product of capital equals the user cost of capital, which consists of the required equity return $\theta(g,t)$ and the depreciation rate δ. Assuming the same Cobb-Douglas production function, $f(k(g,t)) = k(g,t)^\alpha$ with $\alpha \in (0,1)$, the optimal investment strategy leads to the following desired capital stock:

$$k(g,t) = \left(\frac{\theta(g,t) + \delta}{\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{B.8}$$

In our baseline model, the firm's dividend and investment decisions are jointly determined by its sources-and-uses-of-funds identity. In the main text, we cast the firm's problem as the

choice of a dividend payout policy that maximizes the firm's equity value—an alternative but equivalent representation to the investment-based formulation detailed in this appendix.

Appendix B.2. Solve Households' Optimization Problem

Appendix B.2.1. Advocate Household

Advocate optimizes to maximize lifetime utility derived from the consumption stream $c^A(t)$ and the sustainability of its asset portfolio $\mathcal{G}(t)$. The total pecuniary wealth $a^A(t)$ is the state variable in household A's optimization problem:

$$a^A(t) = \int_0^1 z^A(g,t)\, q(g,t)\, \phi(g)\, \mathrm{d}g, \tag{B.9}$$

where $z^A(g,t)$ represents Advocate's stock holdings in firm g at time t, $q(g,t)$ is the price of stocks in firm g at time t, and $\phi(g)$ is the density of firms with sustainability score g. To derive the evolution of state $a^A(t)$ over time, we take the time derivative of this accounting expression on both sides. Using the Leibniz integral rule:

$$\begin{aligned} \dot{a}^A(t) &= \frac{\mathrm{d}}{\mathrm{d}t} \int_0^1 z^A(g,t) q(g,t) \phi(g)\, \mathrm{d}g \\ &= \int_0^1 \left[\dot{z}^A(g,t) q(g,t) + z^A(g,t) \dot{q}(g,t) \right] \phi(g)\, \mathrm{d}g. \end{aligned} \tag{B.10}$$

Under our borrowing constraint assumption, the household's consumption $c^A(t)$ and any stock purchases (where $\dot{z}^A(g,t) > 0$) must be financed by dividend income from currently held shares and proceeds from selling other stocks (where $\dot{z}^A(g,t) < 0$). That is, any changes in stock holdings (buying/selling) must equal income from dividends minus consumption expenditure to satisfy the budget constraint:

$$\int_0^1 \dot{z}^A(g,t) q(g,t) \phi(g)\, \mathrm{d}g = \int_0^1 d(g,t) z^A(g,t) \phi(g)\, \mathrm{d}g - c^A(t). \tag{B.11}$$

Now substituting equation (B.11) into our expression for $\dot{a}^A(t)$ above, we obtain

$$\begin{aligned}\dot{a}^A(t) &= \int_0^1 d(g,t)z^A(g,t)\phi(g)\,\mathrm{d}g - c^A(t) + \int_0^1 z^A(g,t)\dot{q}(g,t)\phi(g)\,\mathrm{d}g \\ &= \int_0^1 \left[d(g,t) + \dot{q}(g,t)\right] z^A(g,t)\phi(g)\,\mathrm{d}g - c^A(t) \\ &= \int_0^1 \underbrace{\frac{d(g,t)+\dot{q}(g,t)}{q(g,t)}}_{=\theta(g,t)} q(g,t)z^A(g,t)\phi(g)\,\mathrm{d}g - c^A(t),\end{aligned}$$

which is the law of motion for Advocate's wealth $a^A(t)$ presented in the main text.

The current-value Hamiltonian for this problem is

$$\begin{aligned}\mathcal{H}(a^A, c^A, z^A, \lambda^A, t) =& \ln c^A(t) + \mathcal{V}(\mathcal{G}(t)) \\ &+ \lambda^A(t)\left[\int_0^1 \theta(g,t)q(g,t)z^A(g,t)\phi(g)\,\mathrm{d}g - c^A(t)\right],\end{aligned} \tag{B.12}$$

where $\lambda^A(t)$ is the costate variable associated with Advocate's wealth accumulation. To incorporate the wealth constraint and short-sale constraints, we form the Lagrangian:

$$\begin{aligned}\mathcal{L}(a^A, c^A, z^A, \lambda^A, \mu^A, \underline{\eta}^A, t) =& \mathcal{H}(a^A, c^A, z^A, \lambda^A, t) \\ &+ \mu^A(t)\left[a^A(t) - \int_0^1 z^A(g,t)q(g,t)\phi(g)\mathrm{d}g\right] \\ &+ \int_0^1 \underline{\eta}^A(g,t)z^A(g,t)\phi(g)\mathrm{d}g,\end{aligned} \tag{B.13}$$

where $\mu^A(t)$ is the Lagrange multiplier for the wealth constraint (B.9), and $\underline{\eta}^A(g,t)$ is the Lagrange multiplier for the constraint $z^A(g,t) \geq 0$.

