Coordinating Technological Change: A Role for Institutions in Economic Growth*

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Abstract

This paper characterizes the role of a class of institutions in fostering technological progress and economic growth. The institutions considered are those that can coordinate among firms that use an input that is anticipated to improve repeatedly over time. Using an endogenous growth model, the average growth rate in the free market and the socially optimal growth rate are derived and it is shown how the institutions considered can reduce the gap between the two. The mechanism considered in the paper offers an explanation for the acceleration in output growth in the US economy during 1993-2003. The paper discusses the changes in institutional arrangements within the semiconductor and electronics industries that facilitated better coordination among firms, leading to a technological acceleration in these industries and faster output growth in the US economy. The model and its implications are applicable to key emerging technologies.

1 Introduction

Endogenous growth models developed in Romer (1990), Aghion and Howitt (1992) and Helpman and Grossman (1991) provide a framework in which profit-driven firms undertake R&D investments that fuel technological progress and economy-wide output growth. Separately, a substantial body of literature, including North (1991) and

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Acemoglu et al. (2005), highlight the critical role of institutions in driving economic growth. A natural progression in growth theory would be to extend endogenous growth models to examine how institutions influence firms' profit motives and thereby shape technological progress and growth of aggregate output. Where such connections between institutions and technological progress have been studied, the focus has primarily been on the role of the legal system and property rights in creating an environment where investments are channeled toward enhancing productive capacity rather than safeguarding against expropriation. Yet, institutions have a more direct and active role in promoting technological change than this indirect framework suggests. This paper seeks to address this gap by explicitly linking the rate of technological progress to the influence of a specific class of institutions, highlighting their role in driving innovation and growth.

The institutions considered in this paper are those that can facilitate coordination among firms that use a technology that is expected to undergo repeated improvements over time. The link between aggregate output growth and these institutions arises from the role that repeated innovations in a few technologies have played in driving output growth. Bresnahan and Trajtenberg (1995) referred to such technologies as General Purpose Technologies (GPTs).[1]

The results in this paper hinge on four attributes of such technologies, three of which have previously been discussed in the literature. First, these technologies are used across a wide range of the firms in the economy; second, they are capable of repeated improvements; and third, improvements in such technologies require complementary investments to adapt the input before it can be incorporated into the production processes of firms. This paper introduces a fourth attribute: the knowledge required to adapt the input is shared between the inventing firm and the downstream firms that use the input. While this aspect has received little attention in the literature, it is intuitive. The supplier of a general purpose input naturally possesses knowledge about the input's construction and operation, while the downstream firms understand the intricacies of their own production processes and how the input must be adapted to fit them.

[1] While the classification of technologies as GPTs has been a topic of debate, the contribution of a few such technologies to aggregate output growth has been widely acknowledged. For example, see Crafts (2004) for the role of steam engine technology in driving growth in Britain during 1850-1870 and David and Wright (1999) for the role of electricity in the US economy during 1919-1929.

If the burden of making these complementary investments falls primarily on the supplier, it reduces the supplier's return to inventing higher quality versions of the input, potentially slowing technological progress. Conversely, if downstream firms can shoulder a greater share of the complementary investments, it enhances the supplier's incentive to innovate, increasing the rate of innovation and growth rate of output in the economy. The reasoning can be made clear using historical examples of two general purpose inputs, steam engine and electricity.

Steam engines were used across multiple sectors, each with unique requirements for engine speed, power and stability. Innovations that improved engine efficiency (reducing fuel consumption for a given amount of mechanical work) benefited all users of steam engines. However, adapting innovations that increased engine efficiency to various sectors posed challenges. For instance, the invention of high-pressure engines, which used increased pressure to reduce fuel consumption, was readily adopted in mining, and paved the way for railroads and steamships, but could not be easily used in textile mills.

This was because, as Rosenberg and Trajtenberg (2004) explain, high-pressure steam engines produced unsteady motion, which did not pose a problem in mining or railroads, but textile machinery required steady, consistent motion to avoid tearing of threads.[2] The necessary adaptations for using high-pressure engines in textile manufacturing could have been undertaken by steam engine suppliers, who could modify the high-pressure engine to provide steadier motion, or by textile equipment manufacturers who could adapt textile machinery to be less sensitive to speed variations. Although there was some effort on the part of textile machinery manufacturers, much of the effort to adapt high pressure engines were undertaken by steam engine manufacturers themselves.[3]

A similar dynamic was prevalent in the early development of electricity generation

[2]See also Hill (1989) page 141, and von Tunzelman (1978) pages 82-83.

[3]The power driven textile machinery like self-acting mules and power looms underwent some improvements in 1840s that made them less sensitive to speed variations, as described in von Tunzelman (1978) (page 187 and page 199). But the most of the innovations for adaptation came from steam engine manufacturers, including clever efforts like McNaughting (von Tunzelman (1978) page 86), where low and high pressure versions were coupled together. It was the invention of Corliss steam engine, with its system of automatic regulators which reduced speed variations, that led to widespread adoption of high-pressure steam engines in the textile industry (see Rosenberg and Trajtenberg (2004)).

and transmission, a good example being the invention of alternating current (AC) technology, which offered significant advantages over the initially developed direct current (DC) systems. In AC, the electric current's direction cycles at a specific frequency, and different frequencies suited different applications (see David and Bunn (1988)).[4]

In the early years of the electricity industry, the necessary adaptation investments to shift to a common frequency were not undertaken by firms that produced electricity driven products, who could have modified their equipment such as electric appliances or industrial motors, to accommodate a standardized frequency. Instead, electricity suppliers had to support multiple frequencies, investing in the design and production of alternative versions of equipment like generators, transformers and meters. This burden reduced the returns electricity suppliers could achieve from new innovations.

Such adaptation investments that could have been made by downstream firms constitute what is referred to as complementary investments in this paper. Firms that use inputs like steam engine and electricity underinvest in complementary investments for two reasons, both related to the general purpose nature of these inputs. First, because such inputs are used pervasively across firms in the economy, each firm may consider itself too small for its investment to have any impact, and decide not to invest. Coordinating institutions overcome this market inefficiency by developing mechanisms that induce downstream firms to make these complementary investments.

Second, each downstream firm has the ability to generate the knowledge, protocols, components or interfaces that can facilitate the integration of the new version of the general purpose input into their own production process, or how it can be embedded within the capital equipment that they are producing. For example, a textile manufacturer would know best about how a new version of the steam engine can be best fit into the textile mill, a railroad firm about how it can fit into a train, and a ship manufacturer about how to integrate it into a ship. However, they would need to pool their knowledge to understand the commonalities among the requirements that they have, and to identify their differences, so that they can jointly report to the supplier of the most efficacious way of meeting their diverse needs.

[4]For example, the electricity market for lighting in the early 1900s adopted high frequencies, such as 133 cycles per second, to minimize flickering in light bulbs. However, industrial applications using electric motors benefited from lower frequencies like 25 or 30 cycles per second. Other applications required frequencies between these extremes. By 1924, there had been over 17 different frequencies in operation in the UK and more than 10 in the US (see Hannah (1979) for UK and Lamme (1918) for US).

The essential role of the institutions considered in this paper is then to bring together downstream firms and the supplier, or to pool together knowledge that is available to them, to understand what part of the technology is common across different users. The need for coordination, ironically, arises from the need to partition responsibilities in the most efficient way. Institutions thus play a synergistic or integrative role in these situations, in contrast to the stabilizing or protective role often assumed in the literature.

Institutions that perform such a function include organizations like industry trade associations, and umbrella groups of trade associations representing different using industries, that articulate the needs of their member firms, and mediate across them. They include professional associations like associations of civil, mechanical, electrical and computer engineers, who organize technical conferences where the requirements are presented, and commonalities and differences understood.[5] They include standards committees, where knowledge from various downstream firms are pooled together to arrive at standards, or standardized interfaces, that can reduce the supplier's investment needed to adapt an innovation across the users. They include research consortia where common problems that face downstream firms are isolated and studied. They include regulatory bodies, like the state level electric utility commissions in the US, who invites comments and promotes discussion from various stakeholders before making rules regarding the introduction of new innovations into the electricity grid.

While institutions mentioned above are effective in solving this coordination issue for immediate problems, university academic departments have performed the same function over the longer term. Rosenberg (1998) explains the role that technically focused academic departments, especially those with close connections to the industry, have played in developing a formal body of thought that can articulate clearly what is common across different uses of a general purpose technology. Such formalization facilitates the development of standardized interfaces or processes, hence reducing the adaptation costs to the supplier.[6]

[5]Hughes (1983) describes the role of the two leading electrical trade associations in the UK, the Incorporated Municipal Electrical Association and Incorporated Associated of Electric Power Companies, in standardizing frequencies and voltages to interconnect the various local electrical grids across the U.K (page 290). Similarly the Institution of Electrical Engineers (IEE) in the UK organized discussions, lectures, conferences and expert testimonies to explore and articulate the advantages of an interconnected electrical grid with a common frequency and voltage. (page 233 and 291).

[6]Rosenberg (1998) highlights the role of the Chemical Engineering Department at MIT, especially

In order not to dampen the supplier's incentive to innovate, coordinated complementary investments by downstream firms must occur prior to the supplier's innovation, signaling to the supplier that the cost of adaptation would not fall on the supplier. In other words, such activities must be pre-competitive, before the downstream firms adopt the innovation and compete among themselves. These pre-competitive complementary investments are not restricted to those focused on standardization. There are often common problems of adaption that are a-priori known before the invention, where the users can coordinate to make these investments. This was the case with the semiconductor industry, examined in more detail in the next section.

One might argue that the discussion here is equivalent to evaluating the benefits of experimenting with new innovations versus coordinating in a common fashion. But the point of this paper is that irrespective of the advantages of experimentation versus coordination, if the experimentation by users of the input results in the supplier of the input bearing the burden of adapting the input across its various uses, then this reduces the supplier's incentive to innovate.

This paper connects together and contributes to the literature on endogenous growth, general purpose technologies, complementary investments and institutions. The model developed in this paper combines elements from the economic growth focused quality ladder model in Aghion and Howitt (1992) with the industrial organization focused model of general purpose inputs in Bresnahan and Trajtenberg (1995), and the adaptation focused model with complementary investments in Helpman and Trajtenberg (1998). A single model is necessary to understand the full role of general purpose technologies in the process of economic growth. This paper also responds to the critique in David and Wright (1999) that although history recognizes institutional changes as crucial drivers of growth shaped by general purpose technologies, economic models have

of Arthur D. Little, in organizing the principles of chemical manufacturing into a set of unit processes that constitute the knowledge that is common across the manufacture of different chemical products: "A critical feature of this concept of unit operations is that it went far beyond the mere descriptive approach of industrial chemistry by calling attention to a critical few distinctive processes that were common to a number of industries. This act of intellectual abstraction, which Little initiated, laid the foundations for a more rigorous and, eventually, more quantitative discipline. Little looked at the large number of unique vertical sequences which had until then described the manufacturing steps of individual chemical products and then looked across these sequences, horizontally, to draw together the small number of common elements in each of them."

largely failed to account for their significance.

The next section describes a specific instance of a surge in output and productivity growth in the US economy. This event is considered in some detail because it provides a clear historical example of the market inefficiencies considered in this paper, and of the kind of institutions and policies that can reduce these inefficiencies and increase aggregate output growth rate.

2 Semiconductor Technology Acceleration and US Output Growth in the 1990s

Output and productivity growth in the US economy accelerated in the mid 1990s, after two decades of lackluster growth. An extensive literature has documented this acceleration including Jorgenson (2001), Oliner and Sichel (2000) and Fernald (2014). This literature has also found that the a surge in productivity in the Computer and Electronics industry (see Figure 1) played an important role in the economy-wide pickup in output and productivity.

The surge in productivity in computer and electronics industry was, in turn, driven by a technological acceleration in semiconductor microchips used in computer and electronic products. Figure 2a shows the acceleration in performance of microprocessor semiconductor chips, which forms the central processing units (CPUs) of computers.[7] Further, Byrne et al. (2017) estimate that the surge in productivity in the computer industry can be accounted for by the productivity acceleration in semiconductors, and Aizcorbe et al. (2002) show that the acceleration in quality adjusted price declines in computers can be fully accounted for by quality adjusted price declines in the semiconductor chips used in computers. The data and analysis in this literature thus make a strong case that technological acceleration in semiconductor chips was a key driver of the output and productivity surge in the US.

During this period, sources within the semiconductor industry documented an acceleration in the rate of miniaturization of electronic components used within semicon-

[7]The technological acceleration in semiconductor chips has been documented using other methods. For example, Aizcorbe (2006) and Jorgenson (2001) document a sharp acceleration in the quality adjusted prices of microprocessor chips and memory chips used in computers, with magnitudes similar to that seen in performance acceleration in Figure 2a.

