On the Elasticity of Substitution between Labor and ICT and IP Capital and Traditional Capital

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Abstract

I estimate a nested CES production function for a panel of European countries, distinguishing between information and communication technologies (ICT), intellectual property (IP) capital, and traditional capital. I allow for differences in the elasticities of substitution between labor, an aggregate of ICT and IP capital, and traditional capital. Results indicate that ICT and IP together are gross substitutes for labor, whereas traditional capital is a gross complement. Rapid technological progress and capital accumulation in ICT and IP capital largely explain the decline in labor income share. The imputed labor-aggregate capital elasticity of substitution is below 1 but increases over time.

Keywords: CES Production Function; Elasticities of Substitution; System of Equations; ICT; IP Capital; Traditional Capital

JEL classification: E22; E25; J23; O33

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1 Introduction

Macroeconomic models typically incorporate explicit production technologies that combine labor and capital. The appropriateness of these models depends on the assumptions regarding the production technology, including the elasticity of substitution between labor and capital and the direction of technological change.

I use data from 9 European countries and the 1996-2020 period and estimate a nested CES production function for total industrial value added, together with the corresponding first-order conditions. I assume that production technology utilizes labor, an aggregate of information and communication technologies (ICT) and intellectual property (IP) capital, and the remainder of capital that I call traditional capital. ICT capital includes computers and peripheral equipment, IP capital includes computer software, databases, and research and development, while traditional capital comprises machinery, transport equipment, and structures. The assumption that ICT and IP capital enter production jointly is motivated by, for example, that computers and software have joint and complementary use in generation of productive capital services. Moreover, a significant share of innovation generated through research and development is related to ICT and its applications in the sample countries and period, as evidenced by patent data, for example. In the baseline specification, ICT and IP aggregate is nested with labor, and then this bundle is nested with traditional capital. This nesting structure is motivated by the automation and task-based production literature (e.g., Acemoglu and Autor, 2011, Acemoglu and Restrepo, 2018, Aum, Lee and Shin, 2018). It depicts a production environment where ICT and IP capital, as a distinct automation bundle, can perform a set of tasks otherwise carried out by labor (e.g., routine tasks). In turn, machinery, equipment, and structures require either humans or automation technologies to operate them.

The estimate of the elasticity of substitution between labor and ICT and IP capital is greater than 1, implying that labor and ICT and IP capital are gross substitutes. The estimate of the elasticity of substitution between labor and traditional capital is below 1, implying that labor and traditional capital are gross complements. Similarly, the estimate of the elasticity of substitution between ICT and IP capital is below 1.

These results can help explain, for example, the dynamics of labor income share in sample countries. I simulate a counter-factual scenario where there is no ICT- and IP-related technological progress and no changes in ICT and IP capital investment prices. The results from this exercise imply that most of the fall in labor share can be attributed to the fast technological progress and extensive accumulation of ICT and IP capital. In turn, labor income share would have been much higher absent technological progress and accumulation in ICT.

Finally, I derive the elasticity of substitution between labor and aggregate capital from the nested CES production function that incorporates multiple types of capital. Using the estimated parameters, I compute the corresponding elasticity values. The values of the implied elasticity of substitution between labor and aggregate capital are below unity using the estimates from the baseline specification, suggesting that the two inputs are gross complements, although the elasticity increases over the sample period.

For comparison, I estimate two alternative nested CES specifications. In the first, labor and traditional capital are grouped together, while ICT and IP capital form a separate nest. I then combine these two nests (Aum and Shin, 2024, Lashkari, Bauer and Boussard, 2024, employ similar specifications). In the second specification, ICT and IP capital are first nested together, then combined with traditional capital, and finally nested with labor. Under both alternative specifications, the findings consistently suggest that ICT and IP capital are gross substitutes for labor and largely responsible for the decline in labor income share. However, the elasticity of substitution between traditional capital and labor in these models is substantially higher than in the baseline specification, and the implied elasticity of substitution between labor and aggregate capital exceeds unity. As an additional comparison, I estimate the elasticity of substitution between labor and aggregate capital following a similar methodology used for the estimation of the nested CES production function. The estimated value of the elasticity of substitution between labor and aggregate capital appears to be below 1 but significantly higher than the imputed values of this parameter from the main specification. These results imply that the specification of the production function and aggregation of capital can play an

important role in determining the value of the elasticity of substitution between labor and aggregate capital.

The values of the elasticity of substitution and the direction of technological change are important for explaining, for example, movements in factor income shares (e.g., Caballero and Hammour, 1998, Karabarbounis and Neiman, 2014). A large number of studies that focus on labor share document that it has fallen. The literature offers competing explanations for this. Karabarbounis and Neiman (2014) use cross-country data and find that labor and capital are gross substitutes. They attribute the fall in labor income share to the rapid fall in prices of capital and capital deepening. Glover and Short (2020) use similar data and suggest that these estimates may be upward biased due to omitted variables. Their estimates indicate that labor and capital are gross complements.[1] The estimates of Glover and Short (2020) suggest that alternative explanations might be in order for the fall in labor income share such as, for instance, the rise in product-market concentration and import competition. The results of, for example, Barkai (2020) and Autor, Dorn, Katz, Patterson and Van Reenen (2020) support this view. In turn, Oberfield and Raval (2021) use plant-level data to estimate production functions using data from the US. They also find that labor and capital are gross complements and suggest that differences in factor-augmenting technological change are important contributors to the fall in labor income share. Moreover, Oberfield and Raval (2021) show that the aggregate elasticity of substitution between labor and capital is a convex combination of the plant-level elasticities of substitution and the elasticity of demand, and it has fallen during their sample period, 1972-2007 (see Grossman and Oberfield, 2022, Karabarbounis, 2024, for detailed reviews of the literature).

A few recent studies provide an in-depth analysis of the fall in labor income share by differentiating types of capital (Aum and Shin, 2024, Eden and Gaggl, 2018, 2019, Koh, Santaeulàlia-Llopis and Zheng, 2020). Eden and Gaggl (2018, 2019) attribute the fall in labor income share to the uptake of information and communication technologies (ICT)

[1] Herrendorf, Herrington and Valentinyi (2015) also estimate a below one elasticity of substitution between labor and capital. Gechert, Havranek, Irsova and Kolcunova (2022) corroborate this evidence in their meta-analysis of 121 studies.

and the potential high substitutability of these technologies with labor because of, for example, the ease that routine tasks yield to automation (e.g., Acemoglu and Autor, 2011, Autor, Levy and Murnane, 2003, Jerbashian, 2019). ICT is a sum of ICT equipment and software in their study. They estimate the elasticity of substitution between labor and ICT using the first-order conditions resulting from canonical firm's optimization problem similarly to, for example, Antràs (2004) and Karabarbounis and Neiman (2014).[2] In turn, Koh et al. (2020) perform an accounting exercise and show that the fall in labor income share can be attributed to the capitalization and the rise of compensation of intellectual property (IP) capital, R&D before 1980 and software after 1980.

A recent study by Lashkari et al. (2024) utilizes firm-level data from France to estimate the parameters of a nested, non-homothetic production function incorporating multiple types of capital. They find that the elasticity of substitution between ICT capital and labor is significantly below 1. Furthermore, Lashkari et al. (2024) argue that a significant part of the decline in labor income share in France can be attributed to the reallocation of economic activity from firms with low intensity of ICT capital use to those with high intensity use as the price of ICT capital falls. This reallocation contributes to the observed trend because firms that use ICT more intensively tend to have lower labor income shares. In turn, Aum and Shin (2024) use data from South Korea to estimate the parameters of a nested CES production function that, in the first step, nests labor with equipment and, in the second step, nests this nest with software capital. Their baseline estimates suggest gross complementarity between equipment and labor and gross substitutability between software and labor, as well as gross substitutability between software and equipment. Moreover, falling software prices and increased use of software are largely responsible for the fall in labor income share in South Korea. According to Aum and Shin (2024), two equally important factors drive this result. First, software increasingly replaces labor. Second, production shifts toward firms that use software more intensively, as these firms

[2] Krusell, Ohanian, Ríos-Rull and Violante (2000) and Ohanian, Orak and Shen (2023) estimate a nested CES production function with capital and labor inputs, and 2 levels of skills. They show that changes in factor inputs can account for most of the changes in skill premium in the US. Ohanian et al. (2023) also show that the incorporation of ICT capital in the production function can help explain some of the movements in labor share. The current study abstracts from levels of skills because of data limitations.

become effectively more productive. These firms have higher markups and lower labor shares in their data. As a result, this reallocation further raises the aggregate markup and reduces the aggregate labor share.

This paper differs from and contributes to these studies in multiple ways. First, I follow the approach developed and implemented by De La Grandville (1989), Klump, McAdam and Willman (2007), and León-Ledesma, McAdam and Willman (2010) and jointly estimate a normalized CES production function and first-order conditions. The normalization is motivated by the observation that the elasticity of substitution is defined as a point elasticity and its identification needs benchmark values for the level of production and factor inputs and incomes. It represents the production function in a consistent, indexed number form and facilitates the identification of parameters. León-Ledesma et al. (2010) use Monte Carlo simulations to provide comprehensive evidence regarding the superiority of this estimation method for identifying elasticities of substitution together with factor-biased technological change as compared to, for example, the estimation of first-order conditions only and a translog function. The use of this estimation method can then be especially relevant for this study because it attempts to identify these parameters for ICT that have been subject to exceptionally rapid technological progress.

Motivated by the joint use of computers and software, I also assume that a CES aggregate between ICT equipment and IP capital is a separate capital input in the production function.[3] I nest this aggregate with labor and then nest that with traditional capital in the baseline specification. This allows me to estimate value added production function. Moreover, it allows me to compute the implied elasticity of substitution between aggregate capital and labor from this production function, which accounts for multiple types of capital. The value of this elasticity is lower than 1 though the results indicate that the fall in labor income share is primarily driven by technological progress and capital accumulation in ICT and IP capital. This is consistent with the juxtaposition of the results of Aum and Shin (2024), Eden and Gaggl (2018, 2019), Karabarbounis and Neiman

[3]This aggregation is consistent with practices in related works. For example, Eden and Gaggl (2018) construct ICT capital by combining data on hardware, software, and services. The EU KLEMS database also often reports results for an aggregate of ICT hardware and software.

(2014), Glover and Short (2020), and Oberfield and Raval (2021). I demonstrate that the elasticity of substitution between aggregate capital and labor depends on the underlying elasticity of substitution parameters and factor compensation shares, and it has slightly increased over the sample period. In addition, unlike Lashkari et al. (2024), who assume unitary elasticities and equal technological change parameters for traditional capital and labor, as well as for ICT and IP capital, I estimate these parameters directly from the data.

Finally, I draw on panel data from European countries in the EU KLEMS database for my estimations (Bontadini, Corrado, Haskel, Iommi and Jona-Lasinio, 2023). Computations based on these data corroborate the findings of Koh et al. (2020), indicating that the share of capital (and labor) compensation would have remained constant if not for the increased compensation share of IP capital.

The next section describes the model. Section 3 describes the estimation framework. Section 4 describes the data and their sources. Section 5 summarizes the results. Section 6 concludes.

2 The Model

I consider a continuum of identical, infinitely lived firms of measure one, each producing a homogeneous good Y in a market with no entry costs. The representative firm discounts future profits at the rate of r and has the following production technology:

$$Y_t = \left[\varpi_{LIJ}^{\frac{1}{\varepsilon_1}} LIJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{1}$$

where

$$LIJ_t = \left[\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{IJ}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$IJ_t = \left[\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}},$$

and ϖ-s are share parameters, ε-s are Allen-Uzawa elasticity of substitution parameters, γ-s are technological progress parameters, L is labor, I is ICT capital, J is IP capital, and K is traditional capital.

The firm operates in a perfectly competitive market. It chooses labor input and investments in I, J, and K, taking as given the wage rate w, the prices of investments p_I, p_J, and p_K, and the interest rate r. The firm solves the following problem:

$$\max_{\{L_t, X_{I,t}, X_{J,t}, X_{K,t}\}} \sum_{t=0}^{+\infty} \prod_{\kappa=0}^{t} \left(\frac{1}{1+r_\kappa}\right) \left(Y_t - w_t L_t - p_{I,t} X_{I,t} - p_{J,t} X_{J,t} - p_{K,t} X_{K,t}\right) \tag{2}$$

$$s.t.$$

$$X_{I,t} = I_{t+1} - (1-\delta_I) I_t,$$

$$X_{J,t} = J_{t+1} - (1-\delta_J) J_t,$$

$$X_{K,t} = K_{t+1} - (1-\delta_K) K_t,$$

where X_i is investment in capital type $i \in \{I, J, K\}$, and $\delta_i \in (0,1)$ is the corresponding depreciation rate.

I denote by S_{LIJ_t,Y_t} the share of labor, IP capital, and ICT capital compensation in value added and use S_{IJ_t,LIJ_t} and S_{I,IJ_t} to denote the share of compensation of ICT and IP capital out of the compensation of labor and ICT and IP capital and the share of ICT capital compensation out of the compensation of ICT and IP capital:

$$S_{LIJ_t,Y_t} = \varpi_{LIJ}^{\frac{1}{\varepsilon_1}} \left(\frac{LIJ_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}.$$

$$S_{IJ_t,LIJ_t} = \varpi_{IJ}^{\frac{1}{\varepsilon_2}} \left(\frac{IJ_t}{LIJ_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}},$$

$$S_{I_t,IJ_t} = \varpi_{I}^{\frac{1}{\varepsilon_3}} \left(\frac{e^{\gamma_I t} I_t}{IJ_t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}.$$

The first-order conditions that follow from the firm's problem are given by

$$\frac{r_{I,t}I_t}{Y_t} = S_{I_t,IJ_t}S_{IJ_t,LIJ_t}S_{LIJ_t,Y_t}, \tag{3}$$

$$\frac{r_{J,t}J_t}{Y_t} = (1 - S_{I_t,IJ_t})\, S_{IJ_t,LIJ_t}S_{LIJ_t,Y_t}, \tag{4}$$

$$\frac{w_tL_t}{Y_t} = (1 - S_{IJ_t,LIJ_t})\, S_{LIJ_t,Y_t}, \tag{5}$$

$$\frac{r_{K,t}K_t}{Y_t} = 1 - S_{LIJ_t,Y_t}, \tag{6}$$

where

$$r_{i,t} = (1 + r_t)\, p_{i,t-1} - (1 - \delta_i)\, p_{i,t} \tag{7}$$

is the rental price of capital type $i \in \{I, J, K\}$. This equation is essentially a non-arbitrage condition that states that the payoff from purchasing one unit of capital type i at price $p_{i,t-1}$, using it for production, and then reselling the depreciated asset at price $p_{i,t}$, must match the payoff from earning the aggregate return r_t.

3 The Estimation Framework

The primary focus of this study is on the estimation of Allen-Uzawa elasticity of substitution parameters, ε, jointly with the technological change parameters, γ. I follow the approach developed by De La Grandville (1989), Klump et al. (2007) and León-Ledesma et al. (2010) in the estimation methodology. Specifically, this involves the joint estimation of normalized versions of equations (1) and (3)-(6), where normalization is based on the sample averages of the variables (I use geometric averages as in Herrendorf et al., 2015).

I use α_1, α_2, and α_3 to denote the geometric averages of (1) the share of labor, IP capital, and ICT capital compensation in value added, (2) the share of labor compensation out of the compensation of labor and ICT and IP capital, and (3) the share of ICT capital

compensation out of the compensation of ICT and IP capital:

$$\alpha_1 = \varpi_{LIJ}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{LIJ_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}, \tag{8}$$

$$\alpha_2 = \varpi_{L}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{e^{\gamma_L t} L_t}{LIJ_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{9}$$

$$\alpha_3 = \varpi_{I}^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_I t} I_t}{IJ_t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{10}$$

where $\overline{z}$ represents the geometric average of the variable z over time.

Using the expressions for α_1, α_2, and α_3, I write the logarithm of the normalized equation for output in the following way:

$$\ln \frac{Y_t}{\overline{Y_t}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_1 \left(\frac{LIJ_t}{\overline{LIJ_t}} \right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + (1-\alpha_1) \left(e^{\gamma_K \hat{t}} \frac{K_t}{\overline{K_t}} \right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \right], \tag{11}$$

where

$$\frac{LIJ_t}{\overline{LIJ_t}} = \left[\alpha_2 \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}} \right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + (1-\alpha_2) \left(\frac{IJ_t}{\overline{IJ_t}} \right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$

$$\frac{IJ_t}{\overline{IJ_t}} = \left[\alpha_3 \left(e^{\gamma_I \hat{t}} \frac{I_t}{\overline{I_t}} \right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\alpha_3) \left(e^{\gamma_J \hat{t}} \frac{J_t}{\overline{J_t}} \right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3-1}},$$

and $\hat{t}$ is demeaned trend,

$$\hat{t} = t - \frac{1}{T} \sum t. \tag{12}$$

I further use the expressions for S_{LIJ_t,Y_t} and S_{IJ_t,LIJ_t} and write the normalized first-order

conditions in the following way:

$$\ln r_{I,t} - \ln \overline{r_{I,t}} = \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LIJ_t,Y_t}}{\overline{S_{LIJ_t,Y_t}}}\right) + \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_1}\frac{\varepsilon_2}{\varepsilon_2 - 1}\right) \ln\left(\frac{S_{IJ_t,LIJ_t}}{\overline{S_{IJ_t,LIJ_t}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_I \hat{t} - \frac{1}{\varepsilon_3} \ln\left(\frac{I_t/\overline{I_t}}{Y_t/\overline{Y_t}}\right), \tag{13}$$

$$\ln r_{J,t} - \ln \overline{r_{J,t}} = \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LIJ_t,Y_t}}{\overline{S_{LIJ_t,Y_t}}}\right) + \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_2}{\varepsilon_2 - 1}\right) \ln\left(\frac{S_{IJ_t,LIJ_t}}{\overline{S_{IJ_t,LIJ_t}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_J \hat{t} - \frac{1}{\varepsilon_3} \ln\left(\frac{J_t/\overline{J_t}}{Y_t/\overline{Y_t}}\right), \tag{14}$$

$$\ln r_{K,t} - \ln \overline{r_{K,t}} = \frac{\varepsilon_1 - 1}{\varepsilon_1}\gamma_K \hat{t} - \frac{1}{\varepsilon_1} \ln\left(\frac{K_t/\overline{K_t}}{Y_t/\overline{Y_t}}\right), \tag{15}$$

$$\ln w_t - \ln \overline{w_t} = \left(1 - \frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LIJ_t,Y_t}}{\overline{S_{LIJ_t,Y_t}}}\right) + \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_L \hat{t} - \frac{1}{\varepsilon_2} \ln\left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}}\right). \tag{16}$$

I use the system of equations (11) and (13)-(16) in the empirical estimations. The values of r_t are needed for the estimations. I obtain its values from the zero profit condition,

$$1 + r_t = \frac{Y_t - w_t L_t + (1 - \delta_I)\, p_{I,t} I_t + (1 - \delta_J)\, p_{J,t} J_t + (1 - \delta_K)\, p_{K,t} K_t}{p_{I,t-1} I_t + p_{J,t-1} J_t + p_{K,t-1} K_t}, \tag{17}$$

assuming the data contain information on real value added, labor compensation, and prices of investments, stocks, and depreciation rates of ICT, IP, and traditional capital.[4]

4 Data

The data are from the 2023 version of the EU KLEMS database for 9 European countries from the Euro Area and the 1996-2020 period for most of the countries. Panel A.1 of Table 1 offers the averages and initial and final sample values of the key variables used in the estimations. A few notable observations are in order. The prices of ICT investments relative to value added prices have fallen markedly during the sample period,

[4]Intellectual property capital might not readily satisfy the usual replication arguments that justify the constant returns to scale assumption. I maintain this assumption together with perfect competition to facilitate normalization and to keep the estimation of the system of equations tractable and comparable with the common practice in the literature. While relaxing this assumption for IP capital can be an interesting extension, it lies beyond the scope of this paper.

with an average annual growth rate of -2.9%. This pronounced decline likely reflects the substantial technological advancements in ICT and contrasts with the dynamics observed for IP and traditional capital. Specifically, the prices of IP capital investments have fallen only marginally, at an average annual rate of -0.2%, while the prices of investments in traditional capital have increased slightly at an average annual rate of 0.2%. Moreover, the amount of ICT capital has increased at a much higher rate than IP and traditional capital.[5]

Table 1: Basic Statistics

Panel A.1

Variable	(1) Mean	(2) 1996	(3) 2020
Value Added, per hours worked (Y/L)	38667	34831	40576
Labor Income, per hours worked (wL/L)	25560	23686	27021
ICT Capital, per hours worked (I/L)	913	514	1335
IP Capital, per hours worked (J/L)	5911	4375	7790
Traditional Capital, per hours worked (K/L)	132810	116376	151653
Total Capital, per hours worked (TK/L)	138194	116460	162379
Hours Worked, mln (L)	23825	23582	18551
ICT Investment Price (p_I)	1.568	3.304	0.910
IP Investment Price (p_J)	1.008	1.019	0.982
Traditional Capital Investment Price (p_K)	1.002	0.971	1.016
Total Capital Investment Price (p_{TK})	1.013	1.013	1.001

Panel A.2

Parameter	
δ_I	0.200
δ_J	0.241
δ_K	0.036

Panel B.1

Derived Variable	(1) Mean	(2) 1996	(3) 2020
r_I	0.518	1.313	0.237
r_J	0.295	0.291	0.280
r_K	0.085	0.081	0.075

Panel B.2

Derived Parameter	
α_1	0.708
α_2	0.923
α_3	0.212

Note: This table presents average as well as sample initial and final values for the variables used in the estimations. All figures are in real terms. The values of value added, labor income, and the real volume of the services provided by each type of capital (productive capital stock) are per hours worked. Price indices are normalized to 1 in 2015 and are expressed relative to the value added price index. The sample includes Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Portugal, and Spain, covering the period from 1996 to 2020, except for Germany (1996–2019) and Portugal (2001–2020). Panels A.2 and B.1 offer the values of depreciation rates and rental prices across the different types of capital. Panel B.2 reports the computed values of α_1, α_2, and α_3 from equations (8), (9), and (10) averaged across countries. ICT capital includes information technologies and communication technologies (e.g., computers). IP capital includes software and databases, research and development, and organizational capital. Table A in the Appendix - Data Description offers detailed descriptions and sources of variables. Table I in the Data Appendix presents the basic statistics for each sample country.