The maximum principle for an interior solution requires first-order conditions:

$$[\ c^A(t)\ ] \quad \frac{1}{c^A(t)} - \lambda^A(t) = 0 \Rightarrow \lambda^A(t) = \frac{1}{c^A(t)}, \tag{B.14}$$

$$[z^A(g,t)] \quad \mathcal{V}'(\mathcal{G}(t))v(g)q(g,t) + \lambda^A(t)\theta(g,t)q(g,t) - \mu^A(t)q(g,t) + \underline{\eta}^A(g,t) = 0, \tag{B.15}$$

$$[\ \lambda^A(t)\ ] \quad \dot{a}^A(t) = \int_0^1 \theta(g,t)q(g,t)z^A(g,t)\phi(g)\,\mathrm{d}g - c^A(t), \tag{B.16}$$

for all $g \in [0,1]$ and $t \geq 0$, with complementary slackness conditions:

$$\mu^A(t) \geq 0, \quad \mu^A(t)\left[a^A(t) - \int_0^1 z^A(g,t)q(g,t)\phi(g)\mathrm{d}g\right] = 0; \tag{B.17}$$

$$\underline{\eta}^A(g,t) \geq 0, \quad \underline{\eta}^A(g,t)z^A(g,t) = 0. \tag{B.18}$$

The costate condition is

$$\rho\lambda^A(t) = \dot{\lambda}^A(t) + \mu^A(t) \tag{B.19}$$

from taking the first-order derivative with respect to the state variable $a^A(t)$. Substituting the consumption first-order condition into equation (B.19), the evolution of the costate variable can be rewritten as

$$\begin{aligned} \rho\frac{1}{c^A(t)} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{c^A(t)}\right) + \mu^A(t) \\ &= -\frac{\dot{c}^A(t)}{(c^A(t))^2} + \mu^A(t) \\ \Rightarrow \quad \frac{\dot{c}^A(t)}{c^A(t)} &= \mu^A(t)c^A(t) - \rho. \end{aligned} \tag{B.20}$$

Again, we specify $\mathcal{V}(\mathcal{G}(t)) = \mathcal{G}(t)$ in the fully dynamic model setup. If $z^A(g,t) > 0$ (i.e., the asset is partially or fully held by household A), then $\underline{\eta}^A(g,t) = 0$, which yields

$$v(g)c^A(t) + \theta(g,t) = \mu^A(t)c^A(t). \tag{B.21}$$

If $\underline{\eta}^A(g,t) > 0$ (i.e., the short-sale constraint binds), then $z^A(g,t) = 0$ (the asset is not held by Advocate), and

$$v(g)c^A(t) + \theta(g,t) < \mu^A(t)c^A(t). \tag{B.22}$$

The consumption first-order condition is used in both cases to eliminate the costate variable $\lambda^A(t)$.

Appendix B.2.2. Benchmark Household

Benchmark maximizes lifetime utility derived solely from the consumption stream $c^B(t)$, without any non-pecuniary preferences for sustainable wealth. Similar to household A, the total pecuniary wealth $a^B(t)$ serves as the state variable:

$$a^B(t) = \int_0^1 z^B(g,t)\, q(g,t)\, \phi(g)\, \mathrm{d}g, \tag{B.23}$$

where $z^B(g,t)$ represents Benchmark's stock holdings in firm g at time t. The evolution of Benchmark's wealth follows the same derivation as for Advocate:

$$\dot{a}^B(t) = \int_0^1 \theta(g,t) q(g,t) z^B(g,t) \phi(g)\, \mathrm{d}g - c^B(t). \tag{B.24}$$

Correspondingly, we formulate the current-value Hamiltonian for Benchmark's problem as

$$\begin{aligned} \mathcal{H}^B(a^B, c^B, z^B, \lambda^B, t) =& \ln c^B(t) \\ &+ \lambda^B(t) \left[\int_0^1 \theta(g,t) q(g,t) z^B(g,t) \phi(g)\, \mathrm{d}g - c^B(t) \right], \end{aligned} \tag{B.25}$$

and the Lagrangian as

$$\begin{aligned} \mathcal{L}^B(a^B, c^B, z^B, \lambda^B, \mu^B, \underline{\eta}^B, t) =& \mathcal{H}^B(a^B, c^B, z^B, \lambda^B, t) \\ &+ \mu^B(t) \left[a^B(t) - \int_0^1 z^B(g,t) q(g,t) \phi(g)\, \mathrm{d}g \right] \\ &+ \int_0^1 \underline{\eta}^B(g,t) z^B(g,t) \phi(g)\, \mathrm{d}g. \end{aligned} \tag{B.26}$$

Maintaining consistent notation, $\lambda^B(t)$ is the costate variable associated with Benchmark's wealth accumulation; $\mu^B(t)$ is the Lagrange multiplier for the wealth constraint (B.23); and $\underline{\eta}^B(g,t)$ is the Lagrange multiplier on the non-negativity constraint $z^B(g,t) \geq 0$.