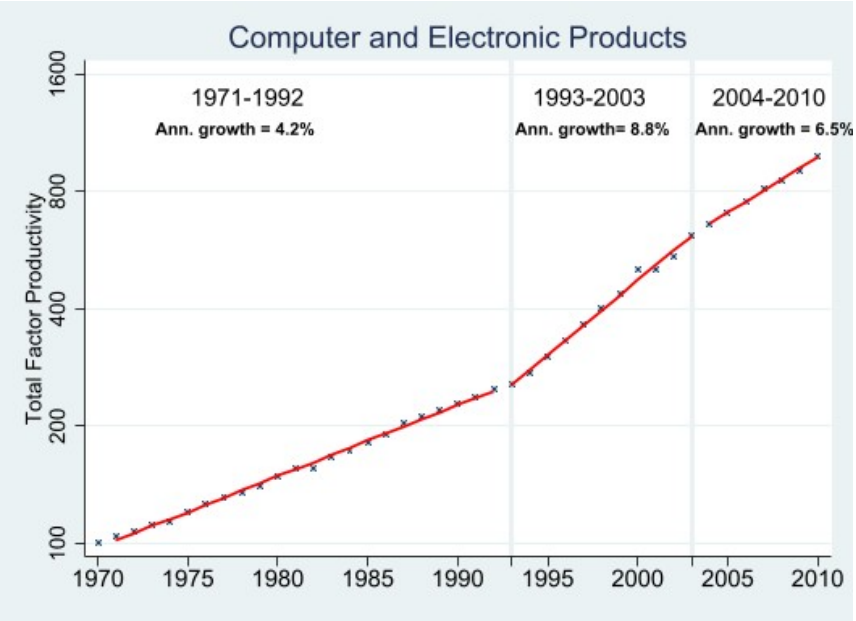
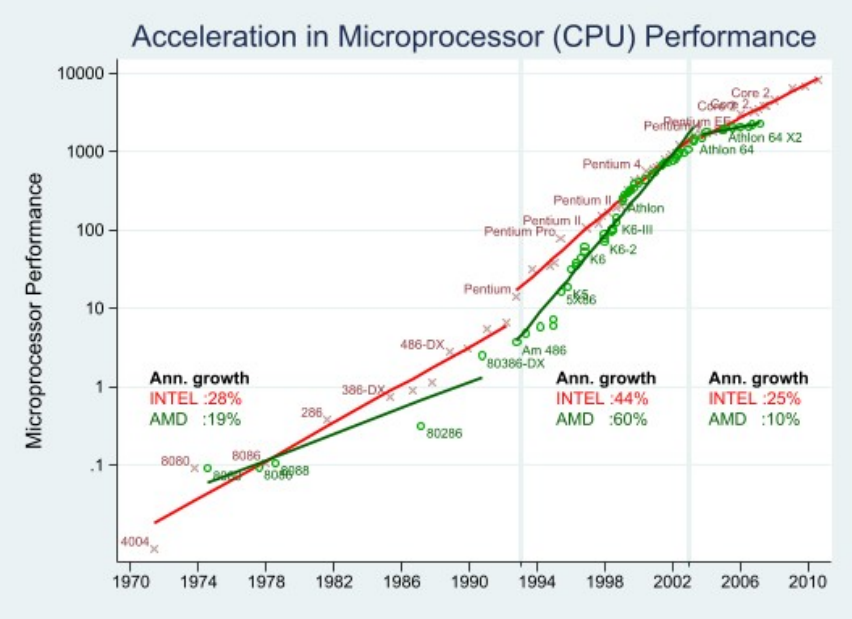
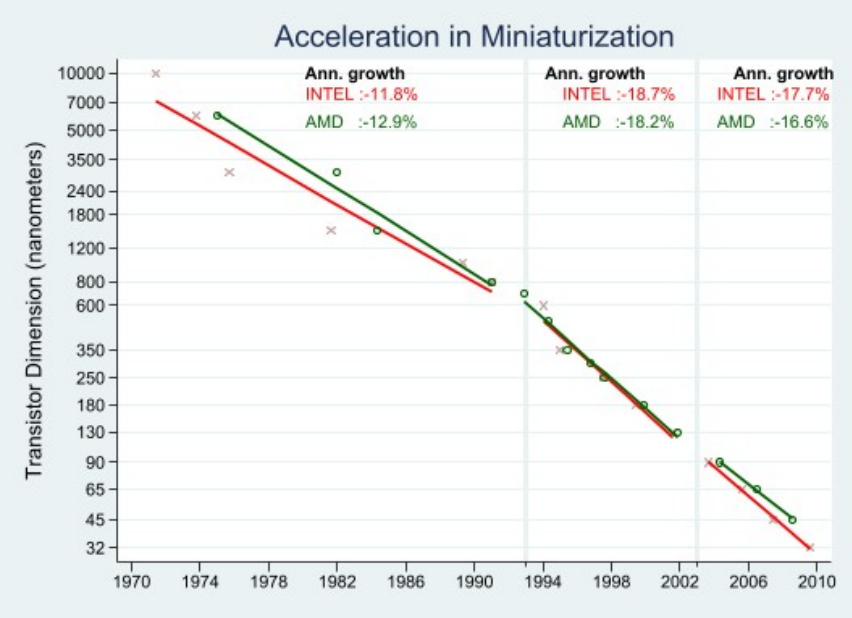


Figure 1: TFP Growth in the Computer and Electronics Industry.

Notes: The graph is constructed by author using data from the BEA-BLS Integrated Industry Level Production Account (1947-2016). The methodology for calculating TFP follows the one suggested in Eldridge et al. (2020), the technical paper accompanying the database, with the exception that R&D investment was not capitalized in author's calculations.



(a) Technological Acceleration in Microprocessor (CPU) semiconductor chips.



(b) Acceleration in the rate of miniaturization in CPU chips. (Source: Author)

ductor chips.[8] The acceleration can be seen in Figure 2b, which shows the evolution of physical dimension (in nanometers) of transistors, the fundamental electronic components used in semiconductor chips.[9]

Using an economic model that incorporated key engineering details of the semiconductor chip production, Pillai (2013) showed that the technological acceleration in CPU performance seen in Figure 2a can be fully explained by the acceleration in miniaturization shown in Figure 2b. But that paper left unanswered the question of what caused the acceleration in miniaturization seen in Figure 2b. The model in the current paper offers an explanation, based on organizational changes within the US semiconductor industry.

Two pieces of indirect evidence support the search for an explanation that centers on changes within the US semiconductor industry.[10] First, in contrast to the US, there was no comparable acceleration in miniaturization among semiconductor firms in Japan, the other leading manufacturer of semiconductor chips during this period. Second, the global market share of US firms in both the semiconductor chip industry and the semiconductor process tools industry increased substantially during this period. The changing fortunes of the US semiconductor firms in the global markets suggests a US centric explanation for the technology acceleration in semiconductors, computers and electronic products.

The application of the model developed in the paper towards explaining the acceleration starts with the premise that miniaturization of electronic components has been the main driver of progress in computers and electronic products.[11] The miniaturization of electronic components is enabled by advancements in capital equipment, called

[8] See Aizcorbe et al. (2008) and the references provided there.

[9] Before 1993, the transistor dimensions reduced at a rate of roughly 12% per year, which accelerated to around 19% per year during 1993-2003. A reduction in the transistor dimension by a factor of 0.7 is considered as a new technology cycle in the semiconductor industry, because this reduces the area of each transistor by $0.7^2 \approx 0.5$, hence allowing for a doubling of the number of transistors on the same area of chip. The change in miniaturization rate depicted in Figure 2b, from 12% to 18% corresponds roughly to a decrease in the technology cycle from 3 years to 2 years (since $\ln(0.7)/3 \approx 12\%$ and $\ln(0.7)/2 \approx 18\%$), which was how it was noted within the semiconductor industry.

[10] See Data Appendix for data supporting these two pieces of indirect evidence.

[11] Flamm (2021) provides an economist's view of the central role of miniaturization of electronic circuits in driving the IT and computer revolution. The role of miniaturization is also made clear in a mathematical model of profit maximization of a microprocessor manufacturer in Pillai (2013).

semiconductor *process tools*.[12] While the process tools perform the same generic function no matter which type of chip they are manufacturing, there are variations in the manufacturing processes for different chips, and the process tools have to be adapted according to the needs of the different chips.

Tracing the origins of the semiconductor process tools suppliers during the 1970s, Pillai (2023) provides an overview of how the process tools suppliers leveraged their expertise in fields as diverse as camera lens manufacturing to nuclear engineering, to enter the semiconductor industry as process tools suppliers. The expansion in the use of semiconductor chips during the 1970s and 1980s to diverse products like automobiles, aircrafts, medical equipment and industrial machinery placed new demands on the nascent process tools industry. The process tools suppliers, who often had little or no expertise in semiconductor manufacturing per se, was confronted with the challenge of meeting the diverse requirements of the expanding set of downstream chip and electronics manufacturers, in the same way that steam engine manufacturers in the early 19th century and the electricity suppliers of the early 20th century were challenged to meet the demands of their own downstream users, as noted in Section 1.

It was in this context that the US government, in response to the drop in global market share of US companies in the semiconductor industry, supported the formation of a research consortium by leading semiconductor and electronic companies in 1987, called SEMATECH. As explained in Stowsky (1987), USGAO (1990) and Flamm (1996), it was widely believed within SEMATECH and the broader US semiconductor industry, that the lack of coordination between semiconductor chip manufacturers and their upstream process tools suppliers, and the lack of standardization in the requirements placed by chip manufacturers on tools suppliers, had begun to weigh down the continued miniaturization of electronic components. As a result, SEMATECH, which was set up to facilitate horizontal R&D collaboration among chip manufacturers, changed its direction and focused on facilitating vertical collaboration between chip manufacturers and tools suppliers, with the aim of accelerating the development of new

[12]Making any chip requires many different kinds of chemical, mechanical, optical and electrical processes, and each of these processes is enabled by a different tool. These tools are manufactured by a set of suppliers to the microchip manufacturing firms. Some of the prominent tool manufacturers are Applied Materials, ASML, Nikon, Tokyo Electron, Lam Corporation and KLA Tencor. The process tool manufacturers, often called semiconductor equipment suppliers, have a separate 6-digit NAICS industrial code of their own, 333242 (Semiconductor Machinery Manufacturing).

technology generations with smaller electronic components (see USGAO (1990), Katz and Ordover (1990), Grindley et al. (1994) and Gibson (1994).)[13]

The model in the next section illustrates how coordination efforts led by SEMATECH can account for the acceleration observed in Figures 1, 2a and 2b. The basic logic of the model is that because general purpose inputs are used by a large number of firms, using firms tend to underinvest in complementary inputs that could lower the supplier's cost of adapting its innovations in the general purpose input across the downstream firms. This lack of complementary investment diminishes the supplier's incentive to innovate, trapping the economy in an equilibrium where advancements in the general purpose input fall below the socially optimal level. Institutions capable of coordinating complementary investments across firms, such as consortia like SEMATECH, can disrupt this equilibrium, accelerating innovation and output growth.

3 Model

The model is in continuous time. There is a single final consumption good in the economy. The consumption good, which is taken as the numeraire, is produced under perfect competition using labor and a unit continuum of capital goods, with the production technology,

$$Y = L_Y^{1-\alpha} \int_0^1 K(i)^\alpha, \quad 0 < \alpha < 1. \tag{1}$$

where L_Y is the quantity of labor and $K(i)$ is the quantity of i^{th} variety capital good.

The capital goods are assumed to depreciate fully after use. The market for capital goods is monopolistically competitive, with each variety produced by a different firm. The production of every variety requires a general purpose input.[14] The quality (or productivity) of the general purpose input can be improved through an R&D process, described in the next section. If $x(i)$ units of the general purpose input of quality q is

[13] See Appendix for a detailed description of the programs initiated by SEMATECH to enhance vertical collaboration between chip/electronics manufacturers and process tools suppliers.

[14] The general purpose input could be an intermediate component, e.g. steam engine embedded in capital goods like trains and ships, or electric motors embedded in machine tools, lawn mowers or robots. The general purpose input could also be a capital good (or service) used in the manufacture of capital goods used for final good production, in which case it is assumed that the input also depreciates fully after use.

used to manufacture capital good i, then,

$$K(i) = qx(i). \tag{2}$$

$K(i)$ is thus to be interpreted as a quality adjusted input of capital good i.

The general purpose input is produced by a monopolist. One unit of labor can produce one unit of the general purpose input of any quality, in unit time. To highlight the role of market structure in the results, the terms *supplier* and *downstream firms* are used in the paper where appropriate, to refer to the monopolist and capital goods firms respectively. The supplier undertakes the R&D investment necessary to invent the higher quality versions of the input, and the adaptation investment to enable their integration into the production processes of downstream firms, as described in the next section.

3.1 Innovation and Adaptation

The possible qualities of the general purpose input are arranged along a geometric ladder with qualities $1, \delta, \delta^2, \delta^3$..., with $\delta > 1$. The arrival of innovations that improve quality is governed by a Poisson process. To generate an average arrival rate of innovation λ, the general purpose input supplier has to employ $R(\lambda)$ units of labor in R&D. If the R&D is successful, then the supplier gets the blueprints to make the next higher quality input. Before the supplier can sell the higher quality input to downstream firms, it needs to adapt the input to meet the needs of the firms.

The adaptation requires further supplier investment at the rate of ϕ units of labor for an interval of duration τ after the invention. It is assumed that the supplier cannot invest in R&D during the adaptation phase. The assumption captures the limitations on the management span of the supplier, i.e. the difficulties in carrying on innovation and adaptation efforts simultaneously. Thus, there are alternating periods of innovation and adaptation, with every innovation period of stochastic duration being followed by an adaptation period of duration τ.

3.2 Complementary Investments

Downstream capital good firms can make complementary investments that generate knowledge which helps the supplier in adapting a new version of the general purpose

input. Assume that such knowledge useful for the supplier can be captured using a variable $h_i \in (0, \frac{\bar{\phi}}{\eta})$, where $\frac{\bar{\phi}}{\eta}$ is the upper limit whose economic meaning is clarified in Section 3.5. To generate the knowledge h_i, firm i incurs a cost $A(h_i)$.[15]

3.3 Institutional Capacity

While individual downstream firms make complementary investments $A(h_i)$, the capability of institutions that can combine and process the individual h_i to understand the commonalities (and differences) among these requirements, will determine the impact of the complementary investments on supplier's adaptation cost rate. Assume that the capacity of institutions to perform this synergistic role can be captured with a function F,

$$\phi = F(\{h_i\}, \bar{\phi}), \tag{3}$$

where $\bar{\phi}$ represents an upper limit of the costs that supplier will incur. For brevity and ease of exposition, we can refer to F as the *institutional capacity function* in the rest of the paper.

To close the model, assume that the economy consists of consumers with identical preferences over the consumption good, captured with the instantaneous utility function $u(C) = C$. The rate of time preference is ρ, which is also equal to the interest rate in the economy. The total supply of labor in the economy is fixed at L, and is distributed across five uses: in the production of final goods, in the production of the general purpose input, in R&D and adaptation investments by the supplier, and in complementary

[15]These complementary investments by downstream firms can be in many forms. It could investment in experiments, simulations or other investigations that generate technical results or specifications that would be useful for the supplier in making suitable adaptations to the new version of the general purpose input. It could be in making blue prints for creating components or interfaces that would allow supplier to integrate the new innovation into downstream production. In the example in Section 1 regarding the adaptation of high-pressure steam engines for textile production, complementary investments might include investigations by textile manufacturers to determine the acceptable range of engine speed variations that can be tolerated without causing thread breakage. The investment $A(h_i)$ would be cost to the textile firm of doing the investigations, and h_i would be the technical results or tolerance specifications from the investigations. In the example of electricity supply mentioned in Section 1, complementary investments could be investigations undertaken by electrical appliance manufacturers to understand what range of frequencies are usable in their own appliances. It could also be investments needed to participate and present results in committees and conferences focused on standardization of frequency.

investments by downstream firms.

3.4 Equilibrium

There is no change in the economy between any two instants of time within an innovation period or an adaptation period. The arrival of a higher quality version of the general purpose input changes the labor allocation, as labor is shifted from R&D to adaptation, with accompanying changes in labor that is used in production of final goods and the general purpose input. There is a unique stationary equilibrium in the economy where the labor allocated to innovation is constant across all innovation periods, and labor allocated to adaptation is constant across all adaptation periods.

To characterize the quality and output growth in this equilibrium, the profits within a period will be derived as function of quality prevalent in that period, and then these functions are used to solve the dynamic decision of the supplier of how much to invest in R&D, which in turn determines the evolution of q and all other variables in the economy.