[5] I use real volumes of the services provided by capital types (i.e., productive capital stock) in the empirical analysis. This relies on the assumption that the flow of capital services is directly proportional to the capital stock, effectively abstracting from time-varying utilization rates. This assumption is maintained in the EU KLEMS database and, for example, by Antràs (2004), Eden and Gaggl (2018), Herrendorf et al. (2015), and Lashkari et al. (2024).

Panel A.2 of Table 1 reports the computed annual rates of depreciation across the different types of capital. ICT and IP capital depreciate at very high rates of around 20% and 24%, respectively, whereas traditional capital depreciates at the rate of 3.6%. I use these figures and prices of investments in the types of capital to compute the aggregate return and the rental prices for each type of capital from non-arbitrage condition (7) and the zero profit condition (17) and report the results in Panel B.1 of Table 1. In turn, Panel B.2 of Table 1 reports the computed values of α_1, α_2, and α_3 averaged across sample countries. The average share of labor, IP capital, and ICT capital compensation in value added is close to 71%. This value, together with the usual values of labor income share, suggests that labor compensation is much larger than the compensation of IP and ICT capital. The value of the average share of labor compensation out of the compensation of labor and ICT and IP capital α_2 reflects this as it is close to 92%. In turn, the average share of ICT capital compensation out of the compensation of ICT and IP capital is close to 21%, implying that the compensation of ICT capital is much smaller than the compensation of IP capital.[6]

Finally, Figure 1 illustrates labor income share and income shares of ICT, IP, and traditional capital in sample countries. I compute the latter using the rental prices of ICT, IP, and traditional capital. Table 2 reports the values of these shares at the beginning of the sample period and the end.

Table 2: Sample Initial and Final Values of Factor Income Shares

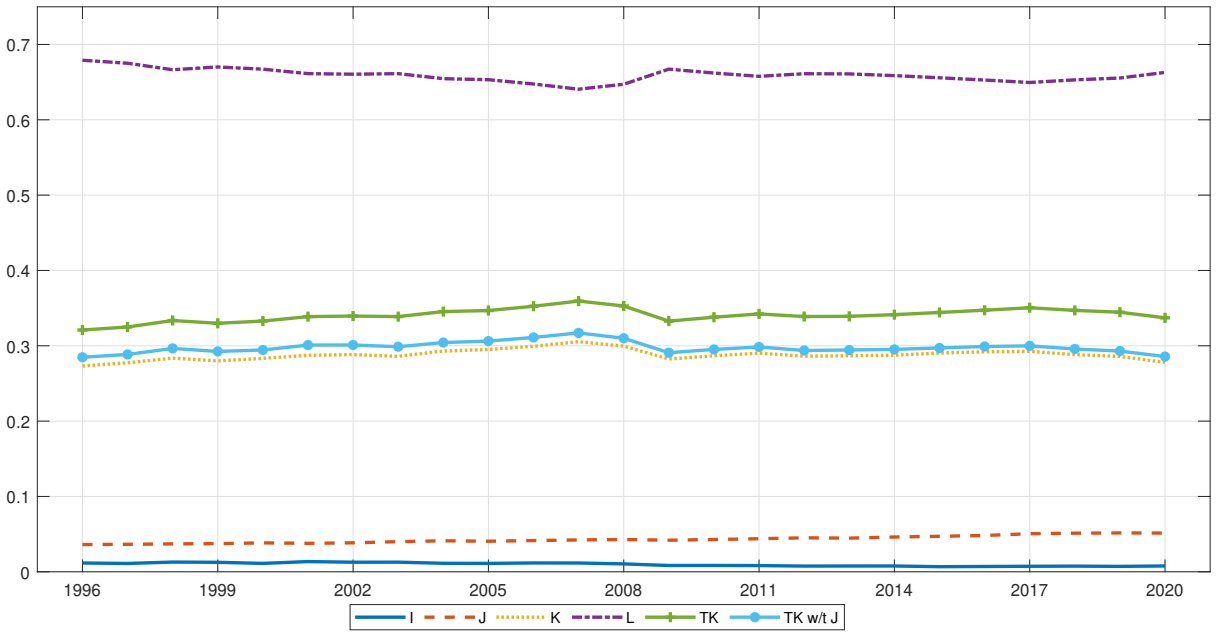
Year	I	J	K	L
1996	0.013	0.034	0.273	0.679
2020	0.010	0.052	0.281	0.657

Note: This table presents the average income shares of factor inputs, including ICT capital I, IP capital J, traditional capital K, and labor L, in sample countries at the beginning and end of the sample period. See Table A in the Appendix - Data Description for complete descriptions and sources of variables.

The average labor income share has fallen during the sample period from 67.9% to 65.7% in the sample European countries. This fall can be largely accounted for by the

[6]The lower average hours worked in 2020 compared to 1996 are due to the exclusion of Germany from the 2020 average and Portugal from the 1996 average. Notably, excluding 2020, the year of the pandemic-induced recession, from the sample does not alter the main results.

Figure 1: The Shares of Income of Factor Inputs



Note: This figure illustrates the average income shares of factor inputs, including ICT capital I, IP capital J, traditional capital K, labor L, total capital TK, and total capital without IP capital (TK w/t J) in the sample countries.

rise in the compensation share of IP capital. Labor income share would be virtually constant absent this raise, i.e., if either IP capital compensation or the raise in it could be attributed to labor income. These results corroborate the results of Koh et al. (2020) that the compensation share of labor/capital is virtually flat in the US absent the rise in the compensation of IP capital.[7] Nevertheless, the share of joint compensation of ICT and IP does not increase as much as the compensation share of IP capital. Labor income share would still fall in European countries absent changes in these two.[8]

5 Results

I perform all estimations using the feasible generalized nonlinear least squares estimator for seemingly unrelated regressions, allowing for arbitrary heteroskedasticity and serial

[7]The inclusion of research and development in IP capital connects the paper to the intangibles literature that documents the role of software, data, and R&D in factor-share dynamics and productivity growth (e.g., Barkai, Nallareddy and Ogneva, 2025, Bontadini et al., 2023, Corrado, Haskel, Jona-Lasinio and Iommi, 2022, Koh et al., 2020, McGrattan, 2020). It also mitigates inconsistencies arising from taxonomy, where software and databases developed within R&D processes are classified under R&D.

[8]Similar results obtain when using weighted averages across sample countries. Figure I and Table II in the Data Appendix show that these findings also hold for the United States.

correlation in the residuals. I replace α_1, α_2, and α_3 in the normalized production function (11) with their corresponding values in the data and use the values of the variables along with the computed shares of compensation in the estimations. Further, I use within-country averages of variables for normalizations in equations (11)-(16) and the country-year-level shares of hours of employment out of the total hours of employment across sample countries as weights in these regressions. Consequently, parameter identification relies on within-country and over-time variation, which is analogous to a fixed-effects estimation in linear models, and the results apply to a weighted average across sample countries. Finally, I use multiple starting/initial points in estimations for the elasticity of substitution parameters and select the results that yield the best fit based on Log Likelihood [the Appendix - Empirical Specifications provides additional details about the empirical specifications].[9]

Column 1 of Table 3 presents the results from the estimation of the system of equations (11)-(16). The estimated elasticity of substitution between labor and traditional capital, ε_1, is 0.745 and it is statistically significantly below 1, implying that labor and traditional capital are gross complements. The estimated elasticity of substitution between labor and ICT and IP capital, ε_2, is 1.187 and it is statistically significantly above 1 implying that labor and ICT and IP capital are gross substitutes.[10] In turn, the estimate of the elasticity of substitution between ICT and IP capital, ε_3, is 0.961 and it is significantly below 1, implying that these two types of capital are gross complements.

The estimate of the ICT capital augmenting technical change parameter γ_I is positive and large. The high value of γ_I can possibly reflect the rapid technological progress in information and communication technologies. The estimate of IP capital augmenting technical change parameter γ_J is negative and its absolute value is smaller than γ_I. In turn, the estimate of the traditional capital augmenting technical change parameter γ_K is negative though small and statistically insignificant, while the estimate of the labor

[9] In these estimations, I disregard results that produce excessively high values for the elasticities of substitution, as well as those with significantly negative values for γ_I.

[10] Eden and Gaggl (2018) have similar a finding for ICT capital that includes software using US data and single equation/first-order condition with no biased technological progress parameters. Antràs (2004) shows that this can introduce an upward bias in the estimates of the elasticity of substitution.

Table 3: Estimation Results

A. Estimates	(1) Main	(2) Main IR	(3) Eq. (18)	(4) Eq. (18) IR	(5) Eq. (19)	(6) Eq. (19) IR
ε_1	0.745***	0.754***	1.582***	1.666***	1.212***	2.367***
	(0.009)	(0.016)	(0.142)	(0.180)	(0.045)	(0.247)
$p_{\varepsilon_1=1}$	0.000	0.000	0.000	0.000	0.000	0.000
ε_2	1.187***	1.344***	0.993***	1.001***	1.127***	1.162***
	(0.010)	(0.047)	(0.004)	(0.003)	(0.015)	(0.019)
$p_{\varepsilon_2=1}$	0.000	0.000	0.767	0.767	0.000	0.000
ε_3	0.961***	0.955***	0.954***	0.962***	0.969***	0.969***
	(0.003)	(0.005)	(0.004)	(0.003)	(0.003)	(0.003)
$p_{\varepsilon_3=1}$	0.000	0.000	0.000	0.000	0.000	0.000
γ_I	0.725***	0.626***	0.663***	0.724***	0.880***	0.856***
	(0.065)	(0.078)	(0.045)	(0.050)	(0.099)	(0.086)
γ_J	-0.111***	-0.156***	-0.167***	-0.189***	-0.101***	-0.137***
	(0.018)	(0.024)	(0.014)	(0.016)	(0.022)	(0.023)
γ_K	-0.002	-0.007***	-0.103***	-0.111***	-0.021***	-0.015***
	(0.002)	(0.002)	(0.006)	(0.007)	(0.002)	(0.002)
γ_L	0.003**	0.008***	0.051***	0.055***	0.009***	0.008***
	(0.001)	(0.001)	(0.003)	(0.003)	(0.001)	(0.000)
Obs. (per eq.)	219	219	219	219	219	219
B. Measures of Fit						
Log Likelihood	2262	2043	2456	2247	2177	2362
R2						
$\ln\left(Y_t/\overline{Y}\right)$	0.889	0.883	0.960	0.958	0.909	0.893
$\ln\left(r_{I,t}/\overline{r_I}\right)$	0.864	0.901	0.872	0.906	0.861	0.904
$\ln\left(r_{J,t}/\overline{r_J}\right)$	0.212	-0.038	0.295	0.120	0.020	-0.188
$\ln\left(r_{K,t}/\overline{r_K}\right)$	0.638	0.274	0.617	0.621	0.968	0.948
$\ln\left(w_t/\overline{w}\right)$	0.968	0.639	0.873	0.814	0.867	0.910

Note: This table offers the results from the estimation of the normalized and logarithmed production function together with the normalized first-order conditions, where the normalizations are based on within-country averages. Panel *A* offers the estimates of the parameters and the corresponding number of observations in each equation. Panel *B* offers various measures of fit, including Log Likelihood and R-squared of each equation. Column 1 offers the main results from the estimation of the system of equations (11) and (13)-(16). Column 2 offers the results from the estimation of the system of equations (11) and (13)-(16) using instrumented regressors. Columns 3-6 offer results from the estimation of systems of equations corresponding to other CES nests, (18) and (19) [see Technical Appendix for the first-order conditions in these columns]. Rows denoted $p_{\varepsilon_i=1}$ report the p-values associated with tests of the null hypothesis $\varepsilon_i = 1$ for $i = 1, 2, 3$. Negative R-squared in an equation means that the residual sum of squares is larger than the total sum of squares in that equation. Figures II, III, and IV show the fit for specifications (1), (18), and (19). All regressions use the feasible generalized nonlinear least squares estimator for seemingly unrelated regressions and the country-year-level shares of hours of employment out of the total hours of employment across sample countries as weights. Initial/starting points for estimations are $\varepsilon_i = \{0.5, 1.5\}$ for $i = 1, 2, 3$ and $\gamma_j = 0.02$ for $j = I, J, K, L$. The reported results correspond to the estimations with the best log-likelihood fit, excluding cases where the estimated elasticity of substitution between a type of capital and labor is implausibly high and/or the estimate of γ_{ICT} is negative. Standard errors are in parentheses and are robust to arbitrary heteroskedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

augmenting technical change parameter γ_L is small and positive.

Several studies estimate a negative technical change parameter for (total) capital (e.g., Herrendorf et al., 2015, Mućk, 2017). Nevertheless, the negative values of γ_J and

γ_K remain challenging to interpret. Such values suggest that the estimates of γ_J and γ_K might capture processes beyond technological progress. In this regard, a potential explanation can be that some parts of the accumulated IP and traditional capital may not be fully utilized in the near term, though the rental prices of these types of capital continue adhering to the non-arbitrage condition (7). Admittedly, equations (1) and (3)-(6) do not readily support this interpretation. Jiang and León-Ledesma (2018), drawing on data from De Loecker, Eeckhout and Unger (2020), incorporate variable markups into their estimation of value added function and first-order conditions for labor and total capital. They show that this can reverse the negative sign of the estimate of (total) capital augmenting technological change. While promising, applying this strategy in the multi-country context of the current study is challenging due to data limitations. Moreover, the results of this study suggest the need to measure markups that vary across different types of capital, such as IP and traditional capital. A further possible drawback is that the data used in this study come from a single source that ensures consistency across variables and introducing markup measures from external sources could raise additional measurement concerns.[11]

A potential threat to identification in this (factor) demand-side system arises from the joint variation in local supply and demand for factor inputs. Several considerations indicate that this concern may be limited. First, most of the variation in ICT and IP (software) investment prices originates outside the sample countries, primarily from the United States. Second, the estimation exploits returns and wages within countries that belong to a block with highly integrated capital and labor markets. Nevertheless, to further alleviate potential concerns about this issue, I use data from the US to construct counterparts for ICT, IP, and traditional capital investment prices, as well as returns r and wages w. I use these variables as instruments for the corresponding variables in the sample countries. Column 2 of Table 3 reports the results based on these instrumented regressors. While some of the estimated parameters are statistically different from those in Column 1, the differences are economically small and do not alter the main conclusions.

[11] Task gains by capital, such as those resulting from automation, could also generate negative values of γ_J and γ_K (Aghion, Jones and Jones, 2019).

Moreover, the estimated coefficients using the instrumented regressors tend to be noisier, as they exhibit larger standard errors.

5.1 The Elasticity of Substitution between Labor and ICT and IP Capital: Alternative Nests

I follow the literature on automation and labor demand and write the CES nests in equation (1) so that the production function permits a difference in the elasticities of substitution between labor and traditional capital and labor and ICT and IP capital (e.g., Acemoglu and Autor, 2011, Acemoglu and Restrepo, 2018, Aum et al., 2018). By construction, the elasticity of substitution between labor and traditional capital and the elasticity of substitution between ICT and IP capital and traditional capital are the same in equation (1). A rationale for this is that the aggregate of ICT and IP capital, as an automation bundle, performs tasks that could otherwise be carried out by labor (e.g., routine tasks). I explore two alternative nests in this section to assess the elasticity of substitution between labor and ICT and IP capital. To save space, I will reuse the letters ε, γ, and ω and their indices, albeit this involves an abuse of notation. In particular, ε_i for $i = 1, 2, 3$ will correspond to the i-th nest starting from the most outer nest, while γ_j for $j \in I, J, K, L$ continues to denote the technological change parameter pertaining to the corresponding factor input and ω-s are share parameters.

First, I consider a specification where the elasticity of substitution between labor and ICT and IP capital is equal to the elasticity of substitution between traditional capital and ICT and IP capital. In this specification, I combine labor and traditional capital in one nest, ICT and IP capital in another, and aggregate these two nests at the final stage (see Aum and Shin, 2024, Lashkari et al., 2024, for similar specifications). In particular, I assume that the production function is given by

$$Y_t = \left(\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right)^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{18}$$

where

$$LK_t = \left[\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$
$$IJ_t = \left[\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}}.$$

I estimate the parameters using normalized and logarithmed Y_t from equation (18) and the corresponding normalized and logarithmed first-order conditions [see equations (32)-(36) in the Technical Appendix]. Columns 3 and 4 of Table 3 report the results using raw regressors and their instrumented counterparts. The estimated elasticity of substitution between labor, as well as traditional capital, and the aggregate of ICT and IP capital, ε_1, is large and significantly above 1. Consistent with the reported results in columns 1-2, the estimates of ε_3 imply that ICT and IP capital are gross complements. However, unlike columns 1–2, the estimates of the elasticity of substitution between labor and traditional capital, ε_2, are not statistically different than 1. The estimates of the parameters γ are also in line with the results from column 1, with a few notable differences. The estimate of γ_L is now close to 0.05, while the estimate of γ_K is negative and large in absolute value, close to -0.1.

Another specification nests first the different types of capital and then nests these with labor. It is given by

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1-1}}, \tag{19}$$

where

$$IJK_t = \left[\varpi_{IJ}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$
$$IJ_t = \left[\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}}.$$

This specification can be thought to be a generalization of a specification that combines total capital, TK, with labor as it permits imperfect substitutability between different

types of capital as well as technological change parameters specific to each type of capital.[12]

Columns 5-6 of Table 3 report the results from the estimation of the normalized and logarithmed Y_t from equation (19) and the corresponding normalized and logarithmed first-order conditions using raw regressors and their instrumented counterparts [see equations (42)-(46) in the Technical Appendix]. The estimated elasticity of substitution between labor and the CES aggregate of ICT, IP and traditional capital, ε_1, is large and significantly above 1. The estimates of ε_2 imply that traditional capital and the aggregate of ICT and IP capital are gross substitutes and that ICT and IP capital are gross complements. Similarly to the results in columns 2-3, the estimates of the parameters γ are in line with the results from columns 1-2. The estimate of ε_1 exceeds 2 in column 6, a magnitude that may be regarded as atypically large. However, it is accompanied by a large standard error, indicating substantial imprecision in the estimate.

The results from estimations of these two CES nests, similar to the baseline results, suggest that ICT and IP capital are gross substitutes for labor. While informative, these alternative nests may be somewhat less aligned with the literature on automation and labor demand and imply a gross substitutability between traditional capital and the aggregate of ICT and IP capital.[13]

Taken together, these results can have important implications for the dynamics of labor income share. They can also have implications regarding the elasticity of substitution between labor and aggregate capital.

5.2 Labor Income Share

The labor income share has fallen during the sample years in the sample European countries according to Panel A of Table 4. This fall is also visible in Figure 2.

[12] The challenge of aggregating heterogeneous capital into a single measure has remained unresolved since the Cambridge–Cambridge controversies (see Cohen and Harcourt, 2003, for a discussion of the issue).