Likewise, the maximum principle for an interior solution requires the following first-order

conditions:

$$[\ c^B(t)\ ] \quad \frac{1}{c^B(t)} - \lambda^B(t) = 0 \Rightarrow \lambda^B(t) = \frac{1}{c^B(t)}, \tag{B.27}$$

$$[z^B(g,t)] \quad \lambda^B(t)\theta(g,t)q(g,t) - \mu^B(t)q(g,t) + \underline{\eta}^B(g,t) = 0, \tag{B.28}$$

$$[\ \lambda^B(t)\ ] \quad \dot{a}^B(t) = \int_0^1 \theta(g,t)q(g,t)z^B(g,t)\phi(g)\,\mathrm{d}g - c^B(t), \tag{B.29}$$

for all $g \in [0,1]$ and $t \geq 0$, with complementary slackness conditions:

$$\mu^B(t) \geq 0, \quad \mu^B(t)\left[a^B(t) - \int_0^1 z^B(g,t)q(g,t)\phi(g)\mathrm{d}g\right] = 0; \tag{B.30}$$

$$\underline{\eta}^B(g,t) \geq 0, \quad \underline{\eta}^B(g,t)z^B(g,t) = 0. \tag{B.31}$$

Therefore, if $z^B(g,t) > 0$ (i.e., the asset is partially or fully held by Benchmark), then $\underline{\eta}^B(g,t) = 0$, which yields

$$\theta(g,t) = \mu^B(t)c^B(t). \tag{B.32}$$

If $\underline{\eta}^B(g,t) > 0$ (i.e., the short-sale constraint binds), then $z^B(g,t) = 0$ (the asset is not held by Benchmark), and

$$\theta(g,t) < \mu^B(t)c^B(t). \tag{B.33}$$

Finally, the costate condition in Benchmark's optimization problem can be expressed as

$$\rho\lambda^B(t) = \dot{\lambda}^B(t) + \mu^B(t) \tag{B.34}$$

$$\Rightarrow \ \frac{\dot{c}^B(t)}{c^B(t)} = \mu^B(t)c^B(t) - \rho. \tag{B.35}$$

In equilibrium, the distinction between $\mu^A(t)$ and $\mu^B(t)$ is crucial for understanding the dynamics of wealth distribution and stock trading between advocate and benchmark investors over time.

Appendix B.3. Equilibrium Characterization

Given the market-clearing condition, in equilibrium, there must be

$$\begin{cases} z^A(g,t) = z(g,t) \geq 0 \\ z^B(g,t) = 1 - z(g,t) \geq 0 \end{cases} \tag{B.36}$$

for all $g \in [0,1]$ and $t \geq 0$.

Comparing Benchmark's asset holding condition in equation (B.32) with the corresponding condition for Advocate in equation (B.21), we observe that household B requires equal returns across all assets it holds, whereas Advocate's required returns are adjusted by the non-pecuniary term $v(g)c^A(t)$. This distinction in portfolio allocation criteria is critical for characterizing the threshold equilibrium defined by $g^*(t)$.

In this section, we provide a rigorous characterization of the equilibrium in the full dynamic model. We begin by establishing the existence of a threshold-based equilibrium structure, and then examine its dynamic properties.

Lemma 1 (Threshold Equilibrium Structure)**.** *In equilibrium, there exists a threshold sustainability score $g^*(t) \in [0,1]$ at each time t such that:*

- *For all $g \geq g^*(t)$, Advocate holds all shares: $z^A(g,t) = 1$ and $z^B(g,t) = 0$;*
- *For all $g < g^*(t)$, Advocate A holds no shares: $z^A(g,t) = 0$ and $z^B(g,t) = 1$.*

Proof. Consider the asset holding conditions for Advocate and Benchmark derived from their respective optimization problems. Suppose that, at time t, both types of households hold strictly positive amounts of shares issued by the same firm g, i.e., $z^A(g,t) > 0$ and $z^B(g,t) > 0$. In this case, we simultaneously have

$$v(g)c^A(t) + \theta(g,t) = \mu^A(t)c^A(t),$$

from equation (B.21), and

$$\theta(g,t) = \mu^B(t)c^B(t),$$

from equation (B.32). This implies

$$\mu^A(t)c^A(t) - \mu^B(t)c^B(t) = v(g)c^A(t). \tag{B.37}$$

For a given time t, the left-hand side of equation (B.37) is independent of g, while the right-hand side varies with g through $v(g)$. Since $v(g)$ is monotonically increasing in g by assumption, the equality can only hold for at most one value of g, which we denote as $g^*(t)$.