To avoid notational clutter, only the variables that are relevant for the dynamic decision of the supplier are written as functions of q.

3.4.1 Static Decision Problems

Consider a period (innovation or adaptation), when the quality of the general purpose input used in capital goods production is q. Perfect competition in final goods sector implies that factors are paid their marginal products, and hence the choice of labor and capital in final goods production should satisfy,

$$(1-\alpha)L_Y^{-\alpha}\int_0^1 K(i)^\alpha = w(q), \tag{4}$$

$$\alpha L_Y^{1-\alpha}K(i)^{\alpha-1} = p(i), \tag{5}$$

where $w(q)$ is the wage rate and $p(i)$ is the price of capital good i.[16] This gives the demand for capital good i,

$$K(i) = L_Y\left(\frac{p(i)}{\alpha}\right)^{-\frac{1}{1-\alpha}}. \tag{6}$$

[16]Note that $p(i), K(i)$ and $x(i)$, are all functions of q. The dependence is not explicitly stated only to avoid notational clutter.

The firm producing capital good i gets a revenue of $p(i)K(i)$ and incurs a variable cost $p_g x(i)$, where p_g is the price of the general purpose input and $x(i)$ is the quantity of the input used by firm i. Firm i chooses price $p(i)$ that maximizes its profit, given the constraints on demand and production imposed by equations (6) and (2), i.e it solves the problem:

$$\begin{aligned} &\max_{p(i)} \; p(i)K(i) - p_g x(i) \\ s.t \;\; &K(i) = L_Y \left(\frac{p_i}{\alpha}\right)^{-\frac{1}{1-\alpha}} \\ &K(i) = qx(i), \end{aligned}$$

Given the constant elasticity demand function, the optimal price is a markup over the marginal cost, $p(i) = \frac{1}{\alpha}\frac{p_g}{q}$. Note that since $K(i)$ is the quality adjusted units of capital good, $p(i)$ is the quality adjusted price, and hence the markup is over the quality adjusted unit cost, equal to $\frac{p_g}{q}$. The optimal $p(i)$ does not depend on i, i.e all capital good firms charge the same price. Let p denote this price, i.e.,

$$p \equiv p(i) = \frac{1}{\alpha}\frac{p_g}{q}. \tag{7}$$

Since $p(i)$ is the same across all i, by equation (6) $K(i)$ will also same for all i. Denoting this by K, equations (6) and (7) gives,

$$K = L_Y \left(\frac{1}{\alpha^2}\frac{p_g}{q}\right)^{-\frac{1}{1-\alpha}}. \tag{8}$$

The quantity of the general purpose input demanded by every capital good firm, $x(i)$, will also be the same. Since there is a unit measure of firms, the firm level demand will also equal the total demanded across all firms. Using equations (2) and (8), the aggregate demand for the general purpose input is,

$$x \equiv \int_{i=0}^{1} x(i)di = x(i) = \frac{K}{q} = L_Y \left(\frac{\alpha^2 q^\alpha}{p_g}\right)^{\frac{1}{1-\alpha}}. \tag{9}$$

The general purpose input supplier takes the demand function above and sets the price p_g to maximize its profit. The optimal choice of p_g is the solution to the problem,

$$\begin{aligned} &\max_{p_g} p_g x - w(q)x \\ &s.t\, x = L_Y \left(\frac{\alpha^2 q^\alpha}{p_g}\right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

It can be seen from the first order condition with respect to p_g that the profit maximizing price is again a markup over the unit cost,

$$p_g = \frac{w(q)}{\alpha}. \tag{10}$$

Since demand function for each capital good firm has constant price elasticity equal to $\frac{1}{1-\alpha}$, the profit of each firm is a fraction $1-\alpha$ of total revenue, and hence equal to $(1-\alpha)pK$.[17] Further, from equations (4) and (5), the total revenue of the capital goods firms is,

$$pK = \frac{\alpha}{1-\alpha} w(q) L_Y. \tag{11}$$

Hence the profit of each capital good firm (which is also the total profit of the capital goods sector) is,

$$\pi(q) = (1-\alpha)pK = \alpha w(q) L_Y, \tag{12}$$

where the second equality follows from equation (11). Since the input supplier also has a demand function with elasticity $\frac{1}{1-\alpha}$, its profit is also a fraction $1-\alpha$ of its revenue. Denoting this profit by π_g,

$$\pi_g(q) = (1-\alpha)p_g x = (1-\alpha)\alpha pqx = (1-\alpha)\alpha pK = \alpha^2 w(q) L_Y. \tag{13}$$

The relationships represented in equations (4)-(13) holds both for innovation and adaptation periods, the only difference being that the labor allocated to final good production, L_Y, will vary between innovation and adaptation periods. Let L_{YI} denote the the value of L_Y for innovation period, and L_{YA} for adaptation period. Hence the profits of the capital goods firms and input supplier are $\alpha w(q) L_{YI}$ and $\alpha^2 w(q) L_{YI}$ during innovation periods, and $\alpha w(q) L_{YA}$ and $\alpha^2 w(q) L_{YA}$ during adaptation periods. The labor used in general purpose input production, which is equal to the quantity of the general purpose input produced, also varies across innovation and adaptation periods, and these will be denoted by x_I and x_A respectively.

3.4.2 Dynamic Decision Problems

Each downstream firm can make an investment to reduce the adaptation cost that the supplier will face when the next jump in quality is made. The downstream firm anticipates that if the adaptation cost is changed, the supplier will change its own R&D

[17] To see this, note that profit is, $pK - p_g x = pK - \alpha qpx = pK - \alpha pK = (1-\alpha)pK$.

investment and achieve a new average innovation rate λ. Denote the belief that capital good firms have about the value of λ that the supplier will choose when facing an adaptation cost ϕ by the function $\beta(\phi)$. Since the downstream firms are identical, we can restrict attention to the case where they hold the same belief, $\beta(\phi)$.

The supplier observes the investments h_i made by the downstream firms and chooses λ to maximize its profits, taking account of how F translates the complementary investments h_i into the adaptation cost rate ϕ that it will have to pay if it makes a quality jump. The dynamic problem of the supplier can be captured with the following Bellman equation:

$$\begin{aligned} V_g(q) = \max_{\lambda}(\alpha^2 L_{YI} w(q) - R(\lambda)w(q))\Delta t \qquad (14) \\ + e^{-\rho\Delta t}\Big((1-\lambda\Delta t)V_g(q) + \lambda\Delta t\left[\frac{1-e^{-\rho\tau}}{\rho}(\alpha^2 L_{YA}w(q) - \phi w(q)) + e^{-\rho\tau}V_g(\delta q)\right]\Big) \\ s.t \quad \phi = F(\{h_i\}, \bar{\phi}). \end{aligned}$$

The first term, $(\alpha^2 L_{YI} w(q) - R(\lambda)w(q))$, represents the instantaneous payoff to the supplier, which is the gross profit, $\alpha^2 L_{YI} w(q)$, less the R&D investment, $R(\lambda)w(q)$. The remaining terms capture the capital gains that will result if the supplier makes an innovation, an event with probability $\lambda\Delta t$. If innovation happens, then the economy moves to adaptation mode for interval τ, which results in a discounted payoff of $\frac{1-e^{-\rho\tau}}{\rho}(\alpha^2 L_{YA}w(q) - \phi w(q))$ to the supplier. At the end of the adaptation period, the economy moves into the innovation mode with downstream firms shifting production to the input with quality δq, hence giving a discounted payoff to the supplier of $e^{-\rho\tau}V_g(\delta q)$.

For each downstream firm, the instantaneous payoff is $(\alpha L_{YI} w(q) - A(h_i)w(q))$, and the firm expects an innovation to arrive with probability $\beta(\phi)\Delta t$, instead of $\lambda\Delta t$ for the supplier. Hence the Bellman equation for each downstream firm is;

$$\begin{aligned} V_i(q) = \max_{h_i}\,(\alpha L_{YI} w(q) - A(h_i)w(q))\Delta t + e^{-\rho\Delta t}\Big((1-\beta(\phi)\Delta t)V_i(q) \qquad (15) \\ + \beta(\phi)\Delta t\left[\frac{1-e^{-\rho\tau}}{\rho}\alpha L_{YA}w(q) + e^{-\rho\tau}V_i(\delta q)\right]\Big) \\ s.t. \quad \phi = F(\{h_i\}, \bar{\phi}). \end{aligned}$$

Finally, the labor markets have to clear in equilibrium, as given below.

3.4.3 Market Clearing

The labor market should clear for innovation periods and adaptation periods.

$$L_{YI} + x_I + R(\lambda) + \int_0^1 A(h_i)di = L \tag{16}$$

$$L_{YA} + x_A + \phi = L. \tag{17}$$

The stationary equilibrium is a set of following objects: (i) a path of prices and quantities; (ii) a set of complementary investments for capital good firms $\{h_i\}$; (iii) an innovation rate for the supplier, λ^*; (iv) an adaptation cost rate for the supplier, ϕ^*; (v) a belief function $\beta(\phi)$; such that: (i) marginal productivity conditions are satisfied in the final good sector, as given in equations (4) and (5); (ii) each capital good firm chooses price $p(q)$ to maximize its gross profit, taking as given the price and quality of the general purpose input; (iii) the input supplier choose its price $p_g(q)$ to maximize its gross profit; (iv) each downstream firm's choice of complementary investment, h_i^*, solves the Bellman equation in (15) given the investments by the other firms, and the belief function of supplier innovation, $\beta(\phi)$; (v) supplier's innovation rate, λ^*, solves the Bellman equation in (14), given complementary investments $\{h_i^*\}$ of downstream firms; (vi) the downstream firm's beliefs are consistent with supplier's choice of innovation rate, $\beta(\phi^*) = \lambda^*$ (vii) the labor market clearing conditions in equations (16) and (17) are satisfied.

The next four propositions show the existence of a unique stationary equilibrium, and characterizes it.

3.4.4 Existence and Properties of Equilibrium

Proposition 1. *The equilibrium wage in the economy depends only on the quality of the general purpose input used, and is given by*

$$w(q) = w_0 q^\alpha, \tag{18}$$

where w_0 is the wage at $t = 0$, given by $w_0 = \alpha^{3\alpha}(1-\alpha)^{1-\alpha}$.

Proof. Equations (8) and (10) together give $K = L_Y \left(\frac{1}{\alpha^3}\frac{w(q)}{q}\right)^{-\frac{1}{1-\alpha}}$. Substituting this expression for K into equation (4), and noting that $K(i) = K$, gives the result. □

Next, to bring out the market inefficiencies clearly, a particular form of the function F is used, as given below.

Assumption 1. *The institutional capacity function is assumed to be,*

$$F(\{h_i\}, \bar{\phi}) = \bar{\phi} - \eta \int_{i=0}^{1} h_i di, \quad \eta \geq 0. \tag{19}$$

In the above assumption, the supplier's ability to influence ϕ is assumed to be fixed at ϕ. The term η represents the efficiency of the institution in translating downstream firm's inputs, h_i, into reductions in adaptation cost for the supplier. The maximum adaptation cost rate that the supplier can incur is $\bar{\phi}$, the best that the the supplier can achieve on its own, and occurs either when the institutions are non-existent or totally inefficient (i.e. $\eta = 0$), or when downstream firms do not make any complementary investment (i.e $h_i = 0 \, \forall i$). The assumption immediately leads to the following proposition.

Proposition 2. *The equilibrium complementary investment by each capital good firm is equal to zero, and hence the equilibrium value of supplier's adaptation cost is* $\phi = \bar{\phi}$.

One can see that the proposition is true by noting that firm i's investment h_i affects supplier's adaptation cost (ϕ) through the integral term $\int_0^1 h_i di$. The effect of any change in h_i for any particular i when all other h_i's are unchanged, is zero. Hence every firm will set $h_i = 0$.

Given this result, we have $h_i = 0$, $A(h_i) = 0$, and $\phi = \bar{\phi}$. With $h_i = 0$ as the optimal policy solving downstream firms' Bellman equation (equation (15)), one can restrict attention to the supplier's Bellman equation as the only pertinent dynamic decision making problem, and the next proposition shows that a unique solution to this problem (and hence a unique stationary equilibrium) exists under very minimal assumptions. To provide intuition behind the assumptions, define three new constants as below;

$$\bar{\lambda} \equiv \frac{\rho}{e^{-\rho\tau}\delta^{\alpha} - 1}; \quad \mu \equiv \bar{\lambda}\frac{1 - e^{-\rho\tau}}{\rho}; \quad \tilde{\alpha} \equiv 1 + \frac{\alpha^2}{1 + \frac{\alpha^3}{1-\alpha}}. \tag{20}$$

Here $\bar{\lambda}$ is the upper limit on the average innovation rate permissible in the model, and μ is the expected discounted value of a unit of profit (or consumption/production in the

appropriate context) over adaptation period τ at the innovation rate $\bar{\lambda}$. The constant $\tilde{\alpha}$ captures the effect of two market power induced inefficiencies in the model, which are explained later in the paper.[18] The assumption required for the existence of stationary equilibrium is the following.

Assumption 2. *The R&D cost function is twice continuously differentiable and satisfies the following conditions: (i)* $R(\lambda) \geq 0, R'(\lambda) > 0$ *and* $R''(\lambda) > 0$*; (ii)* $R(0) = 0$ *and* $\lim_{\lambda \to 0} R'(\lambda) = 0$*; (iii)* $R(\bar{\lambda}) > L + \mu(L - \bar{\phi})$.

Conditions (i) and (ii) are commonly used, and condition (iii) puts an upper limit on the innovation rate that can be achieved, and ensures that even if all the labor in the economy were devoted to R&D, the average innovation rate λ cannot equal or exceed $\bar{\lambda}$.

Assumption 3. *The duration of adaptation period,* τ*, satisfies* $\tau < \frac{\alpha \ln(\delta)}{\rho}$.

The above assumption ensures that the supplier has an incentive to invest in R&D. If the adaptation duration is above the given limit, the present discounted value of innovations would not cover the R&D costs for the supplier.

Proposition 3. *There is a unique stationary equilibrium characterized by constant innovation rate (*λ^**), constant adaptation rate (*$\phi^* = \bar{\phi}$*), no complementary investments (*$h_i = 0 \; \forall i$*), and* $\beta(\bar{\phi}) = \lambda^*$.