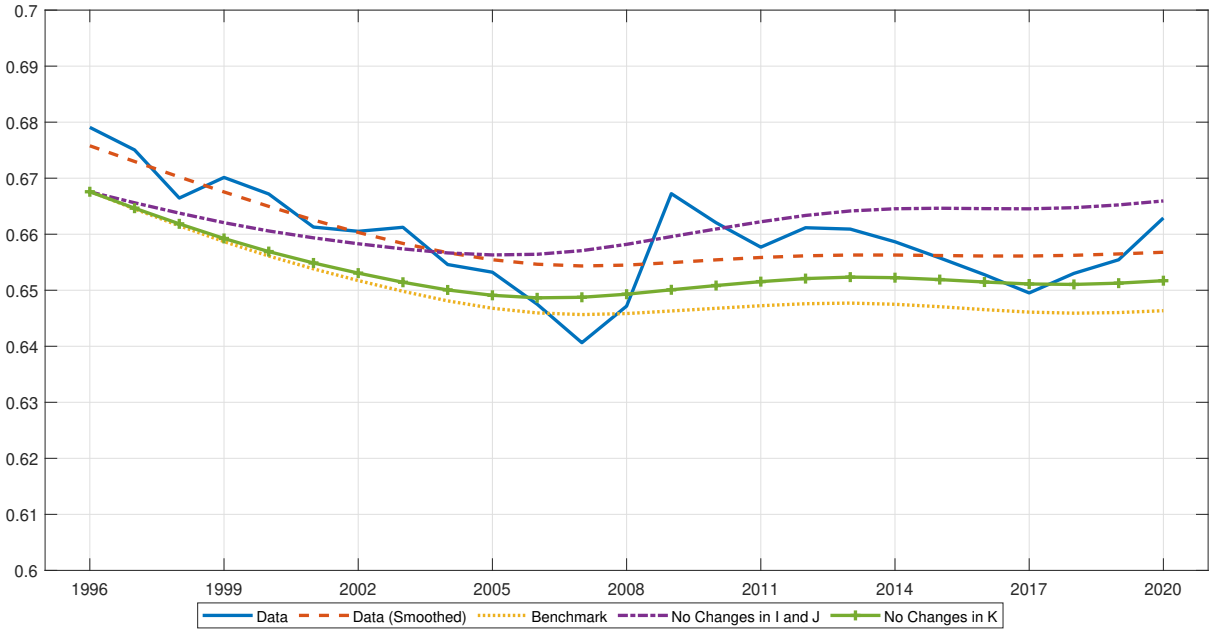
[13] The Appendix - Robustness Checks and Additional Results presents a range of robustness checks, including alternative calculations of rental prices and wages, specifications of the technical change process, and definitions of IP capital. It also reports the estimation results from a specification that first nests L and J, then nests this with I, and finally with K. In total, there are 16 distinct nest specifications with 4 factor inputs. The EU KLEMS database also provides data on additional capital types. Estimating the remaining nests and further disaggregating capital inputs are beyond the scope of the present paper.

I use the parameter values from Table 3 and the first-order conditions implied by the firm's problem to predict labor income share and to conduct counterfactual analysis. In these simulations, I begin with an initial set of investment price levels that solve for the corresponding initial values of normalized capital stocks, given the computed price growth at the beginning of the sample. I then impose the paths of both prices and normalized hours worked and solve for optimal investments in capital types under the assumption of a perfect foresight equilibrium. Further, I consider the estimated trends in the production function and the paths in investment prices as sources of exogenous variation. The latter is justified to the extent that, in the current context, the prices of capital can be interpreted as the transformation rate of output into capital. In turn, these sources of variation capture changes in the "effective" price of capital from the firm's point of view.

The first row in Panel B.1 of Table 4 reports the predicted labor income share at the start and end of the sample period using parameter estimates from columns 1 and 2 of Table 3. While these predictions slightly overstate the decline in labor share for European countries, they are very close to the data. Panel B.1 of Table 4 and Figure 2 also offer counterfactual predictions for cases when there is no technological progress and changes in investment prices in ICT and IP capital, I and J, and traditional capital, K. I fix the corresponding trend index to its sample initial value to have no technological progress and set the value of the investment price equal to its sample initial value to have no changes in it while allowing the firm to optimally select investments.

The counterfactual exercises based on parameter estimates from columns 1 and 2 of Table 3 indicate that, in the absence of technological progress and changes in investment prices in I and J, the labor share in 2020 would be substantially higher than observed. Moreover, without technological progress and the fall in investment prices in ICT capital I, the labor income share would actually increase over the sample period. The implied importance of the combination of ICT and IP capital, I and J, and K for explaining changes in the labor income share is sensitive to the estimated parameter values according to columns 1-2 and 3-4. Nevertheless, technological progress and shifts in investment

Figure 2: Labor Income Share: Data, Predicted, and Counterfactual



Note: This figure illustrates the labor income share computed using the data from the EU KLEMS database. It also illustrates the predicted labor income share using equation (5) and parameter estimates from column 1 of Table 3. The counterfactual predictions are for cases when there is no technological progress and changes in investment prices in ICT and IP capital, I and J, and traditional capital K. The corresponding trend index is fixed to its sample initial value to have no technological progress in the type of capital, and the value of the investment price is set equal to its sample initial value. The averages across sample countries are weighted by the country-level shares of employment hours out of the total hours of employment across sample countries.

prices in I and J emerge as key determinants of the observed fall.[14]

Panel B.2 of Table 4 presents predictions for the labor income share and counterfactual exercises based on parameter estimates from columns 3–4 of Table 3 that correspond to the production function specified in equation (18). As in Panel B.1, the predicted decline in the labor income share aligns well with the data. The counterfactual analysis indicates that, absent technological progress and changes in investment prices in ICT and IP capital, I and J, the labor share would remain virtually constant throughout the sample period. Moreover, the labor share would increase over the sample period without technological progress and the decline in investment prices for ICT capital I alone. According to the parameter estimates from columns 3–4, traditional capital K has little to no effect on the labor income share during the sample period.

[14]The marked rise in labor income share for the counterfactual with respect to I and the fall in it for the counterfactual with respect to J in column 4 of Panels B.2–B.3 primarily stem from the higher estimated value of ε_1 in those specifications. Both effects become substantially less pronounced when lower values of ε_1 within the associated confidence interval are used.

Table 4: Labor Income Share: Data, Predicted, and Counterfactual

A. Data	(1) 1996	(2) 2020
Data	0.679	0.663
Data (Smoothed)	0.676	0.657

B. Predictions and Counterfactuals

			IR	
	(1)	(2)	(3)	(4)
B.1 Main	1996	2020	1996	2020
Prediction	0.668	0.645	0.671	0.649
I and J	0.668	0.666	0.671	0.661
I	0.668	0.679	0.671	0.688
J	0.668	0.613	0.671	0.539
K	0.668	0.651	0.671	0.660
B.2 Eq (18)	1996	2020	1996	2020
Prediction	0.667	0.646	0.664	0.649
I and J	0.667	0.660	0.664	0.663
I	0.667	0.683	0.664	0.689
J	0.667	0.389	0.664	0.193
K	0.667	0.651	0.664	0.649
B.3 Eq (19)	1996	2020	1996	2020
Prediction	0.656	0.651	0.657	0.626
I and J	0.656	0.684	0.657	0.767
I	0.656	0.698	0.657	0.840
J	0.656	0.620	0.657	0.148
K	0.656	0.617	0.657	0.410

Note: Panel A of this table reports the initial and final sample values of the labor income share, together with the HP-filtered series ($\lambda = 100$) for the sample European countries, computed using data from the EU KLEMS database. Panel B presents predicted labor income shares and the results from counterfactual exercises based on estimates from columns 1–6 of Table 3. The counterfactual exercises consider cases with no technological progress and no changes in investment prices for: (1) ICT and IP capital, I and J, (2) ICT capital I, (3) IP capital J, and (4) traditional capital K. For each exercise, the corresponding trend index is fixed at its initial sample value to eliminate technological progress, and investment prices are held constant at their initial levels. Country-level averages are weighted by the share of country-level employment hours relative to total employment hours across sample countries.

Panel B.3 of Table 4 presents predictions for the labor income share and counterfactual exercises based on parameter estimates from columns 5–6 of Table 3 that correspond to the production function specified in equation (19). In this specification, the predicted labor income share falls very little as compared to the data and is not well aligned with it. Nevertheless, the counterfactual analysis still indicates that technological progress and changes in investment prices in ICT and IP capital, I and J, are important determinants for the dynamics of labor income share. Without these, the labor share would be much larger in 2020 than the observed value.[15]

These counterfactual exercises suggest that rapid technological progress and the accumulation of ICT and IP capital have been key factors in the decline of the labor income share. This is consistent with the results of, for example, Aum and Shin (2024), Eden and Gaggl (2018), Ohanian et al. (2023), and Koh et al. (2020).

5.3 The Elasticity of Substitution between Labor and Aggregate Capital

What do the estimates of the elasticities of substitution in 3 imply regarding the elasticity of substitution between labor and total/aggregate capital? In an attempt to answer this question, I write the level of aggregate capital in the following way

$$TK = \frac{r_I}{r_{TK}} I + \frac{r_J}{r_{TK}} J + \frac{r_K}{r_{TK}} K, \tag{20}$$

where r_{TK} is its rental price. I further assume that the output with aggregate capital is represented as a function $f(L, TK)$ with the following properties

$$f(L, TK) = Y(L, I, J, K), \tag{21}$$

[15] Allowing investment prices to deviate from their sample-initial levels alters the counterfactuals only modestly, indicating these results are driven primarily by factor-augmenting technical change.

and

$$\frac{\partial f}{\partial L}L = \frac{\partial Y}{\partial L}L, \tag{22}$$

$$\frac{\partial f}{\partial TK}TK = \frac{\partial Y}{\partial I}I + \frac{\partial Y}{\partial J}J + \frac{\partial Y}{\partial K}K. \tag{23}$$

Given these assumptions, it is straightforward to show that, when Y is given by either (1) or (18), the elasticity of substitution between labor and aggregate capital is:

$$\mathcal{E}_{L,TK} = \frac{1}{\tilde{S}\frac{1}{\varepsilon_2} + \left(1-\tilde{S}\right)\frac{1}{\varepsilon_1}} \tag{24}$$

where $\tilde{S} \in (0,1)$. It is defined as

$$\tilde{S} = \begin{cases} \frac{S_{IJ_t,LIJ_t}}{1-S_{LIJ_t,Y_t}+S_{IJ_t,LIJ_t}S_{LIJ_t,Y_t}} & \text{if } Y \text{ in (1)}, \\ \frac{1-S_{L_t,LK_t}}{1-S_{L_t,LK_t}+\left(1-S_{LK_t,Y_t}\right)S_{L_t,LK_t}} & \text{if } Y \text{ in (18)}, \end{cases} \tag{25}$$

where the term S_{IJ_t,LIJ_t} is the share of IJ in the compensation of labor and IJ, S_{L_t,LK_t} is the share of L in the compensation of LK, while S_{LIJ_t,Y_t} and S_{LK_t,Y_t} are the shares compensation of labor L and IJ and labor L and traditional capital K out of value added, correspondingly.

The equation for $\mathcal{E}_{L,TK}$ (24) implies that if the elasticities of substitution between labor and capital are the same for the traditional capital and the aggregate of ICT and IP capital, $\varepsilon_1 = \varepsilon_2$, then the elasticity of substitution between aggregate capital is not different than these two, $\mathcal{E}_{L,TK} = \varepsilon_1 = \varepsilon_2$. More generally, $\mathcal{E}_{L,TK}$ lies between ε_1 and ε_2 and is increasing in both ε_1 and ε_2. It also increases with S_{IJ_t,LIJ_t} and S_{LIJ_t,Y_t} when Y is given by (1) and $\varepsilon_1 < \varepsilon_2$. It increases with S_{L_t,LK_t} and declines with S_{LK_t,Y_t} when Y is given by (18) and $\varepsilon_1 > \varepsilon_2$.

I compute the elasticity of substitution between labor and aggregate capital $\mathcal{E}_{L,TK}$ using equations (24) and (25) for Y specified by (1) and the parameter estimates reported in columns 1 and 2 of Table 3. The results, presented in columns 1 and 2 of Table 5, indicate that $\mathcal{E}_{L,TK}$ is below unity throughout the sample period. This is in line

with the estimates reported by Herrendorf et al. (2015) and Oberfield and Raval (2021), for example. Repeating the exercise for Y specified by (18) and using the parameter estimates in columns 3 and 4 of Table 3, I obtain the values shown in columns 3 and 4 of Table 5. In this case, $\mathcal{E}_{L,TK}$ is greater than 1 throughout the sample years. In both cases, the computed elasticity of substitution $\mathcal{E}_{L,TK}$ is considerably lower than the estimated ε_1 in columns 5 and 6 of Table 3, which can be interpreted as the elasticity of substitution between labor and aggregate capital, where aggregate capital is a CES composite of traditional, ICT, and IP capital (the value in column 5 aligns with the estimates reported by Karabarbounis and Neiman, 2014). Moreover, in both cases $\mathcal{E}_{L,TK}$ varies over time due to changes in compensation shares, exhibiting an upward trend over the sample period in the European countries. The differences in the values of $\mathcal{E}_{L,TK}$ in columns 1–4 of Table 5 and the estimated ε_1 in columns 5 and 6 of Table 3 highlight that the assumed aggregation of capital can influence the inference about the magnitude of the elasticity of substitution between labor and aggregate capital. Importantly, it can affect the inference regarding whether these inputs are gross complements or substitutes.[16]

For comparison, I assume that the representative firm has the following production technology:

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{26}$$

where TK denotes the total capital stock encompassing all capital types and $\mathcal{E}_1$ is allowed to vary over time.[17] The first-order conditions that follow from profit maximization in this case are:

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}},$$

$$\frac{r_{TK,t} TK_t}{Y_t} = \frac{\varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}.$$

[16] Table III in the Data Appendix section reports the values of the share variables S_{IJ_t,LIJ_t}, S_{LIJ_t,Y_t}, S_{L_t,LK_t}, and S_{LK_t,Y_t}. The Appendix - Imputed Elasticity of Substitution provides additional results for the imputed elasticities of substitution.

[17] The Technical Appendix describes the environment in greater detail.

Table 5: The Imputed Elasticity of Substitution between Labor and Aggregate Capital

Year	(1) Main	(2) Main IR	(3) Eq (18)	(4) Eq (18) IR
1996	0.799	0.822	1.025	1.034
1997	0.800	0.824	1.026	1.035
1998	0.802	0.825	1.027	1.036
1999	0.803	0.827	1.028	1.036
2000	0.804	0.828	1.029	1.037
2001	0.805	0.829	1.030	1.038
2002	0.806	0.831	1.031	1.038
2003	0.808	0.832	1.032	1.039
2004	0.809	0.833	1.033	1.039
2005	0.810	0.834	1.033	1.040
2006	0.810	0.834	1.034	1.040
2007	0.811	0.835	1.034	1.040
2008	0.812	0.835	1.035	1.041
2009	0.814	0.837	1.035	1.041
2010	0.814	0.837	1.036	1.041
2011	0.815	0.837	1.036	1.042
2012	0.816	0.838	1.036	1.042
2013	0.817	0.838	1.036	1.042
2014	0.817	0.838	1.036	1.042
2015	0.818	0.838	1.036	1.043
2016	0.818	0.837	1.036	1.043
2017	0.818	0.837	1.035	1.043
2018	0.819	0.836	1.035	1.043
2019	0.819	0.836	1.035	1.043
2020	0.822	0.838	1.035	1.044

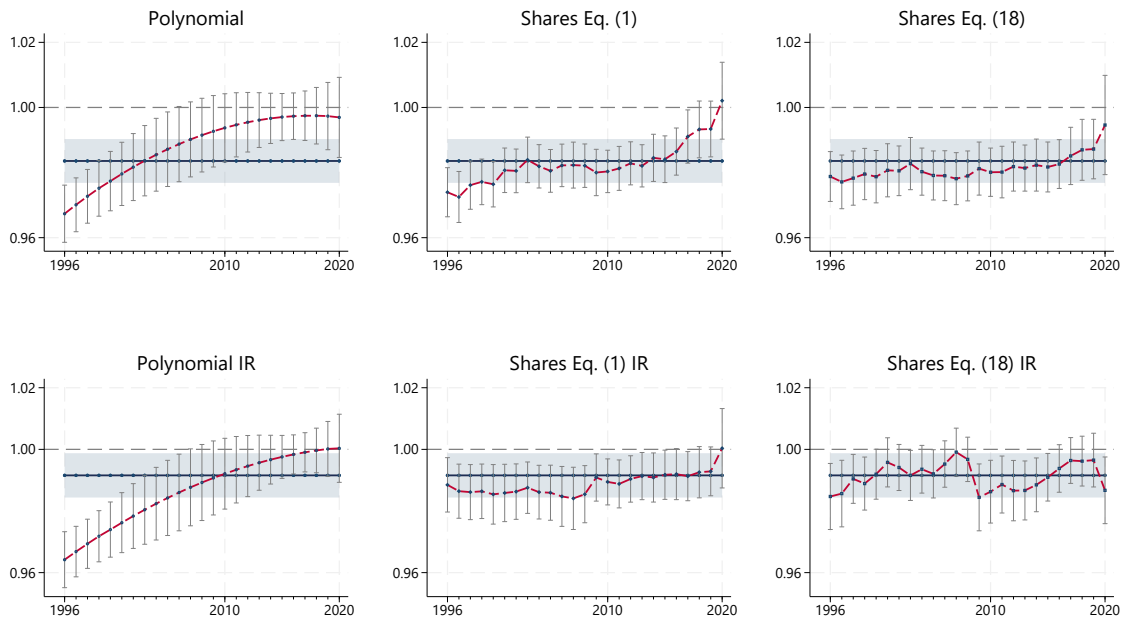
Note: Columns 1 and 2 of this table offer the values of $\mathcal{E}_{L,TK}$ computed using equations (24) and (25) for Y defined by equation (1) and estimated values of parameters from columns 1 and 2 of Table 3. Columns 3 and 4 of this table offer the values of $\mathcal{E}_{L,TK}$ computed using equations (24) and (25) for Y defined by equation (18) and estimated values of parameters from columns 3 and 4 of Table 3. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across the sample European countries. Table III in the Data Appendix reports the values of S_{IJ_t,LIJ_t}, S_{LIJ_t,Y_t}, S_{L_t,LK_t}, and S_{LK_t,Y_t}. The Appendix - Data Description offers further details about the data.

I consider 3 cases for $\mathcal{E}_1$: 1. $\mathcal{E}_1 \equiv \varepsilon_1$ is constant (canonical 2-factor CES), 2. it is a quadratic polynomial of time, $\mathcal{E} = \varepsilon_1 + \varepsilon_\tau t + \varepsilon_{\tau\tau} t^2$, and 3. it linearly depends on $\tilde{S}$, $\mathcal{E}_1 = \varepsilon_1 + \varepsilon_\sigma \tilde{S}$. I estimate the parameters in equation (26) using normalized and logarithmed Y from equation (26) and the corresponding normalized and logarithmed first-order conditions [see equations (48)-(50) in the Technical Appendix].

Table 6 reports the results. The estimation results in column 1 are for the case when $\mathcal{E}_1$ is assumed to be constant. This corresponds to the CES production function commonly employed and estimated in the literature (e.g., Antràs, 2004, Herrendorf et al., 2015, Klump et al., 2007, Glover and Short, 2020, Karabarbounis and Neiman, 2014).

The estimates of $\mathcal{E}_1$ are below 1 but close to it. They are significantly different from 1, however. They are also significantly higher than the values of $\mathcal{E}_{L,TK}$ in columns 1 and 2 of Table 5 and lower than the values of $\mathcal{E}_{L,TK}$ in columns 3 and 4. Columns 2-4 report the estimation results for the case when the elasticity of substitution $\mathcal{E}_1$ is assumed to be either a quadratic polynomial of time or a linear function of $\tilde{S}$. These estimates suggest that allowing for time variation in the elasticity of substitution between labor and aggregate capital provides a somewhat better fit to the data.[18] Finally, columns 5–8 report results obtained when instrumenting r_{TK} and w with their US counterparts. The findings remain very similar.

Figure 3: The Estimated Elasticity of Substitution between Labor and Aggregate Capital



Note: This figure illustrates the estimated values of $\mathcal{E}_1$ based on Table 6. The solid line shows the case where $\mathcal{E}_1$ is constant, with the shaded area indicating its 95% confidence interval. Dashed lines represent specifications where $\mathcal{E}_1$ varies over time, either as a quadratic function of time or linearly with $\tilde{S}$, and the error bars denote 95% confidence intervals. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across the sample European countries.

Figure 3 illustrates the values of $\mathcal{E}_1$ computed based on estimates from Table 5. The solid line shows the case where $\mathcal{E}_1$ is constant, with the shaded area representing its 95%

[18] Koh and Santaeulàlia-Llopis (2022) estimate time-varying elasticity of substitution between labor and capital at business cycle frequencies and find that it is countercyclical.

confidence interval. In turn, the dashed lines correspond to specifications where $\mathcal{E}_1$ varies over time, either as a quadratic function of time or linearly with $\tilde{S}$, with bars indicating 95% confidence intervals. In these cases, the estimated elasticity tends to increase over time. It is often significantly below 1 at the beginning of the sample period, but becomes statistically indistinguishable from 1 toward the end. The increase in $\mathcal{E}_1$ is in line with the changes in imputed elasticity of substitution $\mathcal{E}_{TK}$ in Table 5.

The elasticity of substitution between labor and aggregate capital exceeds 1 when using the production function specification in equations (18) and (19), but falls below 1 when using the production functions and capital aggregations in equations (1) and (26). Moreover, in order for the labor income share to fall, labor augmenting technical progress should be larger than capital augmenting technical change in the setting described by the production function (26). What could explain the differences in findings across the production function specifications (1), (18), (19), and (26)? In an attempt to shed light on this question, I re-estimate the systems of normalized equations corresponding to the production function specifications in (1), (18), and (19), along with their first-order conditions, imposing the restriction that technological parameters are identical across capital types ($\gamma_I = \gamma_J = \gamma_K$). Table 7 reports the estimation results using both raw and instrumented regressors. Across all specifications, the measures of fit are substantially worse than those in regressions reported in Table 3.[19] Moreover, ε_2 is now below 1 in columns 1 and 2, and the estimated elasticity parameter ε_1 is below 1 in columns 2–6. These estimates imply that the elasticity of substitution between labor and aggregate capital is less than 1. These findings suggest that accounting for differences in technical change across capital types can be important in determining the estimated elasticity of substitution between labor and aggregate capital.[20]

Finally, I compute the aggregate capital-specific technological progress parameter from the production function in equation (1), using parameter estimates from column 1 of Table 3 [see Appendix - Robustness Checks and Additional Results for further details]. I then

[19] The first-order condition for J exhibits a large negative R-squared across all estimations, indicating that predicted rental prices do not align well with the data and likely reflecting issues with the specification.

