Ambiguity vs Risk in Investment Decisions: An Illustration from Green Finance

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Does ambiguity (Knightian uncertainty) or risk provide a greater discouragement to investment? There is general agreement in the financial press that uncertainty discourages investment, with uncertainty here meaning any situation where important future policy variables (such as tariffs or tax rates) are not known. And it seems intuitively plausible that situations of ambiguity are "more uncertain" than those of risk: under risk, although the outcome is not known, its expected value is, whereas under ambiguity we have a distribution of possible expected values. In this paper we show that under quite general circumstances it is the case that ambiguity deters investment more than an equivalent risk. We show that there will always be investments opportunities that are rejected when characterized by ambiguity but are accepted when characterized by risk with an equivalent compound lottery, and that the converse is not true: there are no investments that would be made under ambiguity that would not be made under equivalent risk. We develop the analysis in the context of the emerging field of green finance.

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1 Introduction

Does ambiguity (Knightian uncertainty) or risk provide a greater discouragement to investment? There is general agreement in the financial press that uncertainty discourages investment, with uncertainty here meaning any situation where important future policy variables (such as tariffs or tax rates) are not known. And it seems intuitively plausible that situations of ambiguity are "more uncertain" than those of risk: under risk, although the outcome is not known, its expected value is, whereas under ambiguity we have a distribution of possible expected values. In this paper we investigate this issue, and show that under quite general circumstances ambiguity deters investment more than an equivalent risk. We define an equivalent risk as one characterized by a compound lottery reflecting the ambiguous situation, but without ambiguity aversion. We show that there will always be investments opportunities that are rejected when characterized by ambiguity but are accepted when characterized by risk with an equivalent compound lottery, and that the converse is not true: there are no investments that would be made under ambiguity that would not be made under equivalent risk. This conclusion is not obvious: in a companion paper on regulatory ambiguity in green investments [Heal and Lucchetta(2025)], we show that when ambiguity affects returns to both investment options but in opposite ways, ambiguity can actually encourage investment.

We develop the analysis in the context of the emerging field of green finance, and in particular the financing of opportunities for biodiversity conservation and for greenhouse gas mitigation. Billions are being invested in green bonds, renewable energy, and carbon markets; however, the pace of this transition lags climate targets

- a puzzle that standard economic models struggle to explain. Risk-based frameworks predict a robust shift toward green investments when they offer competitive expected returns (Nordhaus 2018; Acemoglu et al. 2012). Yet firms hesitate, capital stalls, and brown assets persist. While our companion paper (Heal and Lucchetta, 2025) explores similar distortions in green investment under regulatory ambiguity, here we focus on biodiversity conservation, where Knightian uncertainty over yields and exchange rates creates a unidirectional drag against high-impact projects. This sluggishness can be exacerbated by political factors, as Besley and Persson (2023) demonstrate in their model of green transitions, where lack of regulatory commitment in democracies - due to electoral cycles and lobbying - creates policy uncertainty that hinders sustainable shifts, amplifying the ambiguity firms face in regulatory environments.

This paper proposes a theoretical explanation rooted in ambiguity, a deeper uncertainty where outcomes cannot be readily quantified (Ellsberg 1961). Unlike risk, where outcomes carry quantifiable likelihoods, ambiguity captures the unmeasurable uncertainty of future policies such as carbon taxes or green incentives (Knight 1921). In green investments, this is pervasive: Will carbon taxes rise to punitive levels or remain symbolic? Will subsidies endure or vanish with political changes? Traditional risk models assume agents maximize expected utility, but ambiguity prompts either a maxmin response - maximizing utility under the worst-case prior within a convex set (Gilboa and Schmeidler 1989 GS) or maximizing a non-linear weighted average of utilities following the approach of Klibanoff Marinacci and Mukherji (KMM). This reflects ambiguity aversion's precautionary pull, yet its implications for mod-

ern markets remain underexplored. Classical works like Becker (1957) show how biases distort allocation, but ambiguity acts as an endogenous wedge via rational pessimism.