See appendix for proof. It is shown in the proof of the proposition, that the optimal choice of λ implies the first order necessary condition,

$$R'(\lambda^*) = \frac{(e^{-\rho\tau}\delta^\alpha - 1)(\alpha^2 L_{YI} - R(\lambda^*)) + \frac{1-e^{-\rho\tau}}{\rho}(\alpha^2 L_{YA} - \bar{\phi})}{\rho - \lambda^*(e^{-\rho\tau}\delta^\alpha - 1)}.$$

The necessary condition above equates the marginal cost of innovation to the supplier, $R'(\lambda^*)$, on the left hand side of the equation, to the marginal benefit of innovation on the right. The marginal benefit is the increase in expected present discounted value of net profits to the supplier from a marginal increase in λ. A marginal increase in λ affects the expected timing of all future increments to quality, and hence the expected increments in profits to supplier from such updates. The marginal benefit is thus the

[18] See the discussion after Proposition (5). Note that since $0 < \alpha < 1$, it follows that $1 < \tilde{\alpha} < 2$.

discounted sum of all such changes, which includes the net profits from the innovation periods and adaptation periods, captured with the terms $\alpha^2 L_{YI} - R(\lambda^*)$ and $\alpha^2 L_{YA} - \bar{\phi}$, on the right hand side of the equation.

The cumulation of these changes is done through the effective discount factor in the denominator, $\rho - \lambda^*(e^{-\rho\tau}\delta^\alpha - 1)$. The effective discount factor consists of two terms, with the first term being the usual time preference factor ρ (which is also the interest rate in the model). The second factor, $\lambda^*(e^{-\rho\tau}\delta^\alpha - 1)$, captures the stochastic nature of the increases to net profits, which depends on the average arrival rate of new innovations, λ^*. From equations (13) and (18) it can be seen that supplier's gross profit is $\pi_g = \alpha^2 w_0 L_Y q^\alpha$. Hence each time the supplier makes an innovation (i.e q jumps up a factor δ), the supplier's gross profit jumps up by a factor δ^α after the adaptation period τ, which leads to the term $(e^{-\rho\tau}\delta^\alpha - 1)$ in the effective discount factor.

Using the definitions of λ, μ and $\tilde{\alpha}$, the first order condition above can be written concisely as;

$$R'(\lambda) = \frac{\alpha^2 L_{YI} - R(\lambda^*) + \mu[\alpha^2 L_{YA} - \bar{\phi}]}{\bar{\lambda} - \lambda}. \tag{21}$$

Using the labor market clearing conditions in equations (16) and (17) to substitute for L_{YI} and L_{YA}, we can rewrite the necessary condition as;

$$R'(\lambda) = \frac{(\tilde{\alpha} - 1)L - \tilde{\alpha}R(\lambda) + \mu\left[(\tilde{\alpha} - 1)L - \tilde{\alpha}\bar{\phi}\right]}{\bar{\lambda} - \lambda}. \tag{22}$$

In Section 3.5, the above equation will be used to compare the market equilibrium with the socially optimal allocation of resources. Before looking at social optimality, the growth rate of different variables in the stationary equilibrium is derived below.

Proposition 4. *Let g_z denote the mean growth rate of variable z in stationary equilibrium. Then,*

$$g_q = \frac{\ln \delta}{\frac{1}{\lambda^*} + \tau}. \tag{23}$$

Further, $g_w = g_{p_g} = \alpha g_q$, $g_p = -(1 - \alpha)g_q$, $g_K = g_q$ and $g_Y = \alpha g_q$.

See appendix for proof.

The expression for growth rate in equation (23) is intuitive. With equilibrium Poisson arrival rate λ^*, the time interval between innovations is distributed exponentially with mean $\frac{1}{\lambda^*}$. Since there is an adaptation interval of duration τ after an innovation,

the mean time interval between successive updates to quality is $\frac{1}{\lambda^*} + \tau$. Since each innovation raises quality by the same factor δ, the mean growth rate in quality is simply $\ln(\delta)$ divided by the mean interval between quality updates, $\frac{1}{\lambda^*} + \tau$.

3.5 Socially Optimal Allocation of Resources

To evaluate the social optimality of the market's allocation of resources, we can look for a socially optimal allocation that is comparable to a stationary equilibrium. In such an allocation, the labor assigned to production, to R&D, to adaptation, and to complementary investments would all be constant for every innovation period, and for every adaptation period. Consequently, the innovation rate, λ, and complementary investment, h, are also constant. Since all capital goods firms are identical, the socially optimal level of $x(i)$, $K(i)$ and $h(i)$ would be the same for all i.

Let $U(\lambda)$ be the expected present discounted value of consumption when average innovation rate is λ. The social planning problem is to choose the labor allocation so that $U(\lambda)$ is maximized. The next lemma derives the function $U(\lambda)$.

Lemma 1. $U(\lambda) = \frac{L_{YI}^{1-\alpha} X_i^{\alpha} + \lambda \frac{1-e^{-\rho\tau}}{\rho} L_{YA}^{1-\alpha} X_a^{\alpha}}{\rho - \lambda(e^{-\rho\tau}\delta^{\alpha} - 1)}$.

See appendix for proof.

The expression is intuitive. It is the expected present discounted value of an infinite series, where the numerator is the sum of expected consumption in the first innovation and adaptation periods of the series, and the denominator is the effective discount factor, explained in Section 3.4.4. The social planning problem is to maximize the expected utility above subject to the resource constraints in the economy, i.e

$$\begin{aligned} \max_{L_{YI}, x_I, L_{YA}, x_A, \lambda, h} \quad & U(\lambda) \\ s.t \qquad & L_{YI} + x_I + R(\lambda) + A(h) = L \\ & L_{YA} + x_A + \phi = L \\ & \phi = \bar{\phi} - \eta h. \end{aligned}$$

It is shown in the appendix that the social planning optimum exists under the minimal assumptions below.

Assumption 4. *The adaptation cost function is twice continuously differentiable and satisfies the following conditions: (i)* $A(h) \geq 0$, $A'(h) > 0$, $A''(h) > 0$*; (ii) A(0)=0,* $\lim_{h\to 0} A'(h) = 0$*, (iii)* $R''(h)A''(h) > \left(\frac{\eta\mu}{\bar{\lambda}}\right)^2$.

Parts (i) and (ii) are commonly used assumptions, and part (iii) is needed to ensure that the second order sufficient conditions for a maximum are satisfied. As is shown in the appendix, the planning solution (λ^s, h^s) is the solution to the pair of first order conditions for λ and h:

$$R'(\lambda^s) = \frac{L - R(\lambda^s) - A(h^s) + \mu(L - \bar{\phi} + \eta h^s)}{\bar{\lambda} - \lambda^s} \tag{24}$$

$$A'(h^s) = \frac{\lambda^s}{\bar{\lambda}}\eta\mu. \tag{25}$$

The first condition above equates the marginal social cost of innovation ($R'(\lambda^s)$) to the marginal social benefit of innovation, and the second condition equates the marginal social cost of complementary investments ($A'(h^s)$) to the marginal social benefit from these investments.

To compare the market equilibrium with the social optimum and to understand the origins of the market inefficiencies in this environment, it is helpful to define two functions. The solution to necessary condition in equation (24) gives the socially optimal λ as a function of h. Let $\hat{\lambda}(h)$ denote this function, which we can call as the *Social Innovation Curve*. The solution to the necessary condition in equation (25) gives the optimal h as a function of λ, which we can denote by the function $\hat{h}(\lambda)$, and call as the *Social Adaptation Curve*. The solution to the planning problem is intersection of the Social Innovation Curve and the Social Adaptation Curve. The following proposition characterizes the two curves and the social optimum.

Proposition 5. *(i) The social innovation curve* $\hat{\lambda}(h)$ *has a global maximum at* $h = h_m$ *such at* $A'(h_m) = \eta\mu$*, and is strictly increasing for all* $h < h_m$*, and strictly decreasing for all* $h > h_m$*. (ii) The adaptation curve,* $\hat{h}(\lambda)$ *is monotonically increasing, and* h_m *is the socially optimal level of complementary investment only in the limit that the innovation rate tends to the maximum innovation rate possible in the economy, i.e* $\lim_{\lambda\to\bar{\lambda}} \hat{h}(\lambda) = h_m$*. (iii) The socially optimal innovation and adaptation rates,* (λ^s, h^s)*, satisfy,* $\lambda^* < \hat{\lambda}(0) < \lambda^s < \hat{\lambda}(h_m) < \bar{\lambda}$*, and* $0 < h^s < h_m$.

See appendix for proof.

Using the results above, the market inefficiencies in the economy can be illustrated in the diagram below, which shows the Social Innovation an Adaptation Curves, the socially optimal pair (λ^s, ϕ^s) at the point S, and the market equilibrium at the point E.

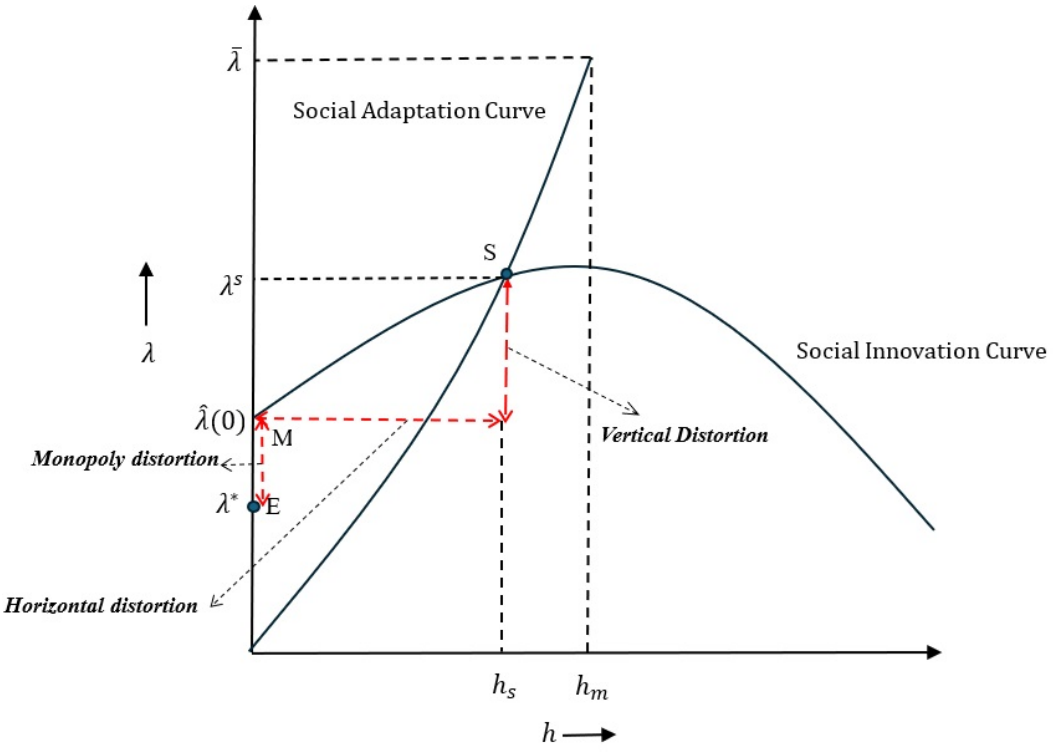


Figure 3: Innovation and Adaptation in Market Equilibrium and at Social Optimum.

Discussion:There are five points about the economy that can be seen in Figure 3. First, the social optimum (point S in the figure) involves a positive level of complementary investment, h^s, compared to zero in market equilibrium (point E). The starkness of the result stems from the specific functional imposed by Assumption (1), which ensures that each downstream firm's contribution, h_i, has a negligible individual impact on reducing the supplier's adaptation cost. But the result brings out with clarity the underlying market inefficiency, the fact that for a general purpose input that is used pervasively across firms, there will be underinvestment in complementary inputs. We can call this as the *horizontal distortion*, since it stems from the market's failure to induce horizontal coordination among capital goods producers.

Second, even at the point labeled M in the figure where h is restricted to be zero, the socially optimal innovation rate is higher than the market innovation rate of λ^* (i.e $\hat{\lambda}(0) > \lambda^*$ as stated in Proposition 5 (iii)). This is because of the presence of $\tilde{\alpha}$ term in the equation (22) which is not there in equation (24). The term encapsulates the effect of two distortions. The first distortion is due to the markups of the supplier and downstream firms. Because of these markups, supplier's profit varies as $\alpha^2 L_Y$, while the socially optimal solution aims to maximize consumption (which is equal to output of final good), which varies as L_Y. The second distortion is that the social planner internalizes the fact that every unit of labor allocated to R&D reduces the labor available for final good production (and hence consumption), whereas the supplier takes L_{YI} as given when choosing λ. The term $\tilde{\alpha}$ captures the combined effects of both these factors, which we can call as the *monopoly distortion*.

Third, even after accounting for the monopoly distortion, the socially optimal innovation rate is higher than the market rate by the distance labeled as *vertical distortion* in the figure. The vertical distortion arises purely from the horizontal distortion. The market's failure to induce complementary investments reduces the supplier's incentive to innovate. This can be seen in the presence of the new term $\mu\eta h^s$ in equation (24), which is not present in equation (22). The term $\mu\eta h^s$ is the increase in marginal social benefit of innovation as a result of reduction in adaptation cost enabled by complementary investment, h^s. Note that the increase in marginal social benefit is higher if institutions are efficient (higher η).

Fourth, the socially optimal innovation rate increases with the level of complementary investment until h^s_m, and then decreases. This is because complementary investment has two opposing effects on the marginal benefit to innovation, captured in the expression $-A(h^s) + \mu\eta h^s$. The positive effect, captured by $\mu\eta h^s$ was explained before. During the innovation period, however, complementary investments reduces the labor available for final good production by the amount $A(h)$, reducing current consumption and the social marginal benefit to innovation. Because the negative effect $A(h)$ is convex in h, while the positive effect $\mu\eta h$ is linear in h, the former effect eventually dominates. The shape of the innovation curve thus results from assumptions in the model which captures the increasing difficulties in continually increasing h. For future reference, we can call this as *diminishing marginal returns to complementary investments*.

Finally, there is a tradeoff between growth and welfare in the economy, for complementary investments in the range (h^s, h^s_m). The level of complementary investments at social optimum, h_s, is less than h_m where the Social Innovation Curve reaches its maximum, although growth rate of the economy is highest when $h = h_m$. This again is a result of the diminishing marginal returns to complementary investments. By increasing h beyond h^s, one can increase the growth rate of the economy, but doing so will require more reduction in consumption during innovation periods than can be compensated for by the combined effect of increase in consumption during adaptation period and increase in the innovation rate.

The monopoly distortion, horizontal distortion and vertical distortion each contribute to making the market's innovation rate and aggregate output growth rate lower than is socially optimal.