[20] It is worth noting that none of these estimation results are directly comparable to those obtained using the production function specification in (26), due to differences in the aggregation of capital inputs.

use this value, along with γ_L and $\varepsilon_{L,TK}$, to impute labor income share. Labor income share counterfactually trends upward with these parameter values.

Taken together, these results indicate that the choice of production function and the aggregation of capital play an important role in the value of the estimated elasticity between labor and aggregate capital and, consequently, inferences about whether labor and capital are gross complements or substitutes. Moreover, the results from the production-function specification in equation (1) suggest that even when the aggregate elasticity is below 1, the dynamics of the labor share can be explained by a relatively small component of capital that is substitutable for labor.

6 Conclusion

This study explores the elasticities of substitution between labor, information and communication technologies (ICT) and intellectual property (IP) capital, and traditional capital. In particular, I estimate a nested CES production technology that utilizes labor, an aggregate of ICT and IP capital, and traditional capital, allowing for heterogeneity in substitution elasticities across these inputs. I use data from the EU KLEMS database for a panel of 9 European countries and the estimation methodology developed and applied by De La Grandville (1989), Klump et al. (2007) and León-Ledesma et al. (2010).

The estimate of the elasticity of substitution between labor and ICT and IP capital exceeds 1 in the baseline specification, implying that labor and ICT and IP capital are gross substitutes. The estimate of the elasticity of substitution between labor and traditional capital is below 1, implying that labor and traditional capital are gross complements and underscoring the heterogeneous functional roles of different capital types within production. In turn, the estimate of the elasticity of substitution between ICT and IP capital is below 1. Similar results hold for the elasticity of substitution between labor and ICT and IP capital, and the elasticity of substitution between ICT and IP capital, in two alternative nested CES specifications. Though the elasticity of substitution between labor and traditional capital turns out to be sensitive to the specification of the

production function and the aggregation of capital.

These findings offer insights into the decline in labor income share in European countries observed since the mid-1990s. Counterfactual simulations reveal that most of the fall in labor share would not have occurred absent the technological progress embodied in ICT and IP capital and the decline in their relative prices. Moreover, labor income share would be significantly higher absent the rapid technological progress and the fall in the relative price of ICT capital.

These results also have implications for the elasticity of substitution between labor and aggregate capital. I derive and quantify the implied elasticity of substitution between labor and aggregate capital from the nested CES production technology and the corresponding estimates of the parameter values. The imputed elasticity is an outcome of the underlying elasticity of substitution parameters and factor compensation shares, and it varies over time as these shares evolve. In turn, whether it is below or above unity depends on the adopted specification of production technology. The imputed elasticity remains below unity throughout the sample period under the baseline specification, indicating gross complementarity between labor and aggregate capital. It also exhibits a gradual increase over time. For comparison, I estimate the elasticity of substitution between labor and aggregate capital using data from the EU KLEMS database and following a similar methodology. The estimated value of the elasticity of substitution between labor and aggregate capital is lower than 1 but greater than the imputed values of the elasticity of substitution from the baseline specification. This implies that the specification of the production function and aggregation of capital can play an important role in determining the value of the elasticity of substitution between labor and aggregate capital. In turn, the substitutability between ICT and IP capital and labor, along with the imputed elasticity of substitution from the baseline specification and the implied labor share dynamics, aligns with the juxtaposition of findings by Aum and Shin (2024), Eden and Gaggl (2018, 2019), Karabarbounis and Neiman (2014) and Oberfield and Raval (2021).

Table 6: The Estimated Elasticity of Substitution and Aggregate Capital using Eq. (26)

A. Estimates	(1) Constant	(2) Polynomial	(3) Shares Eq. (1)	(4) Shares Eq. (18)	(5) Constant IR	(6) Polynomial IR	(7) Shares Eq. (1) IR	(8) Shares Eq. (18) IR
ε_1	0.984***	0.988***	0.983***	0.981***	0.992***	0.987***	0.989***	0.991***
	(0.003)	(0.004)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)
$\mathrm{p}_{\varepsilon_1=1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
γ_L	0.052***	0.054***	0.053***	0.051***	0.058***	0.060***	0.058***	0.059***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)
γ_K	-0.091***	-0.096***	-0.093***	-0.090***	-0.103***	-0.107***	-0.105***	-0.105***
	(0.008)	(0.007)	(0.008)	(0.007)	(0.006)	(0.006)	(0.006)	(0.006)
ε_τ		0.003*				0.003		
		(0.001)				(0.001)		
$\varepsilon_{\tau\tau}$		-0.000				-0.000		
		(0.000)				(0.000)		
ε_σ			0.492***	-0.512*			0.269	-0.187**
			(0.119)	(0.252)			(0.145)	(0.072)
Obs. (per eq.)	219	219	219	219	219	219	219	219
B. Measures of Fit								
Log Likelihood	1977	1990	1986	1979	1601	1622	1605	1608
R2								
$\ln\left(Y_t/\overline{Y}\right)$	0.940	0.917	0.940	0.942	0.937	0.913	0.938	0.913
$\ln\left(w_t/\overline{w}\right)$	0.672	0.631	0.691	0.670	0.666	0.630	0.692	0.642
$\ln\left(r_{TK,t}/\overline{r_{TK}}\right)$	0.860	0.863	0.867	0.863	0.869	0.899	0.877	0.905

Note: This table reports estimates of the normalized and log-transformed function (26) together with the corresponding normalized first-order conditions [see equations (48), (49), and (50) in the Technical Appendix]. Normalizations are based on within-country averages. Panel *A* presents parameter estimates and the number of observations for each equation. Panel *B* offers measures of fit, including Log Likelihood and R-squared of each equation. Column 1 assumes $\mathcal{E}$ is constant, $\mathcal{E}_1 \equiv \varepsilon_1$ (canonical 2-factor CES). Column 2 models $\mathcal{E}_1$ as a quadratic polynomial in time, $\mathcal{E}_1 = \varepsilon_1 + \varepsilon_\tau t + \varepsilon_{\tau\tau} t^2$, using a demeaned time index. Columns 3 and 4 assume ε_1 depends linearly on demeaned $\tilde{S}$ for Y defined by equation (1) and for Y defined by equation (18), respectively, such that $\mathcal{E}_1 = \varepsilon_1 + \varepsilon_\sigma \tilde{S}$. Columns 5-8 report similar regressions where rental price of r_{TK} and wages w are instrumented using US data. Rows labeled $\mathrm{p}_{\varepsilon_1=1}$ report p-values for tests of $\varepsilon_1 = 1$. All regressions use feasible generalized non-linear least squares and weight observations by country-year employment shares. Initial values are $\varepsilon_1 = \{0.5, 1.5\}$ and $\gamma_L, \gamma_{TK} = 0.02$. The reported results correspond to the estimations with the best log-likelihood fit, excluding cases where the estimated elasticity of substitution between capital and labor is implausibly high. Standard errors, shown in parentheses, are robust to heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

Table 7: Restricted Estimation Results with $\gamma_I = \gamma_J = \gamma_K$

A. Estimates	(1) Main	(2) Main IR	(3) Eq. (18)	(4) Eq. (18) IR	(5) Eq. (19)	(6) Eq. (19) IR
ε_1	0.668***	0.873***	0.234***	0.248***	0.234***	0.747***
	(0.007)	(0.020)	(0.054)	(0.066)	(0.054)	(0.016)
$p_{\varepsilon_1=1}$	0.000	0.000	0.000	0.000	0.000	0.000
ε_2	0.652***	0.815***	1.014***	1.012***	1.014***	0.749***
	(0.004)	(0.027)	(0.004)	(0.013)	(0.004)	(0.016)
$p_{\varepsilon_2=1}$	0.000	0.000	0.000	0.355	0.000	0.000
ε_3	0.870***	0.954***	0.857***	0.869***	0.857***	0.939***
	(0.010)	(0.008)	(0.022)	(0.023)	(0.022)	(0.006)
$p_{\varepsilon_3=1}$	0.000	0.000	0.000	0.000	0.000	0.000
γ_K	-0.004***	-0.007**	0.014	0.017	0.014	-0.002
	(0.001)	(0.002)	(0.008)	(0.011)	(0.008)	(0.001)
γ_L	0.008***	0.008***	-0.005	-0.006	-0.005	0.004***
	(0.001)	(0.001)	(0.004)	(0.005)	(0.004)	(0.001)
Obs. (per eq.)	219	219	219	219	219	219
B. Measures of Fit						
Log Likelihood	1926	1850	2096	1852	2096	2064
R2						
$\ln\left(Y_t/\overline{Y}\right)$	0.917	0.917	0.899	0.896	0.899	0.909
$\ln\left(r_{I,t}/\overline{r_I}\right)$	0.656	0.679	0.683	0.740	0.683	0.689
$\ln\left(r_{J,t}/\overline{r_J}\right)$	-2.124	-3.105	-2.433	-4.910	-2.433	-3.106
$\ln\left(r_{K,t}/\overline{r_K}\right)$	0.625	0.377	0.612	0.631	0.612	0.864
$\ln\left(w_t/\overline{w}\right)$	0.680	0.686	0.875	0.818	0.875	0.689

Note: This table offers the results from the estimation of the normalized and logarithmed production function together with the normalized first-order conditions, where technological progress parameters are assumed to be the same across ICT, IP and traditional capital ($\gamma_I = \gamma_J = \gamma_K$). The normalizations are based on within-country averages. Panel *A* offers the estimates of the parameters and the corresponding number of observations in each equation. Panel *B* offers various measures of fit, including Log Likelihood and R-squared of each equation. Columns 1, 3, and 5 offer the results from the estimation of production function specifications given by equations (1), (18) and (19). Columns 2, 4, and 6 offer the results from the estimations of these production function specifications using instrumented regressors. Rows labeled $p_{\varepsilon_i=1}$ report p-values for tests of $\varepsilon_i = 1$ for $i = 1, 2, 3$. Negative R-squared in an equation means that the residual sum of squares is larger than the total sum of squares in that equation. All regressions use the feasible generalized nonlinear least squares estimator for seemingly unrelated regressions and the country-year-level shares of hours of employment out of the total hours of employment across sample countries as weights. Initial/starting points for estimations are $\varepsilon_i = \{0.5, 1.5\}$ for $i = 1, 2, 3$ and $\gamma_K = 0.02$. The reported results correspond to the estimations with the best log-likelihood fit, excluding cases where the estimated elasticity of substitution between a type of capital and labor is implausibly high. Standard errors are in parentheses and are robust to arbitrary heteroskedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

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A Appendix - Data Description

Table A: Definitions and Sources of Variables

Variable Name	Definition and Source
Value Added (Y)	Real total industrial value added in billions of national accounts currency. Source: EU KLEMS database.
Hours Worked (L)	Total hours worked by persons engaged in millions. Source: EU KLEMS database.
ICT Capital (I)	Real volume of the services provided by ICT capital (i.e., its productive capital stock), measured in billions. ICT capital comprises computers, communication devices, and peripheral equipment. It is calculated as the ratio of the nominal volume of ICT capital to the ICT investment price. Source: EU KLEMS database.
IP Capital (J)	Real volume of the services provided by IP capital (i.e., its productive capital stock), measured in billions. IP capital (or intangible capital) comprises software, databases, reseach and development, and other forms of intellectual property. It is calculated as the ratio of the nominal volume of IP capital to the IP investment price. Source: EU KLEMS database.
Traditional Capital (K)	Real volume of the services provided by traditional capital (i.e., its productive capital stock), measured in billions. Traditional capital comprises all types of capital but ICT and IP capital, such as machinery, transport, and construction equipment and structures. It is calculated as the ratio of the nominal volume of traditional capital to the price of investments in traditional capital. Source: EU KLEMS database.
Total Capital (TK)	Real volume of the services provided by total capital (i.e., its productive capital stock), measured in billions. It is calculated as the ratio of the nominal volume of total capital to the price of investments in total capital. Source: EU KLEMS database.

Table A – (Continued)

Variable Name	Definition and Source
ICT Investment Price (p_I)	Price of investments in ICT capital, relative to value added price. It is computed using a weighted average of prices of investments in information technology (IT) capital and communication technology (CT) capital. The weights are the averages of the shares of IT capital and CT capital out of the sum of IT and CT capital. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database.
IP Investment Price (p_J)	Price of investments in IP capital, relative to value added price. It is computed using a weighted average of prices of investments in software and databases, research and development, and other types of intellectual property (IP) capital. The weights are the averages of the shares of different types of IP capital out of the total sum of IP capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database.
Traditional Capital Investment Price (p_K)	Price of investments in traditional capital, relative to value added price. It is computed using a weighted average of prices of investments in different types of traditional capital. The weights are the averages of the shares of different types of traditional capital out of the total sum of traditional capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database.
Total Capital Investment Price (p_{TK})	Price of investments in total capital, TK, relative to value added price. Source: The EU KLEMS database.
δ_I	Annual depreciation rate of ICT capital. It is computed using a weighted average of depreciation rates of information technology (IT) capital and communication technology (CT) capital. The weights are the averages of the shares of IT capital and CT capital out of the sum of IT and CT capital. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database.

Table A – (Continued)

Variable Name	Definition and Source
δ_J	Annual depreciation rate of IP capital. It is computed using a weighted average of depreciation rates of software, R&D capital/patents, and other types of intellectual property (IP) capital. The weights are the averages of the shares of different types of IP capital out of the total sum of IP capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database.
δ_K	Annual depreciation rate of traditional capital K. It is computed using a weighted average of depreciation rates of machinery, transport, and construction equipment and structures. The weights are the averages of the shares of different types of traditional capital (including machinery, transport, and construction equipment and structures) out of the total sum of traditional capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database.
r_I	Real rental price of ICT capital. Source: Authors' calculations using data from the EU KLEMS database and equation (7).
r_J	Real rental price of IP capital. Source: Authors' calculations using data from the EU KLEMS database and equation (7).
r_K	Real rental price of traditional capital. Source: Authors' calculations using data from the EU KLEMS database and equation (7).
r_{TK}	Real rental price of aggregate capital, TK. Source: Authors' calculations using data from the EU KLEMS database and equation $r_{TK} = (Y - wL)/TK$.
w	Real hourly wages. It is computed from the ratio of real labor compensation and hours of work. Source: Authors' calculations using data from the EU KLEMS database.

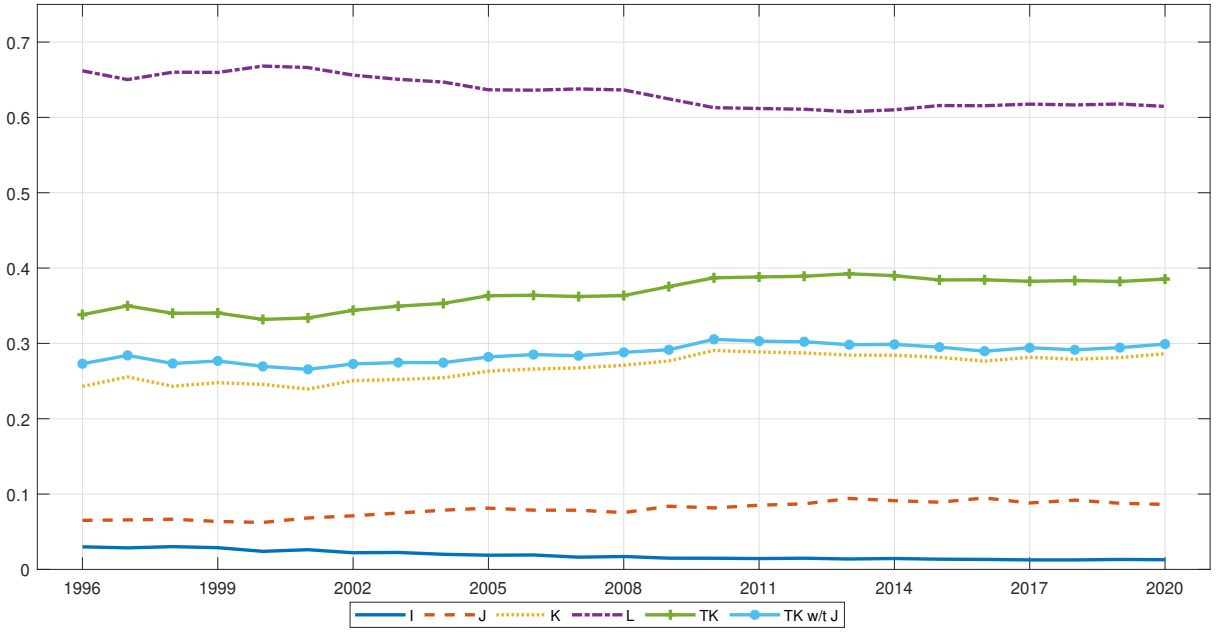
Data Sources: All data are from the 2023 release of the EU KLEMS database by the Luiss Lab of European Economics at Luiss University in Rome, Italy, accessible at https://euklems-intanprod-llee.luiss.it/ (last accessed: 15.10.2024). This database includes information on investment in capital across both tangible and intangible assets. It is often used to study productivity and output, employment, and labor

income dynamics.

Countries: The main sample countries are Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, and Spain, and the period is from 1996-2020 except for Germany (1996-2019) and Portugal (2001-2020). The sample also includes data for the US (1996-2020).

B Data Appendix

Figure I: The Shares of Income of Factor Inputs in the US



Note: This figure illustrates the income shares of factor inputs, including ICT capital I, IP capital J, traditional capital K, labor L, total capital TK, and total capital without IP capital (TK w/t J) in the US.

Table I: Country-Level Basic Statistics

Panel A	Austria			Belgium			Finland			France		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Variable	Mean	1996	2020	Mean	1996	2020	Mean	1996	2020	Mean	1996	2020
Value Added, per hours worked (Y/L)	40791	33485	46494	48007	42344	53504	42986	34752	46763	44325	37888	48908
Labor Income, per hours worked (wL/L)	26951	23589	31666	30094	27481	32216	27806	23731	29085	30239	26187	33796
ICT Capital, per hours worked (I/L)	3264	2432	4167	2056	1165	3092	726	141	1327	589	269	1052
IP Capital, per hours worked (J/L)	7195	3776	12281	6934	4089	11078	8510	5184	8761	7833	5933	11335
Traditional Capital, per hours worked (K/L)	166713	136587	208179	148550	137557	173964	142401	126766	169185	148655	120757	173850
Total Capital, per hours worked (TK/L)	174999	137741	227132	156413	137311	189302	151797	129993	180123	155057	123415	188064
Hours Worked, mln (L)	6834	6508	6709	6899	6108	7083	4015	3624	4148	40570	37982	39692
ICT Investment Price (p_I)	1.188	1.488	0.911	1.272	1.871	0.864	2.192	5.525	0.956	1.606	2.628	0.947
IP Investment Price (p_J)	1.032	1.061	0.970	0.992	1.015	0.983	0.955	0.887	1.008	1.035	1.063	0.962
Traditional Capital Investment Price (p_K)	0.974	0.934	1.006	0.992	0.957	0.994	1.012	0.954	1.047	0.964	0.929	1.008
Total Capital Investment Price (p_{TK})	0.994	0.982	0.991	1.003	1.005	0.985	1.010	0.971	1.040	0.983	0.966	0.995

Panel B												
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Derived Variable	Mean	1996	2020	Mean	1996	2020	Mean	1996	2020	Mean	1996	2020
r_I	0.239	0.345	0.169	0.407	0.650	0.275	0.724	2.103	0.238	0.440	0.660	0.203
r_J	0.289	0.286	0.255	0.294	0.285	0.270	0.265	0.224	0.274	0.321	0.318	0.284
r_K	0.066	0.058	0.053	0.101	0.094	0.100	0.089	0.075	0.088	0.077	0.080	0.067
r_{TK}	0.079	0.072	0.065	0.114	0.108	0.112	0.100	0.085	0.098	0.092	0.095	0.080

Note: This table presents average and sample initial and final values for the main variables in each sample European country and in the US. All figures are in real terms. The values of value added, labor income, and the volume of the services provided by capital types are per hours worked. Price indices are normalized to 1 in 2015 and are expressed relative to the value added price index. See Table A in the Appendix - Data Description for detailed descriptions and sources of variables.