Our contributions are twofold. Firstly, in contrast with risk models (Nordhaus 2018; Acemoglu et al. 2012), we quantify the gap between risk and uncertainty and align with calls for nuanced uncertainty (Hansen and Marinacci 2016). Secondly, we generalize: ambiguity skews any policy-sensitive choice (relating perhaps to trade, tax, or labor), echoing Becker (1957). We extend the ambiguity framework to biodiversity conservation and blended finance, where ambiguity aversion (Knightian uncertainty) depresses investment more than risk, building on conceptual and empirical analyses showing that blended structures - via concessional capital, bundling public/private goods, and fact-finding to reduce ambiguity - de-risk biodiversity projects and expand the three-dimensional efficient frontier of return, risk, and biodiversity impact (Flammer, Giroux, and Heal 2025a, 2025b). This blends micro choice with macro sustainability. Our results allow us to compute the welfare costs of ambiguity and hence the gain to resolving it. the GS and KMM frameworks give slightly different results, with the cost depending on the amount of ambiguity and in the KMM case the degree of aversion to ambiguity.

1.1 Literature Review

Green finance, rife with regulatory opacity, is an ideal testing ground. Nordhaus (2018) and Weitzman (2009) model climate policy under risk or fat tails, overlooking pure ambiguity. Besley and Persson (2023) incorporate political dynamics, align-

ing with our view of ambiguity as endogenously amplified by commitment failures. Empirical hints exist (e.g., Cook et al. 2018, on biases in gig economy allocations), but theoretical frameworks for green investment under pure ambiguity are scarce. Cai et al. (2022) explore green bond pricing under hybrid uncertainty, while Hansen and Marinacci (2016) link ambiguity to model misspecification in decision-making, foreshadowing our micro-firm application.

Recent literature on ambiguity aversion highlights related dynamics. Huo, Pedroni, and Pei (2024) show how ambiguity under dispersed information leads to pessimistic bias and increased signal sensitivity, which can amplify underinvestment in uncertain environmental projects. Auster, Che, and Mierendorff (2024) explore prolonged learning and hasty stopping in Wald problems with ambiguity, suggesting investors may experiment excessively or quit prematurely in biodiversity contexts. Heimer et al. (2025) document dynamic inconsistency in risky choice, extended to ambiguity, where agents cut gains early and chase losses, patterns that could exacerbate sluggish conservation funding. Chen and Zhong (2025) model information acquisition with time-risk preferences, proposing Greedy Exploitation (max time-risk) and Pure Accumulation (min) strategies. This ties to fact-finding in blended finance, optimizing ambiguity reduction in our setup.

These works provide general insights, but our paper is unique in applying them to biodiversity conservation, where Knightian uncertainty on ecosystem returns leads to a quantifiable investment wedge. This aligns with Heal and Millner's emphasis on precautionary approaches in environmental ambiguity (Heal and Millner 2014; Millner et al. 2013), where resolving ambiguity via information yields significant

welfare gains.

The paper proceeds as follows: Section 2 presents the risk case. Section 3 analyzes ambiguity using GS and KMM approaches. Section 4 concludes with a discussion of extensions and welfare implications.

2 Risk

We consider investing in a biodiversity conservation project in a foreign country. The project yields a return in the local currency of $r \in [\underline{r}, \bar{r}]$. The exchange rate between the currency in which the investment is made, and the home currency of the investor, is $x \in [\underline{x}, \bar{x}]$. Hence the overall return to the investor is $R = rx$. We assume r and x to be independently randomly distributed. This implies that $E\{R\} = E\{r\}E\{x\}$ where $E\{\cdot\}$ is the expectation operator. The investor can earn a return of M (hurdle rate) on market investments. In the notes that follow we analyze the conditions under which an investor would invest in this conservation project in two cases: when the probability distributions over r and x are known, so the problem is characterized by risk, and when they are unknown, and it is characterized by ambiguity. A comparison of the two cases shows that ambiguity discourages investment relative to risk: the conditions for investment to be attractive under risk are weaker than their equivalents under ambiguity.