Comparison with Aghion and Howitt (1992) (AH): A meaningful intuition about innovation rate in the model can be gained by comparing the model with the results in AH. In AH, innovators discount future payoff from innovation with an effective discount factor of $\rho + \lambda$, which is increasing in λ, as opposed to the effective discount factor of $\rho - \lambda(e^{-\rho\tau}\delta^{\alpha} - 1)$ in the current paper, which is decreasing in λ. The difference highlights the creative-destruction aspect of AH, where increase in innovation rate λ makes it more likely that a new invention will destroy the current innovator's profit flow. In contrast, in the current paper, the profits from all innovations accrue to the same supplier, and the increment to the profit flows from successive innovations is what matters to the supplier when makign the R&D investment decision, resulting in the effective discount factor, $\rho - \lambda(e^{-\rho\tau}\delta^{\alpha} - 1)$, being decreasing in λ. The supplier thus has an incentive to increase λ.

For the same reason, it can be seen by comparing equations (22) and (24), the economy in this paper does not have the market inefficiencies caused by the failure to internalize the intertemporal spillover effects or business stealing effects that arise in AH, because the supplier internalizes these effects in exactly the same way as the social planner does.

The planning equation in equation (24) is similar to the one in AH, which can be obtained by putting $\tau = 0$, $\alpha = 1$ and $h = 0$, in equation (24). Similarly, the expression for mean growth of quality in equation (23) is a generalized version of the growth rate in AH, whose growth rate can be obtained by putting $\tau = 0$ in equation (23).

3.6 Institutions, Complementary Investments and Growth

The results above raise the question of what mechanisms can reduce the market inefficiencies. Effective mechanisms should bring capital goods firms together to make complementary investments. While the model in this paper assumes identical capital goods firms, real-world users of general purpose inputs are scattered across different industries, and vary in size and market power, making it difficult to coordinate on the objective of maximizing joint profits. However, all downstream firms benefit from faster transitions to higher quality versions of the general purpose input. The next section analyzes a consortium in the semiconductor industry that aimed to accelerate these quality transitions, and its impact on the US economy. For using the model for that purpose, the two propositions below describe the innovation rate and output growth rate that would prevail with such a consortium.

3.6.1 Model Extension with Consortium

Suppose that there exists a consortium funded by contributions from downstream capital goods firms, each firm paying a fee equal to a fraction θ of its gross profits. The consortium uses the fees to make complementary investments during each innovation period that will reduce the supplier's adaptation cost for the next quality update. One could think of this arrangement as firms as providing the manpower directly to the consortium (for example deputizing some of its employees to work for the consortium), or as the consortium fees being used to hire knowledgable employees from downstream firms who can facilitate the complementary investments.

Let η^c denote consortium's efficiency, with $\eta^c < \eta$, where η is the efficiency of the social planner. Let h^c denote the value of h chosen by the consortium, and let ϕ^c denote the resulting adaptation cost for the supplier, i.e $\phi^c = \bar{\phi} - \eta^c h^c$. The funding needed to support the consortium to invest h^c when quality q is operational is $w(q)A(h^c)$. Since the total downstream profit is $w(q)\alpha L_{YI}$, a fraction θ being made available to the consortium would imply that $w(q)A(h^c) = \theta w(q)\alpha L_{YI}$, i.e,

$$A(h^c) = \theta \alpha L_{YI}. \tag{26}$$

The earlier definition of the stationary equilibrium would extend to this case with the consortium, with just one modification. Instead of downstream firms basing their complementary investment decisions on their beliefs about how the supplier would respond,

the consortium simply invests the total resources available to it. The supplier observes the complementary investments made by the consortium, and chooses λ anticipating an adaptation cost of $\phi^c = \bar{\phi} - \eta^c h^c$. From equation (26) it can be seen that h^c is constant as long as L_{YI} is constant, and hence a unique stationary equilibrium would exist, similar to the proof in Proposition (3).

To derive the equilibrium conditions with the consortium, first note that the static decisions of all agents in the economy remain the same as was described in the model without the consortium.[19] There are two changes in the case with the consortium; to the supplier's dynamic decision problem and to the labor market clearing conditions. The supplier's first order necessary condition in equation (27), $\bar{\phi}$ is replaced by $\bar{\phi} - \eta^c h^c$, which yields the following condition;

$$R'(\lambda) = \frac{\alpha^2 L_{YI} - R(\lambda) + \mu\left[\alpha^2 L_{YA} - (\bar{\phi} - \eta^c h^c)\right]}{\bar{\lambda} - \lambda}. \tag{27}$$

The labor market clearing conditions with the consortium are;

$$\begin{aligned} L_{YI} + x_I + R(\lambda) + A(h^c) &= L \\ L_{YA} + x_A + \bar{\phi} - \eta^c h^c &= L. \end{aligned}$$

Substituting the labor market clearing conditions above into the supplier's first order condition in equation (27) gives;

$$R'(\lambda) = \frac{(\tilde{\alpha} - 1)[L - A(h^c)] - \tilde{\alpha} R(\lambda) + \mu\left[(\tilde{\alpha} - 1)L - \tilde{\alpha}(\bar{\phi} - \eta^c h^c)\right]}{\bar{\lambda} - \lambda}. \tag{28}$$

The first order condition is similar to equation (22) which determines λ without the consortium, the only difference being the addition of the new term, $\tilde{\alpha}\mu\eta^c h^c - (\tilde{\alpha} - 1)A(h^c)$. Similar to the social innovation curve in Section 3.5, we can define a *Consortium Innovation Curve*, $\hat{\lambda}^c(h^c)$, which gives the solution to the above equation for each value

[19] The competitive producers are unaffected by the consortium investments. The downstream firms pay a constant fraction of their profits to the consortium, hence their profit maximization problem becomes $(1 - \theta)[pK - p_g x]$ which results in the same demand and production constraints as before, as given in equations (6) and (2). Hence their choice of price and quantity, p and K, as well as their demand for the general purpose input, x is same as in the case without consortium, as given in equations (7), (8) and (9). Hence the supplier's demand function for the general purpose input does not change and consequently, supplier's choice of p_g does not change, nor the supplier's gross profit, which remains at $\alpha^2 w(q) L_Y$. The only change is that capital goods firms' profit under consortium is now $(1 - \theta)\alpha w(q) L_Y$.

of h^c, i.e the supplier's optimal choice of λ for every h^c set by the consortium. The following proposition characterizes the stationary equilibrium with consortium.

Proposition 6. *(i) The innovation rate under consortium is higher than under market for all $h^c \in (0, \bar{h}^c)$, where $\bar{h}^c$ is the solution to the equation: ($\frac{A(h^c)}{h^c} = \frac{\tilde{\alpha}}{\tilde{\alpha}-1}\eta^c\mu$). For h^c in this range, the vertical distortion will be lower, and the equilibrium innovation rate and aggregate output growth rate will be higher, than in the case without the consortium. (ii) The Consortium Innovation Curve has a global maximum when the consortium sets complementary investment level at h^c_m, given by the solution to the equation: $A'(h^c_m) = \frac{\tilde{\alpha}}{\tilde{\alpha}-1}\eta^c\mu$, and is strictly increasing for $h < h^c_m$ and strictly decreasing for $h > h^c_m$. (iii) The maximum innovation rate under consortium, $\lambda^c_m = \hat{\lambda}^c(h^c_m)$, will be the equilibrium outcome when the fraction of downstream profits paid to consortium is $\theta = \frac{\tilde{\alpha}-1}{\frac{L-R(\lambda^c_m)}{A(h^c_m)}-1}$.*

The next proposition characterizes what happens when institutional efficiency increases.

Proposition 7. *For two or more institutional arrangements with different levels of efficiency,η^c, the one with higher efficiency will have higher innovation rate and aggregate output growth rate.*

Discussion: Figure 4 shows the market equilibrium without any consortium at point E, and the solution to the social planning problem at point S, the intersection of the social innovation and adaptation curves. The figure also shows consortium innovation curves for two values of institutional efficiency, η^1 and η^2 with $\eta^2 > \eta^1$. For each value of η, the downstream firms can attain any point on the corresponding consortium innovation curve by choosing the appropriate value of θ, the fraction of their profits that they contribute to the consortium. The figure illustrates the following points made in the two propositions above regarding the equilibrium with the consortium.

First, for points like C_1 and C_2 in the figure, the consortium reduces the horizontal and vertical distortions, i.e the values of h and λ are closer to the social optimum of h^s and λ^s when compared to the equilibrium without the consortium at point E. The horizontal distortion is lower by construction, since it was assumed that the consortium that makes complementary investments exists. However the consortium's impact on reducing the vertical distortion brings out the connection between complementary investments and innovation rate in the economy.

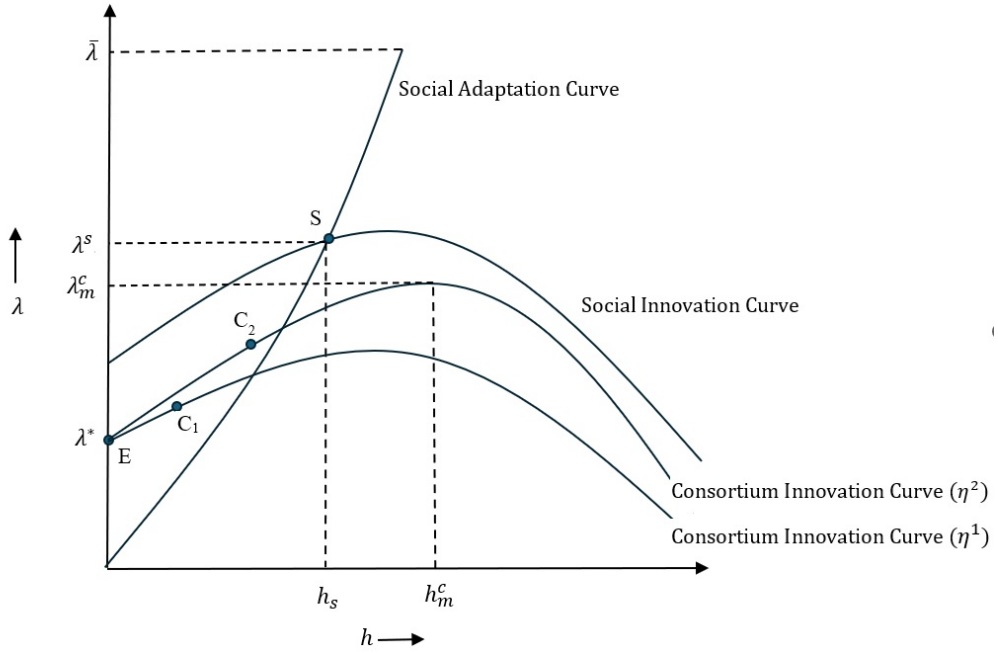


Figure 4: Innovation Possibilities with Consortium

Second, a consortium will always choose a point on the upward sloping section of the consortium investment curve, since any λ on the downward sloping section of the curve can be achieved with smaller complementary investment at some point on the upward sloping portion of the curve. The reason why λ reduces when h exceeds a certain value is again because of diminishing returns to adaptation explained earlier.

Third, for any value of h, the institutional framework with higher efficiency (η_2) will result in higher equilibrium rate of innovation, highlighting the role of institutional efficiency in technological progress and aggregate output growth.

Fourth, if the objective of a consortium is to reach the highest innovation rate, λ^c_m,then the level of complementary investment needed is h^c_m, given as the solution to the equation $A'(h^c_m) = \frac{\tilde{\alpha}}{\tilde{\alpha}-1}\eta^c\mu$. With this value of h^c_m, the innovation rate λ^c_m can be calculated from equation (28). The contribution required from capital good firms to achieve λ^c_m can then be calculated from part (iii) of the proposition; i.e $\theta = \frac{\tilde{\alpha}-1}{\frac{L-R(\lambda^c_m)}{A(h^c_m)}-1}$.

Finally, all Consortium Innovation Curves will lie below the social innovation curve, because of the monopoly distortion that will be present even with the consortium. As

a result, the highest innovation rate possible in the economy, which is the maximum of the Social Innovation Curve, is unattainable with a consortium. Whether the socially optimal innovation rate λ^s is attainable under the consortium depends on the shapes of the cost functions $R(\lambda)$ and $A(h)$. But even if it were attainable, the complementary investments required to achieve λ^s under consortium would be higher than h^s, the value required in the social planner's solution.

Comparison with Bresnahan and Trajtenberg (1995) (BT): One can get intuition about the model results by comparing with results in BT.[20] Similar to the Social Innovation Curve and the Social Adaptation Curves here, the model in BT yields dynamic reaction functions for investment levels for the supplier and downstream firms. Both the curves are upward sloping in BT, while the innovation curve (social planner's and consortium's) in the current paper has a hump-shape, and this a direct consequence of the general equilibrium framework adopted in the current paper. Increasing complementary investment reduces the labor available for final good production, thus reducing demand for the general purpose input and supplier's profit - an effect that does not arise in the partial equilibrium framework used in BT.

The model in BT also generate the vertical and horizontal distortions (which they call externalities) that lead to market inefficiencies. The current paper separate the two components of the vertical externality that are considered together in BT; the monopoly distortion that is caused by appropriability issues, and the vertical distortion that is caused by underinvestment in complementary inputs. This allows one to analyze separately the factors that determine each of these distortions. The extent of monopoly distortion is determined by α, which determines the price elasticity of demand for capital goods, and hence the markups of the downstream firms and the supplier.

BT undertake simulations to show that vertical distortion would be lower (and the equilibrium quality levels would be higher) with higher discount rates, and they associate higher discount rates with a reduction in uncertainty, because of more efficient institutional arrangements. By separating out the effect of institutions, the current paper brings out clearly the role of institutional efficiency in reducing the horizontal and vertical distortions, and in improving welfare.

[20] BT adopts a broader notion of complementary investments, which includes the pre-competitive complementary investments considered in this paper, as well as product specific innovation enabled by higher quality versions of the input.

More efficient institutions (higher η) achieve a higher reduction in adaptation cost for a given level of complementary investment, and this has a direct and indirect effect on consumption (welfare). The direct effect is that it frees up labor for final goods production thus increasing consumption during adaptation periods. The indirect effect can be seen clearly in Figure 4, the effect of complementary investments on increasing the innovation rate, and hence consumption, is higher if institutions are more efficient.

BT points to various mechanisms that have been used to enhance information flows that facilitate coordination and hence induce a rise in institutional efficiency, η^c.[21] The next section describes how such an increase institutional efficiency, resulting from the policies initiated by SEMATECH, can explain the acceleration of the 1990s.

4 Institutional Efficiency and the Acceleration of 1993-2003

To understand the role of institutional efficiency and SEMATECH in the US aggregate output and productivity growth, we can map the model to the semiconductor and electronics industry as follows. The semiconductor process tools that enables continued miniaturization of electronic components, and hence quality improvement in computers and other electronic capital goods, maps to the general purpose input in the model.[22] The quality of the general purpose input has a clear physical meaning in this context, it the dimension (length) of transistors that can be manufactured with a given vintage of the process tool. Denote this length by ℓ.