Table I: Country-Level Basic Statistics — (Continued)

Panel A	Germany			Italy			Netherlands			Portugal		
	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
Variable	Mean	1996	2019	Mean	1996	2020	Mean	1996	2020	Mean	2001	2020
Value Added, per hours worked (Y/L)	41987	35936	46872	34653	32894	36958	45875	38737	48430	17206	15568	18650
Labor Income, per hours worked (wL/L)	28215	24841	31780	22132	21563	23340	31201	27049	35057	11054	10828	12053
ICT Capital, per hours worked (I/L)	968	531	1197	690	385	1011	799	128	1305	619	306	857
IP Capital, per hours worked (J/L)	7321	5393	9714	3782	2867	5422	8213	5539	10810	1156	664	1768
Traditional Capital, per hours worked (K/L)	144577	127888	155715	120527	102853	144878	142003	128389	150733	67515	55681	78355
Total Capital, per hours worked (TK/L)	149215	124559	169009	124230	103940	152343	149710	123881	164898	68583	54416	83663
Hours Worked, mln (L)	58759	57723	62127	43034	41139	38507	12331	11104	13475	9195	9767	8564
ICT Investment Price (p_I)	1.987	4.023	0.922	1.275	1.773	0.958	2.452	7.344	0.828	1.470	2.654	0.899
IP Investment Price (p_J)	1.009	1.004	0.985	1.014	1.061	0.946	1.034	1.075	1.034	1.027	1.048	0.976
Traditional Capital Investment Price (p_K)	0.979	0.962	1.030	0.983	0.969	0.986	1.005	0.961	1.021	1.016	1.026	1.045
Total Capital Investment Price (p_{TK})	1.012	1.049	1.016	0.993	0.995	0.978	1.021	1.051	1.008	1.031	1.078	1.009
Panel B												
	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
Derived Variable	Mean	1996	2019	Mean	1996	2020	Mean	1996	2020	Mean	1996	2020
r_I	0.614	1.177	0.249	0.380	0.565	0.244	1.089	4.521	0.309	0.414	0.846	0.223
r_J	0.276	0.253	0.282	0.317	0.369	0.283	0.303	0.319	0.288	0.308	0.293	0.304
r_K	0.078	0.071	0.077	0.092	0.098	0.082	0.083	0.073	0.065	0.082	0.077	0.075
r_{TK}	0.092	0.089	0.090	0.102	0.109	0.089	0.098	0.094	0.081	0.090	0.087	0.079

Note: This table presents average and sample initial and final values for the main variables in each sample European country and in the US. All figures are in real terms. The values of value added, labor income, and the volume of the services provided by capital types are per hours worked. Price indices are normalized to 1 in 2015 and are expressed relative to the value added price index. See Table A in the Appendix - Data Description for detailed descriptions and sources of variables.

Table I: Country-Level Basic Statistics — (Continued)

Panel A	Spain			US		
	(25)	(26)	(27)	(28)	(29)	(30)
Variable	Mean	1996	2020	Mean	1996	2020
Value Added, per hours worked (Y /L)	29042	27457	31766	59093	47140	71346
Value Added, per hours worked (Y/L)	29042	27457	31766	59093	47140	71346
Labor Income, per hours worked (wL/L)	18536	18194	20942	37341	31209	43728
ICT Capital, per hours worked (I/L)	810	630	1231	1864	318	4152
IP Capital, per hours worked (J/L)	2796	1672	4610	16921	9260	24474
Traditional Capital, per hours worked (K/L)	103421	87500	132139	168157	128134	206751
Total Capital, per hours worked (TK/L)	108285	91367	139504	174503	115706	235136
Hours Worked, mln (L)	31202	24467	30234	252145	233350	260700
ICT Investment Price (p_I)	1.329	1.776	0.916	3.016	9.252	0.772
IP Investment Price (p_J)	0.979	0.986	0.974	1.069	1.206	0.952
Traditional Capital Investment Price (p_K)	1.091	1.105	1.020	0.994	0.967	1.036
Total Capital Investment Price (p_{TK})	1.076	1.088	1.006	1.055	1.151	0.986
Panel B						
	(25)	(26)	(27)	(34)	(35)	(36)
Derived Variable	Mean	1996	2020	Mean	1996	2020
r_I	0.338	0.484	0.234	1.068	4.453	0.220
r_J	0.284	0.274	0.284	0.288	0.332	0.251
r_K	0.092	0.097	0.070	0.095	0.090	0.099
r_{TK}	0.098	0.101	0.078	0.128	0.138	0.117

Note: This table presents average and sample initial and final values for the main variables in each sample European country and the US. All figures are in real terms. The values of value added, labor income, and the volume of the services provided by each type of capital are per hours worked. Price indices are normalized to 1 in 2015 and are expressed relative to the value added price index. See Table A in the Appendix - Data Description for detailed descriptions and sources of variables.

Table II: Sample Initial and Final Values of Factor Income Shares in the US

Year	I	J	K	L
1996	0.030	0.065	0.243	0.662
2020	0.013	0.086	0.286	0.615

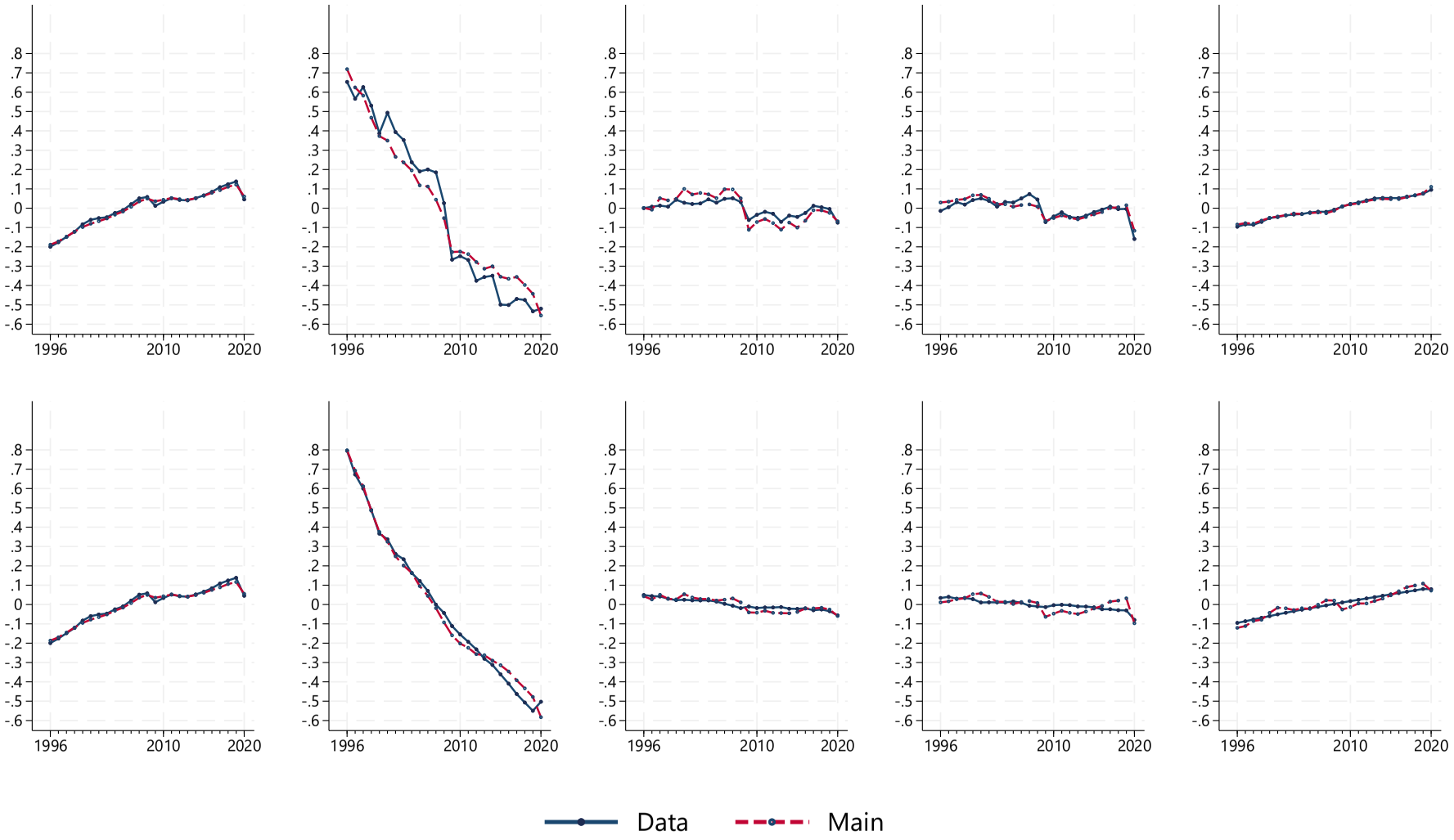
Note: This table presents the income shares of factor inputs, including ICT capital I, IP capital J, traditional capital TK, and labor L, in the US at the beginning and end of the sample period. See Table A in the Appendix - Data Description for complete descriptions and sources of variables.

Table III: Shares and Ratios

Year	(1) $S_{LIJ,Y}$	(2) $S_{IJ,LIJ}$	(3) $S_{LK,Y}$	(4) $S_{L,LK}$
1996	0.727	0.066	0.952	0.713
1997	0.723	0.066	0.953	0.709
1998	0.716	0.070	0.950	0.702
1999	0.720	0.069	0.950	0.705
2000	0.717	0.069	0.951	0.702
2001	0.713	0.072	0.949	0.697
2002	0.712	0.072	0.949	0.696
2003	0.714	0.074	0.947	0.698
2004	0.707	0.074	0.948	0.691
2005	0.705	0.073	0.948	0.689
2006	0.701	0.076	0.947	0.684
2007	0.695	0.078	0.946	0.677
2008	0.701	0.076	0.947	0.684
2009	0.718	0.070	0.950	0.703
2010	0.713	0.072	0.949	0.698
2011	0.710	0.073	0.948	0.694
2012	0.714	0.074	0.947	0.698
2013	0.713	0.073	0.948	0.697
2014	0.713	0.076	0.946	0.696
2015	0.710	0.076	0.946	0.693
2016	0.708	0.078	0.945	0.691
2017	0.707	0.082	0.942	0.689
2018	0.712	0.083	0.941	0.694
2019	0.714	0.082	0.941	0.696
2020	0.722	0.082	0.941	0.705

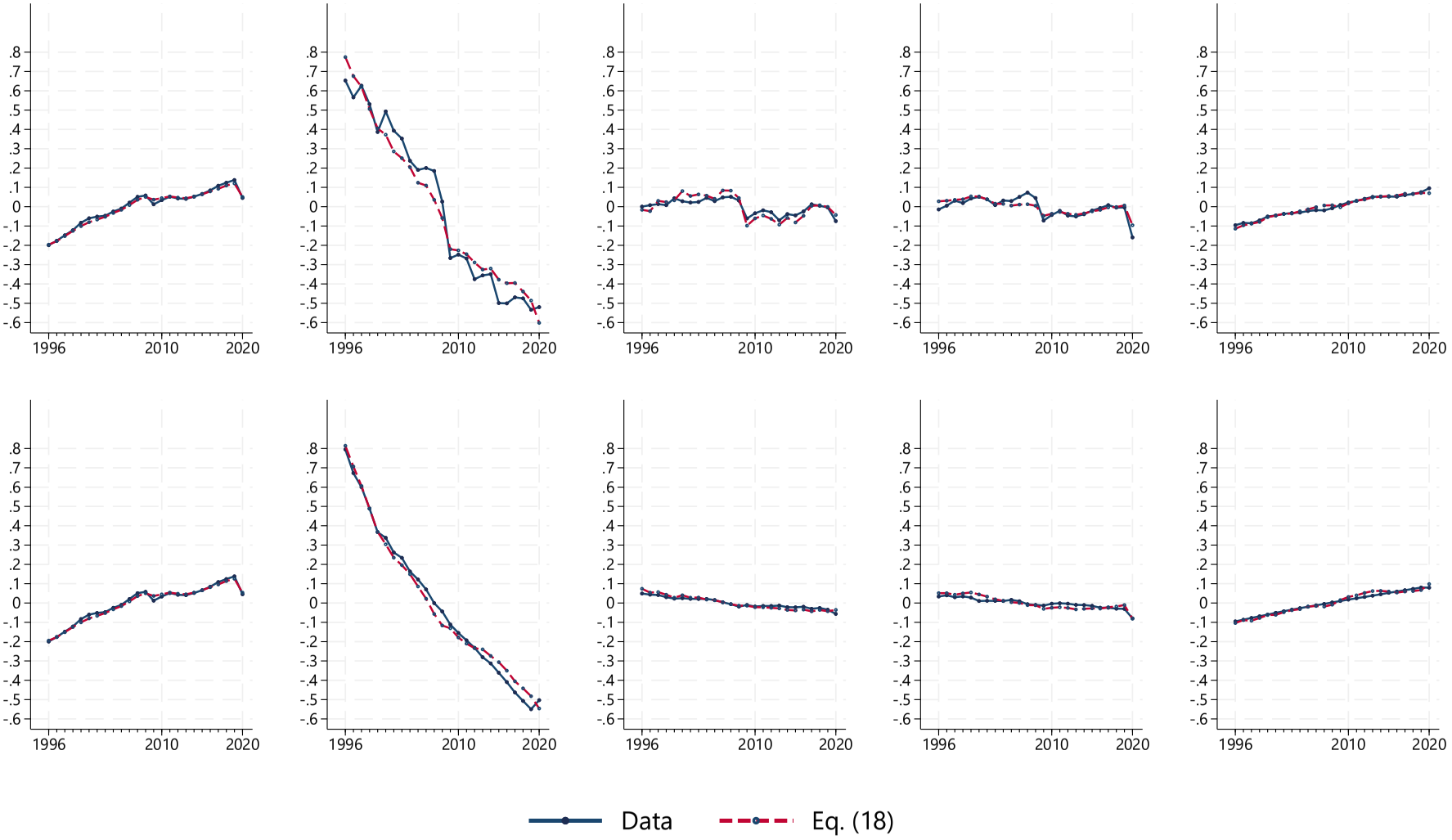
Note: This table offers the values of the shares S_{LIJ_t,Y_t}, S_{IJ_t,LIJ_t}, S_{LK_t,Y_t}, and S_{L_t,LK_t}. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across the sample European countries.

Figure II: Data and Prediction - Main and Main IR



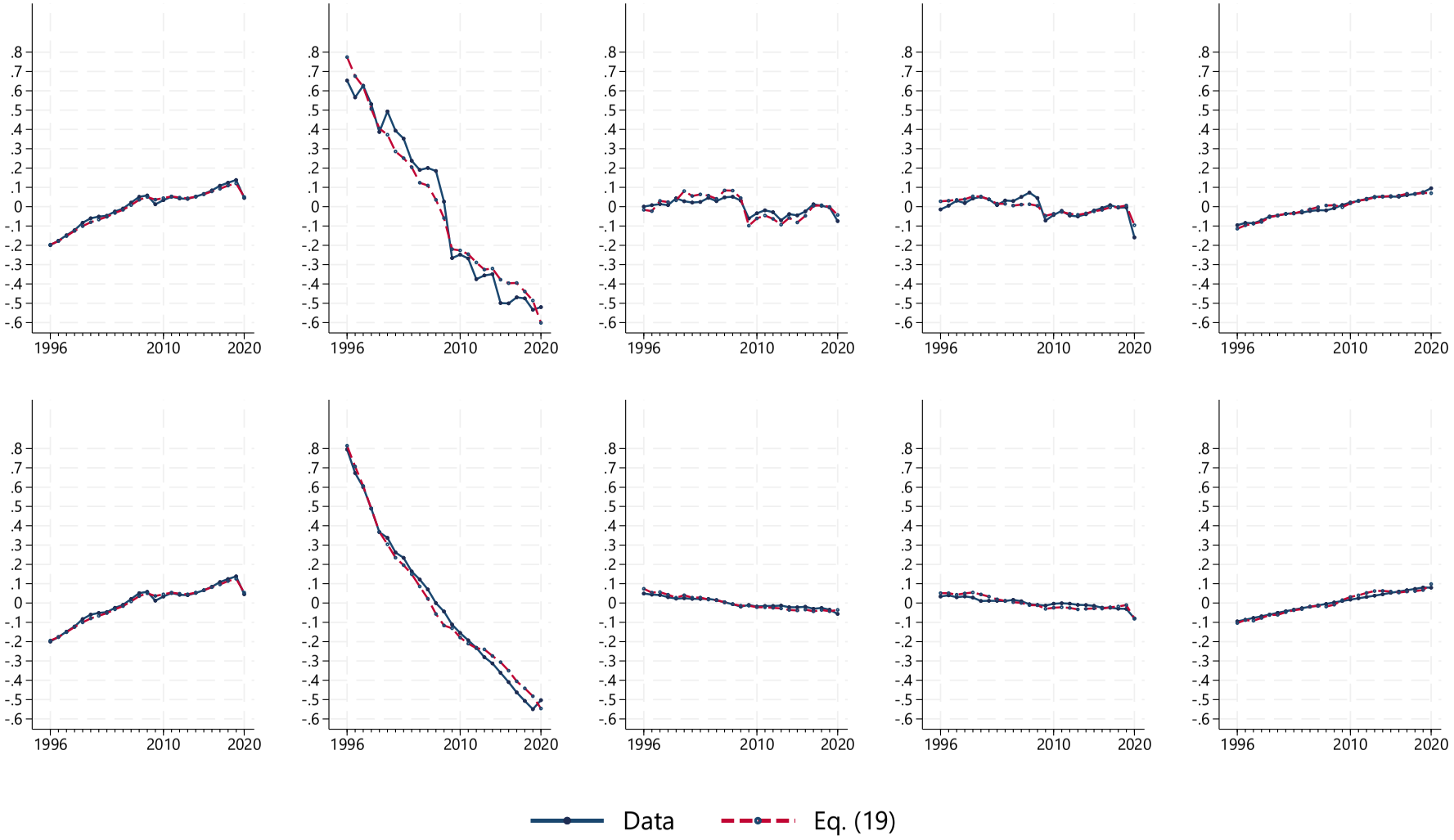
Note: This figure illustrates the evolution of dependent variables in the system of equations (11) and (13)-(16) together with their predicted values, averaged across sample countries using country-year employment-hour shares as weights. The first row presents results based on raw regressors, while the second row displays results using instrumented regressors. The first column illustrates logarithmized and normalized value added, and the remaining columns show normalized and logarithmized rental prices for I, J, and K, followed by wages, in that order.

Figure III: Data and Prediction - Eq. (18) and Eq. (18) IR



Note: This figure illustrates the evolution of dependent variables in the system of equations corresponding to the production function specification (18) [equations (32)-(36)] together with their predicted values, averaged across sample countries using country-year employment-hour shares as weights. The first row presents results based on raw regressors, while the second row displays results using instrumented regressors. The first column illustrates logarithmized and normalized value added, and the remaining columns show normalized and logarithmized rental prices for I, J, and K, followed by wages, in that order.

Figure IV: Data and Prediction - Eq. (19) and Eq. (19) IR



Note: This figure illustrates the evolution of dependent variables in the system of equations corresponding to the production function specification (19) [equations (42)-(46)] together with their predicted values, averaged across sample countries using country-year employment-hour shares as weights. The first row presents results based on raw regressors, while the second row displays results using instrumented regressors. The first column illustrates logarithmized and normalized value added, and the remaining columns show normalized and logarithmized rental prices for I, J, and K, followed by wages, in that order.

C Technical Appendix

In this section, I offer the systems of estimated equations for alternative specifications of the production function (18), (19), and (26).

C.A Separate Nests for Labor and Traditional Capital and for ICT and IP

I assume that the representative firm solves the problem (2) where production function is now given by

$$Y_t = \left(\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right)^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{27}$$

and

$$LK_t = \left[\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t} K_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$
$$IJ_t = \left[\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

The first-order conditions that follow from this problem are given by

$$\frac{r_{I,t} I_t}{Y_t} = \frac{\varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \frac{\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \tag{28}$$

$$\frac{r_{J,t} J_t}{Y_t} = \frac{\varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \frac{\varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \tag{29}$$

$$\frac{r_{K,t} K_t}{Y_t} = \frac{\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \frac{\varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t} K_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t} K_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}, \tag{30}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_{LK}^{\frac{1}{\varepsilon_1}} LK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IJ}^{\frac{1}{\varepsilon_1}} IJ_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \frac{\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t} K_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}. \tag{31}$$

I denote by S_{LK_t,Y_t} and S_{IJ_t,Y_t} the shares of compensation of labor and traditional

capital and ICT and IP capital in value added:

$$S_{LK_t,Y_t} = \varpi_{LK}^{\frac{1}{\varepsilon_1}} \left(\frac{LK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$S_{IJ_t,Y_t} = \varpi_{IJ}^{\frac{1}{\varepsilon_2}} \left(\frac{IJ_t}{Y_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}.$$

I also use $\alpha_{1,A1}$, $\alpha_{2,A1}$, and $\alpha_{3,A1}$ to denote the following expressions:

$$\alpha_{1,A1} = \varpi_{LK}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{LK_t}{Y_t}\right)}^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$\alpha_{2,A1} = \varpi_{L}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{e^{\gamma_L t} L_t}{LK_t}\right)}^{\frac{\varepsilon_2-1}{\varepsilon_2}},$$
$$\alpha_{3,A1} = \varpi_{I}^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_I t} I_t}{IJ_t}\right)}^{\frac{\varepsilon_3-1}{\varepsilon_3}}.$$

I use $\alpha_{1,A1}$, $\alpha_{2,A1}$, and $\alpha_{3,A1}$ and write the logarithm of the normalized equation for output in the following way:

$$\ln \frac{Y_t}{\overline{Y_t}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,A1} \left(\frac{LK_t}{\overline{LK_t}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + (1 - \alpha_{1,A1}) \left(\frac{IJ_t}{\overline{IJ_t}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \right], \tag{32}$$

where

$$\frac{LK_t}{\overline{LK_t}} = \left[\alpha_{2,A1} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + (1 - \alpha_{2,A1}) \left(e^{\gamma_K \hat{t}} \frac{K_t}{\overline{K_t}}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$
$$\frac{IJ_t}{\overline{IJ_t}} = \left[\alpha_{3,A1} \left(e^{\gamma_I \hat{t}} \frac{I_t}{\overline{I_t}}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + (1 - \alpha_{3,A1}) \left(e^{\gamma_J \hat{t}} \frac{J_t}{\overline{J_t}}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3-1}},$$

and $\hat{t}$ is demeaned trend.