Let the investor's wealth be W and $f \in [0, 1]$ be the fraction of this that is invested in the conservation project, with the balance being invested in the market and earning a certain rate M. The investor therefore seeks to

$$\max_f \int_r \int_x U(fWrx + (1-f)MW)\, dr\, dx$$

where $U(\cdot)$ is a concave increasing function. For simplicity we let $R = rx$ so this can be written

$$\max_f \int_R U(fWR + (1-f)MW)\, dR$$

The first order condition here is

$$\int_R U'(fWR + (1-f)MW)W(R-M)\, dR = 0$$

(with appropriate corner conditions as $f \in [0,1]$).

If the investor does not invest in the project, then $f = 0$. In this case the FOC becomes

$$U'(MW)W \int_R (R-M)\, dR = 0$$

as $U'(MW)$ and W are constants. Clearly $\int_R R\, dR = E\{R\}$. If $E\{R\} > M$, this equation cannot hold and therefore we cannot have $f = 0$: the investor must have a positive investment in the conservation project. We conclude that under conditions of risk, the investor will have a positive investment in the project if its expected return is greater than or equal to the market rate.If $E\{R\} < M$ the investor will not invest.

3 Ambiguity

We will investigate the implications of both the Gilboa-Schmeidler MaxMin Expected Utility (GS) approach and Klibanoff-Marinacci-Mukherji (KMM) smooth ambiguity approach.

3.1 G-S Approach

Following the Gilboa-Schmeidler approach, there is a convex set P of prior probabilities $p \in P$ over returns and the objective is

$$\max_f \min_p E_p U(fWR + (1-f)MW)$$

Since U is concave and increasing, and the argument is linear in R, the minimizing p is a Dirac measure on the worst-case realization of the random variables x and r, with $xr = R$, which is $\underline{rx}$ (assuming positive returns).[1] Thus, the objective simplifies to

$$\max_f U(fW(\underline{rx}) + (1-f)MW)$$

The derivative of this with respect to f is

[1]Given U concave, by Jensen's inequality, for any p with support on multiple points, $\min E_p U(fWR + (1-f)MW) \geq U(fWE_pR + (1-f)MW)$, but since we minimize, the worst is at the boundary, specifically the point where the linear argument is minimized ($\min R$), as U increasing means a lower argument gives lower utility. For a convex set C of priors on the rectangle, the min over p in C of E_pU is achieved at extreme points, but since U composed with a linear function is concave, the min is at the boundary, specifically the Dirac on $\min R$.

$$U'(\cdot)W(\underline{rx} - M)$$

Since U' is positive, the sign is that of $(\underline{rx} - M)$. Hence, if $\underline{rx} > M$, $f^* = 1$; if $\underline{rx} < M$, $f^* = 0$; if equal, the investor is indifferent.

We want to compare outcomes under ambiguity and risk. Under the former, the decision-maker considers multiple probabilities, but under risk, only one. If we select just one probability distribution for the risk case, which is the one that is appropriate for comparison? We argue that this is the uniform distribution over R: we call this the **equivalent distribution**. Two arguments support this. One is that this represents an uninformative prior, which is appropriate for the comparison with ambiguity. The second is more complex. Consider the set of all distributions over R, and form a compound lottery by giving equal weight to each distribution. Then the probability representing this compound lottery is the uniform Lebesgue measure on R. We provide a rigorous justification in footnote 2.[2]

We will show that also for the uniform distribution, there is an open set of parameters for which investment occurs under risk but not ambiguity, and that the opposite is not true. Alternatively, we can show that there are many distributions