The y-axis of Figure 2b, which shows the values of ℓ in nanometers, thus maps to the geometric quality ladder for the industry. Each new vintage of process tool shrinks ℓ by a factor of 0.7, thus $\delta = \frac{1}{0.7}$ for the industry. The rate of quality improvements is thus, $\frac{\dot{q}}{q} = -\frac{\dot{\ell}}{\ell}$, the rate of miniaturization, equal to the slope of the graph in 2b.

The downstream firms in the model is taken as a composite of semiconductor chip

[21] They mention strategic alliances, formal standard setting processes, consortia, software missionaries and manipulation of trade press as some of the methods that can facilitate information flow.

[22] Because these tools are complements, and each tool segment is highly concentrated, there is not much loss of generality in considering the collection of process tools as a single tool provided by a common supplier, as is done in the model. Many of the leading tools have just one supplier, and almost all of them have a dominant supplier.

manufacturers and producers of capital equipment that embed semiconductor chips. The acceleration in the rate of miniaturization during 1993-2003 seen in Figure 2b, can be explained as a result of the activities and programs initiated by SEMATECH, the consortium of downstream firms, during this period.

By using its budget on complementary investments, SEMATECH clearly increased h in the model. In addition, consider two following programs initiated by SEMATECH:[23] (i) SEMATECH set up a centralized process tool qualification facility, which certified tools made by suppliers based on requirements placed jointly by downstream firms (see Grindley et al. (1994)). (ii) SEMATECH, in collaboration with other industry trade associations, started releasing periodic roadmaps for the evolution of semiconductor technology for the next 15 years. These roadmaps provided the consensus view of downstream firms about the requirements that the process tools suppliers had to meet for future versions of their tools. The roadmaps became a central feature of the semiconductor industry in the 1990s.

By bringing together all downstream firms to formulate the requirements for a centralised tool qualification facility, or to formulate a jointly agreed upon roadmap for future tool requirements, SEMATECH's activities guided downstream firms to process their individual requirements to arrive at a consensus on what is common across their production processes, and articulate it to the supplier. This corresponds to an increase in institutional efficiency, η, in the model.

SEMATECH's efforts thus pushed the industry from a point like C_1 in Figure (4), to a point like C_2, with a higher h and η. As shown in the model, and as can be seen in the graph, the result was an increase in innovation rate λ, leading to the acceleration in miniaturization seen in Figure (2b), and consequently an increase in aggregate output growth rate by Proposition (4).

If we assume the value of capital's share of aggregate income is $\alpha = \frac{1}{3}$, then the change in miniaturization rate ($\frac{\dot{\ell}}{\ell}$) from -12% to -18% (see Figure 2b), would suggest an increase in GDP growth rate from 4% to 6%, an overestimation compared to the actual observed growth rate. This discrepancy arises from two simplifying assumptions in the model; first, that the general purpose input is the only factor in the production of all capital goods (equation 2), and second, that all capital goods contribute identically

[23] See Appendix for a detailed description of the programs initiated by SEMATECH to enhance vertical collaboration between chip/electronics manufacturers and process tools suppliers.

to final good production (equation 1). These assumptions were useful in bringing out the role of general purpose technologies and the associated market inefficiencies with clarity. Modifying these assumptions to better reflect the structure of the US economy would improve the accuracy of the model's GDP growth predictions.

5 Application of the model to other institutions

While the discussion until now has focused on the role institutions that can coordinate across downstream firms, comparative static analyses using the model can provide intuition about how other kinds of institutions can affect innovation and output growth rate. For example, the analysis in Goldin and Katz (1998) on technology-skill complementarity suggests that workers with certain vocational skills are necessary to fully realise the gains from new technologies. Such workers may include machinists during the steam era, electrical technicians and engineers during electricity era, and computer engineers during the IT era. David and Wright (1999) goes one step further and argue that it was not necessarily vocational education, rather the overall changes in educational system which embedded some of these skills across the whole labor force that enabled the full utilization of general purpose technologies like steam or electricity.

While impact of technology-skill complementarity on occupation (skill) specific wages has been extensively studied, the model in the paper brings out a channel through which technology-skill complementarity affects aggregate output growth. An increase in technology specific skills across the labor force reduces the effort needed on the part of the suppliers of general purpose inputs to adapt new higher quality versions of the input across its various uses. For example, one can imagine that if the textile workers of the early 19th century had basic machining skills and understood the mechanics of operating steam engines, it would have been easier to upgrade from low-pressure steam engines to high-pressure ones (see Section 1). Or if the labor force of the early 20th century had basic knowledge of electricity, it would have made it much more easier for electricity suppliers to upgrade the outdated direct current (DC) systems at various factories to the better alternating current (AC) systems.

In the model, this would amount to a reduction in adaptation cost rate $\bar{\phi}$ of the supplier. A reduction in $\bar{\phi}$ would lead to an increase in mean innovation rate λ and

hence an increase in the mean growth rate of aggregate output.[24] The reason for the increase is the mechanism at the heart of the paper, that a reduction in adaptation cost increases the supplier's marginal benefit to innovation.

It is instructive to compare the role of skill upgrades in the labor force with the role of a consortium in increasing output growth rate. With the adaptation cost rate for the supplier being given by $\phi = \bar{\phi} - \eta h$, in the case with the consortium it was the increase in η or h (or both) that leads to reduction in ϕ, whereas for the improvement in the technology specific skills of the labor force, it is the reduction in $\bar{\phi}$ that leads to increase in the innovation rate. The latter can clearly happen, with or without the consortium.

6 Conclusion

This paper showed that many factors that growth theorists and economic historians have argued are important for growth, namely, profit-driven R&D, general purpose technologies, complementary investments, and institutions, can be bought together in a single tractable framework.

The paper demonstrated that a model incorporating the concept of General Purpose Technologies (GPTs) and their associated attributes can provide insights into the drivers of productivity and output growth that go beyond what can be obtained using existing endogenous growth models. The industrial organization dynamics related to the allocation of investment responsibilities between the suppliers and users of GPTs, can influence the rate of technological progress and economy-wide output growth. Institutions that can shape or coordinate these investment decisions may therefore have a meaningful impact on output growth. These findings underscore the value of the GPT framework in understanding the sources of economic growth.

The paper also bought out a role for institutions that growth literature has not given attention to, that institutions play a synergistic role in driving technological progress and economic growth, as opposed to the stabilizing or protective role usually considered in the literature. The paper also provided a notion of institutional efficiency that is relevant in this context.

[24] This follows simple by applying Lemma (2)(iii), to equation (21), noting that an increase in $\bar{\phi}$ corresponds to an increase in the parameter b in the Lemma.

From a policy perspective, the result in this paper support the formation of communities of interest of users of an anticipated series of innovations who can make pre-competitive complementary investments that can reduce adaptation investments required by the supplier of a technology. Two recent examples of such policies would be the sections of the National Quantum Initiative Act (2017) and CHIPS Act (2022) that directs and funds the National Institute of Technology and Standards (NIST) to form communities of interest comprising parties interested in the future implementation of quantum technology and advanced semiconductor technology.[25] Quantum technology and advanced semiconductor technology are examples of the type of technologies considered in this paper. The model developed in the paper showed that policies such as these that will induce pre-competitive complementary investments can accelerate progress in these technologies and foster economy-wide output growth.

Finally, the paper points to the need to modify national accounting methods to use quality adjusted prices for semiconductor process tools. Without this modification, the whole contribution of semiconductor process tools industry is attributed to the computer and semiconductor chip industries, where quality adjusted prices pick up the technological progress in these products (see Figure 1). Making this modification would enable the national accounts to provide a clearer picture of sources of economic growth in the last half century. It would also bring out more clearly the connection between economic growth and fundamental advances in science, since continued miniaturization abilities enabled by semiconductor process tools are intimately connected to fundamental advances in physics and chemistry.

[25]The relevant section are Section 201 of the National Quantum Initiative Act and Section 15.USC.4656 of the CHIPS Act.

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A Mathematical Appendix

The following lemma is used in the proofs of propositions, given below.

Lemma 2. *Consider the equation* $(\bar{\lambda}-\lambda)R'(\lambda)+aR(\lambda)=b$*, where* $a>1$ *and* $b>0$*.* $R(\lambda)$ *is twice continuously differentiable and satisfies parts (i) and (ii) in Assumption (2), and* $R(\bar{\lambda})>b$*. Then the following are true: (i) the equation has a unique solution for* λ *in* $(0,\bar{\lambda})$*; (ii) the* λ *that solves the equation is decreasing in* a*; (iii)* λ *that solves the equation is increasing in* b*.*

Proof. Let $f(\lambda)$ denote the left hand side of the equation, i.e $f(\lambda)=(\bar{\lambda}-\lambda)R'(\lambda)+aR(\lambda)$. Then $f(\lambda)$ is continuous and differentiable, with $f'(\lambda)=(\bar{\lambda}-\lambda)R''(\lambda)+(a-1)R'(\lambda)$, which positive over the range $(0,\bar{\lambda})$, since $R''(\lambda),\ R'(\lambda)>0$ by Assumption 2, and $a>1$. Further $f(0)=0$ and $f(\bar{\lambda})=aR(\bar{\lambda})$. Hence if $aR(\bar{\lambda})>b$, then the equation has a unique solution in $(0,\bar{\lambda})$. Since $a>1$, the condition above that $R(\bar{\lambda})>b$ ensures that $aR(\bar{\lambda})>b$, and hence a unique solution for λ exists in the interval $(0,\bar{\lambda})$. For proving parts (ii) and (iii), let $\tilde{\lambda}(a,b)$ be the solution to the equation for various values of a and b. By the Implicit Function Theorem, $\tilde{\lambda}(a,b)$ is differentiable in a and b. Implicitly differentiating the equation w.r.t a gives: $\frac{d\tilde{\lambda}}{da}=-\frac{R(\lambda)}{R''(\lambda)(\bar{\lambda}-\lambda)+(a-1)R'(\lambda)}$, which is less than zero, since $R(\lambda),R'(\lambda),R''(\lambda)>0,\ \lambda<\bar{\lambda}$, and $a>1$. Similarly, implicitly differentiating the equation w.r.t b gives, $\frac{d\tilde{\lambda}}{db}=\frac{1}{R''(\lambda)(\bar{\lambda}-\lambda)+(a-1)R'(\lambda)}$, which is greater than zero. Hence, the λ that solves the equation is decreasing in a and increasing in b. □

Proof of LEMMA 1: Let t_n be the duration of the n^{th} innovation period. Hence quality δ^n is operational with output rate $L_{YI}^{1-\alpha}(\delta^N x_I)^\alpha$ during time interval $(\sum_{k=1}^{n}t_k+n\tau,\sum_{k=1}^{n+1}t_k+n\tau)$, and with output $L_{YA}^{1-\alpha}(\delta^N x_A)^\alpha$ during $(\sum_{k=1}^{n+1}t_k+n\tau,\sum_{k=1}^{n+1}t_k+(n+1)\tau)$. Since L_{YI},L_{YA},X_I,X_A are constant over time, the expected present discounted value of production (consumption) is

$$\frac{L_{YI}X_I^\alpha}{\rho}\sum_{n=0}^{\infty}\delta^n\left[Ee^{-\rho\left(\sum_{k=1}^{n}t_k+n\tau\right)}-Ee^{-\rho\left(\sum_{k=1}^{n+1}t_k+n\tau\right)}\right]$$
$$+\frac{L_{YA}X_A^\alpha}{\rho}\sum_{n=0}^{\infty}\delta^n\left[Ee^{-\rho\left(\sum_{k=1}^{n+1}t_k+n\tau\right)}-Ee^{-\rho\left(\sum_{k=1}^{n+1}t_k+(n+1)\tau\right)}\right],$$

where $Ee^{-\rho\left(\sum_{k=1}^{n}t_k+n\tau\right)}$ is the expectation over the distribution of $\sum_{k=1}^{n}t_k+n\tau$. Since t_k is an i.i.d draw from exponential distribution with mean $\frac{1}{\lambda}$, it follows that $\sum_{k=1}^{n}t_k$ is

distributed according to the Gamma distribution $G(n,\lambda)$. Further, it is shown in Pillai (2013), LEMMA 5, that if $T \sim \mathbf{G}(n,\lambda)$, then $E\left[e^{-\rho T}\right] = \left(\frac{\lambda}{\rho+\lambda}\right)^n$. Using this result to substitute for the expected values above, it can be seen that the above expression becomes,

$$\frac{L_{YI}X_I^{\alpha}}{\rho}\left[\sum_{n=0}^{\infty}(\delta e^{-\rho\tau}\frac{\lambda}{\rho+\lambda})^n - \sum_{n=0}^{\infty}\frac{\lambda}{\rho+\lambda}(\delta e^{-\rho\tau}\frac{\lambda}{\rho+\lambda})^n\right]$$
$$+\frac{L_{YA}X_A^{\alpha}}{\rho}\left[\sum_{n=0}^{\infty}\frac{\lambda}{\rho+\lambda}(\delta e^{-\rho\tau}\frac{\lambda}{\rho+\lambda})^n - \sum_{n=0}^{\infty}e^{-\rho\tau}\frac{\lambda}{\rho+\lambda}(\delta e^{-\rho\tau}\frac{\lambda}{\rho+\lambda})^n\right]$$

Noting that the terms in each of the summation constitute an infinite geometric series, and putting in their sums, gives the expression for $U(\lambda)$ in LEMMA (1).