At the threshold $g^*(t)$, the following equalities hold:

$$\mu^A(t)c^A(t) = v(g^*(t))c^A(t) + \theta(g^*(t), t), \tag{B.38}$$

$$\mu^B(t)c^B(t) = \theta(g^*(t), t). \tag{B.39}$$

We will later use these two equations to eliminate the Lagrange multipliers $\mu^A(t)$ and $\mu^B(t)$ from the equilibrium conditions, allowing us to express the relevant conditions directly in terms of the threshold $g^*(t)$.

For any $g > g^*(t)$, we have $v(g) > v(g^*(t))$ and

$$\mu^A(t)c^A(t) - \mu^B(t)c^B(t) = v(g^*)c^A(t) < v(g)c^A(t)$$

due to monotonicity, which implies

$$v(g)c^A(t) + \mu^B(t)c^B(t) > \mu^A(t)c^A(t).$$

Since Benchmark's ownership of any equity in firm g requires $\theta(g, t) = \mu^B(t)c^B(t)$, it follows that:

$$v(g)c^A(t) + \theta(g, t) > \mu^A(t)c^A(t).$$

This outcome contradicts Advocate's optimality condition, leading us to rule out the possibility that Benchmark is the sole shareholder of firm g for all $g > g^*(t)$. On the other hand, if Benchmark does not hold equity in firm g when $g > g^*(t)$, the inequality $\theta(g, t) < \mu^B(t)c^B(t)$ is permissible and remains consistent with Advocate's optimality condition. Hence, we conclude that $z^B(g, t) = 0$ for any firm $g > g^*(t)$, and by market clearing, $z^A(g, t) = 1$ for all such firms.

Similarly, for any $g < g^*(t)$, we have $v(g) < v(g^*(t))$, and

$$\mu^A(t)c^A(t) - \mu^B(t)c^B(t) = v(g^*)c^A(t) > v(g)c^A(t)$$

due to monotonicity, which implies

$$v(g)c^A(t) + \mu^B(t)c^B(t) < \mu^A(t)c^A(t).$$

This means that if Advocate were to hold any of firm g's equity, we would also have $\theta(g,t) > \mu^B(t)c^B(t)$, which contradicts Benchmark's optimality condition. Therefore, Advocate cannot own shares of firm g when $g < g^*(t)$, which means $z^A(g,t) = 0$ and $z^B(g,t) = 1$.

Thus, the equilibrium is characterized by a threshold $g^*(t)$ that divides equity ownership between the two types of households. □

Lemma 2 (Consumption Growth Dynamics)**.** *In the threshold equilibrium:*

- *Advocate's consumption growth rate is given by*

$$\frac{\dot{c}^A(t)}{c^A(t)} = \theta(g^*(t),t) + v(g^*(t))c^A(t) - \rho.$$

- *Benchmark's consumption growth rate is given by*

$$\frac{\dot{c}^B(t)}{c^B(t)} = \theta(g^*(t),t) - \rho.$$

Proof. From the costate evolution equations (B.20) and (B.35), we have consumption growth rates:

$$\frac{\dot{c}^A(t)}{c^A(t)} = \mu^A(t)c^A(t) - \rho,$$
$$\frac{\dot{c}^B(t)}{c^B(t)} = \mu^B(t)c^B(t) - \rho.$$

As shown in the proof of Lemma 1 (equations (B.38) and (B.39)), the asset holding conditions at the threshold $g^*(t)$ imply

$$\mu^A(t)c^A(t) = v(g^*(t))c^A(t) + \theta(g^*(t),t)$$

for household A, and

$$\mu^B(t)c^B(t) = \theta(g^*(t),t)$$

for household B. Substituting these expressions into the right-hand side of the corresponding consumption growth equations completes the proof.

□

Lemma 3 (Threshold Dynamics)**.** *The evolution of the threshold $g^*(t)$ is governed by*

$$\dot{g}^*(t) = \frac{c^A(t) - \int_{g^*(t)}^1 d(g,t)\phi(g)\,\mathrm{d}g}{q(g^*(t),t)\phi(g^*(t))},$$

or, equivalently,

$$\dot{g}^*(t) = \frac{\int_0^{g^*(t)} d(g,t)\phi(g)\,\mathrm{d}g - c^B(t)}{q(g^*(t),t)\phi(g^*(t))}. \tag{B.40}$$