[2]To make this rigorous, consider a discretization of the interval $[\underline{R}, \bar{R}]$ into n equally spaced points $R_i = \underline{R} + (i - 0.5)\frac{\bar{R}-\underline{R}}{n}$ for $i = 1, \ldots, n$. The set of all probability distributions over these points is the $(n-1)$-dimensional simplex $\Delta^{n-1} = \{\mathbf{p} \in [0,1]^n \mid \sum_{i=1}^n p_i = 1\}$.Giving equal weight to each distribution corresponds to integrating with respect to the uniform probability measure on this simplex, which is equivalent to the Dirichlet distribution with parameters $(1, 1, \ldots, 1)$, whose density is constant: $(n-1)!$ on the simplex. Under this measure, the marginal expected value for each p_i is $E[p_i] = \frac{1}{n}$. Therefore, the compound lottery assigns probability $\frac{1}{n}$ to each point R_i.As $n \to \infty$, this discrete uniform distribution over equally spaced points converges weakly to the continuous uniform distribution on $[\underline{R}, \bar{R}]$, with density $\frac{1}{\bar{R}-\underline{R}}$.Thus, in the continuous limit, the probability measure representing the compound lottery is the uniform Lebesgue measure (normalized to a probability measure) on $[\underline{R}, \bar{R}]$.

for which there is investment under risk but not ambiguity.

Proposition 1. *Under ambiguity using the GS framework, and assuming a uniform distribution under risk, there is no investment in the ambiguity case if* $\underline{rx} < M$*, even if* $E\{R\} > M$ *and there is investment under risk. However we cannot have a case in which there is investment under ambiguity but not under risk. The converse does not hold.*

Proof: As derived above, under ambiguity investment (positive f) requires $\underline{rx} \geq M$ for $f^* = 1$. However, it is possible for the uniform distribution that $\underline{rx} < M$ while $E\{R\} = E\{r\}E\{x\} > M$.[3] Thus, parameters exist where risk yields $f > 0$ but GS-ambiguity yields $f = 0$. Clearly there are also many other distributions for which $E\{R\} > M$ while $\underline{rx} < M$. The converse does not hold: as $\underline{rx} > M$ implies $E\{R\} > M$ by independence and positivity.

Another way of seeing the impact of uncertainty relative to risk, is to ask by how much the hurdle rate would have to be increased if the choices under risk with the new hurdle rate were to be the same as those under uncertainty with the original rate. This characterizes the impact of uncertainty as an increase in the hurdle rate - the uncertainty premium. In the GS framework we have just studied, for parameter values where $R < M < E\{R\}$(implying no investment under uncertainty but investment under risk), the uncertainty premium P is $P = E\{R\} - M$. If the risk distribution is uniform, then this simplifies to $P = 0.5\left[\underline{R} + \bar{R}\right] - M$.

[3]This also true for many other distributions of R.

3.1.1 Numerical Example for GS

To illustrate the wedge, assume uniform distributions for r and x over [0.8, 1.2], so $E\{r\} = E\{x\} = 1$ and $E\{R\} = 1$. Let $M = 0.9$.

Under risk: $E\{R\} = 1 > 0.9$, so the FOC implies $f > 0$ - the investor allocates positively to conservation.

Under ambiguity (GS): The worst-case prior assigns probability 1 to the minimum $rx = 0.8 \times 0.8 = 0.64 < 0.9$, so $f = 0$ - no investment, despite favorable expected returns.

This example shows how ambiguity deters investment even when risk encourages it, consistent with sluggish biodiversity funding.

To extend this to a real-world biodiversity context, consider emerging market data where local yields and exchange rates exhibit greater volatility (e.g., based on conservation project returns from Flammer et al. (2025a) and IMF (2024) data on currency fluctuations in countries like Brazil). Assume uniform distributions for $r \sim [0.7, 1.3]$ (local yield, $E\{r\} = 1.0$) and $x \sim [0.6, 1.4]$ (exchange rate, $E\{x\} = 1.0$), so $E\{R\} = 1.0$. Let $M = 0.9$.