I write the normalized equations corresponding to the first-order conditions in the

following way:

$$\ln r_{I,t} - \ln \overline{r_{I,t}} = \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{IJ_t,Y_t}}{\overline{S_{IJ_t,Y_t}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_I \hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{I_t/\overline{I_t}}{Y_t/\overline{Y_t}}\right), \quad (33)$$

$$\ln r_{J,t} - \ln \overline{r_{J,t}} = \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{IJ_t,Y_t}}{\overline{S_{IJ_t,Y_t}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_J \hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{J_t/\overline{J_t}}{Y_t/\overline{Y_t}}\right), \quad (34)$$

$$\ln r_{K,t} - \ln \overline{r_{K,t}} = \left(1 - \frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LK_t,Y_t}}{\overline{S_{LK_t,Y_t}}}\right) + \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_K \hat{t} \quad (35)$$
$$- \frac{1}{\varepsilon_2}\ln\left(\frac{K_t/\overline{K_t}}{Y_t/\overline{Y_t}}\right),$$

$$\ln w_t - \ln \overline{w_t} = \left(1 - \frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LK_t,Y_t}}{\overline{S_{LK_t,Y_t}}}\right) + \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_L \hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}}\right). \quad (36)$$

In this case, the estimations use the system of equations (32) and (33)-(36).

C.B One Nest for Capital Types

I assume that the representative firm solves the problem (2) where the production function is now given by

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \quad (37)$$

and

$$IJK_t = \left[\varpi_{IJ}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$IJ_t = \left[\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t} I_t\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

The first-order conditions that follow from this problem are given by

$$\frac{r_{I,t}I_t}{Y_t} = \frac{\varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{IJ}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t}K_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \times \frac{\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t}I_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t}I_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t}J_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{38}$$

$$\frac{r_{J,t}J_t}{Y_t} = \frac{\varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \frac{\varpi_{IJ}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{IJ}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t}K_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \times \frac{\varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t}J_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_I^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_I t}I_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t}J_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{39}$$

$$\frac{r_{K,t}K_t}{Y_t} = \frac{\varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \frac{\varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t}K_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{IJ}^{\frac{1}{\varepsilon_2}} IJ_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_K^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_K t}K_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{40}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IJK}^{\frac{1}{\varepsilon_1}} IJK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}. \tag{41}$$

I denote by S_{IJK_t,Y_t} and S_{IJ_t,IJK_t} the share of total capital compensation out of value added and the share of compensation of ICT and IP capital out of total capital compensation:

$$S_{IJK_t,Y_t} = \varpi_{IJK}^{\frac{1}{\varepsilon_1}} \left(\frac{IJK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$S_{IJ_t,IJK_t} = \varpi_{IJ}^{\frac{1}{\varepsilon_2}} \left(\frac{IJ_t}{IJK_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}.$$

I also use $\alpha_{1,A2}$, $\alpha_{2,A2}$, and $\alpha_{3,A2}$ to denote the following expressions:

$$\alpha_{1,A2} = \varpi_L^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{e^{\gamma_L t}L_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}},$$
$$\alpha_{2,A2} = \varpi_{IJ}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{IJ_t}{IJK_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}},$$
$$\alpha_{3,A2} = \varpi_I^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_I t}I_t}{IJ_t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}.$$

I further use $\alpha_{1,A2}$, $\alpha_{2,A2}$, and $\alpha_{3,A2}$ and write the logarithm of the normalized equation

for output in the following way:

$$\ln \frac{Y_t}{\overline{Y_t}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,A2} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \alpha_{1,A2}) \left(\frac{IJK_t}{\overline{IJK_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right], \tag{42}$$

where

$$\frac{IJK_t}{\overline{IJK_t}} = \left[\alpha_{2,A2} \left(\frac{IJ_t}{\overline{IJ_t}} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1 - \alpha_{2,A2}) \left(e^{\gamma_K \hat{t}} \frac{K_t}{\overline{K_t}} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$\frac{IJ_t}{\overline{IJ_t}} = \left[\alpha_{3,A2} \left(e^{\gamma_I \hat{t}} \frac{I_t}{\overline{I_t}} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1 - \alpha_{3,A2}) \left(e^{\gamma_J \hat{t}} \frac{J_t}{\overline{J_t}} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

I write the normalized equations corresponding to the first-order conditions in the following way:

$$\ln r_{I,t} - \ln \overline{r_{I,t}} = \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IJK_t,Y_t}}{\overline{S_{IJK_t,Y_t}}} \right) + \tag{43}$$
$$\left(1 - \frac{\varepsilon_2}{\varepsilon_2 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IJ_t,IJK_t}}{\overline{S_{IJ_t,IJK_t}}} \right) + \frac{\varepsilon_3 - 1}{\varepsilon_3} \gamma_I \hat{t} - \frac{1}{\varepsilon_3} \ln \left(\frac{I_t / \overline{I_t}}{Y_t / \overline{Y_t}} \right),$$

$$\ln r_{J,t} - \ln \overline{r_{J,t}} = \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IJK_t,Y_t}}{\overline{S_{IJK_t,Y_t}}} \right) + \tag{44}$$
$$\left(1 - \frac{\varepsilon_2}{\varepsilon_2 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IJ_t,IJK_t}}{\overline{S_{IJ_t,IJK_t}}} \right) + \frac{\varepsilon_3 - 1}{\varepsilon_3} \gamma_J \hat{t} - \frac{1}{\varepsilon_3} \ln \left(\frac{J_t / \overline{J_t}}{Y_t / \overline{Y_t}} \right),$$

$$\ln r_{K,t} - \ln \overline{r_{K,t}} = \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IJK_t,Y_t}}{\overline{S_{IJK_t,Y_t}}} \right) + \frac{\varepsilon_2 - 1}{\varepsilon_2} \gamma_K \hat{t} \tag{45}$$
$$- \frac{1}{\varepsilon_2} \ln \left(\frac{K_t / \overline{K_t}}{Y_t / \overline{Y_t}} \right),$$

$$\ln w_t - \ln \overline{w_t} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_L \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{L_t / \overline{L_t}}{Y_t / \overline{Y_t}} \right). \tag{46}$$

In this case, the estimations use the system of equations (42) and (43)-(46).

C.C One Type of Capital

I consider a continuum of identical, infinitely lived firms of measure one, each producing a homogeneous good Y in a market with no entry costs. The representative firm discounts

future profits at the rate of r and has the following production technology:

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{47}$$

where TK includes all types of capital and $\mathcal{E}_1$ can vary over time.

The firm operates in a perfectly competitive market and decides how much labor to hire and how much to invest in TK, taking the wages w, the prices of investments p_{TK}, and the interest rate r_{TK} as given. It solves the following problem:

$$\max_{\{L_t, X_{TK,t}\}} \sum_{t=0}^{+\infty} \prod_{\kappa=0}^{t} \left(\frac{1}{1 + r_\kappa} \right) \left(Y_t - w_t L_t - p_{TK,t} X_{TK,t} \right)$$

$$s.t.$$

$$X_{TK,t} = TK_{t+1} - (1 - \delta_{TK}) TK_t,$$

where X_{TK} is investment in TK, and $\delta_{TK} \in (0, 1)$ is its depreciation rate.

The first-order conditions that follow from this problem are given by

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}},$$

$$\frac{r_{TK,t} TK_t}{Y_t} = \frac{\varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}.$$

I use $\alpha_{1,A3}$ to denote

$$\alpha_{1,A3} = \varpi_L^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{A_{L,t} L_t}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} = \overline{\frac{w_t L_t}{Y_t}}.$$

The estimations use the following system of normalized equations:

$$\ln \frac{Y_t}{\overline{Y}} = \frac{\mathcal{E}_1}{\mathcal{E}_1 - 1} \ln \left[\alpha_{1,A3} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}} \right)^{\frac{\mathcal{E}_1 - 1}{\mathcal{E}_1}} + (1 - \alpha_{1,A3}) \left(e^{\gamma_{TK} \hat{t}} \frac{TK_t}{\overline{TK_t}} \right)^{\frac{\mathcal{E}_1 - 1}{\mathcal{E}_1}} \right], \tag{48}$$

$$\ln w_t - \ln \overline{w_t} = \frac{\mathcal{E}_1 - 1}{\mathcal{E}_1} \gamma_L \hat{t} - \frac{1}{\mathcal{E}_1} \ln \left(\frac{L_t / \overline{L_t}}{Y_t / \overline{Y_t}} \right), \tag{49}$$

$$\ln r_{TK,t} - \ln \overline{r_{TK,t}} = \frac{\mathcal{E}_1 - 1}{\mathcal{E}_1} \gamma_{TK} \hat{t} - \frac{1}{\mathcal{E}_1} \ln \left(\frac{TK_t / \overline{TK_t}}{Y_t / \overline{Y_t}} \right). \tag{50}$$

The values of r_{TK} are computed as:

$$r_{TK,t} = \left(Y_t - w_t L_t \right) / TK_t. \tag{51}$$

D Appendix - Empirical Specifications

In Table 3, I present results from the estimation of three different nested CES production functions together with first-order conditions given by systems of equations: 1. (11)-(16), 2. (32)-(36), and 3. (42)-(46). Let the function $F_m\left(\right)$ represent the production function for the system of equations $m = 1, 2, 3$ and let c index countries. The empirical specifications for these cases are:

$$\ln \frac{Y_{c,t}}{\overline{Y_{c,t}}} = \ln F_m \left(e^{\gamma_I \hat{t}} \frac{I_{c,t}}{\overline{I_{c,t}}}, e^{\gamma_J \hat{t}} \frac{J_{c,t}}{\overline{J_{c,t}}}, e^{\gamma_K \hat{t}} \frac{K_{c,t}}{\overline{K_{c,t}}}, e^{\gamma_L \hat{t}} \frac{L_{c,t}}{\overline{L_{c,t}}} \right) + \nu_{Y,m,c,t}, \tag{52}$$

$$\ln r_{I,c,t} - \ln \overline{r_{I,c,t}} = \ln \frac{\partial F_m}{\partial I_{c,t}} - \ln \frac{\overline{\partial F_m}}{\partial I_{c,t}} + \nu_{I,m,c,t}, \tag{53}$$

$$\ln r_{J,c,t} - \ln \overline{r_{J,c,t}} = \ln \frac{\partial F_m}{\partial J_{c,t}} - \ln \frac{\overline{\partial F_m}}{\partial J_{c,t}} + \nu_{J,m,c,t}, \tag{54}$$

$$\ln r_{K,c,t} - \ln \overline{r_{K,c,t}} = \ln \frac{\partial F_m}{\partial K_{c,t}} - \ln \frac{\overline{\partial F_m}}{\partial K_{c,t}} + \nu_{K,m,c,t}, \tag{55}$$

$$\ln w_{c,t} - \ln \overline{w_{c,t}} = \ln \frac{\partial F_m}{\partial L_{c,t}} - \ln \frac{\overline{\partial F_m}}{\partial L_{c,t}} + \nu_{L,m,c,t}, \tag{56}$$

where $\hat{t}$ is demeaned time trend as given by equation (12), ν-s are error terms, and $\overline{z_{c,t}}$ is the geometric average of variable $z_{c,t}$ for country c over sample period,

$$\overline{z_{c,t}} = \left(\prod_{t}^{T_c} z_{c,t} \right)^{\frac{1}{T_c}}.$$

Additionally, I estimate a production function together with first-order conditions given by equations (48)-(51). The empirical specification in this case is given by

$$\ln \frac{Y_{c,t}}{\overline{Y_{c,t}}} = \ln F_4 \left(e^{\gamma_L \hat{t}} \frac{L_{c,t}}{\overline{L_{c,t}}}, e^{\gamma_{TK} \hat{t}} \frac{TK_{c,t}}{\overline{TK_{c,t}}} \right) + \nu_{Y,4,c,t}, \tag{57}$$

$$\ln r_{TK,c,t} - \ln \overline{r_{TK,c,t}} = \ln \frac{\partial F_4}{\partial TK_{c,t}} - \ln \overline{\frac{\partial F_4}{\partial TK_{c,t}}} + \nu_{TK,4,c,t}, \tag{58}$$

$$\ln w_{c,t} - \ln \overline{w_{c,t}} = \ln \frac{\partial F_4}{\partial L_{c,t}} - \ln \overline{\frac{\partial F_4}{\partial L_{c,t}}} + \nu_{L,4,c,t}, \tag{59}$$

where the exponent of $F_4()$ is the function in equation (48).

I employ the feasible generalized non-linear least squares estimator for seemingly unrelated equations (STATA command: nlsur) and allow for arbitrary correlation and heteroskedasticity in the error terms in the estimation of the systems of equations. The CES functions and their first-order conditions are normalized using geometric means over time within each country, so that the logarithms of all variables average to zero at the country level in each equation. Consequently, parameter identification relies on within-country and over-time variation. This is analogous to a fixed-effects estimation in linear models. To account for the relative importance of each country-year observation, I apply weights based on the share of hours worked in each country-year relative to the total hours worked across all sample countries. The results thus apply to a weighted average across countries.

For initialization, I set $\gamma_j = 0.02$ for all input types $j = I, J, K, L$ and use multiple starting values for the elasticity parameters in all estimations: $\varepsilon_i \in \{0.5, 1.5\}$ for $i = 1, 2, 3$. I select the results that yield the best Log-Likelihood fit, discarding cases where the estimated elasticity of substitution between capital and labor is implausibly high and where the technological change parameter pertaining to ICT capital γ_I is negative.

A potential challenge for identification in this demand-side framework stems from the simultaneous variation in local supply and demand for factor inputs. To alleviate this concern, I use data from the US to construct counterparts for p_i for $i = I, J, K$, r_t, and w_t. These variables serve as instruments for their counterparts in the sample countries, allowing for unrestricted country-level differences in exposure. I then estimate the following instrumental variable regressions:

$$z_{c,t} = \beta_{1,c} + \beta_{2,c} z_{US,t} + \beta_{3,c} t + \nu_{c,t},$$

where $z_{c,t}$ is one of these variables. There are a few motivations to use US data. First, the over-time variation in p_I and p_J originates primarily from the United States. Furthermore, applying the same estimation procedure to US data produces estimates that are qualitatively and quantitatively similar to the main results reported in the text. This similarity in estimates is consistent with comparable demand conditions for factor inputs across the United States and the sample countries.[21]

E Appendix - Imputed Elasticity of Substitution

I write the level of aggregate capital in the following way

$$TK = \frac{r_I}{r_{TK}} I + \frac{r_J}{r_{TK}} J + \frac{r_K}{r_{TK}} K. \tag{60}$$

I further assume that the output with aggregate capital at the firm level is represented as a function $f(L, TK)$ with the following properties

$$f(L, TK) = Y(L, I, J, K), \tag{61}$$

$$\frac{\partial f}{\partial L} L = \frac{\partial Y}{\partial L} L, \tag{62}$$

$$\frac{\partial f}{\partial TK} TK = \frac{\partial Y}{\partial I} I + \frac{\partial Y}{\partial J} J + \frac{\partial Y}{\partial K} K, \tag{63}$$

[21] In estimations, the F-statistics for the block containing only $\beta_{2,c} z_{US,t}$ range from 5.1 to 388.3 depending on z. For the joint test of $\beta_{2,c} z_{US,t}$ and $\beta_{3,c} t$, the F-statistics range from 32.4 to 1681.5. In all cases, the p-values are very close to 0.

Equations (63) and (60) coincide with the assumption about the market structure maintained in this paper and imply that

$$dTK = \frac{r_I}{r_{TK}} dI + \frac{r_J}{r_{TK}} dJ + \frac{r_K}{r_{TK}} dK. \tag{64}$$

The general expression for the elasticity of substitution between labor L and aggregate capital TK is

$$\mathcal{E}_{L,TK} = \frac{\frac{\partial f}{\partial L}\frac{\partial f}{\partial TK}\left(L\frac{\partial f}{\partial L} + TK\frac{\partial f}{\partial TK}\right)}{L \times TK \times \left[-\frac{\partial^2 f}{\partial L^2}\left(\frac{\partial f}{\partial TK}\right)^2 + 2\frac{\partial^2 f}{\partial L \partial TK}\frac{\partial f}{\partial L}\frac{\partial f}{\partial TK} - \frac{\partial^2 f}{\partial TK^2}\left(\frac{\partial f}{\partial L}\right)^2\right]}. \tag{65}$$

It follows from the definition of TK in equation (60), and equations (62) and (63) that

$$f = \frac{\partial f}{\partial L} L + \frac{\partial f}{\partial TK} TK \tag{66}$$

since Y has constant returns to scale in equations (1) and (18). This, in turn, implies that

$$\frac{\partial^2 f}{\partial L^2} L + \frac{\partial^2 f}{\partial TK \partial L} TK = 0, \tag{67}$$

$$\frac{\partial^2 f}{\partial L \partial TK} L + \frac{\partial^2 f}{\partial TK^2} TK = 0. \tag{68}$$

Therefore, $\mathcal{E}_{L,TK}$ in equation (65) can be rewritten as

$$\mathcal{E}_{L,TK} = \frac{\frac{\partial f}{\partial L}\frac{\partial f}{\partial TK}}{\frac{\partial^2 f}{\partial TK \partial L} f}.$$

The expressions for the elasticity of substitution between labor and aggregate capital (24) and (25) can be easily derived using this expression and the partial derivatives of the function $f(L, TK)$ from equations (62) and (63). The share variables in (25) are

given by

$$S_{IJ_t,LIJ_t} = \frac{r_{I,t}I_t + r_{J,t}J_t}{w_tL_t + r_{I,t}I_t + r_{J,t}J_t}, \tag{69}$$

$$S_{LIJ_t,Y_t} = \frac{w_tL_t + r_{I,t}I_t + r_{J,t}J_t}{Y_t}, \tag{70}$$

and

$$S_{L_t,LK_t} = \frac{w_tL_t}{w_tL_t + r_{K,t}K_t}, \tag{71}$$

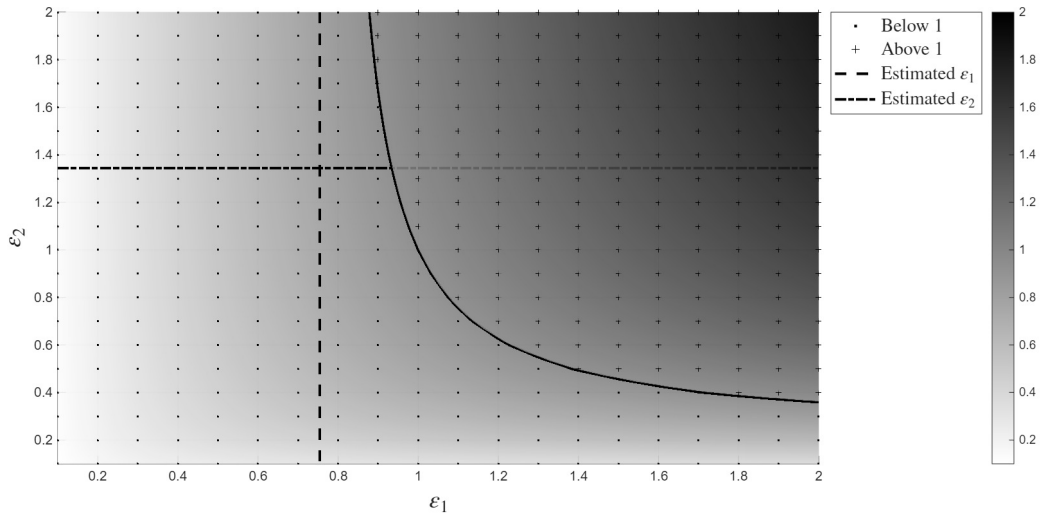
$$S_{LK_t,Y_t} = \frac{w_tL_t + r_{K,t}K_t}{Y_t}. \tag{72}$$

I compute the values of $\mathcal{E}_{L,TK}$ using equation (24) for Y defined in equation (1), considering a range of ε_1 and ε_2 values from the interval $(0.1, 2)$. These computations use the averages of S_{IJ_t,LIJ_t} and S_{LIJ_t,Y_t}. Figure V illustrates the results, highlighting whether $\mathcal{E}_{L,TK}$ exceeds or falls below 1. For most combinations of ε_1 and ε_2, $\mathcal{E}_{L,TK}$ is below 1. Moreover, at the estimated value of ε_1, $\mathcal{E}_{L,TK}$ is below 1 for all values of ε_2. In turn, I compute $\mathcal{E}_{L,TK}$ for a grid of values of S_{IJ_t,LIJ_t} and S_{LIJ_t,Y_t} from the interval $(0.01, 0.99)$, using the estimated ε_1 and ε_2 from column 1 of Table 3. Figure VI presents these results, showing that $\mathcal{E}_{L,TK}$ remains below 1 across a wide range of values of these shares.