Proof. From the threshold equilibrium structure (Lemma 1), the differential equation governing the evolution of Advocate's wealth (equation (18) in the main text) can be rearranged as follows:

$$\begin{aligned} c^A(t) &= \int_0^1 \big[d(g,t) + \dot{q}(g,t)\big] z(g,t)\phi(g)\,\mathrm{d}g - \dot{a}^A(t) \\ &= \int_{g^*(t)}^1 \big[d(g,t) + \dot{q}(g,t)\big]\phi(g)\,\mathrm{d}g - \frac{\mathrm{d}}{\mathrm{d}t}\left[\int_{g^*(t)}^1 q(g,t)\phi(g)\,\mathrm{d}g\right]. \end{aligned}$$

Applying the Leibniz integral rule, we obtain

$$\begin{aligned} c^A(t) &= \int_{g^*(t)}^1 \big[d(g,t) + \dot{q}(g,t)\big]\phi(g)\,\mathrm{d}g - \left[0 - \dot{g}^*(t)q(g^*(t),t)\phi(g^*(t)) + \int_{g^*(t)}^1 \dot{q}(g,t)\phi(g)\,\mathrm{d}g\right] \\ &= \int_{g^*(t)}^1 \big[d(g,t) + \dot{q}(g,t)\big]\phi(g)\,\mathrm{d}g + \dot{g}^*(t)q(g^*(t),t)\phi(g^*(t)) - \int_{g^*(t)}^1 \dot{q}(g,t)\phi(g)\,\mathrm{d}g \\ &= \int_{g^*(t)}^1 d(g,t)\phi(g)\,\mathrm{d}g + \dot{g}^*(t)q(g^*(t),t)\phi(g^*(t)). \end{aligned} \tag{B.41}$$

Isolating $\dot{g}^*(t)$ from equation (B.41), we express the law of motion for $g^*(t)$ in terms of $c^A(t)$ as

$$\dot{g}^*(t) = \frac{c^A(t) - \int_{g^*(t)}^1 d(g,t)\phi(g)\,\mathrm{d}g}{q(g^*(t),t)\phi(g^*(t))}.$$

Since goods market clearing requires

$$c^A(t) + c^B(t) = \int_0^1 d(g,t)\phi(g)\,\mathrm{d}g, \tag{B.42}$$

the law of motion for $\dot{g}^*(t)$ can alternatively be written in terms of $c^B(t)$ as

$$\begin{aligned}\dot{g}^*(t) &= \frac{\left[\int_0^1 d(g,t)\phi(g)\,\mathrm{d}g - c^B(t)\right] - \int_{g^*(t)}^1 d(g,t)\phi(g)\,\mathrm{d}g}{q(g^*(t),t)\phi(g^*(t))} \\ &= \frac{\int_0^{g^*(t)} d(g,t)\phi(g)\,\mathrm{d}g - c^B(t)}{q(g^*(t),t)\phi(g^*(t))}.\end{aligned}$$

Equation (B.40) can also be derived directly from the differential equation governing the evolution of Benchmark's wealth (equation (20) in the main text):

$$\begin{aligned}c^B(t) &= \int_0^1 \left[d(g,t) + \dot{q}(g,t)\right]\left[1 - z(g,t)\right]\phi(g)\,\mathrm{d}g - \dot{a}^B(t) \\ &= \int_0^{g^*(t)} \left[d(g,t) + \dot{q}(g,t)\right]\phi(g)\,\mathrm{d}g - \frac{\partial}{\partial t}\left[\int_0^{g^*(t)} q(g,t)\phi(g)\,\mathrm{d}g\right] \\ &= \int_0^{g^*(t)} \left[d(g,t) + \dot{q}(g,t)\right]\phi(g)\,\mathrm{d}g - \left[\dot{g}^*(t)q(g^*(t),t)\phi(g^*(t)) - 0 + \int_0^{g^*(t)} \dot{q}(g,t)\phi(g)\,\mathrm{d}g\right] \\ &= \int_0^{g^*(t)} d(g,t)\phi(g)\,\mathrm{d}g - \dot{g}^*(t)q(g^*(t),t)\phi(g^*(t)) \\ \Rightarrow \quad \dot{g}^*(t) &= \frac{\int_0^{g^*(t)} d(g,t)\phi(g)\,\mathrm{d}g - c^B(t)}{q(g^*(t),t)\phi(g^*(t))}.\end{aligned}$$

□

Lemma 4 (Cost of Capital Differential)**.** *In the threshold equilibrium characterized by $g^*(t)$:*

- *For all $g < g^*(t)$, the instantaneous return $\theta(g,t) = \theta(g^*(t),t)$*
- *For all $g > g^*(t)$, the instantaneous return $\theta(g,t) = \theta(g^*(t),t) - [v(g) - v(g^*(t))]c^A(t)$*