Under risk: $E\{R\} = 1 > 0.9$, implying positive investment ($f > 0$).

Under ambiguity (GS): The worst-case is $\underline{rx} = 0.7 \times 0.6 = 0.42 < 0.9$, so no investment ($f = 0$). This illustrates how ambiguity deters funding in volatile markets, where blended finance (e.g., public guarantees) could raise the lower bound to encourage investment.

3.2 Welfare Cost of Ambiguity in the G-S Framework

In the baseline GS model, the uncertainty premium $P = E\{R\} - M$ (for $\underline{R} < M < E\{R\}$) characterizes the increase in the hurdle rate due to ambiguity, deterring investment relative to risk. However, this does not directly measure the *welfare cost* of ambiguity—the ex-ante value of resolving uncertainty by learning the true distribution. We derive this as follows. The ambiguous objective is:

$$V^A = \max_f \min_{p \in \mathcal{P}} E_p \left[U(f\tilde{W} + (1-f)MW) \right],$$

where $\tilde{W} = RW$, $R \in [\underline{R}, \overline{R}]$, and $\mathcal{P}$ is the convex set of priors over R. As established, this simplifies to the worst-case: $V^A = \max_f U(f\underline{R} + (1-f)M)$, yielding $f^* = 0$ if $\underline{R} < M$ (no investment) or $f^* = 1$ if $\underline{R} > M$. Now, suppose uncertainty is resolved: we learn the true distribution is $p_k \in \mathcal{P}$. The value becomes the risk-case optimum:

$$V_k^R = \max_f E_{p_k} \left[U(fR + (1-f)M) \right].$$

Under our assumptions (concave U, linear in R), investment occurs iff $E_{p_k}\{R\} \geq M$, with $V_k^R = U(E_{p_k}\{R\})$ if investing (assuming $W = 1$ for simplicity; full derivations in baseline). The *conditional* value of resolving ambiguity (given true p_k) is $\Delta_k = V_k^R - V^A$. However, ex-ante, we do not know p_k. To compute the expected welfare cost, take the expectation over a reference measure on $\mathcal{P}$. A natural choice is a uniform (uninformative) prior over $\mathcal{P}$, or the compound lottery equivalent (uniform over $[p, \bar{p}]$ for binary simplification). Assume $\mathcal{P}$ corresponds to priors with support on $[\underline{R}, \overline{R}]$, and take the expectation under a uniform density μ over possible means

$E_p\{R\}$. The ex-ante welfare cost of ambiguity C^{GS} is:

$$C^{GS} = \int_{\mathcal{P}} \mu(p)\Delta_p \, dp = \int_{\mathcal{P}} \mu(p) \left(V_p^R - V^A\right) dp.$$

For the no-investment region under ambiguity ($\underline{R} < M < E\{R\}$), $V^A = U(M)$. Under resolution, if $E_p\{R\} \geq M$, $V_p^R = U(E_p\{R\})$; else $V_p^R = U(M)$. Thus:

$$C^{GS} = \int_{p:E_p\{R\}\geq M} \mu(p) \left[U(E_p\{R\}) - U(M)\right] dp.$$

Proposition 2 (Welfare Cost in GS)**.** *Assuming uniform μ over $E_p\{R\} \in [\underline{R}, \overline{R}]$ and CRRA utility $U(w) = \frac{w^{1-\gamma}}{1-\gamma}$ ($\gamma > 0$), the normalized cost simplifies to:*

$$C^{GS} \approx \frac{1}{1-\gamma} \int_M^{\overline{R}} \frac{1}{\overline{R} - \underline{R}} \left[r^{1-\gamma} - M^{1-\gamma}\right] dr,$$

which integrates to a closed form depending on γ, M, and the ambiguity interval width. For $\gamma \to 1$ (log utility), $C^{GS} \approx \frac{1}{\overline{R}-\underline{R}} \left[(\overline{R} - M) \ln\left(\frac{\overline{R}+M(\ln M - \ln \overline{R})-M}{\overline{R}-\underline{R}}\right)\right]$ (approximation; full derivation in the appendix).