I repeat the analysis for Y defined by equation (18) and parameter values from column 3 of Table 3. Figure VII shows the regions of ε_1 and ε_2, taken from the interval $(0.1, 2)$ using the averages of S_{L_t,LK_t} and S_{LK_t,Y_t} and highlighting whether $\mathcal{E}_{L,TK}$ is above or below 1. The computed value of $\mathcal{E}_{L,TK}$ is particularly sensitive to ε_2. A slightly lower ε_2 would make $\mathcal{E}_{L,TK}$ fall below 1.[22] Finally, Figure VIII reports results for a grid of S_{L_t,LK_t} and S_{LK_t,Y_t} values from $(0.01, 0.99)$, using the estimated values of ε_1 and ε_2. In this case, $\mathcal{E}_{L,TK}$ exceeds 1 for most of the values of values of S_{L_t,LK_t} and S_{LK_t,Y_t}.

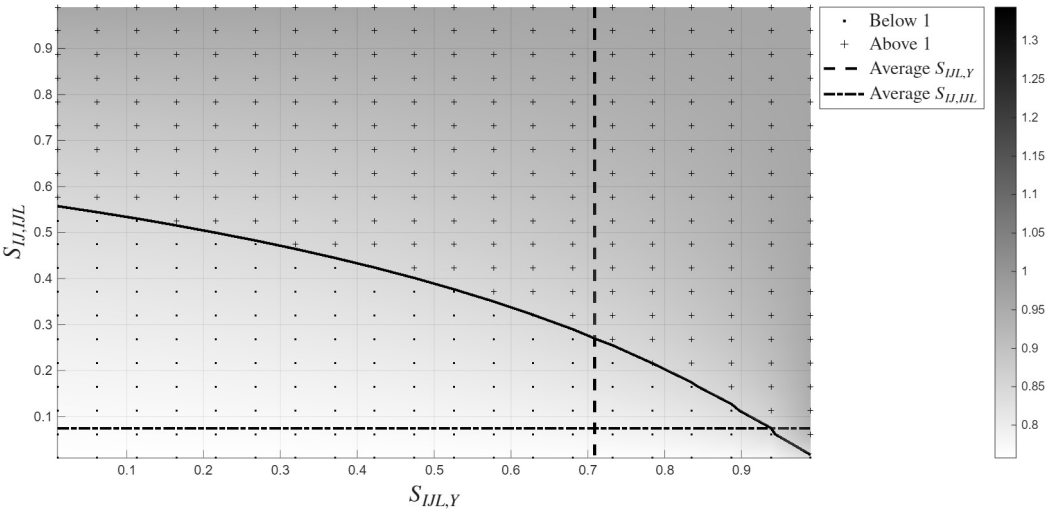
[22] The value of $\mathcal{E}_{L,TK}$ remains above 1 for the lowest values of ε_1 and ε_2 in their respective confidence intervals.

Figure V: Values of $\mathcal{E}_{L,TK}$ Y from Eq. (1) across ε_1 and ε_2



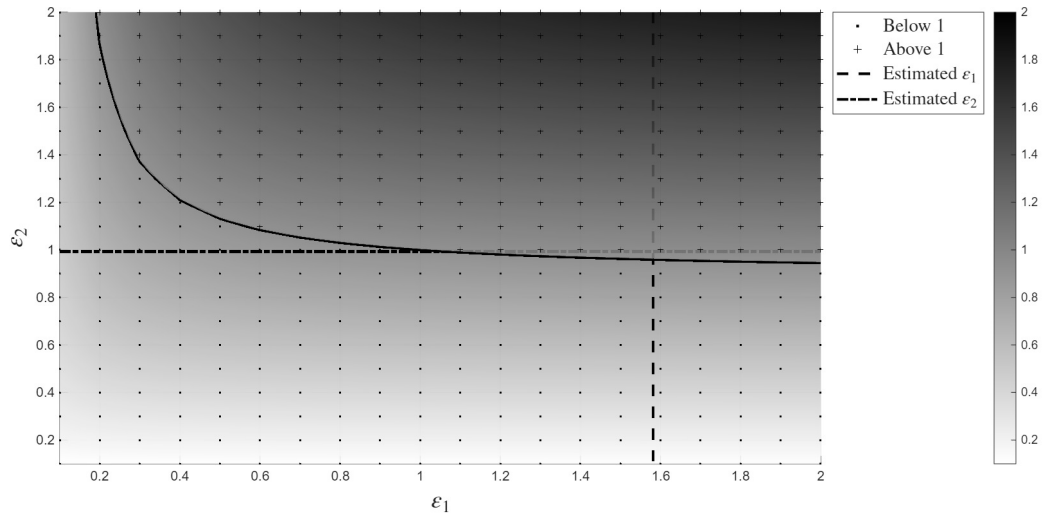
Note: This figure illustrates the values of $\mathcal{E}_{L,TK}$ computed from equation (24) for Y from equation (1), varying ε_1 and ε_2 in $(0.1, 2)$ and using average values of S_{IJ_t,LIJ_t} and S_{LIJ_t,Y_t}.

Figure VI: Values of $\mathcal{E}_{L,TK}$ for Y from Eq. (1) across S_{IJ_t,LIJ_t} and S_{LIJ_t,Y_t}



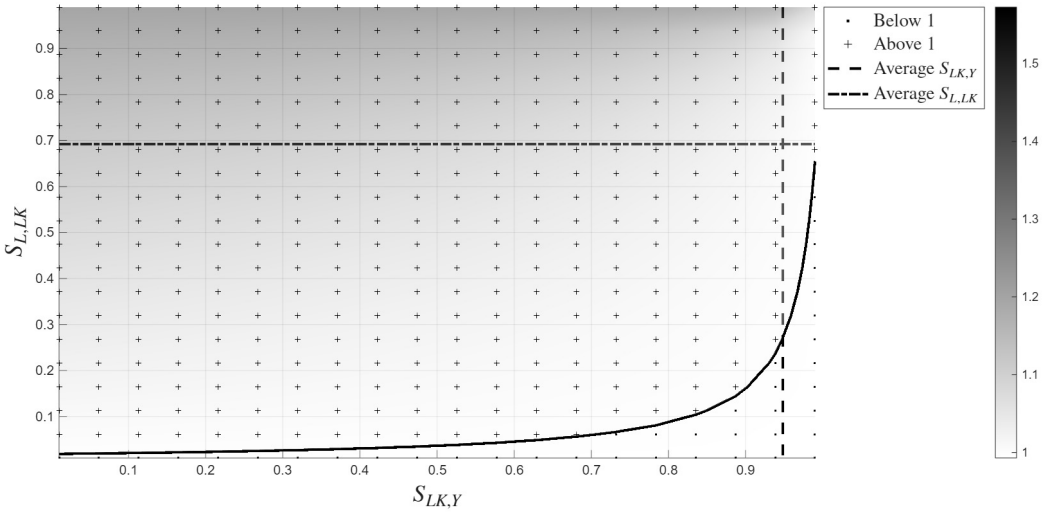
Note: This figure shows $\mathcal{E}_{L,TK}$ computed from equation (24) for Y from equation (1), varying S_{IJ_t,LIJ_t} and S_{LIJ_t,Y_t} in $(0.01, 0.99)$ and using estimated ε_1 and ε_2 from column 1 of Table 3.

Figure VII: Values of $\mathcal{E}_{L,TK}$ for Y from Eq. (18) across ε_1 and ε_2



Note: This figure illustrates the values of $\mathcal{E}_{L,TK}$ computed from equation (24) for Y from equation (18), varying ε_1 and ε_2 in $(0.1, 2)$ and using average values of S_{L_t,LK_t} and S_{LK_t,Y_t}.

Figure VIII: Values of $\mathcal{E}_{L,TK}$ for Y from Eq. (18) across S_{L_t,LK_t} and S_{LK_t,Y_t}



Note: This figure shows $\mathcal{E}_{L,TK}$ computed from equation (24) for Y from equation (18), varying S_{L_t,LK_t} and S_{LK_t,Y_t} in $(0.01, 0.99)$ and using estimated ε_1 and ε_2 from column 3 of Table 3.

F Appendix - Robustness Checks and Additional Results

This section presents robustness checks and additional results. I do not report the estimation results based on instrumented regressors for brevity. These results are consistent

with those discussed here, unless noted otherwise.

Wages in the main sample include both the wages of employees and the income of proprietors. As a robustness check, I subtract proprietors' income from wages and rerun the estimations of systems of equations corresponding to production function specifications (1), (18), and (19). Columns 1-3 of Table IV offer the results. These are very similar to the main results offered in Table 3.

In the main estimations, I compute r_t using equation (17) and derive rental prices of different types of capital using its values (e.g., Herrendorf et al., 2015, use an analogous approach for computing r_t in a similar estimation). As an alternative, I consider annual real returns on "risk-free" assets, denoted r_t^f, defined as the nominal yield on 10-year government bonds minus the 10-year moving average of inflation, using data from the OECD database.[23] Columns 4-6 of Table IV offer the results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19) using this alternative measure for r_t. When the fit of the empirical specifications to the data significantly worsens in all cases, the results remain similar to those in Table 3 except for the results corresponding to specification (18).

There is considerable variation in rental prices across different types of capital, primarily driven by fluctuations in corresponding investment prices. I rescale the investment prices for ICT and IP capital by a constant factor such that the average of the rental price, computed using equation (7), is equalized across these capital types and with the average of the rental price of traditional capital within each country. To perform this rescaling, I select $\zeta_{1,c}$ and $\zeta_{2,c}$ so that the average rental price within each country satisfy:

$$\begin{aligned}
&\frac{1}{T}\sum_{t=1}^{T}\left[\left(1+r_{c,t}\right)\zeta_{1,c}p_{I,c,t-1}-\left(1-\delta_I\right)\zeta_{1,c}p_{I,c,t}\right]= \\
&\quad\frac{1}{T}\sum_{t=1}^{T}\left[\left(1+r_{c,t}\right)\zeta_{2,c}p_{J,c,t-1}-\left(1-\delta_J\right)\zeta_{2,c}p_{J,c,t}\right]= \\
&\quad\frac{1}{T}\sum_{t=1}^{T}\left[\left(1+r_{c,t}\right)p_{K,c,t-1}-\left(1-\delta_K\right)p_{K,c,t}\right].
\end{aligned}$$

[23] The correlation between r_t from equation (17) and r_t^f is about 0.17 and statistically significant at the 5% level.

Columns 7-9 of Table IV offer the results from estimations of the systems of equations corresponding to production function specifications (1), (18), and (19) using these measures for $r_{i,t}$ for $i = I, J, K$. In all cases, the results are similar to the main results in Table 3.[24]

I compute the rental prices of different types of capital using equation (7). This method follows closely the seminal works by Jorgenson and Siebert (1968a,b), Christensen and Jorgenson (1969) and has been used recently, for example, by Caselli and Feyrer (2007) and Eden and Gaggl (2018).[25] It assumes perfect foresight. A more general formulation of the equation (7) allowing stochasticity is

$$r^e_{i,t} = r^e_t p_{i,t-1} + \delta_i p^e_{i,t} + \left(p_{i,t-1} - p^e_{i,t}\right), \tag{73}$$

where superscript e stands for the expected value, and I have separated the expected capital gains at the end of the equation for convenience.

I HP-filter $p_{i,t}$ ($\lambda = 100$) and use the smoothed series of $p_{i,t}$ as measures for the expected value of the corresponding price series in equation (7), as well as in equation (17) for computing r_t. Columns 1-3 of Table V offer the results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19) using these alternative measures for the rental prices of different types of capital. In all cases, the results are similar to the main results in Table 3.

Further, two alternative approaches assume either static expectations or that producers anticipate no capital gains (e.g., similar to Jorgenson, 1963). The rental prices of different types of capital $i \in \{I, J, K\}$ are expressed as follows in these cases:

$$r_{i,t} = r_t p_{i,t} + \delta_i p_{i,t}, \tag{74}$$

$$r_{i,t} = r_t p_{i,t-1} + \delta_i p_{i,t}. \tag{75}$$

[24] I have checked that the results do not change when I equalize the rental prices on different types of capital, also across countries.

[25] The EU KLEMS database also employs exactly this method, for example, when constructing capital services within the Törnqvist aggregation of capital types.

Columns 4-9 of Table V offer the results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19) using these alternative measures for the rental prices of different types of capital. In all cases, the results are similar to the main results in Table 3.

In the main estimations, I assume that technological parameters γ_i for $i = I, J, K, L$ are the same across countries. I relax this assumption allowing for country-level differences in γ_i, and estimate the systems of equations corresponding to the production function specifications in equations (1), (18), and (19). Table VI reports the estimates of the elasticities of substitution together with the measures of fit, while Table VII presents the estimates of the technical change parameters.

The results obtained under the specification in equation (1) remain very similar to those reported in Table 3. Although some cross-country variation in the estimated technical change parameters emerges, the magnitude of these differences is modest. By contrast, the estimates of the elasticity of substitution ε_1 under the specification in equation (18) deviate from those presented in Table 3. This specification also produces very large point estimates and standard errors for the technical change parameters. Finally, under equation (19), the estimated parameters are also very similar to those reported in Table 3 with some variation in technical change parameters across countries.

These estimation issues suggest that introducing country-specific technological parameters substantially increases the flexibility of the model, but at the cost of weakened identification. In particular, the large and imprecise estimates of the technical change parameters under specification (19) indicate that the data provide limited independent variation to jointly identify both substitution patterns and heterogeneous technical change in this specification.[26]

I extend the model to allow for time-varying technical change using a Box–Cox trans-

[26]The Bayesian Information Criterion (BIC) is −4874 for the estimation reported in column 2 of Table 3. With an additional 32 parameters, the BIC falls to −5389 in the corresponding estimation reported in column 2 of Table VI.

formation as proposed by Klump et al. (2007). The functional form is given by:

$$\Gamma_i(t) = \frac{\gamma_i}{\lambda_i}\left(\frac{1}{T}\sum t\right)\left[\left(\frac{t}{\frac{1}{T}\sum t}\right)^{\lambda_i} - 1\right], \tag{76}$$

which simplifies to $\Gamma_i(t) = \gamma_i \hat{t}$ when $\lambda_i = 1$. I replace $\gamma_i \hat{t}$ with $\Gamma_i(t)$ for $i = I, J, K$ in the normalized analogues of equations (1), (18), and (19), along with their corresponding first-order conditions, and estimate the resulting normalized systems of equations.

Columns 1-3 of Table VIII present the estimation results. The results for the production function specification corresponding to equations (1) and (19) are broadly consistent with Table 3, and λ_i for $i = I, J, K$ is not statistically different from 1 in most cases, though estimates tend to be noisy. In some instances, λ_K, λ_J, and λ_L are not statistically different from 0.

In turn, the estimated values of the elasticity of substitution parameters, as well as the estimated values of γ_i for $i = I, J, K$ are not that different from the estimated values reported in Table 3 for production function specification as given by equation (18). The estimated values of λ_i for $i = I, J, K$ are often statistically different than 1. Moreover, the estimated values of λ_I are statistically significantly higher than 1 while the estimated values of λ_K and λ_L are statistically significantly lower than 1. When coupled with positive estimates of parameter γ the latter implies deceleration in factor-augmenting technical change for K and L, the former implies an acceleration in factor-augmenting technical change for I, a pattern that is difficult to justify and may reflect specification issues.

I measure IP capital as the sum of computer software and databases, research and development (RD), and other intellectual property products (OIPP). I bundle this aggregate with ICT capital, which comprises computer hardware, to form the composite input IJ. A principal motivation for this is that computer hardware, software, and databases jointly generate productive capital services. Moreover, a significant share of innovation generated through research and development is related to ICT and its applications in the sample countries and period, as evidenced by patent data. This aggregation

also addresses a classification feature of the EU KLEMS database whereby software and databases developed within the research and development process are recorded under RD rather than under software and databases. This is largely a taxonomy decision rather than an economic one, and aggregating these components avoids the artificial separation of assets that share distinct economic properties (Corrado et al., 2022). Finally, this treatment connects the paper to the intangibles literature that documents the role of software, data, R&D, and other intellectual property products in factor-share dynamics and productivity growth (e.g., Barkai et al., 2025, Bontadini et al., 2023, Corrado et al., 2022, Koh et al., 2020, McGrattan, 2020).

To assess the sensitivity of the results, I re-estimate the systems of equations corresponding to production function specifications (1), (18), and (19) under three alternative IP capital definitions: (1) excluding OIPP; (2) excluding OIPP and 50% of RD; and (3) excluding OIPP and all RD (retaining only software and databases). In each case, any IP components excluded from the IJ bundle are reassigned to traditional capital K to ensure total capital accounting is preserved.[27]

Table IX reports the results. The estimates remain robust under the first two definitions of IP capital (columns 1-6), though the log-likelihood decreases compared to the baseline in Table 3. The measure of fit in the equations corresponding to the first-order condition with respect to IP capital also falls. The columns 7-9 report the result when I exclude RD and OIPP from IP capital entirely and add them to traditional capital. The estimation results from the systems of equations corresponding to the baseline specification (1) and the specification (18) are sensitive to this exclusion. Under specification (1), the estimate of ε_1 exceeds unity while other elasticities fall below 1, implying that traditional capital substitutes for labor while the IJ bundle complements it.[28] In turn, the estimate of ε_1 is imprecise and implausibly large when estimating the system of equations corresponding to the production function specification (18). These estimates though, still suggest that labor and ICT and IP capital bundle are substituting for labor while tradi-

[27] Further disaggregating capital types and estimating nests that treat RD or OIPP as separate inputs is beyond the scope of the this paper.

[28] Notably, the counterfactual exercise with these parameter values reveals that ICT and IP capital remain a dominant driver of labor share dynamics.

tional capital complements it, in line with the results in Table 3. However, in this case, ICT and IP capital are now substitutes. The estimate of ε_1 corresponding to specification (19) is very close to its counterpart in Table 3 while the estimate of ε_2 is now below 1, implying that aggregate capital substitutes for labor, but now ICT and IP capital and traditional capital are complements. The R-squared in the first-order condition with respect to J is large and negative in columns 7-9. All in all, the results from this more extreme reclassification are suggestive of misspecification and weakened identification, as evidenced by degraded fit, implausible elasticity estimates, and negative R-squared in the first-order condition for J.

Taken together, these robustness checks indicate that the estimates from the baseline specification of the production function (1) remain relatively stable across a wide range of alternative assumptions and data treatments. This production function specification is aligned with the literature on automation (e.g., Acemoglu and Autor, 2011, Acemoglu and Restrepo, 2018, Aum et al., 2018). It also delivers an inference that is consistent with the juxtaposition of the findings in the literature on the elasticity of substitution between labor and aggregate capital and on labor income share dynamics (e.g., Aum and Shin, 2024, Eden and Gaggl, 2018, 2019, Karabarbounis and Neiman, 2014, Glover and Short, 2020, Oberfield and Raval, 2021). Estimates obtained from specification (19) are likewise generally robust to alternative assumptions and data treatments. By contrast, the estimates derived from specification (18) exhibit greater sensitivity and can yield imprecise or implausible estimates, particularly when allowing for country-specific or time-varying technical change or when excluding RD and OIPP from IP capital. These results also reaffirm the inference that the aggregation of capital, as well as the specification of the production function, can matter for determining the estimated elasticity of substitution parameters.

Many papers examine how intangible capital complements labor and enhances its productivity. A production function specification that might further align with this literature is given by:

$$Y_t = \left(\varpi_{LJI}^{\frac{1}{\varepsilon_1}} LJI_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} K_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right)^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{77}$$

and

$$LJI_t = \left[\varpi_{LJ}^{\frac{1}{\varepsilon_2}} LJ_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_I^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_I t} I_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$

$$LJ_t = \left[\varpi_L^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_J^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_J t} J_t\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}}.$$

I use this production function specification to derive the first-order conditions of the representative firm that solves problem (2). Next, I normalize the production function and the first-order conditions and estimate the resulting system of equations.[29] Column 4 of Table VIII reports the estimation results. The estimates of ε_1 and ε_2, consistent with other specifications, indicate that traditional capital is a complement to labor, while ICT capital is a substitute. The factor-specific technical change parameters for labor, traditional capital, and ICT capital also align with previous specifications. One notable difference, however, is that in this case labor and IP capital appear as complements, and the estimate of γ_J is positive.

The estimation of this specification is sensitive to the use of instrumented regressors, as it produces very large point estimates for ε_1 and ε_2. Moreover, the estimates of this specification imply that ICT capital substitutes for IP capital and that IP capital compensation share falls over time. This is not consistent with the results reported in Figure 1 and the results of Koh et al. (2020). An indicator of these discrepancies is the large negative R-squared value in the estimated equation for r_J under this specification.

F.A Imputed Aggregate Capital-Specific Technological Progress

I consider the production function in equation (1) and use the values of γ_I, γ_J, and γ_K from column 1 of Table 3 and compute the aggregate capital-specific technological progress parameter in the following way

$$\gamma_{L,TK} = \frac{r_I I \gamma_I + r_J J \gamma_J + r_K K \gamma_K}{r_I I + r_J J + r_K K}.$$

[29] The system of equations (52)-(56) in the Appendix - Empirical Specifications provides the general form of the estimated system.