Proof of PROPOSITION 3: First step in the proof is to show that if λ and ϕ are constant over time, then L_{YI}, L_{YA}, x_I and x_A will be constant as well. If λ and ϕ are constant, the labor market clearing conditions in equations (16) and (17) implies that L_{YI} and L_{YA} will be constant as long as the ratio $\frac{x}{L_Y}$ is constant during innovation and adaptation periods. Using equations (9) and (18), it can be seen that,

$$\frac{x}{L_Y} = \frac{\alpha^3}{1-\alpha}, \tag{29}$$

Hence L_{YI}, L_{YA}, x_I and x_A will be constant in a stationary equilibrium. Next, it is shown below that if L_{YI} is constant, then $V_g(q)$ is proportional to $w(q)$, and hence homogenous of degree α. The first order condition with respect to λ for the supplier's dynamic problem gives,

$$w(q)R'(\lambda) = \frac{1-e^{-\rho\tau}}{\rho}(\alpha^2 L_{YA} - \phi)w(q) + e^{-\rho\tau}V_g(\delta q) - V_g(q). \tag{30}$$

The Bellman equation should be satisfied at the optimal λ. Taking the limit $\Delta t \to 0$ in equation (14) gives,

$$\rho V_g(q) = (\alpha^2 L_{YI} - R(\lambda))w(q) + \lambda\left(\frac{1-e^{-\rho\tau}}{\rho}(\alpha^2 L_{YA} - \phi)w(q) + e^{-\rho\tau}V_g(\delta q) - V_g(q)\right). \tag{31}$$

The preceding two equations together imply $V_g(q) = \frac{w(q)}{\rho}\left[\alpha^2 L_{YI} - R(\lambda) + \lambda R'(\lambda)\right]$. Since L_{YI} and λ are constant by the above arguments, $V_g(q)$ is proportional to $w(q)$,

and hence can be written as $V_g(q) = vq^\alpha$, in accordance with Proposition 1. With $V_g(q) = vq^\alpha$ and $w(q) = w_0 q^\alpha$, the first order condition in equation (30) becomes,

$$w_0 R'(\lambda) = w_0 \frac{1 - e^{-\rho\tau}}{\rho}(\alpha^2 L_{YA} - \bar{\phi}) + (e^{-\rho\tau}\delta^\alpha - 1)v. \tag{32}$$

Further substituting $V_g(q) = vq^\alpha$ into equation (31),

$$v = w_0 \frac{\alpha^2 L_{YI} - R(\lambda) + \lambda \frac{1-e^{-\rho\tau}}{\rho}(\alpha^2 L_{YA} - \bar{\phi})}{\rho - \lambda(e^{-\rho\tau}\delta^\alpha - 1)}. \tag{33}$$

Substituting the value of v into equation (32), and using the definition of $\bar{\lambda}$ and μ from equation (20) gives equation 21 in text, i.e.,

$$R'(\lambda) = \frac{\alpha^2 L_{YI} - R(\lambda) + \mu(\alpha^2 L_{YA} - \bar{\phi})}{\bar{\lambda} - \lambda}.$$

Next, using equation (29), and noting that these apply for both innovation and adaptation periods, we have $x_I = L_{YI}\frac{\alpha^3}{1-\alpha}$ and $x_A = L_{YA}\frac{\alpha^3}{1-\alpha}$. Substituting these into the labor market clearing condition gives, $L_{YI} = \frac{L - R(\lambda)}{1 + \frac{\alpha^3}{1-\alpha}}$ and $L_{YA} = \frac{L - \bar{\phi}}{1 + \frac{\alpha^3}{1-\alpha}}$. Substituting these into preceding equation, and using the definition of $\tilde{\alpha}$ from equation (20), gives equation (22) in text, i.e.,

$$R'(\lambda) = \frac{(\tilde{\alpha} - 1)L - \tilde{\alpha}R(\lambda) + \mu[(\tilde{\alpha} - 1)L - \tilde{\alpha}\bar{\phi}]}{\bar{\lambda} - \lambda}. \tag{34}$$

Since $1 < \tilde{\alpha} < 2$, it is true that $(\tilde{\alpha} - 1)L + \mu[(\tilde{\alpha} - 1)L - \tilde{\alpha}\bar{\phi}] < L + \mu[L - \phi]$. Hence by Assumption (2)(iii), $R(\bar{\lambda}) > (\tilde{\alpha} - 1)L + \mu[(\tilde{\alpha} - 1)L - \tilde{\alpha}\bar{\phi}]$. Then, using Lemma (2), with $a = \tilde{\alpha}$ and $b = (\tilde{\alpha} - 1)L + \mu[(\tilde{\alpha} - 1)L - \tilde{\alpha}\bar{\phi}]$, gives the result that there is a unique $\lambda^* \in (0, \bar{\lambda})$ that solves the above equation. Hence there is a unique stationary equilibrium in the model.

Proof of PROPOSITION 4: Consider a unit interval of time, with the economy starting with any quality q at the beginning of the interval. Let N be a random variable representing the number of inventions that can occur during this unit time interval, so that $g_q \equiv E\left[\ln\left(\frac{\delta^N q}{q}\right)\right] = \ln(\delta)E[N]$, where E denotes the expectation over N. If $E(N)$ innovations occur during the unit time interval, then these must have occurred during an interval of time equal to $1 - E(N)\tau$, since $E(N)\tau$ would be taken up for adaptation phase during which there is no investment in R&D. But the expected number

of Poisson events during time interval $1 - E(N)\tau$, when the mean arrival rate is λ^*, is equal to $\lambda^*(1 - E(N)\tau)$. Hence it must be that $E(N) = \lambda^*(1 - E(N)\tau)$, which gives $E(N) = \frac{\lambda}{1+\lambda\tau}$, and $g_q = \frac{\ln \delta}{\frac{1}{\lambda^*}+\tau}$. The expressions for $g_w, g_{p_g}, g_p, g_K, g_Y$ follow from equations (18), (10), (7), (8) and (1) respectively.

First and Second Order Conditions for Social Planning Problem: To solve the social planning problem, first note that the first order conditions with respect to L_{YI} and X_I together imply $x_I = \frac{\alpha}{1-\alpha} L_{YI}$. Similarly, $x_A = \frac{\alpha}{1-\alpha} L_{YA}$. Substituting these expressions for x_I and x_A into the resource constraints gives $L_{YI} = (1 - \alpha)(L - R(\lambda) - A(h))$ and $L_{YA} = (1-\alpha)(L-\bar{\phi}+\eta h)$. With these substitutions, the social planning problem reduces to the unconstrained problem of choosing λ and h, i.e

$$\max_{\lambda,h} \frac{\alpha^\alpha}{(1-\alpha)^{1-\alpha}} \frac{L - R(\lambda) - A(h) + \lambda \frac{1-e^{-\rho\tau}}{\rho}(L - \bar{\phi} + \eta h)}{\rho - \lambda(e^{-\rho\tau}\delta^\alpha - 1)}.$$

The first order necessary condition for optimal λ is:

$$R'(\lambda) = \frac{(e^{-\rho\tau}\delta^\alpha - 1)(L - R(\lambda) - A(h)) + (1 - e^{-\rho\tau})(L - \bar{\phi} + \eta h)}{\rho - \lambda(e^{-\rho\tau}\delta^\alpha - 1)} \tag{35}$$

The first order necessary condition for optimal h is:

$$A'(h) = \lambda \frac{1 - e^{-\rho\tau}}{\rho} \eta \tag{36}$$

Using the definitions of $\bar{\lambda}, \mu$ and $\tilde{\alpha}$ in equation (20), the above first order conditions translate into equations (24) and (25) in the text.

The second order condition for a maximum is that the Hessian matrix should be negative definite. It can be seen that at the value of λ and h that satisfy the first order conditions in equations (24) and (25), the Hessian is given by;

$$\begin{bmatrix} \frac{\partial^2 U}{\partial^2 \lambda} & \frac{\partial U}{\partial \lambda \partial h} \\ \frac{\partial U}{\partial h \partial \lambda} & \frac{\partial^2 U}{\partial^2 h} \end{bmatrix} = \begin{bmatrix} -\frac{\bar{\lambda} R''(\lambda)}{\bar{\lambda} - \lambda} & \frac{\bar{\lambda}}{(\bar{\lambda}-\lambda)^2}(\eta\mu - A'(h)) \\ \frac{\bar{\lambda}}{(\bar{\lambda}-\lambda)^2}(\eta\mu - A'(h)) & -\bar{\lambda}\frac{A''(h)}{\bar{\lambda}-\lambda}. \end{bmatrix}$$

The first principal minor is less than zero, since $R''(\lambda) > 0$ and $\lambda < \bar{\lambda}$. Hence all that is required is to show that the determinant of the Hessian is greater than zero. The determinant is $D = \left(\frac{\bar{\lambda}}{\bar{\lambda}-\lambda}\right)^2 R''(\lambda)A''(h) - \frac{\bar{\lambda}^2}{(\bar{\lambda}-\lambda)^4}(\eta\mu - A'(h))^2$. Substituting from equation (25) that $A'(h) = \frac{\lambda}{\bar{\lambda}}\eta\mu$, the determinant reduces to $D = \frac{1}{(\bar{\lambda}-\lambda)^2}\left[\bar{\lambda}^2 R''(\lambda)A''(h) - (\eta\mu)^2\right]$. Hence $D > 0$ if $R''(\lambda)A''(h) > \left(\frac{\eta\mu}{\bar{\lambda}}\right)^2$, which is the condition stated in Assumption (4).

Proof of PROPOSITION 5: Implicity differentiating the social planner's first order condition in equation (24) with respect to h gives,

$$(\bar{\lambda} - \lambda)R''(\lambda)\frac{d\hat{\lambda}(h)}{dh} = \eta\mu - A'(h). \tag{37}$$

Since $(\bar{\lambda}-\lambda) > 0$ and $R''(\lambda) > 0$, it follows that $\frac{d\hat{\lambda}(h)}{dh}$ has the same sign as $\eta\mu - A'(h)$. Since $A'(h) > 0$ and $\lim_{h\to 0} A'(h) = 0$, it follows that there is a unique value of $h = h_m$, such that $A'(h_m) = \eta\mu$, with $A'(h) < \eta\mu$ for $h < h_m$, and $A'(h) > \eta\mu$ for $h > h_m$. For (ii) it can be seen from equation (25), that since $A''(h) > 0$, $\hat{h}(\lambda)$ must be a monotonically increasing function. Further, it can be seen from equation (25), that $A'(h) = \eta\mu$ iff $\lambda = \bar{\lambda}$, hence it must be true that $\lim_{\lambda\to\bar{\lambda}} \hat{h}(\lambda) = h_m$. For (iii) To show that $\lambda^* < \hat{\lambda}(0)$, note first that λ^* is the solution to equation (22), and $\hat{\lambda}(0)$ is the solution to equation (24) with h^s set equal to zero. Hence both are solutions to the general equation $(\bar{\lambda} - \lambda)R'(\lambda) + aR(\lambda) = b$ in LEMMA 2, with $a = \tilde{\alpha}$, $b = (\tilde{\alpha}-1)L + \mu[(\tilde{\alpha}-1)L - \tilde{\alpha}\bar{\phi}]$ giving the solution λ^*; and $a = 1$, b=$L + \mu[L - \bar{\phi}]$ giving the solution $\hat{\lambda}(0)$. Since $\tilde{\alpha} > 1$ and $(\tilde{\alpha}-1)L + \mu[(\tilde{\alpha}-1)L - \tilde{\alpha}\bar{\phi} < L + \mu[L - \bar{\phi}]$ because $\alpha \tilde{<} 2$, it is a direct application of Lemma 2(ii) and (iii) that $\lambda^* < \hat{\lambda}(0)$. To show that, $\hat{\lambda}(0) < \lambda^s$, it suffices to show that $\lambda^s < \hat{\lambda}(h_m)$, since by (i) $\hat{\lambda}(h)$ is increasing in the range (0,h_m). Using equations (25) and (37), it can be seen that the slope of the innovation curve where it hits the adaptation curve hit is given by, $\frac{d\hat{\lambda}(h)}{dh} = \frac{\eta\mu - A'(h)}{(\bar{\lambda}-\lambda)R''(\lambda)} = \frac{\eta\mu - \frac{\lambda}{\bar{\lambda}}\eta\mu}{(\bar{\lambda}-\lambda)R''(\lambda)} = \frac{\eta\mu}{\bar{\lambda}R''(\lambda)}$, which is greater than zero. Hence $\lambda^s < \hat{\lambda}(h_m)$. Further, the result in (ii) that $\lim_{\lambda\to\bar{\lambda}} \hat{h}(\lambda) = h_m$, and $\lambda^s < \hat{\lambda}(h_m)$ together imply that $\lambda^s < \bar{\lambda}$. Finally, $\hat{h}(\lambda)$ is monotonically increasing, together with $\lambda^s < \bar{\lambda}$ and $\lim_{\lambda\to\bar{\lambda}} \hat{h}(\lambda) = h_m$, imply that $h^s < h_m$.

Proof of PROPOSITION 6 and PROPOSITION 7: First rewrite the first order condition with consortium as;

$$R'(\lambda)(\bar{\lambda} - \lambda^c) + \tilde{\alpha}R(\lambda^c) = (\tilde{\alpha}-1)[L - A(h^c)] + \mu\left[(\tilde{\alpha}-1)L - \tilde{\alpha}(\bar{\phi} - \eta h^c)\right] \tag{38}$$

Since $\tilde{\alpha} > 1$, there is a unique solution to the above equation by Lemma 2. For proving part (i) of the proposition, compare equations (22) and (38) for the equilibrium without consortium and with consortium respectively. Both the equations can be written as $(\bar{\lambda} - \lambda)R''(\lambda) + \tilde{\alpha}R(\lambda) = b$, where $b = (\tilde{\alpha}-1)L + \mu[(\tilde{\alpha}-1)L - \tilde{\alpha}\bar{\phi}]$ for the case without consortium, and $b = (\tilde{\alpha}-1)[L - A(h^c)] - \tilde{\alpha}R(\lambda) + \mu\left[(\tilde{\alpha}-1)L - \tilde{\alpha}(\bar{\phi} - \eta h^c)\right]$

for the case with the consortium. Hence by Lemma 2, part (iii), the solution for λ is higher for the case with the consortium if $\frac{A(h^c)}{h^c} - \frac{\tilde{\alpha}}{\tilde{\alpha}-1}\eta\mu > 0$. Since $A(h^c)$ is a convex function with $\lim_{h^c->0} A'(h^c) = 0$, then $\frac{A(h^c)}{h^c}$ is a strictly increasing function that is equal to zero at $h^c = 0$. Hence there exists a unique $\bar{h}^c$ such that $\frac{A(h^c)}{h^c} = \frac{\tilde{\alpha}}{\tilde{\alpha}-1}\eta\mu$, with $\frac{A(h^c)}{h^c} - \frac{\tilde{\alpha}}{\tilde{\alpha}-1}\eta\mu > 0$ for $h^c \in (0, \bar{h}^c)$.