This cost is positive and increasing in the ambiguity set size ($\overline{R} - \underline{R}$), paralleling the result we obtain for the KMM case below. Unlike the premium P, C^{GS} captures the option value of resolution, even when no investment occurs under ambiguity.

3.2.1 Implications for Green Finance

In biodiversity investments, this welfare cost justifies policies to resolve ambiguity (e.g., funding research on project outcomes or standardization via TNFD). For ex-

ample, if ambiguity stems from unknown monetization (x), public guarantees could shrink $\mathcal{P}$, reducing C^{GS} and bridging the \$900bn funding gap.

3.3 KMM Approach

Suppose there are N possible probability distributions over the investment outcomes R indexed by n and denoted by $\theta_n(R)$. Decision-makers are assumed to choose according to the axioms of smooth ambiguity aversion (KMM), so the value they place on the project is given by

$$\sum_{n=1}^{N} \pi_n \phi(E|\theta_n U(R)) \tag{1}$$

where $U(R)$ is the decision-maker's utility function defined on outcomes R, ϕ is a concave real-valued function expressing ambiguity-aversion and $\pi_n \geq 0$ is the likelihood that the decision-maker assigns to the event that distribution θ_n is the true distribution. The expression $E|\theta_n$. is the expectation of the following variable according the the distribution θ_n. This expression is a non-linear weighted average of the expected utilities of the project according to the different distributions, where the weight π_n is the likelihood that distribution θ_n is the true distribution. An important point to note is that in a situation of risk, with a single probability distribution, although the outcome x is unknown, the expected utility of the outcome is known. However with multiple possible distributions (ambiguity) there are as many possible expected utilities as there are distributions, so the decision-maker is faced with a distribution of possible expected utilities. If as a result of fact-finding investigations they are

assured that the correct distribution of possible outcomes is θ_k, then the value that they place on the project is

$$E|\theta_k U(R)$$

and the gain from the extra information is obviously

$$E|\theta_k U(R) - \sum_{n=1}^{N} \pi_n \phi(E|\theta_n U(R))$$

As we do not know which distribution $\theta_k$is the correct one, we have to take the expectation of this over all possible values of k. For that we have to assign probabilities to the $k's$ and the obvious probabilities are the π_k. Hence the expected gain from eliminating uncertainty is

$$\sum_{k} \pi_k \{E|\theta_k U(R)\} - \sum_{n=1}^{N} \pi_n \phi(E|\theta_n U(R)) \tag{2}$$

Here $\sum_n \pi_n \{E|\theta_k U(R)\}$ is a compound lottery over the the distributions θ_n with probabilities π_n: this is the **equivalent distribution** for the KMM case. Hence the expected gain is:

$$\sum \pi_n \{E_n - \phi(E_n)\}$$

where $E_n = E_{\theta_n} U(\cdot)$. Next let $E^* = \sum_n \pi_n E_n$, the mean expected utility across the

different distributions π_n. Then define x as the cost of ambiguity given by

$$\phi\left(E^* - x\right) = \sum_n \pi_n \phi\left(E_n\right) \tag{3}$$

The certain value of ϕ of the mean expected utility minus x is the same as the expected value of ϕ of the distribution of expected utilities arising under ambiguity. Using a second order Taylor's expansion we can write this as

$$\begin{array}{ll} \phi\left(E^* - x\right) - \phi\left(E^*\right) & = \sum_n \left\{\phi\left(E_n\right) - \phi\left(E^*\right)\right\} \pi_n \\ = \sum_n \pi_n \left\{\phi'\left(E^*\right)\left(E_n - E^*\right) + \frac{\phi''}{2}\left(E_n - E^*\right)^2\right\} & = \sum_n \pi_n \frac{\phi"}{2}\left(E_n - E^*\right)^2 = \frac{\phi''}{2}\sigma_E^2 \end{array}$$