Proof. For $g < g^*(t)$, Benchmark is the marginal investor and holds all shares of firm g. The optimal portfolio condition for Benchmark (equation (B.32)) implies that

$$\theta(g,t) = \mu^B(t)c^B(t)$$

must hold for all $g < g^*(t)$, including $g = g^*(t)$, where both households may simultaneously hold shares. Therefore,

$$\theta(g,t) = \theta(g^*(t),t) \quad \text{for all } g < g^*(t).$$

For $g > g^*(t)$, Advocate is the marginal investor and holds all shares of firm g. The optimal portfolio condition for Advocate (equation (B.21)) must hold both at the threshold $g = g^*(t)$:

$$v(g^*(t))c^A(t) + \theta(g^*(t), t) = \mu^A(t)c^A(t),$$

and for all $g > g^*(t)$:

$$v(g)c^A(t) + \theta(g, t) = \mu^A(t)c^A(t).$$

Taking the difference between the two expressions above, we obtain

$$v(g)c^A(t) - v(g^*(t))c^A(t) + \theta(g, t) - \theta(g^*(t), t) = 0$$

$$\Rightarrow \theta(g, t) = \theta(g^*(t), t) - \underbrace{[v(g) - v(g^*(t))]}_{>0} c^A(t) \; < \theta(g^*(t), t) \quad \text{for all } g > g^*(t),$$

which indicates a lower cost of capital for firms with higher sustainability scores when advocate investors are the marginal investor.

□

Now that we have expressed the equity return of any firm in relation to the return at the threshold $\theta(g^*(t), t)$, the evolution of wealth for each investor type can be recast in the following form:

- For Advocate,

$$\begin{aligned}
\dot{a}^A(t) &= \int_0^1 \theta(g, t)q(g, t)z^A(g, t)\phi(g)\mathrm{d}g - c^A(t) \\
&= \int_{g^*(t)}^1 [\theta(g^*(t), t) - (v(g) - v(g^*(t)))c^A(t)]q(g, t)\phi(g)\mathrm{d}g - c^A(t) \\
&= \theta(g^*(t), t)a^A(t) - c^A(t) - c^A(t) \int_{g^*(t)}^1 (v(g) - v(g^*(t)))q(g, t)\phi(g)\mathrm{d}g \\
&= \theta(g^*(t), t)a^A(t) - c^A(t) - c^A(t) \left[\underbrace{\int_{g^*(t)}^1 v(g)q(g, t)\phi(g)\mathrm{d}g}_{=\mathcal{G}(t) \text{ by definition}} - v(g^*(t))a^A(t) \right] \\
&= \theta(g^*(t), t)a^A(t) - [1 + \underbrace{\mathcal{G}(t) - v(g^*(t))a^A(t)}_{>0 \text{ by monotonicity}}]c^A(t).
\end{aligned}$$

- For Benchmark,

$$\begin{aligned}\dot{a}^B(t) &= \int_0^{g^*(t)} \theta(g^*(t),t)q(g,t)\phi(g)\mathrm{d}g - c^B(t)\\ &= \theta(g^*(t),t)a^B(t) - c^B(t).\end{aligned}$$

Appendix B.4. Convergence to Steady State

The dynamic equilibrium converges to a steady state characterized by:

- Zero consumption growth for both households;
- Convergence of the threshold to the neutral score: $\lim_{t\to\infty} g^*(t) = g_n$;
- Advocate holding all firms with $g \geq g_n$, and Benchmark holding all firms with $g < g_n$.

From Lemma 2, setting the time derivative $\dot{c}^A(t) = 0$ yields

$$\theta(g^*(t),t) + v(g^*(t))c^A(t) - \rho = 0; \tag{B.43}$$

similarly, setting $\dot{c}^B(t) = 0$ gives

$$\theta(g^*(t),t) - \rho = 0. \tag{B.44}$$

Together, these two steady-state conditions imply

$$v(g^*_{ss})c^A_{ss} = 0. \tag{B.45}$$

Since $c^A(t) > 0$ in any viable equilibrium, this requires that $v(g^*_{ss}) = 0$ in the steady state, which by definition occurs only at $g^*_{ss} = g_n$. Therefore, the threshold must converge to the neutral score in the steady state.

Setting $\dot{g}^*(t) = 0$ in Lemma 3 further yields

$$c^A_{ss} = \int_{g_n}^1 d_{ss}(g)\phi(g)\,\mathrm{d}g, \tag{B.46}$$

$$c^B_{ss} = \int_0^{g_n} d_{ss}(g)\phi(g)\,\mathrm{d}g. \tag{B.47}$$

This confirms that in the steady state, each household's consumption exactly equals the divi-

dend income from their portfolio holdings, consistent with zero net wealth accumulation.