These computations provide negative values of $\gamma_{L,TK}$. However, these values are small in absolute terms, and labor income share computed using production function from equation (26) counterfactually trends upward with these parameter values.

Table IV: Robustness - Wages, Returns, and Rental Prices

A. Estimates	Wages			Yield on Government Bonds			Equal Average Rental Prices		
	(1) Main	(2) Eq. (18)	(3) Eq. (19)	(4) Main	(5) Eq. (18)	(6) Eq. (19)	(7) Main	(8) Eq. (18)	(9) Eq. (19)
ε_1	0.733***	1.529***	1.210***	0.649***	0.344***	1.381***	0.777***	1.885***	1.450***
	(0.012)	(0.122)	(0.045)	(0.034)	(0.075)	(0.062)	(0.025)	(0.267)	(0.079)
$p_{\varepsilon_1=1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
ε_2	1.194***	0.998***	1.128***	1.175***	0.842***	1.319***	1.200***	0.992***	0.750***
	(0.012)	(0.004)	(0.015)	(0.040)	(0.020)	(0.053)	(0.023)	(0.004)	(0.017)
$p_{\varepsilon_2=1}$	0.000	0.542	0.000	0.000	0.000	0.000	0.000	0.053	0.000
ε_3	0.961***	0.956***	0.969***	0.941***	0.908***	0.942***	0.974***	0.937***	0.956***
	(0.003)	(0.004)	(0.003)	(0.009)	(0.014)	(0.007)	(0.004)	(0.008)	(0.005)
$p_{\varepsilon_3=1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
γ_I	0.720***	0.684***	0.875***	0.582***	0.222***	0.467***	1.193***	0.477***	0.482***
	(0.072)	(0.046)	(0.098)	(0.096)	(0.036)	(0.062)	(0.191)	(0.066)	(0.054)
γ_J	-0.124***	-0.168***	-0.102***	-0.064***	-0.092***	-0.066***	-0.265***	-0.127***	-0.142***
	(0.020)	(0.014)	(0.022)	(0.013)	(0.013)	(0.013)	(0.050)	(0.016)	(0.012)
γ_K	-0.004*	-0.104***	-0.021***	0.024**	0.050**	-0.029***	0.003*	-0.113***	-0.001
	(0.002)	(0.006)	(0.002)	(0.008)	(0.016)	(0.003)	(0.001)	(0.009)	(0.001)
γ_L	0.004***	0.052***	0.009***	-0.003	-0.001	0.012***	0.007***	0.059***	0.010***
	(0.001)	(0.003)	(0.001)	(0.002)	(0.004)	(0.001)	(0.001)	(0.004)	(0.001)
Obs. (per eq.)	219	219	219	219	219	219	219	219	219
B. Measures of Fit									
Log Likelihood	2181	2360	2177	1147	1289	1810	2202	2431	2067
R2									
$\ln\left(Y_t/\overline{Y}\right)$	0.887	0.960	0.909	0.896	0.669	0.930	0.891	0.949	0.829
$\ln\left(r_{I,t}/\overline{r_I}\right)$	0.863	0.871	0.861	0.858	0.857	0.880	0.868	0.870	0.839
$\ln\left(r_{j,t}/\overline{r_J}\right)$	0.241	0.291	0.026	-0.096	0.395	0.205	0.395	0.282	0.186
$\ln\left(r_{K,t}/\overline{r_K}\right)$	0.641	0.615	0.967	0.095	0.363	0.983	0.608	0.617	0.937
$\ln\left(w_t/\overline{w}\right)$	0.967	0.876	0.873	-2.302	-0.272	0.837	0.981	0.873	0.862

Note: This table reports results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19). Panel *A* presents parameter estimates and the number of observations for each equation. Panel *B* offers measures of fit, including Log Likelihood and R-squared of each equation. In columns 1-3, I subtract proprietors' income from labor income when computing wages. In columns 4-6, rental prices are computed using an alternative measure of r_t, defined as the 10-year government bond yield minus the 10-year moving average of inflation from OECD data, r_t^f. In columns 7-9, I use a constant factor to equate the average rental price of each type of capital within countries. The reported results correspond to the estimations with the best log-likelihood fit, excluding cases where the estimated elasticity of substitution between a type of capital and labor is implausibly high and/or the estimate of γ_{ICT} is negative. Rows denoted $p_{\varepsilon_i=1}$ report p-values for tests of $\varepsilon_i = 1$ for $i = 1, 2, 3$. Standard errors, shown in parentheses, are robust to heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

Table V: Robustness - Expectations

A. Estimates	Smoothed Returns			Static Expectations			No Capital Gains		
	(1) Main	(2) Eq. (18)	(3) Eq. (19)	(4) Main	(5) Eq. (18)	(6) Eq. (19)	(7) Main	(8) Eq. (18)	(9) Eq. (19)
ε_1	0.745***	1.806***	1.245***	0.738***	1.327***	1.455***	0.738***	1.322***	1.427***
	(0.010)	(0.238)	(0.056)	(0.008)	(0.045)	(0.100)	(0.008)	(0.044)	(0.093)
$p_{\varepsilon_1=1}$	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ε_2	1.220***	0.994***	1.154***	1.229***	0.994***	1.121***	1.226***	0.994***	1.121***
	(0.012)	(0.004)	(0.021)	(0.008)	(0.004)	(0.013)	(0.008)	(0.004)	(0.013)
$p_{\varepsilon_2=1}$	0.000	0.150	0.000	0.000	0.151	0.000	0.000	0.148	0.000
ε_3	0.964***	0.956***	0.972***	0.967***	0.966***	0.975***	0.966***	0.966***	0.975***
	(0.003)	(0.004)	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.002)
$p_{\varepsilon_3=1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
γ_I	0.691***	0.627***	0.838***	0.723***	0.801***	0.972***	0.725***	0.800***	0.970***
	(0.075)	(0.043)	(0.105)	(0.063)	(0.050)	(0.109)	(0.064)	(0.050)	(0.109)
γ_J	-0.108***	-0.168***	-0.099***	-0.100***	-0.160***	-0.090***	-0.101***	-0.161***	-0.093***
	(0.020)	(0.014)	(0.024)	(0.015)	(0.011)	(0.020)	(0.015)	(0.011)	(0.020)
γ_K	-0.002	-0.104***	-0.019***	-0.000	-0.114***	-0.022***	-0.000	-0.113***	-0.022***
	(0.002)	(0.006)	(0.002)	(0.002)	(0.007)	(0.002)	(0.002)	(0.007)	(0.002)
γ_L	0.003***	0.053***	0.009***	0.003***	0.056***	0.009***	0.003***	0.056***	0.009***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.003)	(0.001)
Obs. (per eq.)	219	219	219	219	219	219	219	219	219
B. Measures of Fit									
Log Likelihood	2200	2456	2123	2463	2640	2358	2464	2638	2359
R2									
$\ln\left(Y_t/\overline{Y}\right)$	0.891	0.961	0.910	0.881	0.959	0.903	0.881	0.959	0.902
$\ln\left(r_{I,t}/\overline{r_I}\right)$	0.852	0.862	0.847	0.865	0.876	0.867	0.868	0.879	0.870
$\ln\left(r_{J,t}/\overline{r_J}\right)$	0.403	0.499	0.287	0.293	0.306	0.030	0.287	0.298	0.027
$\ln\left(r_{K,t}/\overline{r_K}\right)$	0.619	0.611	0.953	0.622	0.617	0.974	0.624	0.617	0.975
$\ln\left(w_t/\overline{w}\right)$	0.954	0.873	0.870	0.958	0.870	0.867	0.960	0.871	0.867

Note: This table reports results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19). Panel *A* presents parameter estimates and the number of observations for each equation. Panel *B* offers measures of fit, including Log Likelihood and R-squared of each equation. In columns 1-3, I use smoothed series for $p_{i,t}$ in equations (7) and (17) to compute the rental prices of different types of capital. In columns 4-6 and columns 7-9, I compute the rental prices of different types of capital using equations (74) and (75), respectively. The reported results correspond to the estimations with the best log-likelihood fit, excluding cases where the estimated elasticity of substitution between any type of capital and labor is implausibly high and/or the estimate of γ_{ICT} is negative. Rows denoted $p_{\varepsilon_i=1}$ report p-values for tests of $\varepsilon_i = 1$ for $i = 1, 2, 3$. Standard errors, shown in parentheses, are robust to heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

Table VI: Robustness - Estimates for Elasticities with Country-Level Technical Change

A. Estimates	(1) Main	(2) Eq. (18)	(3) Eq. (19)
ε_1	0.658***	0.782***	1.192***
	(0.017)	(0.068)	(0.031)
$p_{\varepsilon_1=1}$	0.000	0.000	0.000
ε_2	1.230***	1.004***	1.479***
	(0.018)	(0.002)	(0.078)
$p_{\varepsilon_2=1}$	0.000	0.542	0.000
ε_3	0.952***	0.978***	0.949***
	(0.004)	(0.006)	(0.006)
$p_{\varepsilon_3=1}$	0.000	0.000	0.000
Obs. (per eq.)	219	219	219
B. Measures of Fit			
Log Likelihood	2589	2800	2500
R2			
$\ln\left(Y_t/\overline{Y}\right)$	0.961	0.975	0.959
$\ln\left(r_{I,t}/\overline{r_I}\right)$	0.942	0.941	0.944
$\ln\left(r_{J,t}/\overline{r_J}\right)$	0.658	0.838	0.722
$\ln\left(r_{K,t}/\overline{r_K}\right)$	0.754	0.682	0.975
$\ln\left(w_t/\overline{w}\right)$	0.985	0.897	0.874

Note: This table reports results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19), where the technical change parameters are allowed to vary across countries. Panel *A* presents the estimates of ε_i for $i = 1, 2, 3$ and the number of observations for each equation. Panel *B* offers measures of fit, including Log Likelihood and R-squared of each equation. Rows denoted $p_{\varepsilon_i=1}$ report the p-values associated with tests of the null hypothesis $\varepsilon_i = 1$ for $i = 1, 2, 3$. Initial/starting points for estimations are $\varepsilon_i = \{0.5, 1.5\}$ for $i = 1, 2, 3$ and $\gamma_{j,c} = 0.02$ for $j = I, J, K, L$. The reported results correspond to the estimations with the best log-likelihood fit. Standard errors, shown in parentheses, are robust to heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

Table VII: Estimates for Technical Change Parameters with Country-Level Technical Change

	γ_I			γ_J			γ_K			γ_L		
Country	(1) Main	(2) Eq. (18)	(3) Eq. (19)	(4) Main	(5) Eq. (18)	(6) Eq. (19)	(7) Main	(8) Eq. (18)	(9) Eq. (19)	(10) Main	(11) Eq. (18)	(12) Eq. (19)
AT: Austria	0.764***	1.569***	0.710***	-0.198***	-0.721***	-0.231***	-0.007	1.088**	-0.014***	0.006**	-0.424**	0.012***
	(0.064)	(0.467)	(0.083)	(0.023)	(0.208)	(0.029)	(0.004)	(0.342)	(0.002)	(0.002)	(0.138)	(0.001)
BE: Belgium	0.629***	1.318***	0.604***	-0.143***	-0.633***	-0.193***	-0.005***	0.741**	-0.009***	0.002	-0.356**	0.006***
	(0.055)	(0.379)	(0.076)	(0.025)	(0.180)	(0.028)	(0.001)	(0.239)	(0.002)	(0.001)	(0.118)	(0.001)
DE: Germany	0.981***	2.388***	0.967***	-0.229***	-0.645***	-0.237***	-0.003	1.301***	0.001	0.011***	-0.503***	0.009***
	(0.099)	(0.656)	(0.117)	(0.027)	(0.179)	(0.031)	(0.002)	(0.359)	(0.001)	(0.001)	(0.142)	(0.001)
ES: Spain	0.892***	1.673***	0.809***	-0.160***	-0.757***	-0.208***	-0.013***	0.861***	-0.020***	0.002	-0.427***	0.008***
	(0.096)	(0.473)	(0.119)	(0.032)	(0.209)	(0.038)	(0.004)	(0.216)	(0.002)	(0.002)	(0.108)	(0.001)
FI: Finland	0.481***	1.205***	0.504***	-0.043**	-0.281***	-0.083***	-0.002	0.010	-0.008***	0.006***	0.012	0.011***
	(0.112)	(0.292)	(0.114)	(0.015)	(0.064)	(0.017)	(0.004)	(0.443)	(0.001)	(0.002)	(0.199)	(0.001)
FR: France	0.388***	0.809**	0.361***	-0.032**	-0.148***	-0.043***	0.005**	-0.326*	-0.009***	0.004***	0.136*	0.011***
	(0.062)	(0.281)	(0.068)	(0.010)	(0.045)	(0.010)	(0.002)	(0.150)	(0.001)	(0.001)	(0.057)	(0.000)
IT: Italy	0.317***	0.666**	0.285***	-0.049**	-0.180**	-0.057***	-0.008***	-0.309**	-0.013***	-0.000	0.153**	0.003***
	(0.057)	(0.227)	(0.060)	(0.016)	(0.055)	(0.013)	(0.002)	(0.103)	(0.001)	(0.001)	(0.052)	(0.000)
NL: Netherlands	0.584***	1.402**	0.555***	-0.106***	-0.369**	-0.124***	0.009	-0.680**	-0.005**	0.003	0.270**	0.010***
	(0.070)	(0.456)	(0.080)	(0.018)	(0.116)	(0.016)	(0.005)	(0.259)	(0.002)	(0.003)	(0.099)	(0.001)
PT: Portugal	0.951***	2.046***	0.971***	-0.499***	-1.418***	-0.555***	-0.027***	1.300***	-0.012***	0.015***	-0.653***	0.007***
	(0.097)	(0.597)	(0.140)	(0.061)	(0.398)	(0.078)	(0.007)	(0.357)	(0.002)	(0.003)	(0.181)	(0.001)

Note: This table reports the estimates of γ_j, c for $j = I, J, K, L$ from the systems of equations corresponding to production function specifications (1), (18), and (19), where the technical change parameters are allowed to vary across countries. Standard errors, shown in parentheses, are robust to heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

Table VIII: Robustness - Time Varying Technical Change

A. Estimates	(1) Main	(2) Eq. (18)	(3) Eq. (19)
ε_1	0.726***	1.538***	1.275***
	(0.009)	(0.114)	(0.065)
$p_{\varepsilon_1=1}$	0.000	0.000	0.000
ε_2	1.203***	0.987***	1.119***
	(0.010)	(0.004)	(0.015)
$p_{\varepsilon_2=1}$	0.000	0.003	0.000
ε_3	0.959***	0.956***	0.969***
	(0.003)	(0.003)	(0.003)
$p_{\varepsilon_3=1}$	0.000	0.000	0.000
γ_I	0.715***	0.749***	0.899***
	(0.068)	(0.050)	(0.113)
γ_J	-0.114***	-0.199***	-0.098***
	(0.018)	(0.014)	(0.025)
γ_K	-0.001	-0.108***	-0.022***
	(0.001)	(0.007)	(0.003)
γ_L	0.002*	0.055***	0.009***
	(0.001)	(0.003)	(0.001)
λ_I	1.127***	1.430***	1.288***
	(0.166)	(0.172)	(0.232)
λ_J	0.865*	1.287***	1.263
	(0.345)	(0.191)	(0.744)
λ_K	-1.311	0.615***	1.118***
	(0.753)	(0.106)	(0.197)
λ_L	-0.681	0.710***	0.998***
	(0.381)	(0.098)	(0.187)
Obs. (per eq.)	219	219	219
B. Measures of Fit			
Log Likelihood	2283	2483	2183
R2			
$\ln\left(Y_t/\overline{Y}\right)$	0.888	0.961	0.909
$\ln\left(r_{I,t}/\overline{r_I}\right)$	0.869	0.876	0.868
$\ln\left(r_{J,t}/\overline{r_J}\right)$	0.234	0.300	-0.015
$\ln\left(r_{K,t}/\overline{r_K}\right)$	0.660	0.620	0.969
$\ln\left(w_t/\overline{w}\right)$	0.970	0.873	0.869

Note: This table reports results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19) with time-varying growth rates based on a Box–Cox specification. Panel *A* presents parameter estimates and the number of observations for each equation. Panel *B* offers measures of fit, including Log Likelihood and R-squared of each equation. The reported results correspond to the estimations with the best log-likelihood fit, excluding cases where the estimated elasticity of substitution between a type of capital and labor is implausibly high and/or the estimate of γ_{ICT} is negative. Rows denoted $p_{\varepsilon_i=1}$ report the p-values associated with tests of the null hypothesis $\varepsilon_i = 1$ for $i = 1, 2, 3$. Standard errors, shown in parentheses, are robust to heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.

Table IX: Robustness - IP Capital and Specification

Estimates	No OIPP			Half of RD and no OIPP			No RD and OIPP			
	(1) Main	(2) Eq. (18)	(3) Eq. (19)	(4) Main	(5) Eq. (18)	(6) Eq. (19)	(7) Main	(8) Eq. (18)	(9) Eq. (19)	(10) Eq. (77)
ε_1	0.734***	1.454***	1.195***	0.750***	1.512***	1.133***	1.336***	4.604*	1.327***	0.613***
	(0.010)	(0.088)	(0.036)	(0.009)	(0.107)	(0.019)	(0.030)	(2.292)	(0.037)	(0.013)
$p_{\varepsilon_1=1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ε_2	1.173***	0.991***	1.131***	1.198***	0.989***	1.197***	0.881***	0.974***	0.849***	1.492***
	(0.009)	(0.005)	(0.016)	(0.010)	(0.004)	(0.021)	(0.007)	(0.003)	(0.008)	(0.034)
$p_{\varepsilon_2=1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ε_3	0.959***	0.952***	0.966***	0.963***	0.955***	0.966***	0.965***	1.377***	0.947***	0.941***
	(0.003)	(0.004)	(0.004)	(0.003)	(0.004)	(0.003)	(0.003)	(0.085)	(0.004)	(0.003)
$p_{\varepsilon_3=1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
γ_I	0.710***	0.640***	0.823***	0.678***	0.632***	0.767***	0.529***	-0.120***	0.403***	0.333***
	(0.063)	(0.043)	(0.095)	(0.058)	(0.047)	(0.082)	(0.037)	(0.016)	(0.024)	(0.038)
γ_J	-0.111***	-0.165***	-0.102***	-0.175***	-0.257***	-0.220***	-0.418***	0.052***	-0.376***	0.031***
	(0.018)	(0.013)	(0.021)	(0.028)	(0.022)	(0.036)	(0.039)	(0.014)	(0.025)	(0.004)
γ_K	-0.003	-0.098***	-0.020***	-0.004**	-0.106***	-0.012***	-0.011***	-0.073***	-0.005***	-0.005***
	(0.002)	(0.007)	(0.002)	(0.002)	(0.007)	(0.002)	(0.003)	(0.008)	(0.001)	(0.001)
γ_L	0.003***	0.049***	0.009***	0.005***	0.056***	0.010***	0.013***	0.041***	0.010***	0.000
	(0.001)	(0.003)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)
Obs. (per eq.)	219	219	219	219	219	219	219	219	219	219
B. Measures of Fit										
Log Likelihood	2257	2430	2166	2256	2431	2158	2111	2357	2178	1982
R2										
$\ln\left(Y_t/\overline{Y}\right)$	0.891	0.956	0.911	0.893	0.951	0.889	0.891	0.901	0.900	0.939
$\ln\left(r_{I,t}/\overline{r_I}\right)$	0.867	0.875	0.867	0.903	0.911	0.907	0.934	0.934	0.934	0.811
$\ln\left(r_{j,t}/\overline{r_J}\right)$	-0.002	0.091	-0.223	-1.449	-0.944	-1.343	-1.471	-0.095	-0.522	-0.975
$\ln\left(r_{K,t}/\overline{r_K}\right)$	0.629	0.609	0.966	0.677	0.657	0.961	0.509	0.677	0.963	0.593
$\ln\left(w_t/\overline{w}\right)$	0.972	0.871	0.866	0.953	0.865	0.862	0.945	0.856	0.863	0.895

Note: This table reports results from estimations of systems of equations corresponding to production function specifications (1), (18), and (19) under alternative definitions of IP capital. In columns 1-3, I define it as software and databases and research and development, excluding OIPP. In columns 4-6, I define it as software and databases and half of the research and development, also excluding OIPP, and in columns 7-9 as software and databases only. In each case, the components of intellectual property capital excluded from J are reassigned to traditional capital K. This table also offers estimates from the system of equations corresponding to the production function specification (77) in column 10. Panel A presents parameter estimates and the number of observations for each equation. Panel B offers measures of fit, including Log Likelihood and R-squared of each equation. The results correspond to best log-likelihood fit from the estimations with starting values $\varepsilon_i = \{0.5, 1.5\}$ for $i = 1, 2, 3$ and $\gamma_j = 0.02$ for $j = I, J, K, L$ and excluding cases where the estimate of γ_{ICT} is negative. Rows denoted $p_{\varepsilon_i=1}$ report p-values for tests of $\varepsilon_i = 1$ for $i = 1, 2, 3$. Standard errors, shown in parentheses, are robust to heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Appendix - Data Description and the Appendix - Empirical Specifications provide additional details about the data and the empirical specifications.