where σ_E^2 is the variance of the distribution of the $E_n s$. The LHS of the first line of this expression can be expressed as $-\phi'\left(E^*\right)x - \phi"\left(E^*\right)x^2/2$ so that

$$-\phi'\left(E^*\right)x = \phi''\left(E^*\right)\left\{\sigma_E^2 + x^2\right\}/2$$

and assuming x to be small so that x^2 is negligible,

$$x = -\frac{\phi''\left(E^*\right)\sigma_E^2}{\phi'\left(E^*\right)2} \tag{4}$$

The cost of ambiguity is therefore the index of ambiguity aversion $-\phi''/\phi'$ times half the variance of expected utilities: the more ambiguity-averse the decision-maker, the greater the cost of ambiguity. This adjustment lowers the effective expected utility, making the project less attractive compared to risk.

To see the impact on the choice of f, note that under KMM, the objective is

concave in f due to ϕ concavity, but the effective return is discounted by the ambiguity cost. To derive the first order conditions take the derivative of the objective $V(f) = \sum \pi_n \phi(\int U(fWR + (1-f)MW)d\theta_n)$. This is

$$V'(f) = \sum \pi_n \phi'(E_n) * (dE_n/df),$$

where $dE_n/df = \int U'(\cdot)W(R-M)d\theta_n$. Consider the first order condition $V'(f) = \sum \pi_n \phi'(E_n)U'(MW)W(E_n\{R\} - M) = 0$. We can rewrite this as

$$WU'(MW)\sum_n \pi_n \phi'(E_n)[E_n\{R\} - M = 0]$$

Divide both sides by $WU'(MW)\sum_n \pi_n \phi'(E_n)$ and let

$$\pi'_n = \frac{\pi_n \phi'(E_n)}{\sum_n \pi_n \phi'(E_n)}$$

Then we can write the FOC as

$$\sum_n \pi'_n [E_n\{R\} - M] = 0 \tag{5}$$

Here the π' are ambiguity-adjusted probabilities [Millner et al.(2013)] so this is saying that the expected rate of return calculated at the ambiguity-adjusted probabilities must equal the hurdle rate for indifference between investing and not investing, with the appropriate inequalities for corner solutions. So we now have to compare the above with the equivalent for risk which is just $E\{R\} - M = 0$ with appropriate corner conditions. We need to decide with respect to which probability

distribution this expectation is taken - what is the benchmark distribution in the risk case? The natural case is the equivalent distribution used above, a compound lottery constructed by taking each of the probabilities considered in the ambiguity case with the likelihoods that the decision-maker assigns to them. In this case, the FOC for the risk case is $\sum_n \pi_n E_n \{R\} - M = 0$. Recall that ϕ is strictly concave and that $\sum_n \pi_n = \sum_n \pi'_n = 1$. Hence $\exists\, j :\ \pi_j \neq \pi'_j$. Let $X_> = \{n : \pi'_n > \pi_n\}$ and $X_< = \{n : \pi'_n \leq \pi_n\}$. By the strict concavity of ϕ, the $n \in X_>$ apply to smaller values of E_n than the $n \in X_<$. It follows that if $\sum_n \pi_n E_n \{R\} - M = 0$, then $\sum_n \pi'_n E_n\{R\} - M < 0$. Hence if the investor under risk is indifferent between investing and not investing, then the investor under ambiguity will not invest. And there will be parameter values for which investment occurs under risk but not under ambiguity. The converse is not true, that is, if there is investment with ambiguity so that $\sum_n \pi'_n E_n\{R\} - M > 0$, then $\sum_n \pi_n E_n \{R\} - M > 0$ also and there is investment under risk. Formally,

Proposition 3. *Under ambiguity (KMM), the project value is lower than risk by the cost of ambiguity; parameters exist justifying investment under risk but not ambiguity, but the converse does not hold,*