In the steady state, both wealth accumulation and decumulation cease, and the evolution equations satisfy $\dot{a}^A(t) = \dot{a}^B(t) = 0$. Imposing $g^*(t) = g_n$ and $\theta(g^*(t), t) = \rho$, we have:

$$0 = \dot{a}^A(t) = \rho a^A(t) - \left[1 + \mathcal{G}(t) - v(g_n)a^A(t)\right] c^A(t) \tag{B.48}$$

$$= \rho a^A(t) - \left[1 + \mathcal{G}(t)\right] c^A(t), \tag{B.49}$$

and

$$0 = \dot{a}^B(t) = \rho a^B(t) - c^B(t), \tag{B.50}$$

which implies that Advocate's steady-state consumption satisfies

$$c^A_{ss} = \frac{\rho a^A_{ss}}{1 + \mathcal{G}_{ss}}, \tag{B.51}$$

while Benchmark's consumption must satisfy

$$c^B_{ss} = \rho a^B_{ss}. \tag{B.52}$$

Together, these consumption rules ensure that household wealth remains constant in the steady state.

Appendix B.4.1. Long-Run Implications for Capital Allocation

Proof of ***Proposition 2*** *(Steady-State Capital Allocation).* From the firm's optimal investment condition and the cost of capital differentials in Lemma 4, we have

$$f'(k(g,t)) = \theta(g^*(t), t) + \delta \quad \text{for } g < g^*(t), \tag{B.53}$$

and

$$f'(k(g,t)) = \theta(g^*(t), t) - [v(g) - v(g^*(t))]c^A(t) + \delta \quad \text{for } g \geq g^*(t). \tag{B.54}$$

In the steady state, the threshold satisfies $g^*_{ss} = g_n$, and by definition $v(g^*_{ss}) = 0$, with $\theta(g^*(t), t) = \rho$ from the costate condition. Substituting into the optimal investment conditions,

we obtain the steady-state capital allocation:

$$f'(k_{ss}(g)) = \begin{cases} \rho + \delta & \text{for } g \leq g_n, \\ \rho + \delta - v(g)c_{ss}^A & \text{for } g > g_n, \end{cases}$$

where

$$\begin{aligned} c_{ss}^A &= \int_{g_n}^1 d_{ss}(g)\phi(g)\,\mathrm{d}g \\ &= \int_{g_n}^1 \left[f(k_{ss}(h)) - \delta k_{ss}(h)\right]\phi(h)\,\mathrm{d}h. \end{aligned} \tag{B.55}$$

□

To highlight the impact of Advocate households on capital allocation, consider a counterfactual steady state in which the economy consists solely of Benchmark households. In this case, the cost of capital is uniform across all firms, leading to

$$f'(\hat{k}) = \rho + \delta \quad \text{for all } g \in [0,1]. \tag{B.56}$$

By contrast, in the presence of Advocate households, Proposition 2 implies that

$$k_{ss}(g) > \hat{k} \quad \text{for all } g \geq g_n. \tag{B.57}$$

Furthermore, since $v(g)$ is strictly increasing in g, for any $g_1 > g_2 \geq g_n$ it follows that

$$k_{ss}(g_1) > k_{ss}(g_2). \tag{B.58}$$

For firms with $g < g_n$, capital allocation remains unchanged.

The presence of Advocate households thus induces both scale and composition effects on capital allocation in the long run: total capital in the economy exceeds that in the benchmark-only case, and capital is disproportionately allocated toward firms with higher sustainability scores.

Appendix B.4.2. Direction of Convergence

Proof of ***Proposition 1*** *(Direction of Convergence).* From Lemma 2, the difference in consumption growth rates between Advocate and Benchmark is:

$$\frac{\dot{c}^A(t)}{c^A(t)} - \frac{\dot{c}^B(t)}{c^B(t)} = v(g^*(t))c^A(t) \tag{B.59}$$

When $g^*(t) > g_n$, we have $v(g^*(t)) > 0$, which implies that Advocate's consumption grows faster than Benchmark's. This means Advocate's saving rate is higher, leading to wealth accumulation and expansion of its portfolio. As Advocate accumulates wealth, it gradually acquires stocks with lower sustainability scores, pushing $g^*(t)$ downward toward g_n.

Conversely, when $g^*(t) < g_n$, we have $v(g^*(t)) < 0$, making Advocate's consumption growth slower than Benchmark's. This results in Advocate reducing its savings and eventually selling some of its holdings, starting with the firms having the lowest sustainability scores in its portfolio. This pushes $g^*(t)$ upward toward g_n.

To formalize this intuition, we examine the threshold dynamics from Lemma 3:

$$\dot{g}^*(t) = \frac{c^A(t) - \int_{g^*(t)}^1 d(g,t)\phi(g)\mathrm{d}g}{q(g^*(t),t)\phi(g^*(t))} \tag{B.60}$$

When $g^*(t) > g_n$, the higher saving rate of Advocate means

$$c^A(t) < \int_{g^*(t)}^1 d(g,t)\phi(g)\mathrm{d}g. \tag{B.61}$$

Therefore, $\dot{g}^*(t) < 0$, confirming that the threshold decreases. Similarly, when $g^*(t) < g_n$, the lower saving rate means

$$c^A(t) > \int_{g^*(t)}^1 d(g,t)\phi(g)\mathrm{d}g. \tag{B.62}$$

Therefore, $\dot{g}^*(t) > 0$, confirming that the threshold increases.