For proving part (ii), implicitly differentiating equation (38) w.r.t h^c gives,

$$R''(\lambda)(\bar{\lambda} - \lambda^c) + (\tilde{\alpha} - 1)R'(\lambda)\frac{d\hat{\lambda}^c(h^c)}{dh^c} = \mu\tilde{\alpha}\eta - (\tilde{\alpha} - 1)A'(h^c),$$

or

$$\frac{d\hat{\lambda}^c(h^c)}{dh^c} = \frac{\mu\tilde{\alpha}\eta - (\tilde{\alpha} - 1)A'(h^c)}{R''(\lambda)(\bar{\lambda} - \lambda^c) + (\tilde{\alpha} - 1)R'(\lambda)}.$$

Since the denominator is always greater than zero, $\frac{d\hat{\lambda}^c(h^c)}{dh^c} > 0$ if $A'(h^c) > \frac{\tilde{\alpha}}{\alpha-1}\eta\mu$, $\frac{d\hat{\lambda}^c(h^c)}{dh^c} < 0$ if $A'(h^c) < \frac{\tilde{\alpha}}{\alpha-1}\eta\mu$, and $\frac{d\hat{\lambda}^c(h^c)}{dh^c} = 0$ if $A'(h^c) = \frac{\tilde{\alpha}}{\alpha-1}\eta\mu$, thus proving part (ii). For part (iii), from the innovation period budget constraint,

$$L_{YI}(1 + \frac{\alpha^3}{1-\alpha}) + A(h^c) = L - R(\lambda).$$

Substituting $L_{YI} = \frac{A(h^c)}{\theta\alpha}$ from equation (26) into the budget constraint above gives, $\theta = (\tilde{\alpha} - 1)\frac{A(h^c)}{L - R(\lambda) - A(h^c)}$. This should hold for all h^c. Hence the θ that give the complementary investment h^c_m and corresponding maximum innovation rate of $\hat{\lambda}^c(h^c_m)$ is as given in the proposition. For proving Proposition (7), note that in the FOC above, the right hand side is increasing in η. Hence the Proposition (7) result that innovation rate λ that solves the FOC will be increasing in η, is a direct application of LEMMA (2 (iii)). The growth rate of quality q, growth rate of aggregate output, and other variables in the economy will be the same as given in Proposition (4), since the proof of the proposition applies without any change for the case with the consortium as well.

B Data Appendix

B.1 Acceleration in Miniaturization during 1993-2003: US vs Japan

In contrast to the US, there was no acceleration in the rate of miniaturization in semiconductor chips manufactured by companies from Japan. Figure 5 shows the rate of

miniaturization in semiconductor chips manufactured by two leading semiconductor chip producers in Japan during the 1971-2010 time period, Nippon Electric Company (NEC) and Toshiba Corporation.

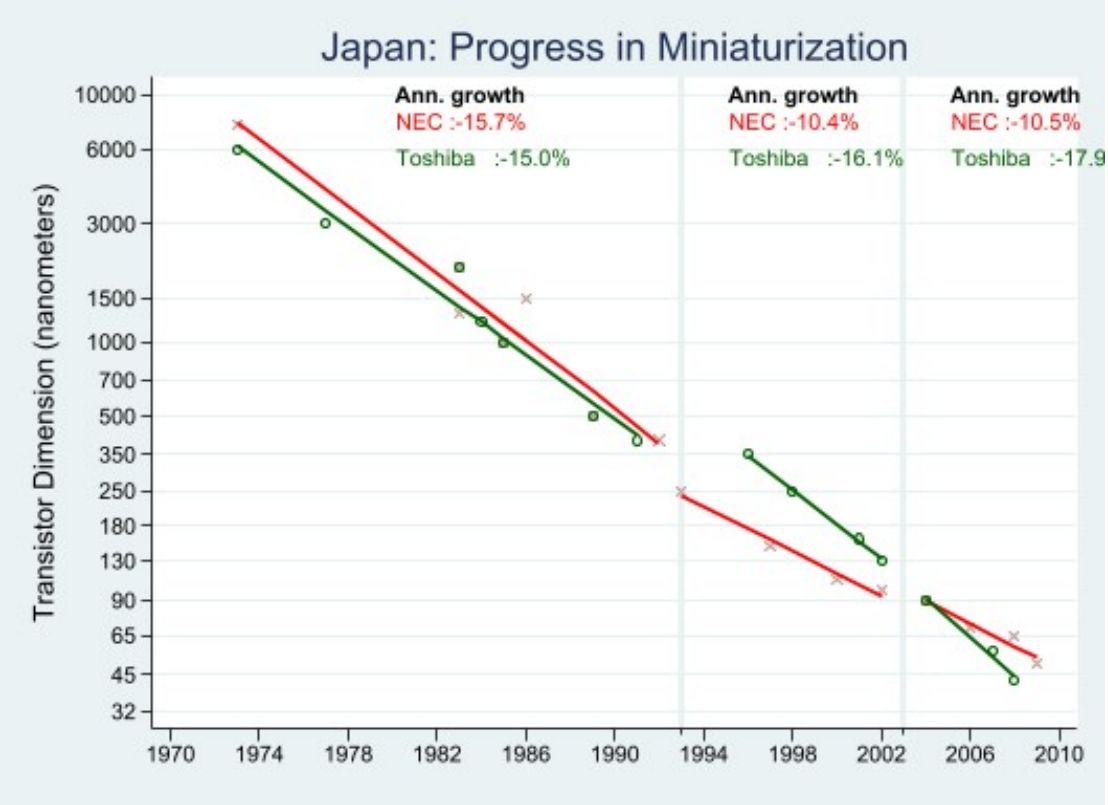


Figure 5: Progress in Miniaturization in Japan.

Each point in the graph shows the year that a new vintage of process tools was used for the first time by the company, with the corresponding transistor dimension given on the y-axis. The data for the graph was collected from multiple sources, including from IC Knowledge (a market research company in the semiconductor industry), from news articles, and from technical articles published in IEEE journals. The first company, NEC, spun off its semiconductor operations towards the end of the year 1999, as a new company, Elpida. The data points for NEC in the graph from 2000 on are the ones for Elpida.

As can be seen from the graph, the rate of miniaturization actually decreased for

NEC for the period 1993-2003, when compared to the period 1971-1992. Toshiba's data shows a small increase in the annual miniaturization rate by around 1%, much less than the 6% change observed for US companies (Figure 2b in the main paper).

B.2 Increase in US Semiconductor Market Share

The market share of US companies in the global semiconductor chip market and the process tool market increased substantially during 1993-2003, when compared to 1971-1992 (see Figure 6. The US share in the chip market decreased at an average annual rate of -1.73% per year during 1971-1992, but increased at the rate of 0.75% per year during 1993-2003. There was similar, but slightly smaller, increase for the US semiconductor process tool manufacturers.

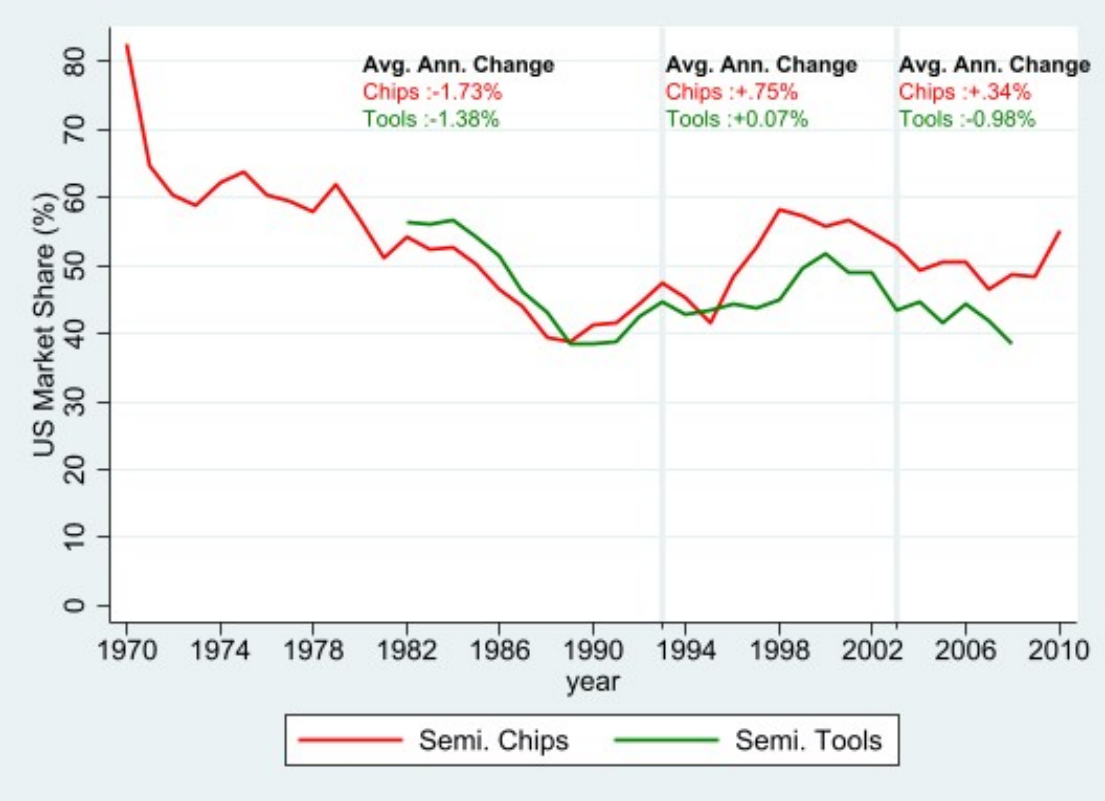


Figure 6: Progress in Miniaturization in Japan.

The data for semiconductor chip market share was collected from two sources; Gart-

ner Inc. (a market research company in the semiconductor industry), and the Semiconductor Industry Association (SIA). The data for semiconductor process tool data was purchased from Gartner. The process tools considered for the data in the graph includes only the tools used inside the fabrication plant (which are the ones that determine transistor dimensions, and are also called "Wafer Fab Equipment" in the industry), and does not include the tools for testing, assembling and packaging the chips after they are produced.

B.3 SEMATECH Programs to strengthen the coordination between US semiconductor chip producers and US semiconductor process tool manufacturers

The reports by U.S. General Accounting Office (GAO) (USGAO (1990) and USGAO (1992)) to the House of Representatives Committee on Science, Space, and Technology provides a comprehensive account of how SEMATECH enhanced the vertical relationship between chip manufacturers and process tool suppliers. Since SEMATECH received partial funding from the U.S. government, its members were required to undergo evaluation by the GAO as part of the reporting requirements. The GAO report is based primarily on direct interviews and surveys with senior executives, managers, and engineers from SEMATECH, its member companies, and process tool suppliers. Similar findings have also been documented in other academic studies on SEMATECH, including Katz and Ordover (1990), Grindley et al. (1994), and Gibson (1994). USGAO (1992) summarizes that, "An important element of SEMATECH'S program has been to change relationships between semiconductor manufacturers and their suppliers from a traditional adversarial approach to a collaborative approach...".

The following are some of the activities that SEMATECH undertook that are relevant to the paper.

(1) In September 1987, SEMATECH and the trade association of process tool suppliers (Semiconductor Equipment and Materials International, SEMI) set up a collaborative organization called SEMI/SEMATECH. SEMATECH and SEMI/SEMATECH together established seven supplier technical advisory boards (TABs) to act as forums for suppliers' representatives to exchange technical information with members of SEMATECH'S technical advisory boards, who are senior technical managers from SE-

MATECH'S member companies.

(2) In July 1989, SEMATECH setup a Department of Supplier Relationships to develop a comprehensive strategy to improve the relationship between chip manufacturers and process tools suppliers. A common forum was important, as stated in USGAO (1990): "According to SEMATECH'S Vice President for Supplier Relations, SEMATECH could not effectively develop a comprehensive strategy to improve relations between manufacturers and suppliers prior to the department's creation because the various SEMATECH groups dealing with suppliers were spread over a number of different departments."

(3) In June 1990, SEMATECH set up a Partnership for Quality program which provided "guidelines for improving manufacturer-supplier relations. The guidelines, which were developed by two task forces of managers from SEMATECH members and SEMI/SEMATECH suppliers, highlight the elements necessary for long-term cooperative relationships and generally call for semiconductor manufacturers to work more closely with their key U.S. suppliers by (1) sharing strategic goals and plans; (2) giving them greater access to information about the long-term performance of their equipment in a fabrication facility, including the equipment's reliability and any recurring problems; (3) providing them with competitive analysis information; and (4) supporting their product development work. Each of SEMATECH'S member companies agreed to have a high-level executive responsible for implementing the partnering guidelines. According to SEMATECH'S Vice President for Supplier Relations, SEMATECH'S Supplier Relations Action Council-which is composed of senior purchasing executives from the consortium's 14 members-will periodically review each company's progress in implementing the partnering guidelines.

The guidelines also provide (1) standards that both manufacturers and suppliers can use to continually improve the quality of products and services and (2) a total cost of ownership model for procuring equipment and materials that includes factors in addition to the purchase price, such as customer service, equipment failures, and factory yields. The cost model is *"intended to reduce uncertainties that result from manufacturers' emphasizing different factors in their procurement decisions"* (see USGAO (1990)).

As summarized in the investigation in USGAO (1992): "Most of SEMATECH'S member companies identified the partnering guidelines as one of SEMATECH'S most

important initiatives...Executives from 23 of 26 SEMI-SEMATECH members interviewed stated that relations between their companies and SEMATECH'S member companies had improved since the guidelines were adopted. Furthermore, executives from 20 of the 26 members believed that relationships between semiconductor manufacturers and their suppliers would continue to improve."

(4) Developing standardized interfaces for process tools: Grindley et al. (1994) explain the contribution of SEMATECH in developing standardized interfaces for tools: "A related SEMATECH effort to facilitate systemic innovation entails the development of standards for equipment interfaces and for evaluation of equipment. For example, SEMATECH has initiated a Computer Integrated Manufacturing (CIM) program to standardize hardware and software interfaces across equipment, in an effort to reduce problems in the insertion of improved tools into existing production lines."

(5) Centralized Tool Qualification: SEMATECH also set up a centralized *process tool qualification* facility in Austin, Texas, which certified tools made by suppliers based on requirements placed jointly by chip makers. Qualification is a lengthy process where the tools supplier demonstrates to chip manufacturer (buyer) that the process tool has met the required specifications.

Having a central tool qualification facility greatly reduced the effort required on the part of the suppliers to adapt and qualify their tools for each chip maker. As stated in USGAO (1992):"Because its member companies have accounted for about 80 percent of the U.S. semiconductor manufacturing capacity, SEMATECH is uniquely able to develop industry wide practices and standards that can improve the industry's overall efficiency. For example, SEMATECH has developed common methods for evaluating, improving, and qualifying equipment and associated software.... Industry wide standards benefit both semiconductor manufacturers and equipment and computer software suppliers, lowering costs by reducing the extent to which suppliers customize products for individual customers."

(6) Identifying common downstream problems: "Furthermore, several member companies have stated that one of SEMATECH'S most important contributions is as a forum for communications, enabling members to discuss manufacturing problems and solutions. *In many cases, member companies have found that they were protecting similar trade secrets and experiencing similar problems. Through discussions, members have better identified the nature of manufacturing problems and the success or failure of*

approaches taken to fix them."

(7) Roadmaps: Beginning in 1992, SEMATECH, in collaboration with other industry trade associations, started releasing periodic roadmaps for the evolution of semiconductor technology for the next 15 years. These roadmaps provided the consensus view of microchip manufacturers about the requirements that the tools suppliers had to meet. The roadmaps became a central feature of the semiconductor industry in the 1990s, thus effectively allowing the tools suppliers to focus and streamline their R&D projects.