There is a more intuitive way of approaching this result. Carry out a thought experiment in which the degree of ambiguity-aversion is gradually reduced to zero. If $\phi(E_n) = \frac{E_n^{1-\eta}}{1-\eta}$ giving an index of relative ambiguity aversion of η, think of η going to zero from above, so that $\phi(E_n) \to E_n$. Then the cost of ambiguity given in equation (2) goes to zero. The expression for the ambiguity objective in (1) is now the compound lottery $\sum_n \pi_n E_n$, identical to the objective in the risk case. In the

argument for proposition 2, $\pi_n = \pi'_n$ and there is no difference between the risk and KMM FOCs, and so no impact of ambiguity on the decision about whether to invest.

3.3.1 Numerical Example for KMM

Assume the same uniform distributions as above, with $\sigma_E^2 = 0.04$ (variance of expected utilities under $N = 2$ priors), and aversion index $-\phi''/\phi' = 2$. The cost of ambiguity is approximately $2 \times (0.04/2) = 0.0267$, reducing the project value below the risk threshold, leading to no investment despite $E\{R\} > M$.

Extending to multiple priors with emerging market inspiration, assume $N = 3$ distributions over $R = rx$, with priors weighted by likelihoods reflecting ambiguity (e.g., pessimistic on currency volatility per IMF 2024).

Prior 1 (pessimistic, $\pi_1 = 0.4$): uniform over [0.7, 1.1], $E_1\{R\} = 0.9$.

Prior 2 (neutral, $\pi_2 = 0.4$): uniform over [0.8, 1.2], $E_2\{R\} = 1.0$.

Prior 3 (optimistic, $\pi_3 = 0.2$): uniform over [0.9, 1.3], $E_3\{R\} = 1.1$.

The weighted expected return under risk (compound lottery) is $\sum \pi_n E_n\{R\} = 0.98 > M = 0.9$, suggesting investment. Variance of expected utilities $\sigma_E^2 \approx 0.05$ (assuming normalized utilities). With relative ambiguity aversion $\eta = 1.5$, the cost of ambiguity is approximately $(\eta/2) \times 0.05 = 0.0375$, reducing the effective value below M, leading to no investment. Fact-finding could reduce N or σ_E^2, aligning with blended finance strategies.

4 Conclusion

This paper demonstrates that ambiguity aversion creates a distinct barrier to investment in biodiversity conservation projects, beyond what traditional risk models predict. Our key results reveal that while expected returns exceeding market rates suffice to encourage investment under risk, ambiguity imposes stricter conditions: in the Gilboa-Schmeidler maxmin framework, worst-case scenarios deter allocation even when averages are favorable, and the same is true in the Klibanoff-Marinacci-Mukherji smooth model. In this framework the cost of ambiguity - quantified as approximately half the variance of expected utilities times an aversion index - lowers project value, leading to underinvestment. Numerical examples illustrate this wedge.

The novelty of our work lies in applying these ambiguity frameworks to environmental economics, specifically biodiversity conservation, where Knightian uncertainty from foreign exchange rates and local yields is pervasive. By integrating recent literature on bias, dynamic inconsistency, and information acquisition, we highlight how ambiguity amplifies political and regulatory frictions, as in Besley and Persson (2023), and extends to welfare implications: the resulting underinvestment leads to foregone social benefits like ecosystem services, with quantifiable losses that policy interventions can mitigate. This bridges microeconomic decision-making with macroeconomic sustainability, emphasizing precautionary effects aligned with environmental ambiguity. Our results suggest a possible reframing of green finance policies: rather than relying solely on incentives, it may pay to resolve ambiguity through the allocation of funds to fact-finding. Although we have presented our results in the context of investment in biodiversity conservation, they are definitely

not limited to this context. They apply to any situation where an investor has to compare an uncertain and ambiguous return to an investment with an exogenously given market rate of return.

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