This bidirectional convergence mechanism ensures that, regardless of initial conditions, the system eventually reaches the steady state where $g^*(t) = g_n$.

□

Appendix B.5. Numerical Solution Method

This section outlines a numerical solution method for computing the steady state and the equilibrium path of the model.

The numerical solution involves the following steps:

1) Discretize the state space (time and sustainability score grid)

2) Specify functional forms for preferences, production, and the distribution of firms

3) Initialize the system with given initial conditions

4) Compute the steady state as a reference point

5) Solve for the transition path using backward induction

Step 1: Discretize the State Space. Define a grid for sustainability scores: $\{g_1, g_2, ..., g_N\}$ with $g_1 = 0$ and $g_N = 1$. Define a time grid: $\{t_0, t_1, ..., t_T\}$ with $t_0 = 0$ and t_T being sufficiently large to approximate infinity.

Step 2: Specify Functional Forms.

- Production function: $f(k) = k^\alpha$ with $\alpha \in (0, 1)$
- Utility from sustainable wealth: $\mathcal{V}(\mathcal{G}) = \mathcal{G}$
- Sustainability preference function: $v(g)$ (as specified in the main text)
- Distribution of firms: $\phi(g)$ (empirical or uniform)

Step 3: Initialize the System. Specify initial capital stocks $k_0(g)$ for all $g \in [0, 1]$ and initial portfolio allocations $z_0(g)$.

Step 4: Compute the Steady State.

1) Set $g^* = g_n$ (the neutral score)

2) For firms with $g < g_n$, compute $k_{ss}(g)$ by solving:

$$f'(k_{ss}(g)) = \rho + \delta \tag{B.63}$$

3) Compute steady-state output, prices, and dividends for these firms

4) For firms with $g \geq g_n$, iteratively solve for $k_{ss}(g)$ and c_{ss}^A that satisfy:

$$f'(k_{ss}(g)) = \rho + \delta - v(g)c_{ss}^A \tag{B.64}$$

$$c_{ss}^A = \int_{g_n}^1 d_{ss}(g)\phi(g)\mathrm{d}g \tag{B.65}$$

5) Compute steady-state wealth levels a_{ss}^A and a_{ss}^B

Step 5: Solve for the Transition Path. To solve for the transition path from initial conditions to the steady state, we use a backward-shooting algorithm:

1) Start with the steady-state values for all variables

2) Discretize time into T periods, with period T representing the steady state

3) For each period $t = T-1, T-2, ..., 0$, solve backwards:

   (a) Given g_{t+1}^*, use the threshold dynamics equation to determine g_t^*:

$$g_t^* = g_{t+1}^* - \Delta t \cdot \frac{c_t^A - \int_{g_t^*}^1 d(g,t)\phi(g)\mathrm{d}g}{q(g_t^*,t)\phi(g_t^*)} \tag{B.66}$$

   (b) Determine the cost of capital for each firm using:

$$\theta(g,t) = \theta(g_t^*,t) \quad \text{for } g < g_t^* \tag{B.67}$$

$$\theta(g,t) = \theta(g_t^*,t) - [v(g) - v(g_t^*)]c_t^A \quad \text{for } g \geq g_t^* \tag{B.68}$$

   (c) Solve for capital allocation based on:

$$f'(k(g,t)) = \theta(g,t) + \delta \tag{B.69}$$

   (d) Update prices and dividends for all firms

   (e) Compute consumption levels using:

$$\frac{c_{t+1}^A}{c_t^A} = 1 + \Delta t \cdot [\theta(g_t^*,t) + v(g_t^*)c_t^A - \rho] \tag{B.70}$$

$$\frac{c_{t+1}^B}{c_t^B} = 1 + \Delta t \cdot [\theta(g_t^*,t) - \rho] \tag{B.71}$$

(f) Update portfolio holdings and wealth levels

4) Adjust initial guess of steady state if necessary to match initial conditions at $t = 0$

5) Iterate until convergence

This numerical approach allows us to compute the entire transition path for all key variables, including the threshold g_t^*, capital allocation $k(g,t)$, consumption levels, and wealth distribution.

Note that the numerical algorithm needs special handling for potential corner solutions:

- When g_t^* approaches 0 or 1, adjust the update rule to respect these bounds
- Ensure that consumption remains positive throughout the transition
- Verify that the short-sale constraints ($z^A(g,t) \geq 0$ and $z^B(g,t) \geq 0$) are satisfied at each step